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A COMPARISON OF APPROACHES TO STEPWISE REGRESSION ON VARIABLES SENSITIVITIES IN BUILDING SIMULATION AND ANALYSIS

Mengchao Wang[1], Jonathan Wright[1], Alexander Brownlee[2], Richard Buswell[1]

[1]School of Civil &Building Engineering, Loughborough University, Loughborough, LE11 3TU, UK.
[2]Computing Science &Mathematics, University of Stirling, Stirling, FK9 4LA, UK.

Highlights:

- The robustness of stepwise regression is irrespective of selection approach.
- The linear regression model constructed by AIC has a high risk of overfitting, especially when the sample size is small.
- A mixture of discretized-continuous and categorical variables can be used for global SA.
- For stepwise regression, increasing sample size can identify more sensitive variables, but the importance of highly sensitive variables remains the same.
- The importance of variables for design objectives and constraints is better to be classified through more than one sensitivity indexes.

Abstract

Developing sensitivity analysis (SA) that reliably and consistently identify sensitive variables can improve building performance design. In global SA, a linear regression model is normally applied to sampled-based solutions by stepwise manners, and the relative importance of variables is examined by sensitivity indexes. However, the robustness of stepwise regression is related to the choice of procedure options, and therefore influence the indication of variables' sensitivities. This paper investigates the extent to which the procedure options of a stepwise regression for design objectives or constraints can affect variables global sensitivities, determined by three sensitivity indexes. Given that SA and optimization are often conducted in parallel, desiring for a combined method, the paper also investigates SA using both randomly generated samples and the biased solutions obtained from an optimization run. Main contribution is that, for each design objective or constraint, it is better to conclude the categories of variables importance, rather than ordering their sensitivities by a particular index. Importantly, the overall stepwise approach (with the use of bidirectional elimination, BIC, rank transformation and 100 sample size) is robust for global SA: the most important variables are always ranked on the top irrespective of the procedure options.

Keywords: Global sensitivity analysis, Stepwise regression, Sensitivity indexes, Standardized (rank) regression coefficients.

1. Introduction

Reducing the energy consumption in the building sector have a critical role in meeting energy and emission reduction targets in both developing and developed countries [15, 21]. In order to improve building performance by implementing the optimal design solutions at a reasonable investment, both sensitivity analysis and model-based optimization can be used to inform design decisions. Compared to model-based optimization that is applied to find the combinations of variables values that optimize the objectives while satisfying the design constraints, Monte Carlo sensitivity analysis (SA) can support decision making by providing insight into the input variables that most influence design objectives, such as operational energy use or comfort metrics of the final building [16, 24]. This makes SA a useful tool for designers. Consequently, developing SA methodologies that reliably and consistently identify sensitive variables is an important area of research.

SA can be generally grouped into local and global forms [23]. Global SA based on a linear regression is normally adopted to evaluate the relative importance of input variables [2, 3, 12]. When many input variables are involved, stepwise regression provides an alternative. Saltelli et al. [23] state that, when the input variables are ideally uncorrelated, the importance of variables sorted by any sensitivity indexes should be the same. This applies whether sorting by: order of addition to the regression model;

size of the R^2 changes attributable to individual variables; absolute values of variables' standardized (rank) regression coefficients (SRCs/SRRCs); absolute values of correlations coefficients (CCs); or absolute values of partial correlation coefficients (PCCs). Therefore, in previous researches, the global sensitivities of variables are normally evaluated by a particular index, e.g. De Wilde et al. [7] using SRRCs to determine the major contributors for heating energy use; Hopfe and Hensen [16] using both SRRCs and the change of R^2 to explore variables influence on building performance simulation.

However, according to our previous researches [27, 28], the ordering of variables' importance with respect to design objectives and constraints could be switched by applying the same global SA method to different sets of random samples. Furthermore, the robustness and effectiveness of a stepwise regression depends on the choice of procedure options, as each option has its advantages and weakness [26]. For instance, the F-test is often used as default criterion to stop the stepwise regression process, but it has been shown to perform poorly relative to other criteria, e.g. the corrected AIC [17]. It has also been shown that it is possible to conduct global SA in parallel with optimization by an evolutionary algorithm [25]. This further supports decision making by providing suggested optimal solutions as well as variables' sensitivities. Solutions generated during the optimization run can be reused for the global SA to save computational efforts, despite the bias in the samples arising from operation of the evolutionary algorithm.

Therefore, this paper explores the impact of procedure options in a global SA for design objectives or constraints, providing insights that will enable more robust and more accurate assessments of variables' sensitivities to be made, through the relative magnitudes of variables sensitivity indexes. The procedure options explored are:

different approaches to obtaining samples (i.e. randomly generated samples or the biased solutions obtained at the start of an optimization process, with a sample size of 100 or 1000); data forms of input variables (i.e. raw data with categorical variables, or rank-transformed data); selection approaches (i.e. the results in this paper are based on bidirectional elimination); and selection criteria (i.e. F-test, AIC or BIC).

2. Stepwise Regression Methodology

Stepwise regression analysis is usually used as an alternative of linear regression to do global SA. Since, when no impacted or correlated variables are included in the same regression model, it can avoid misleading regression of variables importance. It can also avoid overfitting of the data, as all of input variables are arbitrarily forced into the same regression model [23]. Overfitting occurs when the regression model in essence 'chases' the individual observations rather than following an overall pattern in the data, which can produce a spurious model, giving poor predictions of variables importance [23]. Thus, the overfitting is used as an important standard, to evaluate how well the linear model constructed by stepwise regression can fit to the data (generated from different samples).

Therefore, the global SA adopted here is based on a linear regression model in the stepwise manner, which is performed by R statistic software [20]. The idea is to add or remove variables in a linear model: at each iteration, selecting the variable which most increases the R^2 coefficient of the model. The robustness of stepwise regression analysis is dependent on the choice of procedure options, including the sampling method, sample size, data form of input variables, selection approach and selection criterion, with the accuracy being evaluated through the R^2 (coefficient of determination) and PRESS (predicted error sum of squares) values in the linear regression model.

In a stepwise regression analysis, the relative importance of the variables for a given output can be evaluated through sensitivity indexes, including variables' entry-order to the model, SRCs (standardized regression coefficients)/SRRCs (standardized rank regression coefficients, for rank-transformed data), and R^2 change attributable to the individual variables. The more important (sensitive) the variable is, the earlier it is selected into the linear model, the larger its SRC/SRRC is, the greater it is attributable to R^2 change [23].

2.1 Samples and sample size

The robustness of a sensitivity method is related to the choice of sample size and the manner in which the samples are generated [23]. For a sample size of 100 and above, the difference in the results from different sampling methods is decreased; thus, it is feasible to use a random sampling method and 100 samples in a Monte Carlo analysis for typical building simulation applications [19, 22]. In this paper, the conclusion is further validated by comparing the SA resulting from a 100 random samples with those from a 1000 random samples; the 100 random samples are taken as being the first 100 samples of the 1000 randomly generated samples. The repeatability of the approach is investigated by repeating the analysis for two sets of random samples (Random Sample A and Random Sample B). Moreover, the first 100 solutions (being consistent with the smaller sample size in random samples) obtained from a multi-objective optimization process (based on NSGA-II) have also been used to do global SA, for design objectives and constraints (See Section 3.1). The aim is to explore the extent to which the biased samples can affect the robustness of variables global sensitivities, determined by the same method based on stepwise regression.

