

STATE AND PRIVATE PENSIONS, RETIREMENT BEHAVIOUR
AND PERSONAL CAPITAL ACCUMULATION

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INTRODUCTION

A pension scheme is an arrangement whereby an individual contributes a regular sum of money to the State or a private institution during his working life and, in return, receives a regular income during his retirement years.

The distinction between State and private pension schemes is essential to the whole analysis of pensions since membership of the former is compulsory for all workers whilst although it is true that, in some cases, the membership of private schemes is voluntary in general it is compulsory but the creation of the pension arrangement is a voluntary act of the employer. This poses three interesting questions: why is there a compulsory scheme organised through the State; why might individuals voluntarily join a pension scheme; and why would an employer make pension provision for his workers?

Consider firstly the compulsory state scheme. With the slowing down of the population growth rate and the increase in life expectancy this century has seen an increase in the proportion of population who are elderly from less than 5% at the turn of the century to 12% in 1966. These percentages refer to an age specific definition of the elderly i.e., in the U.K. it is men aged 65 and over and women aged 60 and over. In the analysis of pensions a more appropriate definition of elderly would be those unable to work as a result of old age. In that the above ages constitute official retirement dates in the U.K. the administrative definition can be viewed as an average, ability to work varying with the type of occupation.

Combined with this increase in the proportion of the population who can be regarded as elderly the changing industrial and occupational environment, in particular the growth of mechanisation and specialisation, has made it increasingly difficult for the elderly to remain in the workforce. Additionally, in labour surplus economies, unemployment will tend to fall disproportionately on the lower productivity and geographically less mobile elderly. In the absence of a pension scheme those elderly who would like to work will find themselves involuntarily without a source of income, assuming that in most cases any capital income is small.

Apart from those who face involuntary retirement there will also be a group of people who, late in their working life, find they have developed a preference for retirement - unfortunately in many cases financial support will not be available to facilitate this retirement.

Having explained the existence of a class of elderly non-workers in society without financial provision for retirement it then has to be asked why there is an absence of a pension arrangement. Perhaps the most important reason is that individuals are myopic in that they foresee neither the possibility of enforced retirement, as a result of unemployment or illness, nor a preference for a period of retirement at the end of the working life. As a result of this they have neither saved explicitly nor made some intergenerational arrangement of the type suggested by a Hobbes-Rousseau social contract as discussed in Samuelson (52). Alternatively, the explanation could lie in the structure of the capital market, imperfections in which make it difficult to arrange suitable long-term contracts.

The absence of a pension arrangement implies that, for individuals who are involuntarily retired, the burden of their financial support has to fall on somebody. For a long time this was regarded as the responsibility of children but just as the work prospects for the elderly have been changing so has family life, in terms of a shift away from children's obligation towards their parents. As regards those who would like to retire but cannot afford to, they probably realise, albeit too late, the advantages of pension provision and would have liked a compulsory arrangement. To make available cover for these contingencies has been seen as a social responsibility and therefore the role of the State.

Prior to 1925 Government policy in the U.K. did not distinguish between the above reasons for intervention - pensions were integrated into the National Assistance Scheme, being paid out of general tax revenue and subject to a means test. In 1925 it was decided that groups of people who were likely to be future recipients of state pensions ought to contribute towards them during their working lives - thus the Contributory Pensions Act of that year incorporated pension provision into the system of social insurance that had been in operation since 1911. The principle employed was that of a flat rate contribution in return for a flat rate benefit. The individual contribution was augmented by employer and Exchequer contributions - a fund was set up and the Exchequer contribution varied according to whether the fund made a profit or loss from year to year. Endorsement of this arrangement was contained in the Beveridge Report¹, where it was argued that the insurance principle should guarantee a basic income for subsistence whilst provision for a higher standard would be the responsibility of the

individual and require voluntary insurance. The basic scheme was to be made available to all the workforce and coverage was thus extended in 1946. Nevertheless National Assistance still had a supporting role to play since most people who retired between 1946 and 1966 had not built up full pension rights, which were based on 20 years' membership, whilst others had not satisfied the full entitlement conditions, in that they had breaks in their payments.

Since then there have been only two major pension changes². In 1961 a graduated scheme³ was introduced which would operate alongside the flat rate scheme - in addition to the flat rate contributions, one which was earnings related was payable in return for which individuals received a contribution related pension as well as that guaranteed them under the flat rate scheme. The aim of the scheme was threefold. The Exchequer contribution to the flat rate scheme was growing and it was thought that relating benefits to contributions would reverse this trend. Private earnings related schemes were also available to many people and the graduated scheme offered similar arrangements to those not covered privately. It was also hoped that the scheme would encourage the growth of private schemes since employers who offered schemes that satisfied certain requirements could 'contract out' of the graduated scheme⁴, meaning that they had the option of withdrawing from the scheme.

This scheme was relatively short-lived and was discontinued, along with the flat rate scheme in 1975⁵, when a scheme based on proportional contributions and earnings related benefits was introduced. Such a scheme was the only logical choice given that the benefits from the flat rate scheme had continually fallen below the National Assistance/Supplementary Benefit subsistence levels

whilst under all assumptions concerning the final incidence of employers' contributions, the contribution system was regressive. In addition, Exchequer contributions were continuing to rise.

Turning now to the consideration of private pension schemes, the Beveridge Report, as mentioned above, had argued that individuals who required relatively high levels of retirement income and/or a certain degree of financial independence would have to make their own pension arrangements. This would also be the case if a flexible retirement date was desired. But neither of these is the most important reason individuals will voluntarily make pension provision. Its most important role is that it constitutes a hedge against unanticipated longevity. It is quite clear that the major problem in planning to live off accumulated savings during retirement is that when the date of death is uncertain it is difficult to determine the rate at which wealth ought to be consumed. A pension arrangement alleviates the consequences of an incorrect plan.

It has already been mentioned that pension schemes are often initiated by employers. In some cases these are non contributory: in most cases they are contributory and participation is compulsory. This type of arrangement has little to do with the uncertain length of life of employees and it seems unlikely that the employer is so benignant as to be concerned with the retirement welfare of his employees. It is not then surprising that employers themselves stand to gain from setting up a pension scheme. If pension claims are not transferable between occupations then an employer can maintain a fairly stable labour force and not incur costs of labour turnover (i.e. training costs). If a scheme encourages early retirement then

the employer can gain since he can plan an orderly pattern of recruitment and training and possibly reap efficiency gains as a result of replacing older workers by higher productivity younger workers. Provision of a pension scheme has also proved an important collective bargaining weapon and is a relatively cheap concession when employer contributions can be passed on in wages and prices as well as there existing the possibility of contracting out of the graduated scheme.

As well as individual and employer benefits from private pension schemes another beneficiary is the Government. This is the result of the financial burden of ageing being shifted from the Government to individuals and employers. This is explained by the reduced dependence on State pensions and the transfer of administration costs as a result of contracting out.

Although private schemes have existed in the public sector since the middle of the last century, it is only since the First World War that continuous growth has taken place. At the end of the inter-war period there were 4 million employees covered by public sector pension schemes. This figure has remained constant ever since. Between 1936 and 1971 the coverage of non public sector private schemes rose from 1.6 million to 7.0 million workers.

From the individual point of view there are two types of pension arrangement available. Formula plans refer to schemes where, although savings are generated, the pension paid is not determined by savings alone, if at all. Typical of these types of scheme are flat rate schemes where the contribution/benefit formula is as with the state scheme - this arrangement is popular with works schemes, 57% of works members having such a pension in 1971. Amongst staff

members the most popular type of arrangement, enjoyed by some 76%, is a terminal salary scheme where the pension is related to salary at retirement. The other major type of scheme is an average salary scheme in which case the pension is a function of average lifetime earnings, or the latter part thereof. About 10% of staff and works members are in such a scheme. In the previous two schemes the pension contributions are a fixed proportion of salary or earnings, whilst employers contribute a sum related to individual employee contributions. Savings plans involve capital accumulation in some form by or on behalf of the individual. The pension benefit is a function of accumulated savings and any interest payments. Such a scheme will be referred to as a money purchase scheme. Approximately 10% of staff and works members contribute to this type of scheme.

Schemes can be capital reserve or pay-as-you-go. Capital reserve schemes operate on the basic principle of risk pooling, losses being incurred on individuals whose life span is longer than average whilst gains are made on those who die relatively early. The average contributor just covers the cost of his pension after allowance is made for employer's contributions and interest earned on contributions. These schemes can themselves be divided into those which are funded and those that are insured. The difference between the two is simply the distinction between implicit and explicit insurance. A funded scheme relies on mutual insurance as a result of risk pooling whilst an insured scheme involves payments based explicitly on life contingencies. One purchases a life insurance policy which on retirement is converted to an annuity.

A pay-as-you-go arrangement is one where the pension is paid directly out of current resources including employee and employer contributions. The scheme has no investment income as there are neither interest payments on a fund nor insurance company profits.

Another important factor, which affects the attractiveness of a scheme from the employer's point of view, is the distinction between vested and unvested rights. A scheme is vested in a member when he becomes absolutely entitled to a future pension or cash refund of his contributions. Vested rights when first included in schemes often had age and loyalty restrictions attached to them. More recently encouragement towards unconditional vested rights has met with partial success although examples of this form of industrial feudalism are still not unknown.

This has been only a brief review of institutional pension arrangements in as much as they are relevant to the discussion of the economic aspects of pensions to follow. Indeed pensions raise many interesting, although underinvestigated, questions from an economic standpoint. The relationship between pensions and the distribution of wealth has only been investigated by Atkinson (8) (9) although interest should be increased in the aftermath of the Diamond Report (16). The role of pension funds in a growing economy, in particular the optimal combinations of funded and pay-as-you-go schemes, has recently been analysed by Praag and Poeth (48). How the State ought to decide the appropriate intergenerational redistribution rule in designing a social security pension scheme has been discussed in Asimokopoulous and Weldon (4). Other problems which have received some although not excessive attention are the flexibility of pension funds in creating credit and the effect of pension schemes on labour mobility.

In the opinion of the author, perhaps the most interesting question is the relationship between pension savings and other forms of personal capital accumulation. Although this aspect has been investigated in detail for the U.S. no corresponding work has been attempted for the U.K. In the first chapter of this thesis existing work in this area will be reviewed - the remaining chapters will contain a theoretical and empirical investigation of the relationship in the U.K.

NOTES

1. Cmnd. 6404 (1942). 'Social Security and Allied Services'.
2. Others were prepared. In particular two Labour party proposals contained in Labour Party (1957), 'National Superannuation' and Cmnd. 3883 (1969), 'National Superannuation and Social Insurance' were subject to much detailed discussion (see Atkinson (5) (6), Black (12) and Prest (49)).
3. Cmnd. 538 (1958), 'Provision for Old Age'.
4. The conditions of and issues relating to contracting out are discussed in Lynes (42).
5. Cmnd. 5713 (1974), 'Better Pensions'.

CHAPTER ONE

PENSIONS AND PERSONAL SAVINGS:

A REVIEW OF RESEARCH

1.1 INTRODUCTION

Pension saving is one of many possible methods of personal capital accumulation but, for the most part, it distinguishes itself from the alternatives by being automatic and passive. Only individuals who voluntarily join a pension scheme, which will normally be of the money purchase variety, can be said to be involved in active saving. Although it is unlikely that the behaviour within other financial intermediaries will affect the level of pension saving this does not imply that pension saving will not alter the rate of other types of saving¹.

A priori the effect of pension savings on other alternatives is difficult to determine since there exist reasons why savings could either increase or decrease. Savings might increase if participation in a pension scheme induces early retirement - any retirement saving then has to be spread over a longer period and an individual's reaction to this might be to intensify his savings effort. On the other hand, if savings were left unaltered, participation in a pension scheme would involve a fall in consumption - individuals might be more interested in maintaining the level of consumption and substitute pension saving for other forms of personal savings. The net outcome of these 'induced retirement' and 'savings replacement' effects, in the absence of further information relating to the

determinants of individual behaviour, is clearly ambiguous². It may be possible to say a little more if attention is focussed on the effects on total savings including retirement savings. The combination of the two effects mentioned above will generally result in an increase in total savings since the substitution effect leaves the level of savings constant, except in the case of state schemes where the pension contributions are not necessarily saved.

This introduction has served to illustrate how pension plans might affect savings although so far little has been learnt about the sign of the effect and nothing about its magnitude. A number of empirical studies have tackled this problem and the purpose of this chapter is to report their results; in particular those conducted in the U.S.

Given the findings of Kuznets (37), Modigliani and Brumberg (46), and Goldsmith (25) concerning the long run constancy of the savings ratio one would expect that a pension asset would be a substitute for other assets in the individual's portfolio. Indeed a general study of savings behaviour undertaken by Friend and Jones (24), using BLS-Wharton data for the U.S., found that in the case of private pensions, in common with other forms of contractual savings, this was the position although the substitution was far from perfect. Evidence relating specifically to the relationship between private pension schemes and personal savings in the U.S. is contained in studies by Cagan (15) and Katona (35), whilst the role of social security, both in the U.S. and internationally, is examined by Feldstein (19) (20). It is this research which is to be reported in detail.

1.2 CAGAN

Cagan, on behalf of the NBER Pension Group, investigated the savings behaviour of an 11% sample (11513) of members of the U.S. Consumers Union for the year 1958-9. The Consumers Union has a membership with a higher than average mean income and level of educational attainment.

Cagan first tested the substitution hypothesis between discretionary savings and pension savings and found that for those covered by pension plans their average pension saving ratio was 2.8% as was their average discretionary ratio³. For those not covered the average savings ratio was 2.1%. Although it is clear that pension savings and other forms of discretionary savings do not appear to be substitutes the Friend and Jones result, that discretionary savings and other forms of contractual savings were substitutes, was confirmed. As regards the relationship between pension and other forms of contractual savings individuals with pension plans had average contractual savings ratios of 5.9% whilst for those not covered the average was 5.7%.

In the U.S. social security coverage is not universal and if some households take anticipated social security benefits into account in making retirement plans this could affect their behaviour. Cagan therefore presented estimates of the average savings ratios of households with and without social security coverage but all of whom had a private pension. The average contractual savings ratio was 5.9% for both groups whilst the average pension savings ratio was 3.7% for those with social security and 2.6% for those without. This latter group had a higher average discretionary savings ratio of 3.1% as opposed to 1.7%. It would appear that the evidence suggests that

being covered by social security as well as a private pension increases the degree of complementarity.

Cagan had a number of reservations about this analysis. Firstly it could be that those covered by private pensions and social security save more because they have higher incomes and are older. To examine the extent of these variabilities the savings ratios of arbitrarily chosen sub-samples divided according to age and income were calculated and on the basis of these results the full sample estimates were adjusted. It was found that the 'pension effect' did not vary with either age or income, except in the case of the relatively small extreme income classes, although the range of error was larger. Secondly, it might be that the savings ratio is positively correlated with occupation and educational attainment. Although the data confirmed this it was assumed that as income, occupation and educational attainment are in general highly correlated the above results for income variability probably hold in these cases. Lastly it might be that the causation runs the opposite way to that assumed and individuals with high savings ratios choose occupations offering pension coverage. The results already cited suggest that this is not the case - if jobs with pension plans are more accessible to younger workers, which is likely to be the case, then one would expect the 'pension effect' to be age specific, which it is not.

An attempt was made to test whether there were any extraneous differences between covered and non-covered groups by inferring quasi-life cycle savings ratios from the cross-section data. The survey contained information as to how long people had been covered by private pensions and given their age it was then possible

to work out how long they were not covered if an assumption was made about the age of entry into the workforce. If this was taken to be 22 then the wealth of a person aged A would be given by

$$W_A = W_{22} + \sum_{i=23}^{i=P-1} s_i + \sum_{j=P}^{j=A} s_j \quad (1.2.1)$$

where s is annual savings (discretionary plus contractual) and P is the age at which an individual joined the private scheme. The following regression equation has then estimated

$$\frac{W_A}{Y_A} = \alpha \frac{W_{22}}{Y_{22}} + \beta_0 \sum_{i=23}^{i=P-1} S_i + \beta_1 \sum_{j=P}^{j=A} S_j \quad (1.2.2)$$

where S_i and S_j are the average savings ratios while not covered and covered by a private pension respectively. The regression estimates⁴ revealed that for those covered the average savings ratio rose from 6.2% to 8.2% on entering a pension scheme. For those not covered by the average savings ratio was 6.3%. This analysis provided confirmation of the earlier results in qualitative terms. The fact that the latter ratios were all higher probably reflects the different estimation procedure used.

Cagan explained his results in terms of a recognition effect - he argued that on joining a private pension scheme an individual is made aware that a 'reasonable' income is attainable during the retirement years. The way to achieve this is through additional savings.

1.3 KATONA

Katona surveyed a cross-section of the continental U.S. population over 1962-3 - he confined his analysis to 1853 households in a 'crucial' group which were complete families with a head aged 35-65 and earning at least \$3000 per annum. Young persons were excluded since they would not have formulated specific retirement expectations whilst low income families were excluded owing to their assumed lack of discretion over consumption/saving decisions. Although Katona concerned himself with a number of aspects of the relationship between retirement, pension plans and income only those connected with the relationship between the participation in a private pension plan and savings behaviour will be reported.

Less than half of the crucial group were covered by a private pension plan and Katona compared the savings ratios of this group with those not covered. It was found that 7% of those not covered and 14% of those covered had savings ratios greater than or equal to 10%. For the savings ratios 5-9% and 2-4% the corresponding percentages were 6%/9% and 6%/12% respectively. These results support those of Cagan.

Katona also performed a series of regressions to determine the relative significance of pension plan participation in explaining savings. The method was to regress savings - defined in the following three ways; savings ratio greater than 5%, those who had saved in the last 24 months (savings behaviour) and the proportion of an anticipated income increase saved (attitudinal behaviour) - on age, income and a series of retirement variables each added successively into the equation. Retirement was represented by pension plan participation (yes/no dummy variable), ratios of expected retirement income to current income and expected retirement needs (high/low dummy variables and

a subjective estimate of the adequacy of expected retirement income (enough/not enough dummy variable). Analysis of t values revealed that pension plan participation explained savings consistently better than other retirement variables except in the case of savings behaviour where the subjective estimate of the adequacy of retirement income came out best. It also proved more significant than age and compared favourably with income. As with the Cagan regressions, these confirmed that the relationship between savings and pension plan participation was one of complementarity.

Katona explained his findings in terms of a goal gradient hypothesis arguing that as an individual nears his aspiration level, which in this case will be a desired level of retirement income, his savings effort is intensified.

1.4 FELDSTEIN

Feldstein (19) examined the effect of the growth of social security pensions on aggregate savings in the U.S. He argued that if the combination of social security taxes and benefits has no income effect and if the pattern of retirement and income is fixed a social security programme will reduce savings by just enough to leave consumption during retirement unchanged. This he called the savings replacement effect. Operating in the opposite direction will be an induced retirement effect. This comes about because in the case of those who, in the absence of social security⁵, would have worked after the official retirement date there is now an incentive to reduce the supply of labour. If younger workers correctly anticipate

this reduction in their working lives then they will increase their savings. The net outcome of these two effects will be ambiguous. This possibility, which is one that is also mentioned in the introduction to this chapter, is arrived at not because of a recognition or goal gradient effect but because of the endogeneity of the retirement decision. Feldstein demonstrated this possibility theoretically by a simple extension of the two period life cycle model⁶.

Consider the case of an individual who, in the absence of social security would retire at the official retirement age. In Figure 1.4.1 his first period earnings are Y_0 - he would choose equilibrium A, consuming C_0 and saving $(Y_0 - C_0)$. Second period consumption would be $(1+r)(Y_0 - C_0)$, the interest factor, $(1+r)$, being given by the slope of the budget line tangential to I. Introducing the social security pension a tax $(Y_0 - Y_0^1)$ is imposed and benefits in the second period are $(1+r)(Y_0 - Y_0^1)$. The individual's budget line is not altered and he maintains equilibrium A - the individual reduces savings by exactly $(Y_0 - Y_0^1)$.

This result can be compared with that for the case where, in the absence of social security, the individual would continue to work after the official retirement date. In Figure 1.4.2 the individual's initial position is given by C with the same first period earnings Y_0 and positive second period earnings. Equilibrium would be at B with consumption C_0^1 in the first period and $(1+r)(Y_0 - C_0^1) + Y_1$ in the second period. If the introduction of the social security induces retirement at the official date then the initial position will shift to a point such as D with first period earnings falling by the amount of the tax $(Y_0 - Y_0^1)$ and lower second

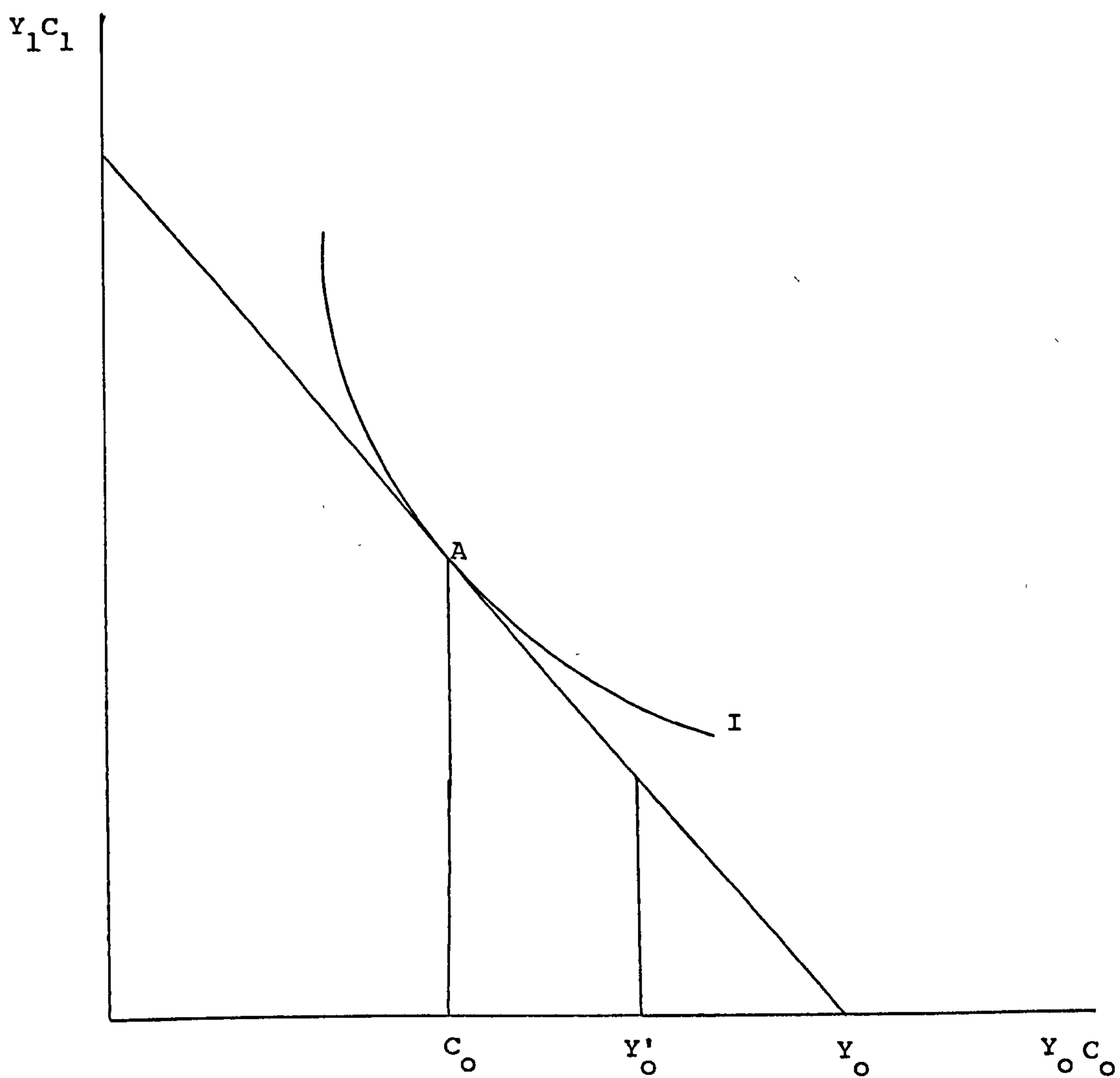


Figure 1.4.1

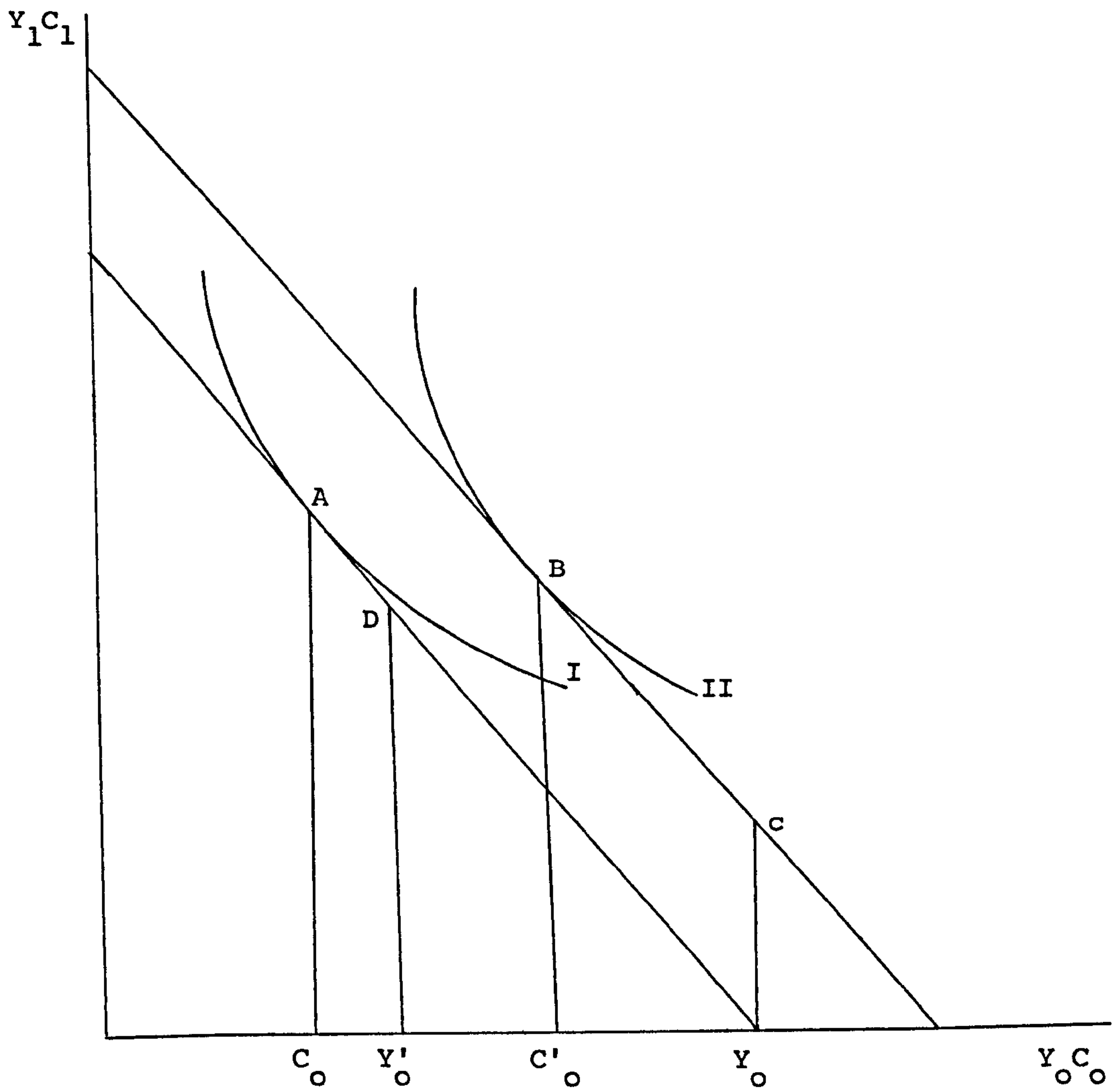


Figure 1.4.2

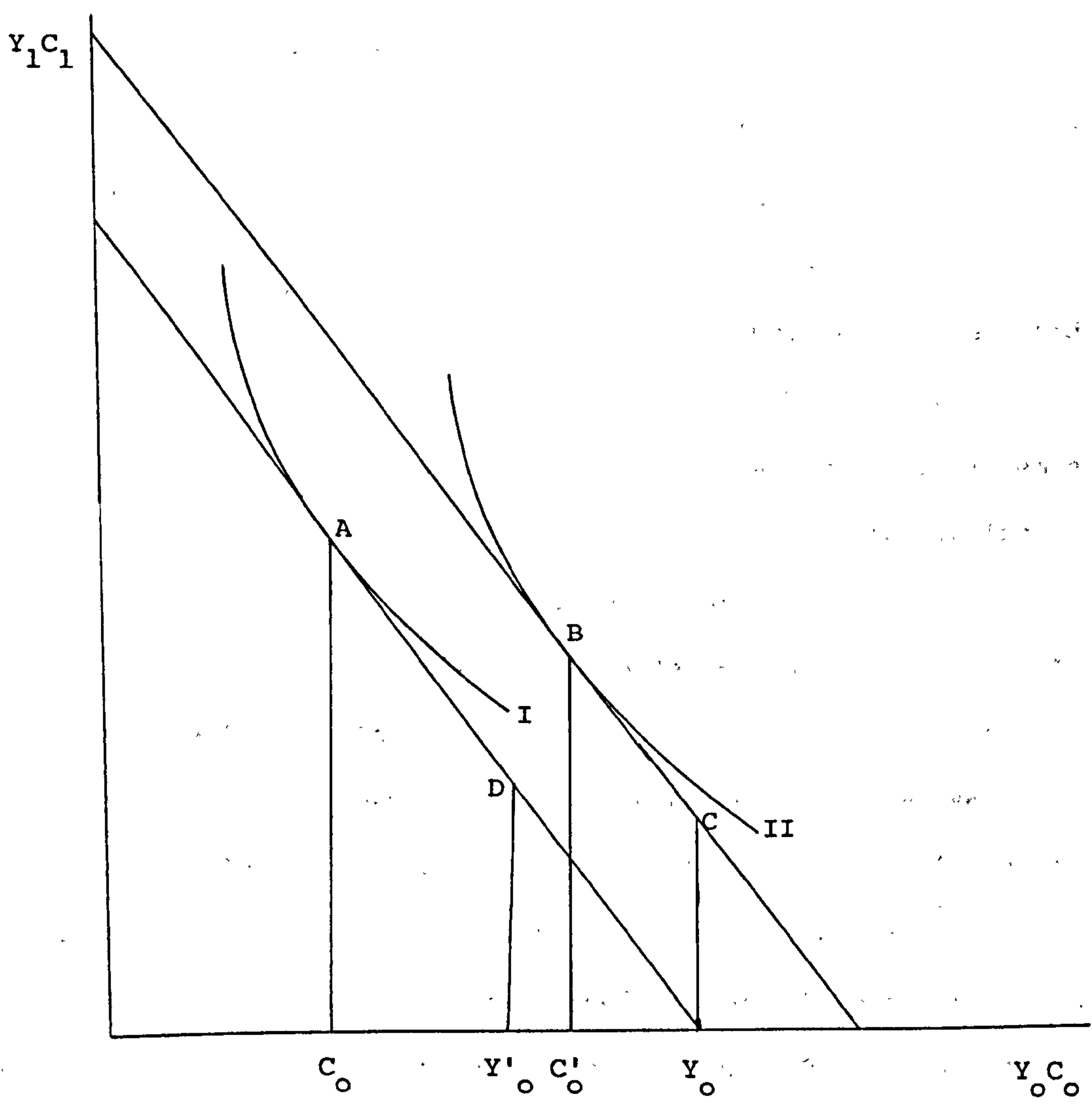


Figure 1.4.3

period income as earnings are not fully replaced by social security. Starting from initial position D the individual's equilibrium is at A - this choice implies a reduction in savings since $(Y_0 - C_0^1) > (Y_0^1 - C_0)$. It is quite clear though that a different expansion path of consumption could yield $(Y_0 - C_0^1) < (Y_0^1 - C_0)$. Figure 1.4.3 is constructed to demonstrate such a result.

