Mathematics and General Education

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Table of Contents

Part I: A Mathematics Curriculum for General Education

Chapter 1: Democracy and General Education

My purpose in Part I is to develop a model of general mathematical education: that is, to identify aims appropriate to a course of mathematical education which forms part of a programme of general education. To do so presumes, of course, that it is possible to justify both the inclusion of mathematics-related aims and content in the curriculum, and their organisation around a unit entitled 'mathematics'. I will offer arguments for both these presuppositions, as well as for my model of general mathematical education.

I regard it as particularly important that these arguments should be anchored in a global theory of general education, rather than an *ad hoc* theory which professes to deal satisfactorily with a small part of general education in isolation. For it is only from the viewpoint of a global theory that it is possible to evaluate the conflicting and competing claims for the inclusion of individual curricular units. Nonetheless, my strategy of exposition anticipates, to a certain extent, my conclusions. Where appropriate, I will illustrate and discuss global arguments with particular reference to mathematics. In this way I hope to indicate both the general criteria underlying my global theory of general education, and their application to the particular case of mathematics.

(1) Educational change and the idea of democracy

The long public debate over the democratisation of our school system has focused predominantly on its selective function, in particular on the way in which the organisation of schooling helps to reproduce a stratified and differentiated social and occupational structure. Discussion of the content and aims of the education which schools provide has not been prominent.[1]

The dominant reformist view has sought equality of opportunity, a shift from ascribed to achieved status, from aristocracy to meritocracy. This view stems from a conception of democracy which champions the right of each individual to improve his social and economic status. It demands not that the school should relinquish its selective role, but that it should exercise it rationally, effectively and equitably. There has, of course, been increasing disagreement over the rationality, effectiveness and equity of different forms of selection. As a result, there has been a shift from sponsored to contest mobility, leading to a delayed ascription of roles, and from overt to hidden selection. In particular, patterns of curriculum have become more diffuse, and curricular differentiation has become the instrument rather than the outcome of selection.

A more recent, but less influential view seeks equality of outcome as the necessary prerequisite of a more even distribution of wealth and status, either within society as a whole, or between particular groups within society.[2] It suggests that the school should abandon, or at least adapt its selective role as part of a programme of positive intervention aimed at diminishing variations in wealth and status within society. The protagonists of this view welcome the deferment of selection and the diminution of differentiation, although they believe that these processes have not yet advanced sufficiently.

Both these views are based on primarily economistic conceptions of democracy: that is, they define democracy principally in terms of some ideal principle underlying the just distribution of wealth and, to a lesser extent, status within society. Their principal concern is the <u>selective function</u> which the school fulfils, and which influences the distribution of wealth and status. Their concern with the <u>educational purpose</u> of the school is subordinate to their concern with its selective function.

Over the last century these reformers have successfully challenged first, the restricted availability of education, and then, differentiation within the educational system. This century of educational change has led to the comprehensive school, mixed-ability teaching, proposals for common systems of examinations at 16 plus in both England and Scotland, and a certain measure of positive discrimination in the allocation of educational resources.

I am not convinced that democracy in its full sense can be reduced, either in principle or practice, to economic democracy, admirable as that concept may be in its own right. In principle, a conception of democracy which asserts people's equal rights in society has not just an economic, but a political and cultural dimension as I hope to make clear at a later point. And in practice these three aspects of democracy are interdependent and mutually sustaining. In a society in which the influence of the principle of economic democracy, at least in its stronger forms, is notably absent from other important social institutions, and in which the principle itself is far from generally accepted, the effectiveness of educational change in advancing economic democracy is likely to be limited if that change ignores other aspects of democracy.

For these reasons I believe that, despite the desirability of their aims, the predominant influence of economistic conceptions of democracy on the debate about the democratisation of our school system, and on the ensuing changes has been unfortunate.

The extension of educational provision and opportunity has been rationalised and supported, however, not only in terms of economic democracy, but as instrumental to the maintenance of social order and to the encouragement of economic growth in a changing and increasingly complex society. Indeed, this second argument fits neatly with that of economic democracy, and the resulting economistic view of education has exerted a profound influence on public debate since the Second World War.[3] For the individual, education is seen as the instrument of personal economic and social advancement through the access it offers to more skilled employment: for the economic manager, education is the instrument of manpower planning which provides the 'human capital' to sustain economic growth. Not surprisingly then, many of the proponents of the extension of educational opportunity have allied themselves with a technocratic model of vocationally-oriented education, consistent with the economic presuppositions of their argument.[4]

Now, an awareness of the role of education in advancing economic democracy has long had a place in radical thought. We find, for example, Thelwall, a leading radical of the 1790's, arguing in his 'The Rights of Nature', that all children should be educated so that,

> "if they have the virtue and talent [they should be able] to improve their condition and mount to their intellectual level, though it be from the lowest to the very highest station of society."[5]

But there is another aspect of democracy, and another view of the role of education in advancing democracy which deserve our attention. In the first half of the nineteenth century, the self-education of the working classes through the Corresponding Societies, Secular Sunday Schools, Hampden Clubs, Owenite Halls of Science, and Chartist Halls and Schools was seen by the theorists of the Radical movement as an instrument of self-realisation and political emancipation.[6] This flowering of independent working class education was strongly influenced by the views of education advanced by Radical writers such as Paine, Godwin, Owen, Carlile, Thompson and Lovett.[7]

The first characteristic of the Radical tradition was a rejection of theories which saw man's patterns of behaviour as innate, and a belief in the formative power of education and the ultimate perfectibility of man. The second characteristic was an emphasis on science and scientific education as a means to truth. Paine, in his widely read and influential 'The Age of Reason', argued for a secular education based on science, which would enable man to be free to realise himself, to understand his place in the universe and to act accordingly. He attacked the mythology of Christianity as a barrier to science, and the central place of classical languages in contemporary education as irrelevant to enlightenment and understanding. This argument was taken up and developed, notably by Carlile who argued that scientists should make their discoveries known to all, in order to drive out superstition and dogma: scientific truth about the universe was the condition for human progress and enlightenment. Finally, as their emphasis on science and attack on religion suggests, the members of this tradition argued for a secular education, and the development of a secular morality.

The content and aims of education were of the highest significance to the early Radicals. The independent working class education of the first half of the nineteenth century centred on the physical, natural, social and political sciences, and encouraged discussion and argument about the relation of their ideas to contemporary issues. Its intentions were liberal and general, and were closely linked to the struggle for political democracy.

The pre-eminent bearer of this tradition in our time is Williams.[8] He argues that we are living through 'a long revolution' which has three aspects. The democratic revolution by which people shall come to,

> "govern themselves, and make their own decisions, without concession of this right to any particular group, nationality or class"[9]

is at an early stage. This achievement of political democracy is made more difficult by the increasingly complex social organisation created by the continuing industrial revolution. Beside and beyond these lies the cultural revolution which will extend the active process of learning and the ability to communicate in varied and effective ways throughout society.

Williams is particularly concerned that education should prepare individuals to participate fully in democratic decision making, and that it should help to develop, and give access to, a common intellectual culture. He criticises the lack of attention given by the traditional grammar-school curriculum to social studies, to non-literary and popular art forms and to the history of scientific discovery and its social effects. The curricular model he advances is as follows.

(a) Extensive practice in the fundamental languages of English and mathematics;

(b) General knowledge of ourselves and our environment, taught at the secondary stage not as separate academic disciplines but as general knowledge drawn from the disciplines which clarify at a higher stage, i.e.

(i) biology, psychology,

(ii) social history, law and political institutions, sociology, descriptive economics, geography including actual industry and trade,

(iii) physics and chemistry;

(c) History and criticism of literature, the visual arts, music, dramatic performance, landscape and architecture;

(d) Extensive practice in democratic procedures, including meetings, negotiations, and the selection and conduct of leaders in democratic organizations. Extensive practice in the use of libraries, newspapers and magazines, radio and television programmes, and other sources of information, opinion and influence;

(e) Introduction to at least one other culture, including its language, history, geography, institutions and arts, to be given in part by visiting and exchange.[10]

While Williams' curricular model is at a high level of generality and lacks an articulate rationale, it builds on the Radical tradition with its concern that pupils should come to see and understand the world through the eyes of science, and should develop the ability to participate articulately and effectively in political decision making. To this Williams adds a concern with the arts in their widest sense, and with different patterns of life and culture.

This is the tradition of educational theorising which I wish to extend in arguing for a democratic model of general education. While its proponents have often been over-optimistic about the role which education might play in advancing democracy - attributing to it a primacy or autonomy which it does not possess - I believe that they are fundamentally correct in arguing that a certain form of education is a necessary prerequisite for a fully democratic society.

(2) The idea of general education

The very vagueness of the terms 'liberal' and 'general' education which recommends them to the debater, renders them inadequate for the planner. For the common kernel of the different formulations of these concepts - beyond which many extend no further - is essentially negative. General education is not vocational or specialist education. Many of the positive definitions are very weak, making general education synonymous with little more than the study of a 'broad' or 'balanced' selection of subjects. There are three levels for such definitions. Weakest is that where general education is defined solely in terms of the number of subjects that pupils study. Next come those which insist on a certain range of subjects. Finally, there are those which prescribe the inclusion of certain subjects or areas of experience in a curriculum for general education.

For example, Hunter[11] presents the Scottish Ordinary Arts degree as an example of general education on the grounds that students have to study at least five subjects. Drever[12] argues from the second level, observing that a certain range of subjects is required, thus preventing undue concentration on cognate subjects, and notes that previously it was obligatory to include in the curriculum for the degree, a philosophical subject, a foreign or classical language, and a mathematical or scientific subject (the third level).

Thus, in these senses, both Scottish and English schools offer a general education up to age 16, in that current practice is for pupils to study a large number (first level) and a wide range (second level) of subjects. The recent Munn Report[13] proposes a slightly stronger definition by prescribing the inclusion of seven subjects, or subject-types in the Scottish curriculum (third level).

Nonetheless, definitions of general education which focus solely on the organisation and structure of the curriculum are relatively weak. For however many subjects a pupil may study, however exhaustive and wide-ranging they may be, such a definition ignores the possibility that, within each subject, the perspective is that of the specialist: that the subject unit is conceived as part of the formation of the future specialist, and aims to impart the knowledge, skill and understanding appropriate to this end, rather than that which will be of value to the non-specialist. That this is the case with current mathematics curricula will become more fully apparent at a later point.

In recent years 'general education' has acquired another connotation. It has become a euphemism for non-certificate education. One example is the recently developed 'Mathematics for General Education': the title is justified by its authors on the grounds that they believe that it will encourage a more positive attitude towards the education of non-certificate pupils than the originally proposed 'Non-certificate Mathematics'.[14] Now, while the sentiments implicit in this justification are entirely laudable, I doubt the wisdom of arrogating the title 'general' to a form of mathematical education which is not grounded in any theory of general education, and which, it transpires, is general neither in the view of mathematics and mathematical activity which it promotes, nor in terms of the group of pupils at which it is aimed.

What all these definitions lack is a clear, positive rationale to act as a guide to the aims, content and organisation of a curriculum for general education, and of the units which it comprises.

(3) Democratic general education

It is man's intellectuality which makes human society possible. Through it man builds systems of ideas which enable him to interpret and intervene in the world. Medical treatment, economic planning, the adaptation of physical environment, religious observance, government and education are examples of human interventions based on these systems of ideas. This is not to argue that ideas are autonomous or asocial, nor that man's intellectuality is the motor of social and historical change. It is simply to draw attention to the fact of man's intellectuality, and the possibility of understanding, and thereby regulating and transforming the world, which it holds out to him, limited as it may be.

The positive formulation of general education which I wish to advance starts from the belief that the overriding aim of general education should be to give access to man's intellectuality, to the systems of ideas through which he makes sense of, and modifies his experience. The need for general education arises from the richness, variety and extent of these systems. Certainly no individual could expect to master more than a small fraction of them during his lifetime. The division of labour, and the associated specialisation of knowledge, skill and understanding is a social reality which the school cannot ignore.

It is here that questions of political and cultural democracy arise. For the possession ~~fo~~ of an appropriate framework of understanding is a prerequisite of participation in any activity. Any educational process equips its subjects to participate in society in certain ways.

Political decisions are decisions about the kind of society that we will live in. A political decision is one that can be seen to influence or effect change - or the absence of change - in the world in which we live. Of course, the social significance of an issue, and thus the legitimacy of its inclusion in the political arena, may itself be a matter of political controversy. Arguments over the role of 'political' considerations in sport and education, or over the degree of public scrutiny to which certain planning decisions should be subject - for example, those relating to the construction of motorways and industrial complexes, or the development of nuclear power - exemplify how political disagreement may reflect different demarcations of the domain of politics.

Nonetheless, although views may differ over the extent to which decisions have political significance, or should be subject to public control, in all societies, and particularly in an industrial society such as ours where innovation is in many senses institutionalised, decisions which clearly have the potential to change the nature of the society have frequently to be made. Such decisions are made, not only within the formal institutions of national and local government, but within the network of institutions around which social life is organised; institutions such as industrial and commercial enterprises, financial institutions, professional bodies, trade unions, and the mass-communications media. Effective political democracy - the collective control of the processes by which the natural and social world is regulated and transformed - depends on the ability of members of society to recognise and comprehend the issues involved in making such decisions, as well as their right to participate in the process of decision-making.

The role of general education in advancing political democracy is a modest one. It is clear that in a complex and changing society not even the specialist can expect to have at his finger-tips the detailed knowledge and understanding needed to resolve the many issues which arise in deciding - say - Britain's constitutional future, or on the development of a nuclear power programme, or on the ratification of an international trade agreement. It would be most misguided to expect that general education could anticipate those issues which were likely to exercise society over the coming half century, let alone provide the detailed knowledge and understanding - much of it still undeveloped - needed to make a satisfactory decision on these issues. What general education can offer is an insight into the fundamental principles underlying the complex systems of ideas which man has developed, and into the significance of these systems. It provides a foundation on which more particular understanding can be built as the occasion arises.

For example, a curriculum for general education is likely to deal - if not in name in substance - with concepts such as the atomic structure of matter, radioactivity, mutation, pollution, renewable and non-renewable resources, exponential growth and decay, and extrapolation on which a more specific understanding of the issues surrounding the development of nuclear power can be built; the processes by which nuclear power can be produced, the forms of environmental pollution which may result, the feasibility of using alternative energy sources, the methods of projecting future power needs, and so on.

The part which general education plays, then, in advancing and securing political democracy is to provide a basic understanding of the ways in which man interprets and intervenes in the world.

The basis of the democratic ideal is the belief that all people have the same rights in society. Economic democracy asserts their right to share the wealth that society produces, political democracy their right to shape the development of society. But social life cannot be reduced simply to its economic and political dimensions. Cultural democracy is a more complex phenomenon. Ultimately it asserts the right to 'belong' in society. Under that rather vague rubric can be grouped the right to have access to, and to develop or reject man's systems of ideas in order to examine and make sense of the world, and the right to develop a sensibility and to find a form of life within society. These two rights are linked to the extent that the choice of a form of life is based on some view of the world in which that life takes place.

This definition of cultural democracy may appear to be social rather than personal, to place society before the individual. Such an interpretation would be mistaken. Certainly this view dismisses the romantic fiction of the individual as apart from, or above society: instead it is based on the recognition that personal fulfilment is fulfilment within society, even if it involves changing rejecting or distancing oneself from society. This is what Heidegger means when he talks of the hermit as "being with others in a deficient mode". Effective cultural democracy depends on the accessibility of different world-views and forms of life, not just to allow individuals or groups to choose a form of life, but to make possible a common understanding of different world-views and forms of life, and the political questions which these may raise.

White[15] lists a number of forms of life, including those devoted to the pursuit of truth, to artistic creativity, to others' good, to physical prowess and adventure, to physical pleasure, to religious devotion, the acquisition of goods, the acquisition of power over others, and, of course, the ever-present alternative, the surrender of choices about one's life to others. These are clearly 'ideal' types. In practice, the life of any individual is likely to reflect some combination of such ideals. Similarly the view that individuals or groups <u>choose</u> a form of life is idealised. Many aspects of a form of life may be traditional - inherited, or adopted without scrutiny.

Again the role of general education is a modest one. It cannot conceivably examine all potential world-views and forms of life.

What it can do is to provide an intellectual framework within which different kinds of sensibility and forms of life, and the world-views on which they are based can be understood or investigated, whether they have been consciously chosen, or simply inherited by members of society.

Democratic general education, then, attempts to provide a framework of understanding which opens up human intellectuality, and through it human endeavour to all, as a prerequisite of cultural and political democracy. It aims to provide a key to the systems through which man makes sense of, and modifies his experience.

Clearly, if it is a prerequisite of effective democracy, such an education should be available to all, regardless of the specialised and differentiated social roles that they will play. Considerations of aptitude and ability, or motivation do not provide legitimate grounds for restricting the availability of such an education. On the contrary, these considerations, if applicable, point to the necessity of identifying methods of making the framework which general education offers intelligible to the less able, and meaningful to the unmotivated. Given a commitment to democracy, this is a simple corollary of our common humanity and our social existence.

There is a further argument of a rather different kind for such a general education. It points to the uncertainty and unpredictability of the future in a complex and changing society such as ours, and suggests that an education which focuses on particulars, or on specialised knowledge and understanding is likely to become rapidly redundant. An education which deals with broad principles over a wide area of understanding is more likely to enable the members of such a society to adapt effectively to their changing environment.

These are not simply arguments for general education which can now be quietly discarded. They generate criteria against which a proposed programme of general education must be evaluated. The arguments from political and cultural democracy suggest that it is not enough for a curriculum for general education to familiarise pupils with the basic principles of systems of ideas. It is essential also, that the nature and significance of these systems of ideas, and the ways in which they help man to interpret and intervene in the world should be considered within the curriculum. The argument from change suggests that general education should aim to convey a framework of understanding which can accommodate the dynamic of social and intellectual change.

The view that general education should give access to man's intellectuality runs, to some extent, counter to currently fashionable notions such as 'community education' and 'total education' (which I understand to comprise social, personal, moral, and leisure education). Certainly it is a reassertion of the intellectual purpose of the school, from which many of these notions seem to be a flight, encouraged both by the ossified and largely academic goals set by traditional subject curricula, and the failure of many schools to achieve these intellectual goals with the majority of their pupils. There is no reason why schools should not pursue aims additional to those of general education. There is certainly a place in curriculum planning for more directly utilitarian aims related to pupils' everyday needs, and for more specialised aims which take account of pupils' individual interests.

What is important is that general educational aims should not be subordinated to other aims, or entirely driven from the curriculum by such aims. Further, there is no reason why general education, as I have described it, should not use the local community as a resource. Nor is it antagonistic to the aims of social, personal, moral or leisure education. Indeed, it offers access to the basic intellectual frameworks within which the more specialised issues that these topics raise can be understood. The only prescription implicit in my argument is that general educational aims should be pursued by the school, and that they should be pursued for all pupils.

The view that general education should give access to man's intellectuality is one which commands fairly wide support among curriculum theorists, as does the view that such an education should be available to all. Theorists such as Phenix[16] and Schwab[17] in the United States, and Hirst, Peters[18] and White[19] in the United Kingdom all broadly support such a view: where there is less agreement is over the form and content of a curriculum for general education.

Chapter 2: Knowledge and the Disciplines

The argument so far has been rather abstract. While it may be hard to dissent from the conclusion that all pupils should be offered a general education which makes human intellectuality accessible, it would have rather more significance if the form and content of general education, and their relation to the argument in favour of such an education could be made more concrete.

But a coherent programme of general education must be based on an analysis of the systems of ideas which man has developed, an analysis which will influence the more specific aims, content and organisation of the curriculum. The path to a more concrete model of general education is through an analysis of these systems of ideas.

(1) The disciplines thesis

I have already observed that ideas underpin man's interpretation of his world and his intervention in it. The distinction between the two processes is reflected, in some measure, in the way in which systems of ideas are structured. Some systems of ideas are organised around issues of interpretation, others centre on problems of intervention. For example, geometry, as we commonly understand it, is primarily concerned with interpretation: surveying, by contrast, focuses on intervention. Astronomy, geography and aesthetics are primarily interpretative, while aerospace engineering and environmental planning and design are primarily concerned with intervention.

Most contemporary curriculum theorists restrict their analysis of systems to interpretative systems, on the grounds that the developed systems of intervention are all based on knowledge locatable within some interpretative system. This is the approach of the two analyses which have been particularly influential in curriculum theory in recent years, those of Phenix and Hirst.[1] It is in the works of these two theorists that the thesis that knowledge falls into a limited number of logically distinct categories - the disciplines thesis - can be found in its most developed forms.

Hirst argues that there are seven logically distinct forms of knowledge; mathematics, the physical sciences, the human sciences, and aesthetic, moral, religious and philosophical knowledge. Phenix suggests that knowledge falls into six logically distinct realms of meaning; synoetics, aesthetics, symbolics, empirics, ethics and synoptics. However, much of his argument is framed in terms of disciplines which, he argues, exhibit distinctive logical structures and patterns of meaning which enable them to be grouped in realms. Phenix's argument is not entirely clear here. His realms seem to cut across his disciplines to some extent: parts of the disciplines of philosophy and religion belong to the realm of synoetics, others to synoptics; parts of literature to synoetics, others to aesthetics. Indeed, Hirst has criticised the loose and unconvincing relation between disciplines and realms in Phenix's argument.[2]

I propose to examine the thesis in relation to mathematics. There are a number of reasons for this choice of approach. First, while there are fairly substantial disagreements between Hirst and Phenix at a more general level, their characterisation of mathematics, and their arguments for its status as a discipline are remarkably similar. Second, mathematics is an area in relation to which the disciplines thesis has been taken as particularly clear cut. To test it here is to test it where it is strong. Finally, I am particularly concerned with the practical implications of the thesis for curriculum design in mathematics.

(2) Mathematics as a discipline: exposition

Mathematics is one of Hirst's seven 'forms'. He describes a form as

> "a distinct way in which our experience becomes structured round the use of accepted public symbols"[3]

and argues that for an area of knowledge to be a form it must possess

(1) certain central concepts that are peculiar in character to it,

(2) a distinctive logical structure ordering its concepts, and relations between them,

(3) a distinctive way in which propositions are tested against experience,

(4) distinctive methods of enquiry.

He identifies (3) as the crucial criterion.

> " .. the central feature .. (to which these criteria point) .. is that disciplines can be distinguished by their dependence on some particular kind of test against experience for their distinctive expressions."[4]

Although Hirst's description of mathematics is fragmentary and his work lacks a systematic argument for regarding mathematics as a discipline, it is possible to construct the form of such an argument from his writings. We are told that,

" .. number, integral and matrix (are distinctive concepts) .. in mathematics,"[5]

and, crucially, in view of his account of the criteria for demarcating disciplines, that,

" .. (the validity of mathematics) depends on deductive demonstrations from certain sets of axioms."[6]

More particularly he states that,

" .. the truth that the lengths of the sides of a right angled triangle satisfy the equation $a^2 = b^2 + c^2$ rests on the truth of a sequence of earlier propositions which, in turn, depend on the axioms of Euclidean geometry."[7]

Mathematics is one of the 'disciplines' that make up Phenix's 'realm' of symbolics. He argues that a discipline is identifiable by its representative ideas and their structure (Hirst's (1) and (2)), the methods of enquiry and testing that it employs (Hirst's (3) and (4)), and its subject matter (effectively reducible to Hirst's (1) at some theoretical level).

Phenix's description of mathematics is more compact and detailed than that of Hirst. He argues, as Hirst appears to, that,

" .. The method of mathematics is essentially postulational. This means that certain postulates, or axioms, are arbitrarily chosen as part of the foundation of a given mathematical system. These postulates are not "self-evident truths," as, for example, the axioms of Euclidean geometry were formerly thought to be. They are assumptions taken as a starting point for the development of a chain of deductive inferences."[8]

He argues further that:

> "The subject matter of mathematics is .. formal (abstract) symbolic systems within which all propositions are consistent with each other."[9]

Phenix identifies certain concepts as central to mathematics.

> "These ideas of elements, sets, equality, sum, product, and difference comprise basic terms from which all other mathematical concepts can be developed, provided certain basic logical concepts are also presupposed."[10]

(3) Criticism: the nature of a discipline

But throughout the work of Phenix there is ambiguity about whether the definition of a discipline which is being used is indeed a logical one, based on distinctions between the truth criteria used to evaluate propositions and theories, rather than a social one which identifies a discipline with some historical tradition of enquiry and activity, or a commonsense one which tacitly reflects elements of both.

On those occasions when Phenix deals explicitly with the problem of demarcating disciplines he argues for a logical definition which makes distinctions between disciplines on the basis of the truth-criteria, and the associated concepts and methods which characterise a particular area of knowledge. On other occasions, however, Phenix appears to use a commonsense definition of a discipline which is not strictly founded in the logical criterion. Passages such as the following seem to conflate social and logical definitions.

> "The general test for a discipline is that it be the characteristic activity of an identifiable organized tradition of men of knowledge, that is, of persons who are skilled in certain specified functions that they are able to justify by a set of intelligible standards."[11]

While Hirst consistently asserts the priority of the logical definition, he sees social and logical definitions as coinciding, at least in the disciplines as they are currently constituted.

> "The development of mind has been marked by the progressive differentiation in human consciousness of some seven or eight distinguishable cognitive structures, each of which involves the making of a distinctive form of reasoned judgement and is, therefore, a unique expression of man's rationality. This is to say that all knowledge and understanding is logically locatable within ... mathematics, the physical sciences, knowledge of persons, literature and the fine arts, morals, religion and philosophy. These would seem to me to be the logically distinct areas."[12]

The case of arithmetic points to the weakness of Hirst's position, and to the seriousness of the confusion in that of Phenix. Both regard arithmetic as clearly part of mathematics. But in what sense are the arithmetical propositions we learn, construct and use dependent on deductions from axioms? To make statements about numbers we do not make deductions from axioms in any sense which preserves the distinctiveness of that conceptual scheme. Rather we use a number of geometrical and physical analogies (such as the number line), and rules of calculation, to construct and test arithmetical statements.

Of course it might be argued that, while we do not actually construct and test arithmetical propositions in such a manner, their truth is in some ultimate sense dependent on their deducibility from some set of axioms. But, as we shall see in a later section, this argument can only be sustained at the expense of the meaningfulness and applicability of arithmetical propositions. Further, such an argument would appear to be incompatible with Hirst's claim that

> "It is quite impossible to learn facts, to know them as facts, without acquiring the basic concepts and criteria for truth involved."[13]

Now it is part of our commonsense knowledge that arithmetic is part of mathematics. But arithmetic does not satisfy the logical criterion for inclusion in the discipline of mathematics. Here logic conflicts with commonsense.

The confusion between logical and commonsense definitions of mathematics is then a serious one. Indeed, I hope to show that to adopt the logical definition of mathematics which Hirst and Phenix advance is to exclude virtually all of what is, and has been, commonly termed mathematics, whether we interpret 'deduction from axioms' as a characterisation of the method of procedure adopted by mathematicians, the form in which they present their conclusions, or the epistemological basis of mathematical knowledge.

(4) Criticism: a socio-historical perspective

Interpreted as a characterisation of the concerns, or methods of procedure of mathematicians, or of the form in which their conclusions are expressed the logical definition of mathematics adopted by Hirst and Phenix admits little of what, in the commonsense terms of mathematician and layman alike, passes as mathematics. The great mass of it must be excluded as based on d'Alembert's optimistic credo;

Allez de l'avant: la foi vous viendra.

Quite simply the majority of mathematicians spend their time producing interesting and plausible guesses, supported by informal reasoning, in order to develop mathematical systems, or to apply mathematical ideas and procedures to problems elsewhere. The methods they employ do not approach the degree of rigour demanded by Hirst and Phenix. Full-blooded deductive rigour is the goal of only a small number of mathematicians.

Clearly this view raises even greater problems in relation to the history of mathematics. Phenix would be compelled to agree with Russell that the history of mathematics starts in 1854.

> "Pure mathematics was discovered by Boole, in a work which he called the 'Laws of Thought' (1854). This book abounds in asseverations that it is not mathematical, the fact being that Boole was too modest to suppose his book the first ever written on mathematics. ... His book was concerned with formal logic, and this is the same thing as mathematics."[14]

Hirst would be slightly less exclusive. Euclid's geometry and Cauchy's analysis are on the right lines, but riddled with errors and omissions.

Enquiry governed by other conceptions of mathematics is dismissed, and the knowledge it produces not recognised unless, and until it has been put into deductive form. This dogmatic viewpoint excludes from 'mathematics' the mathematics of ancient Egypt, India and China, a great deal of Greek mathematics, and Arab algebra. The mathematics of the 17th and 18th centuries, which included book-keeping, ballistics, navigation, astronomy and optics, guided by Descartes' view of mathematics as the science of quantity, is excluded by this dogmatism, as are the topological and algebraic enquiry of the 19th century, and the metamathematical enquiry of the 20th century which have led some mathematicians to conceive of their work as the study of structure. Dogmatism conceals diversity, dissent and change in mathematics as we commonly understand it, by excluding that which is anomalous through its definition of the discipline.

Lakatos has pointed out the basic weakness of this logical definition.

> "Formalism denies the status of mathematics to most of what has been commonly understood to be mathematics and can say nothing about its growth. None of the 'creative' periods and hardly any of the 'critical' periods of mathematical theories would be admitted into the formalist heaven, where mathematical theories dwell like seraphim, purged of all the impurities of earthly uncertainty. Formalists, though, usually leave open a small back door for fallen angels; if it turns out that for some 'mixtures of mathematics and something else' we can find formal systems 'which include them in a certain sense', then they too may be admitted. On these terms Newton had to wait four centuries until Peano, Russell, and Quine helped him into heaven by

formalising the Calculus. ... Perhaps we should mention here the paradoxical plight of the metamathematician: by formalist, or even by deductivist standards, he is not an honest mathematician. Dieudonne talks about 'the absolute necessity imposed on any mathematician who cares for intellectual integrity' to present his reasonings in axiomatic form."[15]

(5) Criticism: a philosophical perspective

Having established that an interpretation of Hirst and Phenix's logical definition of mathematics as a characterisation of the procedure of mathematicians, or of the form in which they present their conclusions, excludes much of what we commonly understand as mathematical activity, I now intend to show that, interpreted in epistomological terms sufficiently rigorous to preserve its power of demarcation, their logical definition excludes virtually all of mathematics as we conventionally understand it. In order to do so it will be necessary to explain the meaning of some of the technical terms which mathematicians use to distinguish different kinds of mathematical theories.

(i) Informal mathematics and truth

In informal mathematics - that is the kind of mathematics that most of us are familiar with - terms and propositions have specific meanings. The fundamental question is whether or not a proposition or a theory is true, what Godel terms 'correct as regards content'. Axiomatising a theory reduces the problem of the truth of its theorems to that of its axioms, for if the axioms are true (and the rules of inference preserve truth) then the deductive method transmits truth down to the theorems. By axiomatising a theory mathematicians hope to reduce the truth of its propositions to that of a set of axioms. Clearly, then, the deductive method does not solve the problem of truth, it only transfers it. At some point we still become dependent on some direct test of truth in terms of, for example, arithmetic, logical or spatial 'intuition'.

(ii) <u>Formal derivation and formal systems</u>

There are certain rules of inference which allow us to draw immediate conclusions from suitable propositional forms. For example modus ponens

$$\frac{\begin{array}{c}\text{If P then Q}\\ \text{P}\end{array}}{\text{Q}}$$

and conversio simplex

$$\frac{\text{Some A's are B}}{\text{Some B's are A}}$$

A proposition is formally derivable from a set of axioms if we can by manipulating the axioms in accordance with certain rules of inference obtain the proposition. The important aspect of this process is that the meaning of terms or propositions is immaterial to the drawing of the conclusion. That is why we call it a formal derivation. A formal system is defined by a set of axioms and a set of rules of inference. It is simply an axiomatic system in which terms and propositions are uninterpreted.

(iii) Deduction and Proof

An axiomatised informal theory (a deductive system) shorn of its meaning is, then, just a formal system. We say that the informal theory is a model of the formal system. A deduction of a theorem from the axioms in the deductive system runs parallel to a formal derivation of the identical propositional form in the formal system.

But in informal mathematics 'proof' is a wider concept than 'deduction from axioms'. The admissible methods of proof include not just the syntactic (meaning-independent) techniques of derivation, but semantic (meaning-dependent) techniques such as the use of counter examples.

(For example we can prove that the proposition, For all natural numbers n, strictly greater than 1, 2^n-1 is a prime is false, by use of the counterexample, $2^4-1 = 15$). To show such results deductively we would have to demonstrate that no formal derivation yielded the appropriate propositional forms.

It was in the hope of eliminating such semantic techniques and developing a more rigorous concept of proof that modern mathematicians turned to the axiomatisation of informal mathematics. They hoped that by axiomatising a mathematical theory they could conclusively reduce its theorems to a set of self-evident axioms. But self-evidence is elusive! When Russell deduced a contradiction from Frege's axioms for set theory, Frege revealingly confessed of the guilty axiom of abstraction,

> "I have never disguised from myself its lack of the self-evidence that belongs to the other axioms and that must be properly demanded of a logical law."[16]

Such an admission, whether genuine or a manouvre to preserve the theory, undermines the claims of self-evidence as a guide to certainty. To avoid these awkward questions at the very start of their enterprise mathematicians neatly inverted the problem. They developed a formal theory, and then asked whether it had any models.

Clearly a minimal condition for a formal theory to have some model is that the theory be consistent (loosely, free from contradictions). Then it can be argued that the theory characterises some structural pattern. The mathematicians' hope was still of course that this structural pattern could be shown to be a familiar one, essentially that of, say, arithmetic or Euclidean geometry.

The now well-known results of Gödel[17] (and those that followed) destroyed such hopes by showing that the axiomatic method has severe and unavoidable limitations. The area Gödel chose to demonstrate this was that of the arithmetic of whole numbers, the foundation of classical mathematics. The axiomatisation he considered was that of Russell and Whitehead, the lynchpin of the argument that mathematics can be reduced to logically self-evident propositions. Although for his main result Gödel chose a particular axiomatisation of a particular area, he showed how his argument would apply to other axiomatisations of set-theory and of arithmetic.

First Gödel (in a result later strengthened by Rosser) showed that for any consistent axiomatisation of the arithmetic of whole numbers there is some true proposition which is not deducible from the axioms. That is to say that consistent axiomatisations of arithmetic are necessarily incomplete. Another way to express this is to say that any formalisation of arithmetic has non-standard models - that is models essentially different in structure from the intended one. We express this by saying that the axiomatic system is not categorical. Arithmetic and other similar mathematical theories cannot be reduced to a consistent set of axioms from which it is possible to deduce all truths of the system. Any consistent axiomatisation misses some truths of the system. Even worse, Gödel showed that there are certain consistent axiomatisations in which we can deduce propositions which are false. Another way to express this is to say that for such axiomatisations none of the models is the intended one. Finally Gödel demonstrated that no system can be proved consistent by methods formalisable within the system itself.

One way to avoid the force of Gödel's conclusions might seem to be to attempt to circumvent them by proving the consistency of axiomatisations, and filling the gaps shown by the incompleteness theorem, in some meta-theory. But this merely moves the problems of proof and consistency back one level and leads to an infinite hierarchy of increasingly obscure and decreasingly plausible theories; moreover it still leaves the problem of the truth of the axioms untouched.

Hirst and Phenix are caught in a logical fork. They must either abandon the conventional notion of truth entirely and replace it with 'deducibility from axioms' - in which case mathematics so redefined becomes meaningless triviality - or they must admit some combination of deductive and critical methodologies.

Phenix appears at times to take the first course. Certainly he sees mathematics as concerned with the deductive relations between propositional forms within arbitrarily chosen axiom systems. But such a view is inconsistent with his claim that mathematics is about numbers, points, and lines, or even sets. For as we have seen no such conceptual system is unambiguously defined, either in meaning or structure, by any axiomatic system. 'Mathematics' in this first sense is purely about the deductive relations between propositional forms which could as justifiably be interpreted in scientific, moral or aesthetic terms as in mathematical terms.

To follow the second course is to lose the claimed demarcation of the disciplines, and thus entails the abandonment of the disciplines thesis in its present form. It is not clear in what way 'mathematics' in this sense is either a coherent unit or epistemologically distinguishable from science.

The definition advanced by Hirst and Phenix fails, then, to demarcate mathematics as we commonly understand it, either epistemologically or methodologically. Certainly mathematics has developed a concern for logical structure and become popularly associated with that concern. But it has not been, and is not exclusively concerned with logical structure. This developing concern is part of a process of social change, reflected in the continuing methodological diversity and complexity of mathematics. Indeed, many would argue that it is the sustained methodological diversity of mathematics, the counterpoint of criticism and deduction, that confers its power and interest.

(6) Disciplines as traditions of enquiry

This view can be developed to give an account of the disciplines which is both more consistent with our common understanding of them, and restores to them their social and historical identity, while acknowledging the role of logical and intellectual considerations in shaping their development. It sees a discipline as a growing and changing system of ideas within a tradition of enquiry, but recognises that one feature of such traditions is the attempt to impose a simplifying rationality on the growing system of ideas, to develop superordinate principles which help to summarise the results of past enquiry, and to guide future enquiry.

Sometimes a single powerful and economical framework may come to dominate the field. The conception of mathematics as 'the science of quantity' is an example of a conceptualisation which dominated a discipline, and is still not without influence, both as a summary of mathematical knowledge, as a principle regulating mathematical enquiry, and as a criterion used to demarcate mathematics from other disciplines.[18]

As well as periods of relative stability, however, there are periods of dissent and change in the development of a discipline. Several frameworks may coexist or compete within the discipline. For example, the late eighteenth and early nineteenth centuries saw a renewed concern with rigour among some mathematicians, and an attempt to resolve the intellectual problems associated with the concept of the infinite, in order to give the calculus firmer foundations. Other mathematicians were attempting to establish that, in geometry - what we would now call Euclidean geometry - the parallel postulate could be deduced from the other postulates and axioms. The persistent problems and paradoxes of the infinite, and the invention of anomalous 'non-Euclidean' geometries stimulated new conceptions of the nature of mathematics and mathematical truth, new conceptions of proof and its role in mathematics, and new programmes for the development of mathematics, among more philosophically inclined mathematicians.

The development of geometries radically different from Euclidean geometry, but still capable of describing the physical world, seriously undermined existing theories of mathematical knowledge which accorded Euclid's geometry an absolute status. It catalysed a reassessment of the relationship of mathematical knowledge to knowledge of the physical world, and a search for theories to replace the discredited idealist and empiricist theories of mathematics typified by those of Kant and Mill respectively.

This search led mathematicians to reconsider the relation of logic to mathematics. Not only did certain mathematicians start to work in the area of logic in an attempt to illuminate and resolve problems about the foundations of mathematics, they developed conceptions of mathematics which included logic, or even tried to reduce mathematics to logic. Thus an area which had been considered for centuries to be quite distinct from mathematics, and had lain stagnant outside the discipline, was assimilated to mathematics and developed within the discipline, under the influence of these new conceptions.[19]

During the nineteenth century there was a shift from informal to formal methods of mathematical criticism. Attention moved from the semantics of mathematical systems to their logical syntax. As a result many mathematical concepts were 'stretched': so indeed were some logical concepts. The extension of the applicability of 'all' from non-empty classes to all classes removed the 'existential import' of the term and opened up the possibility of vacuous satisfaction, thus changing the very meaning of truth.[20]

The optimism of some mathematicians about the power of formal methods gave rise to a number of programmes which aimed to formalise mathematical knowledge in order to establish that it was reducible to a small number of axioms which were self-evident, or failing that consistent and complete. Gödel's results put an end to this attempt to restore mathematics to its pedestal of absolute truth.

Of course, in the nineteenth century as nowadays most mathematicians were not in the slightest bit interested in foundational problems. They were interested simply in mathematics for itself, or as a useful tool, whatever its ontology or epistemology. Foundational research interested them only to the extent that it threw up interesting new ideas and methods which they could adapt to their own concerns and purposes. Similarly, questions about the foundations of the mathematics which they developed and used did not inhibit them: they were quite happy to pass these problems over to their more philosophically-minded fellows, and to proceed as before.

But here too there were changing frameworks which sought to summarise knowledge and to guide enquiry. The revolutionary definition of geometry as the study of the invariant properties of figures - which underlay Klein's Erlanger programme[21] - assimilated topology to the geometric tradition, and opened both metric and topological geometry to the powerful methods of the structural algebra developing at that time. This reconceptualisation of algebra as concerned with structure rather than quantity is another example of a changing governing framework.[22]

There are cases of frameworks which have had a profound influence at both the philosophical and practical levels - the Greek synthesis of arithmetic and geometry through the theory of proportions,[23] and the Cartesian synthesis of algebra and geometry[24] spring readily to mind - and which have dominated mathematics for considerable periods. By and large, however, mathematics has been characterised by diversity, dissent and change at this level. Mathematicians are heirs to a continuing debate, which has a prescriptive as well as a descriptive aspect, about the nature of mathematical knowledge and enquiry, rather than the passive inheritors of established conclusions.

By contrast, mathematicians, whatever their philosophical or aphilosophical persuasion, are broadly agreed about the subject of the debate. All seek to rationalise, extend or apply the same corpus of knowledge. As I have already observed, different programmes for mathematics exist symbiotically. The application and adaptation of mathematical ideas and procedures to problems outside the discipline raises mathematical problems which are pursued for their own sake, while the continuing enlargement of the corpus enhances the repertoire of concepts and techniques available to the applied mathematician.

Theories, originally developed without regard to use, have found applications - conics in describing planetary motion, non-Euclidean geometry in relativistic mechanics, Boolean algebra in circuit design. Conversely theories developed with applications in mind have been pursued and extended for themselves - calculus has produced a superstructure of analysis which would have bewildered Newton: the theory of groups originated with a problem in the theory of equations and blossomed, via problems in crystallography and atomic physics. The process of reinterpretation and restructuring, the assimilation of new ideas and the synthesis of old ones, and the interplay between different programmes, all help to create broad assent in the demarcation of mathematics. At any one time its extremities may be fluid and controversial, but its interior commands agreement.

Even the philosophers broadly agree. An intuitionist might quibble as to whether parts of mathematics had been properly justified or were justifiable, but he would agree that they were at least mathematical conjectures. Similarly a formalist might be reluctant to accord much of mathematics full status in the absence of adequate formalisation, but he would aspire to give it this status by formalising it. While characterised in different ways and subjected to different methodological demands, the content and boundaries of mathematics are, in general, agreed.

Finally, mathematics possesses a distinctive - if evolving - methodological repertoire which is generalisable over the different branches of the discipline. The same notions of thought-experiment and proof underlie the development, extension and systematisation of different areas of mathematics, and the common notions of modelling influence the development of mathematics in its applicable role. While, during the 17th and 18th centuries, mathematics came close to assimilation to science, since then it has reaffirmed its methodological independence.[25]

This evidence suggests that any conceptualisation which is to take account of diversity, dissent and change in mathematics must admit the logical complexity of the discipline, or at least the controversiality of claims about its logical status. It suggests that a more plausible and accomodating conceptualisation is of mathematics as a changing and growing system of knowledge within a tradition of enquiry.

This socio-historical conceptualisation of a discipline as a tradition of enquiry is still more compelling than the logical alternative proposed by Hirst and Phenix, when we consider those areas where diversity, dissent and change are more explicitly in evidence, even central to the coherence of the unit. In the humanities, for example, disciplines might be better defined in terms of traditions of disagreement, than by any cumulative agreement. The disciplines thesis, reformulated in these terms, appears considerably more plausible, if less pleasingly exact, than in its original logical formulation. In this new version the boundaries between disciplines are potentially more diffuse: indeed, there may be several equally plausible demarcations of the organisation of knowledge and enquiry. From this perspective there is no clear-cut or enduring map of the disciplines. But, as the example of mathematics and the topographical analogy suggest, neither is organisation arbitrary and change anarchic. Change builds on existing rational structures of knowledge and, in general, proceeds within established disciplinary boundaries. It is easy to overestimate the extent of revolution within man's organisation of knowledge and enquiry.

(7) The relativist critique

There is, however, a currently influential school of thought which, at least in some of its more extreme versions, argues that,

> "knowledge is not disinterested and that the construction of a corpus of knowledge is inextricably linked to the interests of those who produce it."[26]

and that,

> "the implications of treating what counts as knowledge as problematic is inevitably to abandon notions of formal logic and to offer no explicit epistemology or truth criteria."[27]

> "Knowledge at all levels, common sense, theoretical and scientific, thereby becomes thoroughly relativised, and the possibility of absolute knowledge is denied."[28]

> "Knowledge and human thought are reversibly one and the same thing. 'Knowledge' is the external face of subjective reality."[29]

From this it is concluded that there are no grounds for claiming that educational or academic knowledge is superior or preferable to the everyday commonsense knowledge which people possess.[30] The teacher is compared to the colonist and the missionary.

> "One group may impose its logic or 'truth' on another and this is a form of colonization, be it the 'truth' a missionary imposes on darkest Africa or a middle class white on an Indian reservation, black-Harlem or lower working-class child."[31]

Further, it is argued that,

> "subjects are mystifications which arbitrarily differentiate and objectify the physical and symbolic universes."[32]

The argument for this position seems to be based on the assumption that claims to knowledge must be either absolute or arbitrary. Evidence of changing systems of ideas - in particular from Kuhn's accounts of scientific change - is taken to exclude the first possibility, and it is concluded that systems of ideas are arbitrary social constructs.

Unfortunately the dichotomy between the absolute and the arbitrary is implausible. Indeed, Young, Esland and Gorbutt's selective deployment of Kuhn's arguments ignores those parts which cast doubt on this view. While they emphasise rupture and paradigm change, the assimilation of existing knowledge to the new paradigm, the rarity of paradigm shifts, and the role of reason in paradigm change are passed over.[33]

The members of this relativist school set up an absolutist straw-man as the representative of objectivism. It is not surprising then, that both Young and Jenks attribute to Hirst an absolutism which he has specifically repudiated, rather than countering the arguments he offers for his position.

Jenks writes,

> "/Hirst/ is legislating for the permanent indubitable status of his 'forms' as the final, inevitable and indisputable paradigm...It is as if the philosopher has placed limits upon the perception of mind and knowledge through the revelation of his objectivist 'forms'."[34]

This certainly runs counter to Hirst's statement that,

> "As distinct from a Kantian approach, it is not my view that in elucidating the fundamental categories of our understanding we reach an unchanging structure that is implicit, indeed *a priori* in all rational thought in all times and places. That there exist any elements in

> thought that can be known to be immune to change, making transcendental demands on us, I do not accept."[35]

Young makes the same accusation of absolutism,

> "The problem with this kind of [philosophical] critique is that it appears to be based on an absolutist conception of a number of distinct forms of knowledge which correspond closely to the traditional areas of the academic curriculum and thus justify, rather than examine, what are no more than the historical constructs of a particular time. It is important to stress that it is not 'subjects', which Hirst recognizes as the socially constructed ways that teachers organize knowledge, but forms of understanding, that it is claimed are 'necessarily' distinct. The point I wish to make here is that unless such <u>necessary</u> distinctions or intrinsic logics are treated as problematic, philosophical criticism cannot examine the assumptions of academic curricula."[36]

But to treat Hirst's position as problematic, and to observe that it provides a basis for supporting what are seen as conservative patterns of curricular organisation, is not to refute it: at least, not as we conventionally argue. It is not clear whether here Young is intentionally following his dictum that,

> "it is in the end personal commitments that are the grounds for action, whether that action is deciding what to do in the classroom or *the 'adequacy' of a researcher's* account. The point is not to ask whether particular

> research methods are, of themselves, 'good' or 'bad', but to ask for what and for whom are we providing accounts."[37]

Intentionally or not, Young points to the Achilles heel of this extreme form of relativism, whose proponents, in Popper's words,

> "invite the application of their own methods to themselves with an almost irresistible hospitality."[38]

Nonetheless, there are eminently <u>reasonable</u> grounds for rejecting the attack of this relativist school on rationality. Quite simply, rationality does not entail an absolutist view of knowledge, nor does the abandonment of an absolutist view necessitate a refusal to judge between competing systems of ideas.