2.2 Input variables and rank-transformation

The input variables considered in most sensitivity analyses are real-valued quantities [14, 16]. However, in this paper, the categorical variables for construction types are applied with others having physical representations (see Section 4.1). Such variables frequently appear in building design problems, so it is important to consider them. Furthermore, a non-linear relationship between the input variables and the output is possible, whether the input variables have real-valued quantities or not. A rank transformation of the variables based on a monotonic relationship can mitigate the problems associated with fitting linear models to nonlinear data [23]. The rank transformation is defined according to Spearman's rank correlation coefficients: raw data are replaced by their corresponding ranks, and then the ranks of input variables and outputs are used to do regression analysis. Particularly, the smallest rank *1* is assigned to the smallest value of each variable, and then the rank *2* is assigned to the next larger value, and so on until the largest rank *m* assigned to the largest value (i.e. *m* indicates the number of observations for each variable).

Thus, two alternative representations of the input variables are considered here:

- The input variables in their raw form.
- A rank-transformation of the variables (and outputs).

2.3 Selection approach

There are three model-selection approaches [6] as below. Due to identical results in this case study, the results from bidirectional elimination are only discussed here:

- Forward selection: which starts from an 'empty' model with no input variable but an intercept, and then adds the variable most improving the model one-at-a-time until no more added variables can significantly improve the model. This approach is based on a pre-defined selection criterion.

- Backward elimination: which starts from a 'full' model with all predictive input variables and an intercept, and then deletes the variable least improving the model one-at-a-time until no more deleted variables can significantly improve the model. This approach is based on a pre-selected selection criterion.
- Bidirectional elimination: which is essentially a forward selection procedure but with the possibility of deleting a selected variable at each stage, as in the backward elimination. This approach is commonly applied for stepwise regression, particularly when there are correlations between variables.

2.4 Selection criterion

The selection criterion is used to stop the construction of the stepwise regression. It is also used to determine when an already-selected variable should be deleted from the linear regression model. The commonly applied selection criteria that are adopted in this research are:

- F-test: which is often used as a default criterion for stepwise regression. To avoid overfitting the model, Saltelli et al. [23] suggest using the α-value of 0.01 or 0.02 for a F-test, rather than the conventional choice of 0.05.
- AIC (Akaike information criterion): which is based on a penalty of maximum log likelihood (the maximum likelihood is defined as a general technique to estimate the parameters and draw statistical inferences in various situations, especially in non-standard ones), to balance the linearity of the model with the model size [13].
- BIC (Bayesian information criterion): which is an alternative method to AIC. In comparison to AIC, it penalizes larger model models more heavily, aiming to avoid the overfitting problem.

This research focuses on comparing the robustness of stepwise regression driven by AIC and BIC. The linear regression models selected by F-test (with different α-values of 0.01, 0.02 or 0.05) are primarily used to examine the influences of the selection criterion on the overfitting problem.

2.5 Model accuracy

The adequacy of a linear regression model's fit can be assessed through R^2 (coefficient of determination) and PRESS (predicted error sum of squares) [23], which are both considered in this paper:

- R^2 (coefficient of determination): is a simple way to test the linear fitness of a model, having a value between zero and unity: the larger the R^2 value is, the better the model fits the data. In this paper, a fairly strong linear regression model is valid for the value of $R^2 > 0.7$.
- PRESS (predicted error sum of squares): the value of which can be used to check the model 'overfitting' to the data. According to Satelli et al. (2008), overfitting could occur when the regression model involving more variables, in essence it 'chases' the individual observations rather than following an overall pattern in the data, which can produce a spurious model, giving poor predictions of variables importance. In particular, when a variable has been added into the model, resulting in an increased PRESS value, the model is overfitted. Therefore, the linear regression model with the lowest PRESS value is preferred when choosing between two competing regression models.

3. Experimental Approach

3.1 Multi-objective optimization algorithm

Evolutionary algorithms (EAs) have been shown to perform well for many building optimization problems [11]. Their origins are based on Darwinian principles: survival of the fittest population of solution, passing of characteristics from parents to offspring, and eliminating of the poorest solution during each generation. Building design problems are inherently multi-objective, where the designer seeks to find an optimal trade-off between two or more conflicting design criteria. Consequently, this work adopts the well-known non-dominated sorting genetic algorithm II (NSGA-II) [8], which is used widely to solve bi- and multi-objective building optimization problems [4, 11]. The algorithm is an EA which seeks to approximate the Pareto front of solutions that represents the trade-off in the optimization objectives. The specific implementation of NSGA-II is:

- Gray-coded bit-string encoding of the problem variables (163 bits);
- uniform crossover (100% probability of chromosome crossover with 50% probability of gene crossover);
- single bit mutation (a probability of 1 bit per chromosome);
- passive archive of solutions, allowing a Pareto front to be constructed from all solutions visited during the search rather than only those in the final population;
- population size of 20 with the search stopped after 5000 unique simulations.

These parameters were chosen empirically.

3.2 Example Building and HVAC Systems

The example building is based on a mid-floor of a commercial office building with 5 zones located in Birmingham, England (Figures 1 and 2). The size of two end zones and three middle zones are 24m x 8m and 30m x 8m separately, with floor to ceiling height of 2.7m. Each zone has typical design conditions of 1 occupant per 10m^2 floor area and equipment loads of 11.5 W/m^2 floor area. Maximum lighting loads are set at 11.5 W/m^2 floor area, with the lighting output controlled to provide an illuminance of 500 lux at two reference points located in each of the perimeter zones. Infiltration is set at 0.1 air change per hour and ventilation rates at 8 l/s per person. The heating and cooling is modelled by an idealised system that can provide sufficient energy to offset the zone loads and meet the zone temperature setpoint during hours of operation (from 9am to 5pm all year around). The internal zone is treated as a passive unconditioned space. The operational energy use, building comfort, and the capital costs related to equipment size, are determined by EnergyPlus [9], with the weather data based on the CIBSE reference year [5].

4 Input Variables, Objective Functions and Design Constraints

4.1 Input variables characteristics

Table 1 gives 16 **input variables**, and specifies their bounds, discrete increment, and the total number of unique value that each can take, according to model characteristics, previous experiments and handbooks [1, 10]. Those input variables associated with perimeter zones are related to building geometry (i.e. *orientation* and *window-wall ratios*), the properties of constructions (i.e. *window/wall/ceiling-floor types*), and operating conditions (i.e. *heating and cooling setpoint* (via the *dead band*), system operating hours (per day) in 'winter' months (November to April) and 'summer' months (May to October) (via *winter/summer start and stop time*)).

