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ANALYSIS OF PARTICLE ATOM COLLISION PROCESSES, USING POLARIZED BEAMS

by

S. M. KHALID

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Institute of Atomic Physics
University of Stirling

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S.M. Khalid

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ABSTRACT

Theory of measurement for particle atom collision processes using polarized beams is presented. The density matrix formulism is applied to derive expressions for the Stokes parameters describing light emitted in particle photon coincidence experiments. These Stokes parameters in general depend on the spin polarization of the colliding particles and atoms. It is shown that an almost complete determination of scattering and target parameters is possible if one collides polarized electrons with light polarized one electron atoms. Starting with polarized electrons and polarized heavy alkalis (Cs etc.) and applying the formulism of Burke and Mitchell[8] it is shown how to find the collision matrix. Some observable consequences of this formulism are also deduced.

The theory of measurement for electron capture on atomic targets with polarized bare nuclei as projectiles is described in terms of Stokes parameters. This process with polarized protons is discussed as an illustration of the theory. When the scattered hydrogen atom is not registered in coincidence with the Lyman-α resulting from the decay of H(2P)*, it is shown that circular polarization of the Lyman-α is directly proportional to the spin polarization of the protons. This relationship becomes the basis for an optical detector of the proton spin polarization.

We derive expressions for the Stokes parameters in case of steady state excitation for electron atom collisions and also describe the threshold and pseudo-threshold behaviour. Some numerical calculations are plotted to illustrate the usefulness of the theory of measurement.

INTRODUCTION

Most of what we know about the microworld has been established by means of collision processes. The outcome of a particle atom collision process is studied through scattering experiments in which a well defined beam of particles is fired on the atomic target and the products of the collision (photon, scattered atom, ion etc.) are detected in some suitable arrangement. The target is usually supposed to be so thin that the probability that any particle in the beam is scattered by more.than one atom in the target can be neglected. It is assumed that the density of projectiles in the beam is so low that they do not interact with each other. It is further supposed that each projectile in the beam has the same definite value of energy and momentum. Under these physical conditions, the outcome of a particle atom collision process is usually predicted by application of the quantum theory of scattering incorporating some sort of approximation scheme.[1] We are, however, not concerned with this kind of approach and instead work on theoretical analysis of experimental measurements (the so called 'Theory of the Measurement') for particle-atom collision processes. This type of theory of the measurement was first developed for nuclear processes.[2] For particle-atom collision processes, Macek and Jaecks[3] developed a theory of atomic photon-particle coincidences which provides excellent information about the atomic excited state produced by the collision. Later Fano and Macek (1973)[4] presented a new formulism for the angular distribution and the polarization of light excited by atomic and electronic collisions and which may be modulated in time by the action of internal and external fields. Their formulism disentangled geometrical and dynamical effects and

stressed the extraction of data on the alignment and orientation of radiating atoms from observations of the emitted light. Thereafter, Blum and Kleinpoppen (1979)[5] presented a theory of electron-photon angular correlation in atomic physics in the more elegant framework of density matrices and state multipoles. More recently Berezhko and Kabachnik[6] have calculated the alignment of hydrogenlike atoms produced by electron capture in collisions of heavy charged particles with target atoms. All the above theories of measurement consider unpolarized projectiles colliding with unpolarized atomic targets.

Recent advances in technology, however, have provided reliable sources of polarized particles (electrons, protons, etc.) and polarized atoms. This paves the way for studying the spin dependence of the particle atom collisions. As pointed out by Hanne,[7] particle-photon coincidence technique may be combined with the spin analysis of the reactants before and after the collision to extract almost complete information on all scattering and target parameters (scattering parameters consist of scattering/excitation amplitudes and their phases; target parameters as such are orientation, alignment and multipole moments of collisionally excited atoms).

In Chapter 1, we lay down the basis of the theory of measurement and discuss concepts which are fundamental to this work. The second chapter is devoted to the theory of measurement for electron atom collisions, neglecting spin-dependent interactions during the collision. In Chapter III we work out a scheme to extract scattering amplitudes for elastic electron atom scattering when spin orbit interaction is included in the description of the collision from the formulism of Burke and Mitchell.[8] Theory of measurement for electron capture on atomic targets with bare nuclei as projectiles

is presented in Chapter IV. We discuss some implications of our work and illustrate their usefulness through numerical calculations in Chapter V.

CHAPTER I

FRAMEWORK OF THE THEORY OF MEASUREMENT

We sum up some important concepts which form the basis of the theory.

I.1 PURE AND MIXED STATES

When the eigenvalues of a complete set of commuting observables describing a quantum mechanical system are known, it is said to be in a pure state. A pure state can be described by a single state vector $|\chi\rangle$ which is a simultaneous eigenstate of the complete set of commuting observables. Unfortunately, however, in many practical situations, the system is in a state for which the eigenvalues of a complete set of commuting observables are not all known. Certain predictions about such a system can still be made by methods of statistics. One then assigns certain probabilities $w_i (i = 1, 2, \ldots)$ for the system being in a set of pure states $|\chi_i\rangle$. The state of this system is then a statistical mixture of these pure states, and is termed as a mixed state. It is usually convenient to describe a mixed state by the density operator[9]

$$\rho = \sum_i |\chi_i\rangle w_i \langle\chi_i| \qquad \text{(I.1-1)}$$

I.2 THE DENSITY MATRIX

For convenient description of a quantum mechanical system, the statistical methods are useful in two ways. Firstly, the uncontrollable perturbation of states by the process of measurement is easily accounted for. Secondly, when dealing with mixed states, it is only known that the system can be in any one of the several pure states with some probability. It is, therefore, reasonable

to apply a statistical description because of the lack of information available on the system. It was primarily for this reason that the density operator was introduced by J.V. Neumann (1927)[10] to describe statistical concepts in quantum mechanics. The matrix representation of the density operator can be found by choosing a convenient orthonormal basis $|\psi_j\rangle$ (j = 1, 2, ...) . Then we have

$$|\chi_i\rangle = \sum_j a_{ji} |\psi_j\rangle , \quad \langle\psi_l|\psi_m\rangle = \delta(l, m)$$

where

$$a_{ki} = \langle\psi_k|\chi_i\rangle ;$$

the matrix representation of the density operator (called density matrix) turns out to be

$$\rho_{lm} = \langle\psi_l|\rho|\psi_m\rangle = \sum_i a_{li} \, w_i \, a^*_{mi} \qquad \text{(I.2-1)}$$

The main virtue of the density matrix is its analytical power in the construction of general formulae and in the proving general theorems. The evaluation of averages and probabilities of the physical quantities characterizing a given system becomes straightforward by the use of density matrix techniques. The representation of quantum mechanical states by density matrices enables the maximum information available on the system to be expressed in a compact manner and hence, avoids the introduction of unnecessary variables.

To sum up, the density matrix in a way is the counterpart of the distribution function of classical statistical mechanics, since it considers how many systems are in an ensemble with given wave functions. Sometimes, one is only interested in a few of many degrees of freedom of a quantum system, for instance, when only one of several interacting systems is to be observed. It is then

impossible to find a wave function which depends only on the variables of the system of interest and not on those of all other systems as well. For such an open quantum mechanical system, the so called "reduced" density matrix is found by averaging over all unobserved degrees of freedom, which describes the behaviour of the system of interest. As a compliment to the density matrix, we can say that the density matrix contains all physically significant information on the system.

The basic properties of the density matrix are found in textbooks on quantum mechanics and its applications are extensively discussed in literature.[11] [12] For our purpose we list some properties of the density matrix as follows:

(a) The density matrix is a positive definite hermitian matrix.

(b) The density matrix is related to observable quantities by the way in which it gives the expectation value, $\langle Q \rangle$, of any observable Q :

$$\langle Q \rangle = \frac{\text{tr } \rho Q}{\text{tr } \rho} \qquad \text{(I.2-2)}$$

Eq. (I.2-2) is true for all representations of ρ.

(c) The following inequality holds for the traces:

$$\text{tr } \rho^2 \leq (\text{tr } \rho)^2 \qquad \text{(I.2-3)}$$

Note that for a pure state, the relation

$$\text{tr } \rho^2 = (\text{tr } \rho)^2,$$

holds, whereas for a mixed state, we have the relation

$$\text{tr } \rho^2 < (\text{tr } \rho)^2 \ .$$

I.3 POLARIZATION DENSITY MATRIX OF PHOTONS AND STOKES PARAMETERS

A monochromatic electromagnetic wave with angular frequency ω, wave-vector $\bar{\kappa} = \frac{2\pi}{\lambda} \hat{\eta}_\gamma$ ($\hat{\eta}_\gamma$ is a unit vector in the direction of propagation and λ is the wave-length) and polarization vector $\hat{\varepsilon}$ is represented by

$$\bar{E}(\underline{r}, t) = E_0 e^{i(\bar{\kappa}.\bar{r} - \omega t)} \hat{\varepsilon} \qquad \text{(I.3-1)}$$

Taking the direction of propagation $\hat{\eta}_\gamma$ as quantization axis (Z_γ-axis of the coordinate system), it is obvious that $\hat{\varepsilon}$ is perpendicular to $\hat{\eta}_\gamma$ due to the transverse nature of electromagnetic waves. If $|X_\gamma\rangle$ ($|Y_\gamma\rangle$) denotes photon polarization state along the X_γ (Y_γ) axis it is obvious that

$$|\hat{\varepsilon}\rangle = \alpha(X_\gamma) \; |X_\gamma\rangle + \alpha(Y_\gamma) |Y_\gamma\rangle \quad .$$

When angular momentum conservation is important in calculations it is convenient to describe the photon-polarization state in the 'helicity coordinate system' (helicity-frame) defined by the basis vectors

$$|\pm 1\rangle = \mp \frac{1}{\sqrt{2}} (|X_\gamma\rangle \pm \imath |Y_\gamma\rangle) \quad , \quad \imath = \sqrt{-1} \qquad \text{(I.3-2)}$$

which describe the states of circularly polarized photons with helicity $\lambda = \pm 1$. In this new basis we write

$$|\hat{\varepsilon}\rangle = \sum_{\lambda=\pm 1} \alpha(\lambda) |\lambda\rangle \qquad \text{(I.3-3)}$$

Hereafter we always use (I.3-2) as our basis for the description of photon polarization.

A mixture of pure states $|\varepsilon_j\rangle$ with statistical weights $w(j)$ is described by

$$\rho = \sum_j |\varepsilon_j\rangle w(j) \langle\varepsilon_j| \qquad \text{(I.3-4)}$$

In 'helicity frame' we have

$$\rho = \sum_{j\lambda'\lambda} \alpha_j(\lambda')\ \alpha_j(\lambda)^*\ |\lambda'>w(j)<\lambda| \ , \ (\lambda', \lambda = \pm 1) \qquad \text{(I.3-5)}$$

and the relevant matrix elements are given by

$$\rho_{\lambda'\lambda} = <\lambda'|\rho|\lambda> = \sum_j \alpha_j(\lambda')w(j)\ \alpha_j(\lambda)^* \qquad \text{(I.3-6)}$$

where we assume the normalization $\mathrm{tr}\ \rho = I$, the intensity of the photon beam. It is convenient to parameterize ρ in terms of the Stokes parameters $(I, \eta_1, \eta_2, \eta_3)$ in the following way[13]

$$\rho_{\lambda'\lambda} = \begin{pmatrix} \rho_{1,1} & \rho_{1,-1} \\ \rho_{-1,1} & \rho_{1,-1} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1+\eta_2 & -\eta_3 + i\eta_1 \\ -\eta_3 - i\eta_1 & 1-\eta_2 \end{pmatrix} \qquad \text{(I.3-7)}$$

In general ρ has four complex elements (eight real parameters) but due to hermiticity of $\rho_{\lambda'\lambda}(\rho^*_{\lambda\lambda'} = \rho_{\lambda'\lambda})$ and the normalization $\mathrm{tr}\ \rho = I$, only three of these (η_1, η_2, η_3) are independent.

The Stokes parameters are defined such that I is the total intensity, η_2 is the degree of circular polarization defined by

$$I\eta_2 = I_{RHC} - I_{LHC} \ ,$$

η_3 is the degree of linear polarization with respect to orthogonal axes $\hat{\varepsilon}(\theta_\gamma)$, $\hat{\varepsilon}(\phi_\gamma)$ i.e.

$$I\eta_3 = I[\hat{\varepsilon}(\theta_\gamma)] - I[\hat{\varepsilon}(\phi_\gamma)]$$

and η_1 is the degree of linear polarization with respect to two orthogonal axes orientated at 45° to the right of $\hat{\varepsilon}(\theta_\gamma)$, $\hat{\varepsilon}(\phi_\gamma)$. The unit vectors $\hat{\varepsilon}(\theta_\gamma)$, $\hat{\varepsilon}(\phi_\gamma)$ point in the directions $(\theta_\gamma + 90°, \phi_\gamma)$, $(\theta_\gamma, \phi_\gamma + 90°)$ where $(\theta_\gamma, \phi_\gamma)$ are the polar angles of $\hat{n}_\gamma$ in the 'collision frame' (Fig. 2).

I.4 IRREDUCIBLE TENSOR OPERATORS

When the angular symmetries of the physical system are important it is always convenient to expand the density matrix in terms of a basis set (of operators) which have simple transformation properties under rotations. Such a set of operators are the irreducible tensor operators. This method provides a well developed and efficient way of using the inherent symmetry of the system. It also enables the consequences of angular momentum conservation to be simply accounted for. One can separate the dynamical and geometrical factors in the equation of interest without much effort (e.g. by the application of Wigner-Eckart theorem).

The physicist is now quite familiar with these irreducible tensor operators and many detailed accounts[14] can be found in the theory of angular momentum. We, however, restrict ourselves to the definition and listing of some useful relations. An irreducible tensor operator T_K of rank K is a set of $2K + 1$ operators $T_{KQ}(Q = -K, -K + 1, \ldots, K - 1, K)$ which transform under the $2K + 1$-dimensional representation of the rotation group according to the relation

$$R\, T_{KQ} R^{-1} = \sum_q D(\alpha\beta\gamma)^{(K)}_{qQ}\, T_{Kq} \qquad \text{(I.4-1)}$$

where R is the rotation operator and transforms a wave function Ψ into $R\Psi$ and an operator Ω into $R\Omega R^{-1}$. The Euler angles (α, β, γ) describe the rotation R and $D(\alpha\beta\gamma)^{(K)}_{qQ}$ are the matrix elements of R in the KQ representation. The tensor T_K transforms like the spherical harmonic of order K.

The spherical tensor operators defined by Eq.(I.4-1) can be constructed from the angular momentum states by the application of

the angular momentum addition rules, in the form (all the fixed quantum numbers are collectively represented by β):

$$T(J', J)_{KQ} = \sum_{MM'} (-1)^{J'-M'} (2K + 1)^{\frac{1}{2}} \begin{pmatrix} J' & J & K \\ M' & -M & -Q \end{pmatrix} |\beta J'M'\rangle\langle\beta JM| \qquad (I.4\text{-}2)$$

The 3j symbol puts the necessary restrictions due to angular-momentum coupling rules ($|J'-J| \leq K \leq J' + J$; $-K \leq Q \leq K$) and limits the number of possible operators for given J' and J. The matrix element of $T(J', J)_{KQ}$ between any pair of desired states is given by

$$\langle\beta J'M'|T(J'J)_{KQ}|\beta JM\rangle = (-1)^{J'-M'} (2K+1)^{\frac{1}{2}} \begin{pmatrix} J' & J & K \\ M' & -M & -Q \end{pmatrix} \qquad (I.4\text{-}3)$$

The operator $T(J'J)_{KQ}$ transforms under rotation $\omega \equiv (\alpha, \beta, \gamma)$ from KQ representation [(X, Y, Z) coordinate system] to Kq representation [(x, y, z) coordinate system] according to the relation

$$T(J'J)_{KQ} = \sum_{q} T(J'J)_{Kq} D(\omega)^{(K)}_{qQ} \qquad (I.4\text{-}4)$$

We now list some useful properties[15] of the spherical tensor operators as follows:

(a) The hermitian adjoint is defined by the relation

$$T(J'J)^{\dagger}_{KQ} = (-1)^{J'-J+Q} T(J'J)_{K,-Q} \qquad (I.4\text{-}5)$$

and the normalization used here is

$$\text{tr } T(J'J)_{KQ} T(J'J)_{kq} = \delta(K, k)\delta(Q, q)\delta(J', J)$$

In particular

$$\text{tr } T(J)_{KQ} = \frac{1}{\sqrt{2J+1}} \delta(K, 0)\delta(Q, 0) \qquad (I.4\text{-}6)$$

(b) These operators satisfy the Wigner-Eckart theorem:[14]

$$\langle j'm'|T(j'j)_{KQ}|jm\rangle = (-1)^{j'-m'} \begin{pmatrix} j' & j & K \\ m' & -m & -Q \end{pmatrix} \langle j'||T_K||j\rangle \ ;$$

$$\langle j'||T_K||j\rangle = (2K+1)^{\frac{1}{2}} \qquad (I.4\text{-}7)$$

The "reduced" matrix element $\langle j' || T_K || j \rangle$ is a scalar and is independent of all the magnetic quantum numbers. The conservation of angular momentum is contained in the 3j symbol and in a way reflects on the geometry of the interaction whereas the "reduced" matrix element embodies the dynamics of the interaction. These two aspects of the interaction are nicely separated in Eq.(I.4-7).

I.5 STATE MULTIPOLES

The set of operators $T(J', J)_{KQ}$ is complete and, therefore, any operator which describes the angular properties of a system can be expanded in terms of this set. In particular, the density operator ρ, which describes the angular properties of a given system, can be expanded as

$$\rho = \sum_{J'J\,KQ} \langle T(J'J)^{\dagger}_{KQ} \rangle \, T(J', J)_{KQ} \qquad \text{(I.5-1)}$$

where the expansion coefficients

$$\langle T(J'J)^{\dagger}_{KQ} \rangle = \mathrm{tr}\ \rho\, T(J', J)^{\dagger}_{KQ}\,, \quad \mathrm{tr}\ \rho = 1 \qquad \text{(I.5-2)}$$

are called the "state multipoles" or "statistical tensors" (Fano, 1953).[16] The matrix element of the density operator ρ between any two arbitrary states is given by

$$\langle J'M'|\rho|JM\rangle = \sum_{KQ} (-1)^{J'-M'} (2K+1)^{\frac{1}{2}} \begin{pmatrix} J' & J & K \\ M' & -M & -Q \end{pmatrix} \langle T(J', J)^{\dagger}_{KQ} \rangle \qquad \text{(I.5-3)}$$

and the converse relation is given by

$$\langle T(J'J)^{\dagger}_{KQ} \rangle = \sum_{M'M} (-1)^{J'-M'} (2K+1)^{\frac{1}{2}} \begin{pmatrix} J' & J & K \\ M' & -M & -Q \end{pmatrix} \langle J'M'|\rho|JM\rangle \qquad \text{(I.5-4)}$$

All the relations in this section are indispensible for our work, on the theory of measurement. In the following we lay down some basic properties[15] of state multiples for application in our formulism:

(a) The complex conjugate is defined according to

$$\langle T(J'J)^{\dagger}_{KQ}\rangle^{*} = (-1)^{J'-J+Q}\langle T(J'J)^{\dagger}_{K,-Q}\rangle \qquad \text{(I.5-5)}$$

In particular for states with sharp values of angular momentum ($J' = J$), eq (I.5-5) becomes

$$\langle T(J)^{\dagger}_{KQ}\rangle^{*} = (-1)^{Q}\langle T(J)^{\dagger}_{K,-Q}\rangle \qquad \text{(I.5-5a)}$$

(b) The counterpart of eq.(I.4-4) for the state multipoles is

$$\langle T(J'J)^{\dagger}_{KQ}\rangle = \sum_{q}\langle T(J'J)^{\dagger}_{Kq}\rangle D(\omega)^{(K)*}_{qQ} \qquad \text{(I.5-6)}$$

which means that the state multipoles transform as irreducible tensors of rank K and component Q. The transformation converse to (I.5-6) is

$$\langle T(J'J)^{\dagger}_{Kq}\rangle = \sum_{Q}\langle T(J'J)^{\dagger}_{KQ}\rangle D(\omega)^{(K)}_{qQ} \qquad \text{(I.5-6a)}$$

For sharp angular momentum states ($J' = J$) the state multipoles have simple physical interpretation in terms of orientation and alignment[4] of the excited state. The detailed discussion can be found in Blum and Kleinpoppen (1979).[5]

I.6 SPIN TENSORS

We describe only spin-$\frac{1}{2}$ particles characterized by a density matrix ρ_s with elements $\langle \frac{1}{2}m'|\rho_s|\frac{1}{2}m\rangle$. This can be expanded in terms of a set of state multipoles $\langle T(s)^{\dagger}_{K_s q_s}\rangle$, the so called spin-tensors, by means of eq.(I.5-1). The relevant state multipole is found from eq.(I.5-4) in the form

$$\langle T(\tfrac{1}{2})^{\dagger}_{K_s q_s}\rangle = \sum_{m'm}(-1)^{\frac{1}{2}-m'}(2K+1)^{\frac{1}{2}}\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & K_s\\ m' & -m & q_s\end{pmatrix}\langle \tfrac{1}{2}m'|\rho_s|\tfrac{1}{2}m\rangle \qquad \text{(I.6-1)}$$

The angular momentum addition rules allow only terms with $K_s = 0$ and $K_s = 1$ in the above expression. The spin density matrix

can be written as[17]

$$\rho_s = \tfrac{1}{2}(\underline{1} + \underline{P}.\underline{\sigma}) \equiv \tfrac{1}{2}\begin{pmatrix} 1 + P_z & P_x - iP_y \\ P_x + iP_y & 1 - P_z \end{pmatrix} ; \quad \mathrm{tr}\,\rho = 1 \qquad \text{(I.6-2)}$$

where $\underline{P}$ is the spin polarization vector and $\underline{\sigma}$ is the Pauli's spin-vector. Then in our normalization [eq.(I.4-6)] we get

$$\langle T(\tfrac{1}{2})^{\dagger}_{00} \rangle = \frac{1}{\sqrt{2}} \qquad \text{(I.6-3)}$$

and

$$\langle T(\tfrac{1}{2})_{1q_s} \rangle = \frac{P_{q_s}}{\sqrt{2}} \quad \text{with} \quad q_s = 0, \pm 1 \qquad \text{(I.6-4)}$$

and where P_{q_s} are the spherical components of the polarization vector. These spherical components are defined according to the relation

$$P_{\pm 1} = + \frac{1}{\sqrt{2}}(P_x \pm iP_y) \ , \ P_0 = P_z \ . \qquad \text{(I.6-5)}$$

The expansion of ρ_s in terms of spin-tensors can, therefore, be written as

$$\rho = \sum_{K_s q_s} \langle T(s)^{\dagger}_{K_s q_s} \rangle T(s)_{K_s q_s} = \tfrac{1}{2}\{\underline{1} + \sum_{q_s} \langle T(s)^{\dagger}_{1q_s} \rangle\} \qquad \text{(I.6-6)}$$

where $\underline{1}$ is the 2×2 unit matrix.

1.7 RESTRICTIONS ON STATE MULTIPOLES DUE TO SYMMETRY PROPERTIES OF THE EXCITATION

Blum and Kleinpoppen (1979)[5] have given a "picturesque" description of the symmetry properties of the atomic systems. They have discussed very exhaustively restrictions imposed by those symmetries on the relevant density matrix elements and the state multipoles. We supplement their discussion with a few remarks which are relevant when particle-photon coincidence experiments are done with spin polarized incident beams. In coincidence experiments

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Blum and Kleinpoppen (1979)[5] have given a "picturesque" description of the symmetry properties of the atomic systems. They have discussed very exhaustively restrictions imposed by those symmetries on the relevant density matrix elements and the state multipoles. We supplement their discussion with a few remarks which are relevant when particle-photon coincidence experiments are done with spin polarized incident beams. In coincidence experiments

without spin-selection before or after the scattering, the geometry of the experiment possesses reflection invariance in the scattering plane.[3] In general, this no more holds when we work with spin polarized particles and spin polarized atoms. As is to be seen later, the state multipoles characterizing the excitation in general depend upon the spin polarizations of the incident beams. If the polarization vectors of both the beams are perpendicular to the scattering plane then reflection invariance holds [see Fig. (3b)] for the state multipoles.

When one or both the initial polarization vectors have some component in the scattering plane, then the reflection invariance breaks down as a consequence of the initial conditions [see Fig. (3a)]. By using spin polarized initial beams and inverting one of the incident polarization vectors, more information on scattering amplitudes can be extracted. This is illustrated in a later chapter.

In case the reflection symmetry no longer holds, the residual symmetry is the hermiticity of the state multipoles expressed by eq.(I.5-5). These restrictions, however, do not apply on the scattering amplitudes due to reflection invariance of the interaction and the transformation properties under reflection of the initial and the final states.

CHAPTER II

THEORY OF MEASUREMENT FOR ELECTRON ATOM COLLISIONS WHEN SPIN DEPENDENT INTERACTIONS ARE NEGLIGIBLE DURING COLLISION

The electron-photon coincidence technique enabled the experimentalists to study the coherent excitation of atoms by electron impact. Recent development of reliable sources of polarized electrons and polarized atoms has made it feasible to study the spin dependence of electron atom collisions. These two aspects of electron-atom collisions are utilized to make a nearly complete determination of scattering amplitudes for elastic scattering and the excitation of the ^2p state of light one electron atoms (for hydrogen only H(2p) state). We can describe the electron atom collision in terms of the relevant set of quantities in the form

$$e(\bar{p}_0,\ \tfrac{1}{2}\ m_0) + A(n_0 s_0 m_{s_0}) \rightarrow e(\bar{p}_1,\ \tfrac{1}{2}\ m_1) + A^*(n_1 L_1 M_1 s_1 m_{s_1}) \quad \text{(II.1)}$$

the incident atoms are assumed to be in their ground state. For elastic collision we can write $A(n_0 s_1 m_{s_1})$ instead of $A^*(n_1 L_1 M_1 s_1 m_{s_1})$. The excited atom A^* later decays to a lower state $A(n_2 L_2 M_2 s_2 m_{s_2})$. We are interested to study the transitions

$$\Gamma_0(\bar{p}_0\ \tfrac{1}{2}\ m_0,\ n_0 s_0 m_{s_0}) \rightarrow \Gamma_1(\bar{p}_1\ \tfrac{1}{2}\ m_1,\ n_1 L_1 M_1 s_1 m_{s_1}),$$

$$\Gamma_0(p_0\ \tfrac{1}{2}\ m_0,\ n_0 s_0 m_{s_0}) \rightarrow \Gamma_1(\bar{p}_1\ \tfrac{1}{2}\ m_1,\ n_0 s_1 m_{s_1}) \quad \text{(II.2)}$$

referring to inelastic and elastic collisions respectively. Both the transitions are characterized by the scattering amplitude $a(\Gamma_1 \Gamma_0)$ for the sake of convenience. The normalization to the relevant differential cross-section is chosen by the relation:

$$\sigma(\Gamma_1\Gamma_0) = |a(\Gamma_1\Gamma_0)^2| \qquad \text{(II.3)}$$

When unpolarized electrons and atoms collide, the bilinear products $a(\Gamma'_1\Gamma_0)a(\Gamma_1\Gamma_0)^*$ which are relevant to express the results of such experiments must be averaged over the initial and summed over the final spins. These spin averaged bilinear products are denoted by

$$\langle a(M'_1)a(M_1)^*\rangle = \frac{1}{2(2S_0+1)} \sum_{\substack{m_0 m_{s_0} \\ m_1 m_{s_1}}} a(\Gamma'_1\Gamma_0)a(\Gamma_1\Gamma_0)^* \qquad \text{(II.4)}$$

for the inelastic case and

$$\langle a'a^*\rangle = \frac{1}{2(2S_0+1)} \sum_{\substack{m_0 m_{s_0} \\ m_1 m_{s_1}}} a(\Gamma'_1\Gamma_0)a(\Gamma_1\Gamma_0)^* \qquad \text{(II.4a)}$$

for the elastic case. We normalize to the unpolarized differential cross section for the magnetic sublevel M_1 by the relation

$$\sigma_{M_1} = \langle |a(M_1)|^2\rangle = \sum_{\substack{m_0 m_{s_0} \\ m_1 m_{s_1}}} \sigma(\Gamma_1\Gamma_0)/2(2S_0+1) \qquad \text{(II.5)}$$

for the inelastic scattering. The corresponding relation for elastic scattering is

$$\sigma = \langle |a|^2\rangle = \sum_{\substack{m_0 m_{s_0} \\ m_1 m_{s_1}}} \sigma(\Gamma_1\Gamma_0)/2(2S_0+1) \qquad \text{(II.5a)}$$

Starting with electrons and atoms spin polarized along the same direction $\hat{n}_s$ [Fig.(2)] and assuming that final spins are not observed, we call the polarized differential cross section for

magnetic sublevel M_1 as $\mathcal{S}_{M_1}$. The polarized differential cross section $\mathcal{S}_{M_1}$ can be defined in a similar notation by the relation

$$\mathcal{S}_{M_1} = \langle |a_p(M_1)|^2 \rangle \ , \quad \text{and} \quad \mathcal{S}_1 = \langle |a_p|^2 \rangle \quad \text{(elastic scattering)} \tag{II.6}$$

The subscript 'p' on the scattering amplitude distinguishes it from the corresponding scattering amplitude for the case of unpolarized initial beams.

The relevant differential cross-sections summed over all the magnetic sublevels are then given by the relations:

$$\sigma = \sum_{M_1} \sigma_{M_1} \qquad \text{and} \qquad \mathcal{S} = \sum_{M_1} \mathcal{S}_{M_1} \tag{II.7}$$

The theory in this chapter is mainly addressed to the light alkalis and 2p state of hydrogen for low energy scattering. The term high and low energy refers to electron energies high enough for electrons to pass the atomic shells and where the electrons experience a pure coulomb field and to energies so low that the electrons are scattered by a strongly screened coulomb field, respectively. In case of light atomic targets collided by low energy electrons, exchange collisions predominantly alter spin polarization of incident electrons and atoms whereas at high energies of about 100 kev with heavy targets, the changes in relevant spin polarizations are also due to spin-orbit interaction of the electrons in the pure coulomb field (spin-flip due to electromagnetic interaction). In short, starting with light targets and low energy electrons, it is a good approximation to neglect spin dependent interactions during collision time ($\sim 10^{-6}$ns) so that

changes in spin polarizations are entirely due to exchange collisions. In case of elastic scattering, the measurement of spin polarization of electrons and atoms is relatively easy, whereas for inelastic scattering, the spin polarization of scattered atoms (which move relatively slowly towards the spin analyzer) changes during the spin orbit relaxation time ($\sim 10^{-3}$ ns) via fine structure and hyperfine structure interactions. It is also affected by the finite life time (~10 ns) of the excited state in consideration and the rules governing transitions to the lower states. The scattered electron reaches the Mott detector almost immediately. It is, therefore, reasonable to rely on the spin measurements performed on the scattered electron in case of inelastic scattering.

As we neglect spin dependent interactions during the collision, total spin of the electron plus atom system and its projection on the quantization axis is conserved during the collision. The coupled spin S (channel spin) and (its projection on the quantization axis) M_S, therefore, take on the following values

$$S = \begin{matrix} s_0 + \tfrac{1}{2} \text{ to } |s_0 - \tfrac{1}{2}| \\ s_1 + \tfrac{1}{2} \text{ to } |s_1 - \tfrac{1}{2}| \end{matrix} \quad \text{and} \quad M_S = \begin{matrix} m_{s_0} + m_0 \\ m_{s_1} + m_1 \end{matrix} \quad \text{i.e.} \quad s_0 = s_1$$

Assuming that $\bar{P}_0$, $\bar{P}_1$, n_0, n_1, L_1 are sharply defined (in case of hydrogen target, only H(2p)* state) and suppressing their dependence hereafter, application of simple scattering theory gives [15]

$$a(\Gamma_1\Gamma_0) = a(M_1 m_{s_1} m_1, m_{s_0} m_0) = \sum_{SM_S} (2S+1) \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & S \\ m_{s_1} & m_1 & -M_S \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & S \\ m_{s_0} & m_0 & -M_S \end{pmatrix} a(\Gamma_1\Gamma_0)^{(S)} \qquad \text{(II.8)}$$

where the scattering amplitude $a(\Gamma_1\Gamma_0)^{(S)}$ gives the cross section

for scattering in the channel with total spin S. For light one electron atoms in consideration ($s_0 = \frac{1}{2} = s_1$), there are two channels with $S = 0$ (singlet) and $S = 1$ (triplet). The singlet and the triplet scattering amplitudes can be written in terms of scattering amplitudes for direct and exchange scattering. For instance for $^2S \rightarrow {}^2P$ excitation,

$$\left.\begin{aligned} a(M_1)^{(0)} &= f_{M_1} + g_{M_1} \\ \text{and} \qquad & \\ a(M_1)^{(1)} &= f_{M_1} - g_{M_1} \end{aligned}\right\} \qquad \text{(II.9)}$$

are scattering amplitudes for magnet sublevels in case of singlet and triplet channels respectively [f_{M_1} and g_{M_1} being the relevant scattering amplitudes for magnetic sublevels referring to a 'direct' or 'exchange' collision, respectively].[18] The cross sections for direct, exchange and triplet scattering are given by the following relations

$$\sigma^d = \sum_{M_1} \sigma^d_{M_1} = \sum_{M_1} |f_{M_1}|^2 \qquad \text{(II.10)}$$

$$\sigma^{ex} = \sum_{M_1} \sigma^{ex}_{M_1} = \sum_{M_1} |g_{M_1}|^2 \qquad \text{(II.10a)}$$

$$\sigma^{int} = \sum_{M_1} \sigma^{int}_{M_1} = \sum_{M_1} |f_{M_1} - g_{M_1}|^2 \qquad \text{(II.10b)}$$

The corresponding expressions for elastic scattering can be written as

$$\left.\begin{aligned} a^{(0)} &= f + g\,, \qquad a^{(1)} = f - g \\ \sigma^d &= |f|^2\,, \qquad \sigma^{ex} = |g|^2\,, \qquad \sigma^{int} = |f - g|^2 \end{aligned}\right\} \qquad \text{(II.11)}$$

For convenience, we work in three different coordinate systems in different stages of formulation, namely 'collision frame', 'spin polarization frame' and 'photon detector frame'. These coordinate systems are defined by taking the incident projectile direction $\hat{p}_0$, the spin polarization direction of the scattered atom $\hat{n}_s$ and the axis of the photon detector $\hat{n}_\gamma$, as quantization axis [Fig.(2)] respectively.

II.1 CORRELATION OF FINAL AND INITIAL SPIN POLARIZATIONS

When spin-orbit coupling is neglected during the collision, Blum and Kleinpoppen[19] have given a general theory of spin polarization phenomena in collisions between electrons and atoms with arbitrary initial spin polarization state. Without going into the details of their formalism, we rewrite their eq.(28) which correlates initial spin tensors of the electron plus atom system with those after the collision (for elastic as well as inelastic scattering) with a little change in their notation as:

$$\sigma(\Gamma_1\Gamma_0)\langle T'(s_1)^{\dagger}_{K_{s_1}Q_{s_1}} \otimes t'^{\dagger}_{k_1q_1}\rangle = \sum_{\bar{S}S} \{(2\bar{S}+1)(2S+1)\}a(\Gamma_1\Gamma_0)^{(\bar{S})}\, a(\Gamma_1\Gamma_0)^{(S)*}$$

$$\times \sum_{\substack{K_{s_0}Q_{s_0},k_0q_0 \\ jM}} \langle T(s_0)^{\dagger}_{K_{s_0}Q_{s_0}}\rangle\langle t^{\dagger}_{k_0q_0}\rangle (-1)^{K_0+k_0+K_1+k_1+M}$$

$$\times (2j+1)\{(2K_{s_0}+1)(2k_0+1)(2K_{s_1}+1)(2k_1+1)\}^{\frac{1}{2}}$$

$$\times \begin{pmatrix} K_{s_0} & k_0 & j \\ Q_{s_0} & q_0 & -M \end{pmatrix}\begin{pmatrix} K_{s_1} & k_1 & j \\ Q_{s_1} & q_1 & -M \end{pmatrix}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & k_0 \\ s_0 & s_0 & K_{s_0} \\ S & \bar{S} & j \end{Bmatrix}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & k_1 \\ s_1 & s_1 & K_{s_1} \\ S & \bar{S} & j \end{Bmatrix} \qquad \text{(II.1-1)}$$

$\langle T(s_0)^{\dagger}_{K_{s_0} Q_{s_0}} \rangle$, $\langle t^{\dagger}_{k_0 q_0} \rangle$ are spin tensors referring to the initial atoms and electrons respectively, whereas $\langle T'(s_1)^{\dagger}_{K_{s_1} Q_{s_1}} \rangle$, $\langle t'^{\dagger}_{k_1 q_1} \rangle$ are spin tensors referring to the final atoms and electrons respectively.

To give the above expression a simple appearance, we used the normalization (I.4-6) for the spin tensors and normalized tr ρ_{out} to $\sigma(\Gamma_1\Gamma_0)$ instead of their normalizations for these quantities.

