Prospective elementary teachers' beliefs about collaborative problem solving and dialogue in mathematics

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Prospective elementary teachers' beliefs about collaborative problem solving and dialogue in mathematics

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This study is concerned with prospective elementary teachers' beliefs about collaborative problem solving and dialogue in mathematics classrooms. Participants (n=16) attended an undergraduate module titled *Problem Solving in Primary Mathematics*, which was specifically designed to provide them with opportunities in collaborative problem solving and dialogic activities. Students were invited to answer an open-ended questionnaire at the beginning and end of the course. In addition, at the end of the semester, students wrote individual reflective reports commenting on their experiences. Qualitative analyses revealed three areas of beliefs for which positive changes were observed: (a) dialogue and its relation to school mathematics, (b) the characteristics of a 'good' interlocutor, and (c) the importance of collaborative problem solving. This paper closes with a discussion concerning the implementations of our findings and suggestions for future research.

Keywords · prospective teachers · beliefs · collaborative problem solving · dialogue

Introduction

The idea of mathematics learners working collaboratively to solve problems has been active for some decades, yet in recent years it has attracted particular research interest. It is, therefore, not surprising that the Programme for International Student Assessment (PISA) has shifted its focus from Analytical Problem Solving in PISA 2003 (that is, domain-specific skills and competences in mathematics, science, and literacy), to Interactive Problem Solving in PISA 2012 (i.e. domain-general and content-free skills and competences, measured with the use of computer-based tests), to Collaborative Problem Solving (CPS) in PISA 2015 (Greiff, Holt, & Funke, 2013). While a number of recent papers examine prospective teachers' beliefs in relation to mathematical problem solving (i.e. Stylianides & Stylianides, 2014; Xenofontos & Andrews, 2012, 2014), little is known about prospective mathematics teachers' beliefs regarding the importance of collaboration in mathematical problem solving (Xenofontos, 2014, 2015) and effective classroom talk (Lee & Kim, 2016).

Mathematics teacher education is often challenged by the questions of how to prepare prospective teachers to teach mathematics in ways different to those they experienced as learners (Chapman, 2012), and how to provide them with opportunities to change their beliefs, in order to accommodate pedagogical approaches that promote a deeper conceptual understanding and problem solving. Teachers' beliefs are often defined as conceptions, personal ideologies, world views and values that shape practice and orient knowledge (Aguirre & Spear, 1999) and have a significant impact on what gets taught, how it gets taught, and what gets learnt in classrooms (Chapman, 2002; Middleton, 1999; Thompson, 1984). The majority of studies in this area follow Ernest's (1989) triadic model for mathematics teachers' beliefs regarding the nature of mathematics, mathematics teaching, and mathematics learning.

Belief change in pre-service teacher education is restrained by many factors and is not always successful (Bauml, 2009; Ng, Nicholas, & Williams, 2010; Potari, 2001). In the relevant literature, various reasons are provided to explain this. For example, trainees bring deep-rooted beliefs about mathematics teaching and learning to teacher education, which are influenced by prior schooling experiences. There is also a lack of opportunity for university instructors to challenge such beliefs. This paper is set to investigate prospective elementary teachers' beliefs before and after a course designed to actively engage them in CPS activities in mathematics. More specifically, our work here is guided by the following research question:

> What are prospective elementary teachers' beliefs about CPS and dialogue before and after a course designed to provide them with such experiences?

Before presenting and discussing our findings, we turn our focus on studies related to CPS and talk in the mathematics classroom, which have been helpful in framing our work.

Literature Review

Collaborative problem solving

CPS is concerned with the engagement of individuals "in a process whereby two or more agents attempt to solve a problem by sharing the understanding and effort required to come to a solution and pooling their knowledge, skills and efforts to reach that solution" (OECD, 2013, p. 7). In CPS activities, team members are confronted with a situation that challenges everyone. They then work hard together to make sense of the problem and to plan, implement and evaluate solution approach(es), as well as collaboratively evaluating the appropriateness of the solution (Kroll, Masingila, & Mau, 1992). Such activities are highly structured (Albert & Kim, 2013; Barron, 2000), meaning that they require a careful coordination of all members' behaviour, knowledge, participation, and understanding. Along these lines, PISA 2015 has developed a specific framework for assessing CPS skills (see OECD, 2013). These skills are perceived as the crossing of four individual problem solving processes (exploring and understanding, representing and formulating, planning and executing, monitoring and reflecting) and three CPS competences (estimating and maintaining shared understanding, taking appropriate action to solve the problem, and estimating and maintaining team organisation).