To assess the effect empirically Feldstein estimated the Modigliani-Brumberg-Ando life cycle consumption function with an additional term, the discounted present value of anticipated pension benefits weighted by survival probabilities, included. This was called social security wealth and was defined in both gross (excluding liabilities) and net (including liabilities) terms. It was evaluated on the assumption that individuals would not work beyond the official retirement date and reflects, therefore, both the savings replacement and induced retirement effects. For the period 1929-71 (excluding 1941-6), using both definitions of social security wealth, discounting future pension income at both 1% and 5% and considering a number of exact specifications of the consumption function it has found that the marginal propensity to consume out of social security wealth was both larger and more significant than that out of ordinary wealth. It took significant values of between .010 and .075⁷ - the higher marginal propensities corresponded to the net definitions of social security wealth but changing the definition did not affect the coefficients of the other income and wealth variables. To test the possibility that the significant coefficient on social security wealth variable was only a reflection of the shift in consumption behaviour in moving from the pre- to the post-war period the consumption functions were re-estimated for the period 1947-71.

Although the coefficients on social security wealth were much the same, ranging from .014 to .035⁸, and Feldstein argued that they supported the conclusion that social security depressed the level of personal savings, it has to be added that these coefficients were statistically insignificant.

Turning now to the implications of this study, a marginal propensity to consume personal disposable income of .650 and that to consume social security wealth of .021⁹ imply that in 1971 social security halved the rate of personal saving¹⁰. Feldstein felt that this was not a surprising result since for many low and middle income families the savings replacement effect would be large whilst the induced retirement effect was likely to be relatively small. The evidence available appeared to confirm this since before the introduction of social security about 45% of men over 65 were retired whilst in 1965 the figure had only risen to 73%. Those who had not retired would typically be workers on lower incomes.

In a second paper, Feldstein (20) presented an international cross-section analysis of the relationship. He argued that in a stationary economy the standard life cycle model implies that there will be no net aggregate savings and there will exist an equilibrium aggregate ratio of wealth to income which is a function of the interest rate, the rate of discount and the life cycle earnings profile. A growing economy on the other hand will exhibit positive net aggregate savings and require continuing increases in wealth to maintain the equilibrium ratio of wealth to income. If there is a system of social security then the adjustment of wealth to maintain this equilibrium will have to take account of wealth embodied in social security pensions. Specifically if the maintenance of the

wealth income ratio requires an increase in wealth then savings has to increase but if social security benefits increase this can be substituted for an equal amount of private savings. This wealth replacement effect can be seen to be formally identical to the savings replacement effect. Social security also reduces the number of older workers which implies an increase in the equilibrium wealth to income ratio and private savings has to increase. This is the induced retirement effect.

In a situation of steady state growth, where the savings ratio is approximately proportional to the rate of growth of income¹¹ this hypothesis could be tested by estimating a savings function of the form

$$\frac{S}{Y} = (\alpha + \beta_0 \frac{B}{Y} + \beta_1 LPA)G \quad (1.4.1)$$

where S is per capita savings, Y per capita income, B is the average social security pension, LPA the labour force participation of the aged and G the rate of growth of national income. The bracketed term, which contains both the wealth replacement and induced retirement effects reflects the equilibrium ratio of wealth to income. When dealing with cross-section data the above steady state specification has to be modified since the data will reflect disequilibrium situations. This was achieved by adopting a linear form of the equation with G as a linearly related independent variable and introducing further variables used by Modigliani (45) in his international cross-section test of the life cycle hypothesis, in particular the ratio of minor dependants to working population (non-aged) and the ratio of old individuals to working population (non-aged)

and life expectancy at the official retirement date. In addition variables representing the proportion of the aged receiving pensions and the average benefit per aged person with and without an earnings test were included in some of the estimated equations. All ratios were included in percentage terms.

The savings functions were estimated¹² by two stage least squares with labour force participation of the aged endogenous and the results confirmed that social security pensions had considerable impact on personal savings. The results suggested that the savings ratio was increasing in the rate of growth of income and life expectancy at retirement whilst being negatively related to the labour force participation of the aged. It was also found to be decreasing in both the level of social security benefits and coverage¹³. These findings accord with the discussion of the theory.

To discover the net impact on savings Feldstein estimated a series of equations with the labour force participation of the aged as the dependent variable and a number of these included in the previous regressions as dependent variables. The most interesting result was that labour force participation was revealed to be a decreasing function of both the level of social security pension benefits and the ratio of pension recipients to the total population over the official retirement age¹⁴. Combining estimated equations relating to the determination of the savings ratio and the labour force participation of the aged suggested that the net impact of the wealth replacement and induced retirement effects was to reduce savings¹⁵.

1.5 CONCLUSION

It has been shown in this review that the effect of pension scheme participation on personal savings is both theoretically and empirically ambiguous. Using cross section data for the U.S. both Cagan and Katona found that the relationship between personal savings and pension plan participation was one of complementarity. In a development context this result was confirmed by Reviglio (51). This positive net effect on savings was explained by a recognition or goal gradient hypothesis. Feldstein (19) (20) argued that this result could be arrived at without resorting to these hypotheses: an induced retirement effect explains the same outcome. His U.S. time series and international cross section empirical analyses suggested that the net impact on savings was negative. If there was an induced retirement effect it was outweighed by a savings replacement effect. The overall relationship was therefore one of substitutability. This result has also been arrived at in the Canadian context by Schoepflein (53).

So far no analysis of this relationship has been undertaken for the U.K. - this will be the aim of this thesis. Specifically it will attempt to

- i) extend Feldstein's two period life cycle model to a multi-period analysis. A continuous time formulation will be used.
- ii) include both state and private pensions in the theoretical and empirical analysis.

NOTES

1. This will now be called simply savings except where this proves grammatically clumsy. In such cases the nature of saving will be made explicit.
2. From what has been said this is clearly the only conclusion but one might argue that it is unlikely that increases in savings due to induced retirement will ever outweigh the reduction necessary to compensate for lower disposable income. The feasibility of such an outcome is discussed in detail in the next chapter.
3. Excluding those with extreme savings ratios.
4. The regressions ignored changes in savings ratios with age and used current income to deflate wealth, therefore understating the ratio W/Y as one moves back through time.
5. Social security always means the social security pension scheme.
6. Feldstein also presents his model in mathematical form. An abbreviated version of this model is presented in the Appendix to this chapter.

7. Using a net rate of discount of 1%, typical results were

$$C_t = 228 + .530 Y_t^d + .120 Y_{t-1}^d + .356 RE_t + .014 W_{t-1} + .021 SSW \text{ (gross)}$$

(31) (.047) (.035) (.074) (.004) (.006)

SSR = 3618 DW = 1.82

$$C_t = 218 + .528 Y_t^d + .137 Y_{t-1}^d + .376 RE_t + .013 W_{t-1} + .032 SSW \text{ (net)}$$

(27) (.047) (.034) (.073) (.004) (.009)

SSR = 3548 DW = 1.85

There are results for regressions using per capita values of all variables at constant prices. C_t is consumer's expenditure, Y_t^d personal disposable income, RE_t corporate retained earnings, W_{t-1} net wealth and SSW social security wealth. The inclusion of the term $Y_{t-1}^{\hat{d}}$ reflects expected income. The justification for including RE_t is that it proved a significant determinant of consumption in testing the capital income hypothesis (see Feldstein and Fane (22)). This hypothesis is considered in some detail in Section I of Chapter 4.

8. Corresponding to previous results one has

$$C_t = 193 + .535 Y_t^d + .139 Y_{t-1}^d + .414 RE_t + .015 W_{t-1} + .014 SSW \text{ (gross)}$$

(159) (.097) (.097) (.163) (.009) (.030)

SSR = 3086 DW = 1.63

$$C_t = 232 + .535 Y_t^d + .119 Y_{t-1}^d + .349 RE_t + .014 W_{t-1} + .035 SSW \text{ (net)}$$

(104) (.084) (.085) (.170) (.007) (.030)

SSR = 2926 DW = 1.78

9. See note 7 above.

10. The decrease consists of two components. Firstly savings is lost due to the fact that social security contributions reduce disposable income - this loss is equal to $(1 - .650)(1 - \text{the rate of income tax}) \times \text{aggregate contributions}$. Secondly there is a positive marginal propensity to consume out of social security wealth which implies a reduction in savings of $.021 \times \text{social security wealth}$.
11. The proportionality is only approximate because a growing population will change the age structure which implies a different equilibrium ratio of wealth to income.
12. The data used was a sub-sample of that used by Modigliani - although it consisted of only 15 of his 36 countries it included 62% of the total sample by numbers and 97% of those in the developed countries.
13. The significant coefficients on the labour force participation of the aged varied between $-.24$ and $-.38$, those on benefits per retired pension between $-.101$ and $-.108$, and those on benefits per elderly person between $-.077$ and $-.104$. On the rate of growth they varied between 1.17 and 1.69 and on life expectancy between 1.60 and 2.20. The effect of coverage was also negative, the coefficient varying between $-.056$ and $-.083$.
14. The significant coefficients on the level of benefits vary between $-.25$ and $-.37$ and on coverage between $-.1$ and $-.2$.

15. Feldstein considered the effect of a 25% difference in the benefit level and coverage. The net effect consists of a pure wealth replacement effect, which if the coefficient on benefits per aged person is $-.108$ implies a reduction in savings of 2.7%, and an induced retirement effect, which if the relevant coefficient is $-.25$ implies an increase on the savings rate of 1.49%. The net effect is a decrease in savings of 1.21%. A similar calculation for coverage yielded a net decrease in savings of 1.48%.

APPENDIX 1

Appendix 1 THE FELDSTEIN MODEL Model

For purposes of comparison with results in the next chapter this Appendix contains the mathematical version of Feldstein's model for the case where an individual is free to choose how much work he does in the second period, when he is entitled to a pension.

The individual has a two period utility function

$$U = u(c_0, l_0) + v(c_1, l_1) \quad (1.A.1.)$$

Denoting labour income in each period, i , $(1-l_i)$ we can write

$$c_0 = (1-t)(1-l_0) - s \quad (1.A.2.)$$

where t is the rate of social security tax, s denotes savings, and

$$c_1 = (1+r)s + (1-t-u)(1-l_1) + b \quad (1.A.3.)$$

where r is the rate of interest, u is the earnings test rate of tax and b is pension income.

Assuming that l_1 is fixed the first order conditions for utility maximisation are

$$\partial u / \partial s = -u_1 + v_1(1+r) = 0 \quad (1.A.4.)$$

$$\partial u / \partial l_1 = -v_1(1-t-u) + v_2 = 0 \quad (1.A.5.)$$

Totally differentiating these equations and employing the assumption that the pension scheme operates according to the constraint

$$b = (1+r)t(1-l_0) + (t+u)(1-l_1) \quad (1.A.6.)$$

yields two equations for ds and $d\ell_2$ from which dt can be eliminated.

These equations are given by

$$A \begin{pmatrix} ds \\ d\ell_2 \end{pmatrix} = \underline{a} du + \underline{b} db \quad (1.A.7.)$$

where

$$A = \begin{pmatrix} (u_{11} + (1+r)^2 v_{11}) & (1+r)(v_{12} - v_{11}) \\ (1+r)(v_{12} - v_{11}) & (v_{11} + v_{22} - 2v_{12}) \end{pmatrix}$$

$$\underline{a} = \begin{pmatrix} \left[((1+r)(1-l_0) + (1-l_1))^{-1} (1-l_0)(1-l_1) (u_{11} + (1+r)^2 v_{11}) \right] \\ - \left[((1+r)(1-l_0) + (1-l_1))^{-1} (1+r)(1-l_0) (v_1 + (v_{11} - v_{12})(1-l_1)) \right] \end{pmatrix}$$

$$\underline{b} = \begin{pmatrix} \left[((1+r)(1-l_0) + (1-l_1))^{-1} (1-l_0) (u_{11} + (1+r)^2 v_{11}) \right] \\ \left[((1+r)(1-l_0) + (1-l_1))^{-1} (v_1 - (1+r)(1-l_0)(v_{11} - v_{12})) \right] \end{pmatrix}$$

which can be solved to obtain

$$|A| \frac{ds}{du} = \left[(1+r)(1-l_1) \right]^{-1} (1-l_0) \left\{ |A| (1-l_1) - (1+r)^2 (v_{11} - v_{12}) v_1 \right\} \quad (1.A.8.)$$

$$|A| \frac{ds}{db} = - \left[(1+r)(1-l_0) + (1-l_1) \right]^{-1} \left\{ |A| (1-l_0) + (1+r)(v_{11} - v_{12}) v_{11} \right\} \quad (1.A.9.)$$

$$|A| \frac{d\ell_1}{du} = - \left[(1+r)(1-l_0) + (1-l_1) \right]^{-1} \left\{ (1+r)(1-l_0) \left[v_{11} + (1+r)^2 v_{11} \right] v_1 \right\} \quad (1.A.10.)$$

$$|A| \frac{d\ell_1}{db} = - \left[(1+r)(1-l_0) + (1-l_1) \right]^{-1} \left\{ u_{11} + (1+r)^2 v_{11} \right\} v_1 \quad (1.A.11.)$$

The second order conditions for utility maximisation imply that $|A| > 0$

and $\left[u_{11} + (1+r)^2 v_{11} \right] < 0$. Thus from (1.A.10.) and (1.A.11)

$$d\ell_1/du < 0 \quad (1.A.12.)$$

$$d\ell_1/db < 0 \quad (1.A.13.)$$

Increases in the rate of tax on second period earnings as a result of the earning rule and the pension make second period leisure relatively attractive. If it is assumed that $v_{11} - v_{12} < 0$ then from (1.A.8.)

$$ds/du > 0 \quad (1.A.14.)$$

which is understandable since the earnings test reduces second period labour income. Unfortunately this assumption does not enable ds/db to be signed.

CHAPTER TWO

A LIFE CYCLE MODEL OF ACCUMULATION AND RETIREMENT BEHAVIOUR INCORPORATING A STATE PENSION SCHEME

2.1 INTRODUCTION

The purpose of this chapter is to extend Feldstein's two period life cycle model of accumulation and retirement behaviour to a multiperiod analysis. The features of the U.S. social security retirement plan included in his analysis will be replaced by those of the flat rate state pension scheme as it has operated in the U.K. until fairly recently.

In section 2.2 a simple version of the two period model will be reviewed. This will then be extended employing a continuous time formulation and a life cycle accumulation/consumption plan will be derived.

Section 2.3 will see the incorporation of the state pension scheme into the model. The optimal retirement date will be considered.

The results of sections 2.2 and 2.3 will be combined in section 2.4 when the effect of changes in pension contributions and benefits on retirement behaviour and lifecycle savings is analysed.

In section 2.4 certain modifications of the analysis are presented.

2.2 THE OPTIMUM CONSUMPTION PLAN

In the simplest version of the life cycle model of consumer behaviour¹ it is assumed that

A.1. the individual knows all price and income variables with certainty

A.2. the individual knows how long he will live

A.3. the individual neither receives nor desires to leave any bequest

A.4. there is a perfect capital market.

In the two period model the individual's problem is to choose a consumption vector (c_0, c_1) to maximise his lifetime utility subject to a budget constraint determined by his income in each period. Assume the utility function to be of the form

$$U = u(c_0) + u(c_1)(1/1+\delta) \quad (2.2.1.)$$

where u is the same one period utility function with the properties $u'(\cdot) > 0$, $u''(\cdot) < 0$. δ is the pure or subjective rate of time preference. If it is assumed that there are no second period earnings then (2.2.1.) is maximised subject to the constraint that

$$y_0 - c_0 - c_1(1/1+r) = 0 \quad (2.2.2.)$$

where r is the market rate of interest, which is used to discount future income and consumption.

From the first order conditions for a maximum

$$\partial \pi / \partial c_0 = u'(c_0) - \lambda = 0 \quad (2.2.3.)$$

$$\partial \pi / \partial c_1 = u'(c_1)(1/1+\delta) - \lambda(1/1+r) = 0 \quad (2.2.4.)$$

where \mathcal{L} is the appropriate Lagrangian and λ the Lagrange multiplier, it can be shown that

$$u'(c_1)/u'(c_0) = (1+\delta)/(1+r) \quad (2.2.5.)$$

which implies that if

$$(a) \quad r > \delta \text{ then } c_1 > c_0$$

$$(b) \quad r = \delta \text{ then } c_1 = c_0$$

$$(c) \quad r < \delta \text{ then } c_1 < c_0$$

Diagrammatically these results are summarised in Figure 2.2.1.

This model is easily extended to the multiperiod case where the individual is assumed to live for L years. The first order condition in each period will be

$$u'(c_i) - \lambda(1/1+r)^i = 0 \quad (i = 0 \dots L) \quad (2.2.6.)$$

whilst the marginal rate of substitution between consumption in any two periods is given by

$$u'(c_j)/u'(c_i) = \left[(1+\delta)/(1+r) \right]^{j-i} \quad (j \geq i) \quad (2.2.7.)$$

(2.2.7.) again implies that consumption is growing at a steady rate from period to period, decreasing at a steady rate, or constant through time depending on the relative magnitude of r and δ . This result has straightforward economic meaning. If the market rate of interest is greater than the rate of time preference the individual can always increase the value of his lifetime utility by delaying consumption and investing his income. On the other hand, if the rate of interest is less than the rate of time preference lifetime utility will be raised by borrowing and bringing forward consumption.

To analyse the life cycle model in more detail a continuous

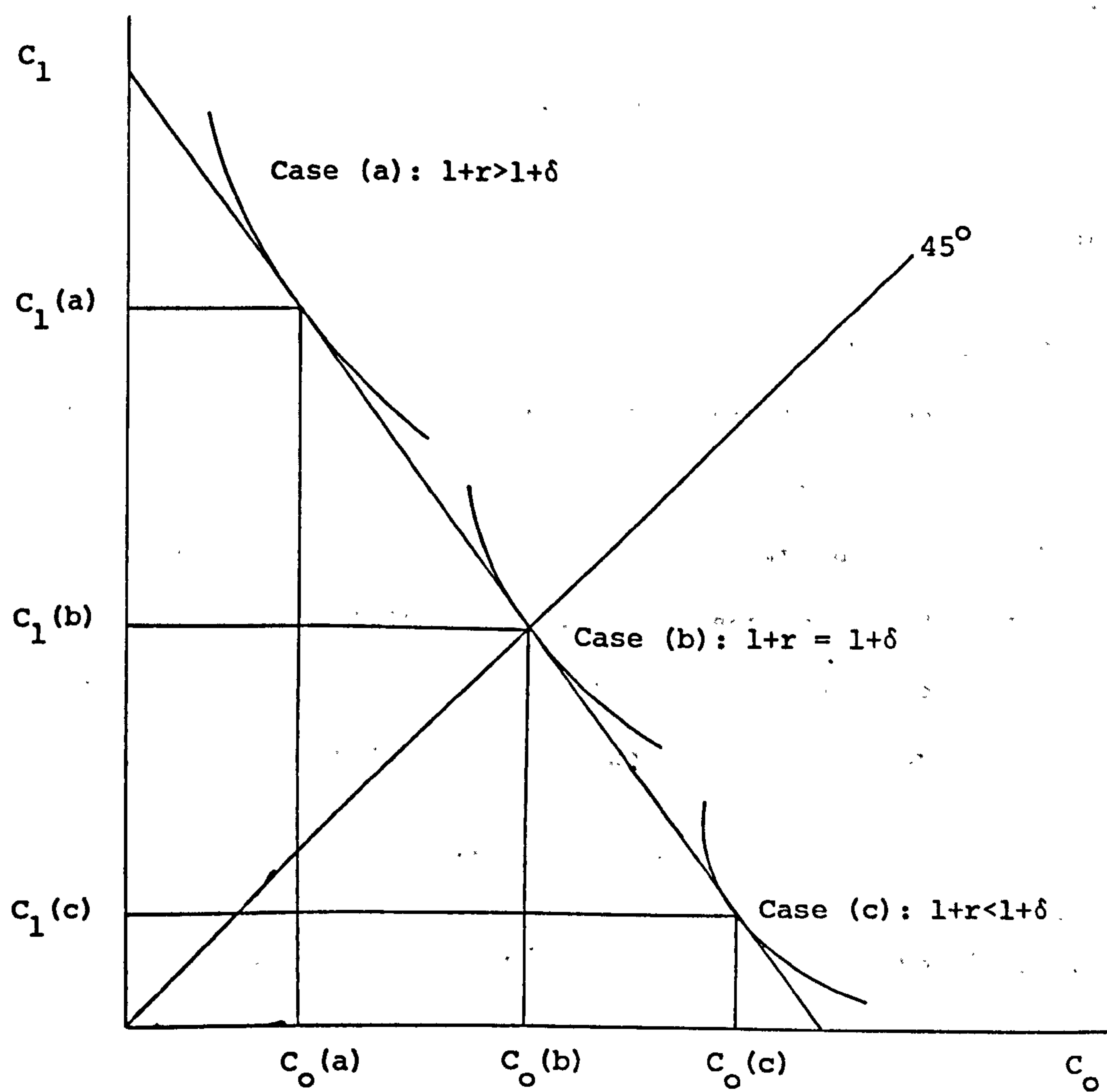


Figure 2.2.1

time formulation will be used. This has the advantage that it accords more closely with economic theory than its discrete time analogue in that consumption/accumulation decisions are assumed to take place continuously over time.

In such a model the individual is assumed to maximise the intertemporal utility function

$$U = \int_0^L u(c(t)) e^{-\delta t} dt \quad (u'(0) = \infty) \quad (2.2.8.)$$

subject to the accumulation and terminal constraints

$$\dot{k}(t) = rk(t) + y(t) - c(t) \quad (2.2.9.)$$

and $k(L) \geq 0 \quad (2.2.10.)$

where $y(t)$ is non-capital income and $k(t)$ is non-human wealth at the beginning of period t . Both the rate of interest, r , and the subjective rate of time preference, δ , are known constants - this assumption is to later be replaced by that of r and δ being known functions of time².

The maximisation of (2.2.8.) subject to (2.2.9.) and (2.2.10.) is a straightforward problem in the calculus of variations³. Introducing the Lagrange multiplier $\lambda(t)$ one obtains the expression

$$\mathfrak{F} = \int_0^L u(c(t)) e^{-\delta t} dt + \int_0^L \lambda(t) (rk(t) + y(t) - c(t) - \dot{k}(t)) dt \quad (2.2.11.)$$

Integrating $\lambda(t)\dot{k}(t)$ by parts and substituting back into (2.2.11.) yields

$$\mathfrak{F} = \int_0^L u(c(t)) e^{-\delta t} dt + \int_0^L \lambda(t) (rk(t) + y(t) - c(t)) dt +$$

$$\int_0^L \dot{\lambda}(t)k(t) dt - \lambda(L)k(L) - \lambda(0)k(0) \quad (2.2.12.)$$

Differentiating (2.2.12.) with respect to $c(t)$, $k(t)$ and $k(L)$ yields

$$u'(c(t))e^{-\delta t} - \lambda(t) = 0 \quad (2.2.13.) \text{ i}$$

$$\lambda(t)r + \dot{\lambda}(t) = 0 \quad (2.2.13.) \text{ ii}$$

$$\lambda(L) \geq 0 \quad k(L) \geq 0 \quad (2.2.13.) \text{ iii}$$

the last condition holding with complementary slackness. From (2.2.13.) i and (2.2.13.) ii

$$-r u'(c(t))e^{-\delta t} = (d/dt) \{u'(c(t)) e^{-\delta t}\} \quad (2.2.14.)$$

which is the standard Euler equation result in the calculus of variations. In addition, (2.2.13.) i and (2.2.13.) iii yield the terminal constraint

$$\lambda(L) = u'(c(L))e^{-\delta L} \quad (2.2.15.)$$

Differentiating (2.2.14.) and setting the elasticity of the marginal utility of consumption $-u''(c(t)) \cdot c(t) / u'(c(t)) = \theta(c(t))$ yields the time path of consumption

$$\dot{c}(t)/c(t) = (r-\delta)/\theta(c(t)) \quad (2.2.16.)$$

If it is assumed that the elasticity of the marginal utility of consumption is constant the implications for the growth rate of consumption of the relative magnitudes of r and δ , as given in (2.2.7.) are again confirmed.

One can allow for the possibility that the growth rate of consumption changes over the life cycle by making the assumption that $r(\cdot)$ and $\delta(\cdot)$ are known functions. Repeating the maximisation will reveal that (2.2.10.) becomes⁴

$$\dot{c}(t)/c(t) = (r(t)-\delta(t))/\theta \quad (2.2.17.)$$

Having now derived the optimal time path of consumption with $y(\cdot)$, $r(\cdot)$ and $\delta(\cdot)$ all unspecified functions little can be said about the accumulation path. Although it can be said that it will be the difference between consumption and income in every period one cannot say, a priori, whether the time path of accumulation will exhibit the Harrod 'hump', will be rapidly rising or diminishing, or be constant. In any case, as interest is only to be focussed on qualitative changes in accumulation its exact specification is not crucial. For this reason the consumption model will be used in the perfectly general form described above.

Before going on to consider the introduction of a state pension into the above model and the effect this will have on the accumulation/consumption path the following point can provide an introduction to the analysis. The individual's budget constraint (2.2.9.) can be written

$$\int_0^L c(t)e^{-rt} dt = k_0^{(0)} + \int_0^L y(t)e^{-rt} dt \quad (2.2.18.)$$

where the left hand side is total lifetime consumption and the right hand side total lifetime wealth. Changes in income affect consumption only through changes in total wealth: specifically, a higher value of total wealth will result in a higher value of lifetime consumption. The effect on consumption in every period will be given by solving (2.2.17.) for c_0 and deriving a relationship between this and (2.2.18.). This problem is held over until section 2.4.

2.3 STATE PENSIONS AND OPTIMAL RETIREMENT

From what has just been said it should be fairly clear what the significance of introducing a state pension scheme into the above model is going to be. Interest is to be focussed on how the introduction of a pension scheme will affect capital accumulation or consumption. There are two important points. The first is that depending on the relationship between pension contributions and benefits lifetime income and therefore total wealth can rise or fall. Secondly, if retirement is endogenous to the individual, then the pension scheme allows the income stream to become a variable to be chosen by the individual through the choice of the retirement date. These are the problems to be analysed.