When the incident electrons and atoms are polarized along the same direction $\hat{n}_s$, they would still be polarized along this direction just after the collision. This statement can be checked by taking $\hat{n}_s$ as quantization axis and putting $q_0 = 0 = Q_0$, $k_1 = 0 = q_1$ in eq.(II.1-1) to get the atomic polarization after collision. The electronic polarization after collision is similarly obtained by putting $q_0 = 0 = Q_{s_0}$, $K_{s_1} = 0 = Q_{s_1}$ in eq.(II.1-1). If $\underline{P}^a$, $\underline{P}^e$ are initial spin polarization along the direction $\hat{n}_s$ and $\underline{P}^{a'}$, $\underline{P}^{e'}$ are final spin polarizations along this direction (taken as quantization axis) we then get the following simplified expressions (one electron light atomic targets and elastic as well as inelastic collisions):

$$\sigma(\Gamma_1\Gamma_0)P^{e'}/\sqrt{2} = \sum_{\bar{S}S} (-1)^{\bar{S}+1}\, a(\Gamma_1\Gamma_0)^{(\bar{S})*} a(\Gamma_1\Gamma_0)^{(S)} F_1(\bar{S},S) \qquad \text{(II.1-2)}$$

$$\sigma(\Gamma_1\Gamma_0)P^{a'}/\sqrt{2} = \sum_{\bar{S}S} (-1)^{S+1} a(\Gamma_1\Gamma_0)^{(\bar{S})} a(\Gamma_1\Gamma_0)^{(S)*} F_1(\bar{S},S) \qquad \text{(II.1-3)}$$

The quantity $F_i(\bar{S},S)$ is given by the relation

$$F_i(\bar{S},S) = (2\bar{S}+1)(2S+1)\sum_{K_{s_0}k_0}\{3(2K_{s_0}+1)(2k_0+1)\}^{\frac{1}{2}}$$

$$\begin{Bmatrix} K_{s_0} & k_0 & i \\ 0 & 0 & 0 \end{Bmatrix} \begin{Bmatrix} S & \bar{S} & i \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix}$$

$$\times \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & k_0 \\ \frac{1}{2} & \frac{1}{2} & K_{s_0} \\ S & \bar{S} & i \end{Bmatrix} \langle T(\tfrac{1}{2})^{\dagger}_{K_{s_0}0}\rangle\langle t^{\dagger}_{k_{s_0}0}\rangle \; ; \quad i = 0, 1$$

Note that we use superscripts 'a' and 'e' to denote atoms and electrons respectively and a prime on various quantities after scattering. Similarly the relevant polarized differential cross-sections are given by the relation

$$\sigma(\Gamma_1\Gamma_0)/\sqrt{2} = \sum_{\bar{S}S}(-1)^{S+1}a(\Gamma_1\Gamma_0)^{(\bar{S})}a(\Gamma_1\Gamma_0)^{(S)*}F_0(\bar{S},S)$$

on simplification we get

$$\sigma(\Gamma_1\Gamma_0) = \sum_{Sk_0}(-1)^{S+1}(2S+1)\left|a(\Gamma_1\Gamma_0)^{(S)}\right|^2 \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & k_0 \\ \frac{1}{2} & \frac{1}{2} & S \end{Bmatrix} \langle T(\tfrac{1}{2})^{\dagger}_{k_0 0}\rangle\langle t^{\dagger}_{k_0 0}\rangle$$

(II.1-4)

Starting with atoms and electrons polarized along direction $\hat{p}_0$, we get from eqs. (II.1-2) and (II.1-3) for elastic scattering and $2_S \rightarrow 2_P$ excitation, the final spin polarizarions by the relations

$$P_z^{a'} = \{(P_z^a + P_z^e)\sigma - P_z^e\sigma^d - P_z^a\sigma^{ex}\}/\mathcal{S} \qquad \text{(II.1-2a)}$$

$$P_z^{e'} = \{(P_z^a + P_z^e)\sigma - P_z^a\sigma^d - P_z^e\sigma^{ex}\}/\mathcal{S} \qquad \text{(II.1-3a)}$$

where

$$\mathcal{S} = \sigma - P_z^a P_z^e \zeta\,, \qquad \text{(II.1-4a)}$$

$$\zeta = \zeta_0 + 2\zeta_1\,,\ \zeta_0 = \text{Re } f_0 g_0^*\,,\ \zeta_1 = \text{Re } f_1 g_1^* \quad (^2S \to {}^2P \text{ excitation})$$

and (II.1-4b)

$$\zeta = \text{Re } fg^* \qquad \text{(elastic scattering)}$$

The relevant differential cross sections σ, σ^d, σ^{ex} and $\mathcal{S}$ have already been defined for the elastic as well as the inelastic case.

II.2 DETERMINATION OF SCATTERING AMPLITUDES FOR ELASTIC SCATTERING ON LIGHT ONE ELECTRON ATOMS

Starting with unpolarized electrons and unpolarized atoms, the differential cross section for elastic scattering on light one electron atoms is given by the relation[18]

$$\sigma(E, \theta_e) = \tfrac{1}{4}|f + g|^2 + \tfrac{3}{4}|f - g|^2 = \tfrac{1}{2}(\sigma^d + \sigma^{ex} + \sigma^{int}) \qquad \text{(II.2-1)}$$

where E is the energy of the electrons and θ_e is the scattering angle. The polarized differential cross section and the final polarizations in terms of initial polarizations (if we start with electrons and atoms polarized in the direction $\hat{n}_s$ and have the same E and θ_e) are given by the relations (II.1-2a) to (II.1-4a) i.e.

$$\mathcal{S}P^{a'} = (P^a + P^e)\sigma - P^e\sigma^d - P^a\sigma^{ex} \qquad \text{(II.1-2b)}$$

$$\mathcal{S}P^{e'} = (P^a + P^e)\sigma - P^a\sigma^d - P^e\sigma^{ex} \qquad \text{(II.1-3b)}$$

$$\mathcal{S} = \sigma - P^a P^e \zeta,\ \zeta = \text{Re } fg^* \qquad \text{(II.1-4b)}$$

where we have dropped the subscript 'z' for the sake of convenience

in this section. If we solve eqs.(II.1-2b) and (II.1-3b) for σ^d and σ^{ex}, we get

$$\sigma^d = \sigma - \frac{S(P^e P^{a'} - P^{'e} P^a)}{(P^e)^2 - (P^a)^2} \qquad \text{(II.2-2)}$$

and

$$\sigma^{ex} = \sigma - \frac{S(P^e P^{e'} - P^a P^{a'})}{(P^e)^2 - (P^a)^2} \qquad \text{(II.2-3)}$$

Substituting the values of σ^d and σ^{ex} in (II.2-1) we find σ^{int} in the form

$$\sigma^{int} = S\left(\frac{P^{e'} + P^{a'}}{P^e + P^a}\right) \qquad \text{(II.2-4)}$$

From eq.(II.2-1) it is trivial to find that

$$\zeta = \text{Re } fg^* = \tfrac{1}{2}(\sigma^d + \sigma^{ex} - \sigma^{int}) \qquad \text{(II.2-5)}$$

and from (II.1-4b) we get

$$\zeta = \left(\frac{\sigma - S}{P^a P^e}\right) = \tfrac{1}{2}(\sigma^d + \sigma^{ex} - \sigma^{int}) \qquad \text{(II.1-4c)}$$

Substituting the values of σ^d, σ^{ex} and σ^{int} from eqs. (II.2-2) to (II.2-4) into eq.(II.1-4c) we obtain

$$P^{a'} + P^{e'} = \left(\frac{1}{P^a} + \frac{1}{P^e}\right)\left\{\frac{\sigma}{S}(P^a P^e - 1) + 1\right\} \qquad \text{(II.2-6)}$$

If one makes measurements P^a, P^e, σ, S and $P^{a'}$ (or $P^{e'}$), $P^{e'}$ (or $P^{a'}$) can be found by making use of eq.(II.2-6). Eq. (II.2-6) is of considerable importance with regard to the experimental situation, e.g. if we start with both electrons and atoms polarized along the same direction (for convenience say $\bar{P}_0$ direction), then the final polarizations are correlated by eq.(II.2-6) so that spin polarization measurements are only necessary on any one of the

scattered particles. This gives one a greater degree of choice suitable for the experimental situation. In expressions (II.2-2) to (II.2-4) $p^{e'}$ or $p^{a'}$ can be eliminated by making use of eq.(II.2-6). In this way these expressions are modified according to the requirements of the experimentalist.

The scattering amplitudes 'f' and 'g' are found by the following set of measurements.

(a) The unpolarized differential cross-section $\sigma(E, \theta_e)$ is measured by starting with unpolarized initial electrons and atoms.

(b) Starting with electrons and atoms polarized along the same direction (say $\bar{p}_0$) , the polarized cross section $\mathcal{S}(E, \theta_e)$ and one of the final polarizations $p^{a'}$ or $p^{e'}$ is measured.

Then if we put $f = |f|$ and $g = |g|e^{i\delta}$ where δ is the relative phase between f and g , we get $\sigma^d = |f|^2$, $\sigma^{ex} = |g|^2$ from eqs.(II.2-2) and (II.2-3) whereas δ is found by the relation

$$\delta = \cos^{-1}\left[\frac{1}{p^a p^e}\ \frac{1 - (\mathcal{S}/\sigma)}{\sqrt{\sigma^d/\sigma}\sqrt{\sigma^{ex}/\sigma}}\right] \qquad \text{(II.2-7)}$$

The relative phase δ can also be found by inverting one of the initial spin polarizations in eq.(II.1-4b) to get

$$\mathcal{S}_i = \sigma + p^a p^e\ \zeta \qquad \text{(II.1-4d)}$$

Subtracting eqs.(II.1-4d) and (II.1-4b) we get ζ and finally the phase δ .

Theoretical data for $\sigma^d/2\sigma$, $\sigma^{ex}/2\sigma$, $\sigma^{int}/2\sigma$ and $Refg^*/\sigma$ shows an interesting structure in their angular distribution, as can be seen in Figs.(Li,Na.1) for low energy elastic scattering on

Na and Li (note that the method of approximation applied is mentioned on the various figure captions). Following the above-mentioned procedure, these quantities are measurable and may prove to be a substantial test ground for the theoretical predictions.

It is trivial to verify that all the relations in this section (except eq.(II.2-7)) are also true for the inelastic scattering, provided we substitute the relevant expressions for σ, σ^d, σ^{ex}, σ^{int}, $\mathcal{S}$ and ζ in the above equations. In particular for $^2S \rightarrow {}^2P$ excitation of light one electron atoms (when initial electrons and atoms are polarized along $\bar{p}_0$ direction) we substitute

$$\sigma = \sigma_0 + 2\sigma_1,\ \sigma^d = \sigma_0{}^d + 2\sigma_1{}^d,\ \sigma^{ex} = \sigma_0{}^{ex} + 2\sigma_1{}^{ex}$$

$$\sigma^{int} = \sigma_0{}^{int} + 2\sigma_1{}^{int} \quad \text{and} \quad \mathcal{S} = \mathcal{S}_0 + 2\mathcal{S}_1,\ \zeta = \zeta_0 + 2\zeta_1$$

($\mathcal{S}$ and ζ are given by eqs.(II.1-4a) and (II.1-4b)).

II.3 ELECTRON-PHOTON ANGULAR CORRELATION USING SPIN POLARIZED ELECTRONS AND SPIN POLARIZED ATOMS

The theory of angular correlations was first developed and practised successfully in nuclear physics.[2] Following the successful application of the theory of electron-photon coincidences with unpolarized electrons and unpolarized atoms[5] it is reasonable to extend its applicability for polarized initial beams. To start with it is a good practice to lay down the following assumptions which are applicable to the electron impact excitation of light atoms at low energies.

(a) All explicit spin dependent terms are neglected in the Hamiltonian describing the inelastic collision so that total spin

for electron plus atom system S and its projection M_S on the quantization axis is conserved during the collision. It means that changes in the spin variables are entirely caused by <u>electron exchange</u> collisions. It is also justified that we neglect the fine and hyperfine coupling inside the atom during the collision. We try to explain this S, L uncoupling physically in the following way. In the excited atomic states the orbital angular momentum L and the spin S couple under the influence of the fine structure interaction and precess around the total angular momentum J of the atom. The collision time ($\tau_c \sim 10^{-6}$ ns) is, however, much shorter than this precession period ($\tau_{J_1' J_1} = \frac{1}{E_{J_1'} - E_{J_1}}$) so that the spin vector does not have appreciable time (i.e. 10^{-6} ns) to precess around during the collision. Therefore, L and S can be considered to be uncoupled during the collision. The above arguments are even stronger for the coupling of isotopic spin since hyperfine structure interaction in most physical situations is much weaker than the fine structure interaction [i.e.

$$\tau_{J_1 F_1' F_1} \left(= \frac{1}{E_{J_1 F_1'} - E_{J_1 F_1}} \right) \gg \tau_{J_1' J_1} \quad] \ .$$

The assumption that $\tau_c \ll \tau_{J_1' J_1}$ (or $\tau_{J_1 F_1' F}$) implies that the atoms can be considered as instantaneously excited just after the collision which is, therefore, chosen as zero of the time scale in our formulism.

(b) We are going to describe a new generation of coincidence experiments where we start with polarized electrons (with sharply defined initial momentum $\bar{p}_0$) and polarized atoms and electrons scattered (with sharply defined final momentum $\bar{p}_1$) in the direction

(θ_e, ϕ_e) (with respect to the initial direction $\bar{p}_0$) are detected in coincidence with photons emitted in the direction $\hat{n}_\gamma = (\theta_\gamma, \phi_\gamma)$ (in the subsequent decay of the excited atomic states) without spin selection of the final electrons and atoms. The geometry of such an experiment is shown in Fig.(1). The essential point for the understanding of electron-photon-coincidence experiments is that the observation is restricted to radiation emitted by those atoms only which "scattered" the electrons in the direction (θ_e, ϕ_e) and emitted the photons in the direction $(\theta_\gamma, \phi_\gamma)$. In other words, a certain subensemble of excited atoms is "selected" in the experiment, and it is the state of this subensemble only which is relevant in the theory of coincidence experiments.

In view of the above assumptions, the description of the coincidence experiment is divided into three steps:

A.

The characterization of the atomic subensemble of interest just after the collision.

B.

The time evolution of the excited states under the perturbations due to the fine structure and hyperfine structure interactions.

C.

The description of the photons observed at a later time instant 't' just after the collision.

These three steps are the subject of the next three sub-sections.

II.3A DESCRIPTION OF SPIN-POLARIZED EXCITED ATOMIC ENSEMBLE JUST AFTER THE COLLISION

The density matrix characterizing the excited atomic ensemble just after the collision (at instant $t = 0$) when collision is describable in LS-coupling approximation is given by the relation

$$\rho(0) = \rho_{s_1}(0) \circledast \rho_{L_1}(0)$$

where the subscripts s_1 and L_1 refer to the spin system and the orbital system (of the excited atomic ensemble of interest) respectively. The symbol $\circledast$ stands for a Kronecker-product (direct product). The density matrices $\rho_{s_1}(0)$ and $\rho_{L_1}(0)$ are expanded in terms of the spherical tensor operators according to the relation (I.5-1) in the 'collision frame'. In particular (subscript 'p' emphasizes polarization dependence)

$$\rho_{L_1}(0) = \sum_{K_{L_1} Q_{L_1}} \langle T_p(L_1)^\dagger_{K_{L_1} Q_{L_1}} \rangle \, T_p(L_1)_{K_{L_1} Q_{L_1}}$$

where according to eq.(I.5-4) [for sharp values of L_1 only]

$$\langle T_p(L_1)^\dagger_{K_{L_1} Q_{L_1}} \rangle = (2K_{L_1} + 1)^{\frac{1}{2}} \sum_{\bar{M}_1 M_1} (-1)^{L_1 - \bar{M}_1} \begin{pmatrix} L_1 & L_1 & K_{L_1} \\ \bar{M}_1 & -M_1 & -Q_{L_1} \end{pmatrix} \langle \bar{M}_1 | \rho_{L_1}(0) | M_1 \rangle$$

(II.3-1)

If $\rho(i)$ is the initial density matrix describing the state of electron plus atom system before collision, we have

$$\rho(0) = T \, \rho(i) T^\dagger$$

where T is the transition operator for the transition $\Gamma_0 \to \Gamma_1$. The matrix element of $\rho(0)$ between the desired pair of final states is given by the relation

$$\langle \bar{M}_1 \bar{m}_{s_1} \bar{m}_1 | \rho(0) | M_1 m_{s_1} m_1 \rangle = \sum_{\substack{m_{s_0} m_0 \\ m_{s_0} m_0}} \langle \bar{M}_1 \bar{m}_{s_1} \bar{m}_1 | T \rho(i) T^\dagger | M_1 m_{s_1} m_1 \rangle$$

Making use of the completeness relation $\sum_{m_{s_0} m_0} |m_{s_0} m_0\rangle\langle m_{s_0} m_0| = 1$,

twice, we get:

$$= \sum_{\substack{m_{s_0} m_0 \\ m_{s_0} m_0}} \langle \bar{M}_1 \bar{m}_{s_1} \bar{m}_1 | T | \bar{m}_{s_0} \bar{m}_0 \rangle \langle \bar{m}_{s_0} \bar{m}_0 | \rho(i) | m_{s_0} m_0 \rangle \langle m_{s_0} m_0 | T^\dagger | M_1 m_{s_1} m_1 \rangle$$

$$= \sum_{\substack{m_{s_0} m_0 \\ m_{s_0} m_0}} a(\bar{M}_1 \bar{m}_{s_1} \bar{m}_1, \bar{m}_{s_0} \bar{m}_0) a(M_1 m_{s_1} m_1, m_{s_0} m_0)^* \langle m_{s_0} m_0 | \rho(i) | m_{s_0} m_0 \rangle \quad \text{(II.3-2)}$$

To transform the scattering amplitudes to the coupled spin space of electron plus atom system eq.(II.8) is used. The state vectors $|s_0 m_{s_0}, \frac{1}{2} m_0\rangle$ are transformed to the state vectors $|(s_0 \tfrac{1}{2}), SM_S\rangle$ by a Clebsch-Gordon transformation of the type

$$|(s_0 \tfrac{1}{2}) m_{s_0} m_0\rangle = \sum_{SM_S} (2S+1)^{\frac{1}{2}} (-1)^{\frac{1}{2} - s_0 - M_s} \begin{pmatrix} s_0 & \frac{1}{2} & S \\ m_{s_0} & m_0 & -M_S \end{pmatrix} |(s_0 \tfrac{1}{2}) S M_S\rangle \quad \text{(II.3-3)}$$

After transforming eq.(II.3-2) to the SM_S-representation, we sum up the resulting expression over unobserved final spins of the atoms and electrons to get (after applying the orthogonality of the relevant 3j symbols) the reduced density matrix[13] describing only the orbital state of the excited atomic ensemble in the form

$$\langle \bar{M}_1 | \rho_{L_1}(0) | M_1 \rangle = \langle a_p(\bar{M}_1) a_p(M_1)^* \rangle = \sum_{SM_S} a(\bar{M}_1)^{(S)} a(M_1)^{(S)*} \langle SM_S | \rho(i) | SM_S \rangle \quad \text{(II.3-4)}$$

where we write '$\langle a_p(\bar{M}_1) a_p(M_1)^* \rangle$' for the reduced density matrix

element of $\rho(0)$ for the sake of convenience. The matrix element $\langle SM_S|\rho(i)|SM_S\rangle$ for polarized initial beams is found from Blum's eq.(24)[19] and in our normalization is given by the expression

$$\langle SM_S|\rho(i)|SM_S\rangle = \sum_{\substack{K_{s_0} Q_{s_0} \\ k_0\ q_0}} (2S+1)\{(2K_{s_0}+1)(2k_0+1)\}^{\frac{1}{2}} \langle T(s_0)^{\dagger}_{K_{s_0} Q_{s_0}} \rangle \langle t^{\dagger}_{k_0 q_0} \rangle$$

$$\times (-1)^{K_{s_0}+k_0+S-M_S} \sum_j (2j+1) \begin{pmatrix} S & S & j \\ M_S & -M_S & 0 \end{pmatrix} \begin{pmatrix} K_{s_0} & k_0 & j \\ Q_{s_0} & q_0 & 0 \end{pmatrix}$$

$$\times \begin{Bmatrix} K_{s_0} & k_0 & j \\ s_0 & \frac{1}{2} & S \\ s_0 & \frac{1}{2} & S \end{Bmatrix}. \qquad \text{(II.3-5)}$$

When one starts with unpolarized initial beams, then eq.(II.3-4) gives

$$\langle a(\bar{M}_1) a(M_1)^* \rangle = \frac{1}{2(2S_0+1)} \sum_S (2S+1) a(\bar{M}_1)^{(S)} a(M_1)^{(S)*} \qquad \text{(II.3-4a)}$$

which is in agreement with Blum and Kleinpoppen.[5]

We now specialize eq.(II.3-4) for $s_0 = \frac{1}{2}$ (light one electron atoms) and for both the initial beams polarized along the Z direction. In this physical situation the reduced density matrix element (II.3-4) looks like

$$\langle a_p(\bar{M}_1) a_p(M_1)^* \rangle = \langle a(\bar{M}_1) a(M_1)^* \rangle - \tfrac{1}{2} P_z^a P_z^e \{ f_{\bar{M}_1} g^*_{M_1} + g_{\bar{M}_1} f^*_{M_1} \}$$

(II.3-4b)

One gets a similar expression if the initial beams are polarized along X(Y) directions. In that case the subscript Z is replaced by X(Y) with the initial polarizations in eq.(II.3-4b).

The reduced density matrix elements in eqs. (II.3-4) and (II.3-4b) explicitly depend upon the initial spin polarization of electrons and atoms. This dependence is transferred to the state multipoles (describing only the orbital system of the excited atomic ensemble) defined by eq.(II.3-7). This causes a break down of reflection symmetry in the scattering plane for the excitation process when any one of the spin polarization vectors have a non-zero component in the scattering plane.[21] This symmetry, however, survives when the spin polarization vectors are normal to the scattering plane. In particular when electrons and atoms are polarized along the direction Z (or X) [ZX is the scattering plane defined by the vectors $\bar{p}_0$ and $\bar{p}_1$] the residual symmetry is the hermiticity of the state multipoles defined by eq.(I.5-5a), i.e.

$$\langle T_p(L_1)^{\dagger}_{K_{L_1}-Q_{L_1}} \rangle = (-1)^{Q_{L_1}} \langle T(L_1)^{\dagger}_{K_{L_1}Q_{L_1}} \rangle^* \qquad \text{(II.3-6)}$$

For the sake of convenience, the relevant state multipoles can be decomposed into spin polarization independent and spin polarization dependent parts as follows

$$\langle T_p(L_1)^{\dagger}_{K_{L_1}Q_{L_1}} \rangle = \langle T(L_1)^{\dagger}_{K_{L_1}Q_{L_1}} \rangle + \langle T_s(L_1)^{\dagger}_{K_{L_1}Q_{L_1}} \rangle \qquad \text{(II.3-7)}$$

where a subscript 's' denotes the spin polarization dependent part. A complete set of state multipoles for a ^2P state of light one electron atoms is listed below for utilization afterwards (the subscript 's' takes on values X, Y, Z).

$$\langle T_p(1)^{\dagger}_{00} \rangle = \mathcal{S}/\sqrt{3} = \frac{\sigma}{\sqrt{3}} + \left(-\frac{1}{\sqrt{3}} P_s^a P_s^e \zeta\right)$$

$$\langle T_p(1)^{\dagger}_{10} \rangle = 0 + \sqrt{3}\, P_s^a P_s^e (\zeta_0 - 2\zeta_1) , \qquad s \neq Y$$

$$\langle T_p(1)^{\dagger}_{11} \rangle = -\frac{i}{\sqrt{2}} \mathrm{Im}(2f_1 f_0^* + 2g_1 g_0^* - f_1 g_0^* - g_1 f_0^*) + \frac{i}{\sqrt{2}} P_s^a P_s^e \mathrm{Im}(f_1 g_0^* + g_1 f_0^*)$$

$$\langle T_p(1)^{\dagger}_{1,-1} \rangle = \langle T_p(1)^{\dagger}_{11} \rangle$$

$$\langle T_p(1)^{\dagger}_{20} \rangle = \sqrt{\frac{2}{3}}(\zeta_1 - 2\zeta_0) = \sqrt{\frac{2}{3}}(\sigma_1 - 2\sigma_0) + \{(-)P_s{}^a P_s{}^e(\zeta_1 - 2\zeta_0)\}$$

$$\langle T_p(1)^{\dagger}_{21} \rangle = (-)\frac{1}{\sqrt{2}} \mathrm{Re}(2f_1 f_0^* + 2g_1 g_0^* - f_1 g_0^* - g_1 f_0^*)$$

$$+ \frac{1}{\sqrt{2}} P_s{}^a P_s{}^e \mathrm{Re}(f_1 g_0^* + g_1 f_0^*)$$

$$\langle T_p(1)^{\dagger}_{2,-1} \rangle = (-)\langle T_p(1)^{\dagger}_{21} \rangle \qquad \text{(II.3-8)}$$

$$\langle T_p(1)^{\dagger}_{22} \rangle = (-)\zeta_1 = (-)\sigma + P_s{}^a P_s{}^e \zeta_1$$

$$\langle T_p(1)^{\dagger}_{2,-2} \rangle = \langle T_p(1)^{\dagger}_{22} \rangle$$

II.3B PERTURBATION OF THE SPIN POLARIZED EXCITED ATOMIC ENSEMBLE BY FINE STRUCTURE AND HYPERFINE STRUCTURE INTERACTIONS

We have explained earlier that during the excitation the orbital and spin angular momenta of the atoms are uncoupled. Therefore, the atomic states immediately after the excitation are represented in the uncoupled representation $|L_1 M_1, s_1 m_{s_1}\rangle$ to a good degree of approximation (especially for the light atoms under consideration). These states are the eigenstates of the unperturbed Hamiltonian H_0. After the excitation the excited atomic ensemble is assumed to evolve under the influence of the total Hamiltonian of the form

$$H = H_0 + V(f) + V(hf)$$

where $V(f)$ and $V(hf)$ are the extremely weak perturbations due to fine structure and hyperfine structure interactions. The time development of the atomic state is governed by the operator

$$U(t) = \exp(-\imath Ht/h_1 - \Gamma t)$$

where the operator Γ describes the decay of excited state by photon emission.

Since the orbital system and the spin systems are uncorrelated at time $t = 0$, the density matrix describing the excited atoms is given by the relation

$$\rho(0) = \rho_{s_1}(0) \otimes \rho_{L_1}(0) \otimes \rho_I(0) \qquad \text{(II.3.9)}$$

Note that in subsection (A) we did not include $\rho_I(0)$ which describes the nuclear spin system at time $t = 0$, since it was irrelevant during atomic excitation. Assuming that the atomic spin and nuclear spin is polarized along the same direction $\hat{n}_s$ [Fig.(2)], then, the density matrix $\rho(0)$ (which describes a coupled system comprising of orbital system and spin systems of the excited atomic ensemble) is expanded in terms of spherical tensor operators taking spin polarization direction $\hat{n}_s$ as quantization axis (due to rotational invariance of the perturbation tensor around $\hat{n}_s$) in the form[15]

$$\rho(0) = \sum_{\substack{K_{s_1} K_i \\ K_{L_1} \chi_{L_1}}} \langle T(s_1)^{\dagger}_{K_{s_1} 0} \otimes T(L_1)^{\dagger}_{K_{L_1} \chi_{L_1}} \otimes T(I)^{\dagger}_{K_{i0}} > T(s_1)_{K_{s_1} 0} \otimes T(L_1)_{K_{L_1} \chi_{L_1}} \otimes T(I)_{K_i 0} \qquad \text{(II.3-10)}$$

In most of the practical situations the fine structure interaction is by far stronger than the hyperfine structure interaction (i.e. the energy splitting due to fine structure interaction $E_{J'_1} - E_{J_1} >> E_{J_1 F'_1} - E_{J_1 F_1}$ the energy splitting due to hyperfine structure interaction) so that J_1 can be treated as a good quantum number for the purpose of estimating the effect of these perturbations during decay. Let $\tau_{J'_1 J_1}$, $\tau_{J_1 F'_1 F}$ be the relevant precession periods and τ the mean life time of the excited state in consideration, then the following physical situations are treated.

(a) $\tau_{J'_1 J_1} \sim \tau$

The perturbation due to fine structure interaction must be considered whereas hyperfine structure interaction has no appreciable effect before the excited state has already decayed. In this case, we need not couple $\rho_i(0)$ in eq. (II.3-9), i.e.

$$\rho(0) = \rho_{s_1}(0) \otimes \rho_{L_1}(0) = \sum_{K_{s_1} K_{L_1} \chi_{L_1}} \langle T(s_1)^{\dagger}_{K_{s_1} 0} \otimes T(L_1)^{\dagger}_{K_{L_1} \chi_{L_1}} \rangle T(s_1)_{K_{s_1} 0} \otimes T(L_1)_{K_{L_1} \chi_{L_1}} \quad \text{(II.3-10a)}$$

The density matrix $\rho(t)$ describing the state of the excited atoms at a later time 't' after the collision is given by the relation

$$\rho(t) = U(t)\rho(0)U(t)^{\dagger} \quad \text{(II.3-11)}$$

or

$$\rho(t) = \sum_{K_{s_1} K_{L_1} \chi_{L_1}} \langle T(s_1)^{\dagger}_{K_{s_1} 0} \rangle \langle T(L_1)^{\dagger}_{K_{L_1} \chi_{L_1}} \rangle U(t) \left\{ T(s_1)_{K_{s_1} 0} \otimes T(L_1)_{K_{L_1} \chi_{L_1}} \right\} U(t)^{\dagger} \quad \text{(II.3-12)}$$

For electric dipole transitions it can be assumed that the spins do not change on decay.[3] Then, one is only interested to find the reduced density matrix $\rho(L_1; t)$ describing the state of the orbital system of the atomic ensemble at time 't' . This is given by the relation[13]

$$\rho_{L_1}(t) = \frac{1}{2s_1 + 1} \sum_{K\chi} \langle \underline{1} \otimes T(L_1; t)^{\dagger}_{K\chi} \rangle \underline{1} \otimes T(L_1; t)_{K\chi} \quad \text{(II.3-13)}$$

where

$$\langle T(L_1; t)^{\dagger}_{K\chi} \rangle = \sum_{K_{L_1} \chi_{L_1}} \langle T(L_1)^{\dagger}_{K_{L_1} \chi_{L_1}} \rangle G(L_1; t)^{\chi_{L_1} \chi}_{K_{L_1} K} \delta(\chi, \chi_{L_1}) \quad \text{(II.3-14)}$$

and the perturbation coefficients[21] $G(L_1;\ t)^{\chi_{L_1}\chi_{L_1}}_{K_{L_1}K}$ are defined by the relation

$$G(L_1;\ t)^{\chi_{L_1}\chi_{L_1}}_{K_{L_1}K} = \sum_{K_{s_1}} \langle T(s_1)^{\dagger}_{K_{s_1}0} \rangle \mathrm{tr}\{U(t)(T(s_1)_{K_{s_1}0} \otimes T(L_1)_{K_{L_1}\chi_{L_1}}) U(t)^{\dagger} \underline{1} \otimes T(L_1;\ t)_{\bar{K}\chi}\} \qquad \text{(II.3-15)}$$

The elements of $U(t)$ are diagonal in the eigen-state representation $|(L_1 s_1)J_1 M_{J_1}\rangle$ of the total Hamiltonian H, the matrix representation of $U(t)$ in this representation has elements

$$\langle (L_1 s_1)J'_1 M'_{J_1} | U(t) | (L_1 S_1)J_1 M_{J_1} \rangle = \exp(-iE_{J_1} t/\hbar)\delta(J'_1,\ J_1)\delta(M'_{J_1},\ M_{J_1}) \qquad \text{(II.3-16)}$$

Making use of eq.(II.3-16) and some standard techniques of coupling spherical tensor operators and finding their matrix elements,[22] we get the following expression for the perturbation coefficient.

$$G(L_1;\ t)^{\chi_{L_1}\chi_{L_1}}_{K_{L_1}K} = \sum_{K_{s_1}J'_1J_1} \langle T(s_1)^{\dagger}_{K_{s_1}0} \rangle (-1)^{(s_1+L_1+J'_1)+(K_{s_1}+K_{L_1}+K)+\chi_{L_1}} \times (2J'_1+1)(2J_1+1) \times \{(2K_{s_1}+1)(2K_{L_1}+1)(2K+1)\}^{\frac{1}{2}} \begin{pmatrix} K_{s_1} & K_{L_1} & K \\ 0 & \chi_{L_1} & -\chi_{L_1} \end{pmatrix} \begin{Bmatrix} J'_1 & J_1 & K \\ L_1 & L_1 & s_1 \end{Bmatrix} \times \begin{Bmatrix} J_1 & L_1 & s_1 \\ J'_1 & L_1 & s_1 \\ K & K_{L_1} & K_{s_1} \end{Bmatrix} \exp\{-i(E_{J'_1} - E_{J_1})t/\hbar - \tfrac{1}{2}(\gamma_{J'_1} + \gamma_{J_1})t\} \qquad \text{(II.3-17)}$$

Note that we have also included the effect of the decay of the fine structure states ($\gamma_{J'_1}$, γ_{J_1} are the relevant decay widths). On substituting $K_{s_1} = 0$ in eq.(II.3-17) we recover Blum's eq.(4.3.3) in Phys. Repts. (1979).[5]

(b) $\tau_{J'_1 J_1} \ll \tau$

The perturbation due to hyperfine structure interaction can be included by assuming that fine structure states are almost instantaneously populated just after the collision and later precess extremely repidly about their mean value with a period $\tau_{J'_1 J_1} \ll \tau$. We, therefore, couple $\rho_i(0)$ with the stationary value of $\rho_{s_1}(0) \otimes \rho_{L_1}(0)$ under the influence of the Hamiltonian $H = H_0 + V(f)$ and then consider the time evolution of $\rho(0)$ under the total Hamiltonian $[H = H_0 + V(f) + V(hf)]$. Proceeding exactly as in case (a) (assuming that spins do not change on decay) the reduced density matrix $\rho_{L_1}(t)$ is given by the relation (t is the time of photon emission)

$$\rho_{L_1}(t) = \frac{1}{(2s_1 + 1)(2I + 1)} \sum_{K\chi} \langle \underline{1} \otimes T(L_1; t)_{K\chi} \otimes \underline{1} \rangle \, \underline{1} \otimes T(L_1; t)_{K\chi} \otimes \underline{1} \qquad \text{(II.3-18)}$$

where $\langle T(L_1; t)^{\dagger}_{K\chi} \rangle$ is given by the relation (II.3-14). The perturbation coefficient

$$G(L_1; t)^{\chi_{L_1} \chi_{L_1}}_{K_{L_1} K}$$

however is given by

$$G(L_1;\, t)^{\chi_{L_1}\chi_{L_1}}_{K_{L_1}K} = e^{-\gamma t} \sum_{K_{s_1}K_i} \langle T(s_1)^{\dagger}_{K_{s_1}0}\rangle \langle T(I)^{\dagger}_{K_i 0}\rangle \delta(J'_1,\, J_1)$$

$$\times \sum_{J_1 F'_1 F_1 K_{J_1}} (-1)^{(K_{s_1}+K_{L_1}+K_{J_1}+K_i)+(L_1-s_1+I+F_1)}$$

$$\times (2J_1+1)^2 (2F'_1+1)(2F_1+1)$$

$$\times \{(2K_{s_1}+1)(2K_{L_1}+1)(2K_{J_1}+1)^2(2K_i+1)(2K+1)\}^{\frac{1}{2}}$$

$$\begin{Bmatrix} K_{s_1} & K_{L_1} & K_{J_1} \\ 0 & \chi_{L_1} & -\chi_{L_1} \end{Bmatrix} \begin{Bmatrix} K_{J_1} & K_i & K \\ \chi_{L_1} & 0 & -\chi_{L_1} \end{Bmatrix}$$

$$\times \begin{pmatrix} J_1 & J_1 & K \\ L_1 & L_1 & s_1 \end{pmatrix} \begin{pmatrix} F'_1 & F_1 & K \\ J_1 & J_1 & I \end{pmatrix} \begin{Bmatrix} K_{s_1} & K_{L_1} & K_{J_1} \\ s_1 & L_1 & J_1 \\ s_1 & L_1 & J_1 \end{Bmatrix} \begin{Bmatrix} K_{J_1} & K_i & K \\ J_1 & I & F'_1 \\ J_1 & I & F_1 \end{Bmatrix}$$

$$\times \exp\{-i(E_{J_1F'_1} - E_{J_1F_1})t/\hbar - \gamma_{J_1} t\} \qquad \text{(II.3-19)}$$

The quantities K_{J_1} and K take on the relevant values in accordance with the angular momentum coupling rules, i.e.

$K_{J_1} = |K_{s_1} - K_{L_1}|$ to $K_{s_1} + K_{L_1}$ and $K = |K_{J_1} - K_i|$ to $K_{J_1} + K_i$

If we substitute $K_{s_1} = 0 = K_i$ om (II.3-19) we get Blum's eq.(4.7.22) in his book.[15]

We next specialize eq.(II.3-19) for the case when either nuclear spin is unpolarized ($K_i = 0$) or atomic spin is unpolarized ($K_{s_1} = 0$).

For $\underline{K_i = 0}$ we obtain

$$G(L_1;\ t)^{\chi_{L_1}\chi_{L_1}}_{K_{L_1}K} = \frac{e^{-\gamma t}}{2I+1} \sum_{\substack{K_{s_1}J_1 \\ F'_1F_1}} \langle T(s_1)^{\dagger}_{K_{s_1}0}\rangle (-1)^{(s_1+L_1+J_1)+\chi_{L_1}}$$

$$\times (2J_1+1)^2$$

$$\times (2F'_1+1)(2F_1+1)\{(2K_{s_1}+1)(2K_{L_1}+1)\}^{\frac{1}{2}}$$

$$(2K+1)^{\frac{1}{2}} \times \begin{pmatrix} K_{s_1} & K_{L_1} & K \\ 0 & \chi_{L_1} & -\chi_{L_1} \end{pmatrix}$$

$$\times \begin{Bmatrix} J_1 & J_1 & K \\ L_1 & L_1 & s_1 \end{Bmatrix} \begin{Bmatrix} F'_1 & F_1 & K \\ J_1 & J_1 & I \end{Bmatrix} (2) \begin{Bmatrix} K_{s_1} & K_{L_1} & K \\ s_1 & L_1 & J_1 \\ s_1 & L_1 & J_1 \end{Bmatrix} e^{-i\omega_{F'_1F_1}t}$$

(II.3-19a)

where we assume that

$$\gamma_{J'_1} = \gamma_{J_1} = \gamma = \frac{1}{\tau} \quad .$$

For $\underline{K_{s_1} = 0}$ we obtain

$$G(L_1;\ t)^{\chi_{L_1}\chi_{L_1}}_{K_{L_1}K} = \frac{e^{-\gamma t}}{2s_1+1} \sum_{\substack{K_iJ_1 \\ F'_1F_1}} \langle T(I)^{\dagger}_{K_i0}\rangle (-1)^{(J_1+I+F_1)+K_i+\chi_{L_1}}$$

$$\times (2J_1+1)^2(2F'_1+1)(2F_1+1)$$

$$\times \{(2K_{L_1}+1)(2K_i+1)(2K+1)\}^{\frac{1}{2}} \begin{pmatrix} \chi_{L_1} & K_i & K \\ \chi_{L_1} & 0 & -\chi_{L_1} \end{pmatrix}$$

$$\begin{Bmatrix} J_1 & J_1 & K_{L_1} \\ L_1 & L_1 & s_1 \end{Bmatrix} \begin{Bmatrix} J_1 & J_1 & K \\ L_1 & L_1 & s_1 \end{Bmatrix}$$

$$\times \begin{Bmatrix} F'_1 & F_1 & K \\ J_1 & J_1 & I \end{Bmatrix} \begin{Bmatrix} K_{L_1} & K_i & K \\ J_1 & I & F'_1 \\ J_1 & I & F_1 \end{Bmatrix} e^{-i\omega_{F'_1F_1}t} \qquad \text{(II.3-19b)}$$

where

$$\omega_{F'_1F_1} = \left(\frac{E_{J_1F'_1} - E_{J_1F_1}}{\hbar} \right) ;$$

we will need eq. (II.3-19b) in Chapter (IV) where we will discuss electron capture by polarised protons on atomic targets.

