Taylor & Francis
Taylor & Francis Group

International Journal of Mathematical Education in Science and Technology

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tmes20

Estimation in the primary mathematics curricula of the United Kingdom: Ambivalent expectations of an essential competence

Paul Andrews , Constantinos Xenofontos & Judy Sayers

INTERNATIONAL JOURNAL OF MATHEMATICAL EDUCATION IN SCIENCE AND TECHNOLOGY
https://doi.org/10.1080/0020739X.2020.1868591

Estimation in the primary mathematics curricula of the United Kingdom: Ambivalent expectations of an essential competence

Paul Andrews [a], Constantinos Xenofontos [b] and Judy Sayers [c]

[a]Department of Mathematics and Science Education, Stockholm University, Stockholm, Sweden; [b]Faculty of Social Sciences, University of Stirling, Stirling, UK; [c]School of Education, University of Leeds, Leeds, UK

ABSTRACT

In this paper, we examine the national curricula for primary mathematics for each of the four constituent nations of the United Kingdom (England, Northern Ireland, Scotland and Wales) for the estimation-related opportunities they offer children. Framed against four conceptually and procedurally different forms of estimation (computational, measurement, quantity and number line), the analyses indicate that computational estimation and measurement estimation were addressed in all four curricula, albeit from a skills-acquisition perspective, with only the Scottish offering any meaningful justification for their inclusion. The process of rounding, absent in the Northern Ireland curriculum, was presented as an explicit learning objective in the English, Scottish and Welsh curricula, although it was only the Scottish that made explicit the connections between rounding and computational estimation. In all curricula, both quantity estimation and number line estimation were effectively absent, as was any explicit acknowledgement that learning to estimate, irrespective of its form, has a developmental role in the learning of other mathematical topics.

ARTICLE HISTORY
Received 7 June 2020

KEYWORDS
Computational estimation; measurement estimation; number line estimation; quantity estimation; England; Northern Ireland; Scotland; Wales; primary mathematics curriculum

1. Introduction

Over the last decades, much attention has been paid to so-called twenty-first century skills. Generally, these refer to broad competences frequently related to, for example, communication, collaboration or technological literacy (Binkley et al., 2012). However, as we show below, a skill that has been meaningfully employed in human societies for millennia is typically missing from such repertoires. Estimation 'is a pervasive activity in the lives of both children and adults' (Booth & Siegler, 2006, p. 189), a core skill of everyday life and a determinant of later arithmetical competence (Sasanguie et al., 2013; Schneider et al., 2009), particularly in respect of novel situations (Booth & Siegler, 2008; Holloway & Ansari, 2009).

Unfortunately, an ongoing failure of textbooks to address estimation (Grossnickle, 1935; Hong et al., 2018) may have contributed to many teachers having an underdeveloped

CONTACT Paul Andrews paul.andrews@mnd.su.se Department of Mathematics and Science Education, Stockholm University, 106 91, Stockholm, Sweden.

conception of the topic (Alajmi, 2009; Subramaniam, 2014) and uncertainty as to why and how they should teach it (Joram et al., 2005). That said, an issue missing in the estimation-related literature is the role played by curricula. Therefore, it seems judicious to examine whether concerns raised forty years ago remain today; that the 'cursory treatment given to estimation in most mathematics programs is insufficient to build any appreciable estimation' (Bestgen et al., 1980, p. 124). In this paper we examine qualitatively the role of estimation in the mathematics curricula of the four educational jurisdictions of the United Kingdom – England, Northern Ireland, Scotland and Wales – with the aim of understanding how these different but closely related systems conceptualize this important topic.

2. What is estimation and why is it important?

In broad terms, children's mathematical learning is justified in two ways. The first, the psychological, concerns the developmental role of estimation in children's learning of mathematics in general and its potential for identifying developmental problems in particular. The second, the pragmatic, refers to the significance of various forms of estimation in different real-world contexts. In the following, while conscious of the latter, we focus principally on the former. Also, estimation has historically taken three forms; *computational estimation, measurement estimation* and *quantity (or numerosity) estimation* (Sowder, 1992). Today, a fourth form, *number line estimation,* has become a familiar sight in the fields of cognitive psychology, mathematics education and special needs education (Sayers et al., 2020a). In the following, we examine, albeit briefly, these four forms of estimation, their characteristics and relevance to the teaching and learning of school mathematics.

2.1. Computational estimation

Of all the forms of estimation, computational estimation seems to be the least well-defined. On the one hand, for example, Dowker (1992, p. 45) describes it as the process of 'making reasonable guesses as to the approximate answers to arithmetic problems, without or before actually doing the calculation'. On the other hand, Ainsworth et al. (2002, p. 28) write that computational estimation refers to 'the process of simplifying an arithmetic problem using some set of rules or procedures to produce an approximate but satisfactory answer through mental calculation'. In similar vein, Siegler and Booth (2005, p. 199) assert that it involves 'approximating the correct magnitude rather than calculating the exact answer'. In short, while all three definitions equate estimation with approximation, the latter two avoid words like guess and allude to forms of systematic procedure, whereby two individuals applying the same procedures to the same problem will always arrive at the same estimation.

Computational estimation is a process that draws on a wide range of strategies (Dowker, 1992; Levine, 1982; Mildenhall, 2009). For Reys et al. (1982), these strategies include reformulation, (a process whereby numerical data may be changed to create a more manageable form but which leaves the structure of the problem intact), translation (a process in which the structure of the problem is changed to produce a more manageable form), and compensation (a process whereby the estimator makes numerical adjustments prompted by translations or reformulations). These three broad strategies have been consistently identified across cultures and ages (Alajmi, 2009; Boz & Bulut, 2012; LeFevre et al., 1993; Reys et al., 1991; Sekeris et al., 2019; Sowder & Wheeler, 1989).

Computation estimation is frequently 'more important and practical than precise calculation for many everyday uses of mathematics' (Bestgen et al., 1980, p. 124), because 'it takes less time and attentional resources than exact calculation, and thus can be used in circumstances where time or attention resources are limited' (Ganor-Stern, 2018, p. 2). Thus, computational estimation is not only an essential life skill (Ganor-Stern, 2016; Sekeris et al., 2019) but also, despite teacher scepticism concerning its relevance (Alajmi, 2009), an important facilitator of, for example, children's understanding of place value and standard algorithms (Dowker, 2003; Sowder, 1992). Children's computational estimation competence develops with age, particularly with respect to strategy choice (Ganor-Stern, 2016, 2018; Lemaire & Brun, 2014; Lemaire & Lecacheur, 2011), because maturation brings with it a range of experiences supportive of a wider range of appropriate strategies (LeFevre et al., 1993). The emergence of computational estimation is unlikely before the age of eight (Siegler & Booth, 2005), is influenced by the complexity of the task (Dowker, 1997) and draws on children's arithmetical competence, becoming less accurate the further the task strays from their base-competence (Dowker, 1997; Seethaler & Fuchs, 2006). Children often misunderstand the function of estimation, tending to abandon rounding strategies for adaptations of written algorithms as they get older (Liu, 2009). In sum, computational estimation is a core competence with implications for both the learning of mathematics and real-world functionality and amenable to interventions (Ainsworth et al., 2002; Bobis, 1991; Mildenhall & Hackling, 2012; Palaigeorgiou et al., 2018).

2.2. Measurement estimation

Measurement estimation, which can be construed as measuring without measurement tools, invokes various forms of mental referents to provide a measure of the entity under scrutiny (Sowder, 1992), typically takes one of three forms. The first, *unit iteration*, occurs when individuals iterate mentally a standard unit of measure to achieve the desired goal. The second, *reference points*, occurs when people compare the quantity to be estimated against familiar, and therefore meaningful, objects. Indeed, such context familiarity improves estimates (Jones et al., 2012). The third, *decomposition*, occurs when the estimator splits the object into smaller quantities before applying either unit iteration or reference points (Joram et al., 1998). That said, the most important strategy seems to be reference points, not only because children who employ reference points are more accurate than those who do not (Joram et al., 2005) but because the strategy has been linked with mathematics achievement more generally (Kramer et al., 2018). Moreover, reference points are everyday tools of professional users of mathematics (Jones & Taylor, 2009).

Both children and adults appear to be making poor measurement estimations (Joram et al., 1998). This is not surprising, given that this estimation form is often avoided to be taught by teachers (Ruwisch et al., 2015), perhaps due to uncertainty about its teaching (Joram et al., 2005; Pizarro et al., 2015). This problem is exacerbated by a relatively poor inclusion of measurement estimation in textbooks (Hong et al., 2018; Mengual et al., 2015). It may also be a consequence of teachers construing measurement and estimation as distinct domains, with estimation located in the realm of the hypothetical – tied to notions of vagueness, inexactness and guesswork – while measurement is located in the realm of the real – tied to notions of correctness, exactitude and no guesswork (Forrester & Pike, 1998).

Research on children's measurement estimation competence is not entirely consistent. For example, children in the age range nine through eleven have been found to be poor estimators of lengths, although their estimates improve with age (Desli & Giakoumi, 2017). Alternatively, across the age range eight through eleven, little evidence was found of estimation maturation (Forrester & Shire, 1994). That being said, young children are more accurate when they use non-standard units (Desli & Giakoumi, 2017). From the perspective of strategies, secondary-aged children typically base their measurement estimations on some form of individual frame of reference, which is one element of a complex interaction with the nature of the estimation activity and the physical context in which it occurs (Gooya et al., 2011). Middle school children typically do not spontaneously use reference points (Lipovec & Ferme, 2017), although unit iteration and reference points were the strategies typically found with primary-aged children (Desli & Giakoumi, 2017). Overall, it is likely that classroom interventions can improve measurement estimates, as Taiwanese students are more accurate estimators of length than German (Hoth et al., 2019).

2.3. Number line estimation

Typically, number line estimation entails 'translating a number into a spatial position on a number line' or, less commonly, 'translating a spatial position on a number line into a number' (Siegler et al., 2009, p. 144). In many respects, while its role in a person's real-world functionality is limited, its impact on mathematical learning is profound. Broadly speaking, the accuracy of young children's number line estimations, which typically involve the marking of a target number on a number line with two orientation points labelled, and the strategies they employ improve with age and experience (Praet & Desoete, 2014; White & Szűcs, 2012), although, unsurprisingly, children's number line estimations of fractions are typically much less successful than their estimations of integers (Siegler et al., 2011).