Much research has focused on the cognitive and/or metacognitive processes and skills observed and developed during CPS. For instance, Artzt and Armour-Thomas (1992) identified eight types of episodes during CPS activities (read, understand, analyze, explore, plan, implement, verify, watch and listen) and connected each episode to a predominant cognitive and/or metacognitive level. Poor metacognitive decisions, due to lack of critical engagement with other group members' thinking, are more likely to lead to unsuccessful problem solving, while successful outcomes are more likely in cases where group members challenge and discard unhelpful ideas and actively endorse useful strategies (Goos, Galbraith, & Renshaw, 2002). More recently, when working with 10-year old pupils, Iiskala et al. (2011) found that difficult and challenging problems trigger significantly more and longer metacognitive episodes than easier tasks. Other studies are concerned with the social conditions under which CPS is successful. From this perspective, Forman (1989) argues that collaborative pupils must have a mutual respect of their peers' viewpoints on the task, while, at the same time, there must be an equal distribution of power and knowledge. In a similar vein, Barron (2000) claims that successful group interaction is characterised by mutuality of exchanges, joint attentional engagement, and alignment of group members' goals. From their analyses of group discussions in CPS, Tatsis and Koleza (2008) identify a number of social (e.g. justification, avoidance of thread) and socio-mathematical norms (e.g. mathematical justification, non-ambiguity, validation), the existence or absence of which affects the problem solving process. Nevertheless, recent research evidence (Langer-Osuna, 2016) suggests that the construction of social and intellectual authority relations during CPS in mathematics is directly associated with how pupils' ideas and behaviors are evaluated by the teacher in the classroom.

The use of CPS in the mathematics classroom challenges the '*tell, show and do*' model of more traditional, teacher-centered instruction (Albert & Kim, 2013). However, many teachers avoid incorporating activities of this kind in their teaching for various reasons, such as seeing themselves as weak problem-solvers or lacking the pedagogical knowledge of communicating mathematics, as well as not being familiar with collaborative grouping as a pedagogical approach (Wadlington, 1993). Because of teachers' reluctance to incorporate CPS, a number of scholars highlight the need for mathematics teachers' professional development and support in how to introduce CPS in class and evaluate its outcomes (Albert & Kim, 2013; Barron, 2000; Kroll et al., 1992). Bridging the gap between pre- and in-service teacher education is important, not least because many prospective teachers who are introduced to CPS during their preparatory programmes return to university from their school placements believing that CPS does not seem to work in action, influenced by the beliefs and practices of their mentors (Xenofontos, 2014). A few studies discuss the use of CPS in the pre-service education of prospective elementary teachers. In such cases, participants typically work in small groups and experience CPS from the learners' perspective. For example, Bjuland (2004) focuses on student-teachers who have limited mathematical backgrounds and their reflections regarding their own work in CPS activities. The students in that study argued that their experiences of getting stuck during the problem solving process enabled them to better understand the frustration pupils experience while working on unfamiliar problems in the classroom. In Xenofontos (2015), prospective teachers who worked collaboratively in solving unusual geometry problems expressed positive attitudes while reflecting upon two issues: (a) their experience of working collaboratively and (b) their experience of working with unusual, challenging geometry problems that did not require advanced mathematical knowledge. For both issues, the participants commented on the absence of similar experiences in their prior schooling. In the study of Shilling-Traina and Stylianides (2013), positive belief changes concerning the nature of mathematics were observed among the participating prospective elementary teachers during the implementation of activities favouring group collaboration. More recently, Zsoldos-Marchis (2015) presented a project aiming at changing prospective teachers' attitudes towards mathematics through CPS of non-routine problems. Her findings indicate that students who worked collaboratively had significantly more positive change regarding how much they liked mathematics, compared to those who worked individually.

Talk in classrooms

In classrooms, language plays a significant role in most kinds of interactions (Joshua, 2008, Nystrand, Wu, Gamoran, Zeiser, & Long, 2003). It constitutes teachers' main pedagogic tool (Mercer & Littleton, 2007) and pupils' means for building their understanding (Zuengler & Cole, 2005). Research has established that the verbalization of mathematical ideas and thinking improves mathematical understanding (Bills & Grey, 2001; Carpenter, Franke, & Levi, 2003; Pirie & Schwarzenberger, 1988; Smith, 2010). Therefore, teachers need to provide learners with opportunities to talk about mathematics during classes, across all levels of education. However, while raising the quality of classroom talk has been, for some decades, a target for many educational systems around the world (Clarke, 2013), there is no consistent evidence to indicate its success or how it can be successful (Kyriakou, 2015).

In our work, we adopt the Vygotskian premise of social constructivism. According to this perspective, knowledge is constructed through social interaction, while higher mental functions are developed through interactions either with adults or more capable peers (Vygotsky, 1978). For Vygotsky, the use of language as externalized thought acts both at the social (intermental) and self-directing (intramental) levels, eventually remaining within the mind as inner speech. The view of language as externalized thought underlines the link between thinking and talking, which both act upon learning (Smith, 2010). Drawing on Vygotsky, Mercer (2000) perceives language used collectively by partners involved in problem-solving activities as *interthinking*, the main purposes being effective communication and collective development of ideas. Therefore, improving understanding through managing classroom talk can provide more insight into thinking in the classroom.