When $\tau_{J_1F'_1F_1} \sim \tau$, the effect of hyperfine structure is definitely included and the oscillatory terms $(F'_1 \neq F_1)$ in the above eqs. are not discarded. It may sometimes happen that we still have $\tau_{J_1F'_1F_1} >> \tau$, then the effect of hyperfine structure interaction is neglected and eq.(II.3-17) is used, retaining only $J'_1 = J_1$ terms. For some metastable states it is possible that $\tau_{J_1F'_1F_1} << \tau$, then the oscillatory terms $(F'_1 \neq F_1)$ average out during the comparatively long life time and are neglected in the above equations.

(c) $(\tau_{J_1F'_1F_1} \sim \tau_{J'_1J_1}) >> \tau$

The excited state in question decays before these perturbations have any appreciable effect.

In case fine structure and hyperfine structure splittings are comparable $(\tau_{J_1F'_1F_1} \sim \tau_{J'_1J_1})$ and $\tau_{J'_1J_1} << \tau$ or $\tau_{J'_1J_1} \sim \tau$; J_1 is no more a good quantum number and more elaborate procedure is necessary.[4] Luckily we seldom come across this type of situation in atomic dipole transitions under consideration. We, therefore, do not attempt this problem.

The time dependent exponential factor in the perturbation coefficients has to be integrated over the resolving time τ_R of the apparatus. In case $\tau_R >> \tau$, it is a reasonable approximation to integrate this factor from $t = 0 \to \infty$ with negligible error. To distinguish this situation, we put a bar on the relevant perturbation coefficients.

For instance if the atomic spin just after the collision is polarized in the direction $\bar{P}_0$, the 'spin polarization frame' then coincides with the 'collision frame' and in all the expressions for perturbation coefficients we substitute $\chi_{L_1} = Q_{L_1}$ and $\chi = Q$ (χ's are the components of the state multipoles in the 'spin polarization frame' and Q's the corresponding components in the 'collision frame'). For convenience, the perturbation coefficient is decomposed into spin polarization independent ($\bar{G}_{K_{L_1}}$) and spin polarization dependent $\bar{G}(L_1)^{Q_{L_1} Q_{L_1}}_{K_{L_1} K}$ parts. Then for the case of $E_{J'_1} - E_{J_1} >> \gamma (\tau_{J'_1 J_1} << \tau)$ and when hyperfine structure interaction is neglected, the following perturbation coefficients are relevant for a ^2p state of light one electron atom:

$$\bar{G}_0 = \frac{25}{24}\tau , \qquad \bar{G}_1 = \frac{7}{9}\tau , \qquad \bar{G}_2 = \frac{1}{3}\tau$$

$$\bar{G}(1)^{00}_{10} = 0 , \qquad \bar{G}(1)^{00}_{01} = 0.363\ \tau\ P'^{a}_{z}$$

$$\bar{G}(1)^{00}_{21} = 0.074\ \tau\ P'^{a}_{z} , \qquad \bar{G}(1)^{11}_{21} = 0.065\ \tau\ P'^{a}_{z}$$

$$\bar{G}(1)^{00}_{12} = 0.577\ \tau\ P'^{a}_{z} , \qquad \bar{G}(1)^{11}_{12} = 0.500\ \tau\ P'^{a}_{z} \qquad \text{(II.3-20)}$$

II.3C RADIATIVE DECAY OF SPIN POLARIZED EXCITED ATOMS

The radiative decay of an excited atomic ensemble with unpolarized spins is discussed very extensively in literature.[3 4 5 15] We describe the decay of a spin polarized excited atomic ensemble. The following additional assumptions are made in addition to those laid down at the beginning of section (II.3).

(i) Irrespective of the excitation mechanism, all the details of the excitation process are given by $\rho(0)$ which is assumed to be known.

(ii) The perturbations due to fine structure and hyperfine structure interactions are weak and of little relevance to excitation and decay processes which can be treated as completely independent processes. These perturbations, however, change the state multipoles describing the excited ensemble between excitation ($t = 0$) and decay by photon emission (time 't') via the perturbation coefficients (subsection B).

(iii) The excitation and decay times are sharply defined.

(iv) The atomic and nuclear spins do not change on excitation and decay.

(v) The excited ensemble in consideration is in general a statistical mixture of states $|\gamma_1 J_1 M_{J_1}>$ where γ_1 denotes collectively the set of quantum numbers which are necessary to describe these states in addition to the angular momentum quantum. numbers $J_1 M_{J_1}$. These excited atoms then decay to the lower levels $|\gamma_2 J_2 M_{J_2}>$ by emitting photons. We assume that γ_1 and γ_2 are fixed and suppress the dependence of the state vectors on

these quantum numbers hereafter.

(vi) The lower level is either the ground state (resonance-transitions) or it decays very slowly.

In view of the above assumptions we consider the time evolution of $\rho(0)$ under the interaction between the excited ensemble and the electromagnetic field of the virtual photons $[V_\gamma(t)]$ independent of the simultaneous (extremely weak) perturbations due to fine structure and hyperfine structure interactions. These effects are included at a later stage via the perturbation coefficients.

Immediately after the excitation $\rho(0)$ evolves under the influence of the total Hamiltonian $H = H_0 + V_\gamma(t)$. This time evolution is described by the time evolution operator $\bar{U}(t)$ so that at time 't' we obtain

$$\bar{\rho}(t) = \bar{U}(t)\rho(0)\bar{U}(t)^\dagger \qquad \text{(II.3-21)}$$

The density matrix $\bar{\rho}(t)$ describes the entire ensemble of atoms and photons at time t, that is, the atoms which are still in the excited state, the atoms in the lower levels, and the photons emitted in the time interval $(0, t)$. The decay process can be described in the first order perturbation theory. In this approximation the operator $\bar{U}(t)$ is given by the relation

$$\bar{U}(t) = U_0(t)\{1 - \frac{i}{\hbar}\int_0^t U_0(t)^\dagger V_\gamma(t) U_0(t)dt\} \qquad \text{(II.3-22)}$$

where $U_0(t)$ is the free time evolution operator corresponding to the unperturbed Hamiltonian H_0.

To be concise, we omit all the preliminary details of these calculations which can be found in ref.(15). We are interested in the reduced density matrix describing the state of only those

photons emitted at instant 't' after the excitation in the direction $\hat{\eta}_\gamma$ with frequency ω and helicity λ in the 'photon detector frame' (when level $|J_1M_{J_1}>$ decays to level $|J_2M_{J_2}>$ by photon emission). This is found by differentiating eq.(5.1.11) of ref. (15) with respect to time i.e.,

$$\rho^{\cdot}(\hat{\eta}_\gamma, t)_{\lambda'\lambda} = c(\omega) \sum_{\substack{J_2M_{J_2} \\ J'_1M'_{J_1}J_1M_{J_1}}} \{<J_2M_{J_2}|r_{-\lambda'}|J'_1M'_{J_1}><J'_1M'_{J_1}|\rho(0)|J_1M_{J_1}> \times <J_1M_{J_1}|r^{+}_{-\lambda}|J_2M_{J_2}>\}$$

$$\times \exp\{-i(E_{J'_1}-E_{J_1})t/\hbar - \tfrac{1}{2}(\gamma_{J'_1}+\gamma_{J_1})t\} \qquad \text{(II.3-23)}$$

where $\lambda', \lambda = \pm 1$ (± 1 for RHC/LHC photons).

The quantities $r_{-\lambda'}$, $r_{-\lambda}$ are the spherical components of the dipole vector $\underline{r}$ in the 'helicity-frame' spanned by the unit vectors

$$\hat{\varepsilon}_{\pm 1} = \mp \frac{1}{\sqrt{2}} \{\hat{\varepsilon}(\theta_\gamma) \pm \hat{\varepsilon}(\phi_\gamma)\}, \quad \hat{\varepsilon}_0 = \hat{\eta}_\gamma$$

Then,

$$\bar{r} = r^*_{+1}\,\hat{\varepsilon}_{+1} + r^*_{-1}\,\hat{\varepsilon}_{-1} + r^*_0\,\hat{\eta}_\gamma. \qquad \text{(II.3-24)}$$

When we normalize according to $\mathrm{tr}\,\rho^{\cdot}(\hat{\eta}_\gamma, t)_{\lambda'\lambda} = I(\hat{\eta}_\gamma, t)$, the intensity of the radiation, the leading factor $c(\omega)$ is given by the relation

$$c(\omega) = \frac{e^2\omega^4}{2\pi c^3}\, d\Omega_\gamma$$

e being the electron charge, c the velocity of light in vacuum and $d\Omega_\gamma$ the solid angle into which photons are collected.

Since the light emission depends on the electronic variables alone (as assumed earlier, the spins do not change on decay), therefore, to include the effect of extremely weak perturbations

due to fine structure and hyperfine structure interactions we substitute the reduced density matrix $\rho_{J_1}(t)$ (describing only the electronic states of the excited atomic ensemble at time t) for $\rho(0)$ in eq.(II.3-23) and do not repeat the resulting exponential factor of the perturbation coefficient [Blum[15] §5.4]. The density matrix representing the atomic states at the time of decay (after evolving under the influence of weak perturbations due to fine structure and hyperfine structure interactions) in the uncoupled representation[13] ($|J_1M_{J_1}\rangle|IM_I\rangle$ of H_0) and in the photon detector frame is given by

$$\rho(t) = \sum_{\substack{K\,q \\ K_i q_i}} \langle T(J_1;\,t)^{\dagger}_{Kq} \otimes T(I;\,t)^{\dagger}_{K_i q_i} \rangle T(J_1;\,t)_{Kq} \otimes T(I;\,t)_{K_i q_i} \tag{II.3-25}$$

To find the reduced density matrix $\rho_{J_1}(t)$ we put $K_i = 0$ in the above expression and obtain

$$\rho_{J_1}(t) = \sum_{Kq} \frac{\langle T(J_1;\,t)_{Kq} \rangle}{(2I+1)}\; T(J_1;\,t)_{Kq} \otimes \underline{1} \tag{II.3-26}$$

Substituting $\rho(0)$ by $\rho_{J_1}(t)$ in eq. (II.3-23) (dropping the exponential) we obtain

$$\begin{aligned}
\overline{\rho}^{\cdot}(n_\gamma,\,t)_{\lambda'\lambda} = \frac{c(\omega)}{2I+1} \sum_{\substack{Kq,J_2M_{J_2} \\ J'_1M'_{J_1},J_1M_{J_1}}} & \{\langle J_2M_{J_2}|r_{-\lambda'}|J'_1M'_{J_1}\rangle \\
& \times \langle J'_1M'_{J_1}|T(J_1;\,t)_{Kq} \otimes \underline{1}|J_1M_{J_1}\rangle\langle J_1M_{J_1}|r^{\dagger}_{-\lambda}|J_2M_{J_2}\rangle\} \\
& \times \langle T(J_1;\,t)^{\dagger}_{Kq}\rangle \\
= \frac{c(\omega)}{2I+1} \sum_{Kq} & \mathrm{tr}\{r_{-\lambda'}(T(J_1;\,t)_{Kq} \otimes \underline{1})\, r^{\dagger}_{-\lambda}\}\langle T(J_1;\,t)^{\dagger}_{Kq}\rangle
\end{aligned} \tag{II.3-27}$$

Since vector $\underline{r}$ is a tensor of rank one, we apply the Wigner Eckart theorem to get the relevant matrix elements.

$$\langle J_2 M_{J_2} | r_{-\lambda'} | J'_1 M'_{J_1} \rangle = (-1)^{J_2 - M_{J_2}} \begin{pmatrix} J_2 & 1 & J'_1 \\ -M_{J_2} & -\lambda' & M'_{J_1} \end{pmatrix} \langle J_2 || \underline{r} || J'_1 \rangle$$

$$\langle J_2 M_{J_2} | r_{-\lambda} | J_1 M_{J_1} \rangle^* = (-1)^{J_2 - M_{J_2}} \begin{pmatrix} J_2 & 1 & J_1 \\ -M_{J_2} & -\lambda & M_{J_1} \end{pmatrix} \langle J_2 || \underline{r} || J_1 \rangle^*$$

$$\langle J'_1 M'_{J_1} | T(J'_1 J_1)_{Kq} | J_1 M_{J_1} \rangle = (-1)^{J'_1 - M'_{J_1}} \begin{pmatrix} J'_1 & J_1 & K \\ M'_{J_1} & -M_{J_1} & -q \end{pmatrix} (2K+1)^{\frac{1}{2}}$$

By simple application of the following[22] Sum-Rule on 3j-symbols,

$$\sum_{n_1 n_2 n_3} (-1)^{l_1 - n_1 + l_2 - n_2 + l_3 - n_3} \begin{pmatrix} l_1 & j_1 & l_2 \\ n_1 & m_1 & -n_2 \end{pmatrix} \begin{pmatrix} l_2 & j_2 & l_3 \\ n_2 & m_2 & -n_3 \end{pmatrix} \begin{pmatrix} l_3 & j_3 & l_1 \\ n_3 & m_3 & -n_1 \end{pmatrix}$$

$$= (-1)^{j_1 + j_2 + j_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{Bmatrix} j_1 & j_2 & j_3 \\ l_3 & l_1 & l_2 \end{Bmatrix}$$

and some simplification we obtain

$$\mathrm{tr}\{r_{-\lambda'} T(J'_1 J_1, t)_{Kq} r^{\dagger}_{-\lambda}\} = \sum_{J'_1 J_1 J_2} (-1)^{J_1 + J_2 + \lambda} (2K+1)^{\frac{1}{2}} \langle J_2 || \underline{r} || J'_1 \rangle \langle J_2 || \underline{r} || J_1 \rangle^* \times \begin{pmatrix} 1 & 1 & K \\ -\lambda' & \lambda & q \end{pmatrix} \begin{Bmatrix} 1 & 1 & K \\ J_1 & J'_1 & J_2 \end{Bmatrix} \qquad \text{(II.3-28)}$$

We next transform the state multipole $\langle T(J_1; t)^{\dagger}_{Kq} \rangle$ from the 'photon detector frame' to the spin polarization frame by the relation

$$\langle T(J_1; t)^{\dagger}_{Kq} \rangle = \sum_{\chi} \langle T(J_1; t)^{\dagger}_{K\chi} \rangle D(\hat{n}_{\gamma} \to \hat{n}_s)^{(K)}_{q\chi}$$

and obtain

$$\bar{\rho}^{\cdot}(\hat{n}_\gamma,\ t)_{\lambda',\lambda} = c(\omega) \sum_{Kq\chi} \mathrm{tr}\{r_{-\lambda'}, T(J_1;\ t)_{Kq} r^{\dagger}_{-\lambda}\} \langle T(J_1;\ t)^{\dagger}_{K\chi} \rangle D(\hat{n}_\gamma \to \hat{n}_s)^{(K)}_{q\chi}$$

(II.3-29)

To specialize eq.(II.3-29) for the present case of 'LS-coupling holding during the collision', we substitute L_1 for J_1 in eq. (II.3-29) and use the appropriate reduced density matrix $\rho_{L_1}(t)$ instead of $\rho_{J_1}(t)$ in eq.(II.3-26) from subsection (B) in the photon detector frame. Eq.(II.3-29) in this case becomes

$$\bar{\rho}^{\cdot}(n_\gamma,\ t)_{\lambda',\lambda} = c(\omega) \sum_{Kq\chi} \mathrm{tr}\{r_{-\lambda'}, T(L_1;\ t)_{Kq} r^{\dagger}_{-\lambda}\}\ T(L_1;\ t)^{\dagger}_{K\chi} \rangle D(\hat{n}_\gamma \to \hat{n}_s)^{(K)}_{q\chi}$$

(II.3-30)

Substituting the value of the state multipole $\langle T(L_1;\ t)^{\dagger}_{K\chi} \rangle$ from eq.(II.3-14) in terms of the relevant perturbation coefficients of subsection (B) we obtain

$$\bar{\rho}^{\cdot}(\hat{n}_\gamma;\ t)_{\lambda',\lambda} = \sum_{\substack{K \\ K_{L_1}\chi_{L_1}}} \sum_{q} \mathrm{tr}\{r_{-\lambda'}, T(L_1;\ t)_{Kq} r^{\dagger}_{-\lambda}\} \langle T(L_1)^{\dagger}_{K_{L_1}\chi_{L_1}} \rangle G(L_1;\ t)^{\chi_{L_1}\chi_{L_1}}_{K_{L_1}K}$$

$$\times D(\hat{n}_\gamma \to \hat{n}_s)^{(K)}_{q\chi_{L_1}} \qquad \text{(II.3-30a)}$$

As a final step we transform the state multipole $\langle T(L_1)^{\dagger}_{K_{L_1}\chi_{L_1}} \rangle$ from the 'spin polarization frame' to the 'collision frame', by application of the transformation

$$\langle T(L_1)^{\dagger}_{K_{L_1}\chi_{L_1}} \rangle = \sum_{Q_{L_1}} \langle T(L_1)^{\dagger}_{K_{L_1}Q_{L_1}} \rangle D(\hat{n}_s \to \hat{p}_0)^{(K_{L_1})}_{\chi_{L_1}Q_{L_1}} \qquad \text{(II.3-31)}$$

to obtain

$$\bar{\rho}^{\cdot}(\bar{n}_\gamma,\ t)_{\lambda',\lambda} = c(\omega) \sum_{Kq} \sum_{K_{L_1}\chi_{L_1}\varrho_{L_1}} \mathrm{tr}\{r_{-\lambda'}T(L_1;\ t)_{Kq}r^{\dagger}_{-\lambda}\}\, D(\hat{n}_\gamma\to\hat{n}_s)^{(K)}_{q\chi_{L_1}} D(\hat{n}_s\to\hat{p}_0)^{(K_{L_1})}_{\chi_{L_1}\varrho_{L_1}}$$

$$\times \langle T(L_1)^{\dagger}_{K_{L_1}\varrho_{L_1}}\rangle G(L_1;\ t)^{\chi_{L_1}\chi_{L_1}}_{K_{L_1}K} \qquad \text{(II.3-30b)}$$

The state multipoles $\langle T(L_1)^{\dagger}_{K_{L_1}\varrho_{L_1}}\rangle$ are functions of the excitation amplitudes, as, for example, can be noticed in the set of eqs.(II.3-8). It is convenient for the experimentalist if he relates eq.(II.3-30b) to the Stokes parameters (introduced in Section I.3) via eq.(I.3-7).

When the excited atom is spin polarized along the direction $\hat{p}_0$ just after the collision (then since $\hat{n}_s \equiv \hat{p}_0$, the rotation matrix element $D(\hat{p}_0\to\hat{p}_{0^-})^{(K_{L_1})}_{\chi_{L_1}\varrho_{L_1}}$ is unity and $\chi_{L_1} \equiv \varrho_{L_1}$) eq.(II.3-30b) simplifies to

$$\bar{\rho}^{\cdot}(\hat{n}_\gamma,\ t)_{\lambda',\lambda} = c(\omega) \sum_{Kq,K_{L_1}\varrho_{L_1}} \mathrm{tr}\{r'_{-\lambda'}T(L_1;\ t)_{Kq}r^{\dagger}_{-\lambda}\}D(\hat{n}_\gamma\to\hat{p}_0)^{(K)}_{q\varrho_{L_1}}$$

$$\times \langle T(L_1)^{\dagger}_{K_{L_1}\varrho_{L_1}}\rangle G(L_1;\ t)^{\varrho_{L_1}\varrho_{L_1}}_{K_{L_1}K} \qquad \text{(II.3-30c)}$$

To illustrate our formulism, we find Stokes parameters for the resonance transition $^2S \to {}^2P$ in light one electron atoms when $E_{J'_1} - E_{J_1} >> \gamma(\tau_{J'_1 J_1} << \tau)$ and the hyperfine structure interaction is neglected. The relevant perturbation coefficients are

given in the set of eqs.(II.3-20) [assuming $\tau_R >> \tau$] . When the initial electrons and atoms are polarized along the direction $\bar{P}_0$, the Stokes parameters are found by comparing the density matrices in eqs.(II.3-30c) and (I.3-7) in the form

$$I = c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\Big(\Big[\tfrac{2}{3\sqrt{3}}\,\bar{G}_0\langle T_p(1)^{\dagger}_{00}\rangle + \tfrac{1}{3}\,\bar{G}_2\Big\{\tfrac{1}{2\sqrt{3}}\,(3\cos^2\theta_\gamma - 1)\langle T_p(1)^{\dagger}_{20}\rangle - \sin 2\theta_\gamma\cos\phi_\gamma\langle T_p(1)^{\dagger}_{21}\rangle + \sin^2\theta_\gamma\cos 2\phi_\gamma\langle T_p(1)^{\dagger}_{22}\rangle\Big\}\Big]$$
$$+ \Big\{\tfrac{2}{3\sqrt{3}}\,\bar{G}(1)^{00}_{10} + \tfrac{1}{3}\sqrt{\tfrac{2}{3}}(3\cos^2\theta_\gamma - 1)\bar{G}(1)^{00}_{12}\Big\}\langle T_p(1)^{\dagger}_{10}\rangle$$
$$- \tfrac{1}{3}\,\bar{G}(1)^{11}_{12}\sin 2\theta_\gamma\sin\phi_\gamma\, i\langle T_p(1)^{\dagger}_{11}\rangle\Big)$$

$$In_2 = \tfrac{\sqrt{2}}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\Big(\Big[-\sqrt{2}\,\bar{G}_1\sin\theta_\gamma\sin\phi_\gamma\, i\langle T_p(1)^{\dagger}_{11}\rangle\Big]$$
$$+ \bar{G}(1)^{00}_{01}\cos\theta_\gamma\langle T_p(1)^{\dagger}_{00}\rangle$$
$$+ \bar{G}_1\cos\theta_\gamma\langle T_p(1)^{\dagger}_{10}\rangle + \bar{G}(1)^{00}_{21}\cos\theta_\gamma\langle T_p(1)^{\dagger}_{20}\rangle$$
$$- \sqrt{2}\,\bar{G}(1)^{11}_{21}\sin\theta_\gamma\cos\phi_\gamma\langle T_p(1)^{\dagger}_{21}\rangle\Big)$$

$$In_3 = -\tfrac{1}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\Big(\Big[\bar{G}_2\big(\tfrac{\sqrt{3}}{2}\sin^2\theta_\gamma\langle T_p(1)^{\dagger}_{20}\rangle + \sin 2\theta_\gamma\cos\phi_\gamma\langle T_p(1)^{\dagger}_{21}\rangle$$
$$+ (1 + \cos^2\theta_\gamma)\cos 2\phi_\gamma\langle T_p(1)^{\dagger}_{22}\rangle\big)\Big]$$
$$+ \tfrac{\sqrt{3}}{2}\,\bar{G}(1)^{00}_{12}\sin^2\theta_\gamma\langle T_p(1)^{\dagger}_{10}\rangle + \bar{G}(1)^{11}_{12}\sin 2\theta_\gamma\sin\phi_\gamma\, i\langle T_p(1)^{\dagger}_{11}\rangle\Big)$$

$$In_1 = \tfrac{1}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\Big(\Big[2\bar{G}_2\big(\sin\theta_\gamma\sin\phi_\gamma\langle T_p(1)^{\dagger}_{21}\rangle$$
$$+ \cos\theta_\gamma\sin 2\phi_\gamma\langle T_p(1)^{\dagger}_{22}\rangle\big)\Big]$$
$$- 2\bar{G}(1)^{11}_{12}\sin\theta_\gamma\cos\phi_\gamma\, i\langle T_p(1)^{\dagger}_{11}\rangle\Big) \qquad \text{(II.3-32)}$$

where

$$|\langle 0||\underline{r}||1\rangle|^2 = 1$$

and we have put those terms which are independent of the polarization of the scattered atom $p_z^{a'}$ in the square brackets. It is worthwhile to point out that our theory has agreement with Blum and Kleinpoppen (1979)[5] when unpolarized initial electrons and atoms are used.

II.4 COMPLETE DETERMINATION OF SCATTERING AMPLITUDES OF A ^2p STATE OF LIGHT ONE ELECTRON ATOMS

From a practical point of view the resonance transition $^2p - {}^2s$ in light atoms is quite useful and can be tackled now by the above theory. The six state multipoles in the relations of eq.(II.3-8) can be determined by suitable experimental measurements. For instance if the spin polarizations P_e, P_a and P'_a are known then these state multipoles can be determined in the following two ways:

(a) by measuring $I(\theta_\gamma \phi_\gamma)$ at, at least six different photon-detector positions, (keeping the electron-analyzer fixed, a so-called angular correlation experiment), and by simultaneously inverting the spin polarization of one of the incident beams. In this way the components of the state multipoles dependent on spin polarization can be separated from those which are independent.

(b) Stokes parameters (II.3.32) can be measured at two suitable photon-detector positions with simultaneous inversion of spin polarization of one of the incident particles. This set of measurements produces sixteen simultaneous linear equations from which the required set of state multipoles can be determined.

We list all the state miltipoles in terms of scattering amplitudes for magnetic sublevels for direct and exchange scattering.

$$\langle T(1)^{\dagger}_{00}\rangle = \frac{\sigma}{\sqrt{3}}$$

$$\langle T(1)^{\dagger}_{10}\rangle = 0$$

$$\langle T(1)^{\dagger}_{11}\rangle = -\frac{i}{\sqrt{2}}\ \mathrm{Im}(2f_1f_0^* + 2g_1g_0^* - f_1g_0^* - g_1f_0^*)$$

$$\langle T(1)^{\dagger}_{20}\rangle = \sqrt{\frac{2}{3}}\,(\sigma_1 - 2\sigma_0)$$

$$\langle T(1)^{\dagger}_{21}\rangle = -\frac{1}{\sqrt{2}}\,\mathrm{Re}\{2f_1f_0^* + 2g_1g_0^* - f_1g_0^* - g_1f_0^*\}$$

$$\langle T(1)^{\dagger}_{22}\rangle = -\sigma_1$$

and

$$\langle T_s(1)^{\dagger}_{00}\rangle = -\frac{P^aP^e}{\sqrt{3}}\,\zeta$$

$$\langle T_s(1)^{\dagger}_{10}\rangle = \sqrt{3}\ P^aP^e(\zeta_0 - 2\zeta_1)$$

$$\langle T_s(1)^{\dagger}_{11}\rangle = \frac{i}{\sqrt{2}}\,P^aP^e\ \mathrm{Im}(f_1g_0^* + g_1f_0^*)$$

$$\langle T_s(1)^{\dagger}_{20}\rangle = \sqrt{\frac{2}{3}}\,P^aP^e(\zeta_1 - 2\zeta_0)$$

$$\langle T_s(1)^{\dagger}_{21}\rangle = \frac{1}{\sqrt{2}}\,P^aP^e\ \mathrm{Re}(f_1g_0^* + g_1f_0^*)$$

$$\langle T_s(1)^{\dagger}_{22}\rangle = P^aP^e\,\zeta_1$$

Only eight out of these eleven quantities are independent since it can be checked that

$$\langle T_s(1)^{\dagger}_{10}\rangle = -1.8\langle T_s(1)^{\dagger}_{00}\rangle + 1.2\sqrt{2}\langle T_s(1)^{\dagger}_{20}\rangle$$

$$\langle T(1)^{\dagger}_{22}\rangle = -\frac{2\sqrt{3}}{5}\langle T(1)^{\dagger}_{00}\rangle - \frac{1}{5}\ \frac{\sqrt{3}}{2}\langle T(1)^{\dagger}_{20}\rangle$$

$$\langle T_s(1)^{\dagger}_{22}\rangle = \frac{3}{4}\,(\frac{\sqrt{3}}{5} - 1)\langle T_s(1)^{\dagger}_{00}\rangle - \frac{\sqrt{6}}{10}\langle T_s(1)^{\dagger}_{20}\rangle$$

From the eight independent quantities it is trival to find σ, σ_0, σ_1, ζ, ζ_0, ζ_1, $\mathcal{S}$, $\mathcal{S}_0$, $\mathcal{S}_1$ together with $(f_1f_0^* + g_1g_0^*)$ and $(f_1g_0^* + g_1f_0^*)$. We write

$f_0 = |f_0|$, $f_1 = |f_1|e^{i\chi_1}$, $g_0 = g_0 e^{i\psi_0}$, $g_1 = |g_1|e^{i\psi_1}$

and we have

$$\sigma_0 = |f_0|^2 + |g_0|^2 - \zeta_0$$

or $$|f_0|^2 + |g_0|^2 = \sigma_0 + \zeta_0$$

$$|f_1|^2 + |g_1|^2 = \sigma_1 + \zeta_1 \quad ;$$

we substitute

$$|f_0|^2 = x_0, \quad |f_1|^2 = x_1, \quad \text{and} \quad |g_0|^2 = y_0, \quad |g_1|^2 = y_1$$

then we write eqs.(II.2-2) and (II.2-3) for $^2S \rightarrow {}^2P$ excitation in consideration in the form:

$$x_0 + 2x_1 = c_1 \quad \text{(known)} \qquad \text{(II.4-1)}$$

$$y_0 + 2y_1 = c_2 \quad \text{(known)} \qquad \text{(II.4-2)}$$

and from above

$$x_0 + y_0 = c_3 \quad \text{(known)} \qquad \text{(II.4-3)}$$

$$x_1 + y_1 = c_4 \quad \text{(known)} \qquad \text{(II.4-4)}$$

Further

$$\mathrm{Re}(f_1 g_0^* + g_1 f_0^*) = |f_1||g_0|\cos(\chi_1 - \psi_0) + |g_1||f_0|\cos\psi_1 \qquad \text{(II.4-5a)}$$

$$\mathrm{Im}(f_1 g_0^* + g_1 f_0^*) = |f_1||g_0|\sin(\chi_1 - \psi_0) + |g_1||f_0|\sin\psi_1 \qquad \text{(II.4-5b)}$$

and

$$\mathrm{Re}(f_1 f_0^* + g_1 g_0^*) = |f_1||f_0|\cos\chi_1 + |g_1||g_0|\cos(\psi_1 - \psi_0) \qquad \text{(II.4-6a)}$$

$$\mathrm{Im}(f_1 f_0^* + g_1 g_0^*) = |f_1||f_0|\sin\chi_1 + |g_1||g_0|\sin(\psi_1 - \psi_0) \qquad \text{(II.4-6b)}$$

squaring and adding (II.4-5a), (II.4-5b) and then (II.4-6a), (II.4-6b) we obtain

$$|f_1|^2|g_0|^2 + |g_1|^2|f_0|^2 + 2|f_0||g_0||f_1||g_1|\cos\{(\chi_1 - \psi_1) - \psi_0\} = C_5 \qquad \text{(II.4-5)}$$

$$|f_1|^2|f_0|^2 + |g_1|^2|g_0|^2 + 2|f_0||g_0||f_1||g_1|\cos\{(\chi_1 - \psi_1) + \psi_0\} = C_6 \qquad \text{(II.4-6)}$$

since

$\zeta_1 = |f_1||g_1|\cos(\psi_1 - \chi_1)$ and $\zeta_0 = |f_0||g_0|\cos\psi_0$

we get, on taking positive square roots only

$$\sin(\psi_1 - \chi_1) = + \sqrt{1 - \zeta_1^2/|f_1|^2|g_1|^2} \qquad \text{(II.4-7)}$$

and

$$\sin\psi_0 = + \sqrt{1 - \zeta_0^2/|f_0|^2|g_0|^2} \qquad \text{(II.4-8)}$$

Subtracting (II.4-5), (II.4-6) and using (II.4-7), (II.4-8) we obtain

$$(x_1 - y_1)(x_0 - y_0) + 4\sqrt{(x_1y_1 - \zeta_1^2)(x_0y_0 - \zeta_0^2)} = C_7$$

But $y_0 = C_3 - x_0$ and $y_1 = C_4 - x_1$ we obtain

$$(2X_1 - C_4)(2X_0 - C_3) + 4\sqrt{\{X_1(C_4 - X_1) - \zeta_1^2\}\{X_0(C_3 - X_0) - \zeta_0^2\}} = C_7$$

Furthermore with

$$2X_1 = C_1 - X_0$$

we get

$$(C_1 - X_0 - C_4)(2X_0 - C_3) + 4\sqrt{\{\tfrac{1}{2}C_1 - \tfrac{1}{2}X_0)(C_4 - \tfrac{1}{2}C_1 + \tfrac{1}{2}X_0) - \zeta_1^2\}} \times$$
$$\times\sqrt{\{X_0(C_3 - X_0) - \zeta_0^2\}} = C_7$$

or

$$[(X_0 - C_1 + C_4)(2X_0 - C_3) - C_7]^2 = 4[\{(X_0 - C_1)(X_0 + 2C_4 - C_1) + 4\zeta_1^2\}$$
$$\{X_0(X_0 - C_3) + \zeta_0^2\}]$$

This is a cubic equation in X_0 of the form

$$a_0X_0^3 + a_1X_0^2 + a_2X_0 + a_3 = 0$$

The physically acceptable root can be chosen such that

$$X_0 = |f_0|^2 = \text{positive and } X_0 \leq \sigma_0$$

Substituting X_0 in eqs. (II.4-1) to (II.4-4) we get X_1, Y_0, Y_1. Then ψ_0 is found from eq.(II.4-8). Now eqs.(II.4-5a), (II.4-5b) and (II.4-6a), (II.4-6b) take the simple form

$$A_1\cos\chi_1 + B_1\cos\psi_1 = D_1$$

$$A_2\sin\chi_1 + B_2\sin\psi_1 = D_2$$

$$A_3\cos\chi_1 + B_3\cos\psi_1 = D_3$$

$$A_4\sin\chi_1 + B_4\sin\psi_1 = D_4$$

Taking these equations and using suitable pairs, we get χ_1 and ψ_1 . In this way all the quantities $|f_0|$, $|f_1|$, $|g_0|$, $|g_1|$ and the three relative phases χ_1, ψ_0, ψ_1 are determined.

CHAPTER III

THEORY OF MEASUREMENT FOR ELECTRON ATOM ELASTIC SCATTERING WHEN SPIN ORBIT INTERACTION IS INCLUDED IN THE COLLISION

Spin polarization analysis[23 24 17] in electron atom collisions is important for increasing understanding of the detailed mechanism of scattering in atomic physics. We have illustrated this for elastic as well as inelastic electron atom scattering (neglecting spin dependent interactions during collision) in Chapter II.

The early theoretical studies on electron spin polarization by elastic scattering from unpolarized targets were carried out for high-energy scattering by Mott[25] on the basis of the Dirac equation (assuming the scattering process as a deflection in the pure Coulomb field of the nucleus at relativistic electron energies ~ 100 Kev). Later Massey and Mohr[26] considered the polarization effects in electron atom scattering down to lower energies of 100 eV, taking into account the screening of the nuclear Coulomb field by the atomic electrons. Burke and Schey[27] applied density matrix fomulism for the spin polarization correlation of initial and final electrons and atoms for low energy elastic scattering on hydrogen. The extensive study of spin-orbit effects in polarized electrons scattering on unpolarized heavy atoms was reviewed by Kessler.[23] Farago also dealt with the subject of electron spin polarization in detail.[24] Kleinpoppen[18] carried out detailed calculations on spin polarized electrons colliding with spin polarized light alkalis, whereas Blum and Kleinpoppen[19] dealt with spin analysis of electron atom collisions (elastic as well as inelastic) excluding spin orbit interaction during the collision in the more elegant framework of spin tensors.

All the above theoretical attempts treat only the limiting cases where either exchange interaction (a consequence of Pauli's exclusion principle for identical particles) only or the spin orbit interaction only is taken into account during the collision. It is, however, well known that spin polarization effects are largest when either the scattering amplitude is small or is rapidly varying so enhancing the importance of the spin orbit interaction in the collision.[28] Afterwards Burke and Mitchell[8] included both electron exchange and relativistic effects (spin orbit interaction) in their calculations which are very suitable in electron scattering by the heavier alkalis such as Cs . They, however, did not include electron-electron spin and electron-nuclear spin interactions in their theory. Although electron exchange plays a dominant role in changing the initial spin polarizations in such collisions at relatively low energies, yet there is hard experimental evidence from Wilmers (1972)[29] to suggest that spin orbit interaction is also important. He found that when unpolarized electrons and unpolarized K atoms collide, the elastically scattered electrons are polarized, indicating that spin orbit interaction is also indispensible for a reliable theory on such processes.

When spin orbit interaction is included in the description of the collision, Burke and Mitchell[8] have shown that six independent scattering amplitudes are necessary to completely describe the polarized electron, polarized S-state spin-$\frac{1}{2}$ target elastic collision at all energies. This obviously means that at least eleven independent measurements are necessary for a complete determination of the collision matrix at given energy and electron scattering direction (θ , ϕ) . They, however, had not indicated how these amplitudes can be extracted experimentally. In this chapter we find a scheme for extracting these amplitudes from experiment.

Some observable consequences of their theory are also given. Throughout this chapter all the quantities refer to a fixed electron energy E and scattering direction (θ , ϕ). We use superscripts e and a for electrons and atoms respectively, a prime for various quantities after scattering and subscript '0' (zero) when the relevant initial electron/atom beam is unpolarized. For any quantity Q_{xy}, subscripts x and y represent initial polarization components of atoms and electrons, respectively.