Up to the age of around eight years, children tend to construe small numbers as more widely spaced than large, leading some scholars to perceive a logarithmic progression that tends to linear (Ashcraft & Moore, 2012; Siegler & Booth, 2004). Others, however, argue that a bilinear representation is at least as accurate an account for this shift (Dackermann et al., 2015; Ebersbach et al., 2008). Successful number line estimation draws on an understanding of ordinality (Van 't Noordende et al., 2018) and the use of reference points (Sullivan & Barner, 2014a). The latter, particularly the midpoint, not only facilitates estimation but encourages the development of proportional reasoning (Bicknell & Young-Loveridge, 2015; Rouder & Geary, 2014).

Importantly, from the perspective of this paper, number line estimation competence is a strong predictor of both later mathematical learning difficulties (Andersson & Östergren, 2012; Wong et al., 2017) and mathematical achievement across all ages of compulsory school (Fuchs et al., 2010; Schneider et al., 2018; Simms et al., 2016), to the extent that 'number line estimation at age 16 was significantly related to mathematics … at each age, beyond variance explained by other cognitive abilities at that age' (Tosto et al., 2017, p. 1934). In particular, whole number line estimation competence has been implicated in children's arithmetical development across the years of primary education (Dietrich et al., 2016; Fuchs et al., 2010; Träff, 2013) in ways suggesting that number line estimation and arithmetical competence may be reciprocally related (Friso-van den Bos et al., 2015). Number line estimation accuracy is a predictor of fractions knowledge in general (Bailey et al., 2014;

Hansen et al., 2015; Vukovic et al., 2014), particularly in the upper primary and lower secondary age ranges (Fazio et al., 2014; Torbeyns et al., 2015; Van Hoof et al., 2017). Finally, the ability to estimate accurately the position of fractions on the number line is a strong predictor of algebraic readiness (Booth & Newton, 2012) and equation solving competence (Booth et al., 2014). Further, decimal number line estimation competence is a better predictor of algebraic competence than either integer or fraction number line estimation (DeWolf et al., 2015).

2.4. Quantity estimation

Quantity (or numerosity) estimation refers to the ability to discern or produce the number of objects in a set without recourse to counting (Crites, 1992). It is a skill reciprocally dependent on the ability to count (Barth et al., 2009) and one that diminishes in accuracy as the numerosity of the set of objects grows (Smets et al., 2015). In broad terms, when researching quantity estimation, researchers typically exploit three forms of task: perception tasks, where participants estimate the quantity of a collection of dots shown them; production tasks, where participants produce under pressure of time a set of dots equal to a symbolically presented numerical input; reproduction tasks, where participants reproduce a set of dots equal in numerosity to a set of dots presented to them.

Research has shown that young children, up to the age of around eight years, tend to construe smaller numbers as being more widely spaced than larger numbers, with the result that quantity estimation in young children follows a logarithmic pattern before, with increasing age, tending towards linear (Berteletti et al., 2010; Siegler et al., 2009; Sullivan & Barner, 2013). Others, however, have argued that the logarithmic model for young children may be better construed as two linear forms, the first reflecting children's relatively accurate representation of small numbers and the second their lack of knowledge of large numbers (Stapel et al., 2015).

The provision of reference points helps children with their estimations, although kindergarten children are more successful when the reference is larger than the target and that two reference points are less helpful than one (Baroody & Gatzke, 1991). Young children tend to overestimate small quantities and underestimate large, although both are mediated by age (Ebersbach & Wilkening, 2007; Stapel et al., 2015). Also, irrespective of their estimation accuracy, young children estimate in an ordinal fashion; that is, they estimate in the correct direction relative to previous estimates (Sullivan & Barner, 2014b). There is a strong relation between children's ability to count and the accuracy of their quantity estimations (Barth et al., 2009), with both uniquely predicting later arithmetical competence (Bartelet et al., 2014). Moreover, a study examining the relationship between measures of quantity estimation, number line estimation and arithmetical competence found, with respect to six years-old children, not only both measures of estimation correlating significantly with each other but also independently predicting arithmetical competence (Wong et al., 2016).

Interestingly, groups of people estimate quantities more accurately than individuals (Bonner et al., 2007; Laughlin et al., 2003), while quantity estimation is negatively associated with sensation seeking (Ginsburg, 1996). Finally, from a didactical perspective, recent years have seen scholars turning attention to Fermi problems as a means of facilitating students', across all school years, competence with large numbers (Albarracín & Gorgorió, 2019; Ärlebäck & Bergsten, 2013).

3. The current study and its methods

Acknowledging the above, and evidence of a paucity of estimation-related opportunities in school textbooks international (Sayers et al., 2020b), our goal in this paper is to examine how estimation is conceptualized in the four curricula of the countries of the United Kingdom. In so doing, we are conscious that the curriculum, as an object of analysis, may be construed differently in different cultural contexts. For example, in countries like Cyprus, state-mandated learning outcomes are manifested in government-produced textbooks that all teachers are obliged to follow (Xenofontos, 2019). In such a context, curriculum analyses would sensibly focus on the content of these textbooks as they represent the 'contract' between the state and the individual teacher. In other countries, like the United States, there is no centrally-produced curriculum with, historically, each school district identifying both curricular goals and the textbooks to address them (Reys, 2001). Here, depending on the extent to which such curricular goals are represented in some form of steering document, analyses might focus on the steering documents themselves or the textbooks chosen to reify them. In other systems, government-mandated goals are specified in steering documents, hereafter national curricula, that explicate for schools, teachers and parents not only what is to be taught but when it is to be taught. In these systems, textbooks, typically produced by commercial publishers subject to varying degrees of systemic regulation, fall outside the 'contract' between the state and the teacher. Consequently, analyses of curricula and analyses of textbooks are qualitatively different enterprises.

The four countries of the United Kingdom fall into this latter group, whereby government expectations are manifested in centrally-produced documents that outline, in varying degrees of detail, both the content and the timing of the mathematics to be taught. In this paper, we examine qualitatively the ways in which the primary mathematics curricula, including the final year of pre-school, of England, Northern Ireland, Scotland and Wales structure opportunities for children to develop estimation-related competence. In so doing, as indicated above, we focus on the curriculum documents themselves as they, rather than any textbooks used by teachers in those countries, form the 'contract' between the state and individual schools and teachers. Indeed, across the UK, schools' inspectors are not concerned with teachers' adherence to the content of a textbook but whether their practices fulfil the expectations specified in the respective curricular documents.

In each of the four countries, the national curriculum and any supporting documents are available electronically as searchable pdf files. In this respect, and to simplify the narrative for the reader, the documents scrutinized for each country can be found in Tables 1 through 4 respectively, each of which can be found in the relevant results section. At this stage, it is important to note not only that all documents have been produced by government departments or agencies but also that the four curricular authorities present both expected learning outcomes and any exemplification in different ways. Consequently, the number of documents analyzed varied from one country to another. Also, several documents were excluded from the analyses due to their discussing curricula at too general a level or repeating the content of other scrutinized documents. Summaries of such matters can be seen in the respective tables.

Each downloaded pdf was subjected to the following procedure. First, searches were undertaken for any occurrences of the words, *estimation, approximation*, *check*, *round* and their variants. Searches were also undertaken four any occurrences of the four key terms

Table 1. Documents scrutinised for the English analysis.

Documents included in the analyses
Department for Education (2013a). Early years outcomes: A non-statutory guide for practitioners and inspectors to help inform understanding of child development through the early years. London, DfE. Hereafter, DfE (2013a), this document provides exemplificatory support for teachers working with pre-school children and addresses expectations with respect to estimation-related learning.
Department for Education (2013b). Mathematics programmes of study: key stages 1 and 2. London, DfE. Hereafter, DfE (2013b), this document is the statutory primary curriculum document and contains the statutory goals teachers are expected to address alongside illustrative non-statutory examples.
Standards and Testing Agency (2019) Early years foundation stage profile: 2020 handbook. London, STA. Hereafter, STA (2019), this document, which provides non-statutory support for teachers, 'has been produced to help practitioners make accurate judgements about each child's attainment' (STA, 2019, *p*. 6) and has a limited but unique connection to estimation.
Documents not included in the analyses
Department for Education (2014) The national curriculum in England framework document. London, DfE. This document brings together all subject-related expectations and replicates the material found in mathematics-specific primary curriculum above
Department for Education (2017). *Statutory framework for the early years foundation stage: Setting the standards for learning, development and care for children from birth to five*. London, DfE. This document, which represents the legal framework for teachers of early years children, addresses the generalities of such work but offers nothing with respect to the particularities of estimation in children's learning.

of *computation*, *measurement*, *number line* and *quantity*, their variants, as well as alternatives like *calculation* or *numerosity*. Relevant results were copied and pasted into a single text document for each country. By way of illustration, English curricular expectations that children will check the results of their calculations were manifested in various statements, some of which were included in the analysis and others not. On the one hand, expectations that year-two children will recognize 'the inverse relationship between addition and subtraction and use this to check calculations' was rejected as unrelated to estimation due to its focus on precision. On the other hand, expectations that year-six children will be seen 'rounding answers to a specified degree of accuracy and checking the reasonableness of their answers' was included due to its focus on imprecision. Second, each statement in each text document was read and categorized against the four forms of estimation discussed in the introduction. A small number of such statements fell outside these categories and these are discussed individually in each of the country narratives below. Third, categorized statements were synthesized into a summary narrative for each form of estimation across the years of each of the four countries' curricula. It is on these country narratives, presented alphabetically, that the following is based.

4. Results: estimation in the primary national curriculum for England

English pre-school, or the early years foundation stage, covers the years birth through five, while primary education covers the ages five through eleven in two phases. These are key stage one (KS1) for school years one and two (ages 5–7) and key stage two (KS2) for school years three through six (ages 7–11). Table 1 outlines the documents that were implicated in the analysis, alongside those that were not. In each case, a short summary is offered to contextualize the relevance of the document concerned.

4.1. Computational estimation

While there are no references to computational estimation in the early years' materials, the statutory curriculum for key stages one and two includes several references we construe as related to computational estimation. It emerges as a specific goal in years three and four, when children should be able to 'estimate the answer to a calculation' (DfE, 2013b, p. 19) and 'estimate and use inverse operations to check answers to a calculation' (DfE, 2013b, p. 25). By the end of year six children should 'use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy' (DfE, 2013b, p. 40). However, clarity with respect to the curriculum's construal of estimation emerges only after a reading of the requirements with respect to rounding. Here, children in year four are expected to 'round any number to the nearest 10, 100 or 1000' (DfE, 2013b, p. 24) and 'round decimals with one decimal place to the nearest whole number' (DfE, 2013b, p. 26). This emphasis on rounding continues. In year five children should be able to 'round any number up to 10,00,000 to the nearest 10, 100, 1000, 10,000 and 1,00,000' (DfE, 2013b, p. 31) and 'round decimals with two decimal places to the nearest whole number and to one decimal place' (DfE, 2013b, p. 34). By year six, the single statutory statement concerning rounding is that children should be able to 'round any whole number to a required degree of accuracy' (DfE, 2013b, p. 38), with other references occurring in the non-statutory sections. That being said, the solitary reference to rounding with an explicit connection to calculation is found in year five, where children should be able to 'use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy' (DfE, 2013b, p. 32).

