# How Can We Diversify Portfolios?

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# Abstract

This thesis aims to provide new insights into how investors can diversify their portfolios by studying the benefits of different types of diversified portfolios, comparing the performance of various diversification strategies, and estimating the effect of U.S. monetary policy on portfolio diversification. The basic concept of portfolio diversification was proposed by American economist Harry Markowitz in his paper "Portfolio Selection" in 1952, which also laid the foundation for Modern Portfolio Theory. Thus, most investors understand the importance of diversification. However, due to the market's volatility and unpredictability, asset selection and allocation in portfolios have proven to be a challenging task. As a result, researchers and investors have consistently focused their research on this topic.

This thesis contains three complete empirical studies, presented in Chapters two, three, and four, respectively, each with different objectives. Chapter two explores how U.S. investors can benefit from various types of portfolio options, including a stock (60%)-bond (40%) portfolio, an internationally diversified portfolio, an asset-diversified portfolio, or only investing in U.S. stocks. In this chapter, we discover that since 2009, U.S. investors are less likely to benefit from an internationally diversified portfolio due to the strong performance of the S&P 500 index. Chapter three compares the performance of various portfolio diversification strategies, including the naive diversified strategy (1/N rule), market capitalisation-weighted strategy, risk parity (equally weighted risk contribution) strategy, mean-variance (MV) strategy, Black-Litterman (BL) strategy, and three types of the Parametric Portfolio Policy (PPP) diversified strategies. Out of these, the naive diversified strategy (1/N rule), the market capitalisation-weighted strategy, and the risk parity (equally weighted risk contribution) strategy are three

benchmarks, while the mean-variance (MV) strategy, the Black-Litterman (BL) strategy, and three types of the Parametric Portfolio Policy (PPP) diversified strategy are portfolio optimisation strategies. The mean-variance (MV) and Black-Litterman (BL) strategies consistently do better than the three benchmarks in terms of Sharpe ratio. The market capitalisation-weighted portfolio does better than the 1/N rule and risk parity portfolios among the three benchmarks. Chapter four investigates the impact of changes in the U.S. monetary policy on portfolio diversification. In this chapter, our results show that an unexpected Fed funds target rate cut (negative surprise) triggers an increase in the return of portfolios.

# Dedication

To my grandmother, who profoundly influenced me since my childhood. Throughout her life, she faced numerous challenges, but she overcame them with her perseverance and determination. When I was young, she told me that the only way to achieve any goal is through hard work and determination. Therefore, my motto is:

*Never be lazy and lost on the path to truth and career.*

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# Table of Content

# **Chapter One**: Introduction

Portfolio diversification plays a crucial role in modern investing markets, and helps investors minimise the overall risks of their investments without reducing their potential returns (Markowitz, 1952). However, some voices argue (*e.g.,* Holton, 2009; Fabozzi et al., 2014) that portfolio diversification fails when investors need it most, especially in the context of financial and economic crises. With the COVID-19 health crisis outbreak, portfolio diversification has once again been heatedly discussed by investors, policymakers, and researchers. During this pandemic, as the world moves into an uncertain phase with lockdown and economic shutoffs, financial markets have been reeling under huge pressure of higher volatility, with nearly a third of market capitalisation being wiped out during these times (Ali et al., 2020). Therefore, two significant questions are raised: First, does portfolio diversification truly fail when investors need it most? Second, do different crises have a different impact on the performance of portfolio diversification?

At present, the United States is one of the most developed economies in the world, and the stock market in the United States is also one of the most developed all over the world. Therefore, the third question arises: Is the international diversification necessary to U.S. investors?

When investors construct their portfolios, approaches and models they use to select assets and allocate the weight to each asset also matter. Brandt et al. (2009) propose a novel approach called the Parametric Portfolio Policy (PPP). Their research proves that this approach has a favourable performance in portfolio selection and asset allocation and prompting a fourth question: Does the Parametric Portfolio Policy (PPP) approach generate a superior in- and out-of-sample portfolio than other popular portfolio diversification approaches and models, such as the 1/N rule, market capitalisation weighted approach, risk parity (equally weighted risk contribution) model, mean-variance (MV) optimisation, and Black-Litterman (BL) approach?

Given the U.S. market's leadership role, any new information on the Fed's interest rate policy will have direct and indirect effects on the rest of the world's stock markets, other asset prices, and portfolios. Starting in the second half of 2008, the United States maintained low interest rates for over a decade until the end of 2021 to boost the economy that brings yet another question to consider. How does the U.S. monetary policy influence the performance of portfolio diversification?

With these five questions, this thesis studies the benefits of portfolio diversification with various types of portfolio options across three major crises (Dot-com bursting, Great Recession, and COVID-19), compares the benefits of portfolio diversification for different diversification strategies, and estimates the effect of U.S. monetary policy on the performance of portfolio diversification.

The following sections of this introduction specify the rationale for the research, present the historical background of portfolio diversification, categorise types of diversification, provide quantitative methodologies and instruments for portfolio diversification, discuss factors affecting the performance of portfolio diversification, introduce the structure of this thesis, and outline three empirical chapters, two, three, and four, along with the contributions.

## 1.1. Rationale for the Research

Portfolio diversification is defined as a strategy that spreads investments across various financial assets to reduce exposure to any single asset, aiming to minimise the overall risks in the investment portfolio without reducing potential returns (Markowitz, 1952). It is one of the main components of investment decision-making under volatility or uncertainty. The idea of diversifying investment can be traced back to the 18th and 19th centuries. At that time, investors recognised that the investment risk can be mitigated by holding a variety of assets (Kindleberger, 1986), but there was no systematic theory that could guide investors to diversify their investments until 1952, when the American economist, Harry Markowitz, built Modern Portfolio Theory (MPT) in his thesis "Portfolio Selection," which is a significant milestone in the field of portfolio diversification. Modern Portfolio Theory (MPT) is a practical method that helps investors allocate their wealth among alternative assets while maximising their overall return within an acceptable level of risk (Elton and Gruber, 1997).

A vital component of Modern Portfolio Theory (MPT) is diversification. Markowitz (1952) states that the basic principle of diversification is that a portfolio's total risk can be reduced by holding a variety of assets that are not perfectly correlated. Theoretically, he highlights that the correlation between asset returns is significant when diversifying portfolios and believes that the combination of assets with low or negative correlations might mitigate total portfolio risk (Markowitz, 1952). With the development of the Modern Portfolio Theory, most investors have come to understand the importance of diversifying their investments, but so far, the question of how to allocate available assets when they construct portfolios has consistently been a topic of research for both researchers and investors.

There are several underlying rationales why portfolio diversification has maintained its critical significance in both academia and the financial sector since Markowitz (1952) published Modern Portfolio Theory:

1) Financial markets are characterised by uncertainty and unpredictability and researching portfolio diversification helps investors manage risks associated with market volatility. The uncertainty and unpredictability in the financial market have a significant effect on portfolio diversification in the following ways. First, with the market conditions evolving, the correlation between stock markets or other assets is changing, which increases the risk of volatility in portfolio performance. Goetzmann et al. (2001) examine the major world equity markets and find that correlations vary considerably over time, and, because of the time-varying nature of correlations, diversification benefits are also time-varying. Markowitz (1952) points out that the combination of assets with low or negative correlations might mitigate total portfolio risk. However, some literature (*e.g.,* Roll, 1988; Bertero and Mayer, 1990; King and Wadhwani, 1990; Solnik et al., 1996; Butler and Joaquin, 2002; Guidi and Ugur, 2014) find that the correlation between stock markets in crisis periods is higher than that in non-crisis periods. Moreover, empirical evidence (e.g., Longin and Solnik, 1995; Driessen and Laeven, 2007; Koch and Koch, 1991) shows that the benefits of international investment portfolio diversification are declining because of the increasing correlation of national stock markets.

Second, with the market conditions evolving, the risk characteristics of certain assets may shift, which affects their role in a diversified portfolio. Ang and Bekaert (2002) discuss how regime shifts in economic conditions can change the risk and return profiles of assets, thus affecting diversification strategies. They find the high volatility regime mostly induces a switch towards the lower volatility assets, which are cash (if available), U.S. equity, and German equity if available.

Third, given the unpredictability of financial markets, investors may need to adopt more dynamic and tactical asset allocation strategies rather than static ones, which might increase the transaction costs. French (2008) finds when investors adopt a dynamic strategy, like re-balancing daily or monthly, the transaction cost is a main consideration for investors since the transaction costs may negate the advantages of diversification. He also finds that, under reasonable hypotheses, if investors switched their dynamic portfolio strategy to the passive portfolio strategy, their average annual return could rise by 0.67% between 1980 and 2006.

2) The emergence of new asset classes such as financialised commodities (such as gold, crude oil, copper, and aluminium), crypto-assets, and alternative investments necessitates ongoing research to understand their impact on portfolio diversification. The financialisaton of commodities refers to the transformation of commodities from physical goods to financial assets as financial markets and investors become more involved. The financialisaton process of commodities began significantly around 2004 (Bicchetti and Maystre, 2013). During this period, financial investors poured into the commodity market, leading to a significant increase in the financial attributes of commodities. Financial markets and instruments began to play a significant role in determining commodity prices, overshadowing traditional economic factors like supply and demand (Cheng and Xiong, 2014). The financialisaton of commodities has significant implications for portfolio diversification. Commodities have traditionally been seen as a hedge against inflation and a source of diversification due to their low correlation with traditional asset classes like stocks and bonds (Gorton and Rouwenhorst, 2006). The inclusion

of commodities in a portfolio can reduce overall risk and improve returns by providing exposure to different market dynamics. However, the financialisaton process has altered these diversification benefits. The increasing correlation between commodities and financial markets has raised questions about their role as diversifiers. According to Modern Portfolio Theory, if the correlation between assets is low, investors can gain diversification benefits by spreading their investment between them (Markowitz, 1952; Levy and Sarnat, 1970; Brown and Kapadia, 2007). Studies like those by Tang and Xiong (2012) highlight how increased financial investor presence can lead to greater price volatility and correlation with equities, potentially diminishing the diversification advantages. Recently, the potential de-financialisaton and shifts in market dynamics might restore some traditional diversification benefits as commodity prices become less influenced by financial markets and more by fundamental supply-demand factors (Bianchi et al; Natoli, 2021.).

Crypto assets, such as bitcoins, offer several potential benefits as innovative and efficient payment system and portfolio diversification. Guesmi et al. (2019) find that hedging strategies incorporating gold, oil, equities, and Bitcoin significantly lower the portfolio's risk compared to a portfolio consisting solely of gold, oil, and equities. However, at the same time, they are the source of potential risks that could harm investors, consumers, businesses, financial systems and even the national security.

3) The prevailing shift in behavioural insights has a significant influence on investment decisions and portfolio construction (Barberis and Thaler, 2003). Behavioural finance reveals that psychological biases, like herding behaviour, loss aversion, and home bias, significantly impact investors' decision-making processes, often leading to sub-optimal diversified

portfolios (A sub-optimal portfolio is one that fails to achieve the best risk-return balance for a given risk level or fails to effectively meet the investor's investment objectives).

Herding behaviour is the behaviour of investors who tend to follow other investors to make investment decisions without conducting a fundamental and independent analysis first (Bikhchandani and Sharma, 2000; Fityani and Arfinto, 2015). This bias can result in a lack of diversification as investors flock to popular stocks or trends, ignoring independent analysis and the benefits of spreading investments across various asset classes.

The concept of loss aversion is introduced by Daniel Kahneman and Amos Tversky in 1979 (Kahneman and Tversky, 1979). Loss aversion denotes the tendency for people who prefer avoiding losses rather than acquiring equivalent gains (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992; Kahneman, 2011). Loss-aversion can significantly influence portfolio diversification in several ways. First, investors with loss aversion show a tendency to avoid assets that they perceive as risky, which can lead to a lack of diversification (Kahneman and Tversky, 1979). Second, loss-aversion investors may exhibit a tendency to hold onto losing investments due to the pain of realising a loss, leading to an under-diversified portfolio. This behaviour is further explored in the context of myopic loss aversion, where investors prioritise short-term losses over long-term gains. Third, loss aversion contributes to the disposition effect, where investors are more likely to sell winning investments quickly while holding onto losing ones and this can result in a concentrated portfolio that lacks proper diversification (Shefrin, 2000; Shefrin, 2002). Forth, loss-averse investors may engage in herd behaviours, leading to panic selling during market downturns and this behaviour disrupts a diversified portfolio as investors may sell off diverse holdings to avoid perceived losses (Baker and Nofsinger, 2002).

Home bias is indeed a significant psychological bias that affects portfolio diversification and investment behaviour. It refers to the tendency for investors to favour domestic assets over foreign ones, often leading to a lack of proper diversification in their portfolios. A concentrated portfolio, overly exposed to the domestic economy and specific market risks, can result from home bias. Investors overlooking international assets may miss opportunities for higher returns and risk reduction that come from diversifying across different geographical regions. Home bias has consistently been observed in most countries. The seminal paper by French and Poterba (1991) finds that U.S. investors held a significantly larger proportion of their portfolios in domestic stocks compared to foreign stocks, indicating a clear home bias; they also find that the home bias has decreased over the last few decades due to increased financial globalization. Financial globalisation refers to the increasing integration of financial markets across the globe, characterised by the cross-border flow of capital, investments, and financial services.

The relationship between financial globalisation and home bias is complex and has evolved over time, influenced by historical, economic, and psychological factors. After World War II, many countries adopted protectionist policies and capital controls, limiting cross-border investments. As a result, investors primarily focused on domestic markets. Especially, the Bretton Woods system established fixed exchange rates and restricted capital mobility, reinforcing home bias (Eichengreen, 2019). In the 1980s and 1990s, there was a wave of financial liberalisation and deregulation, with many countries removing capital controls and opening their markets. This period marked the beginning of significant financial globalisation. However, despite increased opportunities for international investment, home bias remained prevalent. Investors often feel more comfortable investing in familiar domestic markets due to perceived risks and information asymmetries associated with foreign investments (French and Poterba, 1991). Investors often possess more information about domestic markets, leading to a preference for local investments (Ahearne et al., 2004). Transaction costs are also a primary consideration for investors in the addiction domain. Although financial globalisation has reduced some barriers to international investment, transaction costs, including fees and taxes, still play a role in perpetuating home bias. Investors may find it more cost-effective to invest domestically, despite the potential benefits of diversification (Bekaert and Harvey, 2000). The early 2000s witnessed a surge in investments in emerging markets as globalisation facilitated access to diverse asset classes. However, the global financial crisis of 2007-2008 led to a re-evaluation of investment strategies. During this period, many investors returned to home markets as a response to heightened uncertainty and perceived risks in foreign investments (Gourinchas and Rey, 2007). Since 2009, U.S. investors have shown an increased interest in investing abroad, although this trend has evolved over time and varies by asset class and market conditions. Certainly, in the aftermath of the 2007-2008 financial crisis, the U.S. Federal Reserve implemented a prolonged period of low interest rates to stimulate the economy, which facilitated investors to seek higher yields abroad as domestic investments often offered lower returns (Kuttner, 2014). However, the home bias remains prevalent in most countries (Coeurdacier and Rey, 2013; Mishra, 2015; Hu, 2020). For example, Hu (2020) finds that the historical foreign ownership shares of the US and China relative to the optimal portfolio are roughly 33% and 5%, respectively.

4) The emergence of new portfolio diversification strategies, such as the Parametric Portfolio Policy by Brandt et al. (2009), and technologies such as big data analytics and machine learning are reshaping portfolio management. The Parametric Portfolio Policy (PPP) approach parameterises the asset weights as a function of their characteristics, thereby estimating those parameters in a way that maximises the investor's average utility. Big data analytics is defined as the process of analysing massive amounts of data to uncover hidden patterns, unknown

relationships, market trends, customer preferences, and other valuable insights, and it is an essential tool for investors to make high-stakes investment decisions (Boubaker et al., 2023). To succeed in today's data-driven world, investors will have a greater chance of success only if they have superior analytical skills, the ability to manage and interpret massive data sets, and the ability to evaluate and effectively implement insights (Monino, 2021). Machine Learning (ML) is attracting considerable attention among academics and financial markets. Gu et al. (2020) highlight the potential of machine learning to improve asset allocation and diversification strategies, thereby optimising risk-return profiles.

5) The financial regulatory landscape is constantly evolving, which influences how investors make diversification decisions. Research by Allen and Carletti (2013) discusses how regulatory changes can influence the structure and risk exposure of diversified portfolios.

6) Global economic dynamics, like globalisation and the interconnection between markets, necessitate a thorough understanding of international diversification. As noted by some literature (*e.g.,* Longin and Solnik, 1995; Forbes and Rigobon, 2002; Kim et al., 2005; Morana and Beltratti, 2008; Christoffersen et al., 2014), correlations between national stock markets have been increasing in recent years. Moreover, some empirical work (*e.g.,* Roll, 1988; Bertero and Mayer, 1990; King and Wadhwani, 1990; Solnik et al., 1996; Butler and Joaquin, 2002; Guidi and Ugur, 2014) find that the correlation between stock markets in crisis periods is higher than in non-crisis periods.

7) It is important to provide basic and updated financial education (education needs) on effective diversification strategies that allow investors to better navigate complex financial

markets, especially economic crises. Research by Lusardi and Mitchell (2014) highlights the importance of financial literacy in making informed investment decisions.

Overall, due to the never-ending market fluctuations caused by various factors, the emergence of new asset types, the continuous emergence of new portfolio strategies, and so on, conducting portfolio diversification research is essential to adapt to the complexity of modern financial markets, understand investor behaviours, and integrate new investment opportunities.

### 1.2. Historical Background

The concept of portfolio diversification has a rich historical background that reflects the evolution of investment strategies and financial theories. Below is an overview of key milestones in the history of portfolio diversification. The notion of diversifying investments across different assets can be traced back to early financial practices. In the 18th and 19th centuries, investors recognised that by spreading their investment in a variety of assets could mitigate risk. Notably, the Dutch East India Company provided a model for diversification in its financing practices by investing in various trade ventures (Kindleberger, 1986).

The formalisation of portfolio diversification is largely credited to Harry Markowitz, an American economist and Nobel Prize winner, who introduced the theory of portfolio selection in his seminal paper “Portfolio Selection” in 1952 (Markowitz, 1952). He demonstrated that investors could construct portfolios that optimise expected returns for a given level of risk by diversifying across uncorrelated assets. The theory he proposed in his research later formed the basis for Modern Portfolio Theory (MPT). Markowitz's concept has been developed by different researchers.

Building on MPT, the Capital Asset Pricing Model (CAPM) was developed in the 1960s by Sharpe (1964), Treynor (1962), Lintner (1965a, b) and Mossin (1966), providing a framework for understanding the relationship between risk and expected return. CAPM emphasises the importance of diversification in reducing unsystematic risk and highlighted the role of systematic risk in asset pricing (Perold, 2004). In the 1980s, Eugene Fama and Kenneth French expanded on CAPM by introducing multi-factor models that included additional risk factors beyond market risk, such as size and value (Fama and French, 1993). Their work reinforced the idea that diversified portfolios could achieve better risk-adjusted returns. During the same period, research in behavioural finance began to explore how psychological biases affect investment decisions and risk perceptions (Barberis and Thaler, 2003). This body of work highlighted the fact that, despite the theoretical benefits of diversification, many investors still concentrated their portfolios due to biases such as overconfidence and familiarity.

After the year 2000, the rise of technology has transformed portfolio management and diversification strategies. Advanced data analytics and algorithmic trading systems allow investors to analyse vast amounts of data to continually optimise their portfolios. These tools enhance the ability to identify diversification opportunities across global markets (Lo, 2007). The global financial crisis (2007-2008) revealed vulnerabilities in diversified portfolios, particularly those heavily invested in correlated asset classes, such as mortgage-backed securities (Acharya and Richardson, 2009). This led to a re-evaluation of traditional diversification strategies and a renewed focus on risk management and asset correlation. Since 2010, the growing interest in Environmental, Social, and Governance (ESG) investing has prompted investors to diversify their portfolios based on sustainability criteria. Research indicates that incorporating ESG factors can enhance long-term performance while promoting responsible investing (Friede et al., 2015).

Conclusively, the historical background of portfolio diversification showcases its evolution from early investment practices to sophisticated modern theories. The contributions of key figures, the emergence of new investment paradigms, and the impacts of technological advancements and behavioural insights have all shaped the understanding and application of diversification in investment strategies today. As markets continue to evolve, ongoing research and adaptation will be essential for optimising portfolio diversification.

### 1.3 Types of Diversification

Portfolio diversification is a fundamental strategy used by investors to manage risk and enhance returns. Rational, risk-averse investors realise that not all investments simultaneously perform well (indeed, some may never perform well). Moreover, since no one can accurately predict which investments will perform and which will not, investors can minimise investment risk by spreading their investments across a broad range of assets to form a diversified portfolio. Four broad aspects of portfolio diversification can be considered, and each serves different purposes and offers unique benefits:

1) Cross-asset diversification: Investors should spread their funds across different asset classes, such as equities, fixed income (bonds), funds, real estate, commodities, cash equivalents and alternative assets (such as hedge funds, private equity, or cryptocurrencies) and so forth. The idea is that different asset classes react differently to market conditions, which can help investors reduce overall portfolio volatility (*e.g.,* McDonald and Solnik, 1977; Lean and Wong,

2015; Guesmi et al., 2019). This can include diversification across assets of different maturities. Different types of assets have different maturities. Investors can arrange the maturity structure of their investments to achieve a high degree of uniformity in profitability, liquidity, and risk (*e.g.,* Levy and Lerman, 1988; Hatemi-J and Roca, 2006; Guidi and Ugur, 2014).

2) Cross-geography diversification: Investors can diversify their portfolios by investing in assets from different geographic regions or countries (*e.g.,* Solnik, 1974; Levy and Lerman, 1988; Hatemi-J and Roca, 2006; Guidi and Ugur, 2014). Different global regions have different economic conditions, so the degree of investment risk is also different. Investors should diversify their investments in different countries and regions to avoid major losses due to the deterioration of the political and economic environment in a certain region. This can also include diversification across different currencies. Different currencies can help mitigate risks related to currency fluctuations (Jorion, 1991). This strategy is particularly relevant for investors with exposure to global markets.

3) Cross-sector diversification: This involves spreading investments across various sectors of the economy, such as technology, healthcare, finance, consumer goods, and energy (Fama and French, 1993). By diversifying across sectors, investors can reduce the impact of a downturn in any single sector on their overall portfolio. When the price or interest rate of securities in one sector falls, the price or interest rate of securities in another sector rises, or vice versa (*e.g.,* Meric and Meric, 1989; Moerman, 2008; Balli et al., 2013). The rise and fall of various securities in the securities portfolio offset each other, reducing the risk of the securities portfolio.

4) Investment style diversification: Investors can diversify based on investment styles, such as growth vs. value investing or large-cap vs. small-cap stocks (Fama and French, 1992). This type of diversification acknowledges that different styles can perform better under varying market conditions.

Overall, each type of portfolio diversification serves to enhance risk management and maximise returns by spreading investments across various dimensions, but how investors decide the best diversification strategy for their portfolios should consider their individual risk tolerance, investment goals, and market conditions.

### 1.4. Quantitative Approaches and Models

Quantitative approaches and models to portfolio diversification involve the use of mathematical and statistical models to analyse risk and return, optimise asset allocation, and enhance overall investment performance. Below are the main approaches and models commonly used in quantitative portfolio diversification:

1) 1/N Rule: It is a well-known investment strategy (DeMiguel et al., 2009a) that allocates an equal proportion of the investment budget to each available asset. We also refer to it as the Equally Weighted Portfolio (EWP). Usually, we use the 1/N rule or equally weighted portfolio (EWP) as a benchmark (Bessler et al., 2017; Hsu et al., 2018). This rule is unique in that it completely ignores historical information and assigns time-invariant portfolio weights.

2) Mean-Variance Optimisation (MVO): Harry Markowitz developed the mean-variance portfolio optimisation framework in the 1950s (Markowitz, 1952), which has gained

widespread use in academic literature as one of the most popular portfolio optimisation approaches. Mean-variance optimisation is a method of portfolio optimisation based on Modern Portfolio Theory (MPT), and it seeks to construct portfolios that maximise the expected return for a given level of risk. This is done by calculating the expected return and volatility of each asset class or security and using these estimates to construct portfolios that maximise returns while minimising risk (Markowitz, 1952; Kim et al., 2021).

3) Capital Asset Pricing Model (CAPM): The CAPM is built on the model of portfolio selection developed by Harry Markowitz (1959) and is used to determine the expected return on an asset based on its systematic risk, measured by beta (Sharpe, 1964). It helps investors understand the trade-off between risk and expected return, aiding in asset selection for diversification.

4) Multi-Factor Models: Multi-factor models extend the CAPM by incorporating multiple factors that may affect asset returns, such as size, value, momentum, and macroeconomic indicators. The Fama-French three-factor model (Fama and French, 1993) and the Fama-French five-factor model (Fama and French, 2015) are popular examples.

5) Black-Litterman Model: The Black-Litterman (BL) model is one of the prevailing portfolio optimisation models out there, which is developed by Black and Litterman (1992), it combines Capital Asset Pricing Theory (CAPM) with Bayesian statistics and Markowitz's modern portfolio theory (Mean-Variance Optimisation) to produce efficient estimates of the portfolio weights (Bessler et al., 2017). The model starts with an investor's views on the expected returns of different asset classes or securities and then uses these views to construct portfolios that

maximise expected returns while minimising risk (Black and Litterman, 1992; Bessler et al., 2017). This model is particularly useful for investors who have strong views on the expected performance of specific asset classes or securities.

6) Risk Parity Approach: Risk Parity is an investment technique that has garnered considerable attention recently. It seeks to allocate portfolio risk equally among various asset classes or securities rather than capital (Costa and Kwon, 2020; Maillard et al., 2010; Fabozzi et al., 2021). Under this approach, investments are allocated according to the volatility or risk of each asset class or security, rather than their expected return. A significant advantage of Risk Parity weighting compared to mean-variance optimisation is that investors do not need to formulate expected return assumptions for portfolio construction (Kolm et al., 2014; Fabozzi et al., 2021).

7) Monte Carlo Simulation: Monte Carlo simulation is a statistical technique used to model the probability of different outcomes in a portfolio (Glasserman, 2004). It helps investors assess the impact of various scenarios on portfolio performance, allowing for better risk management and diversification decisions.

8) Market Capitalisation-weighted Approach: It is a type of investment approach in which the allocation to each asset is proportional to its market capitalisation (Bodie et al., 2014). This approach is commonly used in constructing indices and mutual funds, reflecting the relative size of companies in the market. For instance, if a company has a market capitalisation of $100 billion and the total market capitalisation of all companies in the portfolio is $1 trillion, the company's weight in the portfolio would be 10%.

9) Parametric Portfolio Policy (PPP): Brandt and Santa-Clara (2006) first propose the characteristic portfolio approach for portfolio optimisation. Brandt et al. (2009) further improve and clarified this strategy, naming it the Parametric Portfolio Policy. This approach parameterises the portfolio's weights of each asset as a function of the asset's characteristics and then maximises the investor's average utility by choosing optimally the coefficients of this function. The advantages of this approach include its ease of implementation, its good in-and out-of-sample performance, and its ability to incorporate some of the methods used to optimise the Markowitz model, such as portfolio constraints, shrinkage estimates, and combining investors' prior beliefs with return history data (Barroso and Santa-Clara, 2015; Fletcher, 2017; Joenväärä et al., 2021).

Overall, quantitative approaches and models for portfolio diversification provide investors with robust tools to analyse risk, optimise asset allocations, and enhance return potential. Each approach or model has its advantages and disadvantages. By utilising these methodologies while understanding their pros and cons, investors can make more informed decisions and build a portfolio that matches their risk tolerance and investment objectives.

### 1.5. Factors Affecting the Performance of Portfolio Diversification

The performance of portfolio diversification is influenced by various factors that can impact both the risk and return of diversification portfolios. Understanding these factors is essential for investors seeking to optimise their diversification strategies. Here are key factors affecting the performance of portfolio diversification:

1) Correlation between assets: Correlation is a statistical measure of how two or more variables are related to one another. The degree to which asset returns move in relation to one another is crucial for effective portfolio diversification (Elton and Gruber, 1997). Low or negative correlations between assets can enhance diversification benefits, as they reduce overall portfolio risk (Markowitz, 1952). There is now considerable evidence that the correlation of returns between assets has varied substantially over time. For instance, Goetzmann et al. (2001) examine the major world equity markets and find that correlations vary considerably through time, and, because of the time-varying nature of correlations, diversification benefits are also time-varying. Evidence also indicates that inter-asset correlations typically rise during financial crises and, more broadly, in bear markets (*e.g.,* Longin and Solnik, 2001; Ang and Bekaert, 2002; Ang et al., 2006).

2) Asset allocation: The distribution of investments across various asset classes (*e.g.,* stocks, bonds, real estate), across geographies, or industries and so on is fundamental to diversification. Strategic asset allocation determines the overall risk-return profile of the portfolio (Ibbotson and Kaplan, 2000). Studies (*e.g.,* Markowitz, 1952; Brinson et al., 1986; Malkiel, 2003; Fama and French, 2010) show that asset allocation is one of the most significant determinants of a portfolio's performance, often explaining a substantial portion of its returns.

3) Economic and market conditions: Economic and market conditions, such as interest rates, inflation, and overall economic growth, influence asset performance, correlations, and investor behaviours (Ang and Bekaert, 2002). Different market environments can impact the effectiveness of diversification strategies. For example, during market downturns, correlations between asset classes may increase, reducing the effectiveness of diversification (*e.g.,* Longin and Solnik, 2001; Ang and Bekaert, 2002; Ang et al., 2006). One of the most important factors

that influences the economic and market conditions is monetary policy. For instance, an increase in interest rates by the Federal Reserve can result in higher borrowing costs, lower bond prices, and a decrease in the attractiveness of equity. This, in turn, can impact asset correlations and the effectiveness of diversification (Bernanke and Kuttner, 2005). For example, expansionary monetary policy (*e.g.,* lowering interest rates or quantitative easing) increases liquidity in the financial system, which can drive up asset prices and change the dynamics of risk and return across different asset classes (Stark and Croushore, 2002; Bernanke, 2004; Bernanke and Kuttner, 2005; Gagnon et al., 2011; Barro and Redlick, 2011; Krishnamurthy and Vissing-Jorgensen, 2011; Borio and Zhu, 2012). Conversely, contractionary policy can tighten liquidity and lead to increased correlations among assets during market downturns (Bernanke and Kuttner, 2005; Baker and Wurgler, 2006; Adrian and Shin, 2010; Rigobon and Sack, 2004; Borio, 2014). In addition, monetary policy also shapes investor expectations and market sentiment, impacting risk appetite and the allocation of capital across various asset classes, which in turn affects diversification strategies (Shiller, 2000; Bernanke, 2004; Baker and Wurgler, 2006; Cohen and Frazzini, 2008; Friedman and Schwartz, 2008; Barroso and Santa-Clara, 2015).

4) Investment horizon: The time frame for holding investments influences diversification strategies. Longer investment horizons may allow for greater risk-taking and a focus on higher-return assets (Statman, 1987). Research indicates that over longer periods, diversified portfolios may yield better risk-adjusted returns due to the compounding of returns and the ability to weather market fluctuations.

5) Behavioural factors: Psychological biases, including overconfidence, loss aversion, and home bias, might affect how investors make investment and diversification decisions (Barberis and Thaler, 2003). Such biases may cause investors to focus their investments instead of diversifying adequately. Research in behavioural finance indicates that these biases may lead to suboptimal investing decisions, adversely affecting total portfolio performance (Kahneman and Tversky, 2013).

6) Liquidity of assets: The ease with which assets can be bought or sold without significantly affecting their price is a critical factor (Amihud and Mendelson, 1986). Assets with strong liquidity can be bought or sold without significantly affecting their price, while illiquid assets can pose risks to diversification, especially in volatile markets. A portfolio containing illiquid assets may be harder to rebalance or liquidate during market downturns, which can hinder performance.

7) Transaction costs, management fees, and taxes can diminish the returns of a diversified portfolio. Higher costs may negate the advantages of diversification (French, 2008). Investors should evaluate the cost structure of their investment vehicles, since high transaction costs and fees may diminish overall portfolio performance, even with effective diversification.

Conclusively, the performance of portfolio diversification is influenced by various interrelated factors, including asset correlation, allocation strategies, market conditions, investment horizons, behavioural biases, liquidity, and costs. Understanding these factors can help investors make informed decisions to optimise their portfolios and achieve better risk-adjusted returns.

## 1.6. Structure and Overview of the Thesis.

After this introduction, the remaining chapters are organised as follows. Chapters two, three, and four are empirical studies, each with distinctive objectives. Chapter five summarises and concludes the whole thesis and provides recommendations for future research. Here we provide an overview for each of the empirical studies (chapters two, three, and four).

### *1.6.1 An Overview of Chapter Two*

Chapter two investigates the diversification benefits of portfolio choices of U.S. investors, given the three major crisis periods and the apparent dominance of the U.S. market. More specifically, this chapter examines which of these four investment options, including three portfolio diversification options (a stock (60%)-bond (40%) portfolio, an international diversification portfolio, and an asset-class diversified portfolio) and a U.S.-only investment strategy, is more beneficial to U.S. investors over the sample period from January 1995 to December 2021. This chapter solves for the following questions:

1) Do U.S. investors need to diversify internationally because the US has one of the world's most developed economies and stock markets?

2) Does portfolio diversification theory fail when investors need it most?

3) Do the three portfolio diversification options (a stock (60%)-bond (40%) portfolio, an international diversification portfolio, and an asset-diversified portfolio) perform better during the crises than a U.S.-only investment strategy?

4) Does the health crisis have a different impact on the performance of portfolio diversification than the financial and economic crises have on that?

The data frequency of this chapter is monthly. The whole period is divided into six sub-sample periods to estimate and compare the four investment options for the full sample periods and for the six sub-sample periods. We classify these sub-sample periods into two broad categories: three crisis periods and three non-crisis periods. The crisis periods include the Dot-com bursting period from April 2000 to December 2002, the Great Recession period from December 2007 to June 2009, and the COVID-19 health crisis period from December 2019 to December 2021. The non-crisis periods include the Dot-com booming period from January 1995 to March 2000, the 2003-2007 period, and the 2009-2019 period. We consider three types of diversification opportunities for U.S. investors, which are compared against a U.S.-only position that involves the S&P 500 index as the portfolio. The first one is a stock (60%)-bond (40%) portfolio, which is composed of the S&P 500 index and U.S. 10-year Treasury note using a 60/40 weighting (Markowitz, 1952). The second portfolio is an internationally diversified stock portfolio, comprising the S&P 500 index, the EAFE index, and the EM index. The third portfolio is an asset-class diversified portfolio, which is constructed across different asset classes and is constituted of the S&P 500 index, gold, oil, and the 10-year Treasury-note. The currency of all series is the U.S. dollar. The data used in this chapter consists of six variables, including three stock indexes (S&P 500 index, MSCI EAFE index (developed market index), and MSCI EM index (emerging market index), three assets (gold (Gold Bullion), oil (Brent Oil), and bonds (U.S. 10-year Treasury note).

This chapter undertakes research from three distinct perspectives. First, we compared the performance of different portfolios over the sample period of January 1995–December 2021 from the perspective of U.S. investors. Second, this chapter compares the correlation trend of each portfolio in crisis and non-crisis periods and evaluates the performance of each portfolio.

Third, this chapter compares whether the health crisis and the financial and economic crises have different effects on portfolio diversification.

### *1.6.2 An Overview of Chapter Three*

Chapter Three optimises portfolio selection for an investment universe of developed and emerging market stock indexes using the Parametric Portfolio Policy (PPP) approach of Brandt et al. (2009) for the period from December 2004 to December 2023, and compares the results to the performances of naïve diversified portfolios (1/N-rule), market capitalisation weighted, risk parity (equally weighted risk contribution), mean-variance (MV), and Black Litterman (BL) optimised portfolios. This chapter solves for the following questions:

1) Which kinds of asset index characteristics matter when optimising portfolio selection using the Parametric Portfolio Policy (PPP) approach for internationally diversified portfolios?

2) Is it feasible to optimise portfolio selection using the Parametric Portfolio Policy (PPP) approach for internationally diversified portfolios?

3) Does the Parametric Portfolio Policy (PPP) approach generate superior in- and out-of-sample portfolio performance compared to the naïve diversified portfolios (1/N-rule), market capitalisation weighted, risk parity (equally weighted risk contribution), mean-variance (MV), and Black-Litterman approaches?

To empirically test the in- and out-of-sample performance of our portfolio strategies, our modelled sample set comprises seven global indexes from developed economies (i.e., USA, Japan, UK, Italy, France, Germany, and Canada, known as the G7), and five global indexes from emerging economies (i.e., Brazil, Russia, India, China, and South Africa, known as the

BRICS). In total, we model 12 global indexes with each monthly price series covering the period from December 2004 to December 2023.

### *1.6.3 An Overview of Chapter Four*

Chapter four investigates the impact of changes in the U.S. monetary policy on portfolio diversification. The investing options are consistent with Chapter two, including a stock (60%)-bond (40%) portfolio, an international diversification portfolio, an asset-class diversified portfolio) and a U.S.-only investment strategy. This chapter solves for the following questions:

1) How do monetary policy surprises affect the relationship between assets and indexes?

2) Do monetary policy surprises cause assets and indexes to move together or apart?

3) Does the effect of monetary policy shocks on portfolios challenge the Modern Portfolio Theory (MPT)?

To answer these three questions, we follow three steps we present next. First, we measure the monetary policy shocks based on the Federal Funds futures rate which is an approach conducted by Kuttner (2005). Second, we use a fixed-coefficient technique to evaluate the effect of U.S. monetary policy on asset prices and portfolios. Third, we estimate the effects of U.S. monetary surprise on the time-varying correlation between all six indexes, and four portfolios by modelling the heteroscedasticity.

## 1.7. Contributions

This thesis contributes to the existing literature in the following areas, separating them by chapters featuring empirical studies:

Chapter two compares the impact of the financial crisis and the health crisis on the benefits of investors' diversification portfolios. Financial markets are characterised by uncertainty and unpredictability. Financial crises are one of the main reasons that cause substantial volatility on the financial market, leading to a change in the connection between stock markets and between assets and a change in the risk characteristics of certain assets, and it in turn affects investors' investment allocation strategies and the performance of portfolio diversification. Some literature (*e.g.*, Holton, 2009; Ilmanen and Kizer, 2012; Miccolis and Goodman, 2012; Statman, 2013; Fabozzi et al., 2014) investigates the effect of the financial crisis on portfolio diversification, but there is no work comparing how different the impact of the financial crisis on portfolio diversification is from the impact of the health crisis on that. Chapter two also examines which of these four investment options, including three portfolio diversification options (a stock (60%)-bond (40%) portfolio, an international diversification portfolio, and an asset-diversified portfolio), and a U.S.-only investing option, is more beneficial to U.S. investors, and to the authors' greatest knowledge, there is no other work conducting this examination in the literature.

Chapter three contributes to the literature by optimising portfolio selection for an investment universe of developed and emerging market stock indexes using the Parametric Portfolio Policy (PPP) approach of Brandt et al. (2009) for the period from December 2004 to December 2023 and compare the in- and out-of-sample results to naïve diversified portfolios (1/N-rule), market capitalisation weighted, risk parity (equally weighted risk contribution), mean-variance (MV), and Black Litterman (BL) optimised portfolios. Parametric Portfolio Policy (PPP) is novel to the field of portfolio diversification; thus, so far there has been no work in the literature to comprehensively test its performance and compare it to the performances of other popular portfolio optimisation approaches and models.

Chapter four examines how different portfolios respond to surprises from FOMC announcements over the period February 2000 to December 2021. Our sample pool contains four main types of security portfolios. These portfolios consist of six different indexes. Our empirical analysis provides insight into how these four main types of portfolios are affected by surprises from FOMC announcements. To the author's best knowledge, this is also the first work that studies how different portfolios respond to monetary surprise and the most thorough analysis of how U.S. monetary policy shocks affect global asset markets. Several studies have examined how U.S. monetary policy affects global stock markets and asset prices (*e.g.,* Ehrmann and Fratzscher, 2004; Bernanke and Kuttner, 2005; Wongswan, 2006; Andersen et al., 2007; Ehrmann and Fratzscher, 2009; Wongswan, 2009; Hausman and Wongswan, 2011). However, these studies focus on a limited number of nations and a specific asset classification and there is no study examining how U.S. monetary policy affects portfolio diversification. This chapter examines the effects of changes in U.S. monetary policy on various portfolios by analysing their impact on bond prices, bullion prices, oil prices and equity markets. Compared to the existing literature, this should provide more exhaustive and reliable results.

**Chapter Two:** Is Portfolio Diversification Still Effective? Evidence Spanning Three Crises from the Perspective of U.S. Investors

## Abstract

This chapter uses data of over twenty years to examine diversification benefits for U.S. investors through assessing different portfolio options, including a stock (60%)-bond (40%) portfolio, an internationally diversified stock portfolio, and a cross-asset diversified portfolio compared with investing only in the U.S. stock market. Our data set consists of three stock indexes (S&P 500, MSCI EAFE, and MSCI EM) and three assets (gold, oil, and bonds). First, using the ARMA (0,0)-DCC-GARCH (1,1) model, we find that the S&P 500 index and the other five variables (the MSCI EAFE index, MSCI EM index, gold, oil, and bonds) have an interaction relationship. Second, by combining the time-varying correlation and fixed correlation, we reinforce the existing argument that correlations between national stock markets have been increasing in recent years and we also support the existing argument that the correlation between stock markets in crisis periods is higher than in non-crisis periods. Portfolios are built using both equal- and mean-variance efficient-weights and are compared primarily using the Sharpe ratio. The results indicate that before 2009, U.S. investors could benefit from an internationally diversified stock portfolio. However, since 2009, this international stock portfolio is less likely to benefit U.S. investors. In contrast, the cross-asset

diversified portfolio does provide greater benefit and outperforms the U.S-only, the stock-bond portfolio, and the international stock portfolio over different time periods. Of note, the efficient portfolio weighting outperforms the equal-weighted portfolio. Overall, a portfolio consisting of the S&P 500 index, gold, oil, and U.S. 10-year Treasury-note is the preferred option for U.S investors.

## 2.1. Introduction

Modern portfolio theory (MPT) is a practical approach that helps investors allocate their wealth among alternative assets within an acceptable level of risk to maximise overall returns (Elton and Gruber, 1997). The foundation of the Modern portfolio theory (MPT) is provided by an American economist, Harry Markowitz, in his 1952 paper "Portfolio Selection" (Rubinstein, 2002). An important principle of the MPT at work here is portfolio diversification. Most investments are either high-return with high risk or low-risk with low return. Markowitz (1952) believes that investors can choose the best combination of the two based on their assessment of individual risk tolerance to obtain the best results. So, what are the most important factors driving the risk and return of a portfolio? Just as any food consists of a bundle of nutrients that sustain us, we can view all assets or indexes in a portfolio as a bundle of factors that reflect the deeper risks and rewards of that portfolio. However, the extent to which the risk can be reduced depends on the correlation between these assets (Markowitz, 1952). If the correlation between them is zero, then the firm-specific risk can be eliminated in theory (Levy and Sarnat, 1970; Brown and Kapadia, 2007). Therefore, most investors know that they should follow the theory of not putting all their eggs in the same basket when investing, but how to allocate these eggs has always been the focus of research by researchers and investors. With the COVID-19 health crisis outbreak, the diversification of international investment portfolios has once again been heatedly discussed by investors, policymakers, and researchers. During this pandemic, as the world moves into an uncertain phase with lockdown and economic shutoffs, the financial markets have been reeling under the pressure with higher volatility and nearly a third of market capitalisation being wiped out during these times (Ali et al., 2020; Davis et al., 2021). It is worth considering how the impact of a health crisis on portfolio diversification differs from that of financial and economic crisis. The purpose of this chapter is to investigate the diversification benefits of portfolio choices of U.S. investors, given the three major crisis periods and the apparent dominance of the U.S. market. More specifically, this examines which of these four portfolios, including a stock (60%)-bond (40%) portfolio, an international diversification portfolio, an asset-diversified portfolio, and a U.S.-only portfolio, is more beneficial to U.S. investors.

As noted by Markowitz (1952), the correlation between all components of a portfolio is key to its portfolio. Theoretically, if the correlation between them is zero, then the firm-specific risk can be eliminated in theory (Levy and Sarnat, 1970; Brown and Kapadia, 2007). Empirical evidence (*e.g.,* Koch and Koch, 1991; Longin and Solnik, 1995; Driessen and Laeven, 2007) shows that the benefits of international investment portfolio diversification are declining because of the increasing correlation of national stock markets. Karolyi and Stulz (1996) argue that increasing correlations are detrimental to the benefits of international diversification and increase shock transmission between financial markets. De Roon et al. (2001) find that once the transaction costs and short-selling restrictions are considered, the international diversification gains of U.S. investors are small. Some other literature (*e.g.,* Longin and Solnik, 1995; Forbes and Rigobon, 2002; Kim et al., 2005; Morana and Beltratti, 2008) also show that correlations between national stock markets have been increasing in recent years.

Conversely, some literature (*e.g.,* Gilmore and McManus, 2002; Hatemi-J and Roca, 2006) shows that although the correlation between the stock markets of various countries increases over time, if investors can measure the correlation between the stock markets of various countries through technical means and combine them into an optimal investment portfolio within an acceptable risk range, international portfolio diversification remains. At present, the United States is one of the most developed economies in the world, and so is the U.S. stock market. Therefore, the first and second research questions in this chapter arise: Is it still necessary for U.S. investors to diversify their portfolios internationally under this context? If the benefit of the international diversification is truly diminishing, can U.S. investors benefit from other types of portfolios, such as the stock (60%)-bond (40%) portfolio and the cross-asset diversified portfolio? Some voices argue (*e.g.,* Holton 2009; Fabozzi et al. 2014) that portfolio diversification fails when investors need it most, especially in the context of financial and economic crises. With the outbreak of the COVID-19 health crisis, the third and fourth research questions arise: Does portfolio diversification theory fail when investors need it most? Do different crises have a different impact on the performance of portfolio diversification?

This chapter presents research from three dimensions. First, it compares the performance of different investment options, including a U.S.-only, a stock (60%)-bond (40%) portfolio, an international diversification portfolio, and a cross-asset diversified portfolio, over the sample period of January 1995-December 2021 from the perspective of U.S. investors. The most important comparison is whether the portfolio diversification options, which include the stock (60%)-bond (40%) portfolio, the international diversification portfolio, and the cross-asset diversified portfolio, have better performance than only investing in the U.S. market (U.S.-only). In addition, assessing which of the three portfolio diversification options has a better performance over the sample period is also another important purpose. The following four points explain how each portfolio is built in this chapter: 1) As we analyse and compare the benefits of different investment options, including three portfolio diversification opportunities and a strategy only investing in the U.S. market (U.S.-only), from the perspective of U.S. investors, a representative indicator as a benchmark to represent and evaluate the U.S. stock market is necessary. The Standard & Poor's 500 (S&P 500) index is a representative indicator of the U.S. stock market, so, this chapter uses the Standard & Poor's 500 index as a benchmark to evaluate the performance of U.S. investors who only diversify portfolios in the U.S. stock market. 2) Pension funds typically recommend the 60-40 stock-bond rule to reduce risk, as bonds tend to rise during periods of stock market decline (Ziemba, 2013). So, a stock (60%)-bond (40%) portfolio consisting of the S&P 500 index and U.S. 10-year Treasury note is constructed. 3) International diversification theory suggests that if international stock markets do not correlate perfectly, investors can benefit from international diversification (Li et al., 2003). Thus, a representative international portfolio is constructed to estimate whether the benefit of international diversification still exists for U.S. investors. The international diversified portfolio consists of the S&P 500 index, the MSCI EAFE index, and the MSCI EM index. The MSCI EAFE index (excluding the U.S. market) and MSCI EM index represent the developed countries and emerging countries, respectively. 4) Alternatively, investors can diversify across assets, in addition to bonds, including commodities, such as gold and oil. Some literature (*e.g.,* Capie et al., 2005; McCown and Zimmerman, 2006; Baur and McDermott, 2010) finds that gold not only protects investors from inflation but also has hedging properties. Hamoud et al. (2011, 2013) find that when oil is combined with precious metals in a diversified portfolio, it has the property of increasing returns and reducing risk. Therefore, we build a cross-asset diversified portfolio consisting of the S&P 500 index, gold, oil, and bonds.

Second, this chapter compares the correlation trend of each portfolio in both crisis and non-crisis periods and evaluates the performance of each portfolio. Empirical evidence (*e.g.,* Roll, 1988; Bertero and Mayer, 1990; King and Wadhwani, 1990; Solnik et al., 1996; Butler and Joaquin, 2002; Guidi and Ugur, 2014) suggests that the correlation between stock markets in crisis periods is higher than in non-crisis periods. Certain literature (*e.g.,* Holton 2009; Fabozzi et al. 2014) critiques that diversification inadequately safeguarded against losses during the Great Recession, as correlations typically surge during down markets. So, the fixed (Pearson correlation) and time-varying (ARMA (0,0)-DCC-GARCH (1,1) model) methodologies are conducted to study the trend of correlation between indexes and between assets in this chapter.

Third, this chapter compares if the health crisis and the financial and economic crises have different effects on portfolio diversification. After the COVID-19 health crisis outbreak, to control the spread of the epidemic, countries have adopted a policy of isolation, which has made economic and financial exchanges between countries challenging. Therefore, it is worth examining whether the health crisis has a different impact on four different types of diversified portfolios compared with the Dot-com bubble crisis and the Great Recession, which opens new perspectives on the impact of different types of crises on financial markets.

From the perspective of American investors, this chapter spans three major crises, including the Dot-com bubble crisis starting from April 2000, the Great Recession starting from December 2007, and the COVID-19 health crisis starting from December 2019, to compare four different types of diversified portfolios: First, this chapter assesses the benefits of diversification for investors that only invest in the U.S. stock market. Second, this chapter examines the benefits of portfolio diversification through the stock-bond allocation. Third, this chapter evaluates the benefits investors might obtain by investing in international stock markets. Fourth, compared with holding only U.S. stocks, this chapter is trying to work out whether U.S. investors will benefit more from holding a diversified portfolio consisting of stocks, bonds, and commodities (gold and oil).

Our main findings are as follows. First, since 2009, compared with the MSCI EAFE index, and the MSCI EM index, the S&P 500 index has been the best performer, with a higher average monthly real return and a higher Sharpe ratio. Second, the ARMA (0,0)-DCC-GARCH (1,1) model shows that the S&P 500 index and the other five variables have an interaction relationship. Third, by combining the time-varying correlation and fixed correlation, we reinforce the existing argument that correlations between national stock markets have been increasing in recent years (Longin and Solnik, 1995; Forbes and Rigobon, 2002; Kim et al., 2005; Morana and Beltratti 2008; Christoffersen et al., 2014) and we also support the existing argument that the correlation between stock markets in crisis periods is higher than in non-crisis periods (Roll, 1988; Bertero and Mayer, 1990; King and Wadhwani, 1990; Solnik et al., 1996; Butler and Joaquin, 2002; Guidi and Ugur, 2014). Fourth, for both sub-sample periods and the full sample period in this chapter, the cross-asset portfolio comprising the S&P 500 index, gold, oil, and the 10-year U.S. Treasury Note can offer significant diversification benefits to U.S. investors. This is true whether investors choose an equally weighted or mean-variance optimally diversified asset portfolio. Fifth, the cross-asset diversified portfolio outperformed the U.S only, the stock (60%)-bond (40%) portfolio, and the international diversified portfolio, so, the asset-diversified portfolio consisting of the S&P 500 index, gold, oil, and U.S. 10-year Treasury Note is the best choice for U.S investors. Sixth, before 2009, U.S. investors could benefit from the international-diversified portfolio consisting of the S&P 500 index and MSCI EM index. However, since 2009, the international-diversified portfolio is less likely to benefit U.S. investors. There are two possible reasons behind it. The first one is that since 2009, compared to the MSCI EAFE index, and the MSCI EM index, the S&P 500 index has been the best performer. Another reason might be that the correlation between international stock markets has been increasing recently, which may eliminate the benefits of international diversification. Seventh, compared with the Dot-com bursting crisis and the Great Recession, the COVID-19 health crisis did not have an evident impact on the return of the four portfolios, although it increased the volatility of each variable.

This chapter expands the existing literature library from the following aspects. First, it compares the impact of the financial crisis and the health crisis on the benefits of investors' diversification portfolios. Financial markets are characterised by uncertainty and unpredictability. Financial crises are one of the main reasons that cause substantial volatility on the financial market, leading to a change in the connection between stock markets and between assets and a change in the risk characteristics of certain assets. This, in turn, affects investors' investment allocation strategies and the performance of portfolio diversification. Some literature (*e.g.*, Holton 2009; Ilmanen and Kizer 2012; Miccolis and Goodman 2012; Statman 2013; Fabozzi et al. 2014) investigates the effect of the financial crisis on portfolio diversification, but there is no work comparing how different the impact of the financial crisis on portfolio diversification is from the impact of the health crisis on that. Second, this chapter also examines which of these four investment options, including three portfolio diversification options (a stock (60%)-bond (40%) portfolio, an international diversification portfolio, and an asset-diversified portfolio), and a U.S.-only investing option, is more beneficial to U.S.

investors. To the authors' greatest knowledge, there is no other work conducting this examination in the literature.

After this introduction, the rest of the chapter is organised as follows: Section 2.2 provides a literature review; Section 2.3 presents the data and descriptive statistics for the variables in this chapter; Section 2.4 presents the methodology; Section 2.5 analyses the main empirical results, and Section 2.6 concludes this chapter.

## 2.2. Literature Review

The diversification theory is proposed by Markowitz (1952). Rational, risk-averse investors realise that not all investments perform well simultaneously, some investments may never perform well, and few may perform well. Since no one can accurately predict which investments will perform well and which will not perform well, investors can minimise investment risk by spreading their investments across a broad range of investments to form a diversified portfolio (Markowitz, 1952). There are four broad types of portfolio diversification, including cross-asset diversification, cross-geography diversification, cross-sector diversification, and investment style diversification. In this chapter, cross-asset, and cross-country diversification are considered:

1) Cross-asset diversification: Investors should spread their funds across different asset-classes,

such as equities, fixed income (bonds), funds, real estate, commodities, cash equivalents and alternative assets (such as hedge funds, private equity, or cryptocurrencies) and so forth. The idea is that different asset classes react differently to market conditions, which can help investors reduce overall portfolio volatility (*e.g.,* McDonald and Solnik, 1977; Lean and Wong, 2015; Guesmi et al., 2019). This can include diversification across assets of different maturities. Different types of assets have different maturities. Investors can arrange the maturity structure of their investments to achieve a high degree of uniformity in profitability, liquidity, and risk (*e.g.,* Levy and Lerman, 1988; Hatemi-J and Roca, 2006; Guidi and Ugur, 2014).

2) Cross-country diversification: Cross-country diversification is within the Cross-geography category. Investors can diversify their portfolios by investing in assets from different geographic regions or countries (*e.g.,* Solnik, 1974; Levy and Lerman, 1988; Hatemi-J and Roca, 2006; Guidi and Ugur, 2014). Different global regions have different economic conditions, so the degree of investment risk is also different. Investors should diversify their investments in different countries and regions to avoid major losses due to the deterioration of the political and economic environment in a certain region. This can also include diversification across different currencies. Different currencies can help mitigate risks related to currency fluctuations (Jorion, 1991). This strategy is particularly relevant for investors with exposure to global markets.

### *2.2.1 Research on Portfolio Diversification across International Stock Markets*

There is a large amount of literature investigating the benefits of international diversified portfolios and examining the correlation between international stock markets, as the correlation between the global stock markets determines to what extent the risk in an international diversified portfolio can be diversified away (Markowitz,1952).

#### *2.2.1.1 Benefits of international diversification*

Grubel (1968) and Levy and Sarnat (1970) are the first to show that combining foreign stocks with domestic stocks can improve portfolio diversification. Solnik (1973) finds that by adding international securities to a portfolio of U.S. securities, the systematic risk of the portfolio can be partially diversified. Solnik (1974) shows that the market risk of a U.S. portfolio is much higher than the market risk of an internationally diversified portfolio. Bergstrom (1975) finds that international portfolio diversification can reduce portfolio volatility by up to 40% without affecting returns. Levy and Lerman (1988) examine whether U.S. investors could obtain portfolio diversification by holding stocks in the Czech, Hungarian, and Polish stock markets over the 21 years from 1960 to 1980, by using the cointegration process and Granger causality test. Their results shows that U.S. investors could obtain international diversification benefits by investing in these three Central European stock markets. Odier and Solnik (1993) examine the potential benefits of international diversification in the world's 15 largest stock markets between 1980 and 1990. They conclude that investments in foreign assets provided attractive diversification and profit opportunities during the period under observation. De Santis and Gerard (1997) use the conditional CAPM to estimate that the expected return that U.S. investors would receive from international diversification averaged 2.11% per year between 1970 and 1994.