III.1 DETERMINATION OF COLLISION MATRIX IN TERMS OF SPIN TENSORS

If the collision frame is defined by the unit vectors

$$\underline{n} = \frac{\bar{P}_0 \times \bar{P}_1}{|\bar{P}_0 \times \bar{P}_1|} \quad , \quad \underline{p} = \frac{\bar{P}_0 + \bar{P}_1}{|\bar{P}_0 + \bar{P}_1|} \quad , \quad \underline{q} = \frac{\bar{P}_0 - \bar{P}_1}{|\bar{P}_0 - \bar{P}_1|}$$

where $\bar{P}_0$ and $\bar{P}_1$ are, respectively, electron momenta before and after collision, the collision-matrix can then be expressed in terms of Pauli spin vectors for the atom and the scattered electron in the form [30]

$$M = Ia_1 + (\sigma^a.n)a_2 + (\sigma^e.n)a_3 + (\sigma^a.n)a_4 + (\sigma^a.p)(\sigma^e.p)a_5 + (\sigma^a.q)(\sigma^e.q)a_6 \qquad \text{(III.1-1)}$$

These a_s's $(s = 1, \ldots, 6)$ being the scattering amplitudes, are complex functions of (θ, E) the scattering angle θ and the initial electron energy E. If we put

$$a_s = |a_s| e^{i\gamma_s} \qquad s, t = 1, 2, \ldots 6 \qquad \text{(III.1-2)}$$

and

$$\gamma_{st} = \gamma_s - \gamma_t$$

and take up eqs. (40 and (41) of Burke and Mitchell[8] we have the differential cross section for the scattering of unpolarized electrons on unpolarized atoms:

$$\sigma(0,\ 0,\ 0,;\ 0,\ 0,\ 0) = \sigma_{00} = |a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 + |a_5|^2 + |a_6|^2 \qquad \text{(III.1-3)}$$

where

$$\sigma(\underline{P}^a,\ \underline{P}^e) = \sigma(P_n^a,\ P_p^a,\ P_n^a;\ P_n^e,\ P_n^e,\ P_n^e)$$

is the differential cross-section with partially polarized atom and partially polarized electron beams. The relevant components of the polarization refer to the $(\underline{n},\ \underline{p},\ \underline{q})$ frame as introduced above. The components of the depolarization-tensor D are given by:*

$$\sigma_{00}D^a_{nn} = \sigma_{00}D^e_{nn} = |a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 - |a_5|^2 - |a_6|^2 \qquad \text{(III.1-4)}$$

$$\sigma_{00}D^a_{pp} = |a_1|^2 - |a_2|^2 + |a_3|^2 - |a_4|^2 + |a_5|^2 - |a_6|^2 \qquad \text{(III.1-5)}$$

$$\sigma_{00}D^a_{qq} = |a_1|^2 - |a_2|^2 + |a_3|^2 - |a_4|^2 - |a_5|^2 + |a_6|^2 \qquad \text{(III.1-6)}$$

$$\sigma_{00}D^e_{pp} = |a_1|^2 + |a_2|^2 - |a_3|^2 - |a_4|^2 + |a_5|^2 - |a_6|^2 \qquad \text{(III.1-7)}$$

$$\sigma_{00}D^e_{qq} = |a_1|^2 + |a_2|^2 - |a_3|^2 - |a_4|^2 - |a_5|^2 + |a_6|^2 \qquad \text{(III.1-8)}$$

If the incident atom beam is partially polarized in the i direction and incident electron beam is partially polarized in the j direction then we denote the polarizations of the atoms and electrons after collision, measured in k and l directions respectively, by P'^{ak}_{ij} and P'^{el}_{ij} (i, j, k, l = n, p, q). Note that these symbols refer to unique experimental measurements. Remembering that all tensor components containing just one subscript n are zero,[8] the remaining relations are:

$$\sigma_{00}P'^{an}_{00} = 2[|a_1||a_2|\cos\gamma_{12} + |a_3||a_4|\cos\gamma_{34}] \qquad \text{(III.1-9)}$$

$$\sigma_{00}P'^{en}_{00} = 2[|a_1||a_3|\cos\gamma_{13} + |a_2||a_4|\cos\gamma_{24}] \qquad \text{(III.1-10)}$$

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* The tensors C, D and K have been explicitly defined in reference 8.

The components of the spin-correlation tensor C have the relations:*

$$\sigma_{00}C_{nn} = 2[|a_1||a_4|\cos\gamma_{14} + |a_2||a_3|\cos\gamma_{23} - |a_5||a_6|\cos\gamma_{56}] \quad \text{(III.1-11)}$$

$$\sigma_{00}C_{pp} = 2[|a_1||a_5|\cos\gamma_{15} - |a_4||a_6|\cos\gamma_{46}] \quad \text{(III.1-12)}$$

$$\sigma_{00}C_{qq} = 2[|a_1||a_6|\cos\gamma_{16} - |a_4||a_5|\cos\gamma_{45}] \quad \text{(III.1-13)}$$

$$\sigma_{00}C_{pq} = 2[|a_3||a_5|\sin\gamma_{35} - |a_2||a_6|\sin\gamma_{26}] \quad \text{(III.1-14)}$$

$$\sigma_{00}C_{qp} = 2[|a_2||a_5|\sin\gamma_{25} - |a_3||a_6|\sin\gamma_{36}] \quad \text{(III.1-15)}$$

The remaining components of depolarization-tensor are:

$$-\sigma_{00}D^a_{qp} = \sigma_{00}D^a_{pq} = 2[|a_1||a_2|\sin\gamma_{12} + |a_3||a_4|\sin\gamma_{34}] \quad \text{(III.1-16)}$$

$$-\sigma_{00}D^e_{qp} = \sigma_{00}D^e_{pq} = 2[|a_1||a_3|\sin\gamma_{13} + |a_2||a_4|\sin\gamma_{24}] \quad \text{(III.1-17)}$$

The components of polarization transfer tensor K are given by:*

$$\sigma_{00}K^e_{nn} = \sigma_{00}K^a_{nn} = 2[|a_1||a_4|\cos\gamma_{14} + |a_2||a_3|\cos\gamma_{23} + |a_5||a_6|\cos\gamma_{56}] \quad \text{(III.1-18)}$$

$$\sigma_{00}K^e_{pp} = \sigma_{00}K^a_{pp} = 2[|a_1||a_5|\cos\gamma_{15} + |a_4||a_6|\cos\gamma_{46}] \quad \text{(III.1-19)}$$

$$\sigma_{00}K^e_{qq} = \sigma_{00}K^a_{qq} = 2[|a_1||a_6|\cos\gamma_{16} + |a_4||a_5|\cos\gamma_{45}] \quad \text{(III.1-20)}$$

$$-\sigma_{00}K^e_{qp} = \sigma_{00}K^a_{pq} = 2[|a_2||a_5|\sin\gamma_{25} + |a_3||a_6|\sin\gamma_{36}] \quad \text{(III.1-21)}$$

$$-\sigma_{00}K^e_{pq} = \sigma_{00}K^a_{pq} = 2[|a_2||a_6|\sin\gamma_{26} + |a_3||a_5|\sin\gamma_{35}] \quad \text{(III.1-22)}$$

The matrix of eqs.(III.1-3) to (III.1-8) which are linear in $|a_s|^2$ ($s = 1, 2, \ldots, 6$) is singular ($|A| = 0$) and is of rank 5. It means that one of the $|a_s|^2$ remains arbitrary. After some calculations we obtain

$$|a_5|^2 = \frac{\sigma_{00}}{4}[(1 - D^a_{nn}) + (D^a_{pp} - D^a_{qq})] = \frac{\sigma_{00}}{4}[(1 - D^e_{nn}) + (D^e_{pp} - D^e_{qq})] \quad \text{(III.1-23)}$$

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* The tensors C, D and K have been explicitly defined in reference 8.

and

$$|a_6|^2 = \frac{\sigma 00}{4}\,[(1 - D^a_{nn}) - (D^a_{pp} - D^a_{qq})] = \frac{\sigma 00}{4}\,[(1 - D^e_{nn}) - (D^e_{pp} - D^e_{qq})] \qquad \text{(III.1-24)}$$

It turns out that

$$D^a_{pp} - D^a_{qq} = D^e_{pp} - D^e_{qq} \qquad \text{(III.1-25)}$$

$$|a_1|^2 - |a_4|^2 = \frac{\sigma_{00}}{2}\,[D^a_{pp} + D^e_{qq}] = \frac{\sigma_{00}}{2}\,[D^a_{qq} + D^e_{pp}] \qquad \text{(III.1-26)}$$

$$|a_3|^2 - |a_2|^2 = \frac{\sigma_{00}}{2}\,[D^a_{pp} - D^e_{pp}] = \frac{\sigma_{00}}{2}\,[D^a_{qq} - D^e_{qq}] \qquad \text{(III.1-27)}$$

$$|a_2|^2 + |a_4|^2 = \frac{\sigma_{00}}{4}\,[(1 + D^a_{nn}) - (D^a_{pp} + D^a_{qq})] \;;\; D^a_{nn} = D^e_{nn} \qquad \text{(III.1-28)}$$

To solve eqs. (III.1-26), (III.1-27) and (III.1-28) for $|a_1|$, $|a_2|$, $|a_3|$ and $|a_4|$ we apply eqs.(III.1-9) to (III.1-22) to obtain

$$|a_4||a_5|\cos\gamma_{45} = \frac{\sigma_{00}}{4}\,(K^a_{qq} - C_{qq}) \qquad \text{(III.1-29)}$$

$$|a_5||a_6|\cos\gamma_{56} = \frac{\sigma_{00}}{4}\,(K^a_{nn} - C_{nn}) \qquad \text{(III.1-30)}$$

$$|a_4||a_6|\cos\gamma_{46} = \frac{\sigma_{00}}{4}\,(K^a_{pp} - C_{pp}) \qquad \text{(III.1-31)}$$

$$|a_1||a_6|\cos\gamma_{16} = \frac{\sigma_{00}}{4}\,(K^a_{qq} + C_{qq}) \qquad \text{(III.1-32)}$$

$$|a_2||a_6|\sin\gamma_{26} = \frac{\sigma_{00}}{4}\,(K^a_{qp} - C_{pq}) \qquad \text{(III.1-33)}$$

$$|a_3||a_6|\sin\gamma_{36} = \frac{-\sigma_{00}}{4}\,(K^a_{pq} + C_{qp}) \qquad \text{(III.1-34)}$$

It may be useful to mention the following relations

$$|a_1||a_5|\cos\gamma_{15} = \frac{\sigma_{00}}{4}\,(K^a_{pp} + C_{pp}) \qquad \text{(III.1-35)}$$

$$|a_3||a_5|\sin\gamma_{35} = \frac{\sigma_{00}}{4}\,(K^a_{qp} + C_{pq}) \qquad \text{(III.1-36)}$$

$$|a_2||a_5|\sin\gamma_{25} = \frac{\sigma_{00}}{4}\,(C_{qp} - K^a_{pq}) \qquad \text{(III.1-37)}$$

$$|a_1||a_4|\cos\gamma_{14} + |a_2||a_3|\cos\gamma_{23} = \frac{\sigma_{00}}{4}\,(K^a_{nn} + C_{nn}) \qquad \text{(III.1-38)}$$

$$\frac{\sigma^2_{00}}{4}\,(P'^{an}_{00}{}^2 + D^{a^2}_{pq}) = |a_1|^2|a_2|^2 + |a_3|^2|a_4|^2 + 2|a_1||a_2||a_3||a_4| \times \cos(\gamma_{12} - \gamma_{34}) \qquad \text{(III.1-39)}$$

$$\frac{\sigma^2_{00}}{4}\,(P'^{en}_{00}{}^2 + D^{e^2}_{pq}) = |a_1|^2|a_3|^2 + |a_2|^2|a_4|^2 + 2|a_1||a_2||a_3||a_4| \times \cos(\gamma_{13} - \gamma_{24}) \qquad \text{(III.1-40)}$$

Since $\gamma_{12} - \gamma_{34} = \gamma_{13} - \gamma_{24}$, and using eqs.(III.1-39), (III.1-40), and (III.1-26), (III.1-27) we get

$$P'^{2}_{00}{}^{en} - P'^{2}_{00}{}^{an} = (D^a_{pq} - D^e_{pq})(D^a_{pq} + D^e_{pq}) + (D^a_{pp} - D^e_{pp})(D^a_{pp} + D^e_{qq}) \qquad \text{(III.1-41)}$$

Further

$$\frac{\sigma^2_{00}}{4}\,(P'^{2}_{00}{}^{an} - D^{a^2}_{pq}) = |a_1|^2|a_2|^2\cos 2\gamma_{12} + |a_3|^2|a_4|^2\cos 2\gamma_{34} + 2|a_1||a_2||a_3||a_4|\cos(\gamma_{12} + \gamma_{34}) \qquad \text{(III.1-42)}$$

$$\frac{\sigma^2_{00}}{4}\,(P'^{en}_{00}{}^2 - D^{e^2}_{pq}) = |a_1|^2|a_3|^2\cos 2\gamma_{13} + |a_2|^2|a_4|^2\cos 2\gamma_{24} + 2|a_1||a_2||a_3||a_4|\cos(\gamma_{13} + \gamma_{24}) \qquad \text{(III.1-43)}$$

Returning to the solution of $|a_t|^2$ (t = 1, 2, 3, 4), we find $|a_4|^2$ by the following procedure.

Substituting the values of $|a_5|$ and $|a_6|$ in eq.(III.1-30) we obtain

$$\gamma_{56} = \cos^{-1}\left[\frac{K^a_{nn} - C_{nn}}{\sqrt{(1 - D^a_{nn})^2 - (D^a_{pp} - D^a_{qq})^2}}\right] \qquad \text{(III.1-44)}$$

Division of (III.1-31) by (III.1-29) results in

$$\gamma_{45} = \tan^{-1}\left[\cot\gamma_{56} - \left(\frac{K^a_{pp} - C_{pp}}{K^a_{qq} - C_{qq}}\right)\operatorname{cosec}\gamma_{56}\right] \qquad \text{(III.1-45)}$$

Substituting (III.1-45) in (III.1-29) we obtain

$$|a_4|^2 = \frac{\sigma_{00}}{4}\left[\frac{(K^a_{qq} - C_{qq})^2\operatorname{cosec}^2\gamma_{45}}{(1 - D^a_{nn}) + (D^a_{pp} - D^a_{qq})}\right] \qquad \text{(III.1-46)}$$

$|a_1|^2$, $|a_2|^2$ and $|a_3|^3$ are determined by substituting (III.1-46) in eqs. (III.1-26) to (III.1-28). The quantum mechanical phases of the complex functions a_s ($s = 1, \ldots, 6$) relative to γ_6 (i.e. γ_{16}, γ_{26}, γ_{36}, γ_{46}, γ_{56}) are determined by eqs.(III.1-30) to (III.1-36). In this way the collision matrix in the form (III.1-1) is completely determined.

To transform the collision matrix into other suitable forms (eqs. (9), (11) and (20 of Burke and Mitchell[8]) we are going to apply the following set of relations (Eqs.(10) and (18) of Burke and Mitchell[8])

$$\tfrac{1}{2}f_1 + \tfrac{1}{4}f_2 + \tfrac{1}{4}g + 0h_1 + 0h_2 + 0k + 0m = a_1$$

$$0f_1 + 0f_2 + 0g + \frac{i}{2\sqrt{2}}h_1 + \frac{i}{2\sqrt{2}}h_2 + \frac{i}{\sqrt{2}}k + 0m = a_2$$

$$0f_1 + 0f_2 + 0g + \frac{i}{2\sqrt{2}}h_1 + \frac{i}{2\sqrt{2}}h_1 + \frac{i}{\sqrt{2}}k + 0m = a_3$$

$$0f_1 + \tfrac{1}{4}f_2 - \tfrac{1}{4}g + 0h_1 + 0h_2 + 0k - \tfrac{1}{2}m = a_4$$

$$\frac{1 + \sec\theta}{4}f_1 - \frac{\sec\theta}{4}f_2 - \tfrac{1}{4}g + 0h_1 + 0h_2 + 0k + \frac{1 - \sec\theta}{4}m = a_5$$

$$\frac{1 - \sec\theta}{4}f_1 + \frac{\sec\theta}{4}f_2 - \tfrac{1}{4}g + 0h_1 + 0h_2 + 0k + \frac{1 + \sec\theta}{4}m = a_6$$

$$f_1 - f_2 + 0g + \sqrt{2}\cot\theta h_1 - \sqrt{2}\cot\theta h_2 + 0k - m = 0$$

where θ is the scattering angle.

By Cramer's rule or otherwise we obtain

$$f_1 = a_1 + a_5 \cos^2 \frac{\theta}{2} + a_6 \sin^2 \frac{\theta}{2}$$

$$f_2 = (a_1 + a_4) - (a_5 - a_6)\cos\theta$$

$$g = a_1 - a_4 - a_5 - a_6$$

$$h_1 = -\frac{1}{\sqrt{2}} [(a_5 - a_6)\sin\theta + i(a_2 + a_3)]$$

$$h_2 = \frac{1}{\sqrt{2}} [(a_5 - a_6)\sin\theta - i(a_2 + a_3)]$$

$$k = \frac{i}{\sqrt{2}} (a_2 - a_3)$$

$$m = \mp a_4 + a_5 \sin^2 \frac{\theta}{2} + a_6 \cos^2 \frac{\theta}{2} \qquad \text{(III.1-47)}$$

When there is no spin orbit interaction we use Eq.(12) of Burke and Mitchell, i.e.

$$f_1 = f_2 = \text{triplet amplitude}$$

$$g = \text{singlet amplitude}$$

$$h_1 = h_2 = k = m = 0$$

and by inspection of (III.1-47) we obtain

$$a_2 = a_3 = 0$$

$$a_4 = a_5 = a_6 \qquad \text{(III.1-48)}$$

which is eq.(2) of Farago.[31]

III.2 SCHEME FOR EXPERIMENTAL DETERMINATION OF SPIN TENSORS

With the modifications in notation already used in previous sections we make use of Burke and Mitchell's[8] eqs.(35) to (39) for differential cross-section with polarized beams and relevant polarizations after the collision. Now the spin-correlation tensor is given by the relation

$$\sigma(\underline{P}^a, \underline{P}^e) = \sigma_{00}[1 + P_n^a P'^{an}_{00} + P_n^e P'^{en}_{00} + \sum_{ij} P_i^a P_j^e C_{ij}] \qquad \text{(III.2-1)}$$

and we obtain

$$\sigma_{nn}(E,\theta) = \sigma(P_n^a, 0, 0; P_n^e, 0, 0) = \sigma_{00}[1 + P_n^a P'^{an}_{00} + P_n^e P'^{en}_{00} + P_n^a P_n^e C_{nn}] \qquad \text{(III.2-1a)}$$

P'^{an}_{00} and P'^{en}_{00} are polarizations of the recoiled atom and the scattered electron, respectively, which are necessarily perpendicular to the scattering plane when both incident beams are unpolarized. These symbols are consistent with the notation already introduced in section (III.1) and stand for a unique experimental measurement.

From (III.2-1a)

$$C_{nn} = \frac{(\sigma_{nn}/\sigma_{00}) - (1 + P_n^a P'^{an}_{00} + P_n^e P'^{en}_{00})}{P_n^a P_n^e} \qquad \text{(III.2-1a)}$$

Similarly

$$\sigma_{pp} = \sigma(0, P_p^a, 0, 0, P_p^e, 0)$$

or

$$C_{pp} = \frac{(\sigma_{pp}/\sigma_{00}) - 1}{P_p^a P_p^e} \qquad \text{(III.2-1b)}$$

$$\sigma_{qq} = (0, 0, P_q^a; 0, 0, P_q^e) = \sigma_{00}(1 + P_q^a P_q^e C_{qq})$$

or

$$C_{qq} = \frac{(\sigma_{qq}/\sigma_{00}) - 1}{P_q^a P_q^e} \qquad \text{(III.2-1c)}$$

$$\sigma_{pq} = \sigma(0, P_p^a, 0; 0, 0, P_q^e) = \sigma_{00}(1 + P_p^a P_q^e C_{pq})$$

or

$$C_{pq} = \frac{(\sigma_{pq}/\sigma_{00}) - 1}{P_p^a P_q^e} \qquad \text{(III.2-1d)}$$

$$\sigma_{qp} = \sigma(0, 0, P_q^a : 0, P_p^e, 0) = \sigma_{00}(1 + P_q^a P_p^e C_{qp})$$

or

$$C_{qp} = \frac{(\sigma_{qp}/\sigma_{00}) - 1}{P^a_q P^e_p} \qquad \text{(III.2-1e)}$$

Since $C_{np} = C_{pn} = C_{nq} = C_{qn} = 0$, we also get from (III.2-1)

$$P'^{an}_{00} = \frac{(\sigma_{np}/\sigma_{00}) - 1}{P^a_n} = \frac{(\sigma_{nq}/\sigma_{00}) - 1}{P^a_n} = \frac{(\sigma_{n0}/\sigma_{00}) - 1}{P^a_n} \quad , \qquad \text{(III.2-1f)}$$

$$\sigma_{np} = \sigma_{nq} = \sigma_{n0}, \ \sigma_{0p} = \sigma_{0q} = \sigma_{00} \ , \qquad \text{(III.2-1g)}$$

$$P'^{en}_{00} = \frac{(\sigma_{pn}/\sigma_{00}) - 1}{P^e_n} = \frac{(\sigma_{qn}/\sigma_{00}) - 1}{P^e_n} = \frac{(\sigma_{0n}/\sigma_{00}) - 1}{P^e_n} \qquad \text{(III.2-2)}$$

and

$$\sigma_{pn} = \sigma_{qn} = \sigma_{0n} \ , \quad \sigma_{p0} = \sigma_{q0} = \sigma_{00} \qquad \text{(III.2-2a)}$$

Substituting the values of C_{nn}, C_{pp}, C_{qq}, C_{pq} and C_{qp} in (III.2-1) we obtain

$$\sigma(\underline{P}^a, \underline{P}^e) = (\sigma_{nn} + \sigma_{pp} + \sigma_{qq} + \sigma_{pq} + \sigma_{qp}) - 4\sigma_{00} \qquad \text{(III.2-3)}$$

Relations (III.2-1g) and (III.2-2a) can also be obtained from (III.2-3) by mathematical induction.

The polarization of the recoiled atoms after unpolarized electrons collide with polarized atoms is given in terms of the depolarization tensor

$$P'^{aj}_{\underline{x}0} = (P'^{an}_{00}\ \sigma_{jn} + \sum_i P^a_i\ D^a_{ij})/(1 + P^a_n\ P'^{an}_{00}) \qquad \text{(III.2-4)}$$

$\underline{x}$ being the spin polarization direction of the incident atoms. We get

$$P'^{an}_{n0} = (P'^{an}_{00} + P^a_n\ D^a_{nn})/(1 + P^a_n\ P'^{an}_{00}); \quad P'^{ap}_{n0} = 0 = P'^{aq}_{n0} \qquad \text{(III.2-4a)}$$

or

$$D^a_{nn} = \frac{(1 + P^a_n P'^{an}_{00})\, P'^{an}_{n0} - P'^{an}_{00}}{P^a_n}$$

$$P'^{an}_{p0} = P'^{an}_{00}\ ,\ P'^{ap}_{p0} = P^a_p D^a_{pp}\ ,\ P'^{aq}_{p0} = P^a_p D^a_{pq} \qquad \text{(III.2-4b)}$$

or

$$D^a_{pp} = P'^{ap}_{p0}/P^a_p \quad \text{and} \quad D^a_{pq} = P'^{aq}_{p0}/P^a_p$$

$$P'^{an}_{q0} = P'^{an}_{00}\ ,\ P'^{ap}_{q0} = P^a_q D^a_{qp}\ ,\ P'^{aq}_{q0} = P^a_q D^a_{qq} \qquad \text{(III.2-4c)}$$

or

$$D^a_{qp} = P'^{ap}_{q0}/P^a_q \quad \text{and} \quad D^a_{qq} = P'^{aq}_{q0}/P^a_q$$

The relevant relations for the polarization of the scattered electrons after polarized electrons collide with unpolarized atoms and all components of the depolarization tensor D^e_{ij} are obtained by replacing the superscript $a \to e$ and interchanging the subscripts n, p, q with the subscript 0 throughout the family of eqs. (III.2-4).

The polarization of the scattered electrons after unpolarized electrons collide with the polarized atoms is given in terms of the polarization transfer tensor as:

$$P'^{ej}_{\underline{x}0} = (P'^{en}_{00}\, \delta_{jn} + \sum_i P^a_i K^e_{ij})/(1 + P^a_n P'^{an}_{00}) \qquad \text{(III.2-5)}$$

we obtain

$$P'^{en}_{n0} = (P'^{en}_{00} + P^a_n K^e_{nn})/(1 + P^a_n P'^{an}_{00})\ ;\ P'^{ep}_{n0} = 0 = P'^{eq}_{n0} \qquad \text{(III.2-5a)}$$

or

$$K^{e}_{nn} = \frac{(1 + P^{a}_{n} P'^{an}_{00}) P'^{en}_{00} - P'^{en}_{00}}{P^{a}_{n}}$$

$$P'^{en}_{p0} = P'^{en}_{00} \; , \; P'^{ep}_{p0} = P^{a}_{p} K^{e}_{pp} \; , \; P'^{eq}_{p0} = P^{a}_{p} K^{e}_{pq} \qquad \text{(III.2-5b)}$$

or

$$K^{e}_{pp} = P'^{ep}_{p0}/P^{a}_{p} \quad \text{and} \quad K^{e}_{pq} = P'^{eq}_{p0}/P^{a}_{p}$$

$$P'^{en}_{q0} = P'^{en}_{00} \; ; \; P'^{ep}_{q0} = P^{a}_{q} K^{e}_{qp} \; ; \; P'^{eq}_{q0} = P^{a}_{q} K^{e}_{qq} \qquad \text{(III.2-5c)}$$

or

$$K^{e}_{qp} = P'^{ep}_{q0}/P^{a}_{q} \quad \text{and} \quad K^{e}_{qq} = P'^{eq}_{q0}/P^{a}_{q}$$

A corresponding set of relations is obtained for the polarization of the recoiled atoms when polarized electrons collide with unpolarized atoms, if we interchange $e \leftrightarrow a$ in superscripts and n, p, q with 0 in the subscripts throughout the family of eqs. (III.2-5). Since $K^{e}_{ii} = K^{a}_{ii}$ and $K^{e}_{ij} = - K^{a}_{ji}$ for $i \neq j$ we obtain

$$\frac{(1 + P^{e}_{n} P'^{en}_{00}) \cdot P'^{an}_{0n} - P'^{an}_{00}}{P^{e}_{n}} = \frac{(1 + P^{a}_{n} P'^{an}_{00}) \cdot P'^{en}_{n0} - P'^{en}_{00}}{P^{a}_{n}} \qquad \text{(III.2-6)}$$

$$\frac{P'^{ap}_{0p}}{P^{e}_{p}} = \frac{P'^{ep}_{p0}}{P^{a}_{p}} \; ; \; \frac{P'^{aq}_{0q}}{P^{e}_{q}} = \frac{P'^{eq}_{q0}}{P^{a}_{q}} \; ; \; \frac{P'^{aq}_{0p}}{P^{e}_{p}} = - \frac{P'^{eq}_{p0}}{P^{a}_{p}} \qquad \text{(III.2-7)}$$

If we scrutinize eqs. (III.2-1g), (III.2-2a), (III.2-4a), (III.2-4b), (III.2-4c) and (III.2-5a), (III.2-5b), (III.2-5c) we find that the components of the spin polarization of the colliding partners (p, q) components) in the scattering plane do not affect those perpendicular (n components) in a collision involving Coulomb and spin orbit interactions. This is the physical interpretation

which corresponds to the vanishing components of the spin tensors involving one subscript n ($C_{np} = C_{pn} = C_{nq} = C_{qn} = D_{np} = D_{nq} = D_{qn} = K_{np} = K_{nq} = K_{qn} = 0$) . If experimental evidence contradicts this, then interactions other than Coulomb and spin orbit must be taken into account. For example if eqs. (III.2-1f) and (III.2-2) do not fit to experimental test, then we must look for some interaction hitherto not included in the theoretical framework.

Finally from the formulism in section (III.1) we see that for complete determination of the complex amplitudes a_s (s = 1, 2, ..., 6), one possible set of tensor components is C_{nn}, C_{pp}, C_{qq}, C_{pq}, C_{qp} ; D^a_{nn}, D^a_{pp}, D^a_{qq} ; D^e_{pp}. D^e_{qq} ; K^a_{nn}, K^a_{pp}, K^a_{qq}, K^a_{pq}, K^a_{qp} which respectively correspond to the set of experimental measurements

$$\sigma_{nn},\ \sigma_{pp},\ \sigma_{qq},\ \sigma_{pq},\ \sigma_{qp};\quad P'^{an}_{n0}, P'^{ap}_{p0},\ P'^{aq}_{q0}\ ;\ P'^{ep}_{0p},\ P'^{eq}_{0q}\ P'^{an}_{0n}.$$

$$P'^{an}_{0n},\ P'^{ap}_{0p},\ P'^{aq}_{0q},\ P'^{aq}_{0p},\ P'^{ap}_{0q}$$

and the measurements

$$\sigma_{00},\ P'^{an}_{00},\ P'^{en}_{00}\ .$$

These are eighteen measurements altogether at the same energy and scattering angle!! The minimum of eleven is exceeded due to complex interdependence of some of them.

III.3 OBSERVABLE CONSEQUENCES

The observable aspects of the formulism can be easily deduced from relations which hold amongst spin tensors and are tabulated below for convenience:

Eq. No.	Spin Tensor Relation	Observable Relation
III.3-1	$C_{np} = 0 = C_{nq}$	$P'^{an}_{00} = \dfrac{(\sigma_{np}/\sigma_{00}) - 1}{P^a_n} = \dfrac{(\sigma_{nq}/\sigma_{00}) - 1}{P^a_n} = \dfrac{(\sigma_{n0}/\sigma_{00}) - 1}{P^a_n}$
	$C_{pn} = 0 = C_{qn}$	$P'^{en}_{00} = \dfrac{(\sigma_{pn}/\sigma_{00}) - 1}{P^e_n} = \dfrac{(\sigma_{qn}/\sigma_{00}) - 1}{P^e_n} = \dfrac{(\sigma_{0n}/\sigma_{00}) - 1}{P^e_n}$
		$\sigma(\bar{P}^a, \bar{P}^e) = (\sigma_{nn} + \sigma_{pp} + \sigma_{qq} + \sigma_{pq} + \sigma_{qp}) - 4\sigma_{00}$
III.3-2	$D^a_{nn} = D^e_{nn}$	$P^a_n[(1 + P^a_n P'^{an}_{00})P'^{an}_{n0} - P'^{an}_{00}] = [(1 + P^e_n P'^{en}_{00})P'^{en}_{0n} - P'^{en}_{00}]P^e_n$
III.3-3	$D^a_{np} = 0 = D^a_{nq}$	$P'^{ap}_{n0} = 0 = P'^{aq}_{n0}$
	$D^e_{np} = 0 = D^e_{nq}$	$P'^{ep}_{0n} = 0 = P'^{eq}_{0n}$
III.3-4	$D^a_{pn} = 0 = D^a_{qn}$	$P'^{an}_{p0} = P'^{an}_{00} = P'^{an}_{q0}$
	$D^e_{pn} = 0 = D^e_{qn}$	$P'^{en}_{0p} = P'^{en}_{00} = P'^{en}_{0q}$

Eq. No.	Spin Tensor Relation	Observable Relation
III.3-5	$D^a_{pp} - D^a_{qq} = D^e_{pp} - D^e_{qq}$	$(P'^{ap}_{p0}/P^a_p) - (P'^{aq}_{q0}/P^a_q) = (P'^{ep}_{0p}/P^e_p) - (P'^{eq}_{0q}/P^e_p)$
	$P'^{en\,2}_{00} - P'^{an\,2}_{00} = (D^a_{pq} - D^e_{pq})*$	
	$*(D^a_{pq} + D^e_{pq}) + (D^a_{pp} - D^e_{pp})*$	$P'^{2\,en}_{00} - P'^{2\,an}_{00} = \left(\frac{P'^{aq}_{p0}}{P^a_p} - \frac{P'^{eq}_{0p}}{P^e_p}\right)\left(\frac{P'^{aq}_{p0}}{P^e_p} - \frac{P'^{eq}_{0p}}{P^e_p}\right) + \left(\frac{P'^{ap}_{p0}}{P^a_p} - \frac{P'^{ep}_{0p}}{P^e_p}\right)\left(\frac{P'^{ap}_{p0}}{P^a_p} + \frac{P'^{eq}_{0q}}{P^e_q}\right)$
	$*(D^a_{pp} + D^e_{qq})$	
III.3-6	$K^e_{np} = 0 = K^e_{nq}$	$P'^{ep}_{n0} = 0 = P'^{eq}_{n0}$
	$K^a_{np} = 0 = K^a_{nq}$	$P'^{ap}_{0n} = 0 = P'^{aq}_{0n}$
III.3-7	$K^e_{pn} = 0 = K^e_{qn}$	$P'^{en}_{p0} = P'^{en}_{00} = P'^{en}_{q0}$
	$K^a_{pn} = 0 = K^a_{qn}$	$P'^{an}_{0p} = P'^{an}_{00} = P'^{an}_{0q}$

The observable effects of $K^{e}_{ii} = K^{a}_{ii}$ and $K^{e}_{ij} = - K^{a}_{ji}$ $(i \neq j)$ are given by (III.2-6) and (III.2-7).

From observable relations (III.3-3), (III.3-6) and (III.3-4), (III.3-7) we deduce respectively

$$P'^{ap}_{nn} = P'^{aq}_{nn} = P'^{ep}_{nn} = P'^{eq}_{nn} = 0 \quad \text{and}$$

$$P'^{an}_{pp} = P'^{an}_{qq} = P'^{an}_{pq} = P'^{an}_{qp} = P'^{an}_{00'}, \quad P'^{en}_{pp} = P'^{en}_{qq} = P'^{en}_{pq} = P'^{en}_{qp} = P'^{en}_{00'},$$

which again substantiates our previous claim that spin orbit interaction does not cause coupling of the spin states in the scattering plane with those normal to the scattering plane during elastic collision. We conclude that spin states in the scattering plane and normal to the scattering plane interact independently with the Coulomb and exchange interactions in elastic electron-atom scattering.

For the convenience of the experimentalist, taking ZX as the scattering plane (Z being the incident electron beam direction $\overline{p_0}$ and for elastic scattering $|p_1| = |p_0|$) we get the following transformations connecting the $\hat{n}$, $\hat{p}$, $\hat{q}$ frame with the XYZ frame

$$P_p = \sin \frac{\theta}{2} P_x + \cos \frac{\theta}{2} P_z$$

$$P_q = \cos \frac{\theta}{2} P_x + \sin \frac{\theta}{2} P_z$$

$$P_n = P_y \qquad \text{(III.3-8)}$$

and the inverse transformations

$$P_x = \sin \frac{\theta}{2} P_p - \cos \frac{\theta}{2} P_q$$

$$P_z = \cos \frac{\theta}{2} P_p + \sin \frac{\theta}{2} P_q$$

$$P_y = P_n \qquad \text{(III.3-8a)}$$

III.4 IMMEDIATE IMPLICATIONS

Although collision experiments with spin polarized electron and atom beams are quite expensive, cumbersome and challenging, yet impressive progress[32] has been and is being reported in recent years in comparatively simple laboratory situations. For example in Bederson's[32] experimental arrangement the following measurements can be suggested.

Since $\hat{n} = \dfrac{\bar{p}_0 \times \bar{p}_1}{|p_0 \times p_1|}$, $\theta \to -\theta$ means

$$P_n^a = \underline{P}^a . \hat{n} \to - P_n^a \; ; \quad P_n^e = \underline{P}^e . \hat{n} \to - P_n^e \; ,$$

we get from (III.2-1a)

$$\sigma_{nn}(\pm\theta) = \sigma_{00}[1 \pm P_n^a P'^{an}_{00} \pm P_n^e P'^{en}_{00} + P_n^a P_n^e C_{nn}] \qquad \text{(III.2-1a)''}$$

if we define

$$\mathcal{S}_{nn}(\theta) = \sigma_{nn}(\theta) + \sigma_{nn}(-\theta) \quad \text{and} \quad A_{nn}(\theta) = \sigma_{nn}(\theta) - \sigma_{nn}(-\theta)$$

then

$$C_{nn} = \frac{\frac{1}{2}(\mathcal{S}_{nn}/\sigma_{00}) - 1}{P_n^a P_n^e} \qquad \text{(III.2-1a)'''}$$

and

$$A_{nn}(\theta) = 2\sigma_{00}(P_n^a P'^{an}_{00} + P_n^e P'^{en}_{00}) \qquad \text{(III.4-1)}$$

we deduce that

$$A_{n0}(\theta) = 2\sigma_{00} P_n^a P'^{an}_{00}, \; A_{0n}(\theta) = 2\sigma_{00} P_n^e P'^{en}_{00} \qquad \text{(III.4-1a)}$$

Accounting for the change of polarization of the incident atomic beam in the field-free region of the interaction region due to hyperfine-coupling we get

$$A_{n0}(\theta) = 2\sigma_{00} P_n^a P'^{an}_{00}/(2I + 1) \qquad \text{(III.4-2)}$$

To verify (III.4-2) the following experiments are necessary:

(a) Differential cross section σ_{00} when both incident beams are unpolarized and the scattered atomic polarization P'^{an}_{00} perpendicular to the scattering plane caused by spin orbit interaction, in Bederson's arrangement.

(b) Collide atomic beam partially polarized in the $\hat{n}$ direction with unpolarized electronic beam (upward normal to the scattering plane) and measure left-right assymetry $A_{n0}(\theta)$.

The following observable relation

$$A_{0n}(\theta) = 2\sigma_{00}\, P^{e}_{n}\, P'^{en}_{00}$$

can also be verified if in step (a) P'^{en}_{00} is measured instead of P'^{an}_{00} and in step (b) starting with unpolarized atoms and electrons polarized normal to the scattering plane and by measuring the left-right asymmetry $A_{0n}(\theta)$ after scattering.

CHAPTER IV

THEORY OF MEASUREMENT FOR ELECTRON CAPTURE ON ATOMIC TARGETS WITH BARE NUCLEI AS PROJECTILES

Electron capture has been one of the most widely studied atomic collision processes. It either controls or plays an important part in the evolution of many physical processes encountered in astrophysics, atmosphere physics, laser physics and plasma physics. The investigations of the capture into excited states is also important in connection with the problem of impurities in thermonuclear plasma. Electron capture, however, has proved to be one of the most difficult collision processes to predict reliably.