4.2. Measurement estimation

By the end of the EYFS, children are expected to be able to 'estimate, measure, weigh and compare and order objects and talk about properties, position and time' (STA, 2019, p. 54). These expectations continue into key stage one, whereby year-two children should be able to 'choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels' (DfE, 2013b, p. 14),while by the end of year three, such goals include being able to 'estimate and read time with increasing accuracy (DfE, 2013b, p. 21). By the end of year five, children should be able to 'estimate the area of irregular shapes (and) volume' (DfE, 2013b, p. 36), as well as 'know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles' (DfE, 2013b, p. 37). By year six, they should be able to 'calculate, estimate and compare volume of cubes and cuboids' (p. 43). Interestingly, the only reference for pupils in year four is an ambiguous expectation, presented under the heading of measurement, that year-four pupils should be taught to 'estimate, compare and calculate different measures, including money in pounds and pence (DfE, 2013b, p. 27). This, it seems to us is ambiguous, because money is more about computational estimation than measurement.

4.3. Number line estimation

Expectations with respect to number line estimation are not only limited but lack clarity. For example, while there are various references, both statutory and non-statutory, to children placing integers, decimals and fractions on a number line, there is only one reference connecting number line to estimation. Here, year two children should be able to '… estimate numbers using different representations, including the number line' (DfE, 2013b, p. 11).

4.4. Quantity estimation

Within the early years' outcomes document is a single estimation-related expectation, whereby a child between 40 and 60 months 'estimates how many objects they can see and checks by counting them' (DfE, 2013a, p. 24). This is augmented by an assertion in the foundation stage profile document that children should be able to 'estimate a number of objects and check quantities by counting up to 20'. That said, once a child moves into key stage one, there are no further references to quantity estimation.

In sum, while computational estimation dominates English curricular expectations, it is mediated by extensive reference to rounding, almost to the extent that it has become an end in itself. In comparison, the situation with respect to measurement estimation seems clear. Finally, both number line estimation and quantity estimation are, de facto, absent.

5. Results: estimation in the primary national curriculum for Northern Ireland

The Northern Ireland national curriculum for primary mathematics, first published in 2007 and revised in 2019, is split into three phases, focused on ages 4–6, 6–8 and 8–11 respectively. Its expectations are summarized in the single document shown in Table 2. The first phase, known as the foundation stage, comprises a small number of broad goals and relatively few specific statements. The second and third phases, known as key stage one and key stage two respectively, comprise similar broad goals alongside sets of specific statements.

Of relevance to this particular analysis is that the curriculum comprises a number of statements concerning estimation at levels of generality that cannot be pigeonholed. For example, during years one and two, children 'should begin to estimate and make simple predictions in all areas of mathematics' (CCEA, 2007, p. 23), while in the years three through six, children's understanding of the world around them should be facilitated by their 'interpreting statistical data and using it to solve problems using measurement, shape, space and estimation in the world around them' (CCEA, 2007, p. 58). In neither case can estimation be construed as anything other than a general goal. That being said, there is additional evidence supportive of three of the four forms of estimation identified in the literature.

Table 2. Document scrutinised for the Northern Irish analysis.

Document included in the analyses
Council for the Curriculum, Examinations and Assessment (2007). The Northern Ireland Curriculum Primary. Belfast, CCEA. Hereafter, CCEA (2007), this document includes both statutory expectations and non-statutory illustrations for both preschool and primary phases.

5.1. Computational estimation

For children in years 3 and 4 there are broad goals that they 'should be given opportunities, on a regular basis, to develop their skills in mental mathematics, to estimate and approximate', as well as 'explore how a calculator works … , check calculator results by making an estimate … ' (CCEA, 2007, p. 59). This latter goal is supported by the requirement that 'pupils should be enabled to … estimate and approximate to gain an indication of the size of a solution to a calculation or problem' (CCEA, 2007, p. 65).

5.2. Measurement estimation

For children in years 1 and 2 the curriculum includes the broad goal that 'children should develop much of their early mathematical understanding during play, where the activities provided offer opportunities for them to estimate size, weight, capacity, length and number' (CCEA, 2007, p. 23). However, apart from two statements concerning pupils being enabled to compare lengths, weights, times and other measures from the physical world, there is no explicit expectation that pupils should estimate measures. In years 3 and 4, however, there are requirements that pupils 'should be enabled to … make estimates using arbitrary and standard units' (CCEA, 2007, p. 63), while in years 5 and 6 these are extended to include the requirement that they should 'develop skills in estimation of length, weight, volume/capacity, time, area and temperature' (CCEA, 2007, p. 66).

5.3. Number line estimation

There is no reference, in any context, to the expression number line.

5.4. Quantity estimation

Quantity estimation is only obliquely referenced. For example, in years 1 and 2, in statements redolent of subitizing, children 'should be enabled to … state, without counting, quantities within 5' and 'make a sensible guess of quantities within 10' (CCEA, 2007, p. 24). In years 3 and 4, 'pupils should be enabled to … make a sensible estimate of a small number of objects and begin to approximate to the nearest 10 or 100' (CCEA, 2007, p. 62).

In sum, although there are some general expressions concerning the importance of estimation in children's learning, the details are focused on checking calculations, whether manually or with a calculator. The lack of reference to rounding suggests that the details of computational estimation are left for teachers to decide. As far as measurement estimation is concerned, there seems to be a clear cognitive progression with respect to the properties of physical objects. Finally, quantity estimation is only obliquely referenced and number line estimation is missing.

6. Results: estimation in the primary national curriculum for Scotland

In Scotland the curriculum for compulsory school is structured in five phases, the first three of which refer to primary aged learners. The first of these refers to children aged four or five in what is known as year P1. The second refers to children aged five to eight in school years P2–P4, while the third refers to children aged eight to eleven in school years P5–P7.

Table 3. Documents scrutinised for the Scottish analysis.

Documents included in the analyses
Education Scotland (2016a) National numeracy and mathematics progression framework. Edinburgh, Education Scotland. Hereafter, ES (2016), this document provides extensive non-statutory support for teachers and includes relevant references to estimation in different forms.
Education Scotland (2019a) Curriculum for excellence: Numeracy and mathematics experiences and outcomes. Edinburgh, Education Scotland. Hereafter, ES (2019a), this document summarises the content that teachers are encouraged to address. Material to be covered is typically presented in the in the form of 'I can … ' or 'I have … ' statements for children.
Education Scotland (2017) Benchmarks numeracy and mathematics. Edinburgh, Education Scotland. Hereafter, ES (2017), this document, while including all the learning goals specified in ES (2019a), offers assessment guidance through statements prefaced with the phrase 'learners need to … '. In particular, frequent reference is made to estimation in different forms.
Documents not included in the analyses
Education Scotland (2016b) Curriculum for excellence: A statement for practitioners from HM Chief Inspector of Education. Edinburgh, Education Scotland. This document offers non-statutory curriculum-related support for teachers at general levels but nothing with respect to the particularities of estimation.
Education Scotland (2019b) Curriculum for excellence: Mathematics principles and practice. Edinburgh, Education Scotland. This document outlines the curricular justification of mathematics and, while including repetitions of the broad headings found in other documents, offers nothing with respect to estimation.

The Scottish educational authorities do not specify a statutory set of learning outcomes but a series of learner entitlements. This distinction, as is apparent below, is reflected in the manner in which learning goals and support materials, shown in Table 3, are presented.

6.1. Computational estimation

From the curricular perspective, year-one children should be able to 'share ideas with others to develop ways of estimating the answer to a calculation or problem, work out the actual answer, then check (their) solution by comparing it with the estimate' (ES, 2019a, p. 2), while in year two they should be able to use 'knowledge of rounding to routinely estimate the answer to a problem then, after calculating, decide if (their) answer is reasonable, sharing my solution with others' (ES, 2019a, p. 2). Both goals are supported by a range of statements found elsewhere. For example, the benchmarks for phase one add that a child 'uses strategies to estimate an answer to a calculation or problem, for example, doubling and rounding' and 'rounds whole numbers to the nearest 10 and 100 and uses this routinely to estimate and check the reasonableness of a solution' (ES, 2017, p. 13). By the end of the second phase it is expected that a child 'rounds whole numbers to the nearest 1000, 10,000 and 1,00,000 … rounds decimal fractions to the nearest whole number, to one decimal place and two decimal places (and) applies knowledge of rounding to give an estimate to a calculation appropriate to the context' (ES, 2017, p. 21), while by the end of the third phase, he or she 'rounds decimal fractions to three decimal places (and) uses rounding to routinely estimate the answers to calculations' (ES, 2017, p. 32). By way of justification, the progression document suggests that 'as this skill (estimation) becomes more refined, learners will be able to predict solutions and check the accuracy of calculations' (ES, 2016a, p. 7). Finally, the progression document, connecting estimation to simple monetary calculations, presents an expectation that children should be 'able to estimate and calculate costs', 'estimate and calculate change', 'estimate or calculate the total cost of goods or services', adding that children's 'mental strategies can involve rounding' (ES, 2016a, p. 104).

On one occasion, presented independently of estimation, there is a curricular expectation that by the end of phase two, children will be able to 'round a number using an appropriate degree of accuracy, having taken into account the context of the problem' (ES, 2019a, p. 2). This single curricular reference to rounding, it could be argued, is warranted by a statement in the progression document, whereby the 'ability to round supports the development of mental agility. It also allows for quick estimations to be made in calculations and to check the reasonableness of a solution' (ES, 2016a, p. 8), before asserting that 'rounding accurately is an essential component of determining the reasonableness of a solution' (ES, 2016a, p. 10). Also, the progression document includes six statements relating to rounding, most of which we construe as referring to computational estimation. Among these are expectations that children will 'understand that a rounded value is not equal to the original value', 'explain what rounding means using vocabulary of estimation', 'determine the reasonableness of an outcome' and 'understand that there are acceptable degrees of accuracy required in calculations' (ES, 2016a, p. 11).