In this spirit, Alexander (2008) used analyses of data from different countries around the world to identify five types of classroom talk. The first type is called *rote* (teacher-class) and is the drilling of facts, ideas and routines through constant repetition. The second is *recitation* (teacher-class or teacher-group) and is the accumulation of knowledge and understanding through questions to stimulate recall or to cue pupils to work out the answer from clues provided in the question. *Instruction/exposition* (teacher-class, teacher-group or teacher-individual) is the third type and is described as the teacher telling the pupil what to do, and/or imparting information and/or explaining facts, principles or procedures. The fourth type is *discussion* (teacher-class, teacher-group or pupil-pupil) and has to do with the exchange of ideas in view of sharing information and solving problems. Finally, the fifth type is *dialogue* (teacher-class, teacher-group, teacher-individual, or pupil-pupil), which is concerned with achieving common understanding through structured, cumulative questioning and discussion which guide and prompt, reduce choices, minimize risk and error, and expedite the 'handover' of concepts and principles.

In the relevant literature, the terms *discussion* and *dialogue* are often used interchangeably, while those who choose to distinguish them argue that in the latter, participants reach a common understanding, something that is not necessarily achieved in the former (see, for example, Howe & Abedin, 2013; Mercer & Littleton, 2007; Pirie & Schwarzenberger, 1988). Nevertheless, perceived as distinct or synonymous, these two types of talk are found less frequently within primary classrooms, while the first three types constitute the basic oral teaching repertoire (Alexander, 2008). In their relatively recent systematic review of studies from 1972 onwards, Howe and Abedin (2013) conclude that the situation has remained static for over 40 years, as classroom talk has not yet departed from traditional patterns of talking where the teacher is the one asking the questions and there is a focus on short and predictable answers by a single pupil. Of course, no lesson can be characterized by a single type of talk (Wells, 1999), as the boundaries among types of talk are permeable (Teo, 2013).

Discussion and dialogue have their merit within a larger oral repertoire that might as well include rote, recitation and exposition (Alexander, 2008). Yet, research needs to find more ways of bringing discussion and dialogue to the fore. In their research attempting to promote dialogue in classrooms across settings, Hennessy et al. (2016) use the term Dialogic Teaching-and-Learning (DLT) instead of 'dialogic teaching' to refer to classroom activities as indivisible units that do not segregate teaching from learning. As they summarise, DLT:

> a) harnesses the power of language to stimulate and extend students' understanding, thinking and learning; b) is collective, reciprocal, supportive, cumulative and purposeful; c) engages in

> 'social modes of thinking' where possibilities can be explored collectively through creative problem solving framed by open-ended or authentic questions/tasks and reasoning can be made visible to others; d) encourages inquiry and equitable participation, where all, including teachers, are seen as co-learners who construct knowledge jointly; e) is open to new ideas and critically constructive, where negotiation of perspectives allows joint problem solving; f) promotes the creation of environments where diverse voices can be expressed, explored, contrasted, challenged, cumulatively built upon each other and synthesised, allowing analysis, transformation and reconciliation of underlying points of view; and g) brings into question the widely observed predominance of traditional and 'monologic' educational practices where only one voice (primarily the teacher's) tends to be heard, legitimised and sometimes imposed. (Hennessy et al., 2016, p. 18)

In recognizing the important role of talk in mathematics classrooms in cultivating and promoting pupils' deeper understanding, many recent studies focus on how dialogic discourse patterns can be developed in class. These include, among others, engaging pupils in whole-class discussions (Black 2004; Duffin, 1986), teachers' use of appropriate questioning strategies (Bennett, 2010; Bills, 2000; Brown & Hirst, 2007), and teachers' use of purposeful pauses (Cohrssen, Church, & Tayler, 2014; Tobin, 1986). Along these lines, our work adopts the view that learners' active involvement in CPS activities in the classroom can raise the quality of their talk (Kersaint, Thompson, & Petkova, 2013; Mercer, 1996), which, in turn, provides opportunities for more effective learning of mathematics (Yackel, Cobb, & Wood, 1991).

Our Study

The participants of this study were 16 undergraduate students (11 female, 5 male) reading for a degree in elementary education with a qualified teacher status at a university in the Republic of Cyprus. All participants attended a class titled *Problem Solving in Primary Mathematics*, taught by the first author. The class lasted 12 weeks, during which the students and the instructor met once a week for three hours. The language of instruction was Greek.