The longest façades of the building face the 'true north/south', when the variable of *orientation* is set at 0^o. For the example building, the valid value range of *orientation* is -90^o to 90^o. In order to avoid an overlap of the *heating and cooling setpoint*, *dead band* is used instead of the *cooling setpoint*, even the heating and cooling have the potential to be run all year around [10]. The *window-wall* ratio refers to the window area of 6 equally sized windows placed in three groups against the wall area in each façade (see Figures 1 and 2): its value range for this case-study building is between 0.2 and 0.9. The *façade-1/2/3/4 window-wall ratio* reflects its position in each perimeter zone, corresponding to Zone N/S/E/W. Both heating and cooling are available all year around, although the operating hours (between start and stop time) are different for the 'winter' months (November to April) and 'summer' months (May to October) [10]. Three construction types are available for *external wall and ceiling-floor constructions*: heavy weight, medium weight and lightweight. Similarly, there are two *internal wall types* (heavy weight and light weight), and two double-glazed *windows types* (plain glass and low-emissivity (Low-E) glass). For categorical construction variables, the heavy weight construction corresponds to a value of 0, with the construction weight decreasing with increasing variable value; for *window type*, the normal plain and Low-E glasses are corresponding to the values of 0 and 1 separately [1]. Since, based on previous researches [25], the construction variables having physical representation or having not barely affect the performance of stepwise regression (i.e. the identification of variables importance to output), and in a real case, the combination of variables is normally mixed with different data types (continued, discrete or categorical variables). In this paper, no prior knowledge of variable sensitivity is assumed. Thus, in a set of randomly generated samples, the frequency distributions of variables are uniform and the correlations between any pairs of variables are less than 0.1.

4.2 Design objectives and constraints

The **design objectives**, to be minimised by a multi-objective optimization process, are the building annual energy demand (as determined by EnergyPlus (V7) for heating, cooling and artificial lighting), and the capital costs (using a model derived from cost estimating data in London, UK [18]):

- **Energy demand:** the annual energy demand of the heating, cooling and artificial lighting. As the HVAC system of the example building is an ideal load air system, it is not like the real case where the cooling energy could be reduced to 'free', due to the free cooling ventilation. The 'energy consumption' is more correctly defined as 'energy demand' in this paper.
- **Capital costs:** is known to be a linear function of most of the variables, although the cost of the HVAC system is a function of the peak heating and cooling capacity, the capacity being a non-linear function of some variables.

The **design constraints** are that the thermal comfort in each perimeter zone should not exceed 20% of predicted percentage dissatisfied (PPD), for no more than 150 working hours per annum. The constraint functions are configured to return the number of hour above 150, or zero if the constraint is feasible. The **solution infeasibility** combines the separate constraint violations into a single metric. It is taken as the sum of the squares of each constraint violation (with an entirely feasible solution having an infeasibility of zero). Thus, the infeasibility only has a non-zero value when one or more constraints are violated (this forming a discontinuity in the function space, with the infeasibility of the infeasible solutions changing with the variable values, but all feasible solutions having a constant infeasibility of zero). For randomly generated solutions, most of the solutions are infeasible.

5. Results and Analysis

5.1 The impact of selection approach

In this paper, for each set of samples (random and slightly biased samples from optimization), the linear regression models driven by different selection approaches are identical, represented in terms of the number of variables involved in the model (i.e. the size of the model), and the R^2 and PRESS values. Therefore, the selection approach (forward, backward, or bidirectional) has no impact on the identification and determination of variables sensitivities (represented by sensitivity indexes of variables entry-orders, SRCs/SRRCs and the change of R^2), for design objectives and constraints, although variables could be moderately correlated (i.e. the maximum correlation coefficients between variables are approximate 0.55, when the first 100 optimization solutions are taken from the NSGA-II search). As the linear regression model constructed by the backward elimination only removes variables with no impact from the full model, it is meaningless to sort variables importance based on their entry-orders or individual contributions to R^2 change. In this paper, the results from stepwise regression with the use of bidirectional elimination are only presented as it is a combination of the two other selection approaches.

5.2 The impact of selection criterion

Figure 3 compares the fitness of the linear regression models for energy demand, capital costs and solution infeasibility, that were constructed by different selection criteria (i.e. AIC, BIC, and F-test with α-level of 0.05, 0.02 or 0.01) using global SA with raw and rank-transformed data from 5 different sets of samples. These were the 100 and 1000 randomly generated solutions from Random Sample A and Random Sample B, and the first 100 biased solutions obtained at the beginning of NSGA-II. In order to protect against overfitting, the regression models with minimum PRESS values are also included in this figure. Columns exceeding that of minimum PRESS are classed as overfitted (containing redundant variables). Thus, for a given set of samples (irrespective of sampling methods and sample size), the number of variables selected into a linear regression model by a stepwise manner depends on the choice of selection criterion. The linear regression model constructed by AIC always contains the largest number of input variables, which increases the risk of overfitting, particularly in the case of a smaller sample size (e.g. a sample size of 100). In contrast, the performance of F-test is relevant to the choice of significance level (α-value): the global SA driven by F-test with α-value of 0.01 or 0.02 is close to that found by BIC, but the F-test with α-value of 0.05 performs close to that of AIC. In this case study, the regression models constructed by BIC, F-test with α-value of 0.01, 0.02 or 0.05, and AIC with larger sample size (i.e. 1000) do not cause overfitting problems in the global SA.

Furthermore, the differences between AIC and BIC on the determination of variables' sensitivity have also been evaluated, and are given in Tables 2 and 3. These show the percentage changes in variables sensitivity indexes; that is, the percentage changes of the sensitivity indexes for variables identified by BIC, due to the application of AIC. The length of the bars indicates the relative magnitude of the percentage change: the larger the bar, the more significant changes due to different selection criteria. It can conclude that the choice of selection criterion barely impacts on variables' entry-orders and contributions to R^2 changes, but it does have some influence on the absolute values of SRCs/SRRCs (particularly for moderately important variables with medium entry-orders), which may be due to the mathematical assumption of SRCs/SRRCs calculation; further research is required. Taking the case of Random Sample B as example, with a sample size 1000 and rank-transformed data for capital costs (Table 3 column 5), most medium-ranked variables have approximately 30% variation in their relative magnitudes of SRRCs due to the choice of different selection criteria. In contrast, the R^2 of the linear model is almost constant.

It can conclude from these results as that, for a given set of samples, while different selection criteria lead to the linear regression models having different numbers of variables (that could, in turn, further affect the representation of variables sensitivities), the selection criterion only has a slight influence onmodel's fitness (the relative magnitude of R^2). This indicates that those newly added-in variables (due to different selection criteria, e.g. AIC) account for only limited uncertainty of the design objectives or constraints. Thus, for computer efficiency and model accuracy (to avoid overfitting problem), it is better to use BIC as selection criterion to stepwise regression analysis.

5.3 The impact of data type of input variables

Based on previous results, to avoid the overfitting problem, the global SA driven by BIC and bidirectional elimination is used here to further explore the influences of procedure options (i.e. data representation and sets of samples) of a stepwise regression, for design objectives and constraints. Tables 4 to 6 state the global sensitivities of variables, based on different sensitivity indexes (i.e. the order of variables entry into the linear regression model, the absolute value of SRCs/SRRCs, and the size of R^2 changes attributable to individual variables), for energy demand (Table 4), capital costs (Table 5) and solution infeasibility (Table 6). In each case, the variables are represented by rank-transformed data or raw data with categorical variables. Each data type of input variables applied to global SA has 5 sets of samples, including 100 and 1000 solutions from Random Sample A and also Random Sample B, as well as the first 100 biased solutions obtained at the beginning of NSGA-II. In those tables, the length of the bar indicates the relative magnitude of variable's importance. The more important (sensitive) the variable, the longer the bars of the SRCs/SRRCs and the size of R^2 change, and correspondingly, the shorter the bar of entry-order.