6.2. Measurement estimation

With respect to the curricular goals, by the end of phase one, children should be able to 'estimate how long or heavy an object is, or what amount it holds, using everyday things as a guide, then measure or weigh it using appropriate instruments and units' and 'estimate the area of a shape by counting squares or other methods' (ES, 2019a, p. 8). By the end of the second phase, they should use 'knowledge of the sizes of familiar objects or places … when making an estimate of measure'. From the perspective of time, by the end of phase two, children should 'using simple time periods … give a good estimate of how long a journey should take, based on … knowledge of the link between time, speed and distance'. By the end of the third phase this expectation is repeated, but with the insertion of a reference to 'the speed travelled at or distance covered' (ES, 2019a, p. 7).

By way of justification, the progression document suggests that 'measuring and estimating with non-standard units develops understanding of why standard units are necessary and help to provide an estimation of size' (ES, 2016a, p. 166), before adding that the 'ability to estimate volume is built on an understanding of how to estimate other properties of shapes … ' and reaffirms 'that an estimated value is not exact' (ES, 2016a, p. 176). These justifications are matched by various assessment-related outcomes whereby, by the end of phase one, a child 'estimates, then measures, the length, height, mass and capacity of familiar objects using a range of appropriate non-standard units' (ES, 2017, p. 10), 'uses knowledge of everyday objects to provide reasonable estimates of length, height, mass and capacity' (ES, 2017, p. 16) and 'compares measures with estimates' and 'uses square grids to estimate then measure the areas of a variety of simple 2D shapes to the nearest half square' (ES, 2017, p. 17). By the end of the second phase he or she 'uses the comparative size of familiar objects to make reasonable estimations of length, mass, area and capacity' and 'estimates to the nearest appropriate unit, then measures accurately', by means of various standard units, 'length, height and distance … mass … and capacity' (ES, 2017, p. 25).

From the perspective of time, the progression document argues not only that children should 'be able to round appropriately' (ES, 2016a, p. 135) but also develop the 'ability to estimate how long an event took or will take, using non-standard or standard units of time (and) a sense of how long a task will take, by using familiar benchmarks' (ES, 2016a, p. 141). It also suggests, with respect to travel, that 'calculating journey times is an introduction to establishing the relationship between time, speed and distance and sets the foundation for more complex calculations and estimation' (ES, 2016a, p. 146), where the use of 'timetables helps to develop mental agility in relation to time calculations and develops skills in estimation and in rounding' (ES, 2016a, p. 137), not least because 'estimations are used in daily situations to determine either an approximate arrival time, speed or distance for a journey' (ES, 2016a, p. 154). From the perspective of assessment, the benchmarks document asserts that by the end of the second phase, a child 'estimates the duration of a journey based on knowledge of the link between speed, distance and time' (ES, 2017, p. 24).

In sum, benchmark document summarizes the estimation-related aims of the first phase by asserting that a child 'demonstrates skills of estimation in the contexts of number and measure using relevant vocabulary, including less than, longer than, more than and the same' (ES, 2017, p. 8), while the progression document writes that tolerance 'relates to acceptable margins of error when measuring, estimating or calculating measurements. Understanding of tolerance in measurement is appreciation of accuracy when making calculation' (ES, 2016a, p. 193). Also, despite no reference to the estimation of angle in the curricular expectations, the benchmarks include an expectation that by the end of the first phase an individual 'uses informal methods to estimate, compare and describe the size of angles in relation to a right angle' (ES, 2017, p. 19).

6.3. Number line estimation

There are no explicit references to number line estimation in any of the documents. There are allusions, as with the single curricular expectation that by the end of phase one children know 'where simple fractions lie on the number line' (ES, 2019a, p. 5). By way of justification, the progression document suggests that 'comparing size and amount supports the development of appropriate language relating to quantities (and) supports an understanding of where numbers sit on a number line' (ES, 2016a, p. 4). It adds, more specifically, that children should be 'able to place fractions, decimal fractions and percentages on a number line' (ES, 2016a, p. 88) or, more generally, be able 'to place different forms in order on a number line and know the relative value of each one' (ES, 2016a, p. 89). The benchmark document suggests that by the end of the early phase, a child 'finds missing numbers on a number line within the range 0–20' (ES, 2017, p. 10) and, by the end of phase one, 'compares the size of fractions and places simple fractions in order on a number line ' (ES, 2017, p. 14). However, none of these statements are explicitly tied to number line estimation.

6.4. Quantity estimation

Like number line estimation, quantity estimation is barely addressed in any document, although there is a curricular expectation that year-one children should develop 'a sense of size and amount by observing, exploring, using and communicating with others about things in the world around' (ES, 2019a, p. 2). This somewhat vague statement, which is only tangentially connected to estimation, is supported by various statements in the other documents, typically alluding to subitizing. For example, the progression document describes subitizing as 'recognising a quantity without counting, simply by looking' (ES, 2016a, p. 28). The same document adds that by learning how to round, children would be able to 'give an increasingly accurate estimation of the quantity of a given set' (ES, 2016a, p. 11). Also, implicitly offering a didactical perspective, the materials suggest that children should be able to 'identify quantities and patterns to make quick estimates' (ES, 2016a, p. 29) before adding that they should be able 'recognise the amount of objects in a group and use this information to estimate the amount of objects in a larger group' (ES, 2016a, p. 30). Finally, the benchmarks suggest that by the end pre-school, a child 'recognises the number of objects in a group, without counting (subitising) and uses this information to estimate the number of objects in other groups', 'checks estimates by counting' and 'identifies "how many?" in regular dot patterns, for example, arrays, five frames, ten frames, dice and irregular dot patterns, without having to count (subitising)' (ES, 2017, p. 8).

In sum, the Scottish curricular expectations with respect to estimation, as evidenced in the many occurrences of keywords, appear extensive, driven not only by a sense of pragmatics but also a view that estimation enhances mental agility. Computational estimation is broadly construed as a check to calculation in various contexts, a process supported by rounding, which also seems to be an end in itself. Measurement estimation permeates the expected experiences of all children, particularly its emphases on the physical properties of objects and time, while acknowledging the role of non-standard units in the development of such learning. Number line estimation is effectively absent, although invocations to place different forms of number on a number line could, under particular circumstances, involve estimation. Finally, the extent to which children are expected to engage with quantity estimation is limited, with young children being expected to estimate the number of objects in a group.

7. Results: estimation in the primary national curriculum for Wales

The statutory expectations of Welsh primary education are structured in two phases. The first, the foundation stage, covers children from pre-school until the end of year two – effectively ages 3 through 7. Of the three analyzed documents shown in Table 4, one, CW (2016b), repeats much of the material found in CW (2016a) and is referenced only when it offers additional insights into how the Welsh authorities construe estimation as part of a general education. While most statements relating to estimation can be categorized against the four forms, occasionally estimation is presented as a general competence. This is particularly the case for the foundation stage, where there is a broad goal that children 'explore, estimate and solve real-life problems in both the indoor and outdoor environment' (CW, 2015, p. 27) and that

> Before children can begin to make estimations based upon approximate calculations of number, they need to develop an understanding of the process of making a reasonable guess of a visual or physical measure, and checking their estimation using appropriate methods (CW, 2017, p. 82).

Table 4. Documents scrutinized for the Welsh analysis.

Documents included in the analyses
Curriculum for Wales (2015). Foundation phase framework. Cardiff: Department for Education and Skills. Hereafter, CW (2015), this document outlines the statutory learning goals for children in the first phase of education and incorporates material relevant to estimation.
Curriculum for Wales (2016a) Programme of study for mathematics key stages 2–4. Cardiff: Department for Education and Skills. Hereafter, CW (2016a), this document outlines the statutory learning goals for all children in compulsory school, including those related to estimation.
Curriculum for Wales (2016b) *Numeracy framework*. Cardiff: Department for Education and Skills. Hereafter CW (2016b), this document, while repeating much of what is written in the previous two, offers some additional statements indicating how the Welsh authorities construe number-related learning in general and estimation in particular.
Documents not included in the analyses
Curriculum for Wales (2017) *Foundation phase profile handbook*. Cardiff: Department for Education and Skills. This document offers extensive assessment-related advice to teachers, including much material related to estimation. However, this material essentially replicates that found in the programme of study.
Curriculum for Wales (2020). *Curriculum for Wales guidance*, Cardiff: Department for Education and Skills. This document, in addition to presenting a range of broad aims of no relevance to estimation, repeats the goals found in the programme of study as a set of 'I can … ' statements. This repetition led to its being excluded from the analysis.

In similar vein, across all years from reception to year two, the numeracy framework expects children to be able to 'use knowledge and practical experience to inform estimations' (CW, 2016b, p. 1).

7.1. Computational estimation

With respect to the foundation stage, a series of statutory statements, typically beneath a broad heading of *estimation and checking*, allude to computational estimation. These include, presented as a progression of expected learning outcomes, that children will initially 'use estimation and checking with calculations', 'use a variety of estimation and checking strategies that are appropriate to calculations' and 'use finer estimations and checking strategies including inverse addition/subtraction and halving/doubling' (CW, 2017, p. 82). With respect to key stage two, the national curriculum, under the broad heading of *use number facts and relationships*, expects children to be able to 'compare and estimate with numbers up to 100' and 'compare and estimate with numbers up to 1000' by the ends of years three and four respectively (CW, 2016a, p. 3). A similar set of goals is presented under the broad heading of *estimate and check*. Here, children in years four, five and six should be able to 'estimate by rounding to the nearest 10 or 100', 'estimate by rounding to the nearest 10, 100 or 1,000'and 'estimate by rounding to the nearest 10, 100, 1,000 or whole number' respectively (CW, 2016a, p. 5).

7.2. Measurement estimation

By the end of year one, the foundation curriculum asserts that children should 'make a sensible estimate of measurement in length, height, weight and capacity that can be checked using non-standard measures' and, by the end of year two, be able to do the same with respect to 'standard measures' (CW, 2015, p. 33). Also, within the hierarchy of level descriptors, there is an expectation that children should 'use a variety of estimation and checking strategies that are appropriate to … measurements' (CW, 2015, p. 55). Across all years of key stage two there is a broad statutory expectation that children should be able to 'estimate and visualise size when measuring and use the correct units' (CW, 2016a, p. 2). More particularly, by the ends of years three, four, five and six they should 'use standard units to estimate and measure' length, weight/mass and capacity, 'select and use appropriate standard units to estimate and measure length, weight/mass and capacity', 'make estimates of length, weight/mass and capacity based on knowledge of the size of real-life objects' and 'make estimates of length, weight/mass and capacity based on knowledge of the size of real-life objects, recognising the appropriateness of units in different contexts' (CW, 2016a, p. 6) respectively. Also, by the end of year five, children should be able to 'calculate, estimate and compare the area of squares and rectangles using standard units' (CW, 2016a, p. 8).