De Roon et al. (2001) find that once transaction costs and short-selling restrictions are taken into account, international diversification benefits for U.S. investors are small. Gilmore and McManus (2002) examine the short- and long-term relationship between the U.S. stock market and three central European markets (Czech, Hungarian, Polish markets). Their data include weekly closing price indexes of the Czech, Hungarian, Polish, and U.S. stock markets from July 1, 1995, to August 1, 2001. They provide evidence that U.S. investors can benefit from international diversification in these markets. Hatemi-J and Roca (2006) study international portfolio diversification among the world's three largest financial markets (i.e., the United States, Japan, and the United Kingdom) from the perspective of U.S. investors during the period 1970 to 2000. Their results suggest that international diversification among the world's three largest financial markets (the United States, the United Kingdom, and Japan) can improve risk-adjusted returns. Coeurdacier and Guibaud (2011) study whether investors properly hedge their excessive domestic risk by investing in foreign stock markets that are less correlated with their home stock markets. They found that, all else being equal, investors do tend to tilt their overseas holdings toward countries that offer better diversification opportunities. Rezayat and Yavas (2006) use daily closing price data for the United States, the United Kingdom, France, Germany, and Japan from January 1999 to February 2002 to study the joint impact of any four stock markets on the fifth market. Their findings suggest that despite the strong interdependence among markets, there is still room for international portfolio diversification. In particular, they find that European stock markets are highly correlated. On the other hand, the Japanese stock market has almost no significant impact on the movements of other markets. Their results show that the S&P 500 has the most significant impact on the Nikkei, and the European index has a greater impact on the S&P 500 than the S&P 500 has on the European index. However, the Nikkei has no significant impact on the European index or the S&P 500 index during this period. Meric et al. (2008) investigate the impact of industry index co-movement on portfolio diversification in bull and bear markets in the US, UK, German, French and Japanese stock markets and find that in bull markets, even if investors invest in the same industry in different countries, rather than in different industries in the same country, investors can obtain greater benefits through global diversification than domestic diversification.

By using Johansen's (1988) cointegration methodology, Zafaranloo and Sapian (2013) investigate the long-run relationships between five Asian markets (Malaysia, Thailand, Indonesia, China and India) and the United States during the period from 2006 to 2012. They use Granger's (1969) causality methodology to capture short-run relationships between these markets. They find that there were no long run relationships among all five Asian markets and the US market over the sample periods, while they provide evidence that short-run relationships exist between Malaysia, Indonesia, and Thailand with U.S market. They believe that long-term investment in emerging Asian markets could provide some diversification benefits to U.S. investors. Christoffersen et al. (2014) study time-varying correlation patterns and trends using weekly returns from 16 developed and 16 emerging markets over the period 1973-2012. They find a significant upward trend in correlations for both developed and emerging markets, and they also find some evidence that adding emerging markets to a portfolio containing only developed markets can increase diversification benefits. Guidi and Ugur (2014) investigate whether the Southeast European (SEE) stock markets of Bulgaria, Croatia, Romania, Slovenia, and Turkey were comparable to the developed stock markets of Germany, the United Kingdom, and the United States from November 8, 2000, to June 26, 2013. They find that Southeast European markets offer diversification advantages to international investors with investment horizons of less than three years. Oloko (2018) adopts the VAR-BEKK-GARCH model and used the conditional variance and covariance in the model to estimate the optimal portfolio weight and the optimal hedge ratio to examine the period from January 2004 to 2015. This study provides evidence that US and UK investors may gain potential benefits from diversifying their portfolios through Nigerian stocks and that financial risks or financial bubbles may be transmitted from US and UK stock markets to Nigerian stock markets. Tai (2018) estimates the Dot-com bursting crisis, and the Great Recession influence the benefits of international diversification from the perspective of US investors. He finds that over the whole sample period, international diversification could provide investors an average return of 1.253 percent per year. During the Great Recession, the average return falls to approximately 0.567 percent per year, but it rises to 2.829 percent per year during the Dot-com bursting crisis.

Attig and Sy (2021) show that after the millennium, international diversification continues to outperform industrial diversification, especially when investors consider emerging markets. Viceira and Wang (2022) conclude that diversification benefits have not declined for long-term investors (despite the long-term rise in global stock correlations). Based on a large sample of nearly 42,000 stocks, Attig et al. (2023) find that international diversification still dominates industrial diversification over the past quarter century.

*2.2.1.2 The trend of correlation between international stock markets*

The correlation between international markets is one of the most important factors affecting the performance of the internationally diversified portfolios. Granger and Morgenstern (1970) study the correlation between seven European indexes and the market indexes of the New York, Tokyo, and Sydney Stock Exchanges. Using weekly data from 1961 to 1964, they conclude that the market indexes moved independently of each other during the sample period. The only cases where relationships were detected were between the New York and Amsterdam markets and between the German and Amsterdam markets. Hilliard (1979) examines the co-movement between six European indexes and the New York, Toronto, Sydney, and Tokyo Stock Exchange indexes over a 10-month period before and after the 1973 oil embargo (July 1973 to April 1974). He finds some very strong relationships between these indexes. Kaplanis (1988) studies the stability of the correlation and covariance matrices of monthly returns for ten markets over a 15-year period (1967-82). She compares the matrices estimated for 46 monthly subperiods using the Box (1949) and Jenrich (1970) tests. The null hypothesis that the correlation matrix remains constant over two consecutive subperiods could not be rejected at a 15% confidence level. The stability of the covariance matrix was much worse (rejected at a 5% confidence level for most subperiods). She argues that this result could be due to changes in the conditional variance in the absence of constant international conditional correlations. Ratner (1992) also claimed that international correlations remained constant between 1973 and 1989. Von Furstenberg and Jeon (1989) reach similar conclusions using the VAR approach for four markets and a very short period (1986-88).

A large amount of literature (*e.g.,* Longin and Solnik, 1995; Solnik et al., 1996; Forbes and Rigobon, 2002; Kim et al., 2005; Morana and Beltratti, 2008) show that correlations between national stock markets have been increasing in recent years. Empirical evidence (*e.g.,* Koch and Koch, 1991; Longin and Solnik, 1995; Driessen and Laeven, 2007) shows that the benefits of international investment portfolio diversification are declining because of the increasing correlation of national stock markets. Karolyi and Stulz (1996) argue that increasing correlations are detrimental to the benefits of international diversification and increase shock transmission between financial markets. Levy and Lerman (1988) show that the correlation between the Czech, Hungarian, and Polish stock markets and the U.S. stock market has increased over the 21-year period from 1960 to 1980. Using high-frequency data surrounding the crash of 1987, King and Wadhwani (1990) and Bertero and Mayer (1990) find that international correlation tends to increase during the stock market crisis. Koch and Koch (1991) look at the correlation of eight markets using daily data for three separate years (1972, 1980 and 1987) and conclude from simple Chow tests that international markets have recently grown more interdependent. King et al. (1992) claim that this is only a transitory increase caused by the 1987 crash. Indeed, a question often raised is whether the international correlation increases in periods of high turbulence. The international correlation increases when global factors dominate domestic ones and affect all financial markets. The dominance of global factors tends to be associated with very volatile markets (the oil crises, the Gulf war, etc.). Longin and Solnik (1995) study the correlation of monthly excess returns for seven major countries over the period 1960-90. They find that the international covariance and correlation matrices are unstable over time. They also find that the correlation rises in periods when the conditional volatility of markets is large.

Butler and Joaquin (2002) use three basic models to evaluate the correlation between the US, UK, Japan, Australia and European stock market indexes and the corresponding MSCI Global ex-Domestic index, and divided the market into observable bear markets, calm markets and bull markets. Their results show that the correlation observed during bear markets is significantly higher than that in calm markets and bull markets. The higher-than-normal correlation exhibited by bear markets causes the monthly returns of domestic and international stocks and other weighted portfolios to be on average more than 2% lower than predicted by the normal distribution. On the other hand, Li et al. (2003) use Bayesian inference to test the impact of short-selling restrictions on the international diversification benefits of US investors by establishing a sample set of stock indexes of the G7 developed countries and eight emerging market countries from January 1976 to December 1999. They found that when US stock investors were prohibited from short selling in emerging markets, their international diversification benefits were still considerable. The integration of global stock markets reduces (but does not eliminate) the diversification benefits of investing in emerging markets under short-selling restrictions. Driessen and Laeven (2007) use the standard mean-variance framework of Markowitz (1952) to construct a monthly data set of stock market index returns for 23 developed and 29 developing countries over the period 1985-2002 to examine how the benefits of international portfolio diversification from the perspective of local investors vary across countries. They find that for investors in developing countries, the benefits of investing abroad are greatest, controlling currency effects, while the benefits of international portfolio diversification appear to be greatest for countries with high country risk. They also provide evidence that diversification benefits vary over time as country risk changes. They find that diversification benefits have declined for most countries over the sample period. This is mainly due to an increase in the correlation of local market returns with the global index and a decrease in the variance of the local index. Bekaert et al. (2009) examine the co-movement of international stock returns for 23 developed markets over the period 1980-2005 and find that only a subsample of European stock markets shows an increasing trend in return correlation, while North American and East Asian markets do not.

Guidi and Ugur (2014) investigate whether the Southeast European (SEE) stock markets of Bulgaria, Croatia, Romania, Slovenia, and Turkey were comparable to the developed stock markets of Germany, the United Kingdom, and the United States from November 8, 2000, to June 26, 2013. They find that over the sample period, the Southeast European (SEE) stock markets of Bulgaria, Croatia, Romania, Slovenia, and Turkey was cointegrated with the German and UK markets, but not with the US market. Their results indicate the existence of time-varying cointegration and increasing conditional correlations from the onset of the financial crisis in September 2007 to May 2010.

Recent evidence suggests that correlations between international security markets are associated with greater volatility in those markets, thereby reducing the effectiveness of international diversification. Longin and Solnik (2001) show that correlations across countries are asymmetric in that they tend to increase sharply during market downturns, when investors are eager to see the benefits of diversification. Meric et al. (2008) find that in a bear market, the sectors of different countries tend to be more closely correlated, and country diversification opportunities are limited. The 2007–2009 financial crisis has raised a large number of questions about the capability of diversification to protect well against loss. Guidi and Ugur (2014) find that the existence of increasing conditional correlation from the onset of the financial crisis in September 2007 until May 2010. Holton (2009) and Fabozzi et al. (2014) argue that diversification failed to adequately protect against loss during the 2007–2009 financial crisis, because (Pearson) correlations tend to peak during bear markets. James et al. (2022) confirm that financial crises are characterised by a high degree of collective behaviour of equities, whereas periods of financial stability exhibit less collective behaviour.

Conversely, some literature (*e.g.,* Gilmore and McManus, 2002; Hatemi-J and Roca, 2006) shows that although the correlation between the stock markets of various countries increases over time, if investors can measure the correlation between the stock markets of various countries through technical means and combine them into an optimal investment portfolio within an acceptable risk range, international portfolio diversification remains. There is also some literature (*e.g.,* Ilmanen and Kizer, 2012; Miccolis and Goodman, 2012; Statman, 2013) that argues that although correlations between stock markets increase during financial crises, investors can still benefit from internationally diversified portfolios during crises.

### *2.2.2 Research on Portfolio Diversification across Different Industries or Sectors*

The cross-sector diversification is one of the most common diversification types. Some literature (Roll, 1992; Balli et al., 2013) conclude that the cross-sector diversification is superior than the cross-country diversification (international diversification). Using daily data for 24 country indexes for the period from April 1988 to March 1991, Roll (1992) concludes that a significant portion of the international structure of return correlations among countries can be ascribed to the industrial compositions of the country indexes. According to Roll (1992), parts of the benefits of international diversification are the result of industrial diversification. Moerman (2008) tests whether sector-based diversification strategies applied to the euro area stock markets obtain higher diversification benefits than country-based strategies for a sample period of industry and country indexes of the euro area from January1995 to December 2004. It finds strong evidence that diversification over industries yields more efficient portfolios than diversification over countries. By comparing the efficiency frontiers of the portfolios created using the sector equity indexes with those of the GCC (Gulf Cooperation Council) national equity indexes, Balli et al. (2013) investigate whether investing in sectoral equity markets provides more diversification opportunities than investing in stocks across the national borders. They document that portfolios diversified across GCC-wide sectors perform better than portfolios diversified across GCC national equity markets. They also reveal that portfolios diversified with a mix of GCC-wide sector and national equities produce higher returns than portfolios made up of pure GCC national equity indexes or GCC-wide sector indexes. James et al. (2022) analyse 20 years of US stock price data, which includes the global financial crisis (GFC) and the COVID-19 market crash, as well as periods of financial stability, to determine the "all weather" nature of equity portfolios. They confirm that financial crises are characterised by a high degree of collective behaviour of equities, whereas periods of financial stability exhibit less collective behaviour. Using hierarchical clustering, they discover a "best value" equity portfolio for diversification consisting of 36 equities sampled uniformly from 9 sectors. They further show that it is typically more beneficial to diversify across sectors rather than within.

However, there is still controversy about whether the cross-sector diversified portfolio has a better performance than the cross-country diversified portfolio. Some literature (Solnik, 1974; Meric and Meric, 1989; Heston and Rouwenhorst, 1994) provide different conclusion. Solnik (1974) assumes that the cross-industry diversification might have a better performance than the cross-country diversification. However, he finds little evidence to support the sevtor hypothesis, and thus he concludes that cross-industry diversification is inferior to cross-country diversification. Meric and Meric (1989) find empirical evidence that diversification across countries reduces risk more than diversification across industries. By using a sample of individual stocks from twelve European countries, Heston and Rouwenhorst (1994) find that country-specific effects dominate, and that differences in the industrial structure can only contribute very little to the explanation of inter-country correlations. Based on monthly return data from January 1980 to June 2001, Hauser and Vermeersch (2002) found that cross-country correlations are lower than cross-sector correlations, and therefore the diversification benefits of cross-country investing are more important.

### *2.2.3 Research on Portfolio Diversification across Different Asset Classes*

In addition to building a diversified portfolio through stocks, portfolio managers have also tried to find other asset classes to obtain the benefits of a diversified portfolio, such as bonds, commodities and alternative asset classes. Levy and Lerman (1988) find that the benefits of such diversification are substantial. With the same level of risk, U.S. investors who diversify into global bond markets are likely to earn an average return more than double that of a U.S. bond portfolio.

Some literature (*e.g.,* Baur and McDermott, 2010; Baur and Lucey, 2010; Sari et al., 2010; Coudert and Raymond-FeinGold, 2011; Hood and Malik, 2013; Gurgun and Unalmis, 2014; Ciner et al., 2013; Bekiros et al., 2017) conclude that gold is a stock hedge and a safe haven during extreme stock market conditions. Other studies (*e.g.,* McDonald and Solnik, 1977; Sherman, 1982; Sherman, 1986; Jaffe, 1989; Chua et al., 1990; Hillier et al., 2006; Soytas et al., 2009; Sarafrazi et al., 2014; Lean and Wong, 2015; Guesmi et al., 2019; Alkhazali and Zoubi, 2020) show that including gold in a stock portfolio can enhance overall returns and provide benefits of portfolio diversification. McDonald and Solnik (1977) conduct an empirical study of gold in portfolio diversification. They find that both gold and gold mining stocks can be beneficial for portfolio diversification. Sarafrazi et al. (2014) argue for the benefits of a portfolio that is diversified with the commodities gold, silver and oil. Lean and Wong (2015) find that gold is beneficial for stock portfolio diversification but not for bond portfolios. Guesmi et al. (2019) find that hedging strategies involving gold, oil, emerging stock markets and Bitcoin reduce a portfolio's volatility, as compared to the volatility of a portfolio composed of gold, oil and stocks from emerging market stocks only. Recently, Alkhazali and Zoubi (2020) suggest that risk-averse investors in Islamic stock indexes should include gold in their portfolios to maximise their expected utility.

However, here are some opposite conclusions. Cotter et al. (2017) re-examine diversification benefits of investing in commodities and currencies by considering a risk-averse investor with mean-variance preferences who exploits the possibility of predictable time variation in asset return means, variances, and covariances. They find that, for all portfolio strategies, commodities and currencies do not improve the investment opportunity set of the investor with an existing portfolio of stocks, bonds and T-bills, and an investment horizon of one month. Their results are in line with Daskalaki and Skiadopoulos (2011), and also consistent with the empirical evidence that the financialisaton of the commodity markets has weakened the diversification potential of commodities (*e.g.,* Domanski and Heath, 2007; Tang and Xiong, 2012).

After experiencing several major financial and economic crises, investors are eager to find alternative investment tools that provide diversification and/or hedging advantages besides gold, oil, and other common commodities. Cryptocurrency, like Bitcoins, as a novel asset class attracts the attention of investors, researchers, and policymakers. Some literature (*e.g.,* Dyhrberg, 2016a; Dyhrberg, 2016b; Dimpfl, and Kuck, 2018; Bouoiyour and Selmi, 2017; Guesmi et al., 2019) investigates the properties of cryptocurrencies in the financial markets and the performance of portfolios that contain cryptocurrencies as one of the components. Dyhrberg (2016a) identifies that Bitcoin shares several characteristics with both the U.S. dollar and gold. Dyhrberg (2016b) additionally notes that Bitcoin provides similar hedging and safe-haven opportunities as gold, positioning it in a space that is intermediate between gold and the U.S. dollar. Dimpfl and Kuck (2018), on the other hand, challenge the conclusions drawn by Dyhrberg (2016a, b), contending that Bitcoin has notably different time series characteristics when compared to other assets such as gold and the U.S. dollar. Meanwhile, Bouoiyour and Selmi (2017) claim that Bitcoin demonstrates weak safe-haven properties in both the short and long term. Guesmi et al. (2019) investigates the property of Bitcoin in the financial markets. Specifically, they explore the conditional cross effects and volatility spillover between Bitcoin and financial indicators using different multivariate GARCH specifications. The nature of interaction between Bitcoin and financial variables and their transmission mechanisms are taken into account when analysing the diversification and hedging effectiveness across gold asset and stock market. Their results show that a short position in the Bitcoin market allows hedging risk investment against all different financial assets. Moreover, they find that hedging strategies involving gold, oil, emerging stock markets and Bitcoin reduce considerably a portfolio's risk (variance), as compared to the risk of a portfolio composed of gold, oil and stocks from emerging stock only.

Overall, it can be seen from the above literature that across different portfolio combinations and constituents as well as sample periods, different results arise. This chapter seeks to evaluate these results again in the context of a more recent period, including COVID-19 and a U.S. bull market.

### 2.3. Data and Descriptive Statistics

We use monthly returns over the sample period from January 1995 to December 2021. The monthly data used in this chapter addresses several key aspects: First, while daily data is valuable for identifying short-term trends, patterns, and price movements, relying on it can lead to significant transaction costs when rebalancing. Second, yearly data is more effective for understanding long-term trends, company performance, and growth potential. In contrast, monthly data strikes a balance between the volatility of daily data and the broader trends captured in yearly data, and it allows investors to identify medium-term trends without being excessively influenced by short-term fluctuations, thereby helping to mitigate the substantial transaction costs associated with daily trading. The full sample period is divided into six sub-sample periods as detailed in Table 2.1. The choice of 1995 as a starting point is motivated by looking to provide a balance between crisis and non-crisis periods, thus, starting at the beginning of the Dot-com run period and before the crash. In addition, the MSCI EM index underwent notable changes (additions) during the mid-1990s, where the number of markets included doubled. This, equally, may affect the behaviour of the index before this period. We classify these sub-sample periods into two broad categories: three crisis periods and three non-crisis periods. The crisis periods include the Dot-com bursting period from April 2000 to December 2002, the Great Recession period from December 2007 to June 2009, and the COVID-19 health crisis period from December 2019 to December 2021. The non-crisis periods include the Dot-com booming period from January 1995 to March 2000, the 2003-2007 period, and the 2009-2019 period. We determine the start and end dates of the three crises based on the literature. For the Dot-com bursting crisis, we reviewed a significant amount of literature, which only documents that it happened after the peak of the Dot-com bubble in March 2000 (Chen et al., 2018). So, we mark April 2000 as the official start of the bursting crisis. We consider December 2007, when the stock markets began to decline, as the start of the Great Recession and June 2009 as its end. We mark December 2019 as the start of the COVID-19 crisis, following the initial outbreak of the coronavirus in Wuhan. We mark December 2021 as the end of COVID due to the large-scale vaccination and that most countries led by the United

States have cancelled most of the epidemic prevention measures by the end of 2021.

**Table 2.1.** Sub-sample periods.

| | Starting Date | Ending Date |
|---|---|---|
| Dot-com booming | 1995:01 | 2000:03 |
| Dot-com bursting | 2000:04 | 2002:12 |
| 2003-2007 | 2003:01 | 2007:11 |
| Great Recession | 2007:12 | 2009:06 |
| 2009-2019 | 2009:07 | 2019:11 |
| COVID-19 | 2019:12 | 2021:12 |

The full sample period is from January 1995 to December 2021. The Dot-com bursting period from April 2000 to December 2002, the Great Recession period from December 2007 to June 2009, and the COVID-19 health crisis period from December 2019 to December 2021 are crisis periods. The Dot-com booming period from January 1995 to March 2000, the 2003-2007 period, and the 2009-2019 period are non-crisis periods.

Next, we describe how the benefits of diversification for U.S. investors are measured. We consider three types of diversification opportunities for U.S. investors, which are listed in Table 2.2 and are compared against a U.S.-only position, which involves the S&P 500 index as the portfolio. The first diversified portfolio (Stock-Bond Portfolio) consists of the S&P 500 index and U.S. 10-year Treasury-note (U.S. 10-year T-Note) using a 60/40 weighting (Markowitz, 1952). The second (Portfolio 2), is an internationally diversified stock portfolio. This portfolio is constituted of the S&P 500 index, the EAFE index, and the EM index. The third portfolio (Portfolio 3) is constructed across different asset classes and is constituted of the S&P 500 index, gold, oil, and the 10-year T-Note. The currency of all series is the U.S. dollar. The data used in this chapter consists of six variables, including three stock indexes (the S&P 500 index, MSCI EAFE index (developed market index), and MSCI EM index (emerging market index)), three assets (gold (Gold Bullion), oil (Brent Oil)) and bonds (U.S. 10-year Treasury-note)). The U.S. monthly inflation rate is from Eikon DataStream.

**Table 2.2.** Portfolio Types.

| | Components | | | |
|---|---|---|---|---|
| U.S.-only | S&P 500 | | | |
| Portfolio 1 | S&P 500 | 10-year T-Note | | |
| Portfolio 2 | S&P 500 | MSCI EAFE | MSCI EM | |
| Portfolio 3 | S&P 500 | Gold | Oil | 10-year T-Note |

U.S.-only is the portfolio only investing in the U.S. market. Portfolio 1 is the 60/40 stock/bond portfolio. Portfolio 2 is the internationally diversified portfolio. Portfolio 3 is the cross-asset portfolio. The data frequency for each index is monthly.

The MSCI EAFE (Developed Markets) index comprises 21 developed market country indexes: Australia, Austria, Belgium, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, and the United Kingdom. The MSCI EM (Emerging Markets) index is a free float-adjusted market capitalisation index designed to measure emerging markets' equity market performance. The MSCI Emerging Markets index consists of the following 23 emerging market country indexes: Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Peru, Philippines, Poland, Qatar, Russia, South Africa, Taiwan, Thailand, Turkey, and the United Arab Emirates. The currency of all series is the U.S. dollar. The data for the S&P 500 index comes from the Capital I.Q., while the MSCI EAFE index and the MSCI EM index come from the Morgan Stanley Capital International stock market (MSCI) database. The data for the U.S. 10-year Treasury note comes from the Investing database, and the data for gold and oil comes from the Bloomberg database. In addition, we use the U.S. 3-month Treasury-Bill to represent the risk-free rate ($R_f$), which is sourced from the Bloomberg database. Table 2.3 displays all variables we use in this chapter.

**Table 2.3.** List of Variables

| Variables | Definition | Abbreviation | Frequency | Source |
|---|---|---|---|---|
| S&P 500 | The Standard and Poor's 500 index | S&P 500 | Monthly | Capital I.Q. |
| MSCI EAFE | A developed market index | MSCI EAFE | Monthly | MSCI |
| MSCI EM | An emerging market index | MSCI EM | Monthly | MSCI |
| Gold | Gold Bullion | Gold | Monthly | Bloomberg |
| Oil | Brent Oil | Oil | Monthly | Bloomberg |
| 10-year Treasury-Note | The U.S. 10-year Treasury-Note index | 10 YTN | Monthly | Bloomberg |
| $R_{S\&P\ 500}$ | The simple return of S&P 500 (nominal return) | / | Monthly | |
| $R_{MSCI\ EAFE}$ | The simple return of MSCI EAFE (nominal return) | / | Monthly | |
| $R_{MSCI\ EM}$ | The simple return of MSCI EM (nominal return) | / | Monthly | |
| $R_{Gold}$ | The simple return of gold (nominal return) | / | Monthly | $\frac{\mathrm{P_t - P_{t-1}}}{\mathrm{P_{t-1}}}$ |
| $R_{Oil}$ | The simple return of oil (nominal return) | / | Monthly | |
| $R_{10\ YTN}$ | The simple return of 10-year Treasury-Note (nominal return) | / | Monthly | |
| $\mathrm{R}_f$ | The nominal U.S. 3-month Treasury-Bill Rate (the risk-free rate) | / | Monthly | Bloomberg |
| $\mathrm{R}_{Inflation}$ | The U.S. inflation rate | / | Monthly | Eikon DataStream |

| | | | | |
|---|---|---|---|---|
| $r_{S\&P\ 500}$ | The real return of S&P 500 after inflation | / | Monthly | |
| $r_{MSCI\ EAFE}$ | The real return of MSCI EAFE after inflation | / | Monthly | |
| $r_{MSCI\ EM}$ | The real return of MSCI EM after inflation | / | Monthly | |
| $r_{Gold}$ | The real return of gold after inflation | / | Monthly | |
| $r_{Oil}$ | The real return of oil after inflation | / | Monthly | $= \frac{1 + \mathrm{R_t}}{1 + \mathrm{R}_{Inflation}} - 1$ |
| $r_{10\ YTN}$ | The real return of 10-year Treasury-Note after inflation | / | Monthly | |
| $r_f$ | The real U.S. 3-month Treasury-Bill Rate after inflation | / | Monthly | |
| C | Transaction cost for each trade | / | Per trade | |

The MSCI (Morgan Stanley Capital International) is a leading provider of critical decision support tools and services for the global investment community. $\mathrm{P_t}$ denotes the index price at time t, and $\mathrm{P_{t-1}}$ denotes the index price at time t-1. $\mathrm{R_t}$ means the nominal return of each index at month t. The nominal return is the stated rate of return on an investment, as shown on a fund factsheet or statement. We calculate the real return by subtracting the inflation rate from the nominal return.

Figure 2.1 demonstrates the price movements of the six selected variables within one plot, and Figure 2.2 displays the price movements of the six selected variables individually. Table 2.4 reports the summary statistics for the six selected variables. In Figures 2.1 and 2.2, we find that during the Dot-com booming period, both the S&P 500 index and the MSCI EAFE index all has an obvious rise. More importantly, we can see that during the Dot-Com Bursting period (2000–2002) and the Great Recession (from 2007–2009), the S&P 500 index, the MSCI EAFE index, and the MSCI EM index all has a dramatic drop. They all have a fall at the beginning of COVID-19, but they all recover immediately. It is worth noting that since 2009, the S&P 500 index has shown a significant upward trend, despite a momentary drop at the beginning of the COVID-19, while the MSCI EAFE index has shown a slight upward trend with fluctuation since then. However, from 2009 to 2011, the MSCI EM index increased, but after that, it has not shown an apparent upward trend but fluctuated.

Regarding the non-index series, we observe a rise for the 10-year Treasury note during the Dot-com crash period, followed by a slight decrease from 2003 to 2007, and then a sharp rise during the Great Recession. We again observe a rise during the COVID-19 period, a time when the three stock index series (the S&P 500, the MSCI EAFE, and the MSCI EM indexes) experienced a decline. Gold has a slight fall during the Dot-com booming period (1995–early 2000), an obvious upward trend from 2000 to 2012, and then a slight decline until 2016 before rising at the COVID-19 period. The oil has no apparent upward trend but fluctuated considerably during the whole sample period.

**Figure 2.1.** The price movements of all selected variables within one plot.

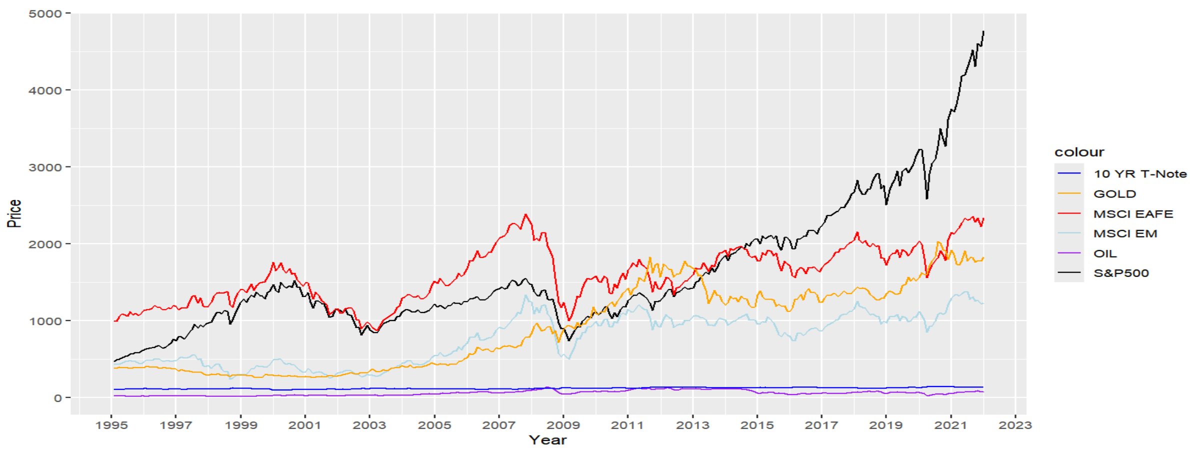



**Figure 2.2.** Price movements of all individual variables.

Figure 2.3 demonstrates the real return movements of the six selected variables. We calculate the real return by subtracting the inflation rate from the nominal return. The nominal return is the stated rate of return on an investment, as shown on a fund factsheet or statement. For simplicity, we call the real returns of the selected variables as return series. The plots indicate that the gold volatility, oil volatility, and U.S. 10-year Treasury note volatility are obviously different from the volatility of the three index variables. As shown in the figure, in 1998, the return pattern for all indexes has a big down spike, with exceptions of gold volatility and oil volatility. Interestingly, at the end of 1999, there is a big up spike in the return of gold, while there is a big down spike in the return of the U.S. 10-year Treasury note. It is notable that the U.S. 10-year Treasury note appears to be less volatile than the other selected variables, although it has a notable outlier.

**Figure 2.3.** Return movements of all selected assets.

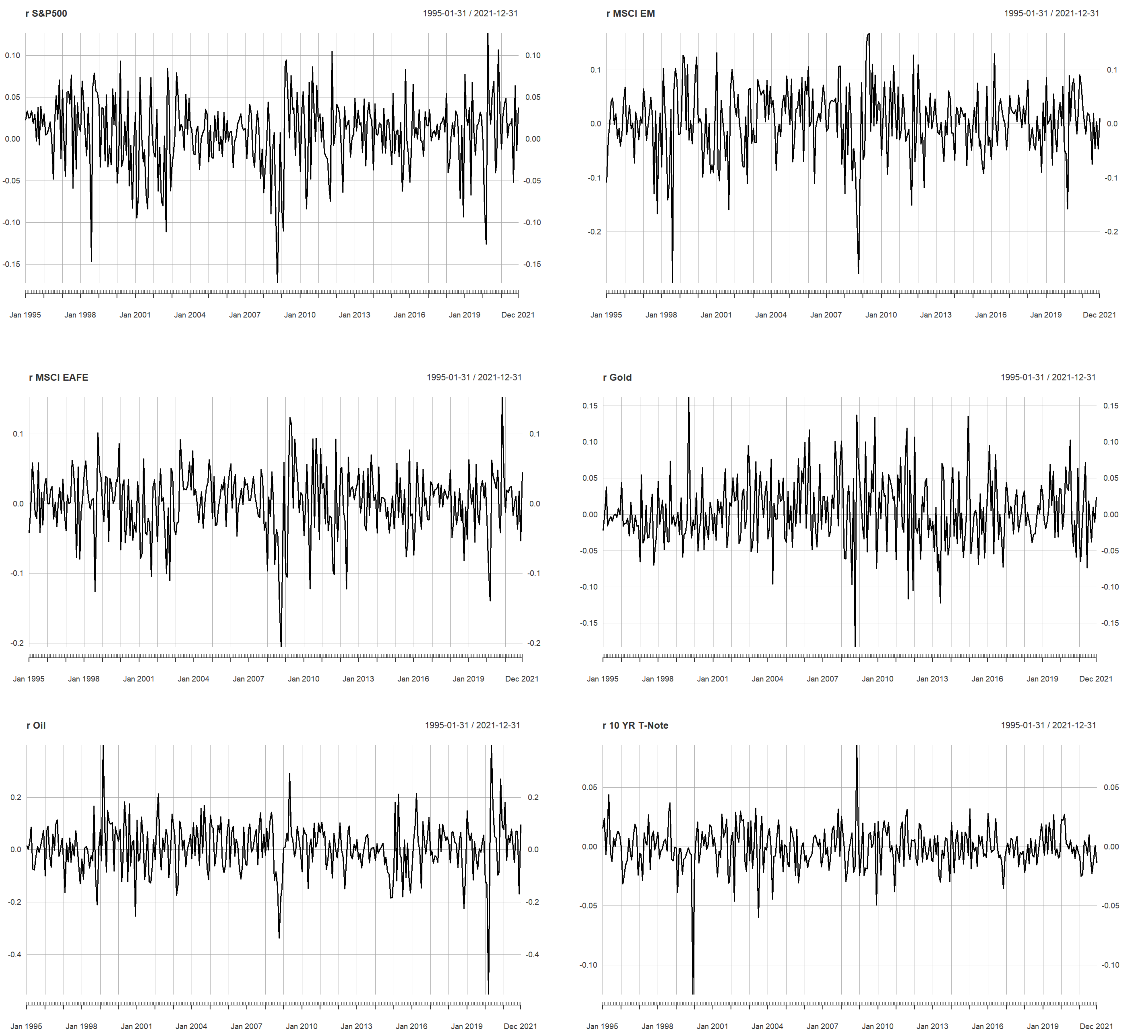


Table 2.4 illustrates descriptive statistics of the real return of all the six selected variables for the whole period. The results in Table 2.4 show that the oil return series has the highest standard deviation with 0.098, which means, compared with the other five return series, the oil is the most volatile series, while the 10-year Treasury-Note return series appears to be the least risky series with a standard deviation of 0.018. The S&P 500 return series, MSCI EAFE return series, MSCI EM return series, oil return series and U.S. 10-year Treasury note return series are skewed to the left, while the gold return series is skewed to the right. The values of Kurtosis statistics are positive and high for all the return series. The results of the Skewness and Kurtosis statistics are consistent with the Jarque-Bera test results, justifying the rejection of the normality. Among the six-return series, only the U.S. 10-year Treasury note has an average negative monthly real return of -0.001, and one of the explanations is that its return could not compensate for inflation. Furthermore, among the three stock return series, the S&P 500 is the best performer, producing an average monthly real return of 0.006, while among the three asset return series, oil produces the highest average monthly real return with 0.008.

**Table 2.4.** Summary Statistics.

| | Stock return series | | | Asset return series | | |
|---|---|---|---|---|---|---|
| | $r_{S\&P\ 500}$ | $r_{MSCI\ EAFE}$ | $r_{MSCI\ EM}$ | $r_{Gold}$ | $r_{Oil}$ | $r_{10YTN}$ |
| Nobs | 324 | 324 | 324 | 324 | 324 | 324 |
| Minimum | -0.172 | -0.205 | -0.294 | -0.183 | -0.550 | -0.125 |
| Maximum | 0.127 | 0.153 | 0.168 | 0.162 | 0.399 | 0.086 |
| 1. Quartile | -0.018 | -0.026 | -0.030 | -0.026 | -0.049 | -0.010 |
| 3. Quartile | 0.034 | 0.032 | 0.042 | 0.030 | 0.068 | 0.009 |
| Mean | 0.006 | 0.002 | 0.003 | 0.004 | 0.008 | -0.001 |
| Median | 0.011 | 0.006 | 0.005 | -0.001 | 0.011 | -0.001 |
| Stdev | 0.043 | 0.047 | 0.064 | 0.046 | 0.098 | 0.018 |
| Skewness | -0.639 | -0.574 | -0.690 | 0.200 | -0.358 | -0.808 |
| Kurtosis | 1.191 | 1.534 | 2.143 | 1.061 | 4.179 | 7.958 |
| JB Test | 42.243 | 50.891 | 89.830 | 18.127 | 247.740 | 905.170 |
| P-value | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

The data used here for each index is the simple return after inflation, called the "real return." $r_{S\&P\ 500}$ = The real return on the S&P 500 index, $r_{MSCI\ EAFE}$ = The real return on the MSCI EAFE (Developed Market) index, $r_{MSCI\ EM}$ = The real return on the MSCI EM (Emerging Market) index, $r_{Gold}$ = The real return on gold, $r_{Oil}$ = The real return on oil, $r_{10YTN}$ = The real return on 10-year treasury note. Nobs represents the number of observations. Stdev means the standard deviation. The JB test means the Jarque-Bera test. The data frequency here is monthly.

As noted above and in Table 2.1, we consider different sample periods. Table 2.5 presents the average monthly real return and standard deviation of each series over the six sub-periods and, for comparison, the full sample period. Only during crisis periods of the Dot-com bubble crash and Great Recession do the three stock index series have a negative average (real) monthly return. For the COVID-19 health crisis, no stock series has an average negative return, and this is consistent with the short-lived nature of the associated stock price fall. Indeed, the return for this period is similar to non-crisis periods. For the oil and gold series, the average monthly return is positive in each sub-period (except the Dot-com boom for the former and the Great Recession for the latter series). The average monthly return on the 10-year Treasury note is positive during the Dot-com crash and the post-Great Recession recovery but is otherwise negative. This same series always exhibits the lowest standard deviation for all sub-periods, while oil is always the most volatile series across all periods.

**Table 2.5.** Mean and standard deviation across all periods.

| | $r_{S\&P\ 500}$ | | $r_{MSCI\ EAFE}$ | | $r_{MSCI\ EM}$ | |
|---|---|---|---|---|---|---|
| | Mean | Stdev | Mean | Stdev | Mean | Stdev |
| Whole period | 0.006 | 0.043 | 0.002 | 0.047 | 0.003 | 0.064 |
| Dot-com booming | 0.018 | 0.042 | 0.007 | 0.041 | 0.001 | 0.072 |
| Dot-com bursting | -0.017 | 0.053 | -0.019 | 0.047 | -0.016 | 0.070 |
| 2003-2007 | 0.007 | 0.025 | 0.013 | 0.031 | 0.024 | 0.050 |
| Great Recession | -0.024 | 0.070 | -0.028 | 0.086 | -0.021 | 0.117 |
| 2009-2019 | 0.009 | 0.036 | 0.003 | 0.043 | 0.002 | 0.050 |
| COVID-19 | 0.016 | 0.055 | 0.006 | 0.056 | 0.006 | 0.058 |

| | $r_{Gold}$ | | $r_{Oil}$ | | $r_{10YTN}$ | |
|---|---|---|---|---|---|---|
| | Mean | Stdev | Mean | Stdev | Mean | Stdev |
| Whole period | 0.004 | 0.046 | 0.008 | 0.098 | -0.001 | 0.018 |
| Dot-com booming | -0.006 | 0.035 | 0.008 | 0.092 | -0.002 | 0.022 |
| Dot-com bursting | 0.005 | 0.035 | 0.008 | 0.105 | 0.003 | 0.019 |
| 2003-2007 | 0.012 | 0.044 | 0.020 | 0.080 | -0.002 | 0.017 |
| Great Recession | 0.010 | 0.079 | -0.005 | 0.140 | -0.001 | 0.027 |
| 2009-2019 | 0.004 | 0.048 | 0.001 | 0.078 | 0.000 | 0.014 |
| COVID-19 | 0.006 | 0.045 | 0.024 | 0.174 | -0.002 | 0.013 |

The Dot-com bursting period from April 2000 to December 2002, the Great Recession period from December 2007 to June 2009, and the COVID-19 health crisis period from December 2019 to December 2021 are crisis periods. The non-crisis periods include the Dot-com booming period from January 1995 to March 2000, the 2003-2007 period, and the 2009-2019 period. The data frequency here is monthly.

## 2.4. Methodology

We divide the methodology section into two parts. In the first part, we utilise the DCC-GARCH model to examine potential interaction relationships among the six variables, as well as to obtain the time-varying correlation and correlation pattern for each pair of variables. In the second part, we investigate the benefit of portfolio diversification using two different portfolio strategies based on alternative portfolio choices and risk measures, and we also build in-sample rolling windows to simulate out-of-sample performance.

### *2.4.1 Dynamic Conditional Correlation GARCH Model*

Correlation is an essential input for asset allocation and risk assessment. In an international portfolio, portfolio managers typically minimise portfolio risk by seeking out markets or assets that are less correlated. Some recent literature shows that, with further globalisation, further consolidation among stock markets leads to higher levels of correlation between stock markets and a narrower range of diversification benefits (*e.g.,* Koch and Koch, 1991; Longin and Solnik, 1995; Kearney and Lucey, 2004; Driessen and Laeven, 2007). To understand changes in the level of correlation between sample markets, we used the dynamic conditional correlation GARCH model (DCC-GARCH Model) proposed by Engle (2002). The advantage of this model is that it has the flexibility of a multivariate GARCH model and can directly parameterise conditional correlations (Engle, 2002). We use this model to assess the interaction relationships between related variables and capture trends in correlations over time.

The DCC-GARCH model can be broken down into two main steps:

Step 1: Univariate GARCH Model

For each asset *i*, the return at time *t* can be expressed as:

$$\mathrm{r}_{i,\mathrm{t}} = u_i + \varepsilon_{i,\mathrm{t}} \tag{2.1}$$

Where $\mathrm{r}_{i,t}$ is the return of asset *i* at time t; $u_i$ is the mean return of asset *i*; $\varepsilon_{i,\mathrm{t}}$ *is* the error term for asset *i*.

The error term can be expressed as:

$$\varepsilon_{i,\mathrm{t}} = \sigma_{i,t} z_{i,\mathrm{t}} \tag{2.2}$$

Where $z_{i,\mathrm{t}}$is a sequence of i.i.d. (independent and identically distributed) random variables (commonly assumed to be normally distributed or t-distributed); $\sigma_{i,\mathrm{t}}$ is the conditional standard deviation (volatility) of asset *i* at time t.

The conditional variance $\sigma_{i,\mathrm{t}}^2$ is modelled using a GARCH (p, q) specification (p=1, q=1 is common):

$$\sigma_{i,\mathrm{t}}^2 = \alpha_{0,i} + \sum_{\mathrm{j}=1}^{p} \alpha_{\mathrm{j},i}\ \varepsilon_{i,\mathrm{t-j}}^2 + \sum_{k=1}^{q} \beta_{k,i} \sigma_{i,t-k}^2 \tag{2.3}$$

Where, $\alpha_{0,i} > 0,\ \alpha_{\mathrm{j},i} \geq 0,\ \beta_{k,i} \geq 0$ ; p and q are the orders of the GARCH model; $\alpha_{0,i}$ is a constant term that represents the long-term average variance of asset *i*; j is used to denote the lagged squared residuals (or shocks) from previous time periods, so $\varepsilon_{i,\mathrm{t-j}}^2$ is the squared residuals (shocks) of asset *i* from previous time periods t–j; ; $\alpha_{\mathrm{j},i}$ is the coefficients that measure the impact of past squared residuals (or shocks) on the current conditional variance

(ARCH terms); $\beta_{k,i}$ is the coefficients that measure the influence of past conditional variances on the current conditional variance (GARCH terms); k is used for lagged conditional variances, so $\sigma_{i,t-k}^{2}$ represents the conditional variances from previous time periods t−k; p is the order of the ARCH terms, indicating how many past squared shocks are included in the model; q is the order of the GARCH model, indicating how many past conditional variances are included.

Step 2: Dynamic Conditional Correlation

Once the univariate GARCH models are estimated, the next step is to model the dynamic correlations among the assets.

For each asset *i*, the return must be standardized as:

$$\hat{r}_{i,t} = \frac{r_{i,t} - u_i}{\sigma_{i,t}} \tag{2.4}$$

The Dynamic Conditional Correlation model, introduced by Engle (2002), can be represented as:

$$V_t = (1 - \alpha - \beta)\,\overline{V} + a(\hat{r}_{i,t-1}\hat{r}'_{i,t-1}) + \beta V_{t-1} \tag{2.5}$$

where $\overline{V}$ is the unconditional correlation matrix of the standardised returns; α and β are parameters that control the weights of the previous correlations and the lagged standardised returns.

The conditional covariance matrix $H_t$ is then constructed using the correlation matrix and the volatilities:

$$H_t = D_t V_t D_t \qquad D_t = \mathrm{diag}(\sigma_{1,t}, \sigma_{2,t}, \ldots \sigma_{n,t}) \tag{2.6}$$

where $H_t$ denotes the conditional variance-covariance matrix of returns at time t; $V_t$ defines the (k×k) time varying correlation matrix, which means $V_t$ varies over time; $D_t$ denotes the diagonal matrix of conditional standard deviations for each return series; The first instance of $D_t$ scales the correlation matrix $V_t$ to account for the volatilities of the assets ($D_t V_t$); The second instance of $D_t$ scales the resulting matrix from the previous operation ($D_t V_t D_t$).

The conditional covariance matrix $H_t$ can be represented as:

$$H_t = \begin{bmatrix} \sigma_{11,t} & \sigma_{12,t} & \cdots & \sigma_{1n,t} \\ \sigma_{21,t} & \sigma_{22,t} & \cdots & \sigma_{2n,t} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{n1.t} & \sigma_{n2,t} & \cdots & \sigma_{nn,t} \end{bmatrix} \tag{2.7}$$

The DCC process relies on the decomposition of the conditional co-variances as the product of conditional standard deviations and conditional correlations between two markets *i* and j such that:

$$\sigma_{ij,t} = \rho_{i,j,t}\sigma_{ii,t}\sigma_{jj,t} \tag{2.8}$$

$\sigma_{ij,t}$ denotes the conditional covariance between assets *i* and j at time t.

Therefore, for a pair of markets *i* and j, their conditional correlation at time *t* can be written as

$$\rho_{i,j,t} = \frac{(1-\alpha-\beta)\,\overline{\sigma}_{ij} + \alpha\varepsilon_{i,t-1}\varepsilon_{j,t-1} + \beta\sigma_{ij,t-1}}{[(1-\alpha-\beta)\,\overline{\sigma}_{ii} + a\varepsilon_{i,t-1}^2 + b\sigma_{ii,t-1}]^{1/2}[(1-\alpha-\beta)\,\overline{\sigma}_{jj} + \alpha\varepsilon_{j,t-1}^2 + \beta\sigma_{jj,t-1}]^{1/2}} \tag{2.9}$$

where $q_{i,j}$ is the element on the *i-th* line and *j-th* column of the matrix $V_t$. $\rho_{i,j,t}$ is the conditional correlation between assets *i* and j at time t; $\overline{\sigma}_{ij}$ is the unconditional correlation between the returns of assets *i* and j; $\sigma_{jj,t-1}$ is the conditional covariance between assets *i* and j at time t−1; $\sigma_{ii,t}$ and $\sigma_{jj,t}$ are the conditional variances of assets *i* and j, respectively; $\varepsilon_{i,t-1}$ and $\varepsilon_{j,t-1}$ are the standardised residuals (shocks) of assets *i* and j at time t−1; $\sigma_{ij,t-1}$ refers to the conditional covariance between the returns of assets *i* and j at time t−1.

### *2.4.2 Portfolio Design*

We consider four types of diversification opportunities for U.S. investors, including the U.S.-Only, Stock-Bond Portfolio (Portfolio 1), Portfolio 2, and Portfolio 3. The first one is the U.S.-only investment, which can be considered as domestic diversification and uses the S&P 500 index. Stock-Bond Portfolio consists of the S&P 500 index and the U.S. 10-Year Treasury Note. Here We adopt the well-known pension fund distribution principle, allocating 60% of the weight to the S&P 500 index and 40% to the U.S. 10-Year Treasury Note. Portfolio 2 is an international diversification portfolio constituted of the S&P 500 index, MSCI EAFE index, and MSCI EM index. Portfolio 3 is a cross-asset diversified portfolio consisting of the S&P 500 index, the U.S. 10-Year Treasury Note, and commodities (gold and oil). For Portfolios 2 and 3, we consider two investment strategies. The first one is the equally weighted portfolio (EWP) strategy. As a first portfolio strategy, we use the EWP as the benchmark, investing the same proportion of the investment budget in each portfolio component. The second one is the mean-variance portfolio (MVP) strategy, which aims to identify the portfolio that provides the highest returns for a given level of risk. Table 2.6 shows the two portfolio strategies for Portfolios 2 and 3.

**Table 2.6.** Portfolio strategies for Portfolios 2 and 3.

| | |
|---|---|
| 1 | Equally weighted portfolio (EWP) |
| 2 | Mean-variance portfolio (MVP) |

The equally weighted portfolio (EWP) and mean-variance portfolio (MVP) are two strategies for the internationally diversified portfolio (Portfolio 2) and asset diversified portfolio (Portfolio 3).

For the international diversified portfolio (Portfolios 2), we initially assume that investors invest equally to the S&P 500 index (1/3), MSCI EAFE index (1/3), and MSCI EM index (1/3), called the equally weighted Portfolio 2. Then we use the mean-variance portfolio strategy to obtain an optimised portfolio, called the mean-variance optimised Portfolio 2. This is mainly to determine whether American investors should diversify their investment portfolios internationally. For the cross-asset portfolio (Portfolio 3), we first assume that U.S. investors allocate equal weights to the S&P 500 index (1/4), the U.S. 10-Year Treasury Note (1/4), and gold (1/4) and oil (1/4), called the equally weighted Portfolio 3. Then, we use the equally weighted Portfolio 3 as the benchmark to find the mean-variance optimised Portfolio 3. This is for two primary purposes: First, to find an optimised cross asset portfolio with the highest returns for a given level of risk. Second, we are trying to estimate whether U.S. investors will benefit more from holding an asset-diversified portfolio, including stocks, bonds, and commodities.

*2.4.2.1 Portfolio calculations*

We use the inflation rate ($R_{\text{inflation}}$), which is used for deflating the nominal return on equity index ($R_{S\&P\ 500}$, $R_{MSCI\ EAFE}$, and $R_{MSCI\ EM}$ ), bonds ($R_{10\ YTN}$), gold ($R_{Gold}$), oil($R_{Oil}$) and T-bill ($R_f$) series to obtain the real index return ($r_{S\&P\ 500}$, $r_{MSCI\ EAFE}$, and $r_{MSCI\ EM}$), real bond return ($r_{10\ YTN}$), real gold return ($r_{Gold}$), real oil return ($r_{Oil}$) and real T-bill return ($r_f$). The monthly returns for each series are calculated in the usual way:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{2.10}$$

where $R_t$ denotes the simple monthly return (nominal return) for each index at time t, $P_t$ denotes the index price at time t, and $P_{t-1}$ denotes the index price at time t-1.

We then deflate the nominal return ($R_t$) into real return ($r_t$):

$$r_t = \frac{1+R_t}{1+R_{Inflation}} - 1 \tag{2.11}$$

The expected return on each investment portfolio is calculated as:

$$E(r_P) = \sum_{i=1}^{N} w_i r_{i,t} \tag{2.12}$$

With the excess return calculated as:

$$E(r_e) = r_t - r_f \tag{2.13}$$

where $r_t$ is the real monthly return for all variables and $r_f$ is the real 3-monthly Treasury-bill return. The Sharpe ratio (SR) is then calculated as:

$$\mathrm{SR} = \frac{\mathrm{E}(\mathrm{r_e})}{\sigma_\mathrm{r}} \tag{2.14}$$

where $\sigma_r$ is the standard deviation of the real return.

*2.4.2.2 Equally weighted and mean-variance portfolios*

In constructing the portfolios for Portfolio 2 and 3, we consider two strategies. First, an equally weighted portfolio (EWP) in which the same proportion is invested in each asset within the portfolio. In an EWP strategy, each asset in the portfolio holds a weight $w_i$= 1/N. Second is the mean-variance portfolio (MVP), which aims to identify the portfolio that provides the highest returns for a given level of risk.

The EWP strategy can be expressed as the solution of the following equations.

$$w = \begin{pmatrix} w_1 \\ w_2 \\ \cdots \\ w_\mathrm{N} \end{pmatrix} \tag{2.15}$$

where w is the $\mathrm{N} \times 1$ vector of portfolio weights. In the equally weighted portfolio, $w_1$=$w_2$=⋯=$w_\mathrm{N}$.

$$\mathrm{E}(\mathrm{r}) = \begin{pmatrix} \mathrm{E}(\mathrm{r}_1) \\ \mathrm{E}(\mathrm{r}_2) \\ \cdots \\ \mathrm{E}(\mathrm{r}_\mathrm{N}) \end{pmatrix} \tag{2.16}$$

where $\mathrm{E}(\mathrm{r})$ is the expected return and:

$$\mathrm{E(r_P)} = (\mathrm{E(r)})^{\top} w \tag{2.17}$$

where $\mathrm{E(r_P)}$ is expected return on portfolios and $(\mathrm{E(r)})^{\top}$ is the transpose of the expected return on assets.

After calculating the expected return for the equally weighted portfolio, we write the variance-covariance matrix of the return as follows:

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \cdots & \sigma_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \sigma_{N3} & \cdots & \sigma_{NN} \end{pmatrix} \tag{2.18}$$

where $\Sigma$ is the variance-covariance matrix of the asset returns. The elements on the leading-diagonal of $\Sigma$ are the variances of each of the component asset returns. The off-diagonal elements are the corresponding co-variances.

The MVP strategy can be expressed as the solution of the following equations:

$$\max \left( \frac{(\mathrm{E(r)})^{\top} w - \mathrm{r_f}}{\sqrt{w^{\top} \Sigma w}} \right) \tag{2.19}$$

where w represents the weight invested in each asset, $w^{\top}$ is the transpose of the weight on assets, $(\mathrm{E(r)})^{\top}$ is the transpose of the expected return, and $\Sigma$ represents the corresponding covariance matrix of the returns. The numerator of the objective function denotes the excess returns of the investment over that of a risk-free rate ($\mathrm{r_f}$) and the denominator represents the risk of the investment. The objective is to maximise the Sharpe ratio.

In both EWP and MVP strategies, we exclude short sales by assuming the following general constraint:

$$\sum_{i=1}^{N} w_i = 1, \; 0 \leq w_i \leq 1 \qquad (2.20)$$

### *2.4.3 In-sample Rolling Windows and Out-of-sample Simulation*

*2.4.3.1 In-sample rolling windows*

To examine the performance of the portfolios, we consider two approaches to enhance the robustness of the results. We first build portfolios for the full sample and each sub-period for each of our strategies (i.e., four fixed weight and two optimised portfolios) and compare their results. Second, to account for any "look-ahead" bias and to add robustness to the results, we generate out-of-sample portfolios. Here, we utilise 24-month rolling windows to construct a portfolio over the sample period for each portfolio strategy, thereby creating 300 rolling windows across our sample set. At the end of the month T ($T = t + 23$, and $t = 1 \ldots n$), we use the return series from the month t to month t + 23 (i.e., the previous 24 months) to derive the in-sample estimates of the parameters for each strategy. This allows calculation of the in-sample performance for the previous 24 months. Using the in-sample values, including the calculated optimal portfolio weight ($w_{i,T}$), we then construct a portfolio for the next, out-of-sample, month. For example, in our sample, the first in-sample estimation window is from January 1995 to December 1996, and we use the optimal weight derived from this in-sample to estimate the out-of-sample portfolio results for January 1997. This rolling procedure operates through the rest of the sample period.