Firstly, even though the categorical variables have fewer options than others, in the case of applying the global SA to raw data, the categorical variables are not always ranked highest for design objectives and constraints. From this, it can be determined that any error in variables' importance due to the large changes in values for categorical variables is not substantial. Thus, in this paper, the discretised-continuous and categorical variables can be used together, avoiding the overrated problem for categorical variables.

Furthermore, for a given set of samples, whether or not rank transformationis applied to the variables, the global SA for a particular design objective leads to similar linear regression models with very close R^2, with only a few switches in the sensitivity of medium-importance variables. For example, in the case of Random Sample B with a sample size of 1000 for energy demand (Table 4), the entry-order of rank-transformed variables into the linear model is as same as that with raw data (except the last two add-in variables, *orientation* and *façade-4 window-wall ratio*), with a small difference in R^2 of 0.006. However, for the first 100 NSGA-II solutions for capital costs (Table 5), apart from the top-three variables, the remainder barely have the same entry-orders, which may be due to the different characteristics of design objectives: the distributions of the random samples for energy demand and capital costs have patterns; further research is required.

Finally, in the case of applying the global SA to randomly generated samples for solution infeasibility (Table 6), using rank transformation in stepwise regression can mitigate against the problems associated fitting the linear regression model to non-linear data, which is indicated by the significantly increased R^2 (from about 0.6 to above 0.8) and the enlarged number of identified variables. This is because the regression model with rank-transformed data is based on a monotonic relationship rather than a linear relationship. However, for the top two most important variables, *heating setpoint* and *dead band*, the indexes of their global sensitivities to solution infeasibility are slightly affected by the use of rank transformation. This is represented by the similar entry-orders, the relative magnitudes of SRRCs and the contributions to R^2 changes. Note that this is not the case with the first 100 NSGA-II solutions for infeasibility, as the biased solutions at the beginning of NSGA-II have already converged, indicated by the R^2 less than 0.5 (smaller than the acceptance level of 0.7). This is caused by the strong convergence properties of NSGA-II, which rapidly directs the search towards feasible solutions (i.e. those meeting the constraints).

5.4 The impact of samples and sample size

From Tables 4 to 6, it can be seen that, in relation to the choice of sample size from a set of randomly generated samples, enlarging the sample size can bring more variables into the linear regression model. This only slightly changes R^2, indicating that those additional selected-in variables have limited influence (less importance). It does, however, result in some changes to the relative magnitudes of variables' global sensitivities, determined by different sensitivity indexes, for each of design objectives and constraints. For instance, when the size of a set of rank-transformed data from Random Sample B is enlarged from 100 to 1000 for capital costs (Table 5 columns 4 and 5), seven more variables identified with an increased R^2 (from 0.963 to 0.978), the entry-order of *façade-3 window-wall ratio*is raised from No.6 to No.4, its SRRC (0.09) and contribution to R^2 change (0.0067) are increased to 0.14 and 0.0187 respectively, and the rank-order of its importance is reduced as well. This is also the case of applying the same global SA for both energy demand and solution infeasibility.

Furthermore, applying different sets of randomly generated samples but with the same sample size (100 or 1000) and data-form for global SA, results in a slightly different linear regression model, in terms of variables identification, sensitivity indexes, and the linear-fitness (R^2 value) of the model. However, the variables accounting for most of the uncertainty in the output can be identified almost entirely, even with a smaller size of samples. For example, three more variables (ranked as medium important) are identified from Random Sample B with sample size of 100 and the use of rank transformation, compared to those from Random Sample A with the same sample size and data form, for energy demand (Table 4 columns 2 and 4). Moreover, it also confirms that a smaller sample size of 100 can be used to identify the most important variables for design objectives and constraints. The importance of these tests is to illustrate the repeatability of the stepwise regression to rank variable importance for design objectives and constraints.

Finally, in the case of applying the global SA to the first 100 solutions from the NSGA-II run for design objectives (energy demand and capital costs; there are some correlations between variables, about 0.55), the same important variables can be identified. They have a similar magnitude of sensitivity indexes, compared to those obtained from random samples. Thus, it is not necessary to re-generate a separate random samples to do global SA, saving the computer efforts.

6 Categorisation and Behaviour of Variables Importance for Design Objectives and Constraints

According to previous results in this paper, in a given global SA of design objectives or constraints to the changes of variables values, the relative importance (sensitivity) of variables determined by different sensitivity indexes is related, but not identical, where the ordering of importance for each variable normally varies within a small range. Furthermore, the identification of variables importance depends on the choice of procedure options for a stepwise regression, e.g. selection criterion, data form (particularly for the objective having weak linearity), the sampling method and sample size. However, the most important variables are always ranked on the top: for our example building these are *heating setpoint* and *dead band* for energy demand and solution infeasibility, and the *ceiling-floor type* and *window type* for the capital costs.

Therefore, it is better to use more than one sensitivity indexes, to provide robust orderings of variables importance for design objectives and constraints. In addition, re-generating different sets of samples could avoid misleading variables importance, particularly when the sample size is smaller. For example, in this paper, the categories of variables importance for energy demand, capital costs or solutions infeasibility, are determined through both the variables' entry-orders and their relative magnitudes of SRRCs, during a stepwise regression analysis with the use of bidirectional elimination, BIC, rank-transformed data and randomly generated samples with a size of 100 (Table 7). The entry-order of variables is the direct outcome from a stepwise regression, but it is meaningless when variables have equal or very close magnitudes of importance to outputs. In this situation, the absolute value of variables SRCs/SRRCs can provide quality determination about how much the variations in output are related to the variations in each input variable.

In the Table 7, the importance of variables is categorised as below:

- **The most important variables selected earliest into the linear regression model (marked by 'yellow')**: those variables SRRCs are normally above 0.4, their identification and ordering of importance are irrespective of the procedure options of stepwise regression.
- **The most important variables selected in the medium orders with switches (marked by 'green')**: those variables SRRCs are normally between 0.1 and 0.2, most of them can be identified through a smaller sample size (a sample size of 100), but their ordering of importance are switched, due to different sensitivity indexes (variables entry-orders or SRRCs).
- **Less important variables selected latest into the linear regression model (marked by 'white')**: those variables SRRCs are only around 0.05, their

identification and ordering of importance are strongly dependent upon the procedure options of stepwise regression, i.e. different sets of samples, sample size (e.g. a larger sample size of 1000) and selection criterion (e.g. AIC).

- **No impact variables (marked by 'blue')**: those variables are never selected into the linear regression models for outputs (their SRRCs are always zero), irrespective of the procedure options of stepwise regression.

Thus, for the example building and its performance model, *heating setpoint* and *dead band* are identified as the top two most important variables for both energy demand and solution infeasibility; meanwhile, *ceiling-floor type* is the dominant variable for the capital costs, followed by the variables related to *window-wall ratios* and other *construction types*, which are consistent with the correlation coefficients of variables. Moreover, *orientation*, and *façade-4 window-wall ratio* and *internal wall type* are considered as no impact variables for capital costs and solution infeasibility separately. This makes sense: *orientation* of the building has no impact on the simple linear model used for calculating capital costs; in this building, solution infeasibility, as design constraint, is strongly related to the performance of design objectives, especially to energy demand, thus, less important variables *façade-4 window-wall ratio* and *internal wall type* account for limited impacts on solution infeasibility. The categories of variables importance for design objectives and constraints from the random samples could be further used as benchmark to explore variables convergence characteristics during an optimization process.