In addition to measurements of the properties of physical objects, key stage two also includes an emphasis on estimation in relation to time. For example, by the ends of year four, five and six, children should be able to 'estimate the number of minutes everyday activities take to complete', 'estimate the length of time everyday activities take to complete, extending to hours and quarters of hours' and 'estimate the length of time everyday activities take to complete with increasing accuracy' respectively (CW, 2016a, p. 7). In related vein, year six children should be able to 'estimate how long a journey takes' (CW, 2016a, p. 7). Finally, the only measurement estimation statements found in the numeracy framework mirror those above.

7.3. Number line estimation

Although there are no explicit references to estimation in relation to the number line in any of the documents, there is an oblique key stage two expectation that by the end of year four, children will be able to 'identify negative whole numbers on a number line' (CW, 2016a, p. 5). Our view is, depending on the context of such activities, that elements of estimation may be inferred. However, as this is the only (out of two number line references in the whole document) reference of this type, it is not unlikely that this may be an inference too far.

7.4. Quantity estimation

With respect to the foundation stage, the hierarchical list of outcomes suggests, respectively, that children will 'begin to make a sensible estimate of up to five objects', an objective clearly not unconnected to subitizing, 'make a sensible estimate up to 10 and understand that this can be checked by counting' and be able to 'make sensible estimates of larger groups of objects' (CW, 2017, p. 82). There are no quantity estimation expectations explicitly presented in the key stage two curriculum, although the numeracy framework asserts that year-one children should be able to 'make a sensible estimate of a number of objects that can be checked by counting' (CW, 2016b, p. 1).

In sum, while the Welsh national curriculum includes clear expectations of estimation permeating children's learning of mathematics at all levels, these expectations relate primarily to computational and measurement estimation alongside limited quantity and, effectively, no number line estimation. With respect to computational estimation, the documents offer clear year-on-year expectations concerning the rounding of numbers but fail to specify, beyond broad statements, how estimates facilitate the checking of calculations. With respect to measurement estimation, the goals are clearly tied to the properties of various physical objects and time. Finally, quantity estimation is given a very limited treatment, and only in the foundation stage, while number line estimation is effectively absent.

8. Discussion and implications

In the above, we presented a summary of the literature concerning the form and function of different forms of estimation in order to frame qualitative analyses of the estimation-related opportunities found in the national curricula of the four education systems of the UK. Such a study is necessary, we believe, for two main reasons. On the one hand, research has increasingly highlighted the importance of estimation as both an end in itself and a predictor of later mathematical achievement. On the other hand, the competence of both adults and children with respect to a core competence of adult life is historically poor (Joram et al., 1998). In the following, we discuss the opportunities found in the four curricula from the perspectives of their similarities, differences, strengths and weaknesses.

First, all four curricula emphasize computational estimation as a way of checking calculations. This emphasis is clear in the following statements: English children should 'use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy' (DfE, 2013b, 2014, p. 40), Northern Irish children should 'estimate and approximate to gain an indication of the size of a solution to a calculation or problem' (CCEA, 2007, p. 65), Scottish children should 'predict solutions and check the accuracy of calculations' (ES, 2016, p. 7), while their Welsh peers should 'use a variety of estimation and checking strategies that are appropriate to calculations' (CW, 2017, p. 82). The similarity of these statements indicates a common view of computational estimation. However, nowhere in any of the curricula is there an indication that computational estimations may take 'less time and attentional resources than exact calculation, and thus can be used in circumstances where time or attention resources are limited' (Ganor-Stern, 2018, p. 2). That is, although the Northern Ireland document refers explicitly to calculators in this context, the role of computational estimation as an essential life skill (Ganor-Stern, 2016; Sekeris et al., 2019) is left implicit. Moreover, none of the documents offered any indication that the skills of computational estimation are implicated in the later learning of mathematics, whether in respect of particular topic areas (Ganor-Stern, 2018; Sowder, 1992), mathematics in general (Sekeris et al., 2019) or problem solving (Star & Rittle-Johnson, 2009). If there were any such expectations, they too have been left implicit. Furthermore, when viewed alongside the lack of estimation-related opportunities in school mathematics textbooks (Sayers et al., 2020b), the lack of curricular attention to the processes of computational estimation, particularly reformulation, translation and compensation (Alajmi, 2009; Boz & Bulut, 2012; LeFevre et al., 1993; Sekeris et al., 2019), may lead to a generation of adults whose low levels of numeracy leave them susceptible, for example, to misinformation about Covid-19 (Roozenbeek et al., 2020).

Second, the role of rounding in computational estimation distinguishes curricula in a number of ways. On the one hand, the curriculum for Northern Ireland makes no reference to rounding, perhaps implying that teachers should decide for themselves how computational estimation should be undertaken. On the other hand, there is a superficial similarity to be seen in the extent to which the English, Scottish and Welsh curricula emphasize rounding as a process. That is, statements like *round to the nearest 100* are found repeatedly in all three documents, warranting a conclusion that rounding is construed as an end in itself. That being said and despite the single assertion that year-five English children should be able to 'use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy' (DfE, 2013b, p. 32), the generality in both England and Wales was that teachers were left to make such necessary connections for themselves. The major contrast concerned the Scottish curriculum, which discusses rounding as part of the process of computational estimation. For example, it asserted that the 'ability to round supports the development of mental agility. It also allows for quick estimations to be made in calculations and to check the reasonableness of a solution' (ES, 2016, p. 8). Moreover, the document adds that children should be able to 'explain what rounding means using vocabulary of estimation' (ES, 2016, p11). In short, the Scottish curriculum makes a strong case for connecting the process of rounding to that of estimation, an emphasis that should prevent children acquiring a misconception with respect to the purpose of estimation (Liu, 2009). Overall, however, all curricula fail to acknowledge that for both adults and children the process of rounding is cognitively demanding and negatively influenced by distractions (Ardiale & Lemaire, 2013; Taillan et al., 2015).

Third, measurement estimation was a common thread across all four curricula, typically focusing on the physical properties of objects. In this respect, expectations concerning, for example, length and area were common across curricula. However, estimations concerning other properties varied. Weight was mentioned in Northern Ireland and Wales, while mass was discussed in Wales and England, implying, at least from the perspective of estimation, such matters were not part of the Scottish discourse and that only the Welsh distinguished between the two measures. Volume was mentioned only in Scotland and England, although 'volume/capacity' was mentioned in Northern Ireland, and capacity was mentioned independently in England and Wales. Thus, only in England could be found a distinction between the two forms of measure, while the implication for Northern Ireland is that they are synonymous. However, despite evidence that the properties to be estimated were typically presented in sequences indicative of a developmental progression (length coming before area and so on), the lack of coherence across the four curricula is unlikely to overcome earlier findings that children and adults make poor measurement estimations (Joram et al., 1998) and may contribute to teacher uncertainty about its teaching (Joram et al., 2005; Pizarro et al., 2015). Moreover, the lack of any reference to the processes of measurement estimation, particularly the use of mental referents (Jones et al., 2012; Joram et al., 2005; Kramer et al., 2018; Sowder, 1992) is likely to contribute to a continuation of such uncertainty. Finally, despite vague allusions in both the English and the Scottish curricula, estimation of angle was effectively absent in all four curricula. This, in light of evidence highlighting adults' inability to estimate the angles of real-world slopes (Creem-Regehr et al., 2004; Proffitt et al., 2001), seems a disappointing omission.

Fourth, also pertaining to measurement estimation, was the extent to which curricula differ with respect to children's use of non-standard units, which research has shown to be a powerful underpinning of conceptual and procedural competence (Anestakis & Desli, 2014; Chang et al., 2011). Here, the Scottish and Welsh curricula see a role for non-standard units, with the Scottish asserting that 'measuring and estimating with non-standard units develops understanding of why standard units are necessary and help to provide an estimation of size' (ES, 2016, p. 166), while the Welsh alludes to the same notion by expecting children to 'make a sensible estimate of measurement in length, height, weight and capacity that can be checked using non-standard measures' (CW, 2015, p. 33). However, while the curriculum for Northern Ireland asserts that children should be able to 'use non-standard units to measure and recognise the need for standard units' (CCEA, 2007, p. 63), nothing is said with respect to estimation. In similar vein, while the non-statutory material for England expects pupils to 'move from using and comparing different types of quantities and measures using non-standard units … to using manageable common standard units' (DfE, 2013b, p. 9), nothing is said with respect to estimation. The omissions in the English and Welsh curricula seem disappointing, as young children's estimates are known to be more accurate when first they use non-standard units (Desli & Giakoumi, 2017).

Fifth, expectations with respect to time estimation (measurement) varied across curricula. For example, in both the English and the Northern Irish curricula, time is presented as one element of a list of properties, like length or area, to be estimated. In Wales, there is the addition of context, as in the expectation that children will be able to 'estimate the length of time everyday activities take to complete with increasing accuracy' (CW, 2016a, p. 7), while in Scotland, the curriculum offers extensive contexts and warrants for time estimation, as in the statement 'using simple time periods, I can give a good estimate of how long a journey should take, based on my knowledge of the link between time, speed and distance' (ES, 2017, p. 25). However, acknowledging research showing that educated adults underestimate time (Koivula, 1996), children with mathematical learning difficulties overestimate time (Hurks & van Loosbroek, 2014) and that time estimation accuracy is a predictor of general mathematical competence (Kramer et al., 2011, 2018), it seems that Scotland may have addressed the issue more effectively than its neighbours.

Sixth, quantity estimation, which is reciprocally linked to the ability to count (Barth et al., 2009) and a predictor of later arithmetical competence (Bartelet et al., 2014; Wong et al., 2016) receives only nominal attention in Northern Ireland, Scotland and Wales and is effectively absent in England.

Seventh, number line estimation is effectively absent in all four countries' curricula. Such an omission seems unfortunate, particularly as number line estimation is a strong predictor of later mathematical learning difficulties (Andersson & Östergren, 2012; Wong et al., 2017) and all manner of later learning (Fuchs et al., 2010; Schneider et al., 2018; Simms et al., 2016; Tosto et al., 2017). Our interpretation of the failure of the different curricular authorities to included number line estimation may be a consequence of their failing to keep up with important trends in mathematics education research. Indeed, of all the forms of estimation we discuss, it is the one with the most significant implications for the later learning of mathematics.