*2.4.3.2 Transaction costs*

In addition to the out-of-sample performance, we take the effect of transaction cost (C) into consideration. In the stock market, the transaction costs can vary widely based on several factors, including the type of broker you use, the size of your transactions, and the specific stock or asset being traded. In our out-of-sample simulation, as our purpose is to observe how the transaction cost affects the return of each type of portfolios, we implement a one-way transaction cost (C) of 0.05% for each trade to all types of portfolios, according to Campbell and Thompson (2008) and Hsu et al. (2018). We define $\mathrm{r}_{i,\mathrm{T}}$ as the real return of the $i$-*th* asset in month T, and set $\sum_{i=1}^{N} r_{i,T} w_{i,\mathrm{T}}$ as the real portfolio return before re-balancing at the end of month T. When the portfolio is re-balanced in the beginning of month T + 1, it yields a trade in each asset with a magnitude of $|w_{i,\mathrm{T}+1} - w_{i,\mathrm{T}}|$, where $w_{i,T}$ is the optimal portfolio weight of each asset in the end of month T, $w_{i,\mathrm{T}+1}$ represents the calculated optimal portfolio weight in each asset in the beginning of month T + 1. We set C as the proportional transaction costs (0.05%), and then the trading costs for all assets are $C \times \sum_{i}^{N} |w_{i,\mathrm{T}+1} - w_{i,\mathrm{T}}|$. Therefore, the net return after the transaction costs for each portfolio strategy in month T + 1 is calculated as:

$$\mathrm{E(r)}^{\mathrm{p}} = (1 + \sum_{i=1}^{N} r_{i,\mathrm{T}+1} w_{i,\mathrm{T}+1})(1 - C \times \sum_{i}^{N} |w_{i,\mathrm{T}+1} - w_{i,\mathrm{T}}|) - 1 \qquad (2.21)$$

where $r_{i,T+1}$ is the real return in month T + 1 for each asset. We consider the gross return as the situation when then transaction cost (C) is zero.

## 2.5. Empirical Results

### ***2.5.1 Dynamic Conditional Correlation Results and Fixed Correlation Results***

#### *2.5.1.1 Dynamic conditional correlation results*

The trend in correlations between markets over time is critical for investors. Therefore, the ARMA (0,0)-DCC-GARCH (1,1) model is employed to detect whether there is an interaction relationship between variables. Second, we obtain trend graphs of the correlation between markets over time through model estimation.

To capture trends in correlations across related markets over time, we establish the ARMA (0,0)-DCC-GARCH (1,1) model and conduct a model evaluation. The model estimation results are shown in Table 2.7 and Table 2.8. The results show a spillover effect between each pair of return series. We do not explain the spillover effect in detail here because the model's primary purpose is to obtain the time-varying correlation between each time series and the change trend graph of the time-varying correlation. Therefore, after obtaining the model evaluation results, we extract the time-varying correlation data between each series and draw the trend graph of the time-varying correlation between each series. Figure 2.4 and Figure 2.5 show the trend graphs of the correlation between each return series over time. As can be seen from the figure, the correlation between each return series changes over time. We now analyse the correlation trend between each pair of return series in detail. We focus on and compare the changing trends of the correlations between the series during the crisis and non-crisis periods.

Figure 2.4 illustrates a deep V-shaped correlation between the S&P 500 index and the MSCI EAFE index during the Dot-com booming period. It goes down from 1995 to the middle of 1997, and it goes up dramatically from the middle of 1997 to the end of 1998. It is worth noting that before the Dot-com bubble bursts, there is a momentary drop from 1999 to the beginning of 2000, and after the bubble bursts, it rises again until the middle of 2003. There is a drop before the Great Recession, and after that, there is a sharp rise from early 2008 to the end of 2009. In addition, the correlation between the S&P 500 index and MSCI EAFE index has had a fluctuating upward trend since the middle of 1997. We notice an intriguing pattern in the correlation between the S&P 500 index and MSCI EAFE index during the COVID-19 health crisis period, i.e., it has a momentary sharp rise in early 2020, but then it falls sharply until later 2021. Now we turn to the correlation between the S&P 500 index and the MSCI EM index. We observe a significant increase from the middle of 1998 to the end of 1998, followed by a decline until the beginning of the Dot-com crisis and a significant increase during the Dot-com Bursting period and the Great Recession, respectively. Before the Dot-com bubble bursts, it also shows a short fall. We also observe a slight upward trend in the correlation between the S&P 500 index and MSCI EM index. Conclusively, the existing empirical evidence (*e.g.,* Roll, 1988; Bertero and Mayer, 1990; King and Wadhwani, 1990; Solnik et al., 1996; Butler and Joaquin, 2002; Guidi and Ugur, 2014) suggests that the correlation between stock markets in crisis periods is higher than in non-crisis periods. Our results of the time-varying correlation also support this argument.

In Figure 2.5, we can see that the correlation between the S&P 500 index and gold is remarkably fluctuating and interesting. From the graph, we can see that during the Dot-com booming period, it shows a deep V pattern, and after that, it shows a sharp downward during the Dot-com bust period. It also shows a deep V pattern during the Great Recession. Interestingly, during the first half of the Great Recession, there is a sharp downward trend, but during the second half of the Great Recession, there is an obvious upward trend that continues until the end of 2009.

The correlation between the S&P 500 index and oil has a downward trend before the Dot-com bubble bursts and an upward trend during the Dot-com Bursting period. The same trend is found during the Great Recession and the COVID-19 pandemic. It is worth noting that after the rise in the Great Recession, it has not fallen back to pre-recession levels. Furthermore, the correlation between the S&P 500 index and the U.S. 10-year Treasury note is typically negative.

More specifically, it exhibited a downward trend during the Dot-com Bursting period, followed by an apparent upward trend from 2003 to 2007. However, during the Great Recession, it experienced a period of initial decline followed by a subsequent rise. From 2009 to 2019, it had no obvious upward or downward trend despite some fluctuations. During the C0VID-19 crisis, it exhibits a V-shaped pattern, with a sudden decline during the onset of the pandemic, followed by an immediate recovery.

**Table 2.7.** Estimates of ARMA (0,0)-DCC-GARCH model for return series from the international diversified portfolio.

| | Estimate | Std. Error | t value | P-value |
|---|---|---|---|---|
| [$r_{S\&P\ 500}$].mu | 0.007 | 0.002 | 3.793 | 0.000 *** |
| [$r_{S\&P\ 500}$]. omega | 0.000 | 0.000 | 1.504 | 0.133 |
| [$r_{S\&P\ 500}$]. alpha1 | 0.201 | 0.061 | 3.292 | 0.001 *** |
| [$r_{S\&P\ 500}$]. beta1 | 0.758 | 0.081 | 9.376 | 0.000 *** |
| [$r_{MSCI\ EAFE}$].mu | 0.004 | 0.002 | 1.513 | 0.130 |
| [$r_{MSCI\ EAFE}$]. omega | 0.000 | 0.000 | 1.201 | 0.230 |
| [$r_{MSCI\ EAFE}$]. alpha1 | 0.135 | 0.059 | 2.302 | 0.021 ** |
| [$r_{MSCI\ EAFE}$]. beta1 | 0.791 | 0.106 | 7.443 | 0.000 *** |
| [$r_{MSCI\ EM}$]. mu | 0.005 | 0.003 | 1.501 | 0.133 |
| [$r_{MSCI\ EM}$]. omega | 0.000 | 0.000 | 2.209 | 0.027 ** |
| [$r_{MSCI\ EM}$]. alpha1 | 0.164 | 0.053 | 3.086 | 0.002 *** |
| [$r_{MSCI\ EM}$]. beta1 | 0.756 | 0.058 | 13.129 | 0.000 *** |
| [Joint] dcca1 | 0.036 | 0.012 | 3.085 | 0.002 *** |
| [Joint] dccb1 | 0.938 | 0.023 | 41.623 | 0.000 *** |
| Log-Likelihood | 1941.328 | | | |

The ARMA (0,0)-DCC-GARCH (1,1) model is utilised to examine the interaction relationships between asset real returns. The mu parameter denotes the mean or average real return of the analysed asset. Omega serves as the constant term in the GARCH model, representing the baseline level of volatility and indicating the long-term average volatility of the returns. The alpha1 parameter gauges the short-term response of volatility to shocks in asset returns. The beta1 parameter reflects the persistence of volatility over time, capturing how past volatility influences current volatility. Std. Error denotes the standard error. The dcca1 parameter reflects the dynamic conditional correlation between the asset real returns. The dccb1 indicates strong persistence in the correlation between the asset real returns. Log-Likelihood indicates the goodness of fit of the model. The p-values associated with these estimates are reported below them, with *, **, and *** denoting significance levels of 10%, 5%, and 1%, respectively. The data frequency here is monthly.

**Table 2.8.** Estimates of ARMA (0,0)-DCC-GARCH model for return series from the cross-asset portfolio.

| | Estimate | Std. Error | t-value | P-value |
|---|---|---|---|---|
| [$r_{S\&P\ 500}$]. mu | 0.007 | 0.002 | 3.787 | 0.000 *** |
| [$r_{S\&P\ 500}$]. omega | 0.000 | 0.000 | 1.523 | 0.128 |
| [$r_{S\&P\ 500}$]. alpha1 | 0.201 | 0.061 | 3.266 | 0.001 *** |
| [$r_{S\&P\ 500}$]. beta1 | 0.758 | 0.081 | 9.407 | 0.000 *** |
| [$r_{Gold}$]. mu | 0.004 | 0.002 | 1.515 | 0.130 |
| [$r_{Gold}$]. omega | 0.000 | 0.000 | 0.977 | 0.328 |
| [$r_{Gold}$]. alpha1 | 0.103 | 0.060 | 1.704 | 0.088 * |
| [$r_{Gold}$]. beta1 | 0.834 | 0.112 | 7.429 | 0.000 *** |
| [$r_{Oil}$l]. mu | 0.010 | 0.004 | 2.219 | 0.026 ** |
| [$r_{Oil}$]. omega | 0.001 | 0.001 | 1.651 | 0.099 * |
| [$r_{Oil}$]. alpha1 | 0.294 | 0.163 | 1.803 | 0.071 * |
| [$r_{Oil}$]. beta1 | 0.651 | 0.116 | 5.627 | 0.000 *** |
| [$r_{10YTN}$]. mu | -0.001 | 0.001 | -0.834 | 0.405 |
| [$r_{10YTN}$]. omega | 0.000 | 0.000 | 0.000 | 1.000 |
| [$r_{10YTN}$]. alpha1 | 0.000 | 0.000 | 18.331 | 0.000 *** |
| [$r_{10YTN}$]. beta1 | 0.999 | 0.000 | 357550.000 | 0.000 *** |
| [Joint]dcca1 | 0.020 | 0.006 | 3.083 | 0.002 *** |
| [Joint]dccb1 | 0.959 | 0.015 | 65.018 | 0.000 *** |
| Log-Likelihood | 2332.627 | | | |

The ARMA (0,0)-DCC-GARCH (1,1) model is utilised to examine the interaction relationships between asset real returns. The mu parameter denotes the mean or average real return of the analysed asset. Omega serves as the constant term in the GARCH model, representing the baseline level of volatility and indicating the long-term average volatility of the returns. The alpha1 parameter gauges the short-term response of volatility to shocks in asset returns. The beta1 parameter reflects the persistence of volatility over time, capturing how past volatility influences current volatility. Std. Error denotes the standard error. The dcca1 parameter reflects the dynamic conditional correlation between the asset real returns. The dccb1 parameter indicates strong persistence in the correlation between the asset real returns. Log-Likelihood indicates the goodness of fit of the model. The p-values associated with these estimates are reported below them, with *, **, and *** denoting significance levels of 10%, 5%, and 1%, respectively. The data frequency here is monthly.

**Figure 2.4.** Patterns of time-varying correlation between S&P 500 index and other two stock series.

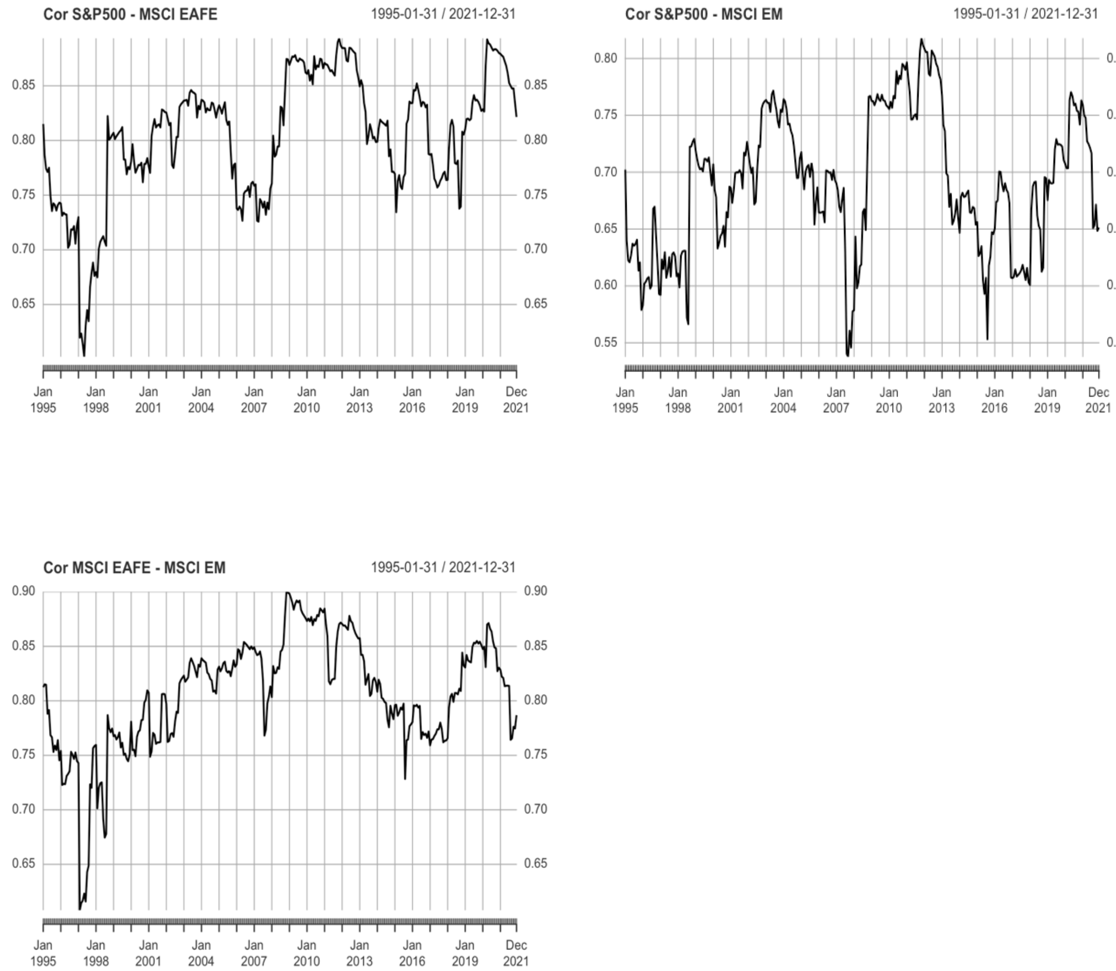


**Figure 2.5.** Patterns of the time-varying correlation between the S&P 500 index and the three non-stock series.

### *2.5.1.2 Fixed correlation results*

In computing optimal, mean-variance, efficient portfolios, correlations between assets are needed. Table 2.9 provides the fixed correlations both for the full sample and each sub-sample period. The results in the table show that the correlation between the S&P 500 index, EAFE index, and EM index is higher than the correlation between the S&P 500 index, gold, oil, and 10-year Treasury note, for both the full sample and each sub-sample period. Further, we find that the correlation between S&P 500 and EAFE is lower during the 1995-2000 period and then jumped from 0.40 to 0.68 with the Dot-com bubble burst. Afterwards, it declines before increasing through the remainder of the sample. A similar pattern is observed between the S&P 500 and EM indexes, although the correlation plateaus more after the Great Recession period.

The correlation between the S&P 500 index and gold is low throughout the sample and is negative for two of the sub-periods. Notwithstanding, there is a notable increase in value in the last period. The correlation between the S&P 500 index and oil is also low at the start of the sample (but greater than with gold) and demonstrates a notable change from the Great Recession period onwards. The correlation between the S&P 500 index and the U.S. 10-Year Treasury Note, except during the 1995-2000 period, is negative throughout. As noted above, the correlation with gold and the 10-Year Treasury-Note indicates the potential to hedge against stock market risks.

The results in Table 2.9 support the argument of a time-varying correlation and for which the correlation between stock markets in a crisis period is higher than in a non-crisis period (Roll, 1988; Bertero and Mayer, 1990; King and Wadhwani, 1990; Solnik et al., 1996; Butler and Joaquin, 2002; Guidi and Ugur, 2014). We also observe a general upward trend in the correlation between the S&P 500, EAFE, and EM indexes, while correlations also appear to strengthen between the S&P 500 index and the alternative assets, albeit negatively with the 10-Year Treasury-Note.

**Table 2.9.** Fixed Correlations.

| | Cor S&P 500 - MSCI EAFE | Cor S&P 500 - MSCI EM | Cor MSCI EAFE - MSCI EM |
|---|---|---|---|
| Whole period | 0.592 | 0.476 | 0.603 |
| Dotcom booming | 0.398 | 0.358 | 0.413 |
| Dot-com bursting | 0.682 | 0.633 | 0.617 |
| 2003-2007 | 0.564 | 0.496 | 0.654 |
| Great Recession | 0.789 | 0.661 | 0.825 |
| 2009-2019 | 0.623 | 0.488 | 0.632 |
| COVID-19 | 0.693 | 0.467 | 0.587 |

| | Cor S&P 500 - Gold | Cor S&P 500 - Oil | Cor S&P 500 - 10YTN |
|---|---|---|---|
| Whole period | 0.006 | 0.145 | -0.106 |
| Dotcom booming | 0.004 | -0.088 | 0.165 |
| Dot-com bursting | -0.057 | 0.087 | -0.299 |
| 2003-2007 | 0.116 | -0.139 | -0.115 |
| Great Recession | -0.088 | 0.287 | -0.076 |
| 2009-2019 | 0.037 | 0.302 | -0.145 |
| COVID-19 | 0.220 | 0.427 | -0.060 |

| | Cor Gold - Oil | Cor Gold - 10YTN | Cor Oil - 10YTN |
|---|---|---|---|
| Whole period | 0.124 | 0.142 | -0.109 |
| Dotcom booming | 0.055 | 0.023 | 0.010 |
| Dot-com bursting | 0.235 | 0.167 | -0.068 |
| 2003-2007 | 0.163 | 0.030 | 0.016 |
| Great Recession | -0.006 | 0.263 | -0.275 |
| 2009-2019 | 0.131 | 0.213 | -0.193 |
| COVID-19 | 0.220 | 0.267 | -0.247 |

The correlation here is the Pearson correlation. The data used here for each index is the return after inflation, called the "real return." The correlation is calculated by real returns of each asset. The data frequency here is monthly.

### *2.5.2 Portfolio Comparisons*

We build three diversified portfolios for U.S. investors to compare against the U.S.-only portfolio. The stock-bond portfolio consists of the S&P 500 index and U.S. 10-year Treasury Note. For the stock-bond portfolio, we use a well-known portfolio allocation that invests 60% in the S&P 500 index and 40% in the U.S. 10-year Treasury Note. Portfolio 2 invests in the S&P 500 index, MSCI EAFE index (a developed market index), and MSCI EM index (emerging markets index). Portfolio 3 consists of the S&P 500 index and three commodity assets, including gold, oil, and the U.S. 10-year Treasury Note. For the international diversified portfolio and the cross-asset diversified portfolio, we adopt two strategies, namely the equally weighted strategy and the mean-variance optimised strategy. We use the standard deviation to measure the risk. To compare the performance of portfolio options and strategies, we consider the Sharpe ratio (SR) a performance measure. The Sharpe ratio is introduced by Nobel laureate William F. Sharpe in 1966 (Sharpe, 1966), which is used to help investors assess the return of an investment compared to its risk. Table 2.10 presents the results for the equally weighted cross- country portfolio and equally weighted cross-asset portfolio against the U.S.-only and the stock (60%)-bond (40%) portfolio.

#### *2.5.2.1 Performance of the equally weighted portfolios*

Table 2.10 presents the performance results for the U.S.-only with the three diversified portfolios, where the stock-bond portfolio allocates 60% of its weight to stocks and 40% to bonds, and the cross-market (Portfolio 2) and cross-asset (Portfolio 3) are equally weighted. Panel A of Table 2.10 provides the results for the U.S.-only. The U.S.-only produces 0.63% average monthly real return, 0.64% average monthly excess return, and 4.32% standard deviation for the whole period. It is worth noting that the monthly average real excess return in the U.S.-only is higher than the average monthly real return over the whole sample period because the average monthly real risk-free rate is negative. During the three non-crisis periods, average monthly real returns, real excess returns, and the Sharpe ratios of the U.S.-only are all positive. Among the three crisis periods, during the Dot-com bursting crisis period and the Great recession period, the average monthly real returns, real excess returns, and Sharpe ratios of the U.S.-only are all negative, while during the COVID-19 crisis, they are all positive. More precisely, during the Dot-com bursting period, compared to the previous period (the Dot-com booming period), the average monthly return in the U.S.-only falls by 194.05%, and the standard deviation increases by 26.55%. During the Great Recession, the average monthly real return in the U.S.-only falls by 459.81%, compared to the previous period (2003-2007), and exhibits tremendous volatility, with a 6.99% standard deviation, which is a dramatic increase of 178.91% from the previous period. In contrast, during the COVID-19, the average monthly real returns in the U.S.-only is 1.58%, increasing by 73.45% compared to the previous period (2009-2019). Another phenomenon during COVID-19 is that although the average monthly real return increased dramatically, the Sharpe ratio is slightly higher than that in the previous period (2009-2019) due to the sharply increased standard deviation with 5.52%.

Panel B of Table 2.10 provides the results for the stock (60%) - bond (40%) portfolio (Stock-Bond Portfolio). Over the whole sample period, Stock-Bond Portfolio delivered a 0.34% average monthly real return, a 0.35% real excess return, and a 2.45% standard deviation. Due to the negative average monthly risk-free rate, the real excess return is also higher than the average monthly real return.

During the three non-crisis periods, average monthly real returns, real excess returns, and the Sharpe ratios of Portfolio1 are all positive. Among the three crisis periods, during the Dot-com bursting crisis period and the Great recession period, the average monthly real returns, real excess returns, and Sharpe ratios are all negative, while during the COVID-19 crisis, they are all positive again. More precisely, during the Dot-com bursting period, compared to the previous period (the Dot-com booming period), the average monthly return in the U.S.-only fall by 190.11%, and the standard deviation increases only by 3.24%. During the Great Recession, the average monthly real return falls by 579.29%, compared to the previous period (2003-2007), and exhibits tremendous volatility, with a 6.99% standard deviation, which is a dramatic increase of 168.51% from the previous period. However, it displays a different pattern during the COVID-19 health crisis. Its monthly real return is 0.87%, increased by 64.15% compared to the period 2009-2019, while its standard deviation during this period also increases significantly compared to the previous period (2009-2019), by 51.80%, so its Sharpe ratio only rises by 15.40% compared to the previous period.

Panel C of Table 2.10 provides the results for the equally weighted internationally diversified portfolio (Portfolio 2). It produces 0.37% average monthly real return, and 4.74% standard deviation for the whole period. During the three non-crisis periods, its average monthly real returns, real excess returns, and Sharpe ratios are positive. Among the three crisis periods, during the Dot-com bursting crisis period and the Great recession period, its average monthly real returns, real excess returns, and Sharpe ratios are all negative, while during the COVID-19 crisis are positive. In detail, during the Dot-com bursting period, its average monthly return falls by 300.19%, and the standard deviation increases by 15.57%, compared to the previous period (the Dot-com booming period). During the Great Recession, its average monthly real return decreases by 267.21% compared to the previous period (2003-2007). It also exhibits considerable volatility, with a standard deviation of 8.64%, which is a dramatic increase of 167.29% from the previous period. However, it displays a different pattern during the COVID-19 health crisis. Although its standard deviation during this period slightly increases compared to the previous period (2009-2019), its average monthly real return is 0.92%, increasing by 93.46% compared to the previous period (2009-2019).

Panel D of Table 2.10 provides the results for the equally weighted cross-asset portfolio (Portfolio 3). We can see that over the whole sample period, it produces an average of 0.43% monthly real return, 0.44% monthly excess return, and 3.29% standard deviation. During the three non-crisis periods, the average monthly real returns, real excess returns, and Sharpe ratios of the Equally weighted Portfolio 3 are positive. Among the three crisis periods, during the Dot-com bursting crisis period and the Great recession period, its average monthly real return, real excess return, and Sharpe ratios are negative, while during the COVID-19 crisis, they are positive. More accurately, during the Dot-com bursting period, its average monthly return falls by 105.20%, and the standard deviation increases by 14.83% compared to the previous period (the Dot-com booming period). During the Great Recession, its average monthly real return falls by 153.97% compared to the previous period (2003-2007). It also exhibits substantial volatility, with a standard deviation of 5.34%, a dramatic increase of 111.16% from the previous period. During COVID-19, its average monthly real return is 1.10%, increasing dramatically by 234.73% compared to the previous period (2009-2019), and its standard deviation is 5.4%, increasing by 88.83% compared to the previous period (2009-2019).

Overall, in Table 2.10, we have the following findings. First, over the full sample period, all four portfolios have positive average real monthly returns and positive Sharpe ratios. Second, during the three non-crisis periods, the U.S-only, the stock (60%) - bond (40%), the equally weighted Portfolio 2, and equally weighted Portfolio 3 all has positive average real monthly returns and positive Sharpe ratios. Third, during the Dot-com bursting crisis and the Great Recession, all four benchmark portfolios have negative average real monthly returns and negative Sharpe ratios. In contrast, during the COVID-19 health crisis, all four portfolios have positive average real monthly returns and Sharpe Ratios, and their Sharpe ratios all increase compared with the previous period, although their volatility rise compared with the period 2009-2019.

**Table 2.10.** The results for the four portfolio benchmarks over different periods.

**Panel A. U.S.-only**

| | Weight Allocation | Real risk- | | | | | Changes from the last period | | |
|---|---|---|---|---|---|---|---|---|---|
| | S&P 500 | free rate | Ave.ret. | Real excess | Std Dev | Sharpe ratio | Ave.ret. | Std Dev | Sharpe ratio |
| Whole period | 100.00% | -0.01% | 0.63% | 0.64% | 4.32% | 14.76% | | | |
| Dot-com booming | 100.00% | 0.23% | 1.78% | 1.55% | 4.16% | 37.21% | | | |
| Dot-com bursting | 100.00% | 0.08% | -1.67% | -1.75% | 5.27% | -33.18% | -194.05% | 26.55% | -189.18% |
| 2003-2007 | 100.00% | 0.01% | 0.68% | 0.67% | 2.50% | 26.71% | | | |
| Great Recession | 100.00% | -0.12% | -2.44% | -2.32% | 6.99% | -33.19% | -459.81% | 178.91% | -224.25% |
| 2009-2019 | 100.00% | -0.09% | 0.91% | 1.01% | 3.62% | 27.78% | | | |
| COVID-19 | 100.00% | -0.22% | 1.58% | 1.81% | 5.52% | 32.68% | 73.45% | 52.51% | 17.64% |

**Panel B. Portfolio 1**

| | Weight Allocation | | Real risk- | | | | | Changes from the last period | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | S&P 500 | 10 YTN | free rate | Ave.ret. | Real excess | Std Dev | Sharpe ratio | Ave.ret. | Std Dev | Sharpe ratio |
| Whole period | 60.00% | 40.00% | -0.01% | 0.34% | 0.35% | 2.54% | 13.71% | | | |
| Dot-com booming | 60.00% | 40.00% | 0.23% | 0.98% | 0.76% | 2.70% | 28.06% | | | |
| Dot-com bursting | 60.00% | 40.00% | 0.08% | -0.89% | -0.96% | 2.78% | -34.64% | -190.11% | 3.24% | -223.44% |
| 2003-2007 | 60.00% | 40.00% | 0.01% | 0.31% | 0.30% | 1.53% | 19.70% | | | |
| Great Recession | 60.00% | 40.00% | -0.12% | -1.49% | -1.37% | 4.11% | -33.27% | -579.29% | 168.51% | -268.85% |
| 2009-2019 | 60.00% | 40.00% | -0.09% | 0.53% | 0.62% | 2.05% | 30.39% | | | |
| COVID-19 | 60.00% | 40.00% | -0.22% | 0.87% | 1.09% | 3.12% | 35.07% | 64.15% | 51.80% | 15.40% |

**Panel C. Equally weighted Portfolio 2**

| | Weight Allocation | | | Real risk- | | Real | | | Changes from the last period | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | S&P 500 | EAFE | EM | free rate | Ave.ret. | excess | Std Dev | Sharpe ratio | Ave.ret. | Std Dev | Sharpe ratio |
| Whole period | 33.00% | 33.00% | 33.00% | -0.01% | 0.37% | 0.37% | 4.74% | 7.90% | | | |
| Dot-com booming | 33.00% | 33.00% | 33.00% | 0.23% | 0.86% | 0.63% | 4.50% | 14.08% | | | |
| Dot-com bursting | 33.00% | 33.00% | 33.00% | 0.08% | -1.73% | -1.80% | 5.21% | -34.65% | -300.19% | 15.57% | -346.01% |
| 2003-2007 | 33.00% | 33.00% | 33.00% | 0.01% | 1.45% | 1.44% | 3.23% | 44.61% | | | |
| Great Recession | 33.00% | 33.00% | 33.00% | -0.12% | -2.43% | -2.31% | 8.64% | -26.68% | -267.21% | 167.29% | -159.81% |
| 2009-2019 | 33.00% | 33.00% | 33.00% | -0.09% | 0.48% | 0.57% | 4.02% | 14.20% | | | |
| COVID-19 | 33.00% | 33.00% | 33.00% | -0.22% | 0.92% | 1.14% | 5.18% | 22.09% | 93.46% | 28.98% | 55.58% |

**Panel D. Equally weighted Portfolio 3**

| | Weight Allocation | | | | Real risk- | | Real | | Sharpe | Changes from the last period | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | S&P 500 | Gold | Oil | 10 YTN | free rate | Ave.ret. | excess | Std Dev | ratio | Ave.ret. | Std Dev | Sharpe ratio |
| Whole period | 25.00% | 25.00% | 25.00% | 25.00% | -0.01% | 0.43% | 0.44% | 3.29% | 13.25% | | | |
| Dot-com booming | 25.00% | 25.00% | 25.00% | 25.00% | 0.23% | 0.44% | 0.21% | 2.75% | 7.81% | | | |
| Dot-com bursting | 25.00% | 25.00% | 25.00% | 25.00% | 0.08% | -0.02% | -0.10% | 3.15% | -3.17% | -105.20% | 14.83% | -140.55% |
| 2003-2007 | 25.00% | 25.00% | 25.00% | 25.00% | 0.01% | 0.92% | 0.91% | 2.53% | 35.90% | | | |
| Great Recession | 25.00% | 25.00% | 25.00% | 25.00% | -0.12% | -0.49% | -0.37% | 5.34% | -7.00% | -153.97% | 111.16% | -119.51% |
| 2009-2019 | 25.00% | 25.00% | 25.00% | 25.00% | -0.09% | 0.33% | 0.42% | 2.86% | 14.81% | | | |
| COVID-19 | 25.00% | 25.00% | 25.00% | 25.00% | -0.22% | 1.10% | 1.33% | 5.40% | 24.55% | 234.73% | 88.83% | 65.70% |

The table above shows the performance results for the U.S.-only with three different diversified portfolios throughout the entire period and in six sub-sample periods. In the stock-bond portfolio, 60% is invested in stocks and 40% in bonds, while the cross-market (Portfolio 2) and cross-asset (Portfolio 3) portfolios have equal weights in the table above. Ave.ret. means the average real return. Real excess is the real excess return. Std Dev is the standard deviation. The data frequency here is monthly. EAFE = MSCI EAFE, EM = MSCI EM.

### *2.5.2.2 Comparison between the four portfolio benchmarks*

Table 2.11 presents a more direct comparison between the four portfolios over the different periods, with a ranking based on the Sharpe ratio. Each panel within the table presents results for the full, and different sub-sample periods. Panel A of Table 2.11 displays the results of the four portfolio benchmarks for the whole sample period. In the whole sample period, U.S.-only has the highest average real monthly return and the highest Sharpe ratio among the four portfolio benchmarks, while the stock-bond portfolio has the lowest real return and smallest standard deviation that makes it rank number 2. In contrast, the cross-market portfolio has the highest volatility among four benchmarks. As we can see from the table, in the full sample period, none of the other portfolios outperforms the U.S.-only in terms of the real return and Sharpe ratio.

Panel B of Table 2.11 presents the performance of each portfolio opportunity for six sub-sample periods one by one. In Panel B-1 of Table 2.11, during the Dot-com booming period, compared with the other three investment opportunities, U.S.-only still has the highest real return with 1.78% and Sharpe ratio with 37.21, while the stock - bond portfolio still has the lowest standard deviation 2.70% and still ranks in the second. However, during this period, the equally weighted cross-asset portfolio has the lowest standard deviation and the lowest real return that makes it the worst performer compared to the other investing options. In this period, the U.S.-only is still the best performer.

From Panel B-2 of Table 2.11, we can see that the average monthly real returns and Sharpe ratios for all investment opportunities are negative during the Dot-com bursting period. The stock-bond portfolio has the lowest standard deviation among all investing options, but the best performer during this period is the equally weighted Portfolio 3 whose real return is not as negative as other investing options. In contrast, the equally weighted Portfolio 2 is the worst performer, with the lowest monthly real return and Sharpe ratio but a high volatility. The U.S.-only and the Stock-Bond Portfolio are ranked number 2 and number 3, respectively.

During the non-crisis period from 2003 to 2007, the performance of all four investment opportunities improves compared to the Dot-com bursting period. In Panel B-3 of Table 2.11, the equally weighted Portfolio 2 wins with the highest average monthly return and highest Sharpe ratio compared to the other three investment opportunities, even though it also has the highest standard deviation. The equally weighted Portfolio 3 has a slightly lower average monthly return and Sharpe ratios than the equally weighted Portfolio 2, which make it rank in second. Furthermore, during this period, although the stock-bond portfolio has the smallest standard deviation, it still has the lowest Sharpe ratio among four investing options because its real monthly return is much lower than other investing options.

During the Great Recession, we find that all four investment opportunities have negative average monthly real returns, as they do during the Dot-com bubble. As we can see in Panel B-

4 of Table 2.11, the best performer is the equally weighted Portfolio 3, mainly because the average monthly return of the Equally weighted Portfolio 3 is not as negative as that of the other investment opportunities, and also the standard deviation is 23.45% lower than the U.S.-only. Surprisingly, the U.S.-only has the most negative return among all four investing options, and its standard deviation is also high with 6.99%. The Stock-Bond portfolio is the worst performer during this period, although it has the lowest standard deviation, but its Sharpe ratio is still the lowest.

From Panel B-5 of Table 2.11, we find that during the non-crisis period from 2009 to 2019, the U.S.-only outperforms other investment opportunities in terms of average monthly real return and Sharpe ratio. In addition, the performance of the stock-bond portfolio is also better than the equally weighted cross-market portfolio and the equally weighted cross asset portfolio, and it benefits from its low volatility, while the equally weighted cross-market portfolio is the worst performer during this period compared to other investment opportunities.

During the COVID-19 health crisis period, we find that the average monthly real returns for all four investment opportunities are positive, which delivers the fact that the performance of these four investment opportunities during the COVID-19 health crisis period is completely different from the previous two crisis periods.

The Sharpe ratios in Stock-Bond Portfolio is 7.30% higher than that in the U.S.-only, while the Sharpe ratios in the equally weighted Portfolio 2 and the equally weighted Portfolio 2 are 45.66% and 24.89% lower than that in the U.S.-only, respectively. During this period, the U.S.-only has the highest average monthly real return among the four investment opportunities, but Stock-Bond Portfolio has the highest Sharpe ratio, while the equally weighted Portfolio 2 is the worst performer again, which produces the lowest Sharpe ratio, among the four investment opportunities.

Overall, in this part, we summarise the following findings. Here we can see that the U.S.-only stock portfolio is the best performer (achieves the highest Sharpe ratio) across the full sample period. This is also the case for the dot-com run-up period and the post-Great Recession period of 2009-2019. The U.S.-only portfolio is ranks second during the dot-com crash and COVID-19 period and never ranks last. Stock-Bond Portfolio (the traditional stock-bond portfolio) does not provide a diversification benefit compared to the U.S.-only, performing worse in each sub-period with the exception of the COVID-19 period (although the Sharpe ratio is similar in the Dot-com bursting and Great Recession crisis period). The cross-market stock Portfolio 2 often performs the worst (including worse than the U.S.-only portfolio). This is the case for the full sample period and three of the six sub-sample periods. However, it does provide the best performance during the Dot-com bursting recovery period of 2003-2007. The cross-asset Portfolio 3 also typically performs poorly (ranking 3 or 4) but does achieve the highest Sharpe ratio during the Dot-com bursting and the Great Recession periods, suggesting advantages during crisis periods.

In considering the crisis periods, the average monthly real returns for all four portfolios are negative during the periods of the Dot-com busting and the Great Recession but positive during the COVID-19 period, this perhaps reflects the economic support mechanisms that governments implemented as well as the fact that certain sectors (e.g., technology and pharmaceuticals) performed well during this period. Notwithstanding this, we can see that market volatility (standard deviation) is similar to that in previous crisis periods.

**Table 2.11.** Comparisons between the two benchmarks with two equally weighted portfolios.

**Panel A. The full period**

| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Changes from the U.S.-only: Ave.ret. | Std Dev | Sharpe ratio |
|---|---|---|---|---|---|---|---|
| U.S.-only | 0.63% | 4.32% | 14.76% | 1 | | | |
| Portfolio 1 | 0.34% | 2.54% | 13.71% | 2 | -45.82% | -41.19% | -7.10% |
| Equally Weighted Portfolio 2 | 0.37% | 4.74% | 7.90% | 4 | -41.59% | 9.86% | -46.46% |
| Equally Weighted Portfolio 3 | 0.43% | 3.29% | 13.25% | 3 | -31.80% | -23.67% | -10.24% |

**Panel B. The sub-sample periods**

**Panel B-1** The Dot-com booming period

| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Changes from the U.S.-only: Ave.ret. | Std Dev | Sharpe ratio |
|---|---|---|---|---|---|---|---|
| U.S.-only | 1.78% | 4.16% | 37.21% | 1 | | | |
| Portfolio 1 | 0.98% | 2.70% | 28.06% | 2 | -44.60% | -35.25% | -24.58% |
| Equally Weighted Portfolio 2 | 0.86% | 4.50% | 14.08% | 3 | -51.47% | 8.19% | -62.15% |
| Equally Weighted Portfolio 3 | 0.44% | 2.75% | 7.81% | 4 | -75.10% | -34.01% | -79.02% |

**Panel B-2** The Dot-com bursting period

| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Changes from the U.S.-only: Ave.ret. | Std Dev | Sharpe ratio |
|---|---|---|---|---|---|---|---|
| U.S.-only | -1.67% | 5.27% | -33.18% | 2 | | | |
| Portfolio 1 | -0.89% | 2.78% | -34.64% | 3 | 46.92% | -47.18% | -4.39% |
| Equally Weighted Portfolio 2 | -1.73% | 5.21% | -34.65% | 4 | -3.30% | -1.21% | -0.02% |
| Equally Weighted Portfolio 3 | -0.02% | 3.15% | -3.17% | 1 | 98.62% | -40.12% | 90.86% |

**Panel B-3** The 2003-2007 period

| | | | | | Changes from the U.S.-only | | |
|---|---|---|---|---|---|---|---|
| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Ave.ret. | Std Dev | Sharpe ratio |
| U.S.-only | 0.68% | 2.50% | 26.71% | 3 | | | |
| Portfolio 1 | 0.31% | 1.53% | 19.70% | 4 | -54.19% | -38.88% | -26.23% |
| Equally Weighted Portfolio 2 | 1.45% | 3.23% | 44.61% | 1 | 114.05% | 29.06% | 67.02% |
| Equally Weighted Portfolio 3 | 0.92% | 2.53% | 35.90% | 2 | 35.19% | 0.93% | 34.40% |

**Panel B-4** The Great Recession period

| | | | | | Changes from the U.S.-only | | |
|---|---|---|---|---|---|---|---|
| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Ave.ret. | Std Dev | Sharpe ratio |
| U.S.-only | -2.44% | 6.99% | -33.19% | 3 | | | |
| Portfolio 1 | -1.49% | 4.11% | -33.27% | 4 | 38.98% | -41.16% | -0.25% |
| Equally Weighted Portfolio 2 | -2.43% | 8.64% | -26.68% | 2 | 0.53% | 23.69% | 19.60% |
| Equally Weighted Portfolio 3 | -0.49% | 5.34% | -7.00% | 1 | 79.72% | -23.58% | 78.90% |

**Panel B-5** The 2009-2019 period

| | | | | | Changes from the U.S.-only | | |
|---|---|---|---|---|---|---|---|
| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Ave.ret. | Std Dev | Sharpe ratio |
| U.S.-only | 0.91% | 3.62% | 27.78% | 1 | | | |
| Portfolio 1 | 0.53% | 2.05% | 25.78% | 2 | -41.95% | -43.32% | -7.21% |
| Equally Weighted Portfolio 2 | 0.48% | 4.02% | 14.20% | 4 | -47.82% | 10.90% | -48.89% |
| Equally Weighted Portfolio 3 | 0.33% | 2.86% | 14.81% | 3 | -63.88% | -21.00% | -46.67% |

**Panel B-6** The COVID-19 period

| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Changes from the U.S.-only Ave.ret. | Std Dev | Sharpe ratio |
|---|---|---|---|---|---|---|---|
| U.S.-only | 1.58% | 5.52% | 32.68% | 2 | | | |
| Portfolio 1 | 0.87% | 3.12% | 35.07% | 1 | -45.06% | -43.58% | 7.30% |
| Equally Weighted Portfolio 2 | 0.92% | 5.18% | 17.76% | 4 | -41.80% | -6.21% | -45.66% |
| Equally Weighted Portfolio 3 | 1.10% | 5.40% | 24.55% | 3 | -30.30% | -2.18% | -24.89% |

The table above provides a direct comparison of the four portfolios across various periods, ranked according to their Sharpe ratios. The calculations for each portfolio are consistent with those used in Table 2.9. This table reorganizes the results to facilitate a clearer comparison of each portfolio's performance throughout the overall period and within each sub-period. Ave.ret. means the average real return. Std Dev means the standard deviation. The data frequency here is monthly.

*2.5.2.3 Comparison between the equally weighted portfolios and optimised portfolios*

In the above analysis, Portfolios 2 and 3 are constructed using equal weights. Here, the mean-variance approach is used to optimise each portfolio for obtaining the asset weights, with the results presented in Table 2.12 (only these new results are presented as those for the U.S.-only and Stock-Bond Portfolio already discussed).

In Panel A of Table 2.12, we find that in the optimised Portfolio 2, across both the full and sub-periods, no weight is allocated to the EAFE market, apart from the post-dot-com recovery period (2003-2007). We can also observe that, with optimisation, the Sharpe ratios for Portfolio 2 are increased over the equal-weight Portfolio 2 (see the last three columns for a comparison) across the full and sub-sample periods. Considering the results more specifically, over the full sample period, the optimised Portfolio 2 allocates all the weight to S&P 500, with an average monthly real return and Sharpe Ratio that are 71% and 87% higher than for the equal-weight Portfolio 2, respectively. During the dot-com boom (1995-2000), the post-Great Recession recovery and the COVID-19 periods, again all the portfolio weight is allocated to the S&P 500. In contrast, for the dot-com crash and the Great Recession periods, all the portfolio weight is allocated to the EM index, while a 72% weight is allocated to EM during the 2003-2007 period (with 28% to EAFE). Notwithstanding the different weights, the portfolio continues to exhibit negative returns and Sharpe ratio during the Dot-com bursting and Great Recession.

Panel B of Table 2.12 presents the results for the optimised cross-asset portfolio (Portfolio 3). Over the full sample period, the allocated weights are 61% to the S&P 500 index, 33% to gold, and 5% to oil but with no allocation to the U.S. 10-year Treasury Note. The Sharpe ratio of the optimised Portfolio 3 is higher than that of the equal-weight Portfolio 3 both over the full sample and each of the six sub-sample periods. This can be mainly attributable to the higher average monthly real returns generated by optimising Portfolio 3, while the standard deviations are both higher and lower across different sample periods. The notable exception is the COVID-19 period, where the portfolio return is lower, but so is the standard deviation, which still leads to a higher Sharpe ratio. Through optimisation, over the full sample period, the average monthly real return of the optimised Portfolio 3 is 0.57%, which is 31% higher than the equal-weight Portfolio 3, with its Sharpe ratio 30% higher. In considering the sub-samples, during both the Dot-com bursting and Great Recession crisis periods, the optimised Portfolio 3 allocates no weight to the S&P 500 index. However, during the COVID-19 crisis, a weight of 40% is allocated to the S&P 500 index. Furthermore, the average monthly return and Sharpe ratio are positive through all periods. In comparing the two optimised portfolios presented in Table 2.12, we can observe that Portfolio 3 outperforms Portfolio 2 over the full and each sub-period, except during 2003-2007, where there is a minimal difference.

**Table 2.12**. Performance for the optimised Portfolio 2 and optimised Portfolio 3.

**Panel A. Optimised Portfolio 2**

| | Portfolio allocation | | | Real risk-free | | Real excess | | | Changes from Equally weighted Portfolio 2 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | S&P 500 | MSCI EAFE | MSCI EM | rate | Ave.ret. | return | Std Dev | Sharpe ratio | Ave.ret. | Std Dev | Sharpe ratio |
| Whole period | 100.00% | 0.00% | 0.00% | -0.01% | 0.63% | 0.64% | 4.32% | 14.76% | 71.21% | -8.97% | 86.77% |
| Dot-com booming | 100.00% | 0.00% | 0.00% | 0.23% | 1.78% | 1.55% | 4.16% | 37.21% | 106.07% | -7.57% | 164.21% |
| Dot-com bursting | 0.00% | 0.00% | 100.00% | 0.08% | -1.58% | -1.65% | 6.89% | -24.03% | 8.61% | 32.31% | 30.65% |
| 2003-2007 | 0.00% | 28.18% | 71.82% | 0.01% | 2.07% | 2.06% | 4.29% | 47.94% | 42.37% | 32.71% | 7.48% |
| Great Recession | 0.00% | 0.00% | 100.00% | -0.12% | -2.06% | -1.94% | 11.40% | -17.01% | 15.08% | 31.97% | 36.25% |
| 2009-2019 | 100.00% | 0.00% | 0.00% | -0.09% | 0.91% | 1.01% | 3.62% | 27.78% | 91.64% | -9.83% | 95.67% |
| COVID-19 | 100.00% | 0.00% | 0.00% | -0.22% | 1.58% | 1.81% | 5.52% | 32.68% | 71.83% | 6.62% | 47.96% |

**Panel B. Optimised Portfolio 3**

| | Portfolio allocation | | | | Real risk- | | Real excess | | | Changes from Equally weighted Portfolio 3 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | S&P 500 | Gold | Oil | 10 YRN | free rate | Ave.ret. | return | Std Dev | Sharpe ratio | Ave.ret. | Std Dev | Sharpe ratio |
| Whole period | 61.31% | 33.33% | 5.36% | 0.00% | -0.01% | 0.56% | 0.57% | 3.31% | 17.16% | 30.54% | 0.42% | 29.56% |
| Dot-com booming | 90.31% | 0.00% | 9.69% | 0.00% | 0.23% | 1.69% | 1.46% | 3.78% | 38.57% | 281.11% | 37.64% | 394.15% |
| Dot-com bursting | 0.00% | 29.30% | 6.14% | 64.57% | 0.08% | 0.39% | 0.31% | 1.84% | 16.90% | 1782.25% | -41.83% | 633.94% |
| 2003-2007 | 62.81% | 18.68% | 18.52% | 0.00% | 0.01% | 1.03% | 1.02% | 2.29% | 44.47% | 11.91% | -9.58% | 23.89% |
| Great Recession | 0.00% | 100.00% | 0.00% | 0.00% | -0.12% | 1.00% | 1.12% | 7.70% | 14.51% | 301.53% | 44.28% | 307.13% |
| 2009-2019 | 48.39% | 3.60% | 0.00% | 48.01% | -0.09% | 0.43% | 0.53% | 1.70% | 30.97% | 31.45% | -40.48% | 109.06% |
| COVID-19 | 39.38% | 8.68% | 0.00% | 51.94% | -0.22% | 0.57% | 0.80% | 2.19% | 36.38% | -48.08% | -59.48% | 48.19% |

The table above compares the results of optimised Portfolio 2 and optimised Portfolio 3, both of which are constructed using the mean-variance optimisation approach. The weight allocations for these portfolios are derived from this optimisation method. Ave.ret. means the average real return. Std Dev means the standard deviation. The data frequency here is monthly.

Table 2.13 displays comparisons between the U.S.-only, Stock-Bond Portfolio and two optimised portfolios (the mean-variance Portfolio 2 and the mean-variance Portfolio 3) for different periods. Over the full sample period, Portfolio 3 performs the best both and it also performs the best when it is equally weighted, while Stock-Bond Portfolio is the worst performer. The optimised Portfolio 2 and the U.S.-only have the same performance, because that after optimisation. The Portfolio 2 allocates all its weights in the S&P 500 index.

As we can see in Panel B-1 of Table 2.13, during the Dot-com booming period and the non-crisis period 2009-2019, the optimised Portfolios 3 outperforms the U.S.-only, the stock-bond portfolio, and even the optimised Portfolio 2. In addition, after optimisation, Portfolio 2 allocates all its weights to the S&P 500 index, while Stock-Bond Portfolio is the worst performer, during these two periods. However, during the non-crisis period 2003-2007, the optimised portfolio 2 is the best performer, with a real return of 2.07% that increases 204.74% from the U.S.-only, and a Sharpe ratio of 47.94% that is 79.50% higher than the U.S.-only. During this period, the Portfolio 3 still performs well, with a higher Sharpe ratio than the U.S.-only and Stock-Bond Portfolio.

During the Dot-com bursting and the Great Recession, the optimised Portfolios 3 is still the best performer, while Stock-Bond Portfolio is still the worst performer among four investing options. Interestingly, during these two crisis periods, the Portfolio 2 performs better than the

U.S.-only after optimisation. Furthermore, during the COVID period, the optimised Portfolio 3 still have the best performance compared with other portfolio opportunities, but this time the U.S.-only and the optimised Portfolio 2 are both in the last place as the optimised Portfolio 2 allocates all its weights in the S&P 500 index.

From this we can conclude the following main findings. First, the optimised Portfolio 3 (cross-asset) not only ranks first over the whole sample period but also in five sub-sample periods, with only the 2003-2007 period, where it is second preferred. Second, only the optimised Portfolio 3 has a positive average real monthly return and Sharpe ratio during the Dot-com bursting and the Great Recession periods, which could help U.S investors hedge risks during these two crisis periods. Third, the optimised Portfolio 2 typically outperforms the U.S only portfolio, whereas the equal-weight Portfolio 2 does not, although performance is lower than the optimised Portfolio 3.

Overall, by comparing the results in Tables 2.11 and 2.13, we conclude the following findings. First, the cross-asset diversified portfolio (Portfolio 3) offers substantial diversification benefits for U.S investors over both the full sample and individual sub-sample periods regardless of whether investors chose equal-weighting or mean-variance optimisation. Second, across the full sample, the traditional stock-bond approach (Stock-Bond Portfolio) does not provide much in terms of diversification benefit compared to the U.S only position with a similar Sharpe ratio.

Third, the equal-weighted Portfolio 2 (internationally diversified stock portfolio) outperforms the U.S.-only (S&P 500 index) only over a small number of selected sub-samples. Fourth, when considering the whole sample period, the optimised Portfolio 2 does provide better diversification benefits for U.S investors. However, when examining the six sub-sample periods, we find that while before 2009 the optimised Portfolio 2 does benefit U.S investors, this is no longer the case after 2009. This arises due to the performance of the S&P 500 index compared to the EAFE and EM indexes and thus the weight of the former in the portfolio.

**Table 2.13.** Comparisons between the U.S.-only, Portfolio 1 and two optimised portfolios.

**Panel A. The full period**

| | | | | | Changes from the U.S.-only | | |
|---|---|---|---|---|---|---|---|
| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Ave.ret. | Std Dev | Sharpe ratio |
| U.S.-only | 0.63% | 4.32% | 14.76% | 2 | | | |
| Portfolio 1 | 0.34% | 2.54% | 13.71% | 4 | -45.82% | -41.19% | -7.10% |
| Optimised Portfolio 2 | 0.63% | 4.32% | 14.76% | 2 | 0.00% | 0.00% | 0.00% |
| Optimised Portfolio 3 | 0.56% | 3.31% | 17.16% | 1 | -10.96% | -23.34% | 16.29% |

**Panel B. The sub-sample periods**

**Panel B-1** The Dot-com Boom period

| | | | | | Changes from the U.S.-only | | |
|---|---|---|---|---|---|---|---|
| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Ave.ret. | Std Dev | Sharpe ratio |
| U.S.-only | 1.78% | 4.16% | 37.21% | 2 | | | |
| Portfolio 1 | 0.98% | 2.70% | 28.06% | 4 | -44.60% | -35.25% | -24.58% |
| Optimised Portfolio 2 | 1.78% | 4.16% | 37.21% | 2 | 0.00% | 0.00% | 0.00% |
| Optimised Portfolio 3 | 1.69% | 3.78% | 38.57% | 1 | -5.10% | -9.17% | 3.66% |

**Panel B-2** The Dot-com Bursting period

| | | | | | Changes from the U.S.-only | | |
|---|---|---|---|---|---|---|---|
| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Ave.ret. | Std Dev | Sharpe ratio |
| U.S.-only | -1.67% | 5.27% | -33.18% | 3 | | | |
| Portfolio 1 | -0.89% | 2.78% | -34.64% | 4 | 46.92% | -47.18% | -4.39% |
| Optimised Portfolio 2 | -1.58% | 6.89% | -24.03% | 2 | 5.59% | 30.71% | 27.59% |
| Optimised Portfolio 3 | 0.39% | 1.84% | 16.90% | 1 | 123.15% | -65.17% | 150.93% |

**Panel B-3** The 2003-2007 period

| | | | | | Changes from the U.S.-only | | |
|---|---|---|---|---|---|---|---|
| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Ave.ret. | Std Dev | Sharpe ratio |
| U.S.-only | 0.68% | 2.50% | 26.71% | 3 | | | |
| Portfolio 1 | 0.31% | 1.53% | 19.70% | 4 | -54.19% | -38.88% | -26.23% |
| Optimised Portfolio 2 | 2.07% | 4.29% | 47.94% | 1 | 204.74% | 71.28% | 79.50% |
| Optimised Portfolio 3 | 1.03% | 2.29% | 44.47% | 2 | 51.28% | -8.73% | 66.50% |

| **Panel B-4** The Great Recession period | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | Changes from the U.S.-only | | |
| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Ave.ret. | Std Dev | Sharpe ratio |
| U.S.-only | -2.44% | 6.99% | -33.19% | 3 | | | |
| Portfolio 1 | -1.49% | 4.11% | -33.27% | 4 | 38.98% | -41.16% | -0.25% |
| Optimised Portfolio 2 | -2.06% | 11.40% | -17.01% | 2 | 15.53% | 63.23% | 48.75% |
| Optimised Portfolio 3 | 1.00% | 7.70% | 14.51% | 1 | 140.86% | 10.26% | 143.71% |

| **Panel B-5** The 2009-2019 period | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | Changes from the U.S.-only | | |
| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Ave.ret. | Std Dev | Sharpe ratio |
| U.S.-only | 0.91% | 3.62% | 27.78% | 2 | | | |
| Portfolio 1 | 0.53% | 2.05% | 25.78% | 3 | -41.95% | -43.32% | -7.21% |
| Optimised Portfolio 2 | 0.91% | 3.62% | 27.78% | 2 | 0.00% | 0.00% | 0.00% |
| Optimised Portfolio 3 | 0.43% | 1.70% | 30.97% | 1 | -52.52% | -52.98% | 11.48% |

| **Panel B-6 The** COVID-19 period | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | Changes from the U.S.-only | | |
| | Ave.ret. | Std Dev | Sharpe ratio | Rank | Ave.ret. | Std Dev | Sharpe ratio |
| U.S.-only | 1.58% | 5.52% | 32.68% | 3 | | | |
| Portfolio 1 | 0.87% | 3.12% | 35.07% | 2 | -45.06% | -43.58% | 7.30% |
| Optimised Portfolio 2 | 1.58% | 5.52% | 32.68% | 3 | 0.00% | 0.00% | 0.00% |
| Optimised Portfolio 3 | 0.57% | 2.19% | 36.38% | 1 | -63.81% | -60.36% | 11.30% |

The table above displays comparisons between the U.S.-only (only investing in the S&P 500 index), Portfolio 1 (the stock (60%)–bond (40%) portfolio), and two optimised portfolios (the mean-variance optimised international diversified Portfolio 2, and the mean-variance optimised cross-asset Portfolio 3) for different periods. The rankings are based on the size of the Sharpe ratio. Ave.ret. means the real average return. Std Dev represents the standard deviation. The data frequency here is monthly.