To take an extreme, but illuminating example, philosophers such as Quine and Putnam who adopt a pragmatist perspective on logic have suggested that there could be <u>reasons</u> for revising the (classical) logic we employ.[39] That is, unlike the realist Intuitionists, such as Brouwer, who maintain that classical logic is mistaken, they argue that a choice of logic is to be made on grounds of convenience, simplicity and economy. Indeed Putnam has proposed that the distributive laws of classical logic be abandoned to enable quantum mechanics to be simplified.[40] Such philosophers maintain neither that logic is absolute, nor that it is arbitrary: their project is summarised in Neurath's analogy of 'rebuilding our raft while afloat on it.'[41] The absolutist straw-man bears little resemblance to current theories defending the rationality of knowledge, notably Popper's theory of objective knowledge.[42] Further, even a historical relativist like Kuhn is prepared to defend rationality.[43]

Moreover, claims about the differences between, or the incommensurability of ideational systems - for example 'academic' and 'commonsense' - need to be treated with caution. To stay with the example of logic, to establish that a system of logic is a genuine rival to classical logic, it is necessary to show that the difference between the systems is not simply one of notation or trivial meaning variance. And even if the systems are rivals they may share a substantial semantic and structural core. The relationship between ideational systems has to be analysed more deeply before their incommensurability can be accepted. Indeed Keddie acknowledges this indirectly in observing that Labov's work on nonstandard English established that such speech,

> "can be shown to conform to the strictest principles of Aristotelian syllogisms."[44]

The case for 'academic' or 'educational' knowledge is, first, that in general, it is both more plausible and powerful than 'commonsense' alternatives - although always open to criticism - and second, that without access to this knowledge one is in no position to participate effectively in a society in which, for better or worse, this knowledge provides the grounds for action. Finally, the boundaries of disciplines, while not absolute, are certainly not capricious, but reflect the evolving rationality through which man makes sense of his world. They are fallible, but equally they are defensible.

Despite the extravagance of many of its claims, this relativist school has served a valuable purpose in drawing attention to the tendency of teachers and schools to differentiate and stratify knowledge, and to <u>present</u> it as absolute rather than reasonable.[45]

Freed of its extreme epistemology, the work of this school, like that of Freire,[46] plausibly asserts the importance of relating learning to the experience of the learner. Keddie, for example, emphasises this point.

> "The learning of any 'logic' is a highly situated activity which cannot be treated as though it were context-free if it is to become part of the life world of the learner."[47]

The relativists sound a salutary warning against the reification of human knowledge and enquiry, while illustrating the dangers of an excessive subjectivism.

(8) Conclusion

Despite the disagreements among philosophers and curriculum theorists about the nature of the distinctions between disciplines, there is a considerable degree of congruence between the maps which they draw. Bellack,[48] Schwab,[49] Hirst and Peters,[50] and White[51] all recognise the commonly-made distinctions between mathematics, the natural sciences, the social sciences and the humanities. Hirst, Peters and White wish, of course to make more refined distinctions within the humanities, subdividing that area into aesthetic, religious, moral, philosophical and possibly historical knowledge. Similarly, one could argue for subdivisions of the other areas into units such as geometry, algebra, analysis and statistics, or physics, chemistry, astronomy, geology and biology, or economics, sociology, anthropology and psychology. As far as mathematics goes, the evidence which has been offered of recurrent, and largely successful attempts to synthesise and unify the branches of mathematics, and of their common methodological repertoire, points to the greater plausibility of the larger unit in summarising these traditions of knowledge and enquiry. This is a relatively uncontroversial judgement: the corresponding judgement in relation to natural sciences is more contentious, and is very definitely so in the social sciences and humanities. I make no claims for these areas, but observe that to acknowledge their meaningfulness, as most theorists do, at least reduces a global problem to a number of localised problems. Even the more speculative schemes of Phenix[52] and Broudy[53] start from commonsensically familiar disciplines, which are then grouped into their, sometimes idiosyncratic, organising categories.

This fundamental agreement over a map of man's interpretative systems is manifested in many millions of words. By comparison, little attention has been paid to the analysis of systems of intervention. Broudy's curricular scheme, and Tykociner's[54] analysis of knowledge from which it is derived, are notable in encompassing systems of intervention as well as interpretation. In Broudy's scheme this is confined to a component labelled 'Social Problems'. This he relates to Tykociner's categories of pronoetics (sciences providing for the future; agriculture, medicine, technology and national defence), regulative sciences (social cybernetics; jurisprudence, economics, management and government) and disseminative sciences (education, educational psychology, library science, journalism, and sciences of mass communication).

The absence of any sustained concern with systems of intervention on the part of curriculum theorists, and the lack of a developed analysis of them is to be regretted. At first sight certainly, there seems little doubt that the socio-historical conceptualisation of discipline could fruitfully be extended to cover systems of intervention. While drawing heavily on the interpretative disciplines, areas such as agriculture, engineering, medicine, law and government could be plausibly interpreted as evolving traditions of intervention. On the other hand, while logical conceptions of a discipline, in striving to establish its purity, tend to turn their backs on its applications, the socio-historical conception which I have advanced welcomes this aspect of knowledge, and tends to blur the distinction between interpretation and intervention, which in the development of a discipline are often closely related.

Chapter 3: A Curriculum for General Education

(1) The structure of the curriculum

The practical significance of the disciplines thesis lies in its use in justifying or criticising patterns of curricular organisation. While both Hirst and Phenix are cautious about the implications of the disciplines thesis for curriculum design, it is clear that both are sympathetic to the argument that it is in general desirable to base curricular units on distinct logically defined disciplines such as mathematics. Hirst writes:

> "The logical distinctness of the different forms of knowledge and the close inter-relation of the various elements within a form or sub-division of it, would seem to suggest that the most rational way in which to develop the modes of understanding, would be by direct organisation of the curriculum in units corresponding to the forms."[1]

Phenix and Hirst are agreed on the basic argument for discipline-based curricular units. It is, in the words of Phenix , that,

> "The difficulty with cross-disciplinary studies is that they offer a temptation to shallow, nondisciplined thinking because of the mixture of methods and concepts involved. They require more knowledge and skill, greater care, and better mastery of materials than do studies within a particular discipline, where the lines of productive thought may be kept more directly and continually in view."[2]

While I share with Hirst and Phenix the belief that curricular units which possess a clear and powerful rationale are more likely to be educationally effective, I am sceptical about the particular <u>logical</u> rationale that they propose. This argument that a discipline-based curriculum structure is least likely to lead to confusion between the concepts and criteria which belong to the different disciplines loses its force if, as has been argued in the case of mathematics, any meaningful conceptualisation of the discipline has to admit logical complexity.

Further this argument sits uneasily with Hirst's claim that

> "There is no obvious reason why a form of knowledge based school subject should not deal with many practical applications of the knowledge falling under the form,"[3]

and Phenix's that,

> "It should be possible to teach fundamental studies in such a way as to capture the interest of the student, particularly if ample use is made of examples of applications."

For even if we accept uncritically Hirst's and Phenix's claims about the basis of mathematical knowledge, it is clear that, in applying this knowledge, our reasoning becomes of necessity logically complex. To represent a physical situation in mathematical terms, or to follow a moral argument, requires a synthesis of the conceptual structures that underlie different kinds of judgement.

A further objection to a curriculum organised around discipline-based units is that such units are likely to be introverted, focusing on a single system of interpretation and thus ignoring both the inter-relation of different systems of interpretation, and the role of intellectuality in human intervention. This is a charge which could justly be levelled at some of the discipline-based curricula developed during the sixties which displayed a dogmatic commitment to a single, narrow conceptualisation of the discipline amounting to intellectual imperialism. In propagating a narrow, often introverted view of the discipline these curricula reified and decontextualised it. Small wonder that, for instance, many mathematicians scarcely recognised their discipline in the offerings of the new mathematics curricula.[5]

This is an objection of which Hirst is certainly aware. Indeed, in meeting it, he undermines still further his argument for logically defined curricular units by suggesting that a looser conceptualisation of discipline-based 'subjects' can resolve such problems.

> "But do such interconnections ... necessitate a new non-subject type of curriculum unit? Not if the term 'subject' is taken as widely as it has traditionally been ... What we need are units ... which do not seek to 'integrate' the forms of knowledge, or cut across them for no real reason, but which are true to the dependence of some elements of knowledge on knowledge of other kinds."[6]

It is apparent, in the light of these observations, that the project of basing curricular units on logically defined disciplines is ill founded. Even Hirst abandons a strictly logical definition of disciplines in favour of a more diffuse commonsense one when it comes to planning curricular units.

Criticism of the introversion of discipline-based units has led to a search for 'relevance'. One recent manifestation of this approach can be seen in the demands for educational reform advanced by the New Left-oriented student movement of the late sixties, although many of their ideas were taken, rather uncritically, from Neill and the Progressive movement. It is one of history's ironies that while authority was strenuously opposing these ideas in higher education, they were being promulgated in official pronouncements on primary education. If, the student argument went, discipline-based curricula fragment knowledge and fail to treat society as a totality, thus acting as instruments of mystification, what is needed are 'relevant' units which recognise no intellectual boundaries and focus on human problems.[7] The appeal of 'relevant' units, however, now extends further than the left. In recent years there has been widespread discussion, if rather less adoption, of 'integrated studies' and 'multidisciplinary' courses.

The main kind of alternative unit championed by the critics of the discipline-based curriculum is the issue or problem-centred unit, which examines a particular problem, or class of problems confronting man. This approach seeks to organise the curriculum around the problems which confront man and demand his intervention. Certainly, issue-centred units have much to recommend them. They involve pupils in a direct encounter with the kind of questions which democratic general education aims to help them to resolve. Such units give pupils an opportunity to become familiar with, and to synthesise different systems of ideas, and to relate these ideas to 'real' problems.

A unit focused on 'the energy crisis', for example, might examine the following topics:

(1) What is energy? What forms does it take?

(2) The generation of energy from:

Coal and the hydrocarbons,

Wind, wave, river, sun, tide, earth,

Nuclear fission and fusion.

(3) The uses of energy in:

Home,

Industry,

Transport and communications.

(4) The environmental impact of the generation and use of energy.

(5) What is the energy crisis?

The growing consumption of energy in a changing world;

Population growth,

Urbanisation and industrialisation.

The projection of future trends;

Prediction,

Renewable and non-renewable resources,

Energy conservation.

The politics of energy resources;

Nuclear power and nuclear weapons,

OPEC and the international economy.

(6) Alternative futures and energy policy;

Continued economic growth,

The steady-state economy,

The low-technology society.

Other examples of issue-centred units might be: Industrial democracy, State subsidy of the arts, Science and religious belief.

Despite their undoubted strengths such units have a number of serious weaknesses. First, such units may fail to make clear the generalisability to other issues of the systems of ideas which have been used to analyse and resolve the particular issue at the centre of the unit. Man's understanding of radiation and the structure of matter has enabled him not just to build bombs and to produce electricity on a large scale, but to date archeological and geological finds, to diagnose and treat various medical complaints, to detect metal, whether the hijacker's gun or some ancient buried artefact, and to take photographs by night. Models of exponential growth and decay can be used not just in predicting population, energy needs, and the decline of a radioactive source, but in describing and predicting phenomena as diverse as the loss of dye from clothes, the volume of traffic on roads, the cooling of bathwater, the increase in value of an investment, and the bouncing of a ball. Similarly, the basic economic and political concepts used in analysing 'the energy crisis' are equally applicable to the issues surrounding the exploitation of any natural resource and the production of any commodity.

Certainly these ideas may recur in other units and thus in other contexts, but such repetition is likely only to exacerbate a second weakness, the difficulty of ensuring that the issue-centred curriculum gives sufficient coverage of the fundamental general frameworks of interpretation, and the general frameworks of intervention based on them. We want pupils to understand, for example, the concepts of cell and molecule, as well as that of atom, to understand the relation between these concepts, and the analogies between their analytic functions. We want pupils to be aware that the exponential model is not the only possible model of growth and decay. There are, for example, linear, polynomial, inverse, and periodic models, each of which has distinctive characteristics. And, at a more general level, we want pupils to be familiar not just with the approaches of economics and politics to the study of social life, but with those of anthropology, psychology and sociology.

Third, any curriculum must take account of the dependence of certain ideas upon others, and the grouping of ideas in related clusters. To understand nuclear fission one must understand something of atomic structure: to take another example, the ideas of limit, infinite and asymptote are closely related.

An issue-centred curriculum is at a clear disadvantage when compared to a discipline-based curriculum, both in ensuring a satisfactory coverage of the fundamental interpretative systems, and in taking account of their structure. Indeed, even to attempt to do so would require a second, tacit structure underpinning the overt organisation of the issue-centred curriculum. Certainly a unit focused on 'Mathematical models of growth and decay' or 'Radiation' is much more likely to develop a coherent understanding of these systems of ideas than a number of issue-based units which deal only obliquely with parts of these topics.

Bellack draws attention to previous experience with issue-centred curricula during the Progressive era of the 1920's, 1930's and 1940's.

> "Difficulties in this approach soon become apparent, not the least of which was the students' lack of firsthand acquaintance with the disciplines that were the source of the concepts and ideas essential to structuring

problems under study. Without adequate understanding of the various fields of knowledge, students had no way of knowing which fields were relevant to problems of concern to them. Indeed, without knowledge of the organised fields it was difficult for them to ask the kinds of questions about their problems that the various disciplines could help them answer."[8]

Issue-centred units, then, are unsatisfactory vehicles for the development of an understanding of the interpretative systems on which human intervention and problem solving are founded. They are unlikely to do justice to the structure of these systems, or to their general applicability. They are, on the other hand, particularly valuable in showing how these systems are used and combined in solving problems.

A second possible approach is offered by the <u>activity-centred unit</u>. Activity-centred units focus on some aspect of man's intervention in the world: communication, technology, environmental planning, industry and education are examples of forms of intervention on which such units might be based. Such units are in many ways close to issue-centred units. They are ultimately addressed, if more generally, to the same kind of problem solving, and are equally eclectic in drawing on the fundamental interpretative systems. Hence they share the major disadvantages and advantages of issue-centred units. For this reason they are no substitute for discipline related units. As an alternative to issue-centred units they offer a more general approach which inter-relates different problems, but are open to the criticism that they can deal only in a very fragmented manner with broad issues such as 'the energy crisis' which involve considerations related to several kinds of intervention; government, environmental and economic planning, and technology in this case. Their main advantage over issue-centred units is that they offer a potentially more coherent and economical account of problem solving.

Thus, at closer examination it becomes clear that, while they may be of considerable value in supplementing discipline related units in a curriculum for general education, issue and activity-centred units cannot supplant discipline related units. It seems that, even if the disciplines cannot be distinguished in purely logical terms, they have, as systems of interpretation and traditions of enquiry, an intellectual and social coherence which a curriculum for general education cannot ignore.

Indeed, the shift from the dogmatic, and ultimately implausible logical conception of a discipline to the more flexible socio-historical one points the way to a discipline related unit which is capable of countering the accusations of introversion and absolutism levelled at the conventional discipline-based unit. For the socio-historical conception of a discipline enables us to build an awareness of diversity, dissent and change into discipline related units, rather than evading or suppressing them. Similarly, from this perspective it becomes quite appropriate for a discipline related unit to investigate the relations between that discipline and other systems and traditions of interpretation and intervention. It offers a principled, rather than pragmatic justification for Hirst's strategy of adapting the discipline-based unit in preference to abandoning it. While we must beware of reifying the relation between traditions of enquiry and systems of interpretation and the resulting disciplinary structure, no curriculum can afford to ignore the parsimonious but powerful framework that the disciplines offer for the analysis of the fundamental interpretative aspect of human intellectuality.

(2) The discipline-centred unit and the Scottish democratic tradition

It is against this background that I want to argue for discipline-centred, rather than discipline-based units in a curriculum for general education. This terminology is intended to distinguish between a unit which confines itself to the kinds of knowledge and understanding falling within some system or tradition of interpretation - discipline-based - and a unit which starts from the discipline but considers its relation to other systems and traditions of interpretation and intervention - discipline-centred. The discipline-centred unit makes possible a coordinated exposition of the related systems of ideas which make up the interpretative framework offered by the discipline. At the same time it attends to the application of these ideas to problems of intervention, and to problems of interpretation occurring within other disciplines.

Whereas conventional discipline-based units tend to convey a static and reified view of the nature and methods of the discipline, the discipline-centred unit aims to develop an understanding of the discipline within its changing social, intellectual and historical context. The unit is a reflexive one in which the nature, method and purpose of the discipline is open to investigation. The discipline-centred unit adds to the technical perspective of the discipline-based unit, a cultural perspective.

A precedent for the discipline-centred unit as the basis for a curriculum for general education is provided by the curriculum associated with the 'democratic' intellectual tradition which achieved its fullest expression in the Scottish universities of the eighteenth and early nineteenth centuries.[9] At this time it was normal for students to enter university directly from the parish schools at the age of 15 or 16, to spend four years following a general course in the classics, philosophy and the exact sciences, before proceeding to specialist or professional training. This course included classes in Latin (or Humanity). Greek, Mathematics, Natural Science (or Natural Philosophy). Logic and Metaphysics, and Moral Philosophy. The perspective of philosophy - a 'philosophy' which included the fledgling social sciences as well as metaphysics and ethics - coloured the treatment of the non-philosophical subjects. Great attention was paid to the first principles and metaphysical basis of each of the disciplines, and to their social and cultural context.

In Natural Science, consideration of the principles of scientific enquiry and the experimental method, and the applications of scientific understanding to the development of technology and the analysis of practical problems, was a major element of the course.

Of the mathematical and classical courses Davie writes,

> "The Professors of Mathematics found ... that the best way to render their task of imparting the elements of geometry, algebra and arithmetic interesting to themselves and their youthful pupils was to concentrate on the philosophy and history of the branches of mathematics in question, and to treat the mathematics class as a cultural course, concerned with the relations

of the subject to social life and to the plain man. So, too, something similar happened in relation to Greek and Latin, and, in the process of teaching, the emphasis was much more on the aesthetic value of the poetry than on its grammatical peculiarities, and Professors preferred rather to give some understanding of ancient civilisation than to insist on the business of textual emendation."[10]

Similarly, philosophy, as taught at this time, was as much concerned with the application of philosophical principles to literary, historical, economic, social, legal, mathematical and scientific questions as with the pure study of ethics and the theory of knowledge.

In each class, then, the emphasis was not so much on technical detail as on the philosophical foundations and the 'commonsense' of the subject - its origins and development, and its relation to society. Indeed, the contrary tendencies of the ancient English universities which, in science for example, made,

> "the facts of nature mere pegs on which to suspend festoons of algebraic drapery",[11]

or where, in classics,

> "words are more carefully studied than things",[12]

and,

> "an accurate knowledge of the niceties of ancient languages is often found accompanied by little study of enlarged investigations",[12]

were deplored by the defenders of the Scottish tradition.

For Jardine, the leading ideologue of the Scottish 'democratic' tradition in education,[13] the well-educated man was one capable of making his special or professional concerns publicly comprehensible. The emphasis in the Scottish tradition on the elucidation of general principles, and their application to theoretical and practical problems was designed to produce both an intelligent public, and articulate and readily intelligible specialists.

This Scottish example shows how a curricular unit starting from the ideas of a discipline can radiate outwards to consider the social and intellectual context in which the development of these ideas took place, their intellectual foundation, and their application to practical and theoretical problems outside the discipline. The discipline-centred unit is a compromise between the discipline-based and activity-centred units which can hope to evade the sterile introversion of the former and the eclectic disorder of the latter, while preserving the intellectual structure of the former and incorporating the awareness of the world displayed by the latter.

For these reasons I believe that the discipline-centred unit is basic to a satisfactory curriculum for general education. Discipline-centred units need to be complemented by activity or problem-centred units, but they cannot be dispensed with. For they offer an understanding of the fundamental interpretative systems of thought on which the resolution of practical and theoretical problems, and an understanding of different forms of life are based, without isolating these systems of ideas from the very issues and aspects of the world which give them importance to the common man, and thus underpin the argument for a general education which aims to help pupils sustain and advance political and cultural democracy.

As I argued earlier, it is not sufficient for a curriculum which espouses this aim merely to familiarise pupils with the basic principles of these interpretative systems in isolation: it must actively examine the nature and significance of these ideas, and the ways in which they help man to interpret and intervene in the world.

While it offers a precedent for the discipline-centred unit, the particular form and content of the curriculum of the Scottish democratic tradition are, in some ways, outdated. Indeed an examination of this curriculum and the way in which it was subsequently modified provides an illustration of changing conceptions of disciplines. In the humanities, it reflects an age when literary and aesthetic sensibilities were dominated by the models of Greek and Roman civilisation, and developed in a course which taught the rudiments of the classical languages as a means of access to the classical writers, and to native authors writing in the classical languages. During this period, however, the dominance of classical models was increasingly challenged: this challenge is reflected in the development of the academic study of English literature in the nineteenth century.[14]

While the scope of the humanities was enlarged during this period, that of philosophy was narrowed. While natural science had long since become demarcated from philosophy (although still termed 'natural philosophy' in the curriculum) it was the work of the teachers of this period, Smith, Ferguson, Millar and Robertson which helped to lay the foundations for the development of an autonomous social science.

Similarly, leaving aside the distinctive emphasis on first principles and applications, the actual discipline-based content of the mathematics and physics courses, and the rather rigid separation between them, reflected a classical view of the nature of mathematical and scientific knowledge which was at odds with the developing algebraic-analytic methods in mathematics and their application to physics. Eventually both the humanistic perspective and the classically influenced content disappeared from the Scottish curriculum under the influence of this new mathematical movement and increasing pressure for more specialised university education.

Nonetheless, much of the change which a curriculum plan for the present must recognise has been within disciplinary boundaries. With the important addition of social science, the form of the Scottish democratic curriculum still remains fundamentally true to the summation of knowledge and the organisation of enquiry in our present society. As the example of mathematics suggests, knowledge may have grown immensely, and conceptions of the disciplines changed markedly, but current patterns of knowing and enquiring can be seen to be directly related to those of two centuries ago.

(3) A curricular pattern for general education

This argument points to five discipline-centred units, mathematics, natural science, social science, philosophy, and arts, as the basis for general education. These represent the distinctive intellectual traditions which man can bring to bear on the problems which confront him. The nature and demarcation of the curricular units beyond mathematics is, however, not essential to my argument: I am prepared to accept that further units, or different demarcations may be needed. The important claim which I make, and the one which is central to my argument, is that a satisfactory curriculum for general education should include a discipline-centred unit focusing on mathematics.

At this point it is appropriate to interject a supplementary argument to counter the objection that while these traditions may be distinctive, some of them, and here mathematics or philosophy might be cited, are far from the immediate problems of the world, and deserve little or no place in a curriculum for general education. This argument is, I believe, mistaken. First and most important, it elevates a largely unexamined principle of 'practicality', 'utility' or 'relevance' to a position of unjustified influence on the curriculum. Human aspiration and achievement cannot be reduced to the resolution of immediate or practical problems: indeed there is even dissent about what kind of problems fall into these categories. The imposition of such a narrow view of what is valuable on the curriculum would prevent it from offering the insight into different world-views and forms of life, which, it was argued in Chapter 1, ought to be one of the principal aims of a democratic general education.

Secondly, even if we accept the premise that 'relevance', however defined, ought to be the principal criterion for the inclusion of content in the curriculum, it is certainly the case that parts of _all_ the disciplines are apparently remote from practical concerns. Equally, it is possible to point to parts of all the disciplines which bear directly even on everyday problems; concepts of number, shape and size from mathematics, or of truth, right and responsibility from philosophy, for example.

Finally, while certain systems of ideas, or forms of enquiry may appear at present to have little bearing on practical problems, that does not mean that they will not find such uses in the future. Mathematicians, for example, have been surprised time and time again at the way in which seemingly 'pure' parts of abstract mathematics have found applications and uses. Fifty years ago, few people can have imagined the role that abstract algebra would play in the design of computers, or anticipated the development of operational research, which draws on much of what was then 'pure' mathematics to resolve a huge variety of planning and managerial problems. Indeed, the development of the electronic computer has catalysed an explosion in the application of mathematics which shows little respect for traditional notions of which parts of mathematics are, or are not applicable.

This is no argument for abandoning 'utility' altogether as a criterion: a curriculum for general education which made no reference to it would be indefensible. But it suggests that the sovereignty of this criterion may lead to a selection of curriculuar content which is narrow, and insensitive to social change.

The main deficiencies, however, of popular conceptualisations of relevance are that they emphasise the particular and the concrete, to the exclusion of the general and the abstract; and that they place little or no emphasis on understanding the processes through which knowledge is produced, structured and applied, and the human context of these processes.

In applauding relevance, we imply that education should help pupils to understand and participate in the world, where before it has seemed to turn its back on that world. I have argued that to understand the world we cannot escape the abstraction and generalisation of the disciplines; and that to understand the relation of the disciplines to the world, and the relevance of their abstractions and generalisations to it, we must step back and reflect on the nature and context of the discipline. The fundamental criticism is not of the disciplines themselves, but of their introverted and self-regarding presentation in the discipline-based curriculum.

For these reasons I would insist that the kind of general education that I envisage is highly relevant. But a satisfactory conceptualisation of relevance which is consistent with my argument from democracy would place a strong emphasis on understanding intellectual processes, and the structure of individual and social purpose and consequence within which these processes take place. Each curricular unit should seek to answer certain fundamental questions about the relation to the world of the discipline on which it centres;

What kind of aims does the discipline profess, and how might these aims be justified and criticised?

How do the practitioners of the discipline go about achieving these aims, and why do they do it in these ways?

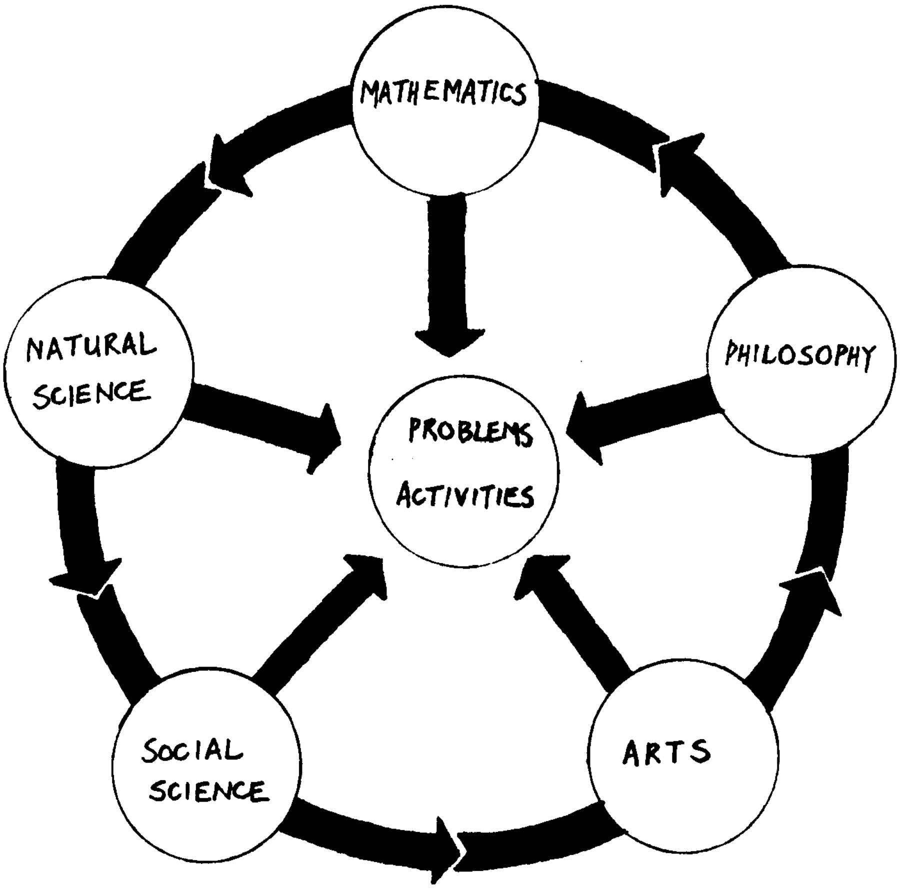
How does what they do affect or reflect a wider society?

It is clearly important, then, that each of the distinctive intellectual traditions should be represented in the curriculum, and taught in a way which illustrates their relation, actual or potential, with the world - this is implicit, in any case, in the idea of the discipline-centred unit - but neither the form nor the content of the curriculum should be restricted to conform to some narrow and dogmatic view of what is practical or useful.

The inclusion of discipline-centred units corresponding to the distinctive intellectual traditions would seem, then, to be a <u>necessary</u> prerequisite of an effective curriculum for general education of the type which I have advocated. The question remains as to whether this is a <u>sufficient</u> condition for democratic general education, bearing in mind the likely limitations of discipline-centred units, and the importance of giving pupils experience in tackling the kind of problems and issues which can only be understood and resolved from a multidisciplinary perspective.

I have already hinted that I do not consider a curriculum consisting solely of discipline-centred units to be sufficient. It seems to me that, while the discipline-centred unit is perfectly adequate to deal with the relations between that discipline and others, and to show how the particular systems of ideas which the discipline offers bear on a variety of problems of human intervention, it does not provide a satisfactory framework within which to consider the synthesis of ideas from several disciplines in solving problems. For such a task any single discipline-centred unit is likely to be a lopsided and ineffective vehicle. There is also the danger that the strong unifying framework which the concept of discipline offers may be undermined by the attempt to include this kind of 'social problem solving' in a discipline-centred unit.

Such considerations support a curriculum plan for general education in which discipline-centred units are complemented by activity or problem-centred units. The function of the discipline-centred units is to give a broad understanding of the disciplines themselves: their purposes, methods, concepts, rationales, development, mutual interaction and interpenetration, and the applicability of the systems of ideas that each offers to a variety of problems. The function of the activity or problem-centred units is to show how to approach particular problems or issues, drawing on the intellectual resources which the disciplines make available.



A Curricular Model for General Education

The outer ring contains the basic discipline-centred units. The arrows round the outer ring indicate that the inter-relations between the disciplines are considered within the discipline-centred units: i.e. the relation of Mathematics with Natural Science, Social Science, Arts and Philosophy will be considered in Mathematics, and so on. The inward pointing arrows indicate that the discipline-centred units examine the application of their distinctive intellectual frameworks to problems of intervention, and provide the ideas and concepts which will be used in the activity or problem-centred units.

Chapter 4: Mathematics in the General Curriculum

I want now to relate the curricular theorising of the preceding chapters more directly to the mathematics curriculum as it is, and as it might be. The argument for democratic general education leads to a demand for the inclusion of a particular kind of mathematical education in the curriculum; one which is concerned to illuminate the purposes, methods, concepts, rationale and development of mathematical activity and argument, the interaction of mathematics with other disciplines, and the mutual influence of mathematics and its social context.

The questions which are central to a general mathematics course are as follows:

What are mathematicians trying to do?

What is the point in doing it?

How do they go about doing it?

Why do they do it that way?

How does what they do affect, or reflect the rest of social activity?

(1) The current mathematics curriculum

The first weakness of existing mathematics curricula is their methodological narrowness. This is a criticism which applies equally as regards the suitability of the curriculum as a training in technical skills, and has been advanced by many who have very different views of general education, or who are solely concerned with the mathematical education of future specialists.

Hirst, for example, may have mathematics curricula in mind when he sounds a note of caution to those who would use his arguments to justify existing curricular units and structures,

> "Many well established courses need to be critically re-examined both philosophically and psychologically before they can be accepted as suitable for liberal education. Superficially at least most of them would seem to be quite inappropriate for the purpose."[1]

Certainly, little of school mathematics is concerned with proof in any meaningful sense: none of it approaches the degree of formal rigour implied by the phrase 'deducibility from axioms'. It is not until the last years of school, by which time the great majority of pupils have left school, or abandoned the subject, that proof comes to play any great role in school mathematics.

The dominant concern of current mathematics curricula is that pupils should become familiar with a collection of informally and loosely justified conceptual systems, and an associated network of standard problems and procedures. The overarching aim is that pupils should become competent users of these conceptual systems, in particular that they should be capable of matching procedures to standard problems, or minor variations on them, and executing the appropriate procedures. So, for example, pupils learn to solve simultaneous linear equations by the techniques of elimination, substitution and matrix inversion, and to recognise and solve 'simultaneous equations problems' such as

> "Twelve expensive flower bulbs and eight cheap ones cost £3.80. Nine of the expensive ones and four of the cheap ones cost £2.65. Find the price of each kind of bulb."[2]

The mathematical argument and activity which pupils encounter in school generally takes the form of simple rule-citing and rule-following. Strictly speaking it may be deductive, but it has little to do with proof as the mathematician understands it.

Of course, one of the original aims of the 'modern mathematics' movement was to counter the lack of concern for method and structure in traditional curricula. It was argued that mathematics ought to be presented to pupils in the same sequence as that used by constructivist logicians in building up mathematics from set theory.[3] But, in practice, this concern that the structure of the curriculum should conform to that adopted by certain of the programmes which aim to give mathematics a unified foundation has had only a slight impact.[4] While set theory has been included as a new topic, it has penetrated the treatment of the remaining curricular content only at a superficial level. There has been a rather half-hearted attempt to anticipate structural algebra through ritualistic mention of the commutative, associative, distributive, identity and inverse laws in the presentation of the ideas of number, matrix and vector. But the new curricula - and this may be no bad thing - lack a coherent deductive thread running through their presentation of material.

The second proposed innovation, a move to 'discovery' or 'active' learning has been little more successful than the first. It is interesting to note however, that the <u>inductive</u> model of discovery-learning which the curriculum planners imported from the revised natural science curricula ran directly counter to the <u>deductivist</u> ideology which underpinned the proposed emphasis on structure in mathematics. So one modern curriculum, for example, allows pupils to 'discover' Pythagoras' Theorem by asking them to find trios of squares which will 'fit' exactly to form right-angled triangles.[5] Indeed, if anything, modern curricula, shorn of the traditional deductive presentation of Euclidean geometry place even less emphasis on proof than their predecessors - however debased a form proof may have taken there.

But, as I have indicated earlier, while proof is a central part of the mathematician's methodological repertoire, it is doubtful whether its role can be properly understood without an awareness of the other parts of this repertoire. This is an argument which has been developed well by Polya[6] and Lakatos;[7] by Polya from the commonsense viewpoint of the practitioner, by Lakatos from the viewpoint of the philosopher. Both are concerned to show that there is more to mathematical method than proof and deduction: in addition, Lakatos offers a novel and powerful insight into the role of proof in mathematics, and its relation to other parts of the mathematician's methodological repertoire.

Polya starts by pointing out that it is not merely the form in which a mathematician finally presents his argument which is of methodological interest, but also the process of enquiry through which the result was formulated and a proof constructed. He refers to this process as plausible reasoning, and to the formal canons governing proof as demonstrative reasoning.

> "Finished mathematics presented in finished form appears as purely demonstrative, consisting of proofs only. Yet mathematics in the making resembles any other human knowledge in the making. You have to guess a mathematical theorem before you prove it; you have to guess the idea of the proof before you

> carry through the details. You have to combine observations and follow analogies; you have to try and try again. The result of the mathematician's creative work is demonstrative reasoning, a proof; but the proof is discovered by plausible reasoning, by guessing."[8]

Polya describes the informal methods which mathematicians use to extend their knowledge, the <u>mathematical heuristic</u>. His main concern is with guessing, and with distinguishing a more reasonable guess from a less reasonable one. He outlines the ways in which the processes of specialisation, generalisation, analogy and the examination of consequences can help to generate and criticise conjectures.

Polya's argument has been developed by Lakatos who argues,

> "that informal, quasi-empirical, mathematics does not grow through a monotonous increase of the number of indubitably established theorems but through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutations."[9]

Lakatos considers that, while Polya has done full justice to the place of guessing, or <u>naive conjecturing</u> in the mathematical heuristic, he has ignored the important role of proof. Lakatos observes, following mathematicians such as Hardy, Littlewood and Wilder, that proofs seldom actually prove. Rather than dismissing proof entirely, however, he argues that its significance lies in the way that it forces the improvement of naive conjectures, and throws up new <u>deductive conjectures</u>. Thus, for Lakatos, the presentation of a mathematical theory in demonstrative form is merely a convenient, if opaque summary of the criticism of previous conjectures and proofs. The mathematical enquiry of Lakatos, like Popper's scientific enquiry, "begins and ends with problems".[10]

Polya and Lakatos allude to the different kinds of judgement which may enter into mathematical enquiry: profundity, generalisability, simplicity, economy, applicability and beauty are examples. Of course, in the kind of enquiry within established conceptual frameworks with which Polya deals, such criteria are subordinate to those of validity. On the other hand, Lakatos suggests that at the leading edge of mathematical enquiry where conceptual frameworks are in the process of development, such considerations may be paramount.

Although Polya and Lakatos are predominantly concerned with 'pure' mathematics, the case of 'applied' mathematics is in many ways analogous. First note, however, that the name is unfortunate: it suggests the application of a mathematics that is already there. Very often, of course, quite the reverse is the case: some elusive 'real' phenomenon or problem inspires a new piece of mathematics. Whatever the case, the plausible reasoning surrounding the development of mathematical models and techniques involves processes of conjecture and criticism similar to those already discussed in the 'pure' case. In particular, whether the applied mathematician draws on existing mathematical structures or develops new ones to meet a particular problem, he seeks models or techniques which 'fit' well, and are simple, generalisable, clear and reliable.

These views lead Polya[11] and Lakatos[12] to be critical not only of the deductivist approach to the presentation of mathematics -

where convoluted axioms and definitions spring from the mathematical conjuring box to lead inexorably to seemingly unguessable theorems by means of proofs, full of apparently arbitrary twists and turns, from which the final result finally, and often unexpectedly emerges - but of any authoritarian presentation where the reasoning behind the final edifice is passed over.

Polya argues that pupils ought to learn plausible reasoning in their mathematics course. Here his main argument is from the needs of the future mathematician. He suggests, however, that the non-specialist will find that the strategies of plausible reasoning in mathematics are applicable in other areas.

Both these arguments need to be treated with caution. I have already argued that the needs of the future specialist ought not to shape a curriculum for general education: it may be that the general curriculum turns out to meet specialist needs, but these should not be allowed to distort its purpose. And, given the lack of plausibility of the classical 'transfer of training' theory, experience does not portend well for the second argument, which implies a transfer of plausible, rather than demonstrative reasoning from mathematics.

The work of Polya and Lakatos, in combination with the learning theories of psychologists such as Bruner and Piaget, has encouraged, and been used to legitimise, 'active' or 'discovery' learning approaches to the teaching of mathematics. Again, there are a number of dangers in this argument.

First, it has helped to encourage a naive conflation of cognitive theories of learning, heuristic theories of mathematical enquiry and deductivist theories of mathematical knowledge, which has led to the phenomenon I have noted already - a pedagogy based on a deductivist ordering of content within which new material is introduced from an inductivist perspective: a pedagogy ultimately false to all three theories.

Second, for this approach to lead pupils to knowledge of the conceptual systems and procedures institutionalised in our society, activity or discovery must be guided. Even Dawson[13] who cites Lakatos at length seems to miss the fundamental point of Lakatos' argument that ultimately man creates mathematics. If this is indeed the case, then it is not sufficient, as Dawson suggests, simply that pupils should possess the skills and attitudes needed to attack problems in a rational and critical fashion. In order to recreate our mathematics they must have access to the past judgements institutionalised in our mathematical systems. Of course, very little discovery learning is unguided, and even less recreates mathematical activity authentically. Certainly, to produce authentic mathematical enquiry it would be necessary to disengage it completely from the acquisition of mathematical content. The two aims are incompatible in the same learning activity.

Third, the argument for active learning often runs ahead of itself. Although primarily an argument about means, it is often not clear whether active learning is being proposed simply as a means to existing ends, or whether it encompasses completely new ends, or a mixture of old and new. The argument would benefit from a clarification of ends before active learning is considered as a means to these ends.

The criticisms of the methodological narrowness of the conventional mathematics curriculum offered by Polya, Lakatos and the proponents of active learning, although they need to be handled with care, offer important insights which can be used in building a mathematics curriculum for general education. In particular, they draw attention to certain aspects of the hidden curriculum of present courses, the tacit framework of assumptions and beliefs which these courses transmit to pupils.

The lack of attention to the process of mathematical enquiry and the authoritarian presentation of mathematical knowledge encourage a view of that knowledge as unquestionable, and fundamentally unchanging - the development, reinterpretation and restructuring of knowledge, and the reasoning which sustains it and gives it coherence are passed over. The highly structured tasks generally set for pupils, promote a view of mathematical activity as the routine application of techniques, and capricious juggling with conceptual systems. This, and the occasional encounter with proof, where the outcome is known in advance, and line follows line with military precision, paints a picture of mathematical enquiry as a relentless march towards a pre-existent truth. Such a state of affairs is hardly desirable in any mathematics course; certainly not in one which aims to open up the discipline to the nonspecialist, to give him an insight into mathematics as a form of enquiry, a way of understanding.

Methodological narrowness is not the only weakness of existing mathematics curricula as instruments of general education. Present mathematics curricula and the great majority of their critics hold in common the assumption that the over-riding aim of a mathematics course should be to teach pupils to 'do maths', to develop mathematical knowledge and expertise. Where they differ is either in their conception of what counts as 'doing maths', or over the kind and extent of knowledge and expertise which pupils require.

It may seem perverse to draw attention to this assumption, let alone to question it. Certainly such an assumption would be fully justified in a course with directly utilitarian ends in mind, equipping pupils with the technical expertise needed in everyday life, in some vocation, or for further specialist education in mathematics or mathematically-based subjects. All these ends require, primarily, that pupils should, in some sense, be capable of 'doing maths'. Nonetheless, the readiness of our assent to this assumption is only an index of the extent to which utilitarian and specialist conceptions of education take priority over generalist conceptions in mathematical education. Indeed, only a weak conception of general education which identifies it with diluted or discontinued specialist education is capable of coexisting with the currently dominant, utilitarian and specialist models of mathematical education.

This is not to suggest that utilitarian and specialist aims have no place in the school curriculum, but simply that they are no substitute for generalist aims, and that the latter should not be subordinated to them. Nor is it to suggest that a mathematics course based on a strong conception of general education, such as the one I have advanced, can, or should not aim to give pupils some measure of technical knowledge and expertise. For general education, however, this aim is not paramount, and entails a broad understanding of central mathematical concepts rather than a detailed knowledge and a developed manipulative competence. The development of a measure of technical expertise is, in a general mathematics course, instrumental to the attainment of a wider understanding of mathematics and mathematical activity.

The reason why developing the ability to 'do maths' cannot be the over-riding aim of a general mathematics course becomes more apparent if we imagine a course incorporating the changes recommended by Polya, Lakatos and the proponents of active learning - one incorporating a heuristic presentation of content, and aiming to develop the skills of plausible reasoning - and note that it too has an undesirable aspect to its hidden curriculum, particularly if it ignores, as the forementioned critics of present curricula do, applied mathematics. Quite simply, such a course presents mathematics as socially disembodied, ignoring its past and present interactions with the society which nurtures it. While such a course acknowledges the dynamic nature of mathematics, it presents it as an autonomous area of knowledge and tradition of enquiry, isolated from a wider society.

(2) Designing a general mathematics curriculum

I am now in a position to outline the kind of mathematics course which would satisfy the criteria for general education advanced in previous chapters, and take account of the criticisms I have made of existing courses and commonly proposed alternatives.

(i) Aims

Briefly, such a course has four interdependent and over-riding aims:

(a) to familiarise pupils with the central principles and rationale of the major conceptual systems of mathematics,

(b) to give pupils an insight into the nature of mathematical enquiry, argument and knowledge,

(c) to give pupils an insight into the development of mathematics and its relationship to a wider society,

(d) to develop the ability of pupils to apply this knowledge and understanding to their concerns as individuals and as citizens of a democratic society.

(ii) Technical skills

Before I amplify this brief statement of aims I want to say something about the relationship of general mathematical education to the kind of education in mathematics which aims to develop the technical knowledge and expertise which will be of use to pupils in their everyday lives, or in work or further study. There is certainly a core of technical mathematical expertise which is likely to be of direct use to the majority of pupils, although, inevitably, there is some disagreement about its precise boundaries. It is clearly desirable, on these grounds of direct utility, that all pupils should become familiar with some core of technical mathematical expertise, as well as pursuing the general mathematical aims for which I have argued. In general, however, existing curricula go far beyond any definition of an essential utilitarian core into areas of technical expertise which will conceivably be of direct use only to a small minority of pupils. It is material of this sort which must be revised or excised in planning a common course in mathematics.

A common course in mathematics should aim to familiarise pupils with a widely useful core of technical mathematics. This need not detract from, nor impede its over-riding general aims. Indeed it is likely to be instrumental to their achievement. And while a common course is not directly concerned with specialist aims, it can be designed so as to provide a firm foundation for subsequent specialist courses. Other things being equal, material of value to the intending specialist can be chosen as a vehicle for general aims. While such a course is unlikely to produce the level of technical sophistication at which present courses aim, to compensate for this it has a wider methodological compass, and offers the potential mathematician a better grasp of the nature of his subject. And it aims, of course, to familiarise him with the central concepts of his specialism.

It is important that the material of the 'everyday' technical core should not be presented in a way which encourages a mistaken view of mathematics, and thus prejudices the attainment of general aims. The core, as it is usually and, I believe, correctly conceived, focuses on the basic conceptual systems of number, shape and size, and the techniques to which they give rise: the aim is to develop the ability of pupils to apply these systems to commonly encountered situations, and to correctly and easily execute the appropriate mathematical procedures. Here the immediate social significance of the ideas is something of which pupils are already likely to be aware: what is not apparent to many pupils, and not made clear by current approaches is the rationale behind the mathematical systems and techniques which we all use in everyday life.

(iii) Rationalisation

For example, in the current curriculum to learn about percentages is usually simply to learn to use a collection of procedural rules - changing between percentage, decimal and fractional form, and calculating a given percentage of some quantity.

At best these procedures will be embedded in the conceptual framework of fraction, ratio and proportion. But this still leaves many questions unasked and unanswered;

Why do mathematicians use decimal <u>and</u> vulgar fractions <u>and</u> percentages to represent parts and relative sizes? Why not have a single system? And if it is worth having three systems, in what situations, or for what purposes is each preferable?

Does the common denominator for the percentage-idea have to be 100? Would any denominator do? What numbers might be rivals to 100? What considerations led to the choice of 100? Are any other systems used in similar or analogous situations - weights and measures, angle measure?

To answer these questions is to reconstruct the rationale behind the development and continuing use of percentages.

Often such questions can be built into a heuristic presentation of a topic. For example, one approach to the exposition of methods of measuring and calculating area is through what Lakatos would term the <u>rational reconstruction</u> of their development. Starting from an intuitive notion of area, and the simple case of comparing the size of two shapes where one can be fitted inside the other, the complex superstructure of ideas and techniques which mathematicians have built on this can be made articulate and rational. A fuller outline of this example can be found in Appendix 1.