All lessons included practical workshop elements during which students worked on solving mathematical problems as learners. For about half of the classes the lessons were designed to include the study of issues from the mathematical problem-solving literature (i.e. heuristic strategies, affective factors and problem solving, problem solving and mathematics teaching), while the rest were entirely practical. During the latter part of the course, students spent the entire class working in small groups of four, solving non-routine mathematical problems. These problems were carefully chosen so that they could be solved by several alternative strategies (Borasi, 1986) in order to enable Alexander's (2008) last two types of talk, discussion and dialogue, to emerge and be promoted. Figure 1 presents a problem requiring arithmetical/algebraic manipulations, while in Figure 2, a problem from geometry is illustrated.

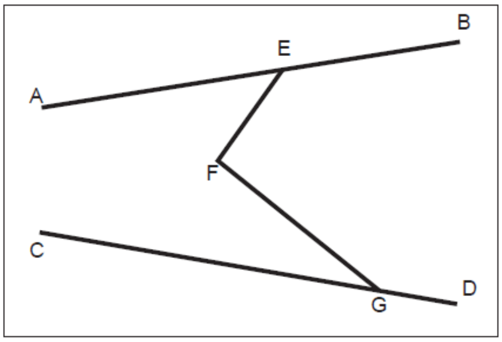
The two buckets problem

A painter has two buckets, the first containing one litre of black and the second containing one litre of white paint. The painter is using a small container of 1/8 L to transfer paint from one bucket to the other. He fully fills the container with white paint, transfers this quantity to the bucket with the black paint (assume that there is enough space for this extra quantity) and agitates well. Then, he fully fills the small container with this mixture, transfers this quantity to the bucket with the white paint, and agitates well. Now, both buckets have one litre of paint each. Can you calculate the percentages of the white paint in the predominantly black bucket and the percentages of black paint in the predominantly white bucket?

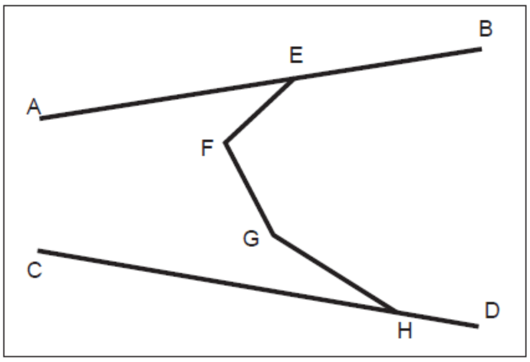
Figure 1. An arithmetical/algebraic problem given to the students

The farmers' field boundary problem

The illustration below shows the boundary, EFG, between two fields. Each field is owned by each of the two farmers and both agree that their lives would have been easier if the boundary were straight. Where might we draw a straight boundary, in order to preserve the areas of both fields?



What if the boundary was as shown below? Where would the straight boundary be now?



What general geometrical principle is outlined in this problem?

Figure 2. A geometrical problem given to the students

In the lectures, the topic and literature of CPS and classroom dialogue were purposely not discussed with the students, as one of our goals was to see whether any possible changes in their beliefs could be attributed to their practical experiences in such activities, rather than being a result of the lectures and notes/guidelines provided by the instructor.

Each student was randomly assigned to a group at the first meeting (see Table 1). The groups did not change throughout the semester, while the instructor's input was kept at a minimum. In such activities, as documented by previous research (Webb, 1991), offering elaborate explanations is found to be positively related to achievement, while receiving nonresponsive feedback, such as asking for an explanation and being told only the correct answer, appears to be negatively related to achievement. At various points, the instructor visited each group to observe its progress, ask questions to clarify ideas and provide guidance where necessary. At other times, during and at the end of each class, students were invited to a whole-class discussion, so that each group shared some of their ideas and approaches with their peers. Group talk was audio-recorded and later transcribed. We analysed the characteristics of the talk of the four groups in another paper (see Xenofontos & Kyriakou, 2016).

Table 1. The participants

	Pseudonym	Gender
Team 1	Stefania	F
	Chrysanthi	F
	Georgina	F
	Soula	F
Team 2	Petros	M
	Iphigenia	F
	Damianos	M
	Natali	F
Team 3	Dorothea	F
	Mary	F
	Theodoros	M
	Emily	F
Team 4	Iakovos	M
	Electra	F
	Andriana	F
	Lefteris	M

During the first and last meeting of the course, participants were given the same open-ended questionnaire, which required that they answer in written form questions about CPS, talk, and their relations to mathematics teaching and learning. The questions were phrased in such a manner that they would not guide students to provide certain answers simply to "please" the instructor. On the contrary, they included questions about participants' own definitions of dialogue, as well questions about the circumstances under which dialogic and