Table 2.14 presents a set of analysis designed to provide robustness to our results from two perspectives. First, we conduct a series of rolling windows to generate out-of-sample values to construct the portfolios. Second, we consider the effect of transaction costs within portfolio performance. More specifically, in Table 2.14 the in-sample results are based on 24-month rolling windows (with 300 windows in total) with average over these windows reported for the return, standard deviation and Sharpe ratio. In the out-of-sample exercise, we use the estimates from the rolling windows to produce the next months (one-step ahead) portfolio, including the estimated correlation. That is, with each rolling window, we optimise based on the in-sample values to build our portfolio for the subsequent month. Moreover, regarding the out-of-sample results, the gross return is the average return based on each period without considering transaction costs, while the net return is the average return that takes into account the transaction costs, where the one-way transaction cost is considered to be 0.05%.

The results reveal two broad conclusions. First, if we consider the in-sample results, they support those previously noted. Specifically, the optimised Portfolio 3 produces the highest Sharpe ratio over the full sample and for each of the sub-samples. Furthermore, it is the only portfolio that achieves a positive Sharpe ratio, including in the crisis periods. Elsewhere, the optimised Portfolio 2 generally performs well, ranking either second or third. Again, the equal-weighted portfolios are outperformed by the optimised ones. The U.S.-only portfolios, both the S&P 500 index and stock and bond only, are lower ranked in term of their Sharpe ratio across the full sample and each sub-sample, with the exception of the post-Great Recession period. Second, if we consider the out-of-sample results, we see differences emerging. Here, we see much less consistency in the preferred portfolio over the different periods. For the full sample period, the optimised Portfolio 2 (cross-international stock markets) achieves the highest Sharpe ratio. Across the sub-samples, we observe the S&P 500 only portfolio achieving a Sharpe ratio ranked in the highest three in the Dot-com boom period (ranked first), the post-Great Recession (second) and COVID-19 (third), while in the remaining periods it ranks in the bottom three. This volatility in performance is matched in the other portfolios. However, it is noticeable that in the two periods of negative returns (Dot-com crash and the Great Recession), it is the diversified (cross-market and cross-asset) portfolios that are preferred. In comparing the gross and net returns, we can observe that the cross-asset portfolio sees the highest amount of transaction costs (and therefore, trading), which damages its performance.

**Table 2.14.** Comparisons of the in- and out-of-sample results.

| | In-sample | | | Out-of-sample | | | |
|---|---|---|---|---|---|---|---|
| **Panel A. The full period** | | | | | | | |
| | Ave.ret. | Std Dev | Sharpe ratio | Gross return | Net return | Std Dev | Sharpe ratio |
| U.S.-only | 0.52% | 4.11% | 12.82% | 0.54% | 0.54% | 4.43% | 12.66% |
| Portfolio 1 | 0.37% | 3.17% | 15.68% | 0.28% | 0.28% | 2.58% | 11.69% |
| Equally Weighted Portfolio 2 | 0.54% | 0.53% | 8.44% | 0.35% | 0.35% | 4.88% | 7.62% |
| Optimised Portfolio 2 | 0.90% | 4.91% | 25.18% | 0.77% | 0.70% | 5.35% | 13.53% |
| Equally Weighted Portfolio 3 | 0.35% | 3.77% | 11.13% | 0.41% | 0.41% | 3.40% | 12.65% |
| Optimised Portfolio 3 | 1.08% | 3.31% | 32.93% | 0.35% | 0.23% | 4.22% | 6.08% |
| **Panel B. The sub-sample periods** | | | | | | | |
| **Panel B-1 The Dot-com Booming period** | | | | | | | |
| | In-sample | | | Out-of-sample | | | |
| | Ave.ret. | Std Dev | Sharpe ratio | Gross return | Net return | Std Dev | Sharpe ratio |
| U.S.-only | 1.79% | 4.32% | 38.54% | 1.77% | 1.77% | 4.97% | 30.68% |
| Portfolio 1 | 1.23% | 3.30% | 31.75% | 0.89% | 0.89% | 3.16% | 20.63% |
| Equally Weighted Portfolio 2 | 0.87% | 4.32% | 21.78% | 1.01% | 1.01% | 5.46% | 14.14% |
| Optimised Portfolio 2 | 1.79% | 4.29% | 38.61% | 1.70% | 1.65% | 4.93% | 28.64% |
| Equally Weighted Portfolio 3 | 0.41% | 2.65% | 7.18% | 0.25% | 0.25% | 3.33% | 0.26% |
| Optimised Portfolio 3 | 1.79% | 4.20% | 39.73% | 1.42% | 1.36% | 4.52% | 24.88% |
| **Panel B-2 The Dot-com Burst period** | | | | | | | |
| | In-sample | | | Out-of-sample | | | |
| | Ave.ret. | Std Dev | Sharpe ratio | Gross return | Net return | Std Dev | Sharpe ratio |
| U.S.-only | -0.45% | 5.03% | -11.78% | -1.67% | -1.67% | 5.27% | -33.18% |
| Portfolio 1 | -0.45% | 3.90% | -15.72% | -0.89% | -0.89% | 2.83% | -34.13% |
| Equally Weighted Portfolio 2 | -0.63% | 4.86% | -10.95% | -1.73% | -1.73% | 5.29% | -34.11% |
| Optimised Portfolio 2 | -0.22% | 6.10% | -5.14% | -1.37% | -1.58% | 5.86% | -28.24% |
| Equally Weighted Portfolio 3 | -0.11% | 3.71% | -6.50% | -0.02% | -0.02% | 3.20% | -3.16% |
| Optimised Portfolio 3 | 1.48% | 7.00% | 18.74% | -1.12% | -1.26% | 7.70% | -17.41% |
| **Panel B-3 The 2003-2007 period** | | | | | | | |
| | In-sample | | | Out-of-sample | | | |
| | Ave.ret. | Std Dev | Sharpe ratio | Gross return | Net return | Std Dev | Sharpe ratio |
| U.S.-only | 0.23% | 3.31% | 14.43% | 0.68% | 2.50% | 26.71% | 26.71% |
| Portfolio 1 | 0.11% | 2.40% | 10.71% | 0.31% | 0.31% | 1.55% | 19.24% |
| Equally Weighted Portfolio 2 | 0.68% | 3.62% | 33.49% | 1.45% | 1.45% | 3.26% | 44.14% |
| Optimised Portfolio 2 | 1.36% | 4.49% | 29.55% | 1.72% | 1.60% | 4.11% | 38.83% |
| Equally Weighted Portfolio 3 | 0.77% | 3.08% | 26.38% | 0.91% | 0.91% | 2.53% | 35.69% |
| Optimised Portfolio 3 | 1.10% | 2.57% | 45.64% | 0.67% | 0.54% | 2.92% | 18.04% |

**Panel B-4 The Great Recession period**

| | In-sample | | | Out-of-sample | | | |
|---|---|---|---|---|---|---|---|
| | Ave.ret. | Std Dev | Sharpe ratio | Gross return | Net return | Std Dev | Sharpe ratio |
| U.S.-only | -0.95% | 4.05% | -17.77% | -2.44% | -2.44% | 6.99% | -33.19% |
| Portfolio 1 | -0.66% | 3.10% | -17.77% | -1.46% | -1.46% | 4.20% | -31.99% |
| Equally Weighted Portfolio 2 | -0.80% | 5.26% | -3.65% | -2.40% | -2.40% | 8.85% | -25.78% |
| Optimised Portfolio 2 | 0.12% | 7.54% | 7.08% | -2.13% | -2.15% | 11.71% | -17.38% |
| Equally Weighted Portfolio 3 | 0.35% | 5.17% | 11.91% | -0.47% | -0.47% | 5.45% | -6.47% |
| Optimised Portfolio 3 | 1.07% | 3.75% | 29.13% | -1.16% | -1.44% | 7.01% | -18.77% |

**Panel B-5 The 2009-2019 period**

| | In-sample | | | Out-of-sample | | | |
|---|---|---|---|---|---|---|---|
| | Ave.ret. | Std Dev | Sharpe ratio | Gross return | Net return | Std Dev | Sharpe ratio |
| U.S.-only | 0.64% | 3.91% | 25.32% | 0.91% | 0.91% | 3.62% | 27.78% |
| Portfolio 1 | 0.27% | 3.01% | 21.76% | 0.53% | 0.52% | 2.06% | 30.15% |
| Equally Weighted Portfolio 2 | 0.40% | 4.31% | 20.04% | 0.47% | 0.47% | 4.02% | 14.07% |
| Optimised Portfolio 2 | 0.80% | 4.48% | 27.70% | 0.86% | 0.83% | 3.92% | 23.69% |
| Equally Weighted Portfolio 3 | 0.22% | 3.96% | 9.68% | 0.33% | 0.32% | 2.86% | 14.62% |
| Optimised Portfolio 3 | 0.86% | 2.71% | 39.25% | 0.51% | 0.40% | 2.90% | 17.20% |

**Panel B-6 The COVID-19 period**

| | In-sample | | | Out-of-sample | | | |
|---|---|---|---|---|---|---|---|
| | Ave.ret. | Std Dev | Sharpe ratio | Gross return | Net return | Std Dev | Sharpe ratio |
| U.S.-only | 1.05% | 5.45% | 20.08% | 1.58% | 1.58% | 5.52% | 32.68% |
| Portfolio 1 | 0.94% | 4.73% | 20.88% | 0.88% | 0.87% | 3.17% | 34.60% |
| Equally Weighted Portfolio 2 | 0.76% | 5.28% | 18.23% | 0.93% | 0.93% | 5.28% | 21.78% |
| Optimised Portfolio 2 | 1.05% | 5.45% | 20.08% | 1.58% | 1.58% | 5.52% | 32.68% |
| Equally Weighted Portfolio 3 | 0.56% | 5.28% | 11.32% | 1.11% | 1.11% | 5.51% | 24.16% |
| Optimised Portfolio 3 | 0.53% | 1.44% | 43.91% | 0.23% | 0.13% | 1.46% | 24.56% |

The in-sample results are the 24-month rolling window's average for each of the portfolio return, standard deviation and Sharpe ratio. The out-of-sample results are obtained as one-step ahead forecasts for the portfolio parameters. Ave.ret. means the real average return. Std Dev represents the standard deviation. The gross return is the average return without considering transaction costs, while the net return takes account of a one-way transaction cost of 0.05%. The data frequency here is monthly.

## 2.6. Conclusion

This chapter investigated how U.S. investors could benefit from diversifying their investment through different portfolio opportunities, including a stock (60%)-bond (40%) portfolio, an international diversification portfolio, and an asset-diversified portfolio, or just invest in the U.S. stock market. This chapter uses monthly data to build the four investment opportunities over the period 1995-2021, and we also segment the whole period into two categories, including crisis periods and non-crisis periods. The crises include the Dot-com bursting crisis, the Great Recession, and the COVID-19 health crisis. In our dataset, there are six variables, including three stock indexes (S&P 500 index, MSCI EAFE index (Developed Market index), and MSCI EM index (Emerging Market index)), three assets (gold (Gold Bullion), oil (Brent Oil)) and bonds (U.S. 10-year Treasury-note)). The stock (60%)-bond (40%) portfolio consists of the S&P 500 index and U.S. 10-year Treasury Note. The international diversification portfolio consists of the S&P 500 index, MSCI EAFE index, and MSCI EM index, while the asset-diversified portfolio consists of the S&P 500 index, gold, oil, and U.S. 10-year Treasury Note. We use S&P 500 representing the U.S. stock market.

Descriptive statistics reveal that since 2009, compared with the MSCI EAFE index, and the MSCI EM index, the S&P 500 index has been the best performer with a higher average monthly real return and a lower standard deviation. This is further confirmed through the portfolio optimisation across international stocks. Before 2009, with optimisation, the international diversification portfolio allocates weights across three index series, which could benefit U.S investors. However, after that year, with optimisation, it allocates all weights to the S&P 500 index. This arises due to the S&P 500's performance being more stable and better since 2009 compared to the EAFE and EM indexes.

The ARMA (0,0)-DCC-GARCH (1,1) model shows that the S&P 500 index and the other five variables have an interaction relationship. Combining the time-varying correlation and fixed correlation, we reinforce the existing argument that correlations between national stock markets have been increasing in recent years (Longin and Solnik, 1995; Forbes and Rigobon, 2002; Kim et al., 2005; Morana and Beltratti, 2008; Christoffersen et al., 2014) and we also support the existing argument that the correlation between stock markets in crisis periods is higher than in non-crisis periods (Roll, 1988; Bertero and Mayer, 1990; King and Wadhwani, 1990; Solnik et al., 1996; Butler and Joaquin, 2002; Guidi and Ugur, 2014).

After we find there are interactional relationships between the S&P 500 index and the other five variables, we test the benefit that U.S investors can obtain from four benchmark investment opportunities and adopt the mean-variance portfolio strategy to optimise the international-diversified portfolio and asset-diversified portfolio.

In examining the performance of the different portfolios, we report several key findings. Most importantly, the cross-asset diversified portfolio consisting of the S&P 500, gold, oil, and the U.S. 10-year Treasury Note results in the best performing portfolio and outperforms alternative portfolio regardless of whether investors choose an equal-weighted or optimised portfolio. This result is consistent with some literature (*e.g.,* Baur and McDermott, 2010; McCown and Zimmerman, 2006; Capie et al., 2005) that supports the inclusion of gold for its hedging properties. Equally, further work (Hamoud et al., 2011, 2013) finds that when oil is combined with precious metals in a diversified portfolio, it has the property of increasing returns and reducing risk. Thus, the portfolio option that is more likely to provide diversification benefits for U.S. investors is the asset-diversified portfolio. In comparing the equal-weighted and optimised portfolios, the latter is preferred, but as noted, the former does outperform the S&P 500 index. It is also of interest to note that in the optimised portfolio, over the full sample period, no weighting is allocated to the U.S. 10-year Treasury Note (similar to the reported in (Hamoud et al., 2011, 2013). However, during two of the crisis periods, it is the dominant asset in the portfolio.

It is notable that the equal-weighted international stock market portfolio often performs worse than the S&P 500 only portfolio. This is especially true over the full sample period and in the Dot-com busting and COVID-19 crisis periods. In the optimised portfolio, it is of interest that the EAFE index is excluded, except for the post-Great Recession recovery period, indicating that adding developed markets to a U.S. portfolio does not help performance. Furthermore, after 2009, the optimised international stock portfolio only includes the S&P 500 (during the great recession, only the EM index is included). In seeking to understand this result, it is notable that since 2009, compared to the EAFE and EM indexes, the S&P 500 index is the best performer. A further reason might be that the correlation between international stock markets has increased in recent years, which may eliminate international diversification benefits and increase shock transmission (*e.g.,* Koch and Koch, 1991; Longin and Solnik, 1995; Karolyi and Stulz, 1996; Driessen and Laeven, 2007).

In seeking to consider the robustness of our results, we utilise rolling windows to reconsider the in-sample evidence and to allow construction of out-of-sample portfolios to avoid look-ahead bias. In addition, we incorporate a transaction cost into the out-of-sample portfolios. The results are broadly confirmed with the in-sample rolling portfolios, suggesting that the shorter formation period (and correlation) does not affect the performance of the portfolio. In the out-of-sample period, however, we find more mixed results. Although the cross-asset portfolio continues to perform well during the crisis periods, its performance is less supported in periods of stronger market behaviours.

The key result here shows that the cross-asset portfolio performs the best across the different sample periods. In general, while the optimised portfolios provide higher diversification benefits than the equal-weighted portfolios, the equal-weighted cross-asset portfolio does outperform the optimised international stock market portfolio across certain periods. A further interesting result is that compared with the Dot-com bursting and the Great Recession, the COVID-19 health crisis did not have an evident impact on the return of the four portfolios, although it increased the volatility of each. However, the caveat to these results is that in the out-of-sample exercise, the performance of the cross-asset portfolios is relatively weaker during periods of market growth and is subject to a larger amount of trading than other portfolios. It remains an avenue for future research to consider further how the in-sample benefits can be obtained out-of-sample.

# **Chapter Three:** Portfolio Selection by Using Six Basic Portfolio Strategies


## Abstract

In this chapter, we optimise portfolio selection for an investment universe of developed and emerging market stock indexes using the Parametric Portfolio Policy (PPP) approach of Brandt et al. (2009) for the period from December 2004 to December 2023 and compare the results to naive diversified portfolios (1/N-rule), market capitalisation weighted portfolios, risk parity (equally weighted risk contribution) portfolios, mean-variance (MV), and Black-Litterman (BL) optimised portfolios. To estimate the performance of the Parametric Portfolio Policy (PPP), we form three types of different characteristics optimised portfolios, including two-characteristic optimised portfolios, three-characteristic optimised portfolios, and six-characteristic optimised portfolios. We also consider the effect of short selling and transaction costs on portfolio construction. The 1/N rule, risk parity and market capitalisation-weighted strategies are benchmarks, while the mean-variance, Black-Litterman and three types of PPP are portfolio optimisation strategies. We find the mean-variance (MV) and Black-Litterman (BL) strategies have more stable and better performance in terms of Sharpe ratio than the 1/N rule, risk parity, and market capitalisation-weighted portfolios. In the in-sample simulation, the mean-variance (MV) and Black-Litterman (BL) strategies consistently beat all other strategies, no matter with or without short-selling constraints. In the out-of-sample simulation, all three types of PPP-

optimised portfolios outperform the three benchmarks (equally weighted portfolio, risk parity portfolio, and market capitalisation-weighted portfolio) before controlling for short selling. In addition, the six-characteristic optimised portfolio, without short selling constraints, outperforms all other selected portfolio strategies before the deduction of transaction costs. Moreover, the six-characteristic-optimised (the market capitalisation, return-to-equity ratio, book-to-market ratio, dividend yield, volume, and the 12-month cumulative return) and the three-characteristic (the market capitalisation, book-to-market ratio, the 12-month cumulative return) optimised portfolios seem to produce more stable and better performance than the two-characteristic (market capitalisation and the 12-month cumulative return) optimised portfolio. Furthermore, the market capitalisation-weighted portfolio performs better than the equally weighted portfolio and the risk parity portfolio within three benchmarks, both before and after the deduction of the transaction costs in our sample period.

### 3.1. Introduction

In Chapter two, we use historical data to determine U.S. investors' optimal diversified portfolio selection. In this process, we estimate the moment of asset return using historical data on returns such as mean, variance and correlation. However, some researchers find that a sample-estimated portfolio based solely on historical data on returns poorly performed out-of-sample (Brandt and Santa-Clara, 2006). Brandt and Santa-Clara (2006) proposes a dynamic strategy for portfolio optimisation according to Hansen and Singleton (1982). In Brandt et al. (2009), they further improve and clarify this strategy and name it the Parametric Portfolio Policy (PPP) approach. This approach parameterises the portfolio's weights of each asset as a function of the asset's characteristics and then maximises the investor's average utility by choosing optimally the coefficients of this function. The implicit assumption is that the characteristics convey all relevant information about the assets' conditional distribution of returns. The advantages of this approach are that: 1) it is easy to implement; 2) it has good in- and out-of-sample performance; 3) some of the methods we use to optimise the Markowitz model can also be used in this framework, which includes using portfolio constraints, shrinkage estimates, and combining investors' prior beliefs with the information contained in return history (Barroso and Santa-Clara, 2015; Fletcher, 2017; Joenväärä et al., 2021).

So far, the academic literature provides little empirical evidence analysing the performance of the Parametric Portfolio Policy (PPP) approach of Brandt et al. (2009). Although several studies apply it to construct portfolios, there is no obvious evidence that the Parametric Portfolio Policy (PPP) approach generates a superior in- and out-of-sample performance relative to other popular portfolio strategies, like the 1/N rule (equally weighted strategy), risk parity (equally weighted risk contribution), the market capitalisation-weighted strategy, mean-variance (MV) optimisation strategy and Black-Litterman optimisation strategy. Thus, in this chapter, we apply all these six portfolio strategies, including the 1/N rule (equally weighted strategy), risk parity (equally weighted risk contribution), the market capitalisation-weighted strategy, mean-variance (MV) optimisation strategy and Black-Litterman optimisation strategy, and the Parametric Portfolio Policy (PPP), to construct portfolios and compare their results with each other. When estimating the performance of the Parametric Portfolio Policy (PPP), we form three types of different characteristics optimised portfolios, including two-characteristic optimised portfolios, three-characteristic optimised portfolios, and all six-characteristic optimised portfolios.

By building a 19-year global index portfolio of developed, and emerging markets, we conduct a comprehensive study that optimises portfolio selection using the Parametric Portfolio Policy (PPP) approach of Brandt et al. (2009) and compare its performance to the performance of the naïve diversified (1/N-rule) portfolio, market capitalisation-weighted, risk parity (equally weighted risk contribution), mean-variance (MV), and Black-Litterman optimised portfolios by using the same data-set. Out of this, the 1/N rule diversified portfolio, the market capitalisation-weighted portfolio and equally weighted risk contribution are benchmarks in this chapter, while the mean-variance (MV) portfolio, Black-Litterman portfolios, and the Parametric Portfolio Policy (PPP) portfolio are optimised portfolios. Precisely, we apply these portfolio strategies to each of the following twelve indexes formed dataset: seven global indexes from developed economies (i.e., USA, Japan, UK, Italy, France, Germany, and Canada, known as the G7), and five global indexes from emerging economies (i.e., Brazil, Russia, India, China, and South Africa, known as the BRICS).

In the academic literature, the 1/N rule, risk parity, and market capitalisation-weighted strategies are treated as benchmarks. In this chapter, we also use all these three benchmarks to compare if the portfolio optimisation approaches, i.e., the mean-variance optimisation approach, the Black-Litterman optimisation approach, and the Parametric Portfolio Policy approach, perform better than these three benchmarks.

The 1/N rule (equally weighted strategy) is a practical portfolio strategy that is easy to implement and has been found to be a strong benchmark (DeMiguel et al., 2009a). The rule itself is unique in that it completely ignores historical information and assigns time-invariant portfolio weights. Some literature (*e.g.,* Bloomfield et al., 1977; Jobson and Korkie, 1980; Benartzi and Thaler, 2001; DeMiguel et al., 2009a; Duchin and Levy, 2009; Gelmini and Uberti, 2024) provides evidence that optimal portfolio strategies (*e.g.,* the mean-variance optimisation and the Black-Litterman optimisation) do not outperform the 1/N rule. However, others (*e.g.,* Kirby and Ostdiek, 2012; Bessler et al., 2017; Hsu et al., 2018) argue that optimisation approaches beat the 1/N rule. Our comparison is motivated by a thought-provoking question raised by DeMiguel et al. (2009a): do those portfolio strategies developed in academia really outperform a naive 1/N rule? When they consider various MV-based portfolio strategies as well as a range of different datasets, they report that none of those strategies consistently outperform the 1/N rule. Since then, the 1/N rule has become a popular benchmark to evaluate portfolio strategies.

The risk parity strategy, also referred to as the equally weighted risk contribution portfolio, has garnered considerable attention recently. Its goal is to build a risk-balanced portfolio with the same risk contribution for all asset components (Qian, 2005; Demey et al., 2010; Hsu et al., 2018; Costa et al., 2019; Costa and Kwon, 2020; Fabozzi et al., 2021; Anis and Kwon, 2022). Investment weights are allocated according to the volatility or risk of each asset class in a portfolio, rather than their expected return. A significant advantage of Risk Parity weighting compared to mean-variance optimisation is that investors do not need to formulate expected return assumptions for portfolio construction (Kolm et al., 2014; Fabozzi et al., 2021). The first risk parity fund launched in 1996, and it has become popular among practitioners and academics since then (Maillard et al., 2010; Mausser and Romanko, 2014; Bai et al., 2016; Costa and Kwon, 2019; Li et al., 2022). Thus, it is key to analyse if portfolio optimisation approaches outperform the equally weighted risk contribution approach.

A market capitalisation-weighted portfolio, also called a cap-weighted portfolio, is one where the weight of each asset is determined by the ratio of its market capitalisation to the total market capitalisation of all assets in the portfolio. The weights of all assets in the portfolio add up to one. The market capitalisation weighted approach is another benchmark in the portfolio construction (Grinold, 1992; Ko et al., 2024) and it is one of the most common used approaches to compile indexes such as S&P 500 (Elton et al., 2009; Branch and Cai, 2010; Bolognesi et al., 2013). The market capitalisation-weighted portfolio is another benchmark in this chapter.

In this chapter, we employ three portfolio optimisation approaches, including the mean-variance optimisation approach, the Black-Litterman optimisation approach, and the Parametric Portfolio Policy (PPP) approach. The mean-variance approach and Black-Litterman approach are popular optimisation frameworks and also have a long history, while the Parametric Portfolio Policy (PPP) optimised portfolio is novel to the portfolio optimisation field.

The mean-variance portfolio optimisation framework (Markowitz, 1952) is widely employed in academic literature, which is one of the most popular portfolio optimisation approaches. It is a method of portfolio optimisation based on Modern Portfolio Theory (MPT) and seeks to construct portfolios that maximise the expected return for a given level of risk. This is done by calculating the expected return and volatility of each asset class or security and using these estimates to construct portfolios that maximise returns while minimising risk (Markowitz, 1952;

Kim et al., 2021). However, in practice, estimates of the necessary input parameters are so imprecise that a naive 1/N strategy that optimises the composition of a risky asset portfolio without using any historical data often leads to better performance. For example, Board and Sutcliffe (1994), Jagannathan and Ma (2003), DeMiguel et al. (2009a), Dickson (2016), Li (2016), Hwang et al. (2018), and Barroso and Saxena (2021) show that the 1/N rule has a bigger Sharpe ratio than the mean-variance optimisation for individual equities. In the existing optimal portfolio selection literature, the superior performance of the 1/N portfolio strategy relative to the optimal portfolio strategy in out-of-sample asset allocation tests is largely attributed to the estimation error of the optimal portfolio strategy (Hanoch and Levy, 1969; Kalymon, 1971; Jobson and Korkie, 1981; Chaves et al., 2011). In order to implement an optimisation model in practice, model parameters such as the expected vector and variance-covariance matrix of asset returns need to be estimated from historical data. However, the future (true) parameters of the asset return distribution estimated based on historical data are uncertain, and trying to predict them is a difficult task. Estimation errors in the optimal portfolio strategy can produce extreme weights that fluctuate widely over time, leading to poor out-of-sample performance.

The Black-Litterman (BL) model aims to enhance asset allocation decisions by overcoming the weaknesses of standard mean-variance (MV) portfolio optimisation. Practitioners frequently try to cope with these problems by implementing constraints on the portfolio weights and turnover. Black and Litterman (1992) propose an alternative approach to deal with the shortcomings of MV and to improve portfolio performance. Their approach has gained increasing attraction among practitioners. The Black-Litterman (BL) model combines Capital Asset Pricing Theory (CAPM) with Bayesian statistics and Markowitz's modern portfolio theory (mean-variance optimisation) to produce efficient estimates of the portfolio weights (Black and Litterman, 1992). The Black-Litterman model starts with an investor's views on the expected returns of different asset classes or securities and then uses these views to construct portfolios that maximise expected returns while minimising risk (Black and Litterman, 1992; Bessler et al., 2017).

Parametric Portfolio Policy (PPP) is also called the characteristic portfolio approach for portfolio optimisation, and it is formally proposed by Brandt et al. (2009). We also call the PPP-optimised portfolio the characteristic-based portfolio. It parameterises the asset weights as a function of their characteristics, thereby estimating those parameters in a way that maximises the investor's average utility. The implicit assumption is that the characteristics convey all relevant information about the assets' conditional distribution of returns. Given the advantages of this approach, we apply it to construct an optimised portfolio and compare its performance to three benchmarks and the other two portfolio optimisation approaches.

In the construction of the Parametric Portfolio Policy (PPP) optimised portfolio, we methodologically extend Brandt et al. (2009), but instead of optimising large-scale stock portfolios, we solve for optimal internationally diversified index portfolios. To accomplish this objective, we adopt a methodology wherein portfolio weights are expressed as a mathematical function of the characteristics of indexes. By solving for these parameters, we aim to optimise the investor's average utility. We optimise internationally diversified index portfolios from the perspective of U.S. investors. To form the characteristic pool for indexes, we rely on empirical literature and theory in economics and finance. In their original paper, Brandt et al. (2009) use momentum, the book-to-market ratio, and market capitalisation. In this research, we have incorporated additional characteristics, such as the return-to-equity ratio, dividend yield, and volume for each selected index, in addition to momentum, book-to-market ratio, and market capitalisation. Overall, we chose the following characteristics: market capitalisation, return-to-equity ratio, book-to-market ratio (also called the price-to-book ratio), 12-month cumulative return (as an indicator of momentum), dividend yield, and volume. We assume that investors have a constant relative aversion (CRRA) of five. Before modelling the characteristic-based portfolios, we run a pre-sample test to determine which characteristics matter for investment purposes. According to the results of the pre-sample test, we form three types of PPP-optimised portfolios: two-characteristic optimised portfolios (PPP-Two), three-characteristic optimised portfolios (PPP-Three), and all six-characteristic optimised portfolios (PPP-Six), all of which include the market capitalisation characteristic.

We use all six diversification strategies in 60 expanding windows to calculate the optimal weight for each asset across the dataset and simulate portfolios that are re-estimated and updated every month. The interpretation is that at the end of each month, we optimise with all the data we have until then and build our portfolio for the next month. Here, we explain the first expanding window as an example. We first construct the first expanding window, starting in January 2006 and ending in December 2018, and estimate the in-sample optimised weight allocation for all 8 diversification strategies. After we get the in-sample weight allocation, we use it to simulate the out-of-sample results for the next month, namely, January 2019. For each portfolio strategy in our dataset, we calculate its in- and out-of-sample Sharpe ratio (SR) as the measurement of the performance. We consider the effect of the short-selling, and we do not place any limitations on the weight when we simulate portfolios without short-selling constraints. However, when we optimise portfolios with short-selling constraints, we limit the portfolio weight to be exactly one.

The main research questions are: 1. Which kinds of asset characteristics matter when optimising portfolio selection using the Parametric Portfolio Policy (PPP) approach for international diversified portfolios? 2. Is it feasible to optimise portfolio selection using the Parametric Portfolio Policy (PPP) approach for international diversified portfolios? 3. Does the Parametric Portfolio Policy (PPP) approach generate superior in- and out-of-sample portfolio performance compared to the naïve diversified portfolios (1/N-rule), market capitalisation weighted, risk parity (equally weighted risk contribution), mean-variance (MV), and Black-Litterman approaches?

Our empirical results offer new insights from the in- and out-of-sample simulation. The in-sample simulation produces the following findings. First, the mean-variance and Black-

Litterman optimised portfolios have the best performance, no matter with or without short-selling constraints, among all six basic strategies and always outperform the three benchmarks (1/N rule, market capitalisation-weighted strategy, and risk parity strategy). Second, all three types of characteristic-optimised portfolios outperform the three benchmarks when short selling constraints are not present. Third, when short selling is controlled for, the performance of all three types of characteristic-optimised portfolios falls short of the 1/N rule and the market capitalisation-weighted portfolio in terms of in-sample performance. Fourth, within three benchmarks, the market capitalisation-weighted portfolio performs better than the 1/N rule and the risk parity strategy, no matter whether it controls for short selling or not.

The out-of-sample simulation leads us to the following conclusions. First, we find that the performances of the mean-variance optimised and Black-Litterman optimised portfolios are always better than the three benchmarks (equally weighted portfolio, risk parity portfolio, and market capitalisation-weighted portfolio), no matter with or without short-selling constraints and no matter before or after the deduction of the transaction costs. Our out-of-sample results are consistent with some studies (*e.g.,* Durand et al., 2011; Han, 2016; Platanakis et al., 2021) that the mean-variance optimisation outperforms the 1/N rule in terms of Sharpe ratio and are consistent with Bessler et al. (2017), who find that BL-optimised portfolios perform better than naïve diversified portfolios in terms of out-of-sample Sharpe ratio.

Second, prior to controlling short-selling, the Black-Litterman (BL) optimised portfolio outperforms the mean-variance (MV) optimised portfolio, regardless of whether transaction costs are deducted before or after. However, once short-selling is controlled, the mean-variance (MV) optimised portfolio outperforms the Black-Litterman model, regardless of whether transaction costs are deducted before or after. This outcome differs slightly from the findings of Bessler et al. (2017), who find that the BL model consistently outperforms the MV model.

Third, all three types of PPP-optimised portfolios without short-selling constraints perform better than the three benchmarks (equally weighted portfolio, risk parity portfolio, and market capitalisation-weighted portfolio), both before and after the deduction of the transaction costs, and even the six-characteristic optimised portfolio without short-selling constraints performs better than all other selected portfolio strategies before the deduction of the transaction costs. This result is new in the literature, as there is no work comparing the performance of PPP-optimised portfolios with other portfolio strategies.

Fourth, the six-characteristic optimised portfolio consistently outperforms the equally weighted portfolio and the risk parity but could not outperform the market capitalisation-weighted portfolio consistently; the three-characteristic optimised portfolio consistently outperforms only the equally weighted portfolio among the three benchmarks, and the two-characteristic optimised portfolio could not consistently outperform any benchmarks. This result is also new to the literature.

Fifth, the market capitalisation-weighted portfolio performs better than the equally weighted portfolio and the risk parity portfolio, both before and after the deduction of the transaction costs. Some studies (*e.g.,* Plyakha et al., 2012; Bolognesi et al., 2013; Malladi and Fabozzi, 2017; Taljaard and Maré, 2021) document that the equal-weighted stock portfolio is more efficient than the market capitalisation-weighted portfolio over the long term.

We contribute to the literature by empirically testing the Parametric Portfolio Policy (PPP) approach, in which we conduct in- and out-of-sample multi-index portfolio optimisations for the period from December 2004 to December 2023. We also implement the naive diversified (1/N-rule) strategy, market capitalisation weighted, risk parity (equally weighted risk contribution), mean variance (MV), and Black-Litterman strategies to construct portfolios and compare the respective portfolio performance with the performance of the Parametric Portfolio Policy (PPP) approach. In addition, to estimate the impact of all six characteristics on the performance of the Parametric Portfolio Policy (PPP), we create three types of optimised portfolios: two-characteristic optimised portfolios (PPP-Two), three-characteristic optimised portfolios (PPP-Three), and all six-characteristic optimised portfolios (PPP-Six). In their original work of Brandt et al. (2009), Brandt, Santa-Clara, and Valkanov use three characteristics, including momentum, book-to-market ratio, and market capitalisation. In this research, we incorporate additional characteristics such as return-to-equity ratio, dividend yield, and volume for each selected index, in addition to momentum, book-to-market ratio, and market capitalisation.

We organise the remainder of this chapter as follows: Section 3.2 provides a literature review of each strategy we use in this chapter. Section 3 presents the methodology. We provide the data and its description in Section 3.4. Section 3.5 presents and discusses empirical results and compares the performance of portfolio strategies. Section 3.6 concludes the whole chapter.

### 3.2. Literature Review

Portfolio construction is one of the central problems in the financial field. In the literature, there are two broad categories for portfolio construction. The first category pertains to basic portfolio allocation and includes benchmarks such as the 1/N rule, which is an equally weighted strategy; the risk parity, which is an equally weighted risk contribution; and the market capitalisation-weighted approach, which allocates investment weights according to the market capitalisation of each asset. Another category is for portfolio optimisation, which includes the mean variance (MV) framework, Black-Litterman optimisation, and the Parametric Portfolio Policy (PPP) approach. Out of this, the Parametric Portfolio Policy (PPP) approach is a novel methodology, developed by Brandt et al. (2009), but its implementation in the literature remains limited, with only a handful of papers employing it for portfolio construction.

### *3.2.1 1/N Rule*

The 1/N rule (equal-weighted portfolio) is a well-known investment strategy (*e.g.,* DeMiguel et al., 2009a) that allocates the same amount to each available asset (from the total number of different assets N), which is unique in that it completely ignores historical information and assigns time-invariant portfolio weights (Hsu et al., 2018). In the literature, most studies use it as a benchmark portfolio against which other portfolio investment strategies are compared. (Clarke et al., 2006; Duchin and Levy, 2009; DeMiguel et al., 2009b; Fletcher, 2009; Jiang et al., 2013; Hsu et al., 2018). The equal-weighting approach has several benefits that make it useful for investors in constructing portfolios:

1) Equal-weighted portfolio can mitigate concentration bias because weights are evenly distributed among all assets in the portfolio.

2) In the equal-weighted asset rebalancing process, effectively follow the "buy low, sell high" strategy, sell expensive assets and buy cheaper assets, thereby investing in the best performing assets.

3) The equal-weight method does not underperform the asset which is the worst performing.

4) Due to the low turnover rate, the transaction cost of the equal-weight method is relatively low compared to other dynamic asset allocation methods (Palit and Prybutok, 2024; Kritzman et al., 2010).

In an early study, Jobson and Korkie (1980) find that “Naïve rules such as the equal-weight rule can outperform the Markowitz rule.” Benartzi and Thaler (2001) provide evidence that the 1/N rule is a popular strategy among private investors, with one-third of direct contribution plan participants dividing their assets equally among investment options. Despite the seemingly simple nature of this portfolio construction approach, research shows that 1/N portfolios outperform out-of-sample optimised portfolios. Tang (2004) argues that simple diversification is a simple but effective method to effectively reduce the risk of a portfolio without sacrificing expected returns. Tang (2004) also finds that, given an infinite number of stocks, portfolio size affects the expected efficiency of naive diversification; on average, a portfolio of 20 stocks is required to eliminate 95% of the diversifiable risk; on average, an additional 80 stocks (i.e., a portfolio size of 100 stocks) is required to eliminate an additional 4% (i.e., a total of 99%) of the diversifiable risk.

Duchin and Levy (2009) compare the 1/N rule with the Markowitz mean-variance optimisation using 30 Fama-French industry portfolios over the period 1991-2007. They conclude that for individual small portfolios, the 1/N rule outperforms the MV optimisation strategy, but for large portfolios (i.e., institutional investors), the MV strategy provides superior results in an out-of-sample framework. Another recent study by DeMiguel et al. (2009a) analyses whether the MV strategy and its variants adopted in the literature outperforms a simple diversified 1/N portfolio in a variety of different asset allocation data sets. Using a variety of performance measures, DeMiguel et al. (2009a) find that none of the various MV strategies consistently outperforms the naive equal-weighted benchmark (1/N) on three criteria (Sharpe ratio, certainty equivalent, and turnover) in out-of-sample applications. Pflug et al. (2012) report that monthly rebalanced equal-weighted (1/N) stock portfolios achieve higher total average returns, four factor alphas (Fama and French, 1993; Carhart, 1997), and Sharpe ratios compared to value-weighted and price-weighted portfolios. Plyakha et al. (2012) observe that monthly rebalanced equal-weighted portfolios outperform the optimised portfolio even after accounting for transaction costs. Murtazashvili and Vozlyublennaia (2013) show that a mean-variance optimal portfolio does not outperform a simple 1/N diversification strategy (out-of-sample) even when securities are grouped into indexes or broad asset classes. Sass and Westphal (2020) extend this observation to continuous-time models. Both results demonstrate the robustness of the equal-weighted strategy, which is difficult to beat when average uncertainty is high. Gelmini and Uberti (2024) confirm the results of DeMiguel et al. (2009a) that the equal-weighted portfolio remains a difficult benchmark to beat.

However, according to Kritzman et al. (2010), the optimised portfolio outperforms the equal-weighted portfolio. In their study, the optimised portfolio produced superior out-of-sample performance compared to the equal-weighted portfolio. Kirby and Ostdiek (2012)'s analysis suggests that the results of DeMiguel et al. (2009a) are mainly driven by their research design and the choice of asset allocation dataset. When transaction costs are considered, the results of

Kirby and Ostdiek (2012) suggest that high turnover weakens the benefits of MV optimisation. These findings may explain why naive diversification methods are gaining attention in academia and among practitioners. Kirby and Ostdiek (2012) also find that market timing skills make mean-variance efficient portfolios often outperform naive diversification (1/N rule).

Continually, Bessler et al. (2017) find that the sample-based MV approach slightly outperforms a naïve 1/N strategy. In line with Kirby and Ostdiek (2012), they find the level of outperformance (after transaction costs) of MV is insignificant. Their findings differ from that of DeMiguel et al (2009a) and Murtazashvili and Vozlyublennaia (2013) who conclude that none of the variations of MV can outperform a naïve 1/N strategy. They explain that the difference is in the employed data set. While both earlier studies (DeMiguel et al., 2009a; Murtazashvili and Vozlyublennaia, 2013) analyse stock-only portfolios, Bessler et al. (2017) additionally include government bonds, corporate bonds, and commodities that should result in broader diversification and might enhance the portfolio optimisation benefits. They suggest that by including government bonds, asset allocation models may outperform naïve strategies if investors actively shift wealth from stocks to bonds during stock market downturns and vice versa. Hsu et al. (2018) assess the out-of-sample performance of 16 portfolio strategies based on traditional MV strategies relative to the naive 1/N rule. They find that some strategies outperform the 1/N rule in conventional tests that do not account for data snooping bias.

However, after they use the new tests that control for such bias, they find that none or very few of these strategies outperform the 1/N rule.

It is well documented that the equal weighted stock portfolio is more efficient than the market capitalisation (cap-weighted) portfolio over the long-term (*e.g.,* Plyakha et al., 2012; Bolognesi et al., 2013; Malladi and Fabozzi, 2017; Taljaard and Maré, 2021). Taljaard and Maré (2021) show that equal-weighted portfolios do outperform market capitalisation-weighted portfolios in the long run, but there is significant underperformance in the short run. Some studies mix the 1/N rule with other approaches like the mean-variance approach and Black-Litterman. For example, Kan and Zhou (2007), Tu and Zhou (2011) and Branger et al. (2019) suggest a combination of mean-variance strategies and the naive portfolio.

Overall, the literature on portfolio construction highlights the ongoing debate between naive strategies, such as the equal-weight (1/N) rule, and more sophisticated approaches, such as Markowitz mean-variance (MV) optimisation. There are some studies (*e.g.,* Jobson and Korkie, 1980; Benartzi and Thaler, 2001; DeMiguel et al., 2009a; Sass and Westphal, 2020; Gelmini and Uberti, 2024) that show that 1/N portfolios can outperform optimised portfolios in out-of-sample testing. Conversely, there is also literature (*e.g.,* Kirby and Ostdiek, 2012; Bessler et al., 2017; Hsu et al., 2018) that shows that optimised portfolios outperform the 1/N rule. There are also some studies (*e.g.,* Kan and Zhou, 2007; Tu and Zhou, 2011; Branger et al., 2019) that suggest combining 1/N with other strategies, showing that hybrid approaches can produce better results.

### *3.2.2 Mean-variance Optimisation Approach*

The mean-variance (MV) optimisation framework, developed by Markowitz (1952), is widely employed by academic literature, and it is one of the most popular portfolio optimisation approaches. The framework is based on Modern Portfolio Theory (MPT), which aims to construct a portfolio that maximises expected return for a given level of risk. This is done by calculating the expected return and volatility of each asset class or security and using these estimates to construct a portfolio that maximises return while minimising risk (Markowitz, 1952). The mean-variance optimisation framework is a simple and elegant portfolio construction method with excellent theoretical properties and remains popular among practitioners and academics (Kolm et al., 2014). The classical theory of mean–variance analysis suggests that the optimal portfolio lies on the efficient frontier (Markowitz, 1952, Tobin, 1958).

Some studies (*e.g.,* Bekaert and Urias, 1996; De Roon et al., 2001; Chiang et al., 2007; Galema et al., 2011; Daskalaki and Skiadopoulos, 2011; Bessler et al., 2012) use the MV framework to test the diversification benefits for including an additional asset class in a multi-asset portfolio context. There are also substantial studies (*e.g.,* Board and Sutcliffe, 1994; Jagannathan and Ma, 2003; Durand et al., 2011; Dickson,2016; Han, 2016; Platanakis et al., 2021) comparing the mean-variance optimisation to other portfolio techniques. The most common comparison is between mean-variance optimisation and the 1/N rule. Some studies (*e.g.,* Durand et al., 2011; Han, 2016; Platanakis et al., 2021) find that the mean-variance optimisation outperforms the 1/N rule in terms of Sharpe ratio. However, in practice, estimates of the necessary input parameters are so imprecise that a naive 1/N strategy that optimises the composition of a risky asset portfolio without using any historical data often leads to better performance. For example, Board and Sutcliffe (1994), Jagannathan and Ma (2003), DeMiguel et al. (2009a), Dickson (2016), Li (2016), Hwang et al. (2018), and Barroso and Saxena (2020) show that for individual stocks, the 1/N rule yields higher Sharpe ratios than mean-variance optimisation.

In the mean-variance framework, the expected risk and return of a portfolio are estimated based on historical data. If asset returns follow a normal distribution and parameter estimates are known, the MV portfolio framework is the optimal strategy in terms of expected utility (Hanoch and Levy, 1969). However, the future (true) parameters of the asset return distribution are uncertain and trying to predict them is a difficult task (Kalymon, 1971). Therefore, expectations about the risk-return structure of a portfolio may not be realised ex post. In fact, an early study by Jobson and Korkie (1981) highlights the large estimation errors when using sample estimates and the poor out-of-sample performance of MV strategies. However, return estimation errors are more critical than estimation errors of the covariance matrix as they have about ten times the impact on the optimisation of portfolio weights (Chopra and Ziemba, 1993). In the mean-variance optimisation framework, assets with the largest estimation errors tend to receive the highest portfolio weights, leading to "estimation error maximisation" (Michaud, 1989). Furthermore, mean-variance approaches tend to produce extreme portfolio allocations and low levels of diversification across asset classes (Broadie, 1993), i.e., the optimised portfolios are often corner solutions. In addition, mean-variance optimal portfolio weights are highly sensitive to changes in input parameters and can lead to radical portfolio reallocations even with small changes in expected return estimates (Best and Grauer, 1991). Barry (1974) and Chopra and Ziemba (1993) show the high sensitivity in particular when estimating the expected returns. Chaves et al. (2011) also argue that the mean–variance optimisation methodology developed by Markowitz (1952) is difficult to implement due to the challenges associated with estimating the expected returns and covariances for asset classes with accuracy.

In summary, the traditional mean–variance approach of Markowitz (1952) requires modelling the expected returns, variances, and covariance of all stocks as functions of their characteristics. As an efficiency paradigm for portfolio selection it has some very attractive features, including ease of application and analysis, and a long history of theoretical understanding and practical experience (Durand et al., 2011). However, when we apply it in practice, estimating the expected return, variance, and co-variance is not only a formidable econometric problem given the large number of moments involved and the need to ensure the positive definiteness of the covariance matrix, but the results of the procedure are also notoriously noisy and unstable (Michaud, 1989; Brandt et al., 2009; Chaves et al., 2011; Costa and Kwon, 2020).

### *3.2.3 Risk Parity Approach*

Risk parity has garnered considerable attention recently. This is an asset allocation strategy that aims for equal risk contribution from each asset class or security within a portfolio (Qian, 2005; Demey et al., 2010; Costa and Kwon, 2020; Fabozzi et al., 2021; Anis and Kwon, 2022). Investments are allocated according to the volatility or risk of each asset class or security, rather than their expected return. A significant advantage of risk parity weighting compared to mean-variance optimisation is that investors do not need to formulate expected return assumptions for portfolio construction (Kolm et al., 2014; Fabozzi et al., 2021). Since its launch in 1996, the first risk parity fund has gained popularity among practitioners and academics (Maillard et al., 2010; Mausser and Romanko, 2014; Bai et al., 2016; Costa and Kwon, 2019; Li et al., 2022).

Merton (1980) argues that the only necessary input in risk parity strategy is the covariance of the asset classes, which is generally more accurate than predicting returns derived from historical data. Maillard et al. (2010) find that, overall, risk parity portfolios appear to be attractive alternatives to minimum variance and 1/N portfolios and can be viewed as a good trade-off between the two approaches in terms of absolute risk level, risk budget, and diversification. Chaves et al. (2011) find that the risk parity portfolio structure does not generally outperform equal weights, or a model pension fund portfolio based on a traditional 60/40 stock-bond allocation in terms of risk-adjusted performance. Nonetheless, they observe that it consistently outperforms the best allocation strategies, including minimum variance and mean-variance efficient portfolios.

However, some strategists (*e.g.,* Foresti and Rush, 2010; Chaves et al., 2011; Fabozzi et al., 2021) argue that the neglect of returns is a significant shortcoming of the model. Their view is that the role of an asset manager is to maximise returns, not minimise risk. Therefore, despite the difficulty of predicting asset class returns, some strategists argue that the weighting of each asset class should be based on its Sharpe ratio, which involves estimating expected returns. The use of covariance matrices in risk parity models brings up a second problem. The implementation of the model may still require reliable variance and covariance forecasts for a large number of assets, thereby posing the same fundamental dimensionality problem as mean-variance optimisation. Even when relying solely on historical data, potential challenges persist, such as the possibility that some assets lack sufficient historical data to be confidently modelled. Therefore, while the pure risk nature of the risk parity model simplifies the modelling challenge, it does not eliminate it entirely. The susceptibility of the input covariance matrix to portfolio outcome instabilities, which plague any optimisation exercise relying on quadratic programming techniques like Markowitz's (1956) critical line algorithm, presents a third related problem. Small changes to the optimisation inputs often result in significantly different portfolios. This phenomenon is especially evident in the fundamental mean-variance framework. Risk parity methods address this problem to some extent by abandoning expected returns, but they do not eliminate it. This is because quadratic programming methods require the inversion of a positive definite covariance matrix.

### *3.2.4 Market Capitalisation-weighted Approach*

A market capitalisation-weighted portfolio, also called a cap-weighted portfolio, is one where the weight of each asset is determined by the ratio of its market capitalisation to the total market capitalisation of all assets in the portfolio. The weights of all assets in the portfolio add up to one. The market capitalisation-weighted approach is another benchmark in the portfolio construction (Grinold, 1992; Ko et al., 2024) and it is one of the most common used approaches to compile indices such as the S&P 500 Index (Elton et al., 2009; Branch and Cai, 2010; Bolognesi et al., 2013). In an efficient market, the market capitalisation-weighted portfolio is considered a proxy for the market portfolio. Both are expected to yield the same risk-return characteristics under the assumptions of the CAPM. Bodie et al. (2013) believe that when we sum over, or aggregate, the portfolios of all individual investors, lending and borrowing will cancel out (because each lender has a corresponding borrower), and the value of the aggregate risky portfolio will equal the entire wealth of the economy. Under this scenario, they provide suggestions that investors can skip the trouble of doing security analysis and obtain an efficient portfolio simply by holding the market portfolio as the market portfolio held by all investors is based on the common input list, thereby incorporating all relevant information about the universe of securities.

However, early works (*e.g.,* Haugen and Baker, 1991; Grinold, 1992) provide evidence that cap-weighted portfolios are not well-diversified portfolios and thus lead to an inefficient risk–return trade-off. Haugen and Baker (1991) contend that the inefficiency and poor diversification of market-cap-weighted portfolios, which heavily concentrate the largest market-cap stocks, may not come as a surprise, and this is due to the construction of these portfolios using a single mechanism that solely takes stock market capitalisation into account. Following such early criticism of cap weighted equity portfolios, more recent papers have documented that cap-weighted portfolios suffer from numerous shortcomings, and various alternative weighting schemes have been proposed to improve on cap weighting (*e.g.,* Amenc et al., 2011; Arnott et al., 2005; Choueifaty and Coignard, 2008; Maillard et al., 2010) to name but a few. Although it is now commonly accepted that moving away from cap weighting tends to enhance diversification and increase risk adjusted performance over long horizons, it must be recognised that each alternative weighting scheme will expose an investor to two related types of risk, namely, model selection risk and relative performance risk (Amenc et al., 2012). Hsu (2004) shows that under a rather innocuous assumption of price efficiency, market capitalisation-weighted portfolios are suboptimal. Arnott et al. (2005) find that capital weighted portfolios tend to be flawed when it comes to pricing resulting into a price drag. It is therefore not prudent to fully rely on them. Managers should rather focus on building mean variance efficient indices than those built on capitalisation weighting.

### *3.2.5 Black-Litterman Optimisation Model*

The Black-Litterman (BL) model is one of the most prevalent portfolio optimisation models out there, which was developed by Black and Litterman (1992). It combines capital asset pricing theory (CAPM) with Bayesian statistics and Markowitz's modern portfolio theory (mean-variance optimisation) to produce efficient estimates of the portfolio weights (Bessler et al., 2017). The model starts with an investor's views on the expected returns of different asset classes or securities and then uses these views to construct portfolios that maximise expected returns while minimising risk (Black and Litterman, 1992; Bessler et al., 2017). This model is particularly useful for investors who have strong views on the expected performance of specific asset classes or securities.

In the academic literature, some works (*e.g.,* Satchell and Scowcroft, 2000; Lee, 2000; Drobetz, 2001; Idzerek, 2005; Mishra et al., 2011; Bessler et al., 2017; Ko et al., 2024) analyse the rationale of the BL model, provide examples for its implementation, and combine it with other models or approaches. Mishra et al. (2011) find that the Black-Litterman portfolio achieve a significantly better return-to-risk performance than the mean-variance optimal approach/strategy. Creamer (2015) proposes a method where investors' expectations are based on either news sentiment using high-frequency data or on a combination of accounting variables, financial analysts' recommendations, and corporate social network indicators with quarterly data. They find their results show promise when compared to a market portfolio.

Bessler et al. (2017) implement the Black-Litterman optimisation model in a multi-asset portfolio setting. They empirically test the out-of-sample portfolio performance of the BL-optimised portfolio using an investment universe of global stock indexes, bonds, and commodities and compared the results with mean-variance (MV), minimum-variance, and equally weighted portfolios (1/N rule) over the period January 1993 to December 2011. They find that the BL-optimised portfolio outperforms the MV and equally weighted portfolios in terms of out-of-sample Sharpe ratios, even after controlling for varying degrees of risk aversion, realistic investment constraints, and transaction costs.

Arisena et al. (2018) find that the ARMA-GARCH model can be used to determine investors' views in the Black-Litterman model but with the assumption that the time series assumption can be fulfilled. Kara et al. (2019) utilise the ARMA-GARCH (1,1) model and learning algorithms to forecast asset returns, thereby generating a base for the application of the Black-Litterman model. Their results reveal better portfolio returns and Sharpe ratios than the index return for different holding periods, as well as better portfolio returns than randomly generated portfolios. Chen and Lim (2020) show how the views of multiple experts can be modelled as a Bayesian graphical model and estimated using historical data, which may be of interest in applications that involve the aggregation of expert opinions for the purpose of decision making. Due to some significant advantages of machine learning algorithms, Min et al. (2021) implement the Black-Litterman model using investor views generated by machine learning algorithms. They find that the BL model with the machine learning algorithm is robust and well performed. Ko et al. (2024) propose a novel asset allocation framework that integrates the Fama-French three-factor model into the Black-Litterman model. They find that the Black-Litterman model combined with the Fama-French three-factor model generates better performance than usual benchmarks (market capitalisation-weighted portfolio and mean-variance portfolio), lowering estimation error, and significantly raising alpha.

However, the model does have its shortcomings. First, the model restricts investors' ability to specify private information. That is, it only allows investors to specify views on asset returns, not on their volatility or market dynamics. Second, and more strictly, the model presupposes a mean-variance approach to portfolio allocation. A large body of theoretical and empirical research suggests that variance may not be a suitable proxy for risk (Meucci, 2005; Giacometti et al., 2007; Martellini and Ziemann, 2007; Bertsimas et al., 2012).