The adoption of a heuristic approach, showing the rationale behind the development and use of mathematical methods, and locating them in their historical and social context means that both utilitarian and general aims can be satisfied within this part of the common course. It is also possible that an understanding of the rationale of techniques will help pupils to use them more adaptably and intelligently.

(iv) Methodological and cultural elaboration

The greater part of a common course needs, however, to be designed with general aims directly in mind. Nonetheless, this part can build on the basic mathematics of number, shape and size. Number theory, for example, offers a rich and readily accessible domain in which to illustrate the mode of enquiry of the pure mathematician. Whereas in traditional Euclidean geometry many of the results proved at great length seem trivial and intuitively obvious, surprising, and often deep conjectures abound in number theory. Furthermore, the familiarity that pupils already have with numbers makes this an ideal area in which to encourage them to formulate, test, and attempt to prove conjectures of their own. And, at the same time, such activity reinforces and enhances basic number skills.

Arithmetic is also a good area in which to illustrate the idea and problems of axiomatisation - although this is likely to be best dealt with later in the course. Nor need arithmetic be unremittingly pure. Sequences provide some of the simplest and most elegant mathematical models which are of very wide applicability. The need for some kind of convenient notation soon becomes apparent in this kind of work and can provide a motivation for introducing simple algebraic notation.

The formal emphasis in traditional geometry may account for its minimal concern with applications, limited, usually, to mensuration, simple constructions, and the elements of scale drawing and navigation. This state of affairs has only been confirmed in the modern change to motion or transformational geometry. Surprisingly, the wide applicability of simple geometric ideas to the design of engineering structures and mechanisms - bridges, buildings, playground toys, household gadgets, bicycles and motor vehicles, drills, cranes and the like - has been ignored.[14] Similarly the relation between geometric ideas of pattern and art and design has been little exploited. The use of tesselation in Celtic and Islamic art and the graphics of Escher, the symmetries of common logograms, the development - and limited realism - of perspective, anamorphic art, op art, and the design of containers and packaging are examples of topics on this interface.[15]

While elementary geometry may not be a good area in which to illustrate proof, the study of pattern in geometry, notably in tilings and tesselations is a valuable precursor to the study of the concept of area.

(v) <u>The choice and presentation of central concepts</u>

Extension and elaboration in the presentation of basic mathematics can help to fulfil general aims. But a course which went little beyond elementary arithmetic and geometry could hardly convey an impression of contemporary mathematics. The question then arises as to what material to include beyond this basic mathematics. There are a number of criteria which can be used.

First, the systems of ideas chosen should be powerful, significant and versatile, and should be pursued to a point where these qualities can be illustrated. One serious criticism of modern mathematics curricula is that pupils spend a great deal of time on material which is of strictly limited mathematical value, and often go no further than learning definitions whose significance will not become apparent until some subsequent course to which the majority will never proceed. It is imperative that a common course be self-sufficient and self-justifying. The material which is included must be both central to mathematics, and developed to a level where its purpose and value can be appreciated. And, of course, the material chosen must be a suitable vehicle for illustrating the methodological and cultural dimensions of mathematics.

The result of applying these criteria is likely to be a more single-minded pursuit of a smaller number of areas than is currently the case, and the use, whenever possible, of 'current interest' material which is relevant to mathematics. Furthermore, by concentrating on a core of central mathematical concepts, a generalist common course can provide a sound foundation for subsequent specialist study.

If any field is central to mathematics it is analysis. This is the fundamental tool of the applied mathematician, and in and around it the pure mathematician has built his most elaborate theoretical edifice. Through an informal, heuristic development of the calculus, it is possible to explore both mathematical modelling and - from Zeno's paradoxes[16] to Peano's and Hilbert's space filling curves, and Von Koch's snowflake curve[17] - examples of the methodological problems which stimulate the development of mathematical systems, and influence our view of mathematics.

Another field which calls for inclusion in a general mathematics course is that of probability and statistics, on the grounds both of its widespread use, and its influence on ideas of mathematics.

Although originating as an adjunct of gambling it is now widely used as a modelling device in the natural and social sciences, and in management and planning.

A third area which no general mathematics course can avoid is that of computing. First its social impact is potentially enormous - we are only starting to appreciate its influence on patterns of employment, and the threat which its unsupervised use presents to civil liberties. Second, it has immensely extended the power and potential of applied mathematics, and - on the evidence of Appel and Haken's recent proof of the four colour conjecture[18] - may have as great an effect on methods in pure mathematics. Finally, work in the theory of automata raises important questions about the nature of mathematical thought and the limitations of mathematical systems, as well as wider questions about the nature of 'intelligence' and 'creativity'.

Beyond this there is likely to be much more dissent. One area which I feel has a great deal to contribute to a general course is that of combinatorial and graph theory: first, it is a readily accessible and fertile area of mathematics in its own right; second, its applications are wide and relatively easily understood; third, many of its ideas are applicable to probability and computing; finally, it provides elementary examples of non-metric geometries to contrast with the metric geometry which the pupil has already met.

Although this content would be an excellent foundation for subsequent specialist training, a common course should present it in a way very different from that appropriate to specialist courses. For, in this area beyond basic mathematics, the acquisition of technical skill is only instrumental to the achievement of the wider generalist aims of understanding mathematics and mathematical activity as a whole, and of relating them to their social, historical and intellectual context. These aims call for a presentation which is informal and heuristic, and refers to the development, use and impact of mathematical ideas to a depth and degree much greater than present courses, where, if history enters at all, it consists solely of attaching a name or anecdote to some theorem or technique. A satisfactory history of mathematics asks why ideas became important and were pursued, and what effects they had inside and outside the discipline.

The difference in approach is likely to be particularly marked in applied mathematics. The aim of the course is not that pupils should memorise and acquire proficiency in the use of, say, the Newtonian square root algorithm, or that they should be capable of solving convoluted problems on confidence limits. It is that they should understand what an algorithm is, why and where they are used, and why some algorithms are preferable to others; or the meaning and importance of confidence limits, and that these limits depend on certain assumptions about the initial data. Clearly, then, considerably less time needs to be spent on acquiring detailed technical knowledge and practising technical skill, and a great deal more on developing an understanding both of the general processes of mathematical reasoning and the general structure of mathematical argument, and of the relation of mathematics to human life as a whole.

It will be apparent from this discussion of content and its presentation that the aims of general mathematical education cannot be usefully separated. The presentation of content and the pupil activity which accompanies it convey, implicitly or explicitly, a view of mathematics and mathematical activity. Recognising this, a general mathematics curriculum uses the presentation of content and related pupil activity to initiate and illustrate reflection on mathematical enquiry and argument, and on the wider social relations of mathematics.

(vi) Pupil activity

While a didactic presentation of mathematics, through rational reconstruction, is possible in all these areas of content, some lend themselves particularly well to actual pupil participation in mathematical activity. Number theory and combinatorial and graph theory are excellent media for pupils to make and test their own conjectures, and to construct and criticise proofs for themselves. Similarly, as their technical expertise in combinatorial and graph theory, probability and statistics, the theory of functions, and finally, analysis accumulates they can start to build and evaluate simple mathematical models. In computing they can quickly start to design and execute algorithms and problem-solving strategies.

This experience of mathematical activity on the part of pupils is an important part of a general mathematics course; first and foremost, because only a very limited understanding of mathematics is likely to be achieved by someone who has never had this experience, but second, because pupils ought to have an opportunity to find out whether they enjoy and value mathematical activity.

(vii) Reflection

On and around this experience of mathematical activity, and the presentation of content much of the reflective work of the course can be built. Often, indeed, heuristic presentation or pupil activity can be built around historical or social themes - 'the development of counting systems and calculating devices', 'methods of presenting information', 'models of growth and decay', or 'paradoxes' - through which the social context and impact of mathematics, and the nature and limitations of mathematical knowledge and argument can be explored. Even where this thematic continuity is not possible, consideration of the development, rationale and implications of different pieces of mathematics is likely to throw light on the broad issues with which the reflective part of the course is concerned. Later such insights can be summarised and synthesised in a lesson dealing more directly with reflective issues.

For example, the contrasting responses of Greek and Babylonian mathematicians to the knowledge that certain numbers could not be expressed in rational form, and the arguments that raged between mathematicians throughout the eighteenth and early nineteenth centuries about the value of establishing rigorous foundations for the calculus, exemplify the persisting coexistence of very different views about what is important in mathematics, and of the nature and purpose of mathematical enquiry. Descartes' search for a universal method, reflected in his synthesis of algebra and geometry, and contemporary work in metamathematics and artificial intelligence demonstrate a similar concern to extend, or at least demarcate the power of mathematical enquiry. And both have significant implications for our use of concepts such as 'intelligence' and 'imagination' in relation to mathematical activity.

Examples of the social impact of mathematics abound, from the role of surveying in the government of Ancient Egypt, through the influence of ballistics on seventeenth and eighteenth century warfare, to the effects of the computer on our society.

The reflective part of a general mathematics course seeks to illuminate mathematics and mathematical activity as an intellectual and social phenomenon, to make pupils aware of the nature and context of mathematical activity, and of its role in resolving - and creating - intellectual and social problems. In this way the course provides a cultural perspective to complement the technical perspective in those parts which deal with central concepts and mathematical activity.

(3) Summary

I will summarise this chapter in terms of a simple model for describing the subject matter of mathematics courses. First, this model distinguishes between, on the one hand, 'doing' or learning to 'do' mathematics - the perspective of the participant - and, on the other, stepping back to examine mathematics from some wider perspective; that is, between the Articulation of the methods and concepts of mathematics, and Reflection on mathematics.

Within Articulation the model distinguishes between a presentation of mathematical methods, concepts and activity which is concerned solely with establishing, or laying down and acting in conformity with a body of 'correct' or conventional systems, rules and relations, and a presentation which goes beyond this to examine other kinds of evaluations and reasons underlying the structure of mathematical methods, concepts, arguments and activity. The first, Standard Articulation, takes the framework of mathematical systems, rules and relations for granted. While it may be concerned with relations within the system, it does not examine the basis of the system, or the intrusion of considerations logically external to it, in its use. Nonstandard Articulation does examine such aspects of mathematical systems, arguments and activity. For example, using a given Newtonian model of motion to resolve a problem about the path of a projectile is Standard Articulation; discussing the appropriateness or accuracy of the model, or the clarity and economy of two alternative methods of solution which employ the model, is Nonstandard Articulation.

This is an important distinction. A course which emphasises Standard Articulation to the exclusion of Nonstandard presents mathematical activity as solely concerned with 'getting the right answer' within some taken-for-granted framework of rules. Now, while this describes certain parts and aspects of mathematical activity, it is, as the arguments of Polya and Lakatos establish, an inadequate one. Further, as Lakatos is aware, such a course tends to encourage a view of mathematics as some kind of ultra-physics - or as an arbitrary and capricious game. Heuristic presentation, or open-ended mathematical activity which aims to give an authentic insight into the growth, development and application of mathematics cannot avoid consideration of Non-standard evaluations and reasons.

Within Reflection, the model distinguishes between Methodological Reflection - discussion of philosophical and psychological questions about the nature of mathematical knowledge, argument, enquiry and thought, and the judgements underlying them - and Cultural Reflection - discussion of social and historical questions about the development of mathematics, and its interaction with a wider society, in particular, its impact on man's world and ideas.

I have argued for a general mathematics course which concerns itself, to a substantial degree, with Reflection of both kinds, and which embeds its Standard Articulation in the Nonstandard

Articulation which underpins and augments it. I have suggested that current courses cannot meet these criteria; that they ignore both kinds of Reflection, and pay little attention to Nonstandard Articulation. This suggestion provides the starting point for the empirical study described in Part II.

Part II: An Empirical Study Of Mathematics Teaching

Chapter 5: The Research Problem

In Part I I argued that a strong conception of general education entailed a commitment to a mathematics curriculum very different from that which is current in Scotland. I will now present empirical evidence that my characterisation of the present Scottish curriculum is justified; that it does indeed differ in certain crucial respects from my model of a general mathematics curriculum.

School attendance in Scotland is compulsory until the age of 16. The great majority of pupils complete at least four years of secondary education. It is in the years S1 to S4, then, that we would particularly hope to detect the influence of general educational aims on the Scottish mathematics curriculum, and it is here that the study which I will describe sought evidence of the influence of such aims.

First, however, one important point must be clarified. In what sense is it meaningful and justifiable to talk of 'a Scottish mathematics curriculum'?

(1) The Scottish mathematics curriculum

The Scottish educational system forms a single administrative structure with the Scottish Education Department (SED), and latterly the Consultative Committee on the Curriculum (CCC) and the Scottish Certificate of Education Examination Board (SCEEB) at its apex. The normal, and officially endorsed organisational pattern divides S1/S4 into a two year common course in S1 and S2, followed, at the start of S3, by the allocation of pupils to two year certificate or noncertificate courses in individual subjects.

The certificate courses lead to presentation in the SCE 'O' Grade at the end of S4. Virtually all pupils follow some kind of mathematics course throughout S1/S4. In S1/S2 all follow a common course, although in the many schools where ability setting is introduced at the end of S1 (or even earlier), the second year of the course is likely to be common only in name. At the start of S3 around 60% of pupils embark on a course aimed at presentation in both Mathematics and Arithmetic at 'O' Grade, and a further 25% on a course aimed at presentation in Arithmetic only. The great majority are still following the same course at the start of S4, although many are not eventually presented, and still fewer are finally successful, as the following table shows.[1]

Career of a cohort of pupils through S3 and S4 (1974-76)

Subject	% studying a certificate course in the subject at the start of S3	% presented in the subject at 'O' Grade	% A-C award	% D-E award
Arithmetic	84	64	40	14
Mathematics	61	35	21	9

Base for %: total number of pupils at start of S3

There are two SCE syllabuses in Mathematics, A and B. Each is associated with a set of nationally prepared curriculum materials specifically designed for that syllabus: the series of textbooks 'Modern Mathematics for Schools' (MMS) is associated with Syllabus A, the 'Modular Mathematics' workcard/sheet system with Syllabus B. The two syllabuses run broadly parallel throughout S1 and S2 before diverging to some extent in the later two years. As a result, a number of schools which only present candidates on Syllabus A make use of Syllabus B materials during the common course. Perhaps around 10% of Scottish schools make use of 'Modular Mathematics' materials during the common course. In the remainder, the common course is almost invariably based on MMS: indeed, almost all use MMS as the class text.

The great majority of S3 and S4 certificate pupils follow courses aiming at presentation on Syllabus A: 98% of presentations are on this syllabus.[2] Those following the double course in both Mathematics and Arithmetic almost invariably use MMS. The more 'able' of those following the single course, if they have a text-book, may use 'Modern Arithmetic for Schools', a compilation of the arithmetic chapters from MMS.

This evidence points to a single dominant mathematics curriculum in S1/S4 which is adapted in various ways to take account of what are perceived as the differing abilities of pupils. In particular, in S3/S4 some pupils study only the Arithmetic part of this standard course, while others on noncertificate courses study a diluted version of this already curtailed course. To this extent it is appropriate and justifiable to talk of a Scottish mathematics curriculum.

(2) The empirical study

The first step in planning the study was to decide what kind of evidence was needed, and how it could be obtained. In fact, this stage of the research took place in parallel with the development and clarification of the theoretical concepts which were introduced in Part I. This first stage combined participant observation in schools with reading in areas - the philosophy, history and sociology of mathematics and science, curriculum theory, and studies of teachers and teaching - which might illuminate both the fundamental theoretical issues raised by general mathematical education, and the ways in which theoretical conjectures could be empirically tested. From this stage came the outlines of the study.

This study, described in the following chapters, is in two parts. The first examines three critical manifestations of the curriculum - classroom talk, curriculum materials and examination papers - in order to provide evidence on which to judge the extent to which the present Scottish mathematics curriculum satisfies the criteria for general mathematical education which have been advanced in the preceding chapters.

Classroom talk and curriculum materials are the major direct influences on the outcomes of pupil learning. National curriculum materials and examinations represent the 'official' curriculum. They are likely to exert a powerful influence on teacher behaviour, and thus indirectly on pupil learning.

The second part of the study examines the views of mathematics teachers on the role of mathematics education in the first four years of the secondary school, and the ways in which they approach the teaching of mathematics in these years.

For convenience I shall refer to the first part as 'the curriculum study', and the second as 'the teacher study'.

Chapter 6: The Curriculum Study - Aims and Methods

(1) The aims of the curriculum study

The principal hypothesis which the curriculum study was designed to test is that the present mathematics curriculum is almost exclusively concerned with technical expertise in mathematics - how to 'do maths' - rather than with the nature of mathematical knowledge and enquiry, and the social, cultural and intellectual context of mathematics.

An auxiliary hypothesis is that the conception of 'doing maths' current in the curriculum centres on using, or working within certain given frameworks of rules which define what is 'correct' or 'appropriate', ignoring the more fundamental issues of validity involved in constructing, and justifying the application of these frameworks, and the use of criteria other than those of validity in evaluating mathematical procedures and constructs.

(2) The development of an analytic instrument

The research strategy adopted for the curriculum study was a systematic content analysis of classroom talk, curriculum materials, and examination question papers. A search of the literature revealed that there was no content analysis instrument available which could be used or suitably adapted to meet the purposes of this study. A new instrument, the Mathematics Topic Handling System (MATHS), was developed which embodies the critical distinctions underlying the hypotheses described above.

The full instrument is described in Appendix 2. In brief it operationalises the theoretical distinctions introduced in Chapter 4, between Methodological Articulation, both Standard (SMA) and Nonstandard (NMA), and Reflection, both Methodological (MR) and Cultural (CR). Topics falling into none of these classes are categorised as Residual (RES).

I will discuss the methodological issues raised in the development of MATHS in the context of the classroom version which is the most complex and difficult to use. The basic rationale is, however, the same for all versions.

The central issue in developing a content analysis system is the definition of a unit of content to which the theoretical distinctions, which the system seeks to operationalise, can be tied. The 'topic' on which MATHS is based, starts from the work of Gallagher.[1] Gallagher's 'topic' is a more flexible unit for content analysis than, for example, the 'venture' of Smith and Meux.[2] In particular, it serves better as a model of loosely structured talk. By allowing the interleaving and interpenetration of themes the 'topic' model can accomodate digression and the parallel development of themes, whereas the 'venture' model tends to make sense of such talk by ignoring it, or by absorbing it into some larger unit.

The 'topic' is also more suitable than units such as the 'episode' of Smith and Meux,[3] the 'incident' of Nuthall and Lawrence,[4] or the 'topical cycle' of Bellack,[5] which only model question-centred dialogue, in view of the likelihood - apparent from the pilot studies - that a high frequency of teacher monologue would be found in the observed classrooms.

Nonetheless MATHS differs from Gallagher's Topic Classification System (TCS) in a number of ways. First MATHS is concerned with a different facet of classroom talk. Gallagher's system aims to describe what he terms the 'cognitive' dimensions of classroom talk. He developed the 'topic' as a 'natural' unit of content which could be multiply coded on three 'cognitive' dimensions; Instructional Intent, Conceptualization, and Style. The details of these dimensions need not detain us here. For MATHS the description of the substantive content itself is the end.

The second difference arises from this difference in aims. Because MATHS is directly concerned with the content of classroom talk, it subdivides and categorises in a single process. Categorical distinctions are incorporated in the subdivision rules. In particular, these analytic distinctions take priority over naturalistic ones.

Third, MATHS codes directly from audiotapes rather than indirectly from transcripts. Gallagher points out that the preparation of transcripts is laborious and time consuming, and that the nuance of spoken language is lost in the process. On the other hand coding from transcripts is probably rather easier and more reliable. Gallagher's study suggests, nonetheless, that satisfactory agreement between observers is likely to be attainable from the coding of audiotapes. Given its clear advantages in other respects, this method was preferred. This particular problem does not arise, of course, in the analysis of textbooks and examination papers where printed matter can be coded directly.

Fourth, while it is not entirely clear from his description of TCS, it seems that Gallagher's system codes only whole class discussion. MATHS codes all substantive talk, including that between the teacher and individual pupils or groups of pupils. In particular, it codes talk occurring during what the TCS would code solely as Activity. Pilot studies suggested that the great majority of substantive talk came from the teacher and was addressed either to the whole class or to individual pupils. In order to ascertain whether the content of discussion differed between these two situations, a distinction between Class and Individual discussion was introduced, with the dividing line set, rather arbitrarily, at four pupils. Again, this is a problem which does not arise in the textbook and examination versions.

Fifth, Gallagher's 'developed' and 'undeveloped' 'topics' and 'themes' have been dropped. The definitions of, and distinctions between these units lack a theoretical basis to sanction their use as indicators of the degree of emphasis or elaboration given to topics in different categories. Indeed, the lack of uniqueness of naturalistic divisions at each of, and between these levels - evident in Gallagher's studies and those of Smith and Meux - suggests that an adequate theoretical basis is unlikely to be forthcoming.

Instead, MATHS uses a simple time measure to quantify the emphasis given to different categories. Such a measure has the advantages of being easy to use, and producing data which are highly reliable and easy to interpret. The basic unit of measurement is a 10 second interval. Given the ambiguity of the opening and closing of individual utterances and topical units, and variations in tape speed, to use a smaller unit would be to claim a spurious accuracy for measurement.

The main function, however, of the developed/undeveloped distinction in Gallagher's TCS is to set a threshold below which topics need not be - on the Conceptualization dimension - or are not - on the Style dimension - categorised. This device smooths out ambiguous or insubstantial talk. The threshold that the TCS sets - 15 lines of typescript, equivalent to around one minute of talk - seems rather high for the purposes of MATHS. MATHS lowers the threshold to 15 seconds: that is, no topic has a measured length of less than two 10 second units. A topic which lasted less than 15 seconds would be extremely insubstantial. It could consist of little more than a single statement; there would certainly be no time for interchange. It seems reasonable to suppose that any topic to which importance is attached will be dwelt on for longer than 15 seconds.

This, then, is a rather more sensitive smoothing device than that of TCS. For consistency, and greater sensitivity of measurement, we rule that periods of silence or chaos within the boundaries of a segment are deleted if they exceed 15 seconds in length. In analysing textbooks and examination papers the problems of defining thresholds and units of measurement are less complex. For textbooks the sentence was chosen as the threshold, and the line or line equivalent as the unit of measurement, for examinations, the question and its mark respectively.

Finally, one weakness of Gallagher's system is its lack of definition of the nature of 'auxiliary' comments grouped under the headings of 'management' and 'structuring'. In this matter MATHS aspires to a rather more precise definition by laying down ground rules for identifying such comments.

(3) <u>The choice of a sample</u>

The other important issue at the planning stage was the choice of particular instances of classroom talk, curriculum materials, and examination papers to be analysed. For this choice might affect the validity of generality of the findings.

It seemed important to analyse a broad sample of classroom talk from mathematics classes in the years S1 to S4. The sampling plan was in two stages. First, a number of schools were to be chosen and asked to cooperate in the study. Then similar classes in each school were to be chosen for observation.

It was decided to sample only from the Central Region of Scotland. There is no evidence to suggest that curriculum and teaching practices in secondary school mathematics vary between regions within Scotland. In addition, because of its geographical position, teachers in Central Region are drawn from a variety of sources, both in terms of university and college training, and of previous teaching experience.

Within each school it was decided to observe classes in S1 and S3. In S1 classes are furthest from the influence of SCE examinations and most likely to be following a common course: here we might expect to find evidence of any strongly held interpretations of general mathematical education. By S3 courses have diverged and differences in their content are likely to be apparent. (Since the study was to take place in April, May and June 1977 - a period which straddled the SCE examinations - it would not have been possible to observe S4 classes. In any case, the S3 classes observed would be well - about half-way - into the S3/S4 course at the time of observation).

After consultation with the Central Region Education Department, and with its approval, eight schools were approached, of which four agreed to cooperate in the study. At least one preliminary visit was paid to each of these schools before observation started, to determine the way in which the mathematics department organised its classes, and to make arrangements for observation.

Two schools were in the same urban, industrial area. Both were fully developed six-year comprehensive schools, one with around 1800 pupils, the other with around 700. The third, situated in a small industrial town, was a fully developed six-year comprehensive with around 900 pupils. The fourth, situated in a small rural, commuter town, was in the process of developing from a four-year junior secondary into a five-year comprehensive. At the time when the observation took place, years S1 to S3 were comprehensive. This school had around 350 pupils.

The organisation of mathematics courses was similar in all four schools. In S1, mathematics classes were mixed-ability: from S2 onwards they were set by ability. Around half the S3 pupils were following a course leading to presentation at SCE 'O' Grade in both Mathematics and Arithmetic, a further quarter were aiming at presentation in Arithmetic only, and the remainder were following noncertificate courses.

As a result it was decided that three classes would be observed in each school; one mixed-ability from S1, one 'above average' (double subject) from S3, and one 'below average' from S3. Note that 'above' and 'below average' are not mere statistical artefacts: they correspond to real organisational and curricular distinctions.

So as to view as many different teachers as possible, and to inconvenience individual teachers as little as possible, each class observed was to have a different teacher. The principal teacher of mathematics in each school was asked to identify those teachers in his department willing to cooperate in the study, and to select, in as unbiased a manner as possible (given the constraints above) the three classes to be observed. The researcher's impression is, however, that in all cases the choice was arrived at through a process of discussion and reasoned choice, rather than one of unguided choice.

The sample is, then, certainly not random. Observation took place only in the four schools prepared to cooperate in the study, and, in each school, only in classes approved by both the principal teacher and the class teacher - in two cases an observed class was taught by the principal teacher. In one school it became clear at the last moment that the teacher of the 'below average' S3 class originally chosen was not prepared to cooperate in the study, and an additional S1 class had to be substituted. But while this was not a random sample from the three populations, it is plausible that the bias was towards those classes and teachers which were seen as satisfactory if not positively successful. For example, in each school it was the top S3 class from the 'above average' population which was chosen to be observed. The schools had 6, 2, 4 and 2 'above average' classes respectively: the probability of this event happening strictly by chance is just over 1%. Any bias at the selection stage is likely to be towards teaching styles and classes that are approved by the teachers themselves.

The second way in which classroom observation data may be biased is related to the actual fact of observation. Teachers may, consciously or unconsciously, alter their behaviour towards models which they believe will bring approval. Nonetheless they remain in a 'real' situation to the extent that the presence of an observer is a marginal addition to existing constraints and influences on their behaviour, which will remain long after the observer has gone. It seems plausible that the largely unknown dispositions of a young graduate student are not likely to exert a great influence on the behaviour of teachers, in the face of existing and less transient influences: furthermore any alterations would have to be sustained over a number of lessons, and corresponding alterations sustained throughout the interview and the informal conversation which the 'teacher study' entailed.

All but one of the 12 classes chosen were observed over at least three 35-40 minute periods. Each observed class was recorded on a 2-track audio-cassette. On one track was a record of classroom talk from a radio-microphone attached to the teacher's person, on the other a commentary on any events or details which might assist coding, given by the observer who sat at the back of the classroom throughout the lesson, monitoring the recording equipment. In the case of one class, equipment failure made it possible only to record two periods. In all, thirty-eight periods were recorded and analysed.

The choice of curriculum materials and examination papers for analysis was much simpler. Previous discussion has pointed to the significance of the series of textbooks 'Modern Mathematics for Schools'. The four schools in the study were no exception to the pattern which has been described. In three of the four the S1/S2 common course was based on MMS and the appropriate volumes issued to pupils. The fourth used 'Modular Mathematics' materials but supplemented these with work from MMS for certain groups of pupils. In all four schools certificate courses were on Syllabus A and the S3/S3 double course was based on MMS, which was issued to all pupils following it.

In all of the schools the work of the department was organised around the standard curriculum materials: the shared assumption was that all teachers of S1 and of 'above average' S3 classes would basically work through these texts. No single text was favoured for S3 'below average' classes. Indeed, many were not issued with, and rarely used texts. Those in use, and in stock for use with such classes, differed little in content from the arithmetic sections of MMS. The main difference lay in the number and simplicity of the examples in the exercises.

This evidence pointed to an analysis of the seven volumes of MMS which cover the full 'O' Grade course.

We have seen how S3/S4 certificate courses in Scottish secondary schools aim towards presentation at SCE 'O' Grade. Although 98% of presentations in Mathematics and Arithmetic are on Syllabus A, it was decided to include Syllabus B examinations in the study in order to test the hypothesis that, in terms of the distinctions of MATHS, there is no difference between the two syllabuses. While the individual items in examinations on a particular syllabus change from year to year, the broad principles underlying the construction of the examination are unlikely to do so without some explicit indication. For these reasons it was decided that an analysis of the 1975 and 1976 'O' Grade examinations in Mathematics and Arithmetic on both syllabuses would be sufficient to provide valid evidence.

(4) <u>The reliability and validity of the data</u>

The MATHS coding system is intended to operationalise a model of the content of mathematics curricula. When we ask whether the data it produces are valid and reliable, we are asking questions about the adequacy of the model and its operationalisation, and the way in which the system has been used.

A model fulfils two functions: it generalises and it simplifies. Strictly, generalisation is a form of simplification; to generalise is to ignore or discard the unique, but theoretically insignificant characteristics of individual phenomena. To theorise, or to build and use models is, then, to commit oneself to simplification of some kind. This is a point which is often misunderstood. Many teachers and some researchers claim that no model can adequately describe individual phenomena. Inasmuch as this observation is correct it is trivial; for while, in principle, there may be no limit to the complexity of the discriminations which can be made about an individual phenomenon, it is clear that, in practice - in particular, in our use of language - we necessarily use such simplifying models and theories, if only tacitly.

There are a number of demands that can be made of an instrument, and corresponding grounds on which it can justifiably be criticised.

First, it should have a sound and articulate theoretical basis. An instrument based on an inconsistent, unjustifiable, or ambiguous theory, can only produce data which are, at the best, meaningless and, at the worst, positively misleading.

Second, the instrument itself should be as simple, clear, and concise as possible, making only those distinctions which have theoretical significance, and making these clearly and adequately.

Third, the instrument must be sensitive to the 'ecology' of the phenomenon under observation. The data it produces should not be a mere statistical artefact; it should reflect the 'reality' of the phenomenon.

Fourth, the process of gathering data should not unduly disturb the 'ecology' of the phenomenon, nor influence the observations which are made.

A number of factors affecting the validity of the data produced by MATHS have already been discussed. Arguments have been advanced for the validity of the conceptual distinctions on which the system is based, and the adequacy with which these distinctions are reflected in the system. It has been argued that the use of the instrument involves no significant distortion of the 'reality' of mathematics curricula, and the possible bias of samples has been explored and delimited. None of these potential flaws is amenable to direct empirical investigation; argument alone can identify and guard against them.

The one aspect of validity which can be examined empirically is the extent to which the distinctions made by MATHS are clear and unambiguous. The purpose of the reliability study which follows is to conduct such an empirical examination.

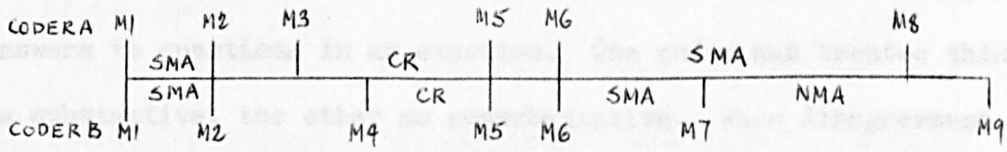
If the distinctions made by MATHS are clear and unambiguous then different observers should agree in their coding of the same situations. In short, coding should be reliable.

Gallagher, to whose Topic Classification System MATHS is close, uses a relatively weak test of reliability; he compares, for a particular lesson, the final percentage distributions between categories which different coders produce.

A stronger test of reliability examines not just the final, aggregated data, but the pattern of raw codings from which it is derived.

The first part of the MATHS reliability study examines the most complex version of the system, the classroom version. If ambiguity or lack of clarity is present in the system it is in the use of this version that this is most likely to show.

The method adopted to establish an index of reliability was as follows. Three single lesson tapes were each analysed independently by two coders. The two resulting codings of each tape were then compared to establish an index of agreement. For each tape the utterances selected by one or both of the coders to mark the boundaries of segments (or deletions) were recorded in sequence on a sheet. These markers defined first, a collection of intervals coded as substantive by at least one of the coders, and a second collection of intervals - contained within the first - where the coders were agreed both on the substantiveness and the specific categorisation of discourse.



In the hypothetical example above the intervals coded as substantive by at least one of the coders are M1-M2, M3-M5 and M6-M9. The intervals over which the coders are entirely agreed are M1-M2, M4-M5 and M6-M7.

Once all the markers and intervals had been determined the tape was played through and the time at which each marker occurred was noted. From this information interval lengths were calculated. Finally, the ratio of the length of the intervals over which there was complete agreement - the second, and smaller, collection - to the length of the intervals coded as substantive by at least one coder - the first collection - was calculated to provide an index of agreement.

In the example above this index would be given by the expression

$$\frac{L(M1-M2) + L(M4-M5) + L(M6-M7)}{L(M1-M2) + L(M3-M5) + L(M6-M9)}$$

where L(A-B) signifies the length of the interval A-B.

Clearly the value of the index must lie between 0 and 1, the first value signifying a complete absence of agreement, the second, complete agreement.

The three analyses which made up the MATHS reliability study gave ratios of 0.75, 0.83, and 0.94. In the first two cases virtually all the disagreement was due to a single difference in interpretation of the coding instructions. In both lessons there was a period of talk which consisted of the teacher soliciting the answers to questions in an exercise. One coder had treated this as substantive, the other as nonsubstantive. When disagreement due to this difference was removed the coefficients rose to 0.97 and 0.99 respectively. On this index, then, there was an extremely high level of agreement between the two coders.

Under normal circumstances this would be impressive and quite adequate evidence for the reliability of coding using the system. But the observations coded were unusual in one respect. As was anticipated in the hypothesis, virtually all topics were coded as belonging to a single category (SMA). Thus, while this first study provides strong evidence of the ability of the coders to distinguish between substantive and nonsubstantive talk, it provides weaker evidence of their ability to distinguish between the substantive categories of the system.

To remedy this weakness a second study was conducted. The same coders independently coded 54 passages of varying length, drawn from a number of printed sources, each of which was taken to constitute a single topic; these can be found in Appendix 4. One coder, the researcher, constructed the item bank in such a manner that, by his judgement, the distribution of topics was different from that found on the classroom tapes; in particular, all the substantive categories were represented to some degree.

The coders agreed on the classification of 53 of the 54 topics which were distributed as follows,

SMA	NMA	MR	CR	RES
12	13	7	14	7

The contested topic was classified as MR by one coder and as CR by the other.

The second study provides the evidence lacking from the first. Combined they suggest that the distinctions made by the MATHS coding system are clear and unambiguous; that coding using the system is indeed reliable.

Chapter 7: The Curriculum Study - Evidence and Conclusions

(1) Classroom talk

Thirty-eight periods of classroom talk, consisting of seven double periods and twenty-four singles,[1] were observed. On average, there was slightly more than 20 minutes of classroom talk per period. Individual figures ranged from just over 9 minutes to just under 39.

The distribution of classroom discussion between the two Group categories, Individual and Class, varied considerably over the observed periods. There were some where all discussion fell into one or other of the categories. Overall, around 30% of talk fell into the Individual category. There was no marked difference, however, in the distribution of topics by Content between the two Group categories.[2] For this reason the Group distinction will be ignored in presenting results.

The results in Tables 1, 2 and 3 give clear evidence that the classroom talk fell almost exclusively into the SMA category. Indeed no MR topics occurred throughout the 38 periods, and in only 2 did CR topics occur, on one occasion only fleetingly. While NMA topics occurred more frequently, they typically occupied only a small fraction of discussion time.

Two lessons stood out as exceptional relative to the typical pattern. In one 81% of the discussion was CR: here the teacher was explaining the purpose of using statistics to a 'below average' S3 class. In the other, 35% of the discussion was RES: here the lesson was on life insurance, and part of the lesson was taken up with discussing the reasons for taking out life insurance, the

Table 1 % of classroom talk in each Content category by school, class type, and subject area

School	Class type	Subject area	Double/ single	SMA	Content NMA	Category MR	CR	RES
	S1	A	S	91	9	0	0	0
		A	D	76	21	0	0	2
	S3U	C	D	98	2	0	0	0
	S3L	A	S	12	7	0	81	0
		A	S	100	0	0	0	0
		A	S	87	10	0	1	1
	S1	M	S	100	0	0	0	0
		A	S	95	5	0	0	0
		M	S	100	0	0	0	0
	S3U	M	D	100	0	0	0	0
		M	D	98	0	0	0	2
	S3L	A	D	98	2	0	0	0
		A	D	100	0	0	0	0
	S1	A	S	98	2	0	0	0
		A	S	100	0	0	0	0
		M	S	98	2	0	0	0
	S3U	M	D	91	9	0	0	0
		A	S	100	0	0	0	0
		M	S	100	0	0	0	0
	S3L	A	S	100	0	0	0	0
		A	S	100	0	0	0	0
		A	S	100	0	0	0	0
	S1	A	S	100	0	0	0	0
		M	S	100	0	0	0	0
		A	S	100	0	0	0	0
	S1	A	S	82	18	0	0	0
		A	S	100	0	0	0	0
		A	S	100	0	0	0	0
	S3U	M	S	100	0	0	0	0
		M	S	100	0	0	0	0
		A	S	65	0	0	0	35

Key to Tables 1, 2 and 3

Class type: S1 = first year mixed-ability

S3U = third year 'above average'

S3L = third year 'below average'

Subject area: A = Arithmetic

M = Mathematics

C = Computer studies

Table 2

Mean % of classroom talk in each Content Category by class type, and subject area

		SMA	NMA	MR	CR	RES
S1	(n=15)	94.4	5.2	0	0	0.3
S3U	(n=13)	95.3	1.7	0	0	3.0
S3L	(n=10)	89.5	2.1	0	8.2	0.1
Arithmetic	(n=23)	91.2	4.0	0	3.3	1.4
Mathematics	(n=13)	98.1	1.6	0	0	0.3
All	(n=38)	93.4	3.2	0	2.2	1.2

Table 3

Median % of classroom talk in each Content Category by class type, and subject area

		SMA	NMA	MR	CR	RES
S1	(n=15)	100	0	0	0	0
S3U	(n=13)	98	0	0	0	0
S3L	(n=10)	100	0	0	0	0
Arithmetic	(n=23)	98	0	0	0	0
Mathematics	(n=13)	100	0	0	0	0
All	(n=38)	100	0	0	0	0

regulations surrounding it, and the way in which life insurance companies operate.

The overall uniformity of the results is striking. The infrequency with which lessons diverge by more than 2% from the stereotypical scores for each Content category indicates the degree of uniformity.

Table 4 Number of periods outside 2% range (n = 38)

SMA < 98	NMA > 2	MR > 2	CR > 2	RES > 2
10	9	0	1	1

These results suggest that, as far as MATHS is concerned, there are no substantial differences between the content of classroom talk in different types of lesson - maths or arithmetic, S1, S3 'above average', or S3 'below average'. There is a common pattern running through all of them.

(2) Textbooks

The results from the textbook analysis are even more clear cut than those from the classroom observation. Again, virtually all content falls into the SMA category. Topics in other categories are infrequent and insubstantial.

Interestingly, the one exception to the overall pattern is computer studies, the one non-examined subject area covered by MMS. The high level of CR in this subject area is due to a chapter in Book 6 which describes uses of computers in business and industry: here CR constitutes nearly 60% of the content. Over the other three computer studies chapters in the series, CR falls to a typical 0.5%

Table 5 <u>% of textbook content in each Content Category by subject area</u>

	SMA	NMA	MR	CR	RES
Algebra	99.6	0.2	0	0.3	0
Geometry	99.2	0.1	0	0.7	0
Trigonometry	99.9	0	0	0.1	0
Arithmetic	96.6	0.8	0	0.9	1.8
Computer studies	85.9	0.1	0	14.0	0
All	98.1	0.3	0	1.0	0.6

(3) <u>Examination papers</u>

The results from the analysis of examination papers could not be more clear cut: in all eight examinations analysed, all the questions sought answers in the SMA category.

(4) <u>Conclusion</u>

These results provide strong evidence for the principal hypothesis of the curriculum study: that the present mathematics curriculum is concerned almost exclusively with technical expertise in mathematics.

They provide equally strong evidence for the subsidiary hypothesis that the conception of 'doing maths' implicit in the curriculum is a limited one which ignores the fundamental methodological issues at the heart of mathematical activity.

Chapter 8: The Teacher Study - Aims and Methods

(1) The aims of the teacher study

The basic purpose of interviewing the class teachers was to discover what kinds of knowledge, skill and understanding they aimed to develop through their teaching: in particular, whether they aimed to develop the kinds of knowledge and understanding which relate to the conception of general mathematical education advanced in Part I.

A second purpose was to discover the extent to which the 'official' curriculum determined what pupils were taught, and to relate teachers' expressed aims to the aims of this curriculum.

Moreover, if carefully designed, the interview might yield further valuable information about the way in which mathematics teachers viewed their subject and the teaching of it. In fact it became apparent during the course of the pilot studies that, however the interview was designed, it was likely to elicit a general discussion of these matters.

The interview schedule was designed to be open with respect to specific aims and objectives. The teachers were to be encouraged to talk about their teaching from a number of different starting points. In this way it was hoped to obtain a full account of the teacher's aims, while avoiding the possibility of influence involved in more direct questioning.

(2) The design of an interview schedule

The interview was in three sections, of which the first and last were common to all teachers. The first dealt with a particular lesson which had recently been observed, and provided an opportunity for the teacher to talk about the way in which an individual lesson had been planned and taught, and to describe the objectives which it sought to realise.

The second section asked teachers of S1 and S3 certificate classes to describe their views on the standard curriculum and any ways in which they deviated from it. Teachers of S3 non-certificate classes were asked to talk about the kind of curriculum that the class followed.

The last section dealt with two background factors which it was felt might influence the aims of individual teachers. These factors were; other subjects that the teacher had studied at post-school level, or taught at school level, and other occupations that the teacher had had. Here the teacher was given an opportunity to describe how such factors - if relevant - influenced his or her teaching of mathematics.

The basic structure of the interview was provided by a number of key statements and questions. Throughout the interview the subjects were probed for the reasons behind their reported beliefs and actions, and for exemplifications of general statements. On occasion key questions and statements were slightly rephrased in order to acknowledge the particular context, or statements made previously by the teacher. It was hoped that this would help to sustain the informal atmosphere in which the fullest account of the teacher's views might be forthcoming.

The pilot studies suggested that some teachers were ill at ease when confronted with questions, phrased in general terms, about the curriculum, and that an interview conducted solely at this level would elicit little information from them. There was also evidence that the answers given to general questions might be a verbal gloss, only loosely related to the actual practices of the teacher. The opening section, tied to a specific lesson, and the probing for examples in the second and third sections fulfil a similar dual function. First, they provide a specific, and possibly more congenial introduction to more general questions: second, they encourage the formulation of generalisations consistent with teaching practices.

The full interview schedule can be found in Appendix 3. Section 1 was dropped from two of the twelve interviews, in one case because the teacher had been returning examination papers during the specified lesson, and in the other because the teacher was pressed for time. All twelve teachers answered Sections 2 and 3.

All the interviews were recorded on audiotape.

(3) The analysis of interview data

Not surprisingly, given the open structure of the questions in the interview schedule, responses to individual questions tended to be discursive, often touching on matters related to previous or later questions. For this reason a flexible method, deriving from the work of Becker,[1] was used to analyse the interview data. Becker's method was developed for use in participant observation. Its value for our purposes is that it makes it possible not only to analyse the rather unstructured statements of the teachers, but to assess their veracity and interpret them.

Adapted to the present problem, the method consists of formulating a hypothesis, then searching the interview tapes for evidence that, directly or indirectly, confirms, makes plausible, or refutes the hypothesis. To exemplify this technique in action I will describe the way in which it was used to test the general hypothesis that teachers of S1 and S3 'above average' classes stick very closely to the set curriculum, against the statements of a particular teacher.

This teacher made a number of statements bearing on the hypothesis. He answered the question (1.2) about his reasons for teaching a particular topic that day,

> "Well simply again, this is the next part of the course: according to the book the next section's leading through."

In answer to question (2.1)(c) on the relation of classwork to the set curriculum he started,

> "What I mainly want to do is teach what is there to be taught properly",

and went on to explain that he was quite happy to teach what "those higher up" had laid down.

Then talking about the problems of teaching classes of uneven ability,

> "Likes of this morning I could have went on to the next part of the lesson or the course without any trouble with maybe twenty-five out of whoever was there. But the other half dozen, they would have been in trouble. And again as far as I'm concerned I've got to get them through an exam, and if I can get those other six through the exam I think I'll have succeeded."

In answer to the questions (2.6) and (2.7) about introducing outside topics he answered simply "No", and, "No again. I stick very rigidly to the work that's involved." Here he proceeded to explain,

> "Again this may be just because I'm into school. I don't know. Well I've been in a year and a half and perhaps I don't feel free enough to do other things. And again plus the fact that it is the certificate class and, as I say, at the end of the day, that's what these pupils want, a certificate, and that's what I want to get them as well really. And at the moment I don't feel I've got time maybe to give them something outside of that."

Probed about whether there were any topics he might have liked to talk about had he had time,

> "No again. As I say again I'm perhaps not outward looking that way myself: it's maybe my personality. As I say I believe there's a job there, I'm involved with that job, and I get on with that job. Again this may come back to my work in industry. You were given a job and that was it. As far as the curriculum's concerned I stick rigidly to it."

He felt that the only university subject he had taken which was of value in teaching mathematics was Education, since it was the only one which had dealt with the maths that was in the school curriculum. Indeed, he had complained to one of his mathematics professors that the university mathematics courses were not relevant to what he was going to be called on to teach. As he put it,

> "Again it was more or less what I've said about the curriculum here: it's there and that's what's to be

done and that's all there is to it."

Finally, everything that this teacher said during the interview was perfectly consistent with the events which took place while he was under observation in the classroom.

In the example above we have not simply direct evidence that this teacher seeks to stick close to the set curriculum - in the form of his statements to that effect - but evidence which ties these statements into a wider view of his role as a teacher. The consistency of the teacher's statements, and their embeddedness in a larger world-view, lends them a plausibility as accurate accounts of his beliefs and actions which a simple answer to a direct question lacks. This is the strength of this method. It uses not just direct, but indirect evidence to test hypotheses: in particular, it tests for consistency throughout the interview, and within a more general framework of belief and action.

There are, of course, other factors which must be considered in assessing the validity of the teachers' accounts. First, the researcher had already seen several lessons taught by the teacher by the time of the interview, and discussing one of these lessons provided a bridge between the classroom observation and the interview. This places pressure to be consistent on the teacher, and at the same time provides additional evidence which the researcher can draw on to test his hypotheses.

Second, the teacher is talking about his beliefs and actions as a teacher to another individual whom he knows to have been a teacher - teachers invariably sought this information at a very early stage in the research encounter - and who, because of his lack of status and power, poses very little threat to the teacher.

Indeed some of the teachers seemed to positively welcome this opportunity to talk about their work, and commented on this fact. A few were much less enthusiastic, possibly because they disapproved of educational research and its practitioners in general. At least one teacher voiced her suspicions,

> "I don't want to sound old fashioned but I think that in some ways the old system of teaching maths for good pupils had something to commend it in comparison with what we do now. What has happened I think is - I have to say it you know - theorists - we're at the mercy of the theorists really - have said that time should be spent on trying to encourage children to understand the reasoning for something."