The electron capture process under investigation is denoted by the following reaction

$$P^{+Z} + A \rightarrow P^{(Z-1)*} + A^{+} \qquad \text{(IV.1)}$$

where P^{+Z} is a fully stripped projectile (polarized or unpolarized) and A is a neutral atomic target in the ground state. After capturing an electron from A, $P^{(Z-1)*}$ in general is in some excited state and emits photons which may be recorded in coincidence with the hydrogenlike atom $P^{(Z-1)}$. We will express the state multipoles characterizing the excited state $P^{(Z-1)*}$ in terms of Stokes parameters. It is reasonable to distinguish the two situations, namely

(A) Collision described in LS coupling

(B) Collision described in jj-coupling,

as for example indicated by Jaecks and Macek[3] (1971).

In section (IV.4) we illustrate our formulism on spin polarized protons capturing an electron from unpolarized atomic targets.

IV.1A DESCRIPTION OF COLLISION IN LS COUPLING

The process in consideration is denoted in terms of the relevant variables as [c stands for the continuous variables of A^+]

$$P^{+Z}(Im_I, \bar{p}_0) + A(n_0L_0M_0, s_0m_{s_0}) \rightarrow P^{+(Z-1)*}(n_1L_1M_1s_1m_{s_1}; Im_I\bar{p}_1) + A^+(nLMsm_s; c) \qquad (IV.1\text{-}1)A$$

where the projectile captures the electron from the $|n_0L_0M_0, s_0m_{s_0}>$ state to the $|n_1L_1M_1, s_1m_{s_1}>$ state and $\bar{p}_0$, $\bar{p}_1$ are the sharply defined momenta of the projectile before and after the collision. Since hyperfine structure interaction is negligibly small as compared to fine structure interaction and Coulomb interaction in almost all practical situations, we suppress dependence of the collision on nuclear spin, however, it is included in the decay of the excited state $P^{+(Z-1)*}$.

The density matrix describing the state of the $(P^{(Z-1)*} + A^+)$ system just after the collision is given by the relation,

$$<n_1L_1M'_1: s_1m'_{s_1}, nLM'sm'_sC|\rho(0)|n_1L_1M_1: s_1m_{s_1}, nLMsm_sc> \qquad (Iv.1\text{-}2)A$$

in the 'collision-frame' (taking $\bar{p}_0$ as quantization axis). But

$$\rho(0) = T\rho(i)T^\dagger , \qquad (IV.1\text{-}3)A$$

T being the relevant transition operator which takes the initial system to the final system in the process (IV.1-1). The density matrix $\rho(i)$ describes the $(P^{+Z} + A)$ system before the collision. If we apply eq.(IV.1-3)A in eq.(IV.1-2)A and then make use of the completeness relation

$$\sum_{n_0 L_0 M_0 m_{s_0}} |n_0 L_0 M_0 m_{s_0}\rangle\langle n_0 L_0 M_0 m_{s_0}| = 1 \qquad \text{(IV.1-4)A}$$

in the resulting expression twice, we obtain

$$\langle n_1 L_1 M'_1: s_1 m'_{s_1},\ nLM'sm'_s C|\rho(0)|n_1 L_1 M_1: s_1 m_{s_1},\ nLMsm_s c\rangle$$

$$= \sum_{n_0 L_0 M_0 M'_0,\ m_{s_0} m'_{s_0}} \langle n_1 L_1 M'_1: s_1 m'_{s_1},\ nLM'sm'_s C|T|n_0 L_0 M'_0 m'_{s_0}\rangle$$

$$\times \langle n_0 L_0 M'_0 m'_{s_0}|\rho(i)|n_0 L_0 M_0 m_{s_0}\rangle$$

$$\times \langle n_0 L_0 M_0 m_{s_0}|T^{\dagger}|n_1 L_1 M_1: s_1 m_{s_1},\ nLMsm_s c\rangle$$

$$= \sum_{n_0 L_0 M_0 M'_0,\ m_{s_0} m'_{s_0}} a(n_1 L_1 M'_1: s_1 m'_{s_1}\ nLM'sm'_s c,\ n_0 L_0 M'_0 m'_{s_0})$$

$$\times \langle n_0 L_0 M'_0 m'_{s_0}|\rho(i)|n_0 L_0 M_0 m_{s_0}\rangle$$

$$\times a(n_1 L_1 M_1: s_1 m_{s_1}\ nLMsm_s C;\ n_0 L_0 M_0 m_{s_0})^*$$

where

$$\langle n_0 L_0 M'_0 m'_{s_0}|\rho(i)|n_0 L_0 M_0 m_{s_0}\rangle = \frac{\delta(M'_0, M_0)\delta(m'_{s_0}, m_{s_0})}{W_0}, \quad W_0 = (2s_0 + 1)(2L_0 + 1)$$

since the atomic target is initially unpolarized.

To find the reduced density matrix describing the state of $p^{+(Z-1)*}$ alone, we sum over the undetected final spins and unobserved discrete states of A^+ and integrate over continous variables of A^+ to obtain

$$\langle n_1 L_1 M'_1|\rho_{L_1}(0)|n_1 L_1 M_1\rangle = \int_c \sum_{\substack{n_0 L_0 M_0 m_{s_0} m_{s_1} \\ nLMm_s}} \frac{1}{W_0}$$

$$a(n_1 L_1 M'_1: s_1 m_{s_1}\ nLMm_s C;\ n_0 L_0 M_0 m_{s_0})$$

$$\times\ a(n_1 L_1 M_1: s_1 m_{s_1}\ nLMm_s C;\ n_0 L_0 M_0 m_{s_0})^*$$

We now suppress dependence on all the fixed variables and write for the sake of convenience

$$\langle n_1 L_1 M'_1 | \rho_{L_1}(0) | n_1 L_1 M_1 \rangle = \langle a(M'_1)\, a(M_1)^* \rangle \qquad \text{(IV.1-5)A}$$

where the sumbol $\langle \ldots \rangle$ has been used to denote averages over initial internal states, sum over final unobserved discrete states and integration over continuous variables. The invariance of the interaction dynamics under reflection in the scattering plane (defined by $\bar{p}_0$ and $\bar{p}_1$) requires that[33]

$$\langle a(M'_1)\, a(M_1)^* \rangle = (-1)^{M'_1 + M_1} \langle a(-M'_1)\, a(-M_1)^* \rangle$$

we normalize according to the relation

$$\sigma(M_1) = \langle |a(M_1)|^2 \rangle$$

where $\sigma(M_1)$ is the relevant differential cross section for magnetic sublevel M_1.

IV.1B DESCRIPTION OF COLLISION IN JJ COUPLING

In this case the process under investigation looks like

$$P^{+Z}(Im_I, \bar{p}_0) + A(n_0 L_0 J_0 M_0) \rightarrow P^{(Z-1)*}(n_1 L_1 J_1 M_1;\ Im_I \bar{p}_1) + A^+(nLJM;\ C) \qquad \text{(IV.1-1)B}$$

Proceeding exactly as in subsection A, we obtain the following reduced density matrix in this case

$$\langle n_1 L_1 J_1 M'_1 | \rho_{J_1}(0) | n_1 L_1 J_1 M_1 \rangle = \int_C \sum_{\substack{n_0 L_0 J_0 M_0 \\ nLJM}} \frac{1}{W_0}\, a(n_1 L_1 J_1 M'_1 :\ nLJMC,\ n_0 L_0 J_0 M_0)$$

$$\times\ a(n_1 L_1 J_1 M_1 :\ nLJMC,\ n_0 L_0 J_0 M_0)^*$$

$$= \langle a(M'_1)\, a(M_1)^* \rangle\ ,\quad W_0 = 2J_0 + 1$$

where the symbol $\langle \ldots \rangle$ has the same meaning as defined earlier. Once again it is trivial to find that

$$\langle a(M'_1)\, a(M_1)^* \rangle = (-1)^{2J_1+M'_1+M_1} \langle a((-M'_1)\, a(-M_1)^* \rangle$$

and the differential cross section for capture to the magnetic sublevel M_1 is given by the normalization condition

$$\sigma(M_1) = \langle |a(M_1)|^2 \rangle$$

IV.2 EXPANSION OF THE REDUCED DENSITY MATRIX IN TERMS OF STATE MULTIPOLES

We expand the reduced density matrices $\rho_{L_1}(0)$ and $\rho_{J_1}(0)$ in terms of the spherical tensor operators according to eq.(I.5-1). The relevant state multipoles (assuming sharply defined angular momentum states) are given by the relation

$$\langle T(J)^\dagger_{KQ} \rangle = (2K+1)^{\frac{1}{2}} \sum_{M'M} (-1)^{J-M'} \begin{pmatrix} J & J & K \\ M' & -M & -Q \end{pmatrix} \langle a(M')\, a(M)^* \rangle \qquad \text{(IV.2-1)}$$

where J stands for L_1 or J_1.

Note that the reduced density matrix elements $\langle a(M'_1)\, a(M_1)^* \rangle$ are independent of the nuclear spin polarization of the projectile so that due to reflection invariance of the atomic ensemble of interest in the scattering plane and also due to the hermiticity of the density matrix, the following relations hold[5]

$$\langle T(J)^\dagger_{K,-Q} \rangle = (-1)^{K+Q} \langle T(J)^\dagger_{KQ} \rangle$$

and

$$\langle T(J)^\dagger_{KQ} \rangle^* = (-1)^K \langle T(J)^\dagger_{KQ} \rangle$$

For instance, the following set of state multiples is required for capture into a P state for case A.

$$\langle T(1)^{\dagger}_{00}\rangle = \frac{\sigma(0) + 2\sigma(1)}{\sqrt{3}}$$

$$\langle T(1)^{\dagger}_{11}\rangle = i\sqrt{2}\ \mathrm{Im}\langle a(0)a(1)^{*}\rangle$$

$$\langle T(1)^{\dagger}_{20}\rangle = \sqrt{\frac{2}{3}}\{\sigma(1) - \sigma(0)\}$$

$$\langle T(1)^{\dagger}_{21}\rangle = -\sqrt{\frac{2}{5}}\ \mathrm{Re}\langle a(0)a(1)^{*}\rangle$$

$$\langle T(1)^{\dagger}_{22}\rangle = -\frac{1}{\sqrt{5}}\langle a(1)a(-1)^{*}\rangle \qquad \text{(IV.2-2)}$$

For case B and for capture into a $P_{\frac{1}{2}}$ state, the relevant state multipoles are

$$\langle T(\tfrac{1}{2})^{\dagger}_{00}\rangle = \frac{\sigma(\frac{1}{2}) + \sigma(-\frac{1}{2})}{\sqrt{2}} = \sqrt{2}\,\sigma(\tfrac{1}{2}), \quad \because \sigma(-M_1) = \sigma(M_1)$$

$$\langle T(\tfrac{1}{2})^{\dagger}_{11}\rangle = -\langle a(\tfrac{1}{2})a(-\tfrac{1}{2})^{*}\rangle \qquad \text{(IV.2-3)}$$

Similarly for capture into a $P_{3/2}$ state (Case B) the set of state multipoles is found to be

$$\langle T(3/2)^{\dagger}_{00}\rangle = \sigma(3/2) + \sigma(\tfrac{1}{2})$$

$$\langle T(3/2)^{\dagger}_{11}\rangle = -\sqrt{\frac{2}{5}}\langle a(\tfrac{1}{2})a(-\tfrac{1}{2})^{*}\rangle$$

$$\langle T(3/2)^{\dagger}_{20}\rangle = \sigma(3/2) - \sqrt{2}\sigma(\tfrac{1}{2})$$

$$\langle T(3/2)^{\dagger}_{21}\rangle = -\sqrt{2}\ \mathrm{Re}\langle a(3/2)a(\tfrac{1}{2})^{*}\rangle$$

$$\langle T(3/2)^{\dagger}_{22}\rangle = \sqrt{2}\ \mathrm{Re}\langle a(3/2)a(-\tfrac{1}{2})^{*}\rangle$$

$$\langle T(3/2)^{\dagger}_{31}\rangle = \sqrt{\frac{3}{5}}\langle a(\tfrac{1}{2})a(-\tfrac{1}{2})^{*}\rangle$$

$$\langle T(3/2)^{\dagger}_{32}\rangle = \sqrt{2}\ i\ \mathrm{Im}\langle a(3/2)a(-\tfrac{1}{2})^{*}\rangle$$

$$\langle T(3/2)^{\dagger}_{33}\rangle = -\langle a(3/2)a(-3/2)^{*}\rangle \qquad \text{(IV.2-4)}$$

When $P^{(Z-1)*}$ is not registered in coincidence with the photons emitted by it, the relevant state multipoles are obtained by integration over the solid angle $d\Omega_p$ subtended by the particle detector on the interaction region, i.e.

$$\langle \tau(J)^{\dagger}_{KQ}\rangle = \int d\Omega_p \langle T(J)^{\dagger}_{KQ}\rangle \qquad \text{(IV.2-5)}$$

IV.3 EFFECT OF FINE STRUCTURE AND HYPERFINE STRUCTURE INTERACTIONS ON THE EXCITED ENSEMBLE $P^{(Z-1)*}$

When the collision is describable in LS coupling, the time evolution of the excited atomic ensemble under the influence of fine structure and hyperfine structure interactions has already been discussed in detail in subsection (II.3)B. The time evolved reduced density matrix $\rho_{L_1}(t)$ ($t = 0$ is the instant just after the collision and the instant 't' is the time when the photon is emitted) has been explicitly described in terms of the state multipoles (characterizing the relevant excited atomic ensemble) and the perturbation coefficients. In particular eq.(II.3-19b) for the perturbation coefficient is applicable when the target atom A is initially unpolarized whereas the projectile P^{+Z} is polarized (e.g. spin polarized protons etc.).

The LS coupling is violated in the excitation in two somewhat different situations[3]

1. $P^{(Z-1)*}$ violates LS coupling

The effect of spin orbit coupling during the collision time is appreciable and causes splitting of the fine structure states during the excitation. In other words the collision time τ_c can no longer be considered extremely short compared to the characteristic times ($\because \tau_c \sim \tau_{J_1 J'_1}$) of the excited state, as was assumed in Chapter II. The scattering amplitudes obviously now refer to different fine structure states. As a result of this the relevant state multipoles (being functions of the scattering amplitudes) also characterize different fine structure states. Even if one does not resolve different fine structure levels, the breakdown of LS coupling implies that the splittings are large compared to the level widths ($\tau_{J'_1 J_1} \ll \tau$, see section (II.3)B, Case (b)). The oscillatory terms

for $J'_1 \neq J_1$ are then neglected in the calculations for the perturbation coefficient. The radiation from different fine structure levels then adds incoherently. Therefore, only the interference of radiation from different hyperfine levels has to be considered. In view of these considerations, the density matrix describing the state of the excited ensemble of interest just after the collision is given by

$$\rho(0) = \rho_{J_1}(0) \otimes \rho_I(0) \qquad \text{(IV.3-1)}$$

2. <u>The Ion A^+ violates LS Coupling</u>

When the target atom A or any atom formed in the collision violates the LS coupling scheme (even though the radiating atom $p^{(Z-1)*}$ obeys them), the total electronic spin of the system (target + projectile) is no longer a good quantum number. For instance the Ly-α radiation emitted by electron capture of protons on heavy atoms originates from the 2P state of the hydrogen atom, which obeys the LS Coupling scheme, whereas the states of the heavy target and the resulting heavy ion do not. One then has three different physical situations as follows:

(a) The relevant transition operator T may depend explicitly on the spin.

(b) The initial states (describing the internal state of both the collision partners before collision) and the final states (describing the internal state of the scattered ion A^+) may not obey LS Coupling scheme.

(c) The state vectors of the above mentioned states may approximately obey LS Coupling, but some of the substates of their multiplets may not be energetically accessible.

In all the above three cases the reduced density matrix elements $\langle a(J_1M'_1)a(J_1M_1)^*\rangle$ can be related to those in the $(L_1M_1s_1m_{s_1})$ representation (i.e. $\langle a(L_1M'_1)a(L_1M_1)^*\rangle$) via a Clebsch-Gordon transformation. As the spin quantum number is no longer a good quantum number, the scattering amplitudes referring to different m_{s_1} states can interfere. As a result of this interference, the number of independent parameters are not reduced on transformation to the $L_1M_1s_1m_{s_1}$ representation. It is, therefore, reasonable to express amplitudes for the radiating atom in J_1M_1 representation. The radiative decay, however, does not change the spins of the excited atom (electronic as well as nuclear spin) so it is convenient to evaluate the dipole matrix element in the LS representation.

Keeping in view the above mentioned arguments, we find the effect of fine structure and hyperfine structure on the initial excited ensemble $P^{(Z-1)*}$ in terms of the perturbation coefficients for different physical situations of interest:

(i) $\underline{\tau_{J'_1J_1} \sim \tau}$: (see section II.3)B)

The scattering amplitudes referring to different fine structure levels interfere since the splittings are of the same order as the level width. The reduced density matrix describing the state of the excited ensemble $P^{(Z-1)*}$ just after the collision is then given by the relation

$$\rho(0) = \rho_{J'_1J_1}(0) \otimes \rho_I(0) \qquad \text{(IV.3-2)}$$

Assuming that the hyperfine structure splittings are small compared to the relevant fine structure splittings $(\tau_{J_1F'_1F_1} >> \tau_{J'_1J})$ i.e. $\tau_{J_1F'_1F_1} >> \tau$, then the excited state under consideration decays

before the hyperfine structure interaction has any appreciable effect, It is, therefore, unnecessary to couple $\rho_I(0)$ in eq.(IV.3-2). The expansion of $\rho(0)$ in terms of a complete set of spherical tensor operators in the 'collision frame' looks like

$$\rho(0) = \rho_{J'_1 J_1}(0) = \sum_{K_{J_1} Q_{J_1}} \langle T(J'_1 J_1)^\dagger_{K_{J_1} Q_{J_1}} \rangle T(J'_1 J_1)_{K_{J_1} Q_{J_1}} \qquad \text{(IV.3-3)}$$

The time evolution of an unperturbed reduced density matrix under the influence of fine structure and hyperfine interactions is discussed in section (II.3)B in great detail. In the present situation the time evolved density matrix $\rho_{J'_1 J_1}(t)$ (t being the time of photon emission) is given by the relation

$$\rho_{J'_1 J_1}(t) = \sum_{KQ} \langle T(J'_1 J_1;\, t)^\dagger_{KQ} \rangle T(J'_1 J_1;\, t)_{KQ}$$

$$= \sum_{K_{J_1} Q_{J_1}} \langle T(J'_1 J_1)^\dagger_{K_{J_1} Q_{J_1}} \rangle U(t)\, T(J'_1 J_1)_{K_{J_1} Q_{J_1}}\, U(t)^\dagger \qquad \text{(IV.3-4)}$$

where $U(t)$ is the time evolution operator corresponding to the total Hamiltonian $H = H_0 + V(f)$. Applying eq.(I.5-2) we obtain,

$$\langle T(J_1 J'_1;\, t)^\dagger_{KQ} \rangle = \mathrm{tr}\{\rho_{J'_1 J_1}(t)\; T(J'_1 J_1;\, t)_{KQ}\}$$

$$= \sum_{K_{J_1} Q_{J_1}} \langle T(J'_1 J_1)^\dagger_{K_{J_1} Q_{J_1}} \rangle \mathrm{tr}\{U(t)\, T(J'_1 J_1)_{K_{J_1} Q_{J_1}}\, U(t)^\dagger\, T(J'_1 J_1;\, t)_{KQ}\}$$

$$= \sum_{K_{J_1} Q_{J_1}} \langle T(J'_1 J_1)^\dagger_{K_{J_1} Q_{J_1}} \rangle G(J'_1 J_1;\, t)^{Q_{J_1} Q}_{K_{J_1} K} \qquad \text{(IV.3-5)}$$

Obviously

$$G(J'_1 J_1;\, t)^{Q_{J_1} Q}_{K_{J_1} K} = \mathrm{tr}\{U(t)\, T(J_1 J'_1)_{K_{J_1} Q_{J_1}}\, U(t)^\dagger\, T(J'_1 J_1;\, t)_{KQ}\} \qquad \text{(IV.3-6)}.$$

Applying eq.(II.3-16) and standard formulae of angular momentum theory for matrix elements of spherical tensor operators and 3j-symbols, the perturbation coefficient becomes

$$G(J'_1 J_1;\ t)^{Q_{J_1} Q}_{K_{J_1} K} = e^{-\gamma t} \sum_{J'_1 J_1} \cos \omega_{J'_1 J_1} t\ \delta(K_{J_1}, K)\,\delta(Q_{J_1}, Q) \qquad \text{(IV.3-7)}$$

where we assume that $\gamma_{J'_1} = \gamma_{J_1} = \gamma$ and $\omega_{J'_1 J_1} = (E_{J'_1} - E_{J_1})/\hbar$.

(ii) $\tau_{J'_1 J_1} \ll \tau$

Contrary to case (i), the scattering amplitudes referring to different fine structure levels do not interfere since the splittings $E_{J'_1} - E_{J_1}$ are large compared to the level width γ . Therefore, the reduced density matrix describing the state of the excited ensemble $P^{(Z-1)*}$ just after the collision is given by eq.(IV.3-1). Assuming that the nuclear spin of the excited ensemble $P^{(Z-1)*}$ is polarized along the direction $\hat{n}_s$, we expand $\rho(0)$ in terms of a complete set of spherical tensor operators in the 'spin polarization frame' as applied in section (II.3)B, i.e.

$$\rho(0) = \sum_{K_I K_{J_1} \chi_{J_1}} \langle T(J_1)^{\dagger}_{K_{J_1} \chi_{J_1}} \otimes T(I)^{\dagger}_{K_I 0} \rangle\ T(J_1)_{K_{J_1} \chi_{J_1}} \otimes T(I)_{K_I 0} \qquad \text{(IV.3-8)}$$

Following a similar procedure as in case (i), the reduced density matrix describing the state of the electronic system alone at time t of the photon emission (remembering that the emission of light depends only on the electronic states at the time of emission) is given by the relation

$$\rho_{J_1}(t) = \frac{1}{2I+1} \sum_{K\chi} \langle T(J_1;\ t)_{K\chi} \otimes \underline{1} \rangle\ T(J_1;\ t)_{K\chi} \otimes \underline{1} \qquad \text{(IV.3-9)}$$

and without going into the details of the procedure, we obtain

$$\langle T(J_1;\, t)^{\dagger}_{K\chi} \rangle = \mathrm{tr}\{\rho_{J_1}(t)\, T(J_1;\, t)_{K\chi} \otimes \underline{1}\}$$

$$= \sum_{K_I K_{J_1} \chi_{J_1}} \langle T(J_1)^{\dagger}_{K_{J_1}\chi_{J_1}} \rangle \langle T(I)^{\dagger}_{K_I 0} \rangle \mathrm{tr}\{U(t)\, T(J_1)_{K_{J_1}\chi_{J_1}} \otimes T(I)_{K_I 0} U(t)^{\dagger}\; T(J_1;\, t)_{K\chi} \otimes \underline{1}\}$$

$$= \sum_{K_{J_1} \chi_{J_1}} \langle T(J_1)^{\dagger}_{K_{J_1}\chi_{J_1}} \rangle G(J_1;\, t)^{\chi_{J_1}\chi}_{K_{J_1}K}\, \delta(\chi,\, \chi_{J_1}) \qquad \text{(IV.3-10)}$$

where

$$G(J_1;\, t)^{\chi_{J_1}\chi_{J_1}}_{K_{J_1}K} = \sum_{K_I F'_1 F_1} (-1)^{(J_1+I+F_1)+K_I+K_{J_1}+K+\chi_{J_1}} (2F'_1+1)(2F_1+1)$$

$$\{(2K_I+1)(2K_{J_1}+1)(2K+1)\}^{\frac{1}{2}}$$

$$\times \begin{pmatrix} K_I & K_{J_1} & K \\ 0 & \chi_{J_1} & -\chi_{J_1} \end{pmatrix} \begin{Bmatrix} F'_1 & F_1 & K \\ J_1 & J_1 & I \end{Bmatrix} \begin{Bmatrix} K_I & K_{J_1} & K \\ I & J_1 & F'_1 \\ I & J_1 & F_1 \end{Bmatrix} \langle T(I)^{\dagger}_{K_I 0} \rangle$$

$$\times \exp\{-\, i(E_{J_1F'_1} - E_{J_1F_1})\, t/\hbar - \gamma_{J_1} t\} \qquad \text{(IV.3-11)}$$

If $\tau_{J_1F'_1F_1} \sim \tau$, the effect of hyperfine structure is definitely included and the time dependent terms $(F'_1 \neq F_1)$ in eq.(IV.3-11) do not average out. For some metastable states it may happen that $\tau_{J_1F'_1F_1} \ll \tau$, then the oscillatory terms $(F'_1 \neq F_1)$ average out during the comparatively long life time and are negligible. In a physical situation where one still has $\tau_{J_1F'_1F_1} \gg \tau$, then the effect of hyperfine structure interaction can be neglected by putting $I = 0$ in eq.(IV.3-11).

IV.4 RADIATIVE DECAY OF THE EXCITED STATE $P^{(Z-1)*}$

The radiative decay of any excited atomic ensemble is described in section (II.3)C in great detail. The formulism therein is applicable in general (subject to the assumptions enumerated there) and is independent of the excitation mechanism. The relevant Stokes parameters can be obtained by substituting the appropriate perturbation coefficient

$$G(L_1;\ t)^{\chi_{L_1}\chi_{L_1}}_{K_{L_1}K}$$

in eq.(II.3-30b) and then comparing with eq.(I.3-7) when the collision is described in LS coupling. In the case that LS coupling is violated in the collision, eq.(II.3-29) is our starting point. Then if spins of $P^{(Z-1)*}$ are polarized in the direction $\hat{n}_s$, the density matrix $\bar{\rho}^{\cdot}(n_\gamma;\ t)_{\lambda'\lambda}$ is given by the relation

$$\bar{\rho}^{\cdot}(n_\gamma,\ t)_{\lambda'\lambda} = c(\omega) \sum_{Kq\chi} tr\{r_{-\lambda'}T(J'_1J_1;\ t)_{Kq}r^{\dagger}_{-\lambda}\}\langle T(J'_1J_1;\ t)^{\dagger}_{K\chi}\rangle D(\hat{n}_\gamma \to \hat{n}_s)^{(K)}_{q\chi} \qquad \text{(II.3-29a)}$$

where for the sake of generalization we substitute $\rho(0)$ by $\rho_{J'_1J_1}(t)$ in eq. (II.3-23) and go through a manipulation identical to that laid out in section (II.3)C. When the spins of the excited ensemble are polarized along the direction $\bar{p}_0$ or are unpolarized the above equation transforms into 'the collision frame' in the form:

$$\bar{\rho}^{\cdot}(n_\gamma,\ t)_{\lambda'\lambda} = c(\omega) \sum_{Kq\Omega} tr\{r_{-\lambda'}T(J'_1J_1;\ t)_{Kq}r^{\dagger}_{-\lambda}\}\langle T(J'_1J_1;\ t)^{\dagger}_{K\Omega}\rangle D(\hat{n}_\gamma \to \hat{p}_0)^{(K)}_{q\Omega} \qquad \text{(IV.4-1)}$$

When $\tau_{J'_1J_1} \sim \tau$, we substitute $\langle T(J'_1J_1;\ t)^{\dagger}_{K\Omega}\rangle$ from eq.(IV.3-5) to get

$$\overline{\rho}^{\cdot}(n_\gamma,\ t)_{\lambda'\lambda} = c(\omega) \sum_{K_{J_1} Q_{J_1} q} \mathrm{tr}\{r_{-\lambda'} T(J'_1 J_1;\ t)_{K_{J_1} q} r^{\dagger}_{-\lambda}\} \times$$

$$\times \langle T(J'_1 J_1)^{\dagger}_{K_{J_1} Q_{J_1}} \rangle G(J'_1 J_1;\ t)^{Q_{J_1} Q_{J_1}}_{K_{J_1} K_{J_1}}$$

$$\times\ D(\hat{n}_\gamma \to \hat{p}_0)^{(K_{J_1})}_{q Q_{J_1}} \qquad \text{(IV.4-7a)}$$

The relevant perturbation coefficient is found in eq.(IV.3-7).

In the case $\tau_{J'_1 J_1} \ll \tau$, we substitute $\langle T(J_1;\ t)^{\dagger}_{K\chi} \rangle (J'_1 = J_1)$ from eq.(IV.3-10) into eq.(II.3-29a) and the relevant perturbation coefficient from eq.(IV.3-11) to obtain

$$\overline{\rho}^{\cdot}(\hat{n}_\gamma,\ t)_{\lambda'\lambda} = c(\omega) \sum_{\substack{Kq \\ K_{J_1} \chi_{J_1}}} \mathrm{tr}\{r_{-\lambda'} T(J'_1 J_1;\ t)_{Kq} r^{\dagger}_{-\lambda}\} \langle T(J_1)^{\dagger}_{K_{J_1} \chi_{J_1}} \rangle \times$$

$$\times G(J_1;\ t)^{\chi_{J_1} \chi_{J_1}}_{K_{J_1} K}\ D(\hat{n}_\gamma \to \hat{n}_s)^{(K)}_{q \chi_{J_1}} \qquad \text{(IV.4-2)}$$

The state multipole $\langle T(J_1)^{\dagger}_{K_{J_1} \chi_{J_1}} \rangle$ is transformed from the 'spin polarization frame' to the 'photon detector frame' by the transformation

$$\langle T(J_1)^{\dagger}_{K_{J_1} \chi_{J_1}} \rangle = \sum_{Q_{J_1}} \langle T(J_1)^{\dagger}_{K_{J_1} Q_{J_1}} \rangle\ D(\hat{n}_s \to \hat{p}_0)^{(K_{J_1})}_{\chi_{J_1} Q_{J_1}} \qquad \text{(IV.4-3)}$$

to give

$$\overline{\rho}^{\cdot}(\hat{n}_\gamma,\ t)_{\lambda'\lambda} = c(\omega) = \sum_{\substack{Kq Q_{J_1} \\ K_{J_1} \chi_{J_1}}} \mathrm{tr}\{r_{-\lambda'} T(J'_1 J_1;\ t)_{Kq} r^{\dagger}_{-\lambda}\} \times$$

$$\times \langle T(J_1)^{\dagger}_{K_{J_1} Q_{J_1}} \rangle D(\hat{n}_s \to \hat{p}_0)^{(K_{J_1})}_{\chi_{J_1} Q_{J_1}}$$

$$\times\ G(J_1;\ t)^{\chi_{J_1} \chi_{J_1}}_{K_{J_1} K}\ D(\hat{n}_\gamma \to \hat{n}_s)^{(K)}_{q \chi_{J_1}}$$

$$\text{(IV.4-2a)}$$

When the atomic and the nuclear spins are unpolarized or polarized along the direction $\hat{p}_0$ we get the simplified form

$$\bar{\rho}\cdot(\hat{n}_\gamma;\ t)_{\lambda'\lambda} = c(\omega) \sum_{\substack{Kq \\ K_{J_1}Q_{J_1}}} \mathrm{tr}\{r_{-\lambda'}T(J'_1J_1;\ t)_{Kq}r^{\dagger}_{-\lambda}\}\times$$

$$\times T(J_1)^{\dagger}_{K_{J_1}Q_{J_1}} > G(J_1;\ t)_{K_{J_1}K}^{Q_{J_1}Q_{J_1}}$$

$$\times D(\hat{n}_\gamma \to \hat{p}_0)^{(K)}_{qQ_{J_1}} \qquad \text{(IV.3-2b)}$$

The relevant Stokes parameters are obtained by comparing $\bar{\rho}\cdot(\hat{n}_\gamma;\ t)_{\lambda'\lambda}$ with eq.(I.3-7).

IV.5 DESCRIPTION OF THE H(2p)* EXCITED STATE FORMED BY THE ELECTRON CAPTURE OF POLARIZED PROTONS ON ATOMIC TARGETS IN TERMS OF STOKES PARAMETERS

In this case the process (IV.1-1) looks like

$$H^+ + A \quad H(2p)^* + A^+$$

the target A is assumed to have unpolarized spins. Now the relevant perturbation coefficients are obtained if one substitutes $I = \frac{1}{2} = s_1$, $K_{s_1} = 0$ and $L_1 = 1$ in eqs. (II.3-17) and (II.3-19b) for the case in sub-section (IV.1)A and $I = \frac{1}{2}$ and $j_1 = \frac{1}{2}$ or $3/2$ in eqs. (IV.3-7) and (IV.3-19) for the case (IV.1)B. Assuming that the resolution time of the photon detector is large compared to the life time of the excited H(2p)* state $\tau_R >> \tau$, we can integrate the exponential factor in the perturbation coefficients from $t = 0 \to \infty$ with negligible error.[15] (The details of other experimental situations can be found in Macek and Jaecks.)[3] The Stokes parameters in different cases then turn out to be

(A): Collision in LS Coupling:

(1) Longitudinal proton-polarization

$$I = c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\Big([\tfrac{2}{3\sqrt{3}}\,\bar{G}_0\langle T(1)^{\dagger}_{00}\rangle + \bar{G}_2\{\tfrac{1}{6}\sqrt{\tfrac{2}{3}}(3\cos^2\theta_\gamma - 1)\langle T(1)^{\dagger}_{20}\rangle - \tfrac{1}{3}\sin 2\theta_\gamma\cos\phi_\gamma\langle T(1)^{\dagger}_{21}\rangle + \tfrac{1}{3}\sin^2\theta_\gamma\cos 2\phi_\gamma\langle T(1)^{\dagger}_{22}\rangle\}] - \tfrac{1}{3}\sin 2\theta_\gamma\sin\phi_\gamma\,\bar{G}_z(1)^{11}_{12}\,i\langle T(1)^{\dagger}_{11}\rangle\Big)$$

$$I\eta_2 = \tfrac{\sqrt{2}}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\Big([-\sqrt{2}\sin\theta_\gamma\sin\phi_\gamma\,\bar{G}_1\,i\langle T(1)^{\dagger}_{11}\rangle] + \cos\theta_\gamma\{\bar{G}_z(1)^{00}_{01}\langle T(1)^{\dagger}_{00}\rangle + \bar{G}_z(1)^{00}_{21}\langle T(1)^{\dagger}_{20}\rangle - \sqrt{2}\sin\theta_\gamma\cos\phi_\gamma\,\bar{G}_z(1)^{11}_{21}\langle T(1)^{\dagger}_{21}\rangle\Big)$$

η_2 being the circular polarization.

The two linear polarizations are given by

$$I\eta_3 = -\tfrac{1}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\Big(\bar{G}_z[\sqrt{\tfrac{3}{2}}\sin^2\theta_\gamma\langle T(1)^{\dagger}_{20}\rangle + \sin 2\theta_\gamma\cos\phi_\gamma\langle T(1)^{\dagger}_{21}\rangle + (1+\cos^2\theta_\gamma)\cos 2\phi_\gamma\langle T(1)^{\dagger}_{22}\rangle] + \sin 2\theta_\gamma\sin\phi_\gamma \times \bar{G}_z(1)^{11}_{12}\,i\langle T(1)^{\dagger}_{11}\rangle\Big)$$

$$I\eta_1 = \tfrac{1}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\Big(2\bar{G}_2[\sin\theta_\gamma\sin\phi_\gamma\langle T(1)^{\dagger}_{21}\rangle + \cos\theta_\gamma\sin 2\phi_\gamma\langle T(1)^{\dagger}_{22}\rangle] - 2\sin\theta_\gamma\cos\phi_\gamma\,\bar{G}_z(1)^{11}_{12}\,i\langle T(1)^{\dagger}_{11}\rangle\Big) \qquad \text{(IV.5-1)}$$

The terms which arise when protons are unpolarized have been inserted in the square brackets.