CPS activities are *not* considered appropriate. The selection of questions, a common decision of the two authors, aimed at providing research evidence that responds to the lack of studies concerning prospective teachers' beliefs around the importance of collaboration in mathematical problem solving (Xenofontos, 2014, 2015) and effective classroom talk (Lee & Kim, 2016). Below, the full set of questions is presented:

1. What does the term "dialogue" mean to you?
2. What do you think about your conversational skills when collaborating with peers for assignments? Justify your answer.
3. When Christina, an elementary school teacher, gives her pupils mathematical problems, they work in small groups and collaboratively. What are, in your opinion, the advantages and disadvantages (if any) of Christina's approach?
4. What characteristics make someone a good interlocutor during whole-class or group-work activities? You may give specific examples from your experience, without necessarily naming specific people.
5. How do you think you could improve your conversational skills?
6. Under which circumstances do your instructors expect you to take part in dialogic activities during classes? When do you think this might not be appropriate?
7. Why would you choose to work *individually* to solve a mathematical problem?
8. Why would you choose to work *collaboratively* to solve a mathematical problem?

Finally, for triangulation purposes (Cohen, Manion, & Morrison, 2011; Robson, 2011), at the end of the semester, the students were asked to prepare an individual report reflecting on their experiences in the CPS activities and evaluating the collaboration between themselves and their peers.

For our analyses, we adopted a data-driven approach, as no predetermined coding scheme was utilized. By exploiting the constant comparison process outlined by Strauss and Corbin (1998), as well as the ideas of coding and categorization (Kvale & Brinkmann, 2009; Miles & Huberman, 1994), the two authors worked individually on all types of data (written responses on the pre- and post-course questionnaire, individual reflecting reports). Later, we brought our individual works together, discussed identified patterns, and reached an agreement on the findings, which are subsequently presented.

Findings

Through analysing students' responses on the pre- and post-course questionnaire, as well as their individual reflective report, we identified three general themes, which illustrate their beliefs on CPS and dialogue in the mathematics classroom. Briefly, these concern (a) their definitions of *dialogue* and its relation to mathematics, (b) what, in their view, characterizes a 'good' interlocutor and, finally, (c) the importance of collaboration in mathematical problem solving activities. Our aim was dual: to examine students' beliefs about these three themes before and after the course and to see which of these could be attributed to their experiences with CPS during the course. Below, the three themes are presented in more detail.

Dialogue and its relation to mathematics

In regard to their own definitions of *dialogue*, an interesting homogeneity was observed in the participants' pre-course responses. None of them attempted to explicitly define what dialogue *is*; they rather described it by presenting what they saw as its ingredients. *All* 16 students indicated that a dialogue takes place between two or more persons, who freely choose to take part in it and respect each other's right for expression. In a typical response, which also sums up those of her peers, Soula wrote:

> In my view, dialogue happens when two or more people share their opinions on something, and everyone has to listen to what others have to say without interrupting. If we disagree with that others say, we can express it at the end. Through dialogue, the concept of respect is embedded because we have to respect all opinions and ideas.

Similar views were expressed in the post-course responses of the students. Nevertheless, it was particularly interesting to observe that seven of them explicitly used the terms "argumentation" and "justification" in regards to the presentation of one's ideas and opinions in a dialogic situation. Theodoros, for instance, commented that in a dialogue, "two or more persons have a conversation on a topic, listen to each other, and, agreeing or disagreeing, present arguments to justify their position." Similarly, Electra wrote that "each participant presents her/his point of view, justifying their arguments. Then, another participant replies, agreeing or, when disagreeing, he or she presents counter-arguments. This is an ongoing process."

At the beginning of the semester, 11 of the students expressed very generic views concerning the circumstances under which effective dialogue can be promoted in class. Natalie's comment is a typical example:

> It is important to promote dialogue in class because this has benefits for both the students and teacher. Through dialogue in class, learners exchange opinions. On appropriate occasions (when the topic of the lesson allows it) the teacher provides an open question as food for thought. Questions of this kind challenge the students, and this is how dialogue is promoted.

In fact, four students claimed that dialogic activities are not appropriate for mathematics classrooms; they believed that, on the contrary, they better suit language and social studies. Stefania, for instance, who viewed mathematics as an applied and practical subject, wrote that "dialogic conversations should take place during more theoretical school subjects, for example language and literature. I don't think this is appropriate for practical subjects, like mathematics. In mathematics, dialogue should be avoided." In a similar manner, Petros argued that dialogue is better for "cases of ethical dilemmas, where a decision needs to be taken, like what we did in psychology class." Chrysanthi was the only student who, at the beginning of the semester, claimed, "dialogue is appropriate for all school subjects, from the arts to mathematics. It improves learners' cognitive and social skills".