7 Conclusions

This paper has explored several factors that impact upon the performance of global SA. In particular, it is concerned with finding methodologies that produce consistent results, to give confidence in the determined variable sensitivities. The extent to which the procedure options of a stepwise regression can affect the identification of variables global sensitivities have been examined, determined by three sensitivity indexes (i.e. variables entry-orders, the relative magnitudes of SRCs/SRRCs, and the size of R^2 changes attributable to individual variables), when using the randomly generated samples and the biased solutions obtained at the start of a multi-objective optimization process (based on NSGA-II), to do global SA for design objectives and constraints. Five key conclusions can be drawn.

First of all, in the experiment of applying the global SA to a given set of samples for design objectives or constraints, an identical linear regression model with the same representation of variables global sensitivities has been found through a stepwise regression. This is irrespective of the choice of selection approach, which is due to the weak or moderate correlations between any pair of variables (less than 0.6). Bidirectional elimination is suggested in this paper, as it is the combination of other selection approaches.

Secondly, the linear regression model constructed by AIC has a high risk ofoverfitting due to the inclusion of redundant variables, especially in the case of a smaller sample size (e.g. 100). Furthermore, the performance of the F-test is dependent on the choice of significance level (α-value). Consequently, the stepwise regression constructed by BIC is suggested in this paper to do global SA for design objectives and constraints. It also concludes that, for a given set of samples, the choice of selection criterion barely has impact on the entry-orders of variables and the size of R^2 change attributable to individual variables, but does cause some variations in the absolute values of variables SRCs/SRRCs, particularly for those having moderate importance.

Thirdly, this paper has confirmed that the discretized-continuous and categorical variables can be used together. Furthermore, different data forms of variables (raw data or rank-transformed data) can lead to similar linear regression models with very close R^2, even though some switches in sensitivity occur for the medium-important variables, when applying the same global SA to a given set of samples for design objectives. Moreover, the use of rank transformation in a stepwise regression tends to mitigate against problems associated fitting the linear regression model to nonlinear data, particularly in the case for solution infeasibility, which results in the model with rank-transformed data having a significantly increased R^2 value (beyond the reliability standard of 0.7).

Fourthly, an increased sample size will bring more variables into the linear regression models. A smaller sample size of 100 can lead to a robust model, with the identification of the most important variables (having earlier entry-orders), for either design objectives or constraints. The repeated tests from random sets of solutions further confirm the robustness of the ordering of variables importance for design objectives and constraints. The importance of highly sensitive variables remains the same between different samples, although the orders of less important variables entry into the linear regression model are slightly changed.

Finally, for design objectives or constraints, the ordering of variables' importance based on one of the sensitivity indexes could be different to that based on others, even when the correlation coefficients between variables from randomly generated samples are around 0.1. Therefore, it has concluded that, the (global) importance of variables for design objectives and constraints is better to be classified, through more than one sensitivity indexes. For example, in this paper, the categorisation of variables importance was considered using both variables' entry-orders and SRRCs, for design objectives and constraints. Even though the entry-order of variables is the direct outcome from a particular stepwise regression, it is meaningless when variables have equal or very close magnitudes of importance to outputs.

Although the case-study building in this paper is a simple model with some input variables of an ideal load air system (not covering all of typical design variables of HVAC systems, such as ventilation related variables). The above five conclusions still form a set of suggestions for those deploying global SA as part of the building design process, which will improve the consistency and robustness of the approach, applied to various building-models. It is crucial that designers consider all of the factors described in this paper will have more confidence in the global SA approach. Assuming that this is the case, global SA, alongside optimization, will be a valuable tool for decision making in building design.

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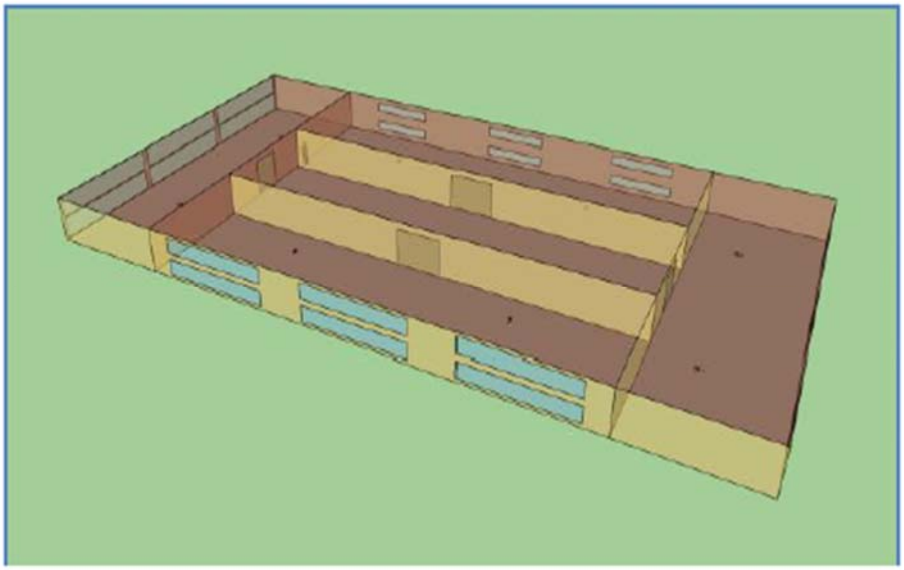

Fig.1. Example building.

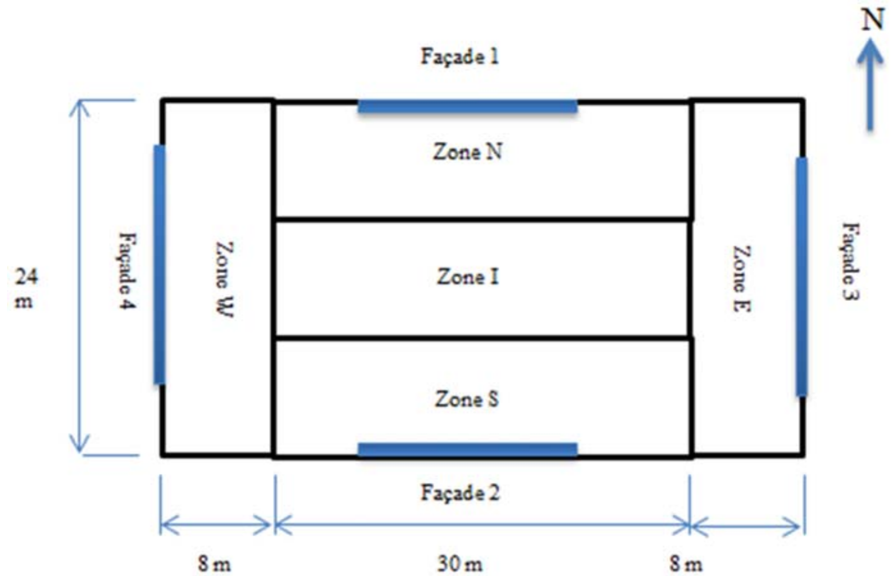

Fig.2. Top view of the five-zone layout of the example building.