Other teachers talked about their "hobby-horse" or "bias", or in similar ways excused their expressing opinions which, it was implied, might bore, or fail to meet the approval of the researcher.

Nonetheless, such evidence suggests that, whatever they imagined the researcher's opinions to be, teachers were quite prepared - or possibly determined - to say what they thought on these professional matters.

Chapter 9: The Teacher Study - Evidence and Conclusions

(1) The role of the set curriculum

The first hypothesis I will examine is that the curriculum of S1 and 'above average' S3 classes is strongly influenced by the nationally standard curriculum. Three of the S1 classes were following a curriculum based on 'Modern Mathematics for Schools'. The remaining two used the 'Modular Mathematics' materials with some additional work in arithmetic drawn from MMS or similar sources.

Four of the five teachers of S1 classes reported themselves as rarely introducing topics of any kind outside the set curriculum. This was consistent both with their other statements and with observations made in their classes. Two of these teachers could think of only one case; when teaching about statistics they asked pupils to bring in examples of charts and graphs from newspapers. Another teacher reported only that she had given her class some number puzzles for entertainment, the fourth only that he mentioned the use of scale drawing in geography when he taught that topic. All those teachers gave direct indications that the basic order and direction in their teaching came from working through the set curriculum.

One teacher reported himself as straying frequently from the set curriculum. By this he meant two things. First, that he occasionally deleted topics which he regarded as unimportant or too difficult, and anticipated future content when an appropriate situation presented itself. Second that he often answered pupils' questions on outside topics: the examples he gave were explaining the meaning of a word pupils had found on the board from a previous

English lesson, and explaining how clouds formed. He also mentioned that he occasionally used examples from geography and physics: for example maps and orienteering to introduce mathematical topics such as angle and gradient.

All four S3 'above average' classes were following a curriculum based on Syllabus A, and used 'Modern Mathematics for Schools' as their textbook. In two cases where it was anticipated that the class would cover the set examination curriculum in a shorter time than that available, it was department policy that computing should be studied.

One teacher reported himself as never introducing topics falling outside the standard curriculum, two as only rarely doing so. In one case the only example that could be recalled was mentioning the difference between the definition of gradient used in mathematics and that used in geography. In the other the teacher gave what he suggested was an exhaustive list: he had mentioned that examples of statistical graphs could be found in newspapers, explained that the angle in a semicircle property could be used in marking a running track, and mentioned that it is possible to use calculus to show that the standard shape of a tin can is not the most economical in use of tin. Again, all these teachers gave evidence that for them teaching maths was basically a matter of working through the set curriculum.

Finally, one teacher reported that he frequently introduced topics not in the set curriculum. These were of two types: first, extensions of the theory being covered - more rigorous proofs, generalisations - and second, problem solving - presenting pupils with non-routine problems in mathematics and logic to solve. In all cases there was substantial direct and indirect evidence for these reports.

There is very strong evidence then, that the majority of these teachers stuck very close to the set curriculum. In only two cases is there evidence of any real concern with topics and approaches lying outside the set curriculum, and in only one of these cases was this discursion related to mathematics.

(2) The aims of the teachers

This conclusion is given added plausibility when we come to test the hypothesis that teachers are not concerned to develop the broader methodological and cultural understanding which is central to the conception of general mathematical education advanced in Part I.

These teachers of S1 and S3 'above average' classes had surprisingly little to say about the purpose of teaching their subject, even in response to the very general questions in the second section of the interview. What they did have to say assumed that the purpose of mathematical education was the transmission of various types of technical skill and knowledge. There was no evidence that any of these teachers had ever entertained the possibility of teaching towards the broader aims which the hypothesis concerns: there was certainly no mention of anything resembling them.

This apparent lack of concern with the purpose of teaching mathematics prompted the formulation of a further hypothesis that these teachers were not so much concerned with the value of curricular aims as with their feasibility.

Two of the teachers did offer evaluations of the aims of the present curriculum. The S3 teacher who introduced problem-solving into his lessons felt that a greater emphasis in this area was desirable as it would give pupils "more facility to think practical problems out". Another teacher made no specific suggestions, but felt that much of the present curriculum was irrelevant to the vocational needs of the majority of pupils.

The remaining teachers gave little evidence of any direct concern with the value of curricular aims: their primary concern was their attainability. One, for example, while prepared simply to accept the curriculum laid down by "others who know what should be taught", considered that the changes of the sixties had improved the mathematics curriculum because,

> "you don't have to be really mathematically minded to tackle some of these [new] problems."

Indeed, the concern that many pupils were unable to master the content of the curriculum was voiced, in varying degrees, by all nine of the S1 and S3 'above average' teachers. Seven suggested that certain parts of the course should be omitted, or given less emphasis with less able pupils.

For example, several teachers questioned the set geometry curriculum on the grounds that many pupils find it difficult, another questioned the emphasis on algebraic manipulation on the same grounds. Others suggested that many pupils were not capable of learning most of mathematics and that for this reason they should be taught only the minimal core of basic arithmetic and mathematical skills which they were likely to be capable of learning and to need after leaving school.

This last example may seem to contradict the earlier statement that the teachers were not primarily concerned with aims. This is only marginally so. The evidence suggests that the purpose of teaching mathematics was not an important issue for these teachers. Mathematics was simply there to be taught. What concerned them was that so many pupils seemed to lack the ability or motivation to learn mathematics. It was this fact which formed the basis of their criticisms of the present curriculum, not beliefs about value or purpose.

While the importance of differences in pupils' ability to learn mathematics was spontaneously mentioned by all these teachers as a reason for changed emphasis on content, it was usually necessary to probe for statements of belief about the purpose or value of learning specific parts of mathematics, or mathematics in general. The eight teachers who expressed views all argued that there were certain arithmetic skills which all pupils would find useful in everyday and working life after they left school. There was also agreement that the present curriculum was of value to the small number of pupils proceeding to study mathematics at university level.

Beyond this there was little agreement or conviction. Four teachers expressed doubt and confusion. As one put it,

> "I have found very little maths of value to me. I just think it's an experience, it's a subject which you study, which some are good at, some are bad at, and I make no wild claims about it at all."

Or another;

> "You see, I think really for an awful lot of children

at school what they're doing has probably got very little value, but maths I don't think is any different from any other subject in that respect. Perhaps it gives them a little, it interests them for a while and it keeps them occupied. And there's nothing else for them to do."

The remaining four advanced with varying degrees of confidence, arguments that the study of mathematics disciplines the mind. Two talked about the subject "stretching the mind": another two about it developing "logical thinking", defined by one as,

"If you obey the rules you get the correct answer, if you don't obey the rules you don't get the correct answer."

Only in one of these four cases - that of the teacher who advocated an emphasis on problem solving - was there evidence that this justification for mathematics actually influenced teaching behaviour, either as observed or reported.

Beyond the recognition of certain basic arithmetic needs, then, arguments about the value or purpose of mathematics had little place in the justifications which teachers offered for their views on the kind of curriculum appropriate to their pupils. The dominant consideration was what pupils were believed to be capable of mastering, rather than what they ought to learn. In short, teachers assumed that pupils should learn simply what they appeared to be capable of learning.

A similar phenomenon was evident in the views of the teachers of S3 'below average' classes. Three such classes were observed. One was aiming at presentation in Arithmetic at 'O' Grade, the others were noncertificate. All three teachers acknowledged the aim of developing basic arithmetic skills. For the teacher of the certificate class this aim was paramount. Answering question (2.6) he explained,

> "Well with this particular class I keep quiet about mathematicians and try to keep it all on a very basic 'that world out there' level. It's arithmetic for use rather than for - well, I have tried to interest them near the end of term, in things that I've thought fascinating; little bits of number theory, things like that. It's like water off a duck's back. So I tend now to stick to my last of trying to get them to calculate reasonably accurately."

He did, however, introduce stories about 'that world out there' in order to motivate the pupils and,

> "to get across to them that out there they won't always have paper and pencil, and that the methods we teach will not always be applicable, and that they've got to find some way to use their common sense to get an answer that's appropriate to the circumstances."

The opening statements of the replies of the two teachers of non-certificate classes to question (2.2NC), asking what kinds of things they did with their classes, offer a fascinating insight into their thinking.

> "It's mainly general arithmetic. They like nothing better than adding up a string of figures or multiplying, so it's mainly that."

> "Anything I think they can absorb, that they may use

> later on, that they may meet later on, although a lot of them won't use mathematics again."

Both teachers mentioned what they termed 'interest based' topics.

> "They like something that seems a bit different to them from maths. Not mathematics, a game."

One of the teachers explained why he taught this way.

> "That type of pupil is the kind of pupil that in a larger school, it's not geared to them, they don't come to school. So it's got to be something that they enjoy doing, and that they can do. They'd be quite happy, and they are quite happy if we do an exercise and show them how to do something - say in algebra - and you give them forty examples of a similar kind. They can do them and they're happy and they like doing it, rather than doing something they can't do. They get very discontented."

These remarks suggest that the primary considerations in planning the curriculum of these noncertificate classes are ones of feasibility. The questions 'Will they understand it?' and 'Will they enjoy it?' seem to take precedence over the questions 'What are the substantive aims of this curriculum?' and 'How does this topic help to realise these aims?'.

Again, with these 'below average' S3 classes there was no evidence of a concern to develop a broader methodological and cultural understanding of mathematics. The only teacher who came near such a conception was the teacher of the certificate

Arithmetic class, with his concern to motivate his pupils by talking about 'the world out there'. He was, incidentally, the only teacher in whose classes CR topics were observed.

(3) The significance of teachers' backgrounds

Although most of the teachers had studied subjects other than mathematics they found that this experience was, in general, of little or no value in teaching mathematics. Indeed a widespread view seemed to be that,

> "Maths goes out [to other subjects] rather than them coming back into maths."

Similarly, the three teachers who had worked in occupations other than teaching found this experience of little value in teaching mathematics, other than in making them more 'realistic' about the standards of arithmetic competence current, and acceptable outside the school.

In addition, none of the twelve teachers referred to any involvement with mathematics outside that directly related to teaching the subject; the preparation of lessons, looking at curriculum materials, attending inservice courses. Indeed, three teachers specifically indicated that they avoided anything to do with mathematics outside their teaching responsibilities. A fourth made a point of explicitly communicating his lack of enthusiasm for mathematics to his pupils.

(4) Conclusion

Both the hypotheses which the teacher study set out to test are supported by the evidence it provides. First, there is no evidence that teachers are concerned to develop the broader methodological and cultural understanding which is central to the model of general mathematical education advanced in Part I. Second, there is a great deal of evidence that these teachers' actions and beliefs are hardly touched by considerations of the value or purpose of the subject they teach. Their goals are, in many ways, not subject goals but organisational goals; to cover the set curriculum, to get pupils certificate passes, to keep pupils occupied and amused, to get pupils to attend classes.

These teachers, then, largely saw and taught mathematics as a static, socially disembodied, taken-for-granted corpus of technical knowledge and skill. They organised their teaching not so much around a framework of beliefs about the purpose and value of including mathematics in the curriculum they taught, as around an organisational framework common to all subjects, and unrelated to the particular content of the mathematics curriculum.

It seems, then, that, to advance our understanding of what goes on in individual classrooms and schools, we must turn to the larger arena within which change in the organisational framework around which teachers structure their activity takes place. For this reason Part III will examine the evolution of the Scottish school mathematics curriculum.

Part III: The Development of the Scottish Mathematics Curriculum, 1887-1977

Chapter 10: The Origins and Development of the 'Traditional' Secondary Mathematics Curriculum, 1887-1962

(1) The Growth of a national system of secondary education

The late 19th century saw radical changes in the structure of the Scottish educational system. At the heart of these changes lay a new conception of the relation of school to university, and of the functions of these institutions.[1]

In the traditional system the university took up where the elementary school left off. There were no formal entrance requirements and the courses of the first Junior year assumed little or no previous knowledge so as to neutralise differences of educational and social background. The four year Arts curriculum of Language, Philosophy and Science held pride of place, and was the precondition of specialist or professional study. While the content of the individual courses was relatively elementary in the specialist sense, each was treated from the distinctive philosophical perspective of the Scottish Enlightenment. It was this characteristic which gave the Arts curriculum its coherence and its intellectual strength. It provided a humanistically-oriented common course of general education, prior to specialist or vocational training.

'Secondary' or 'higher class' education was restricted in availability and varied in standard. In many rural areas it was unknown, while at the opposite pole, many of the private and endowed city schools were in competition with the universities as providers of post-elementary education for the upper middle classes, aspiring to enter the ancient English universities, or the Civil

Service at home and abroad. The changes which took place over this period led to a much sharper differentiation between secondary and university education. The former became for the first time, a central and distinctive part of the Scottish educational system, while the latter changed markedly in character and function. This process is reflected in the development of a national system of secondary education, and the growth of specialist study in the universities.

The 1889 Universities Act, which followed the Reports of Commissions in 1878 and 1889, instituted stiff compulsory matriculation requirements, restructured the degree regulations in the Arts faculties to permit early specialisation and introduced science faculties in all the Scottish universities. The effect of these changes, which became effective from 1892, was to raise the age of university entry by two years from 15 or 16 to 17 or 18, and to promote the growth of specialist study. In the Arts faculty the Ordinary curriculum became an <u>alternative</u> to Honours where before it had been a <u>precondition</u>.

The founding of an independent Scottish Education Department in 1885 marked the start of an era of vigorous growth in secondary education in Scotland. The new conception of the relationship and functions of school and university clearly demanded an extension of secondary education to enable the school to take the place of the university as the instrument of general education. The SED encouraged the extension of secondary education in two ways, with financial assistance from the popularly termed 'Equivalent Grant', made available under the Education and Local Taxation Account (Scotland) Act of 1892, and by establishing a national system of examinations.

Chrystal, Professor of Mathematics at Edinburgh University had conducted a feasibility study for a national examination in mathematics during 1887.[2] His tests were designed on the assumption that completion of the secondary curriculum should equip the pupil to enter the Senior classes at university. This assumption did not, of course, reflect current practice, for many students, even if they had passed through all or part of secondary education, spent a year in the Junior classes at the university, before entering the Senior classes and the three year Arts curriculum proper.

Chrystal's experiment was judged a success. It established that, at least in mathematics, a common examination was feasible, and the variation in standards it revealed gave strength to the argument of the reformers that a national system of examinations was needed to raise the standard of secondary education. In 1888 the SED introduced a school leaving certificate, modelled on those offered by the ancient English universities. The main aim behind the introduction of the certificate was to set standard levels of achievement for secondary education, in order both to raise the standard of existing 'higher class' education, and to bring a halt to the rapid multiplication of examinations set by the universities, the professions and other such bodies.[3]

The SED originally planned to set examinations for the Leaving Certificate at two grades, the First, or Higher, corresponding to the Senior local examinations, and the qualifying examinations for entry to the three year Senior Arts course - both already set by each of the universities - and the Second, or Lower, corresponding to the existing Junior Local Examinations, and the preliminary examinations for entry to the university medical course. But, as we have seen, in the 1880's some of the large city 'higher class' schools were still in competition with the universities in promoting advanced tuition, and for their benefit, a more advanced grade, Honours, corresponding to the entrance examination for the Indian Civil Service, was added. It was, however, short lived. The growth and development of the new Honours courses in the universities and the decision, in 1900, to award the Leaving Certificate on a group basis, rather than in individual subjects, led to the Honours grade being discontinued in 1907.

The new certificate was largely successful in achieving the aims set out for it. After an initial show of reluctance the Scottish universities agreed to recognise it on the condition that the papers in Latin, Greek and Mathematics - the subjects of the university Junior year - were approved by the universities before being administered. The new examination was quickly accepted by the ancient English universities and the major professions. Soon success in the Leaving Certificate became the common aim of pupils in those institutions offering 'higher grade' education.

In the early years of the certificate, then, the examiners were university professors and, although in time replaced by inspectors, the professors left their stamp on the examinations, and through them on the curriculum itself.[4] By the turn of the century the shape of the five year academic secondary curriculum had become clear. In mathematics it was to remain substantially unchanged for more than 60 years.

(2) The School and University Curricula 1887-1924

The secondary mathematics curriculum was then, initially drawn up by university professors. The intention behind its design was that it should prepare a pupil for university study. In the early years of the certificate a precise syllabus was not officially laid down, although the questions set in examinations followed a relatively consistent pattern. It is clear that a process of adaptation and negotiation between the examiners, on one hand, and teachers and pupils, on the other, was taking place. By the time of the 1904 'Note as to Mathematical Papers',[5] however, a stable and explicit curriculum had been reached which did not change until 1924.

The curriculum was in four parts, Arithmetic, Geometry, Algebra and Trigonometry. Arithmetic covered the elementary rules, prime factors, weights and measures, vulgar and decimal fractions, approximate calculation by decimals, and 'practical problems' which included what we now call 'social arithmetic' - profit and loss, insurance, foreign exchange, interest and so on. This was the one part of the curriculum designed with non-university needs in mind. Indeed, until 1905, when the practice was forbidden, (and after 1932, when it was again permitted) it was common to present certain pupils in Arithmetic only in the Leaving Certificate. In Geometry, the emphasis was firmly on Euclid, although some scale drawing and mensuration were included. Algebra consisted of formulae, graphical representation of functions, equations, linear, quadratic and simultaneous, indices and logarithms, surds, the remainder Theorem and progressions. Trigonometry covered the elements, and the solution of triangles.

Until around 1910 a student who completed the Higher course in Mathematics at school would find much of the university Ordinary course in Mathematics familiar.[6] The Ordinary course served a dual purpose as the first course in the Honours degree in mathematics and as the mathematics course for the Ordinary Arts degree, and was pitched mathematically more at the level of the student who might have passed Mathematics at the Lower Grade in the Leaving Certificate, than at that of the intending Honours mathematician with a good pass at the Higher Grade. If he intended to take Honours the student would proceed to the Intermediate Honours class where more advanced plane trigonometry, analytic geometry, conics and the calculus were studied, and finally to Advanced Honours for a fuller treatment of the integral calculus, spherical trigonometry, solid geometry, and differential equations.

The generalist cultural spirit of the traditional Ordinary course in mathematics was at odds with the specialist, technical spirit of the developing Honours course, modelled on those of Cambridge and the continental universities. From this latter perspective the Honours student was merely marking time for a year while he took the Ordinary course, time which could be used to better effect if the Honours course were to be extended and made more ambitious.

Around 1910, then, the universities started to extend the Honours curriculum. They were able to accommodate new topics in the Advanced Honours course by covering the content of the existing Honours course in the first two years of the new course. This entailed the introduction of a Second or Higher Ordinary course which rapidly became the standard first course in mathematics for intending Honours students in mathematics, leaving the (First) Ordinary course for those students taking the Ordinary Arts degree. (Edinburgh, for example, introduced a Second Ordinary in 1909, Glasgow a Higher Ordinary in 1910).

By the early 1920's a new university Honours curriculum, which was to survive, in broad outline, until the fifties, had been established. The Edinburgh curriculum of this time is typical.

Date of Introduction	Arts	Science[7]	Syllabus
1916	2nd Ordinary Mathematics	Mathematics I	Algebra; Interpolation; Differential and Integral Calculus; Spherical Trigonometry; Co-ordinate Geometry.
1916	Intermediate	Mathematics 2	Higher Algebra; Differential and Integral Calculus; Differential Equations; Solid and Descriptive Geometry.
1922	Advanced Honours Mathematics	Mathematics 3 Mathematics 4	<u>Foundations of Analysis</u>: Convergence; Continuity; Uniformity; Integration; Fundamental Theorem of the Calculus. <u>General Analysis</u>: Contour Integration; Gamma Function; Fourier's Theorem; Legendre's and Bessel's Functions; Elliptical Functions, Partial Differential Equations; Calculus of Variations. <u>Higher Algebra and Geometry</u>: Determinants; Matrices; Tensors; Differential Geometry; Relativity; Non-Euclidean Geometry. <u>Mathematical Laboratory</u>.

(3) First Reforms in the School Curriculum 1924-1936

Until 1924 there were no notable changes in the school mathematics curriculum. Between then and 1936 a number of changes were made, primarily in the geometry course: they can be followed in detail through (four) issues of the 'Note as to Mathematics'.[8] There was a progressive reduction in the number of proofs required in plane geometry, solid geometry was removed from the course - first in its formal aspects and then entirely - and the analytical geometry of the straight line, and later the circle, was introduced in place of the deleted material.

The 1931 SED memorandum presents the changes as a readjustment of the balance of the curriculum to take more account of the needs and abilities of the majority of pupils. The opening paragraph reads:

> "It is no doubt a common experience among teachers of mathematics that the difficulty of certain parts of the present course is out of all proportion to their usefulness to the average pupil. The schools have to meet the needs of those who will later proceed to the University as well as of the much larger number who have no such intention, and the general tendency is to give precedence to the claims of the minority."[9]

Now there is no doubt that many pupils found parts of formal geometry hard, and were forced to resort to the memorisation of a large number of proofs without understanding them. The change to a group leaving certificate meant that many pupils who would not formerly have studied mathematics to certificate level were forced to do so. It is also quite conceivable that the curriculum planners believed that these pupils would find analytical geometry easier, or, less plausibly, useful. But we are bound to note that there were other factors which made these changes likely.

In university mathematics classical formal geometry had long been in decline. In Edinburgh, for example, Euclid disappeared from the specialist curriculum in name in 1903, and in spirit in 1916. By contrast the basic ideas of analytic geometry and calculus were now central to large areas of the contemporary mathematics curriculum: by the twenties they constituted a major part of the first year university curriculum in mathematics not just for the Honours mathematician but for the physical scientist. The substitution of the elements of analytic geometry and the calculus for some of the more recondite aspects of classical formal geometry in the school curriculum is the curricular change that one would expect the influence of the universities, direct and indirect, to be promoting at that time. Only the removal of informal solid geometry in 1936 is anomalous from this perspective. But, as we shall see, this was reinstated to the curriculum soon after the war.

(4) The 1947 Report and its aftermath

In 1947, the Advisory Council on Education presented their Report 'Secondary Education' which recommended a number of substantial changes in the mathematics curriculum. It criticised the existing curriculum for its:

> "excessive preoccupation with the inner ordering of mathematical truth as against the application of it to the real world"[10]

and recommended that the curriculum should be more concerned with the practical and applicable aspects of mathematics, and less concerned with the abstract and theoretical.

Their major recommendations for curriculum change were:

(1) The reintroduction of informal solid geometry,

(2) A ruthless reduction in the number of proofs required in the geometry course,

(3) The introduction of calculus for the most able pupils,

(4) A substantial reduction in the content of the algebra course; in particular, for most pupils nothing beyond the simultaneous equation and a graphical treatment of quadratics,

(5) A much greater emphasis on practical work in geometry including a thorough training in mensuration and technical drawing,

(6) The introduction of mechanics.[11]

Recommendation (1) was uncontroversial. (2) - in direction, although perhaps not in scale - and (3) simply confirmed the pre-war pattern of change, and were likely to be broadly acceptable to university and university-influenced opinion. (4), (5) and (6) on the contrary represented a dramatic break with the 'pure' tradition of Scottish academic mathematics.

These proposals would have turned school mathematics into a 'technological' rather than an 'intellectual' subject. Indeed the existing curriculum in technical subjects was closer to these proposals than the existing mathematics curriculum. An academic treatment of mechanics might have found some support in university circles - it was common in English schools - but not the relatively untheoretical and practical treatment proposed, and certainly not at the cost of the traditional academic curriculum in algebra and geometry.

These proposals were perceived as a threat to the preparation of university mathematics students and the continuation of the pure mathematical tradition in Scottish education. It is clear from paragraph 475 of the Report that academic mathematicians had already expressed strong opposition to them.

> "Some of our witnesses whose work has lain within the more academic tradition of Scottish mathematics urged, if we may put it so, that real mathematicians must get busy with real Mathematics before the VIth Form"[12]

To placate this opposition the Council suggested that there should be alternative examination syllabuses, one of which would require the traditional theoretical and logically rigorous approach to mathematics. In the event they were unsuccessful. The SED continued to offer a single syllabus and modified it in accordance with proposals (1), (2) and (3). While some of the more elaborate manipulative work on trigonometry was excised, (4) was largely, and (5) and (6) entirely, ignored.[13]

In passing it is worth noting two other important facets of the 1947 Report. First it firmly repudiated the 'transfer of training' theory which had exerted a strong influence on educational theory and practice in the twenties and thirties, and was frequently used to justify the compulsory study of mathematics. Second the Report provided support for the alternative utilitarian argument for the study of mathematics by promoting the idea that the interest of pupils in the subject is related to the 'usefulness' and 'applicability' of the subject content; this was an idea which had a considerable influence on junior secondary education and which continues to inform much current discussion of the curriculum of the noncertificate pupil.

(5) Conclusion

The changes effected in the school curriculum between 1924 and 1952, then, fall into a simple pattern; a gradual diminution in the coverage of, and emphasis on the formal geometry of Euclid, in favour of the elements of analytic geometry and an informal treatment of the calculus. Where it had occurred change in the school curriculum reflected change in the university curriculum.

The last changes in the 'traditional' curriculum took place between 1960 and 1962 with the introduction of the 'O' Grade examination.[14] In Mathematics the reduction in emphasis on formal geometry was carried still further. Statistics was introduced to the Arithmetic syllabus. This reflected the growing use of statistical techniques throughout industry, commerce and research.

In the decade between 1963 and 1973 the 'traditional' mathematics curriculum vanished, swept out by the international movement commonly known as 'The New Mathematics'. In 1973 the last 'traditional' examination on the Higher Grade was set.

Chapter 11: The Origins and Development of the New Mathematics 1952-1968

To fully understand the changes which took place in the Scottish mathematics curriculum in the sixties, we must digress from our main theme to examine the history of the 'New Mathematics'.

(1) The United States

The 1950's saw the institutionalisation of a new mathematics curriculum in university undergraduate courses, on an international scale. This new curriculum incorporated not simply new topics but a new approach to mathematics, both more formal and more abstract. Abstract algebra, topology, function theory, probability and statistics, and numerical analysis, moved into the curriculum. The ideas of set, relation and function, started to permeate the treatment of old and new topics.

The growing distance between the content and approach of the school curriculum and that of the university generated pressure for change in the school curriculum. Until the school curriculum was reformed university mathematicians could not fully implement the changes that they wished to see in the university curriculum. The United States had taken the lead in curriculum change at university level. It was here that pressures for reform in the school curriculum first built up.

The first institutional expression of the curriculum reform movement in mathematics was the University of Illinois Committee on School Mathematics (UICSM) founded in 1951. UICSM set the pattern for the many curriculum development projects which were to follow over the next decade, not only in mathematics but in science. The Illinois project-workers drew up a new mathematics curriculum, designed materials for teaching it, tested these in pilot schools and revised them in the light of this practical experience. They also trained teachers to use the materials and (unlike a number of projects) only made the materials available to teachers trained in their use.[1]

Throughout the mid-fifties, discussion and small-scale development of mathematics curricula were taking place in academic circles. In particular in 1955 the College Entrance Examination Board (CEEB) appointed a Commission on Mathematics to examine the high school mathematics curriculum. This period, however, saw little discussion or change in the schools themselves. It took an external event, a startling demonstration of the achievements of Soviet technology in late 1957, to disturb the equilibrium of American education.

Sputnik was a catalyst rather than a cause. It acted as a symbol around which a number of previously unrelated or conflicting social forces could realign to produce effective pressure for educational change. For those to whom it was a sign of America's technological backwardness or military weaknesses - in particular those traditionally unsympathetic to educational and social expenditure - education became an arm of economic management or national defence. For the curriculum reformers Sputnik offered a pretext for change; for the articulate middle class parents of college-bound pupils, it became a symbol of the failings of the educational system. In short, Sputnik created a climate of opinion in which finance and support for change were unusually forthcoming.[2]

Early 1958 saw the founding of a number of curriculum development projects, including the government financed School Mathematics Study Group (SMSG) which was to become the leading mathematics project in the US. In late 1958 the National Defense Education Act (NDEA) set the development funds flowing. Funding agencies and project workers were agreed on the broad aim of the projects, to redesign the curriculum of the college-bound pupil to produce mathematicians, scientists, and technologists, both in greater number, and with a higher level of knowledge and skill.

Although the federal government provided most of the funds for curriculum development through the NDEA and the National Science Foundation (NSF), the university-based reformers were careful to avoid federal control. They argued that, as academics and professional educators, they alone could provide the authoritative insights into a subject necessary to design a satisfactory curriculum, and that they should be given a free hand in doing so.

The university perspective can, not surprisingly, be seen in the conclusions of the CEEB Commission on Mathematics which reported in late 1958. It strongly endorsed the ideas which formed the basis of the 'modern mathematics' programme for reform - principally represented by UICSM and SMSG - an emphasis on the role of deduction, a concern for 'structure' in mathematics, and the use of the unifying concepts of set, relation and function.[3]

By 1959 the broad outline of a new school mathematics curriculum was clear. The central problem was now that of implementation. College pressure and financial inducement would help. But the key element, a rationalisation of change, emerged from the Woods Hole Conference of September 1959.

The Conference brought together the leading academic figures associated with the major contemporary curricular development projects in mathematics and science, and a number of prominent psychologists and educationalists, to discuss how science education could be improved in American primary and secondary schools. Here, attention was focused, for the first time, on the processes of teaching and learning. While there had been an implicit awareness of this dimension in the work of many of the projects, until Woods Hole their overriding and explicit concern had been with curricular content.

Behind the conclusions drawn in the conference report[4] was the belief that, to adapt to a future increasingly marked by change, the pupil required not knowledge and skills of particulars, but an understanding of the basic ideas and structure of a subject, and a mastery of the high order cognitive skills which would enable him to apply this understanding to a variety of particular situations. The report asserted that,

> "the curriculum of a subject should be determined by the most fundamental understanding that can be achieved of the underlying principles that give structure to the subject"[5]

and that,

> "it is possible to present the fundamental structure of a discipline in such a way as to preserve some of the exciting sequences that lead a student to discover for himself"[6]

A curriculum based on these principles, the report argued, would facilitate learning and increase motivation among pupils.

The role of the university scholar in the design of such a curriculum was seen as central.

> "Designing curricula in a way that reflects the basic structure of a field of knowledge requires the most fundamental understanding of that field. It is a task which cannot be carried out without the active participation of the ablest scholars and scientists."[7]

> "To decide that the elementary ideas of algebra depend upon the fundamentals of the commutative, distributive, and associative laws, one must be a mathematician in a position to appreciate and understand the fundamentals of mathematics...only by the use of our best minds in devising curricula will we bring the fruits of scholarship and wisdom to the student."[8]

Finally, the report argued, in the now famous dictum,

> "that any subject can be taught effectively in some intellectually honest form to any child at any stage of development."[9]

McClure aptly summarizes the conclusions of Woods Hole as

> "a fusion of nineteenth century education emphasis (academic and solid subject matter elements) with the currents of the early twentieth century (Progressive education and the child-centered curriculum)."[10]

The significance of Woods Hole was in providing a common rationale, apparently founded in psychological theory, for what had previously been a relatively haphazard collection of innovations in different subject areas. First, the formulation and endorsement of this rationale by the authoritative figures who attended the conference gave an increased legitimacy both to the individual projects represented here, and to contemporary curriculum reform in general. Second, it provided a rationalisation of the autonomy that the university-based projects enjoyed, and of the form taken by the curricula that they had developed - a form which, at least superficially, seemed closer to the ideals of the pure mathematician than to the needs of the technologist. Finally, and most significantly, this psychologically-based rationale provided reassuring answers to the questions which teachers ask about any innovation, questions about the feasibility of the innovation given the constraints under which they work, in particular those related to the abilities and attitudes of pupils.

(2) Europe

The unprecedented concern with curriculum reform, manifest in the U.S. in the wake of Sputnik, attracted international attention. The American example was a powerful argument for reformers elsewhere; it provided a means of capturing the interest of government and industry. For, in Europe, outside interest stemmed primarily from economic considerations. Contemporary economic orthodoxy argued that the level of educational provision was a major - perhaps the major - factor behind economic growth. Government encouragement of curriculum reform in mathematics and science was part of a larger policy aimed at ensuring the availability of an adequate supply of skilled manpower to sustain and promote economic growth. Whereas, before the war, virtually all professional mathematicians were employed as teachers in educational institutions, the early fifties saw the start of a steadily growing and unsatisfied demand for mathematicians and the particular skills and abilities they possessed, in business and industry. This was a result of the increasing number of non-military uses which were being found for techniques such as operational research and computer automation and simulation, pioneered, mainly in strategic work, during the war years.[11]

In the U.K. the insufficient supply of scientifically-trained manpower had been a consistent concern of government and industry since the war, a concern reflected in the post-war Percy and Barlow Reports, a stream of reports from the Advisory Council on Scientific Policy throughout the fifties, and the Robbins, Swann, and Dainton Reports of the sixties.[12]

In England there were three major conferences on curriculum reform in mathematics between 1957 and 1961, at the universities of Oxford, Liverpool and Southampton. All were financed, at least in part, by industry. Membership came from schools, universities and industry.[13] Their concerns as assumptions were broadly the same. For example, the Foreword to the Southampton Conference Report, by the then Minister of Education, Sir David Eccles, opens,

> "The schools and industry are both short of mathematicians. The fact of the shortage and its gravity has been recognised in the educational world for some time. We know that the quality of mathematics teaching could and should be improved, the curricula brought up to date, and above all the number of mathematicians with good

qualifications increased"[14]

The introductory chapter starts in similar vein,

"We see little reason to argue the importance of fully trained professional mathematicians to this country... There is scarcely any sense in which we as a country would not be disastrously the poorer were we to neglect the study of mathematics. Yet it can only be studied if the teachers are there to teach it, at all levels, in sufficient quantity and quality. Are we training enough professional mathematicians and teachers of mathematics at this time? That a serious, indeed critical, situation exists in this country is clearly recognised within the teaching professions of science and mathematics."[15]

Government support and encouragement for curriculum reform in the sixties is just one of a number of measures, from expansion of university provision in science in the late forties and early fifties[16] to current attempts to develop elite engineering courses and to encourage industrial sponsorship for students on science-based courses[17] which reflect the continuing policy aim of increasing the supply of scientific manpower.

This concern about the availability of scientific manpower was shared by all the major developed countries. In late 1959 the Organization for European Economic Co-operation (OEEC) (now the Organization for Economic Co-operation and Development (OECD), which includes all the large, non-communist developed economies) held a conference at Royaumount in France which was to play a major role in disseminating the 'New Mathematics'. Indeed, in many ways the conference was little more than a propaganda exercise. The organisers had, it seems, already decided that reform was necessary and that it should broadly follow the American example.

> "Despite the great amount of discussion and study of the problems of mathematics teaching, much of it is not having the desired impact on schools... This lag between the new ideas and their effect on the schools is of course inevitable - and even desirable. Nevertheless, it was felt that the time had come to arrange a well prepared exchange of views between those pioneering new ideas in mathematics teaching, and those with responsibilities for policy and its implementation in this field in OEEC countries"[18]

The conference arose from a coalition of interest between the economic planners and the largely university-based reformers. At a rhetorical level both groups sought to 'improve and modernise mathematics teaching' but beneath this apparent consensus lay different conceptualisations of the 'problems' of existing mathematics teaching.

From the viewpoint of the planners the major 'problem' was that too few pupils were studying too little mathematics at all levels. The economy required an increasing volume of more highly mathematically skilled manpower.

Unfortunately (or for the university pure mathematicians fortunately), the planners lacked a clear idea of the kinds of mathematical skills in which pupils were supposedly deficient. They wanted change which would facilitate learning, and increase motivation and interest among pupils. They viewed mathematics as a ladder, and saw the purpose of change as helping and encouraging pupils to climb higher than they had in the past. Certainly they wanted 'modern' rather than 'traditional' mathematics, but this meant to them simply replacing a rather rusty old ladder with a longer and shinier new model which was easier to negotiate, and reached higher. To them both led in the same upwards direction.

The anology is, of course, false. There are different kinds of mathematics and different kinds of mathematical achievement. The reformers had, by contrast, a very clear idea of the kind of mathematics they wished to see in the school curriculum. Indeed for them the 'problem' was that school mathematics was the wrong kind of mathematics - change in the school curriculum had not kept pace with that of the university. The existing school curriculum was a poor preparation for the reformed university curriculum, and this was, in turn, holding back further reforms at university level. The reformers had what the planners thought they wanted, the prototype of a new, modernised school curriculum. And wishful thinking and serious theorising - such as that of Bruner - had already invested the new curriculum with exactly the properties of clarity and excitement which the planners sought.

Both viewpoints are, however, still apparent in the following formulation of the Conference's task, which comes from the official Report of the Conference, 'New Thinking in School Mathematics'.

> "To decide the nature of the mathematics that all capable youth should learn if they are to go on to further study

of science, engineering or mathematics in the university. To find out what mathematical training and competence the university professor desires in his beginning students. To discover - in view of the shortage of scientifically trained personnel in industry, government, research and teaching - how mathematics can be presented so as to attract larger numbers and produce more secondary-school graduates with high competence in the subject"[19]

This view of school mathematics as essentially a preliminary to university study makes university mathematicians the authoritative arbiters of curriculum content at school level.

"The nature of mathematics - and the designation of the types of mathematics that are important - are rightfully the decisions of mathematicians. What portion of mathematics can be taught below university level, to whom it can be taught and the way it can be taught are then the decisions of educators, teachers and writers of textbooks"[20]

It is not surprising, then, that the main speakers, Dieudonné and Tucker, whose proposals for reform are reproduced in considerable detail in the conference report, were both university professors of mathematics. Nor, in view of its constitution, is it surprising that the conference was strongly influenced by American developments. The chairman of the conference was Stone, a university mathematics professor and prominent American reformer, and four of the remaining fifteen speakers at the conference were American. Indeed the programme recommended in outline in the conference report is very close to that of the major American projects and provided the basis for the influential 'Synopses for Modern Secondary School Mathematics' which the OECD published in 1961.

(3) Scotland

By the early sixties, then, there was strong and influential international support for curriculum change. In Scotland, characteristically, the initiative came from the centre. The S.E.D. like the Ministry of Education south of the border, was particularly concerned about the shortage of mathematics teachers which had grown increasingly acute during the fifties. The annual reports on Education of the Secretary of State over this period mention the gravity of this problem with clockwork regularity.

In 1961 the SED took the initiative and set up a committee to consider this problem. It seems plausible that two of the main factors precipitating this new initiative on what was, by then, a relatively old problem, were the burgeoning international interest in curriculum reform, and its authoritative endorsement at conferences such as Woods Hole and Royaumont, and pressure from a small number of active and influential reformers in the universities and colleges, and in the ranks of the inspectorate.

This impression is strengthened by the report of the Committee on Mathematics, entitled 'Recent Changes in Honours courses in Mathematics' written by several of the University mathematicians on the committee.[21] The main body of the report is an exposition of the 'New Mathematics', which was at that time finding its way into the university curriculum. The introduction offers two arguments for corresponding curriculum reform at school level. First,

> "if mathematics in schools can be made more interesting by the introduction of some modern ideas, this of itself might encourage more boys and girls to continue their study of the subject to a higher level"[22]

Second, given the changes taking place in the university curriculum,

> "Changes... [in the school curriculum] ...are even now necessary and... appear to be inevitable"[23]

The quality of these arguments is not impressive. The first, as its phraseology concedes, is based on optimism rather than experience, the second is an example of value-based historicism.

Nevertheless, the introduction concludes,

> "it is hoped to encourage the establishment in a number of schools of pilot groups to experiment with the introduction into school syllabuses of modern aspects of mathematics. The Committee confidently expects that the work of these groups will lead in a relatively short time to considerable developments in the teaching of mathematics in schools"[24]

In April 1963, the SED appointed the Mathematics Syllabus Committee to,

> "review the school mathematics syllabuses and to initiate in a number of schools experimental work on the introduction of certain aspects of modern mathematics"[25]

The Committee consisted of fifteen principal teachers of mathematics, the principal mathematics lecturers from two of the colleges of education, four inspectors of schools, and three university mathematicians. Robertson describes the work of the committee as follows:

> "The committee noted the trends in the development and teaching of mathematics in a number of countries as well as in the Scottish universities and colleges, and critically examined, assessed and redrafted the school syllabus against this background, with the aim that the related courses should be interesting and relevant, and should form a sound foundation for those pupils who would continue the study of mathematics at a later stage"[26]

A draft 'O' Grade syllabus was discussed with representatives of the universities and other institutions of post-school education in December 1963, and the resulting syllabus was made public in April 1964. The committee started to produce a series of texts to cover the new course. In September 1964, when school trials started, 60 Scottish schools adopted the course.

As the Scottish Mathematics Group (SMG), the teacher and lecturer members of the Committee rewrote the trial materials to produce a series of textbooks, Modern Mathematics for Schools (MMS),[27] which has been by far the most frequently used series of texts in Scottish Schools, since that time. Indeed, it is no exaggeration to say that for many teachers MMS is 'The New Mathematics'. In particular it is used by most teachers as the authoritative commentary on the published SCE syllabuses.

The new curriculum which had emerged by 1965 was a compromise, in many ways a conservative compromise, between traditional and modern ideas. In algebra, calculus, trigonometry and arithmetic, most of the traditional content - including, for example, what the Royaumont conference report had described as the "detrimental" and "deplorable" theory of quadratics, and the "unnecessary burdens" of long multiplication and division - remained, although it was now treated in the new language of set, relation and function.

Even in geometry much of the traditional content survived. So, for example, the new curriculum contained no topology but retained a suitably rephrased, but still relatively exhaustive study of the "irrelevant" triangle and circle.

There were, of course, entirely new topics. Sets, functions, matrices and vectors appeared in their own right as well as in the modern treatment of familiar traditional topics. There was an elementary introduction to probability, and the inclusion of a simple form of linear programming and an iterative algorithm for finding square roots was a bow in the direction of numerical methods.

While the new curriculum was undoubtedly the greatest upheaval in mathematics teaching in Scotland since the establishment of the Leaving Certificate, by comparison with many other contemporary curriculum revisions in mathematics it was distinguished by its caution. The relatively conservative character of the new curriculum was no doubt attributable to the predominance of classroom teachers on the committee which designed the new syllabuses and prepared the related curriculum materials. This caution was not without its benefits. It meant that much of the new curriculum was broadly recognisable to teachers as a 'modern' treatment of familiar material. More generally, the predominance of classroom teachers in the planning of the innovation ensured that the new material was, in general, 'suitable' for teachers and pupils - that the majority of teachers and pupils could cope with it - and recognisably so.

The SCEEB[28] circular containing details of the new curricula is at pains to emphasise the similarities with the traditional curriculum,

> "In general, the new topics do not supplant those in the traditional syllabus; they aim to give a greater understanding of the various algebraic techniques and processes, and consequently to facilitate the appreciation of a situation and the acquisition of the skill necessary to carry out the appropriate operations."[29]

Note the Brunerian conflation of the <u>logical</u> and <u>psychological</u> senses of 'understanding' in the foregoing extract; the assumption that the solution to the logical-philosophical problem 'How can we secure mathematical knowledge?' also answers the psychological-pedagogical problem 'How can we help pupils acquire mathematical knowledge?'

The Circular was also anxious to convince teachers that this new, rather abstract, curriculum would interest pupils, if only because for at least the last thirty years teachers had been told that it was practical, useful mathematics that interested pupils. The attempt to assimilate the new mathematics to the traditional ideology makes interesting reading.

> "While the language of sets links mathematics with the world around the pupil it also gives meaning to the idea of a variable and allows a thorough development of the study of equations and inequations"

"Probability, statistics and iterative methods are intrinsically interesting"

"/Transformational geometry/ should be relevant and interesting for all pupils"[30]

The SED had made curriculum change feasible by sponsoring the preparation of textbooks and ensuring the availability of certificate examinations; now it promoted change through inservice training and inspectorial exhortation. But ultimately it was the schools which faced with the choice between the traditional curriculum and a recognisable new curriculum chose to abandon the traditional. Even at the pilot stage around 20% of the Scottish schools offering certificate courses had adopted the new curriculum. The rapid and universal adoption of the new curriculum certainly contrasts with the experience of England and the United States. The Dainton Report, in early 1968, talked admiringly of

"the speed and comprehensiveness of the changeover to the new syllabuses in Scotland"[31]

By then almost all first year potential certificate candidates in mathematics were following the new curriculum.

The cautious design of the new curriculum and the emphasis placed, at the dissemination stage, on content change, and on the continuities between the traditional and reformed curricula brought fast universal take-up of the innovation by schools. But success on these terms was in many ways self-defeating. The price that had to be paid was counter-reformation in the classroom which, while it left the new content intact, assimilated it to traditional approaches to, and strategies of teaching mathematics. This we shall return to in the following section.

Chapter 12: The Non-Academic Curriculum and the New Mathematics

(1) The Context

So far we have been exclusively concerned with the academic certificate curriculum in mathematics. But it does not represent the only tradition of mathematics teaching which can be found in secondary schools today.

While there was, from 1900 onwards, a gradual extension of post-elementary education, academic secondary education remained restricted to the able and the socially privileged.[1] Between 1903 and 1936 the great majority of pupils entered the two or three year 'supplementary courses' or 'advanced divisions', generally conducted in the elementary schools, rather than a 'secondary' course. These courses had, in general, a vocational bias - commercial, technical, domestic, agricultural, nautical. The teaching of mathematics as a separate subject concentrated on revising and extending the arithmetic course of the elementary school which covered what we nowadays term basic and social arithmetic.

Similarly, in the secondary school those whose vocational aspirations lay in the direction of commercial life, or who lacked ability or interest in the academic mathematics course, were able to follow a course in Arithmetic or Commercial Arithmetic, examinable at the Lower Grade of the Leaving Certificate.

Although the 1936 Education Act conferred the name of 'secondary' on all post-primary education, it was clear that 'junior secondary' education was to be of a different type from 'senior secondary'. Andrew, the Senior Chief Inspector, in his 1936 Report, identifies the nature of the change when he writes that it,

> "admits the right of the individual to the type of post primary education most suited to his needs, without involving him in any terminological discrimination"[2]

The raising of the school leaving age to 15 in 1947 marks the start of some form of secondary education for all. In the same year Circular 108 laid down that virtually all children should transfer to secondary school between the age of 11½ and 12½, and certainly none after 13. This meant that most pupils could now expect to spend at least three years in a secondary school. From then until the late sixties the 60-80% of the age group who failed to gain entry to the traditional academic senior secondary course, received junior secondary education, either in separate institutions, or in separate streams in an omnibus secondary school.

In the senior secondary school there continued to be a significant minority of pupils (often girls) who never embarked on, or rapidly dropped out of, the academic mathematics course, and followed a course similar to that of junior secondary education.

It was the elementary school tradition of mathematics teaching which was carried into junior secondary education. The 1947 ACE Report argues that,

> "the evidence is conclusive that very many children, perhaps even a majority, are incapable of progressing any distance in.. [mathematics] .. or of extracting any substantial benefit from [its] study"[3]

The Report concludes that, of the great majority of junior secondary pupils,

> "little mathematics can be required... beyond simple

> everyday arithmetic, easy mensuration and the veriest elements of graphical work - with the immediate usefulness of what is being done evident at all times... Arithmetic should be treated throughout as a "tool" subject"[4]

The narrow utilitarian approach to mathematics which characterised junior secondary education can be inferred from the fact that even an adventurous and idealistic document of the time, the 1955 Memorandum on Junior Secondary Education, spends the first three of its four paragraphs on the aims of mathematics teaching stressing the everyday and vocational importance of basic arithmetic and mathematical skills, and bases its discussion of the content of courses squarely on the perceived vocational and everyday needs of pupils.