At the end of the semester, a great shift was evident in the students' post-course responses. Most notable was that *all* 16 claimed that dialogic activities can be used in mathematics classrooms. In fact, they all provided detailed accounts on the benefits of such activities on learners' deeper mathematical understanding. In Theodoros' words, "I don't think there is a school subject within which dialogic activities don't fit. I can't believe I'm actually writing this, but it's even appropriate for mathematics and sciences." Before the course, Stefania was one of the students who held strong beliefs against the use of dialogic activities in mathematics. In her individual reflective report at the end of the semester, she wrote, "I strongly believe that when learners have to solve challenging mathematical problems, it's better that they work in small groups. This way, through dialogue, they complement each other's ideas and knowledge, and support each other." However, her views on what constitutes a so-called practical school subject seemed to have changed. As she claimed in her post-course questionnaire, "in cases of more practical subjects, like music, physical education, or the arts, I don't think learners need to be involved in any sort of dialogic conversations."

Characteristics of a 'good' interlocutor

As with the previous theme, students' responses in the pre- and post-course questionnaire shared a number of similarities. The majority of students argued that a 'good' interlocutor has an interest in engaging in a conversation and knows (about) the subject under discussion due to previous experiences yet wants to learn more about it. Finally, according to the students, a 'good' interlocutor respects other people's views. A common response was that of Dorothea:

> In my view, good interlocutors are those who, first of all, have knowledge regarding the topic of discussion. They know how to defend their opinion, without attempting to impose it to others. They know when it's time to talk and when it's time to listen. They respect others, and do not fanaticize.

Interestingly, before the course, 14 out of the 16 students claimed to be 'good' and 'very good' interlocutors, similar to what Electra said for herself:

> I believe I'm a very good interlocutor when collaborating with peers during group work. I've come to this conclusion by evaluating my team spirit so far and my contribution to the teams of which I've been a member. The opinion I have of myself was justified by the feedback we received from our instructors and the good grades for our group assignments.

An exception to this was Iakovos who said he was "not very good at it, when it comes to collaboration. However, I wouldn't say I'm an intransigent person, so I guess no one would be disappointed by collaborating with me." Also, Lefteris labelled himself as an "average interlocutor" and said he feels more comfortable "when I collaborate with people I know. This is when I feel happy to share my views."

As far as the characteristics of 'good' interlocutors are concerned, in a third-person manner that did not involve themselves, the students expressed very similar beliefs in their post-course responses. Nevertheless, *all* responses were more detailed and more elaborate. In addition to the previously mentioned characteristics, students added "patient" (seven students), "good listener" (10 students), "someone who justifies her/his views with appropriate arguments" (seven students) and "someone who's not afraid to be wrong and admit it" (four students).

In regard to themselves as an interlocutor, Iakovos' and Lefteris' views did not seem to have changed after the course. However, four other students, while having labelled themselves as "good" or "very good" at the beginning of the semester, subsequently questioned their initial perceptions of themselves, highlighting the need for improvement. For example, Damianos thought that he was "average, cause sometimes I participate well in a conversation, but other times I don't. This depends on what I have to offer to the group discussion. I definitely need to improve my skills." A similar opinion was expressed by Iphigenia, who said "it depends on what I know about the subject. If I know enough, then I say more. If I don't, I'd rather sit back and listen to those who know more. I guess I could improve with practice." Chrysanthi, in turn, said that "collaboration and discussion with people you don't know well is a big challenge. I think I'm quite good and have never faced any problems in group discussions, although I believe there's room for improvement."

It should be noted, however, that perceptions of oneself do not always resonate with what peers think about that person's communicative skills. Such an example can be seen from the individual reflective reports of the members of Team 3. Mary, for instance, commented on "how happy I am with my collaboration with the other three members. I think we all collaborated well and I felt that I could always express my opinion even if it was wrong. We were a good team." On the other hand, her peers expressed serious complaints about her and her team spirit. For them, Mary "didn't really care and didn't help the team much" (Dorothea), "was very quiet most of the time" (Theodoros), and "rarely took part in any discussions cause she was always on Facebook instead of paying attention to our conversations and attempts to solve the problems" (Emily).