(2) Transverse proton-polarization in the scattering plane:

$$I = c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\,([\text{polarization independent terms}] + \tfrac{1}{3}\,\bar{G}_x(1)^{11}_{12}\,\sin 2\theta_\gamma\,\sin\phi_\gamma\; i\langle T(1)^\dagger_{11}\rangle)$$

$$In_2 = \frac{\sqrt{2}}{3} c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\,([\text{polarization independent terms}] + \sin\theta_\gamma\{\bar{G}_x(1)^{11}_{01}\langle T(1)^\dagger_{00}\rangle$$

$$- \bar{G}_x(1)^{00}_{21}\;\; (\tfrac{1}{2}\langle T(1)^\dagger_{20}\rangle + \sqrt{2}\langle T(1)^\dagger_{21}\rangle - \sqrt{\tfrac{3}{2}}\langle T(1)^\dagger_{22}\rangle)\} - \sqrt{2}\,\cos\theta_\gamma\,\cos\phi_\gamma$$

$$\times\, \bar{G}_x(1)^{11}_{12}\langle T(1)^\dagger_{21}\rangle)$$

$$In_3 = -\tfrac{1}{3}\, c(\omega)\langle|0||\underline{r}||1\rangle|^2\,([\text{polarization independent terms}] - \sin 2\theta_\gamma\,\sin\phi_\gamma$$

$$\times\, \bar{G}_x(1)^{11}_{12}\, i\langle T(1)^\dagger_{11}\rangle)$$

$$In_1 = \tfrac{1}{3}\, c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\,([\text{polarization independent terms}]) + \cos\theta_\gamma\,\cos\phi_\gamma \bar{G}_x(1)^{11}_{12}\, i\langle T(1)^\dagger_{11}\rangle)$$

(IV.5-2)

(3) Transverse proton polarization normal to the scattering plane:

$$I = c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\,([\text{polarization independent terms}] - \tfrac{1}{3\sqrt{3}}\,(3\sin^2\theta_\gamma - 1)$$

$$\times\; \bar{G}_y(1)^{00}_{12}\, i\langle T(1)^\dagger_{11}\rangle)$$

$$In_2 = \frac{\sqrt{2}}{3}\, c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\,([\text{polarization independent terms}] + \sin\theta_\gamma\{\bar{G}_y(1)^{00}_{01}\langle T(1)^\dagger_{00}\rangle$$

$$- \bar{G}_y(1)^{00}_{21}(\tfrac{1}{2}\langle T(1)^\dagger_{20}\rangle + \sqrt{\tfrac{3}{2}}\,\langle T(1)^\dagger_{22}\rangle\}$$

$$In_3 = -\tfrac{1}{3}\, c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\,([\text{polarization independent terms}] + \sqrt{3}\,\cos^2\theta_\gamma\;\bar{G}_y(1)^{00}_{12}\, i\langle T(1)^\dagger_{11}\rangle)$$

$$In_1 = \tfrac{1}{3}\, c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\,([\text{polarization independent terms}] - \sqrt{3}\,\cos^2\theta_\gamma\;\bar{G}_y(1)^{00}_{12}\, i\langle T(1)^\dagger_{11}\rangle)$$

(IV.5-3)

When the scattered H atom is not detected in coincidence with the photons, the physical system of interest has axial symmetry[5] about the incident proton direction $\bar{p}_0$. The only non-zero state multipoles then are $\langle T(1)^{\dagger}_{00}\rangle$ and $\langle T(1)^{\dagger}_{20}\rangle$, which describe the above Stokes parameters. The polarization η_1 in all cases vanishes and the circular polarization η_2 for different cases is given by:

(1)

$$I\eta_2 = \frac{\sqrt{2}}{3}\, c(\omega)\, |\langle 0||\underline{r}||1\rangle|^2 \{\bar{G}_z(1)^{00}_{01}\langle\tau(1)^{\dagger}_{00}\rangle + \bar{G}_z(1)^{00}_{21}\langle\tau(1)^{\dagger}_{20}\rangle\}\cos\theta_\gamma$$

(2)

$$I\eta_2 = \frac{\sqrt{2}}{3}\, c(\omega)\, |\langle 0||\underline{r}||1\rangle|^2 \{\bar{G}_x(1)^{00}_{01}\langle\tau(1)^{\dagger}_{00}\rangle + \bar{G}_x(1)^{00}_{21}\langle\tau(1)^{\dagger}_{20}\rangle\}\sin\theta_\gamma$$

(3)

$$I\eta_2 = \frac{\sqrt{2}}{3}\, c(\omega)\, |\langle 0||\underline{r}||1\rangle|^2 \{\bar{G}_y(1)^{00}_{01}\langle\tau(1)^{\dagger}_{00}\rangle + \bar{G}_y(1)^{00}_{21}\langle\tau(1)^{\dagger}_{20}\rangle\}\sin\theta_\gamma$$

(IV.5-4)

The intensity I and the linear polarization η_3 in all the above cases is

$$I = c(\omega)\, |\langle 0||\underline{r}||1\rangle|^2 \{\frac{2}{3\sqrt{3}}\,\bar{G}_0\langle\tau(1)^{\dagger}_{00}\rangle + \frac{1}{6}\sqrt{\frac{2}{3}}(3\cos^2\theta_\gamma - 1)\bar{G}_2\langle\tau(1)^{\dagger}_{20}\rangle$$

$$I\eta_3 = -\frac{1}{\sqrt{6}}\, c(\omega)\, |\langle 0||\underline{r}||1\rangle|^2 \sin^2\theta_\gamma\, \bar{G}_2\langle\tau(1)^{\dagger}_{20}\rangle \qquad \text{(IV.5-5)}$$

The different perturbation coefficients are

$$\bar{G}_0 = \tau\,, \qquad \bar{G}_1 = \frac{13}{16}\,\tau\,, \qquad \bar{G}_2 = \frac{1}{12}\,\tau\,,$$

$$\bar{G}_{\underline{p}}(1)^{00}_{01} = -\,0.0269153777 \times 10^{-8}\, P_{\underline{p}}(I)\,, \qquad \underline{p} = (x, y, z)$$

$$\bar{G}_{\underline{p}}(1)^{11}_{21} = 0.006634055 \times 10^{-8}\, P_{\underline{p}}(I)\,,$$

$$\bar{G}_{\underline{p}}(1)^{00}_{21} = 0.0076603469 \times 10^{-8}\, P_{\underline{p}}(I)\,,$$

$$\bar{G}_{\underline{p}}(1)^{11}_{12} = 0.0000247824 \times 10^{-8}\, P_{\underline{p}}(I)\,,$$

$P_{\underline{p}}(I)$ is the polarization of the incident protons. Note that by measuring I, $I\eta_2$ and $I\eta_3$ at fixed proton energy and photon detector

orientation θ_γ with $\bar{p}_0$, the multipoles $\langle T(1)^\dagger_{00}\rangle$, $\langle T(1)^\dagger_{20}\rangle$ (or equivalently electron capture cross sections $Q(0)$ and $Q(1)$ and spin polarization $P_p(I)$ of the protons can be found. This fact can also be used to make a spin polarization detector for the protons.

All the state multipoles

$$\langle T(1)^\dagger_{K_{L_1} Q_{L_1}} \rangle$$

can be found from the set of relations (IV.5-1), (IV.5-2) or (IV.5-3) by suitable manipulation in different experimental situations. For the experimental arrangement of Hippler et al[40] in the situation (1) of sub-section (IV.5)A for example, we get from (IV.5-1)

$$I^+ = c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2 \left(\frac{4}{3\sqrt{3}}\,\bar{G}_0\langle T(1)^\dagger_{00}\rangle + \bar{G}_2\left\{\frac{1}{6}\sqrt{\frac{2}{3}}\,(3\cos^2\theta_\gamma - 1)\langle T(1)^\dagger_{20}\rangle + \frac{2}{3}\sin^2\theta_\gamma \cos 2\phi_\gamma \langle T(1)^\dagger_{22}\rangle\right\}\right) \qquad \text{(IV.5-6)}$$

$$I^- = c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2 \left(-\frac{2}{3}\sin 2\theta_\gamma \cos\phi_\gamma\, \bar{G}_2\langle T(1)^\dagger_{21}\rangle - \frac{2}{3}\sin 2\theta_\gamma \sin\phi_\gamma\, \bar{G}_z(1)^{11}_{12}\; i\,\langle T(1)^\dagger_{11}\rangle\right) \qquad \text{(IV.5-1)}$$

where

$$I^{\pm} = I(\theta_\gamma, \phi_\gamma) \pm I(\theta_\gamma, \phi_\gamma + \pi)$$

All the five state multipoles can be found by moving the photon detector to five different directions $(\theta_\gamma, \phi_\gamma)$ keeping the particle detector and the incident projectile energy fixed.

(B) Collision in jj Coupling

We assume sharply defined fine structure states of H(2p)*. Furthermore, since the radiative decay does not change the spin of the excited atom[3] (so that the dipole matrix element can still be evaluated in the LS scheme), the Stokes parameters in different cases of proton polarization can be written as follows:

(B_1) $J_1 = \frac{1}{2}$ Case ($P_{\frac{1}{2}}$ state):

$I(\frac{1}{2})$, $I(\frac{1}{2})\eta_3(\frac{1}{2})$ and $I(\frac{1}{2})\eta_1(\frac{1}{2})$ are independent of the proton polarization, i.e. for all the three cases (1), (2), (3) we get

$$I(\tfrac{1}{2}) = c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\,\frac{2}{3\sqrt{3}}\,\bar{G}_0(\tfrac{1}{2})\,\langle T(\tfrac{1}{2})^{\dagger}_{00}\rangle \qquad \text{(IV.5-8)}$$

$$\eta_3(\tfrac{1}{2}) = 0 = \eta_1(\tfrac{1}{2}) \qquad \text{(IV.5-9)}$$

The circular polarization for the three different cases is

(1)

$$I(\tfrac{1}{2})\eta_2(\tfrac{1}{2}) = \frac{\sqrt{2}}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\big([-\sqrt{2}\,\sin\theta_\gamma\,\sin\phi_\gamma\,\bar{G}_1(\tfrac{1}{2})\;i\langle T(\tfrac{1}{2})^{\dagger}_{11}\rangle] + \cos\theta_\gamma\,\bar{G}_z(\tfrac{1}{2})^{00}_{01}\langle T(\tfrac{1}{2})^{\dagger}_{00}\rangle\big)$$

(2)

$$I(\tfrac{1}{2})\eta_2(\tfrac{1}{2}) = \frac{\sqrt{2}}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\big([\text{polarization independent term}] + \sin\theta_\gamma\,\bar{G}_x(\tfrac{1}{2})^{00}_{01}\langle T(\tfrac{1}{2})^{\dagger}_{00}\rangle\big)$$

(3)

$$I(\tfrac{1}{2})\eta_2(\tfrac{1}{2}) = \frac{\sqrt{2}}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\big([\text{polarization independent term}] + \sin\theta_\gamma\,\bar{G}_y(\tfrac{1}{2})^{00}_{01}\,\langle T(\tfrac{1}{2})^{\dagger}_{00}\rangle\big) \qquad \text{(IV.5-10)}$$

where

$$\bar{G}_0(\tfrac{1}{2}) = 0.1596169194 \times 10^{-8}, \qquad \bar{G}_1(\tfrac{1}{2}) = 0.061933777 \times 10^{-8}$$

$$\bar{G}_{\underline{p}}(\tfrac{1}{2})^{00}_{01} = 0.1589975815 \times 10^{-8}\,P_{\underline{p}}(I), \qquad \underline{p} \equiv (x, y, z)\ .$$

(B_2) $J_1 = \frac{3}{2}$ Case ($P_{3/2}$ state)

(1)

$$I(\tfrac{3}{2}) = c(\omega)\,|\langle 0\|\underline{r}\|1\rangle|^2\Big([\tfrac{2}{3\sqrt{3}}\ \bar{G}_0(\tfrac{3}{2})\langle T(\tfrac{3}{2})^{\dagger}_{00}\rangle + \bar{G}_2(\tfrac{3}{2}) \times \{\tfrac{1}{6}\sqrt{\tfrac{2}{3}}\,(3\cos^2\theta_\gamma - 1)\langle T(\tfrac{3}{2})^{\dagger}_{20}\rangle - \tfrac{1}{3}\sin 2\theta_\gamma\cos\phi_\gamma\langle T(\tfrac{3}{2})^{\dagger}_{21}\rangle + \tfrac{1}{3}\sin^2\theta_\gamma\cos 2\phi_\gamma\langle T(\tfrac{3}{2})^{\dagger}_{22}\rangle\}] - \tfrac{1}{3}\sin 2\theta_\gamma\sin\phi_\gamma\,\bar{G}_z(\tfrac{3}{2})^{11}_{12}\,i\langle T(\tfrac{3}{2})^{\dagger}_{11}\rangle - \tfrac{1}{3}\sin 2\theta_\gamma\sin\phi_\gamma\,\bar{G}_z(\tfrac{3}{2})^{11}_{32}\,i\langle T(\tfrac{3}{2})^{\dagger}_{31}\rangle + \tfrac{1}{3}\sin^2\theta_\gamma\sin 2\phi_\gamma\,\bar{G}_z(\tfrac{3}{2})^{22}_{32}\,i\langle T(\tfrac{3}{2})^{\dagger}_{32}\rangle\Big)$$

$$I(\tfrac{3}{2})\eta_2(\tfrac{3}{2}) = \frac{\sqrt{2}}{3}\,c(\omega)\,|\langle 0\|\underline{r}\|1\rangle|^2\Big([-\sqrt{2}\sin\theta_\gamma\sin\phi_\gamma\,\bar{G}_1(\tfrac{3}{2})\langle T(\tfrac{3}{2})^{\dagger}_{11}\rangle] + \cos\theta_\gamma\{\bar{G}_z(\tfrac{3}{2})^{00}_{01}\langle T(\tfrac{3}{2})^{\dagger}_{00}\rangle + \bar{G}_z(\tfrac{3}{2})^{00}_{21}\langle T(\tfrac{3}{2})^{\dagger}_{20}\rangle\} - \sqrt{2}\sin\theta_\gamma\cos\phi_\gamma\,\bar{G}_z(\tfrac{3}{2})^{11}_{21}\langle T(\tfrac{3}{2})^{\dagger}_{21}\rangle\Big)$$

$$I(\tfrac{3}{2})\eta_3(\tfrac{3}{2}) = -\tfrac{1}{3}\,c(\omega)\,|\langle 0\|\underline{r}\|1\rangle|^2\Big(\bar{G}_2(\tfrac{3}{2})\Big[\sqrt{\tfrac{3}{2}}\sin^2\theta_\gamma\langle T(\tfrac{3}{2})^{\dagger}_{20}\rangle + \sin 2\theta_\gamma\cos\phi_\gamma\langle T(\tfrac{3}{2})^{\dagger}_{21}\rangle + (1+\cos^2\theta_\gamma)\cos 2\phi_\gamma\langle T(\tfrac{3}{2})^{\dagger}_{22}\rangle\Big] + \sin 2\theta_\gamma\sin\phi_\gamma\,\bar{G}_z(\tfrac{3}{2})^{11}_{12}\,i\langle T(\tfrac{3}{2})^{\dagger}_{11}\rangle + \sin 2\theta_\gamma\sin\phi_\gamma\,\bar{G}_z(\tfrac{3}{2})^{11}_{32}\,i\langle T(\tfrac{3}{2})^{\dagger}_{31}\rangle + (1+\cos^2\theta_\gamma)\sin 2\phi_\gamma\,\bar{G}_z(\tfrac{3}{2})^{22}_{32}\,i\langle T(\tfrac{3}{2})^{\dagger}_{32}\rangle\Big)$$

$$I(\tfrac{3}{2})\eta_1(\tfrac{3}{2}) = \tfrac{1}{3}\,c(\omega)\,|\langle 0\|\underline{r}\|1\rangle|^2\Big(2\bar{G}_2(\tfrac{3}{2}) \times [\sin\theta_\gamma.\sin\phi_\gamma\langle T(\tfrac{3}{2})^{\dagger}_{21}\rangle + \cos\theta_\gamma\sin 2\phi_\gamma\langle T(\tfrac{3}{2})^{\dagger}_{22}\rangle] - 2\sin\theta_\gamma\cos\phi_\gamma\,\bar{G}_z(\tfrac{3}{2})^{11}_{12}\,i\langle T(\tfrac{3}{2})^{\dagger}_{11}\rangle - 2\sin\theta_\gamma\cos\phi_\gamma\,\bar{G}_z(\tfrac{3}{2})^{11}_{32}\,i\langle T(\tfrac{3}{2})^{\dagger}_{31}\rangle\Big)$$

Note that once again we have put all the terms indepent of proton polarization inside the square brackets.

(2)

$$I(\tfrac{3}{2}) = c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\,([\text{polarization independent terms}]$$
$$+ \tfrac{1}{3}\sin 2\theta_\gamma \sin\phi_\gamma\, \bar{G}_x(\tfrac{3}{2})^{11}_{12}\, i\langle T(\tfrac{3}{2})^{\dagger}_{11}\rangle$$
$$+ \tfrac{1}{3}\sin 2\theta_\gamma \sin\phi_\gamma\, \bar{G}_x(\tfrac{3}{2})^{11}_{32}\, i(-\tfrac{\sqrt{3}}{4}\langle T(\tfrac{3}{2})^{\dagger}_{30}\rangle$$
$$+ \tfrac{15}{4}\langle T(\tfrac{3}{2})^{\dagger}_{31}\rangle + \tfrac{\sqrt{15}}{4}\langle T(\tfrac{3}{2})^{\dagger}_{33}\rangle)$$
$$- \tfrac{1}{3}\cos^2\theta_\gamma \sin 2\phi_\gamma\, \bar{G}_x(\tfrac{3}{2})^{22}_{32}\, i\langle T(\tfrac{3}{2})^{\dagger}_{32}\rangle)$$

$$I(\tfrac{3}{2})\eta_2(\tfrac{3}{2}) = \tfrac{\sqrt{2}}{3}\, c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2([\text{polarization independent terms}]$$
$$+ \sin\theta_\gamma\,\{\bar{G}_x(\tfrac{3}{2})^{00}_{01}\langle T(\tfrac{3}{2})^{\dagger}_{00}\rangle$$
$$- \bar{G}_x(\tfrac{3}{2})^{00}_{21}(\tfrac{1}{2}\langle T(\tfrac{3}{2})^{\dagger}_{20}\rangle + \sqrt{2}\langle T(\tfrac{3}{2})^{\dagger}_{21}\rangle - \sqrt{\tfrac{3}{2}}\langle T(\tfrac{3}{2})^{\dagger}_{22}\rangle)\}$$
$$- \sqrt{2}\cos\theta_\gamma \cos\phi_\gamma\, \bar{G}_x(\tfrac{3}{2})^{11}_{12}\langle T(\tfrac{3}{2})^{\dagger}_{21}\rangle)$$

$$I(\tfrac{3}{2})\eta_3(\tfrac{3}{2}) = -\tfrac{1}{3}\, c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2([\text{polarization independent terms}]$$
$$- \sin 2\theta_\gamma \sin\phi_\gamma\, \bar{G}_x(\tfrac{3}{2})^{11}_{12}\, i\langle T(\tfrac{3}{2})^{\dagger}_{11}\rangle$$
$$- \sin 2\theta_\gamma \sin\phi_\gamma\, \bar{G}_x(\tfrac{3}{2})^{11}_{32}\,(-\tfrac{\sqrt{3}}{4}\langle T(\tfrac{3}{2})^{\dagger}_{30}\rangle$$
$$+ \tfrac{15}{4}\langle T(\tfrac{3}{2})^{\dagger}_{31}\rangle$$
$$+ \tfrac{\sqrt{15}}{4}\langle T(\tfrac{3}{2})^{\dagger}_{33}\rangle)$$
$$- (1 + \sin^2\theta_\gamma)\sin 2\phi_\gamma\, \bar{G}_x(\tfrac{3}{2})^{22}_{32}\, i\langle T(\tfrac{3}{2})^{\dagger}_{32}\rangle)$$

$$I(\tfrac{3}{2})\eta_1(\tfrac{3}{2}) = \tfrac{1}{3}\, c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\,([\text{polarization independent terms}]$$
$$+ 2\cos\theta_\gamma \cos\phi_\gamma\, \bar{G}_x(\tfrac{3}{2})^{11}_{12}\, i\langle T(\tfrac{3}{2})^{\dagger}_{11}\rangle$$
$$+ 2\cos\theta_\gamma \cos\phi_\gamma\, \bar{G}_x(\tfrac{3}{2})^{11}_{32}\, i(-\tfrac{\sqrt{3}}{4}\langle T(\tfrac{3}{2})^{\dagger}_{30}\rangle$$
$$+ \tfrac{15}{4}\langle T(\tfrac{3}{2})^{\dagger}_{31}\rangle$$
$$+ \tfrac{\sqrt{15}}{4}\langle T(\tfrac{3}{2})^{\dagger}_{33}\rangle)$$
$$+ 2\sin\theta_\gamma \cos 2\phi_\gamma\, \bar{G}_x(\tfrac{3}{2})^{22}_{32}\, i\langle T(\tfrac{3}{2})^{\dagger}_{32}\rangle) \quad \text{(IV.5-12)}$$

(3)

$$I(\tfrac{3}{2}) = c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2 \text{ (polarization independent terms]}$$
$$- \frac{1}{3\sqrt{3}}(3\sin^2\theta_\gamma - 1)\,\bar{G}_y(\tfrac{3}{2})^{00}_{12}\, i\langle T(\tfrac{3}{2})^{\dagger}_{11}\rangle$$
$$- \frac{1}{6\sqrt{6}}(3\sin^2\theta_\gamma - 1)\,\bar{G}_y(\tfrac{3}{2})^{00}_{32}\, i(\sqrt{3}\langle T(\tfrac{3}{2})^{\dagger}_{31}\rangle + \sqrt{5}\langle T(\tfrac{3}{2})^{\dagger}_{33}\rangle)$$
$$+ \frac{1}{4\sqrt{3}}\sin 2\theta_\gamma \cos\phi_\gamma\, \bar{G}_y(\tfrac{3}{2})^{11}_{32}\, i\langle T(\tfrac{3}{2})^{\dagger}_{30}\rangle$$
$$+ \frac{1}{3}\bar{G}_y(\tfrac{3}{2})^{22}_{32}\, i\left(\frac{\sqrt{5}}{2\sqrt{2}}\cos^2\theta_\gamma \cos\phi_\gamma \langle T(\tfrac{3}{2})^{\dagger}_{31}\rangle - \cos^2\theta_\gamma \sin 2\phi_\gamma \langle T(\tfrac{3}{2})^{\dagger}_{32}\rangle\right))$$

$$I(\tfrac{3}{2})\eta_2(\tfrac{3}{2}) = \frac{\sqrt{2}}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2 \text{ ([polarization independent terms]}$$
$$+ \sin\theta_\gamma\{\bar{G}_y(\tfrac{3}{2})^{00}_{01}\langle T(\tfrac{3}{2})^{\dagger}_{00}\rangle - \bar{G}_y(\tfrac{3}{2})^{00}_{21} \times (\tfrac{1}{2}\langle T(\tfrac{3}{2})^{\dagger}_{20}\rangle + \sqrt{\tfrac{3}{2}}\langle T(\tfrac{3}{2})^{\dagger}_{22}\rangle)\})$$

$$I(\tfrac{3}{2})\eta_3(\tfrac{3}{2}) = -\frac{1}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2 \text{ ([polarization independent terms]}$$
$$+ \sqrt{3}\cos^2\theta_\gamma\, \bar{G}_y(\tfrac{3}{2})^{11}_{32}\, i\langle T(\tfrac{3}{2})^{\dagger}_{30}\rangle$$
$$+ \bar{G}_y(\tfrac{3}{2})^{22}_{32}\, i\{\tfrac{1}{2}\sqrt{\tfrac{5}{2}}(1 + \sin^2\theta_\gamma)\cos 2\phi_\gamma \langle T(\tfrac{3}{2})^{\dagger}_{31}\rangle - (1 + \sin^2\theta_\gamma)\sin 2\phi_\gamma \langle T(\tfrac{3}{2})^{\dagger}_{32}\rangle\})$$

$$I(\tfrac{3}{2})\eta_1(\tfrac{3}{2}) = \frac{1}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2 \text{ ([polarization independent terms]}$$
$$- \sqrt{3}\cos^2\theta_\gamma\, \bar{G}_y(\tfrac{3}{2})^{00}_{12}\, i\langle T(\tfrac{3}{2})^{\dagger}_{11}\rangle$$
$$- 2\sin\theta_\gamma \cos\phi_\gamma\, \bar{G}_y(\tfrac{3}{2})^{11}_{32}\, i\langle T(\tfrac{3}{2})^{\dagger}_{31}\rangle$$
$$- 2\cos\theta_\gamma \cos 2\phi_\gamma\, \bar{G}_y(\tfrac{3}{2})^{22}_{32}\, i\langle T(\tfrac{3}{2})^{\dagger}_{32}\rangle) \quad \text{(IV.5-13)}$$

when it is not possible to separate the $P_{3/2}$ and $P_{1/2}$ states optically then the light intensity related to those has to be

summed up. Then for example in case (1) for the H(2p)* state we have

$$I = c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\Big(\Big[\frac{2}{3\sqrt{3}}\,(\bar{G}_0(\tfrac{1}{2})\langle T(\tfrac{1}{2})^{\dagger}_{00}\rangle + \bar{G}_0(\tfrac{3}{2})\ \langle T(\tfrac{3}{2})^{\dagger}_{00}\rangle)$$
$$+ \bar{G}_2(\tfrac{3}{2})\Big\{\frac{1}{6}\sqrt{\frac{2}{3}}(3\cos^2\theta_\gamma - 1)\langle T(\tfrac{3}{2})^{\dagger}_{20}\rangle$$
$$- \frac{1}{3}\sin 2\theta_\gamma \cos\phi_\gamma\ \langle T(\tfrac{3}{2})^{\dagger}_{21}\rangle$$
$$+ \frac{1}{3}\sin^2\theta_\gamma \cos 2\phi_\gamma\ \langle T(\tfrac{3}{2})^{\dagger}_{22}\rangle\Big\}\Big]$$
$$- \frac{1}{3}\sin 2\theta_\gamma \sin\phi_\gamma\ \bar{G}_z(\tfrac{3}{2})^{11}_{12}\ i\langle T(\tfrac{3}{2})^{\dagger}_{11}\rangle$$
$$- \frac{1}{3}\sin 2\theta_\gamma \sin\phi_\gamma\ \bar{G}_z(\tfrac{3}{2})^{11}_{32}\ i\langle T(\tfrac{3}{2})^{\dagger}_{31}\rangle$$
$$+ \frac{1}{3}\sin^2\theta_\gamma \sin 2\phi_\gamma\ \bar{G}_z(\tfrac{3}{2})^{\dot{2}\dot{2}}_{32}\ i\langle T(\tfrac{3}{2})^{\dagger}_{32}\rangle\Big)$$

$$I\eta_2 = \frac{\sqrt{2}}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\Big(\Big[-\sqrt{2}\sin\theta_\gamma \sin\phi_\gamma(\bar{G}_1(\tfrac{1}{2})\langle T(\tfrac{1}{2})^{\dagger}_{11}\rangle$$
$$+ \bar{G}_1(\tfrac{3}{2})\langle T(\tfrac{3}{2})^{\dagger}_{11}\rangle)\Big]$$
$$+ \cos\theta_\gamma\{(\bar{G}_z(\tfrac{1}{2})^{00}_{01}\langle T(\tfrac{1}{2})^{\dagger}_{00}\rangle + \bar{G}_z(\tfrac{3}{2})^{00}_{01}\ \langle T(\tfrac{3}{2})^{\dagger}_{00}\rangle)$$
$$+ \bar{G}_z(\tfrac{3}{2})^{00}_{21}\ \langle T(\tfrac{3}{2})^{\dagger}_{20}\rangle\}$$
$$- \sqrt{2}\sin\theta_\gamma \cos\phi_\gamma\ \bar{G}_z(\tfrac{3}{2})^{11}_{21}\ \langle T(\tfrac{3}{2})^{\dagger}_{21}\rangle\Big)$$

$$I\eta_3 = I(\tfrac{3}{2})\,\eta_3(\tfrac{3}{2}) \qquad \text{and} \qquad I\eta_1 = I(\tfrac{3}{2})\,\eta_1(\tfrac{3}{2}) \tag{IV.5-14}$$

Similar relations can also be found for cases (2) and (3). The perturbation coefficients for $P_{3/2}$ are

$$\bar{G}_0(\tfrac{3}{2}) = 0.1596169194 \times 10^{-8}\ ,$$
$$\bar{G}_1(\tfrac{3}{2}) = 0.119737472 \times 10^{-8}\ ,$$
$$\bar{G}_2(\tfrac{3}{2}) = 0.0399785773 \times 10^{-8}, \qquad \underline{p} = (x, y, z)$$
$$\bar{G}_{\underline{p}}(\tfrac{3}{2})^{00}_{01} = 0.1337320253 \times 10^{-8}\ P_{\underline{p}}(I)\ ,$$
$$\bar{G}_{\underline{p}}(\tfrac{3}{2})^{00}_{21} = 0.0133402972 \times 10^{-8}\ P_{\underline{p}}(I)$$

$$\bar{G}_p\left(\tfrac{3}{2}\right)^{11}_{21} = 0.0115530362 \times 10^{-8}\, P_p(I)\ ,$$

$$\bar{G}_p\left(\tfrac{3}{2}\right)^{00}_{12} = 0.1070077867 \times 10^{-8}\, P_p(I)$$

$$\bar{G}_p\left(\tfrac{3}{2}\right)^{11}_{12} = 0.0926714616 \times 10^{-8}\, P_p(I)$$

$$G_p\left(\tfrac{3}{2}\right)^{00}_{32} = 0.0027675078 \times 10^{-8}\, P_p(I)$$

$$G_p\left(\tfrac{3}{2}\right)^{11}_{32} = -\,0.0026092314 \times 10^{-8}\, P_p(I)\ ,$$

$$G_p\left(\tfrac{3}{2}\right)^{22}_{32} = -\,0.0020627785 \times 10^{-8}\, P_p(I)$$

Note that

$$\bar{G}_0\left(\tfrac{1}{2}\right) = \bar{G}_0\left(\tfrac{3}{2}\right) \quad \text{and} \quad \bar{G}_1\left(\tfrac{1}{2}\right) \neq \bar{G}_1\left(\tfrac{3}{2}\right)$$

By appropriate experimental measurements and using relations (IV.5-11) to (IV.5-13) for the Stokes parameters, all the state multipoles of interest can be found. It is, however, not possible to find all the state multipoles for $P_{3/2}$ and $P_{1/2}$ from eq. (IV.5-14) when both the fine structure lines cannot be resolved optically.

When the target and the projectile happen to be the same atom, the detected photons can result from two competing processes. For instance when protons collide with a hydrogen atom, we have the reactions

$$H^+ + H \rightarrow H^* + H^+ \qquad \text{(Electron capture)}$$

$$H^+ + H \rightarrow H^+ + H^* \qquad \text{(Direct excitation)}$$

Both the excited states may happen to be identical. In this case and for comparatively large relative velocities, the two photons are Doppler shifted and may be distinguished.

At low velocity of the projectile the $m_{L_1} = 0$, sublevel is

preferentially populated.[34] Then the non-zero state multipoles are $\langle T(1)^{+}_{00}\rangle$, $\langle T(1)^{+}_{20}\rangle$; $\langle T(\tfrac{1}{2})^{+}_{00}\rangle$, $\langle T(\tfrac{1}{2})^{+}_{11}\rangle$; $\langle T(3/2)^{+}_{00}\rangle$, $\langle T(3/2)^{+}_{11}\rangle$. The relevant Stokes parameters can be obtained by using this consideration.

The last point is that the investigated state can be populated not only by direct capture of an electron, but also by capture from a higher state followed by decay with transition to the investigated state. This problem is quite tricky and Berezhko and Kabachnik[6] have tried to tackle it. Their eqs.(24) and (27) can be used to account for the effect of cascading.

CHAPTER V

DISCUSSION AND NUMERICAL RESULTS

V.1 ELECTRON-ATOM COLLISIONS

The electron-photon angular correlation without spin analysis has provided a complete determination of all scattering and target parameters only in connection with the excitation of singlet ^1P states of helium[35] (in terms of $\lambda = \frac{\sigma_0}{\sigma}$, the phase difference between a_0 and a_1 and σ the differential cross section). Scattering parameters consist of scattering (or excitation) amplitudes and their phase differences. Target parameters as such are orientation, alignment and multipole moments of collisionally excited atoms. A complete determination of all scattering and target parameters represents maximum possible information on observable quantitues of atomic excitation. The electron photon coincidence technique without spin analysis has, however, furnished complete information on alignment and orientation of the excited atomic states in terms of Stokes parameters. The problems which are related to the coherence of the excitation are also analysed by studying the degree of coherence of the coincident photons emitted from the excited atoms. The "degree of coherence" $|\mu|$ is defined through the "coherence correlation factor" μ by the relation[36]

$$\mu = \frac{\eta_1 + i\eta_2}{\sqrt{1 - \eta_3^2}} = |\mu| \, e^{i\beta}$$

where β is the effective phase difference". For completely coherent excitation (^1P states of He)

$$|\mu| = |P| = 1 , \qquad \text{where} \qquad P = \sqrt{\eta_1^2 + \eta_2^2 + \eta_3^2}$$

In general for partially coherent excitation

$$|\mu| \leq 1 \qquad |P| \leq 1$$

The ^1P state of He is the simplest excited state where there is no spin-orbit interaction in the collision and no fine and hyperfine structure splitting of the excited state. For electrons scattering on light atoms (elastic as well as inelastic) it is a good approximation to neglect spin orbit coupling effects during the excitation but exchange effects are indispensible. One then distinguishes direct and exchange scattering processes and interference between them. For instance a ^2P state of light atoms is described in terms of four scattering amplitudes (f_0, f_1, g_0, g_1), which requires seven real parameters to describe the scattering process (note that ^1P state of helium requires three real parameters only). In the simplest angular correlation experiments which are restricted to "directional correlations" only, the electrons and photons are observed in coincidence without spin analysis. These experiments extract five targe parameters $(\langle T(1)^\dagger_{00}\rangle, \langle T(1)^\dagger_{11}\rangle, \langle T(1)^\dagger_{20}\rangle, \langle T(1)^\dagger_{21}\rangle, \langle T(1)^\dagger_{22}\rangle)$ for a P state in terms of Stokes parameters. It is not possible to find out complete information on scattering amplitudes (f_0, f_1, g_0, g_1) from these five state multipoles. The progress of technology of producing spin polarized electrons and spin polarized atoms has, however, enabled the experimentalist to carry out a new generation of "angular correlation" and "Polarization correlation" experiments with spin polarized particles (electrons, protons etc.) and spin polarized atoms. Bartschat et al (1981)[21] have only recently worked out a theory of measurement on electron-photon-coincidences with polarized electrons and have reported the initiative on such experiments in Münster (West

Germany). D. Hils and H. Kleinpoppen (private communication) are planning to carry out measurements on elastic scattering of electrons on polarized Cs atoms.

In section II.4 we have already shown how the scattering amplitudes (f_0, f_1, g_0, g_1) can be completely determined for a ^2P state of light one electron atom as target if one starts with electrons and atoms polarized along $\bar{p}_0$ direction. Similarly in section II.2 we have worked out a scheme to find the direct ('f') and exchange ('g') amplitudes for elastic scattering of electrons on light one electron atoms by applying spin polarization analysis. The experimental data on target and scattering parameters extracted by the theory of measurement may be compared with the predictions of different theoretical approaches and could prove to be a sensitive test ground for assessment and evaluation of these approximation schemes in finest detail. For illustrating this point we have plotted numerical results on the dimensionless quantities $\sigma^d/2\sigma$, $\sigma^{ex}/2\sigma$, $\sigma^{int}/2\sigma$, Re f*g/σ for elastic scattering and $\sigma_0^d/2\sigma$, $\sigma_1^d/2\sigma$ $\sigma^d/2\sigma$; $\sigma_0^{ex}/2\sigma$, $\sigma_1^{ex}/2\sigma$, $\sigma^{ex}/2\sigma$ and $\sigma_0^{int}/2\sigma$, $\sigma_1^{int}/2\sigma$, $\sigma^{int}/2\sigma$ for $^2S \rightarrow {}^2P$ excitation of light spin one half targets by low energy electrons in adjoining figures. As can be seen theoretical predictions on these quantities show interesting structure with respect to incident electron energy and scattering angle and could draw the attention of the experimentalist to carry out these measurements. This type of investigation may lead to greater insight into electron atom collision physics.

V.1A STEADY STATE EXCITATION

In Chapter II, we described the polarization properties of light detected in coincidence with the scattered electrons (that is, those photons which are only emitted in the direction $\hat{n}_\gamma$ by the subensemble of atoms which scattered the electrons in the direction $\bar{p}_i \equiv (\theta_e, \phi_e)$) in terms of Stokes parameters. We now consider the case where the scattered electrons are not observed. It is assumed that the resolution of the photon detector is sufficient to restrict the observation to photons emitted in a transition between levels with fixed quantum numbers $L_1 s_1 \rightarrow L_2 s_2$. Since the electrons are not observed, however, all photons emitted in this transition must now be taken into account irrespective of the direction in which the electrons are scattered.

If one considers excitation by a steady flux of incoming electrons the time at which the photons are emitted is no longer uniquely defined with respect to the excitation time and the time-integrated form of the polarization density matrix $\rho^\cdot(n_\gamma)_{\lambda'\lambda}$ must be used (the time dependent exponential in the perturbation coefficient is integrated from $t = 0 \rightarrow \infty$ with negligible error).

The excited atomic ensemble is invariant with respect to rotations around the direction of incident electron $\bar{p}_0$, which is imposed by the geometry of the experiment. The state multipoles which are functions of the scattering amplitudes are then integrated over all scattering angles. We define these integrated state multipoles by the relation

$$\langle \tau(j)^\dagger_{KQ} \rangle = \int d\Omega_e \, \langle T(j)^\dagger_{KQ} \rangle \qquad \text{(V.1-1)}$$

where $d\Omega_e$ is the solid angle subtended by the electron analyzer on the interaction region.

For instance, when we start with initial electrons and atoms polarized along the direction $\bar{P}_0$ (initial electron direction), the excitation process is axially symmetric around this direction but not invariant under reflection in planes through this vector (since the initial polarization vectors change their sign under this transformation). Then in particular $\langle\tau(j)^{\dagger}_{10}\rangle \neq 0$.

We normalize according to the relation

$$\int \langle |a_p(M)|^2 \, d\Omega_e = Q_M \qquad \text{and} \qquad Q = \sum_M Q_M \tag{V.1-2}$$

where Q_M is the integrated cross section for the magnetic sublevel M and

$$Q_{-M} = Q_M \tag{V.1-3}$$

The rotational invariance of the excited atomic ensemble about $\bar{P}_0$ direction (Z-axis of the 'collision frame' - Fig.(2)) means that the choice of the X and Y axes perpendicular to Z is arbitrary and the physical properties of the ensemble in consideration must, therefore, be independent of this choise (i.e. the particles cannot 'know' how the X and Y axes of 'collision frame' are chosen). In particular the observables like state multipoles must have the same value in the XYZ system and in the system $X_r Y_r Z_r$ obtained by a rotation about the Z-axis through an arbitrary angle γ , i.e.

$$\langle\tau(j'j)^{\dagger}_{KQ}\rangle = \langle\tau(j'j)^{\dagger}_{KQ}\rangle_{rot.} \tag{V.1-4}$$

But by transformation eq.(I.5-6) we have

$$\langle\tau(j'j)^{\dagger}_{KQ}\rangle = \sum_q \langle\tau(j'j)^{\dagger}_{Kq}\rangle_{rot.} \, D(00\gamma)^{(K)*}_{qQ} \tag{V.1-5}$$

where γ denotes the angle between the X and the X_r axes.

Since

$$D(00\gamma)^{(K)*}_{q\Omega} = \exp(-i\Omega\gamma)\,\delta(q, \Omega)$$

we obtain

$$\langle\tau(j'j)^{\dagger}_{K\Omega}\rangle = \langle\tau(j'j)^{\dagger}_{K\Omega}\rangle_{rot.}\ \exp(-i\Omega\gamma) \qquad (V.1\text{-}6)$$

Because of the axial symmetry eq.(V.1-6) holds for any arbitrary rotation of angle γ about Z-axis. This requires that $\Omega = 0$, i.e. all components of the state multipoles with $\Omega \neq 0$ must vanish as they violate the symmetry condition (V.1-4). Then from eq.(I.5-4) we infer that for an axially symmetric excitation, the states with different components of angular momentum are necessarily incoherently excited (provided the quantization axis coincides with the symmetry axis). Thus a coherent excitation of angular momentum states requires a priori an excitation process which is not axially symmetric (e.g. by coincidence method etc.).