### *3.2.6 Parametric Portfolio Policy (PPP) Approach*

The characteristic portfolio approach for portfolio optimisation is first proposed by Brandt and Santa-Clara (2006). In Brandt et al. (2009), they further improved and clarified this strategy and named it Parametric Portfolio Policy (PPP). This approach parameterises the portfolio's weights of each asset as a function of the asset's characteristics and then maximises the investor's average utility by choosing optimally the coefficients of this function. The advantages of this approach are that 1) it is easy to implement; 2) it has good in and out-of-sample performance; 3) some of the methods we use to optimise the Markowitz model can also be used in this framework, which includes using portfolio constraints, shrinkage estimates, and combining investors' prior beliefs with the information contained in return history (Barroso and Santa-Clara, 2015; Fletcher, 2017; Joenväärä et al., 2021).

Several papers adapt and extend the parameter portfolio approach in the existing literature. Plazzi et al. (2011) apply recent advances in portfolio management (Brandt et al., 2009) to efficiently incorporate the information contained in property-specific conditioning variables to the allocation of commercial real estate portfolios. They find that incorporating property-specific characteristics has the potential to improve the performance of the commercial real estate portfolio. DeMiguel et al. (2013) investigate how the information implied by option prices can be used to improve portfolio selection by applying the "parametric-portfolio methodology," of Brandt et al. (2009) by using model-free implied volatility (MFIV), model-free implied skewness (MFIS), the call-put-implied volatility spread (CPVS), and the implied-realised-volatility spread (IRVS), in addition to the traditional stock characteristics (size, value and momentum), to construct parametric portfolios based on mean-variance utility. They find that using these characteristics to rank stocks and adjusting by a scaling factor the expected returns of the stocks or using these characteristics with the parametric-portfolio methodology of Brandt et al. (2009), leads to a substantial improvement in the Sharpe ratio, even after prohibiting short-sales and accounting for transactions costs.

Barroso and Santa-Clara (2015) test the relevance of technical and fundamental variables in forming currency portfolios using the Parametric Portfolio Policy approach of Brandt et al. (2009). They find that find that a Parametric Portfolio Policy diversified currency portfolio that exploits features such as momentum, yield differentials, and value reversals outperforms the carry trade by a wide margin. This outperformance is reflected in higher Sharpe ratios and fewer severe drawdowns, as value reversals and momentum have large positive returns when the carry trade collapses.

Joenväärä et al. (2021) study the portfolio selection methods of hedge funds separately in the context of investors' overall portfolios. Their approach relies on recent developments in portfolio choice techniques that model the portfolio's weight in each asset as a function of the asset's characteristics (*e.g.,* Brandt et al., 2009; Brandt and Santa-Clara, 2006). From a methodological perspective, they extend the approach of Brandt et al. (2009), but instead of optimising a large equity portfolio, they solve for the optimal hedge fund portfolio. They find that characteristic-based portfolios that minimise risk deliver superior out-of-sample performance.

Overall, each strategy has its advantages and disadvantages, and currently there is no work that comprehensively compares the performance of all strategies we aforementioned. Therefore, in this paper we utilise the Parametric Portfolio Policy (PPP), mean-variance (MV), and Black-

Litterman (BL) strategies to construct portfolios and compare their results to three benchmarks (naïve diversified portfolios (1/N-rule), risk parity (equally weighted risk contribution), and market capitalisation weighted).

### 3.3. Methodology

In this chapter, we use six strategies to build portfolios for our data set and compare their performance for each of them, including the naïve diversified strategy (1/N-rule), market capitalisation weighted strategy, risk parity (equally weighted risk contribution) strategy, mean-variance (MV) strategy, Black Litterman (BL) strategy, and the Parametric Portfolio Policy (PPP) diversified strategies. Table 3.1 displays all strategies we use in this chapter. Among these, the naive diversified portfolio (1/N-rule), market capitalisation weighted portfolio, and the risk parity (equally weighted risk contribution) portfolio serve as benchmarks, while the mean-variance (MV) diversified portfolio, the Black-Litterman (BL) diversified portfolio, and the Parametric Portfolio Policy (PPP) portfolio are optimised strategies. We specify each methodology in this section.

**Table 3.1.** List of portfolio strategies.

| | Strategy | Abbreviation |
|---|---|---|
| 1 | 1/N Rule (Benchmark) | 1/N |
| 2 | Market Capitalisation weighted (Benchmark) | MC |
| 3 | Risk parity (Benchmark) | RP |
| 4 | Mean-variance (Optimisation approach) | MV |
| 5 | Black-Litterman (Optimisation approach) | BL |
| 6 | Parametric Portfolio Policy (Optimisation approach) | PPP |

The 1/N rule, market capitalisation weighted, and risk parity approaches are benchmarks, and the mean-variance, Black-Litterman, and Parametric Portfolio Policy approaches are optimisation approaches.

As the Parametric Portfolio Policy (PPP) approach parameterises the asset weights as a function of their characteristics, the asset characteristics are the key to forming an optimised portfolio. In the original work of Brandt et al. (2009), Brandt, Santa-Clara, and Valkanov use three characteristics, including momentum, book-to-market ratio, and market capitalisation. According to Brandt et al. (2009) and the data availability, we select six characteristics of stock indexes, including the market capitalisation, return-to-equity ratio, book-to-market ratio, dividend yield, volume, and the 12-month cumulative return.

### *3.3.1 Specification of the 1/N Rule and Risk Parity Strategy*

To assess whether the in- and out-of-sample performance of the previously described six strategies yields superior results, we use the 1/N rule, the risk parity, and the market capitalisation-weighted approach as benchmarks. The 1/N rule is an investing strategy that distributes an equal allocation to each accessible asset, whereas the risk parity is an equally weighted risk contribution strategy (ERC) in which each available component contributes the same risk proportion to the portfolio (DeMiguel et al., 2009a). The market capitalisation-weighted portfolio, also called a cap-weighted portfolio, is one where the weight of each asset is determined by the ratio of its market capitalisation to the total market capitalisation of all assets in the portfolio. The mean-variance approach, the Black-Litterman approach, and the Parametric Portfolio Policy approach are portfolio optimisation approaches.

*3.3.1.1 The 1/N rule*

We use an equal-weight approach, namely the 1/N rule or the equal-weighted portfolio (EWP) as one of the three benchmarks. The 1/N rule is a well-known investment strategy that allocates the same proportion of the investment budget to each available asset in the portfolio (DeMiguel et al., 2009a). The rule itself is unique in that it completely ignores historical information and assigns time-invariant portfolio weights. The provocative question posed by DeMiguel et al. (2009a) "Do portfolio strategies developed by academics really perform better than the simple 1/N rule?" provides inspiration for our comparison. In the 1/N strategy, each component in the portfolio holds a weight $w_i = 1/N$. The equally weighted approach can be expressed as the solution of the following equations:

$$w = \begin{pmatrix} w_1 \\ w_2 \\ \cdots \\ w_N \end{pmatrix} \tag{3.1}$$

where $w$ is the $N \times 1$ vector of portfolio weights. In the equally weighted portfolio, $w_1 = w_2 = \cdots = w_n$.

$$E(r) = \begin{pmatrix} E(r_1) \\ E(r_2) \\ \cdots \\ E(r_N) \end{pmatrix} \tag{3.2}$$

where $E(r)$ is the expected return.

$$E(r_P) = (E(r))^T w \tag{3.3}$$

Where, $E(r_P)$ is the expected return on the portfolio; $(E(r))^T$ is the transpose of the expected return.

After calculating the expected return for the equally weighted portfolio, we write the variance-co variance matrix of the return as following:

$$\Sigma = \begin{pmatrix} \delta_{11} & \delta_{12} & \delta_{13} & \cdots & \delta_{1N} \\ \delta_{21} & \delta_{22} & \delta_{23} & \cdots & \delta_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \delta_{N1} & \delta_{N2} & \delta_{N3} & \cdots & \delta_{NN} \end{pmatrix} \tag{3.4}$$

Where, $\Sigma$ is the variance-co variance matrix of the asset returns. The elements on the leading-diagonal of $\Sigma$ are the variances of each of the component assets' returns. The off-diagonal elements are the corresponding covariances.

The variance and the standard deviation of the portfolio are given by

$$\delta_p^2 = w^T \Sigma w \tag{3.5}$$

$$\delta_p = \sqrt{w^T \Sigma w} \tag{3.6}$$

respectively.

*3.3.1.2 Risk parity*

The risk parity strategy, also known as an equal-weighted risk contribution (ERC) approach, is the second benchmark in this chapter. Its goal is to solve the asset allocation problem by allocating wealth so that each asset contributes equally to the overall portfolio risk (Costa et al., 2019). The risk contribution of component *i* is the proportion of the total portfolio risk allocated to that component. This calculation is derived by multiplying the allocation of component *i* by its marginal risk contribution, which is defined as the change in total portfolio risk due to an unlimited increase in the holdings of component *i*. Managing risk contributions has been a traditional strategy for institutional investors and is known as "risk budgeting." Risk budgeting involves analysing a portfolio based on its risk contributions rather than portfolio weights. A significant advantage of risk parity weighting over mean-variance optimisation is that investors do not need to make expected return assumptions about portfolio construction (Kolm et al., 2014; Fabozzi et al., 2021). Qian (2005) points out that risk contributions are not just a (ex-ante) mathematical decomposition of risk; rather, they have financial relevance because they effectively predict the contribution of each position to (ex post) losses, especially for large losses.

Studying the in-sample and out-of-sample risk-return properties of equal-weighted risk contribution (ERC) portfolios is interesting because they mimic the diversification effect of an equal-weighted portfolio while accounting for both the individual and joint risk contributions of the assets. In other words, no single asset contributes more to the total risk of the portfolio than its peers (Maillard et al., 2010). Research has repeatedly shown that diversification can improve returns (Booth and Fama, 1992; Fernholtz et al., 1998). Starting with the framework presented in Markowitz (1952), the portfolio expected return and variance are given by

$$E(r_P) = (E(r))^T w \tag{3.7}$$

$$\delta_p^2 = w^T \sum w \tag{3.8}$$

$$\delta_p = \sqrt{w^T \sum w} \tag{3.9}$$

Where, $E(r_P)$ is the portfolio expected return; $E(r) \in R^n$ is the vector of asset expected returns; $(E(r))^T$ is the transpose of the expected return; $w \in R^n$ is the vector of asset weights (i.e. the proportion of wealth invested in each asset); $\delta_p^2$ is the portfolio variance and $\sum \in R^{n \times n}$ is the asset covariance matrix. The individual risk contribution of each asset can be derived from Equation (3.9) by an Euler decomposition of the portfolio standard deviation. As shown in Maillard et al. (2010), the risk contribution per asset is given by

$$\delta_i = w_i \frac{\partial \delta_p}{\partial w_i} = w_i \frac{(\sum w)_i}{\sqrt{w^T \sum w}} \tag{3.10}$$

where $w_i$ is the proportion of wealth allocated to asset *i* (i.e. the weight of asset *i*), $\partial \delta_p / \partial w_i$ is the marginal risk contribution per asset and $\delta_i$ is the individual risk contribution of asset *i*. The only risk measure we consider in this chapter is the volatility of the portfolio, that is, the portfolio variance or standard deviation. However, any other risk measure that can be decomposed into the marginal risk contribution of each asset can be used. The objective of the ERC method is to find a portfolio where $\delta_i = \delta_j \forall i, j$ , By minimising the squared differences in risk contribution, the optimisation model can be written as

$$\text{Minimise} \sum_{i=1}^{N} \sum_{j=1}^{N} (w_i(\Sigma w)_i - w_j(\Sigma w)_j)^2 \tag{3.11}$$

Subject to:

$$\sum_{i=1}^{N} w_i = 1, \; 0 \leq w_i \leq 1 \tag{3.12}$$

Where the constraint $\sum_{i=1}^{N} w_i = 1, \; 0 \leq w_i \leq 1$ ensures that the weights allocated to available assets sum up to one.

### *3.3.2 Specification of Market Capitalisation-weighted Approach*

A market capitalisation-weighted portfolio, also called a cap-weighted portfolio, is one where the weight of each asset is determined by the ratio of its market capitalisation to the total market capitalisation of all assets in the portfolio. The weights of all assets in the portfolio add up to one. In the market capitalisation weighted approach, we allocate portfolio weight for each asset as follows:

$$w_i = \frac{M_i}{\sum_{i=1}^{N} M_i} \tag{3.13}$$

Subject to:

$$\sum_{i=1}^{N} w_i = 1, \; 0 \leq w_i \leq 1 \tag{3.14}$$

where $M_i$ is the market capitalisation of asset i; $\sum_{i=1}^{N} M_i$ is the total market capitalisation of all assets in a portfolio.

### *3.3.3 The Sharpe Ratio-based Mean-variance Optimisation Approach*

Traditional mean-variance (MV) portfolio optimisation (Markowitz, 1952) plays an important role in modern investment theory and has been widely discussed and tested in the literature. In theory, MV portfolio optimisation guides investors to spread their wealth among different assets given the expected returns and covariance matrices of the assets (Hsu et al., 2018). However, a key point here is how investors determine the optimal weights to ensure that the portfolio obtains maximum returns and minimum risks. Here we introduce the Sharpe ratio developed by Nobel Prize winner William F. Sharpe (Sharpe, 1994). It is an indicator for calculating risk-adjusted returns. It helps investors understand the return of their investments relative to the risk they take. The Sharpe Ratio is defined as

$$\mathrm{S} = \frac{\mathrm{E}(\mathrm{r_p}) - \mathrm{R}_f}{\delta_\mathrm{P}} \tag{3.15}$$

Where $\mathrm{E}(\mathrm{r_p})$ is the expected return of the portfolio; $\mathrm{R}_f$ is the risk-free rate; $\delta_\mathrm{P}$ is the standard deviation of the portfolio.

If a portfolio has a higher Sharpe ratio than other portfolios, it is considered to be superior to other portfolios, so how should an investor invest the assets of that portfolio to ensure that the Sharpe ratio is maximised? Knowing the importance of the Sharpe ratio, we assume that an investor wants to invest the assets in such a way that the Sharpe ratio of the portfolio is as high or maximum as possible to ensure that the Sharpe ratio of the investment is maximised. Given the mean and covariance matrix of the assets under consideration, mean-variance optimisation based on the Sharpe ratio can be expressed as the solution of the following equation:

$$\max\left(\frac{(\mathrm{E(r)})^{\mathrm{T}}\mathrm{w}-\mathrm{R}_f}{\sqrt{\mathrm{w}^{\mathrm{T}}\sum\mathrm{w}}}\right) \tag{3.16}$$

where w represents the weight invested in each asset; $\mathrm{E(r_P)}$ is the expected return on the portfolio; $(\mathrm{E(r)})^{\mathrm{T}}$ is the transpose of the expected return; $\Sigma$ represents the corresponding covariance matrix of the returns. The numerator of the objective function represents the excess returns of an investment over the risk-free rate ($\mathrm{R}_f$) and the denominator represents the volatility or risk of the investment. The objective is to maximise the Sharpe Ratio.

If we exclude short sales, we assume the following general constraint:

$$\sum_{i=1}^{\mathrm{N}}\mathrm{w}_i = 1,\ 0 \le \mathrm{w}_i \le 1 \tag{3.17}$$

### *3.3.4. Black-Litterman Model*

The Black–Litterman (BL) model is a "market-based" shrinkage technique that calculates expected returns as a weighted average of market equilibrium (for example, the CAPM equilibrium) and the investor's perspectives. The weights are influenced by two factors: 1) the volatility of each asset and their correlations with other assets; 2) investors' views and the level of confidence in each forecast (Kolm et al., 2014). A key assumption of the Black-Litterman model is that unless investors have a particular view about the target security, the expected return on that security should be consistent with the market equilibrium. In other words, unconstrained investors who lack a view about the market should mirror the performance of the market.

The Black-Litterman (BL) model utilises two types of information to generate expected return estimates: 1) The implied equilibrium excess return, which is the information from the market about the expected return and which is based on market or benchmark weights and serve as a prior; 2) The "subjective" excess return estimates that are also known to as investor's views or investment manager's views. The core idea is that investors should only diverge from these market or benchmark weights when they have trustworthy information and projections about future returns that differ from the implied expectations. The implied equilibrium excess return is calculated under the assumption that the observed weights of assets in the market or benchmark result from a risk-return optimisation process. Specifically, it is assumed that market participants seek to maximise the utility function U:

$$\max_{w} U = w^T \Pi_e - \frac{A}{2} w^T \sum w \qquad (3.18)$$

Where, where w is a vector of portfolio weights; $w^T$ is the transpose of the portfolio weights; $\Pi_e$ is a vector of the implied equilibrium asset excess return; $\sum$ is a variance-covariance matrix; A is a coefficient of investor's risk aversion.

Maximising the unrestricted utility function yields the optimal portfolio weights:

$$w^* = (A\,\Sigma)^{-1}\ \Pi_e \tag{3.19}$$

Assuming that the observable market weights (w) are the average optimised portfolio weights of investors, the implied equilibrium excess-return estimates of the market can be calculated as:

$$\Pi_e = \begin{bmatrix} \pi_1 \\ \vdots \\ \pi_n \end{bmatrix} = A \sum w \tag{3.20}$$

$$A = \frac{E(r_M) - r_f}{\sigma_M^2} \tag{3.21}$$

where $\Pi_e$ is a vector of the implied equilibrium asset excess return; $\sum$ is a variance-covariance matrix; A is a coefficient of investor's risk aversion; w is the observable market weights.

In the Black-Litterman (BL) framework, the vector of the implied equilibrium asset excess return ($\Pi_e$) is integrated with the investor's views, represented in the vector ($Q_e$), along with the reliability of each view (or uncertainty associated with the view) quantified in the matrix ($\boldsymbol{\Omega}$). To obtain portfolio return estimates, the original Black-Litterman (1992) refers to Theil's mixed estimation model (Theil, 1971).

Here, we explain the logic behind combining return estimates using Theil's mixed estimation method. It assumes that the implied equilibrium asset excess return ($\Pi_e$) and subjective views ($Q_e$) serve as estimators for the true excess return estimates. Therefore, the true excess return estimates ($\mu_e$) can be expressed as the implied equilibrium excess return estimates ($\Pi_e$) plus an error term ($\epsilon$), where (I) represents the identity matrix:

$$\Pi_e = \; I \cdot \mu_e + \epsilon \quad \text{with } \epsilon \sim N(0, \tau\Sigma) \tag{3.22}$$

The error term ($\epsilon$) is assumed to follow a normal distribution with a variance proportional to the historical variance-covariance matrix ($\Sigma$). The proportional factor ($\tau$) indicates the confidence we have in the estimates of the implied excess returns.

The subjective excess return estimates ($Q_e$) can be written as a linear combination with the error term ($\theta$), where (P) is a binary matrix which contains the information for which asset a subjective return estimate is considered:

$$Q_e = \; P \cdot \mu_e + \theta \quad \text{with } \theta \sim N(0, \Omega) \tag{3.23}$$

The matrix ($\Omega$) is the covariance matrix of the error terms and represents the reliability of subjective estimates. The implied excess return and subjective estimates can be combined as:

$$\begin{bmatrix} \Pi_e \\ Q_e \end{bmatrix} = \begin{bmatrix} I \\ P \end{bmatrix} \mu + \begin{bmatrix} \epsilon \\ \theta \end{bmatrix} \tag{3.24}$$

Applying a generalised least squares method yields an estimator for the combined excess return estimates, which can be expressed in a simplified form:

$$\hat{\mu}_{e,BL} = \; \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\pi + p^T\Omega^{-1}q\right] \tag{3.25}$$

The resulting excess return estimate can be seen as a weighted average of implied excess returns and subjective excess return estimates (Lee 2000), taking into account the correlation structure. The weights are determined by the uncertainty factors of implied returns ($\tau$) and subjective return estimates ($\Omega$), which will be explained in the next section.

According to Satchell and Scowscroft (2000), the posterior variance-covariance matrix is expressed as:

$$\Sigma_{BL} = \Sigma + \left[(\tau \Sigma)^{-1} + P^T \Omega^{-1} P\right]^{-1} \tag{3.26}$$

Once the combined return estimate and the posterior variance-covariance matrix are calculated, a standard risk-return optimisation is performed to maximise the investor's utility:

$$\max_{w} U = w^T \mu_{e,BL} - \frac{A}{2} w^T \Sigma_{BL} w \tag{3.27}$$

This is the Black–Litterman model for the market equilibrium combined with the investor's views.

What we need to note here is that if investors have no opinion or have zero confidence in their views (i.e. $Q_e = 0$ or $\Omega = 0$), then the Black–Litterman returns are equal to the equilibrium returns, i.e. $\hat{\mu}_{e,BL} = \Pi_e$. Consequently, with no views the investor will end up holding the market portfolio. Second, we can see that the expected returns of the Black–Litterman model are a ‘‘confidence’’ weighted linear combination of market equilibrium and the investor’s views with the two weighting matrices.

$$w_{\pi} = \left[(\tau \Sigma)^{-1} + P^{T}\Omega^{-1}P\right]^{-1} (\tau \Sigma)^{-1} \tag{3.28}$$

$$w_{q} = \left[(\tau \Sigma)^{-1} + P^{T}\Omega^{-1}P\right]^{-1} P^{T}\Omega^{-1}P \tag{3.29}$$

and

$$w_{\pi} + w_{q} = I \tag{3.30}$$

Specifically, $(\tau \Sigma)^{-1}$ and $P^{T}\Omega^{-1}P$ represent the confidence we have in the estimates of the market equilibrium and views, respectively. Therefore, if investors have a low confidence in these views, then the final expected return will be close to the return implied by the market equilibrium. Conversely, if they have a high confidence in these views, then the final expected return will deviate from the expected return implied by the market equilibrium. We say that we “deviate” from the market equilibrium.

Before implementing the Black-Litterman model, there are two parts needing to be specified. The first part is about the investor’s views. An investor may have views on some or all the assets. The investor expresses these views as P ~ N (q, Ω). Here the matrix $P \in R^{n\times n}$ describes the investor’s views, vector return value expectations according to investor's view q x1 and $\Omega \in R^{k\times k}$ is the covariance matrix of the views (the ‘‘confidence’’). The matrix P is referred to as the ‘‘link matrix’’, and it links securities with the investor’s views. Such views in the

Black-Litterman model can be expressed in either absolute or relative terms. We demonstrate three of our views as an example here. View one: we assume IBOV > SENSEX by 0.05 % in the one-month expected excess return; View two: SENSEX > SPX by 0.02%; View 3: SPX > SHANGHAI by 0.01%. We have 12 indexes in our sample-set. We build the link matrix according to our views. In the link matrix, when we assume IBOV > SENSEX by 0.05 %, we put 1 under IBOV, -1 under SENSEX, and 0 under other indexes, while when we assume SENSEX > SPX by 0.02%, we put 1 under SENSEX, -1 under SPX, and 0 under indexes.

Before implementing the BL model, another part must be specified is the uncertainty parameter of implied excess returns ($\tau$). The uncertainty parameter, $\tau$, plays a key role in determining implied excess returns and reflects the investor's confidence in market efficiency. A smaller $\tau$ indicates greater belief in efficient markets, with less subjective uncertainty. In the extreme case where $\tau$ equals zero, all weight is placed on the market portfolio (or benchmark), effectively ignoring active views. Conversely, larger values of $\tau$ suggest the investor perceives market inefficiencies as exploitable, with increasing confidence in active views and a willingness to accept more active risk. In the literature, $\tau$ is typically chosen within a range of 0.025 to 0.3 (Black and Litterman, 1992; Drobetz, 2001; He and Litterman, 2002; Idzorek, 2005). For very small $\tau$ values approaching zero, combined returns align closely with implied excess returns, and the optimised portfolio approximates the market portfolio. In contrast, for large $\tau$ values approaching infinity, the combined returns reflect the active views, and the optimised portfolio mirrors the mean-variance portfolio based on those views. The $\tau$ parameter thus governs how much the optimised portfolio diverges from the market portfolio or benchmark. It can be calibrated to achieve a desired level of tracking error. In our chapter, $\tau$ is set at 0.1.

### ***3.3.5 Parametric Portfolio Policy (PPP) Approach***

#### *3.3.5.1 Model specification*

This part introduces a novel portfolio selection approach which uses stock index characteristics to estimate optimal portfolio weights. This approach is proposed by Brandt and Santa-Clara (2006). More specifically, the characteristics-based optimisation in the portfolio selection models the portfolio's weight in each asset as a function of the asset's characteristics (Brandt and Santa-Clara, 2006). We adapt their approach to appraise the performance of the internationally diversified portfolio and explore the strength of the characteristics-based optimisation in portfolio choice techniques.

A portfolio consists of weights $W_{i,t}$, representing the proportion of wealth allocated to asset i at time t. In a Markowitz portfolio, these weights are determined by minimising the portfolio variance and targeting a specific return. This approach can be thought of as estimating one weight per asset, where the weights act as parameters in a model. However, when dealing with a large number of assets, such as 100 indexes, this method requires estimating 100 parameters, which is unlikely to yield strong out-of-sample performance (Brandt and Santa-Clara, 2006). This challenge is similar to expecting to get reliable results from a regression model with 100 variables. The Parametric Portfolio Policy (PPP) strategy addresses this issue in high-dimensional settings by linking weights to asset characteristics, allowing only one parameter to be estimated per characteristic.

Suppose that at each date $t$, a large number, $n_t$ , of asset indexes in the investable pool can be chosen by U.S. investors. Each index $i$ has a return of $r_{t,t+1}$ from date $t$ to $t+1$ and is associated with a vector of index characteristics $x_{i,t}$ observed at date $t$. For example, the characteristics could be the market capitalisation, return-to-equity ratio, book-to-market ratio, dividend yield. The investor's problem is to choose the portfolio weights $w_{i,t}$ for each component to maximise the conditional expected utility of the portfolio's return $r_{p,t+1}$,

$$\max_{\{w_{i,t}\}_{i=1}^{N_t}} E_t[\,u(\,r_{p,t+1})] \;=\; E_t[u(\textstyle\sum_{i=1}^{n_t} W_{i,t}\, r_{i,t+1})] \tag{3.31}$$

We parameterise the optimal portfolio weights as a function of stocks' characteristics,

$$W_{i,t} \;= f(x_{i,t}; \theta) \tag{3.32}$$

where $\theta$ represents a vector of coefficients to be estimated.

we can then rewrite the conditional optimisation with respect to the portfolio weights in Equation (3.31) as the following unconditional optimisation with respect to the coefficients $\theta$,

$$\max_{\theta} E_t[\,u(\,r_{p,t+1})] \;= E_t[u(\sum_{i=1}^{n_t} W_{i,t}\, r_{i,t+1})]$$

$$= \; E_t[u(\sum_{i=1}^{n_t} f(x_{i,t};\theta)\, r_{i,t+1})] \qquad (3.33)$$

we utilise the following simple linear function to specify the portfolio weight,

$$W_{i,t} \; = \bar{w}_{i,t} \; + \frac{1}{n}\, \theta^T\, \hat{x}_{i,t} \qquad (3.34)$$

where $\bar{w}_{i,t}$ is the weight of index $i$ at date t in a benchmark portfolio, such as the value-weighted market portfolio, θ is a vector of coefficients to be estimated, and $\hat{x}_{i,t}$ is the characteristics of index $i$, standardise d cross-sectionally to have zero mean and unit standard deviation across all asset indexes at date t. Note that, rather than estimating one weight for each index at each point in time, we estimate weights as a single function of characteristics that applies to all asset indexes over time—a portfolio policy.

We can then estimate the coefficients θ by maximising the corresponding sample simulation:

$$\max_{\theta} \frac{1}{T} \sum_{t=1}^{T-1} u(r_{p,t+1}) \; = \frac{1}{T} \sum_{t=1}^{T-1} u\, (\sum_{i=1}^{n_t} f(x_{i,t};\theta)\, r_{i,t+1}) \qquad (3.35)$$

for some previous specified utility function (*e.g.,* quadratic or constant relative risk aversion (CRRA)). In the linear policy case (3.34), the optimisation problem is

$$\max_{\theta} \frac{1}{T} \sum_{t=1}^{T-1} u(r_{p,t+1}) \; = \frac{1}{T} \sum_{t=1}^{T-1} u\, (\sum_{i=1}^{N_t} (\bar{w}_{i,t} \; + \frac{1}{n_T} \theta^T\, \hat{x}_{i,t})\, r_{i,t+1}) \qquad (3.36)$$

We assume investors to have constant relative risk aversion (CRRA) preferences with a relative risk aversion of five. We can also choose other risk aversion levels, like one and eight.

*3.3.5.2 Selecting asset index characteristics*

In the Parametric Portfolio Policy (PPP) strategy, deciding which characteristics we should choose to optimise portfolios is very important. Brandt et al. (2009) use momentum, book-to-market ratio, and market capitalisation. In this research, we incorporate additional characteristics such as the return-to-equity ratio, dividend yield, and volume for each selected index, in addition to momentum, book-to-market ratio, and market capitalisation. Before modelling the characteristic-based portfolios, we run a pre-sample test to study which characteristics matter for investment purposes. Accordingly, the characteristics to be tested include the following: 1) The market capitalisation. 2) The return-to-equity ratio. 3) The book-to-market ratio, also called the price-to-book ratio. 4) The dividend yield. 5) The volume. 6) The 12-month cumulative return, which is generate a variable that will be used as a momentum characteristic. We lose some observations in this procedure and the above-mentioned characteristics must be adjusted accordingly. For the pre-sample test, we use a multivariate linear regression model as following:

$$R_i = \beta_0 + \beta_1 x_m + \beta_2 x_r + \beta_3 x_b + \beta_4 x_d + \beta_5 x_v + \beta_6 x_c + \varepsilon \quad (3.37)$$

where, $R_i$ is the log-return for each index (the dependent variable); $\beta_0$ is the $R_i$-intercept (value of $R_i$ when all other parameters are set to 0); $\beta_1$ is the regression coefficient of the market capitalisation ($x_m$); $\beta_2$ is the regression coefficient of the return-to-equity ratio ($x_r$); $\beta_3$ is the regression coefficient of the book-to-market ratio ($x_b$); $\beta_4$ is the regression coefficient of the dividend yield ($x_d$); $\beta_5$ is the regression coefficient of the volume ($x_v$); $\beta_6$ is the regression coefficient of the 12-month cumulative return ($x_c$); $\varepsilon$ is the model error term.

### *3.3.6 Correlation Measurements and Performance Measurements*

*3.3.6.1 The measurement of pair-wise correlation*

We use the Pearson correlation to measure the fixed correlation between indexes, while using the DCC-GARCH model to obtain the time-varying correlation between them. This is consistent with what we use to measure the correlation in chapter two.

*3.3.6.2 The performance measurements*

To compare the in- and out-of-sample performance of portfolio strategies (three benchmarks and three optimisation strategies), we consider the Sharpe ratio (SR) as the performance measurement. The Sharpe ratio developed by Nobel Prize winner William F. Sharpe (Sharpe, 1994) is a metric for calculating risk-adjusted returns. It helps investors understand the return of their investments relative to the risk they take. The Sharpe Ratio is defined as

$$S = \frac{E(r_p) - R_f}{\delta_P} \tag{3.38}$$

where $E(r_p)$ is the expected return of a portfolio; $R_f$ is the risk-free rate; $\delta_P$ is the standard deviation of the portfolio.

If a portfolio with higher Sharpe ratio than its counterparts, is considered superior to them, then how does one invest in the assets of the portfolio, to ensure maximal Sharpe Ratio? Having understood the significance of the Sharpe Ratio, let us suppose an investor wishes to make an investment in assets in such a way that the Sharpe Ratio of the portfolio would be the best possible or the maximum, that can be ensured for the investment.

### *3.3.7 In-sample Expanding Windows and Out-of-sample Simulation*

#### *3.3.7.1 In-sample expanding windows*

To examine the performance of the portfolios, we consider two approaches to enhance the robustness of the results. We first build portfolios for the full sample for each of our strategies and compare their results (in-sample results). Second, to account for any "look-ahead" bias and to add robustness to the results, we generate out-of-sample portfolios. Here, we use expanding windows to simulate a portfolio for each portfolio strategy, which results in the creation of 60 expanding windows covering the last five years across our sample set. The starting point of an expanding window is fixed at the beginning of the data, and its width increases with the time moving forward. In our case, at the end of the month T=156+t ($t = 0,1,2 \ldots 59$; in the first expanding window, $t = 0$), we use the return series from the month 1 to month 156+t ($t = 0,1,2 \ldots 59$) to derive the in-sample estimates of the parameters for each strategy. This allows calculation of the in-sample performance for each expanding windows.

Using the in-sample values, including the calculated optimal portfolio weight ($w_{i,\mathrm{T}}$), we then construct a portfolio for the next, out-of-sample, month. For example, in our sample, the first in-sample estimation window is from January 2006 to December 2018, and we use the optimal weight derived from this in-sample to estimate the out-of-sample portfolio results for January 2019; the second in-sample estimation window is from January 2006 to January 2019, and we use the optimal weight derived from this in-sample to estimate the out-of-sample portfolio results for February 2019. This expanding procedure operates through the rest of the sample period.

*3.3.7.2 Transaction costs of out-of-sample simulation*

Similar to Chapter two, we implement a one-way transaction cost (C) of 0.05% for each trade in the out-of-sample simulation, as referenced by Campbell and Thompson (2008) and Hsu et al. (2018). We define $\mathrm{r}_{i,\mathrm{T}}$ as the real return of the $i$-*th* asset in month T, and set $\sum_{i=1}^{N} r_{i,\mathrm{T}} \mathrm{w}_{i,\mathrm{T}}$ as the real portfolio return before re-balancing at the end of month T. When the portfolio is re-balanced in the beginning of month T + 1, it yields a trade in each asset with a magnitude of $|\mathrm{w}_{i,\mathrm{T}+1} - \mathrm{w}_{i,\mathrm{T}}|$, where $\mathrm{w}_{i,\mathrm{T}}$ is the optimal portfolio weight of each asset in the end of month T, $\mathrm{w}_{i,T+1}$ represents the calculated optimal portfolio weight in each asset in the beginning of month T + 1. We set C as the proportional transaction costs (0.05%), and then the trading costs for all assets are $C \times \sum_{i}^{N} |\mathrm{w}_{i,\mathrm{T}+1} - \mathrm{w}_{i,\mathrm{T}}|$. Therefore, the net return after the transaction costs for each portfolio strategy in month T + 1 is calculated as:

$$E(r_P)^n = (1 + \sum_{i=1}^{N} r_{i,T+1} w_{i,T+1})(1 - C \times \sum_{i}^{N} |w_{i,T+1} - w_{i,T}|) - 1 \quad (3.39)$$

Where $r_{i,T+1}$ is the real return in month T + 1 for each asset. We consider the return before the transaction as the situation when then transaction cost (C) is zero.

## 3.4. Data and Data Description

### *3.4.1 Basic Data Description*

To empirically test the performance of our portfolio strategies, our modelled sample set comprises seven global indexes from developed economies (i.e., the USA, Japan, the UK, Italy, France, Germany, and Canada, known as the G7) and five global indexes from emerging economies (i.e., Brazil, Russia, India, China, and South Africa, known as the BRICS). In total, we model 12 global indexes, with each monthly price series covering the period from December 2004 to December 2023. The reason why the monthly data is used refers to Section 2.3 in Chapter two. The return in this chapter is the log-return.

Table 3.2 lists all 12 indexes included in our sample set and other variables we use in this chapter. Methodologically, we optimise portfolio selection for an investment universe of developed and emerging market stock indexes using the Parametric Portfolio Policy (PPP) approach of Brandt et al. (2009) for the period of our sample period and compare the results to naïve diversified portfolios (1/N-rule), market capitalisation-weighted, risk parity (equally weighted risk contribution), mean-variance (MV), and Black-Litterman (BL) optimised portfolios. Among these, the naive diversified portfolio (1/N-rule), market capitalisation weighted portfolio, and the risk parity (equally weighted risk contribution) serve as benchmarks, while the mean-variance (MV) diversified portfolio, the Black-Litterman (BL) portfolio, and the Parametric Portfolio Policy (PPP) portfolio are optimised portfolios.

**Table 3.2.** List of variables

| Variables | Definition | Country group | Frequency | Source |
|---|---|---|---|---|
| SPX | The Standard & Poor's 500 index representing USA | | Monthly | Bloomberg |
| Nikkei 225 | The Nikkei 225 Stock Average representing Japan | | Monthly | Bloomberg |
| FTSE | The Financial Times Stock Exchange 100 index in UK | | Monthly | Bloomberg |
| MIB | FTSE MIB Index in Italy | G7 | Monthly | Bloomberg |
| CAC 40 | The CAC 40 index in France | | Monthly | Bloomberg |
| DAX | The DAX index in Germany | | Monthly | Bloomberg |
| TSX | The S&P/TSX Composite Index in Brazil | | Monthly | Bloomberg |
| IBOV | The Ibovespa index in Brazil | | Monthly | Bloomberg |
| IMOEX | The MOEX index In Russia | | Monthly | Bloomberg |
| SENSEX | The Bombay Stock Exchange Sensitive index in India | BRICS | Monthly | Bloomberg |
| SHANGHAI | The Shanghai Stock Exchange Composite Index in China | | Monthly | Bloomberg |
| JALSH | The JALSH index in South Africa | | Monthly | Bloomberg |
| $R_f$ | The U.S. 3-month Treasury-Bill Rate | | Monthly | |
| $r_t$ | The log return of each index at month t | | Monthly | $= ln\left(\frac{P_t}{P_{t-1}}\right)$ |

In this chapter, the effect of inflation is not considered. G7 is the seven global indexes from developed economies, while BRICS is the five global indexes from emerging economies. $P_t$ denotes the index price at time t, and $P_{t-1}$ denotes the index price at time t-1. $R_t$ means the nominal return of each index at month t.

The Parametric Portfolio Policy (PPP) approach parameterises the asset weights as a function of their characteristics, thereby estimating those parameters in a way that maximises the investor's average utility. The implicit assumption is that the characteristics convey all relevant information about the assets' conditional distribution of returns. According to the literature and the data availability, we select six characteristics of stock indexes, including the market capitalisation, return-to-equity ratio, book-to-market ratio, dividend yield, volume, and the 12-month cumulative return. Table 3.3 illustrates these six characteristics. All our data are sourced by Bloomberg.

**Table 3.3.** Six characteristics.

| | |
|---|---|
| 1 | Book-to-market ratio |
| 2 | 12-month cumulative return |
| 3 | Volume |
| 4 | Return-to-equity ratio |
| 5 | Dividend yield |
| 6 | Market capitalisation |

These six characteristics are chosen for constructing the Parametric Portfolio Policy (PPP) optimised portfolios.

Figure 3.1 illustrates the price movements of the G7 indexes in one plot. We have included a separate plot for each index in Figures 3.2 to 3.8 for easier observation. In these figures, we can see that, during the Great Recession (from 2007-2009), all indexes in G7 countries have a dramatic drop. All indexes in G7 fall at the start of COVID-19, but they all recover immediately. It is worth noting that since 2009, the SPX has shown a significant upward trend, despite a momentary drop at the beginning of COVID-19, while the CAC 40 and DAX have shown a slight upward trend, but with huge fluctuations since then. The Nikkei 225 index has shown an obvious upward trend after the Great Recession. It also has a momentary drop at the beginning of COVID-19, but it also recovers immediately. After that, it shows a sharp rise. Another decline occurrs between the beginning of 2021 and the third season of 2022. Interestingly, the FTSE and MIB share similar trends in that, following a significant decline during the Great Recession, they have not yet returned to their pre-2009 levels.

**Figure 3.1.** Price movement of G7 indexes

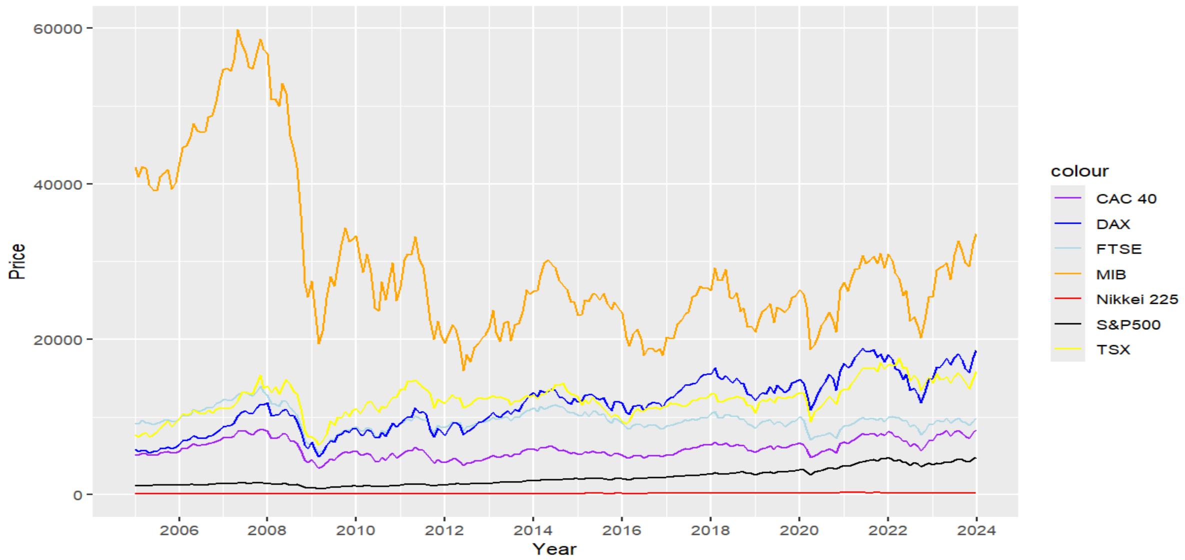



**Figure 3.2.** Price movement of the CAC 40 index

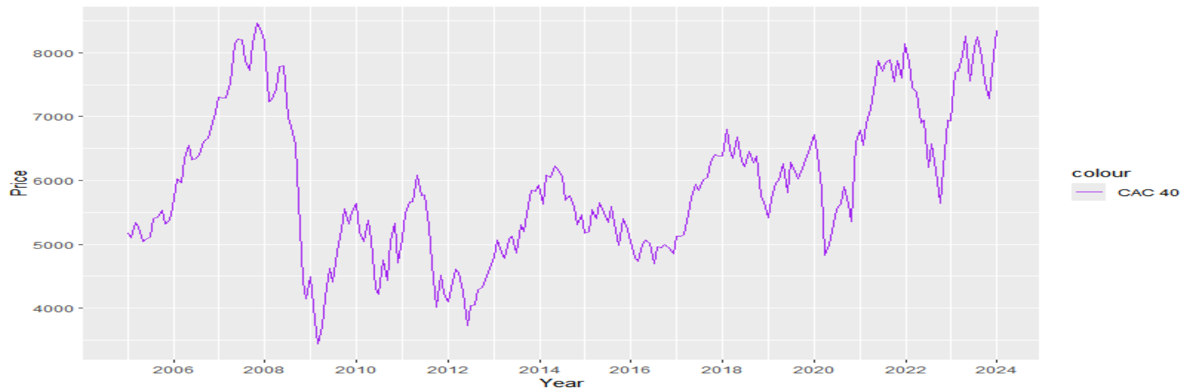


**Figure 3.3.** Price movement of the DAX index

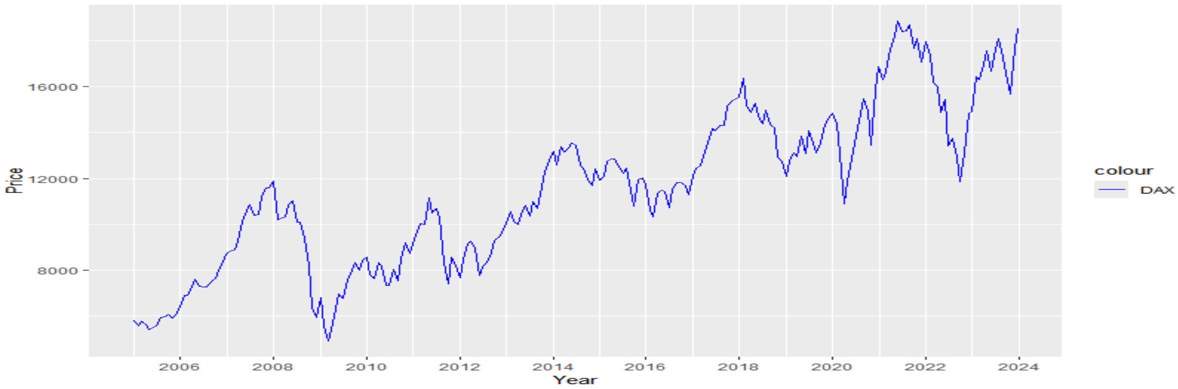



**Figure 3.4.** Price movement of the FTSE index

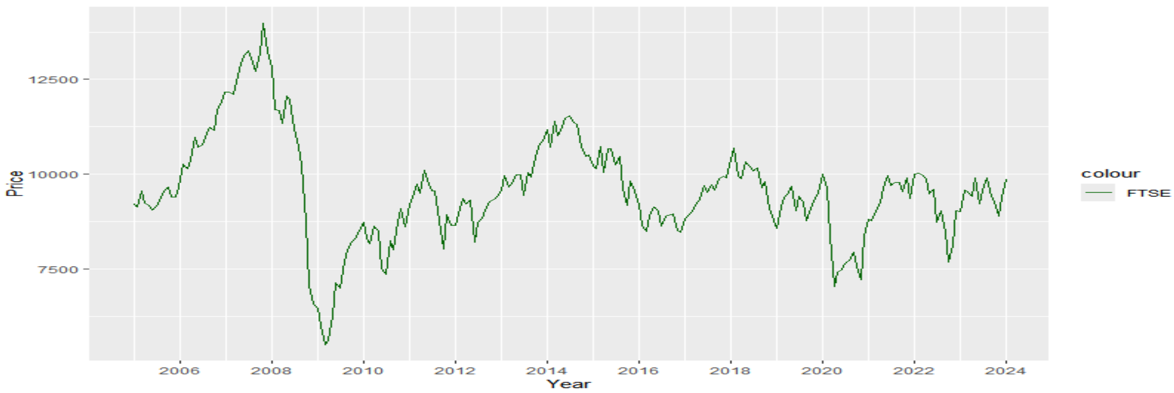



**Figure 3.5.** Price movement of the MIB index.

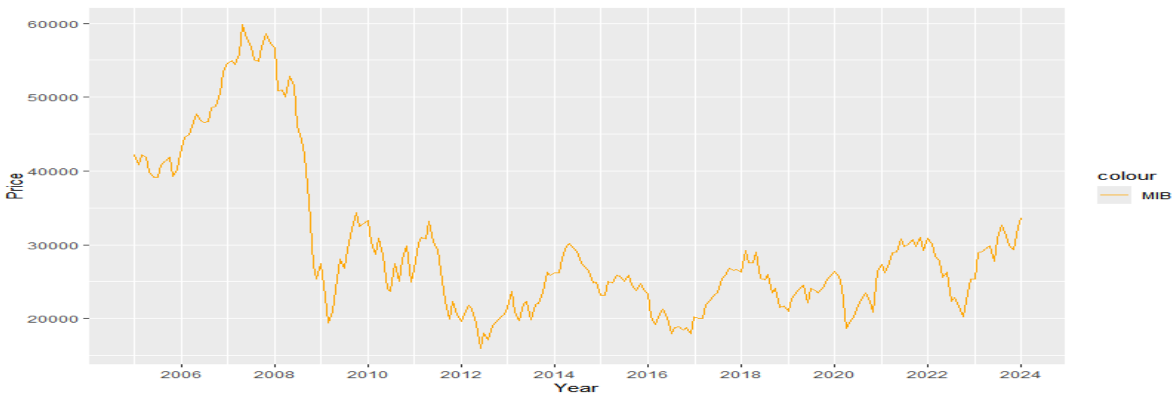


**Figure 3.6.** Price movement of the Nikkei 225 index.

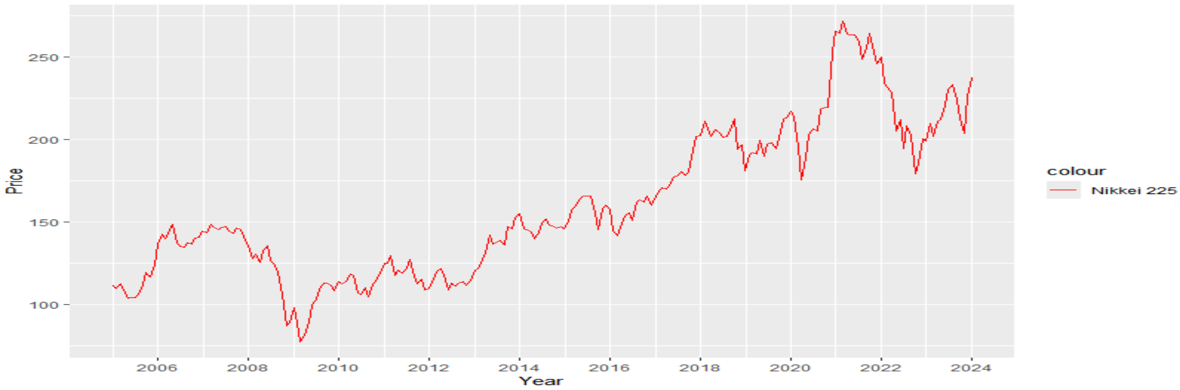



**Figure 3.7.** Price movement of the S&P 500 index.

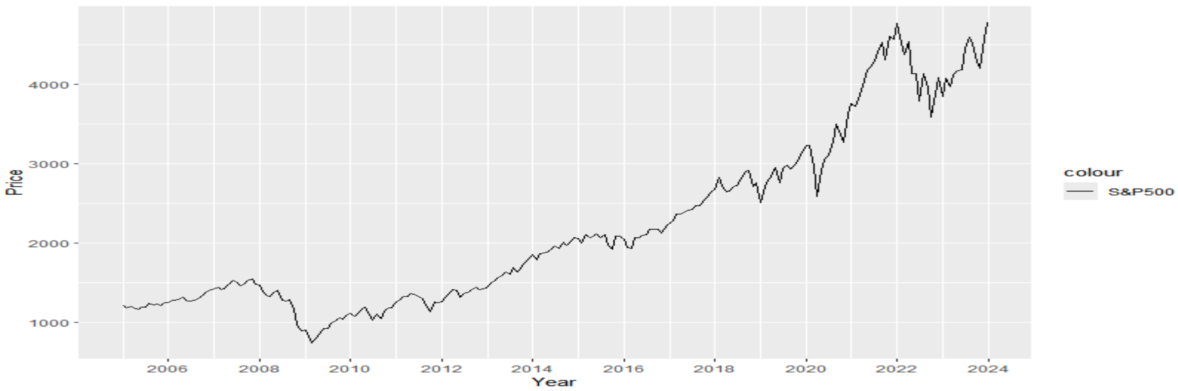



**Figure 3.8.** Price movement of the TSX index.

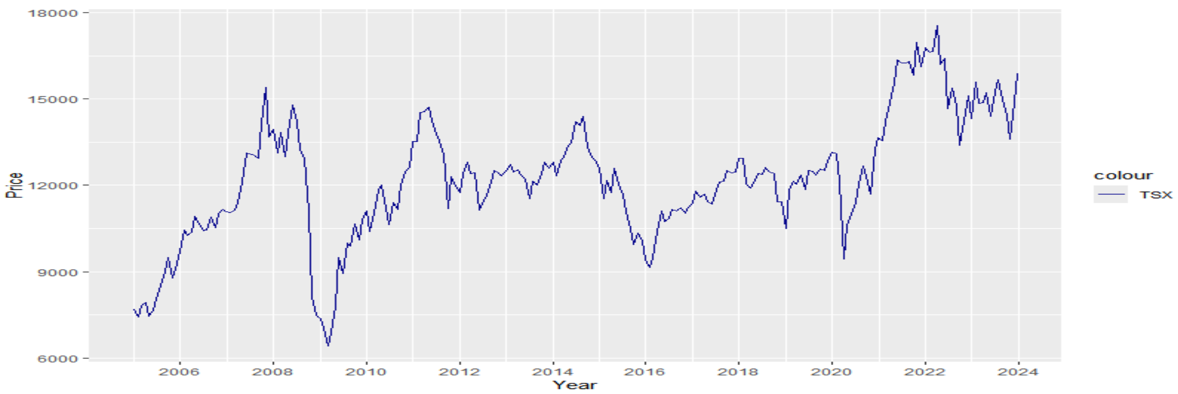


Figure 3.9 displays the price movements of BRICS indexes. Figures 3.10-3.14 display the price movement of BRICS indexes individually. In these figures, we can see that, during the Great Recession (from 2007-2009), all indexes in BRICS countries have a dramatic drop, and they all fall at the start of COVID-19, but they all recover immediately with the exception of the IBOV. In addition, the IBOV and IMOEX show another huge increase from 2011 to 2016. It is worth noting that, among all indexes in BRICS countries, only SENSEX exhibits a distinctive upward trend over our sample period, while others do not show an upward trend.