> "The technical subjects in the school course for boys, and many of the occupations normal for men, demand a degree of competence in geometry and algebra which is not asked for either in the other subjects of the girls' courses or in the posts usually open to women. Further, it is a common experience of many teachers that applications of these branches of mathematics which occur in everyday life tend to do so in circumstances which interest boys more than girls. Courses in geometry and in algebra, therefore, are not regarded as essential in mathematics courses for girls"[5]

For the girls,

> "A large part of the course... must have as its basis the arithmetic of home and shop"[6]

The 1962 Report 'New Ways in Junior Secondary Education' describes the achievement of junior secondary education as follows,

> "Effort has been directed mainly towards ensuring that work is directly related to the pupils' abilities, needs and interests, and that it is permeated by a sense of realism and purpose that makes it a practical preparation for life... In mathematics...the work frequently deals largely with the useful application of mathematics to such matters as timetables and ready reckoners, budgeting, taxation, hire-purchase, everyday formulae, statistical graphs, mensuration and surveying"[7]

For the majority of junior secondary pupils, then, the mathematics course covered little more than 'basic' and 'social' arithmetic, and occasionally more specialised skills related to some anticipated social or vocational role. A small number of junior secondary pupils did study a more complete mathematics course, particularly after the introduction of the SCE 'O' Grade examinations in 1962. In general such a course was modelled on the academic Certificate course with which teachers were familiar, but adapted, where the occasion demanded, to what were seen as the interests and abilities of junior secondary pupils, and their needs in work and further education.

(2) <u>The Interaction</u>

The curriculum of the 'junior secondary pupil' was reformed not by direct design but through the introduction of the comprehensive school and the common course. The process of curriculum reform in the early sixties did not involve, and scarcely touched, the junior secondary schools. It was intended and planned as a reform or the curriculum of the university bound student in the senior secondary school.[8]

In October 1965 when Circular 600 was issued the new curriculum was already well established in senior secondary courses, where it was being followed by around 70% of the first year entrants in Scotland. Schools and teachers were faced with a dilemma. The rhetoric of comprehensivisation - 'equality of opportunity' and 'a grammar school education for all' - pointed to a common course in mathematics based on the new academic curriculum, whereas the received wisdom of mathematics teachers - exemplified in the 1947 ACE Report - suggested that neither teachers nor pupils could cope with such a situation. The dilemma was only exacerbated by the impending raising of the school leaving to 16.

The short term response - and one which continues to be common - was to offer a common course based on the SMG curriculum for an initial period - sometimes as short as a term, typically one school year - and then to set pupils by ability. Under such a plan only the upper sets continue with the full certificate course, while the remainder pursue diluted versions, which for many of the lower sets contain little more than traditional 'basic' and 'social' arithmetic.

Many teachers of certificate classes found that a suitably adapted SMG course which ignored the logical and conceptual structure of mathematics and placed an increased emphasis on the acquisition and practice of standard content - specific skills, was quite adequate for successful presentation for the SCE examinations. This was, after all, a familiar, and, in relative terms, a predictably effective strategy for optimising pupil performance in such examinations.

This adaptation found official sanction, first in the revision of MMS "to cater more adequately for the wider range of pupils now taking certificate courses"... in the light of... "experience gained in the classroom"[9] and second in the production for Glasgow Corporation Education Committee of two rival series of textbooks[10] aimed at non-certificate and certificate pupils, by a group of teachers concerned,

> "that the 'average pupil'... was finding more difficulty with the content of Modern Mathematics syllabuses than was anticipated... [due to]...the oversophisticated treatment encountered in many Modern Mathematics texts.... [and]..the difficulty in extracting the essential features of topics."[11]

In mathematics, as in other subjects, the rhetoric of egalitarianism was translated in practice into the aim of maximising pupils' chances of gaining the tangible rewards of certification. In 1972 the SCEEB decided to band awards at 'O' grade from A to E. Bands A to C were to correspond to the existing pass standard, while D and E were intended as a recognition of achievement for candidates who performed less well in the examination. This change, of course, acted as a 'multiplier'. Its effect was to increase still further the range of pupils who could hope to reap some reward from the SCE examinations. The outcome was a dramatic increase in the proportion of pupils presented for certificate examinations which was not accompanied by a commensurate rise in the number of passes, as the following figures show.[12]

S4 'O' Grade presentations and awards in Arithmetic and Mathematics

	Arithmetic		Mathematics	
Year	Presentations*	Passes*	Presentations*	Passes*
1970	39.7	32.2	25.7	19.2
1971	43.0	34.1	27.1	20.5
1972	44.6	35.3	28.6	21.7
1973	48.7	37.1	31.4	21.6
1974	61.2	38.8	34.9	21.5
1975	62.3	39.1	35.1	20.9
1976	64.5	40.7	36.2	21.2

*as a percentage of the S1 population three years before

One further innovation points to the institutionalisation of this response. In 1972 a working party was set up

"(1) To review and, where necessary, adapt the content of the existing alternative syllabuses for use within a wide range of abilities in S1 and S2

(2) To devise a course in mathematics which is suitable for the needs of pupils who do not in the first instance, propose to continue the study of mathematics beyond the Ordinary grade of the SCE. The course would be primarily concerned with providing a general mathematics education but would be devised in such a way as to permit pupils who so wish to carry the subject further."[13]

The working party produced the Modular Mathematics Scheme for S1/S2 and the 'Syllabus B' curriculum for S3/S4 leading to presentation at 'O' grade. Syllabus B excludes a large amount of the more formal and theoretical content of Syllabus A in algebra and geometry. This has been replaced by a much fuller treatment of statistics along the lines of the existing 'O' grade in Statistics.

Although the main attraction of Syllabus B for teachers, and its original rationale, is that it is more 'suitable' than Syllabus A for 'the average pupil' (a euphemism for 'easier'?), such a curriculum is, in content, considerably more useful for the potential university biological or social scientist, than Syllabus A.

The progress of Syllabus B has been held back by two factors. First, the refusal of the Scottish Universities Council on Entrance (SUCE) to recognise an 'O' grade on Syllabus B as a qualification for university entry has resulted in the new Syllabus being still confined, in S3/S4, to the original pilot schools, and there only to classes of 'less-able' pupils.[14] Second, while at the S1/S2 stage Modular Mathematics has been adopted by a number of non-pilot schools which use it as an alternative to the S1/S2 course, based on MMS 1-4, its higher cost, in a time of financial restrictions, and the uncertainty over the S3/S4 continuation have deterred schools from adopting it.

(3) <u>The outcome</u>

It is possible to talk of a typical current pattern of mathematics education in Scottish schools. While the proportion of pupils in different courses may vary from school to school, the structure of the course is remarkably stable.

All pupils will, in S1, embark on a common course based on MMS, or, less frequently, Modular Mathematics. They will initially be taught in mixed ability groups but by the start of S2 it is likely, and by the start of S3 certain, that they will have been set by ability. Once this setting has taken place the curriculum of different groups rapidly diverges; the lower the set the greater the emphasis on arithmetic.

All pupils will be required to study some 'maths' in S3 and S4. SCMSTE [1977][15] suggests, for example, that around 40% should embark on a course aimed at presentation in Arithmetic and Mathematics (or in a smaller number of cases, statistics). It suggests that a further 30% should aim at presentation in Arithmetic only. Finally 30% will be presented in no examination. In practice, many schools have allocated even larger proportions of pupils to certificate courses in recent years, and there has been a considerable drop-out prior even to presentation. In certificate classes, the tendency is, understandably, to concentrate exclusively on the prescribed content. In noncertificate classes little more than basic and social arithmetic is generally taught. SCMSTE [1976] and SCMSTE [1977] can be seen as attempts to widen the curriculum for the less able 60%, most of whom, as the figures on page 166 show, fail to achieve any certificate passes in mathematical subjects.

In short comprehensivisation and RSLA have had no notable effect on the content of school mathematics, only on the range of pupils following the full academic curriculum. It is the meritocratic, rather than the democratic, aspects of the comprehensive ideology which have influenced schools. Change has been focused on the selective, not the educative, function of schools. The result has been a move away from 'sponsored' towards 'contest' mobility in the secondary school, which has incidentally, rather than intentionally, altered the content of the curriculum for some groups of pupils.

Chapter 13: The Pattern of Curriculum Change

The foregoing sections have offered a largely sequential account of individual changes, or proposals for change, in the certificate curriculum. In this section I will draw on this account in developing a more general understanding of change.

(1) The Perceived Functions of Mathematical Education

To understand curriculum change in mathematics we must look at what are perceived as the purposes and problems of teaching the subject. There is a broad consensus both on the value of mathematics, and on the reasons for its value. This extract from the Norwood Report exemplifies both elements of the consensus.

> "In the first three years of the secondary Grammar school Mathematics should in our opinion be taken by all pupils... first, it is essential that all pupils should gain at least a knowledge of such mathematics as is necessary for everyday affairs and some acquaintance with the most elementary mathematical principles; secondly, full opportunity must be given for mathematical ability or disability to declare itself... We contemplate that for the succeeding two years the majority of pupils should continue a course of Mathematics which would be appropriate to those who need Mathematics for their career"[1]

Here the purpose of teaching mathematics is conceived in direct utilitarian terms as meeting the everyday and vocational needs of pupils for certain arithmetic and mathematical skills and knowledge.

The consensual nature of this belief, at least among parents and pupils, is illustrated clearly in the findings of the 1968 Schools Council Survey,[2] which established that English (including reading, writing and spelling) and mathematics (including arithmetic) are seen by parents as by far the most important subjects for their child to learn at school. Around 95% and 90% of parents rated English and mathematics, respectively, as 'important for their child to learn at school.'[3]

Similarly pupils and recent leavers rated mathematics and English well above other 'nonvocational' subjects in terms of their 'usefulness to learn at school'. Around 93% of pupils and 89% of recent leavers rated mathematics as 'useful to learn at school'.[4,5]

It is clear from the survey that the high valuation accorded to mathematics by parents and pupils is due to its perceived everyday and vocational utility. For example if we examine the reasons given by 15 year old leavers for considering a subject as 'useful to learn at school' the valued characteristics of mathematics emerge strongly.[6]

Leavers were asked to name up to three school subjects which they considered useful. They were then asked to describe the ways in which each of the subjects they had named was useful. This gives a rating for each of the different ways in which a subject was perceived as useful. For example, 70% of the leavers who named mathematics thought that it was vocationally useful, 1% that it provided recreational interest and enjoyment, and so on. In the table below the first three columns indicate the spread of ratings. The first column gives the highest rating that any of the 14 subjects received on each of the particular ways of being useful, the second the median rating, and the third the lowest rating. The fourth column gives the ratings of mathematics on each count. The final column gives the rank order of mathematics among all 14 subjects on each particular count.

		% RATINGS				RANK
Kinds of usefulness		highest	median	lowest	maths	maths
VOCATIONAL	useful in job, helps get a good job	94	44	4	70	4
DOMESTIC	useful in the home, when married	79	2	0	2	7
GENERAL	generally useful, important	29	17	1	22	6
RECREATIONAL	provides recreational interest/ enjoyment	33	10	0	1	13
LIBERAL	develops character, broadens outlook	56	1	0	0	11.5
BASIC	speaking properly, reading, writing, counting	31	0	0	12	2

Mathematics is seen as vocationally useful (only technical and commercial subjects have higher ratings), and as providing basic skills (only English has a higher rating). It is not seen as recreational (only commercial subjects have a lower rating) or as broadening one's outlook (rated on a par with technical and domestic subjects, and PE).

As we established in a previous section, government, business and industry have, since the war, seen mathematics teaching as important because of the manpower needs of the economy. Attention has, in the main, been focused on two 'problems'; ensuring the 'numeracy' of school leavers entering employment or vocational training directly, and increasing the supply of highly trained, specialist mathematicians, scientists and technologists. This concern has been consistently reflected in government policy; for example, the encouragement and financing of inservice training and curriculum development in mathematics, various measures aimed at increasing recruitment, and improving the quality of recruits to mathematics teaching, and the recent institution of the Assessment of Performance Unit (APU) to monitor 'standards' in mathematics and other vocationally important subjects.[7]

This utilitarian view of mathematics is also found in the official SED documents dealing with the mathematics curriculum. Here the problem of determining the curriculum is seen as one of

satisfying the needs of different groups of pupils within the constraints imposed by their differing abilities and interests.

The 1931 Memorandum on the mathematics curriculum reflects this perspective of needs, abilities and interests, when it talks of, "the difficulty of certain parts of the present course... /being/ .. out of all proportion to their usefulness to the average pupil"[8]

It also suggests that both the interest and usefulness of the mathematics course to the average pupil depend on the number of practical applications it contains.

Similarly the 1947 ACE Report argues that the mathematics curriculum should,

"take account of the needs and limitations of the pupils in question"[9]

For example, while the members of the Committee considered the proposed curriculum,

"suitab/le/ and sufficien/t/.. for the average pupil,"[10]

they were concerned as to whether,

"the small minority who have it in them to be mathematics specialists /would/ manage to cover, in two years of VIth form work, all the manipulative practice and the more theoretical, systematic approach to the subject necessary in preparation for the university"[10]

Again they considered that for,

"the girl who combines passable general ability with undeniable weakness in Mathematics ... the sensible course is to attempt little beyond "tool arithmetic", since anything more ambitious is unlikely to be required for the career she will elect to follow"[11]

The Report suggests again that the interest of the 'average' or 'less able' pupil is directly related to the practicality and utility of the subject matter taught.

We have already seen, in an earlier section, how the emphasis in the 1955 Memorandum on Junior Secondary Education is on the everyday and vocational utility of mathematics as both the index of its value and the basis of interest in it. Similarly the Brunton Report of 1963,[12] whose motivational theory centres on the idea of the 'vocational impulse', draws attention to the exceptional vocational importance of English and mathematics among the general subjects, and sets out a minimum requirement of vocationally relevant content in these subjects.

The introductory note to the 1965 SCEEB circular on the modern 'O' Grade Syllabuses, describes the aim of the new curricula in arithmetic and mathematics as,

> "to provide a useful and appropriate study for pupils whose further use of mathematics will be in the home, in business and in industry and commerce, as well as to form sound foundations for those who will proceed to study mathematics at higher levels."[13]

The 1977 Munn Report[14] follows Hirst in identifying a number of distinctive modes of enquiry, and argues that each of these should be represented in the school curriculum. The inclusion of mathematics in the curriculum, along with English, PE, RE, social studies, science and aesthetics, is justified on these grounds. But, in the case of each of these areas, the Report devotes rather more space to specific arguments for the inclusion of each in the curriculum. Mathematics, the report argues, is important because it is a prerequisite for engaging in other important curricular activities, such as science, and because it contains the basic numerical skills which are essential for life in present day society; there is more than a hint of the vocational when the Report writes,

> "The scope of this basic social need is not analysed here, but we believe that is a task to be performed by mathematics teachers in co-operation with industrialists and others in identifying what is required in detail. It is clear that the need exists, and that, for all pupils, mathematical studies will retain a high priority in S3 and S4".[15]

The purpose of school mathematics is seen, then, as providing the technical expertise which pupils will need

(1) In everyday life ,
(2) In studying other subjects at school and post-school level,
(3) In some occupation or differentiated social role,
(4) In studying mathematics at university.

These are, of course, not the only needs which mathematical education might seek to meet. Mathematical education of another kind might make a substantial contribution to liberal or humanistic education, to aesthetic education, or to social education, for example. Nonetheless, it is these directly utilitarian purposes which are consistently cited in arguing for the value of mathematical education and which inform its aims, content and approach.

There is an element of imprecision in this analysis of 'utility'. In one sense 'vocational utility' is all embracing. On one hand much of the arithmetic which is of use in everyday life is essential for many of the occupations that leavers enter directly from school. Further, if we wanted to, we could certainly, for any specific skill of everyday arithmetic, find some occupation in which it was needed. In addition the demand for employees with mathematically-based higher or further education, and the likelihood of students with such an education entering an occupation calling for the use of their specialised skills and knowledge, might lead us to conceptualise needs (2) and (4) as, in some ultimate sense, vocational. In this sense, 'vocational utility' subsumes the other categories. But while the conceptual simplicity which results from this reduction of 'utility' to a single category may suit the level of discussion of the politician, it does not provide an adequate conceptualisation for understanding curriculum change. In any case, in the official reports we have examined 'vocational utility' is used in a more precise and restricted sense; that is 'usefulness in those occupations which school leavers enter directly'. This is the sense in which we shall use it for the present, although, as we shall see later, this sense still has its complications.

(2) Change in the Mathematics Curriculum

We shall now examine the extent to which these different areas of need are reflected in the changing certificate curriculum.

From its inception the curriculum has been split neatly in two, into Arithmetic and Mathematics; this distinction has, as we have seen, been consistently maintained. In particular, the content of the Arithmetic certificate course has traditionally resembled (or rather perhaps influenced) that of noncertificate courses in 'mathematics'. The institutionalization of this distinction in the structure of curriculum and examinations suggests that it is a significant one of which any analysis ought to take account.

In its origins Arithmetic was 'practical': it concentrated on skills which were judged useful in 'everyday life' - the elementary rules, vulgar and decimal fractions, percentages, money, weights and measures, simple mensuration and the like. Specifically 'vocational' skills were left to courses in commercial or technical subjects. (For example, there has, since the start of the Leaving Certificate examinations, been a specialised curriculum in Commercial Arithmetic, or latterly Accounting, which assumes a previous knowledge of the contents of the Arithmetic curriculum.)

Mathematics by contrast was 'intellectual'; it was, in content at least, primarily a preparation for university mathematics. We shall look first at change in the Mathematics curriculum.

The major official proposals for change in the Mathematics curriculum since the start of this century are as follows:

A The introduction of elementary analytic geometry to the curriculum in place of some parts of classical formal geometry (between 1924 and 1936),

B The successful recommendation (in the 1948 ACE Report, implemented in 1950) that calculus should replace some further parts of classical formal geometry ,

C The unsuccessful recommendation (in the 1947 ACE Report) that an alternative curriculum, placing much less emphasis on formal geometry and theoretical algebra, and more on practical aspects of mathematics such as geometrical drawing, mensuration and mechanics should be introduced , D The introduction of the 'Alternative Syllabus' (now 'Syllabus A') (between 1963 and 1968) containing 'modern' topics and treating 'traditional' topics in 'modern' terms,

E The introduction of 'Syllabus B' placing less emphasis on the formal and theoretical parts of algebra and geometry, and more on statistics (between 1971 and 1975).

These changes have been discussed in previous sections. The table below summarises the ways in which these changes brought, or would have brought, the curriculum closer to one appropriate to the different areas of need of which it is commonly argued that the mathematics curriculum ought to take account; (1) everyday need, (2) other subject need, (3) vocational need, (4) university mathematics need. This will also help us to identify those interests which are able to influence the curriculum.

PROPOSAL UTILITY

successful (✓)/ unsuccessful (x)		more (+)/less (-) useful than existing curriculum in area (1)	(2)	(3)	(4)
✓	A		+(university physical science)		+
✓	B		+(university physical science)		+
x	C		+(school & college technical studies)	+(engineering trades)	-
✓	D				+
x	E		+(university biological & social sciences)		-

Undoubtedly, the major change in the Mathematics curriculum since its inception has been the 'New Maths' (case D). Here, as with all the other successful proposals, the contemporary Mathematics curriculum was - and the evidence suggests quite intentionally - brought closer to the changing university mathematics curriculum. Both the unsuccessful proposals would have created an alternative curriculum much further removed from the university curriculum than the established curriculum of the time. Again this was the intention of the proposers. This is strong evidence for the influence of the university mathematics curriculum on the school curriculum. Indeed, it points to this as the major influence.

It is harder to identify the pattern of influence, if any, behind the much slighter correlation between change in the curriculum and the needs of the university physical scientist. One could argue that cases A and B reflect the contemporary influence of what we might call the 'Cartesian' view of mathematics as 'the science of quantity' on the mathematics curriculum of the university and school, as much as the influence of the interests of physical scientists. By contrast many physical scientists disagreed strongly with the 'structural' conception of mathematics underlying the New Maths and protested vehemently against its introduction to the schools, although it is not clear that the watered-down structuralism which found its way into the school curriculum seriously affected the traditional emphasis on the acquisition of manipulative skills, or that the main victim of reform, the Euclidean approach to geometry, was of greater value to the potential student of physical science than the transformational geometry which replaced it. This does suggest, however, that the influence of such needs and of the interests which represent them has been only of marginal significance in determining the pattern of curriculum change.

Case C provides an example of a proposal for change which was specifically intended to come closer to the mathematical needs of the engineering and allied trades. Its failure points to the lack of influence of vocational needs and vocational interests on the curriculum.

Case E is more complex. Perhaps the major purpose behind its design was to provide a curriculum which would be more manageable for the majority of pupils. On the other hand the course itself, with its emphasis on statistics and its eschewal of the more recondite aspects of pure mathematics certainly comes closer to meeting the needs of the potential biological or social scientist than Syllabus A.

The conclusion that we must draw from an examination of these five proposals for curriculum change is that the dominant and consistent influence on the Mathematics curriculum has been the university mathematics curriculum.

There are a number of ways in which this influence is exerted. First, the universities, in particular university mathematicians, have exerted a direct political influence on the content of the school mathematics curriculum. In the 1880's the conditions under which the universities agreed to accept the new Leaving Certificate amounted to direct university control of the course content. Even today the universities can exert direct pressure at several points in the development of a new curriculum. First through their representation on the Consultative Committee on the Curriculum and on its Central Committee on Mathematics, then through their representation on the SCEEB Board and Mathematics Subject Panel, and finally through SUCE which can refuse to recognise a new curriculum or examination for university entrance. The Syllabus B curriculum is an example of an innovation halted at this last ditch.

Second as the origins of both the 'traditional' and 'modern' curricula illustrate, university mathematicians may play a major role in initiating and promoting curriculum change, in the past through informal networks, but now through participation in the official SED structure of subject committees and curriculum development panels.

Finally this overt influence of the universities on the school curriculum is legitimised, and a stronger indirect influence exerted, by the status and authority of the universities as the repositories of learning. University mathematicians are the 'subject experts'. What they choose to teach, or to endorse, is made legitimate as 'mathematics' or more generally as knowledge. Most teachers of mathematics receive the 'mathematical' part of their training in university mathematics departments. Their conception of mathematics is developed through their school and university experience; it is influenced by the selection from knowledge that is transmitted to them, and by the way that this knowledge is made intelligible.

(3) Change in the Arithmetic Curriculum:

We now turn to the Arithmetic curriculum. Here the major proposals for change in the certificate curriculum have taken place since 1960. The changes are as follows:

F The introduction of statistics (between 1960 and 1962),

G The Alternative Syllabus which introduced the study of number systems, number bases, permutations and theoretical probability,

G The change from imperial to metric measures (between 1968 and 1973),

I The change to decimal currency (between 1969 and 1971),

J The introduction of the alternative Syllabus B in Arithmetic. Syllabus B differs principally from Syllabus A in including the study of flow charts and of simple computer programming, in assuming the use of the slide rule and calculator (rather than logarithms) as aids to calculation, and in excluding the more abstract aspects of arithmetic. Some topics treated as Arithmetic in Syllabus A, are, in Syllabus B, treated as part of the Mathematics curriculum.

We now examine these changes, one by one.

F: Since the Thirties the use of statistical methods of analysis and representation, in industry, commerce and the social and natural sciences had been growing rapidly. By the early fifties statistical representation was becoming a relatively common feature of most people's experience, and statistics was moving into the mainstream of the university mathematics curriculum. Thus, while there does not seem to have been strong pressure for the introduction of an elementary treatment of statistics, it was a change to which outside interests - parents, employers, politicians and academics - were unlikely to be unfavourable. In addition, however, experience had shown this to be a topic

Which average and less able pupils were capable of tackling, and enjoyed. It was thus an obvious candidate for filling out the new 'O' Grade Arithmetic curriculum.

G: It is significant that at a time when (in relative terms) cataclysmic changes were taking place in the Mathematics curriculum, the traditional core of the Arithmetic curriculum remained untouched. New topics resulted from the use of the Arithmetic curriculum as a receptacle for the overflow from the brimming Mathematics curriculum, rather than from a re-structuring or extension of the approach and content of the traditional Arithmetic curriculum. A minor, but interesting, exception was the greater attention paid to tolerances, approximations, and errors in the new curriculum. By contrast, this change corresponded to vocational and other-subject needs rather than those of the new university mathematics.

H,I: Here the curriculum planners sought to anticipate, rather than reflect, change in everyday and vocational needs. In both cases change in the school curriculum was linked to a government time-table of change. In case I the timetable was almost universally followed both inside and outside the schools. In case H however, industry and commerce have tended to lag far behind the official programme of change and this has led to a mismatch between the skills of school leavers and the immediate needs of many employers.

J: Like G, J is overshadowed by the Mathematics curriculum with which it is associated; its future hangs in the balance not on its own acceptability, but on that of the Syllabus B 'package' as a whole. It is worth noting however that it includes the main part of the traditional core (logarithms being the only notable exception).

What is striking then, about the Arithmetic curriculum is how little it has changed. The original curriculum, aimed at the 'everyday needs' of the Victorian pedagogue's commonsense stereotype of man, has survived virtually intact. What change there has been has added peripheral topics rather than restructuring or sweeping away the old. A simple index of this stability is the absence of significant change in the curriculum until 1960.

We have already mentioned that there is a substantial common ground between the arithmetic skills which are judged useful in everyday life, in studying other subjects, and in work or vocational training. Even in junior secondary education where the emphasis on vocationalism was extremely high, vocational needs were, in general, seen as being best served by giving pupils a thorough grounding in basic and social arithmetic, rather than in any more vocationally specialised skills. This also seems to accord with the desires of employers then and now. The Brunton Report for example, argues that while employers expect the school to provide a foundation of broadly-based skills, they prefer to train their own employees themselves in the specific techniques which their work requires.

In effect the skills which employers expect the schools to provide are largely those which are, in any case, part of basic and social arithmetic. When employers require recruits to have 'O' grade Arithmetic or Mathematics they do so because they believe that these are relatively reliable indices both of 'general ability' and of competence in the skills on which vocational training is based.

In particular, the vocational value of 'O' grade Mathematics for most school leavers lies not in its content, but in the association between success in that examination and relatively high facility in basic arithmetic skills. In the relatively small number of cases where more specialised skills are required they are provided and certificated not by the mathematics courses and examinations, but by the more directly vocationally oriented technical or commercial courses and examinations.

The stability of the Arithmetic curriculum suggests that it has in general been able to meet the demands of different interests (even if it has often gone beyond what is strictly necessary to meet them) and that these demands have changed little enough to be capable of accommodation within the traditional curriculum.

Chapter 14: The Politics of Curriculum Change

(1) A General Outline

In the Scottish educational system the traditional pattern of innovation, of which the mathematics curriculum provides just one example, is 'centre-periphery'. Until the mid-sixties curriculum change simply 'emerged' from the SED. Since then the SED has delegated, rather than devolved, some of its responsibilities to the SCEEB and the CCC, and their numerous sub-committees.

But, in either form, the centralised process of decision making has been little documented. For this reason most of the conclusions which we draw about this process can be no more than tentative. On the other hand, this very lack of evidence enables us to draw at least one firm conclusion; it testifies to the absence of public participation in the process. The development of the SCEEB and the CCC may have given certain professional groups within the educational system a greater influence, but it still effectively excludes outside influences, as the evidence of the Secretary of the SED to the Commission on the Constitution demonstrates. Asked how the Department reacted to the public at large, and in what way it was sensitive to public opinion, the Secretary replied,

> "To a considerable extent through Parliament. On the more specialised issues, through teacher opinion, reflected either through the teachers' associations or to an increasing extent through the sort of advisory apparatus we have, which very largely now involves teachers, headmasters, principals of colleges, directly"[1]

Whereas most aspects of Scottish life are governed by British legislation and ultimately administered by Whitehall, the SED is one of a small number of specifically Scottish agencies which enjoy a considerable degree of autonomy. Virtually all the legislation governing education in Scotland is Scottish, and on those issues defined as 'educational', rather than 'political', the Department is free to arrive at independent decisions.[2]

On the other hand, the most widely controversial 'political' decisions - those, for example, on the school leaving age, comprehensive education, the price of school meals - are made at a (British) national level. Here it is the DES - which also administers non-university education in England and Wales, and, through the UGC and the research councils, university education and research in Britain - which is most closely involved in decision making. For this reason almost all interest groups outside the educational system - the CBI, the TUC, the political parties, for example - and many within - the NUS and CASE for example - are organised on a (British) national level, to exert pressure at the centre.

Further, whereas the policy of the DES may be influenced by a strong education minister who is free to give almost all of his or her attention to the policy of the Department, an equivalent situation is most unlikely in relation to the SED. The Scottish Secretary and his junior ministers all have multiple responsibilities within a range of London and Edinburgh-based departments.

It is also clear that the SED has little formal contact with the DES other than on the small number of 'political' issues. Kellas writes,

> "Contact between civil servants of the SED and the DES rarely concerns substantive matters of policy. It is assumed that each department has its own educational system to administer, and that the one should not get in the other's way ... in fact, most SED trips to London are to the Treasury, or to Parliament, when legislation is being pushed through."[3]

It is clear that the SED welcomes the administrative convenience which results from its sheltered situation. In a system dominated by educational specialists it is easier to make and implement decisions.[4]

In the Scottish educational system, then, the influence of parents, pupils, politicians and employers is weak. It is the professional groups within the education system, and the civil servants who make policy. And in the area of the curriculum, in particular, professional claims to special expertise strengthen the forces which exclude public influence.

(2) The Mathematics Curriculum

In the matter of the mathematics curriculum, what evidence there is suggests that it is the same interests which exerted influence in the pre-1965 informal system which are represented in the CCC and SCEEB network; the SED - both the administration and the Inspectorate, the universities, and, increasingly in recent years, an atypical group of teachers, experienced, committed to innovation, and promoted (within the school, or as local authority advisers or college of education lecturers). These teachers have the same kind of professional background as the members of the Inspectorate, and have shouldered a part of the development burden which fell on the inspectorate in the early sixties. Similarly the SCEEB has taken up many of the inspectors' administrative responsibilities, leaving them free to make, or influence, general policy.[5]

One interesting example of this important facet of the politics of curriculum change is provided by the precipitate abandonment of Imperial measures in the curriculum. Remarkably, the CCC subcommittee on Decimalisation and Metrication included no representative of industry and commerce: its membership was exclusively drawn from 'inside interests'.[6] It is hardly surprising, then, that the recommendations of this subcommittee failed to take account of the slower pace of change to metric units in industry and commerce.

Within this small circle of decision makers, however, complex considerations come into play. For the universities the matter is relatively simple. They have power but very little responsibility. The SED on the other hand, in deciding whether to promote an innovation will be concerned with how it has been, or will be, received by the teachers and the schools, and by the universities. Similarly the critical factors governing the acceptability of an innovation to ordinary teachers and schools are likely to be its feasibility, and its acceptability, first to the SCEEB (if it relates to a certificate course, as most innovations do), and then to the universities.

It is relatively easy to construct simple and politically significant indices of acceptability to the universities, and to the SED.

SED: Did the SED or SCEEB offer an examination reflecting the curriculum innovation:

Universities: If an exam was offered, was it accepted by the Scottish Universities Entrance Board (SUEB) or SUCE? If not, is there evidence of university opposition?

There is no such simple index of acceptability to schools and teachers. In practice the factors of feasibility and acceptability to ordinary classroom teachers are closely linked. A curriculum innovation may fail because it makes demands on teachers which they are unable to meet, or because teachers are unconvinced of its value, and thus do not adopt it, or adopt it without conviction. An innovation may fail because,

(1) The new content is unfamiliar to teachers,

(2) The innovation requires new teaching methods and/or assumptions about teaching and learning,

(3) The content/teaching methods appear to be, or prove to be, unsuitable (too difficult/uninteresting) for pupils,

(4) The innovation makes much greater demands on resources - teachers' time, materials, space.

On the other hand, all these factors can be manipulated by the SED and local authorities through,

(5) The provision of inservice-training,

(6) The revision of curriculum materials and content,

(7) The provision of additional resources to schools taking up the innovation.

Let us examine the five proposed innovations in the mathematics curriculum in the light of these considerations.

Case A is straightforward. The university Ordinary curriculum had included analytic geometry since the end of the nineteenth century. The new content was therefore familiar to most secondary teachers in the late twenties. As the 1931 Memorandum, and later the 1947 ACE Report testify, traditional Euclidean geometry was generally agreed to be boring and incomprehensible to the majority of pupils. Teachers were unlikely, therefore, to reject this less formal alternative on such grounds. Finally there is no evidence of conflict between the universities and the SED on this matter. The leaving certificate syllabus was altered by the SED and accepted by the universities in four stages between 1924 and 1936.

Proposals B and C, although presented as alternatives, were fundamentally in opposition. While both offered the prospect of a further reduction in the unpopular Euclidean geometry, B represented a continuation of the academic tradition; C offered a quite different practical mathematics. The new material of B, the calculus, had been part of the university Ordinary mathematics curriculum since around 1910. By contrast the university trained teacher was unlikely to have encountered the practical mechanics and geometrical drawing with which C proposed to replace Euclidean geometry and theoretical algebra. In addition it raised questions about resources and teaching styles. For mathematics teachers, then, C would have been a leap in the dark. Moreover, C was, as we have seen, strongly opposed by the universities. Given these conditions its eclipse is not surprising.

Change between 1924 and 1961 moved the curriculum in a direction which was broadly acceptable both to the universities and to teachers, although for rather different reasons. For the former it brought the school curriculum closer to the reformed university curriculum. Of the latter it made few demands, and produced a curriculum which in prospect appeared, and in practice, proved, at least no more boring and no less comprehensible to the majority of pupils.

Proposal D, however, presents us with a case which is much less clear cut. Here change introduced to the curriculum new material, and a new approach to mathematics, which was only familiar to recent Honours graduates. Initially, at least, there were also suggestions of new teaching methods, notably 'discovery learning'. Purely in terms of providing textbooks suitable for the new curriculum there was a clear need for additional resources. All these initial factors, therefore, were likely to predispose teachers against the change. On the other hand it is clear that the SED was determined that this change should be successfully implemented in schools.

The SED funded the preparation and production of textbooks for pilot schools, and supported in-service training on a massive scale. By the end of 1967 around 50% of Scottish mathematics teachers had attended at least one in-service course on 'modern mathematics'.[7] Most local authorities were equally unstinting, making generous funds available to schools who took up the new curriculum. There can be little doubt that the enthusiastic endorsement of the 'New Maths' by the SED and the provision of generous funding for its development, dissemination and take-up were crucial factors in its initial success.

We have seen how initial propaganda was at pains to emphasise both the continuity between the traditional and modern syllabuses, and the relevance, interest and lack of difficulty of the new material and approaches. We have also seen how, as the curriculum developed, certain elements of the original conception diminished in prominence - discovery learning, and the emphasis on 'structure' in particular. Successive versions of the SMG texts retreated back to traditional notions of pedagogy and gave greater emphasis to traditional content and approaches to the subject. This adaptation to 'teachers' problems' was the price that was paid to secure the success of the new curriculum.

As its remit shows the purpose of Syllabus B (case E) was to carry this process of adapting the 'new maths' a stage further to meet the changed circumstances of the common course and RSLA. It was an innovation addressed from the onset to the problem of producing a modification of the existing Syllabus A to the capabilities and interests of a larger group of pupils. Unlike the Syllabus A development committee, the Syllabus B committee, with the exception of a representative of SCEEB, contained only teachers (including in that category ex-mathematics teachers who had moved on to be college curriculum tutors).[8]

Most of the content of their new curriculum was already familiar to teachers, either from Syllabus A or from the 'O' Grade Statistics curriculum introduced in 1967. The design of the S1/S2 Modular Mathematics curriculum materials, however, assumed a significant change in teaching methods away from 'lock-step' class teaching to group and individualised methods.

As might be expected, the propaganda for Modular Mathematics concentrated on reassuring teachers about this aspect of the innovation.

> "The introduction of comprehensive education has had a dramatic effect upon the teaching of mathematics. Mixed ability classes have presented class teachers with the problem of interesting pupils of widely varying levels of ability while presenting useful and challenging work to the more able pupils. Aiming the lesson at the middle of the ability range leaves many of the pupils dissatisfied, and leads to classroom problems. In this situation the mathematics teacher needs help. The Modular Mathematics course is designed to provide a core of content for all pupils with special provision for the less able and supplementary material for the most able pupils. Because of the individual nature of the work each pupil can proceed at a rate which suits his or her own level of ability. The class teacher is able to attend to the difficulties of individuals or small groups knowing that the other pupils are able to work on material which is interesting to them and within their capabilities"[9]

Early experience with the new curriculum, however, established that it was far from 'teacher proof'. Many teachers found ways to adapt the new materials to existing teaching styles. This flexibility, no doubt, worked in favour of the innovation. Teachers could, if they wished, accept it simply as a modification of the <u>content</u> of the existing S1/S2 course. One important factor which inhibited schools from adopting Modular Mathematics was the cost of the new curriculum materials ; in particular the expensive worksheets could be used only once . Although certain local authorities were prepared to offer schools additional funds, the financial cutbacks of the mid-seventies hit this innovation when it was made generally available in 1974.

A second crushing blow, and the one which seems to have halted Syllabus B (of which Modular Mathematics is the first part), was the refusal of SUCE to recognise the 'O' Grade in Mathematics on Syllabus B. Adaptation had gone too far for the universities; they played their trump card.

These two cases suggest two things: First, that given financial and political support by the SED (and possibly local authorities) change in the content of the curriculum which is not immediately acceptable to teachers can be implemented, although it is likely to be adapted in the process. They provide little evidence of the success of attempts to change teaching methods. Second, they show that the universities retain an effective power of veto over curriculum change at certificate level.

There is a common pattern to the three major cycles of innovation in the Scottish mathematics curriculum - the establishment of a national curriculum (1887-1904), the introduction of analytic methods (1924-1950), and the introduction of 'modern' mathematics (1963-   ). In the first stage, new mathematical content and views of mathematics are promoted by the universities; in the second these are taken up and adapted by the schools (within bounds ultimately set by the universities).

The role of the SED is harder to assess. Curriculum change is, formally at least, initiated by the SED; it, and its associated apparatus (SCEEB, CCC), act as the major formal channels of communication between the universities and the schools. But there are times when the SED seems to have a policy of its own on innovation;

the first and third cycles of innovation in the mathematics curriculum can be seen as part of a larger SED endorsed and sponsored programme of change. Given the relative autonomy of the professionals in educational decision making in Scotland, it might be argued that it often takes a purposeful individual or group within the SED (Craik, Secretary from 1885-1905 or Brunton, HMSCI from 1955-66, for example) to promote and guide change.

The pattern which emerges is one in which the universities propose, the SED sponsors, and the schools dispose of change. The universities exert the major influence on the direction that change takes, the SED plays the major role in deciding if and when change is to take place, and the schools control, to a large measure, the extent and speed of change.

Conclusion

Chapter 15: The Political Dynamics of Change

Very briefly, the three main parts of this thesis have (I) argued for a 'democratic general education' in mathematics, emphasising the social, intellectual and cultural context of the discipline; (II) established that the existing pattern of mathematical education fails to meet such generalist aims, and examined the perspectives of those who currently teach the subject; and (III) traced the development of this pattern of mathematical education and the forces influencing it. I intend, in this concluding chapter, to bring together, and build on the arguments of all three parts, in considering the conditions which are likely to lead to effective curriculum change towards the generalist model which I have advocated.

(1) The Politics of Change

It is striking how little the secondary school curriculum has changed over the last century. This stability contrasts vividly with the accelerating growth and change in the content, structure and application of knowledge in the society of which the school is part. The present day curriculum is still firmly set in the mould of the late nineteenth century.

This mismatch between curriculum and society has only been exacerbated by the changing role of the school system. A curriculum for the Victorian intellectual elite has been uncritically preserved as the model for popular secondary education a century later. The only substantial concession to this century of unprecedented intellectual achievement and social change has been the excision of the classical languages from the curriculum.

An examination of this period also displays the limited ability of the school system to reform itself. While change may have found its earliest, and most articulate exponents within the system, it has largely depended on external pressure and support for its realisation. To cite the most prominent example, it has been the commitment of successive governments (with a variety of motivations) to expand educational provision and extend educational opportunity which has led to significant change in the institutional structure of the school system over the past century.

Within the system a sometimes grudging consensus has upheld the main outlines of the traditional curriculum. No group has had the strength to sustain a major initiative; all have been capable of frustrating the intentions of others. The result has been marginal change, the bare minimum judged necessary to accommodate the tremendous structural changes: the basic consensus has been preserved.

It is not unreasonable to suggest that the reluctance of governments over this period to enter the 'secret garden of the curriculum' has allowed this incipient inertia to predominate, despite the changing institutional structure of the school system, and the changing composition of its audience.

This is not so much a Scottish as a British pattern. Kogan, discussing the education system of England and Wales, writes,

> "The British system for the government of education is .. strong, largely continuous and consensual in its working and assumptions.. [Most current educational policies have been] inherited from the first of the public education

systems at the beginning of the twentieth century."[1]

He describes change in the English system as,

"Pluralistic, incremental, unsystematic, reactive."[2]

Indeed, change in both systems has run largely parallel, although in matters of curriculum and examinations England has tended to lead. On the other hand, once started, change has been completed more quickly in Scotland due to the stronger, and more explicitly centralist role of the SED in the Scottish system.

Not that power in the English system is as devolved as it is sometimes taken to be. Kogan argues that during the fifties the then Ministry of Education changed from,

> "being the holder of the ring between the 'real' forces in educational policy making.... to being the enforcer of positive controls, based increasingly on knowledge which the Department itself went out to get."[3]

Reviewing the period 1960-74, the same author concludes that, while the exercise of power is rarely explicit,

> "the only certainty is that the DES wields determinant authority and great power."[4]

Certainly, in matters of curriculum, the supposed autonomy of the headmaster and head of department within the 'pluralist' English system can be exaggerated. Choice here is choice within a relatively narrow spectrum of alternatives.

Thus, while the Scottish and English school systems remain administratively independent, and preserve a number of distinctive characteristics, the pattern of change within each has been broadly similar over the past century. In both systems radical change in the curriculum has been impeded by a similar conservative consensus, upheld by similar interests (although, in Scotland, the lack of pressure from outside groups has subjected the consensus to less strain, and as a result the gradual change that has ocurred here has been even less marked). Moreover, common to both systems is the wider and overweening British society which imposes common economic, political and cultural pressures on the two systems. In particular the assimilation of the Scottish universities to an emerging British university system during the nineteenth century, removed the mainspring of the autonomous Scottish cultural and educational tradition.

In the absence of a Scottish Assembly it is probable that Scottish education will continue to follow broadly British trends, although these may be given a distinctively Scottish elaboration. More specifically, curricular change of the order entailed by my proposals is likely, as I will argue, to require a degree of political sponsorship which makes its isolated emergence in the politically unfocused Scottish context, most unlikely. For these reasons I propose to continue my discussion of curriculum change in the wider British context.

I have argued that a conservative consensus within the school system both reflects and sustains inertia. The one period when curricular consensus visibly faltered was during the sixties. As I have described, the anachronism of the school curriculum, particularly in mathematics and science, had become widely apparent at a time when doctrines of social and economic planning, and, in particular, of the expansion of 'human capital' were gaining influence. It was at this time that Britain saw the first real attempt at government intrusion into what the then Minister of Education, Sir David Eccles, termed 'the secret garden of the curriculum'.

In the early sixties Eccles proposed the creation of an expert Curriculum Study Group within the Ministry of Education as a response to the pressures of rapid change and increase in knowledge. The intention was to ensure that decisions made by the Ministry would be better informed by educational considerations and would be more closely related to more general social and economic plans adopted by the government of the day.[5]

Arousing the fierce opposition of the teacher unions, and lacking political support, the CSG was stillborn: the Schools Council was the compromise which emerged. At much the same time the SED set up the Consultative Committee on the Curriculum, rather different from the Schools Council in constitution, and more limited in purpose, but sharing the aim of stimulating and coordinating curriculum renewal.

The constitutions of both the Schools Council and the CCC originated in a centre-periphery model for the dissemination of educational policy, which seeks to redirect the largely negative power of teachers to impede change, into more positive channels. This concept of teacher-sponsored change was reflected in the predominance of teachers on the committees of these two bodies. It has been styled,

> "a deliberate resort to democracy."[6]

Both bodies, however, have had to operate within the unchanged constraints of the 'internal' balance of power. They have had to legitimise policies and innovations not only in the eyes of university-dominated examination boards, and cost-conscious central and local government, but to the mass of teachers who have little contact with, or knowledge of, these bodies.

The result, particularly in Scotland where most of the development work has been carried out by serving teachers, is a tendency to take a fragmented and gradualist view of curriculum change, a predisposition to work within and preserve consensus. Where articulate criticism of the consensus surfaces, the structural pressures within both bodies tend to suppress it. As MacDonald and Walker put it,

> "The 'cooperative machinery' of the [Schools] Council represents the system, and therefore lacks a mandate to criticise it. It is locked within the protocol of courtesy."[7]

1970, the start of the Thatcher administration at the DES, is a turning point in recent educational policy, marking a break with the educational liberalism common to the Conservative and Labour administrations of the previous decade, under Eccles, Boyle, and Crosland. The optimistic commitment of government to innovation and expansion in education has disappeared, to be replaced by a hardening scepticism, and an increasing assertion of DES authority.

In particular, the period since 1970 has seen increasing government interest and intervention in curricular matters; the setting up of the Assessment of Performance Unit in 1974, the transfer from the Schools Council to the DES, in 1976, of the task of reorganising school examinations, and pressure on the Schools Council to reform itself to reduce the influence of the teachers' unions, and increase that of the DES and interests outside the education system, which has borne fruit in the Council's third

(1978) Constitution. Each of these initiatives testifies to a growing politicisation of decision-making on curriculum and examinations. There is little sign that politicians or public have yet realised either the political significance, or the complexity of the issues underlying curricular change. Both parties have consistently represented curricular decision making as a purely technical matter of finding ways of meeting supposedly uncontentious national needs.

Nonetheless, inasmuch as it points to an important shift in the pattern of decision making, this development is to be welcomed. First, and most important, although the current alignment of political forces has lent support to the traditional curricular consensus, this in itself has made a significant breach in the principle that curricular matters should be kept outside 'politics'. This is an important development, if, as I have argued, thoroughgoing curriculum reform depends on government sponsorship for its success. Certainly, sponsorship for the kind of change which I have advocated depends on a much stronger, and more overtly ideological politicisation of educational decision making. The value of this breach of principle is that, in placing curricular matters on the political agenda, it offers the proponents of democratic curricular change a wider, and potentially more sympathetic constituency, which brings with it the real possibility of a slow realignment of the forces which at present inhibit change.

Second, it is desirable that decisions about change (or its absence) in the educational system, with their far reaching social implications, should be made democratically. Certainly those working within the education system must be party to such decisions, but they have no justification for claiming a monopoly on them. Finally, such a move clearly identifies the issues of value involved in such decisions as political issues: in doing so it offers teachers and other educationalists some protection from unreasonable criticism which originates in the conflict between the demand that the school embody and transmit values, and our society's confusion, or pluralism of values.