The state pension scheme is assumed to operate on a flat rate basis. Irrespective of income the individual contributes an annual sum, a , into the scheme in return for an annual pension p . The pension is payable from period T_1 , ($0 < T_1 < L$). This means that the restrictions on net income in every period can be written

$$y(t) = \begin{cases} w(t) - a & \text{if } 0 \leq t < T \text{ and } T_1 \leq T < L \\ p & \text{if } T \leq t < L \text{ and } T_1 \leq T < L \end{cases} \quad (2.3.1.)$$

Now (2.3.1.) is clearly a constraint in the maximisation problem just outlined. The fact that it has been ignored up until now is not really crucial since one can invoke here the separation theorem which concerns the independence of investment or portfolio decisions and accumulation decisions. The separation theorem can be applied to this analysis since

the assumption that there exists a perfect capital market implies that the individual faces a single budget constraint. When one can assume separability the procedure for optimality is to maximise lifetime income and then maximise utility subject to an investment plan which maximises lifetime income.

Although this analysis has plainly proceeded in the incorrect sequence it is thought to be analytically neater to consider utility maximisation first and then income maximisation. It just has to be remembered that the first plan depends on the second.

To analyse the individual's problem two cases will be considered. Firstly it can be assumed that the individual is free to choose his retirement date, T , and makes a choice which maximises the discounted present value of lifetime income, which will be denoted $Y(T)$. On the other hand it can be assumed that the retirement date is institutionally determined to be T_1 . The former case is similar to that analysed by Feldstein (19) and the results derived will be comparable with those of the Appendix to Chapter 1. The latter case represents a situation without induced retirement as might be the case with a well developed state scheme.

If the individual is free to choose his retirement date then from (2.3.1.) the discounted present value of lifetime income is given

by

$$Y(T) = \int_0^T (w(t) - a) \exp\left(-\int_0^t r(\tau) d\tau\right) dt + p \int_T^L \exp\left(-\int_0^t r(\tau) d\tau\right) dt$$

(2.3.2.)

With the state pension only payable from T_1 , if $T < T_1$ the second expression in (2.3.2.) will only apply from T_1 . This will only occur if $w(t) = 0$ at the end of the individual's working life, an unlikely occurrence.

To reduce the algebraic complexity of the analysis write $R(t) = \exp\left(-\int_0^t r(\tau) d\tau\right)$ - this is then the discounted present value of a pound received in period t . The first order condition on retirement can now be written as

$$Y_T = (w(T) - a - p) R(T) = 0 \quad (2.3.3.)$$

which says that at the optimum the discounted present values of net wage income and pension income must be equal⁵. There will also be the second order condition for an interior maximum

$$Y_{TT} = (w(T) - a - p) R'(T) - w'(T)R(T) < 0 \quad (2.3.4.)$$

which is a requirement that the marginal cost of postponing retirement should be changing more slowly than the marginal benefit from so doing. The marginal cost of postponing retirement is the discounted present value of pension income foregone whilst the marginal benefit is the discounted present value of net wage income. This condition implies that $w'(t) < 0$ for $T_1 \leq t < L$.

A problem arises with this analysis when the optimum retirement date is T_1 since this is a corner solution. Such a situation is shown in Figure 2.3.1. If the state pension is p then the optimal retirement date, as given by (2.3.3.), will be T in the diagram - this retirement date will change for small parameter shifts. When the state pension is p' , T_1 is the optimum retirement date. At T_1 $p' > w(T_1) - a$ and the optimum will in many cases not change for small parameter shifts. This might also be the case if $p' = w(T_1) - a$. The condition for T_1 to be optimal is not then (2.3.3.) but

$$\begin{aligned} pR(T_1) &\geq (w(T_1) - a)R(T_1) \\ &> 0 \end{aligned} \quad (2.3.5.)$$

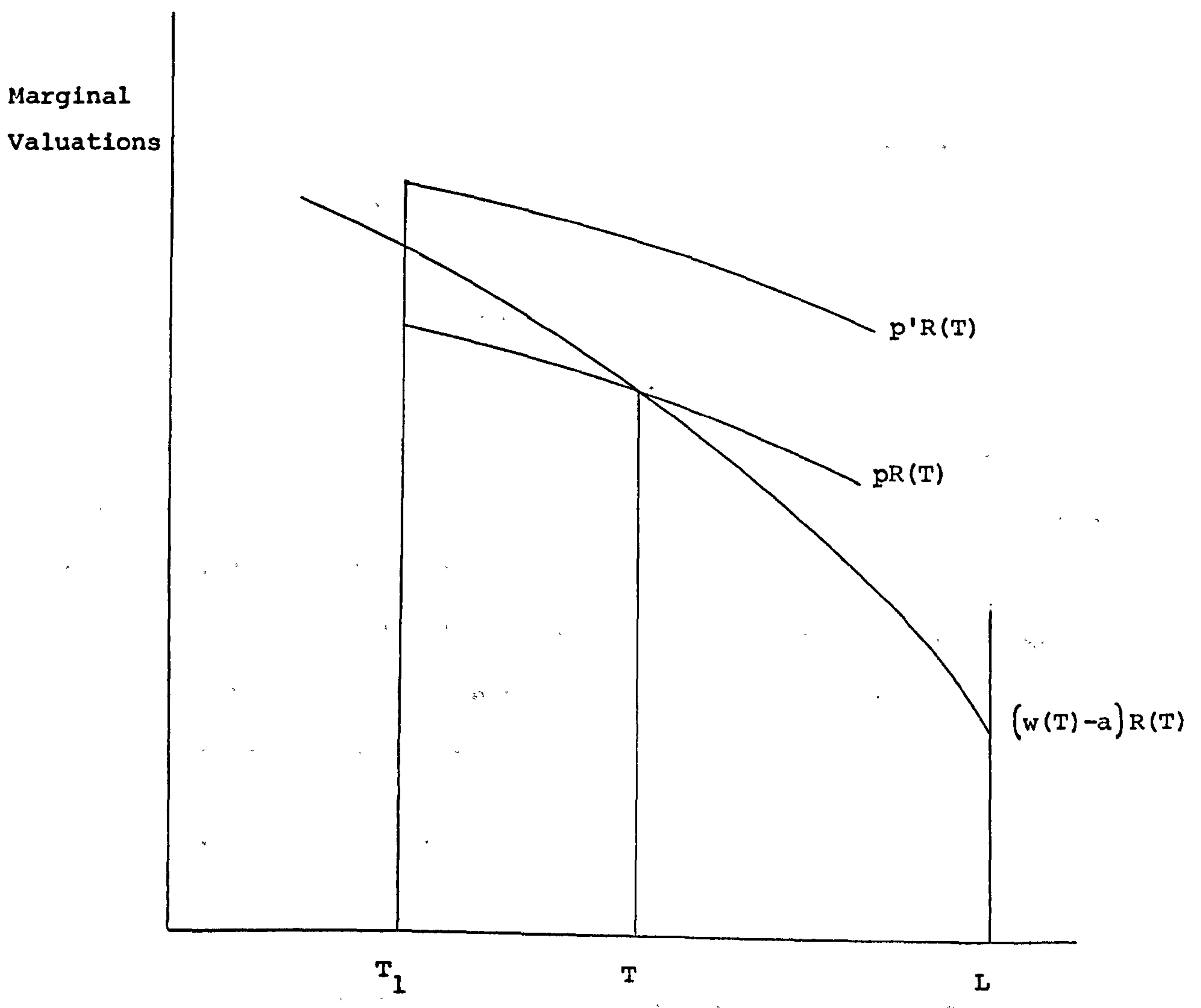


Figure 2.3.1

Implied by this is a modification of the comparative statics effects of parameter changes when $T=T_1$ - in particular the magnitude of the changes will be important. While this problem has been recognised it is not to be considered in any more detail.

The case of the institutionally determined retirement date can be dealt with fairly summarily since (2.3.2.) will now be written

$$Y(T_1) = \int_0^{T_1} (w(t)-a)R(t) dt + p \int_{T_1}^L R(t) dt \quad (2.3.6.)$$

Beyond this little can be said - the individual is faced with this situation and has to accept it.

2.4 STATE PENSIONS AND PERSONAL SAVINGS

The effect of the pension scheme on personal savings can be considered in two parts, the first concerned with the existence of the scheme and the second with changes in pension contributions and benefits.

Consider firstly the impact of the introduction of a scheme with an institutional retirement date. In the absence of a pension scheme lifetime income is given by

$$Y^*(T) = \int_0^T w(t) R(t) dt \quad (2.4.1.)$$

Let

$$Y^*(\bar{T}) = \max\{Y^*(T)\} = \int_0^L w(t) R(t) dt \quad (2.4.2.)$$

since the optimal retirement date will be characterised by $w(T) = 0$.

As outlined earlier one is interested in discovering the effect of the pension scheme on lifetime income. Thus (2.4.2.) has to be compared with

$$Y(T) = Y^*(\tilde{T}) - \int_{T_1}^L w(t)R(t) dt - a \int_0^{T_1} R(t) dt + p \int_{T_1}^L R(t) dt \quad (2.4.3)$$

where T_1 is the official retirement date. Once this retirement date is set the difference between (2.4.2) and (2.4.3) will depend on the values of a and p (or the rate of return to the pension scheme) and the wage possibilities after T_1 . Clearly this difference cannot be determined a priori - it follows that the eventual effect on savings cannot be evaluated. The same result will apply when the scheme allows the individual to retire on or after period T_1 . The relevant comparison is then between (2.4.2) and

$$Y(\tilde{T}) = \max_{T \geq T_1} \{Y(T)\} \quad (2.4.4)$$

To determine the effect of changes in pension parameters on personal savings one can consider the effect of a small increase in pension contributions and benefits on income. Therefore let

$$Y(T,m) = Y(\tilde{T}) - km \int_0^T R(t) dt + m \int_T^L R(t) dt \quad (2.4.5)$$

and

$$Y(m) = \max_{T \geq T_1} \{Y(T,m)\} \quad (2.4.6)$$

where m is the amount by which pension benefits are increased and km the amount by which contributions are increased. k is some exogenous constant. Now if a is increased by a further small amount then the effect on income will be given by

$$\frac{dY}{dm} = \frac{\partial Y}{\partial m} + \frac{\partial Y}{\partial T} \cdot \frac{dT}{dm} \quad (2.4.7)$$

the first component being a direct effect (with T held constant) and the second being an indirect effect operating through induced retirement, if retirement is endogenous.

From (2.3.2) the direct effect of an increase in contributions and benefits is to reduce and increase lifetime income respectively. To relate this to a final impact on savings consider the solution to (2.2.17), which can be written

$$C = c_0 \left\{ \int_0^L \exp \left\{ (r(t) - \delta(t)) / \theta \right\} dt \right\} \quad (2.4.8)$$

where C is the present value of lifetime consumption, and c_0 is consumption in the initial period. From A3 (2.4.8) can be rewritten

$$c_0 = Y(T) / \int_0^L \exp \left\{ \left(r(t) - \delta(t) \right) / \theta \right\} dt \quad (2.4.9)$$

whilst consumption in each period is given by

$$c(t) = c_0 \exp \left\{ \int_0^t \left(r(\tau) - \delta(\tau) \right) / \theta \, d\tau \right\} \quad (2.4.10)$$

An increase in pension contributions, a , results in $Y(T)$ falling which, from (2.4.9) and (2.4.10), results in a lower level of consumption in every period. Pension income is unchanged so that retirement saving is higher in every period (or, more likely, dissaving lower). A3 implies that the present value of lifetime savings is zero. It must then follow that savings during the work period falls. Feldstein's analysis was undertaken on the assumption that there was only a savings replacement effect. In this analysis it is an income effect that is being described, although substitution between pension savings and alternatives is taking place.

An increase in pension benefits, p , increases the value of $Y(T)$ and, using the above argument, consumption in every period will be higher. As net wage income is unaffected savings during the work period falls and there is a compensating increase/decrease in retirement saving/dissaving. This is a fairly easily understood result in that workers will finance a high level of consumption by borrowing out of a higher retirement income.

The indirect component is appropriate when the retirement date is not institutionally determined and there can therefore be an optimal adjustment of the retirement date to the new conditions. (2.4.7) implies that the change in the retirement date will affect lifetime income. It will in fact be the case though that this indirect component is always equal to zero. This can be demonstrated by considering the first order condition for the maximization of (2.4.5) with respect to T , which is

$$Y_T + \lambda = 0 \qquad \lambda \geq 0, \quad T \geq T_1$$

the inequalities holding with complementary slackness.

If $\lambda > 0$, the solution is at a corner and $T = T_1$. $dT/dm = 0$ so that the indirect component in (2.4.7) is zero. If $T > T_1$ then $\lambda = 0$ and $\partial Y/\partial T = 0$ - again the indirect component will be zero. This is an envelope result.

Nevertheless the effect of the change in pension variables on the retirement date still has to be considered. From (2.3.3) the direction of optimal retirement adjustment is given by

$$\left. \begin{array}{l} y_{Ta} \\ y_{Tp} \end{array} \right\} = -R(T) = 0 \quad (2.4.11)$$

which implies early retirement. This is explained by the fact that increases in both a and p make retirement attractive relative to work.

Now, of course, it could be argued that changes in pension variables and, by implication, the introduction of pension schemes has an impact that is scarcely marginal. What the above argument does seem to suggest is that most of the qualitative effect will be captured through the direct effect. Even in the case where pension contributions are changed by a fairly large amount and the indirect effect works in the opposite direction to the direct effect, it will still be the direct effect that dominates. To learn a little more about this consider Figure 2.4.1. If contributions are given by a the optimal retirement date is T and the present value of lifetime income in the area $adfLo$. In the event of contributions increasing to a' , without any retirement adjustment, the present value

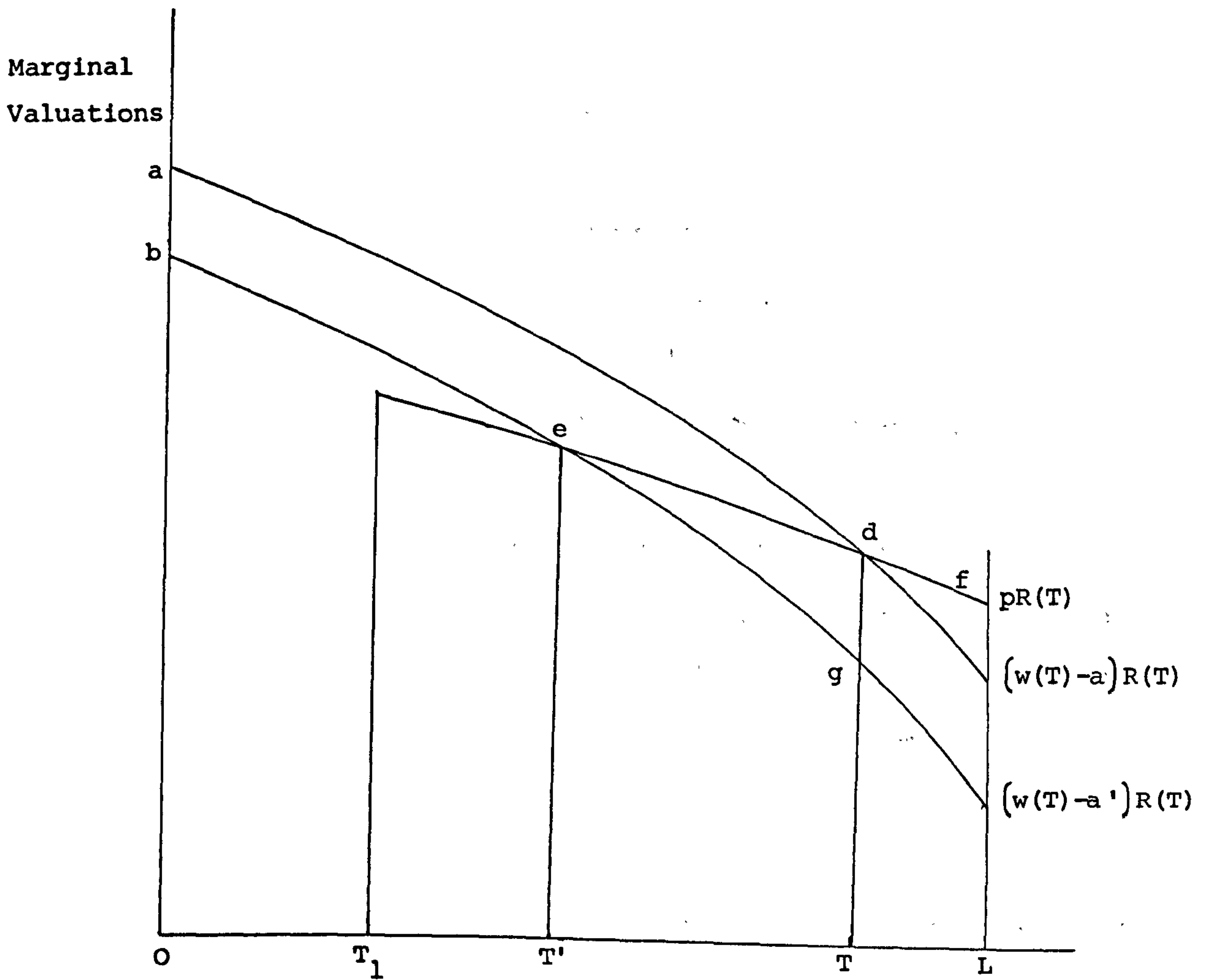


Figure 2.4.1

of lifetime income falls to the area $bgdfLo$. The area $abgd$ is lost. Optimal retirement adjustment produces an optimum at T' with a gain in lifetime income of edg . For early retirement to compensate for the earlier income loss $edg > abgd$. In this model such an outcome is not possible and the net effect on savings is therefore unambiguous.

It is easily checked from Figure 2.4.1 that increasing pension benefits involves no such conflict.

2.5 INCOME TAXATION, LEISURE AND BEQUEST BEHAVIOUR

(a) Income Taxation. So far income taxation, apart from the flat rate pension contribution, has been ignored. As income taxation is to be omitted from the analysis to follow it will be shown in this section how it might be included in the model of the state pension scheme.

Consider the following schema. Non-pension income, net of the pension contribution, is subject to a proportional tax at a rate u^* . The state pension is payable from T_1 even if the individual retires after that date. Over the interval $[T_1, T]$ pension income is subject to a proportion tax at a rate u^{**} . This arrangement can be thought of as a simplified earnings rule.

The discounted present value of lifetime income is now given by

$$Y(T) = \int_0^T (1-u^*) \left[w(t) - a \right] R(t) dt + p \left(\int_{T_1}^T (1-u^{**}) R(t) dt + \int_T^L R(t) dt \right) \quad (2.5.1)$$

Optimal retirement requires

$$Y_T = (1-u^*) \left[w(T) - a \right] R(T) - u^{**} pR(T) = 0 \quad (2.5.2)$$

This can be seen to be similar to (2.3.3) but for the inclusion of a term $(1-u^{**}) pR(T)$ in (2.5.2), reflecting the fact that pension income is payable over the interval $[T_1, T]$.

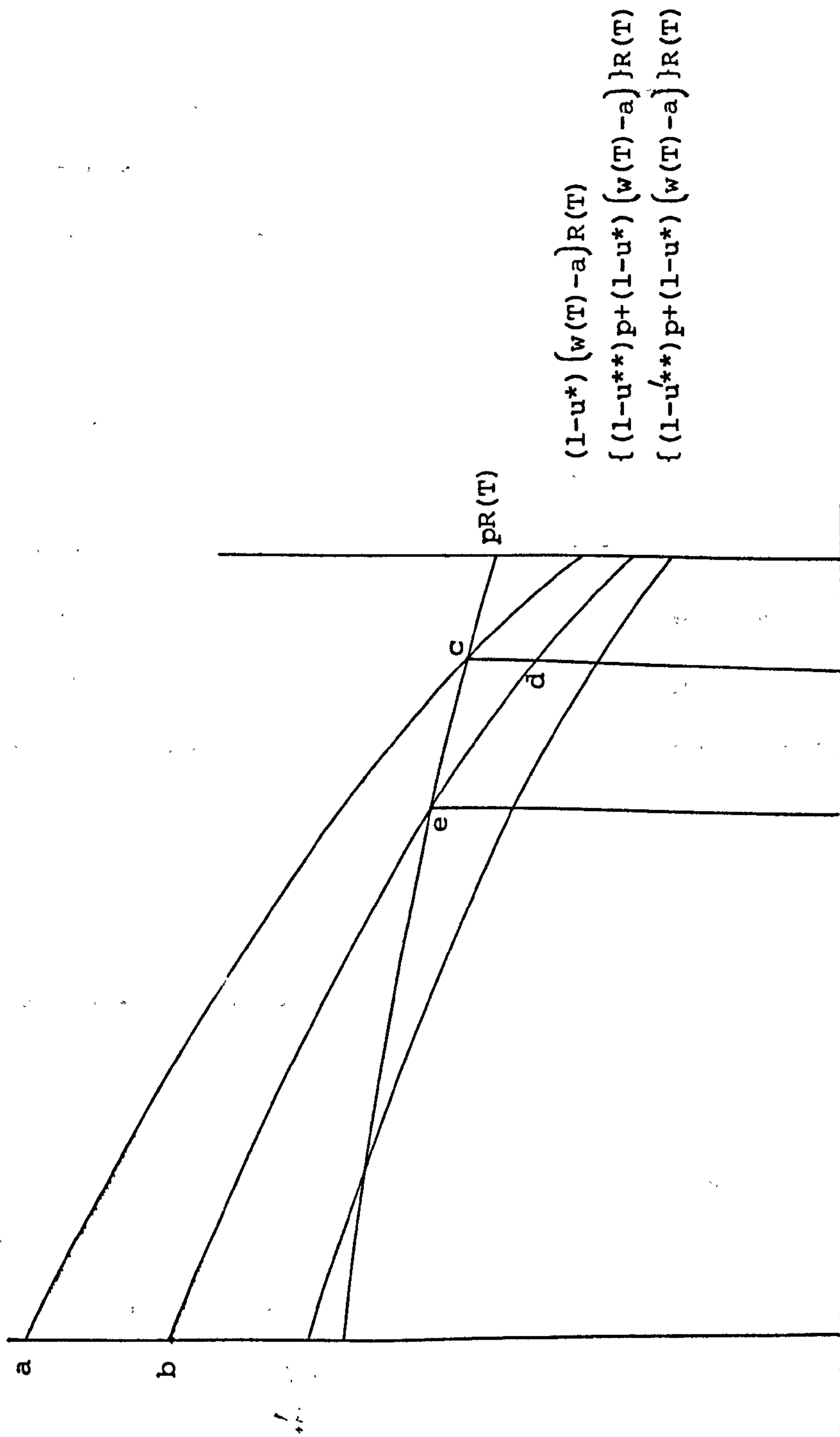
An interesting comparative statics result to consider is the effect of an increase in the pension tax i.e., a toughening of the earnings rule. From (2.5.1) increasing u^{**} reduces $Y(T)$ which implies a lower level of consumption in every period. Over the intervals $[0, T_1]$ and $[T, L]$ income is unchanged and savings therefore higher - it follows that over $[T_1, T]$, when both labour income and pension are payable, savings must fall.

From (2.5.2) optimal retirement adjustment is given by

$$Y_{Tu^{**}} = -p R(T) < 0 \quad (2.5.3)$$

which again implies early retirement, which is understandable since work has become relatively unattractive. Again, for non marginal changes in the tax rate there appears to be the possibility of an ambiguous outcome, since optimal retirement adjustment will increase $Y(T)$. Figure 2.5.1 demonstrates that full compensation is not possible.

Marginal
Valuations



$$(1-u^*) (w(T) - a) R(T)$$

$$\{ (1-u^{**}) p + (1-u^*) (w(T) - a) \} R(T)$$

$$\{ (1-u^{**}) p + (1-u^*) (w(T) - a) \} R(T)$$

Figure 2.5.1

If the rate of pension tax rises from u^{**} to u'^{**} and the retirement date remains at T then the present value of lifetime income falls by an amount given by the area $abcd$. Optimal retirement adjustment can only regain the area cde which will always be less than $abcd$.

This result is the same as the one arrived at by Feldstein outlined in the Appendix to Chapter 1.

(b) Leisure. It has so far been assumed that work and consumption decisions are separable - leisure does not enter the individual's utility function.

A convenient way to analyse the role of leisure is to summarise the standard life cycle model of labour leisure choice and then see what effect the inclusion of the state pension would have. Assume that an individual maximises the additive separable utility function

$$U = \int_0^L \left\{ u(c(t)) + v(l(t)) \right\} e^{-\delta(t)} dt \quad (2.5.4)$$

subject to the accumulation constraint

$$\dot{k}(t) = r(t)k(t) + h(t)\tilde{w}(t) - c(t) \quad (2.5.5)$$

and the terminal constraint (2.2.10) where $v(\cdot)$ is subject to the same restrictions as $u(\cdot)$; $l(t)$ is the amount of time devoted to leisure; $h(t)$ the amount of time spent at work and $\tilde{w}(\cdot)$ the pattern of wage rates.

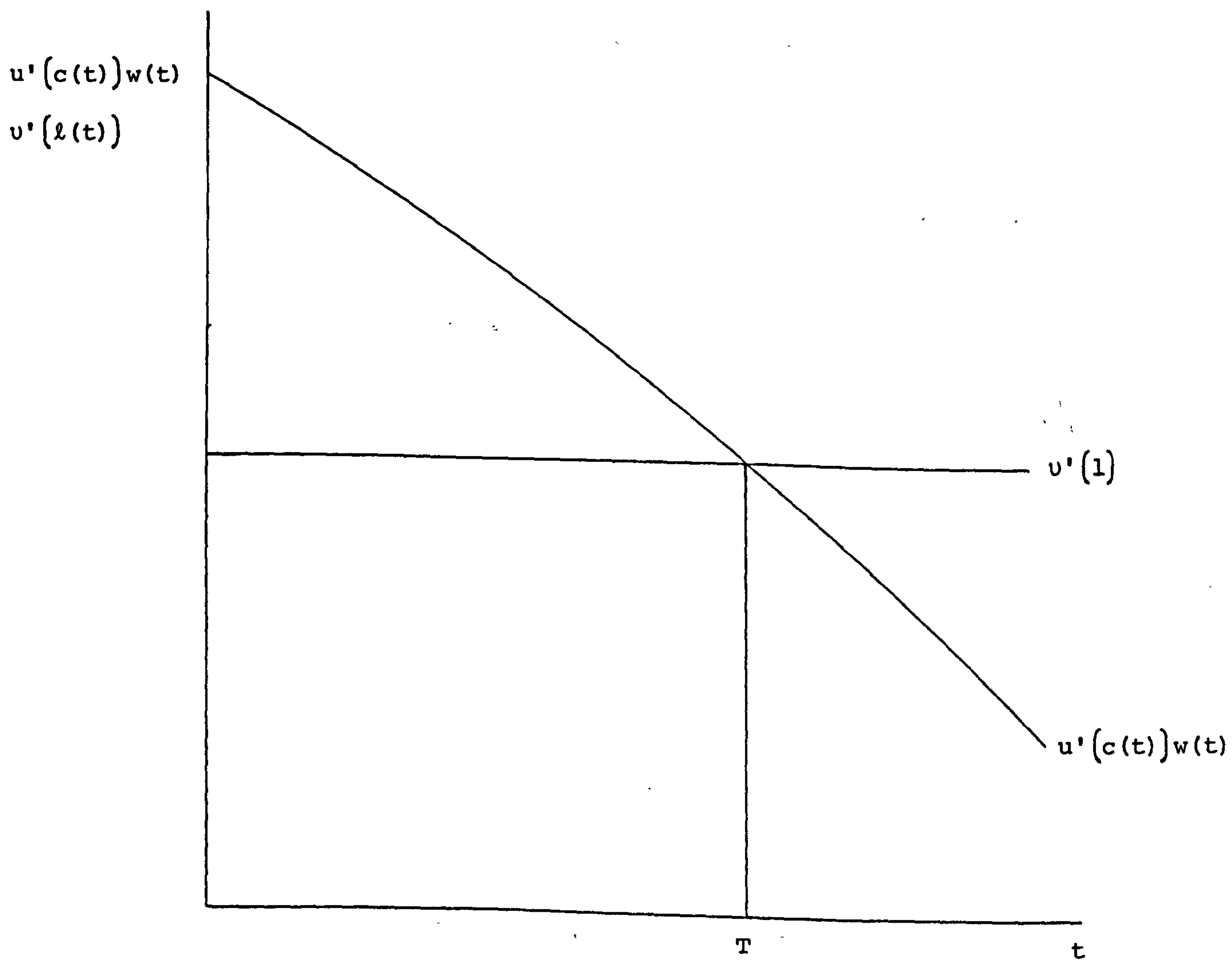


Figure 2.5.1

Given the individual may retire in the absence of any pension arrangement it can now be seen how these results interact with those derived for pension schemes in the absence of leisure considerations.

The analysis differs from that of the previous section in that it is now necessary to compare work and retirement marginal utility streams - where they are equal will give the optimal retirement date. Utility during the work period will be the sum of the marginal utility equivalent of net wage income and the utility derived from leisure whilst retirement utility will be the sum of the marginal utility equivalent of the state pension and the utility derived from spending all ones time in leisure.

A retirement date is chosen to maximise

$$U = \int_0^T \left\{ u' \left[c(t) \right] \left[h(t) \tilde{w}(t) - a \right] + v \left[\ell(t) \right] \right\} e^{-\delta(t)} dt + \int_T^L \left\{ u' \left[c(t) \right] p + \left[1 \right] \right\} e^{-\delta(t)} dt \quad (2.5.10)$$

and this will be the date where

$$\left\{ u' \left[c(T) \right] \left[h(T) \tilde{w}(T) - a \right] + v \left[\ell(T) \right] \right\} e^{-\delta(T)} = \left\{ u' \left[c(T) \right] p + v \left[1 \right] \right\} e^{-\delta(T)}$$

which when written in the form

$$u' \left[c(T) \right] \left[h(T) \tilde{w}(T) - a - p \right] = v \left[1 \right] - v \left[\ell(T) \right] \quad (2.5.11)$$

can be directly compared with (2.3.3).