**Figure 3.9.** Price movement of BRICS.

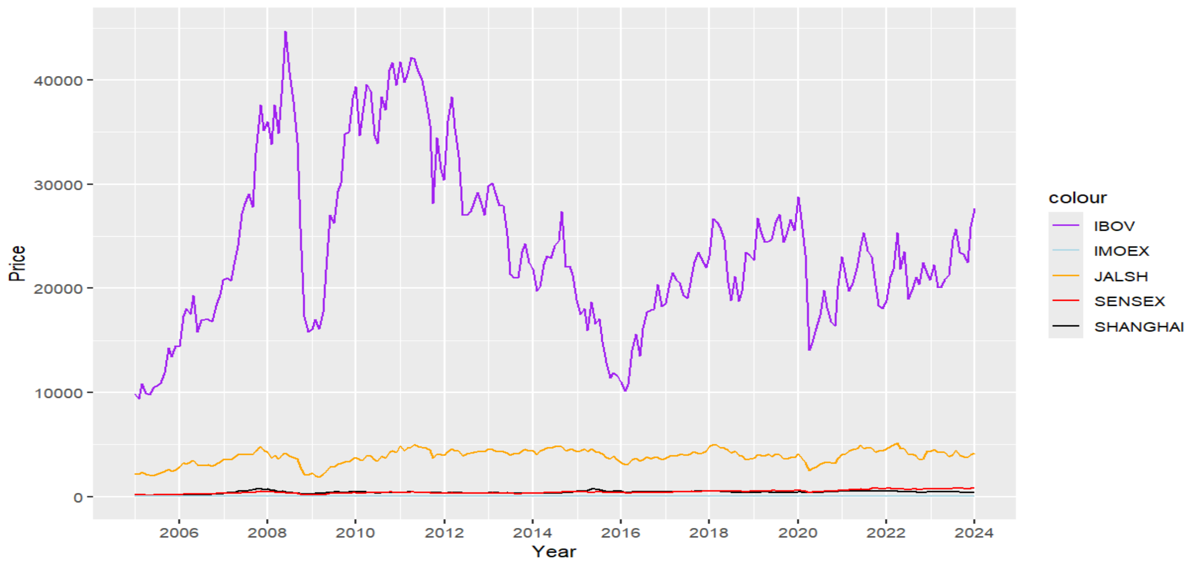


**Figure 3.10.** Price movement of the IBOV index.

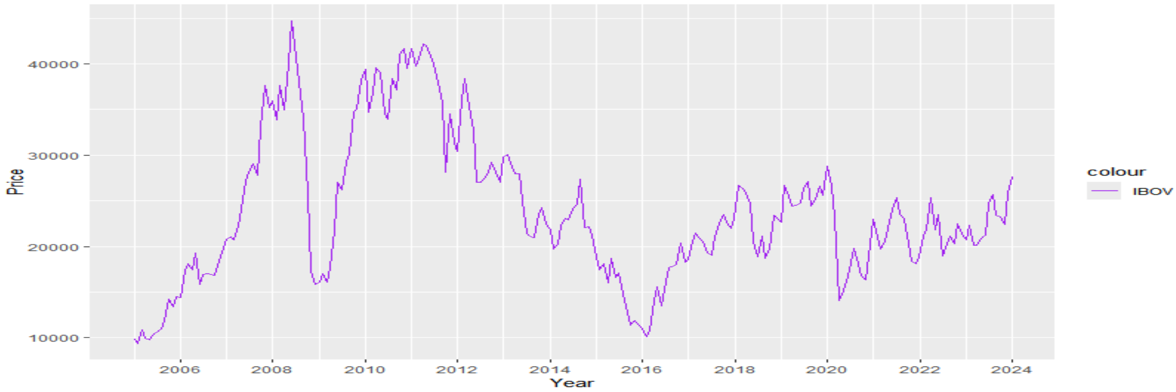



**Figure 3.11.** Price movement of the IMOEX index.

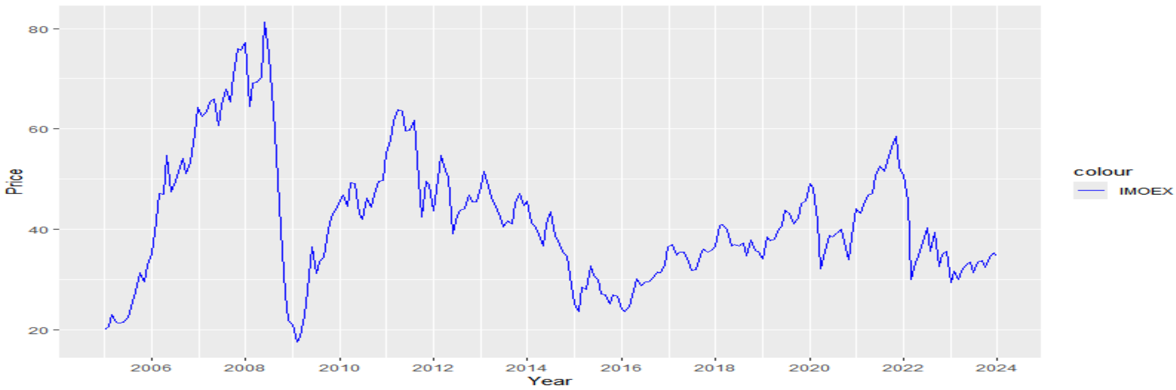



**Figure 3.12.** Price movement of the JALSH index.

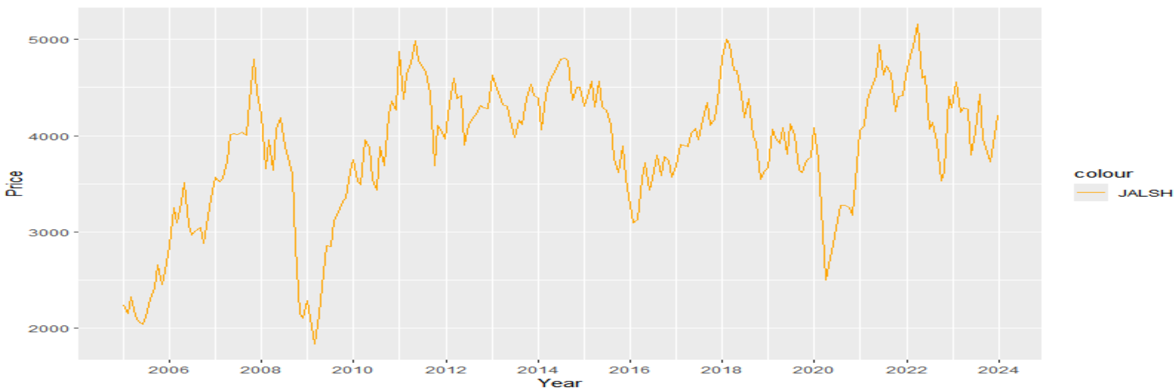


**Figure 3.13.** Price movement of the SENSEX index.

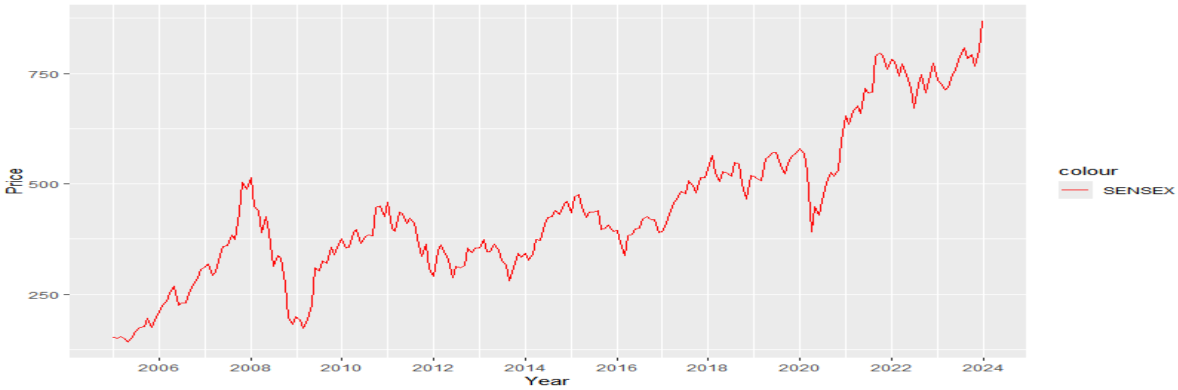
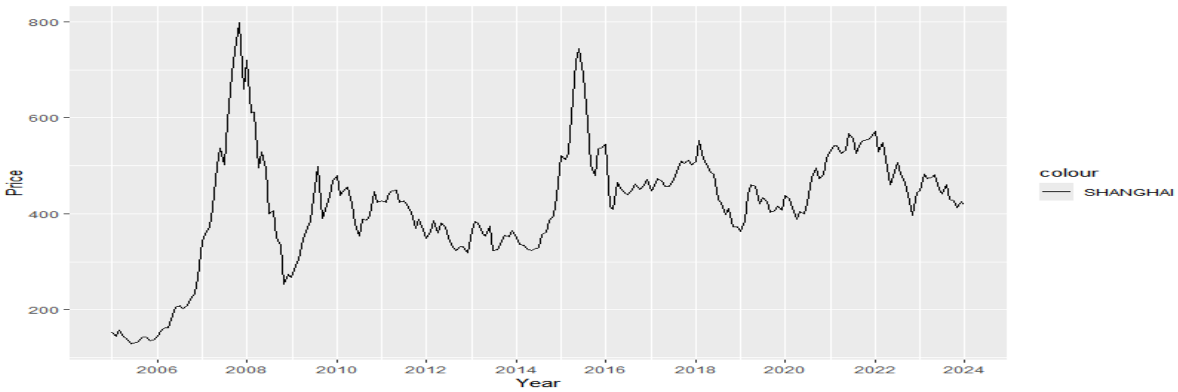



**Figure 3.14.** Price movement of the SHANGHAI index.



Figures 3.15 to 3.26 provide the plots of all six characteristics (market capitalisation, return-to-equity ratio, book-to-market ratio, dividend yield, volume, and the 12-month cumulative return) for all 12 indexes. Figures 3.15 and 16 display the market capitalisation for 12 indexes. As we can see from Figures 3.15 and 3.16, among G7 indexes, the S&P 500 index has the highest market capitalisation over the sample period, and among BRICS indexes, the SHANGHAI index has the highest market capitalisation over the sample period except for the period from the end of 2004 to the end of 2006. Notably, we see an obvious increase pattern in the market capitalisation of both the S&P 500 index and the SHANGHAI index within the sample period, while we could not observe a significant increase in the other 10 indexes' market capitalisation. Figures 3.17 and 3.18 show the volume of 12 indexes. From Figure 3.17, we find that among G7 indexes, the patterns of volumes in the CAC 40, DAX, and TXS indexes are relatively more stable compared to those in the MIB, TSE, Nikkei 225, and S&P 500 indexes. In Figure 3.18, we observe that the pattern of volume in the IMOE index is more volatile than other indexes in BRICS. Especially starting from the beginning of 2022, it became more fluctuated, which may be attributed to the fact that in February 2022, Russia launched a full-scale invasion of Ukraine.  Figures 3.19 and 3.20 show the price-to-book ratio for G7 indexes and BRICS indexes. From Figure 3.19, we find that there was an obvious decrease in all G7 indexes from the end of 2007 to the end of 2008, and thereafter, they all recovered a bit, but only the S&P 500 index had an obvious increase pattern. From Figure 3.20, we observe that after a significant drop in the price-to-book ratio for all BRICS indexes except for the IMOEX index in 2008, they could never recover back to the level before 2008. In the figure, we see that the pattern of the price-to-book ratio for IMOEX is more stable compared to BRICS's other indexes.

Figures 3.21 and 3.22 demonstrate the dividend yield pattern for each selected index. In Figure 3.21, we find that from the end of 2004 to the end of 2008, the dividend yield increased significantly, and after that there was a decrease in 2009, but thereafter we could not observe any increasing or decreasing trend but fluctuated. Figure 3.22 shows that there was not obvious increasing or decreasing pattern for the JALSH index, SENSEX index, and SHANGHAI index, but volatile. Interestingly, from the beginning of 2021 to the middle of 2022, the dividend yield in the IBOV index increased, but after that, it decreased until the end of 2023. In addition, from the end of 2021 to the end of 2022, the dividend yield in the IMOEX index also increased, but after that, it decreased until the end of 2023. Figures 3.23 and 3.24 show the return on equity pattern for each selected index. From Figure 3.23, we find that the return on equity in all G7 indexes dropped significantly from the beginning of 2008 to the end of 2009. In Figure 3.24, we observe an interesting phenomenon that at the beginning of 2014, there was a big drop in the return on equity of the IBOV index. In 2014, Brazil experienced a stalled economy (Leal and Nakane,2025), leading to negative returns on the Ibovespa index, potentially including a negative return on equity (ROE).  Figures 3.25 and 3.26 show the 12-month cumulative return pattern for each selected index. The starting point for it is from January 2006 for the 12-month cumulative return because we originally had 229 observations from December 2004 to December 2023 for each chosen index, but when we calculated the log return, we lost one of them, and when we calculated the 12-month cumulative return, we lost another 12 observations for each index, so at the end, we only have 216 observations for each of the chosen indexes starting from January 2006. It is worth noting that when we model the data, we trim other time series' lengths according to the length of the 12-month cumulative return. Interestingly, all G7 indexes have a similar 12-month cumulative return pattern, and all BRICS indexes also have a similar 12-month cumulative return pattern.

Table 3.4 provides the basic statistics of all 12 indexes' price and log return; Tables 3.5-3.10 show the basic statistics of all six characteristics. In Panel B of Table 3.4, we present the basic statistics of all 12 indexes' log returns. According to panel, SPX has the lowest standard deviation, and its minimum return is higher than that of other indexes, whereas IMOEX has the highest standard deviation. Surprisingly, the average return of MIB is negative. Moreover, SENSEX produces the highest average return, and SPX comes in second among all chosen indexes.

**Figure 3.15.** The plot of market capitalisation in G7.

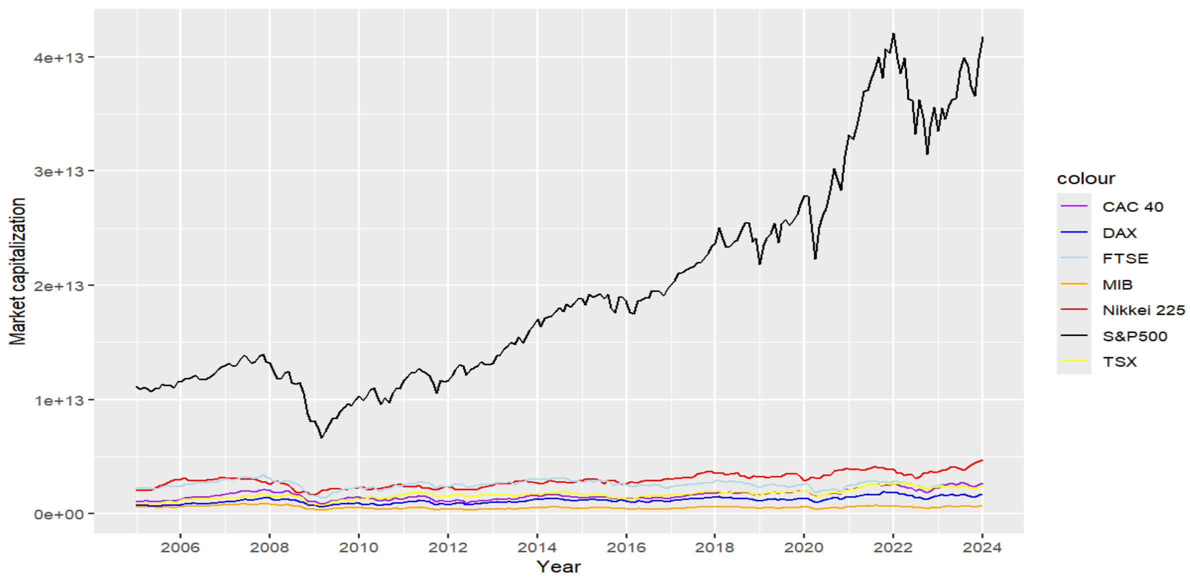



**Figure 3.16.** The plot of market capitalisation in BRICS.

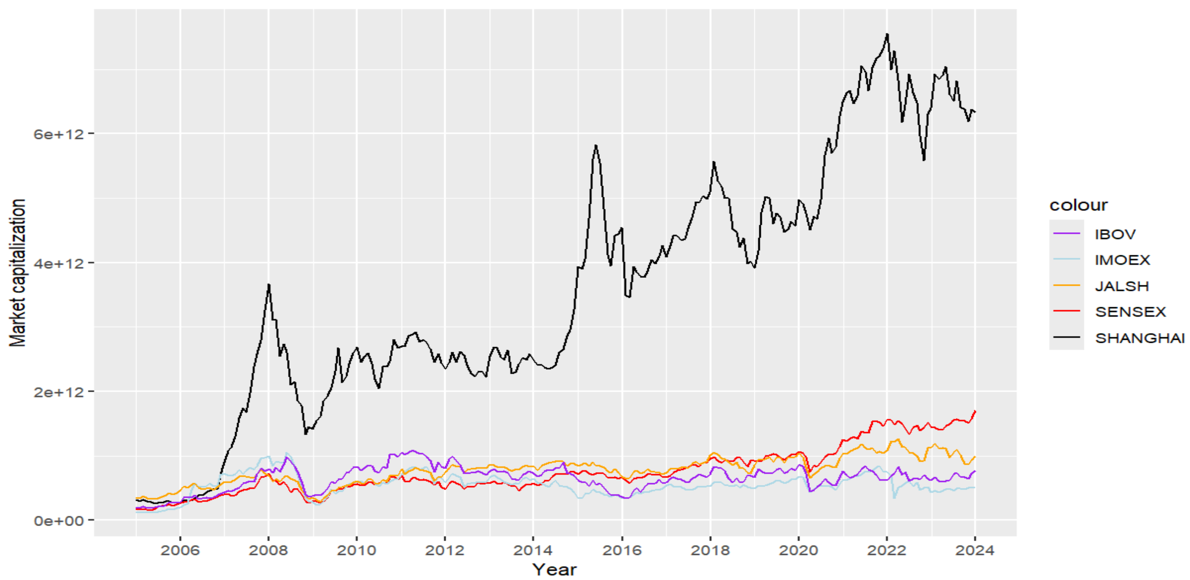


**Figure 3.17.** The plot of the volume in G7.

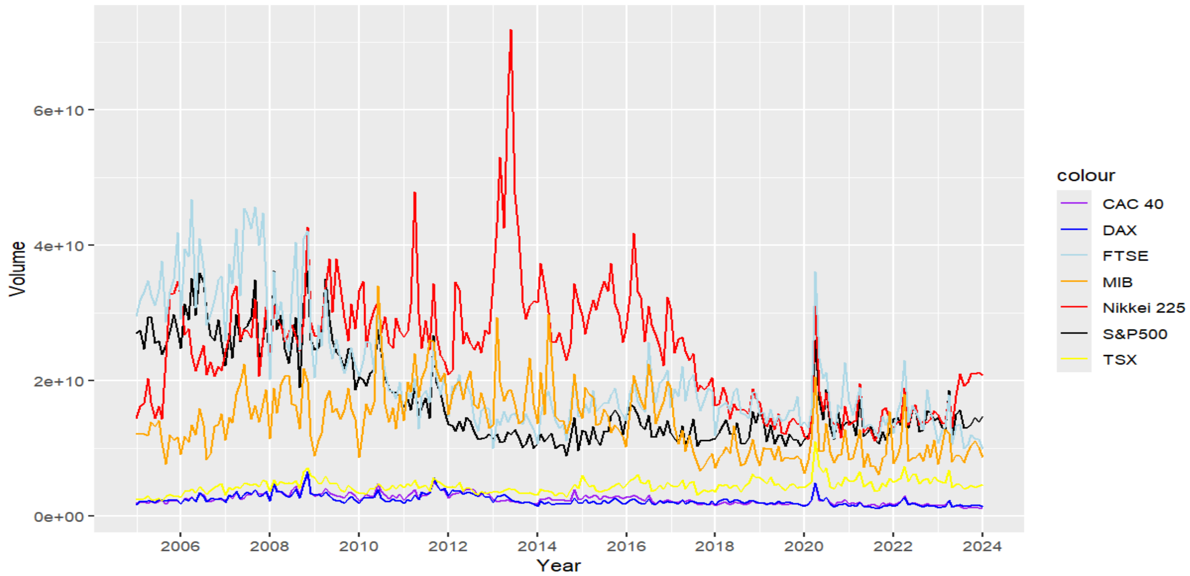



**Figure 3.18.** The plot of the volume in BRICS.

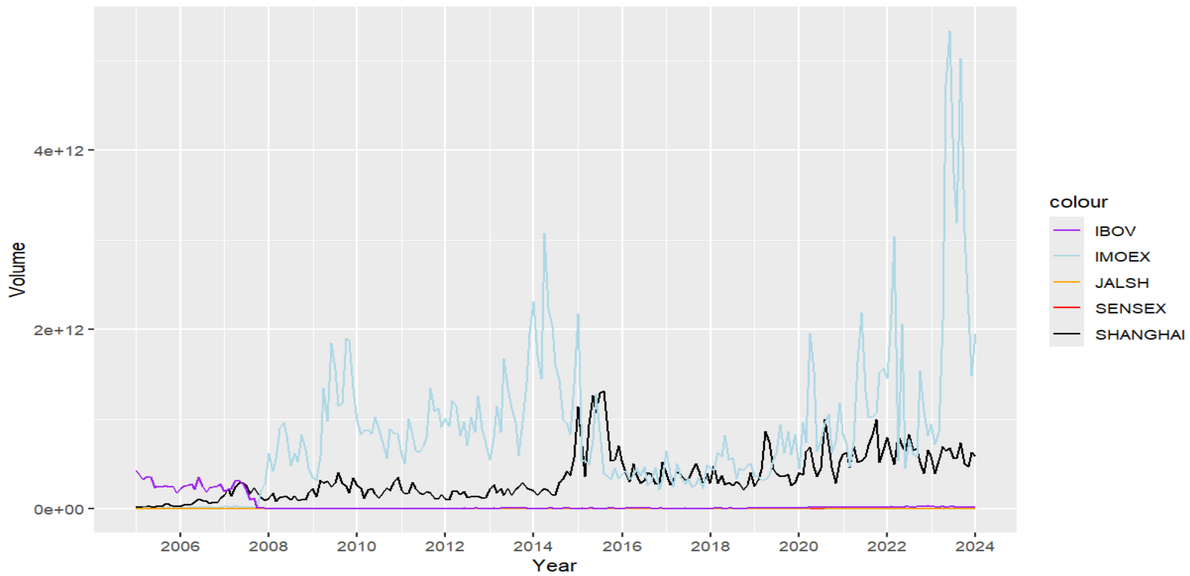


**Figure 3.19.** The plot of the price to book ratio in G7.

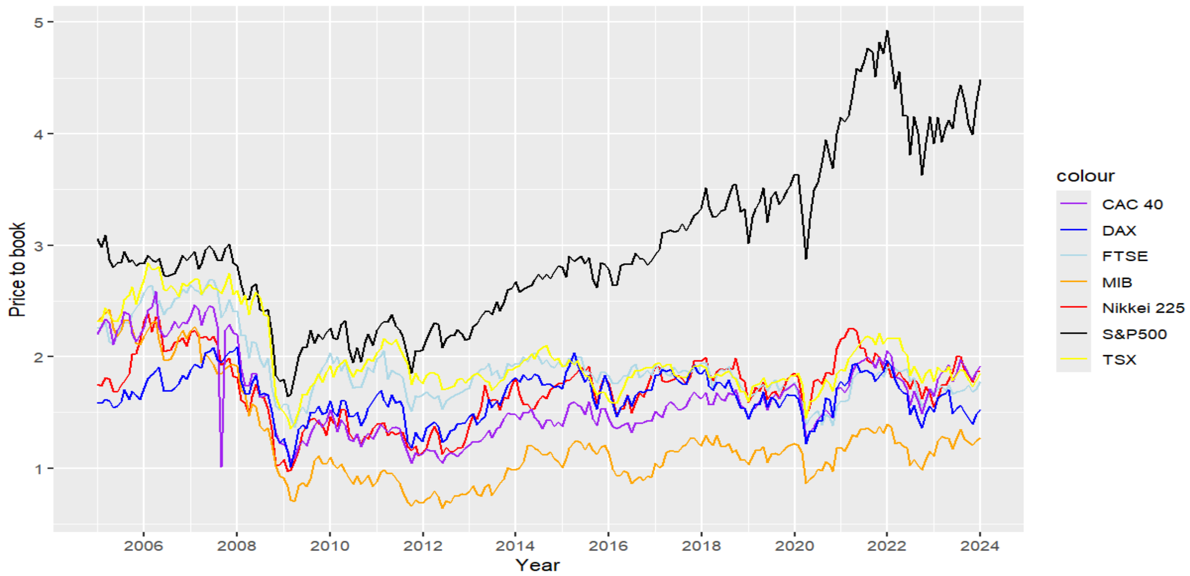



**Figure 3.20**. The plot of the price to book ratio in BRICS.

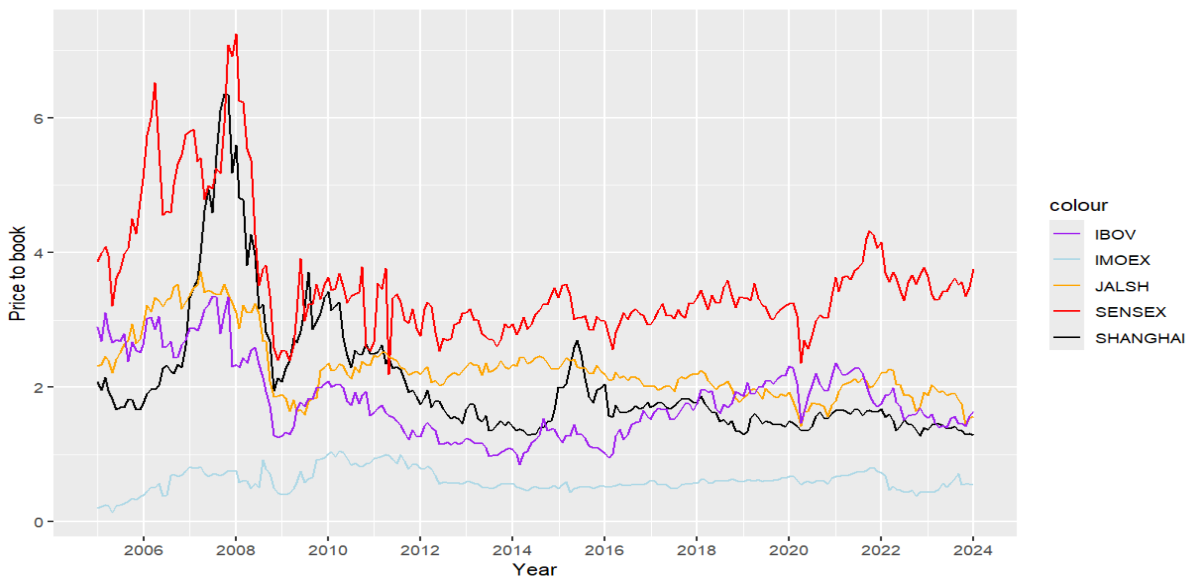


**Figure 3.21.** The plot of the dividend yield in G7.

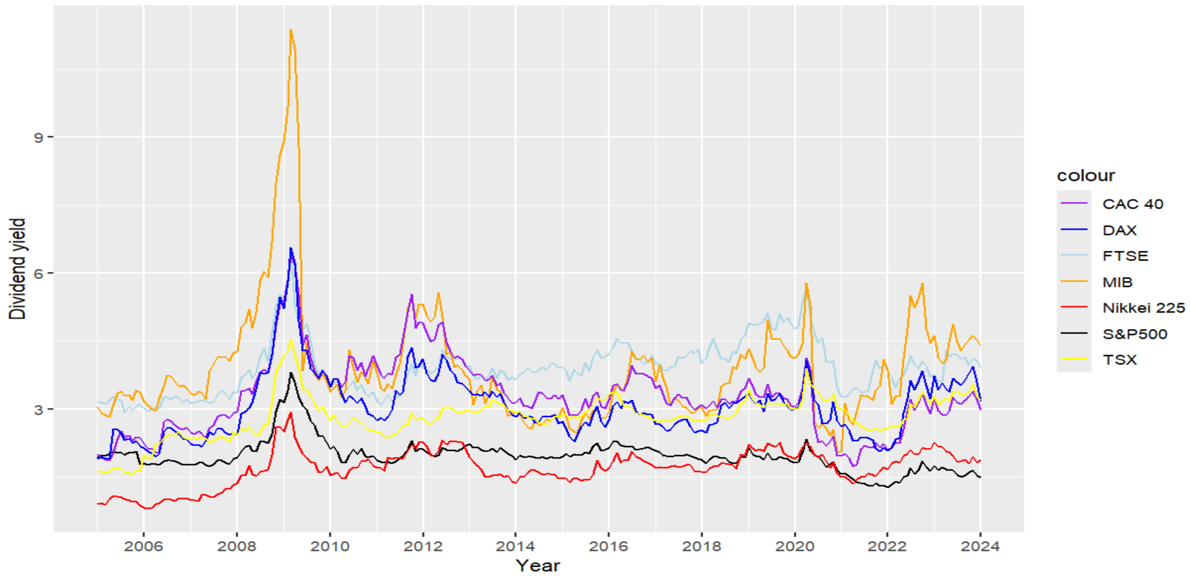



**Figure 3.22.** The plot of the dividend yield in BRICS.

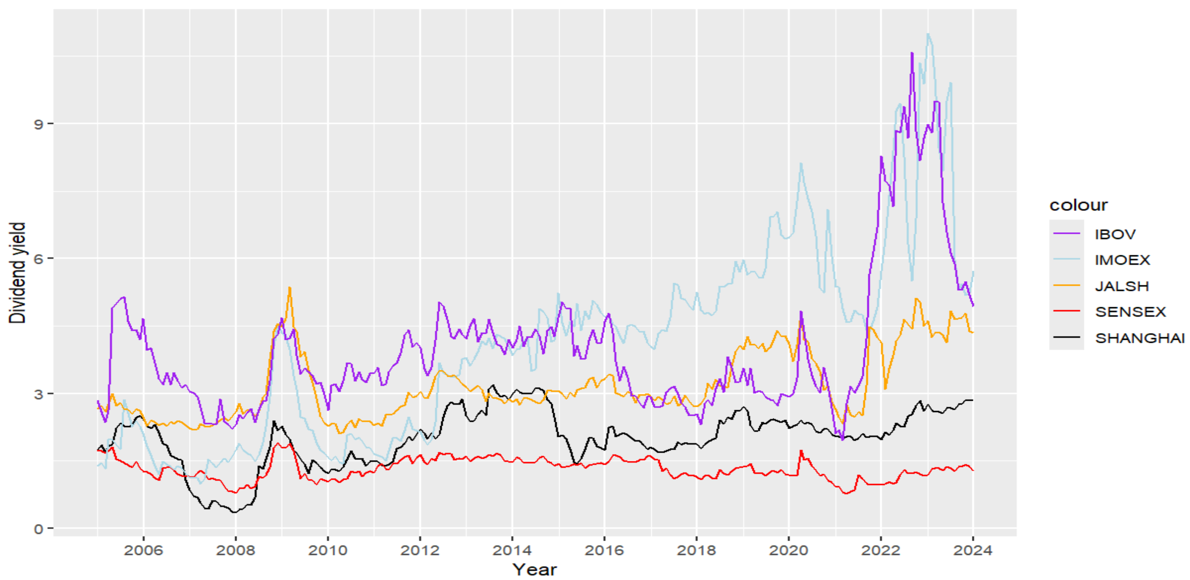


**Figure 3.23.** The plot of the return on equity in G7.

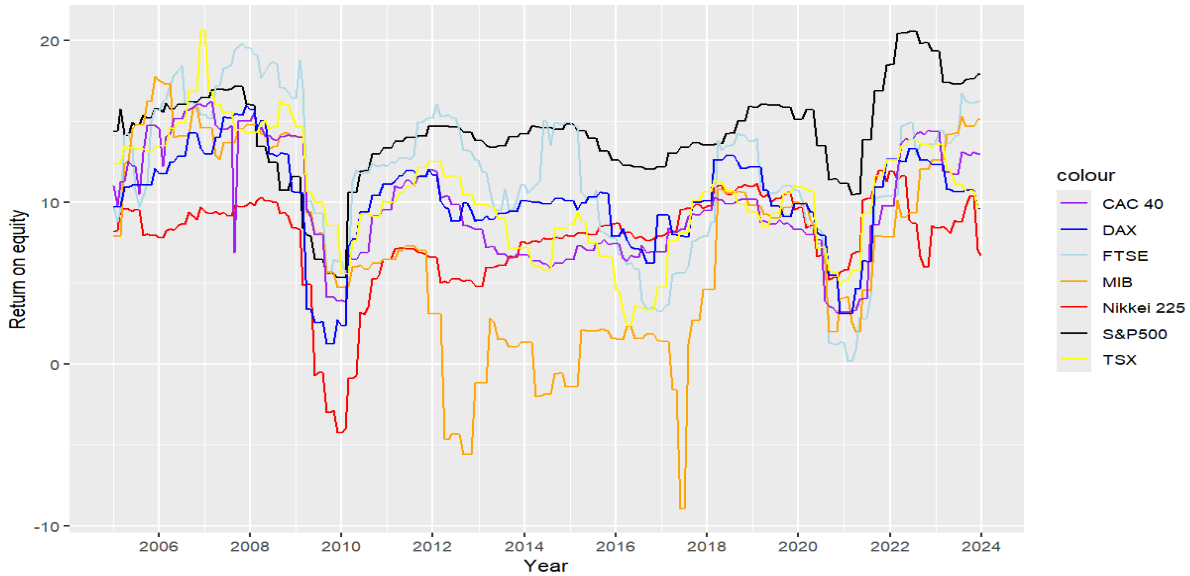



Figure 3.24. The plot of the return on equity in BRICS.

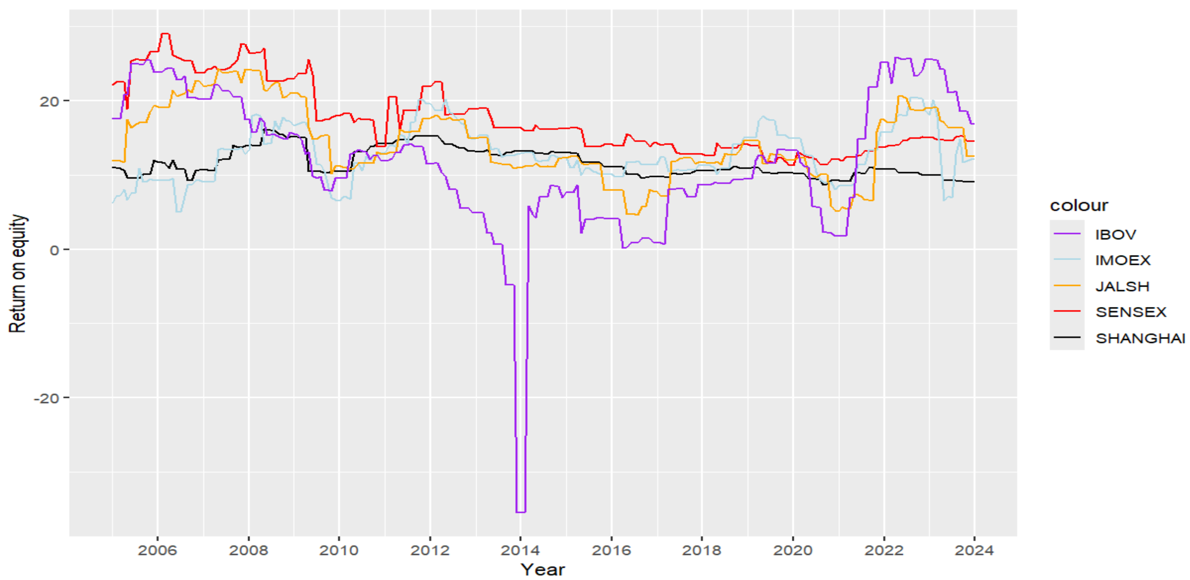


**Figure 3.25.** The plot of the 12-month cumulative return in G7.

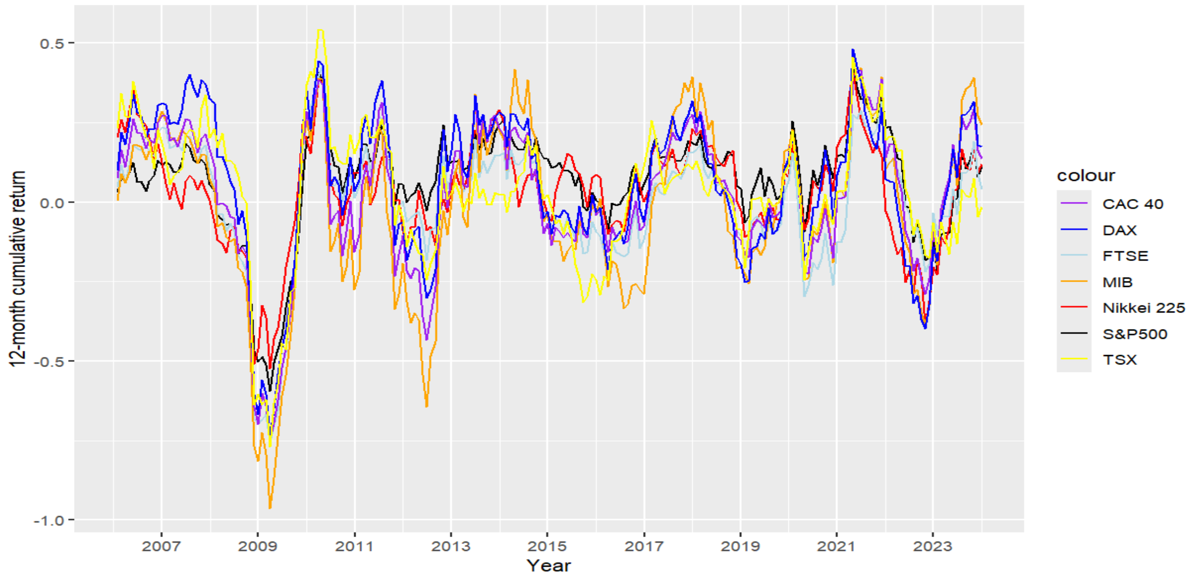



**Figure 3.26.** The plot of the 12-month cumulative return in BRICS.

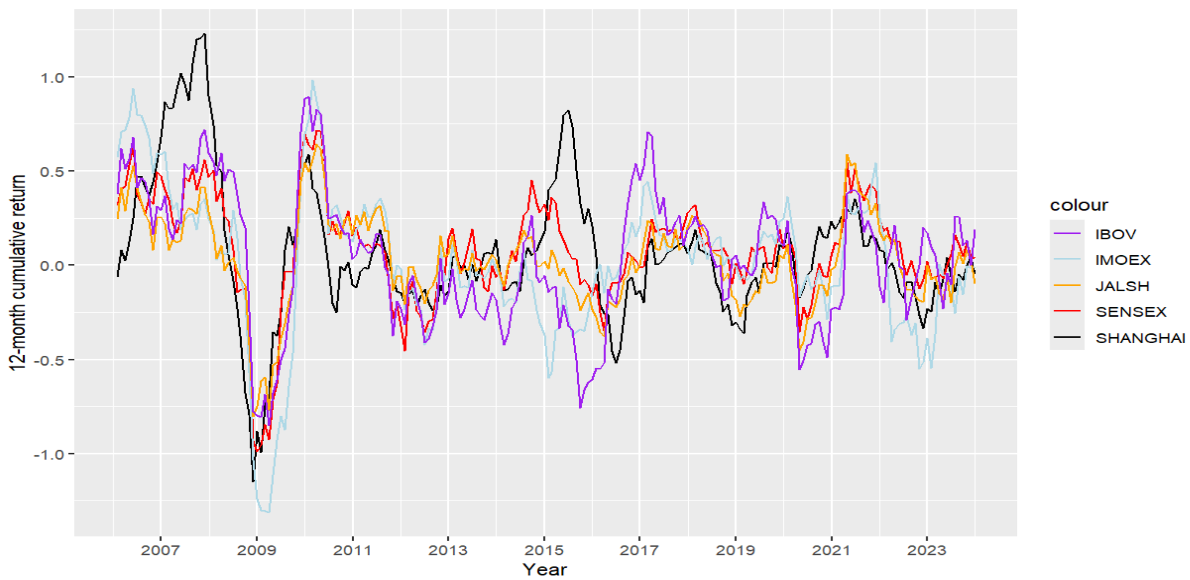


**Table 3.4.** Basic statistics of price and return.

| Panel A. Basic statistic of price | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SPX | Nikkei 225 | FTSE | MIB | CAC 40 | DAX | TSX | IBOV | IMOEX | SENSEX | SHANGHAI | JALSH |
| Nobs | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 |
| Minimum | 735.090 | 77.260 | 5,495.030 | 1,5917.220 | 3436.474 | 4887.700 | 6413.250 | 9335.104 | 17.463 | 141.368 | 128.160 | 1830.153 |
| Maximum | 4769.830 | 271.599 | 1,3974.210 | 5,9804.340 | 8459.060 | 1,8856.960 | 1,7537.380 | 4,4672.310 | 81.248 | 868.036 | 797.798 | 5158.354 |
| Mean | 2193.479 | 160.043 | 9578.153 | 2,9741.710 | 5930.626 | 1,1524.830 | 1,2203.950 | 2,3904.480 | 41.730 | 441.436 | 417.148 | 3858.450 |
| Median | 1923.570 | 147.155 | 9491.832 | 2,6352.990 | 5716.665 | 1,1599.800 | 1,2312.390 | 2,2439.790 | 40.027 | 411.748 | 425.019 | 4002.786 |
| Stdev | 1093.082 | 45.343 | 1387.051 | 10344.690 | 1149.463 | 3462.097 | 2127.166 | 8021.236 | 12.635 | 166.733 | 121.655 | 710.007 |
| Skewness | 0.844 | 0.556 | 0.276 | 1.329 | 0.372 | 0.122 | -0.132 | 0.576 | 0.687 | 0.556 | -0.228 | -0.857 |
| Kurtosis | -0.501 | -0.602 | 1.032 | 0.819 | -0.645 | -0.822 | 0.177 | -0.277 | 0.243 | -0.265 | 0.882 | 0.257 |

| Panel B. Basic statistics of log returns | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SPX | Nikkei 225 | FTSE | MIB | CAC 40 | DAX | TSX | IBOV | IMOEX | SENSEX | SHANGHAI | JALSH |
| Nobs | 228 | 228 | 228 | 228 | 228 | 228 | 228 | 228 | 228 | 228 | 228 | 228 |
| Minimum | -0.186 | -0.195 | -0.211 | -0.278 | -0.245 | -0.256 | -0.31 | -0.503 | -0.423 | -0.32 | -0.281 | -0.285 |
| Maximum | 0.119 | 0.144 | 0.000 | 0.233 | 0.209 | -0.256 | 0.198 | 0.265 | 0.271 | 0.311 | 0.246 | 0.194 |
| Mean | 0.006 | 0.003 | 0.005 | -0.001 | 0.002 | 0.005 | 0.003 | 0.005 | 0.002 | 0.008 | 0.004 | 0.003 |
| Median | 0.012 | 0.007 | 0.005 | 0.005 | 0.006 | 0.013 | 0.003 | 0.008 | 0.012 | 0.008 | 0.009 | 0.006 |
| Stdev | 0.044 | 0.047 | 0.05 | 0.074 | 0.062 | 0.065 | 0.059 | 0.103 | 0.096 | 0.076 | 0.077 | 0.073 |
| Skewness | -0.763 | -0.647 | -0.62 | -0.523 | -0.538 | -0.668 | -0.984 | -0.78 | -0.999 | -0.563 | -0.513 | -0.56 |
| Kurtosis | 1.556 | 1.456 | 1.853 | 1.227 | 1.29 | 1.367 | 4.346 | 2.741 | 2.771 | 2.948 | 1.791 | 1.233 |

**Table 3.5.** Basic statistics of market capitalisation **Unit: Billion**

| | SPX | Nikkei 225 | FTSE | MIB | CAC 40 | DAX | TSX | IBOV | IMOEX | SENSEX | SHANGHAI | JALSH |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nobs | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 |
| Minimum | 7000 | 2000 | 1000 | 300 | 900 | 600 | 800 | 200 | 100 | 200 | 300 | 300 |
| Maximum | 40000 | 5000 | 3000 | 900 | 3000 | 2000 | 3000 | 1000 | 1000 | 2000 | 8000 | 1000 |
| Mean | 20000 | 3000 | 3000 | 600 | 2000 | 1000 | 2000 | 700 | 600 | 800 | 4000 | 800 |
| Median | 20000 | 3000 | 3000 | 500 | 2000 | 1000 | 2000 | 700 | 500 | 700 | 3000 | 800 |
| Stdev | 9000 | 600 | 300 | 100 | 400 | 300 | 400 | 200 | 200 | 400 | 2000 | 200 |
| Skewness | 0.85 | 0.292 | -0.659 | 0.547 | 0.707 | 0.323 | 0.277 | -0.382 | -0.145 | 0.76 | 0.175 | -0.225 |
| Kurtosis | -0.429 | -0.401 | 1.129 | -0.171 | -0.211 | -0.346 | 0.039 | -0.04 | 0.689 | -0.228 | -0.883 | -0.367 |

The unit in this table is billion except for Nobs, skewness, and kurtosis. Market capitalisation is the total dollar value of a company's outstanding shares at the present market price. For each index, the market capitalisation is the total dollar value of each company's market capitalisation in that index. The data frequency here is monthly.

**Table 3.6.** Basic statistics of volume **Unit: Billion**

| | SPX | Nikkei 225 | FTSE | MIB | CAC 40 | DAX | TSX | IBOV | IMOEX | SENSEX | SHANGHAI | JALSH |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nobs | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 |
| Minimum | 9 | 11 | 10 | 6 | 1 | 1 | 2 | 2 | 5 | 0 | 18 | 2 |
| Maximum | 36 | 72 | 47 | 34 | 6 | 7 | 11 | 427 | 5330 | 3 | 1320 | 12 |
| Mean | 17 | 25 | 21 | 14 | 2 | 2 | 4 | 47 | 853 | 1 | 346 | 5 |
| Median | 14 | 25 | 18 | 13 | 2 | 2 | 4 | 8 | 708 | 0 | 284 | 4 |
| Stdev | 7 | 9 | 9 | 5 | 1 | 1 | 1 | 91 | 814 | 0 | 252 | 1 |
| Skewness | 0.991 | 1.166 | 1.227 | 0.832 | 1.09 | 1.568 | 1.414 | 2.31 | 2.406 | 1.691 | 1.3 | 1.306 |
| Kurtosis | -0.224 | 3.553 | 0.597 | 0.775 | 1.732 | 3.959 | 6.768 | 4.004 | 8.543 | 2.056 | 1.89 | 3.927 |

The unit in this table is billion except for Nobs, skewness, and kurtosis. Volume refers to the total number of shares or contracts traded for a specific security or market during a given period, typically measured over a month. It represents the activity level of a stock and provides insights into the liquidity and overall interest in that security. Nobs represent the number of observations. The data frequency here is monthly.

**Table 3.7.** Basic statistics of the price-to-book ratio

| | SPX | Nikkei 225 | FTSE | MIB | CAC 40 | DAX | TSX | IBOV | IMOEX | SENSEX | SHANGHAI | JALSH |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nobs | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 |
| Minimum | 1.642 | 0.971 | 1.37 | 0.639 | 1.015 | 1.014 | 1.357 | 0.847 | 0.133 | 2.192 | 1.278 | 1.422 |
| Maximum | 4.926 | 2.388 | 2.690 | 2.423 | 2.587 | 2.091 | 2.841 | 3.356 | 1.053 | 7.245 | 6.360 | 3.720 |
| Mean | 2.984 | 1.698 | 1.897 | 1.241 | 1.629 | 1.641 | 1.992 | 1.816 | 0.611 | 3.556 | 2.076 | 2.275 |
| Median | 2.849 | 1.744 | 1.854 | 1.151 | 1.542 | 1.654 | 1.885 | 1.706 | 0.583 | 3.303 | 1.703 | 2.200 |
| Stdev | 0.74 | 0.301 | 0.292 | 0.44 | 0.366 | 0.202 | 0.322 | 0.569 | 0.167 | 0.911 | 0.953 | 0.478 |
| Skewness | 0.658 | -0.212 | 1.006 | 1.241 | 0.691 | -0.234 | 0.963 | 0.684 | 0.347 | 1.793 | 2.347 | 1.127 |
| Kurtosis | -0.243 | -0.368 | 0.592 | 0.593 | -0.377 | -0.204 | 0.045 | -0.214 | 0.405 | 3.183 | 5.888 | 0.782 |

The price-to-book ratio (P/B ratio) is a financial metric used to compare a company's market value to its book value. Nobs represent the number of observations. The data frequency here is monthly.

**Table 3.8.** Basic statistics of dividend yield

| | SPX | Nikkei 225 | FTSE | MIB | CAC 40 | DAX | TSX | IBOV | IMOEX | SENSEX | SHANGHAI | JALSH |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nobs | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 |
| Minimum | 1.27 | 0.8 | 2.935 | 2.05 | 1.734 | 1.904 | 1.734 | 1.959 | 0.978 | 0.773 | 0.344 | 2.107 |
| Maximum | 3.814 | 2.927 | 6.432 | 11.375 | 6.422 | 6.567 | 6.422 | 10.597 | 11.006 | 1.898 | 3.203 | 5.365 |
| Mean | 1.974 | 1.703 | 3.92 | 3.906 | 3.29 | 3.064 | 3.29 | 4.004 | 4.071 | 1.317 | 2.008 | 3.178 |
| Median | 1.953 | 1.722 | 3.892 | 3.604 | 3.241 | 2.978 | 3.241 | 3.579 | 4.22 | 1.312 | 2.048 | 2.968 |
| Stdev | 0.358 | 0.4 | 0.598 | 1.302 | 0.827 | 0.719 | 0.827 | 1.589 | 2.178 | 0.234 | 0.629 | 0.751 |
| Skewness | 1.824 | -0.2 | 0.848 | 2.733 | 0.767 | 1.477 | 0.767 | 1.928 | 0.738 | -0.033 | -0.655 | 0.787 |
| Kurtosis | 6.645 | -0.015 | 1.119 | 10.582 | 1.247 | 4.188 | 1.247 | 3.808 | 0.392 | -0.483 | 0.339 | -0.432 |

Dividend yield is a financial ratio that measures the annual dividend income an investor receives from a stock relative to its current market price. Nobs represent the number of observations. The data frequency here is monthly.

**Table 3.9.** Basic statistics of the return on equity ratio

| | SPX | Nikkei 225 | FTSE | MIB | CAC 40 | DAX | TSX | IBOV | IMOEX | SENSEX | SHANGHAI | JALSH |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nobs | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 | 229 |
| Minimum | 5.334 | -4.234 | 0.163 | -8.947 | 3.142 | 1.258 | 2.376 | -35.520 | 5.043 | 11.273 | 8.729 | 4.587 |
| Maximum | 20.567 | 11.929 | 19.819 | 17.763 | 16.222 | 15.993 | 20.682 | 25.867 | 20.453 | 29.01 | 16.167 | 24.427 |
| Mean | 14.235 | 7.719 | 11.681 | 7.023 | 9.772 | 10.134 | 10.42 | 11.849 | 12.796 | 17.525 | 11.65 | 14.382 |
| Median | 14.288 | 8.134 | 12.248 | 7.298 | 9.247 | 10.225 | 10.548 | 11.740 | 12.488 | 16.219 | 10.925 | 12.823 |
| Stdev | 2.85 | 2.961 | 4.515 | 6.003 | 3.406 | 2.952 | 3.444 | 9.396 | 3.539 | 4.724 | 1.911 | 4.936 |
| Skewness | -0.661 | -1.914 | -0.489 | -0.213 | 0.119 | -0.864 | 0.01 | -1.48 | 0.228 | 0.746 | 0.625 | 0.177 |
| Kurtosis | 1.688 | 4.686 | -0.333 | -0.816 | -0.922 | 1.033 | -0.206 | 6.405 | -0.617 | -0.706 | -0.818 | -0.699 |

The return on equity (ROE) ratio is a key financial metric that measures a company's profitability in relation to its shareholders' equity. Nobs represent the number of observations. The data frequency here is monthly.

**Table 3.10.** 12-month cumulative return.

| | SPX | Nikkei 225 | FTSE | MIB | CAC 40 | DAX | TSX | IBOV | IMOEX | SENSEX | SHANGHAI | JALSH |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nobs | 216 | 216 | 216 | 216 | 216 | 216 | 216 | 216 | 216 | 216 | 216 | 216 |
| Minimum | -0.593 | -0.524 | -0.754 | -0.964 | -0.749 | -0.74 | -0.77 | -0.848 | -1.314 | -0.991 | -1.149 | -0.801 |
| Maximum | 0.43 | 0.407 | 0.43 | 0.437 | 0.414 | 0.481 | 0.542 | 0.896 | 0.985 | 0.718 | 1.23 | 0.645 |
| Mean | 0.07 | 0.037 | 0.001 | -0.018 | 0.022 | 0.059 | 0.033 | 0.039 | 0.016 | 0.085 | 0.065 | 0.033 |
| Median | 0.108 | 0.056 | 0.041 | 0.001 | 0.042 | 0.087 | 0.036 | 0.045 | 0.019 | 0.093 | 0.02 | 0.031 |
| Stdev | 0.163 | 0.166 | 0.198 | 0.271 | 0.221 | 0.23 | 0.212 | 0.36 | 0.387 | 0.287 | 0.365 | 0.247 |
| Skewness | -1.55 | -0.735 | -1.355 | -0.785 | -0.949 | -0.883 | -0.849 | -0.065 | -0.625 | -0.882 | 0.563 | -0.333 |
| Kurtosis | 3.745 | 0.964 | 3.092 | 0.772 | 1.274 | 0.853 | 1.93 | -0.319 | 1.914 | 2.454 | 1.893 | 0.969 |

The 12-month cumulative return is a measure of the total return on an investment over the past 12 months, expressed as a percentage. The log return is used to calculate the 12-month cumulative return. Nobs represent the number of observations. The data frequency here is monthly.

### *3.4.2 Pair-wise Correlation*

Table 3.11 displays the pair-wise fixed correlation coefficients. The table shows that correlation coefficients between S&P 500 index and other six G7 indexes are relatively higher than that between S&P 500 index and five BRICS indexes. In addition, as we can see from the table, the correlation coefficients between Shanghai and other chosen indexes are relatively lower than other pair-wise correlation coefficients. The Modern Portfolio Theory suggests that if the correlation between assets is low, investors can get diversification benefits by diversifying their investments in these assets.

We provide the time-varying correlation plot between SPX and other chosen indexes in Figures 3.27–3.37 at the appendix. When we observe all correlation plots, we find that, before the Great Recession, there was a sharp or big fall in the correlation between the S&P 500 index and other indexes, and during the Great Recession, the correlation between the S&P 500 index and other indexes rose dramatically. The existing empirical evidence (*e.g.,* Roll, 1988; Bertero and Mayer, 1990; King and Wadhwani, 1990; Solnik et al., 1996; Butler and Joaquin, 2002; Syllignakis and Kouretas, 2011; Guidi and Ugur, 2014) suggests that the correlation between stock markets in crisis periods is higher than in non-crisis periods. Our results of the time-varying correlation also support this argument.

**Table 3.11.** Pair-wise fixed correlation.

| | SPX | SHANGH | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SPX | 1 | | | | | | | | | | | |
| SHANGH | 0.397 | 1 | | | | | | | | | | |
| NKY | 0.769 | 0.374 | 1 | | | | | | | | | |
| FTSEMIB | 0.759 | 0.382 | 0.684 | 1 | | | | | | | | |
| FTSE | 0.836 | 0.445 | 0.74 | 0.855 | 1 | | | | | | | |
| DAX | 0.842 | 0.425 | 0.771 | 0.895 | 0.877 | 1 | | | | | | |
| CAC | 0.837 | 0.385 | 0.734 | 0.941 | 0.911 | 0.941 | 1 | | | | | |
| TSX | 0.835 | 0.449 | 0.676 | 0.722 | 0.856 | 0.78 | 0.786 | 1 | | | | |
| SENSEX | 0.648 | 0.399 | 0.598 | 0.64 | 0.672 | 0.685 | 0.656 | 0.671 | 1 | | | |
| IMOEX | 0.579 | 0.303 | 0.521 | 0.591 | 0.644 | 0.639 | 0.628 | 0.677 | 0.548 | 1 | | |
| JALSH | 0.743 | 0.458 | 0.7 | 0.733 | 0.844 | 0.791 | 0.799 | 0.829 | 0.701 | 0.634 | 1 | |
| IBOV | 0.591 | 0.386 | 0.53 | 0.608 | 0.668 | 0.623 | 0.623 | 0.765 | 0.642 | 0.623 | 0.767 | 1 |

The correlation here is the Pearson correlation. The data used to calculate the correlation for each index is the log return. The data frequency here is monthly.

### *3.4.3 Selection of Characteristics for the PPP Model*

In the original paper, Brandt, Santa-Clara, and Valkanov use momentum, the book-to-market ratio, and market capitalisation. In this research, we incorporate additional characteristics such as return-to-equity ratio, dividend yield, and volume for each selected index, in addition to momentum, book-to-market ratio, and market capitalisation. Before modelling the characteristic-based portfolios, we run a pre-sample test to determine which characteristics matter for investment purposes. In this process, we decide which characteristics to optimise portfolios using a multivariate linear regression model. Table 3.12 shows the estimates of the pre-sample test. Interestingly, only the results derived from the market capitalisation characteristic are statistically significant for all 12 indexes. This raises the question of which characteristics we should choose.

In their original work of Brandt et al. (2009), Brandt, Santa-Clara, and Valkanov use three characteristics, including momentum, book-to-market ratio, and market capitalisation. The pre-test results show that only the market capitalisation characteristic is statistically significant for all 12 indexes. To estimate the impact of all six characteristics on the performance of the Parametric Portfolio Policy (PPP), we create three types of optimised portfolios: two-characteristic optimised portfolios (PPP-Two), three-characteristic optimised portfolios (PPP-Three), and all six-characteristic optimised portfolios (PPP-Six), all of which include the market capitalisation characteristic. Market capitalisation and the 12-month cumulative return optimise the two-characteristics portfolio; the market capitalisation, book-to-market ratio, and market capitalisation, following Brandt et al. (2009), optimise the three-characteristics portfolio; and all six characteristics, including the market capitalisation, return-to-equity ratio, book-to-market ratio, dividend yield, volume, and the 12-month cumulative return, optimise the six-characteristics portfolio. Table 3.13 shows these three types of optimised portfolios, based on different characteristics.