The natural sponsor, within the wider political system, for the kind of innovation which I have advocated is a party of progressive social reform, a tradition represented in Britain by the Labour Party. The development of democratic general education, as I have argued for it, continues the process of democratising the educational system, by enlarging the focus of change from a primarily economistic concern to diminish inequalities of wealth and status, by reforming the structure of the school system, to a concern to reform the curriculum and pedagogy of the school, motivated by political and cultural considerations.

In Sweden, for example, a succession of Social Democratic governments have, since the Second World War, sponsored a developing programme of educational reform, and notably the reform of curriculum and pedagogy, which has been seen as part of a wider policy of social reform aimed at democratising Swedish society.

In other countries, however, governing parties have assimilated the democratisation of education to rather different ideologies. In France educational reform has arisen partly in response to public discontent, centred on the universities, culminating in the violent events of 1968, and partly as an arm of economic management. The postponement of differentiation until 14 ('promotion collective'), and the development of a common curriculum ('tronc commun') in the early years of the secondary school have been introduced by a government of the right which has incorporated educational planning into a broader middle term plan for economic and social development, and tried to reconcile the conservative respect for tradition with the growing entrepreneurial demand for an intellectually flexible and technically sophisticated labour force.[8]

A recent French Secretary of State summarises this view as follows,

> "The radical transformation of social life has upset the ideas of French education, for it has given rise to two unprecedented phenomena.
>
> The first has been the continuous acceleration of economic activity.. [The increasingly elaborate industrial machine].. is demanding more and more knowledge, and more and more specialization. The framework of education is becoming much too narrow for the acquisition of knowledge which is, at the same time, encyclopaedic yet precise..
>
> The second phenomenon has been the need to make the comprehension of the whole world surrounding us accessible to substantially all the population. No longer can we be content with an initial transmission of knowledge to an elite: to begin with, the demand for democratisation has broken down the social barriers in the universities. But above all, the complex economy of our advanced society calls for greater knowledge from a greater number of people."[9]

The particular direction which this government has sought for curriculum change has been motivated by the consideration that,

> "If culture is, first of all, the understanding of the world in which one lives, then the pupil's comprehension of the technological world is an important aim. To achieve such a comprehension one must allow all children to discover the links between science, technology and economic and social problems."[10]

In the United States, as in France, governments have seen the extension of educational opportunity as an instrument of economic progress, and of social and political stability. Entwhistle compares the viewpoints of British and North American educational conservatives, taking Bantock and Bestor as examples. He contrasts the disdain of Bantock and other Black Paperites for the educational capacity of the majority of people, and their opposition to a common curriculum, with Bestor's belief in the need to transmit intellectual culture throughout all sections of the population.[11]

> "American public schools have the responsibility of raising up a nation of men and women highly literate, accurately informed, and rigorously trained in the process of rational and critical thought. If the schools fail in this, then we may expect to see the collapse or defeat of democratic self-government through the sheer inability of its electorate to grapple intelligently with the complex problems in science, economics, politics and international relations that constantly come up for public decision."[12]

The forces for educational change in the United States have not, of course, been moved solely by such altruistic considerations. As in France, the sometimes violent expressions of public expectation, the demands of an expanding and increasingly technological economy, and strategic considerations have all helped to produce consensus about the desirability of extending educational opportunity.

In Sweden, France and the United States, then, as in Britain, the extension of educational opportunity, and change in the curriculum have largely depended on government sponsorship. Further, in each country, whether in or out of government, parties whose ideals were largely antipathetic to educational expansion and change, have bowed to prevailing social and economic forces, to tolerate, and on occasion encourage reform.

This suggests that, to a certain extent, given different ideological emphases, educational reforms may prove capable of straddling conventional political boundaries. Educational expansion can be conceptualised, on one hand as national investment and private consumption, on the other as extending opportunity and producing social change. However, the progressive nationalism of the American and French right, which endorses educational change as a measured response to social and economic pressure, and appeals in its management of change primarily to apparently apolitical, technocratic considerations, is hardly elastic enough to encompass the kind of change I have advocated.

Moreover, on the British right, this progressive nationalism is overshadowed by a more rigid conservatism, reflected in the emphasis in Conservative education policy on the retention of privilege, and the preservation of traditional values and forms. Educational liberalism has had its defenders within the Conservative Party, but they have been few, and, in general, have found themselves on the defensive. For example, Boyle's tolerance of comprehensive schools, as shadow Education Secretary during the mid-sixties, came under continuous attack from within the party, and this policy did not survive his exit from office. Kogan argues that,

> "[Boyle's liberal] philosophy tells us nothing about the trend of post-war Conservatism, as Mrs Thatcher's decisions and declared policies since his time have since made plainer."[13]

The best that can be expected from the British right is a pragmatic acceptance that the public is unlikely to favour the reversal of successful educational change, which can be seen to present more worthwhile educational aims, and to offer greater opportunity to the majority of school students. Any political sponsorship for democratic educational reform in Britain is likely to come from the left, and the survival and success of such reform is likely to depend on the degree of enthusiasm, and the care in planning which is given to it.

Here, unfortunately, the record is not particularly impressive, as the short account of the introduction of comprehensive education which follows will make clear. Labour education policy since the war has in many ways been no less pragmatic than that of the Conservatives. It has certainly lacked any strong guiding principles, partly, of course, in the hope that change, unencumbered with such principles, would win readier acceptance. In many ways this attempt to make reform uncontroversial has been a double failure, producing undirected, unsystematic and ineffective change, while, at the same time, discrediting, and heightening opposition to the very idea of reform.

There can be no doubt that, despite the growing politicisation of curricular decision making, the proponents of radical reform start from a difficult position, weakened by the consequences of the very lack of principle which they seek to remedy in educational policy. The pragmatic tradition weakens not only their attempt to gain sponsorship, but the transformation of this sponsorship into effective educational change. Here is the greatest political obstacle to radical reform, once the principle of non-intervention in curricular matters has been breached.

(2) The Management of Change

For successful reform depends on more than simple government sponsorship. Creating a dynamic of change within the school system is only the first part of effective innovation. For innovation to be successful, the new aims and practices which it entails must be clarified, and their implementation planned. On the one hand, opposition to change may originate in, and is often articulated as misrepresentation of aims, or criticism of the plans for implementing these aims. On the other, change directed only by ambiguous rhetoric, is likely, at best, to preserve traditional aims and practices under new names, at worst, to undermine the achievement of even traditional aims. A clear contrast in styles of managing innovation, which illustrates these points, is provided by the differing British and Swedish approaches to comprehensivisation.

In Sweden government commitment to a comprehensive school system emerged from a managed process of informed decision-making.[14] Between 1940 and 1947 a government sponsored committee of enquiry, a non-political body of expert educational opinion, made a complete survey and evaluation of the national education system. In 1946 a School Commission consisting of representatives of the five political parties, and a non-political member to represent parental interests, was appointed to make policy recommendations for the development of Swedish education, in the light of the extensive report of the committee of enquiry.

In doing so the Commission had access to expert advice, through the assignation of professional educationalists to advise its specialist subcommittees. The Commission's 1948 Report was a detailed and exhaustive document which clearly located its recommendations for reform within a wider framework of educational and socio-political aims. It made recommendations not only about the extent and structure of compulsory schooling, but also about appropriate patterns of curriculum and pedagogy. Its central recommendation was the establishment of a compulsory, nine-year, comprehensive school. For the first eight years (until the age of 15) pupils were to follow a common course in unstreamed classes (although a small optional element in the curriculum was to be permitted in the seventh and eighth years).

A parliamentary Act of 1950 led to the establishment of a number of experimental comprehensive schools. Throughout the succeeding decade, research, development, and evaluation continued, until, in 1962, an Act was passed making comprehensive reorganisation compulsory throughout Sweden.

In Britain, although the 1944 Education Act permitted the establishment of comprehensive schools, there was little support for them from either party.[15] In 1948 a Labour Education Minister turned down plans for comprehensive reorganisation in Middlesex on the grounds that the tripartite system was 'logical and usual'.[16] The initial drive towards comprehensive education came at local government level, on occasion on ideological grounds, but more often prompted by the problems of providing viable secondary education in areas of dispersed population.

The early development of comprehensive schools was piecemeal and largely pragmatic. These were marginal institutions, lacking a clearly formulated set of alternative values, and under pressure to succeed within the terms of the normative tripartite system. By and large, the new schools adopted traditional models of organisation, curriculum and pedagogy. There were pockets of significant innovation. London developed a common course for the first three years of secondary education, but rejected,

> "the impracticable assumption that teaching groups covering the whole range of ability are suitable or desirable."[17]

By 1965 only 8.5% of secondary age children attended comprehensive schools.[18] It was the return, in 1964, of a Labour government committed to comprehensive reorganisation which gave impetus to change.

The controversiality of change, and confusion and disagreement within the party itself, combined to produce a policy which, while exerting pressure for change, gave it little direction. Before the 1964 election the Labour leader, Harold Wilson, had assured teachers that grammar schools would be abolished 'only over his dead body': as late as 1970 he was representing the comprehensive school as 'a grammar school for all'.[19] This was not mere rhetoric, intended to reassure: it reflects the influence of the arguments which had led an earlier generation to champion the grammar school as an agent of social mobility. Conversely, it illustrates the absence of concern with the nature of the educational process which characterised Labour policy.

Circular 10/65, the Labour government's instrument of change, sought the abolition of selection and segregated secondary schooling. Six distinct schemes were put forward to be considered by individual local authorities as models for reorganisation. These, however, were not the result of government sponsored research or policy-making.

> "It was to local authority practice..that the government and the DES turned when it came to drawing up Circular 10/65. All six of the schemes suggested were either in operation or proposed through local authority initiatives. The 'central guidance' that 10/65 claimed to give in effect amounted to passing around to all authorities what the DES had found in its suggestion box in 1965."[20]

On other matters, such as internal organisation, curriculum and pedagogy, 10/65 was silent.

Government policy, then, perpetuated the pragmatism of the early years. It gave no clear lead in defining new aims or values for comprehensive schools. And Circular 10/65 made it clear that local authorities could expect no special financial assistance with the change. Both the aims of change, and methods of implementing them lacked clarity, coherence and completeness.

The first government-sponsored research into comprehensive schools was the 1966 NFER descriptive survey.[21] Not surprisingly, the report of the NFER team pointed to an assimilation of change to existing values and practices. For example, the great majority of schools continued to group their first year pupils by ability (almost half using streaming), while at most 10% taught any subject to mixed ability groups at this level.[22]

The result of government policy has been to accelerate a process of drift towards a set of diffuse, and imperfectly defined aims. By 1971, 38% of pupils attended at least nominally comprehensive schools:[23] of these schools, about 35% used some form of mixed ability grouping with their first year pupils.[24] While individual schools have developed successful models of internal organisation, curriculum and pedagogy, satisfying strong definitions of comprehensive education, the great majority stick close to traditional models.

In Sweden, by contrast, schools have had the benefit of clearly defined innovatory aims, and considerable professional support in implementing them. The result has been substantial change, disseminated throughout the school system. Marklund and Soderberg summarise the gains of reform as postponed selection, the development of a common course, and the individualisation of instruction.[25]

In both Britain and Sweden the intervention of government was a critical factor in creating a dynamic of change. But in Sweden, the active part played by government in initiating and planning change, produced significant benefits. First, it led to the production of a clear and coherent plan for reform, backed by a strong political mandate, and consistent with the values implicit in that mandate. 'Professional' judgements did not go unchallenged. As a result the professional expertise, which originally resisted the idea of mixed ability grouping, was redirected into developing the individualised pedagogy which now makes this form of organisation feasible.

Second, change was coordinated throughout the system, lessening the internal tensions of innovation, and ensuring a relatively uniform development. In parallel with the introduction of the nine year comprehensive school, the upper school, higher education and teacher training were all reformed, as part of a global plan for 'rolling reform' of the educational system.

Finally, the priority assigned to educational reform in government policy, ensured that adequate resources were made available for the programme of innovation.

(3) The Development and Implementation of Change

The Swedish example illustrates how a purposeful government can initiate, guide and promote innovation within its school system. But there is a point beyond which such sponsorship is of little value. New aims and practices, however clearly conceived, plausibly argued, powerfully sponsored, and well financed, cannot simply be injected into the school system. This is particularly true when change aims to modify not just the organisational structure within which schooling takes place, but the rationale and procedures of teaching itself.

This lesson was learnt by the early protagonists of, what Havelock has termed,[26] the 'research, development, and diffusion' model for innovation, in which a central team of experts designs a new educational package to be distributed to practitioners on the periphery. It was this highly technocratic model, befitting the era, which found favour in the rash of curriculum development during the sixties.

The standard pattern was for a team of subject specialists, often drawn from outside the school system, to develop a comprehensive set of classroom materials, embodying a new curricular philosophy. It was assumed that, once this research and development phase had been completed, diffusion would be a simple matter of distributing the package and its accompanying curricular message to schools.

This assumption proved misconceived. In many cases schools were reluctant to acquire the new packages, or rapidly consigned them to the deepest recesses of their storage cupboards. Even the apparent success of certain projects, measured by their take up and use by schools, often concealed an assimilation of innovatory intention to established practice.

A striking example is described in a research study by McIntyre et al.,[27] which examined science teaching in the early years of the Scottish secondary school. Within two years of the Consultative Committee on the Curriculum suggesting that science should be taught as an integrated subject in S1 and S2, the great majority of S1 and S2 classes were 'integrated'. Within these classes, however, nearly all the content taught could still be clearly identified as 'physics', 'chemistry', or 'biology'. In addition while centrally produced worksheets, aimed at promoting 'guided discovery' and taking account of differences among pupils in 'mixed ability' classes, had been widely adopted, they were rarely used as intended, more frequently being adapted to traditional teaching strategies.

Similarly, I have noted how the emphasis on heuristic explanation and discovery learning, which was a strong element in the curricular philosophy of the New Maths, failed to take root in the classroom. In the case of the Scottish Mathematics Group project, for example, innovatory 'success' was attributable to the weakening or abandonment of this, and other central elements of the reform philosophy, in favour of traditional perspectives.

One response to such failures had been the espousal of, to use Havelock's terminology again, 'social interaction' and 'problem solving' models of innovation. In the former, the central agency merely becomes a clearing house for small-scale peripheral development, without any unifying aims or philosophy. In the latter, the central agency coordinates small-scale peripheral development, guided by what practitioners perceive as their needs, and disseminates the results.

But the available evidence suggests that the adoption of these models is likely to reinforce a reactive, unsystematic, and incremental, pattern of change. If, as the teacher study of Part II concluded, teachers lack an articulate and developed perspective on their subject, and the value and purpose of teaching it, and, in practice, their actions are intended to meet often immediate organisational goals only tenuously related to the subject itself, then their perception of problems, and the solutions they seek are unlikely to challenge the tacit values and purposes of the existing curriculum.

While this response to the failure of the 'research, development, and diffusion' model, in its early versions, does not offer an effective alternative, it does point to the reasons underlying the failure. The real weakness of the curriculum development of the sixties was the subject-mindedness of its analysis, which failed to take account of the complex social ecology of the school and classroom.

As I have argued in Part III, the new curricular aims and methods were initially derived from theories about the subject, and only later legitimised, rather than operationalised, in terms of highly speculative and abstract theories of learning. The implementation of change in the classroom itself was seen as unproblematic, as simply a matter of the teacher using the prepared curriculum materials, and adhering to the new curricular philosophy.

This, of course, reflected the popular view that teaching is a simple and consciously rational activity, and that the knowledge and skills which a teacher requires are primarily those of the subject. The innovators believed that the aims and practices of teachers are guided by their view of the subject. But the evidence suggests that the reality of the classroom is very different. The teacher study of Part II concluded that teachers do not have a highly developed and articulate view of their subject - indeed, that they feel ill at ease in this area - and that their aims and practices are located not in rationalistic theories of knowledge and learning, but in largely tacit structures of social interaction and institutional purpose. The failure of much curriculum development is attributable to an underestimation of the complexity of classroom life, and a fundamental misapprehension about its dynamics. As a result, curriculum developers have omitted to translate new curricular aims and methods into feasible teaching strategies, consistent with the realities of classroom life, and to train teachers to incorporate these strategies into their teaching repertoire.

Any serious attempt to operationalise new curricular aims and methods must take account of the fact that the theoretical models of classroom life which are currently available offer little help in identifying effective strategies for change. Although adequate as descriptions of current practice, beyond this familiar equilibrium they offer little insight into the structure and strength of the constraints on change. This is a major impediment to any radical initiative to reform the curriculum.

How then can the R, D and D model be reformulated to incorporate these insights? In the absence of ready-made 'solutions', in the form of systematic and holistic models of classroom life, the only feasible innovative strategy is to build awareness both of the 'problem' and of the 'reality' it concerns, into the development mechanism.

First, this demands the introduction of social scientific perspectives into research and development, and the establishment of channels of communication between the 'idealistic' curriculum designers and the 'realistic' practitioners, to encourage the feedback which was missing from the original model.

One realisation of this kind of mechanism is provided by an 'action research' model in which researchers, designers and practitioners work together to identify the constraints on change, and to develop structures and strategies which enable agreed innovatory aims to be achieved, within a particular school. This kind of experience in a number of schools then provides a basis for the development of feasible models of organisation and teaching to attain these aims, and of methods of training practitioners in the use of these models.

This latter point is relevant to the second major change which is required to make the R, D and D model effective; the provision of (and the development of strategies for providing) extensive professional support, in particular retraining, to guide the wider diffusion and implementation of change.

For example, the kind of curricular innovation which I have proposed in mathematics is likely to require not just that teachers incorporate new strategies into their teaching, but that they acquire new perspectives on their subject, and develop the ability to apply these new perspectives to the activities which take place in their classrooms. Furthermore, it is likely that an important aim of such innovation would be to counteract the intellectual isolation and stagnation, the lack of involvement in mathematics, which many teachers display, and which is reflected in the image of mathematics conveyed to their pupils.

Behind this approach to innovation lies a revaluation of the role of the teacher, and of the centrality of the process of teaching (in its widest sense) to successful learning. One of the reasons for the resistance of teachers to change based on the technocratic assumptions of the original R, D and D model, has been its devaluation of their role, with its emphasis on 'teacher-proof' packages which cast the teacher as a pedagogical machine minder. Another has been the experience of many teachers asked to implement change without adequate resources, experience, or professional support. Successful change must enhance teachers' self esteem; it must emphasise the value of the teacher's contribution to learning. The provision of opportunities for professional development, and in particular of adequate retraining programmes to meet the demands of innovation, is an essential part of this revaluation of the teacher's role.

(4) The Future

The central problem of effecting the kind of curricular change I have advocated remains a political one. At present the proposal that the secondary school curriculum should be democratised towards a generalist model lacks any powerful constituency. Of course the current curriculum is widely criticised as anachronistic and over-academic; by the major teacher unions on both sides of the Border, by the inspectorate, even, on occasion, by industry. But behind the felicitous phrases there is little real consensus on what change is desirable, and a general inclination to let caution take precedence over commitment.

Furthermore, the main corollary of democratisation, a fundamental change in the relationship between school and university, is certain to antagonise those who see the primary role of the school as the nurture of a future elite. Democratisation of the school curriculum would require universities, at the least, to adapt their courses to take account of the quite different pattern of attainment of their students on entry. Further, the change in values implicit in the reform of the school curriculum would be likely to encourage criticism of the role of the university, and, in particular, of its curriculum. Finally, inasmuch as the majority of secondary school teachers receive most of their higher education within subject-specialised university courses, radical reform in the school curriculum would strengthen demands for change in the university curriculum.

In some ways this pattern of largely conflicting opinion and influence is not dissimilar to that which confronted the early proponents of comprehensivisation: indeed, in view of my contention that democratisation of the curriculum can be seen as an extension of the policy of comprehensivisation, this is unremarkable. Where the two reforms differ significantly is in the absolute priority which curricular democratisation places on reforming the rationale, content, and procedures of classroom teaching, even given the cautious strategy of preserving, in some measure, existing subject boundaries within the school curriculum.

Here, because of the particular influence that they wield over the school examination system, and their more general influence on public conceptions of legitimate knowledge, the universities are in a considerably stronger political position. On the issue of school organisation the universities possessed only indirect influence; in the matter of the school curriculum and examinations they enjoy, and have been prepared to exercise, direct and considerable power.

Teachers, on the other hand, may be rather less resistant to change than they were in the sixties. First, the idea of change is no longer novel: that, if nothing else, is a significant outcome of the last two decades. Second, whereas in the early days of comprehensive reform the main teacher unions remained uncommitted, reflecting the division of opinion among their membership, they now confidently defend the comprehensive system. Teacher opinion has, at the least, come to terms with change. Indeed it could be said that teacher opinion, as expressed through the major trade unions, is relatively sympathetic to carrying change further. Organised teacher opposition to curricular change towards the democratic model is likely to focus on its feasibility, rather than its desirability, and to be concerned to defend, and if possible enhance, the teacher's role and status. That is no bad thing if it helps to ensure the careful planning of change, the provision of resources and re-training: its dangers lie in the arrogation to teachers of unfettered control of curricular decision making.

But, as I have argued, the critical initiating and mediating role must be played by government. The evidence on comprehensivisation is not encouraging. Its proponents had to wait at least twenty years to see it emerge as government policy. Even then it remains highly contentious, not just between, but within parties. It is this failure to create consensus which clearly differentiates the Swedish from the British experience, and points to the danger either of half-hearted innovation, or of a curriculum bending with every turn of the political wind. Further, while curricular change remains politically contentious, the universities may be able to forge effective alliances to impede change. Here a great deal rests on the pattern on which further and higher education develops in the future.

The increasing demand for, and the extension of post-school education must undermine the power and influence of the universities to some degree. Already they are coming under criticism from certain groups within government, business and industry for the highly academic and specialist nature of their courses. They may either choose to diversify to meet new demands (as the American university system has done), or they may stand aside and watch the non-university sector grow. In one case they must, to some extent, compromise in their curricular values, in the other, they must cede some of their influence and power.

At the same time, the inviolability of university entrance standards is likely to be questioned as more and more students successfully complete some form of higher education without high specialist attainments at school. Indeed the success of the Open University has already started this process.

Both these trends will be accentuated by financial considerations. The long run cost to the university system of the expansion of higher education in the sixties has been increased financial dependence on, and accountability to government. The market alternative, the American evidence suggests, offers no general defence against pressure for change in the university curriculum. A minority of institutions may successfully hold out, but the majority must give some ground, or perish.

There is only one certain conclusion to this discussion: the proponents of democratisation of the curriculum cannot expect success in the short run. For the present they must be content to develop and disseminate their ideas in two critical arenas; first, within the educational professions, and second, within those organisations and parties with a commitment to democratising our society. It is here that the seeds of any future advance must be sown.

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1 Hirst (1974), pp.50-51.

2 Scottish Mathematics Group (1971), Vol. 4, p.36.

3 Ormell (1969).

4 This is less true of French and Belgian modern mathematics projects where the structuralist influence was strong. The two main British projects, the Schools Mathematics Project in England, and the Scottish Mathematics Group in Scotland, have taken a very pragmatic approach.

5 Modular Mathematics, Module 26.

6 Polya (1954).

7 Lakatos (1976).

8 Polya (1954), Vol. 1, p.vi.

9 Lakatos (1976), p.5.

10 Ibid. p.105.

11 Polya (1954), Vol. 1, pp.v-vi.

12 Lakatos (1976), pp.142-4.

13 Dawson (1971).

14 The honourable, but little known or used exception being the Schools Council Mathematics for the Majority project. See Mathematics for the Majority (1970).

15 See Pedoe (1976) for an example of a liberal approach to geometry.

16 Kline (1972), pp.34-37.

17 Ibid. pp.1018-21.

18 See Appel and Haken (1977).

Chapter 5

1 Data from SED (1977b), Table A3, p.128.

2 Data for 1976 SCE examinations.

Chapter 6

1 Gallagher (1970).

2 Smith, Meux et al. (1967). See also Dunkin and Biddle (1974), pp.339-48.

3 Smith and Meux (1962). See also Dunkin and Biddle (1974), pp.323-4.

4 See Dunkin and Biddle (1974), pp.324-5.

5 Bellack et al. (1966). See also Dunkin and Biddle (1974), pp.328-30.

Chapter 7

1 Where the class was timetabled for a double period, but the

teacher treated it as two single periods in different subject areas, the analysis adopted the teacher's distinction. There were two such cases.

2 The percentage of total time, over all 38 lessons, in each Content category was as follows:

	SMA	NMA	MR	CR	RES
Individual	98.5	1.1	0	0	0.4
Class	93.3	3.3	0	2.2	1.3
Overall	94.9	2.6	0	1.5	1.0

Chapter 8

1 Becker (1970), Ch. 2, 3 and 5.

Chapter 10

1 A fuller account of these changes can be found in Davie [1961] and Scotland [1969]. Davie is concerned primarily with change in the universities; Scotland is more informative about the schools (see in particular vol. 2. pp.273-274).

2 See SED [1887] pp.119-122, and Dobie [1967].

3 See SED [1888a], SED [1888b].

4 Craik, The Secretary of the Board until 1905, explicitly mentions this each year in his reports on the "Leaving Certificate and Inspection of Higher Class Schools". His successor, Struthers, is, unfortunately, less detailed.

5 SED [1904].

6 This and subsequent information about the university mathematics curricula is drawn from the University Calendars published annually.

7 The same courses were given under different names in the Arts and Science Faculties.

8 SED [1924], SED [1927], SED [1932], SED [1936].

9 SED [1931] par. 1.

10 SED [1947] par. 457, p.94.

11 Ibid. par. 451-469, pp.93-97.

12 Ibid. par. 475, p.98.

13 See SED [1950].

14 See SED [1959], and SED [1961].

Chapter 11

1 See Rosenbaum [1958], and NCTM [1970]. The latter gives a detailed account of the development of the New American mathematics curricula in Chapter 14.

2 For an account of American curriculum reform in general see McClure [1971].

3 See NCTM [1970] pp.260-261.

4 Bruner [1960].

5 Ibid. p.31.

6 Ibid. p.20.

7 Ibid. p.32.

8 Ibid. p.19.

9 Ibid. p.33.

10 McClure [1971], pp.54-55.

11 See Fry [1956], p.71, and Sanderson [1972]. pp.352-3.

12 Ministry of Education [1945], Committee on Scientific Manpower [1946], ACSP [1956], ACSP [1959], ACSP [1961], Ministry of Education [1963], DES [1968a], DES [1968b]

13 See Griffiths and Howson [1974], p.141.

14 Thwaites [1961], Foreword.

15 Ibid. Introduction.

16 See Sanderson [1972], pp.349-352.

17 See THES [9.7.1976] p.1, THES [10.11.1976] p.1, THES [18.2.1977] p.4.

18 OECD [1961a] par. 4-5, p.11.

19 Ibid. par. 199, p.61.

20 Ibid. par. 198, p.61.

21 SED /1962b/. The evidence on the authorship of the Report comes from a personal communication from Mr A.G. Robertson.

22 Ibid. par. 2.

23 Ibid. par. 4.

24 Ibid. par. 5.

25 Robertson /1969/, p.75. This is the fullest published account of the work of the SMG.

26 Ibid. p.75.

27 SMG /1965/.

28 SCEEB /1965/.

29 Ibid. p.3.

30 Ibid. p.3.

31 DES /1968b/, par. 107, p.60.

Chapter 12

1 See Scotland /1969/, vol.2. ch. 4, 5, for a fuller account of schools during this period.

2 SED /1937/.

3 SED /1947/ par. 99, p.20.

4 Ibid. par. 445-6, p.93.

5 SED /1955/ par. 356, pp.138-9.

6 Ibid. par. 359, p.139.

7 SED /1962a/ pp.15-16.

8 For example the schools where the 15 teacher members of the SMG taught in 1965 (according to SMG /1965/ consisted of 8 fee-paying, selective, senior secondaries, 6 selective senior secondaries, and 1 showpiece comprehensive.

9 SMG /1971/, Preface.

10 Glasgow Corporation /1967/, Glasgow Corporation /1970/

11 Glasgow Corporation /1970/, Foreword.

12 SCEEB /1977/.

13 SED [1972] p.15.

14 See SUCE [1977], SCEEB [1976], p.78, SED [1978].
In 1980 the CCC announced that Syllabus B was to be discontinued.

15 p.4.

Chapter 13

1 Board of Education [1943] p.106.

2 Government Social Survey [1968].

3 Ibid. Table II, 3.6, p.77.

4 Ibid. Table II, 3.2, p.73.

5 Only domestic and commercial subjects have higher ratings than mathematics. This statistic is, of course, biased against mathematics and English, tending to overestimate the extent to which pupils as a whole judge a noncompulsory subject useful, since the sample is drawn only from those pupils studying the subject (a group which we would expect to rate the subject unusually high on utility). And whereas virtually all the pupils in the survey were compelled to study English and mathematics, only a fraction of pupils took domestic and commercial subjects - among 16 year old girl leavers, for example, 67% and 30% respectively - and it is likely that at least some of these pupils chose those subjects in preference to others.

6 Ibid. Table II, 3.7, p.78.

7 See DES [1977].

8 SED [1931].

9 SED [1947] par. 442, p.92.

10 Ibid. par. 474, p.98.

11 Ibid. par. 478, p.99.

12 SED [1963].

13 SCEEB [1965].

14 SED [1977a].

15 Ibid. par. 4. 12. p.25.

Chapter 14

1 Commission on the Constitution [1970] par. 785, p.109.

2 Ibid. par. 746-751, p.103.

3 Kellas [1973] p.207.

4 Commission on the Constitution [1970] par. 769, p.106.

5 Select Committee on Education and Science [1968].

6 For the Membership of the Sub-Comittee see SED [1968].

7 DES [1968b], par. 107, p.60.

8 Modular Mathematics publisher's handout [n.d.] p.5.

9 Ibid. p.1.

Chapter 15

1 Kogan (1975) p.23.

2 Ibid. p.238.

3 Kogan (1971) p.30.

4 Kogan (1975) p.238.

5 Ibid. p.142, and Manzer (1970) pp.90-96.

6 Quoted in MacDonald and Walker (1976) p.34.

7 Ibid. p.41.

8 OECD (1971).

9 Ibid. p.155.

10 Ibid. p.151.

11 Entwhistle (1978) pp.79-80.

12 Ibid. p.79.

13 Kogan (1971) p.17.

14 See Husen and Boalt (1967), Marklund and Soderberg (1967), and Paulston (1968) for fuller accounts.

15 See Rubinstein and Simon (1969), Benn and Simon (1972), and Kogan (1975) Ch. 11, for fuller accounts.

16 Quoted in Kogan (1975) p.218.

17 Quoted in Rubinstein and Simon (1969) p.78.

18 Benn and Simon (1972) p.102.

19 Kogan (1975) pp.219-20.

20 Benn and Simon (1972) p.56.

21 Monks et al. (1968).

22 Ibid. App. I, Tables 12A and 12B, pp.102-3.

23 Benn and Simon (1972) p.102.

24 Ibid. p.219.

25 Marklund and Soderberg (1967) pp.115-116.

26 See Becher and Maclure (1978) pp.65-77.

27 Brown and McIntyre (1978).

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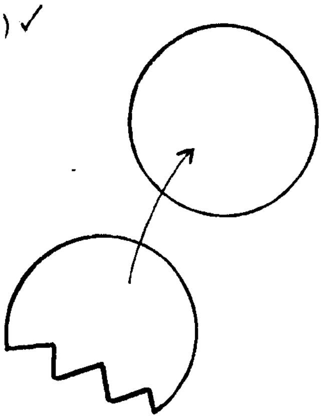
Appendix 1: Lesson-outlines

This Appendix contains four lesson-outlines which exemplify the approach to mathematics education for which I have argued. Each contains the skeleton of a lesson: in two cases I have indicated possible extensions in square brackets.

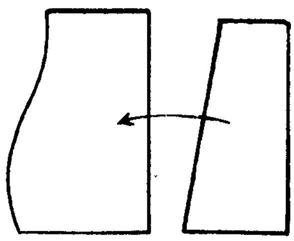
Rationalisation of Area Measurement

Stage 1: Direct comparison of shapes. This is analogous to the method used to compare lengths. But here the method does not cover all cases. It works for some - 1 and 2 - but not for others - 3 and 4.

(1) ✓

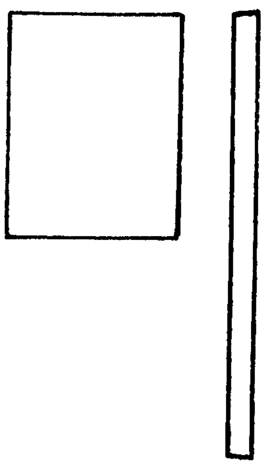


(2) ✓



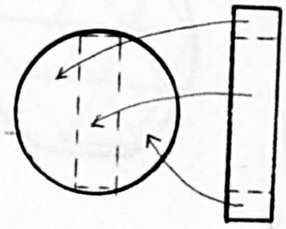
(3) ✗

(4) ✗

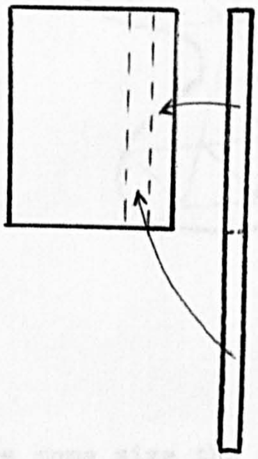


<u>Stage 2</u>: Use of dissection to facilitate comparison. Theoretically tight but complicated in practice

(3) ✓

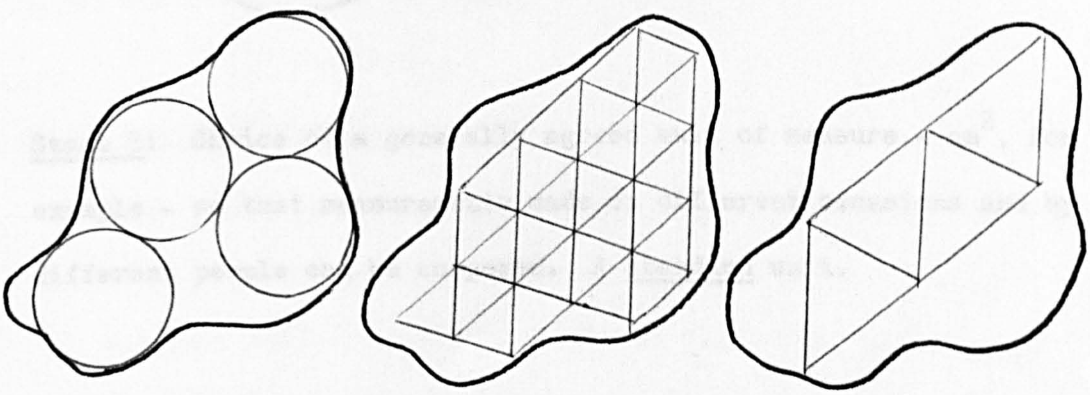


(4) ✓

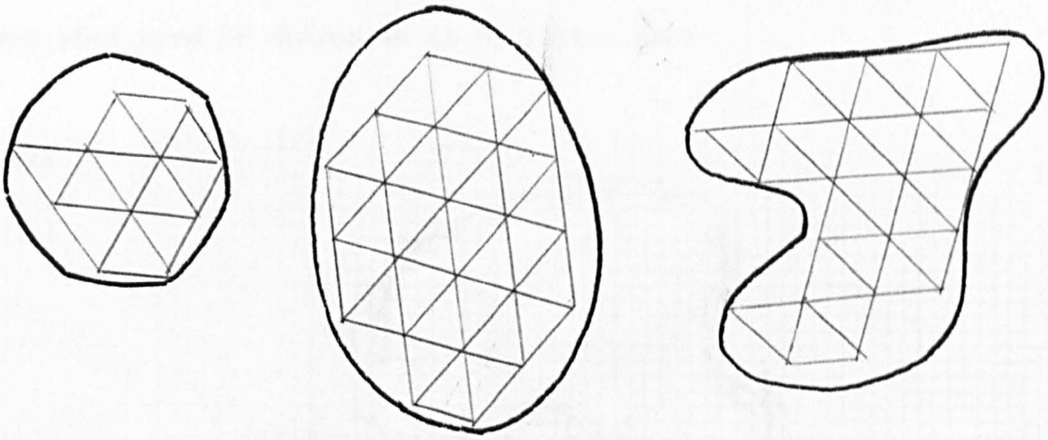


<u>Stage 3</u>: Use of congruent unit shape - see 4. How many times does the sole of a shoe fit into each shape? Or how many penny pieces. Shape must be chosen with care - circles leave gaps, large shapes miss nooks and crannies, small shapes make heavy work. Gives approximate answers only for most shapes, but can be used to compare <u>any</u> number of shapes. We have started to measure.

(5)

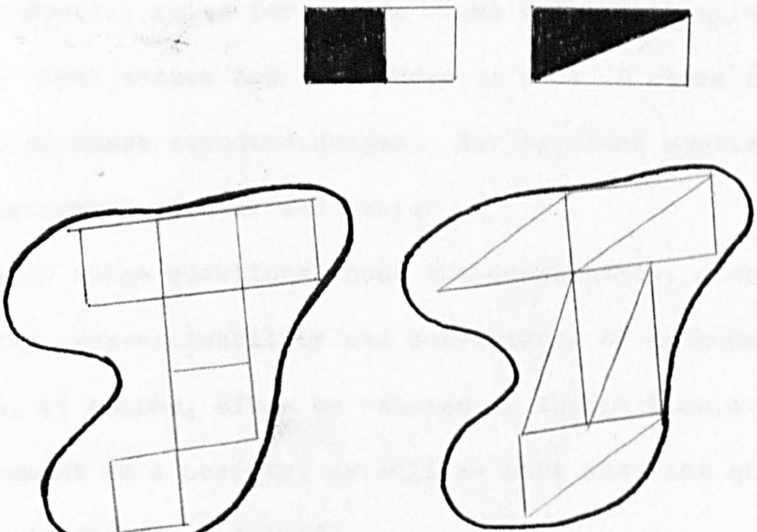


(6)



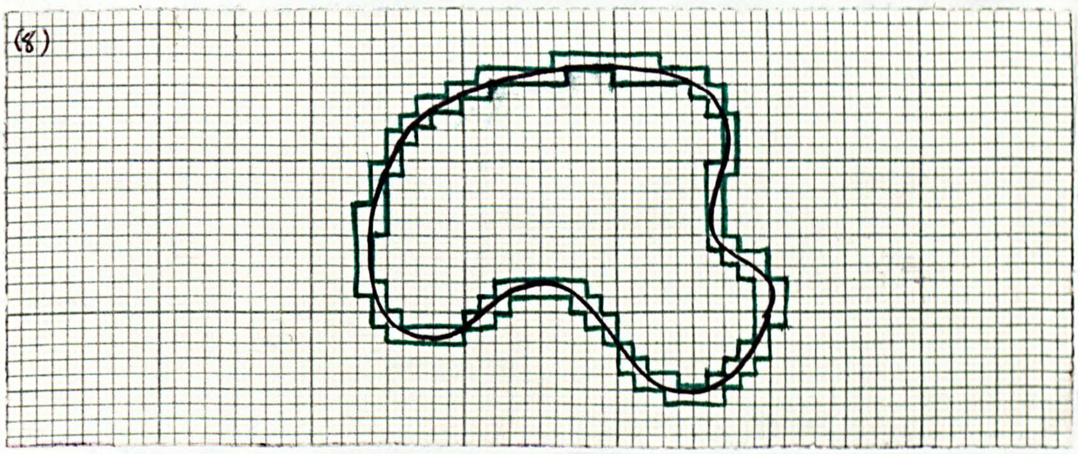
<u>Stage 4</u>: As long as unit shapes are the same <u>size</u> the comparison can be made. Measurement is independent of shape.

(7)



<u>Stage 5</u>: Choice of a generally agreed unit of measure - cm^2, for example - so that measurements made on different occasions and by different people can be compared. A <u>standard</u> unit.

Stage 6: A general strategy for finding approximate areas - the use of a grid. How accurate? Does it always give the same answer? For what kind of shapes is it of little use?



Stage 7: Special rules for common cases - the rectangle and the triangle. Most shapes can be reduced to what is close to an aggregate of these standard shapes. Having these special rules makes measurement quicker and easier.

At each stage questions about the consistency, accuracy, reliability, generalisability and convenience of methods arise. These can, of course, often be related to the particular uses made of measurement in a society, as well as more abstract questions about the structure of methods.

Modelling with geometric sequences

Each time a ball bounces it loses some of its energy and rises to a certain fraction of its previous height.

A tennis ball dropped onto concrete might rise to half its height. If the tennis ball was dropped from a height of 100 centimetres, it would rise to 50, 25, and about 13 centimetres after successive bounces. Mathematically we can express this by the equation,

$$h = 100(0.5)^n$$

where h represents the height to which the ball rises, and n the number of bounces which have taken place.

A superball might rise to 0.9 of its height. If it was dropped from a height of 100 cm. it would rise to 90, 81, and about 73 cm. after successive bounces. Using the same letter conventions,

$$h = 100(0.9)^n$$

A lump of plasticine would not rise at all. If it was dropped from 100 cm. it would remain at 0 cm. on hitting the concrete. By stretching our concept of 'bounce' we could write,

$$h = 100(0)^n$$

Now each of these mathematical descriptions takes the form,

$$h = 100f^n$$

where f is the fraction of its previous height to which the ball rises.

An even more general description of a bouncing ball would be,

$$h = h_o f^n$$

where h_o is the height from which the ball is initially dropped.

Let's take another example.

Each time a pair of jeans is washed it loses a certain proportion of its dye.

A normal pair of jeans might retain about 0.9 of its dye after each wash. If initially the jeans contained 20 g. of dye, after successive washes they would contain about 18, 16, and 15 g. of dye. Mathematically,

$$m = 20(0.9)^n$$

where m is the amount of dye, and n the number of washes.

A pair of colourfast jeans should retain all its dye. So, if initially the pair of jeans contained 20 g. of dye, after successive washes it should contain 20, 20 and 20 g. of dye. Mathematically,

$$m = 20(1)^n$$

Each of these descriptions takes the form,

$$m = 20d^n$$

where d is the fraction of the dye retained after each wash.

An even more general description of the loss of dye would be,

$$m = m_o d^n$$

where m_o is the amount of dye in the jeans initially.

Now, notice that the general descriptions of bouncing balls, and the loss of dye from jeans have the same structure,

number of times
the event has
happened

$$v = v_o r^n$$

current value
of variable

initial value
of variable

proportional
constant

A <u>sequence</u> of values which has this structure is called a <u>geometric sequence</u>. By studying geometric sequences in their abstract generality we can deduce things about bouncing balls the loss of dye from jeans, and the many other specific examples for which this mathematical structure is a <u>model</u>.

For example, by solving the inequation,

$$(0.5)v_0 > v_0(0.9)^n$$

that is $$0.5 > 0.9^n$$

we discover that after 7 bounces the superball will rise to less than half its initial height, and that after 7 washes the jeans will retain less than half their dye - because $0.9^6 \approx 0.53$ while $0.9^7 \approx 0.48$.

Again, because when r is a proper fraction r^n gets smaller and smaller as n gets larger and larger [Try it] we can see that in the long run the superball will come to rest and that the jeans will lose all their dye.

So by studying an abstract mathematical structure such as a geometric sequence we can eventually learn a great deal more about a huge variety of real situations for which the structure is a model. And by matching a particular real problem to a mathematical structure we enable ourselves to use powerful mathematical tools to try and solve the problem.

[Find other situations for which a geometric sequence is a good model: situations where r can be greater than 1, or negative.]

Mathematics in decision-making: an example

This example is directly related to economics and geography, and thence to planning. It concerns the passenger rail network linking the major centres of population in Scotland. Loosely, we are interested in how easy it is to travel in the network, and how different proposals for extending or cutting back the network would affect the ease of travel. Of course such decisions cannot be made from mathematical considerations alone: the role of mathematics here is to help to clarify and operationalise concepts from other disciplines.

The first step is to be more precise about what we mean by 'ease of travel in the network'. We can identify three interlinked aspects:

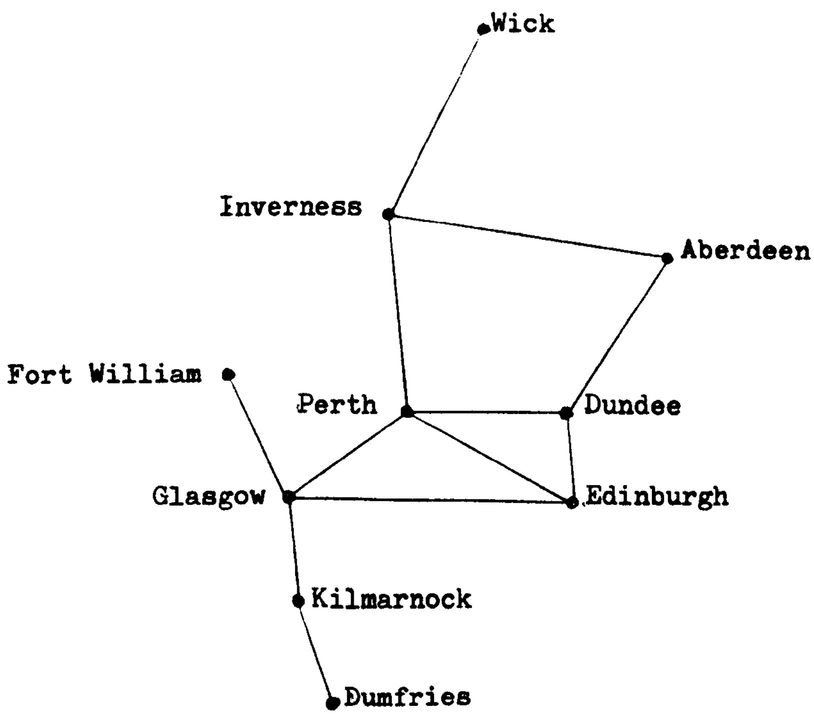
(P) the mutual accessibility of any two centres - pairwise accessibility,

(I) the overall accessibility of a particular centre - individual accessibility,

(G) accessibility within the whole network - gross accessibility.

Our objective, as mathematicians, is to develop indices for these three aspects. To do so we will need to take into account the non-mathematical use to which these indices will be put. We will start with a very simple index system and develop it by criticising its assumptions.

But first we need a way of representing the network. The commonest way is by using a <u>graph</u> or <u>network</u>. We construct it by marking down the centres and drawing in the passenger routes between them. Here is the current passenger network for the ten principal centres I have chosen - this choice is, of course, open to question.



More precisely, each centre is represented by a node or vertex. An edge or link is drawn between these two nodes if there is a passenger service which links the corresponding centres directly; that is without passing through another of the centres on the way. For example, since there is a service linking Edinburgh and Glasgow directly we draw a link between these two centres. On the other hand, to travel from Glasgow to Dundee one must either take the through train which stops at Perth, or travel via Edinburgh and change there. For this reason we do not join Glasgow and Dundee.

[We have established a relation on the set of centres. This is a symmetric relation - if you can travel directly one way between two centres, then you can return directly. This is not true for all networks - one-way traffic systems are a simple counterexample. We could represent such a relation by using directed edges with arrows to indicate the direction of travel.]

We can now make our first attempt at an index for (P). Clearly all the centres are mutually accessible in the sense that it is possible to travel between any two of them - possibly involving intermediate stops and changes. In other words the network is connected. [Think of an example of a disconnected network.] So this does not provide a criterion for discriminating among pairs. At first sight the simplest way to do so is to ask if the pair is directly linked. This gives us the direct access matrix for the network. We mark a 1 in the matrix if there is a link between a pair of nodes, and a 0 otherwise. Notice that there will be 0's on the leading diagonal since there are no links marked between a node and itself. [This is essentially an arbitrary decision. Check whether subsequent arguments would be affected if we had elected to place 1's on the leading diagonal.]