In closing, we return to the earlier concern that the 'cursory treatment given to estimation in most mathematics programs is insufficient to build any appreciable estimation' (Bestgen et al., 1980, p. 124) to warrant our asking, has anything changed? Our view is that the four national curricula of the United Kingdom offer limited perspectives, located in utilitarian expectations that fail to acknowledge the developmental significance of estimation. This seems particularly the case in England and Wales, where the process of rounding, presented as an independent learning objective, cannot be construed as anything but utilitarian. One might concede that the non-statutory Scottish curriculum, which incorporates a limited justification for the curricular inclusion of estimation and its role in the development of mental agility, is a deviation from the skills-driven curricula of the other three nations, particularly as rounding is unequivocally justified as an aid to computational estimation and the importance of estimating time is clearly argued. However, all four curricula effectively fail to acknowledge, particularly with respect to the very limited attention paid to quantity estimation and number line estimation, that learning how to estimate has profound implications for the successful learning of other areas of mathematics (Wong et al., 2016).

This systemic lack of attention to estimation is not unique to the UK. It is known, for example, that estimation is equally poorly addressed in the curricula of the three Scandinavian countries (Sunde et al., Submitted), as it is in the mathematics textbooks of Finland, Singapore and Sweden (Sayers et al., 2020b). Collectively, such findings indicate that children, internationally, may be leaving school with little developed estimation-related understanding or competence. Put bluntly, unless curriculum authorities take appropriate action to address the problem, their failure to facilitate the learning of estimation, in all its forms, may not only hinder children's mathematical growth (Sasanguie et al., 2013; Schneider et al., 2009) but seriously inhibit adults' real-world functioning (Booth & Siegler, 2006; Sekeris et al., 2019), particularly as high levels of education in general (Van Prooijen, 2017) and numeracy in particular (Roozenbeek et al., 2020) are necessary for countering the spread of 'fake news'.

Acknowledgment

The research reported in this paper was funded in part by the Swedish Research Council (Vetenskapsrådet), project grant 2015-01066.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

The research reported in this paper was funded in part by the Swedish Research Council (Vetenskapsrådet), project grant 2015-01066.

ORCID

Paul Andrews http://orcid.org/0000-0003-3679-9187
Constantinos Xenofontos http://orcid.org/0000-0003-2841-892X
Judy Sayers http://orcid.org/0000-0002-9652-0187

References

Ainsworth, S., Bibby, P., & Wood, D. (2002). Examining the effects of different multiple representational systems in learning primary mathematics. *Journal of the Learning Sciences*, *11*(1), 25–61. https://doi.org/10.1207/S15327809JLS1101_2

Alajmi, A. (2009). Addressing computational estimation in the Kuwaiti curriculum: Teachers' views. *Journal of Mathematics Teacher Education*, *12*(4), 263–283. https://doi.org/10.1007/s10857-009-9106-3

Albarracín, L., & Gorgorió, N. (2019). Using large number estimation problems in primary education classrooms to introduce mathematical modelling. *International Journal of Innovation in Science and Mathematics Education*, *27*(2), 45–57. https://doi.org/10.30722/IJISME.27.02.004

Andersson, U., & Östergren, R. (2012). Number magnitude processing and basic cognitive functions in children with mathematical learning disabilities. *Learning and Individual Differences*, *22*(6), 701–714. https://doi.org/10.1016/j.lindif.2012.05.004

Anestakis, P., & Desli, D. (2014). Computational estimation in primary school: Tasks proposed for its teaching. *MENON: Journal of Educational Research, Thematic Issue no. 1*, 75–89. http://www.edu.uowm.gr/site/content/1st-thematic-issue-december-2014

Ardiale, E., & Lemaire, P. (2013). Effects of execution duration on within-item strategy switching in young and older adults. *Journal of Cognitive Psychology*, *25*(4), 464–472. https://doi.org/10.1080/20445911.2013.789854

Ärlebäck, J., & Bergsten, C. (2013). On the use of realistic Fermi problems in introducing mathematical modelling in upper secondary mathematics. In R. Lesh, P. Galbraith, C. Haines, & A. Hurford (Eds.), *Modeling students' mathematical Modeling Competencies* (pp. 597–609). Springer Netherlands.

Ashcraft, M., & Moore, A. (2012). Cognitive processes of numerical estimation in children. *Journal of Experimental Child Psychology*, *111*(2), 246–267. https://doi.org/10.1016/j.jecp.2011.08.005

Bailey, D., Siegler, R., & Geary, D. (2014). Early predictors of middle school fraction knowledge. *Developmental Science*, *17*(5), 775–785. https://doi.org/10.1111/desc.12155

Baroody, A., & Gatzke, M. (1991). The estimation of set size by potentially gifted kindergarten-age children. *Journal for Research in Mathematics Education*, *22*(1), 59–68. https://doi.org/10.5951/jresematheduc.22.1.0059

Bartelet, D., Vaessen, A., Blomert, L., & Ansari, D. (2014). What basic number processing measures in kindergarten explain unique variability in first-grade arithmetic proficiency? *Journal of Experimental Child Psychology*, *117*, 12–28. https://doi.org/10.1016/j.jecp.2013.08.010

Barth, H., Starr, A., & Sullivan, J. (2009). Children's mappings of large number words to numerosities. *Cognitive Development*, *24*(3), 248–264. https://doi.org/10.1016/j.cogdev.2009.04.001

Berteletti, I., Lucangeli, D., Piazza, M., Dehaene, S., & Zorzi, M. (2010). Numerical estimation in preschoolers. *Developmental Psychology*, *46*(2), 545–551. https://doi.org/10.1037/a0017887

Bestgen, B., Reys, R., Rybolt, J., & Wyatt, J. (1980). Effectiveness of systematic instruction on attitudes and computational estimation skills of preservice elementary teachers. *Journal for Research in Mathematics Education*, *11*(2), 124–136. https://doi.org/10.5951/jresematheduc.11.2.0124

Bicknell, B., & Young-Loveridge, J. (2015). Young children's number line placements and place-value understanding. In M. Marshman, V. Geiger, & A. Bennison (Eds.), *Mathematics education in the margins* (pp. 101–108). Mathematics Education Research Group of Australasia.

Binkley, M., Erstad, O., Herman, J., Raizen, S., Ripley, M., Miller-Ricci, M., & Rumble, M. (2012). Defining twenty-first century skills. In P. Griffin, B. McGaw, & E. Care (Eds.), *Assessment and teaching of 21st century skills* (pp. 17–66). Springer Netherlands.

Bobis, J. (1991). The effect of instruction on the development of computational estimation strategies. *Mathematics Education Research Journal*, *3*(1), 17–29. https://doi.org/10.1007/BF03217219

Bonner, B., Sillito, S., & Baumann, M. (2007). Collective estimation: Accuracy, expertise, and extroversion as sources of intra-group influence. *Organizational Behavior and Human Decision Processes*, *103*(1), 121–133. https://doi.org/10.1016/j.obhdp.2006.05.001

Booth, J., & Newton, K. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology*, *37*(4), 247–253. https://doi.org/10.1016/j.cedpsych.2012.07.001

Booth, J., Newton, K., & Twiss-Garrity, L. (2014). The impact of fraction magnitude knowledge on algebra performance and learning. *Journal of Experimental Child Psychology*, *118*, 110–118. https://doi.org/10.1016/j.jecp.2013.09.001

Booth, J., & Siegler, R. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology*, *42*(1), 189–201. https://doi.org/10.1037/0012-1649.41.6.189

Booth, J., & Siegler, R. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*, *79*(4), 1016–1031. https://doi.org/10.1111/j.1467-8624.2008.01173.x

Boz, B., & Bulut, S. (2012). A case study about computational estimation strategies of seventh graders. *Elementary Education Online*, *11*(4), 979–994. http://ilkogretim-online.org.tr/index.php/io/article/download/1460/1316

Chang, K.-L., Males, L., Mosier, A., & Gonulates, F. (2011). Exploring US textbooks' treatment of the estimation of linear measurements. *ZDM*, *43*(5), 697–708. https://doi.org/10.1007/s11858-011-0361-2

Council for the Curriculum, Examinations and Assessment. (2007). *The Northern Ireland curriculum primary*. CCEA.

Creem-Regehr, S., Gooch, A., Sahm, C., & Thompson, W. (2004). Perceiving virtual geographical slant: Action influences perception. *Journal of Experimental Psychology: Human Perception and Performance*, *30*(5), 811–821. https://doi.org/10.1037/0096-1523.30.5.811

Crites, T. (1992). Skilled and less skilled estimators' strategies for estimating discrete quantities. *The Elementary School Journal*, *92*(5), 601–619. https://doi.org/10.1086/461709

Curriculum for Wales. (2015). *Foundation phase framework*. Department for Education and Skills.

Curriculum for Wales. (2016a). *Programme of study for mathematics key stages 2–4*. Department for Education and Skills.

Curriculum for Wales. (2016b). *Numeracy framework*. Department for Education and Skills.

Curriculum for Wales. (2017). *Foundation phase profile handbook*. Department for Education and Skills.

Curriculum for Wales. (2020). *Curriculum for Wales guidance*. Department for Education and Skills.

Dackermann, T., Huber, S., Bahnmueller, J., Nuerk, H.-C., & Moeller, K. (2015). An integration of competing accounts on children's number line estimation. *Frontiers in Psychology*, *6*, 884–884. https://doi.org/10.3389/fpsyg.2015.00884

Department for Education. (2013a). *Early years outcomes: A non-statutory guide for practitioners and inspectors to help inform understanding of child development through the early years*.

Department for Education. (2013b). *Mathematics programmes of study: Key stages 1 and 2*.

Department for Education. (2014). *The national curriculum in England framework document*.

Department for Education. (2017). *Statutory framework for the early years foundation stage: Setting the standards for learning, development and care for children from birth to five*.