**Table 3.12.** The estimates of the pre-sample test.

| Index | Market Capitalisation | Volume | Price-to-book Ratio | Dividend Yield | Return on Equity | 12-month Cumulative Return |
|---|---|---|---|---|---|---|
| SPX | 1.16E-10 | 8.14E-10 | 32.760 | 47.630 | 0.171 | -40.380 |
| | 0.000 *** | 0.080 * | 0.020 ** | 0.002 *** | 0.859 | 0.080 * |
| Nikkei 225 | 3.74E-11 | -5.68E-10 | 77.780 | 58.870 | -0.157 | 45.260 |
| | 0.000 *** | 0.000 *** | 0.000 *** | 0.000 *** | 0.708 | 0.000 *** |
| FTSE | 3.56E-09 | 2.15E-08 | 1051.000 | 190.400 | 30.270 | -387.000 |
| | 0.000 *** | 0.000 *** | 0.000 *** | 0.000 *** | 0.000 *** | 0.001 *** |
| FTSEMIB | 4.67E-08 | 5.51E-07 | 1,3380.000 | 860.600 | 41.520 | -3091.000 |
| | 0.000 *** | 0.000 *** | 0.000 *** | 0.000 *** | 0.492 | 0.008 ** |
| CAC 40 | 1.92E-09 | 1.71E-07 | 1094.000 | 13.020 | 26.030 | 679.700 |
| | 2e-16 *** | 0.000 *** | 0.000 *** | 0.816 | 0.007 *** | 0.000 *** |
| DAX | 1.16E-08 | -2.99E-07 | -2345.000 | -256.300 | -140.100 | 556.200 |
| | 0.000 *** | 0.000 *** | 0.000 *** | 0.095 * | 0.000 *** | 0.070 * |
| TSX | 4.39E-09 | -6.64E-08 | 484.100 | -906.400 | 36.020 | 79.220 |
| | 0.000 *** | 0.145 | 0.107 | 0.000 *** | 0.020 ** | 0.806 |
| IMOEX | 6.62E-11 | -2.62E-13 | -6.354 | -1.435 | 0.115 | 4.661 |
| | 0.000 *** | 0.4002 | 0.003 *** | 0.000 *** | 0.171 | 0.000 *** |
| SENSEX | 4.18E-10 | -9.28E-09 | 13.170 | -26.660 | -0.611 | 26.670 |
| | 0.000 *** | 0.036 ** | 0.000 *** | 0.000 *** | 0.286 | 0.000 *** |

| | | | | | | |
|---|---|---|---|---|---|---|
| SHANGHAI | 4.81E-11 | 6.47E-11 | 50.180 | -19.550 | 12.460 | 53.030 |
| | 0.000 *** | 0.000 *** | 0.000 *** | 0.0214 ** | 0.000 *** | 0.000 *** |
| JALSH | 3.03E-09 | -8.90E-08 | -264.000 | -469.400 | 28.150 | 66.160 |
| | 0.000 *** | 0.000 *** | 0.000 *** | 0.000 *** | 0.000 *** | 0.548 |
| IBOV | 3.98E-08 | 1.09E-08 | -292.000 | -1072.000 | 177.900 | 932.300 |
| | 0.000 *** | 0.040 ** | 0.725 | 0.000 *** | 0.000 *** | 0.284 |

Each characteristic for each index has 216 observations. The table shows the estimates from the regression equation: $R_i = \beta_0 + \beta_1 x_m + \beta_2 x_r + \beta_3 x_b + \beta_4 x_d + \beta_5 x_v + \beta_6 x_c + \varepsilon$, Where, $R_i$ is the log -return for each index (the dependent variable); $\beta_0$ is the $R_i$-intercept (value of $R_i$ when all other parameters are set to 0); $\beta_1$ is the regression coefficient of the market capitalisation ($x_m$); $\beta_2$ is the regression coefficient of the return-to-equity ratio ($x_r$); $\beta_3$ is the regression coefficient of the book to market ratio ($x_b$); $\beta_4$ is the regression coefficient of the dividend yield ($x_d$); $\beta_5$ is the regression coefficient of the volume ($x_v$); $\beta_6$ is the regression coefficient of the 12-month cumulative return ($x_c$); $\varepsilon$ is the model error term. The coefficient shows how much asset index *i* responds to a 100-basis point change in characteristics. The P-value is reported under the estimates. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively. The data frequency here is monthly. The data of market capitalisation, volume, price-to-book ratio, dividend yield, and return on equity data are not multiplied by 100. The reason the way to present the estimates for the market capitalisation and volume is different from other variables is that the estimates for both are too small.

**Table 3.13.** Three types of different characteristics optimised portfolios.

| Strategy | Characteristic |
|---|---|
| Two-characteristic optimised portfolio | 12-month cumulative return<br>Market capitalisation |
| Three-characteristic optimised portfolio | Book-to-market ratio<br>12-month cumulative return<br>Market capitalisation |
| Six-characteristic optimised portfolio | Book-to-market ratio<br>12-month cumulative return<br>Volume<br>Return-to-equity ratio<br>Dividend yield<br>Market capitalisation |

The characteristics-optimised portfolios are under the Parametric Portfolio Policy.

## 3.5. Portfolio Strategy Comparisons

We implement of the 1/N-rule, market capitalisation weighted, risk parity, Black Litterman, mean-variance and Parametric Portfolio Policy (PPP) approaches to optimise portfolios by using all our sample-set. Then we evaluate the respective portfolio performance results and compare the performances of each strategy optimised portfolio. The 1/N-rule portfolio is the benchmark. We use the standard deviation to gauge the volatility, namely risks, and the Sharpe ratio measures compares the return of an investment with its risk. Nobel laureate William F. Sharpe introduced the Sharpe ratio (S.R.) in 1966 (Sharpe, 1966), which is used to help investors assess the return of an investment compared to its risk. It's a mathematical expression of the insight that excess returns over a period of time may signify more volatility and risk, rather than investing skill.

### *3.5.1 In-sample Results*

Table 3.14 compares the in-sample performance of the selected methods' optimised portfolio simulations. Given that the results of each strategy represent the historical average of their monthly return, it would be unreasonable to present the results after deducting the transaction costs. The order in the table is according to the Sharpe ratio. When we display the results in each table, we round normal figures to three decimal places and percentage figures to two decimal places. However, to examine the effect of transaction costs on portfolio performance, we round figures to five decimal places. Panel A of Table 3.14 includes all the performances of the selected methods' optimised portfolio with and without the short-selling constraint, which are simulated using data from the whole sample period. We can see from Panel A of Table 3.14 that the mean-variance strategy without short-selling constraints has the best expected return and Sharpe ratio of all the strategies, at 4.868% and 1.50709. The Black-Litterman strategy without short-selling constraints has the second-best Sharpe ratio of all the strategies. Not surprisingly, the equal weighted portfolio has the lowest Sharpe ratio among all. It is worth noting that the equal weighted, two-characteristic with short-selling constraints, three-characteristic with short-selling constraints, and risk parity portfolios have the worst performance among all strategies, with the monthly average expected return lower than 0.3%. In the table, we split up all results from Panel A into Panel B and Panel C according to whether we place short-selling constraints on the portfolio or not.

Panel B of Table 3.14 shows the in-sample performance of all the strategies without short-selling constraints. We find that the mean-variance portfolio without short-selling constraints produces the highest average expected return with the highest standard deviation, but it still has the highest Sharpe ratio among all strategies in the panel, while the Sharpe ratio of the Black-Litterman portfolio without short-selling constraints still comes in second with the second highest average expected return with the second highest standard deviation. It is worth noting that when there are no short-selling constraints on the Parametric Portfolio Policy (PPP) optimised portfolios, their Sharpe ratios are all higher than both the equal weighted portfolio and risk parity portfolio.

Panel C of Table 3.14 displays the in-sample performance of all strategies with short-selling constraints.

As we can see from the panel, the Sharpe ratios of the MV and Black-Litterman portfolios are still the number one and two, respectively. The MV portfolio still has the highest average expected return with a relatively low standard deviation, although we place the short-selling constraint on it. Surprisingly, when we place the short-selling constraint on the three Parametric Portfolio Policy (PPP) optimised portfolios, the three Parametric Portfolio Policy (PPP) optimised portfolios do not perform well, and their Sharpe ratios are all lower than the equal weighted portfolio (benchmark). It is noticeable that the market capitalisation-weighted portfolio also performs well compared to the three Parametric Portfolio Policy (PPP) with short-selling constraints.

**Table 3.14.** The in-sample performance of the selected methods' optimised portfolio simulations.

| **Panel A. Performance comparison of in-sample results** | | | | |
|---|---|---|---|---|
| Methodology | Average Expected Return | Std Dev | Sharpe Ratio | Order |
| Equal-weighted | 0.00276 | 0.05715 | 0.05715 | 9 |
| Market capitalisation-weighted | 0.00460 | 0.04655 | 0.07520 | 7 |
| Risk parity | 0.00295 | 0.05424 | 0.03404 | 11 |
| Black-Litterman without short-selling constraints | 0.02956 | 0.10349 | 0.27504 | 2 |
| Black-Litterman with short-selling constraints | 0.00606 | 0.04422 | 0.11211 | 6 |
| Mean-variance without short-selling constraints | 0.04868 | 0.15878 | 0.29966 | 1 |
| Mean-variance with short-selling constraints | 0.00617 | 0.04473 | 0.11341 | 5 |
| Two-characteristic without short-selling | 0.00781 | 0.04328 | 0.15507 | 4 |
| Two-characteristic with short-selling constraints | 0.00277 | 0.05711 | 0.02927 | 13 |
| Three-characteristic without short-selling | 0.01049 | 0.05459 | 0.17193 | 3 |
| Three-characteristic with short-selling constraints | 0.00278 | 0.05707 | 0.02949 | 12 |
| Six-characteristic without short-selling constraints | 0.00701 | 0.09449 | 0.06257 | 8 |
| Six-characteristic with short-selling constraints | 0.00349 | 0.05386 | 0.04443 | 10 |
| **Panel B. Performance comparison of in-sample results without short-selling constraints** | | | | |
| Methodology | Average Expected Return | Std Dev | Sharpe Ratio | Order |
| Equal-weighted | 0.00276 | 0.05715 | 0.05715 | 7 |
| Market capitalisation-weighted | 0.00460 | 0.04655 | 0.07520 | 5 |
| Risk parity | 0.00295 | 0.05424 | 0.03404 | 8 |
| Black-Litterman without short-selling constraints | 0.01647 | 0.10349 | 0.14853 | 4 |
| Mean-variance without short-selling constraints | 0.04868 | 0.15878 | 0.29966 | 1 |
| Two-characteristic without short-selling | 0.00781 | 0.04328 | 0.15507 | 3 |
| Three-characteristic without short-selling | 0.01049 | 0.05459 | 0.17193 | 2 |
| Six-characteristic without short-selling constraints | 0.00701 | 0.09449 | 0.06257 | 6 |
| **Panel C. Performance comparison of in-sample results with short-selling constraints** | | | | |
| Methodology | Average Expected Return | Std Dev | Sharpe Ratio | Order |
| Equal-weighted | 0.00276 | 0.05715 | 0.05715 | 4 |
| Market capitalisation-weighted | 0.00460 | 0.04655 | 0.07520 | 3 |
| Risk parity | 0.00295 | 0.05424 | 0.03404 | 6 |
| Black-Litterman with short-selling constraints | 0.00455 | 0.04422 | 0.07805 | 2 |
| Mean-variance with short-selling constraints | 0.00617 | 0.04473 | 0.11341 | 1 |
| Two-characteristic with short-selling constraints | 0.00277 | 0.05711 | 0.02927 | 8 |
| Three-characteristic with short-selling constraints | 0.00278 | 0.05707 | 0.02949 | 7 |
| Six-characteristic with short-selling constraints | 0.00349 | 0.05386 | 0.04443 | 5 |

The table above compares the in-sample performance of the selected methods' optimised portfolio simulations. Panel A of the table compares the performances of the selected methods' optimised portfolio with and without the short-selling constraint comparing to three benchmarks (the equal-weighted, market capitalisation weighted, and risk parity portfolios), which are simulated using data from the whole sample period. Panel B of the table more directly compares the in-sample performance of all the strategies without short-selling constraints to three-benchmarks. Panel C of the table more directly compares the in-sample performance of all strategies with short-selling constraints to three benchmarks. The order in the table is according to the Sharpe ratio. The data frequency here is monthly. Std Dev represent the standard deviation.

Table 3.15 demonstrates the full in-sample weight allocation of each strategy. The table shows that the equal weighted portfolio distributed its weight evenly among 12 indexes. For the three types of Parametric Portfolio Policy (PPP) portfolios, when we simulate the portfolio without the short-selling constraint, they allocate most of their weight into the S&P 500 index, but when we place the constraint, they no longer allocate the majority of their weight into the S&P 500 index. Interestingly, when we observe the table carefully, we find that portfolios that allocate most of their weight into the S&P 500 index outperform those that do not.

Overall, in the in-sample performance comparison, the mean-variance and Black-Litterman optimised portfolios have the best performance, no matter with or without short-selling constraints, and always outperform three benchmarks (1/N rule, market capitalisation-weighted strategy, and risk parity strategy). Furthermore, all three types of characteristic-optimised portfolios outperform the three benchmarks when short selling constraints are not present. However, when short selling is controlled for, the performance of all three types of characteristic-optimised portfolios falls short of the 1/N rule and the market capitalisation-weighted portfolio in terms of in-sample performance. Within three benchmarks, the market capitalisation-weighted portfolio performs better than the 1/N rule and the risk parity strategy, no matter whether it controls for short selling or not.

**Table 3.15**. In-sample weight allocation.

| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% |
| MC | 53.64% | 10.04% | 7.97% | 1.50% | 6.90% | 3.20% | 4.48% | 4.67% | 2.11% | 1.54% | 2.14% | 1.81% |
| RP | 11.45% | 10.13% | 11.86% | 6.88% | 9.49% | 7.44% | 7.87% | 8.52% | 7.54% | 6.32% | 6.93% | 5.58% |
| BL with short selling | 258.30% | 22.39% | -9.02% | -164.11% | -197.20% | 218.99% | -17.52% | -77.12% | 12.74% | -8.61% | 22.76% | 38.41% |
| BL without short selling | 87.89% | 11.92% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.19% | 0.00% | 0.00% | 0.00% |
| MV with short selling | 504.51% | 33.76% | -124.73% | -246.85% | -317.34% | 274.53% | 59.86% | -139.28% | 64.28% | -29.03% | -21.44% | 41.74% |
| MV without short selling | 97.32% | 2.68% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% |
| PPP-Two with short selling | 118.46% | 20.86% | 7.76% | -9.84% | 6.50% | -4.78% | -3.77% | 0.32% | -3.27% | -17.18% | -5.93% | -9.12% |
| PPP-Two without short selling | 8.45% | 8.37% | 8.33% | 8.31% | 8.33% | 8.32% | 8.32% | 8.33% | 8.33% | 8.28% | 8.32% | 8.31% |
| PPP-Three with short selling | 119.50% | 7.90% | 5.75% | -29.02% | 10.51% | -0.29% | -10.74% | 2.83% | 43.59% | -36.83% | 11.81% | -25.01% |
| PPP-Three without short selling | 8.52% | 8.37% | 8.33% | 8.25% | 8.34% | 8.32% | 8.29% | 8.33% | 8.45% | 8.19% | 8.36% | 8.26% |
| PPP-Six with short selling | 54.06% | 16.55% | 80.74% | 67.53% | -58.03% | 20.26% | 17.09% | 12.91% | 37.83% | -58.01% | -11.42% | -79.52% |
| PPP-Six without short selling | 14.50% | 9.48% | 10.31% | 6.91% | 7.24% | 8.30% | 7.06% | 8.76% | 13.29% | 0.33% | 8.87% | 4.97% |

The table above demonstrates the full in-sample weight allocation of each strategy. The data frequency here is monthly.

### *3.5.2 Out-of-sample Results*

In Table 3.16, we present the out-of-sample performances of the 1/N-rule, market capitalisation-weighted, risk parity, Black-Litterman, mean-variance, and Parametric Portfolio Policy (PPP) optimised portfolios before and after the deduction of transaction costs. The 1/N-rule, market capitalisation-weighted, risk parity portfolios are benchmarks, while Black-Litterman, mean-variance, and Parametric Portfolio Policy (PPP) are portfolio optimisation strategies. The order in the table is according to the Sharpe ratio. The out-of-sample results are the average of the predicted performance from 60 expanding windows.

Panel A of Table 3.16 provides all the strategies' out-of-sample performances with and without short-selling constraints before and after the deduction of transaction costs. The results in this panel show that, before the deduction of transaction costs, all other strategies produce a higher average expected out-of-sample return than the 1/N rule-allocated portfolio, but after the deduction of the transaction costs, the two-characteristic optimised portfolio with short selling constraints is the worst performer among all strategies. Form the panel, we also find that the 1/N rule-allocated portfolio, the two-characteristic optimised portfolio with short selling constraints, and the three-characteristic optimised portfolio with short selling constraints have the worst performance, no matter before or after the deduction of the transaction costs. Interestingly, before the deduction of transaction costs, the Black-Litterman optimised portfolio without short-selling constraints has a much better performance than that with short-selling constraints, but it reverses after the deduction of transaction costs. The same situation applies to the mean-variance optimised portfolio; prior to the deduction of transaction costs, the mean-variance-optimised portfolio without short-selling constraints performed significantly better than the one with short-selling constraints, but this performance reverses after the deduction of transaction costs. Moreover, before the deduction of the transaction costs, the six-characteristic optimised portfolio without short-selling constraints yields the highest out-of-sample Sharpe ratio among all strategies in Panel A, followed by the Black-Litterman portfolio without short-selling constraints in second place and the three-characteristic optimised portfolio in third place. After the transaction costs are taken into consideration, the three-characteristic optimised portfolio without short-selling constraints performs the best, followed by the mean-variance with short-selling constraints in second place, and the Black-Litterman with short-selling constraints in third. It is noticeable that the market-weighted portfolio also performs well. According to the panel, we also find that after we add the short-selling constraints, all three PPP-optimised portfolios do not show good performance; even the Sharpe ratios of the two-characteristic and three-characteristic portfolios are lower than those of the risk parity portfolio.

Panel B of Table 3.16 shows the out-of-sample performances of all the strategies with short-selling constraints before and after the deduction of transaction costs. The results in the panel show that, no matter whether before or after the deduction of transaction costs, the mean-variance portfolio with short-selling constraints performs best, followed by the Black-Litterman portfolio with short-selling constraints; the market capitalisation-weighted portfolio comes in third. The results in the panel also show that no matter before or after the deduction of transaction costs, the six-characteristic optimised portfolio with short-selling constraints performs better than the three-characteristic optimised portfolio with short-selling constraints; the three-characteristic optimised portfolio performs with short-selling constraints better than the two-characteristic optimised portfolio with short-selling constraints. Noticeably, no matter before or after the transaction costs, the equally weighted portfolio and the two-characteristic optimised portfolio without short-selling are always the last two.

In Panel C of Table 3.16, we display the out-of-sample performances of all the strategies without short-selling constraints before and after the deduction of transaction costs. As shown in the panel, the six-characteristic optimised portfolio without short-selling constraints is the best performer, followed by the Black-Litterman portfolio with our short-selling constraints; the three-characteristic portfolio without short-selling constraints and the mean-variance portfolio without short-selling constraints come in third and fourth. Moreover, before the deduction of transaction costs, without short-selling constraints, the six-characteristic optimised portfolio performs better than the three-characteristic optimised portfolio; the three-characteristic optimised portfolio performs better than the two-characteristic optimised portfolio. However, after the deduction of transaction costs, the three-characteristic optimised portfolio without short-selling constraints performs better than the six-characteristic optimised and the two-characteristic optimised portfolio without short-selling constraints; the six-characteristic optimised portfolio without short-selling constraints performs better than the two-characteristic optimised portfolio without short-selling constraints.

Overall, the out-of-sample simulation leads us to the following conclusions. First, we find that the performances of the mean-variance optimised and Black-Litterman optimised portfolios are always better than the three benchmarks (equally weighted portfolio, risk parity portfolio, and market capitalisation-weighted portfolio), no matter with or without short-selling constraints and no matter before or after the deduction of the transaction costs. Our out-of-sample results are consistent with some studies (*e.g.,* Durand et al., 2011; Han, 2016; Platanakis et al., 2021) that the mean-variance optimisation outperforms the 1/N rule in terms of Sharpe ratio and are consistent with Bessler et al. (2017), who find that BL-optimised portfolios perform better than naïve diversified portfolios in terms of out-of-sample Sharpe ratio.

Second, prior to controlling short-selling, the Black-Litterman (BL) optimised portfolio outperforms the mean-variance (MV) optimised portfolio, regardless of whether transaction costs are deducted before or after. However, once short-selling is controlled, the mean-variance (MV) optimised portfolio outperforms the Black-Litterman model, regardless of whether transaction costs are deducted before or after. This outcome differs slightly from the findings of Bessler et al. (2017), who find that the BL model consistently outperforms the MV model.

Third, all three types of PPP-optimised portfolios without short-selling constraints perform better than the three benchmarks (equally weighted portfolio, risk parity portfolio, and market capitalisation-weighted portfolio), both before and after the deduction of the transaction costs, and even the six-characteristic optimised portfolio without short-selling constraints performs better than all other selected portfolio strategies before the deduction of the transaction costs. This result is new in the literature, as there is no work comparing the performance of PPP-optimised portfolios with other portfolio strategies.

Fourth, the six-characteristic optimised portfolio consistently outperforms the equally weighted portfolio and the risk parity but could not outperform the market capitalisation-weighted portfolio consistently; the three-characteristic optimised portfolio consistently outperforms only the equally weighted portfolio among the three benchmarks, and the two-characteristic optimised portfolio could not consistently outperform any benchmarks. This result is also new to the literature.

Fifth, the market capitalisation-weighted portfolio performs better than the equally weighted portfolio and the risk parity portfolio, both before and after the deduction of the transaction costs. Some studies (*e.g.,* Plyakha et al., 2012; Bolognesi et al., 2013; Malladi and Fabozzi, 2017; Taljaard and Maré, 2021) document that the equal-weighted stock portfolio is more efficient than the market capitalisation-weighted portfolio over the long term.

**Table 3.16.** The out-of-sample performances.

**Panel A. Performance Comparison of out-of-sample results**

| Methodology | Average Expected Return | | Std Dev | Sharpe Ratio | | Order | |
|---|---|---|---|---|---|---|---|
| | Before | After | | Before | After | Before | After |
| Equally weighted | 0.00529 | 0.00528 | 0.05767 | 0.06396 | 0.06371 | 13 | 12 |
| Market capitalisation-weighted | 0.00759 | 0.00758 | 0.05131 | 0.11659 | 0.11643 | 8 | 7 |
| Risk Parity | 0.00544 | 0.00543 | 0.05528 | 0.06944 | 0.06925 | 10 | 10 |
| Black-Litterman with short selling | 0.03833 | 0.02328 | 0.18464 | 0.19892 | 0.11739 | 2 | 6 |
| Black-Litterman without short selling | 0.00960 | 0.00942 | 0.05460 | 0.14640 | 0.14310 | 6 | 3 |
| Mean-variance with short selling | 0.02618 | 0.01888 | 0.14967 | 0.16419 | 0.11545 | 4 | 8 |
| Mean-variance without short selling | 0.00936 | 0.00918 | 0.05023 | 0.15442 | 0.15086 | 5 | 2 |
| Two-characteristic with short selling | 0.00858 | 0.00791 | 0.05269 | 0.13238 | 0.11962 | 7 | 5 |
| Two-characteristic without short selling | 0.00530 | 0.00521 | 0.05766 | 0.06407 | 0.06249 | 12 | 13 |
| Three-characteristic with short selling | 0.02146 | 0.01065 | 0.05652 | 0.19605 | 0.16000 | 3 | 1 |
| Three-characteristic without short selling | 0.00531 | 0.00531 | 0.05763 | 0.06427 | 0.06427 | 11 | 11 |
| Six-characteristic with short selling | 0.01965 | 0.01194 | 0.07738 | 0.23322 | 0.13362 | 1 | 4 |
| Six-characteristic without short selling | 0.00634 | 0.00599 | 0.05512 | 0.08584 | 0.07951 | 9 | 9 |

**Panel B. Performance comparison of simulated out-of-sample results with short-selling constraints**

| Methodology | Average Expected Return | | Std Dev | Sharpe Ratio | | Order | |
|---|---|---|---|---|---|---|---|
| | Before | After | | Before | After | Before | After |
| Equally weighted | 0.00529 | 0.00528 | 0.05767 | 0.06396 | 0.06371 | 8 | 7 |
| Market capitalisation-weighted | 0.00759 | 0.00758 | 0.05131 | 0.11659 | 0.11643 | 3 | 3 |
| Risk Parity | 0.00544 | 0.00543 | 0.05528 | 0.06944 | 0.06925 | 5 | 5 |
| Black-Litterman | 0.00960 | 0.00942 | 0.05460 | 0.14640 | 0.14310 | 2 | 2 |
| Mean-variance without short-selling | 0.00936 | 0.00918 | 0.05023 | 0.15442 | 0.15086 | 1 | 1 |
| Two-characteristic without short-selling | 0.00530 | 0.00521 | 0.05766 | 0.06407 | 0.06249 | 7 | 8 |
| Three-characteristic without short-selling | 0.00531 | 0.00531 | 0.05763 | 0.06427 | 0.06427 | 6 | 6 |
| Six-characteristic without short-selling | 0.00634 | 0.00599 | 0.05512 | 0.08584 | 0.07951 | 4 | 4 |

| Panel C. Performance comparison of simulated out-of-sample results without short-selling constraints | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Average Expected Return | | | Sharpe Ratio | | Order | |
| Methodology | Before | After | Std Dev | Before | After | Before | After |
| Equally weighted | 0.00529 | 0.00528 | 0.05767 | 0.06396 | 0.06371 | 8 | 8 |
| Market capitalisation-weighted | 0.00759 | 0.00758 | 0.05131 | 0.11659 | 0.11643 | 6 | 5 |
| Risk Parity | 0.00544 | 0.00543 | 0.05528 | 0.06944 | 0.06925 | 7 | 7 |
| Black-Litterman with short-selling | 0.03833 | 0.02328 | 0.18464 | 0.19892 | 0.11739 | 2 | 4 |
| Mean-variance with short-selling | 0.02618 | 0.01888 | 0.14967 | 0.16419 | 0.11545 | 4 | 6 |
| Two-characteristic with short-selling | 0.00858 | 0.00791 | 0.05269 | 0.13238 | 0.11962 | 5 | 3 |
| Three-characteristic with short-selling | 0.02146 | 0.01065 | 0.05652 | 0.19605 | 0.16000 | 3 | 1 |
| Six-characteristic with short-selling | 0.01965 | 0.01194 | 0.07738 | 0.23322 | 0.13362 | 1 | 2 |

The table above compares the out-of-sample performance of the selected methods' optimised portfolio simulations. Panel A of the table compares the performances of the selected methods' optimised portfolio with and without the short-selling constraint comparing to three benchmarks (the equal-weighted, market capitalisation weighted, and risk parity portfolios), which are simulated using data from the whole sample period. Panel B of the table more directly compares the out-of-sample performance of all the strategies without short-selling constraints to three-benchmarks. Panel C of the table more directly compares the out-of-sample performance of all strategies with short-selling constraints to three benchmarks. The order in the table is according to the Sharpe ratio. "Before" means the average expected return before the deduction of the transaction costs; "After" means the average expected return after the deduction of the transaction costs. The order is according to the Sharpe ratio. The data frequency here is monthly. Std Dev is the standard deviation.

### *3.5.3 The Results of 60 Expanding Windows*

Tables 3.17-3.76 show the weight allocation of 60 expanding windows without short-selling constraints. Tables are too many, so we only explain some representatives. From these tables, we can see that for all 60 expanding windows, the MV and BL optimised portfolios allocate the majority of their weight to the S&P 500 index. This likely accounts for the stable and superior performance of these optimised portfolios compared to most of our chosen strategies during our sample period. This reminds us that the S&P 500 index has performed well since 2009. Interestingly, all portfolios - MV, BL, PPP-Two, PPP-Three, and PPP-Six - maintain a long position against SHANGHAI during all expanding windows. We know from the correlation measurement that the pair-wise correlations between SHANGHAI and other selected indexes are relatively lower than pair-wise correlations between other selected indexes.

Tables 3.77-3.136 display the weight allocation and the expected return of 60 expanding windows with short-selling constraints. Each table's columns 1 to 12 provide the in-sample weight allocation for 60 expanding windows, and the last column provides the out-of-sample simulation for the next month. Except for the first two expanding windows, the MV and BL also allocate the majority of their weight to the S&P 500 index. Moreover, after we add short-selling constraints, the MV optimised portfolio only allocates weight to the S&P 500 and SHANGHAI indexes. Similarly, for the first 15 expanding windows, the BL optimised portfolio allocates all weight to the S&P 500, SHANGHAI, and IBOV indexes, but after the expanding window 15, it only allocates all its weight to the S&P 500 and

SHANGHAI indexes. Interestingly, after we add short-selling constraints, the PPP-Two and PPP-Three allocate their weight approximately equal to each of the 12 indexes, while the weight allocation pattern of the PPP-Six is different from the PPP-Two and the PPP-Three.

### 3.6. Conclusion

We optimise portfolio selection for an investment universe of developed and emerging market stock indexes using the Parametric Portfolio Policy (PPP), naïve diversified portfolios (1/N-rule), market capitalisation weighted, risk parity (equally weighted risk contribution), mean-variance (MV), and Black-Litterman (BL) strategies and compare the in- and out-of-sample performance with each other. The Parametric Portfolio Policy (PPP) approach is a novel approach that parameterises the asset weights as a function of their characteristics, thereby estimating those parameters in a way that maximises the investor's average utility. The implicit assumption is that the characteristics convey all relevant information about the assets' conditional distribution of returns. According to the literature and the data availability, we select six characteristics of stock indexes, including the market capitalisation, return-to-equity ratio, book-to-market ratio, dividend yield, volume, and the 12-month cumulative return. To estimate the effect of all six characteristics on the performance of the Parametric Portfolio Policy (PPP), we form three kinds of different characteristics optimised portfolios, including two-characteristic optimised portfolios, three-characteristic optimised portfolios, and all six-characteristic optimised portfolios. We also consider the effect of short selling on portfolio optimisation.

In the in-sample results we have the following findings. First, the mean-variance and Black-Litterman optimised portfolios have the best performance, no matter with or without short-selling constraints, among all six basic strategies and always outperform the three benchmarks (1/N rule, market capitalisation-weighted strategy, and risk parity strategy). Second, all three types of characteristic-optimised portfolios outperform the three benchmarks when short selling constraints are not present. Third, when short selling is controlled for, the performance of all three types of characteristic-optimised portfolios falls short of the 1/N rule and the market capitalisation-weighted portfolio in terms of in-sample performance. Fourth, within three benchmarks, the market capitalisation-weighted portfolio performs better than the 1/N rule and the risk parity strategy, no matter whether it controls for short selling or not.

The out-of-sample simulation leads us to the following conclusions: First, we find that the performances of the mean-variance optimised and Black-Litterman optimised portfolios are always better than the three benchmarks (equally weighted portfolio, risk parity portfolio, and market capitalisation-weighted portfolio), no matter with or without short-selling constraints and no matter before or after the deduction of the transaction costs. Our out-of-sample results are consistent with some studies (*e.g.,* Durand et al., 2011; Han, 2016; Platanakis et al., 2021) that the mean-variance optimisation outperforms the 1/N rule in terms of Sharpe ratio and are consistent with Bessler et al. (2017), who find that BL-optimised portfolios perform better than naïve diversified portfolios in terms of out-of-sample Sharpe ratio.

Second, prior to controlling short-selling, the Black-Litterman (BL) optimised portfolio outperforms the mean-variance (MV) optimised portfolio, regardless of whether transaction costs are deducted before or after. However, once short-selling is controlled, the mean-variance (MV) optimised portfolio outperforms the Black-Litterman model, regardless of whether transaction costs are deducted before or after. This outcome differs slightly from the findings of Bessler et al. (2017), who find that the BL model consistently outperforms the MV model.

Third, all three types of PPP-optimised portfolios without short-selling constraints perform better than the three benchmarks (equally weighted portfolio, risk parity portfolio, and market capitalisation-weighted portfolio), both before and after the deduction of the transaction costs, and even the six-characteristic optimised portfolio without short-selling constraints performs better than all other selected portfolio strategies before the deduction of the transaction costs. This result is new in the literature, as there is no work comparing the performance of PPP-optimised portfolios with other portfolio strategies.

Fourth, the six-characteristic optimised portfolio consistently outperforms the equally weighted portfolio and the risk parity but could not outperform the market capitalisation-weighted portfolio consistently; the three-characteristic optimised portfolio consistently outperforms only the equally weighted portfolio among the three benchmarks, and the two-characteristic optimised portfolio could not consistently outperform any benchmarks. This result is also new to the literature.

Fifth, the market capitalisation-weighted portfolio performs better than the equally weighted portfolio and the risk parity portfolio, both before and after the deduction of the transaction costs. Some studies (*e.g.,* Plyakha et al., 2012; Bolognesi et al., 2013; Malladi and Fabozzi, 2017; Taljaard and Maré, 2021) document that the equal-weighted stock portfolio is more efficient than the market capitalisation-weighted portfolio over the long term.

Overall, the comparison between the mean-variance (MV) and Black-Litterman (BL) strategies, three benchmarks, and all three types of PPP strategies is complicated. We find the mean-variance (MV) and Black-Litterman (BL) strategies have more stable and better performance in terms of Sharpe ratio than the 1/N rule and risk parity, market capitalisation-weighted portfolios. In the in-sample simulation, the mean-variance (MV) and Black-Litterman (BL) strategies consistently beat all other strategies, no matter with or without short-selling constraints. In the out-of-sample simulation, all three types of PPP-optimised portfolios outperform the three benchmarks (equally weighted portfolio, risk parity portfolio, and market capitalisation-weighted portfolio) before controlling for short-selling. In addition, the six-characteristic optimised portfolio, without short-selling constraints, outperforms all other selected portfolio strategies before the deduction of transaction costs. Moreover, the six-characteristic-optimised (the market capitalisation, return-to-equity ratio, book-to-market ratio, dividend yield, volume, and the 12-month cumulative return) and the three-characteristic (the market capitalisation, book-to-market ratio, and the 12-month cumulative return) optimised portfolios seem to produce more stable and better performance than the two-characteristic (market capitalisation and the 12-month cumulative return) optimised portfolio. Furthermore, the market capitalisation-weighted portfolio performs better than the equally weighted portfolio and the risk parity portfolio within three benchmarks, both before and after the deduction of the transaction costs in our sample period.

# **Chapter Four:** The Effects of U.S. Monetary Policy Shocks on Portfolio Diversification


## Abstract

In this chapter, we investigate the impact of changes in U.S. monetary policy on portfolio diversification. We build four different types of portfolios, including a U.S.-only, a stock-bond portfolio, an international diversified portfolio, and an asset-diversified portfolio, using six different assets (including the MSCI EAFE index (developed market index), the MSCI EM index (emerging market index), the S&P 500 index, gold, oil, and 10-year Treasury notes). We not only estimate the fixed response of all six indexes and four portfolios to U.S. monetary policy expected and unexpected changes derived from the 30-day Federal funds futures contract rate, but we also estimate the time-varying correlations of all six indexes and four portfolios to U.S. monetary policy shocks by using the DCC-GARCH model. First, our results show that an unexpected Fed funds target rate cut (negative surprise) triggers an increase in all six variables and four portfolios, but the results for both oil and 10-year Treasury notes are not statistically significant. Second, our findings suggest that the responses of all six variables and four portfolios to the U.S. monetary policy expected change are statistically insignificant. Third, our results show that all stock-related indexes (the MSCI EAFE index, the MSCI EM index, and the S&P 500 index) respond more to U.S. monetary policy surprises than the other three

indexes (gold, oil, and 10-year Treasury notes), while the stock-bond diversified portfolio responds less to U.S. monetary policy surprises than the other three portfolios (U.S.-only, international diversified portfolio, and asset-diversified portfolio). Fourth, we also find that the S&P 500 index and the developed market index exhibit a high degree of co-movement in their response to the U.S. monetary policy unanticipated change, while the S&P 500 index and the emerging market index move apart in their response to the U.S. monetary policy unanticipated change.

## 4.1. Introduction

One of the most important factors affecting economic and market conditions is monetary policy. The ultimate purpose of monetary policy is to achieve macroeconomic goals such as output, employment, and inflation (Karagiannis et al., 2010). However, the impact of monetary policy tools on these variables is, at best, indirect. The most direct and immediate effects of monetary policy actions, such as changes in the Federal funds rate, are on the financial markets. The impact of U.S. monetary policy shocks on global asset prices, especially international equity prices, has always been one of the topics of great interest to the policymakers and the market participants. By affecting asset prices and returns, policymakers try to modify economic behaviours in ways that will help to achieve their ultimate objectives (Karagiannis et al., 2010).

Extensive studies have documented the influence of U.S. monetary policy on U.S. asset prices (*e.g.,* Waud, 1970; Cook and Hahn, 1989; Campbell and Ammer, 1993; Jensen and Johnson, 1995; Patelis, 1997; Kuttner, 2001). Some studies suggest that U.S. monetary policy is a risk factor in U.S. equity markets (*e.g.,* Jensen et al., 1996; Thorbecke, 1997; Bernanke and Kuttner, 2005). There are other studies that have examined the relationship between U.S. monetary policy and foreign asset prices (*e.g.,* Tandon and Urich, 1987; Husted and Kitchen, 1985; Bailey, 1989; Bailey, 1990; Johnson and Jensen, 1993; Kim, 2001; Ehrmann and Fratzscher, 2002). Some studies (*e.g.,* Thorbecke, 1997; Rigobon and Sack, 2001; Rigobon and Sack, 2003; Kalemli-Özcan, 2019) argue that the monetary policy implemented by the United States is a risk factor in global asset markets.

Given the leadership role played by the U.S. market, any new information on the Fed's interest rate policy will have direct and indirect effects on the rest of the world's stock markets and other asset prices and, thus, on security portfolios. We can view all assets or indexes in a portfolio as a bundle of factors that reflect the deeper risks and rewards of that portfolio, just as any food consists of a bundle of nutrients that sustain us. The indirect effects of U.S. monetary policy are the effects of U.S. stock market movements through news announcements on stock returns in other countries. Bernanke and Kuttner (2005) find that U.S. monetary policy shocks impact U.S. equity markets mostly through their effects on risk premiums. The importance of U.S. monetary policy for financial markets is also indicated by the amount of private sector resources devoted to predicting future Federal Open Market Committee (FOMC)'s decisions. The transmission mechanism of U.S. monetary policy shows that it significantly affects asset risks and returns, thereby affecting the risks and returns of security portfolios. For instance, an increase in interest rates by the Federal Reserve can result in higher borrowing costs, lower bond prices, and a decrease in the attractiveness of equity. This, in turn, can impact asset correlations and the effectiveness of diversification (Bernanke and Kuttner, 2005). For example, expansionary monetary policy (*e.g.,* lowering interest rates or quantitative easing) increases liquidity in the financial system, which can drive up asset prices and change the dynamics of risk and return across different asset classes (Stark and Croushore, 2002; Bernanke, 2004; Bernanke and Kuttner, 2005; Gagnon et al., 2011; Barro and Redlick, 2011; Krishnamurthy and Vissing-Jorgensen, 2011; Borio and Zhu, 2012). Thus, understanding how the U.S. monetary policy shocks affect portfolio diversification is crucially important for investors to manage their investment risks and improve their investing profits.

Various methods for estimating monetary policy shocks have been suggested in the literature. There are two main approaches for measuring monetary policy surprises. One of them is by utilising the changes in short-term interest rates (especially the two-year nominal Treasury yield) in a constrained time window around FOMC policy announcements (Gürkaynak et al., 2005; Gilchrist et al., 2015; Hanson and Stein, 2015). Another is based on the Federal Funds futures rate, which has recently drawn much attention among these techniques. By using the Federal Funds futures rate, Bernanke and Kuttner (2005) show that the S&P 500 index returns increase by around 1% following a typical unforeseen Fed rate drop of 25 basis points. They contend that the favourable effects on future dividend streams, decreased discount rate, and increased equity market premium result in a favourable response to such a policy measure. Due to the increased interconnectedness of the global markets as a result of globalisation and the technological revolution, this field of research has been expanded to include foreign stock markets. Ehrmann and Fratzscher (2009) find that in the global context, foreign equity returns react favourably to an unexpected Fed interest rate drop. They put the diversity in reactions between countries down to the degree of financial market integration and the country's amount of exchange rate flexibility. In this chapter, we are going to use the Federal Funds futures rate to measure the U.S. monetary surprise.

A fixed-coefficient method is used in most current studies on the high-frequency response of stock returns to monetary policy shocks. The fundamental assumption is that the reaction of stock returns to monetary policy shocks remains constant across time, contrary to official and anecdotal data which indicate that the responsiveness of stock returns changes over time. For instance, Andersen et al. (2007) discover that the business cycle stage affects how the equity market reacts to macroeconomic news. Similarly, according to Bernanke and Kuttner (2005), equity risk premia explain how stock returns react to monetary policy shocks, which are observed in the literature to change over time. According to Campbell et al. (1998), there are variations in the stock market premium over time, which are relatively significant compared to changes in predicted real interest rates. These arguments show that it is improper to model the stock market response to monetary policy surprises just using a fixed-coefficient methodology, because the responses may change over time due to factors like time-varying financial integration, the state of the business cycles and the time-variation in equity risk premia itself (Kishor and Marfatia, 2013). In this chapter, we use not only the fixed-coefficient technique to evaluate the effect of U.S. monetary policy on asset prices and portfolios but also estimate the time-varying correlation by modelling the heteroscedasticity of the effect of U.S. monetary policy on asset prices and portfolios. This is significant since it has been demonstrated that the bulk of the factors underlying such stock market responses evolve with time.

In this chapter, we are trying to answer the following three questions: 1. How do monetary policy surprises affect the relationship between assets and indexes? 2. Do monetary policy surprises cause assets and indexes to move together or apart? 3. Does the effect of monetary policy surprises on portfolios challenge the Modern Portfolio Theory (MPT)? To answer these three questions, we follow the steps we present next. First, we measure the monetary policy shocks based on the Federal Funds futures rate which is an approach conducted by Kuttner (2005). An effective method for measuring the monetary policy shocks is very important, which directly affects the results of responses of different indexes to U.S. monetary policy surprises. Second, we use a fixed-coefficient technique to evaluate the effect of U.S. monetary policy on asset prices and portfolios. We not only measure the responses of different indexes and portfolios to an unexpected Fed Funds target rate change but also to an expected Fed Funds target rate change. Our results show that an unexpected Fed funds target rate cut (negative surprise) triggers an increase in all six variables and four portfolios, but the results for both oil and 10-year treasury notes are not statistically significant. Third, we estimate the effects of U.S. monetary surprise on the time-varying correlation between all six indexes, four portfolios and monetary policy surprises by modelling the heteroscedasticity. Our results suggest that all stock-related indexes (the MSCI EAFE index, the MSCI EM index, and S&P 500) respond more to U.S. monetary policy surprises than the other three indexes (gold, oil, and 10-year treasury notes), while the stock-bond diversified portfolio respond less to U.S. monetary policy surprises than other three portfolios (U.S only, international diversified portfolio and assets-diversified portfolio). We also find that the S&P 500 index and the developed market index exhibit a high degree of co-movement in their response to the U.S. monetary policy unanticipated change, while the S&P 500 index and the emerging market index move apart in their response to the U.S. monetary policy unanticipated change. Our results can prove that monetary policy shocks are a systemic risk factor, which is in line with Thorbecke (1997), who suggests that monetary policy might be a systematic factor that affects ex-ante returns.

This chapter extends the existing literature in two important ways. First, we examine how different portfolios respond to surprises from FOMC announcements. We examine FOMC announcements from January 2000 to December 2021. Our sample pool contains four main types of security portfolios. These portfolios consist of six different indexes. We all know that Modern Portfolio Theory (MPT) is widely recognised as a pragmatic framework that assists investors in efficiently distributing their capital among various assets with the aim of optimising total returns while maintaining an acceptable degree of risk. As a result, investors often diversify their risk by owning a variety of asset classes and making investments across nations and sectors. Therefore, investors often diversify risk by holding different asset types and investing in different countries and industries. Portfolio channels are derived from theoretical frameworks that prioritise the significance of investor asset holdings in propagating shocks across markets, especially in situations when nations lack shared underlying factors (Kyle and Xiong, 2001). For example, when U.S. investors experience a negative wealth shock from an unexpected change in the Federal funds rate, they may have to liquidate their positions in other countries or adjust the ratio between holdings in different assets. This creates contagion, making the returns of portfolio diversification more volatile and correlated, and raising risk management issues (Kodres and Pritsker, 2002). Our empirical analysis provides insight into how these four main types of portfolios are affected by surprises from FOMC announcements. Second, to the author's best knowledge, this is the first work that studies how different portfolios respond to monetary surprise and the most thorough analysis of how U.S. monetary policy shocks affect global asset markets. Several studies have examined how U.S. monetary policy affects global stock markets and asset prices (*e.g.,* Ehrmann and Fratzscher, 2004; Wongswan, 2006; Andersen et al., 2007; Bernanke and Kuttner, 2005; Ehrmann and Fratzscher, 2009; Wongswan, 2009; Hausman and Wongswan, 2011). However, these studies focus on a limited number of nations and a specific asset classification and there is no study examining how U.S. monetary policy affects portfolio diversification. This chapter examines the effects of changes in U.S. monetary policy on various portfolios by analysing their impact on bond prices, bullion prices, oil prices and equity markets. Compared to the existing literature, this should provide more exhaustive and reliable results. This is, to the best of our knowledge, the first chapter to examine the impact of U.S. monetary policy shocks on security portfolios and the most comprehensive analysis of how U.S. monetary policy shocks influence global asset markets.

After this introduction, the rest of the chapter is organised as follows. Section 4.2 provides the literature review. Section 4.3 presents the data and methodology. In Section 4.4, we estimate the baseline model and analyse its results. Section 4.5 presents how U.S. monetary surprises affect the time-varying correlation between assets, portfolios and Fed monetary policy surprises. Section 4.6 concludes this chapter.

## 4.2. Literature Review

In recent decades, there has been a growing focus on examining the qualitative and quantitative effects of changes in monetary policy on various asset markets, including interest rates, bond rates and stock returns. In the literature, most studies focus on the impact of U.S. monetary policy on asset prices or volatility of asset prices and argue that the monetary policy implemented by the United States may be a risk factor in global asset markets (*e.g.,* Thorbecke, 1997; Rigobon and Sack, 2001; Rigobon and Sack, 2003). However, a few studies suggest that there was a lack of evidence identified about the effects of the Federal Reserve's operations on stock prices (*e.g.,* Tarhan, 1995; Cecchetti, 2003; Hayford and Malliaris, 2004). Additionally, the most frequently used methods in the literature are the simple regression analysis (*e.g.,* Cook and Hahn, 1989; Roley and Sellon, 1995), the vector-regressive analysis (VAR) (*e.g.,* Thorbecke, 1997; Campbell and Ammer, 1993; Bernanke and Blinder, 1992; Laopodis, 2013; Anaya et al., 2017), and the GARCH model analysis (*e.g.,* Bomfim, 2003; Bernanke and Kuttner, 2005).

### *4.2.1 Studies on the Effect of U.S. Monetary Policy on Asset Prices*

#### *4.2.1.1 U.S. monetary policy shocks on U.S. asset prices.*

Extensive studies have documented the influence of U.S. monetary policy on U.S. asset prices (*e.g.,* Waud, 1970; Cook and Hahn, 1989; Campbell and Ammer, 1993; Jensen and Johnson, 1995; Patelis, 1997; Kuttner, 2001). Waud (1970) provided the first rigorous examination of the effects of changes in the discount rate on domestic financial markets. He finds evidence to support this claim: Falling interest rates are viewed as good news by the stock market, while rising interest rates are viewed as bad news by the stock market. Numerous analyses conducted after the Waud study provide evidence supporting the existence of an announcement effect in various U.S. financial markets. Smirlock and Yawitz (1985) identify two channels through which changes in discount rates may alter investor expectations and, therefore, stock prices: 1) changes in discount rates may affect cash flow forecasts for stocks and 2) changes in discount rates may alter interest rate expectations and, therefore, the interest rate used to discount expected cash flows.

Several studies (Cook and Hahn, 1989; Johnson and Jensen, 1993) have concluded that the strength of the market's reaction to changes in the discount rate depends on the Fed's current monetary policy and the Fed's motivation for adjusting interest rates. Cook and Hahn (1989) analyse the one-day reaction of bond rates to movements in the Federal funds target rate over the period from 1974 to 1979 by using an event study. They use a methodology whereby they conducted a regression analysis to regress the change in the bill, note and bond rates on the movements in the Fed funds target rate. They discover that movements in the target results in large movements in short-term rates and smaller but significant movements in intermediate-term rates and long-term rates.

Campbell (1991) breaks unforeseen fluctuations in excess returns into revisions in expectations: 1) revisions in expectations (news) about future dividends, 2) revisions in expectations (news) regarding present and future real rates, and 3) revisions in expectations (news) regarding future excess returns. Bernanke and Blinder (1992) use the vector auto-regression (VAR) methodology to assess monetary policy for the period from July 1969 to December 1989, utilising the Federal funds rate as a key indicator. Their findings derived from variance decomposition and Granger causality tests provide compelling evidence that the funds rate effectively predicts various real variables such as unemployment and industrial output throughout the period spanning from July 1959 to December 1989. This observation aligns with the idea that monetary policy has a significant role in influencing tangible economic factors. Campbell and Ammer (1993) use the same methodology as Bernanke and Blinder (1992) to decompose the variations in excess stock and 10-year bond returns. This decomposition is based on the change in expectations for future stock dividends, inflation, short-term real interest rates, excess stock returns, and excess bond returns. Their research demonstrates that prospective excess stock returns and inflation significantly influence stock and bond returns in monthly postwar U.S. data. The results of this study also indicate that real interest rates have a limited impact on returns. However, they do influence the short-term nominal interest rate and the slope of the term structure.

Johnson and Jensen (1993) examine the impact of U.S. discount rate changes on domestic (United States) and 15 foreign stock markets for the period from October 1979 through December 1991 and find that the Fed plays a key role in the potential influence on economic conditions in both the United States and foreign countries. Roley and Sellon (1995) apply Cook and Hahn (1989)'s event-study approach to analyse the period from 1987 to 1995. Their findings indicate that there was a statistically insignificant increase of four basis points in the bond rate for every percentage point increase in the target funds rate. However, they do observe some evidence suggesting that policy changes were anticipated during the latter period. Krueger and Kuttner (1996) examine the relationship between the one- and two-month Fed funds futures rates and the observed Fed funds rate, and find that over the 1989-94 period, Fed funds futures rates generated very accurate forecasts of the Fed funds rate at one- and two-month horizons.

Thorbecke (1997) employs the vector auto-regression (VAR) approach to investigate the neutrality of money by examining how the response of stock return data to U.S. monetary policy shocks. It provides empirical evidence that monetary policy has a significant impact on both ex-ante and ex-post stock returns. This finding aligns with the hypothesis that monetary policy, particularly in the short run, has substantial and tangible impacts on real variables. The findings derived from the analysis of size portfolios suggest that small enterprises are more significantly impacted by monetary shocks compared to big firms. This evidence supports the hypothesis that monetary policy matters partly because it affects firms' access to credit. The evidence presented above indicates using three different methods that monetary policy has a large and statistically significant effect on ex-post stock returns. Thorbecke (1997) suggests that monetary policy might be a systematic factor that affects ex-ante returns.

Patelis (1997) uses five different indicators, namely the federal funds rate, the spread between the federal funds rate and the yield on the ten-year Treasury note, the spread between the yield on six-month commercial paper and six-month T-Bills, the quantity of non-borrowed reserves, and the portion of non-borrowed reserve growth orthogonal to total reserve growth, to measure the U.S. monetary policy actions. Using long-run regressions and short-run vector autoregressions, this study concludes that monetary policy variables are important predictors of future returns, although they do not fully explain the predictability of observed stock returns.

Kuttner (2001) studies the effects of monetary policy actions on short-term bills, medium-term notes, and bond yields, using data from the federal funds futures market to separate changes in the target funds rate into expected and unexpected components. They reveal that unanticipated changes in target rates elicit a robust and statistically significant response in interest rates, whilst the response of interest rates to predicted changes in target rates is found to be rather weak. The identification approach used by Rigobon and Sack (2003) relies on the presence of heteroskedasticity in stock market return distributions to investigate the relationship between the U.S monetary policy and the stock market. They discover noteworthy policy reactions, whereby a 5% increase (decrease) in the S&P 500 index leads to a roughly 50% increase in the likelihood of a 25-basis point increase (decrease) in monetary tightening (easing).

Ehrmann and Fratzscher (2004) examine the impact of United States monetary policy on the stock market. The authors provide data indicating that individual stocks exhibit significant heterogeneity in their reactions to U.S. monetary policy shocks. Initially, they illustrate the presence of notable industry-specific impacts resulting from U.S. monetary policy. Furthermore, they find that monetary policy has a considerable impact on enterprises within the S&P 500 index. Specifically, companies exhibiting characteristics such as low cash flows, tiny market capitalisation, bad credit ratings, low debt to capital ratios, high price-earnings ratios, or a high Tobin's q are found to be more susceptible to the effects of monetary policy. Hayford and Malliaris (2004) use a forward-looking Taylor rule model to investigate the potential impact of stock market valuation on monetary policy subsequent to the stock market fall of 19 October 1987. By estimating the model using revised and real-time data, their findings do not provide any empirical support that the Federal Reserve policy attempted to moderate stock market valuations during the late 1990s despite the "irrational exuberance" comments by Chairman Greenspan. Their empirical evidence suggests that the Fed accommodated the high valuations of the stock market during this period.

By adapting the methodology established by Campbell (1991) and Campbell and Ammer (1993), Bernanke and Kuttner (2005) conduct a comprehensive analysis to explore the influence of changes in monetary policy on stock prices, with the objectives of both measuring the average response of the stock market and comprehending the underlying economic factors contributing to this response. They discover that, on average, an unforeseen reduction of 25 basis points in the target rate of the Federal funds is linked to an approximate 1% rise in comprehensive stock indexes. The researchers also discover that the impact of unforeseen monetary policy moves on projected excess returns is the predominant component of the stock market reaction. They also find that U.S. monetary policy surprises affect U.S. equity markets mainly through their effects on risk premiums.

Gurkaynak et al. (2005) examines the impact of macroeconomic and monetary policy surprises on the term structure of interest rates. They argue that to comprehensively account for monetary policy shocks, it is necessary to include two factors: the surprise in the present target rate (referred to as target surprise) and the surprise in the anticipated trajectory of future monetary policy (referred to as path surprise). Additionally, their results indicate that the yields on 5- and 10-year treasury notes respond mainly to the path surprise, while U.S. equity indexes respond only to the target surprise. Bjørnland and Leitemo (2009) use structural vector auto-regressive (VAR) methods to quantify the relationship between US monetary policy and the S&P 500. They find a significant association between the interest rate setting and real stock prices. They find that real stock prices immediately decrease by approximately seven to nine per cent due to a monetary policy surprise that increases the Federal funds rate by 100 basis points and that a stock price surprise increasing real stock prices by one per cent causes a rise in the interest rate of nearly to 4 basis points.

Krishnamurthy and Vissing-Jorgensen (2011) evaluate the effect of the Federal Reserve's purchase of long-term Treasuries and other long-term bonds on interest rates. They find that it is inappropriate to focus only on Treasury rates as a policy target because quantitative easing works through several channels that affect particular assets differently. Gospodinov and Jamali (2012) investigate the impact of anticipated and unforeseen components within movements to the Federal funds target rate on both realised and implied volatility for the period from February 4, 1994, to December 11, 2007. They employ Kuttner's (2001) methodology to quantify the impact of monetary policy shocks by analysing Federal funds futures rates. They discover that unanticipated changes in the target rate significantly raise the level of volatility. Consistent with the efficient market hypothesis, their analysis suggests that the expected component of a target rate change as well as the actual target rate change do not significantly affect volatility. They also show that larger than expected decreases in the Federal funds target rate tend to lower the volatility risk premium.

*4.2.1.2 U.S. monetary policy shocks on global asset prices.*

There are some studies that have examined the relationship between U.S. monetary policy and foreign asset prices (*e.g.,* Tandon and Urich, 1987; Husted and Kitchen, 1985; Bailey, 1989; Bailey, 1990; Johnson and Jensen, 1993; Kim, 2001; Ehrmann and Fratzscher, 2002).

Tandon and Urich (1987) present empirical evidence relating to the announcement effects of US money supply to Euro currency interest rates and the foreign currency markets (both spot and forward) for seven industrial countries over the period 1977–1982. Their results indicate that unexpected components of announced changes in money supply have a significant positive effect on Euro currency interest rates and a negative effect (implying dollar appreciation) on the spot exchange rates. Bailey (1989) documents the association between weekly US money supply releases and four price series in Canadian financial markets. He finds that the Toronto stock index, Canadian government bond prices, and Canadian short-term interest rates have moved in response to unexpected changes in the level of US MI releases since the US began targeting money growth in October 1979. These effects do not appear to be transmitted through Canadian monetary variables. He also finds that the Canada/US exchange rate is uncorrelated with US MI surprises. Husted and Kitchen (1985) examine how money supply announcements affect interest parity conditions and how these conditions recover after a disturbance for the sample period from February 8, 1980, to August 27, 1982. They find that U.S. money supply surprises are associated with an increase in forward premiums for the U.S. dollar against the Canadian dollar and the German mark and in short-term European currency deposit rates for these two currencies. Their results also show that the pattern of German and Canadian responses to U.S. money supply surprises is consistent with interest parity conditions and the pattern of responses of these financial variables does differ across countries.