Now if the individual moves smoothly from the work period into the retirement period, in the sense that there is no discontinuity in $l(\cdot)$, then (2.5.11) reduces to (2.3.3), the optimal retirement condition in both cases being $w(T) - a - p = 0$, remembering that $w(T) = h(T)\bar{w}(T)$. However, if there is a discontinuity in $l(\cdot)$, and this can be supported either by recognising the existence of a statutory minimum working week or arguing that it is a feature of a life cycle when there is a pension arrangement, then the above analysis has to be modified. In particular, the possibility then arises that an individual will retire when $w(T) - a > p$ since the discontinuity in the leisure stream can only imply that $v\left[\frac{1}{\cdot}\right] > v\left[l(T)\right]$. Such an outcome is demonstrated in Figure 2.5.2, where the top half of the diagram refers to a comparison of income streams whilst the bottom half compares utility streams.

This graphical representation brings home one point rather clearly in that where the administered value of the state pension is low, and in the previous analysis an interior maximum was thought to be unlikely, the inclusion of leisure in the analysis might increase its likelihood although will still not make it certain.⁹

To establish the effect of the inclusion of leisure on accumulation one would have to know the effect on lifetime income. This reduces to a problem of comparing the endogenous and exogenous wage streams and the optimal retirement dates in each case. In the general model considered here such a comparison is not possible. As for the comparative statics results (2.5.10) and (2.5.11) suggest that there is no reason to believe that changes in pension contributions and benefits will affect optimal accumulation and retirement in any way that is likely to be different to those outlined in the previous section.

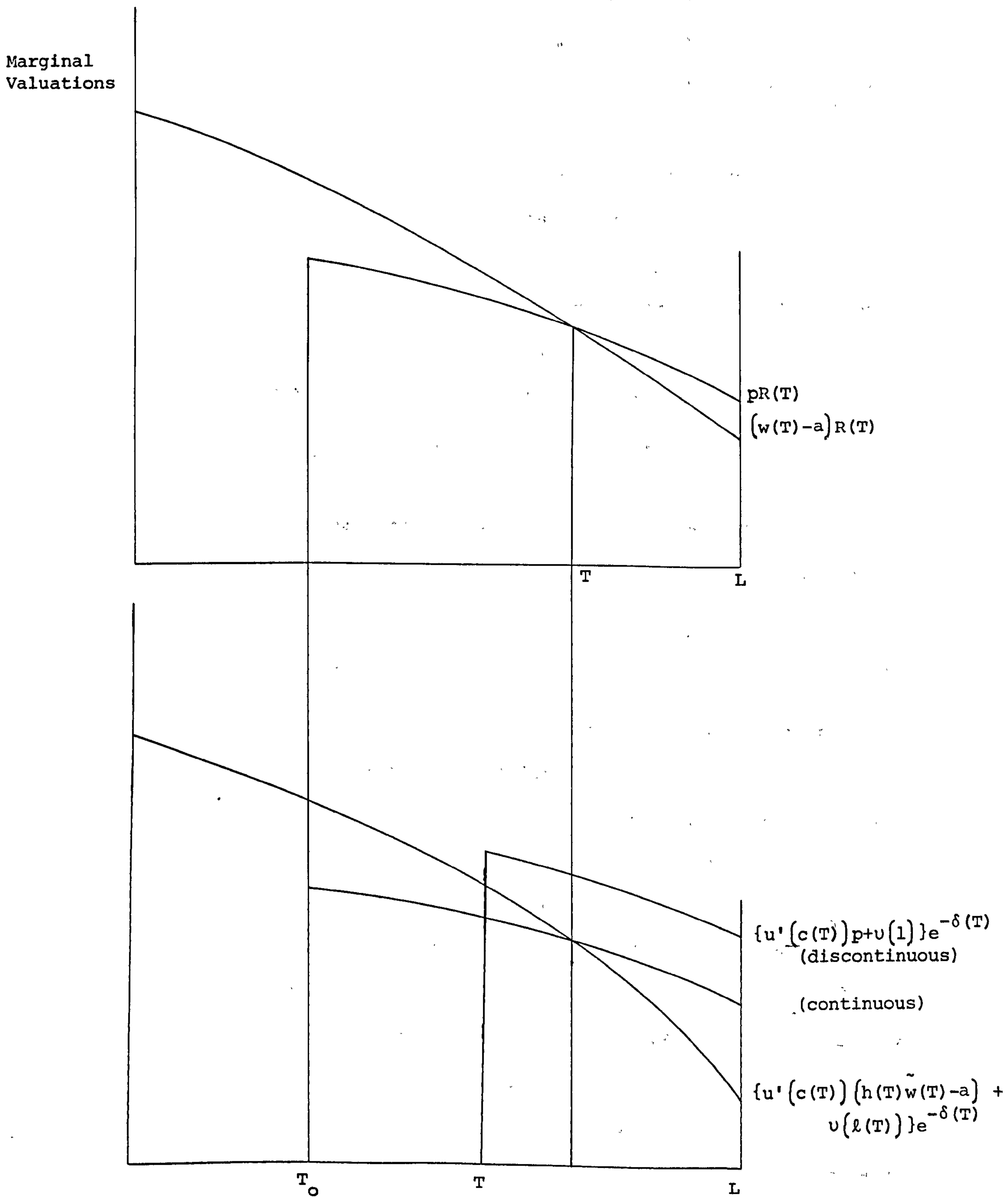


Figure 2.5.2

(c) Bequest Behaviour. There are a number of ways that A3 may be relaxed. It would be easiest to include in (2.3.2) the additional term $\left[\underline{k}(0) - \underline{k}(L) \right]$ representing respectively the discounted present values of bequests received and left. The discounted present value of lifetime savings is then $k(L)$ and as long as bequests are fixed at this level all the earlier results go through. In effect, bequests, as long as they are fixed, are irrelevant.

It is possible to allow for bequests to be variable, dependent on utility considerations. In such a case it can be assumed that the individual maximises a utility function of the form

$$U = \int_0^L u(c(t)) e^{-\delta(t)} dt + \Omega(k(L)) e^{-\delta(L)} \quad (2.5.12)$$

subject to the accumulation constraint (2.1.2). The solution to this problem is the same as that analysed in Section 2.2 but for a change in the terminal constraint which becomes

$$u'(c(L)) = \Omega'(k(L)) \quad (2.5.13)$$

which says that at the optimum the marginal utility of a terminal bequest must equal the marginal utility of terminal consumption.

(2.5.13) defines $k(L)$ as a function of c_0 , the relationship between the two being dependent on the relative elasticities of $u(\cdot)$ and $\Omega(\cdot)$ evaluated at L . It should be noted that the initial level of consumption will not be the same as in (2.4.3) which implied that c_0 has some proportion of $Y(T)$, the proportion depending on length of life, age, the rates of interest and time preference and tastes (the form of the utility function). Unlike (2.4.3) the definition of lifetime income or now, more correctly, net worth has to be modified to include the discounted present values of inheritances received and bequests left, which are unlikely to be equal.

This analysis has been fully exploited in many areas of economic analysis but probably to its greatest advantage in considering problems of wealth distribution (see Meade (44), Atkinson (7) and Ishikawa (32)). Unfortunately it holds only slight interest for this analysis - the qualitative nature of the results outlined are unlikely to be affected. This section has served only to show how the modification to include optimal bequest behaviour might be made.¹⁰

2.6 CONCLUSIONS

This chapter has seen the introduction of a state pension scheme into a continuous time life cycle model of consumption and accumulation. In a model where the results are not in any sense forced it has been shown that

- i) A priori the effect on accumulation of introducing a pension scheme into an economy cannot be determined.
- ii) With the type of pension scheme assumed on optimal retirement date will require wages to be falling in the region of T_1 , unless leisure considerations are taken into account. This optimal retirement date will normally be earlier than in a model without pensions.
- iii) Increasing pension contributions and benefits always induces early retirement. This adjustment can only reinforce the reduction in work period savings brought about when pension benefits increase and work against, though never compensate for, the reduction in work period savings that results from an increase in

pension contributions. To translate these changes into aggregate terms with a stationary population and no wage growth through time aggregate savings does not change. If there is population or wage growth the behaviour of the young (or workers) will always dominate the aggregate.

iv) If there is an earnings rule the tax on pension income implies that savings will be higher over the work period up until T_1 and after retirement. Aggregate savings will therefore be higher.

v) The inclusion of leisure in the utility function world could make an interior solution to the individuals optimum retirement problem more likely, but is unlikely to affect the comparative statics results.

vi) The inclusion of bequests can change the level of consumption, and therefore savings, in every period. The direction of this change is indeterminate. Comparative statics results are not affected.

NOTES

1. This model is fairly well established in the literature - the most thorough expositions are those of Green (26), using a discrete time formulation, and Yaari (61), using the continuous time analogue.
2. For this marginal gain in generality one loses consistency in the sense of Strotz (56) due to the variable discount rate. Of course, there are no clear reasons why one should require consistency in this restrictive sense.
3. This method is outlined in a number of texts. See for example Dixit (17), Hadley and Kemp (28).
4. See Blinder (13), pp.50-54.
5. With $w(t) > 0$ it will always be optimal for $T > T_1$ since $Y(T)$ can always be increased by working up to T_1 .
6. More precisely, given the form of the utility function the functions $r(\cdot)$ and $\delta(\cdot)$ and the value of lifetime consumption one solves for the initial condition. This is done in (2.4.3).
7. In the description of Feldstein's analysis in Chapter 1, the slope of the budget line changes.

8. The problem of initial conditions is rather complex - it is fully analysed in Blinder (13), pp.67-75.
9. It also allows the possibility of a maximum when $w'(t) > 0$.
10. One might also follow Atkinson (7) who suggests that the introduction of bequests permits the relaxation of the assumption of a perfect capital market since if it is assumed $r(t) > \delta(t)$ and the individual can borrow against future bequests it becomes inessential.

CHAPTER THREE

PRIVATE PENSIONS, CAPITAL ACCUMULATION AND RETIREMENT

3.1 INTRODUCTION

In the previous chapter it was assumed that the only pension arrangement available to the individual was arranged through the state. In this chapter a private scheme is introduced from which the individual can receive payment some years before he becomes eligible to receive the state pension.

In the next section the private pension arrangements are described in detail. Two types of scheme are considered, one terminal salary based and one money purchase based. These are analysed separately in sections 3.3 and 3.4.

The relationship between these schemes, capital accumulation and retirement behaviour is considered in section 3.5.

Lastly, in section 3.6, the money purchase scheme will be modified to allow for an uncertain lifetime.

3.2 PRIVATE PENSION ARRANGEMENTS

Four general categories of private pension scheme can be distinguished, flat rate, terminal salary, average salary and money purchase schemes. As the flat rate scheme is similar to the state pension scheme and terminal and average salary schemes are similar only terminal salary and money purchase schemes will be described.

Terminal salary schemes are organised in the following way. During his working life the individual contributes $\alpha w(t)$, ($0 < \alpha < 1$), where $w(\cdot)$ is wage income, into the scheme. In return for these contributions he receives a pension $\beta(T)w(T)$, ($0 < \beta < 1$) where the scale of benefits, given by the function $\beta(\cdot)$ is a function of the retirement date, T . The earliest date of receipt of this pension is $T_0 < T$, and the income it provides will be denoted $\gamma(T)$. If this were an average salary scheme the pension would be based not on the last years salary but an average of salaries received over the last few years of work.

Money purchase schemes give the individual more scope as to the size of his pension contributions and hence the pension he receives. The individual makes payments into the scheme at a rate $f(t)$ and the fund accumulates these payments at a rate $s(t)$ to obtain a gross fund.

$$F = \int_0^T f(t) \exp \left(\int_0^t s(\tau) d\tau \right) dt \quad (3.2.1)$$

On retirement he receives a pension $P=P(F,T)$ where $P(\cdot)$ is an increasing function of both its arguments. Again the earliest date of receipt will be denoted $T_0 < T_1$.

An assumption at the beginning of the last chapter stated that there was a perfect capital market in which the individual can borrow and lend, through the selling and purchase of bonds, at a rate $r(t)$. Something now has to be said about the relationship between the rate of return in this perfect market and the rate of return to pension contributions, $s(t)$.

If $s(t) > r(t)$ for all t , then it would clearly pay the individual to borrow large amounts of money and transfer it into the fund. This is not a desirable feature of equilibrium. There are a number of ways around this problem. One might make some form of institutional assumption about individual behaviour. It is clearly only advantageous for the individual to invest in the pension asset alone if he can withdraw his funds at will. The ability to withdraw could be removed.

A more convenient assumption to make is that $s(t) \leq r(t)$ over the interval $[0, T]$. As it is no longer worthwhile investing in the fund the individual simply transfers a capital sum F into the fund on retirement. The individual will do this either because the fund uses any portfolio advantages it has over the individual to offer a higher stream of income on F than could be obtained in the open market over the period $[T, L]$ or because of the convenience of such an arrangement. For the moment this is the assumption which will be adopted but it is to be discussed further in section 3.6.

3.3 THE TERMINAL SALARY SCHEME¹

If it is assumed that there are no state pension contributions² the restrictions on net income in every period can be written

$$y(t) = \begin{cases} (1-\alpha)w(t) & \text{if } 0 \leq t < T \\ \beta(T)w(T) = \gamma(T) & \text{if } T \leq t < T_2 \\ \gamma(T) + p & \text{if } T_2 \leq t < L \end{cases} \quad (3.3.1)$$

Using these restrictions the present value of lifetime income is given by

$$Y(T) = (1-\alpha) \int_0^T w(t) R(t) dt + \gamma(T) \int_T^L R(t) dt + p \int_{T_2}^L R(t) dt \quad (3.3.2)$$

where $T_2 = \max \{T_1, T\}$. Proceeding as in the previous chapter the first order condition for optimal retirement is

$$Y_T = \left[(1-\alpha)w'(T) - \gamma'(T) - P(dT_2/dT) \right] R(T) + \gamma(T) \int_T^L R(t) dt = 0$$

$$(dT_2/dT) = \begin{cases} 0 & \text{for } T_0 \leq t < T_1 \\ 1 & \text{for } T_1 \leq t < L \end{cases} \quad (3.3.3)$$

which says that at the margin the discounted present value of net wage income plus the gain or minus the loss from delaying retirement $\left((1-\alpha)w'(T)R(T) + \gamma'(T) \int_T^L R(t) dt \right)$ must equal the discounted present value of pension income $(\gamma(T)R(T) + p(dT_2/dT)R(T))$.

The above first order condition will not be appropriate when, as will quite often be the case, the optimal retirement date is exactly T_1 and remains there for small parameter shifts. The problem is that the function $Y(\cdot)$ is not differentiable at T_1 , but has a left hand derivative Y_{T-} and a lower right hand derivative Y_{T+} . The condition for T_1 to be optimal is that $Y_{T-} \geq 0 \geq Y_{T+}$, which reduces to

$$p \geq (1-\alpha)w'(T_1) - \gamma'(T_1) + \gamma'(T_1) \int_{T_1}^L R(t)/R(T_1) dt > 0 \quad (3.3.4)$$

Where the inequality holds T_1 will be optimal and remain so for small parameter shifts if the present value of state pension exceeds the difference between the marginal cost (in terms of private pension only) and marginal benefit of delaying retirement, also evaluated at T_1 . Where the equality holds it will be the case that whilst certain parameter shifts will change the optimal choice of T others will not.

This will also be the case when the inequality holds and

$$(1-\alpha)w'(T_1)R(T_1) + \gamma'(T_1) \int_{T_1}^L R(t) dt = \gamma'(T_1)R(T_1).$$

If $s(t) > r(t)$ for all t , then it would clearly pay the individual to borrow large amounts of money and transfer it into the fund. This is not a desirable feature of equilibrium. There are a number of ways around this problem. One might make some form of institutional assumption about individual behaviour. It is clearly only advantageous for the individual to invest in the pension asset alone if he can withdraw his funds at will. The ability to withdraw could be removed.

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$$y(t) = \begin{cases} (1-\alpha)w(t) & \text{if } 0 \leq t < T \\ \beta(T)w(T) = \gamma(T) & \text{if } T \leq t < T_2 \\ \gamma(T) + p & \text{if } T_2 \leq t < L \end{cases} \quad (3.3.1)$$

Using these restrictions the present value of lifetime income is given by

i) $w'(T) < 0$, $w''(T) > 0$, $\gamma'(T) > 0$. A falling wage structure in the region of retirement would be found where the lifetime income profile is humped. For pension income also to be falling it is necessary that $w'(T)/w(T) > (-) \beta'(T)/\beta(T)$. To determine whether an interior solution is likely the correct procedure would be to substitute $\gamma(T) = \beta(T)w(T)$ into (3.3.5) and determine the values of the parameters of the equations that guarantee such a solution. A more convenient procedure is simply to derive a minimum set of conditions that have to be fulfilled for an interior maximum. Figure 3.3.1 demonstrates that T_2 , where net wage income equals pension income is not a maximum. The first order condition is satisfied at T where the loss in wage income from retiring at T rather than T_2 , over the interval $[T, T_3]$, is exactly equal to the gain in pension income from so doing, over the interval $[T_3, L]$. This case is rather contrived in that the assumption that $\beta < 1$ together with the relatively low administered values of p imply that in many cases T_2 will not be interior. From Figure 3.3.2 it is clear that this does not matter - T can still be interior as long as $\max \{\gamma(T) + p\} > \min \{w(T)(1-\alpha)\}$. This analysis easily extends to the case where $\gamma'(T) > 0$ but in this case T_2 , as just defined, has to be interior since if this were not true then $\max \{\gamma(T) + p\} < \min \{w(T)(1-\alpha)\}$.

ii) $w'(T) > 0$, $w''(T) < 0$, $\gamma'(T) > 0$. From Figure 3.3.3 it is again clear that T_2 has to be interior. It is therefore necessary that $\beta'(T)/\beta(T) > w'(T)/w(T)$. This is a deceptive case since it would appear at first that retirement is never worthwhile since there are always pension and wage gains to be had from delaying retirement.

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Marginal Valuations

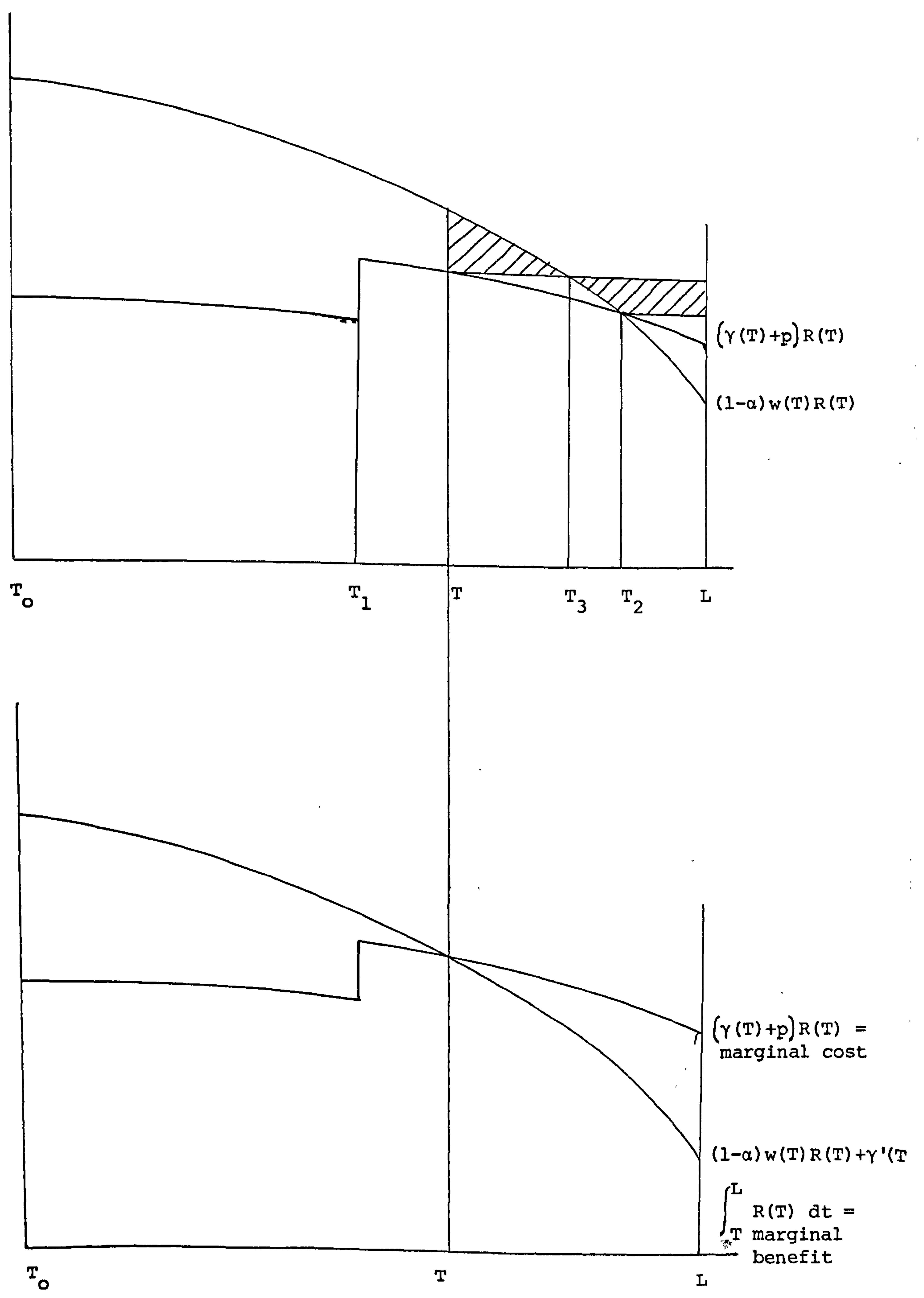


Figure 3.3.1

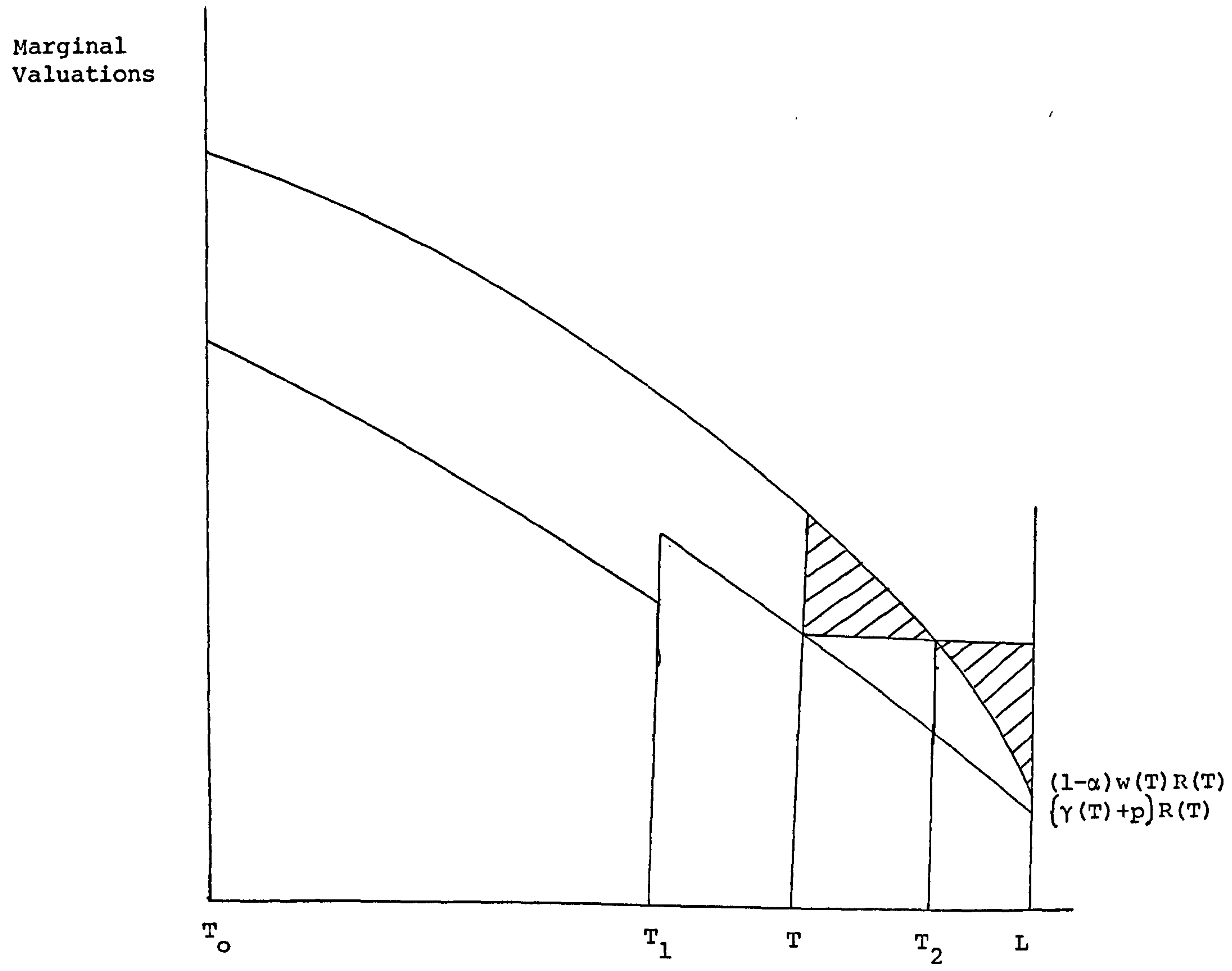


Figure 3.3.2

Marginal
Valuations

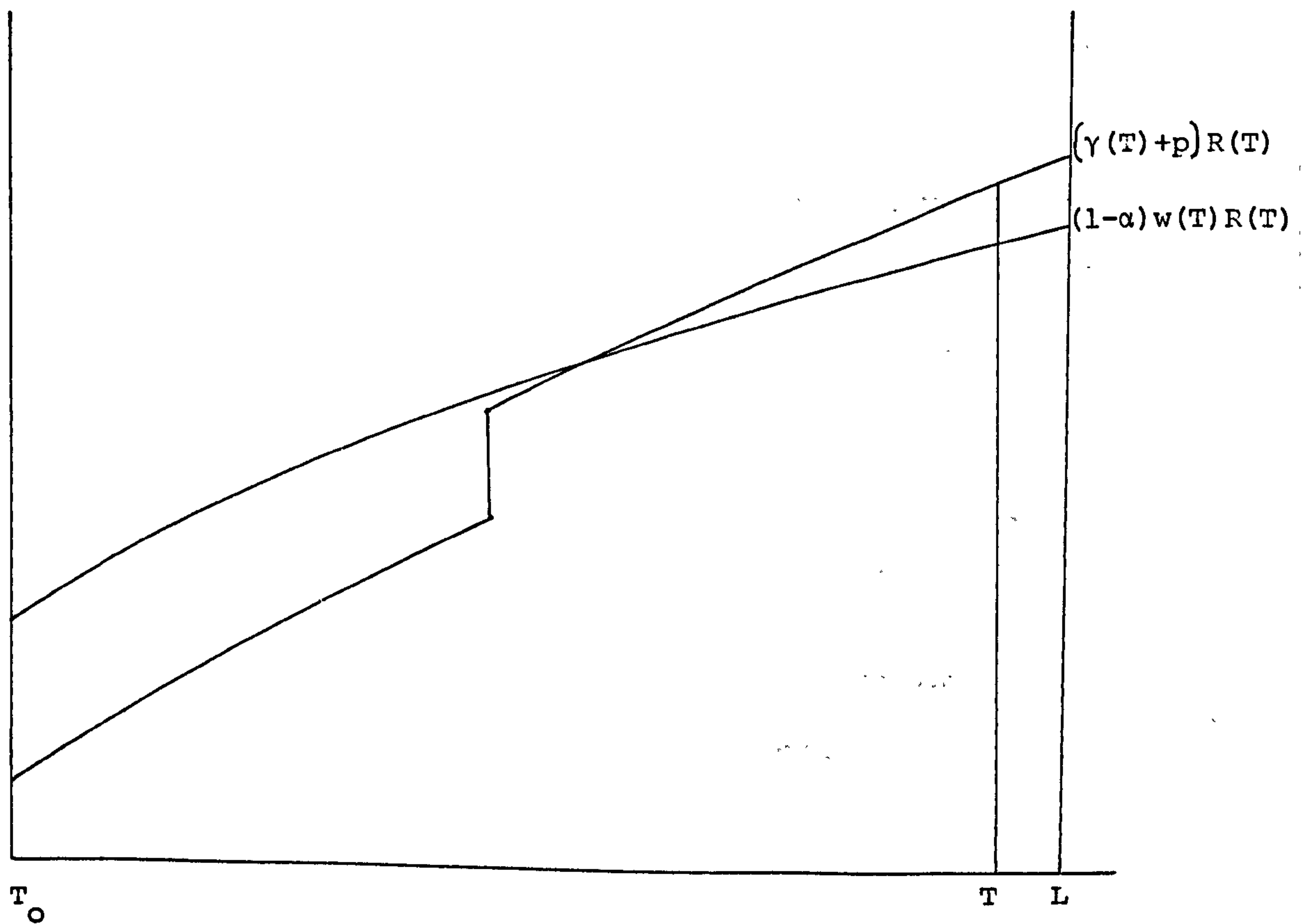
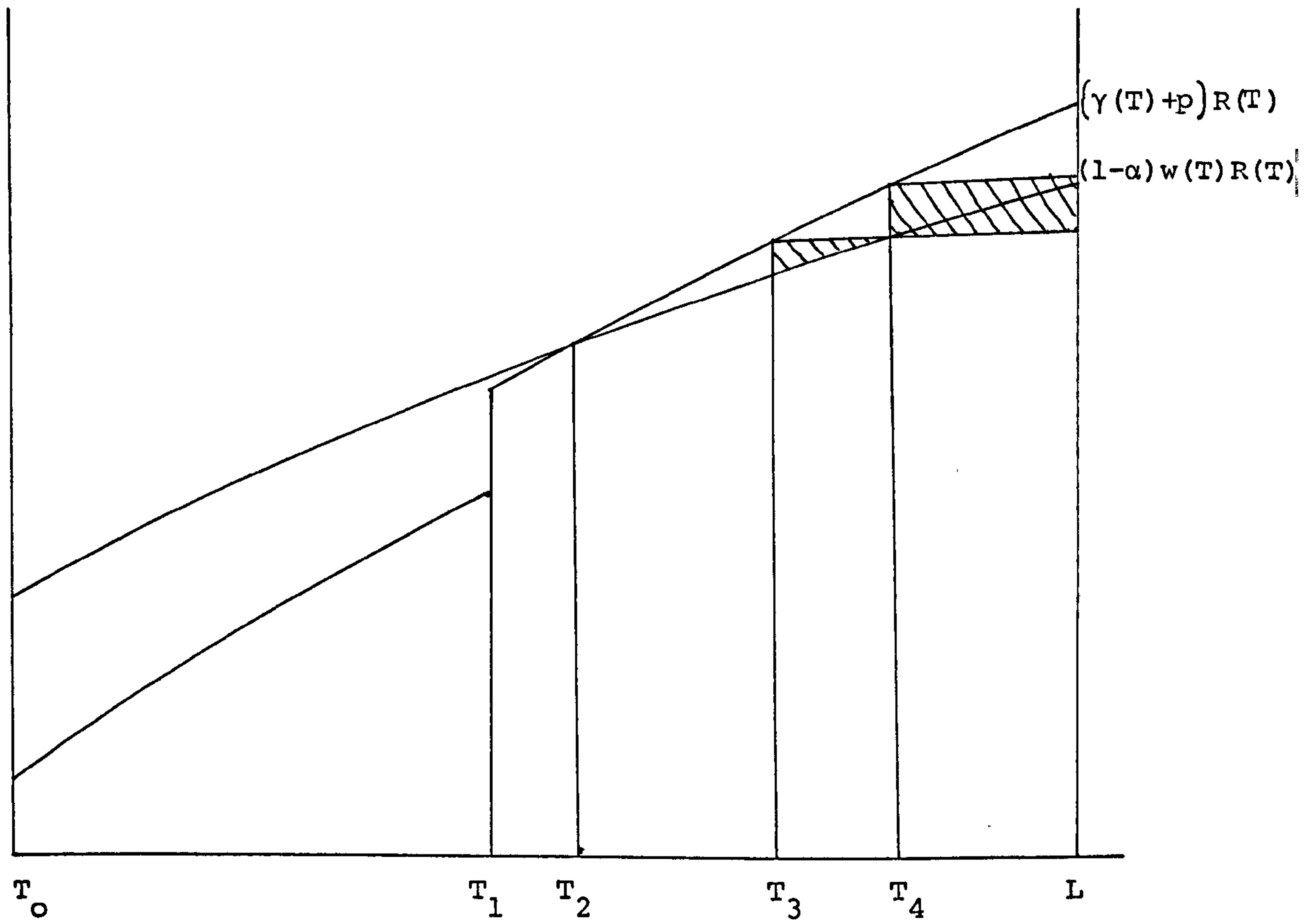