In view of the above deliberations, the Stokes parameters (in eq.(II.3-32)) for the axial symmetry become

$$I = c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\Big(\Big[\frac{2}{3\sqrt{3}}\,\bar{G}_0\langle\tau_p(1)^{\dagger}_{00}\rangle + \frac{1}{6\sqrt{3}}\,\bar{G}_2(3\cos^2\theta_\gamma - 1)\langle\tau_p(1)^{\dagger}_{20}\rangle\Big]$$
$$+ \Big\{\frac{1}{3}\sqrt{\frac{2}{3}}\,(3\cos^2\theta_\gamma - 1)\,\bar{G}(1)^{00}_{12} + \frac{2}{3\sqrt{3}}\,\bar{G}(1)^{00}_{10}\Big\}\langle\tau_p(1)^{\dagger}_{10}\rangle\Big)$$

$$I\eta_2 = \frac{\sqrt{2}}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\,\cos\theta_\gamma\Big(\bar{G}(1)^{00}_{01}\langle\tau_p(1)^{\dagger}_{00}\rangle + \bar{G}_1\langle\tau_p(1)^{\dagger}_{10}\rangle$$
$$+ \bar{G}(1)^{00}_{21}\langle\tau_p(1)^{\dagger}_{20}\rangle\Big)$$

$$I\eta_3 = -\frac{1}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\,\sin^2\theta_\gamma\Big(\sqrt{\frac{3}{2}}\,\bar{G}_2\langle\tau_p(1)^{\dagger}_{20}\rangle +$$
$$\sqrt{\frac{3}{2}}\,\bar{G}(1)^{00}_{12}\langle\tau_p(1)^{\dagger}_{10}\rangle\Big)$$

$$I\eta_1 = 0\ . \qquad (V.1\text{-}7)$$

Note that in relations (V.1-7) the circular polarization is induced by the initial spin polarizations of electrons and atoms.

This is obvious due to the dependence of the relevant state multipoles on the initial spin polarizations and the perturbation coefficients on the final atomic spin polarization. When one or both the initial spin polarizations vanish we get no circular polarization which is in agreement with Blum and Kleinpoppen.[5] The circular polarization also disappears when the photons are observed in a direction normal to the incident spin polarizations.

The state multipoles $\langle\tau_p(1)^\dagger_{00}\rangle$, $\langle\tau_p(1)^\dagger_{10}\rangle$ and $\langle\tau_p(1)^\dagger_{20}\rangle$ are found by the procedure outlined at the beginning of section (II.4). The non-vanishing of the circular polarization η_2 and the orientation $\langle\tau_p(1)^\dagger_{10}\rangle$ in case the excitation is axially symmetric can be explained in a simple way.

Due to spin orbit relaxation during the relatively long life time of the excited state $(\tau>>\tau_c)$ there is a periodic exchange of polarization between atomic spin and atomic orbital systems resulting in polarization of the orbital system and simultaneous depolarization of the spin system. This is reflected in the expression for circular polarization, where $\langle\tau_p(1)^\dagger_{00}\rangle$ and $\langle\tau_p(1)^\dagger_{20}\rangle$ are multiplied by final atomic polarization dependent perturbation coefficients. The non-vanishing of the orientation, however, is a consequence of the initial spin polarization (it vanishes when the initial spins are unpolarized or are polarized normal to the scattering plane).

V.1B POLARIZATION OF THE ELECTRON IMPACT RADIATION

Any polarization state $\hat{\varepsilon}$ of the photons propagating in the direction $\hat{n}_\gamma \equiv (\theta_\gamma, \phi_\gamma)$ can be parameterized as

$$\hat{\varepsilon} = \cos\beta\ \hat{\varepsilon}(\theta_\gamma) + e^{i\delta}\sin\beta\ \hat{\varepsilon}(\phi_\gamma) \qquad \text{(V.1-8)}$$

where δ is the phase which describes elliptical polarization. In the case of linear polarization $\delta = 0$ and β is then the angle between $\hat{\varepsilon}$ and $\hat{\varepsilon}(\theta_\gamma)$; for circular polarization $\delta = \pi/2$ and $\cos\beta = \sin\beta = \frac{1}{\sqrt{2}}$. The intensity of the photons emitted at time t in the direction $\hat{n}_\gamma$ in polarization state $\hat{\varepsilon}$ is then given in terms of the Stokes parameters by the relation[37]

$$I(\hat{\varepsilon}, \hat{n}_\gamma; t) = \frac{I}{2}\{1 + \eta_3 \cos2\beta + \eta_1 \sin2\beta \cos\delta + \eta_2 \sin2\beta \sin\delta\} \qquad \text{(V.1-9)}$$

where I is the intensity of the photons in the absence of the polarizers.

In the experiments with axial symmetry, the photon detector is usually placed normal to the incoming electrons. We choose $\hat{n}_\gamma$ as the X-axis of our 'collision frame' $(\hat{n}_\gamma \equiv (\frac{\pi}{2}, 0))$, then

$$\hat{\varepsilon} = -\cos\beta\ \hat{\varepsilon}(\tfrac{\pi}{2}) - \sin\beta\ \hat{\varepsilon}(0) \qquad \text{(V.1-1)}$$

one measures the polarization P , which is defined as

$$P = \frac{I_{||} - I_{\perp}}{I_{||} + I_{\perp}} \qquad \text{(V.1-11)}$$

$I_{||}$ and $I_{\perp}$ being the intensities of the light polarized parallel $(\beta = 0)$ and perpendicular $(\beta = 90°)$ to the Z-axis. Applying eq.(V.1-9) in eq.(V.1-11) we obtain

$$P = \eta_3(X) \qquad \text{(V.1-11a)}$$

If we substitute $\theta_\gamma = 90°$ in the Stokes parameters (V.1-7) we obtain

$$I(X) = c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\left[\frac{2}{3\sqrt{3}}\,\bar{G}_0\langle\tau_p(1)^{\dagger}_{00}\rangle - \frac{1}{6\sqrt{3}}\,\bar{G}_2\langle\tau_p(1)^{\dagger}_{20}\rangle\right.$$

$$\left. + \left(\frac{2}{3\sqrt{3}}\,\bar{G}(1)^{00}_{10} - \frac{1}{3}\sqrt{\frac{2}{3}}\,\bar{G}(1)^{00}_{12}\right)\langle\tau_p(1)^{\dagger}_{10}\rangle\right] .$$

$$\eta_2(x) = 0$$

$$I(X)\,\eta_3(X) = -\frac{1}{3}\,c(\omega)\,|\langle 0||\underline{r}||1\rangle|^2\left[\sqrt{\frac{3}{2}}\,\bar{G}_2\langle\tau_p(1)^{\dagger}_{20}\rangle + \sqrt{\frac{3}{2}}\,\bar{G}(1)^{00}_{12}\langle\tau_p(1)^{\dagger}_{10}\rangle\right] \qquad \text{(V.1-7a)}$$

Since $\eta_1(X) = 0 = \eta_2(X)$, $P = \sqrt{\eta_1^2 + \eta_2^2 + \eta_3^2}$ is the degree of polarization and is now given by the relation

$$P = -\frac{1}{\sqrt{6}}\,\frac{(\bar{G}_2\langle\tau_p(1)^{\dagger}_{20}\rangle + \bar{G}(1)^{00}_{12}\langle\tau_p(1)^{\dagger}_{10}\rangle)}{\{\frac{2}{3\sqrt{3}}\,\bar{G}_0\langle\tau_p(1)^{\dagger}_{00}\rangle - \frac{1}{6\sqrt{3}}\,\bar{G}_2\langle\tau_p(1)^{\dagger}_{20}\rangle + (\frac{2}{3\sqrt{3}}\,\bar{G}(1)^{00}_{10} - \frac{1}{3}\sqrt{\frac{2}{3}}\,\bar{G}(1)^{00}_{12}$$

$$\times\,\langle\tau_p(1)^{\dagger}_{10}\rangle\} \qquad \text{(V.1-12)}$$

Note that in contrast to unpolarized initial spins case the polarization 'P' depends on the orientation $\langle\tau_p(1)^{\dagger}_{10}\rangle$ and the spin polarizations.

V.2 THRESHOLD AND PSEUDOTHRESHOLD EXCITATION

When all spin couplings are neglected during the collision, orbital and spin angular momenta are separately conserved. This means that

$$m_{s_0} + m_0 = m_{s_1} + m_1 \qquad \text{(conservation of spin)}$$

and

$$M_0 + M_{\ell_0} = M_1 + M_{\ell_1} \qquad \text{(conservation of orbital angular momentum)}$$

where ℓ_0 and ℓ_1 is the orbital angular momentum of the electrons before and after the collision respectively (M_{ℓ_0} and M_{ℓ_1} are the relevant magnetic quantum numbers). The incident electron has no component of its orbital angular momentun along its direction of

motion $\bar{p}_0$ ($M_{\ell_0} = 0$) . After the excitation at threshold the scattered electron has zero energy and hence zero orbital angular momentum ($M_{\ell_1} = 0$) . It follows that $M_0 = M_1$. For excitation from the ground state ($L_0 = 0 = M_0$) only the substate with $M_1 = 0$ can be excited at threshold. The threshold polarization for $^2S \rightarrow {}^2P$ excitation of light one electron atoms (considered at the end of section II.3) then turns out to be

$$P_{thr} = + \frac{1}{3} \left\{ \frac{\bar{G}_2 \bar{S}_0 + \frac{3}{\sqrt{2}} P_z^a P_z^e \bar{G}_2(1)_{12}^{00} \int d\Omega_e \, \mathrm{Re}\, f_0^* g_0}{\frac{2}{9} \bar{S}_0 + \frac{\sqrt{2}}{18} \bar{G}_2 \bar{S}_0 - \frac{1}{2} P_z^a P_z^e \bar{G}_2(1)_{12}^{00} \int d\Omega_e \, \mathrm{Re}\, f_0^* g_0} \right\} \qquad (V.2\text{-}1)$$

Note that we put a bar to indicate the integrated polarized cross section ($\bar{S}_0$) for magnetic sublevel $M_1 = 0$. When the initial electrons and atoms are unpolarized or polarized normal to the scattering plane, one then finds (e.g. for Lyman-α [38])

$$P_{thr} = 43.12\% \qquad (V.2\text{-}2)$$

Note that the threshold polarization in eq.(V.2-2) is independent of cross sections etc; whereas that in eq.(V.2-1) depends on the scattering amplitudes and spin polarizations.

There is considerable interest in the polarization of impact radiation at threshold. The direct measurement of threshold polarization is difficult because of intensity problems and also because of the effects of cascades and resonances in the energy range just above threshold. King et al (1972) [39] have, however, pointed out that polarization measurements made for the subensemble of atoms which have been excited by forward-scattered electrons reproduces threshold conditions as far as polarization is concerned and that errors due to cascade and resonance effects are eliminated. For

the forward scattered electrons we have $M_{\ell_0} = M_{\ell_1}$ (taking $\hat{P}_0 \equiv \hat{P}_1$ as Z-axis of the 'collision frame'). Then exciting the atoms from the ground state and neglecting spin-orbit effects during the collision enables only magnetic substates with $M_1 = 0$ to be excited. The relevant expression for polarization is then found by replacing integrated quantities (cross-sections, state multipoles etc.) by differential quantities in eq.(V.2-1)

$$P_{for.} = + \frac{1}{3} \left\{ \frac{2\bar{G}_2 \mathcal{S}_0 + \frac{3}{\sqrt{2}} P_Z^a P_Z^e \bar{G}_Z(1)^{00}_{12} \text{ Re } f_0 * g_0}{\frac{2}{3\sqrt{3}} \mathcal{S}_0 + \frac{\sqrt{2}}{9} \bar{G}_2 \mathcal{S}_0 - \frac{1}{2} P_Z^a P_Z^e \bar{G}_Z(1)^{00}_{12} \text{ Re } f_0 * g_0} \right\} \qquad \text{(V.2-1a)}$$

In general when initial spin polarizations have a component in the scattering plane

$$P_{thr.} \neq P_{for.} \qquad \text{(V.2-3)}$$

unless we have the condition

$$\langle \tau(j)^{\dagger}_{KQ} \rangle \propto \langle T(j)^{\dagger}_{KQ} \rangle .$$

When the initial beams are unpolarized or polarized normal to the scattering plane one then obtains for the case in consideration

$$P_{thr.} = P_{for.} = 43.12\% \qquad \text{(V.2-4)}$$

V.3 ELECTRON CAPTURE

It is possible to extract valuable information on the complex mechanism of electron capture process by measuring the angular distribution and polarization of the photons emitted by the resulting hydrogenlike excited state $P^{(Z-1)*}$. The angular distribution of these photons is studied in a 'particle-photon angular correlation' experiment where the position of the photon detector is varied while keeping the particle detector (a channeltron) and the energy of the scattered atom (or ion) $P^{(Z-1)*}$ fixed. This measurement gives the coincident intensity $I(\theta_\gamma, \phi_\gamma)$ as a function of the photon detector

position $(\theta_\gamma, \phi_\gamma)$ with respect to the incident projectile direction $\bar{p}_0$. For instance using polarized protons in the experimental set up of Hippler et al,[40] all the state multipoles characterizing the resulting H(2p)* state can be found from eqs.(IV.5-6) and (IV.5-7). Note that the orientation $\langle T(1)^{\dagger}_{11} \rangle$ can not otherwise be extracted from experiment without measurement of circular photon polarization by using unpolarized protons.

It is worthwhile to note that when the scattered atom $P^{(Z-1)*}$ is recorded in coincidence with photons emitted in different polarization states (the so called 'polarization correlation experiment') and by keeping the axes of the particle and photon detectors fixed in space, all the Stokes parameters depend upon the spin polarization of the incident projectile. This dependence of the Stokes parameters on the spin polarization of the projectile can be exploited to construct an optical detector for the proton spin polarization. This optical detection of proton spin polarization is done by measuring the Lyman-α circular polarization without detecting the atoms in coincidence. This becomes obvious when we rewrite eqs.(IV.5-4) and (IV.5-5) in the following simplified form:

$$I(\theta_\gamma) = 1160.114169[24Q - (3\cos^2\theta_\gamma - 1)\{Q(0) - Q(1)\}]\text{ergs/cm}^2/\text{steradian per atom}$$

$$\eta_2 = -P_p(I)\ \begin{matrix}\cos\\ \sin\end{matrix}(\theta_\gamma)\,[4.95653783Q + 1.99499302\{Q(0) - Q(1)\}]/[24Q - (3\cos^2\theta_\gamma - 1)\{Q(0) - Q(1)\}]$$

$$\eta_3 = 3\sin^2\theta_\gamma\{Q(0) - Q(1)\}/[24Q - (3\cos^2\theta_\gamma - 1)\{Q(0) - Q(1)\}]$$

The circular polarization η_2 goes with a cosine for longitudinally polarized protons and with a sine for transversely polarized protons. Note that by measuring these Stoke parameters,

the electron-capture cross sections $Q(1)$, $Q(0)$, $Q = Q(0) + 2Q(1)$ and the spin polarization $P_p(I)$ of the incident protons can be extracted. The circular polarization η_2 is a direct measure of the spin polarization of the incoming proton beam and vanishes for zero spin-polarization of the protons.

When the fine structure states of $H(2p)^*$ can be optically resolved in Case B, state multipoles of rank three can also be found for the $P_{3/2}$ state by applying polarized protons (note that with unpolarized protons, the observation of dipole radiation can not provide information on state multipoles of rank higher than two.)

Another extremely useful aspect of this formulism is that it relates physical measurements with predictions from theoretical approximation schemes. To illustrate this we have plotted linear and circular polarizations of the Lyman-α radiation which results from $H(2p)^*$ excited state produced by electron capture on a hydrogen target {Figs.(4, 5)} . If experimental measurements of the polarizations of Lyman-α radiation are carried out and compared with the relevant theoretical predictions in Figs.(4, 5), these may prove to be a substantial criterion for the evaluation of the various approximation schemes applied. This may also increase our understanding of the detailed mechanism of the electron capture process itself.

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40. Hippler et al (1979) in: Electronic and Atomic Collisions IX - ICPEAC (Kyoto), eds; N. Oda, K. Takayanagi, North-Holland.

Fig.1

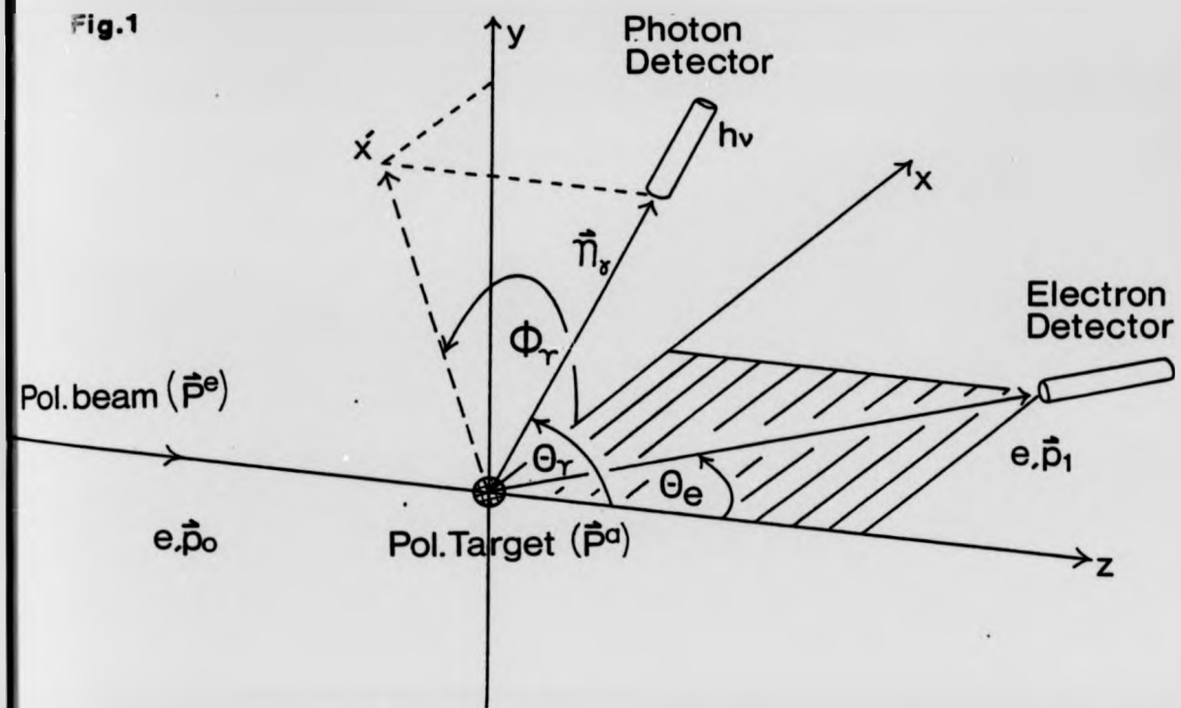

y
Photon
Detector
hν
x'
x
$\vec{n}_\gamma$
Φ_γ
Electron
Detector
Pol. beam ($\vec{P}^e$)
Θ_γ
Θ_e
e,$\vec{p}_1$
e,$\vec{p}_o$
Pol. Target ($\vec{P}^a$)
z

Fig. 2

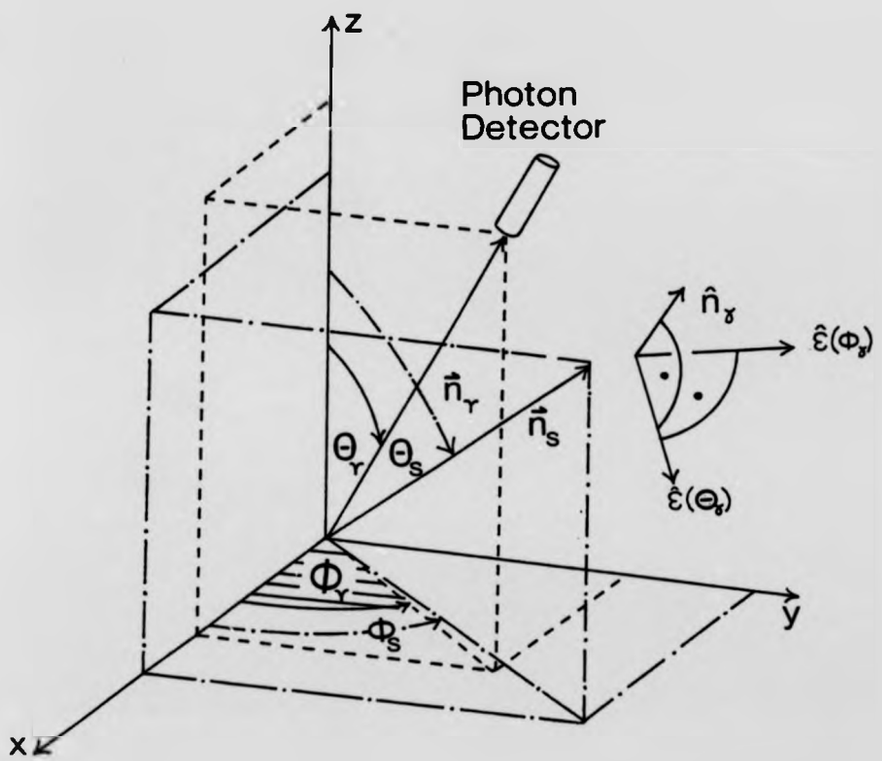

z
Photon
Detector
$\hat{n}_\gamma$
$\hat{\varepsilon}(\Phi_\gamma)$
$\vec{n}_\gamma$
$\vec{n}_s$
Θ_γ
Θ_s
$\hat{\varepsilon}(\Theta_\gamma)$
Φ_γ
Φ_s
y
x

Fig. 3a

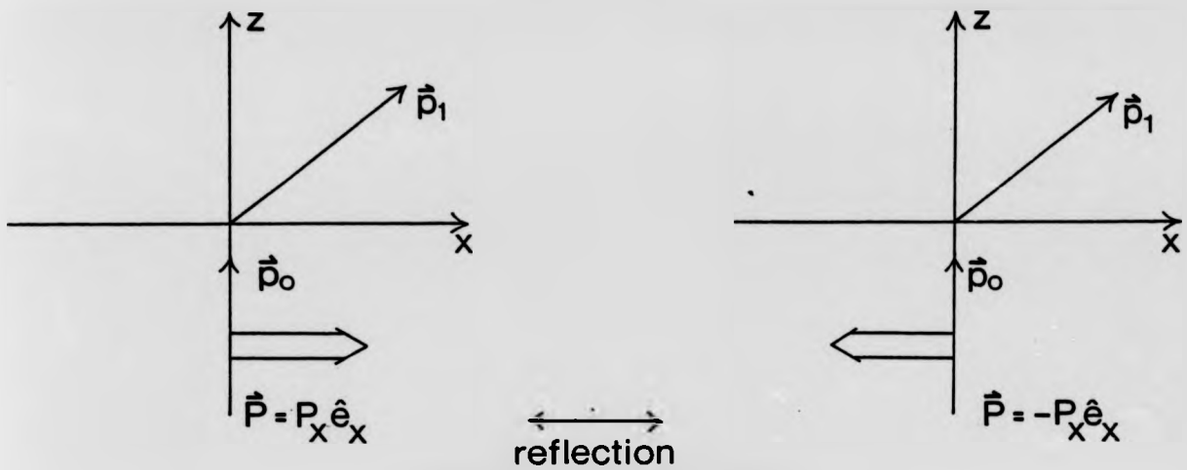
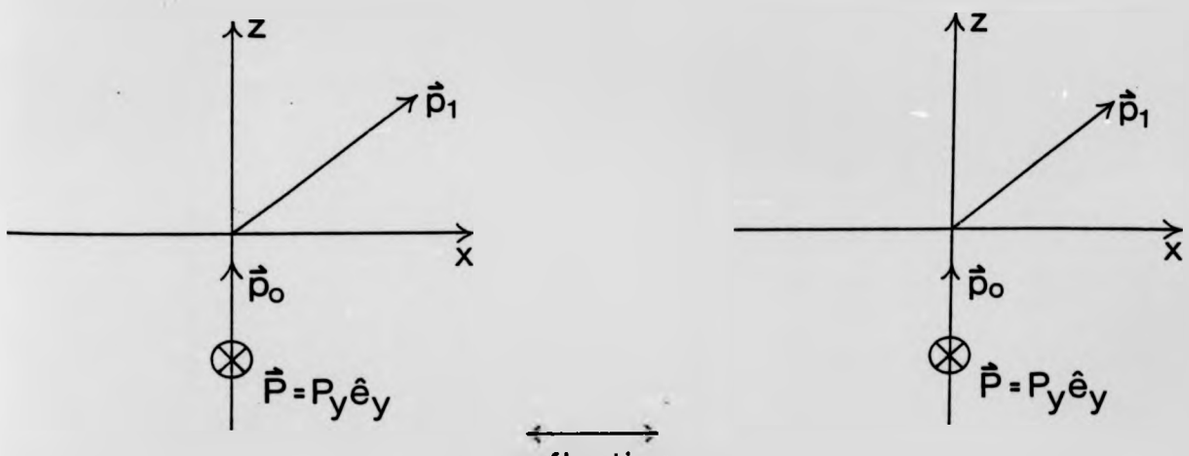
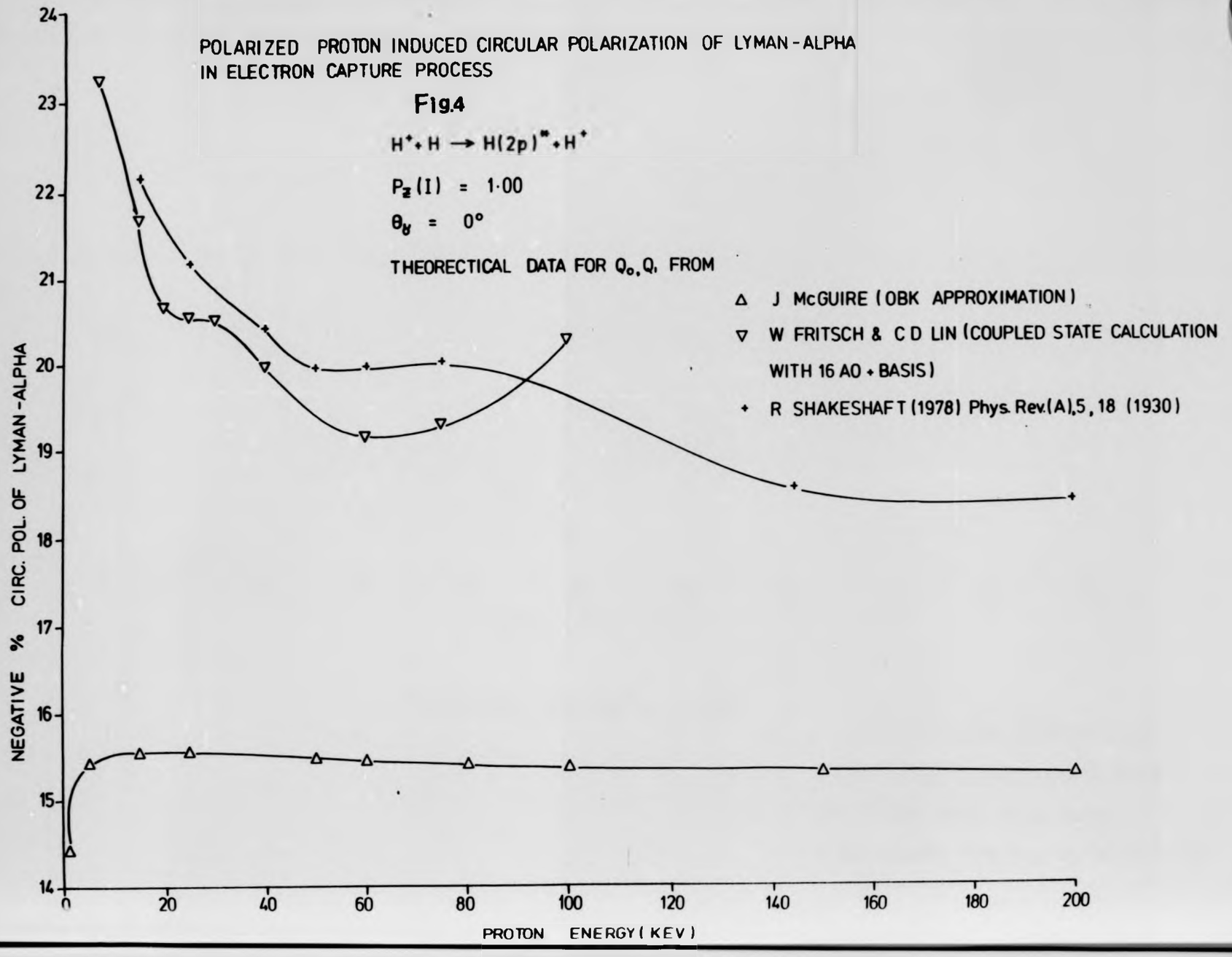

z
$\vec{p}_1$
x
$\vec{p}_o$
$\vec{P} = P_x \hat{e}_x$
reflection
in the scattering plane
z
$\vec{p}_1$
x
$\vec{p}_o$
$\vec{P} = -P_x \hat{e}_x$

Fig. 3b


z
$\vec{p}_1$
x
$\vec{p}_o$
$\vec{P} = P_y \hat{e}_y$
reflection
in the scattering plane
z
$\vec{p}_1$
x
$\vec{p}_o$
$\vec{P} = P_y \hat{e}_y$

POLARIZED PROTON INDUCED CIRCULAR POLARIZATION OF LYMAN-ALPHA IN ELECTRON CAPTURE PROCESS

Fig.4



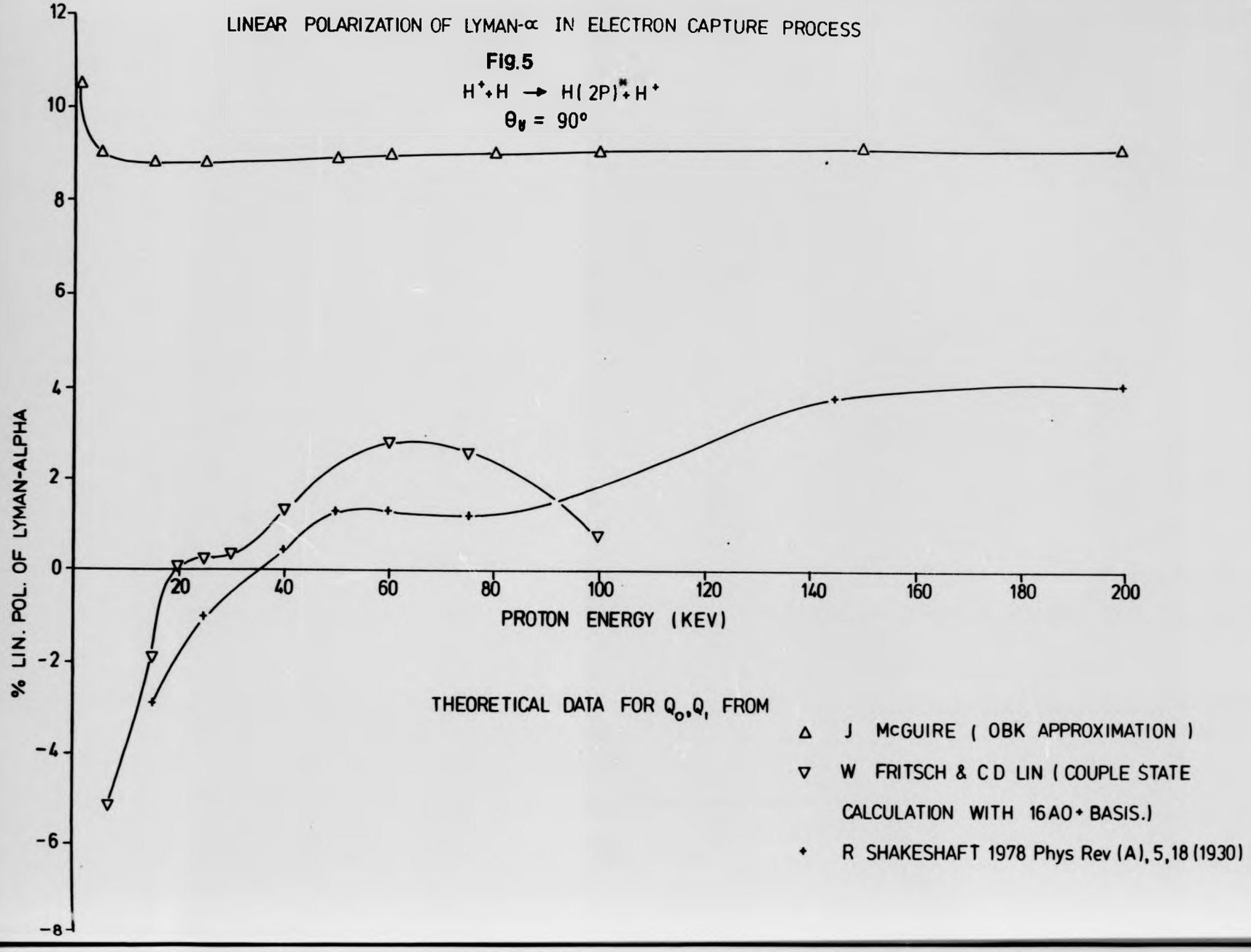
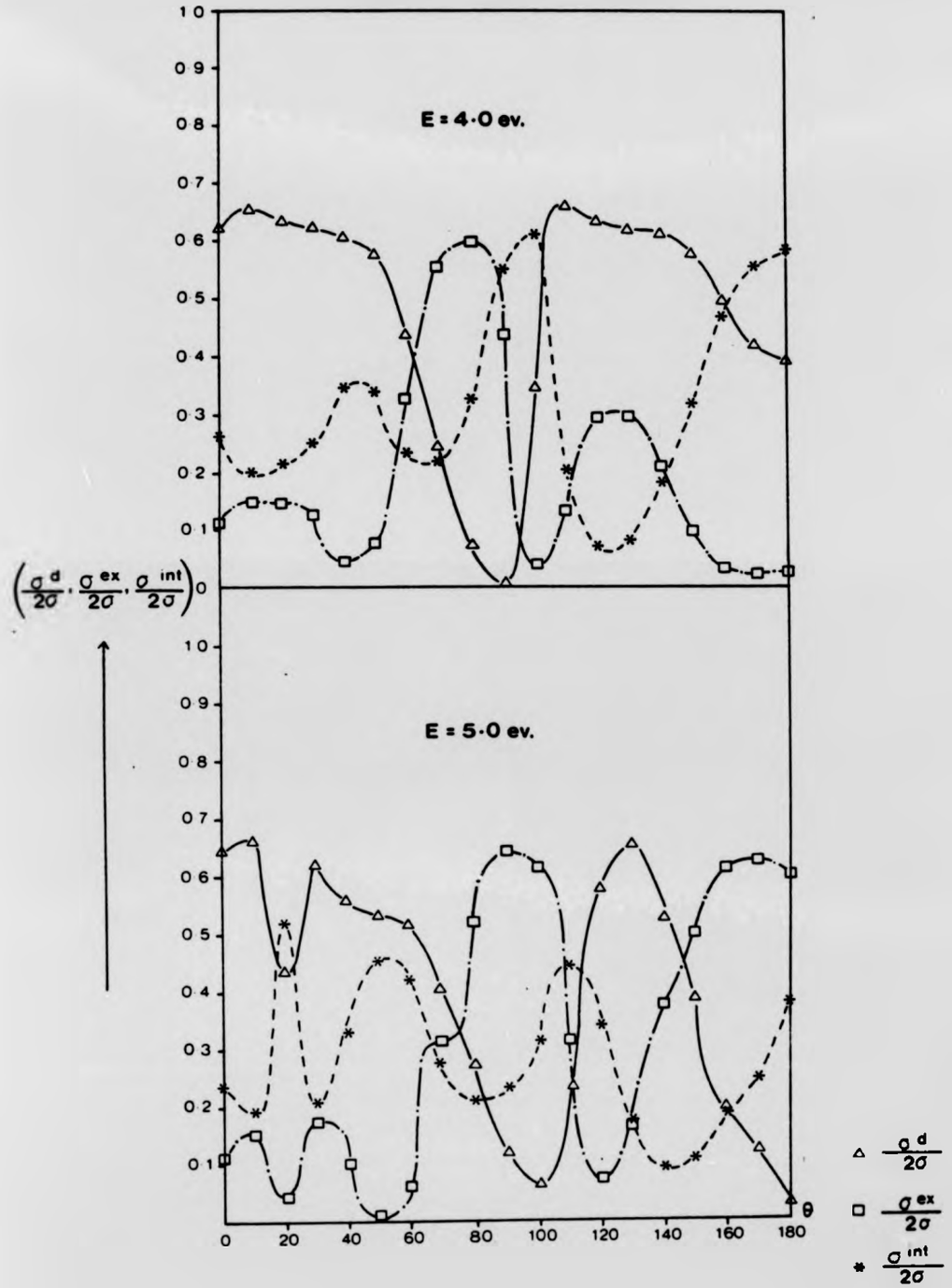
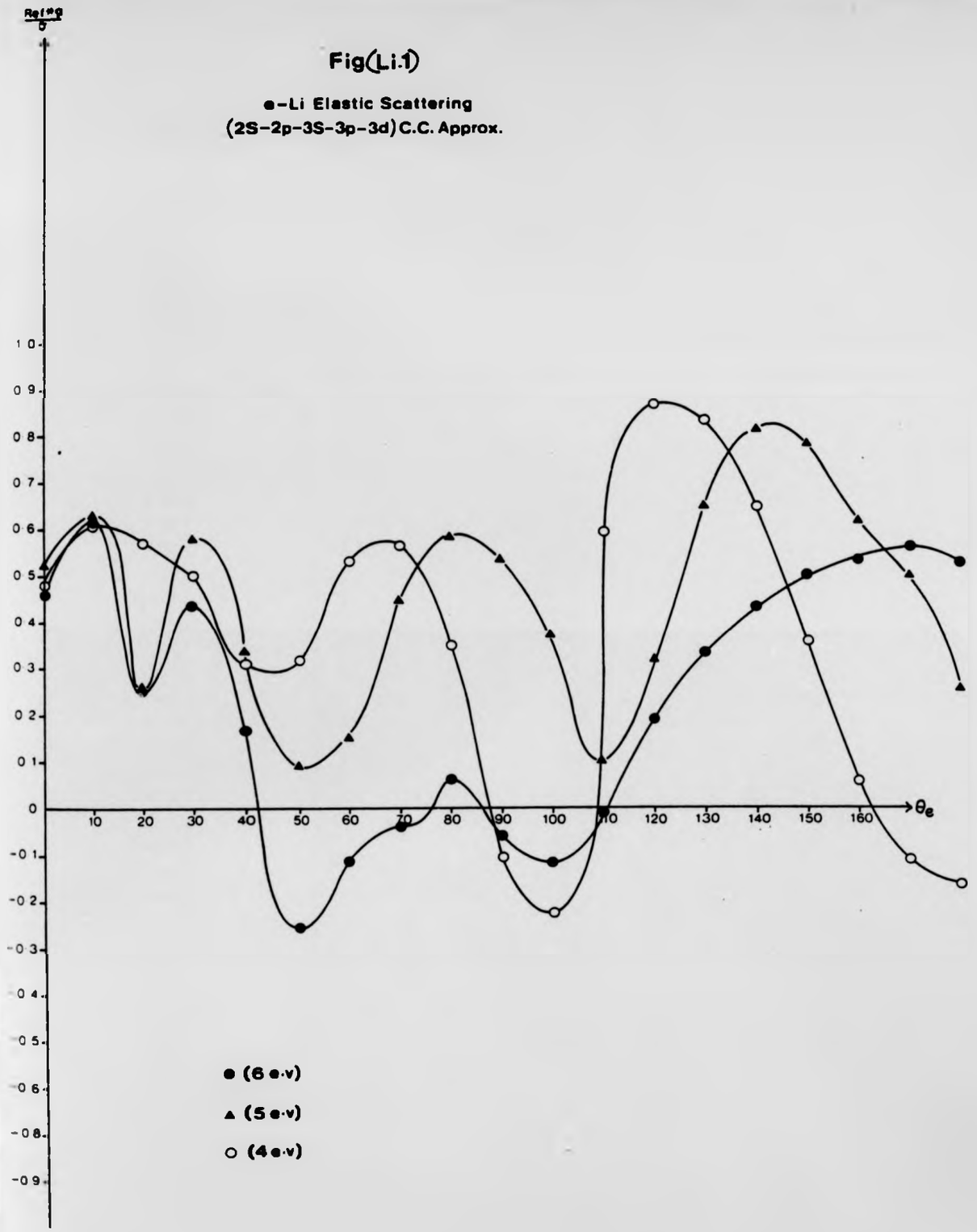

LINEAR POLARIZATION OF LYMAN-α IN ELECTRON CAPTURE PROCESS
Fig.5
$H^+ + H \rightarrow H(2P)^* + H^+$
$\theta_\gamma = 90°$
% LIN. POL. OF LYMAN-ALPHA
PROTON ENERGY (KEV)
12
10
8
6
4
2
0
-2
-4
-6
-8
20
40
60
80
100
120
140
160
180
200
THEORETICAL DATA FOR Q_0, Q_1 FROM
△ J McGUIRE (OBK APPROXIMATION)
▽ W FRITSCH & C D LIN (COUPLE STATE CALCULATION WITH 16AO+ BASIS.)
+ R SHAKESHAFT 1978 Phys Rev (A), 5, 18 (1930)

Fig (Li. 1)

e–Li elastic Scattering
(2S–2p–3S–3p–3d) C.C. Approx.