	W	I	A	F	P	D	G	E	K	Df	I
Wick	0	1	0	0	0	0	0	0	0	0	1
Inverness	1	0	1	0	1	0	0	0	0	0	3
Aberdeen	0	1	0	0	0	1	0	0	0	0	2
Fort William	0	0	0	0	0	0	1	0	0	0	1
Perth	0	1	0	0	0	1	1	1	0	0	4
Dundee	0	0	1	0	1	0	0	1	0	0	3
Glasgow	0	0	0	1	1	0	0	1	1	0	4
Edinburgh	0	0	0	0	1	1	1	0	0	0	3
Kilmarnock	0	0	0	0	0	0	1	0	0	1	2
Dumfries	0	0	0	0	0	0	0	0	1	0	1

From this matrix we can find a simple index of individual accessibility for a particular node by adding the entries in the corresponding row in the matrix. We are just counting the number of nodes to which it is linked. This is called the order of the node. Glasgow, for example, has order 4. A node with only one link, such as Wick, is called a terminal or isolated node. We can rank the nodes in order of accessibility. Clearly Glasgow and Perth are the most accessible since they have the largest number of links.

[What is the highest index that any node could have in a connected network with 10 nodes? and with n nodes? What is the lowest?]

A common way of measuring (G), accessibility within the whole network is the β-index.

$$\beta\text{-index} = \frac{\text{number of links in the network}}{\text{number of nodes in the network}}$$

In this case the β-index is 1.2 since there are 12 links and 10 nodes.

[What is the lowest β-index that a connected network with 10 nodes can have? with n nodes?

Another way to calculate the β-index is to sum the I column beside the direct access matrix and divide by double the number of nodes. Why?]

Our first try at an index for (P) does not really tell us how easy it is to travel between two nodes - only whether or not we can travel directly. We want an index which is more discriminating. One way to discriminate more precisely would be to find the smallest number of links that must be traversed in order to travel between two nodes. For example, the route from Wick to Glasgow via Inverness contains 3 links, while any other route would contain more. In other words we are minimising the number of nodes we must pass through. These measures can be represented in the shortest route matrix.

	W	I	A	F	P	D	G	E	K	Df	i_1
W	0	1	2	4	2	3	3	3	4	5	27
I	1	0	1	3	1	2	2	2	3	4	19
A	2	1	0	4	2	1	3	2	4	5	24
F	4	3	4	0	2	3	1	2	2	3	24
P	2	1	2	2	0	1	1	1	2	3	15
D	3	2	1	3	1	0	2	1	3	4	20
G	3	2	3	1	1	2	0	1	1	2	16
E	3	2	2	2	1	1	1	0	2	3	17
K	4	3	4	2	2	3	1	2	0	1	22
Df	5	4	5	3	3	4	2	3	1	0	30
										g_1	214

Again we can find an index for (I) by adding up the entries in the row of each node. We are adding the number of links that must be traversed in travelling to each of the other centres. This time the node with the lowest index is the most accessible. Why?

[What are the minimal and maximal values for this index in a 10-node connected network? an n-node network?]

Finally we can construct a gross index g_1 by summing the individual index (i_1) column for the network. Our results have a face validity. They correspond fairly well with our intuitive ideas. Although we will criticise this system later we are now going to use it to compare the effects of two alternative extensions to the rail network.

Imagine that the government has decided to devote more resources to rail travel. In Scotland they have proposed the construction of one major new rail link. Two possible links are in contention, an Edinburgh-Dumfries link, and an Inverness-Fort William link. It is our job as mathematicians to evaluate the consequences of each of these two alternatives as they affect accessibility. We use our index system.

Case (1): Edinburgh-Dumfries line

	W	I	A	F	P	D	G	E	K	Df	i_2
W	0	1	2	4	2	3	3	3	4	4	26
I	1	0	1	3	1	2	2	2	3	3	18
A	2	1	0	4	2	1	3	2	4	3	22
F	4	3	4	0	2	3	1	2	2	3	24
P	2	1	2	2	0	1	1	1	2	2	14
D	3	2	1	3	1	0	2	1	3	2	18
G	3	2	3	1	1	2	0	1	1	2	16
E	3	2	2	2	1	1	1	0	2	1	15
K	4	3	4	2	2	3	1	2	0	1	22
Df	4	3	3	3	2	2	2	1	1	0	21
										g_2	196

Case (2): Fort William-Inverness line

	W	I	A	F	P	D	G	E	K	Df	i_3
W	0	1	2	2	2	3	3	3	4	5	25
I	1	0	1	1	1	2	2	2	3	4	17
A	2	1	0	2	2	1	3	2	4	5	22
F	2	1	2	0	2	3	1	2	2	3	18
P	2	1	2	2	0	1	1	1	2	3	15
D	3	2	1	3	1	0	2	1	3	4	20
G	3	2	3	1	1	2	0	1	1	2	16
E	3	2	2	2	1	1	1	0	2	3	17
K	4	3	4	2	2	3	1	2	0	1	22
Df	5	4	5	3	3	4	2	3	1	0	30
										g_3	202

The decision we make will depend on our objectives. If we wish to improve gross accessibility then we will choose the first alternative. The citizens of Wick, on the other hand, will be anxious to improve the communications of that particular centre: it would appear that they should support the second alternative. Dundee, on the contrary, would benefit most from the first alternative.

Note also that our first index system could not give us these results. The orders of the end-nodes of the new links would have increased, but not those of nodes like Dundee and Wick. Similarly, in both cases the β-index increases to 1.3 which does not enable us to choose between alternatives. This second index system is a real advance on our first.

But this index system can still be criticised. It takes no account of the travel-time between each centre, the number of changes of train, the frequency of services, the population of each centre, or the volume of traffic on each route. Our index can be refined in a number of ways to take account of such criticisms, possibly at the expense of simplicity and facility of use. [For reasons of space I will not do so here.]

Finally, note that we cannot escape the reality of the problem. The technical problems and cost of building the two links we have considered are broadly comparable: that is why our mathematical refinement of the problem has a value. Topologically, of course, the optimal link would be Wick-Dumfries: topographically and economically it would be quire unrealistic to build a direct link between these two centres. Here the abstraction is not fruitful, and possibly misleading.

Data-banks and civil liberties

This lesson uses the article 'Your life in their Files' as source material: students should have read it in advance. (A copy is included in the supplementary folder). The lesson should start with the clarification of terms or references in the article with which some of the students may not be familiar - 'main-frame', the Lindop Committee, the National Council for Civil Liberties, for example. It can then proceed around the following central questions;

What is a data-base? How does it work?

What other existing or planned data-bases raise issues of civil liberties?

How has computerisation affected

the amount of information recorded,

the extent to which it is used,

the accessibility of information,

the way that information is recorded and interpreted,

the ways in which information is used?

In what ways can automated information retrieval assist the work of the police?

In what ways may automated information retrieval threaten civil liberties?

How secure are computerised records? What checks are there on the validity and relevance of such records, and the use to which they are put?

Is legislation on data protection necessary? What form should it take? Is it enforceable?

Appendix 2

Mathematical Topic Handling System (MATHS): Classroom version

GENERAL CODING INSTRUCTIONS

1.1(1) The substantive content of the lesson should be divided into one or more topics each having a single overarching concern - that is, a central point, a conclusion to which the verbal exchanges lead, a theme that pervades the exchanges.

(2) The theoretically significant distinctions which the topical structure chosen <u>must</u> respect are those between the content and group categories described below.

(3) Once these systematic constraints have been satisfied it is often convenient to exploit the 'natural' structure of the lesson in order to subdivide it into topical units. The following are useful cues to natural structure.

Management and structuring comments,

Markers such as 'Right' and 'Now',

A teacher or pupil question (may mark a shift in focus),

Summarising or concluding comments.

Strictly, however, it is essential only that theoretically significant distinctions be made.

(4) All topics must be categorised.

(5) In general a topic will consist of a single sustained period of talk, but it is possible that a topic will be interrupted and returned to later in the course of the lesson. It then falls into two or more <u>segments</u>.

1.2(1) The first and last utterances of each segment of a topic should be recorded. Classroom talk is usually quite structured and it is generally easy to pick out the first and last substantive utterances of the segment.

(2) Sometimes, however, structuring utterances run into substantive utterances.

> "Now last week we were looking at certain special kinds of quadrilaterals ... rhombuses .. squares ...."

In such a case the coder should choose the nearest clean break (syntactic or auditory) in the discourse to mark the boundary of the segment.

1.3 (1) The basic unit of measure is the 10 second interval. That is, all measurements are made to the nearest 10 seconds.

(2) Individual measurements smaller than two units (effectively 15 seconds are ignored.

(3) The length of a segment is the time from the opening to the closing utterance with any nonsubstantive interruptions (including silence) subtracted.

(4) Note that, as a corollary of 1.3(2), segments, topics and interruptions lasting less than two units are ignored.

(5) Note that any <u>substantive</u> interruption lasting two units or longer creates a topic in its own right.

(6) Where there is a choice of topical division within the same category, a useful guideline is to aim to create topics lasting between one and five minutes.

1.4 The coding data should be recorded on a chart under the following headings.

SEGMENT			DELETION		
opening utterance	closing utterance	time	opening utterance	closing utterance	time

TOPIC				
label	title	time	group category	content category

IDENTIFYING SUBSTANTIVE TALK

2.1 The following phenomena are regarded as nonsubstantive talk;

(1) silence,

(2) talk which involves someone who is neither a pupil in the class nor the teacher, nor acting for the teacher.

Examples

(a) The headmaster addresses the school over the Tannoy.

(b) A pupil enters and announces that a football match has been cancelled.

(c) A teacher enters the class and converses with the class teacher.

(3) talk which is not monitored by the teacher,

Example

(a) Whispered talk between two pupils.

(4) talk which functions exclusively to manage and structure the learning environment.

- talk concerning school and class management.

Examples

(a) The teacher reads out a circular announcement.

(b) A pupil explains his absence.

(c) A pupil asks when homework is due.

(d) The teacher announces pupils' grades.

(e) The teacher explains how the grades were arrived at.

(f) The teacher hands out rulers.

(g) The teacher shows pupils how to fill in an administrative form.

- talk which links, structures and comments on the activity of the class, or manages pupil learning.

Examples

(a) The teacher outlines the form that a lesson is going to take.

(b) The teacher signals that a change in activity is about to take place.

(c) The teacher alerts pupils to their previous experience of some subject matter.

(d) The teacher advises pupils on the topics they should study for a forthcoming examination.

(e) The teacher asks a pupil what example he is on, or is having difficulty with.

(f) The teacher tells pupils to open their books at a certain page and to do certain examples.

(g) The teacher comments on the performance of the class or individual pupils.

2.2 Talk, other than that covered by the previous rule, is substantive.

2.3 Ground rules

(1) Giving or correcting answers in some codified form in which there is no indication of meaning is nonsubstantive talk.

Example

(a) Giving answers to multiple choice questions.

Teacher: What's the answer to number one?

Pupil: A

Teacher: That's right, A.

(2) Giving a non-content-specific instructions about how work is to be done is nonsubstantive.

Example

(a) "You must write things down clearly..and remember to divide your page in two..put your working at the side..then you'll get on much better."

(3) Instructions which simply refer to topics are nonsubstantive.

Example

(a) "We're going on to inequations today."

GROUP CLASSIFICATION

3.1 Each topic is classified INDIVIDUAL or CLASS depending on whether the teacher is addressing a group of less than four pupils. No topic can mix Group segments. Any shift in audience requires the creation of a new topic.

CONTENT CLASSIFICATION

4.1 Preliminary definitions.

(1) By mathematical methodology we mean the concepts, rules and procedures of the conventionally recognised branches of mathematics - Logic, Algebra, Arithmetic, Geometry, Topology, Mechanics, Probability, Statistics, Actuarial mathematics, and so on.

(2) By a mathematical problem we mean any task or problem framed in terms of mathematical concepts or solved by means of mathematical methods.

(3) We distinguish two kinds of judgement about mathematical methodology or its use. Standard judgements are those made wholly within some conventional framework of rules, or some conventional model, which, in principle, determines their form. They are judgements about whether that framework or model is being used correctly, conventionally, or appropriately. For example, the typical problem in dynamics asks for a situation to be analysed in accordance with a taken-for-granted Newtonian model of motion. Judging whether a proposed solution to the problem conforms to the rules of the model, or follows from a conventional use of the model, is judging in terms of standard criteria. By contrast, to judge the degree of validity of the Newtonian model, or its convenience,

or elegance, is to use a nonstandard criterion.

Again, to judge whether a pupil has given a normal definition of 'prime number', or a definition consistent with or equivalent to a normal definition, is to use standard criteria. To judge whether a definition of 'prime number' which classifies 1 as prime is a good one, is to use nonstandard criteria.

Standard judgements, then, take place within taken-for-granted systems of rules - rules governing the use of concepts (axioms and theorems for example) and rules governing procedure (algorithms, heuristic strategies, and so on). It is the use and manipulation of these given rules which is problematic. Nonstandard judgements are open with respect to systems of mathematical rules, concepts and procedures. Judgements about beauty, simplicity, clarity, and convenience are instances of this type.

4.2 There are five Content categories defined as follows:

SMA Standard methodological articulation

Topics in this category describe, or discuss in terms of standard criteria, mathematical methodology or the formulation and solution of mathematical problems.

Exemplary forms

Conceptual: the description of a set of rules governing the use of some mathematical concept, term or sign (or system of concepts, terms or signs).

the exemplification of some mathematical concept term or sign (or system of concepts, terms or signs).

Relational: the description of some relation (or network of relations) within a mathematical system.

The exemplification of some such relation (or system of relations).

the demonstration of some relation (or network of relations) within a mathematical system.

Procedural: the description of some mathematical tool (such as a calculator or pair of compasses) and how to use it.

the description of (the steps making up) some mathematical procedure (and the rules governing its use).

the execution of some mathematical procedure.

the demonstration of the validity of some mathematical procedure.

Problem solving: the description of some mathematical problem (and the characteristics of the desired outcome).

the solution of some mathematical problem.

the description of general strategies for problem solving.

Critical: the critical discussion (evaluation, explanation, justification) of some aspect of methodology or its use in terms of standard criteria.

Examples

(a) The concept of 'prime number' is defined and examples of prime numbers sought. A list of numbers is presented and the primes identified.

(b) A demonstration of the protractor in use is given.

(c) A quadratic equation is solved.

(d) A proof of Pythagoras Theorem is given.

(e) An algebraic expression is simplified.

(f) The mistakes in a proposed solution are identified.

(g) The conventional way of solving a problem is demonstrated.

NMA <u>Nonstandard methodological articulation</u>

Topics in this category critically discuss methodology or its use in terms of nonstandard criteria.

<u>Exemplary forms</u>

Some aspect or feature of methodology is evaluated in terms of some nonstandard criterion.

Some aspect or feature of methodology is explained or justified in terms of some nonstandard criterion.

<u>Examples</u>

(a) Two alternative methods of solving quadratic equations are compared and evaluated in terms of speed, reliability and easiness.

(b) The case for and against one of the assumptions of a model is discussed weighing accuracy against conciseness and elegance.

(c) A standard method is justified in terms of its convenience compared with possible alternatives.

(d) The plausibility of the Newtonian model of motion is discussed.

(e) Several proofs of Pythagoras Theorem are compared in terms of their clarity, elegance, brevity and rigour.

MR <u>Methodological reflection</u>

Topics in this category describe and discuss mathematical methodology from 'philosophical' perspectives.

<u>Exemplary forms</u>

Discussing the nature of the criteria (such as truth, beauty, and convenience) which are used to evaluate and justify mathematical methods.

Discussing the nature of mathematical thought, mathematical knowledge or mathematical activity.

Describing and discussing standard views on these issues.

Discussing which criteria <u>ought</u> to be used to criticise methodology.

<u>Examples</u>

(a) Discussing what a number of 'beautiful' theorems and proofs have in common.

(b) Discussing what is meant by a 'good' model of a phenomenon.

(c) Discussing whether there is such a thing as a 'correct' model of a physical phenomenon.

(d) Discussing whether we ought to be concerned to produce elegant proofs.

(e) Describing and evaluating the formalist view of mathematics.

CR <u>Cultural reflection</u>

Topics in this category treat mathematics and mathematical activity as a social and human phenomenon: they describe and discuss mathematics in its social, historical and intellectual context.

<u>Exemplary forms</u>

Describing and discussing the characteristics, actions, beliefs, and purposes of mathematicians or users of mathematics.

Describing and discussing the social institutions of mathematics; for example, the IMA or the Royal Society.

Describing and discussing the history and uses of (parts of) mathematics.

Describing and discussing the influences on, and causes of mathematical activity, or some instance of it.

Describing and discussing the consequences, implications and influence of mathematical activity, or some instance of it.

Describing and discussing the purposes, beliefs and actions of mathematicians.

Examples

(a) A biographical account of the life of Descartes.

(b) A historical account of attempts to prove the parallel postulate, and of the influence of the discoveries of Bolyai and Lobachevsky on mathematics and philosophy.

(c) A discussion of the reasons for developing game theory, its current and potential uses, and their moral implications.

(d) A discussion of whether mathematics is of social or personal value.

(e) A brief identification of the inventor of some technique.

(f) A description of the kinds of work that mathematicians do.

(g) A description of the range of mathematics used at different times or in different societies.

RES Residual

A topic which does not fall into one of the previous categories is placed in this category.

4.3 Ground rules

(1) It is permissible for NMA, MR and CR topics to contain sections which, standing independently, would be classified SMA. For example, one part of an NMA topic concerned with comparing two mathematical procedures might consist of actually using the

procedures and then comparing the processes of using them and their actual outcomes.

Such sections are absorbed only when there are explicit cues relating them to the larger unit.

(2) A similar rule is applied to mutual absorption between the categories NMA, MR and CR.

(3) The occasional historical reference woven into an SMA, NMA or MR topic does not transform it into a CR topic. If an independent CR topic cannot be created such references are ignored.

(4) It is important to distinguish between the concretisation of concepts and procedures (classified as SMA) and descriptions of the social application of concepts and procedures (classified as CR).

Merely to offer the big wheel as an example of a circle is SMA: to claim that its designer used geometrical methods is CR. To pose, and solve the problem of the type "if one lollipop and two bags of sweets cost 26p and two lollipops and one bag of sweets cost 22p, what is the price of a lollipop?" is SMA: to suggest that such a technique is used by someone to determine the price of lollipops is CR.

(5) Talk involved in playing games or engaging in practical activities is classified as SMA when it is contextually apparent that it helps to articulate or rehearse some aspect of methodology.

<u>Example</u>

(a) Playing battleships as part of a lesson on co-ordinates.

Appendix 3

Interview Schedule used in teacher study

The interview was preceded by a description of its structure and a brief explanation of the function of probing questioning.

The interview is in three parts. The first part is about the lesson I've just watched, the second part is about the curriculum in general that the class is following, and the third part is about your background as a maths teacher.

Now in the course of the interview I'm likely to say things like "Is there anything else that you think about this?" or "Have you any other reasons?". That doesn't mean that I think you ought to have other reasons or anything else to say. It's just to give you as full an opportunity as possible to say what you think.

SECTION 1

Well first of all I'd like you to think yourself back to the start of the lesson, and I'd like to know what things you intended or expected to talk about or do during the lesson. Of course, you know now what actually happened during the lesson but I'd like you to try and forget that for the moment, and to describe your thoughts before the lesson started.

1.1 Can you tell me the main points that you wanted the pupils to take away from the lesson?

Probe for additional points.

For all classes other than Modular Mathematics continue with:

Now in deciding what to teach this class there are probably various constraints that you have to take into account, but also to some extent there will be considerations which are matters of your personal choice.

1.2 What were your reasons for deciding to teach .... today?
Probe role of set curriculum if mentioned: then any topic which is not part of set curriculum. For all classes continue:

1.3 Now as you actually taught the lesson did you do or talk about anything important that you hadn't intended or expected to talk about during the lesson?
Probe for details and reasons. Then mention any salient topics.

SECTION 2

I'd like now to talk more generally about the curriculum that this class is following.
For S1 and S3 certificate classes only, choosing appropriate options:

2.1(C) The work of this class is based on the SCE Syllabus A/B curriculum in arithmetic/mathematics and arithmetic?

2.2(C) Is there any additional curricular component laid down for this class at department level?
Probe for details.

What I'd like to know is how this curriculum that is laid down for the class - that is ...... - compares with the kind of curriculum that you believe is desirable. Now I know that, in practice, there are all kinds of immediate pressures and constraints that you have to take into account in your day to day teaching. But for the moment I would like you to distance yourself from those everyday pressures and constraints. I'm interested in what you yourself think is important, in your personal opinions about the kind of mathematical education that is desirable for the pupils in this class.

Now to help structure this section I've written down a few questions.

2.3(C) Looking at the set curriculum as a whole do you feel that there are any areas or aspects of mathematics, or ways of looking at or understanding mathematics to which it pays too much attention? Probe specific details, reasons, how teaching is influenced: then probe for further aspects.

2.4(C) Do you, on the other hand, feel that there are any important areas or aspects of mathematics, or ways of looking at or understanding mathematics to which the set curriculum pays too little - and that could mean no - attention?

Again probe specific details, then reasons, how teaching is influenced: then probe further aspects.

If very little has been said continue with next question.

2.5(C) So basically the set curriculum is the kind of curriculum that you think is desirable for the pupils in this class?

Probe reasons for answer.

S3 non-certificate classes only:

2.1(NC) Is the work of the class based on a curriculum laid down at the departmental level?

2.2(NC) Could you describe what kind of things you do, in general, with this class?

Probe for details, reasons, examples. Probe for differences between this curriculum and that of certificate courses.

All classes:

2.6 Have you recently read, heard, seen or done anything related to mathematics in any way - such as a TV programme, or a magazine article, for example - which you have talked to this class about, or intend to?

Probe for details and reasons: then probe further cases.

For S1 and S3 certificate classes only:

2.7(C) Have you recently talked about, or done anything with this class which is not strictly part of the set curriculum?

Probe for details and reasons: then probe further cases.

SECTION 3

I'd like to ask a few questions about your background as a maths teacher now.

3.1 Are you qualified to teach any other subjects?

Probe for details if necessary: then, if relevant:

3.2 Do you currently teach these subjects?

3.3 You've got a degree? ...in? .... What subjects did you study for this degree?

3.4 Which of these subjects, if any, do you see as being of value in teaching maths, either actually or potentially? ..... In what ways?

Probe until reasons and examples are exhausted.

3.5 Have you had any occupation other than teaching? .. What was it?

3.6 Do you see the experience you derived from this occupation as being of value, actually or potentially, in teaching maths?..In what ways?

Probe until reasons and examples seem to be exhausted.

Appendix 4

Item bank used in reliability study

University of Stirling

Department of Education

Kenneth Ruthven

Appendix 4 (and Listener Article) of thesis

"Mathematics and General Education"

3.18 AREA: Surface of a Cone

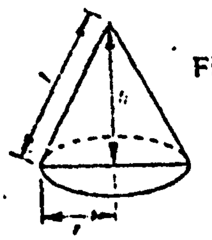


Fig. (i)

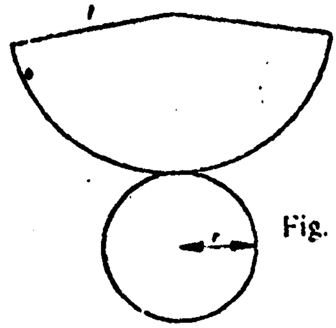


Fig. (ii)

The cone in Fig. (i) has two surfaces, one a flat circular base and one a curved surface with dimensions shown.

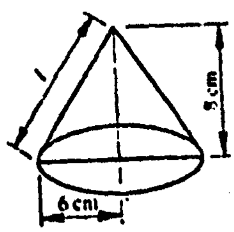
Fig. (ii) shows the net for this cone.

For any cone:
If l is the measure of the slant height, the measure of the area of the base is πr^2 or $\frac{1}{4}\pi D^2$ and the measure of the area of the curved surface is πrl or $\frac{1}{2}\pi Dl$.

The measure of the total surface area is:

$$\pi r^2 + \pi rl \quad \text{or} \quad \pi r(r+l) \quad \text{or} \quad \tfrac{1}{4}\pi D^2 + \tfrac{1}{2}\pi Dl$$

Example. Find the total surface area of a cone whose radius is 6 cm and whose height is 8 cm.



Firstly find l, the measure of the slant height.

$l = \sqrt{6^2+8^2} = 10$

$\pi r^2 + \pi rl = 3{\cdot}14 \times 36 + 3{\cdot}14 \times 6 \times 10$

$= 113{\cdot}04 + 188{\cdot}4 = 301{\cdot}44$

Total surface area of cone is $301{\cdot}4$ cm^2.

In the <u>aircraft industry</u> mathematics helps determine the best shape for an airplane or space ship, and how strong its construction must be. Another kind of mathematics predicts whether a plane will shake itself to pieces as it flies through stormy air at high speeds. Still different forms of mathematics help design the radio and radar devices used to guide the plane and to communicate with other planes and with airfields.

(3)

We have seen that a linear search (for the maximum value of a function, $f(x)$ say, of a single variable x) plays a central rôle in most optimization techniques. How should such a search be conducted? One method would be to evaluate $f(x)$ at regular intervals, say 0·001, until the maximum is reached. However, in practice function evaluation is often a lengthy business, and such a 'brute force' method would be very inefficient, even with the aid of a computer. We are thus led to seek the best search strategy, where we use the term 'best' in the sense of enabling the maximum to be located to a prescribed level of accuracy with the fewest possible evaluations of the function.

(4)

*Precision and Error

In the discussion so far it has been assumed that the exact lengths and widths of the rectangles are known. Actually of course, we have seen this is never the case since no measurement can be made exactly. Thus if we have measured a rectangle and found measurements of $2\frac{1}{2}$ inches and $3\frac{1}{4}$ inches, we must use the "approximately equal" symbol and write $l \approx 3\frac{1}{4}$, $w \approx 2\frac{2}{4}$ and therefore:

$$A = lw$$
$$A \approx (3\tfrac{1}{4})\ (2\tfrac{1}{2})$$
$$A \approx (\tfrac{13}{4})\ (\tfrac{5}{2})$$
$$A \approx \frac{65}{8}$$
$$A \approx 8\tfrac{1}{8}$$

Since A is the number of square inches, we find therefore that the area is approximately $8\frac{1}{8}$ square inches.

A statement concerning a measured quantity should indicate that it is only approximate.

(5)

In studies of astronomy and space flight, especially, we encounter very large numbers. The planet Pluto has a mean dist... from the sun of about 3666 million miles or 3.666×10^9 miles. Distances to the stars are usually measured in "light years." A light year is the distance that light travels in one year. This is a good way to measure such distances. If we expressed them in miles, the numbers would be so large that it would be diffic... to write them, much less understand what they mean.

(6)

7.17 CURRENCY CONVERSION II

Question: Change £25·00 into French francs.

Answer: 297. **Method (a)** £1 ↔ 11·88 francs (from Table 2)

£25 ↔ 25 × 11·88 = 297 francs

Method (b) Using rate of exchange card in Table 2.

£21·04	⟶	250 francs
3·37	⟶	40 francs
0·42	⟶	5 francs
0·16	⟶	2 francs
£24·99	⟶	297 francs

Note. For £25, 297 francs would be received.

7

Scientific Notation

As we remarked, we can write 293 billion as 293×10^9 or as 2.93×10^{11}. These are compact ways of writing the number. Also, it is easy to compare several large numbers written in this form. For example, we can tell at a glance that 4.9×10^{13} is bigger than 9.6×10^{12} without counting decimal places in 49000000000000 and 9600000000000. We shall see later on that it often simplifies calculations with large numbers to work with them in such a standard form. This is especially true of computations by slide rule or by logarithms, as you will learn in high school. For these reasons it is common practice in scientific and engineering work to represent numbers in this way, namely in the form

(a number between 1 and 10) $\times$ (a power of 10).

5 Improving the Model

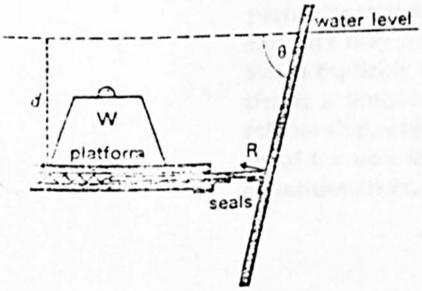


Figure 5.1

A mathematical model of a situation is normally obtained by simplifying a description of the situation. For example, in the case of the diving bell, we treated the platform as a *line* having zero thickness, whereas in the real case the platform would have a definite non-zero thickness z (Figure 5.1). We also treated the platform as being *weightless*, whereas in the real case the platform would almost certainly be made of rigid, probably metallic, materials. Finally we have assumed that there will be no *leakage* of water past the ends of the platform, and in the real case this assumption, too, is likely to be mistaken.

A more accurate model of the proposed diving bell will therefore distinguish the depth of the platform's upper surface d, and its lower surface $d+z$, where z is the thickness of the platform. It will distinguish the weight placed on the platform W, from the total weight of the concrete blocks, platform, loose water, and human operator $W+x+y+w$. It will take into account the force (reaction) R acting between the platform and the sloping sides of the bell.

If we take all these factors into account we obtain a more accurate mathematical model of the weight W needed to produce a platform depth d

$$W = 9810(d+\tfrac{1}{2}z)\{3-\tfrac{1}{4}(d+\tfrac{1}{2}z)\} - x - y - w + 2R \sin \theta.$$

It is reasonable to suppose that the quantity of 'loose water' leaking into the bell depends on the depth at which the platform is set and the length of time it has been operating at that depth. Thus the term y may be replaced by a term of the kind $k(d+\frac{1}{2}z)t$, where k is a constant and t is the time in hours.

So the expression above will become more complicated. But in estimating the effect of the 'loose water' we have simplified again. We have assumed that the bell has been operating with the platform at a fixed depth $d+\frac{1}{2}z$ for a definite number of hours: whereas a graph showing the depth of the platform against time will probably look more like Figure 5.2.

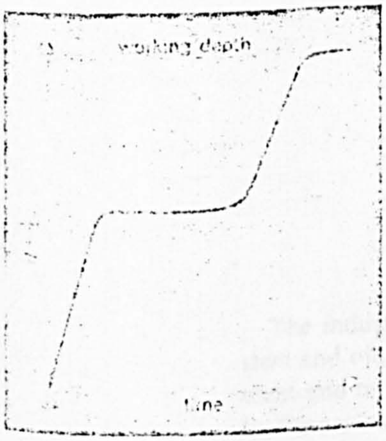


Figure 5.2

The quantity of loose water found at time t (which has leaked past the seals at the ends of the platform) will therefore depend on a *summation* of the water which has leaked past at each instant of time from the moment the operator steps onto the platform. M 36

~~This is rather similar to the problem of summing the horizontal span contribution of each part of the rope which we considered on page 3. To tackle problems of this kind we need to use a general method known as *integration*.~~

(9)

There are basically three aspects involved in any scientific activity, namely: (*a*) the formulation of a hypothesis regarding the relationships between observed data, such as the rate of growth of the national income and the level of employment; (*b*) the collection of statistics relative to the hypothesis and the expression of the hypothesis in concise or mathematical terms; and (*c*) modification or improvement of the hypothesis.

However, even though a particular mathematical form may be found to represent the numerical relationship between observed quantities, this may be no more than a concise description of the particular statistical data and use of the form outside the range of the data may lead to erroneous conclusions. The point can be stated explicitly in that a statistical relationship between quantities is a limited association and is not in any sense a causal relationship, which can only be established by an objective analysis of the way in which the relationship between the particular quantities arises.

(10)

The industrial strength of the USA has been built on coal, steel and oil: and on the readiness of the Americans to experiment and to apply to industry the results of their experiments. In Pennsylvania coal is abundant, and because it lies in thick seams, it is easily mined. At the head of Lake Superior is the Mesabi iron range, an enormous deposit of pure iron ore. In 1856 Bessemer discovered a means of making steel cheaply. In 1875 Andrew Carnegie, who had gone to the USA as a boy from Dunfermline, opened his steel works on the Monongahela River in Pennsylvania. These four things—plentiful coal and iron ore, the Bessemer process and the Carnegie works—enabled the USA to produce steel in abundance for all industrial purposes: for the rails and rolling stock of the expanding railways, for the McCormick reapers which made possible the huge wheat farms of the prairies, for the new steel ships, for the machinery of the textile mills, for all the varied needs of an industrial nation.

(11)

The Intuitionists represent themselves as critics of classical logic, which holds to be true principles to which there are, they claim, counter-examples. But it would be a serious mistake to suppose that their disagreement with certain classical logical principles is the basic tenet of Intuitionism. This disagreement, on the contrary, is a consequence of a more fundamental difference; a difference about the nature and status of logic itself.

While 'classical' logicians no doubt differ among themselves about the status of logic, there is one point on which they are, I think, agreed: that logic is the most basic, the most general, of theories. This idea is crucial to the logicism of Frege and Russell; mathematics is to be reduced to logic, and the epistemological value of the programme lies in the presumed *fundamental* nature of the latter. Even pragmatists, while wishing to treat logic as a theory like others, concede that its extreme generality gives it a special status. But Intuitionists think otherwise. On their view, mathematics is primary and logic secondary: logic is simply a collection of those rules which are discovered, *a posteriori*, to be true of mathematical reasoning. (Intuitionists would therefore regard the logicist programme as hopelessly misconceived.)

But this alone would not account for their claim that certain of the classical logical laws turn out not to be generally true, for the laws of classical logic *do* hold true of classical mathematical reasoning. However, Intuitionists hold, in addition to their unusual views about logic, an unusual view about mathematics. Their view has elements both of psychologism and of constructivism. First, numbers are *mental* entities. They are constructed, according to Brouwer, out of 'the sensation of time'. This seems to mean, from the idea of distinctness or plurality (Brouwer: 'two-ity') acquired thanks to the temporal nature of experience. Mathematics is, thus, a mental activity, and Brouwer stresses that mathematical formalisms are strictly inessential, useful only for communicating the real, mental mathematics. Second, only *constructible* mathematical entities are admitted, so that, for instance, it is not allowed that completed infinite totalities, which are not constructible, exist; and only constructive proofs of mathematical statements are admitted, so that, for instance, a statement to the effect that there is a number with such-and-such a property is provable only if a number with that property is constructible.

This view about the nature of mathematics has a radical consequence: not all of classical mathematics is Intuitionistically acceptable. And from this restriction of mathematics there follows a restriction of logic; some principles of classical logic are found not to be universally valid. The 'principle of excluded third' (LEM) has, for example, counter-instances.

So the structure of the Intuitionist critique of classical logic can be represented as follows:

(1) A subjectivist, constructivist view of mathematics

supports the thesis that

(2) some parts of classical mathematics are unacceptable,

and with

(3) a view of logic as a description of the valid forms of mathematical reasoning

supports the thesis that

(4) some parts of classical logic are mistaken.

The source of the Intuitionists' disagreement with classical logic thus lies deep.

10-11. Historical Note

Some of the geometric ideas in Chapter 10 were discovered by the Egyptians and Babylonians almost 4,000 years ago. For example, they knew how to find the area of a triangle and used this knowledge in surveying and measuring fields.

Thales, mentioned in Section 2, is credited with the discovery that the measures of the base angles of an isosceles triangle are equal. There is some evidence that Thales also knew that the sum of the measures in degrees of the angles in a triangle is 180.

There were many other famous Greek mathematicians. Their work made ancient Greece famous as the "Cradle of Knowledge." We will discuss only a few of these men. Pythagoras (569 ? B.C. - 500 B.C.) organized schools at Croton in southern Italy which contributed to further progress in the study of geometry. You will learn about some of the discoveries credited to him next year. Euclid (365 ? B.C. - 300 ? B.C.) became famous by writing one of the first geometry textbooks called the Elements. This textbook has been translated into many languages. It has been used in teaching geometry classes for some 2,000 years without much change. Its form has been somewhat modernized to fit present needs. All of the properties we have studied in this chapter may be found in the Elements.

From the 7th century until the 13th century very little progress was made in mathematics. From the 13th century, however, the study of geometry and other mathematics spread rapidly throughout Europe. Mathematicians began to examine new ways of studying elementary mathematics. You will learn about the work of men such as Rene Descartes (1596 - 1650, France); Blaise Pascal (1623 - 1662, France); Pierre Fermat (1601 - 1665, France); Carl Friederich Gauss (1776 - 1855, Germany); and others as you continue your studies of mathematics.

(13)

This passage follows a description of two systems of numeration

The Egyptian system was an improvement over the caveman's system because it used these ideas:

1. A single symbol could be used to represent the number of objects in a collection. For example, the heel bone represented the number ten.

2. Symbols were repeated to show other numbers. The group of symbols ??? meant 100 + 100 + 100 or 300.

3. This system was based on groups of ten. Ten strokes make a heelbone, ten heelbones make a scroll, and so on.

(14)

What is frustrating and confusing in network problems is the complete lack of a uniform terminology. 'What's in a name?' is Harary's [2] sarcastic quote from *Romeo and Juliet* as he complains about the 'personalised terminology' of graph theoreticians. Vertex, point, node, junction, are variously used for the same thing and a long glossary of pseudo-botanical names – tree, forest, blossom, branch, vine and so on – is hardly a pretty sight.

The decimal system uses the idea of place value to represent the size of a group. The size of the group represented by a symbol depends upon the position of the symbol or digit in a numeral. The symbol tells us how many of that group we have. In the numeral 123, the "1" represents one group of one hundred; the "2" represents two groups of ten, or twenty; and the "3" represents three ones, or three.

Since we group by tens in the decimal system, we say its <u>base</u> is ten. Because of this, each successive (or next) place to the left represents a group ten times that of the preceding place. The first place tells us how many groups of one. The secondplace tells us how many groups of ten, or ten times one (10×1). The third place tells us how many groups of ten times ten (10×10), or one hundred; the next, ten times ten times ten ($10 \times 10 \times 10$), or one thousand, and so on. By using a base and the ideas of place value, it is possible to write any number in the decimal system using only the ten basic symbols. There is no limit to the size of numbers which can be represented by the decimal system.

To understand the meaning of the number represented by a numeral such as 123 we add the numbers represented by each symbol. Thus 123 means $(1 \times 100) + (2 \times 10) + (3 \times 1)$, or $100 + 20 + 3$. The same number is represented by $100 + 20 + 3$ and by 123. When we write a numeral such as 123 we are using number symbols, the idea of place value, and base ten.

16

In the USSR a special form of Communism has developed. In practice the State is all-important and individuals exist to serve the state. The people's lives, work and ideas must all serve the State and men and women are not free as we are. Secondly, Marx and Lenin taught that our 'capitalist' way of organizing industry must be overthrown, as it has been overthrown in Russia. Communists believe that if everything—including all industries—belongs to the State then everyone will be richer, and the gulf between rich and poor will disappear. (In fact, what happens is that under communist rule some people of the ruling class have many privileges, while the vast majority of the people have few.) Marx and Lenin taught, too, that capitalist countries could not be changed peacefully; there would have to be a world revolution first.

17

Mathematics—Queen of the Possibility-simulating Disciplines

Mathematics has often been described as the 'Queen of the Sciences'. It has gradually become clear, however, that mathematics is not a 'science' in the ordinary sense at all. It is about possibilities; whereas the sciences are about actualities.

A much more apt description of mathematics, at least when seen from the applicable point of view, is that of 'Queen of the Possibility-simulating Disciplines'. 'Queen' because it is free from the awkwardnesses which invariably attach themselves to physical simulations, and because it is by far the most powerful way of simulating possibilities.

In applicable mathematics we do not rely on the properties of one real situation to be used as a substitute for another. We use symbols and formulate rules; then we construct symbolic expressions, the patterns of change in which, will, if *imagined*, mimic the real situation.

(16)

The origin of the idea of zero is uncertain, but the Hindus were using a symbol for zero about 600 A.D., or possibly earlier.

(19)

1.10 Relations

(1)

3	is greater than	2
PQ	is parallel to	YZ
$\angle ABC$	is the complement of	$\angle CDA$

In each case the words is greater than, is parallel to and is the complement of state a relation connecting 3 and 2, PQ and YZ, $\angle ABC$ and $\angle CDA$ respectively.

These relations could be re-written in symbols thus:

$$3 > 2$$
$$PQ \parallel YZ$$
$$\angle ABC + \angle CDA = 90^\circ$$

(2) Now consider the following four relations from everyday language:

Jane	is taller than	Mary
London	is north of	the equator
James	owns	a motorcycle
Television	was invented in	the 20th century

In each of these the relation is expressed in words and not in symbols as there are no recognised mathematical symbols for these expressions.

Note. The above relations are binary relations because in each case two objects are related in a definite order: "bi" is a Latin prefix meaning "two", as in bicycle.

The scientific principles of wireless telegraphy were discovered in 1887 by a German called Hertz, but it was Marconi, an Italian, who developed it for commercial use. He demonstrated the possibilities of wireless telegraphy by sending messages across the English Channel and in 1901 he sent a message consisting of three dots of the Morse code—the letter S—across the Atlantic. Radio telegraphy was publicized in 1910 when it was used to arrest a murderer. Dr Crippen, after murdering his wife, fled by liner to America. He did not realize that messages could be sent by wireless to the ship and he was very surprised when he was arrested. Broadcasting as we know it today, however, would have been impossible without the invention of the thermionic valve by Sir Ambrose Fleming.

The 1914–18 war hastened the development of radio, for it was used by ships and aeroplanes and by artillery. But it was not till after the war that a wireless set became a common sight in people's houses and regular sound broadcasts began. The first wireless sets would seem very strange to you: you listened either through headphones or to a loudspeaker with a large horn.

James Logie Baird, a Scot from Helensburgh, was the pioneer of television. In the 1930s he perfected an apparatus for sending both pictures and sound by radio. It was much improved by American scientists, and now we use the American system of television. After he had discovered how to send black and white pictures Baird went on experimenting to find out how to send pictures in colour. This is now possible but the apparatus is so expensive that it is as yet little used.

(21)

14-6. The Role of Mathematics in Scientific Experiment.

Although the experiment using the lever does not use a great deal of mathematics, it does suggest how mathematics is used in scientific activities. You saw how mathematics was used in measuring, counting, and comparing quantities. You noted how observations of data were recorded in mathematical terms.

You searched for a pattern by studying the numbers in your recorded data. By reasoning from a set of specific cases you developed a general statement to be applied in all similar situations. In Section 14-3 this kind of reasoning is called inductive reasoning. It leads from a necessarily restricted number of cases to a prediction of a general relationship. This general relationship was stated in mathematical symbols in an equation: $WD = wd$. To establish this general principle, further experimentation was performed.

In addition, you drew a graph of $WD = 120$ and of $WD = 96$ to show how these statements tell the complete story in each case. The graph is another instance of the use of mathematics to interpret and to summarize a collection of facts.

The graph also helped to reveal the general pattern which was discovered.

Many scientific facts were undiscovered for thousands of years until alert scientists carefully set up experiments much as you have done and made discoveries on the basis of observations. Some examples of these are the following:

(a) Until the time of Galileo, people assumed that if a heavy object and a light object were dropped at the same time, the heavy one would fall much faster than the light one. Look up the story of Galileo and his experiment with falling objects and see what he discovered.

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(b) From time immemorial, people watched eclipses of the sun and moon and saw the round shadow of the earth but did not discover that the earth was round. Eratosthenes, in 230 B.C., computed the distance around the world by his observations of the sun in two locations in Egypt, yet seventeen hundred years later when Columbus started on his journey, many people still believed the world was flat. Look up in a history of mathematics book or in an encyclopedia the story of Eratosthenes and this experiment.

(c) People had watched pendulums for many centuries before Galileo did some measuring and calculating and discovered the law which gives the relation between the length of the pendulum and the time of its swing. Look up this experiment in a book on the history of mathematics or of science.

Notice that all these experiments are based on many careful measurements and observations in order to discover the scientific law. Then the law is stated in mathematical terms. A great deal of science depends upon mathematics in just this way.

The examples which we have given here describe older fundamental discoveries all of which used relatively simple mathematics. The scientists of today are using more advanced mathematics, and many of the newer kinds of mathematics, in their scientific experiments.

22

Suppose we now wish to calculate the annual premium for an assurance on a person aged 60 to pay £1000 on death before age 65 and £1000 on survival to that age (this is commonly described as an endowment assurance policy). It is clear from the life table that the major part of the premiums will be required to provide the survival benefit and thus the company will be in a position to earn interest on the premiums. If it is assumed that money earns interest at $i\%$ per annum then the present value of a unit due 1 year hence will be $v = 1/(1 + i)$, 2 years hence $v^2 = 1/(1 + i)^2$, etc. It is then a simple matter to calculate the present value of the expected premiums by combining the probability that the person will be alive to pay the premiums with the appropriate present values, i.e., to discount the expected payments. Similarly, the claims outgo can be discounted. By equating these two expectations the required premium can be found.

The calculations, based on an interest rate $i = 4\%$, are shown in Table 5.4. It is assumed that premiums are payable at the beginning of the year, and that claims are paid at the end of the year. Thus the value of expected premiums of P per annum at the outset of the policy in respect of 100 000 persons is

TABLE 5.4

t	$v^t = 1/(1 + i)^t$	$v^t l_{60+t}$	$v^{t+1} d_{60+t}$
0	1·00000	100 000	1414
1	0·96154	94 740	1487
2	0·92456	89 609	1558
3	0·88900	84 605	1628
4	0·85480	79 723	1696
5	0·82193	74 961	

$$\{l_{60} + vl_{61} + \ldots + v^4 l_{64}\} \times P = 448\,677P$$

and the value of the expected claims

$$\{vd_{60} + v^2 d_{61} + \ldots + v^5 d_{64} + v^5 l_{65}\} \times 1000 = 82\,744 \times 1000.$$

Hence $P = £184{\cdot}42$.

25

The view I shall support is the one I called, in ch. 1, a 'pragmatist'[1] conception of logic; according to which logic is a theory, a theory on a par, except for its extreme generality, with other, 'scientific' theories; and according to which choice of logic, as of other theories, is to be made on the basis of an assessment of the economy, coherence and simplicity of the overall belief set. The very existence of arguments in favour of Deviant logics lends some prima facie plausibility to this view. But, of course, the proponents of such logics could be mistaken about the nature of their own enterprise. (The inventors of non-Euclidean geometries, after all, intended to prove the dependence of the parallel postulate.) More argument is necessary.

The pragmatist conception is radically opposed to 'absolutist' views of logic, according to which logical laws are unalterable, because they have a special status which guarantees their certainty. A proponent of a deviant logic *could* take the view that the principles of *his* logic are certain and unalterable, but it is, significantly, much commoner for absolutists to maintain the unalterable certainty of *classical* logical laws.

(24)

Disney Needed the Mathematicians' Help

A number of such probability questions were answered for Walt Disney before Disneyland was built. When he considered setting up Disneyland, Disney wanted to know how big to build it, where to locate it, what admission to charge, and what special facilities to provide for holidays. He didn't want to take a chance on spending $17,000,000 to build Disneyland without knowing something of the probability of success.

What he really wanted answered was this type of question: If I build a certain type of facility, at this particular location, and charge so much admission, then what is the probability that I will make a certain amount of money?