Desli, D., & Giakoumi, M. (2017). Children's length estimation performance and strategies in standard and non-standard units of measurement. *International Journal for Research in Mathematics Education*, *7*(3), 61–84. http://sbem.iuri0094.hospedagemdesites.ws/revista/index.php/ripem/article/view/1381/pdf

DeWolf, M., Bassok, M., & Holyoak, K. (2015). From rational numbers to algebra: Separable contributions of decimal magnitude and relational understanding of fractions. *Journal of Experimental Child Psychology*, *133*, 72–84. https://doi.org/10.1016/j.jecp.2015.01.013

Dietrich, J., Huber, S., Dackermann, T., Moeller, K., & Fischer, U. (2016). Place-value understanding in number line estimation predicts future arithmetic performance. *British Journal of Developmental Psychology*, *34*(4), 502–517. https://doi.org/10.1111/bjdp.12146

Dowker, A. (1992). Computational estimation strategies of professional mathematicians. *Journal for Research in Mathematics Education*, *23*(1), 45–55. https://doi.org/10.5951/jresematheduc.23.1.0045

Dowker, A. (1997). Young children's addition estimates. *Mathematical Cognition*, *3*(2), 140–153. https://doi.org/10.1080/135467997387452

Dowker, A. (2003). Young children's estimates for addition: The zone of partial knowledge and understanding. In A. Baroody, & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 243–265). Erlbaum.

Ebersbach, M., Luwel, K., Frick, A., Onghena, P., & Verschaffel, L. (2008). The relationship between the shape of the mental number line and familiarity with numbers in 5- to 9-year old children: Evidence for a segmented linear model. *Journal of Experimental Child Psychology*, *99*(1), 1–17. https://doi.org/10.1016/j.jecp.2007.08.006

Ebersbach, M., & Wilkening, F. (2007). Children's intuitive mathematics: The development of knowledge about nonlinear growth. *Child Development*, *78*(1), 296–308. https://doi.org/10.1111/j.1467-8624.2007.00998.x

Education Scotland. (2016a). *National numeracy and mathematics progression framework*.
Education Scotland. (2016b). *Curriculum for excellence: A statement for practitioners from HM Chief Inspector of Education*.
Education Scotland. (2017). *Benchmarks numeracy and mathematics*.
Education Scotland. (2019a). *Curriculum for excellence: Numeracy and mathematics experiences and outcomes*.
Education Scotland. (2019b). *Curriculum for excellence: Mathematics principles and practice*.
Fazio, L., Bailey, D., Thompson, C., & Siegler, R. (2014). Relations of different types of numerical magnitude representations to each other and to mathematics achievement. *Journal of Experimental Child Psychology*, *123*, 53–72. https://doi.org/10.1016/j.jecp.2014.01.013
Forrester, M., & Pike, C. (1998). Learning to estimate in the mathematics classroom: A conversation-analytic approach. *Journal for Research in Mathematics Education*, *29*(3), 334–356. https://doi.org/10.5951/jresematheduc.29.3.0334
Forrester, M., & Shire, B. (1994). The influence of object size, dimension and prior context on children's estimation abilities. *Educational Psychology*, *14*(4), 451–465. https://doi.org/10.1080/0144341940140407
Friso-van den Bos, I., Kroesbergen, E., Van Luit, J., Xenidou-Dervou, I., Jonkman, L., Van der Schoot, M., & Van Lieshout, E. (2015). Longitudinal development of number line estimation and mathematics performance in primary school children. *Journal of Experimental Child Psychology*, *134*, 12–29. https://doi.org/10.1016/j.jecp.2015.02.002
Fuchs, L., Geary, D., Compton, D., Fuchs, D., Hamlett, C., & Bryant, J. (2010). The contributions of numerosity and domain-general abilities to school readiness. *Child Development*, *81*(5), 1520–1533. https://doi.org/10.1111/j.1467-8624.2010.01489.x
Ganor-Stern, D. (2016). Solving math problems approximately: A developmental perspective. *PLoS ONE*, *11*(5), 1–16. https://doi.org/10.1371/journal.pone.0155515
Ganor-Stern, D. (2018). Do exact calculation and computation estimation reflect the same skills? Developmental and individual differences perspectives. *Frontiers in Psychology*, *9*, (Article 1316). https://doi.org/10.3389/fpsyg.2018.01316
Ginsburg, N. (1996). Number bias, estimation, and sensation seeking. *Perceptual and Motor Skills*, *83*(3), 856–858. https://doi.org/10.2466/pms.1996.83.3.856
Gooya, Z., Khosroshahi, L., & Teppo, A. (2011). Iranian students' measurement estimation performance involving linear and area attributes of real-world objects. *ZDM*, *43*(5), 709–722. https://doi.org/10.1007/s11858-011-0338-1
Grossnickle, F. (1935). Practice material in the estimation of the quotient in long division found in current arithmetic workbooks. *The Journal of Educational Research*, *28*(9), 668–688. https://doi.org/10.1080/00220671.1935.10880538
Hansen, N., Jordan, N., Fernandez, E., Siegler, R., Fuchs, L., Gersten, R., & Micklos, D. (2015). General and math-specific predictors of sixth-graders' knowledge of fractions. *Cognitive Development*, *35*, 34–49. https://doi.org/10.1016/j.cogdev.2015.02.001
Holloway, I., & Ansari, D. (2009). Mapping numerical magnitudes onto symbols: The numerical distance effect and individual differences in children's mathematics achievement. *Journal of Experimental Child Psychology*, *103*(1), 17–29. https://doi.org/10.1016/j.jecp.2008.04.001
Hong, D., Choi, K., Runnalls, C., & Hwang, J. (2018). Do textbooks address known learning challenges in area measurement? A comparative analysis. *Mathematics Education Research Journal*, *30*(3), 325–354. https://doi.org/10.1007/s13394-018-0238-6
Hoth, J., Heinze, A., Weiher, D., Ruwisch, S., & Huang, H. (2019). Primary school students' length estimation competence—A cross-country comparison between Taiwan and Germany. In J. Novotná, & H. Moraová (Eds.), *Opportunities in learning and teaching Elementary mathematics* (pp. 201–211). Charles University.
Hurks, P., & van Loosbroek, E. (2014). Time estimation deficits in childhood mathematics difficulties. *Journal of Learning Disabilities*, *47*(5), 450–461. https://doi.org/10.1177/0022219412468161
Jones, M., Gardner, G., Taylor, A., Forrester, J., & Andre, T. (2012). Students' accuracy of measurement estimation: Context, units, and logical thinking. *School Science and Mathematics*, *112*(3), 171–178. https://doi.org/10.1111/j.1949-8594.2011.00130.x

Jones, M., & Taylor, A. (2009). Developing a sense of scale: Looking backward. *Journal of Research in Science Teaching*, *46*(4), 460–475. https://doi.org/10.1002/tea.20288

Joram, E., Gabriele, A., Bertheau, M., Gelman, R., & Subrahmanyam, K. (2005). Children's use of the reference point strategy for measurement estimation. *Journal for Research in Mathematics Education*, *36*(1), 4–23. https://doi.org/10.2307/30034918

Joram, E., Subrahmanyam, K., & Gelman, R. (1998). Measurement estimation: Learning to map the route from number to quantity and back. *Review of Educational Research*, *68*(4), 413–449. https://doi.org/10.3102/00346543068004413

Koivula, N. (1996). Estimation of time: Effects of locus of control, mental arithmetic and length of target interval. *Personality and Individual Differences*, *20*(1), 25–32. https://doi.org/10.1016/0191-8869(95)90022-M

Kramer, P., Bressan, P., & Grassi, M. (2011). Time estimation predicts mathematical intelligence. *PLoS ONE*, *6*(12), 1–5. https://doi.org/10.1371/journal.pone.0028621

Kramer, P., Bressan, P., & Grassi, M. (2018). The SNARC effect is associated with worse mathematical intelligence and poorer time estimation. *Royal Society Open Science*, *5*(8), 172362–172362. https://doi.org/10.1098/rsos.172362

Laughlin, P., Gonzalez, C., & Sommer, D. (2003). Quantity estimations by groups and individuals: Effects of known domain boundaries. *Group Dynamics: Theory, Research, and Practice*, *7*(1), 55–63. https://doi.org/10.1037/1089-2699.7.1.55

LeFevre, J.-A., Greenham, S., & Waheed, N. (1993). The development of procedural and conceptual knowledge in computational estimation. *Cognition and Instruction*, *11*(2), 95–132. https://doi.org/10.1207/s1532690xci1102_1

Lemaire, P., & Brun, F. (2014). Effects of strategy sequences and response–stimulus intervals on children's strategy selection and strategy execution: A study in computational estimation. *Psychological Research*, *78*(4), 506–519. https://doi.org/10.1007/s00426-013-0501-0

Lemaire, P., & Lecacheur, M. (2011). Age-related changes in children's executive functions and strategy selection: A study in computational estimation. *Cognitive Development*, *26*(3), 282–294. https://doi.org/10.1016/j.cogdev.2011.01.002

Levine, D. (1982). Strategy use and estimation ability of college students. *Journal for Research in Mathematics Education*, *13*(5), 350–359. https://doi.org/10.5951/jresematheduc.13.5.0350

Lipovec, A., & Ferme, J. (2017). The use of the reference point strategy for measurement estimation. In J. Novotná, & H. Moraová (Eds.), *Equity and diversity in elementary mathematics education* (pp. 311–318). Prague.

Liu, F. (2009). Computational estimation performance on whole-number multiplication by third- and fifth-grade Chinese students. *School Science and Mathematics*, *109*(6), 325–337. https://doi.org/10.1111/j.1949-8594.2009.tb18102.x

Mengual, E., Gorgorió, N., & Albarracín, L. (2015). The mathematical textbook as an obstacle in the learning of measure. *Quaderni Di Ricerca in Didattica*, *25*(Supplement 2), 175–183.

Mildenhall, P. (2009). A study of teachers' learning and teaching of computational estimation: Getting started. In C. Hurst, M. Kemp, B. Kissane, L. Sparrow, & T. Spencer (Eds.), *Mathematics: It's mine* (pp. 153–159). The Australian Association of Mathematics Teachers.

Mildenhall, P., & Hackling, M. (2012). The impact of a professional learning intervention designed to enhance year six students' computational estimation performance. In J. Dindyal, L. Cheng, & S. Ng (Eds.), *Mathematics education: Expanding horizons* (pp. 497–504). Mathematics Education Research Group of Australasia.

Palaigeorgiou, G., Chloptsidou, I., & Lemonidis, C. (2018). Computational estimation in the classroom with tablets, interactive selfie video and self-regulated learning. In M. Auer, & T. Tsiatsos (Eds.), *Interactive mobile communication technologies and learning* (pp. 860–871). Springer International Publishing.

Pizarro, N., Gorgorió, N., & Albarracín, L. (2015). Primary teacher' approach to measurement estimation activities. In K. Krainer, & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for research in mathematics education* (pp. 3227–3233). Charles University in Prague and ERME.