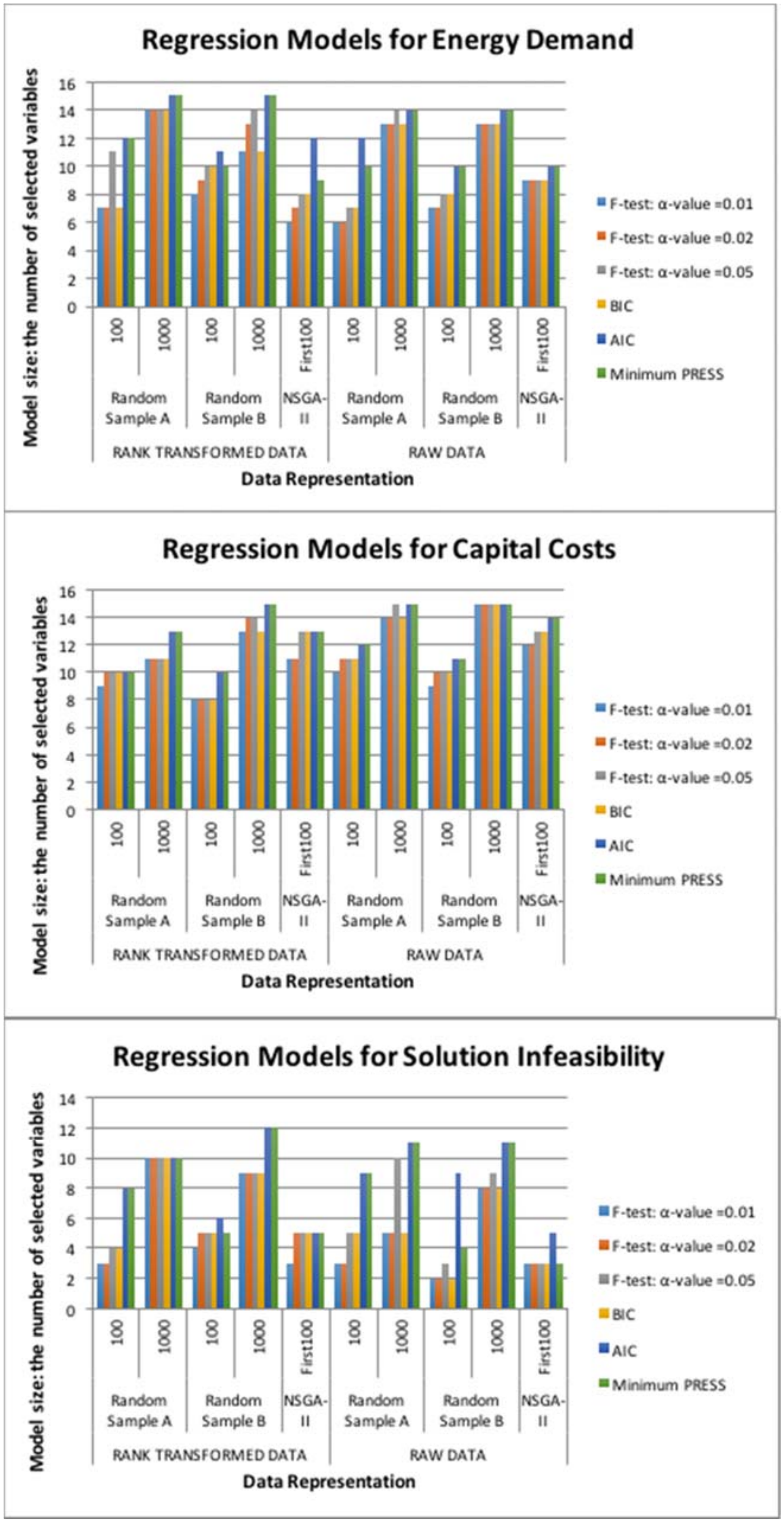

Fig.3. The number of variables selected into the linear regression models by different criteria for design objectives and constraints.

Table 1 Input variables

VARIABLE INDEX	INPUT VARIABLES	UNITS	LOWER BOUND	UPPER BOUND	INCREMENT	THE NUMBER OF VALUE OPTIONS
1	Heating setpoint	(ºC)	18.0	22.0	0.5	9
2	Heating set-back	(ºC)	0.0	8.0	0.5	17
3	Dead band	(ºC)	1.0	5.0	0.5	9
4	Orientation	(º)	-90.0	90.0	5.0	37
5	Façade-1 window-wall ratio	(-)	0.2	0.9	0.1	8
6	Façade-2 window-wall ratio	(-)	0.2	0.9	0.1	8
7	Façade-3 window-wall ratio	(-)	0.2	0.9	0.1	8
8	Façade-4 window-wall ratio	(-)	0.2	0.9	0.1	8
9	Winter start time	(hrs)	1	8	1	8
10	Winter stop time	(hrs)	17	23	1	7
11	Summer start time	(hrs)	1	8	1	8
12	Summer stop time	(hrs)	17	23	1	7
13	External wall type	(-)	0	2	1	3
14	Internal wall type	(-)	0	1	1	2
15	Ceiling-floor type	(-)	0	2	1	3
16	Window type	(-)	0	1	1	2

Table 2 Percentage changes in the sensitivity of energy demand to the variables, due to different selection criteria (AIC and BIC).

Variable Index	Percentage Changes in Variables Entry-Orders									
	Rank Transformed Data					Raw Data				
	RandomSampleA		RandomSampleB		NSGA-II	RandomSampleA		RandomSampleB		NSGA-II
	100	1000	100	1000	First 100	100	1000	100	1000	First 100
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0		0	0
3	0	0	0	0	0	0	0	0	0	0
4		0			0		0		0	0
5		0		0			0		0	0
6		0		0			0		0	
7	0	0	0	0	0	0	0	0	0	0
8		0					0		0	
9		0	0	0			0		0	
10										0
11	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0		0	0	0	0	0
13		0								
14			0					0		
15		0	0	0	0		0	0	0	
16	0	0	0	0	0	0	0	0	0	
Percentage Changes in Variables SRCs/SRRCs										
1	0.065	0.000	0.018	0.047	0.013	0.050	0.000	0.044	0.038	0.004
2	0.150	0.016	0.020	0.140	0.791	0.195	0.000		0.236	0.238
3	0.012	0.002	0.004	0.001	0.053	0.010	0.002	0.051	0.000	0.011
4		0.022			0.121		0.045		0.243	0.037
5		0.019		0.255			0.040		0.523	0.025
6		0.000		0.314			0.000		0.222	
7	0.177	0.000	0.031	0.063	0.171	0.203	0.005	0.041		0.033
8		0.000					0.000		0.610	

9		0.000	0.008	0.539			0.000		0.192	
10										0.123
11	0.149	0.006	0.014	0.277	0.025	0.098	0.005	0.032	0.207	0.077
12	0.093	0.000	0.021	0.402		0.078	0.000	0.111	0.248	0.051
13		0.054								
14			0.033					0.270		
15		0.000	0.041	0.426	0.113		0.000	0.053	0.259	
16	0.128	0.000	0.011	0.093	0.064	0.121	0.005	0.033	0.006	
Percentage Changes in the Size of the R^2 Values Attributable to Individual Variables										
1	1.52E-06	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0		0	0
3	5.62E-07	0	0	0	0	0	0	0.047443	0	0
4		0			0		0		0	0
5		0		0			0		0	0
6		0		0			0		0	
7	0	0	0	0	0	0	0	0	0	0
8		0					0		0	
9		0	0	0			0		0	
10										0
11	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0		0	0	0	0	0
13		0								
14			0					0		
15		0	0	0	0		0	0	0	
16	0	0	0	0	0	0	0	0	0	
Percentage changes in model R^2	0.036	0.001	0.004	0.004	0.005	0.025	0.001	0.015	0.001	0.001

Table 3 Percentage changes in the sensitivity of capital costs to the variables, due to different selection criteria (AIC and BIC).