Disney went to the Stanford Research Institute. There he talked with a group of mathematically trained people who are specialists in applying mathematical reasoning to business problems

The people at Stanford first collected statistics about people--(their income, travel habits, amusement preferences, number of children, etc.). Combining this information by mathematical reasoning they predicted the probability that people would come to a certain location and pay a given price of admission. From reasoning like this they could predict the probability of having a successful Disneyland of a certain type at a given spot. Knowing the chance of success under given conditions, Disney was better able to decide how and where to build Disneyland and how much to charge for admission.

This example is typical of the way probability is often used to give an estimate of the degree of uncertainty of an event or the chance of success of a proposed course of action.

WHAT parts of mathematics are useful?

First, the bulk of school mathematics, arithmetic, elementary algebra, elementary Euclidean geometry, elementary differential and integral calculus. We must except a certain amount of what is taught to 'specialists', such as projective geometry. In applied mathematics, the elements of mechanics (electricity, as taught in schools, must be classified as physics).

Next, a fair proportion of university mathematics is also useful, that part of it which is really a development of school mathematics with a more finished technique, and a certain amount of the more physical subjects such as electricity and hydromechanics. We must also remember that a reserve of knowledge is always an advantage, and that the most practical of mathematicians may be seriously handicapped if his knowledge is the bare minimum which is essential to him; and for this reason we must add a little under every heading. But our general conclusion must be that such mathematics is useful as is wanted by a superior engineer or a moderate physicist; and that is roughly the same thing as to say, such mathematics as has no particular aesthetic merit. Euclidean geometry, for example, is useful in so far as it is dull—we do not want the axiomatics of parallels, or the theory of proportion, or the construction of the regular pentagon.

One rather curious conclusion emerges, that pure mathematics is on the whole distinctly more useful than applied. A pure mathematician seems to have the advantage on the practical as well as on the aesthetic side. For what is useful above all is *technique*, and mathematical technique is taught mainly through pure mathematics.

I hope that I need not say that I am not

OVER

trying to decry mathematical physics, a splendid subject with tremendous problems where the finest imaginations have run riot. But is not the position of an ordinary applied mathematician in some ways a little pathetic? If he wants to be useful, he must work in a humdrum way, and he cannot give full play to his fancy even when he wishes to rise to the heights. 'Imaginary' universes are so much more beautiful than this stupidly constructed 'real' one; and most of the finest products of an applied mathematician's fancy must be rejected, as soon as they have been created, for the brutal but sufficient reason that they do not fit the facts.

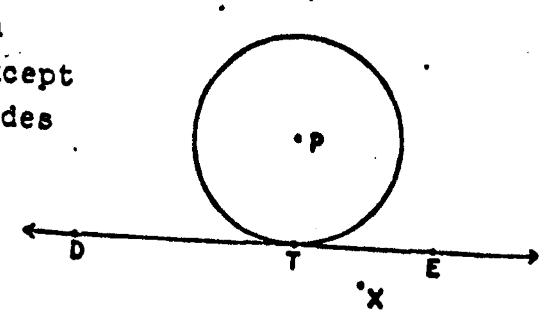
The general conclusion, surely, stands out plainly enough. If useful knowledge is, as we agreed provisionally to say, knowledge which is likely, now or in the comparatively near future, to contribute to the material comfort of mankind, so that mere intellectual satisfaction is irrelevant, then the great bulk of higher mathematics is useless. Modern geometry and algebra, the theory of numbers, the theory of aggregates and functions, relativity, quantum mechanics—no one of them stands the test much better than another, and there is no real mathematician whose life can be justified on this ground. If this be the test, then Abel, Riemann, and Poincaré wasted their lives; their contribution to human comfort was negligible, and the world would have been as happy a place without them.

26

The metric system is a simplified system of weights and measures developed in 1789 by a group of French mathematicians. They decided that, since their system of numeration was a decimal (base 10) system it would be a good idea to have a decimal basis for a system of measures. In such a system the units of length would be some power of ten times a basic unit of length. Then it would be easy to convert from one unit to another. It would only require multiplying or dividing by a power of 10. We shall see that this makes it very much simpler to work with quantities expressed in metric units.

27

The line and the circle in the figure on the right remind us of a train wheel resting on a track, except that the flange (or lip) which guides the train is not shown. How many points are on the circle and also on the line? There is only one such point, the one labeled T.



We say that the line is <u>tangent</u> to the circle. The single point of their intersection is the <u>point of tangency</u>. In this drawing, T is the point of tangency. Another way of describing a point of tangency is to say that it is the only point of the circle which is also on the line. ~~Now answer the following questions.~~

28

10.10 SAVINGS

A wise budget ensures that expenditure is less than income, that is it provides for "saving". Savings may be intended for specific purposes such as holidays, a car, a TV set, . . . Savings may also be intended to form a reserve of money for use in emergencies such as illness or loss of employment, or to provide a reasonable income in old age. In this section we look at some of the ways in which savings may be accumulated.

(a) **Hoarding Cash.** Money saved may simply be stored in some place thought to be secure, but this method has many disadvantages. The money may be stolen, destroyed by fire or some other disaster or it may simply be lost. Furthermore, cash hoarded in this way is "idle". It does not grow by the addition of interest and generally decreases in value with the passage of time. This decrease in the value of money can be seen in the tendency of wages and prices to increase.

(b) **Personal Savings Accounts.** Trustee Savings Banks look after money deposited in them and pay the depositor interest at a stated rate. Money deposited in these accounts may be withdrawn at any time without prior notice and the operation of such accounts is very simple.

(c) **Deposit Accounts.** Banks and Building Societies may accept deposits under the condition that repayment can only be made after an agreed period of notice. The rate of interest paid on deposit accounts is generally higher than that given on the ordinary savings accounts and is generally just below the Bank Rate which is controlled by the government and published by the Bank of England.

7.10 HOLIDAYS ABROAD

More and more people are going abroad for holidays and educational tours. Such holidays may be arranged as follows:

(1) By consulting Travel Agents. They are specialists in arranging holidays. They are paid commission by the hoteliers and transport services with whom they do business. The Agents make arrangements according to your wishes. Such arrangements include accommodation (where you are going to stay) and travel (route and form of travel). Alternatively an Agent may propose a Package Holiday.

(2) Package Holidays. In this case the Agent has himself arranged holidays in units or packages and you select a unit to suit your choice. Such a package holiday is usually cheaper than one specially arranged by the Agents to your own wishes as in (1) above.

(3) Cruises. Shipping companies arrange cruises which allow a visit to more than one country. The company usually arranges excursions ashore to places of interest for sight-seeing and shopping. A cruise is similar to a package holiday in that the journey is decided beforehand.

7.11 PAYMENT FOR HOLIDAYS

It is important to know before you leave what travel or living expenses have been paid through the travel agents and what expenses you still have to meet. In the case of a package holiday or cruise you will already have paid most of the charges before you leave. You will require to take with you only "spending" money. In the case of a holiday arranged in accordance with your own wishes you will require to take with you, in addition to spending money, enough money to cover expenses not already paid through the Agent.

7.12 SOME ADDITIONAL ITEMS

Before visiting another country you require a valid passport. You can obtain this either from the Passport Office or the Department of Social Services. You must also know how much money you can take with you. There may be restrictions on the amount of money you can take out of the country.

A knowledge of the country being visited, its climate, customs, language, and so on would be useful.

(30)

1-4. Kinds of Mathematics

Mathematicians reason about all sorts of puzzling questions and problems. When they solve a problem they usually create a little more mathematics to add to the ever-increasing stockpile of mathematical knowledge. The new mathematics can be used with the old to solve even more difficult problems. This process has been going on for centuries and the total accumulation of mathematics is far greater than most people can imagine. Arithmetic is one kind of mathematics. The trigonometry, algebra, and plane geometry you will study are other kinds.

Today there are more than eighty different kinds of mathematics. No single mathematician can hope to master more than a small bit of it. Indeed the study of any one of these eighty different branches would occupy a mathematical genius throughout his entire life. So don't be surprised if your teacher sometimes fails to know all the answers!

Moreover, hundreds of pages of new mathematics are being created every day of the year -- much more than one person could possibly read in the same day. In fact, in the past 50 years, more mathematics has been discovered than in all the preceding thousands of years of man's existence.

31

Proving a theorem in mathematics is rather like getting a game of patience to 'come out' or checkmating your opponent in chess. But why should we want to prove theorems? What is the purpose of the activity of proving theorems? There are two different answers to these questions.

(1) Because in browsing round the immense number of possibilities in mathematics we stumble across surprising facts. The ancient Babylonians stumbled across the fact that if one takes a triangle with sides of 3, 4, and 5 units, the largest angle turns out to be a right angle. What a coincidence! But perhaps it was not a coincidence after all; perhaps triangles with sides of 4, 5, 6 and 5, 6, 7 and 6, 7, 8 units etc. are right-angled too.

The motive for looking at this kind of thing is curiosity. Can we explain *why* a triangle with sides of 3, 4, and 5 units should have a right angle in it? After all, if we take any other set of three consecutive integers (say, 7, 8, 9), we do not find that a triangle with sides of these unit lengths has a right angle in it.

The difference between thinking that $x^2+y^2 = z^2$ *might* be true (for all right-angled triangles) and knowing that it *must* be true is quite significant. The feeling we get when we move from the first stage to the second stage is rather like 'tuning out' noisy interference on a radio or TV broadcast. To achieve this we have to prove the result to our own satisfaction.

On the other hand one can still ask: Is there a purpose in it? This leads to the second answer.

(2) Proving results consists in looking very carefully at possibilities; making sure that one has considered all the cases; checking each step in the argument; and labelling the results one has taken for granted. Now these are just the mental operations needed in handling applicable mathematics too. In applicable mathematics our aim is mainly *looking into possibilities*: the difference being that here the 'possibilities' are things which might happen in the real world.

The kind of question we can tackle in applicable mathematics is:

◎ If a cable were stretched across the River Mersey from Liverpool to Birkenhead (Figure 1.14) how much would it 'sag' in midstream? How much extra cable would be needed to allow for this sag? How high would the towers have to be on each side of the river? How much force would the cable exert on the towers, tending to pull them over? **B 8**

Probably such a cable will never be thrown across the River Mersey in this way. Nevertheless it is a *possibility*, and by using mathematics we can turn a spotlight of attention onto this possibility. We can, in fact, learn quite definite things about it, such as how high the towers would have to be.

Now the operations needed to understand these possibilities are very similar to the operations needed to understand geometrical possibilities. Proving results helps us to become and to stay 'mathematically fit'; it helps us to tune-in to subtle arguments, and to spot loopholes in fallacious reasoning. In the case of Pythagoras' theorem, the result itself, that $x^2+y^2 = z^2$, where x, y, and z are numbers representing the lengths of the sides of a right-angled triangle, is very useful. It is used in finding the energy of a spinning satellite, for example.

Mathematics = Looking into Possibilities

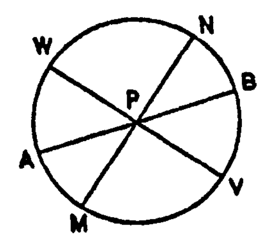
To understand a thing, system or situation is to see its possibilities clearly; to be able to make a good guess at what will happen next; to be aware of what might happen under various conditions.

Mathematics provides us with a sort of microscope bringing into focus the *details* of the predictable aspects of possibilities. This enables us to reach definite conclusions about possibilities.

One advantage of our decimal system is that it has a symbol for zero. Zero is used to fill places which would otherwise be empty and might lead to misunderstanding. In writing the numeral for three hundred seven, we write 307. Without a symbol for zero we might find it necessary to write 3-7. The meaning of 3-7 or 3 7 might be confused.

We have seen that the word "radius" can be used in two different ways. By way of review, a radius of a circle is one of the segments joining a point of the circle and the center. The length of one of these segments is the radius of the circle.

The word diameter is closely associated with the word radius. A diameter of a circle is a line segment which contains the center of the circle and whose endpoints lie on the circle. For the circle represented by the figure at the right, three diameters are shown; $\overline{AB}$, $\overline{MN}$, and $\overline{VW}$. (A diameter of a circle is the longest line segment that can be drawn in the interior of a circle such that its end-points are on the circle.) How many radii are shown in the figure?



A set of points which is a diameter may be described in another way. A diameter of a circle is the union of two different radii which are segments of the same line. How does the length of a diameter compare with the length of a radius?

The length of any diameter of a circle is also spoken of as the diameter of the circle. The diameter is a distance, and the radius is a distance.

The measure of the diameter is how many times the measure of the radius? If we choose any unit of length, and if we let r and d be the measures of the radius and the diameter of a circle respectively, then we have this important relationship:

$$d = 2\ r.$$

What replacement for the question mark makes the following number sentence a true statement?

$$r = ?\ d.$$

Before World War II almost all mathematicians were employed a teachers in schools and colleges. Since then, the world of mathematics and the world of mathematicians have changed tremendously. Today there are more teachers of mathematics than ever before. In junior and senior high school there are about 50,000 people who teach mathematics. There are about 5,000 more teachers employed in colleges and universities. But now (1960), in busine industry, and government there are more than 20,000 persons worki as mathematicians.

The Federal Government hires mathematicians in numerous agencies for many different assignments. Literally thousands of people work with computers and computer mathematics. Industries of all types are hiring mathematicians to solve complex mathematical problems, to help other workers with mathematical difficulties and even to teach mathematics to other employees.

These changes have been brought about by the revolutionary advances in science and technology which we talked about. These changes are still continuing. By the time you are ready for a jo opportunities for a career in mathematics will be even more numerous and varied.

53

Example 2. If the rateable value of a house is £60 and the local rate is £1.20, what are the rates payable by the occupier?

Rates = Rate × the measure of the rateable value
= £1·20 × the measure of the rateable value
= £1·20 × 60 = £72·00

Example 3. If the rateable value of a cinema is £956 and the local rate is £1·25, what are the rates payable by the occupier?

Rates = Rate × the measure of the rateable value
= £1·25 × 956
= £1 195·00

Example 4. If the rates payable on a house are £72 and the local rate is £1·25 what is the rateable value of the house?

Rates = Rate × the measure of the rateable value
£72 = £1·25 × *
where * is the measure of the rateable value.

$$72 = 1{\cdot}25 \times *$$

$$\frac{1}{1{\cdot}25} \times 72 = \frac{1}{1{\cdot}25} \times 1{\cdot}25 \times * \quad \left(\frac{1}{1{\cdot}25} \text{ is the multiplicative inverse of } 1{\cdot}25.\right)$$

$$\frac{72}{1{\cdot}25} = *$$

so, $* = \frac{72}{1{\cdot}25} = 57{\cdot}60$ **The rateable value of the house is £57·60.**

36

It is not hard to see that all the mathematical models we have considered in this book, from the chain curves of Chapter 1 to the patterns of diffusion of dye in the River Rhine, are open to ~~similar~~ improvements. The versions of the models which we have considered have been, in every case, merely 'first approximations' to the truth. But this does not mean that it would be possible finally to obtain models which were *perfectly* accurate, if one's mathematics were sufficiently advanced. The contrary certainly seems to be the case. It seems to be true that we can never get a 'final' version of a mathematical model; there is always room for improvement.

37

This clever idea of place value makes the decimal system the most convenient system in the world.

38

We have seen that it is possible to use a numeration system other than our base ten system. Although we have used five as a base, we could have used any other number. A very important question arises then: "Is there a numeration base better than base ten?" To answer this question, we must consider ways in which a different base could improve upon our present numeration method.

First, it would be practical to select a numeration base having more even divisors than 10 has, for this would simplify work with common fractions. A number like 12 might be a good base, for the even divisors, or factors, of 12 are 2, 3, 4, and 6. The only divisors of 10 are 2 and 5 (besides 10 itself and 1).

It would also be convenient to have a base that is related to some of our common units of measure. Many of our units of measure are based on 12 or multiples of 12; for example, 12 months in a year, 12 hours on the clock face, 60 minutes in an hour, 360 degrees in a circle, 12 eggs in a dozen, and 144 units in a gross.

(37)

Some people think that all—or most—industries should belong to the State. The State should provide the money needed and should run the business as efficiently as possible. Then no money would have to be paid to shareholders or on unnecessary advertising—for in the 'nationalized' industry there would be no shareholders and no competitors. This is the policy of the Socialist Party. After it came to power in 1945 it nationalized the coal industry, road transport, the railways, and the gas and electricity services.

On the other hand, the Conservatives and the Liberals disagree with these arguments. They point out that the amount of money paid out to the shareholders in an industry is very tiny compared with the wages and salaries bill and with the amount of money which is 'ploughed back' into the industry to buy more and newer machinery. They feel, too, that competition is a good thing. When you have a competitor, you are always trying to produce things better and cheaper than he can—which is all to the advantage of the customer or 'consumer'. The huge sums spent in advertising result in very large sales and where you are producing large quantities of an article the cost of producing each one is less and so the selling price can be less. The result is, say the Conservatives and Liberals, that the customer pays less. They are afraid that the big nationalized industries which do not have any competitors will bring an increase in running costs—and, therefore, higher prices. They are also afraid that we will see in Britain government by officials with more power than the ordinary person would like them to have..

Perhaps the answer lies between what the Socialists say on one side and the Conservatives on the other. It may be good for the country to have some of the largest industries nationalized and the others left 'free' if they are being run efficiently. Certainly, short of a revolution, it would be difficult to nationalize everything—and we in this country do not like revolutions.

23

1-7. Mathematics in Other Vocations

Many people who are not primarily mathematicians need to know a lot of mathematics, and use it almost every day. This has long been true of engineers and physicists. Now they find it necessary to use even more advanced mathematics. Every new project in aircraft, in space travel, or in electronics demands greater skills from the engineers, scientists, and technicians.

Mathematics is now being widely used and required in fields such as social studies, medical science, psychology, geology, and business administration. Mathematical reasoning and many kinds of mathematics are useful in all these fields. Much of the use of electronic computers in business and industry is carried on by people who must learn more about mathematics and computing in order to carry on their regular jobs. To work in many such jobs you are required to know a lot about mathematics. Merely to understand these phases of modern life, and to appreciate them enough to be a good citizen, you will need to know about mathematics.

41

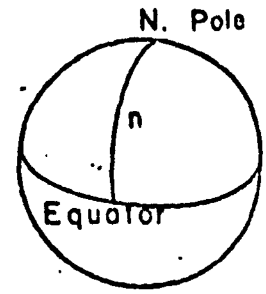
Euclid's theorem tells us that we have a good supply of material for the construction of a coherent arithmetic of the integers. Pythagoras's theorem and its extensions tell us that, when we have constructed this arithmetic, it will not prove sufficient for our needs, since there will be many magnitudes which obtrude themselves upon our attention and which it will be unable to measure; the diagonal of the square is merely the most obvious example. The profound importance of this discovery was recognized at once by the Greek mathematicians. They had begun by assuming (in accordance, I suppose, with the 'natural' dictates of 'common sense') that all magnitudes of the same kind are commensurable, that any two lengths, for example, are multiples of some common unit, and they had constructed a theory of proportion based on this assumption. Pythagoras's discovery exposed the unsoundness of this foundation, and led to the construction of the much more profound theory of Eudoxus which is set out in the fifth book of the *Elements*, and which is regarded by many modern mathematicians as the finest achievement of Greek mathematics. This theory is astonishingly modern in spirit, and may be regarded as the beginning of the modern theory of irrational number, which has revolutionized mathematical analysis and had much influence on recent philosophy.

About two hundred years ago, Georges Buffon, a French naturalist, suggested that a base twelve numeration system be universally adopted. Although the base twelve system is sometimes called the "dozen system," we usually call it the *duodecimal system*. *Duodecimal* is another word for twelve, just as *decimal* is another word for ten.

The fight for base twelve was carried into this century, and duodecimal societies sprang up all over the world. Some mathematicians have urged the adoption of the duodecimal system which would replace the present decimal notation.

symbols and form

Metric Units of Length



$$1 \text{ Meter} = \frac{n}{10,000,000}$$

The French mathematicians began by calculating the distance n from the North Pole to the equator on the meridian through Paris. For the basic unit of length they took $\frac{1}{10,000,000}$ of this distance. By defining the unit in this way the original distance could be measured again if the standard bar of unit length were ever lost.

They named this new standard of length the <u>meter</u> and a standard meter bar was carefully preserved to assure uniformity in future meter units. This definition of the meter was used until October 15, 1960, when a new standard of the meter was agreed upon by delegates from 32 nations. This defines the meter in terms of the orange-red wave-lengths of krypton gas. Precisely, one meter is now defined as:

1 meter = 1,650,763.73 orange-red wave lengths in a vacuum of an atom of the gas krypton 86.

This new definition has the advantage that the unit is easily measured on an interferometer anywhere in the world. Also, it allows an accuracy of one part in one hundred million in linear measurements. Using the old standard bar of platinum-iridium an accuracy of one part in one million was the best attainable.

These modelling approaches have been subjected to severe criticisms. First, as with all computable models, there is a temptation to ignore data and to build on a series of 'what if' questions. It has been remarked that although most of the 22 relationships are plausible, not a single one has been tested empirically. The population model is criticized on the grounds that although it might be appropriate for animal populations, it is generally rejected by demographers and economists for human populations living above subsistence level. Also, contrary to Forrester's assertion that population control of itself will not solve all our problems there are population policies, other than those he studied, which will. A slow decline in population by itself will relieve all the growth pains in world dynamics. There are several similar criticisms of other aspects of the model, and the reader is referred to Nordham's trenchant critique.

However, the most elegant criticism is of a mathematical nature. In such simulations and in the models the flow of time is arbitrary. Suppose therefore that we start from the ultimate steady states adduced by Forrester for the year 2000 and reverse the runs. Will we now return to anything like the conditions of today's starting point? It has been shown by Curnow and Cole that backtracking as a technique can determine four sources of error in dynamic controlling, viz. errors introduced by integration procedures, errors introduced by the imprecision of finite digital computing, transient disturbances and inconsistent starting conditions. But the main criticism of these global models does not rest on back-tracking alone. If the flow diagrams are examined in detail it can be seen that the feedback loops all concentrate on the assumption of the finiteness of the availability of natural resources. The models neglect the possibility of the renewability of natural resources. Even a minor change of the basic assumptions of the global model will defer for centuries a predicted collapse of the economic and material system.

45

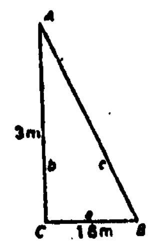
Mathematics began with commonsense and only gradually, over many centuries, crystallized into an 'exact science'. Now it has become clear that it is not so much a 'science' as an 'activity' in which we operate with symbols and diagrams in accordance with rules which we impose ourselves. In other words, moving from step to step in a piece of mathematics is very like making moves in a board game. (The type of moves will change from game to game; compare, for instance, nim, chess, go, ludo.) The basic difference in mathematics is that there are *immense numbers* of possible moves.

45

6.15 Using Pythagoras's Theorem

Example 1. In a triangle ABC, $\angle ACB = 90^\circ$, $AC = 3$ m and $BC = 1{\cdot}6$ m. Calculate the length of AB.

$$a = 1{\cdot}6$$
$$b = 3$$
$$c = ?$$



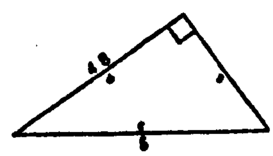
Since $\angle ACB$ is 90°, $c^2 = a^2+b^2$
$= (1{\cdot}6)^2+3^2$
$= 2{\cdot}56+9$
$= 11{\cdot}56$

So, $c = \sqrt{11{\cdot}56} = 3{\cdot}4$

The length of AC is 3·4 m.

Example 2. In a right-angled triangle the hypotenuse is 5 cm and one of the other sides is 4·8 cm. Calculate the length of the third side.

$$b = 4{\cdot}8$$
$$c = 5$$
$$a = ?$$



Since the triangle is right-angled,
$a^2 = c^2 - b^2$
$= 5^2 - 4{\cdot}8^2 = 25 - 23{\cdot}04$
$= 1{\cdot}96$

so, $a = \sqrt{1{\cdot}96} = 1{\cdot}4$

The length of the third side is 1·4 cm.

47

Question: How is a ratepayer notified of the rates he has to pay?

Answer: A rates demand notice showing the amount to be paid is sent by post. This demand notice states the amount of the rates and sets out the method by which they can be paid:

(1) By a single payment,

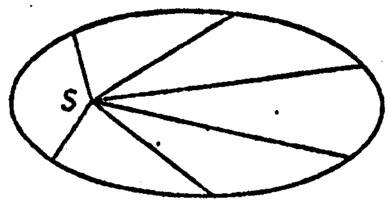
or (2) By a number of instalments (not more than four) as fixed by the Council.

or (3) Houses only, by monthly instalments by special arrangement with the Council.

I hardly suppose that, up to this point, any reader is likely to find trouble with my language, but now I am near to more difficult ground. For me, and I suppose for most mathematicians, there is another reality, which I will call 'mathematical reality'; and there is no sort of agreement about the nature of mathematical reality among either mathematicians or philosophers. Some hold that it is 'mental' and that in some sense we construct it, others that it is outside and independent of us. A man who could give a convincing account of mathematical reality would have solved very many of the most difficult problems of metaphysics. If he could include physical reality in his account, he would have solved them all.

I should not wish to argue any of these questions here even if I were competent to do so, but I will state my own position dogmatically in order to avoid minor misapprehensions. I believe that mathematical reality lies outside us, that our function is to discover or *observe* it, and that the theorems which we prove, and which we describe grandiloquently as our 'creations', are simply our notes of our observations. This view has been held, in one form or another, by many philosophers of high reputation from Plato onwards, and I shall use the language which is natural to a man who holds it. A reader who does not like the philosophy can alter the language: it will make very little difference to my conclusions.

The main spur to the invention of the differential calculus came from the work in astronomy of Kepler (1571–1630), another fascinating intellect. Kepler, adopting the Copernican system, had discovered that each planet moves in an ellipse with the sun at a focus, S, and in such a way that the line joining it to the sun sweeps out equal areas in equal times. It is clear intuitively from the diagram that this implies *varying* speeds, because the planet must travel faster when nearer the sun.



No one had explained any of Kepler's three laws, which involve the idea of *variation* as part of the general problem of *motion*. Newton always relied on his physical intuition to give him an insight into a problem, and it was from this point of view of motion that he began work on variation and variable speed in particular. He was the first 'applied' mathematician, and so successful was he that his law of gravitation remained unchallenged until Einstein's work early this century. To discover the law (or rather, to *prove* it: like Archimedes, he began with his suspicions) he needed the calculus, and so he invented it. Gravitation led to traditional applied mathematics, while the calculus led an outstanding French school to 'analysis' and traditional 'pure' mathematics.

Standard Unit for Angles

Just as there are standard units for measuring a line segment (inch, foot, yard, millimeter, centimeter, meter) so are there standard units for measuring an angle. The one we shall use is determined by a set of one hundred eighty-one rays drawn from the same point. These rays determine 180 congruent angles which, together with their interiors, make a half-plane and the line which determines the half-plane. The rays are numbered in order from 0 to 180, forming a scale. To each ray corresponds a number; that is, there is a number for each ray, and a ray for each number from 0 to 180. Not all 181 rays are shown in the sketch below, but the ray corresponding to 0 and every tenth ray thereafter is drawn. One of these 180 congruent angles is selected as the standard unit. The measurement of this angle is called a degree. The measure of this unit angle, in degrees, is 1.

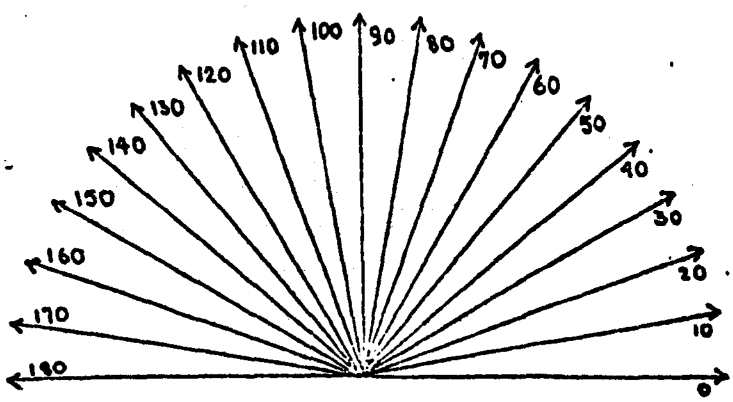


Figure 7-6a

You can use a scale like this to measure an angle. Place the angle on the scale with one side of the angle on the ray that corresponds to zero and the other side on a ray that corresponds to a number less than 180. The vertex of the angle is placed at the intersection of the rays. Then the number which corresponds to that ray is the measure of the angle, in degrees. The size or measurement of the angle is that number of degrees.

The symbol for "degree" is "$^{\circ}$". Thirty-five degrees may be written "35°".

Many attempts have been made to get the United States to adopt the metric system for general use. Thomas Jefferson in the Continental Congress worked for a decimal system of money and measures but succeeded only in securing a decimal system of coinage. When John Quincy Adams was Secretary of State, he foresaw world metric standards in his 1821 "Report on Weights and Measures." In 1866, Congress authorized the use of the metric system, making it legal for those who wished to use it. Finally, in 1893, by act of Congress, the meter was made the standard of length in the United States. The yard and the pound are now officially defined in terms of the metric units, the meter and the kilogram.

A sudden change from our common units (yards, feet, inches, ounces, pounds) to metric units would undoubtedly cause confusion for a time. However, many people think that we will gradually change over to the metric system. Our scientists already use the metric system and people in most foreign countries use it also.

52

Newton was the last of the magicians, the last great mind which looked out on the visible and intellectual world with the same eyes as those who began to build our intellectual inheritance ten thousand years ago. (J. M. KEYNES, *Essays in Persuasion*)

THIS VIEW of Newton suggests a very different figure from the wise, high-domed, silver-haired (it turned grey at thirty) Newton of the portraits, a bulwark of the Age of Reason. But these were painted in the latter phase of his life, when he was Master of the Royal Mint and running it with great administrative skill. Before 1696, in his rooms by the Great Gate at Trinity, the Newton of the calculus and the theory of gravitation was a very different person, 'of the most fearful, cautious, and suspicious temper that I ever knew' according to his successor in the Lucasian chair of mathematics at Cambridge.

During this first phase of his life, when he read the riddle of the heavens, he tried continuously to read other riddles; of health, of immortality, even of the Godhead. His alchemy experiments went on, his assistant wrote, 'about six weeks at spring and six at the fall when the fire in the elaboratory scarcely went out'. After this period came what we should now call a mental breakdown, coinciding with the death of his mother, and he never afterwards quite recovered the incisive edge of his mind. His friends induced him to leave Cambridge for London and his career as an administrator began. He took with him a great trunk packed with records of his alchemical experiments and other mystical speculations. He died with his secret heresy undisclosed; for when in the last century the lid of the trunk was lifted, it appeared that he had abandoned belief in the Trinity since his early twenties.

It is often very helpful to be able to express rational numbers as decimals. When it is necessary to compare two rationals that are very close together, converting to decimal form makes the comparison easier. The decimal form is particularly helpful if there are several rational numbers to be arranged in order. For example, consider the fractions $\frac{13}{25}$, $\frac{27}{50}$, $\frac{3}{8}$, and $\frac{9}{20}$ and their corresponding decimals 0.52, 0.54, 0.375, and 0.45. It is much easier to order the numbers when they are written in decimal form.

2.12 Multiplication of Binomials and Trinomials

Question: What can you write for $(a+b)c$, using the Distributive Law?
Answer: $(a+b)c = ac+bc$.

Question: If you replace c by $(x+y)$, so that you get $(a+b)(x+y)$, what does the answer become?
Answer: $(a+b)(x+y) = a(x+y)+b(x+y)$

Remember: $(a+b).\ c = a.\ c + b.\ c$

Question: In the same way, what can you write for $(3p+2)(2p+5)$?
Answer: $(3p+2)(2p+5) = 3p(2p+5)+2(2p+5)$

Remember: $(a+b)(x+y) = a(x+y)+b(x+y)$

so, $(3p+2)(2p+5) = 3p(2p+5)+2(2p+5)$
$= 6p^2+15p+4p+10 = 6p^2+19p+10$

Question: What can you write for $(3a+2b)(2a+3c)$?
Answer: $(3a+2b)(2a+3c)$
$= 3a(2a+3c)+2b(2a+3c)$
$= 6a^2+9ac+4ab+6bc$

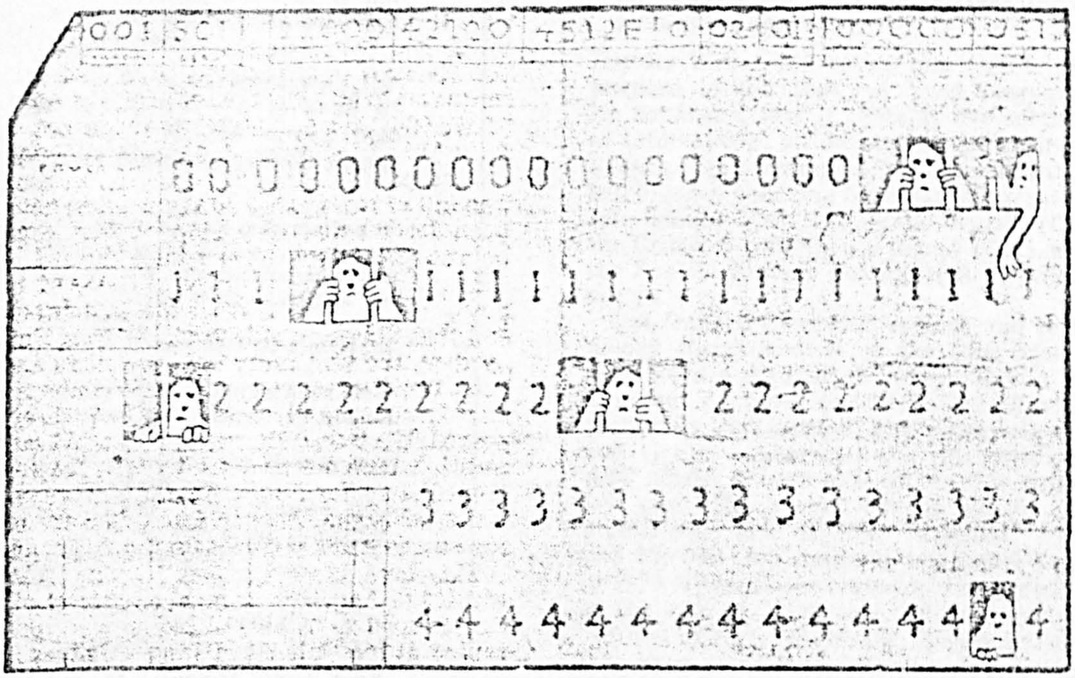
Question: In the same way, what can you write for $(3p+2)(4p^2+p+1)$?
Answer: $(3p+2)(4p^2+p+1)$
$= 3p(4p^2+p+1)+2(4p^2+p+1)$
$= 12p^3+3p^2+3p+8p^2+2p+2$
$= 12p^3+11p^2+5p+2$

Note. When you have expressed the product $(3p+2)(4p^2+p+1)$ as a sum of terms $12p^3-11p^2+5p+2$, you have expanded the product, and your answer is called the expansion of the product.

1	SMA	21	CR	41	CR
2	CR	22	SMA	42	CR
3	NMA	23	MR	43	NMA
4	SMA	24	CR	44	NMA
5	NMA	25	CR	45	MR
6	SMA	26	NMA	46	SMA
7	NMA	27	SMA	47	RES
8	NMA	28	RES	48	MR
9	MR	29	RES	49	CR
10	RES	30	CR	50	SMA
11	MR	31	CR	51	CR
12	CR	32	NMA	52	CR
13	NMA	33	SMA	53	NMA
14	NMA	34	CR	54	SMA
15	SMA	35	SMA		
16	RES	36	MR		
17	MR	37	NMA		
18	CR	38	NMA		
19	SMA	39	RES		
20	RES	40	CR		

Nick Ross

Your life in their files



Necessity is the plea for every infringement of human freedom. It is the argument of tyrants; it is the creed of slaves.

William Pitt, 1788.

The British police are world leaders in the use of computers and already rely heavily on them for day-to-day detective work and crime prevention. Most provincial forces now have their own main-frame computers, or, at least, the use of local authority ones, and, in addition, they are linked by VDUs (visual display units, or computer terminals) to the big Police National Computer (or PNC) in North London. Not surprisingly, because of its size, power, and its network of almost 800 VDUs throughout the country, the PNC worries those concerned with civil liberties who fear its Big Brother implications.

Three smaller force computers have also caused special forebodings: those of Tayside, Thames Valley and Scotland Yard. In Tayside, a wide variety of court and criminal records are held, together with speculative criminal intelligence data, on a computer shared with other, non-police users.

At Thames Valley, an experimental 'collator project' involves the collection and storage of a vast amount of seemingly random information, a 'substantial proportion' of which, according to the *Police Review*, is 'unchecked bunkum' about citizens fewer than half of whom have criminal convictions. At Scotland Yard, the Metropolitan Police have a computer of enormous capacity and with national ramifications, about which they have been so secretive that a recent Home Office report on data protection—the Lindop Committee—specifically excluded it from a general public reassurance about the use to which police computers are put.

Even so, a great deal of information about the Metropolitan Police computer has now been leaked, and assistant commissioner Jock Wilson confirmed for the first time, on *Man Alive*, that at least some of those leaks are accurate. As for the PNC, the Home Office permitted *Man Alive* cameras to take a very rare glimpse at the hardware and allowed reporter Jenny Conway to extract a number of disclosures about its use. But the Metropolitan Police would not agree to their computer being filmed, and the Home Office, as is so often the case nowadays, declined to provide a spokesman for the studio debate.

The PNC is housed in a heavily guarded, unmarked building in Hendon, and the entrance to the computer rooms is safeguarded by a most impressive-looking and elaborate system of security devices. The idea is to give policemen on the beat anywhere in Britain almost instant access to central, national files. The computer can hold up to 40 million records (equivalent to one for every adult in the country) and handles 160,000 messages a day.

Just what these messages and records are all about is still uncertain. According to statements made in Parliament by Home Office ministers, the PNC is supposed to deal only in facts. Philip Knights, president of the Association of Chief Police Officers, confirmed that this is indeed the case:

PHILIP KNIGHTS: It is not the purpose of the PNC to contain what is known more colloquially as 'intelligence'. No, it is facts that we store on the PNC.

JENNY CONWAY: *We've talked to people with no criminal record who claim that their political affiliations and, for example, mem-*

bership of organisations, have been included in the computer alongside information about their cars. Is that true?

I would say they were 100 per cent inaccurate.

Totally inaccurate?

Totally inaccurate.

And that would be impossible, would it?

Oh, anything's possible, but that is not what the PNC is for and it does not store that sort of information.

But that seems to be at variance with the views of Geoffrey Cole, who is head of operations for the PNC. He agreed that data stored on the computer is factual, but concedes that:

GEOFFREY COLE: Some of the information is recording suspicion by the police ...

JENNY CONWAY: *If the information is about suspicions, it can't be then classified as factual. Do you agree?*

Not entirely, no. It is a factual statement of the suspicion.

It may be factual that the police have their suspicions, but their suspicions may not be grounded.

By the nature of suspicion, it may not always be accurate of course.

There are, therefore, some dangers that information may be stored on the computer about totally innocent people?

That's true.

So the computer does contain suppositional, unverified information. It would be surprising if it did not, since police work has always involved hunches. What is surprising is that the authorities are so coy about it. Indeed, the Lindop Committee suggested in its report that it is this secretiveness about police computers which has been the cause of alarm rather than the computers themselves. Charles Read, a member of the committee, amplified the point:

I have no reason to think that the police are doing anything that I wouldn't want them to do. I think, compared with the police forces in most other countries I've seen in action, they adhere to quite extraordinarily high professional standards. I really don't think there is anything they need to hide, and I think it is a great pity that they pretend that there is nothing to hide.

In return, the police point out that they are worried that independent access to their records might frighten away informants (though this has not happened in Sweden where a data protection authority already exists); and that, in any case, computer records are generally more secure than manual ones, since not everyone will have access to a VDU, fewer still will know the appropriate codes, and some computers make a note of the time and origin of information requests.

However, the standard response from the most heavily criticised force, the Metropolitan Police, embodies something of a contradiction. On the one hand, according to Jock Wilson, whose job includes responsibility for their computer, 'we have nothing to hide whatsoever,' and they have co-operated 'in full' with every official request for information, including the provision of 19 pages of written evidence and two hours of oral evidence to the Lindop Committee. On the other hand, when challenged by Charles Read with the fact that most of the most important questions have not been answered, Mr Wilson responded with this:

If you want a police service to do a reasonably good job for you then you must trust the police to do that job with the information they have at their disposal . . . I've been keeping secrets in the Metropolitan Police for 33 years and I don't think [independent scrutiny] is necessary.

But Mr Wilson did reveal one secret: the purpose of the Scotland Yard computer. He confirmed that it contains five sections of information, all devoted to police intelligence. They relate to the Central Drugs Intelligence Unit, the National Immigration Intelligence Unit, the Fraud Squad (C6), the Criminal Intelligence Squad (C11), and the Special Branch. Beyond that he would not go.

But Duncan Campbell, science and technology correspondent of the *New Statesman*, went a great deal farther. It was Mr Campbell's interest in national intelligence gathering that earned him prosecution—and virtual exoneration—in the notorious ABC official secrets trial at the Old Bailey last year. The Special Branch section, he maintained, held 850,000 'nominal' files, and 300,000 'big dossiers' on people back in 1974. These files were said to be growing at the rate of 2,000 new names a month: 'In two or three years, the Special Branch will have something like one and a half million personal files on record.'

What is still not known publicly are the criteria used by the Metropolitan Police for regarding any individual as a criminal suspect, a criminal associate, a political risk or a prospective terrorist. Nor is it clear by which criteria information about people and vehicles is stored on the Police National Computer at Hendon. According to Geoffrey Cole, who runs it:

The information about people consists of an index of all those wanted or who have been reported as being missing from home, and the names of all the people who have a criminal record, together with their fingerprints. The vehicle information consists of an index of all the owners of vehicles in the country [supplied by the Drivers and Vehicles Licensing Centre at Swansea] and also vehicles stolen and suspected of being involved in crimes.

But Duncan Campbell says there are sinister implications. For example, of the 120,000 records contained in the PNC's Suspect and Stolen Vehicles file in 1976, only 30,000 related to stolen vehicles. Three times as many were therefore simply 'suspect', and, according to Duncan Campbell, most of them were, for undisclosed reasons, 'of long-term interest' to the police. These, he pointed out, are therefore:

not vehicles which are stolen, nor even necessarily vehicles which are suspected of use in a crime, which most people would think a perfectly legitimate category for information. These are vehicles which the various police forces simply want kept under surveillance and, as usual, a large proportion of surveillance applications are put on by the national intelligence squads, particularly the Special Branch, who have the largest set of files of political affiliations and so on anywhere in the country.

Once again, Chief Constable Philip Knights is emphatic in his denial:

JENNY CONWAY: *Would it contain people who were under surveillance?*

PHILIP KNIGHTS: No, it wouldn't.

Would it contain information about people who hadn't actually committed any crime, but informants have suggested that they may be about to commit crimes?

That is not the purpose of the PNC.

The net result of all these claims and counter-claims is that the public must draw its own conclusions. No doubt those who habitually think of the police as Good Guys will continue to find no cause for alarm, while groups like the National Council for Civil Liberties will continue to believe that their fears are fully justified.

The police themselves are clearly irked by the suggestion that they are not to be trusted, and loathe the idea of what Jock Wilson calls 'yet another super-body to say, "We're in charge, we'll run the place".' Nor do they understand the objections to unfettered Home Office and police use of their own computers. As West Midlands chief constable, Philip Knights, put it: 'There is nothing in my cupboard that I mind anyone looking at. The only people, it seems to me, who are worried are those who have something to hide. I haven't, and they can computerise my records 50 times a day if they wish.'

In truth, *Man Alive's* researcher, Jill Marshall, and producer, Paul Hamann, toiled almost in vain for over six weeks to find any firm evidence that innocent people had suffered as a direct result of computer data. They found one man who angrily complained that he had been misidentified (though on private, not police, records) as a member of the Communist party, but he declined to appear on the programme because 'television is reformist and I want to see the total overthrow of society'. There was, though, one case which lent weight to the suspicion that political affiliations are sometimes tagged to, for example, supposedly innocuous information about vehicles in the PNC. A member of the NCCL, Roland Jeffery, was waiting for a routine check on the ownership of his motorbike after he had been involved in a minor accident:

> I was waiting in the entrance lobby of the police station, and from there you can't see the computer terminal, but you can hear conversations fairly well. After a brief pause, somebody said, 'Yeah, this seems OK'; then one of the others said, 'You want to watch out here. We've got one of those civil liberties types.'

It is an interesting, perhaps even a disturbing story, but it did not result in Mr Jeffery suffering in any material way. If anything, knowledge of this NCCL affiliation seems to have prompted the police to 'watch out here'. At any rate, they returned his driving licence and promptly bade him on his way.

Nonetheless, it is arguable that the mere holding of this sort of information by the police constitutes an invasion of privacy. Certainly that is the view of Lord Gardiner, the former lord chancellor:

> If you had on one computer everybody's financial position according to information from the Inland Revenue, their health record, including the fact that they had gonorrhoea when they were young, their school and university record, their data credit position and so forth, nobody would have a private life left.

Rory Johnston, a journalist with *Computer Weekly*, does not agree. In his view, much of the disquiet about computers is based on ignorance of their uses and their limitations:

> Records are simply an extension of people's minds. If you say, 'You can think what you like but as soon as you write it down I'm going to start controlling it' that is illogical and doesn't make consistent law. You can't control telephone numbers written down on the back of cigarette packets, and so you'd have to make some arbitrary decisions as to which pieces of paper or which computer records you're going to control. In any case, if the police have got unfounded suspicions about me on their computer, it doesn't affect me because they can't prosecute me without evidence. The whole idea of our legal system is that the police can suspect me until they're blue in the face. The court is impartial; the court will judge whether or not their suspicions are founded.

It is a logic that does not impress Patricia Hewitt of the NCCL. In her view, information about people's politics, sexual predilections, or other details of their private lives, can influence policemen in several potentially damaging ways. The police can (maliciously or with the best intentions in the world) invoke a number of non-judicial 'punishments', such as detaining, tailing or harassing people. She cites as evidence the 30,000 people who have not been charged with any offence, but have been detained, interrogated or expelled under the 'so-called anti-terrorist laws'; or the fact that the police can press or withhold charges or, given any ambiguity in the circumstances surrounding an arrest, can bring charges of greater or lesser severity, —of suspicion under the Vagrancy Acts, or of assaulting a police officer, perhaps, rather than of resisting arrest:

> We're obviously not opposed to the police having information; we're not opposed to the police using computers—they're often the most efficient way. What we're saying is that there needs to be public debate and publicly accountable controls over the kind of information that is kept and how it is used.

Whether or not such controls will be introduced remains to be seen. Those who want them most and who feel most vulnerable without them tend to be of the political left and find it hard to generate widespread public support. It is undoubtedly true that those citizens who are politically orthodox, monogamous, heterosexual and law-abiding have little personal cause for alarm, at least in the short term. Moreover, successive British governments have resisted the introduction of privacy laws for many years (largely at the insistence of the Home Office, according to Lord Gardiner). If precedence is anything to go by, the Lindop Report could well suffer the fate of the Younger Report on privacy which preceded it. Like so much else, it might simply be filed.

Nick Ross reported for 'Man Alive' (BBC2).