Ref*Q
5
Fig(Li.1)
e-Li Elastic Scattering
(2S-2p-3S-3p-3d) C.C. Approx.
1 0
0 9
0 8
0 7
0 6
0 5
0 4
0 3
0 2
0 1
0
-0 1
-0 2
-0 3
-0 4
-0 5
-0 6
-0 8
-0 9
10
20
30
40
50
60
70
80
90
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150
160
θe
● (6 e·v)
▲ (5 e·v)
○ (4 e·v)

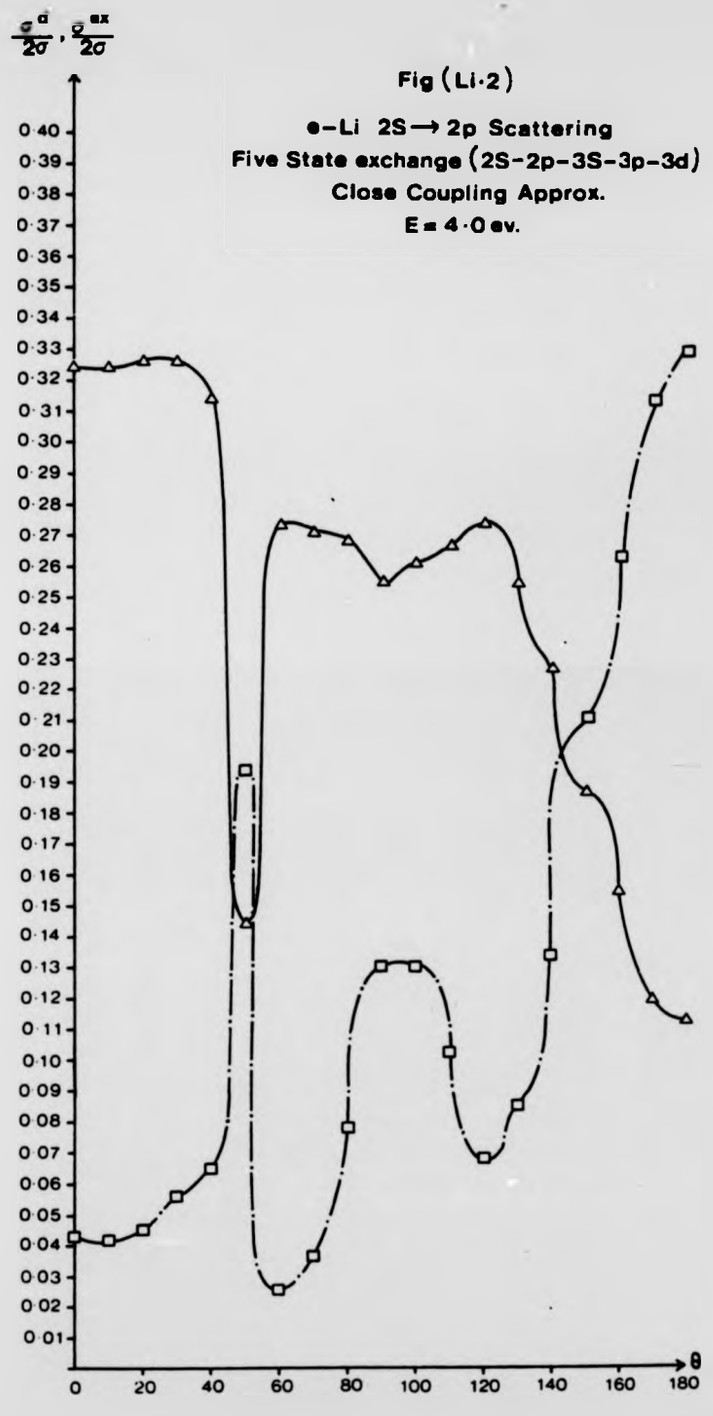
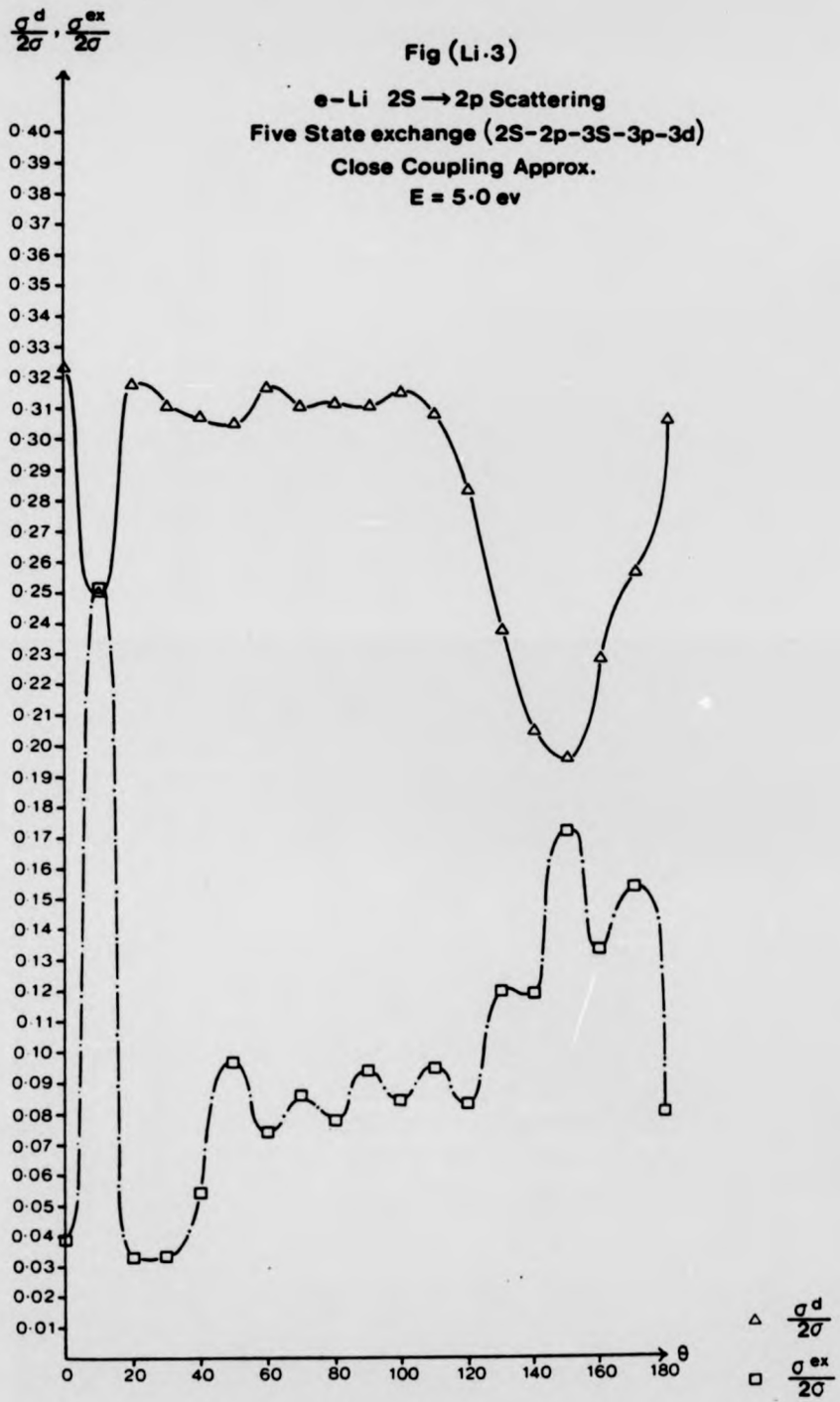


Fig (Li·2)

e–Li 2S → 2p Scattering
Five State exchange (2S–2p–3S–3p–3d)
Close Coupling Approx.
E = 4·0 ev.


$\frac{\sigma^d}{2\sigma}, \frac{\sigma^{ex}}{2\sigma}$
Fig (Li·3)
e–Li 2S → 2p Scattering
Five State exchange (2S–2p–3S–3p–3d)
Close Coupling Approx.
E = 5·0 ev
0·40
0·39
0·38
0·37
0·36
0·35
0·34
0·33
0·32
0·31
0·30
0·29
0·28
0·27
0·26
0·25
0·24
0·23
0·22
0·21
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0·15
0·14
0·13
0·12
0·11
0·10
0·09
0·08
0·07
0·06
0·05
0·04
0·03
0·02
0·01
0
20
40
60
80
100
120
140
160
180
θ
$\frac{\sigma^d}{2\sigma}$
$\frac{\sigma^{ex}}{2\sigma}$

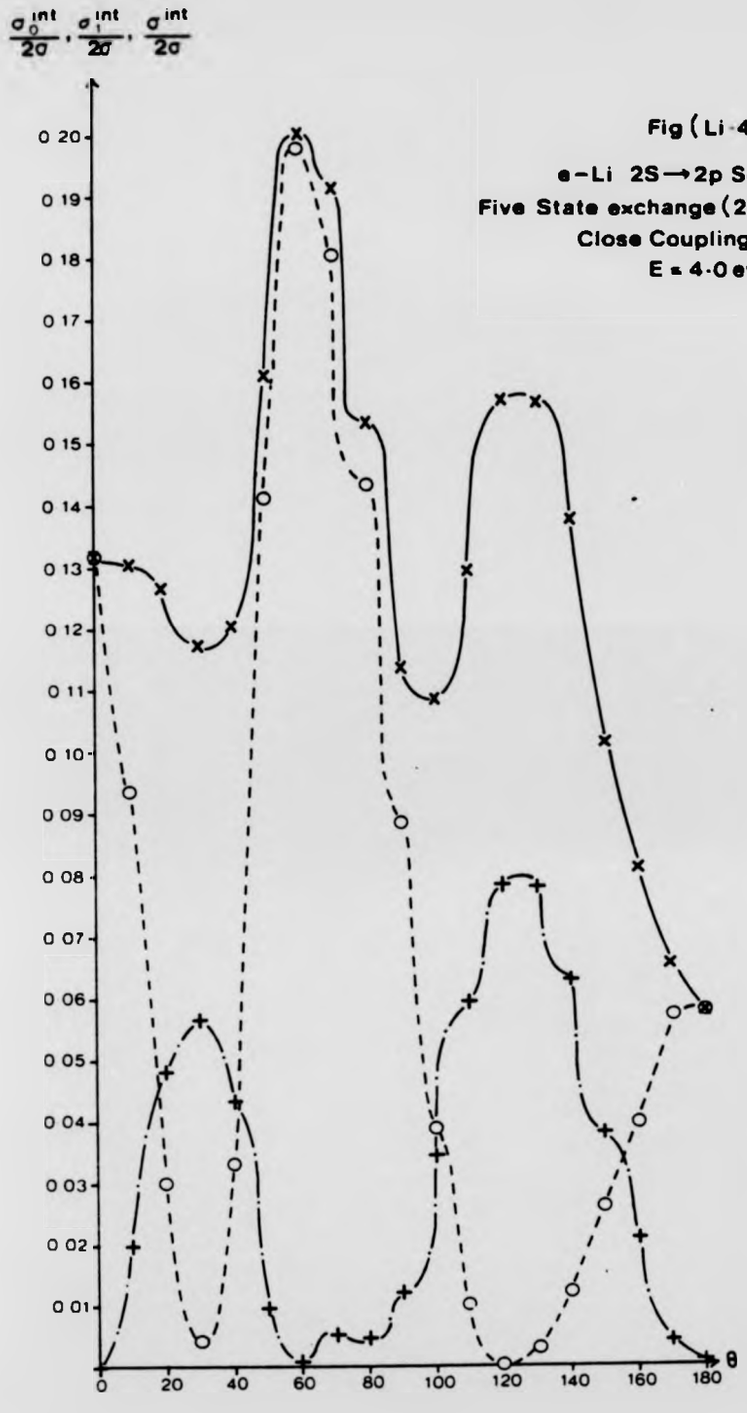


Fig (Li-4)

e–Li 2S → 2p Scattering
Five State exchange (2S-2p-3S-3p-3d)
Close Coupling Approx.
E = 4·0 ev.

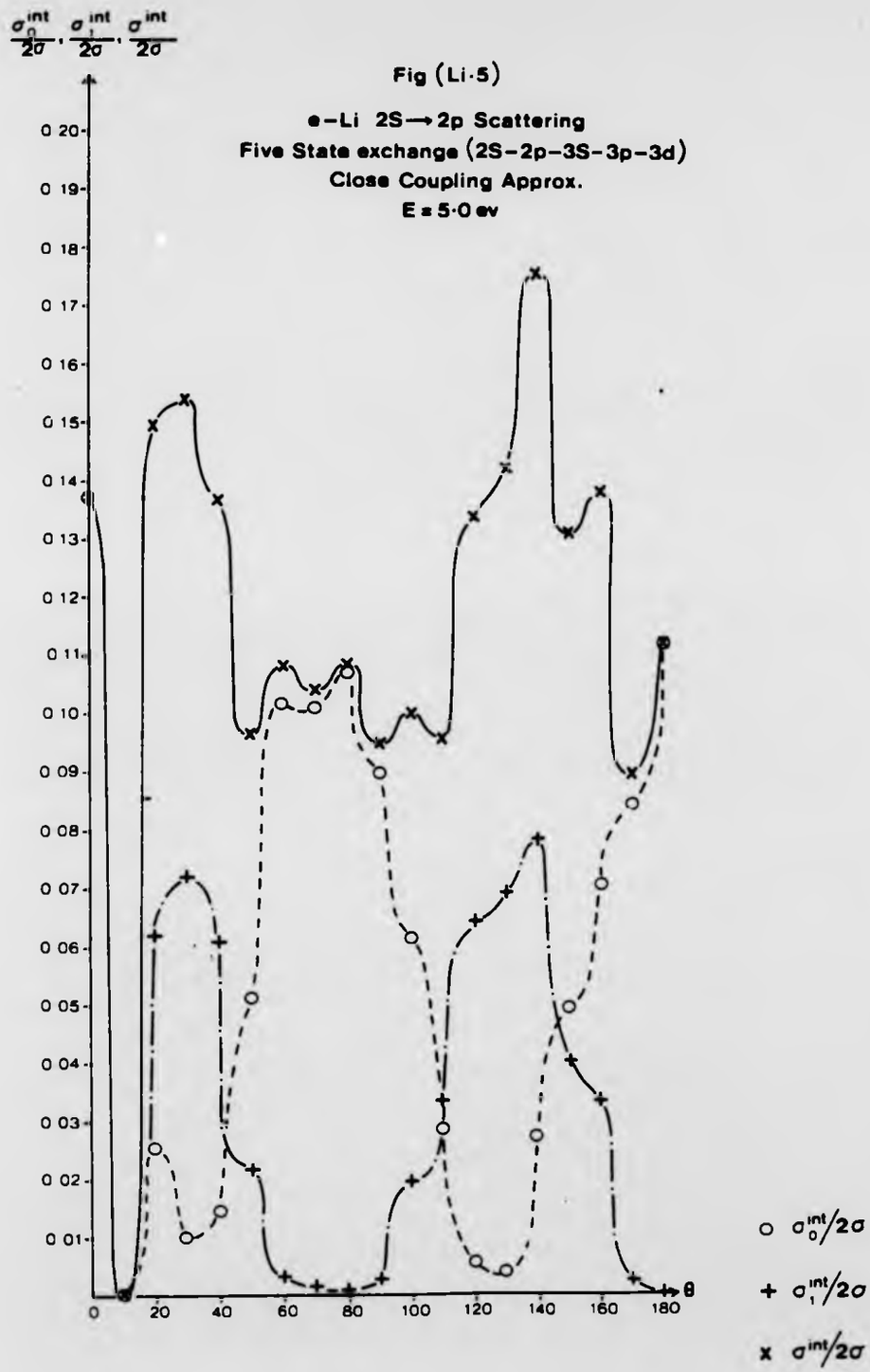
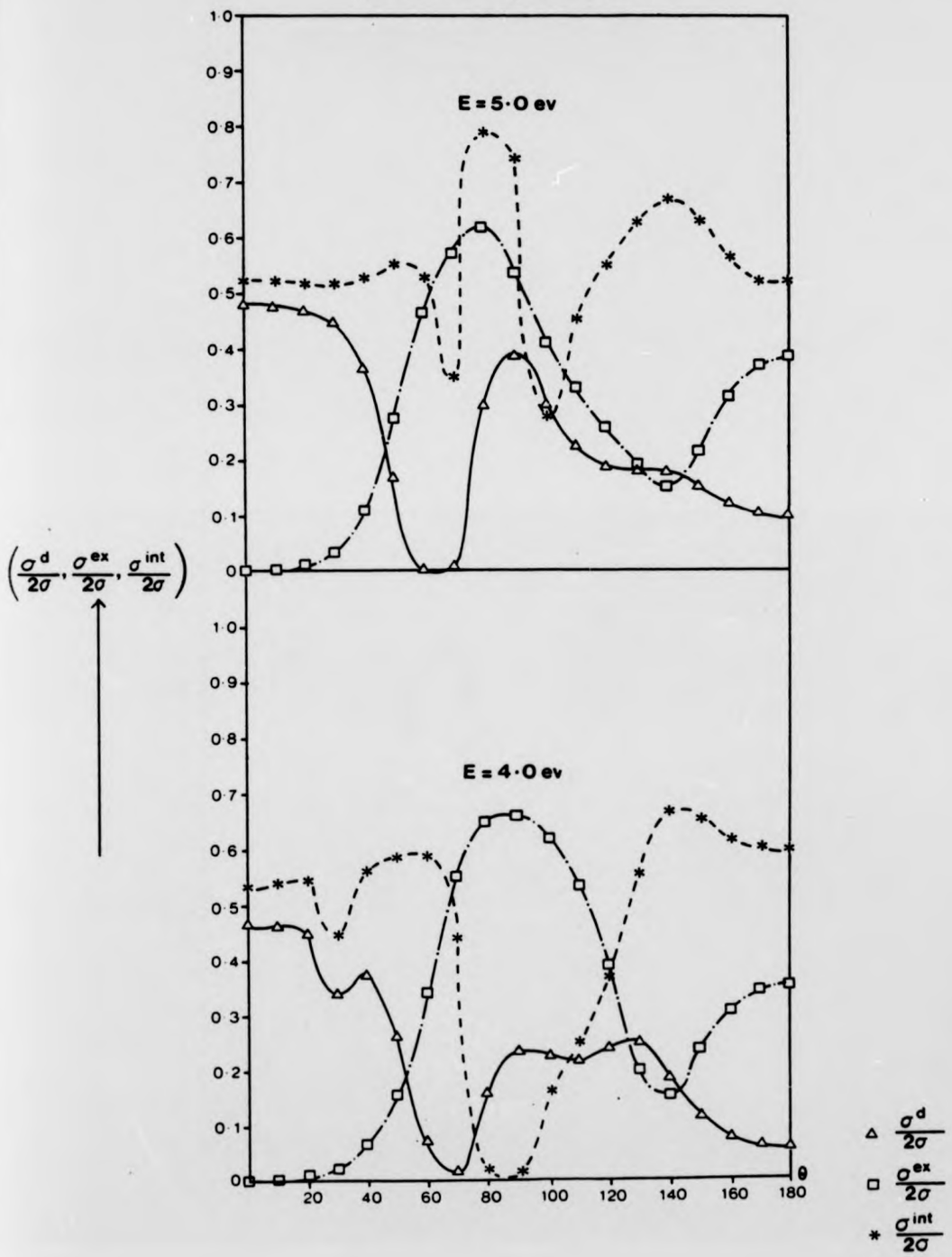

$\frac{\sigma_0^{int}}{2\sigma}, \frac{\sigma_1^{int}}{2\sigma}, \frac{\sigma^{int}}{2\sigma}$
Fig (Li·5)
e–Li 2S⟶2p Scattering
Five State exchange (2S–2p–3S–3p–3d)
Close Coupling Approx.
E = 5·0 ev
0 20
0 19
0 18
0 17
0 16
0 15
0 14
0 13
0 12
0 11
0 10
0 09
0 08
0 07
0 06
0 05
0 04
0 03
0 02
0 01
0
20
40
60
80
100
120
140
160
180
θ
○ $\sigma_0^{int}/2\sigma$
+ $\sigma_1^{int}/2\sigma$
x $\sigma^{int}/2\sigma$

Fig (Na·1)

e–Na elastic Scattering
(3S–3p–4S–3d) C.C. Approx.



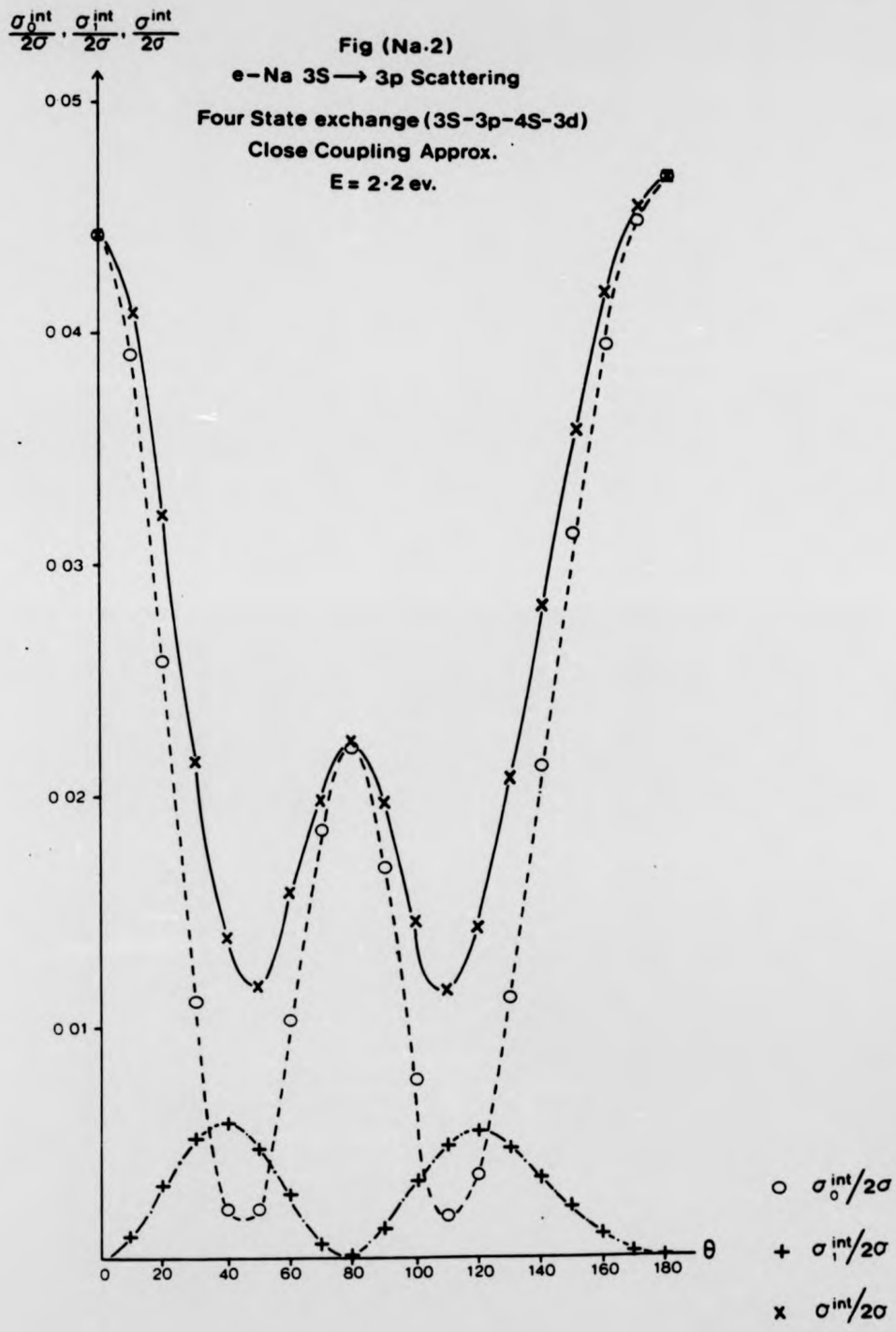

$\frac{\sigma_0^{int}}{2\sigma}, \frac{\sigma_1^{int}}{2\sigma}, \frac{\sigma^{int}}{2\sigma}$
Fig (Na·2)
e–Na 3S ⟶ 3p Scattering
Four State exchange (3S-3p-4S-3d)
Close Coupling Approx.
E = 2·2 ev.
0 05
0 04
0 03
0 02
0 01
0
20
40
60
80
100
120
140
160
180
θ
o $\sigma_0^{int}/2\sigma$
+ $\sigma_1^{int}/2\sigma$
x $\sigma^{int}/2\sigma$

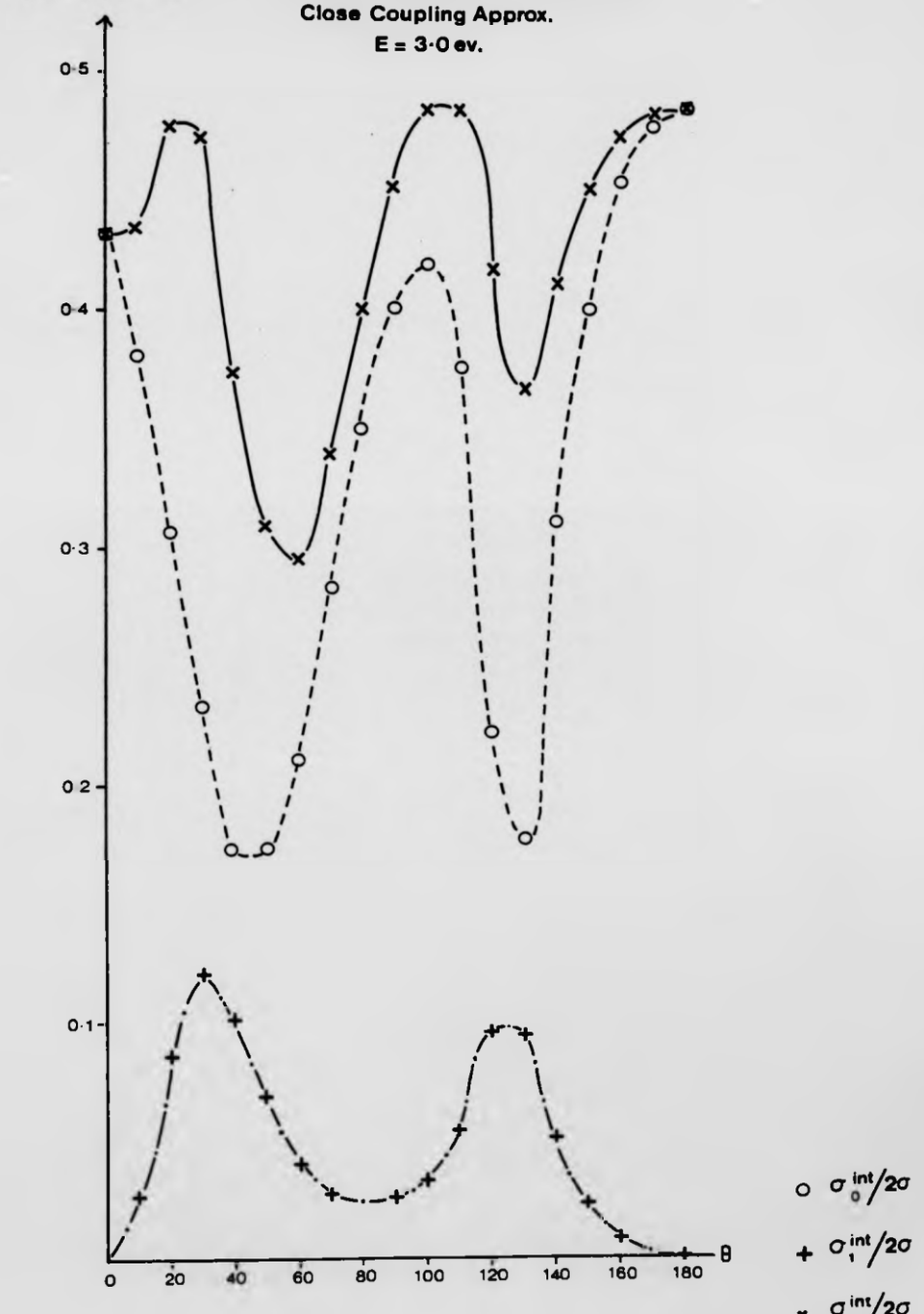
Fig (Na·3)

e–Na 3S⟶3p Scattering

Four State exchange (3S–3p–4S–3d)

Close Coupling Approx.

E = 3·0 ev.



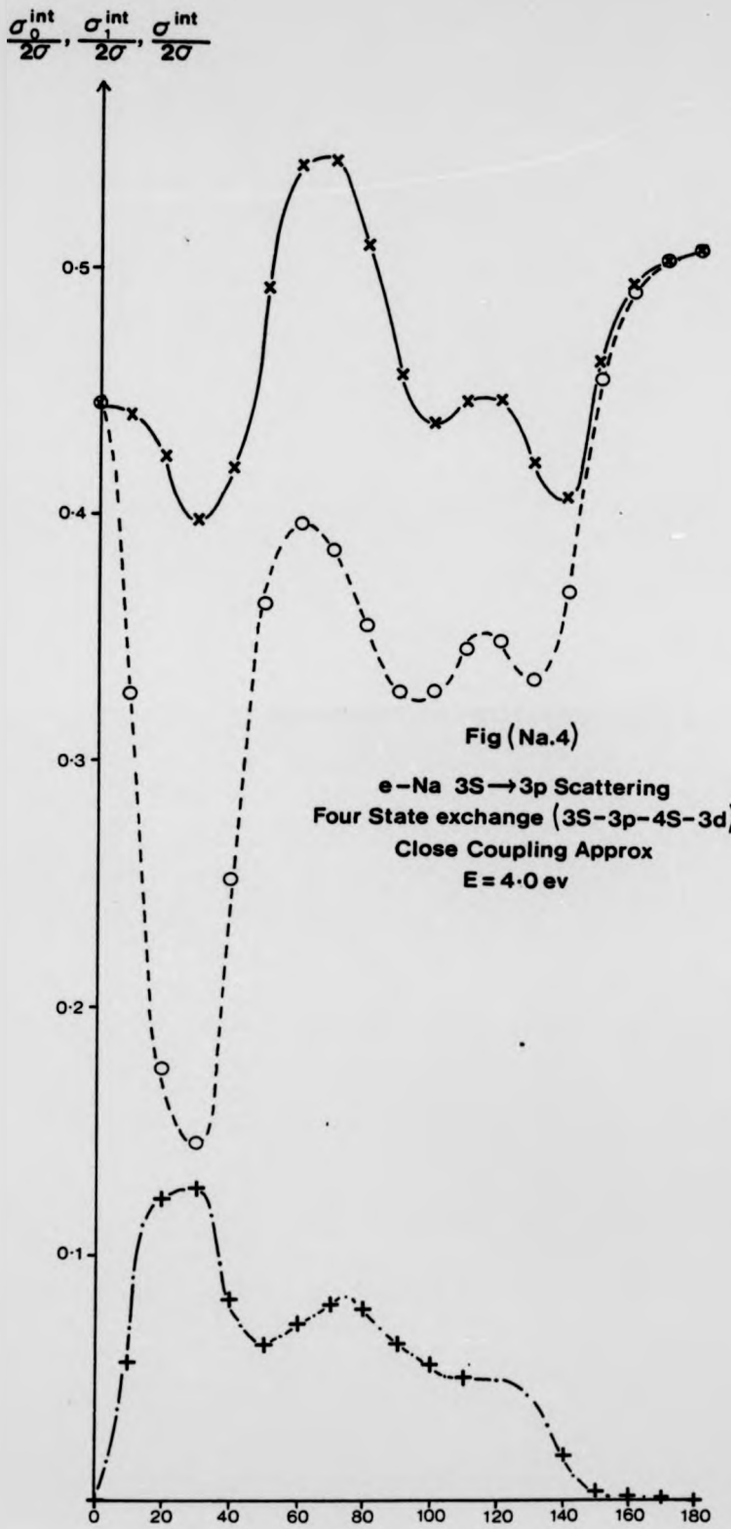


Fig (Na.4)

e–Na 3S⟶3p Scattering
Four State exchange (3S–3p–4S–3d)
Close Coupling Approx
E = 4·0 ev

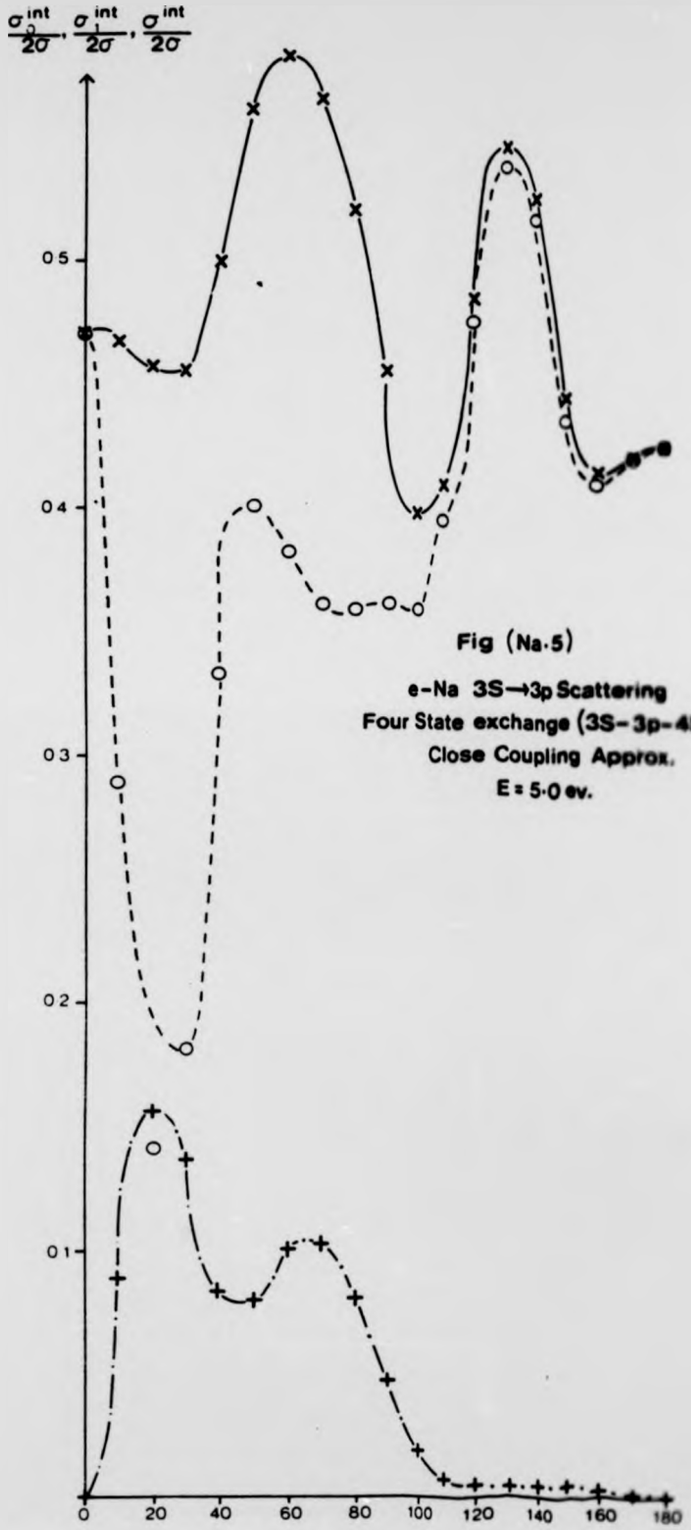


Fig (Na·5)

e-Na 3S→3p Scattering
Four State exchange (3S-3p-4S-3d)
Close Coupling Approx.
E = 5·0 ev.