Variable Index	Percentage Changes in Variables Entry-Orders									
	Rank Transformed Data					Raw Data				
	RandomSampleA		RandomSampleB		NSGA-II	RandomSampleA		RandomSampleB		NSGA-II
	100	1000	100	1000	First 100	100	1000	100	1000	First 100
1	0	0		0	0		0		0	
2				0			0		0	
3	0	0	0	0	0	0	0	0	0	0
4					0					0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0
9				0	0		0	0	0	0
10					0		0		0	0
11		0		0		0	0		0	0
12						0			0	
13	0	0	0	0	0	0	0	0	0	0
14	0	0		0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0
Percentage Changes in Variables SRCs/SRRCs										
1	0.000	0.042		0.684	0.000		0.000		0.652	
2				0.200			0.000		0.133	
3	0.000	0.000	0.000	0.273	0.000	0.077	0.029	0.098	0.056	0.113
4					0.000					0.050
5	0.000	0.006	0.015	0.280	0.000	0.000	0.007	0.006	0.107	0.000
6	0.000	0.007	0.220	0.102	0.000	0.008	0.000	0.008	0.047	0.053
7	0.000	0.007	0.044	0.336	0.000	0.015	0.000	0.015	0.015	0.011
8	0.000	0.011	0.063	0.319	0.000	0.013	0.000	0.000	0.337	0.015
9				0.308	0.000		0.000	0.091	0.900	0.037

10					0.000		0.000		0.667	0.109
11		0.000		0.217		0.095	0.042		0.591	0.125
12						0.063			0.714	
13	0.000	0.000	0.000	0.011	0.000	0.000	0.000	0.013	0.069	0.006
14	0.000	0.000		0.397	0.000	0.000	0.000	0.053	0.035	0.010
15	0.000	0.001	0.001	0.013	0.000	0.002	0.000	0.000	0.022	0.010
16	0.000	0.000	0.021	0.161	0.000	0.005	0.000	0.010	0.005	0.000
Percentage Changes in the Size of the R^2 Values Attributable to Individual Variables										
1	0	0		0			0		0	
2				0	0		0		0	
3	0	0	0	0	0	0	0	0	0	0
4					0					0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0.215451	0	0	0
9				0	0		0	0	0	0
10					0		0		0	0
11		0		0		0	0		0	0
12						0			0	
13	0	0	0	0	0	0	0.177260	0	0	0
14	0	0		0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0
Percentage changes in model R^2	0.000	0.001	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 4 Variables sensitivities for energy demand.

The Entry-Orders of Variables										
Variable Index	**Rank Transformed Data**					**Raw Data**				
	RandomSampleA		**RandomSampleB**		**NSGA-II**	**RandomSampleA**		**RandomSampleB**		**NSGA-II**
	100	**1000**	**100**	**1000**	**First 100**	**100**	**1000**	**100**	**1000**	**First 100**
1	2	2	2	2	2	2	2	2	2	2
2	7	6	10	7	8	7	6		7	8
3	1	1	1	1	1	1	1	1	1	1
4		13			3		13		13	4
5		11		11			12		11	9
6		9		9			9		9	
7	3	5	4	3	4	4	4	4	3	5
8		12					11		12	
9		10	9	10			10		10	
10										7
11	6	4	3	4	5	5	3	3	4	3
12	4	8	7	8		3	7	7	8	6
13		14								
14			8					8		
15		7	5	6	7		8	5	6	
16	5	3	6	5	6	6	5	6	5	
Variables SRCs/SRRCs										
1	0.403	0.438	0.439	0.471	0.537	0.398	0.453	0.428	0.476	0.57
2	0.127	0.128	0.1	0.114	0.067	0.087	0.134		0.123	0.084
3	0.592	0.665	0.682	0.682	0.883	0.609	0.663	0.65	0.68	0.826
4		0.046			0.314		0.044		0.037	0.215
5		0.053		0.051			0.05		0.044	0.079
6		0.117		0.105			0.116		0.117	
7	0.147	0.178	0.223	0.206	0.164	0.138	0.185	0.197	0.205	0.121
8		0.047					0.056		0.041	
9		0.063	0.126	0.076			0.061		0.073	
10										0.106
11	0.134	0.181	0.221	0.173	0.161	0.153	0.191	0.222	0.179	0.156

12	0.205	0.122	0.14	0.102		0.206	0.129	0.162	0.113	0.175
13		0.037								
14			0.091					0.1		
15		0.133	0.195	0.141	0.062		0.125	0.17	0.147	
16	0.187	0.191	0.19	0.172	0.109	0.141	0.183	0.184	0.169	
The Size of the R^2 Values Attributable to Individual Variables										
1	0.1979	0.2023	0.2024	0.2197	0.3158	0.1961	0.2165	0.2082	0.2225	0.3115
2	0.0157	0.0208	0.0081	0.0126	0.0027	0.0073	0.0227		0.0149	0.0066
3	0.5337	0.4308	0.4411	0.4572	0.5009	0.5727	0.4302	0.4703	0.4539	0.5527
4		0.0019			0.0585		0.0019		0.0013	0.0204
5		0.0024		0.0026			0.0024		0.0019	0.0033
6		0.0117		0.0094			0.0114		0.0116	
7	0.0354	0.0342	0.0451	0.0394	0.0416	0.0280	0.0384	0.0362	0.0376	0.0133
8		0.0023					0.0032		0.0015	
9		0.0041	0.0117	0.0056			0.0038		0.0053	
10										0.0084
11	0.0125	0.0349	0.0512	0.0294	0.0181	0.0168	0.0402	0.0441	0.0302	0.0291
12	0.0320	0.0148	0.0198	0.0110		0.0386	0.0151	0.0215	0.0130	0.0126
13		0.0013								
14			0.0142					0.0095		
15		0.0168	0.0285	0.0202	0.0034		0.0164	0.0250	0.0212	
16	0.0278	0.0422	0.0287	0.0302	0.0121	0.0172	0.0369	0.0245	0.0282	
R^2	0.855	0.820	0.851	0.837	0.953	0.877	0.839	0.817	0.843	0.958

Table 5 Variables sensitivities for capital costs.