Figure 3.3.3

Indeed if the individual retires at T_3 lifetime income is higher than if retirement is at T_2 . But if retirement is delayed until T_4 the horizontal shaded area is gained whilst the diagonal shaded area is lost. If at the margin these two areas are equal then an optimal retirement date has been defined. This is denoted T . In this case an optimum will be unlikely - with a rising income stream over the later part of the life cycle it is unlikely that $p > \left[1 - \alpha - \beta(T)\right] w(T)$ which is the necessary condition for T_2 to be interior.

The aim of this brief analysis of interior solutions has been to determine the plausible wage profiles in the region of retirement. If this last point is accepted, and with rising wages it can be assumed that T_2 will not normally be interior, the wage stream will have to be decreasing. Comparative statics analysis will be undertaken on this assumption.

3.4 THE MONEY PURCHASE SCHEME³

From the description of this type of pension arrangement given in section 3.2 the discounted present value of lifetime income will be a function of both the retirement date and the size of the pension fund. Specifically

$$Y(F, T) = \int_0^T w(t)R(t) dt - R(T)F + P(F, T) \int_T^L R(t) dt + p \int_{T_2}^L R(t) dt \quad (3.4.1)$$

where again $T_2 = \max \{T_1, T\}$ and remembering that F is the sum of money transferred into the pension fund on retirement.

There will now be two first order conditions for a maximum

$$Y_F = -R(T) + P_F \int_T^L R(t) dt \leq 0 \text{ for } F \geq 0 \quad (3.4.2)$$

$$Y_T = \left[w(T) - P(F, T) - p(dT_2/dT) \right] R(T) - R'(T)F + P_T \int_T^L R(t) dt = 0$$

$$(dT_2/dT) = \begin{cases} 0 & \text{for } T_0 \leq t < T_1 \\ 1 & \text{for } T_1 \leq t < L \end{cases} \quad (3.4.3)$$

If T_1 is optimal and remains there for small parameter shifts

$$P > w(T_1) - P(F, T_1) - \left[R'(T_1)/R(T_1) \right] F + P_T \int_{T_1}^L R(t)/R(T_1) dt > 0 \quad (3.4.4)$$

The first order condition for optimal retirement has the same interpretation as previously. That for the optimum size of the fund says that at the margin the discounted present value of the marginal pension income from the fund should equal the discounted present value of a bond purchased on retirement. The present value of an additional unit of pension wealth is the present value of the income it yields whilst the present value of an increment of capital is simply $R(T)$. If it is assumed that $P_F(0, T) = +\infty$ for all $T \geq T_0$ then this first order condition will hold with $F > 0$.

To determine whether or not it is worthwhile for the individual to accumulate a fund it can be noted that if $P(\cdot)$ is concave in F then $P(F, T)/F > P_F(F, T)$ for all $F > 0$. Thus from (3.4.2)

$$P(F, T) \int_T^L R(t) dt > R(T)F \quad (3.4.5)$$

which implies that the individual will gain from investing in the pension asset. It will therefore always benefit the individual to accumulate a pension fund.

The second order conditions for an interior maximum are

$$Y_{FF} = P_{FF} \int_T^L R(t) dt < 0 \quad (3.4.6)$$

$$Y_{TT} = \left[w'(T) - 2P_T - p(d^2T_2/dT^2) \right] R(T) + \left[w(T) - P(F,T) - p(dT_2/dT) \right] R'(T) - R''(T)F + P_{TT} \int_T^L R(t) dt < 0 \quad (3.4.7)$$

$$Y_{FF} \cdot Y_{TT} - (Y_{FT})^2 > 0$$

where
$$Y_{FT} = P_{FT} \int_T^L R(t) dt - R(T) P_F - R'(T) \quad (3.4.8)$$

(3.4.6) implies that $P_{FF} < 0$ whilst (3.4.7) is again the standard requirement that the marginal cost function should cut the marginal benefit function from below.

To examine further the implications of these conditions note that $P_{FF} < 0$ guarantees that (3.4.3) will determine a unique value of F satisfying $Y_F = 0$ for all values of T . Denote this value $\phi(T)$. The restrictions that are then of interest are those that guarantee that as $T \rightarrow L$, $\phi(T) \rightarrow 0$ i.e. the shorter the retirement period the smaller the desired fund.

If $Y_F(\phi(T), T) = 0$ on (T_0, L) then $\phi'(T) = -Y_{FT}/Y_{FF}$. From (3.4.6) $Y_{FF} < 0$ and it is therefore only necessary to show that $Y_{FT} < 0$. Consider firstly the last two terms in (3.4.8). These can be written, using (3.4.2)

$$R(T)P_F - R'(T) = \frac{\{R(T)^2 - \left[R'(T) \int_T^L R(t) dt \right]\}}{\int_T^L R(t) dt} \quad (3.4.9)$$

If $r'(t) \geq 0$ for $t \geq T_0$ then (3.4.9) is positive over the interval $[T, L]$. Since $r(T) = R'(T)/R(T)$ then $P_F > r(T)$. Beyond this little can be said. Certainly for $Y_{FT} > 0$ marginal pension income must be rising and interest rates falling over time. Thus for a large number of cases it might not be too unreasonable to assume that $Y_{FT} < 0$ and $\phi'(T) < 0$.

(3.4.7) also reveals that it will often be the case that $w'(T) < 0$ over $[T_0, L]$. To examine this further assume that pension income, after T_0 , is independent of the retirement date. Now define a function

$$g(T) = P \left[\phi(T) \right] - r(T) \phi(T) \quad (3.4.10)$$

which is the marginal cost, in terms of private pension, of delaying retirement one year. Differentiating (3.4.10) with respect to T gives

$$g'(T) = P_F \phi'(T) - r(T) \phi'(T) - r'(T) \phi(T) < 0 \quad (3.4.11)$$

From (3.4.6), (3.4.7) and (3.4.8)

$$R(T) \left[g'(T) - w'(T) \right] = \left[(Y_{FT})^2 - Y_{TT} \cdot Y_{FF} \right] / Y_{FF} > 0 \quad (3.4.12)$$

which with (3.4.11) implies $w'(T) < g'(T) < 0$. For the particular case where pension income is independent of the retirement date and interest rates are rising through time the wage profile must be falling over $[T_0, L]$. Of course this does not guarantee that in general this must be the case. Nevertheless, in keeping with the decision arrived at in the previous section, comparative statics analysis will be undertaken on the assumption that $w'(T) < 0$.

With the terminal salary scheme when an optimum existed it was assumed to be unique. Given a smooth wage function and a typical case where the benefit formula is a linear increasing function of years worked there is no reason to believe that this is unreasonable. But with the money purchase scheme the arbitrariness of many of the functions implies that there might be multiple solutions to the first and second order conditions. In Figure 3.4.1 these solutions are given by intersections of the functions $g(T) + p$ and $w(T)$: T_1 , T_2 , T_3 , T_4 and L all satisfy the first and second order conditions. To derive a global maximum it is necessary to substitute the local maximum values of F and T into (3.4.1) and compare the resulting values of $Y(F, T)$.

3.5 PRIVATE PENSIONS AND PERSONAL SAVINGS

As in Section 2 of Chapter 2 the relationship between pensions and savings divides into two parts, the effect of the introduction of the pension scheme and the effect of changes in pension parameters on savings.

The effect of introducing the schemes is seen by comparing (3.3.2) and (3.4.1) with (2.4.1) with the state pension contributions omitted. Employing the framework of the previous chapter, and assuming an endogenous retirement decision, whether or not lifetime income is raised by the introduction of a scheme will depend on the relative magnitudes of $\alpha \int_0^T w(t)R(t) dt$ and $\gamma(T) \int_T^L R(t) dt$ with a

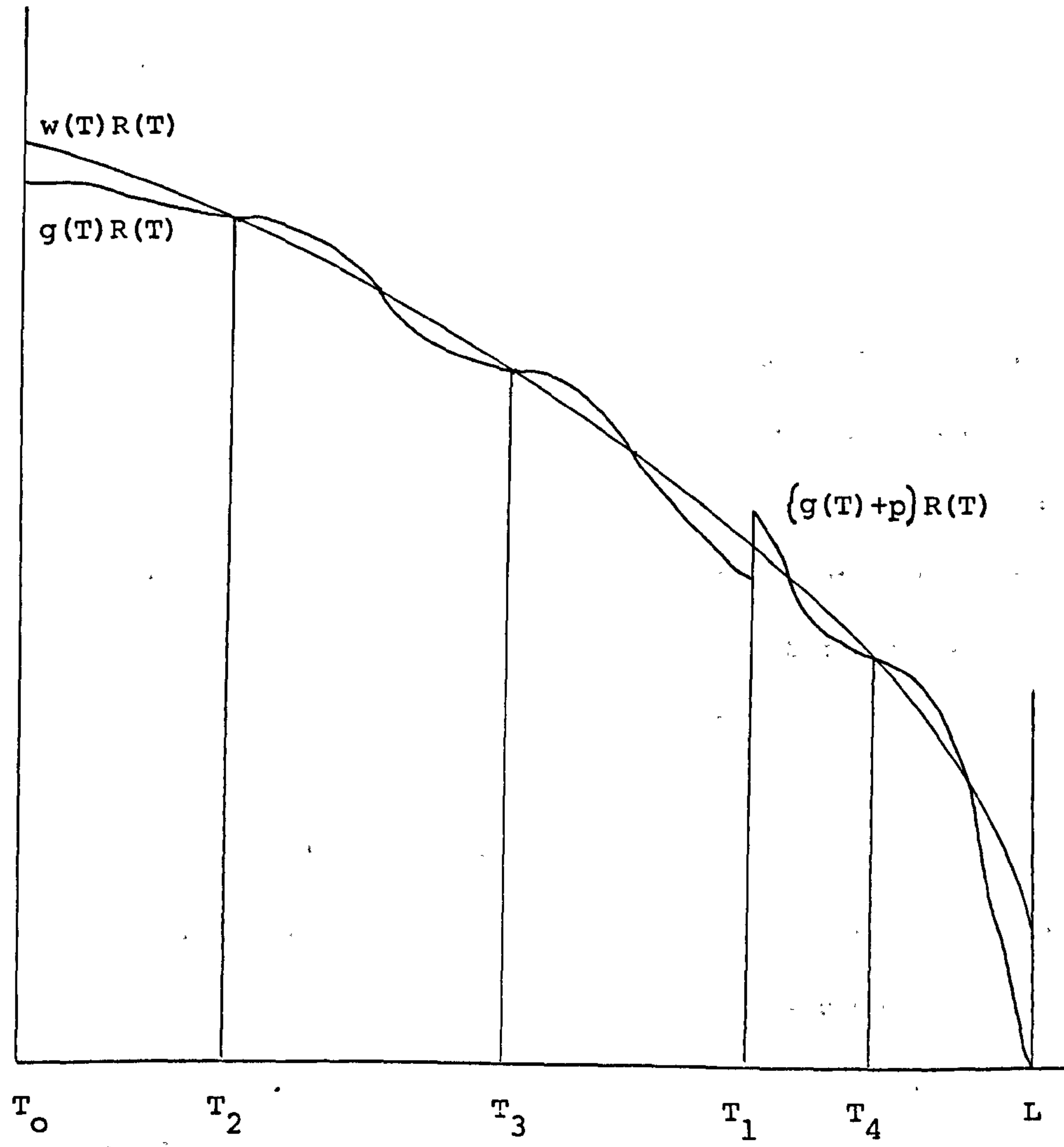


Figure 3.4.1

terminal salary scheme and $R(T)F$ and $P(F,T) \int_T^L R(t) dt$ with a money purchase scheme. In the former case the impact on lifetime income and therefore savings is indeterminate since the lengths of the work and retirement periods, α and the values of $w(T)$ and $\gamma(T)$ are all unspecified. A little more can be said in the latter case. If over the interval $[T_0, L]$ $s(t) > r(t)$ lifetime income is raised; if $s(t) < r(t)$ lifetime income is lowered; if $s(t) = r(t)$ lifetime income is unchanged. The effect on lifetime income and therefore savings can be determined in all these cases but none really describe the type of arrangement outlined. Over $[0, T]$ $r(t) > s(t)$ whilst from the lifetime income maximisation hypothesis lifetime income must be lower in the absence of a pension scheme. It follows that over at least part of $[T, L]$ $s(t) > r(t)$. The higher level of lifetime income implies a lower level of work period, and therefore aggregate, savings as a result of the introduction of the money purchase scheme. This conclusion can be reversed if, as will be shown, the individual might invest in the scheme even if it implies a lower level of lifetime income.

Comparative statics analysis is more straightforward.⁴

Consider firstly an increase in the state pension, p . From (3.3.2) and (3.4.1) this will result in an increase in lifetime income which, using (2.4.2), (2.4.3) and (2.4.4), leads to an increase in consumption in every period. All other things being equal savings falls during the work period. Optimal retirement adjustment, from (3.3.3) and (3.4.2) is given, in both cases, by

$$Y_{TP} = -R(T) (dT_2/dT) < 0 \text{ and } < 0 \text{ if } T_1 < T < L \quad (3.5.1)$$

which implies early retirement.

This result has to be slightly modified in the money purchase case since it will be recalled that it has been assumed that, in a large number of cases, $Y_{FT} < 0$. The induced early retirement will lead to the individual investing more in the private pension fund. Complementarity between the state and private pension is a result that might not have been expected but is fairly readily understood. Retirement is financed out of both state and private pensions. An increase in state pensions does not directly affect the size of the pension fund since, from (3.4.2)

$$Y_{FP} = 0 \quad (3.5.2)$$

but the retirement period is lengthened and it is this which requires a larger private pension fund.

An increase in the terminal salary benefit function, $\beta(\cdot)$, will also raise lifetime income and the resulting decrease in work period savings will be reinforced due to optimal retirement adjustment. The direction of this adjustment, from (3.3.3) is

$$\begin{aligned} Y_{T\beta} &= (d/d\beta) \gamma'(T) \int_T^L R(t) dt - w(T)R(T) \\ &= \left[(d/d\beta) \beta'(T)w(T) + w'(T) \right] \int_T^L R(t) dt - w(T)R(T) \end{aligned} \quad (3.5.3)$$

The sign of this expression is ambiguous. If the analysis is confined to the case of a falling wage stream and the benefit function increasing by a constant absolute amount for all T , then $Y_{T\beta} < 0$. Early retirement is again the outcome.

Similarly, from (3.4.3) and (3.4.4) an increase in the function $P(\cdot)$ changes the size of the pension fund and the retirement date according to

$$Y_{FP} = (d/dP) P_F \int_T^L R(t) dt \quad (3.5.4)$$

$$Y_{TP} = (d/dP) \left(P_T \int_T^L R(t) dt - P(F,T) R(T) \right) \quad (3.5.5)$$

Again assume $P(\cdot)$ is increased by a constant absolute amount. Lifetime income is raised and work period savings falls. $Y_{FP} = 0$ but $Y_{TP} < 0$ - as $Y_{FT} < 0$ early retirement leads to more money being transferred into the pension fund. Although this is the outcome that might have been expected, since the pension investment has been made attractive relative to bonds, this model explains the result only in terms of the longer retirement period. As with the case of an increase in $\beta(\cdot)$ if the simplified assumption about the nature of the change in the function is modified, say to multiplication by a scalar, the effects on savings will be ambiguous.

Lastly, an increase in the private pension contribution, α , in the terminal salary case will reduce lifetime income and work period savings. From (3.3.3)

$$Y_{T\alpha} = -w(T)R(T) < 0 \quad (3.5.6)$$

points to early retirement. With $\gamma'(T) < 0$ this leads to the individual retiring with a higher private pension and a higher lifetime income. Consumption is higher in every period whilst income in the work period is unchanged. The earlier savings changes are therefore confirmed. Less obviously this will also be the case if $\gamma'(T) > 0$ since even though the individual retires with a lower pension lifetime income cannot fall.

3.6 UNCERTAIN LIFETIME⁵

In Section 3.3 some rather forced and contrived assumptions were adopted which led to people investing in the money purchase pension fund. These assumptions can be relaxed once it is realised that one of the most important reasons individuals make a pension arrangement is to provide protection against unanticipated longevity. To incorporate this motive into the analysis will require modification to both the portfolio choice and accumulation aspects of the problem. Consider firstly the portfolio problem.

The assumption that the individual transfers money from other forms of capital to the pension fund on retirement will be maintained. In addition it will be assumed that the individual will live with certainty to T_1 but is unsure as to when he will die thereafter. The maximum length of life will be denoted \tilde{L} . Uncertainty takes the form of a probability density function $\pi(\cdot)$ on $[T_1, \tilde{L}]$ where

$$\pi(t) > 0 \quad \text{for all } t \geq T_1 \quad (3.6.1)$$

and
$$\int_{T_1}^{\tilde{L}} \pi(t) dt = 1 \quad (3.6.2)$$

The corresponding probability distribution is

$$\Omega(t) = \int_t^{\tilde{L}} \pi(\tau) d\tau \quad (3.6.3)$$

which is the probability that the individual will be alive at time t .

To draw any conclusions in this section it is necessary to draw heavily on a paper by Yaari (61) which demonstrates that the uncertain lifetime case is considerably simplified by the introduction of a perfect insurance market in the form of a market in actuarial notes paying out a rate of interest $i(t)$.⁶ Yaari shows that if $i(t)$ is an 'actuarially fair' rate of interest then

$$i(t) = r(t) - \left(\dot{\Omega}(t) / \Omega(t) \right) > r(t) \quad (3.6.4)$$

(3.6.4) defines the rate of return on non-pension assets over $(0, \tilde{L}]$. Beyond T_1 the individual will hold capital only in the form of these notes.⁷

As in the previous sections the individual's problem is to choose F and T to maximise $Y(F, T)$ and although the qualitative solutions go through unhindered they will be quantitatively different for two reasons. The first is that in Figure 3.4.1 the shape of the $g(T)$ function has been changed due to the higher rate of interest after T_1 . Thus the substitution of all the optimal values of F and T that satisfy the first and second order conditions into (3.4.1) may yield a different optimum to that in the certainty case. Secondly it has not been assumed that L in the certainty case is the same as \tilde{L} in the uncertainty case.

The effect of uncertainty on accumulation is analysed by introducing L as a random variable into (2.2.1). This raises a problem in that if L is a random variable then so is the intertemporal utility function. Utility maximisation is now rather imprecise. It is therefore necessary to assume that the individual maximises expected

utility. The only problem remaining is that the terminal wealth constraint, (2.2.3) is probabilistic. Yaari presents two rather attractive ways around this problem, the chance constrained programming and loss function approaches. The former will be described in detail.⁸

The chance constrained programming approach requires that the wealth constraint is met with a probability of at least π . Thus (2.2.3) can be replaced by

$$\text{prob} \left(k(L) \geq 0 \right) \geq \pi \quad (3.6.5)$$

If an admissible plan is defined as one for which $\pi=1$ the optimal plan can then be determined.

The expected utility function is

$$EU = \int_0^{\tilde{L}} \Omega(t) u(c(t)) e^{-\delta(t)} dt \quad (3.6.6)$$

which is maximised subject to

$$\dot{k}(t) = i(t)k(t) + y(t) - c(t) \quad (3.6.7)$$

$$\text{and } k(L) \geq 0 \quad (3.6.8)$$

Applying the calculus of variations Yaari shows that the solution to this problem is

$$\dot{c}(t)/c(t) = \left[i(t) - \delta(t) + \dot{\Omega}(t)/\Omega(t) \right] / \theta \quad (3.6.9)$$

This is formally similar to (2.2.17) - in this case consumption is growing over time if $i(t) > \delta(t) - \dot{\Omega}(t)/\Omega(t)$. From (3.6.4) it becomes clear that not only are (2.2.17) and (3.6.9) formally similar, they are identical. This, of course, only applies to consumption over $[0, L]$.

The introduction of the perfect insurance market reduces the uncertainty problem to that under certainty.

(3.6.9) is also significant in that it implies that the earlier comparative statics outcomes are unchanged since the effects of changes in p and $P(\cdot)$ on consumption in every period is determined from

$$C = Y(F,T) = c_0 \int_0^{\tilde{L}} \exp\left\{i(t) - \delta(t) + \dot{\Omega}(t)/\Omega(t)\right\} / \theta \, dt \quad (3.6.10)$$

which is handled in exactly the same way as (2.4.2).

To simplify the uncertainty problem to the extent that has been achieved it has been necessary to assume the existence of the perfect insurance market. This means that there are three assets in the model, bonds, actuarial notes and the pension - the latter though is clearly inessential since if there are perfect insurance markets then additional insurance in the form of pensions is not needed. The actuarial notes act as the pension asset.

In a recent paper Ulph and Hemming (59) have considered the problem when insurance markets are less complete than assumed by Yaari. Whilst the analysis of the consumers portfolio and accumulation problem turns out to be substantially more complicated most of the comparative statics results arrived at in the earlier part of this chapter are confirmed. The one major difference is that whilst changes in the state pension had no effect on the optimal choice of the retirement date if this occurred before T_1 this result is now changed. In the more complex model a rise in the state pension leads to the postponement of retirement since it induces the individual to

run down private pension income thus making retirement less attractive. To fully understand this result one would have to outline the analysis of the paper in considerable detail - this is not thought to be worthwhile given the implications of the analysis for these results.

3.7 CONCLUSIONS

In this chapter the model developed in Chapter 2 has been modified to include private pension schemes. It has been shown that

- i) The effect on savings of the introduction the schemes described cannot be determined, a priori.
- ii) The effect of increasing the state pension, the functions determining private pension income by a constant absolute amount, and the private pension contributions all induce early retirement and lead to a reduction in work period and therefore aggregate savings. An increase in the function determining private pension with the money purchase scheme leads to a larger fund as does an increase in the state pension scheme.
- iii) Uncertainty about the length of life has been included in the model, albeit in a highly stylised form, with perfect insurance markets which reduce the problem to one of portfolio choice and accumulation under certainty. Now typically one does not observe people trading continuously in actuarial notes. A model in which insurance markets are not complete has been analysed, and found to be tractable, in the paper by Ulph and Hemming. Had this paper been available earlier it would clearly have provided a more realistic framework in which to present the analyses and obtain nearly all of the results of sections 3.4, 3.5 and 3.6.

NOTES

1. This is an expanded version of Hemming (29).
2. These are handled in exactly the same way as in Chapter 2.
3. This section draws heavily on Sections 1 and 2 of Hemming and Ulph (30).
4. With the money purchase scheme there is a problem with comparative statics analysis since changing pension parameter could shift the global maximum from one local maximum to another (see Figure 3.4.1). It is assumed that all comparative statics analysis is carried out at a local level.
5. Again this section draws heavily on Hemming and Ulph (30), Section 3.
6. If the consumer buys such a note he will have an asset which pays out interest at a rate $i(t)$ for as long as he lives, but such will have no value of his death. Similarly, if the consumer sells such a note he will have to pay out interest $i(t)$ as long as he lives, but on death the debt is cancelled.
7. From what has been said the individual can clearly satisfy his terminal constant by always holding actuarial notes. Also there is no apparent limit to lifetime wealth since notes can always be sold and the interest payments financed by selling more notes. To avoid this problem Yaari defines a date $L-\Delta$ by which time the

individual has to have no debt. Over the last $L-\Delta$ years of life the insurance company will not sell notes and this constrains individual behaviour in that there is a positive probability that he will be alive over this period.

8. This approach assumes that the violation of the wealth constraint carries a penalty in the form of a reduction in utility.

CHAPTER FOUR

ESTIMATION : PROBLEMS AND METHODS

4.1 INTRODUCTION

To estimate the relationship between pensions and savings Feldstein's approach will be followed, and pension wealth variables will be introduced into the Modigliani-Brumberg-Ando (M-B-A) formulation of the consumption function.

The only guide to the exact form of the consumption function, under the assumption of perfect certainty, is given by (2.4.2.), (2.4.3.) and (2.4.4.) which show that consumption in each period is related to lifetime income in the following way

$$c_t = \left[\frac{\exp \int_0^t (r(\tau) - \delta(\tau)) / \theta d\tau}{\int_0^L \exp \{ (r(t) - \delta(t)) / \theta \} dt} \right] Y(T) \quad (4.1.1.)$$

It can be seen that the proportion of lifetime income consumed in each period depends on the functions $r(\cdot)$ and $\delta(\cdot)$, age and the form of the utility function, as given by θ . Beyond this though little can be learnt, particularly about the relationship between consumption and resources in each period. It is fortunate then that the analysis is not constrained by (4.1.1.). Indeed, as shown in Hemming (29), the theoretical analysis of the previous two chapters is independent of the analysis of the individual's intertemporal utility maximisation problems. All the results go through if it is assumed that the intertemporal utility function depends only on the intertemporal consumption stream. In addition, utility has to be an increasing function

of consumption expenditure in any one period and consumption expenditure in each period a normal good, increasing as total lifetime income increases.

Even the relaxation of this assumption does not facilitate the a priori specification of the mathematical form of the consumption function. In particular there is no reason to believe the relationship is linear. A linear specification will therefore be compared with some linear transformations of non-linear specifications, chosen for their properties relating to theoretical consumption functions! The following relationships will be estimated

i) linear: $Y = \alpha + \beta X$: constant marginal propensity

to consume;

ii) semi-logarithmic: $\log Y = \alpha + \beta X$: increasing marginal

propensity to consume;

iii) double-logarithmic: $\log Y = \alpha + \beta \log X$: diminishing

marginal propensity to consume if $0 < \beta < 1$.

iv) log-reciprocal: $\log Y = \alpha - \beta/X$: increasing then diminishing

marginal propensity to consume.