Praet, M., & Desoete, A. (2014). Number line estimation from kindergarten to grade 2: A longitudinal study. *Learning and Instruction*, *33*(1), 19–28. https://doi.org/10.1016/j.learninstruc.2014.02.003

Proffitt, D., Creem, S., & Zosh, W. (2001). Seeing mountains in mole hills: Geographical-slant perception. *Psychological Science*, *12*(5), 418–423. https://doi.org/10.1111/1467-9280.00377

Reys, R. (2001). Curricular controversy in the Math Wars: A battle without winners. *Phi Delta Kappan*, *83*(3), 255–258. https://doi.org/10.1177%2F003172170108300315

Reys, R., Reys, B., Nohda, N., Ishida, J., Yoshikawa, S., & Shimizu, K. (1991). Computational estimation performance and strategies used by fifth- and eighth-grade Japanese students. *Journal for Research in Mathematics Education*, *22*(1), 39–58. https://doi.org/10.5951/jresematheduc.22.1.0039

Reys, R., Rybolt, J., Bestgen, B., & Wyatt, J. (1982). Processes used by good computational estimators. *Journal for Research in Mathematics Education*, *13*(3), 183–201. https://doi.org/10.5951/jresematheduc.13.3.0183

Roozenbeek, J., Schneider, C., Dryhurst, S., Kerr, J., Freeman, A., Recchia, G., van der Bles, A., & van der Linden, S. (2020). Susceptibility to misinformation about COVID-19 around the world. *Royal Society Open Science*, *7*(10), Article number 201199. https://doi.org/10.1098/rsos.201199

Rouder, J., & Geary, D. (2014). Children's cognitive representation of the mathematical number line. *Developmental Science*, *17*(4), 525–536. https://doi.org/10.1111/desc.12166

Ruwisch, S., Heid, M., & Weiher, D. F. (2015). Measurement estimation in primary school: Which answer is adequate? In K. Beswick, T. Muir, & J. Fielding-Wells (Eds.), *Proceedings of 39th Conference of the international group for the psychology of mathematics Education* (Vol. 4, pp. 113–120). PME.

Sasanguie, D., Göbel, S., Moll, K., Smets, K., & Reynvoet, B. (2013). Approximate number sense, symbolic number processing, or number–space mappings: What underlies mathematics achievement? *Journal of Experimental Child Psychology*, *114*(3), 418–431. https://doi.org/10.1016/j.jecp.2012.10.012

Sayers, J., Petersson, R., Rosenqvist, E., & Andrews, P. (2020a). Estimation: An inadequately operationalised national curriculum competence. In R. Marks (Ed.), *Proceedings of the British Society for Research into Learning Mathematics* (1st ed., Vol. 40, p. Article 14). https://bsrlm.org.uk/wp-content/uploads/2020/05/BSRLM-CP-40-1-14.pdf

Sayers, J., Petersson, R., Rosenqvist, E., & Andrews, P. (2020b). Opportunities to learn foundational number sense in three Swedish year one textbooks: Implications for the importation of overseas-authored materials. *International Journal of Mathematics Education in Science and Technology*. https://doi.org/10.1080/0020739X.2019.1688406

Schneider, M., Grabner, R., & Paetsch, J. (2009). Mental number line, number line estimation, and mathematical achievement: Their interrelations in grades 5 and 6. *Journal of Educational Psychology*, *101*(2), 359–372. https://doi.org/10.1037/a0013840

Schneider, M., Merz, S., Stricker, J., De Smedt, B., Torbeyns, J., Verschaffel, L., & Luwel, K. (2018). Associations of number line estimation with mathematical competence: A meta-analysis. *Child Development*, *89*(5), 1467–1484. https://doi.org/10.1111/cdev.13068

Seethaler, P., & Fuchs, L. (2006). The cognitive correlates of computational estimation skill among third-grade students. *Learning Disabilities Research & Practice*, *21*(4), 233–243. https://doi.org/10.1111/j.1540-5826.2006.00220.x

Sekeris, E., Verschaffel, L., & Luwel, K. (2019). Measurement, development, and stimulation of computational estimation abilities in kindergarten and primary education: A systematic literature review. *Educational Research Review*, *27*, 1–14. https://doi.org/10.1016/j.edurev.2019.01.002

Siegler, R., & Booth, J. (2004). Development of numerical estimation in young children. *Child Development*, *75*(2), 428–444. https://doi.org/10.1111/j.1467-8624.2004.00684.x

Siegler, R., & Booth, J. (2005). Development of numerical estimation: A review. In J. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 197–212). Psychology Press.

Siegler, R., Thompson, C., & Opfer, J. E. (2009). The logarithmic-to-linear shift: One learning sequence, many tasks, many time scales. *Mind, Brain, and Education*, *3*(3), 143–150. https://doi.org/10.1111/j.1751-228X.2009.01064.x

Siegler, R., Thompson, C., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*(4), 273–296. https://doi.org/10.1016/j.cogpsych.2011.03.001

Simms, V., Clayton, S., Cragg, L., Gilmore, C., & Johnson, S. (2016). Explaining the relationship between number line estimation and mathematical achievement: The role of visuomotor integration and visuospatial skills. *Journal of Experimental Child Psychology*, *145*, 22–33. https://doi.org/10.1016/j.jecp.2015.12.004

Smets, K., Sasanguie, D., Szücs, D., & Reynvoet, B. (2015). The effect of different methods to construct non-symbolic stimuli in numerosity estimation and comparison. *Journal of Cognitive Psychology*, *27*(3), 310–325. https://doi.org/10.1080/20445911.2014.996568

Sowder, J. (1992). Estimation and number sense. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the national Council of teachers of mathematics* (pp. 371–389). NCTM.

Sowder, J., & Wheeler, M. (1989). The development of concepts and strategies used in computational estimation. *Journal for Research in Mathematics Education*, *20*(2), 130–146. https://doi.org/10.5951/jresematheduc.20.2.0130

Standards and Testing Agency. (2019). *Early years foundation stage profile: 2020 handbook.*.

Stapel, J., Hunnius, S., Bekkering, H., & Lindemann, O. (2015). The development of numerosity estimation: Evidence for a linear number representation early in life. *Journal of Cognitive Psychology*, *27*(4), 400–412. https://doi.org/10.1080/20445911.2014.995668

Star, J., & Rittle-Johnson, B. (2009). It pays to compare: An experimental study on computational estimation. *Journal of Experimental Child Psychology*, *102*(4), 408–426. https://doi.org/10.1016/j.jecp.2008.11.004

Subramaniam, K. (2014). Prospective secondary mathematics teachers' pedagogical knowledge for teaching the estimation of length measurements. *Journal of Mathematics Teacher Education*, *17*(2), 177–198. https://doi.org/10.1007/s10857-013-9255-2

Sullivan, J., & Barner, D. (2013). How are number words mapped to approximate magnitudes? *Quarterly Journal of Experimental Psychology*, *66*(2), 389–402. https://doi.org/10.1080/17470218.2012.715655

Sullivan, J., & Barner, D. (2014a). The development of structural analogy in number-line estimation. *Journal of Experimental Child Psychology*, *128*, 171–189. https://doi.org/10.1016/j.jecp.2014.07.004

Sullivan, J., & Barner, D. (2014b). Inference and association in children's early numerical estimation. *Child Development*, *85*(4), 1740–1755. https://doi.org/10.1111/cdev.12211

Sunde, P. B., Petersson, J., Nosrati, M., Rosenqvist, E., & Andrews, P. (Submitted). Estimation in the mathematics curricula of Denmark, Norway and Sweden: Inadequate conceptualisations of an essential competence. *Scandinavian Journal of Educational Research*.

Taillan, J., Ardiale, E., & Lemaire, P. (2015). Relationships between strategy switching and strategy switch costs in young and older adults: A study in arithmetic problem solving. *Experimental Aging Research*, *41*(2), 136–156. https://doi.org/10.1080/0361073X.2015.1001651

Torbeyns, J., Schneider, M., Xin, Z., & Siegler, R. S. (2015). Bridging the gap: Fraction understanding is central to mathematics achievement in students from three different continents. *Learning and Instruction*, *37*, 5–13. https://doi.org/10.1016/j.learninstruc.2014.03.002

Tosto, M., Petrill, S., Malykh, S., Malki, K., Haworth, C., Mazzocco, M., Thompson, L., Opfer, J., Bogdanova, O., & Kovas, Y. (2017). Number sense and mathematics: Which, when and how? *Developmental Psychology*, *53*(10), 1924–1939. https://doi.org/10.1037/dev0000331

Träff, U. (2013). The contribution of general cognitive abilities and number abilities to different aspects of mathematics in children. *Journal of Experimental Child Psychology*, *116*(2), 139–156. https://doi.org/10.1016/j.jecp.2013.04.007

Van Hoof, J., Verschaffel, L., & Van Dooren, W. (2017). Number sense in the transition from natural to rational numbers. *British Journal of Educational Psychology*, *87*(1), 43–56. https://doi.org/10.1111/bjep.12134

Van Prooijen, J.-W. (2017). Why education predicts decreased belief in conspiracy theories. *Applied Cognitive Psychology*, *31*(1), 50–58. https://doi.org/10.1002/acp.3301

Van ‘t Noordende, J., Volman, M., Leseman, P., Moeller, K., Dackermann, T., & Kroesbergen, E. (2018). The use of local and global ordering strategies in number line estimation in early childhood. *Frontiers in Psychology*, *9*, 1562–1562. https://doi.org/10.3389/fpsyg.2018.01562

Vukovic, R., Fuchs, L., Geary, D., Jordan, N., Gersten, R., & Siegler, R. (2014). Sources of individual differences in children’s understanding of fractions. *Child Development*, *85*(4), 1461–1476. https://doi.org/10.1111/cdev.12218

White, S., & Szűcs, D. (2012). Representational change and strategy use in children’s number line estimation during the first years of primary school. *Behavioral and Brain Functions*, *8*(1), 1–12. https://doi.org/10.1186/1744-9081-8-1

Wong, T.-Y., Ho, S.-H., & Tang, J. (2016). Consistency of response patterns in different estimation tasks. *Journal of Cognition and Development*, *17*(3), 526–547. https://doi.org/10.1080/15248372.2015.1072091

Wong, T.-Y., Ho, S.-H., & Tang, J. (2017). Defective number sense or impaired access? Differential impairments in different subgroups of children with mathematics difficulties. *Journal of Learning Disabilities*, *50*(1), 49–61. https://doi.org/10.1177/0022219415588851

Xenofontos, C. (2019). Primary teachers’ perspectives on mathematics during curriculum reform: A collective case study from Cyprus. *Issues in Educational Research*, *29*(3), 979–996. http://www.iier.org.au/iier29/xenofontos.pdf