The Entry-Orders of Variables										
Variable Index	Rank Transformed Data					Raw Data				
	RandomSampleA		RandomSampleB		NSGA-II	RandomSampleA		RandomSampleB		NSGA-II
	100	1000	100	1000	First 100	100	1000	100	1000	First 100
1	10	10		12	12		11		10	
2				11			13		12	
3	9	9	8	9	4	10	9	9	9	11
4					11					9
5	4	3	2	3	6	4	3	2	3	5
6	3	4	5	5	8	3	4	6	4	6
7	5	5	6	4	3	5	5	5	5	3
8	8	7	4	7	5	6	6	4	6	8
9				13	13		12	10	13	10
10					9		14		14	12
11		11		10		11	10		11	13
12						9			15	
13	6	6	7	6	7	7	7	7	7	7
14	7	8		8	10	8	8	8	8	4
15	1	1	1	1	1	1	1	1	1	1
16	2	2	3	2	2	2	2	3	2	2
Variables SRCs/SRRCs										
1	0.037	0.024		0.019	0.005		0.022		0.023	
2				0.02			0.009		0.015	
3	0.042	0.036	0.064	0.044	0.087	0.026	0.035	0.041	0.036	0.053
4					0.059					0.06
5	0.151	0.162	0.2	0.161	0.131	0.137	0.148	0.162	0.149	0.143
6	0.139	0.15	0.1	0.137	0.144	0.124	0.141	0.12	0.128	0.187
7	0.149	0.14	0.09	0.14	0.18	0.133	0.133	0.132	0.13	0.18
8	0.053	0.089	0.128	0.091	0.162	0.078	0.091	0.117	0.086	0.13
9				0.013	0.024		0.011	0.022	0.01	0.027
10					0.075		0.007		0.009	0.046

11		0.02		0.023		0.021	0.024		0.022	0.016
12						0.032			0.007	
13	0.092	0.093	0.094	0.095	0.151	0.064	0.082	0.079	0.087	0.159
14	0.068	0.063		0.063	0.093	0.05	0.058	0.057	0.057	0.104
15	0.884	0.918	0.93	0.915	1	0.919	0.936	0.956	0.935	1.05
16	0.233	0.224	0.187	0.224	0.34	0.198	0.209	0.202	0.202	0.309
The Size of the R^2 Values Attributable to Individual Variables										
1	0.0012	0.0006		0.0003			0.0005		0.0005	
2				0.0004	0.0007		0.0001		0.0002	
3	0.0023	0.0013	0.0040	0.0020	0.0223	0.0007	0.0012	0.0015	0.0013	0.0005
4					0.0015					0.0023
5	0.0231	0.0258	0.0506	0.0236	0.0103	0.0190	0.0216	0.0371	0.0201	0.0111
6	0.0313	0.0243	0.0104	0.0193	0.0072	0.0282	0.0215	0.0113	0.0163	0.0191
7	0.0176	0.0178	0.0067	0.0187	0.0540	0.0147	0.0160	0.0160	0.0165	0.0383
8	0.0030	0.0085	0.0217	0.0088	0.0129	0.0045	0.0070	0.0170	0.0091	0.0118
9				0.0002	0.0006		0.0001	0.0004	0.0001	0.0007
10					0.0025		0.0001		0.0001	0.0009
11		0.0004		0.0005		0.0004	0.0006		0.0005	0.0002
12						0.0014			0.0001	
13	0.0074	0.0087	0.0079	0.0104	0.0078	0.0039	0.0085	0.0064	0.0077	0.0078
14	0.0034	0.0040		0.0042	0.0027	0.0025	0.0035	0.0033	0.0034	0.0197
15	0.8531	0.8365	0.8359	0.8425	0.7254	0.8860	0.8730	0.8690	0.8786	0.7579
16	0.0401	0.0456	0.0261	0.0471	0.1394	0.0327	0.0397	0.0319	0.0387	0.1266
R^2	0.983	0.973	0.963	0.978	0.987	0.994	0.993	0.994	0.993	0.997

Table 6 Variables sensitivities for solution infeasibility

Variable Index	Rank Transformed Data					Raw Data				
	RandomSampleA		RandomSampleB		NSGA-II	RandomSampleA		RandomSampleB		NSGA-II
	100	1000	100	1000	First 100	100	1000	100	1000	First 100
The Entry-Orders of Variables										
1	1	1	1	2		1	2	2	2	
2										
3	2	2	2	1		2	1	1	1	
4					5	4				
5		5	4	6	1					1
6		6		5	4					
7	3	3	3	3	3		4		3	
8										3
9		9							6	
10									8	
11		7	5	7		5	3		4	
12	4	8		8			5		7	
13										
14										
15		10		9						
16		4		4	2	3			5	2
Variables SRCs/SRRCs										
1	0.691	0.629	0.662	0.653		0.563	0.549	0.523	0.553	
2										
3	0.523	0.616	0.534	0.617		0.444	0.55	0.543	0.568	
4					0.378	0.159				
5		0.104	0.141	0.101	0.376					0.42
6		0.09		0.096	0.142					

7	0.165	0.146	0.183	0.151	0.306		0.062		0.095	
8										0.32
9		0.041							0.071	
10									0.054	
11		0.074	0.11	0.087		0.152	0.086		0.085	
12	0.084	0.055		0.044			0.057		0.067	
13										
14										
15		0.041		0.036						
16		0.104		0.105	0.247	0.155			0.074	0.34
The Size of the R^2 Values Attributable to Individual Variables										
1	0.6124	0.3980	0.4784	0.3808		0.3779	0.2972	0.2734	0.2976	
2										
3	0.2330	0.3801	0.2839	0.4062		0.2133	0.3069	0.2872	0.3118	
4					0.0744	0.0205				
5		0.0104	0.0176	0.0097	0.1558					0.0800
6		0.0089		0.0085	0.0437					
7	0.0271	0.0209	0.0397	0.0226	0.0829		0.0039		0.0088	
8										0.1000
9		0.0018							0.0045	
10									0.0029	
11		0.0061	0.0118	0.0072		0.0218	0.0079		0.0070	
12	0.0069	0.0030		0.0019			0.0032		0.0045	
13										
14										
15		0.0016		0.0013						

16		0.01 18		0.01 01	0.090 6	0.03 18			0.00 57	0.090 0
R^2	0.87 9	0.84 0	0.83 2	0.84 8	0.447	0.66 5	0.61 9	0.56 1	0.64 3	0.280

Table 7 Categories of variables importance for design objectives and constraints. In decreasing order of sensitivity, the variables are coloured yellow, green, white, blue.

CATEGORIES OF VARIABLES IMPORTANCE		
ENERGY DEMAND	**CAPITAL COSTS**	**SOLUTION INFEASIBILITY**
Heating setpoint	Heating setpoint	Heating setpoint
Heating set-back	Heating set-back	Heating set-back
Dead band	Dead band	Dead band
Orientation	Orientation	Orientation
Façade-1 window-wall ratio	Façade-1 window-wall ratio	Façade-1 window-wall ratio
Façade-2 window-wall ratio	Façade-2 window-wall ratio	Façade-2 window-wall ratio
Façade-3 window-wall ratio	Façade-3 window-wall ratio	Façade-3 window-wall ratio
Façade-4 window-wall ratio	Façade-4 window-wall ratio	Façade-4 window-wall ratio
Winter start time	Winter start time	Winter start time
Winter stop time	Winter stop time	Winter stop time
Summer start time	Summer start time	Summer start time
Summer stop time	Summer stop time	Summer stop time
External wall type	External wall type	External wall type
Internal wall type	Internal wall type	Internal wall type
Ceiling-floor type	Ceiling-floor type	Ceiling-floor type
Window type	Window type	Window type