The specification giving the best overall fit will be retained.

4.2 WEALTH IN THE CONSUMPTION FUNCTION

Before proceeding to introduce pension wealth variables into the consumption function it is worth while reviewing the evidence, in addition to that of Feldstein already presented, relating to the role of wealth in determining the level of consumers' expenditure.

Prior to the recently published survey by Mayer (43) the role of wealth as a determinant of consumers' expenditure had only been reviewed in detail by Evans (18). He considered the papers by Spiro (55), Ando and Modigliani (2) and Ball and Drake (10).

Spiro suggests that the level of savings is a function of the discrepancy between actual and desired wealth. If desired wealth is a function of the level of income one can write

$$C_t = f(W_t, Y_t, Y_{t-1}, \dots, Y_{t-\infty}). \quad (4.2.1.)$$

which, expressing wealth in terms of income in previous periods, can be written

$$C_t = \sum_{i=0}^{i=\infty} \beta^i Y_{t-i} \quad (4.2.2.)$$

Spiro requires that the coefficients on previous income sum to unity which can be guaranteed if one writes $\beta_i^i = (1-b)b^i$. The final consumption function is

$$C_t = (1-b) \sum_{i=0}^{i=\infty} b^i Y_{t-i} \quad (4.2.3.)$$

Ando and Modigliani follow Modigliani and Brumberg (46) in assuming that individuals maximise a lifetime utility function, defined over present and future consumption, subject to the constraint that lifetime consumption cannot exceed lifetime resources. Lifetime resources are defined as the sum of current earnings, future earnings and current net worth. This is perfectly consistent with the maximisation of (2.2.8.) subject to (2.2.9.) and (2.2.10.). The authors then introduce the specific assumption that the utility function is such that the proportion of total resources devoted to consumption in any year is independent of the size of lifetime

resources. Any increase in lifetime resources will be allocated between present and future consumption in the same proportion as existing resources. This proportion depends on the rate of interest, the rate of discount, age and the form of the utility function².

$$C_t^a = \Omega_t^a V_t^a \quad (4.2.4.)$$

$$\text{where } V_t^a = W_{t-1}^a + Y_t^a + \sum_{\tau=a+1}^T Y_{\tau}^{ea} / (1+r)^{\tau-a} \quad (4.2.5.)$$

where Y^e is expected earnings. Defining average annual expected earnings as

$$\bar{Y}^{ea} = (1/T-a) \sum_{\tau=a+1}^T Y_{\tau}^{ea} / (1+r)^{\tau-a} \quad (4.2.6.)$$

and using (4.2.4.) and (4.2.5.) one can write

$$C_t^a = \Omega_t^a Y_t^a + \Omega_t^a (T-a) \bar{Y}^{ea} + \Omega_t^a W_{t-1}^a \quad (4.2.7.)$$

For ease of exposition it is often assumed that $\Omega_t^a = 1/(L-a)$ and the optimal consumption stream is constant³. Aggregating (4.2.7.) within and over age cohorts produces the consumption function for the whole community

$$C_t = \beta_1 Y_t + \beta_2 \bar{Y}^e + \beta_3 W_{t-1} \quad (4.2.8.)$$

Ball and Drake hypothesise that individual behaviour is myopic and the utility function to be maximised is of the form

$$U_t = f(W_t, C_t) \quad (4.2.9.)$$

which is assumed to be homogeneous of degree 1. Maximising this function subject to the constraint

$$W_t = W_{t-1} + Y_t - C_t \quad (4.2.10.)$$

yields the consumption function

$$C_t = KW_t \quad (4.2.11.)$$

which on substituting into the budget constraint gives

$$C_t = (1/1+K) Y_t + (K/1+K) C_{t-1} \quad (4.2.12.)$$

All these authors would describe the results of their own tests as 'good'. These results though have all been challenged by Evans, who re-estimates all the functions using annual U.S. data for the periods 1929-41 and 1947-62 and quarterly data for the period 1947-62. Specifically his criticisms are:

a) Spiro deflates income by a GNP deflator when a consumers' price index would be more appropriate. Evans shows that the use of a GNP deflator biases the sum of income coefficients towards unity.

b) Of the Ball and Drake estimates Evans shows that they do not hold when income growth is introduced into their consumption function: In particular a realistic growth rate produces an unrealistic marginal propensity to consume.

c) Although Evans was able to confirm that the fit of the Ando-Modigliani function was fairly good using annual data, with post war quarterly data it performs particularly badly. This result suggests that wealth is more important in times of depression.

Thus Evans concludes

"....the time series evidence gives very little reason to believe that wealth should be included in the consumption function, either implicitly or explicitly." (p.349)

Mayer partially confirms these results by showing that for the period 1962-67, using data for the post-war period, only the Ball and Drake function could predict the level of consumption better than a naïve model which suggests that consumption grows at a constant rate (either 0% or 3%).

These findings do not bode well for the analysis to follow, but subsequent work, which suggests extending the definition of wealth, appears to leave some hope. Arena (3)) argues that if one were to include increments of net worth, due to savings and interest, in the consumption function then one ought to include changes in its value due to price movements. Omitting expected income from the Ando-Modigliani model but adding in capital gains, and the lag distribution of all variables, Arena's results supported a wealth model. Bhatia (11) modifies Arena's model by allowing income and capital gains to influence consumption with different geometric lags. Unfortunately Bhatra omits wealth from his function.

The approach of Arena and Bhatia has been modified by Feldstein (21) and Feldstein and Fane (22), who argue that the components of net worth might affect consumption to differing degrees. In addition to net worth, as previously defined, the authors consider the effect of including capital gains, dividends and retained earnings of companies in the consumption function. The marginal propensities to consume out of these components are likely to be different for

two reasons. Firstly, investors are likely to view retained earnings as a more permanent increase in wealth than market revaluations of wealth due to price changes. It is also the case that tax policies in the U.K. and U.S., where the tax rates on retained earnings are below those on dividends, induce companies to retain income rather than distribute it. If changes in retained earnings are tax induced then they might be regarded as good predictors of future changes in retained earnings. Secondly, these components of wealth might react in different ways with savings, subsequent to changes in the rate of interest. Changes in interest rates affect both the value of wealth and the allocation of consumption over the lifecycle. It is well known that the net effect on consumption is ambiguous and it follows that the coefficient on retained earnings can be increased or decreased relative to that on capital gains.

Thus a 'components of capital gains model of the capital income hypothesis', which states that all forms of capital income, whether distributed or not, have a substantial impact on concurrent consumption, has to be tested.

Feldstein proceeds by writing capital gains as the sum of changes in net worth due to retained earnings and changes in the market value of assets

$$G_t = \zeta RE_t + X_t \quad \zeta < 1 \quad (4.2.13.)$$

As ζ cannot be observed the consumption function has to be written in the form

$$C_t = \alpha + \beta_0 Y_t + \beta_1 Y_t^e + \beta_2 W_{t-1} + \beta_3 G_t + \beta_4 RE_t \quad (4.2.14.)$$

where β_4 represents the excess effect on consumption of retained earnings over capital gains in general.

Using U.S. data for the period 1929-66 Feldstein estimates a series of equations all of which support the conclusion that the impact of retained earnings is fairly considerable. The coefficients on capital gains and net worth are not significantly different⁴.

Feldstein and Fane use U.K. data for the period 1948-69. The specification of the consumption function is altered to allow a dynamic lag pattern. They suggest an equation of the form

$$C_t = \alpha + \beta_0 \frac{(1-\lambda)^r}{(1-\lambda L)^r} Y_t + \beta_1 W_{t-1} + \beta_2 G_t + \beta_3 RE_t \quad (4.2.15.)$$

where L is a lag operator such that $L^k Y_t = Y_{t-k}$ and r is the order of the Pascal lag.

The estimated equations suggest that the coefficient on retained earnings is approximately +.25 and again the coefficients on net worth and capital gains are not significantly different⁵. Redefining income to exclude dividends and including dividends explicitly in the consumption function did not change the implications of the analysis. In addition, attempts to estimate separate lag distributions for the for the components of capital gains proved fruitless.

Overall Feldstein and Fane conclude

"Our evidence supports the capital income hypothesis: capital income, including retained earnings as well as dividends, has a substantial effect on concurrent consumption. More specifically the estimates favour the components of capital gains model." (p.410).

The next stage in reviewing the role of wealth in the consumption function would be to examine pension wealth. This has already been carried out in Chapter 1 Section 4 and it should now be clear that Feldstein (19) is a modification of Feldstein (21) to include

pension wealth. A modification of Feldstein (22) to include pension wealth will be included in Section 5 of this chapter. But before doing this another problem has to be tackled - that of 'estimating' expected income.

4.3 THE ESTIMATION OF EXPECTED INCOME

It can be seen from (4.2.8.) that the aggregate consumption function contains a variable 'average annual expected income' which is unobservable. From (4.2.14.) and footnote 4 it can be seen that Feldstein includes disposable income lagged one period to reflect expected income - the reasoning for this is normally that recent income experience is a good guide to future income changes. On the other hand (4.2.15.) implies that expected income is represented by an infinite lag on disposable income - the rationale here is that the complete income experience is important in determining expected income.

To learn a little more about this problem it is probably best to start with the way it was tackled by Ando and Modigliani (2). In this paper the income variables always refer to current non property income. Three methods of measuring expected income were suggested.

i) naive hypothesis: it is assumed that expected income is current income multiplied by a scale factor. Then

$$Y_t^e = \gamma Y_t \quad \gamma \neq 1 \quad (4.3.1.)$$

Thus the relationship between consumption and income variables is written

$$C_t = \beta^1 Y_t \quad \text{where } \beta^1 = \beta_0 + \beta_1 \gamma \quad (4.3.2.)$$

ii) exponentially weighted average of past incomes: this method was not used by the authors because of lack of data. It is to be discussed later in this section.

iii) cyclical weights: it is hypothesised that for those currently employed average expected income is

$$\tilde{Y}_t^e = (\gamma_1/E_t) Y_t \quad (4.3.3.)$$

where E_t is the number of people engaged in production. For those unemployed

$$\tilde{Y}_t^e = (\gamma_2/E_t) Y_t \quad \gamma_1 > \gamma_2 \quad (4.3.4.)$$

Multiplying (4.3.3.) and (4.3.4.) by the numbers employed and unemployed ($L_t - E_t$ where L_t is the total labour force) yields

$$Y_t^e = (\gamma_1 - \gamma_2) Y_t + \gamma_2 (L_t/E_t) Y_t \quad (4.3.5.)$$

The relationship between consumption and income variables is therefore written

$$C_t = \alpha_1 Y_t + \alpha_2 (L_t/E_t) Y_t \quad (4.3.6.)$$

where $\alpha_1 = \beta_0 + \beta_1(\gamma_1 - \gamma_2)$

and $\alpha_2 = \beta_1 \gamma_2$

This discussion raises two points. Consider firstly the definition of income. Although Ando and Modigliani are concerned with labour income estimates are arrived at using personal disposable income due to the difficulties involved in measuring labour income. Feldstein and Fane (p. 402) quite correctly point out that although

Ando and Modigliani think this procedure is perfectly in accord with their original theory it will have different implications for the marginal propensity to consume⁶. Nevertheless it does go a long way towards explaining the use of disposable income in the research reviewed.

Turning now to the lag specification. Ando and Modigliani have possibly suggested two very different procedures. Methods i) and iii) are really very similar in that iii) can be interpreted simply as saying that expected income is proportional to the average current income of those employed. What the two specifications have in common is that they are both theoretical. The exponential weighting method, ii), can be both a theoretical and empirical specification and in discounting it, Ando and Modigliani do not make it clear which they think it is. It is probably fair to say though that in referring to Friedman (23) they believe the data should determine the length of any lag and the specification is therefore empirical.

To consider this distinction in more detail consider the problem of empirically estimating lags - this can be done using three popular methods.

The principle being considered is that consumption is a function of not only current but also expected income. Income expectations are reflected in previous income experience and the problem is to determine how far in the past income experience becomes irrelevant.

Consider then a finite lag structure of the form

$$C_t = \alpha + \beta_0 Y_t + \beta_1 Y_{t-1} + \dots + \beta_k Y_{t-k} + u_t \quad (4.3.6.)$$

If one can make the usual assumptions about the error term i.e. $E(u_t) = 0$, $E(u_t, u_{t+j}) = 0$ for $j \neq 0$ and σ^2 for $j = 0$ then, one can proceed and estimate this equation using ordinary least square (OLS). Unfortunately this method nearly always involves two problems. With time series data over a relatively short period and a large number of lags sampling variances are inevitably large, coefficients tend to be erratic and tests of the significance of parameters are impaired. Secondly successive values of income will be highly multicollinear, and variables which are insignificant will appear significant and vice versa.

Attempts to estimate OLS lags using recent U.K. data⁷ are presented in Table 4.3.1. The main features of these results are that the lag is quite short the first year always having about three quarters of the weight and many of the coefficients are negative and marginally significant which has little economic meaning.

Given these results the procedure is very clear - one has to achieve both a reduction in the number of lagged variables and constrain the coefficients.

Ando and Modigliani suggest using Friedman's method of constraining the coefficients by imposing on the data an exponentially declining pattern of weights. Omitting the constant term, this is written

$$C_t = \beta_0 \sum_{i=0}^k \lambda^i Y_{t-i} + u_t \quad \lambda \in (0,1) \quad (4.3.7.)$$

It has also been suggested that the weights ought to add up to 1 or slightly more than 1 if an allowance is to be made for income growth through time. As $\sum \lambda^i = 1/(1-\lambda) \neq 1$ for $(\lambda \neq 0)$ the lag pattern is normalised to reflect $\sum (1-\lambda)\lambda^i = (1-\lambda)/(1-\lambda) = 1$. Following Friedman, and allowing for income growth at a rate g , (4.3.7.) becomes

Table 4.3.1.

O.L.S. Income Lags : 1949-73 (U.K. annual data).

Dependent Variable : Consumers' Expenditure.

Independent Variable : Personal Disposable Income.

Lag	0	1	2	3	4	5	6	7	8	9	\bar{R}^2	DW
0	.784 (.008)										.997	1.29
1	.789 (.016)	-.004 (.011)									.997	1.36
2	.797 (.016)	.012 (.014)	-.020 (.011)								.997	1.33
3	.799 (.017)	.012 (.014)	-.017 (.014)	-.004 (.011)							.997	1.27
4	.788 (.017)	.013 (.014)	-.017 (.014)	-.018 (.014)	.019 (.011)						.998	1.32
5	.782 (.018)	.012 (.014)	-.017 (.014)	-.018 (.014)	.010 (.014)	.012 (.011)					.998*	1.22
6	.783 (.020)	.012 (.014)	-.017 (.014)	-.018 (.014)	.010 (.014)	.014 (.014)	-.002 (.011)				.997	1.23
7	.786 (.022)	.012 (.015)	-.017 (.015)	-.018 (.015)	.010 (.015)	.014 (.015)	.001 (.015)	-.004 (.012)			.997	1.19
8	.775 (.024)	.012 (.014)	-.017 (.014)	-.018 (.014)	.010 (.014)	.014 (.014)	.001 (.014)	-.014 (.014)	.014 (.012)		.997	1.13
9	.771 (.027)	.012 (.015)	-.017 (.015)	-.018 (.015)	.010 (.015)	.014 (.015)	.001 (.015)	-.014 (.015)	.010 (.015)	.005 (.012)	.997	1.06

Notes: Equations estimated with a constant.

Brackets contain coefficient standard errors.

 \bar{R}^2 is corrected R^2 .

*Minimum SSR.

$$C_t = \beta_0 (1-\lambda) \sum_{i=0}^k \lambda^i (1+g)^i Y_{t-i} + u_t \quad 0 < \lambda < 1 \quad (4.3.8.)$$

Using the U.K. data (4.3.8.) has been estimated - the value of k being dictated, for each value of λ , by that value of $(1-\lambda)\lambda^i$ which falls below 1%. This implies that $\sum (1-\lambda)\lambda^i$ will normally be less than 1, the difference between the two being greater the lower the value of λ .

The results in Table 4.3.2. are based on an assumed income growth rate of 2%. Of the values chosen for λ it can be seen that equations both with and without constants have improved goodness of fits as λ decreases indicating an extremely short lag.

Lastly consider the polynomial lag method as proposed by Almon (1)) and popularised by Jorgenson (34). The method is to assume that the β coefficients in (4.3.6.) can be approximated by a polynomial function of the r^{th} degree. For $r = 2$ the results of fitting a polynomial function to the U.K. data are given in Table 4.3.3. Again it can be seen that the lag is short. In the best fitting equations about 75% of the weight is attached to the first year's income. As with the OLS regressions the fact that the coefficients are not constrained means that a number 'misbehave', particularly with the later values of lagged variables.

Applying these three methods to the U.S. data Mayer (43) produced very similar results - this would appear to justify Feldstein's (19)(21) specification of expected income using simply a one period lag on disposable income.

Having concluded that the empirical lag is short one should now ask what value one can put on this. In particular Griliches (27) has outlined the problems involved in estimating lags in autocorrelated

Table 4.3.2.

Geometric Lags : 1949-73 (U.K. annual data).

Dependent Variable : Consumers' Expenditure.

Independent Variable : As defined in (4.4.8.)

	With Constant				Without Constant			
	M.P.C.	s.e.	\bar{R}^2	D.W.	M.P.C.	s.e.	S.S.R.	D.W.
(1-λ)								
.3	.506	(.039)	.874	.26	.993	(.032)	529 x 10 ⁶	.10
.45	.581	(.042)	.888	.36	.962	(.022)	268 x 10 ⁶	.18
.60	.665	(.038)	.926	.52	.955	(.015)	134 x 10 ⁶	.27
.75	.718	(.028)	.964	.66	.938	(.011)	67 x 10 ⁶	.29
.90	.774	(.014)	.992	.86	.939	(.007)	28 x 10 ⁶	.18
.95	.778	(.010)	.996	.90	.932	(.006)	23 x 10 ⁶	.13

Table 4.3.3.

Polynomial Distributed Lags : 1949-73 (U.K. annual data).

Dependent Variable : Consumers' Expenditure.

Independent Variable : Personal Disposable Income.

Lag	0	1	2	3	4	5	6	7	8	9	\bar{R}^2	DW
YEAR EQN.												
3	.726 (.073)	-.080 (.110)	.149 (.075)								.998*	1.55
4	.684 (.064)	.044 (.063)	-.120 (.060)	.191 (.076)							.998	1.74
5	.563 (.057)	.229 (.010)	.027 (.035)	-.042 (.030)	.022 (.020)						.997	1.71
6	.510 (.045)	.247 (.006)	.069 (.022)	-.025 (.028)	-.035 (.016)	.039 (.020)					.996	1.69
7	.425 (.034)	.249 (.007)	.118 (.013)	.029 (.021)	-.016 (.018)	-.017 (.008)	.024 (.020)				.995	1.40
8	.357 (.026)	.241 (.008)	.146 (.008)	.073 (.014)	.021 (.015)	-.009 (.011)	-.017 (.006)	-.003 (.019)			.994	1.48
9	.347 (.018)	.239 (.007)	.149 (.004)	.079 (.008)	.027 (.010)	-.006 (.009)	-.020 (.006)	-.015 (.006)	.008 (.015)		.994	1.45
10	.339 (.013)	.237 (.006)	.152 (.003)	.084 (.005)	.032 (.007)	-.003 (.007)	-.021 (.006)	-.022 (.004)	-.007 (.006)	.025 (.012)	.994	1.51

Note: Equations estimated with a constant.

*Minimum SSR.

series and one of his conclusions was

"... do not expect the data to give a clear cut answer about the exact form of the lag. The world is not that benevolent. One should try to get more implications from theory about the correct form of the lag and impose it on the data." (p.46.).

The theory outlined in the previous chapters cannot help in determining the lag. Nevertheless it is fairly reasonable to suggest that, if one is going to assume that previous income experience is a good guide to income expectations, the weight one puts on successive lagged values of income is declining. It is probably less reasonable, though still plausible, to suggest the weights decline geometrically. Thus a lag pattern similar to (4.3.7.) can be adopted. A problem still arises in that there is no reason to believe that the lag is of any particular length. A popular solution is to assume that it is infinite. Attention will now be focussed on this case.

4.4 THE THEORETICAL INFINITE LAG : THE KOYCK SCHEME

With an infinite geometric lag (4.3.7.) will become

$$C_t = \beta_0 (1-\lambda) \sum_{i=0}^{\infty} \lambda^i y_{t-i} + u_t \quad (4.4.1.)$$

which can be written in the lag-operator form

$$C_t = \beta_0 \frac{(1-\lambda)}{(1-\lambda L)} Y_t + u_t \quad (4.4.2.)$$

yielding, on inversion,

$$C_t = \beta_0 (1-\lambda) Y_t + \lambda C_{t-1} + (u_t - \lambda u_{t-1}) \quad (4.4.3.)$$

This model has been popularised by Koyck (36) and has been generalised by Solow (54) so that the lag structure becomes $(1-\lambda)r\lambda^i$ - this is the Pascal lag used by Feldstein and Fane. It can be seen that when $r = 1$ this reduces to the geometric lag scheme, whilst $r > 1$ guarantees that the lag structure peaks.

This formulation generally raises four types of problem. Firstly the lagged dependent variable, C_{t-1} , is not independent of the error term $u_t = u_t - \lambda u_{t-1}$ which implies small sample bias in OLS estimates. Secondly the error term is serially correlated. These two problems put together imply that OLS estimates will be inconsistent and inefficient in large samples and that the power of the Durbin Watson statistic to detect serial correlation is seriously impaired.

There are two ways to proceed from here. One can either go back to examine the theoretical justification for an equation like (4.4.3.) or tackle the estimation problems directly. From a theoretical viewpoint two popular models can result in this type of equation.

Cagan's (14) adaptive expectations hypothesis would suggest that consumption is a function of the expected level of income, Y_t^* , and that expectations are formed in each period by updating by a fraction the difference between this period's current income and the previous period's expected income. This yields two equations

$$C_t = \beta_0 Y_t^* + u_t \quad (4.4.4.)$$

$$Y_t^* - Y_{t-1}^* = \delta (Y_t - Y_{t-1}^*) \quad 0 < \delta < 1 \quad (4.4.5.)$$

which setting $1-\delta = \lambda$ can be written

$$C_t = \beta_0 (1-\lambda) Y_t + \lambda C_{t-1} + (u_t - \lambda u_{t-1}) \quad (4.4.6.)$$

which is identical to (4.4.3.)

Nerlove's (47) partial adjustment hypothesis suggests that for a given income level the individual will have an optimal level of consumption, C_t^* , but because income is changing over time and the individual does not have the information, ability or inclination he will not adjust fully to the new optimal consumption level from period to period. Formally

$$C_t^* = \beta_0 Y_t \quad (4.4.7.)$$

$$C_t - C_{t-1} = \delta(C_t^* - C_{t-1}) + u_t \quad 0 < \delta \leq 1 \quad (4.4.8.)$$

which again setting $1 - \delta = \lambda$ implies

$$C_t = \beta_0 (1 - \lambda) Y_t + \lambda C_{t-1} + u_t \quad (4.4.9.)$$

which is this time similar to (4.4.3.) except that the serial correlation in the error term is no longer present. The parameters of the model can therefore be estimated consistently using OLS.

Before suggesting that invoking the partial adjustment hypothesis circumvents all the estimation problems it is worth considering the implications of the model. The crucial point is ought the desired level of consumption to depend only on current income when it is known that income is changing through time. Maybe Johnston (33) is correct in his suggestion that in such a situation it is more logical to base consumption decisions on expected income⁹ - this leads one back to an adaptive expectations model as a logical development of a partial adjustment model.

With specific reference to this analysis it is also clear that a partial adjustment model is generally inapplicable when considering the problem of approximating expected income, although there is no

reason why the partial adjustment could not be made with respect to changes in expected income.

As regards the estimation of these models most econometrics textbooks refer to the paper by Zellner and Geisel (62) who investigate the importance of misspecifying the error term. They consider four error term specifications;

$$i) u's \text{ are } \sim N(0, \sigma_u^2) : E(u_i, u_j) = 0 \text{ which says that the}$$

error term is not serially correlated

$$ii) u's \text{ are } \sim N(0, \sigma_u^2)$$

$$iii) u_t = \rho u_{t-1} + \varepsilon_t : \varepsilon's \text{ are } \sim N(0, \sigma_\varepsilon^2) : E(\varepsilon_i, \varepsilon_j) = 0$$

which says that the u 's follow a first order Markov process. Note that $\rho = \lambda$ implies i) and $\rho = 0$ implies ii).

$$iv) u_t = \psi u_{t-1} + \omega_t : \omega's \text{ are } \sim N(0, \sigma_\omega^2) : E(\omega_i, \omega_j) = 0.$$

This can be rewritten $u_t - u_{t-1} = \psi(u_{t-1} - \lambda u_{t-2}) + \omega_t$ which says that the true disturbance term can be approximated by a first order autoregressive process. Note that if $\psi = 0$, iv) reduces to i).

Using each of these specifications Zellner and Geisel calculate maximum likelihood estimates of the parameters of (4.4.3.), having discussed the estimation problems pertaining to each one. These results are summarised in Table 4.4.1. The lessons to be learnt from the results have two sides. It is clear that the specification of the error term is important in that it does affect the estimated parameters, particularly the lag parameter, λ . On the other hand, β_0 , the long run marginal propensity to consume, is insensitive to the error specification and the sum of the lag coefficient and the short run marginal propensity to consume is remarkably stable. Analysis of the model under assumption iii) suggests that u 's are serially correlated but $\lambda \approx \rho$ in which case iii) reduces to i). Using assumption iv) the estimated value of ψ is

Table 4.3.4.

Zellner and Geisel Estimated Coefficients : 1947-60 (U.S. quarterly data).

Assumption	λ	β_0	$\beta_0(1-\lambda)$	ρ	ψ	$\lambda + \beta_0(1-\lambda)$
i)	.657 (.108)	.89 (.031)	.306 (.095)			.963
ii)	.45 (.029)	.937 (.164)	.515 (n.c.)			.965
iii)	.66 (.085)	.94 (.46)	.321 (n.c.)	.69 (.076)		.981
iv)	.772 (.091)	.96 (.014)	.218 (n.c.)		-.13 (.14)	.990

close to zero in which case this model reduces to i). In addition a Bayesian analysis suggested that assumption i) was a preferred specification to ii).

Despite the fact that it is difficult to justify in terms of economic theory, this analysis suggests that the risk involved in making the simplest assumption is not great. A model will therefore be considered where the composite error term u_t is not serially correlated. It should be emphasised though that this assumption is being made not because it is believed to be true but because the preceding discussion suggests that one does not need to be too careful in specifying the error pattern.

Serially correlated error terms aside there still remains the issue of the correlation between the lagged dependent variable and the error term. To consider the problems raised by this it might be convenient to consider the full model to be estimated.

4.5 THE FULL MODEL

A pension wealth variable is to be introduced into an aggregate life cycle consumption function where expected income is approximated by an infinite geometrically declining lag on previous periods' income. In lag operator form, reintroducing the constant term, this is written

$$C_t = \alpha + \beta_0 \frac{(1-\lambda)}{(1-\lambda L)} Y_t + \beta_1 W_{t-1} + \beta_2 PW_t + u_t \quad (4.5.1.)$$

where PW_t is the value of pension wealth defined in the way described by Feldstein (19). By inversion this becomes

$$C_t = (1-\lambda)\alpha + \beta_0(1-\lambda)Y_t + \lambda C_{t-1} + \beta_2 W_{t-1} - \beta_2 \lambda W_{t-2} \\ + \beta_2 PW_t - \beta_2 \lambda PW_{t-1} + (u_t - \lambda u_{t-1}) \quad (4.5.2.)$$

This is remarkably similar to the inversion of (2.4.15.) and, in the absence of serial correlation in the error term, Feldstein and Fane point out an additional estimation difficulty, in that (4.5.2.) has more coefficients than basic parameters, which are therefore over-identified. As mentioned earlier, OLS will yield biased estimates with small samples, owing to the presence of the lagged dependent variable. An accepted procedure with models of this type is to apply instrumental variables estimation procedures, following the suggestion of Levitan (40) and using lagged values of other explanatory variables as instruments. This though does not solve the problem of biasedness in small samples - it is used to its greatest advantage when the error term is serially correlated as it restores the asymptotic properties of the estimators. The problem of overidentification is alleviated by imposing restrictions on the coefficients λ , β_1 and β_2 in (4.5.2.) and minimising the sum of squared residuals with respect to the values chosen.

This is the method suggested by Feldstein and Fane who refer to it as 'constrained instrumental variables' estimation.

The problem with this method is that, in searching across three parameters, the procedures are likely to be computationally time consuming and therefore expensive.

Thus there are two problems to overcome - small sample bias and computational complexity. A convenient way around the estimation problem is to rearrange (4.5.2.) such that one has a transformed equation

$$C'_t = \alpha' + \beta'_0 Y_t + \beta_1 W'_{t-1} + \beta'_2 PW'_t + u_t \quad (4.5.3.)$$

where $C'_t = C_t - \lambda C_{t-1}$

$$W'_{t-1} = W_{t-1} - \lambda W_{t-2}$$

$$PW'_t = PW_t - \lambda PW_{t-1}$$

$$\alpha' = (1-\lambda)\alpha$$

$$\beta'_0 = (1-\lambda)\beta_0$$

$$u_t = u_t - \lambda u_{t-1} \sim N(0, \sigma_u^2) : E(u_i, u_j) = 0.$$

This is then estimated by OLS searching only over values of λ - these will be generalised least squares (GLS) estimates. Overidentification is no longer a problem, there is no lagged dependent variable and only one variable is involved in the search procedure. The cost incurred is that there will not be separate standard errors for λ , α and β_0 .