The importance of collaboration in mathematical problem solving

Students' responses in regards to group work and collaboration in mathematical problem solving activities shared a lot at the beginning and end of the semester. Both before and after the course, students highlighted a number of advantages and disadvantages of CPS in mathematics. On the one hand, they argued that "collaboration in problem solving develops pupils' social skills" (Soula), as well as "[having] many cognitive benefits, especially for lower ability learners" (Chrysanthi). In particular, ten students argued in their post-course responses that CPS provides opportunities for learners to come across different approaches to a solution and see how peers might think on the same topic. Theodoros, for example, commented, "I discovered new strategies to solve mathematical problems and new ways of thinking that I could also apply later at some point." Damianos shared a very similar view, claiming that when it comes to CPS, "I would prefer group work so that I could become familiar with other ways of thinking which wouldn't be possible to think of by myself." On the other hand, *all* students raised a number of concerns in regard to CPS activities in mathematics. In a typical response, Andriana said, "it's possible that weaker pupils rely on the more able ones, and do nothing, just sit there and let the other kids do the job." Georgina added, "sometimes stronger learners dominate the discussion and because they know what needs to be done, they might not explain what is happening to the others, or, even worse, ignore what others have to say." Emily held the same belief about CPS creating "a noisy classroom" both at the beginning and end of the course, claiming, "this might be difficult for the teacher to control."

Interestingly, a number of issues were observed only in students' post-course responses and their individual reflective reports. Particularly, half of the students commented on how group work and CPS in mathematics helps learners develop their self-esteem and confidence. A characteristic comment is that of Georgina, who, in her reflective report, wrote:

> I'm really happy with how things worked in our team. I believe we collaborated really well. Stefania and Chrysanthi seemed to know more mathematics than Soula and myself. However, they always treated us with respect and our opinions were always valued. They would always explain what I didn't understand. This made me more confident cause I knew I had peers to rely on. I tried my best; it's not that I didn't care. But I think that towards the end of the semester, I felt more confident and more active. I wasn't that scared of failure.

Other issues regarded teachers and teaching. At the end of the semester, five students expressed concerns about how teachers evaluate individual work in CPS activities. Iakovos was one of them. As he wrote, "I don't really know how a teacher is certain of each pupil's individual progress in problem-solving group activities. Such an approach might have a number of benefits but how can we measure them? Parents want to know how their kid is doing at school." His views were similar to Chysanthi's, who stated, "if we want to evaluate a student's progress, we can only do it through individual problem solving."

Finally, 11 of the students wrote in their individual reports that the whole experience in CPS activities was very important to them as prospective teachers. For Damianos, "this whole experience was new to me. I don't think I've ever been asked to collaborate with peers in solving mathematical problems. This is something I'd like to do in the future with my own pupils." Georgina agreed, claiming,

> this was something new to me! What an original idea! It should be promoted in other classes, as well. Not only did we learn to collaborate in solving problems but we also discovered how important it is to collaborate as future teachers. When we will be working in schools, we'll have

to solve problems of various types all the time, and not only mathematical. For me this was a remarkable experience, and I'd like all of our instructors to promote it so we can become better teachers with advanced collaborative skills.

Discussion

Tracing belief changes

In this section, we revisit our initial research question and discuss it in light of our findings and the relevant literature, what this study does and does not inform us about, the educational implementations of our work and the avenues it opens up for future research. Clearly, providing prospective teachers opportunities to experience CPS from the learners' perspective and to be involved in dialogic conversations while attempting to solve challenging, non-routine problems has led to a number of positive changes in their related beliefs. Readers should be reminded that, during the course, students were purposefully not introduced to the literature and ideas of CPS and the importance of dialogue, as one of our main goals was to examine which of the possibly observed changes in their beliefs could be attributed to active, hands-on experiences and not ideas learnt (or memorized) especially for the compulsory final exam of the course.

Although at the beginning of the semester students could not define what dialogue *is,* it was apparent that they brought ideas of a well-structured body of characteristics to class, which, most likely, was intuitive. These characteristics included ideas reminiscent of Alexander's (2008) definitions of *discussion* and *dialogue*. By the end of the course, about half of the students highlighted the importance of *argumentation* and *justification* in dialogic conversations (Hennessy et al., 2016; Nussbaum, 2011), that is, opinions and ideas should be supported by facts, which are not easily contested. In particular, Electra's view that argumentation is an ongoing process is aligned to Ramsey's and Langrall's (2016, p. 414) view that "*mathematical argumentation* [is] a process of dynamic social discourse for discovering new mathematical ideas and convincing others that a claim is true." Furthermore, at the beginning of the semester, *all* students rated themselves as 'good' and 'very good' interlocutors. Yet, one fourth of the students held different beliefs after taking the course, and claimed that they are not as effective as they thought they were, highlighting a need for improvement. Negative changes of prospective teachers' self-efficacy beliefs in mathematics as a result of their teacher education programme have also been observed by other studies (i.e. Charalambous, Panaoura, & Philippou, 2009) and should not necessarily be perceived as a failure of the programme. Nonetheless, in the case of our study, participants seem to have developed an awareness of difficulties in fostering effective dialogue in mathematics (Lee & Kim, 2016), evidenced by their expression of their need for further support and improvement. Furthermore, it is important to point out that the students do not appear to distinguish between the concepts of *discussion* and *dialogue*, the main difference between which is that in dialogue participants reach a common understanding whereas in discussion this is not necessarily achieved (Howe & Abedin, 2013; Mercer & Littleton, 2007; Pirie & Schwarzenberger, 1988). Nonetheless, this may be due to the fact that, as explained earlier, these concepts were purposefully not presented in class. However, we do consider the observed shift in their beliefs encouraging, especially when taking into consideration that the two concepts are often perceived as synonymous by some scholars.