4.6 CONCLUSIONS

This chapter has reviewed some of the problems involved in estimating consumption functions based on a life cycle hypothesis of consumer decision making. The main conclusions are

- i) In the absence of any a priori information as to the form of the consumption function a number of mathematical forms will be estimated. If the non linear specifications do not perform any better than the linear form it will be assumed that the relationship is linear.
- ii) Although Evans argued that there seemed little justification for including wealth in the consumption function more recent research has suggested that an extension of the definition of wealth to include

both the components of capital gains and pension wealth is worth while.

iii) The approximation of expected income by a distributed lag on past income values can be achieved using both empirical and theoretical methods. The empirical estimation of the lag suggests that its mean length is extremely short. One of the models tested will therefore represent expected income by a one period lag on disposable income. Two theoretical lag structures will also be considered. The first will be the simplest form suggested by MBA where only current income is included in the estimated equation. In addition the transformed version of model incorporating an infinite geometrically declining lag structure, as given by (4.6.3.) will be estimated.

NOTES

1. This procedure is suggested by Houthakker and Taylor (31) p.8.
2. In the spirit of M-B-A assume that the individual neither receives nor leaves any bequest. Using the earlier notation this will give

$$k(L) = \int_0^L y(t) \exp \left(-\int_0^t \exp r(\tau) \right) dt - c_0 \int_0^L \exp(r(t) - \delta(t)) / \theta dt = 0$$

Define normal income as the constant income v_0 the discounted present value of which is equal to the discounted present value of lifetime income. This will give

$$v_0 \int_0^L \exp \left(-\int_0^t \exp r(\tau) d\tau \right) dt = c_0 \int_0^L \exp (r(t) - \delta(t)) / \theta dt$$

which solving for c_0 yields

$$c_0 = \chi v_0$$

where χ is fixed and depends on $r(\cdot)$, $\delta(\cdot)$ and the form of the utility function (see Modigliani and Brumberg (46) p.346).

3. From (4.1.1.) it can be seen that the rate at which lifetime resources are consumed depends on age. The special case will be achieved where $r(t) = \delta(t)$ in which case consumption is equal to $C(t) = (1/L)Y(T)$.

4. A typical result is

$$C_t = 41 + .57 Y_t^d + .18 Y_{t-1}^d + .024 W_{t-1} - .12 G_t$$

(.06) (.04) (.008) (2.21)

$$+ .49 RE_t + 2.99 U_t \quad \text{DW} = 2.03$$

$$(.10) \quad (.89) \quad \text{SSR} = 2209$$

where RE_t is defined net of depreciation and the unemployment rate U_t is included to guard against the estimated effect of retained earnings being biased by spurious correlation with cyclical conditions.

5. A typical result is

$$C_t = 17.8 + .77 \frac{.55}{1-.45L} Y_t^d + .019 G_t + .254 RE_t$$

(5.6) (.02) (.007) (.127)

$$+ .017 W_{t-1} \quad \text{SSR} = 65.66$$

(.008)

6. Recent literature reviews (i.e. Timbrell (57)) completely ignore this problem.

7. The data used are Blue Book estimates of disposable income and consumers' expenditure. These are deflated by a price index of consumer goods and services and adjusted for population changes. A major weakness of the results is that they include data for the years 1939-45 - thus some of the long lags on income at the beginning of the sample period depend on data which is unreliable. Nevertheless the problems involved in attempting to justify omitting the

war years altogether are probably no less than including them.

8. Friedman (23) included all lags that explained at least .1% - for the purposes of this exercise 1% is thought to be quite adequate.

9. p.301.

CHAPTER FIVE

EMPIRICAL RESULTS

5.1 INTRODUCTION

In Chapter 4 the methods and problems associated with estimating consumption functions including wealth were described and discussed. The main empirical findings based on the procedures outlined are now to be presented. Before proceeding it should be mentioned that the equations contained in this chapter are not the total number estimated. Where a slight change in the specification of an estimated equation only marginally affects parameter estimates the full equation will not normally be reported.

5.2 THE DATA

The following variables are included in the regressions undertaken -- the data source appears in brackets after the definition except where a detailed description is necessary.

- i) Y_t^d - personal disposable income in year t (National Income and Expenditure Blue Book).
- ii) C_t - consumer's expenditure in year t (as (i)).
- iii) RE_t - retained earnings of companies in year t (as (i)).
- iv) W_t - net worth of the personal sector at the end of year t.

A long series, from any one source, does not exist. It was therefore necessary to combine the shorter period estimates of Langley (38) (39),

Lydall and Tipping (41), Revell (50) and Inland Revenue Statistics. Except for Revell the data refers only to wealth holding in G.B. As the Revell data separates out the wealth held by those in Northern Ireland it was assumed that the residents of this country always held the same average proportion of total wealth as suggested by Revell and the other data was updated accordingly. It was also fortunate that all the short data periods overlapped by one or two years such that the consistency of estimates could be examined. Different studies were clearly unlikely to give the same estimates for the same year even though they were all based on Estate Duty Statistics. The mortality multipliers, and exemption limits were all different. To produce a consistent series the figures were presented as Inland Revenue equivalents, brought about simply by scaling the other estimates. This method essentially assumes that all studies are correct in estimating the proportionate changes in total wealth from year to year but only the Inland Revenue estimate its total level correctly.

v) WF_t - a measure of W_t used by Feldstein and Fane (22).

The 1958 figure in Revell's series is used as a benchmark and year to year changes in net worth are added and subtracted. These annual increments are

- a) savings net of depreciation and the excess of capital taxes minus capital transfers
- b) appreciation of fixed assets owned by the personal sector
- c) net change in the market value of equities due to their price changes
- d) net change in the market value of government bonds due to their price changes.

(c) and (d) both exclude new issues.

vi) SPW_t - the discovered present value of anticipated receipts in the form of the flat rate state pension, evaluated at the beginning of period t . This includes both the wealth of workers and pensioners and is therefore different from Feldstein's measure in that he excludes the wealth of pensioners, presumably because the use of disposable income in the consumption function includes pension payments to those retired. It does seem rather strange though that one has a wealth theory of consumption and then does not allow the consumer who is not working to run down his wealth, even if it is in the form of a state pension where the rate of depletion is exogenously determined. To reconcile this with the use of disposable income is, in theory, not difficult. The first argument is that disposable income with or without some form of lag structure is strictly speaking only a proxy for the discounted present value of expected future net labour income in addition to this period's labour income. An alternative procedure is to subtract state pension benefits from disposable income. The problems involved in doing this are shortly to be reviewed.

vii) NPW_t - SPW_t less the discounted present value of state pension contributions.

An algorithm used to calculate the values of these two variables and the sources of data are described in Appendix I to this chapter.

A point that ought to be discussed at this juncture is how does one choose between the two specifications of state pension wealth. As pension contributions are included in disposable income it might be argued that the present value of expected contributions is included in an expected net income measure based on disposable income. On the other hand personal wealth is normally defined in net worth terms and in the case of pensions this is the individual's accrued right to a pension .i.e.

the pension scheme creates for each individual an amount of wealth equal to the difference between the present value of anticipated benefits and contributions. With this definition the pension contribution in period t is included in the present value of contributions in period t . It therefore probably ought to be added back into disposable income unless one is again to make the assumption that disposable income is simply a proxy for labour income, net of income taxation.

viii) PPW_t - the private sector equivalent of NPW_t . Owing to the diversification of private pension provision the data necessary to calculate such a variable is not currently available. In the Diamond Report (16) it was suggested that the national balance sheet totals of assets of superannuation funds and the pension business of the life assurance funds would at least give an overall figure for accrued rights in the form of private pensions. This would not be the case, because many schemes in which pensions are fixed in relation to salary near retirement are in deficiency as a result of recent rates of inflation and many large unfunded schemes would not be included. The Diamond Report concluded that the only way the necessary data could be collected would be by using sample survey methods and the Government Actuaries Department have agreed to investigate this possibility. Owing to this lack of suitable data for the purposes of this investigation a proxy variable is to be used. It is hoped that the growth of the assets of private and public superannuation funds will reflect the growth of accrued rights in private pension funds. In the Diamond Report it was suggested that in 1972 the value of accrued rights was about £20,000 million, whilst the value of the assets of superannuation funds was £11,447 million. The proxy variable may be a 50% underestimate.

The data for this series comes from Revell and Financial Statistics.

ix) U_t - millions unemployed, excluding school leavers and students, in year t (Economic Trends).

x) BEN_t - aggregate payments in state retirement pensions in year t . If disposable income is to be adjusted to net out pension payments to the retired this is the variable one would have to use. Unfortunately for post 1961 it will be an inaccurate reflection of payments in the form of flat rate pension since it will also include a small amount reflecting graduated pension payments.

xi) CON_t - aggregate receipts in the form of flat rate national insurance contributions in year t . This is the sum of employer and employee contributions and although one can use relative stamp values to determine the aggregate contributions of the employed the component of this contribution which can be thought of as a pension contribution cannot be separated out. Again, where an adjustment in disposable income is required this will only be an approximation to the desired adjustment.

xii) LPA_t - the labour force participation of the aged in period t . This variable reflects retirement trends and is defined as the ratio of those in work who are eligible to retire on the basis of age to the total population eligible to retire.

The data used to calculate the above three series came from the Annual Abstract of Statistics and the ILO Yearbook of Labour Statistics.

All the variables measured in money values are in 1971. £ millions adjusted for population growth.

5.3 CONSUMPTION FUNCTIONS INCLUDING STATE PENSION WEALTH

Of the methods considered in the previous chapter for estimating expected income three are going to be considered; the naive hypothesis where only current disposable income is included in the estimated equation, the empirically justified Feldstein specification with a one period lag and the geometrically declining infinite lag. Unless otherwise stated all the regressions use annual data for the period 1949-73 and the estimates are based on the ordinary least squares (OLS) technique.

i) The naive hypothesis - estimates based on this hypothesis using both the gross and net measures of state pension wealth are given by equations (1) and (2). As one would expect the marginal propensity to consume disposable income is of the order of .75 - .80. Net worth is an insignificant determinant of consumption as is gross state pension worth. The marginal propensity to consume out of net pension wealth is significant and negative. The coefficient of determination is high, the standard errors are low, being less than 1% the mean value of the dependent variable and with $d_u(5\%) = 1.66$ there is no evidence of a serious serial correlation problem.

ii) The Feldstein specification - estimates based on the use of a one period lag in disposable income are given in equations (3) and (4). Although the marginal propensity to consume lagged income is insignificant the other parameter estimates and the overall fit of the functions are not adversely affected although with $d_u(5\%) = 1.77$ equation (3) does show evidence of possible positive serial correlation. The standard errors of equations (3) and (4) are marginally greater than those of equations (1) and (2) respectively and it might therefore be

$$(1) \quad C_t = 4807 + .759 Y_t^d - .003 W_{t-1} + .009 SPW_t$$

(355) (.036) (.004) (.008)

$\bar{R}^2 = .997$ SSR = 1267734
SE = 245.70 DW = 1.60

$$(2) \quad C_t = 4368 + .818 Y_t^d - .000 W_{t-1} - .016 NPW_t$$

(306) (.020) (.003) (.008)

$\bar{R}^2 = .998$ SSR = 1117255
SE = 230.66 DW = 1.67

$$(3) \quad C_t = 4797 + .755 Y_t^d + .009 Y_{t-1}^d - .005 W_{t-1} + .009 SPW_t$$

(363) (.038) (.022) (.007) (.009)

$\bar{R}^2 = .997$ SSR = 1256217
SE = 250.62 DW = 1.58

$$(4) \quad C_t = 4374 + .821 Y_t^d - .011 Y_{t-1}^d + .003 W_{t-1} - .018 NPW_t$$

(312) (.022) (.022) (.007) (.009)

$\bar{R}^2 = .998$ SSR = 1102782
SE = 234.82 DW = 1.77

$$(5) \quad C_t = 3511 + .819 \frac{(1 - .2) Y_t^d + .2 C_{t-1} - .002 (W_{t-1} - .2 W_{t-2}) - .004 (SPW_t - .2 SPW_{t-1})}{1 - .2L}$$

(309) (.029) (.004) (.007)

$\bar{R}^2 = .996$ SSR = 1170059 SE = 236.04 DW = 1.68

$$(6) \quad C_t = 3680 + .823 \frac{(1 - .14) Y_t^d + .14 C_{t-1} - .001 (W_{t-1} - .14 W_{t-2}) - .015 (NPW_t - .14 NPW_{t-1})}{1 - .14L}$$

(281) (.018) (.003) (.007)

$\bar{R}^2 = .997$ SSR = 993596 SE = 217.52 DW = 1.78

argued that the inclusion of the lagged income variable is unjustifiable.

iii) The geometrically declining infinite lag - the estimated equation here is of the form given by (4.6.3.), searching for a value of λ which minimises the sum of squared residuals, given that \bar{R}^2 in the maximum region is high and constant. A search was first done using values of λ between 0 and .9 at .1 intervals. Even using this first approximation it can be seen from equation (5) that the coefficient on gross state pension wealth remains insignificant although its sign is different from equations (1) and (3). The analogous equation using net pension worth confirmed the results of equations (2) and (4) producing a marginal propensity to consume pension wealth of $-.016$ with a standard error of $.008$. Note that in both cases the lag coefficients are small, indicating a short lag.

The next stage in the analysis is to search around a finer grid of λ values in the region of the approximate maximum. This is only done using the specification including the significant net pension worth variable. Values of λ between .01 and .19 constitute the search region. Before proceeding consider the plot of λ against the sum of squared residuals given in Figure 5.3.1. It will be noticed that for values of λ between 0 and .4 the curve is fairly flat, the sum of squared residuals not changing a great deal for changes in λ . The next search is based on the assumption that the best fitting value of λ is in the region of .1. The possibility has to be admitted though that there could be a local maximum anywhere in the region where $\lambda < .4$. Fortunately, the uniform shape of the graph and the evidence from previous regressions suggest that this is unlikely to be the case.

From equation (6) a value of $\lambda = .14$ minimises the sum of squared residuals whilst Figure 5.3.2. tends to confirm that there is

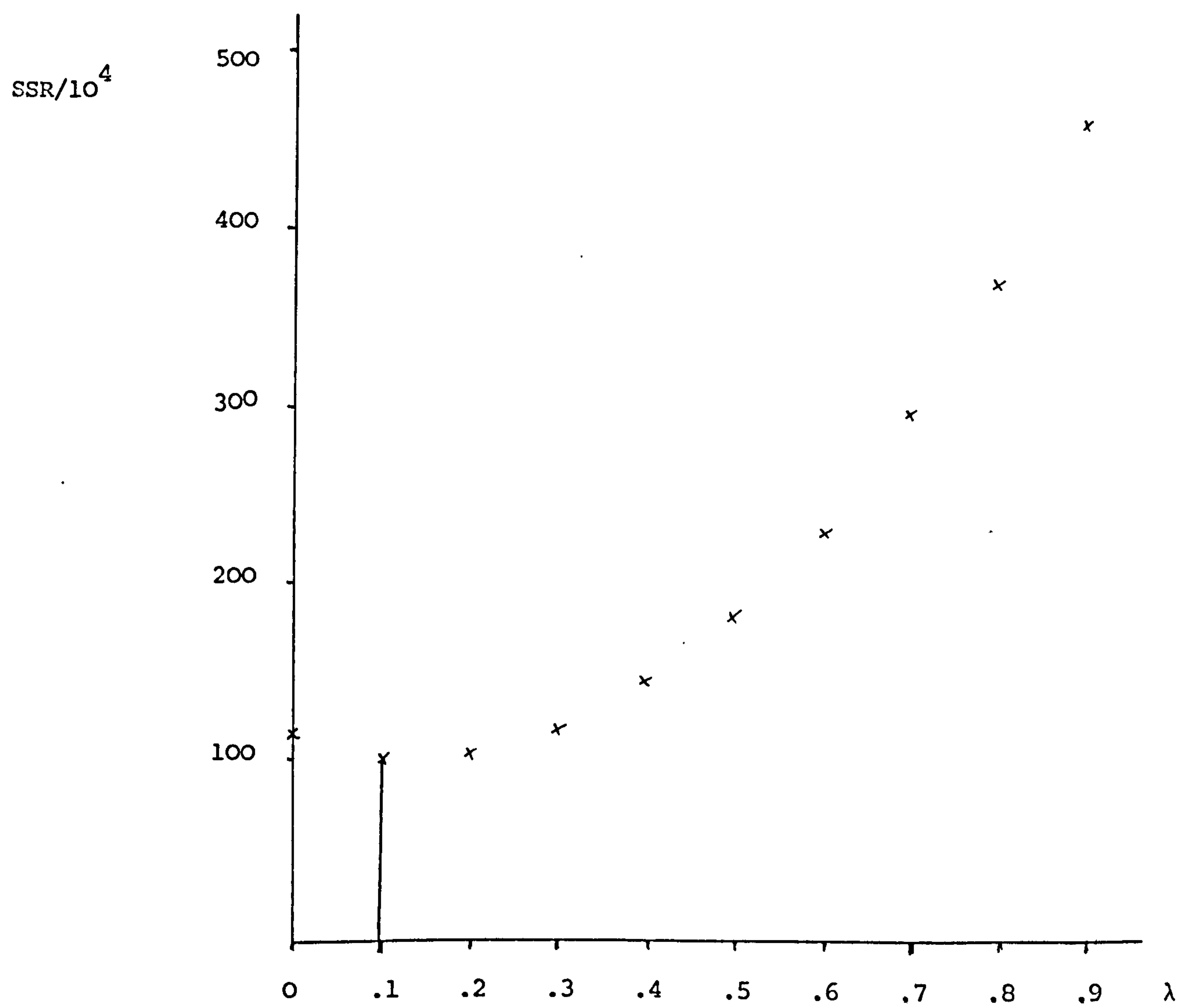


Figure 5.3.1

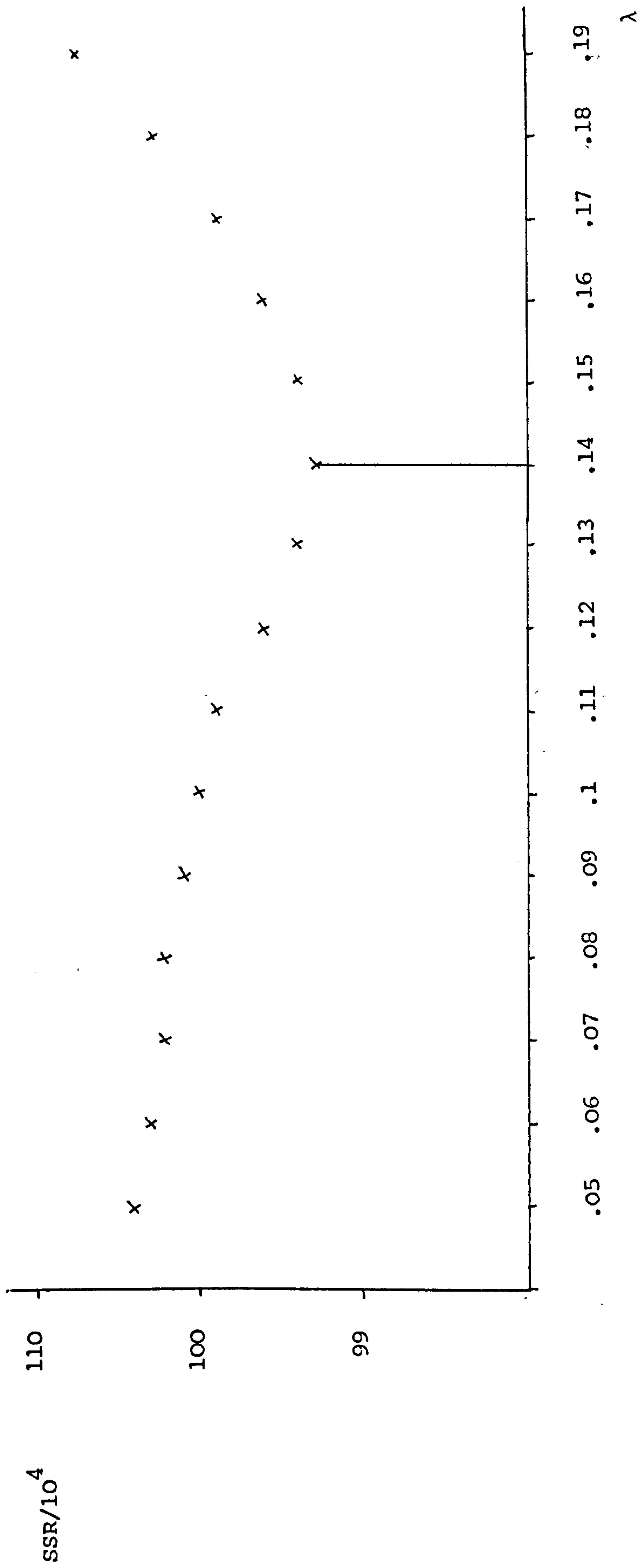


Figure 5.3.2

unlikely to be a better fitting value of λ in the search region. The coefficient on net pension worth remains significant, net worth is insignificant and there is no evidence of serial correlation. It should be noted that as equations (5) and (6) are estimated without a lagged dependent variable in the set of independent variables the Durbin-Watson statistic is not biased towards 2. A low value of lag coefficient justifies the use of the one period lag on disposable income to approximate expected income.

Having examined the consistency of the three hypotheses regarding the specification of expected income three other aspects of the analysis can be examined, the appropriateness of the linear specifications, omitted variables, the definition of the income and net worth variables.

Table 5.3.1. summarises the performance of the linear and non linear equations as outlined in the previous chapter. As the coefficients of determination are not greatly different it can be safely concluded that the performance of the linear specification is certainly no worse than the non linear counterparts. The main features of the non linear equations are that the constant terms, current income and net pension worth, are always significant, lagged income is significant in the log-reciprocal specification, net worth is significant with both the semi-logarithmic and log reciprocal specifications whilst gross pension worth is always insignificant.

Two additional variables were also introduced into the linear equations. Following Feldstein retained earnings was included in the regression and found to be an insignificant determinant of consumption. A time trend variable tended to reduce slightly the significance of all independent variables but the time trend itself

Table 5.3.1

Comparison of linear and non linear specifications based on value of \bar{R}^2 .

Pension Wealth Definition	Linear	Semi-logarithmic	Double-logarithmic	Log-reciprocal
SPW	.997	.994	.997	.989
NPW	.998	.994	.996	.990

was insignificant.

It will be remembered that Feldstein and Fane found that for the U.K. the marginal propensity to consume net worth took a significant value of about .017. The above analysis suggests that this variable is insignificant. Equations (3) and (4) were re-estimated using the Feldstein measure of net worth. Although the coefficient on wealth is in both cases increased, to .006 and .008 respectively, they remain insignificant. The only major effect is to reduce the coefficient on net pension worth to $-.024$ with a standard error of .011. In each case the coefficient of determination is .002 lower. It must therefore be concluded that the choice of net worth variable is not crucial.

Lastly consider the modification of equations (3) and (4) to exclude pension benefits paid to those currently retired from disposable income when using the gross definition of pension worth and the additional adding in of current contributions when using the net definition. The only impact this has is when using the net definition in that the coefficient on net pension worth rises to $-.013$ whilst its standard error remains unchanged. Disposable income appears to be homogenous with respect to its components. Given the difficulties involved in using aggregate benefits and contributions as defined earlier in this context there is probably little danger in continuing with the Blue Book definitions of disposable income.

5.4 CONSUMPTION FUNCTIONS INCLUDING PRIVATE PENSION WEALTH

In this section private pension wealth, defined as in section 5.2, is introduced into the equations estimated in the previous section. The data now refers to the period 1958-73.

The first estimates, given by equations (7), (8), (9) and (10) relate to the introduction of the private pension wealth variable into equations (1), (2), (3) and (4). The interesting features of these results are the general increase in the magnitude and significance of the net worth and gross state pension wealth variables. The net pension wealth variable is smaller and insignificant whilst the private pension wealth variable only approaches marginal significance in equation (7). A significant negative coefficient on net worth is certainly a little surprising since it implies individuals are induced to save as a result of accumulating wealth. The case where expected income is approximated by an infinite geometric lag is considered in equations (11) to (12). The first approximations of the lag coefficients are .3 and .2 respectively compared with .2 and .1 in the earlier equations (5) and (6). Again the general features of equations (7) to (10) are revealed and, for this reason, a more precise value of the lag coefficient has not been estimated. The above differences between the set of equations can be explained by any one or a combination of three things; an omitted variable problem with the equations where private pension wealth is excluded, general collinearity between the wealth variables which is borne out by the correlation matrix, and a change in the relationship between consumption and wealth as between the periods 1949-73 and 1958-73.

$$(7) \quad C_t = 2526 + .818 Y_t^d - .035 W_{t-1} + .014 SPW_t + .205 PPW_t \quad \bar{R}^2 = .996 \quad SSR = 452058$$

$$(1154) \quad (.046) \quad (.013) \quad (.009) \quad (.144) \quad SE = 203.72 \quad DW = 2.10$$

$$(8) \quad C_t = 2650 + .838 Y_t^d - .023 W_{t-1} - .006 NPW_t + .211 PPW_t \quad \bar{R}^2 + .995 \quad SSR = 544203$$

$$(1545) \quad (.049) \quad (.020) \quad (.012) \quad (.202) \quad SE = 222.42 \quad DW = 1.67$$

$$(9) \quad C_t = 2487 + .800 Y_t^d + .035 Y_{t-1}^d - .035 W_{t-1} + .012 SPW_t + .189 PPW_t$$

$$(1220) \quad (.095) \quad (.160) \quad (.014) \quad (.013) \quad (.167)$$

$$\bar{R}^2 = .995 \quad SSR = 449842 \quad SE = 212.09 \quad DW = 2.04$$

$$(10) \quad C_t = 3178 + .727 Y_t^d + .186 Y_{t-1}^d - .023 W_{t-1} - .012 NPW_t + .004 PPW_t$$

$$(1507) \quad (.087) \quad (.124) \quad (.019) \quad (.012) \quad (.236)$$

$$\bar{R}^2 = .995 \quad SSR = 444717 \quad SE = 210.88 \quad DW = 1.74$$

$$(11) \quad C_t = 1971 + .956 \frac{(1 - .3) Y_t^d + .3 C_{t-1} - .034 (W_{t-1} - .3 W_{t-2}) - .006 (SPW_t - .3 SPW_{t-1})}{1 - .3L}$$

$$(758) \quad (.041) \quad (.014) \quad (.009)$$

$$+ .020 (PPW_t - .3 PPW_{t-1})$$

$$(.088)$$

$$\bar{R}^2 = .993 \quad SSR = 400222 \quad SE = 190.74 \quad DW = 1.74$$

$$(12) \quad C_t = 2417 + .909 \frac{(1 - .2) Y_t^d}{1 - .2L} + .2 C_{t-1} - .025 (W_{t-1} - .2 W_{t-2}) - .008 (NPW_t - .2 NPW_{t-1})$$

(858) (.042) (.013) (.008)

$$+ .034 (PPE_t - .2 PPW_{t-1})$$

(.102)

$$\bar{R}^2 = .995 \quad SSR = 385189 \quad SE = 187.13 \quad DW = 1.90$$

$$(13) \quad C_t = 3884 + .818 Y_t^d - .026 W_{t-1} + .018 SPW_t$$

(673) (.048) (.012) (.009)

$\bar{R}^2 = .995 \quad SSR = 535008$
 $SE = 211.15 \quad DW = 1.84$

$$(14) \quad C_t = 4088 + .840 Y_t^d - .006 W_{t-1} - .014 NPW_t$$

(703) (.049) (.013) (.009)

$\bar{R}^2 = .995 \quad SSR = 598078$
 $SE = 223.25 \quad DW = 1.62$

$$(15) \quad C_t = 3433 + .760 Y_t^d + .112 Y_{t-1}^d - .031 W_{t-1} + .011 SPW_t$$

(904) (.090) (.147) (.014) (.013)

$\bar{R}^2 = .995 \quad SSR = 507984$
 $SE = 214.90 \quad DW = 1.81$

$$(16) \quad C_t = 3200 + .727 Y_t^d + .187 Y_{t-1}^d - .022 W_{t-1} - .013 NPW_t$$

(780) (.073) (.096) (.014) (.008)

$\bar{R}^2 = .996 \quad SSR = 444730$
 $SE = 201.07 \quad DW = 1.74$

To test for the possibility of a different relationship over the shorter period equations (1) to (4) were re-estimated using data only for 1958-73. These are the estimated equations (13) to (18). It can be seen that the coefficient on the net worth variable remains fairly large, negative and significant except in equation (14). Where lagged income is omitted gross state pension wealth has a significant and positive effect on consumption. The marginal propensity to consume net pension worth is of the earlier magnitude but less significant. The evidence suggests a greater degree of serial correlation than in earlier equations, the critical values of the Durbin-Watson statistic being $d_u(5\%) = 1.73$ for equations (13) and (14) and $\hat{d}_u(5\%) = 1.93$ for equations (15) and (16). Equations (17) and (18) suggest that these results would be supported by the infinite lag model if λ were estimated to two decimal places. Comparing these with equations (7) to (10) shows that the introduction of private pension wealth only has a marginal effect, its most significant role being that of raising the coefficient on net state pension wealth. Given the insignificance of private pension wealth this is exactly what one would expect when there is a multicollinearity problem.

5.5 RETIREMENT BEHAVIOUR

The preceding theoretical analysis has suggested that pension schemes may have an impact on savings part of which operates indirectly through induced changes in the retirement date. To quantify this effect a variable, the labour force participation of the aged, has been included in equations (3), (4), (9) and (10). As changes in the retirement date are induced the appropriate estimation procedure is two stage least

$$(17) \quad C_t = 2082 + .840 \frac{(1 - .2) Y_t^d}{1 - .2L} + .2 C_{t-1} - .035 (W_{t-1} - .2 W_{t-2}) - .005 (SPW_t - .2 SPW_{t-1})$$

(562) (.038) (.013) (.007)

$$\bar{R}^2 = .993 \quad SSR = 402194 \quad SE = 183.06 \quad DW = 1.69$$

$$(18) \quad C_t = 2626 + .914 \frac{(1 - .2) Y_t^d}{1 - .2L} + .2 C_{t-1} - .025 (W_{t-1} - .2 W_{t-2}) - .008 (NPW_t - .2 NPW_{t-1})$$

(561) (.039) (.013) (.008)

$$\bar{R}^2 = .995 \quad SSR = 389067 \quad SE = 180.06 \quad DW = 1.80$$

$$(19) \quad LPA_t = .132 + .000013 BEN_t^* - .017 u_t$$

(.004) (.000029) (.015) $\bar{R}^2 = .205$ SE = .006 DW = .30

$$(20) \quad LPA_t = .132 + .000019 CON_t^* - .028 u_t$$

(.004) (.000046) (.013) $\bar{R}^2 = .203$ SE = .006 DW = .38

* indicates 'per aged member of the population'.

$$(21) \quad LPA_t = .167 - .000002 Y_t^d + 0 W_{t-1} + 0 SPW_t + .000001 PPW_t \quad \bar{R}^2 = .865 \quad SE = .002 \quad DW = 1.51$$

(.013) (.000001) (0) (0) (.000002)

$$(22) \quad LPA_t = .174 - .000002 Y_t^d + 0 W_{t-1} + 0 NPW_t + 0 PPW_t \quad \bar{R}^2 = .847 \quad SE = .002 \quad DW = 1.30$$

(.017) (.000001) (0) (0) (0)

squares with labour force participation endogenous. Although one would expect the coefficient on this variable to be negative this is not the case in all of the equations, but the coefficient is never significant.

To learn a little more a series of equations were estimated with labour force participation of the aged as the dependent variable. The results are given in equations (19), (20), (21) and (22). Examination of the labour force participation series reveals that it has remained constant at around 13% and this is reflected in the regressions. Apart from the constant term only the income level and unemployment seem to have any effect. The negative coefficient on unemployment is not at all surprising since their labour force participation will reflect the degree to which unemployment falls on those over retirement age. As for the negative coefficient on income this implies a predominance of the income effect of a change in income on the demand for leisure in the form of retirement. The size of the coefficient implies that a 1% point decrease in the labour force participation of the aged requires a doubling of national income.

5.6 PENSIONS AND PERSONAL SAVINGS

To determine the impact of pension schemes on personal savings consider the following simple model. Savings in any period, t , in the absence of a pension arrangement is given by

$$S_t = -a + (1 - b_0)(1 - u_t) Y_t \quad (5.6.1.)$$

where b_0 is the marginal propensity to consume, u_t is the standard rate of income tax and Y_t is gross income. If it is assumed that the individual responds to changes in gross state pension wealth then savings with a

pension plan, given that contributions are tax deductible, is given by

$$S'_t = -a + (1-b_0)(1-u_t)(Y_t - c_t) - b_1 SPW_t - b_2 PPW_t \quad (5.6.2.)$$

where c_t is the pension contribution in period t . The effect on savings of introducing the plans is given by subtracting (5.6.1.) from (5.6.2.) which yields

$$\Delta S = -(1-b_0)(1-u_t)c_t - b_1 SPW - b_2 PPW \quad (5.6.3.)$$

The effect of changing pension parameters is given by differentiating (5.6.2.) with respect to the pension parameters

$$dS = -(1-b_0)(1-u_t)dc_t - b_1 dSPW - b_2 dPPW \quad (5.6.4.)$$

An analogous calculation assuming that individuals react to changes in net state pension worth, remembering that contributions are now included in the wealth variable yields

$$\Delta S = -b_1 NPW - b_2 PPW \quad (5.6.5.)$$

and

$$dS = -b_1 dNPW - b_2 dPPW \quad (5.6.6.)$$

These results can be used to estimate the quantitative impact of pension schemes on personal savings.

Consider firstly the relationship between state pension and savings over the period 1949-73. From (1), (3) and (5) the coefficient on gross state pension wealth is insignificant - savings will only therefore be affected as a result of the reduction in disposable income attributable to pension contributions. The three estimated equations imply marginal propensities to consume disposable income of

.759, .764, and .819 - estimates will be based on an assumed value of .775. The rate of tax used is the standard rate of income tax whilst individual contributions are the employee's share of aggregate contributions to the National Insurance Fund, the relative shares being based on stamp values. Although this will clearly be an over-estimate of pension contributions this is to be used in the analysis to follow. Data relating to aggregate contributions and the components of the National Insurance stamp are published in Social Security Statistics.

Using the gross definition of state pension wealth the impact of the existence of the state pension scheme on personal savings is given in Column 1 of Table 5.6.1. It is clear that the overall effect is to reduce savings but when the magnitude of the change is compared with the figures in Column 5 of the table, which are aggregate contributions to the National Insurance Fund, it can be seen that the substitution is far from perfect. Total savings, including pension savings, clearly increases but as a large proportion of this is paid to the Government, and is thus passed on in the form of concurrent benefits, it cannot in general be assumed that the rate of capital accumulation will be higher.

When the net definition of social security wealth is used only the marginal propensity to consume out of this form of wealth has to be considered. The savings changes in Column 2 of Table 5.6.1 are based on the value of .015 given by equation (6). When compared with actual savings, given in column 4, it can be seen that, for most of the period, saving would have been 30-35% lower in the absence of the state pension scheme but that the degree of complementarity has been increasing over time.

The effect of changing pension parameters can be analysed by considering the effect on savings of a 10% increase in pension benefits and contributions.

Table 5.6.1

The Effect of State Pensions on Personal Savings: 1949-73 (Current £ millions)

	1 *	2	3	4	5
1949	(-) 34.15	154.97		78	409
1950	34.32	155.44		-64	411
1951	35.45	219.48		154	426
1952	28.89	222.46		376	455
1953	31.18	242.65		455	491
1954	31.76	246.01		382	500
1955	38.46	249.60		484	582
1956	39.72	250.35		756	601
1957	39.79	251.10		728	601
1958	46.25	284.91	(-) 1005.23	642	709
1959	50.38	300.72	1008.23	803	724
1960	50.93	301.66	1011.28	1297	729
1961	54.10	447.76	1165.89	1690	784
1962	54.24	507.55	1169.39	1515	786
1963	61.76	528.09	1379.84	1626	895
1964	64.87	538.08	1384.00	1864	940
1965	74.45	566.07	1652.58	2184	1196
1966	73.32	600.85	1657.53	2384	1191
1967	78.98	559.02	1867.97	2340	1271
1968	90.29	623.11	1873.57	2337	1427
1969	91.02	610.90	2088.00	2609	1449
1970	93.83	603.87	2094.12	3150	1494
1971	98.71	900.99	2530.76	3416	1515
1972	96.35	904.56	2540.77	4435	1567
1973	105.77	1440.521	3183.93	5727	1720

* These estimates are based on the assumption (p. 138) that pension contributions are tax deductible - this has not been the case for the latter part of the data period. Nevertheless, the changing of this assumption will not affect the implications of these results.

Using the gross definition of pension wealth the lack of significance of this variable implies that a 10% increase in benefits will have no effect on savings. Evaluated in 1971 a 10% increase in contributions (which would be from 67.2p to 73.9p per week) would depress savings by £4.3 millions or .13% but bring in for the Government £31.5 millions.

Using the net definition of social security wealth it will be assumed that a 10% change in benefits will increase gross state pension wealth by 10% whilst a 10% increase in contributions will increase the difference between the gross and net definitions, i.e., the present value of anticipated liabilities, by 10%. A 10% increase in benefits (which would be from £9.70 to £10.87 per married couple) would cost the Government £182 millions but would bring about an increase in savings of £210.9 millions or 6.2%. If contributions were increased by 10% this would depress savings by £120.8 millions or 3.5%.

It will be remembered from Chapter 2 that in the context of the model described it was found that both increases in state pension benefits and contributions led to a reduction in work period and therefore aggregate savings. As regards the increase in benefits this is not supported by the evidence - gross state pension wealth has no effect on savings whilst changes in net state pension wealth work in the opposite direction. As regards the increasing of contributions the theoretical findings are supported although the magnitude of the effect is vastly different as between definitions.

Although Feldstein felt his results were successful, in that they demonstrated that using either definition of social security wealth, given that they had the same sign, had similar implications for the effect of the existence of a scheme on aggregate savings, this

does not mean that the above results cannot be viewed as at least being moderately successful. Indeed the one thing they do show is the important role played by pension contributions. This is seen in that it is only when these are included in the pension wealth variable that it becomes significant and it is only changes in contributions which have a consistent effect on savings.

For the shorter period 1958-73 where private pension worth was introduced the empirical results were not particularly encouraging. Not only was private pension worth generally insignificant, but also the significance of net state pension worth fell although this could be attributed to collinearity between the pension wealth variables. One feature of the results which is interesting is the increase in significance of the gross state pension wealth variable. In the one case where it is clearly significant, equation (13), the value of the marginal propensity to consume is .018. The saving changes in Column 3 of Table 5.6.1. are based on this figure and to these have to be added the reduction in savings due to the reduction in disposable income attributed to contributions. With a marginal propensity to consume disposable income of .818 the value of this will be about 95% of the figure in Column 1. Although the total reduction in savings is clearly very large these figures have to be treated with extreme caution in that they are based on only 15 observations, there is only a single case when the coefficient on gross state pension wealth is significant and the equation with which this is the case is the simplest form of consumption function estimated, though not necessarily the worst fitting. Changes in contributions and benefits will have the following impact. A 10% increase in pension contributions would have an impact almost the same as that in the previous section. A 10% increase in pension

benefits, which costs the Government £182 millions, will further reduce personal savings by £262.9 millions or 7.7%.

5.7 CONCLUSIONS.

The results of this regression analysis are rather disappointing in that the relationship between pensions and savings is not particularly significant. Nevertheless one might take solace in the fact that for the two data periods considered the parameter estimates, using differing specifications of the consumption function, exhibit no evidence of instability, although this is not the case when comparing the whole period with a sample thereof.

The major findings may be summarised as follows:

i) A non linear specification of the consumption function appears to be in no way superior to a linear alternative.

ii) Three specifications of expected income have been used and no one appears to be preferable to the other two. The use of a short lag on current income to approximate expected income is shown to be appropriate.

iii) For the period 1949-73 the consumption function estimates reveal that net worth in general and gross state pension worth are insignificant determinants of consumption whilst net state pension worth has a significant depressing effect on consumption with a coefficient of around $-.015$. This is thought to reflect the importance of pension contributions as a determinant of consumer behaviour.

The inclusion of the additional variables retained earnings of companies, a time trend and a retirement variable along with the redefinition of the net worth and income variables has no significant impact on these results.

iv) For the period 1958-73 the market value of the assets of superannuation funds, used as a proxy for private pension wealth, was introduced into the consumption function. This results in net worth becoming significant with a negative coefficient, the degree of significance of gross state pension worth increasing (its only significant value was +.018) whilst the significance of net state pension worth is reduced. Private pension worth appears to be insignificant and the above results reoccurred with it omitted from the estimated equations.

v) The regressions with labour force participation of the aged as the dependent variable imply that there has been no induced retirement, or at least none that can be explained by pension changes.

vi) The effect that the existence of a pension scheme has on personal savings is determined by the specification of the pension wealth variables. Using the net definition for the longer period the evidence suggests that in the absence of a state pension scheme personal savings would have been 30-35% lower. As the gross definition is insignificant savings is only affected due to the reduction in disposable income brought about by the pension contributions. In recent years the impact of this has been small relative to the level of personal savings. For the shorter period the significant positive value of the coefficient on gross state pension worth implies that personal savings has been between 50-100% lower than it would otherwise have been.

vii) For the longer period, using the gross definition of social security wealth, a 10% increase in pension benefits, in 1971, has no impact on savings whilst for the shorter period it reduces savings by £262.9 millions. Such a change would cost the Government

£182 millions. A 10% increase in contributions would bring in £31.5 millions for the Government and depress savings by £4.3 millions. Using the net definition of social security wealth over the longer period the benefit increase implies an increase in savings of £210.9 millions whilst the contribution increase would depress savings by £120.8 millions.

APPENDIX I

A TIME SERIES OF ANTICIPATED STATE PENSION WEALTH IN THE U.K.

1. INTRODUCTION

The Diamond Report contained a Government Actuaries Department estimate that the value of accrued rights in the form of state pensions, excluding the graduated scheme, in 1975 was £145,111 millions. A series for 1963-7 has also been estimated by Atkinson (8) but as the purpose of this calculation was to adjust wealth distributions to include pension wealth no aggregate figures were produced. Both calculations were based on the assumption that the accrued right which was a proportion of the individual's total right, the proportion being given by the ratio of the present values of contributions to date and total contributions. Assumptions also had to be made about the growth rates of benefits and contributions to calculate the present values of lifetime benefits and contributions, which were weighted by survival probabilities.

Feldstein (19) observed that over his data period pension receipts varied over time with the per capita disposable income of a particular class of worker around a mean value of .41. To arrive at a value of gross pension wealth Feldstein made projections of the discounted present value of disposable income, weighted by survival probabilities, for all classes of workers and multiplied this figure by .41. Pensions are financed by a proportional tax on disposable income which it is assumed will remain constant. The present value of liabilities are then estimated as above. Net pension wealth is the

difference between these two figures.

The aim of this Appendix is to describe a method of producing a time series of pension wealth for the U.K. The method used will be similar to Feldstein's in that gross and net pension wealth will both be estimated whilst it is much closer to Atkinson's and the Government Actuaries' in that it is to be based on actual values of pension benefits and contributions through time.

2. ASSUMPTIONS

A.A.1.) All persons who for any part of their working lives are unemployed pay national insurance contributions during this period and maintain full pension rights.

A.A.2.) On retirement married women have no pension rights whilst single women have full pension rights.

A.A.3.) There is no divorce or remarriage.

A.A.4.) Pensioners anticipate no change in marital status.

A.A.5.) Of all the men who retire the proportion k will be married - the proportion k of all men not retired will be allocated the married couples pension. Of all the women who retire the proportion k^* will be married - the proportion $(1-k^*)$ of all women not retired will be allocated no pension.

3. THE ALGORITHM

For a single person aged a at time t , who anticipates no change in status, the value of the state pension on retirement is given by

$$p_R(a, t) = p_t (1 + g)^{R-a} \quad (\text{A.3.1.})$$

where p is the annual pension of a single person, R is the retirement age and g is the annual rate of growth of p . At age R the expected value of total pension receipts will be given by

$$\sum_{n=R}^{\bar{L}} \left(\ell(n) / \ell(R) \right) p_R (1+g/1+d)^{n-R} \quad (\text{A.3.2.})$$

where \bar{L} is the maximum length of life, $\ell(x)$ is the proportion of persons born who survive to age x and d is the rate of discount. At age a the pension promise has the value

$$\left(\ell(R) / \ell(a) \right) (1+d)^{a-R} \sum_{n=R}^{\bar{L}} \ell(n) / \ell(R) p_R (1+g/1+d)^{n-R} \quad (\text{A.3.3.})$$

Substituting (A.3.1.) into (A.3.3.) yields

$$\left(\ell(R) / \ell(a) \right) (1+d)^{a-R} \sum_{n=R}^{\bar{L}} \left(\ell(n) / \ell(R) \right) p_t (1+g)^{R-a} (1+g/1+d)^{n-R}$$

which for computational simplicity can be written

$$\left(p_t / \ell(a) \right) (1+g/1+d)^{R-a} \sum_{n=R}^{\bar{L}} \ell(n) (1+g/1+d)^{n-R} \quad (\text{A.3.4.})$$

This is the anticipated state pension wealth of a person aged a at time t . A similar calculation is done for single women and married couples substituting in the different pension values and making proportional adjustments to allow for changes in status. For existing

pensioners anticipated pension wealth is simply

$$\left(p_t / l(a) \right) \sum_{n=a}^{\bar{L}} l(n) (1+g/1+d)^{n-a} \quad (\text{A.3.5.})$$

Once these calculations have been completed for all ages within each pension group and multiplied by the working and retired population figures one has a figure for aggregate anticipated state pension wealth which has been denoted SPW.

An individual's liability is the discounted present value of outstanding contributions. For a single man this is given by

$$\left(c_t / l(a) \right) \sum_{n=a}^{(R-1)} l(n) (1+g/1+d)^{n-a} \quad (\text{A.3.6.})$$

where c_t is the annual contribution of a single man and g its rate of growth. Again this can be applied to all pension groups and all ages. Adjustments for change in status are made in a similar manner to those just outlined. This involves assuming that women who retire married have made no contributions which is clearly unrealistic.

If these figures are aggregated and subtracted from gross state pension wealth one arrives at a value for net pension wealth which has been denoted NPW.

4. THE DATA

The data relates to the period 1948-73 inclusive. The values of state pension benefits and contributions during the period are published in Social Security Statistics. Contributions are the National Insurance contributions and not the proportion that can be viewed solely as a pension contribution - the aggregate value

for liabilities might therefore be an overestimate.

The rate of growth of benefits is the constant annual rate at which benefits have grown over the period and this same growth rate is applied to contributions. It is assumed that individuals discount future receipts at the average rate of interest on 2½% Consols the data for this coming from Paish () and Financial Statistics: The average rate of growth has been approximately 6.5% per annum whilst the average rate of interest has been approximately 5.5%. From equations (1) (6) it can be seen that these two rates always appear in the form $(1+g/1+d)$ implying a net rate of discount of 1.01.

The retirement age, R , is the earliest date at which the individual is eligible to receive the state pension - that is 65 for men and 60 for women. Only the wealth of those over 25 is considered and the maximum length of lifetime, \tilde{L} , is assumed to be 99, the upper limit of accurate values of $l(x)$.

For intervals of five years values of $l(x)$ are available from the Annual Abstract of Statistics. Using the results reported by the Faculty of Actuaries () it can be shown that the survival probability function is smooth within these intervals and values of $l(x)$ within the intervals can safely be interpolated.

Data relating to the population structure and marital structure is contained in the Annual Abstract of Statistics, the ILO Yearbook of Labour Statistics and Social Security Statistics. The marital status of those who retire in any year is only available for a short period in the late 60's and early 70's - the average for those years was applied to the whole data period.

5. RESULTS

The estimated values of SPW and NPW are presented in Table A.5.1.

6. COMMENT

An important point to notice about these estimates is their magnitude. This is clearly seen if the net pension worth series is compared with the net worth series in Table A.5.1. For example, in 1971 the value of net worth was £114,982 millions - in the same year net pension worth was £60,006 millions or more than half the value of net worth in general.

Also, although Feldstein emphasises that calculations of this sort are not actuarially accurate an estimate of net pension worth in 1975, based on this series compares well with the Government Actuaries' calculation reported earlier given that from 1973 to 1975 the basic state pension rose 33%.

Table A.5.1

Aggregate Values of Gross and Net State Pension Wealth, and Net
Worth: 1948-73 (Current £. millions)

	SPW	NPW	NW
1948	28110	11202	29053
1949	28195	10331	29755
1950	28280	10363	29164
1951	32602	14632	32129
1952	35559	14831	33401
1953	35650	16177	29192
1954	35757	16401	32224
1955	44278	16640	31397
1956	44411	16690	33019
1957	44544	16740	35775
1958	55846	18994	40274
1959	56013	20048	46164
1960	56182	20111	52650
1961	64771	29851	55959
1962	64966	33837	59553
1963	76658	35206	64988
1964	76889	35872	73243
1965	91810	39938	76776
1966	92085	40057	78343
1967	103776	37268	85351
1968	104087	41541	89727
1969	116000	40727	93263
1970	116348	40258	98779
1971	140598	60066	114982
1972	141154	60304	141156
1973	176885	96035	167149

CONCLUDING COMMENTS

The aim of this thesis has been to investigate the relationship between state and private pensions and personal savings. A theoretical framework has been developed to facilitate the a priori determination of the relationship and an empirical investigation then undertaken using data for the U.K. in the postwar period. These two aspects of the thesis will firstly be summarised.

Initially, the standard life cycle model of individual accumulation is extended to include a flat rate pension scheme. The introduction of the scheme will have an impact on savings determined by a savings replacement effect, where pension saving is substituted for alternative forms personal saving, and an induced retirement effect, where savings is increased to finance a longer retirement period. The net effect on savings will depend on the rate of return to pension contributions, and therefore lifetime income, implied by the combination of the two effects. As membership of state pension schemes is usually compulsory there exists the possibility that the implied rate of return can be above or below the market rate, indeed it may even be negative, with the result that lifetime income may be higher or lower than prior to the introduction of the pension scheme. The impact on savings cannot therefore be determined at a theoretical level. Increasing pension contributions and benefits always lead to a reduction in work period and, under the usual assumptions, aggregate savings. These changes also induce early retirement.

Having analysed the state pension case the model is further modified to include private pension schemes, in particular a terminal salary scheme, membership of which is assumed to be compulsory, and a money purchase scheme, the joining of which is a voluntary decision. Again, for the reasons outlined above, the effect on personal savings of introducing the private schemes cannot be determined a priori. It is still possible though to say something about the effect of changes in pension contributions and benefits. When considering the increasing of pension contributions and state pension benefits, the outcomes are as in the state pension case alone. If attention is confined to increasing private pension benefit functions by constant absolute amounts then early retirement and a reduction in work period savings are the unambiguous outcomes. In the case of the money purchase scheme early retirement also implies a larger private pension fund.

The empirical estimation of the relationship has been attempted through the inclusion of pension wealth variables in the aggregate life cycle consumption function. Two definitions of pension wealth were used. Gross pension wealth has been defined as the discounted present value of future pension receipts weighted by survival probabilities whilst net pension wealth is gross pension wealth less the discounted present value of future pension contributions weighted by survival probabilities (termed liabilities). Unfortunately these definitions could only be used with state pensions - data was not

available to evaluate these types of wealth variable in the case of private pensions. Over the period 1949-73 gross state pension wealth appears to have no impact on consumption, and therefore savings, whilst net pension wealth depresses consumption. The growth of the market value of the assets of superannuation funds was used as a proxy for the growth of private pension wealth. This data was available for the period 1958-73. Whilst private pension wealth was found to be insignificant in the regressions in these same equations gross state pension wealth becomes significant and positive. If net pension wealth is substituted for the gross alternative its coefficient remains negative but its significance falls. Over both periods there was found to be no evidence of an induced retirement effect. Various estimates of the effect of the existence of pension schemes and changes in pension contributions and benefits were made on the basis of a range of estimated coefficients. As one would expect these estimates varied widely given the different signs of the coefficients on gross and net state pension wealth.

There are four aspects of the work that, at the end of the day, give cause for concern. The two relating to the theoretical work are probably less serious than the two relating to the empirical work.

In the development of the theoretical analysis the individuals work/leisure decision was given only passing consideration. Indeed it was only shown that if the individual moved continuously from the work to the retirement period then leisure considerations would be irrelevant. Nevertheless recognition was accorded to the possibility that if this move is discontinuous, which is highly likely to be the

case, not only might the optimal retirement date be different to that derived in the basic model but also the earlier comparative statics results might no longer apply. In a recent paper Ulph (58) has begun to consider this problem in a model incorporating a terminal salary based private pension scheme. Employing an iso-elastic utility function in consumption and leisure he shows that certain assumptions relating to the parameters of the utility function imply that not all the comparative statics results of earlier chapters necessarily go through. Thus leisure considerations are not irrelevant to the analysis of pensions. The results also imply that analyses of the individual's life cycle problem which do not include pensions, and where the individual retires when the marginal utility of consumption falls below the marginal utility derived from spending all one's time in leisure, might be substantially changed when leisure considerations are taken into account. An obvious extension of pension theory would then be to integrate it into a model of the above type, such as that of Blinder (13).

Another important feature of the pension decision, which again has only been covered rather briefly, is uncertainty about the length of lifetime. It has been shown that if it is assumed that there exists a perfect insurance market then the problem reduces to an analysis under certainty, the insurance asset obviating any problem arising from a probabilistic date of death. Ulph and Hemming (59) have since dropped this last assumption and shown how the results of this thesis are affected. Although this turns out to be to an extent that gives little cause for concern, the new model does produce the results within a framework that is more realistic. The above paper also has important

implications for another aspect of pension analysis. When lifetime is uncertain the purchase of a pension asset or annuity provides insurance and the individual should therefore be prepared to purchase it even if its return were less than actuarially fair. Now in evaluating the value of anticipated state pension wealth the average market rate of interest has been used to discount the expected value of future returns. This is an approximation of the actuarially fair rate of interest. But under uncertainty this is not the appropriate rate of discount - what ought to be used is the lowest rate of return at which the individual is just willing to purchase the annuity. In fact Ulph and Hemming show that whilst the individual is holding non-pension assets the appropriate rate of discount is the rate on those assets. Once the individual has run down his stock of these assets the appropriate rate of discount will be the subjective rate of discount incorporating a risk factor. Thus if the individual is holding the alternative asset the appropriate rate of discount is less than the actuarially fair rate, (see (3.6.4)). Pension wealth will therefore be underestimated. When none of the alternative assets are held the value of pension wealth can be under- or over-estimated depending on the relative magnitudes of the rates of interest and the subjective rate of discount. Not only do these results suggest that pension wealth might be evaluated incorrectly but it could be that the optimal bequest decision, which did not affect the earlier analysis, now becomes important in that individuals will be holding capital through the later part of their life. When they choose to make their bequest will affect their valuation of pension wealth. This interrelationship could produce some interesting results and is worth exploring further.

Turning now to the problems arising on the estimation side one can consider firstly the general insignificance of the pension variables in the regression equations and the inconsistency of the short and long period estimates. In retrospect, it should have been realised that the wealth approach could present a problem in attempting to reconcile the theoretical and empirical results. Recall that increases in both state pension contributions and benefits always lead to a reduction in savings - when using the net state pension wealth variable in the estimated equations, no matter what the sign of its coefficient, both the above predictions cannot be confirmed simultaneously. This follows since increasing contributions and benefits respectively decreases and increases net state pension wealth. Only if the coefficient on gross state pension wealth had been positive and significant could both predictions have been borne out. That net state pension wealth alone should turn out to be significant, and then negative, implies that only the effect on savings of increasing contributions is confirmed. Possibly this result was to have been expected since the only acceptable way of explaining the difference between the coefficients on the two pension wealth variables is through the significance of liabilities. Unfortunately there appears to be no reasonable argument which explains why individual behaviour should be determined by liabilities rather than gross wealth.

One has to be rather surprised about the apparent change in the relationship between the long and short data periods. A somewhat similar case of this arose when Feldstein (19) changed his data period. Since in both analyses the regressors are all income and

wealth variables it is difficult to believe that this is not a classic symptom of the multicollinearity which is quite clearly present rather than some structural change in the relationship over time.

In the theoretical developments an induced retirement effect played a prominent part in the discussion. When it came to estimation the effect was found to be non-existent. Indeed the data suggested that retirement patterns had not changed markedly over the postwar period. This is not surprising. Under the state pension scheme coverage was universal by 1946 and whilst subsequent changes in contributions and benefits have hardly had marginal effects on lifetime income it is unlikely that this will be affected by induced retirement because either many people will retire at the official retirement date or, for those retiring later, the retirement date is not a continuous variable. Even if this were not the case in practice estimation of the effect would be difficult since the retirement date is discrete as far as official statistics are concerned. As far as private pensions are concerned significant growth has taken place over the postwar period but still only half the working population are covered - many of these are younger workers whose retirement behaviour will not yet have been observed.

Indeed this last point is one that applies to the study of private pensions as a whole. The empirical analysis of relationships involving private pensions has been scant and highly unsatisfactory. That no attempt was made to calculate a private pension wealth variable was simply due to data inadequacy: that no attempt was made to collect the data is explained by comparative advantage. As mentioned earlier

the Government Actuaries Department are currently engaged in providing estimates. When these are available and the regression equations re-estimated some more useful results may emerge.

Even when the specification and data problems are overcome there still might exist one reason why the results produced might not be convincing. When a life cycle model of individual behaviour has carefully been built up, and no one could disagree that when considering pension problems this is the appropriate type of model, it will always be disappointing that the model cannot truly be tested with the data currently available. All the methods considered for estimating such a model are only approximations - there can be no substitute for cross-section data relating to the behaviour of the same individuals over fairly long periods of time.

Whilst this pessimism about the reliability of the results is clearly very healthy one is now only left to ponder whether the attitude would have been the same if the estimated coefficients had all been statistically significant and of the theoretically anticipated signs. One has to hope so!

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