In their pre-course responses, all participants presented a number of advantages CPS activities in mathematics have for learners, both cognitive (as discussed in Artzt & Armour-Thomas, 1992; Goos et al., 2002; Iiskala et al., 2011) and social (see, for example, Barron, 2000;

Forman, 1989; Langer-Osuna, 2016; Tatsis & Koleza, 2008). Yet, at the beginning of the course, four of the students held negative beliefs about the applicability of dialogic activities in mathematics, by claiming that such activities are more appropriate of the humanities and social studies. It is apparent that the links between CPS and dialogue were not explicit for all of the participants. However, by the end of the course, all of them could discuss a number of advantages for both concepts (CPS and dialogue), something that reinforces our belief that connections between these concepts were established. However, while these results are optimistic for us, much needs to be done on behalf of teacher education in general, not least because of what Stefania wrote in her pre- and post-course responses. In her first set of answers, Stefania wrote that "dialogic conversations should take place during more theoretical school subjects, as for example language and literature. I don't think this is appropriate for practical subjects, like mathematics." At the end of the semester, she said, "in cases of more practical subjects, like music, physical education, or the arts, I don't think learners need to be involved in any sort of dialogic conversations." Here, we see a positive shift in her beliefs about the relations between CPS and dialogue and how the latter can be fostered in CPS activities relating to mathematics. However, we also observe a negative shift regarding what she calls "practical lessons" and the applicability of dialogic, problem solving activities in those school subjects. For this reason, we strongly believe that closer collaboration between teacher educators of different school subjects, through the adoption of a common approach that promotes dialogue through CPS, could bring more desirable outcomes for-the next generation of teachers.

Another important issue is the fact that five participants raised concerns about the ways teachers can evaluate an individual pupil's contribution to CPS activities. The same concerns have been expressed by in-service teachers in other settings, as discussed by Albert and Kim (2013), Barron (2000), Kroll et al. (1992), Wadlington (1993). Indeed, teachers need appropriate support in this area, both during their preparatory studies and through professional development programmes. In this study, however, as earlier explained, students were intentionally not directly introduced to the literature on CPS, therefore issues of evaluation and assessment in related activities were not discussed. It appears, though, that this is a very important issue, one that requires further work and examination. In the following semesters, we intend to revise the module so that it will explicitly include the topic of how teachers can effectively evaluate and assess pupils' work in CPS activities.

Concluding remarks

Providing prospective teachers with opportunities for experiencing CPS in mathematics as learners, and engaging in dialogic activities, appears to have a positive impact on their related beliefs, especially when their prior schooling experiences do not include activities of this kind (Xenofontos, 2014, 2015). The use of CPS in the mathematics classroom challenges the traditional teacher-centered model of teaching (Albert & Kim, 2013). We, therefore, consider it very important for prospective teachers to develop a deep awareness of CPS and of the importance of dialogue, which will equip them with a set of transferable skills that can be applied in situations other than school mathematics.

Similarly to Zsoldos-Marchis (2015), we'd like to point out an important limitation to our work: the study reported here was carried out during one semester only. Consequently, we do not have further evidence concerning the sustainability of the observed positive changes of the participants' beliefs. Furthermore, we would be particularly interested in examining how the ideas acquired through this module could (or not) be transferred to classroom opportunities for children during the student-teachers' school placements. Another important limitation might be that in this study, we did not consider participants' epistemological beliefs about mathematics, as described by Ernest (1989). Ernest reports three types of mathematics teachers' epistemological beliefs, namely the instrumentalist view (mathematics as a useful and essentially unrelated collection of facts, rules and skills), the Platonist view (mathematics as a static but unified body of knowledge, whose structure and interconnections between various topics are fundamentally important), and the problem-solving view (mathematics as a dynamic and creative human invention, and a problem-driven field of enquiry). In this respect, Shilling-Traina and Stylianides (2013) have observed that group collaboration between prospective elementary teachers has created a movement from the more traditional instrumentalist and Platonist views towards a problem-solving view. Such an additional dimension in future work, that is, the investigation of connections between prospective teachers' epistemological beliefs and their CPS beliefs through the lenses of their talk as interthinking (Mercer, 2000), would probably provide more evidence for a deeper understanding of the limited appreciation of CPS held by some of the student-teachers. Nevertheless, there is opportunity for work to be done in the future, and the study reported here opens new possibilities for further investigation.

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