Bailey (1990) examines the effect of US money supply surprises on Pacific Rim stock markets and presents evidence demonstrating a relationship between U.S. money surprises and reactions in Pacific Rim stock markets. Johnson and Jensen (1993) examine the impact of U.S. discount rate changes on domestic (United States) and 15 foreign stock markets (Australia, Belgium, Canada, France, Germany, Great Britain, Hong Kong, Italy, Japan, Malaysia, Netherlands, Philippines, Singapore, Switzerland, Thailand. Their evidence suggests that there is a sign of the impact of Fed actions on international markets. Using VAR models, Kim (2001) investigates how U.S. monetary policy shocks are transmitted worldwide in the context of a flexible exchange rate system. They discover that U.S. expansionary monetary policy shocks lead to booms in the non-U.S., G-6 countries. They also find that changes in trade balances seem to play a minor role in this transmission, while a fall in the world real interest rate seems important.

According to the study conducted by Ehrmann and Fratzscher (2009), it is evident that the monetary policy of the United States has had a significant role in influencing global equities markets. Upon conducting an analysis of 50 stock markets throughout the globe, they find that, on average, there is a decline of about 3.8% in returns when the US monetary policy tightens by 100 basis points. However, their findings argue that it is important to note that the response to this tightening varies across different nations, with some exhibiting no response while others seeing a reaction of 10% or even more. Additionally, they find that there is notable heterogeneity among sectors in terms of their response to this monetary policy tightening. They observe that the transmission channels may be differentiated, with a particular emphasis on the transmission via US and international short-term interest rates as well as the exchange rate, which are identified as significant factors. They examine the importance of macroeconomic policies and the extent of real and financial integration in determining the effectiveness of transmitting asset prices to specific countries, thus linking the strength of asset price transmission to underlying trade and asset holdings and reveal that the level of global integration among countries, rather than a country's bilateral integration with the United States, plays a crucial role in the transmission.

Wongswan (2009) employs two proxies, namely the target surprise and the path surprise, to represent U.S. monetary policy surprises. This approach differs from the majority of previous studies that utilise a single proxy. The research investigates the impact of unanticipated U.S. monetary policy announcements on 15 equity indexes across Asia, Europe, and Latin America. They find a significant and notable response of global stock market indexes to monetary policy shocks in the United States over short time periods, using high-frequency data. In reaction to an unexpected hypothetical fall of 25 basis points in the Federal funds target rate, it is observed that foreign stocks indexes often experience a notable increase ranging from 12% to 212%.

This study also shows that changes in U.S. interest rates and, consequently, U.S. monetary policy shocks have an impact on overseas equities indexes via their discount rate component. The findings of this research indicate that the monetary policy implemented by the United States may be a risk factor in international equities markets.

Kim (2009) presents a comprehensive analysis of the impact of target interest rate announcements from the U.S. Federal Reserve and the European Central Bank (ECB) on the market returns and return volatility of 12 Asia-Pacific stock markets for the period from 1999 to 2006. The results of their study indicate that a significant number of stock markets experience considerable declines in returns when faced with unexpected rises in interest rates, as shown by the impact of news spillovers on returns. The market exhibited diverse reactions to the news from the Federal Reserve, although the news from the European Central Bank was generally assimilated at a slower pace. The observed rise in return volatility may be attributed to the impact of both sources of interest rate news. Ammer et al. (2010) examine the transmission of United States monetary policy to the global economy via the analysis of intraday fluctuations in stock prices at the company level subsequent to the release of interest rate adjustments. Their results show that, although significant heterogeneity across enterprises, the monetary policy of the United States has a significant influence on the average global stock price. Numerous factors have been identified as contributing to the variability of this particular response. They discover that, based on the demand channels of policy transmission, the sensitivity of a foreign firm to U.S. monetary policy increases when it operates in a sector that is more responsive to economic cycles and produces a greater share of its sales internationally, as seen via the channels of policy transmission related to demand.

Li et al. (2010) conduct an empirical analysis on the effect and transmission of monetary policy shocks on stock prices in Canada and the United States, using structural VAR models with short-run constraints. Their study specifically investigates the relevance of trade and financial market openness in this context. They discover that in Canada, the stock prices exhibit a limited immediate reaction and a brief dynamic response to a contractionary monetary policy shock. Conversely, in the United States, the stock prices demonstrate a comparatively significant immediate response and a relatively prolonged dynamic response to a similar shock.

Hausman and Wongswan (2011) investigate the impact of unanticipated U.S. monetary policy pronouncements on international stock indexes, short- and long-term interest rates, and currency rates across 49 countries. They employ target surprise and path surprise as two proxies to measure monetary policy shocks. They find that a theoretical unexpected fall of 25 basis points in the Federal funds target rate is often associated with a corresponding increase of around 1 percent in overseas equity indexes, along with a decline of 5 basis points in international short-term interest rates. A sudden downward adjustment of 25 basis points in the anticipated trajectory of future policy is linked to a decrease of about 0.5 per cent in the exchange rate of the dollar against foreign currencies and corresponding decreases of 5 and 8 basis points in short-term and long-term interest rates, respectively. Additionally, they discover that the reactions of asset prices to FOMC statements vary significantly between nations and are influenced by the exchange rate system in each one. Equities indexes and interest rates react more strongly to U.S. monetary policy surprises in nations with a less flexible exchange rate framework. Additionally, there is a high correlation between the cross-country variance in the equity market response and the proportion of each country's equity market capitalisation.

Kishor and Marfatia (2013) assess the dynamic response of international stock markets to monetary policy shocks derived from the Federal Funds futures market in the United States, by using the methods conducted by Kuttner (2001). Their results demonstrate a significant time-variation in the response of global equity markets to unanticipated shifts in U.S. monetary policy, with a rise in equity returns following an interest rate fall that was not expected. Additionally, they discover that in times of crisis, foreign stock markets exhibit a heightened sensitivity to monetary policy shocks originating from the United States, and that during the most recent financial crisis, equity markets in both Europe and the United States negatively respond to the Fed's unexpected interest rate reductions.

Barakchian and Crowe (2013) suggest that after 1988 U.S. monetary policy became more forward-looking and suggest this change in approach has rendered the identifying assumptions in traditional ways of evaluating the impacts of monetary policy invalid. Consequently, they argue that the findings obtained during this era using these conventional approaches are deemed spurious and improbable. By using the shock series in a Vector Autoregression (VAR) model, they ascertain the negative impact of monetary tightening on production. Additionally, it has been shown that up to 50% of the fluctuations in production may be attributed to monetary policy shocks. Yang and Hamori (2014) examine the transmission of the spillover impact originating from U.S. monetary policy to specific stock markets in the Association of Southeast Asian Nations (ASEAN) region. They utilise Markov switching models as an analytical framework. They find that the presence of two separate regimes for both US monetary policy and the stock markets is confirmed via the use of univariate Markov-switching models. Through the use of multivariate Markov-switching models, their analysis reveals a discernible inverse relationship between United States interest rates and the chosen ASEAN stock markets specifically during times of economic boom. Nevertheless, this particular phenomenon becomes less prominent in times of economic downturn. The findings of their empirical analysis suggest that the impact of US monetary policy on ASEAN stock markets is seen only during periods of relative calm. The findings of this study have significant significance for understanding the transmission mechanisms of asset prices, including the credit channel, trade channel, and balance sheet channel.

Based on the frequency of media coverage, Baker et al. (2016) created a new indicator of economic policy uncertainty (EPU) based on newspaper coverage frequency for the United States and 11 other major economies. Based on the analysis of firm-level data, they discover that policy uncertainty is associated with heightened levels of stock market volatility as well as a decline in investment and employment in sectors that are susceptible to policy uncertainty, such as defence, healthcare, finance, and infrastructure building. Anaya et al. (2017) use a structural global vector auto-regressive (VAR) model to examine the potential effects of U.S. unconventional monetary policy shocks on financial and economic circumstances in emerging market economies (EMEs). These shocks are recognised via changes in the central bank's balance sheet. In addition, researchers investigate the significance of foreign capital flows as a conduit for transmitting shocks. The researchers discover that the implementation of an expansionary policy shock leads to a substantial rise in portfolio flows from the United States to emerging market economies (EMEs) for a duration of around two quarters. This increase is accompanied by a sustained shift in both real and financial indicators inside the recipient nations. Furthermore, emerging market economies (EMEs) often react to the shock by implementing a more easing monetary policy approach.

Ansari and Sensarma (2019) use the Vector Auto Regression (VAR) model to examine the impact of the United States monetary policy, oil prices, and gold prices on the stock indexes of the BRICS countries. They discover that, with the exception of the Bombay Sensex, the impact of US monetary policy on the stock market indexes of BRICS countries is not significant.

Nevertheless, the research results indicate that the FTSE JSE Johannesburg stock exchange is influenced by fluctuations in oil prices, whilst the RTSI Moscow and the BVSP Sao Paulo stock exchanges are affected to varying extents by changes in gold prices. Albagli et al. (2019) examine the spillover of US monetary policy into international bond yields for 12 developed countries and 12 emerging market economies over the period from January 2003 to December 2016. To identify US monetary shocks, they use the change in short-term Treasuries (two-year maturity in our baseline specification) within a narrow window centred around Federal Open Market Committee meetings. They then test how shocks to US MP affect international bond yields at different maturities using panel data regressions. Their results show that the US monetary policy significantly spillovers to international bond markets in a sample of 24 countries. Kalemli-Özcan (2019) shows that the spillover effects of US monetary policy to the rest of the world work through changes in risk premia. This result is similar to that of Bernanke and Kuttner (2005), who show that US monetary policy surprises affect the US stock market primarily through their impact on risk premia.

By using the Bayesian VAR, Miranda-Agrippino and Rey (2020) provide empirical findings that support the existence of significant financial spillover effects resulting from U.S. monetary policy on global economies. Bhar and Malliaris (2021) formulate the modelling of unconventional monetary policy and critically evaluates its effectiveness to address the Global Financial Crisis. We begin with certain principles guiding general scientific modelling and focus on Milton Friedman's 1968 Presidential Address that delineates the strengths and limitations of monetary policy to pursue certain goals. The modelling of monetary policy with its novelty of quantitative easing to target unusually high unemployment is evaluated by a Markov switching econometric model using monthly data for the period 2002–2015. We conclude by relating the lessons learned from unconventional monetary policy during the

Global Financial Crisis to the recent bold initiatives of the Fed to mitigate the economic and financial impact of the Covid-19 pandemic on U.S. households and businesses. However, Tarhan (1995), Cecchetti (2003), and Hayford and Malliaris (2004) argue that there is a lack of evidence identified about the effects of the Federal Reserve's operations on stock prices.

We summarise the literature above. First, there are two main approaches for measuring monetary policy surprises, namely the target surprise and the path surprise. The target surprise is defined as the difference between the announced target fed funds rate and expectations derived from the fed funds futures contract (Kuttner, 2001). The path surprise is by utilising the changes in short-term interest rates (especially the two-year nominal Treasury yield) in a constrained time window around FOMC policy announcements (Gürkaynak et al., 2005; Hanson and Stein, 2015). Most of the studies just use the target surprise as the proxy to measure monetary policy shocks (*e.g.,* Bernanke and Kuttner, 2005; Ehrmann and Fratzscher, 2009; Bjørnland and Leitemo, 2009; Gospodinov and Jamali, 2012; Kishor and Marfatia, 2013), while a few articles use two proxy to measure monetary policy shocks, namely the target surprise and the path surprise (*e.g.,* Gürkaynak et al., 2005; Wongswan, 2009; Hausman and Wongswan, 2011). Second, some studies in the literature document the effect of U.S. monetary policy only on U.S. asset prices (*e.g.,* Cook and Hahn, 1989; Kuttner, 2001; Rigobon and Sack, 2003), while some studies that have examined the relationship between U.S. monetary policy and foreign asset prices (*e.g.,* Kim, 2001; Yang and Hamori, 2014; Anaya et al., 2017; Ansari and Sensarma, 2019; Miranda-Agrippino and Rey, 2020). Third, some studies suggest that U.S. monetary policy is a risk factor in equity markets (*e.g.,* Thorbecke, 1997; Rigobon and Sack, 2001; Rigobon and Sack, 2003; Wongswan, 2009). However, there is no study which examines the impact of U.S. monetary policy shocks on portfolio diversification and builds the most comprehensive analysis of how U.S. monetary policy shocks influence global asset markets and therefore portfolio diversification.

### *4.2.2 Studies on the Effect of Other Regions' Monetary Policies on Asset Prices*

The study conducted by Bredin et al. (2007) examines the impact of changes in monetary policy in the United Kingdom on the returns of UK stocks, as well as the potential factors contributing to this reaction. The researchers undertake an event study to evaluate the effects of unanticipated shifts in monetary policy on overall and sector-specific stock returns. Additionally, they endeavour to identify the mechanisms that drive the reaction of stock returns to unexpected monetary policy developments. The findings of their study suggest that the monetary policy shock causes a lasting negative response in terms of future excess returns across several sectors.

Fausch and Sigonius (2018) study the impact of unexpected monetary policy actions by the European Central Bank (ECB) on excess stock returns in Germany. First, they conduct an event analysis to assess the impact of conventional and unconventional monetary policy on stock returns. Second, by using the vector autoregression framework of Campbell and Ammer (1993), they decompose excess stock returns into three components: news related to expected excess returns, future dividends, and future real interest rates. In their study, conventional monetary policy shocks are quantified using data from futures markets. The main results of the study show that the overall volatility of German excess stock returns mainly stems from revisions in dividend expectations, while the stock market reaction to monetary policy shocks depends on the existing interest rate regime. In periods of negative real interest rates, an unexpected contraction in monetary policy leads to a reduction in excess stock returns. The channels behind this reaction are news about an increase in expected excess returns and a reduction in future dividends.

Gnabo and Soudant (2022) provide empirical findings about the portfolio rebalancing behaviour of European equities mutual funds in response to both conventional monetary policies (CMP) and unconventional monetary policies (UMP) from the European Central Bank (ECB). Using a panel fixed effect estimator, they provide empirical evidence in favour of the presence of portfolio rebalancing across equity categories subsequent to unconventional monetary policy (UMP) implementation. On average, European equity mutual funds tend to reallocate their assets towards mid-cap and core firms, as well as emerging nations. Conversely, they tend to transfer their investments away from small-cap and value stocks, as well as their home and developed countries. Moreover, mutual funds seem to prioritise and focus on their favoured and established investment techniques. The aforementioned findings indicate that managers exhibit a greater inclination to allocate investments towards equities that are seen as safer and more familiar subsequent to statements about unconventional monetary policy (UMP). This behaviour serves to mitigate the potential risk associated with information asymmetry.

Laopodis (2013) investigates the inter-connectedness between monetary policy and the stock market throughout three separate monetary regimes, namely the Burns, Volcker, and Greenspan eras for the sample period from 1970 to 2005, by using a structural Vector Autoregression (VAR) model. The findings indicate that there was no consistent dynamic relationship between monetary policy and the stock market. Vespignani (2015) investigates the effects of monetary aggregate shocks originating from the United States, China, and Japan on the Euro area throughout the period from 1999 to 2012 and discovers that the expansion of China's monetary policy has a spill-over impact on the Euro area. Its finding suggests that the expansion of China's monetary policy through income absorption and rises in monetary aggregates in China drive the rise in the world price of commodities, the rise in the Euro area CPI, and significant rises in Euro area industrial production and exports.

### *4.2.3 Other Studies Related to Our Study*

The occurrence of pre-announcement effects, which is the first stage of a phenomena referred to as the "calm-before-the-storm" effect by Jones et al. (1998), is a possibility. According to the study conducted by Jones et al. (1998), it has been shown that there is a decrease in conditional volatility in the Treasury market during the days before the release of significant economic data. This phenomenon is referred to as the "calming" or pre-announcement effect. However, on the actual day of the announcement, there is an increase in volatility, often known as the "storm" or news effect. The occurrence of this phenomena is often documented in the financial media and is further substantiated by academic research conducted on several financial markets.

In the study conducted by Bomfim (2003), an investigation is carried out on the impact of pre-announcements and news events from the Federal Open Market Committee (FOMC) on the stock market within the framework of public disclosure of monetary policy choices. The findings indicate that there is a tendency for the stock market to exhibit a relatively calm state, characterised by exceptionally low conditional volatility, in the days leading up to regularly scheduled policy announcements.

Forbes and Chinn (2004) investigate whether actual and financial links across nations may explain why changes in the largest markets have such pronounced impacts on other financial markets and how these cross-market interconnections have evolved over time. They estimate a factor model in which a country's market returns are determined by global, sectoral, and cross-Count. Their results show that both cross-country and sectoral factors significantly influence stock, and bond returns across various countries. Tenreyro and Thwaites (2016) investigate how the response of the US economy to monetary policy shocks depends on the state of the business cycle. The effects of monetary policy are less powerful in recessions, especially for durables expenditure and business investment.

Overall, in the literature, most studies argue that the monetary policy implemented by the United States may be a risk factor in global asset markets (*e.g.* Thorbecke, 1997; Rigobon and Sack, 2001; Rigobon and Sack, 2003). However, a few studies suggest that there was a lack of evidence identified about the effects of the Federal Reserve's operations on stock prices (*e.g.,* Tarhan, 1995; Cecchetti, 2003; Hayford and Malliaris, 2004). Examining whether the U.S. monetary policy is a risk factor in global asset markets is very important because it can affect the way we hedge risks when we invest in the asset markets globally. The Modern portfolio theory (MPT) argues that systematic risks cannot be diversified away, while specific risks can be diversified away as you increase the number of stocks in your portfolio (Beja, 1972), so differentiating whether the risk is a systematic risk or a specific risk is also important. Thus, in this chapter, we are going to examine whether the U.S. monetary policy is a risk factor in global asset markets by checking how it affects four main types of portfolios and all six main assets which we use to construct the four main types of portfolios and provide evidence to prove whether it is a systematic risk or not. Moreover, we find that there are two main approaches for measuring monetary policy surprises, namely the target surprise and the path surprise. In this chapter, we are going to use the most prevailing one, which is the target surprise, because it has been found to generate very accurate forecasts of the Fed funds rate at one- and two-month horizons (Kuttner, 2001). In addition, some studies in the literature have investigated the effect of U.S. monetary policy on U.S. asset prices (*e.g.,* Kuttner, 2001; Rigobon and Sack, 2003), while some studies that have examined the relationship between U.S. monetary policy and foreign asset prices (*e.g.,* Kim, 2001; Anaya et al., 2017; Ansari and Sensarma, 2019; Miranda-Agrippino and Rey, 2020). However, there is no study which examines the impact of U.S. monetary policy shocks on security portfolios and builds the most comprehensive analysis of how U.S. monetary policy shocks influence global asset markets. Therefore, in this chapter, we are not only going to investigate the effect of U.S. monetary policy shocks on different types of portfolios but also build the most comprehensive analysis of how U.S. monetary policy shocks influence international asset markets.

### 4.3. Data and Methodology

FOMC (Federal Open Market Committee) meetings usually take place eight times per year, about every 6 weeks. Beginning in February 1994, the Federal Reserve announced its decisions on the day of the FOMC meetings, a departure from the previous practice of the markets inferring decisions from open market operations. Given three major crises (two economic crises and one health crisis), the sample period for our study is from January 01, 2000, to December 31, 2021. Our data frequency is daily. The daily data used in this chapter to capture the monetary policy shocks, as daily data is valuable for identifying short-term trends, patterns, and price movements. Our sample period includes the Dot-com bursting period from April 2000 to December 2002, the Great Recession period from December 2007 to June 2009, and the COVID-19 health crisis period from December 2019 to December 2021. For the Dot-com bursting crisis, we thoroughly reviewed a large body of literature (Chen et al., 2018), which only documents the bursting of the dot-com bubble in mid-March. So, we marked March 13th, 2000, as the official start date of the bubble crisis. We use the time when stock markets started going down as the beginning of the Great Recession and mark June 30th, 2009, as the end of it. We set January 30, 2020, as the start date of the COVID-19 crisis, when the World Health Organisation (WHO) declared the novel coronavirus outbreak a Public Health Emergency of International Concern. Furthermore, according to the WHO, the novel coronavirus outbreak officially ends in 2023. However, due to the large-scale vaccination of vaccines, most countries led by the United States have cancelled most of the epidemic prevention measures by the end of 2021. In this chapter, we mainly focus on the full sample period, and we may also increase research on sub-samples in future research. We examined all the Federal Open Market Committee (FOMC) meetings for the whole sample period. Our sample contains 180 scheduled meetings and 3 unscheduled meetings. We check the meetings one by one from the official website of the Board of Governors of the Federal Reserve System. This website has real-time updates on both the meeting dynamics and the latest Federal funds rate level. The daily data used in the study included the price of the Fed funds futures rate and four different portfolios of securities composed of different assets or indexes.

The first dataset contains 30-day Fed funds futures market contracts that most closely track the effective overnight Fed funds rate for a given month. The Chicago Board of Trade (CBOT) has offered Fed funds futures contracts since October 1988, with several different deliveries from the current month into the next five months. Even where contracts with longer lead times exist, these contracts are much less liquid. These contracts allow market participants to hedge interest rate risk and have an essential role in revealing market expectations about future monetary policy actions. We use the most liquid spot-month contract to extract monetary policy surprises at each FOMC meeting, which is currently the most popular method for extracting monetary policy surprises in the literature. Among various market-based indicators of monetary policy expectations, the Fed funds futures rate dominates all other tools in predicting the future path of monetary policy in the coming months (Gürkaynak et al., 2007). The data for the 30-day Fed Funds futures rate comes from Bloomberg.

The second dataset contains four types of diversification opportunities consisting of three stock indexes (the S&P 500, MSCI EAFE, and MSCI EM indexes) and three assets (gold, oil, and bonds). The indexes selected here are the same as those discussed in Chapter two, but the frequencies are different. Consideration is given to four categories of diversification opportunities available to U.S. investors. Table 4.1 provides all variables we have in this chapter, and Table 4.2 presents a variety of diversification opportunities for U.S. investors in this chapter, including the U.S.-only, Portfolio 1, Portfolio 2, and Portfolio 3. The first portfolio is limited to the United States and uses the S&P 500 index as its indicator. The second portfolio (Portfolio 1) is a stock-bond portfolio comprised of the S&P 500 index and U.S. 10-year Treasury notes. We adopt the well-known pension funds distribution principle, allocating 60% weight to the S&P 500 index and 40% weight to the U.S. 10-Year Treasury Note. The third portfolio (Portfolio 2) is an international-diversified portfolio made up of the S&P 500 index, the MSCI EAFE index, and the MSCI EM index. The fourth portfolio (Portfolio 3) is an asset allocation portfolio for U.S. investors that consists of the S&P 500 index, gold, gasoline, and the U.S. 10-year Treasury note. For Portfolios 2 and Portfolios 3, we take the equally weighted portfolio (EWP) strategy. All series are denominated in U.S. dollars. The data for the S&P 500 index originates from Capital I.Q., while the MSCI EAFE and MSCI EM indexes are sourced from the Morgan Stanley Capital International stock market (MSCI) database. For how these four portfolios are designed methodologically, please refer to Section 2.4.2 in Chapter Two.

**Table 4.1.** List of variables

| Variables | Definition | Abbreviation | Frequency | Source |
|---|---|---|---|---|
| S&P 500 | The Standard and Poor's 500 index | S&P 500 | Daily | Capital I.Q. |
| MSCI EAFE | A developed market index | MSCI EAFE | Daily | MSCI |
| MSCI EM | An emerging market index | MSCI EM | Daily | MSCI |
| Gold | Gold Bullion | Gold | Daily | Bloomberg |
| Oil | Brent Oil index | Oil | Daily | Bloomberg |
| 10-year Treasury-Note | The U.S. 10-year Treasury-Note index | 10 YTN | Daily | Bloomberg |
| The 30-day Fed funds futures market contracts | / | / | Daily | Bloomberg |
| $R_{S\&P\ 500}$ | The simple return of S&P 500 (nominal return) | / | Daily | $\frac{P_t - P_{t-1}}{P_{t-1}}$ |
| $R_{MSCI\ EAFE}$ | The simple return of MSCI EAFE (nominal return) | / | Daily | |
| $R_{MSCI\ EM}$ | The simple return of MSCI EM (nominal return) | / | Daily | |
| $R_{Gold}$ | The simple return of gold (nominal return) | / | Daily | |
| $R_{Oil}$ | The simple return of oil (nominal return) | / | Daily | |
| $R_{10\ YTN}$ | The simple return of 10-year Treasury-Note (nominal return) | / | Daily | |
| $R_f$ | The nominal U.S. 3-month Treasury-Bill Rate (nominal risk-free rate) | / | Daily | Bloomberg |
| $R_{Inflation}$ | The U.S. inflation rate | / | Daily | Eikon DataStream |
| $r_{S\&P\ 500}$ | The real return of S&P 500 after inflation | / | Daily | $= \frac{1 + R_t}{1 + R_{Inflation}} - 1$ |
| $r_{MSCI\ EAFE}$ | The real return of MSCI EAFE after inflation | / | Daily | |
| $r_{MSCI\ EM}$ | The real return of MSCI EM after inflation | / | Daily | |
| $r_{Gold}$ | The real return of gold after inflation | / | Daily | |
| $r_{Oil}$ | The real return of oil after inflation | / | Daily | |
| $r_{10\ YTN}$ | The real return of 10-year Treasury-Note after inflation | / | Daily | |
| $r_f$ | The real U.S. 3-month Treasury-Bill Rate after inflation | / | Daily | |

| | | | |
|---|---|---|---|
| $\Delta r_t^i$ | The abnormal return for each index | / | Daily |
| $m_s$ | The number of days in the month s | / | Daily |
| $f_{s,t}^0$ | The spot-month futures rate on day $t$ in month s | / | Daily |
| $f_{s,t-1}^0$ | $f_{s,t-1}^0$ denotes the spot-month futures rate on day $t-1$ in month s | / | Daily |
| $\Delta F_t^u$ | The unexpected Fed funds target rate change | / | Daily |
| $\Delta F_{\tilde{t}}^{e}$ | The expected Fed funds target rate change | / | Daily |
| $\Delta F_{\tilde{t}}$ | The actual Fed funds target rate change | / | Daily |
| $\Delta c_t^{\text{index } i \text{ and u}}$ | the correlation between the abnormal return of asset *i* and monetary surprise | / | Daily |

The indexes selected here are the same with Chapter Two, but their frequency differs. For how the nominal return is converted to real return, please refer to Section 2.4.2 in Chapter Two.

**Table 4.2.** Types of diversification opportunities.

| | Components | | | |
|---|---|---|---|---|
| U.S.-only | S&P 500 | | | |
| Portfolio 1 | S&P 500 | 10 YR T-Note | | |
| Portfolio 2 | S&P 500 | MSCI EAFE | MSCI EM | |
| Portfolio 3 | S&P 500 | Gold | Oil | 10 YR T-Note |

U.S.-only exclusively invests in the U.S. market. Portfolio 1 is the 60/40 stock/bond portfolio. Portfolio 2 is an international, diversified portfolio. Portfolio 3 is the asset-diversified portfolio. The portfolio design is the same as Chapter 2. The data frequency for each index is daily. For how these four portfolios are designed methodologically, please refer to Section 2.4.2 in Chapter Two.

### *4.3.1 Measuring Monetary Policy Surprise*

The identification of monetary policy shocks has generated widespread interest in macroeconomics. To measure the policy shock, one needs to capture the market expectation. Several methods have been utilised by researchers to measure unexpected changes in monetary policy. One relatively recent and popular method of estimating monetary policy shocks uses information from the Federal funds futures market. This method is proposed by Kuttner (2001) and has been used by Krueger and Kuttner (1996) among others, who find that Fed funds futures rates generated very accurate forecasts of the Fed funds rate at one- and two-month horizons. Gürkaynak et al. (2007) also show the superiority of the Fed funds futures price among different market-based measures of monetary policy expectations.

The strategy proposed by Kuttner (2001) for quantifying the volatility of monetary target rate by using data from Federal funds futures is a rational way to accomplish this objective. The method used in this study is based on the framework presented by Kuttner (2001). According to this approach, the 1-day surprise is determined as:

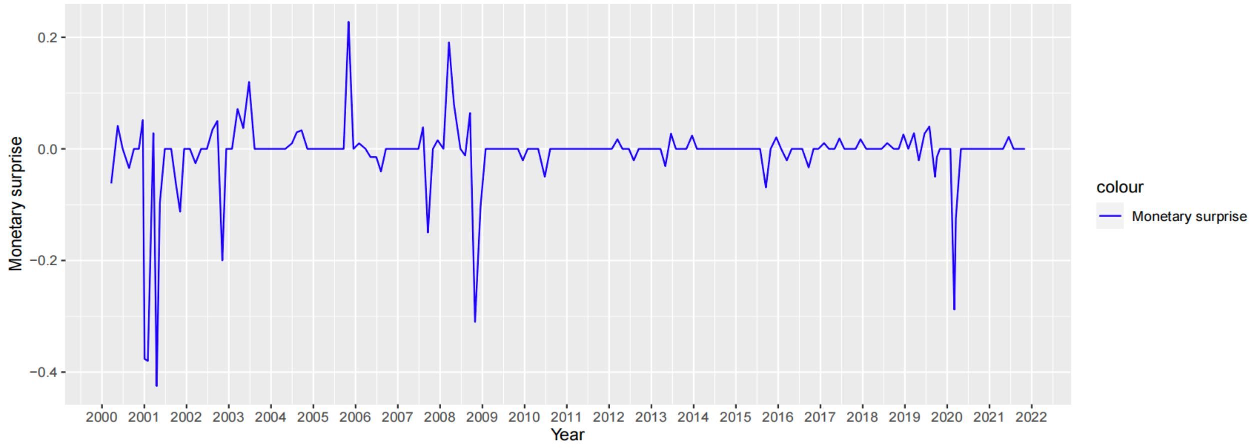
$$\Delta \mathrm{F}_{\mathrm{t}}^{\mathrm{u}} = \frac{\mathrm{m_s}}{\mathrm{m_s - t}}(f_{\mathrm{s,t}}^{0} - f_{\mathrm{s,t-1}}^{0}) \tag{4.1}$$

Where $\Delta \mathrm{F}_{\mathrm{t}}^{\mathrm{u}}$ is the monetary shocks; $\mathrm{m_s}$ represents the number of days in the month s; $\tau$ is is the FOMC announcement day in the month; $f_{s,t}^{0}$ denotes the spot-month futures rate on day $t$ of month s and $f_{s,t-1}^{0}$ denotes the spot-month futures rate on day $t-1$.

When the change occurs on the first day of the month, $f_{s-1,t-1}^{0}$ has been chosen over $f_{s,t-1}^{0}$, because its expectations have already been included into the spot rate of the preceding month. Similarly, when the change occurs on the last day of the month, the change in the 1-month futures rate is used instead. The actual target rate change minus the unexpected target change is then used to compute the expected change in the target rate, $\Delta F_t^e = \Delta F_t^{\sim} - \Delta F_t^u$. Figure 4.1 presents the plot of U.S. Monetary surprise from 2000 to 2021. and Figure 4.1 plots the 57 target rate changes contained in our sample, three of which are not associated with FOMC meetings.

In Figure 4.1, we find that there are some sharp spikes from the beginning of 2000 to the end of 2002, and also some from the late of 2007 to the end of 2009. In addition, there are some sharp spikes from the end of 2019 to the middle of 2020. When we relate these sharp spikes to the big crisis for the last 20 years, we find that almost all of them happened more frequently during the Dot-com Bursting period (2000-2002), the Great Recession (2007-2009), and the COVID-19 health crisis period (2019-2021), respectively. Thus, from Figure 4.1, we suggest that during crisis periods, the U.S. monetary policy tends to be more aggressive, while during normal periods, it tends to be moderate.

**Figure 4.1.** The plot of U.S. monetary surprise.




### *4.3.2 Event Study Approach for Normal and Abnormal Returns*

In this chapter we use event study to check the FOMC meetings one by one. We use 1 day as the event window size in accordance with Bernanke and Kuttner (2005) and Hausman and Wongswan (2011).

Normal and abnormal returns: To analyse the impact of the event, we define anomalous returns as the actual ex-post return (event returns) of the stock market over the event window that exceeds the normal return, or the returns that would have been expected if the event had not occurred (Kishor and Marfatia, 2013). Using a variant of the constant-mean-return model, the normal returns are derived using the average daily real return over the previous 21 trading days, which is essentially equivalent to the average one month prior to the event (FOMC meeting). If the meeting dates are within 21 trading days of each other, the average real returns are adjusted appropriately. This is especially true regarding movements between meetings. Figures 4.2 and 4.3 map the abnormal return for the 6 indexes and 3 portfolios. The return we use here is the real return. See Section 2.4.2 in Chapter Two for the nominal return is converted returns into real returns.

**Figure 4.2.** The plot of abnormal return of in Portfolio 2.

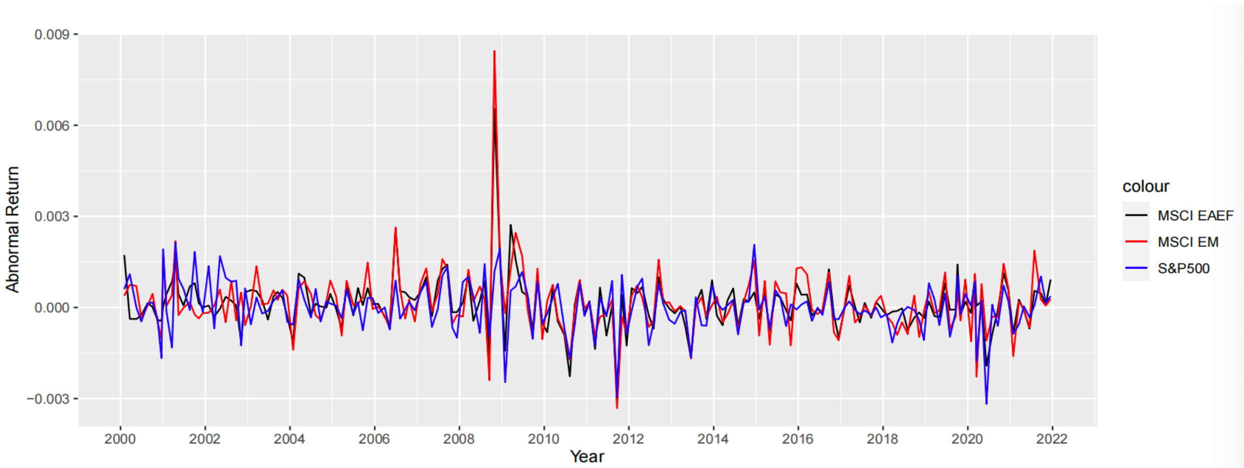
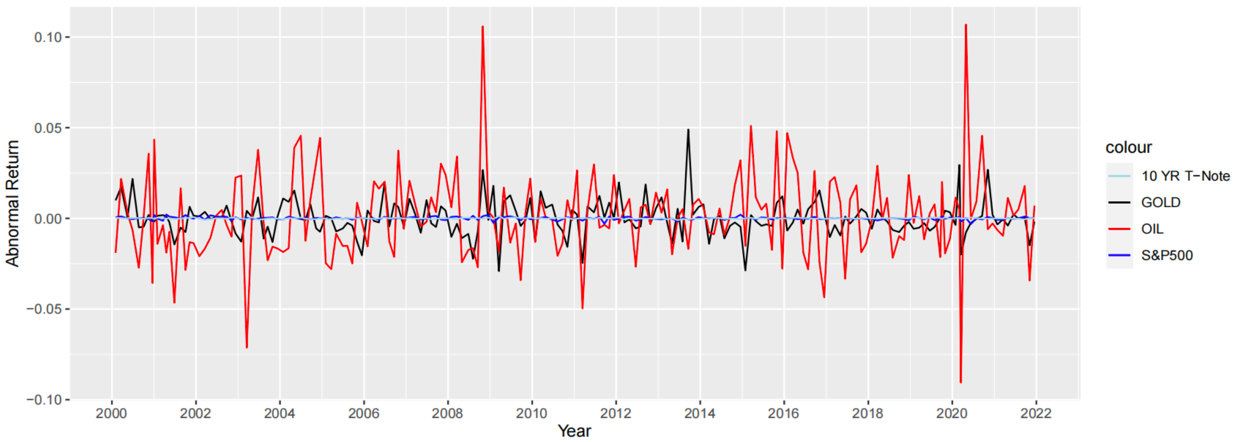



**Figure 4.3.** The plot of the abnormal return of four assets in Portfolio 3.




The accurate determination of event impact dates is crucial owing to variations in time zones across different nations. In the context of our study, it is noteworthy that international portfolios comprise diverse equity markets across the globe, thereby necessitating the traversal of multiple time zones. Hence, the computation of event impact for the international portfolio (Portfolio 2) involves determining the mean returns during the FOMC meeting date and the subsequent date. This practice guarantees uniformity in the comparison process, as well as accounts for the slower assimilation of new information in certain markets. For the S&P 500 index, 10-year Treasury-Note, gold index, oil index, portfolio1 and portfolio3, the relevant closing prices are on the same day as the FOMC announcement, as these markets are relatively advanced and falling at almost the same time zone.

#### *4.3.3 Response of Assets and Portfolios to Unexpected Monetary Policy Surprise*

In this section, we project how different assets and various diversified portfolios respond to monetary policy disruptions in the United States. In our study, the monetary policy disruptions refer to the unexpected Fed funds target rate changes. To accomplish this, we apply the methodology from the previous section of Kuttner (2001) chapter to calculate the monetary policy surprise.

The subsequent step entails determining how the element of astonishment caused by the policy change will impact the abnormal returns of various security portfolios. Following Kuttner (2001) and Bernanke and Kuttner (2005), we use the following regression specification to assess the impact of U.S. monetary policy surprises on the returns of different assets and various diversified portfolios:

$$\Delta \mathrm{r}_{\mathrm{t}}^{i} = \alpha^{i} + \omega^{i} \Delta \mathrm{F}_{\mathrm{t}}^{\mathrm{u}} + \varepsilon_{\mathrm{t}}^{i} \tag{4.2}$$

where $\Delta \mathrm{r}_{\mathrm{t}}^{i}$ represents the abnormal return of asset index $i$ or portfolio $i$ at the event date $\mathrm{t}$ as defined earlier, and $\Delta \mathrm{F}_{\mathrm{t}}^{\mathrm{u}}$ is monetary policy surprise. The coefficient $\omega^{i}$ shows how much the abnormal return of portfolio *i* responds to a 100-basis point unanticipated interest rate increase in the U.S.

The estimated results are shown in Table 4.3. Based on the results from the full sample, an unexpected rate cut by the Fed boosts the stock market indexes across different asset indexes and portfolios. First, we find a surprise rate cut (negative surprise) of 25 basis points by the

Fed triggers a jump in MSCI EAFE by 0.85 per cent, the MSCI EM index by 1.1 per cent, the gold index by 4.7 per cent, and the S&P 500 index by 0.6 per cent. This finding is in line with Krueger and Kuttner (1996), Bernanke and Kuttner (2005) and Bredin et al., (2007) who show that an unanticipated target rate cut leads to an increase in stock returns. However, interestingly, from the table, we can find that the results for both oil and 10Y-TN are not statistically significant. This finding is consistent with Roley and Sellon (1995) that find that the bond rate rose a statistically insignificant four basis points for each percentage point increase in the target funds rate. Second, we find that a hypothetical target rate cut (negative surprise) of 25 basis points by the Fed triggers an increase in Portfolio 1 by 0.85 per cent, Portfolio 2 by 0.46 per cent, and Portfolio 3 by 0.4 per cent. Modern portfolio theory (MPT) argues that it's possible to design an ideal portfolio that will provide the investor with maximum returns by taking on the optimal amount of risk (Markowitz, 1952), and states that the risk for individual stock returns has two components: Systematic Risks: These are market risks that cannot be diversified away (Beja, 1972). Interest rates, recessions, and wars are examples of systematic risks. Unsystematic Risks: Also known as "specific risk," this risk is specific to individual stocks, such as a change in management or a decline in operations (Beja, 1972). This kind of risk can be diversified away as you increase the number of stocks in your portfolio. From the results in Table 4.3, we can see that these three security portfolios are affected by monetary policy to varying degrees. As we stated before, in the Modern portfolio theory, systemic risks cannot be completely dispersed. Our results can prove that monetary policy shocks are a systemic risk factor, which is in line with Thorbecke (1997), who suggests that monetary policy might be a systematic factor that affects ex-ante returns. Third, from Table 4.3, we can also find that among these four portfolios, stock-bond Portfolio 1 and asset-diversified Portfolio 3 are relatively less affected by monetary policy shocks than the U.S.-only and the international-diversified Portfolio 2.

**Table 4.3.** Response of different asset indexes and portfolios to the monetary surprise.

| Index | Full sample | | |
|---|---|---|---|
| | Estimate | R-Sq | Nobs |
| MSCI EAFE | -0.003<br>0.000*** | 0.068 | 178 |
| MSCI EM | -0.004<br>0.000*** | 0.080 | 178 |
| Gold | -0.019<br>0.068* | 0.013 | 178 |
| Oil | -0.014<br>0.576 | -0.004 | 178 |
| S&P 500 | -0.002<br>0.004 *** | 0.040 | 178 |
| 10YTN | -0.000<br>0.208 | 0.003 | 178 |
| Portfolio 1 | -0.002<br>0.002 *** | 0.049 | 178 |
| Portfolio 2 | -0.003<br>0.000 *** | 0.081 | 178 |
| Portfolio 3 | -0.002<br>0.004*** | 0.038 | 178 |

The table shows the estimates from the regression equation for the full sample period (2000–2021). The estimated regression equation is: $\Delta r_t^i = \alpha^i + \omega^i \Delta F_t^u + \varepsilon_t^i$ , where $\Delta r_t^i$ represents the abnormal return of indexes $i$ at the event date t and $\Delta F_t^u$ captures the monetary policy surprise calculated from the Fed funds futures data. The P-value is under the estimates. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively. The data frequency here is daily.

### *4.3.4 Response of Assets and Portfolios to Expected and Unexpected Monetary Policy Surprises.*

After using futures rates to differentiate between expected and unexpected fluctuations in the funds rate target, the natural question to ask is whether the responses of 6 indexes and 3 portfolios to the two components differ for indeed whether rates respond at all to predictable actions. Another purpose of this section is to check the robustness of the results in Section 4.3.3 of this chapter. The Cook and Hahn-style methodology allows for the regression of the change in the interest rate on the two components of the target rate change,

$$\Delta r_t^i = \alpha^i + \omega_1^i \Delta F_t^u + \omega_2^i \Delta F_t^e + \varepsilon_t^i \tag{4.3}$$

where $\Delta r_t^i$ represents the abnormal return of asset index *i* or portfolio *i* at the event date $t$ as defined earlier, and $\Delta F_t^u$ is monetary policy surprise. $\Delta F_t^e$ is the expected change in the Fed funds target rate. The coefficient $\omega_1^i$ shows how much the abnormal return of asset index *i* or portfolio *i* responds to a 100 basis point unanticipated Fed funds target rate increase in the U.S, and the coefficient $\omega_2^i$ shows how much the abnormal return of asset index *i* or portfolio *i* responds to a 100 basis point anticipated Fed funds target rate increase in the U.S.

We estimate the equation (4.3), and the regression results are presented in Table 4.4. As expected, the coefficients on the expected and surprise components are very different: the responses of S&P 500, MSCI EAFE, MSCI EM, gold and all three portfolios to the unanticipated component are large and highly significant, while the responses of S&P 500, MSCI EAFE, MSCIEM, gold and all three portfolios to the anticipated component are small and statistically insignificant. Notably, the responses of oil and 10-YTN to both expected and unexpected components are not statistically significant. The results in Table 4.4 provide evidence that markets are unlikely to respond to policy moves that are already anticipated. Gospodinov and Jamali (2012) conclude the results that the anticipated component of a target rate adjustment does not significantly impact the volatility of the markets. Table 4.4 also provides the robustness to the results in Section 4.3.3, that an unanticipated target rate cut leads to an increase in stock returns and that the response of oil and 10-year Treasury-Note to a cut in the target funds rate is statistically insignificant.

**Table 4.4.** Response of different asset indexes and portfolios to the expected and unexpected Fed funds target rate changes.

| indexes | Full sample | | | | |
|---|---|---|---|---|---|
| | Intercept | Estimate for the unexpected | Estimate for the expected | R-Sq | Nobs |
| MSCI EAFE | 0.00009 | -0.00324 | -0.00043 | 0.07245 | 178 |
| | | 0.00049 *** | 0.20729 | | |
| MSCI EM | 0.00010 | -0.00442 | 0.00016 | 0.07563 | 178 |
| | | 0.00007 *** | 0.68900 | | |
| Gold | -0.00009 | -0.02098 | 0.00471 | 0.01546 | 178 |
| | | 0.0478 ** | 0.23300 | | |
| Oil | 0.00091 | -0.01797 | 0.00878 | -0.00497 | 178 |
| | | 0.49300 | 0.37000 | | |
| S&P 500 | 0.00000 | -0.00234 | -0.00029 | 0.03863 | 178 |
| | | 0.00705 *** | 0.37281 | | |
| 10-YTN | -0.00001 | -0.00039 | -0.00001 | -0.00222 | 178 |
| | | 0.22000 | 0.90500 | | |
| Portfolio 1 | -0.00001 | -0.00156 | -0.00018 | 0.04813 | 178 |
| | | 0.00294 *** | 0.35947 | | |
| Portfolio 2 | 0.00006 | -0.00335 | -0.00017 | 0.07813 | 178 |
| | | 0.00010 *** | 0.59263 | | |
| Portfolio 3 | -0.00002 | -0.00195 | 0.00022 | 0.03794 | 178 |
| | | 0.00344 *** | 0.36592 | | |

The table shows the estimates from the regression equation for the full sample period (2000–2021). The estimated regression equation is: $\Delta r_t^i = \alpha^i + \omega_1^i \Delta F_t^u + \omega_2^i \Delta F_t^e + \varepsilon_t^i$ , where $\Delta r_t^i$ represents the abnormal return of asset index *i* or portfolio *i* at the event date t as defined earlier, and $\Delta F_t^u$ is monetary policy surprise. $\Delta F_t^e$ is the expected change in the Fed funds target rate. The coefficient $\omega_1^i$ shows how much the abnormal return of asset index *i* or portfolio *i* responds to a 100 basis point unanticipated Fed funds target rate increase in the U.S, and the coefficient $\omega_2^i$ shows how much the abnormal return of asset index i or portfolio i responds to a 100 basis point anticipated Fed funds target rate increase in the U.S. The P-value is reported under the estimates. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively. The data frequency here is daily. The results normally round to three decimal places, but here we round the results to five decimal places for clear display estimates of the intercept.

The results presented above assume that the response of the global stock returns to the U.S. monetary policy shock has remained constant over time. To capture how the monetary policy affects the interaction of all indexes and portfolios in our sample set, we are going to model its volatility in the next section.

## 4.4. The Assets' and Portfolios' Return in Response to U.S. Monetary Policy Surprises

### *4.4.1 Methodology and Model Description*

we estimate a DCC-GARCH model to capture how U.S. monetary policy surprises interact with the correlations between all indexes and portfolios, as fixed correlation fails to capture the dynamics of indexes' and portfolios' response to U.S. monetary policy shocks. This enables us to simulate how various assets and portfolios respond to Fed monetary policy changes over time. In the literature, part of them used VAR to identify the interdependence between US monetary policy and other stocks using structural vector auto-regressive (VAR) methodology, but in our chapter, it is very complicated to decompose it, as our portfolios are composed of different types of assets and international indexes, which fall into different categories and regions. Consequently, we estimate the following equation for each index or portfolio i:

$$\Delta \mathrm{r}_{\mathrm{t}}^{i} = \alpha^{i} + \omega^{i} \Delta \mathrm{F}_{\mathrm{t}}^{\mathrm{u}} + \varepsilon_{\tau}^{i} \tag{4.4}$$

The coefficient $\omega^{i}$ assesses the time-varying reaction of abnormal returns of each asset, index and portfolio $i$ to U.S. monetary policy surprises.

Moreover, equity market movements have frequently been observed to exhibit clusters of volatility. In accordance with Sims (1999) and Sims and Zha (2002), we model volatility by allowing heteroskedasticity in the disturbance term $\varepsilon_{i,\mathrm{t}}$. To model heteroskedasticity and extract the correlation between the abnormal returns of assets, we permit the error term to follow the DCC-GARCH (1, 1) process proposed by Engle (2002). The baseline model refers to Session 2.4.1 in Chapter Two. The advantage of this model is that it has the flexibility of a multivariate GARCH model and can directly parameterise conditional correlations (Engle, 2002). We use this model to assess the interaction relationships between related variables and capture trends in correlations over time.

### *4.4.2 Empirical Results for the Baseline Model*

We estimate the time-varying model presented in Section 4.4.1. To model heteroskedasticity and extract the time-varying correlation between the abnormal returns of assets, we permit the error term to follow the DCC-GARCH (1, 1) process proposed by Engle (2002). In our sample, DCC-GARCH (1, 1) is estimated for all six time-series and four types of portfolios are considered in this chapter. The results of the time-varying correlation are presented in Figures 4.4 to 4.5. Figure 4.4 plots the time-varying correlation between each pair of indexes in the international diversified portfolio (Portfolio 2). The pattern in Figure 4.4 suggests that each pair of indexes in the international diversified portfolio is highly correlated. Moreover, we can find that the correlation between the MSCI EAFE index and the MSCI EM index is the highest for most of the time during the period among these three pairs. Figure 4.5 displays the pattern of the time-varying correlation between each pair of indexes in the assets-diversified portfolio (Portfolio 3) within one plot. As can be seen from Figure 4.5, the abnormal returns of gold and oil are also highly correlated, with most of their correlation ranges jumping between 0 to 0.6. we also observe that the correlation between the abnormal returns of gold and the S&P 500 index wanders between positive and negative from 0.4 to -0.4. Nevertheless, a distinct correlation between the abnormal returns of gold and the 10-year Treasury note is presented, which mostly exhibits a negative relationship. In the figure, we can find, mostly, that oil negatively correlates to the S&P 500 index, and the 10-year Treasury note most of the time during the period, respectively. Based on the data shown in Figure 4.5, it can be seen that there exists a link between the anomalous returns of gold and the S&P 500 index. This correlation fluctuates between positive values of 0.4 and negative values of -0.4.

**Figure 4.4.** The time-varying correlation between three stock indexes.

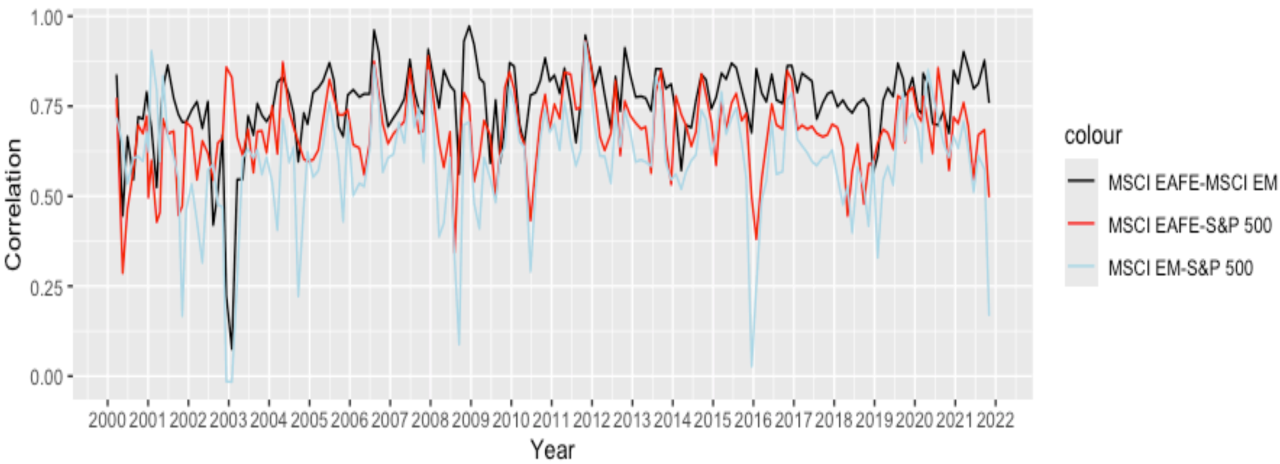
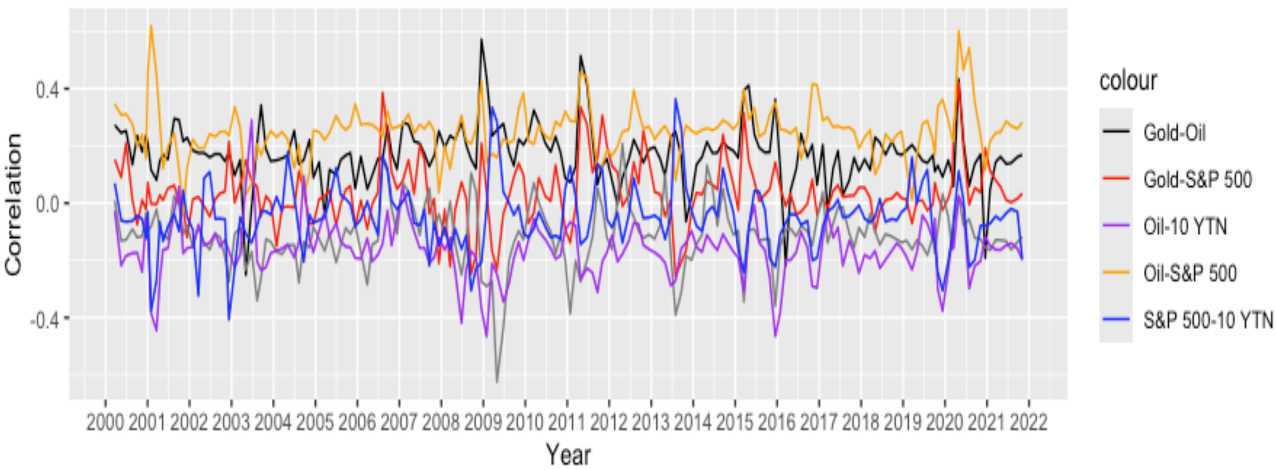



**Figure 4.5.** The time-varying correlation between three asset indexes.



Figures 4.6, 4.7 and 4.8 provide the plot of the time-varying correlation between each variable (including 6 time series and 3 types of portfolios) and U.S. monetary surprise. We find that the correlation between each variable and the unexpected change in the Fed's policy actions does vary significantly over time. Moreover, as we can see in Figure 4.6 all indexes in Portfolio 2 negatively correlate with the U.S. monetary policy surprise most of the time during the sample period, except for some outliers. In detail, correlations between each variable and U.S. monetary policy surprises in the assets-diversified portfolio (Portfolio 3) also negatively correlate with the U.S. monetary policy surprise most of the time during the sample period, except for some outliers. When we look at Figure 4.8, we also find that correlations between each portfolio and U.S. monetary policy surprises exhibit a mostly negative correlation with the U.S. monetary policy surprise throughout the study period, with the exception of a few outliers.

**Figure 4.6.** The plot of time-varying correlation between each stock index and the monetary policy shocks.

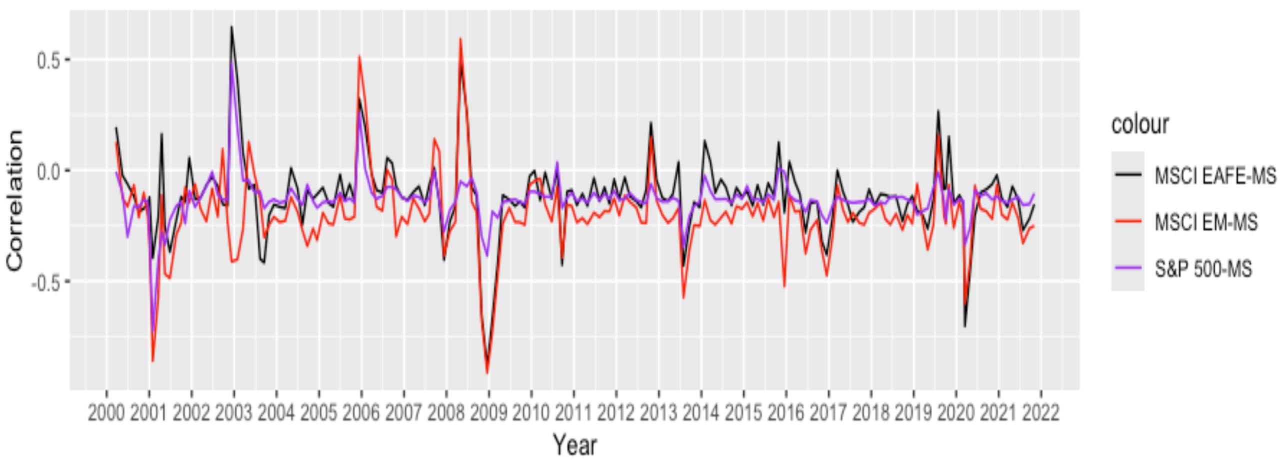
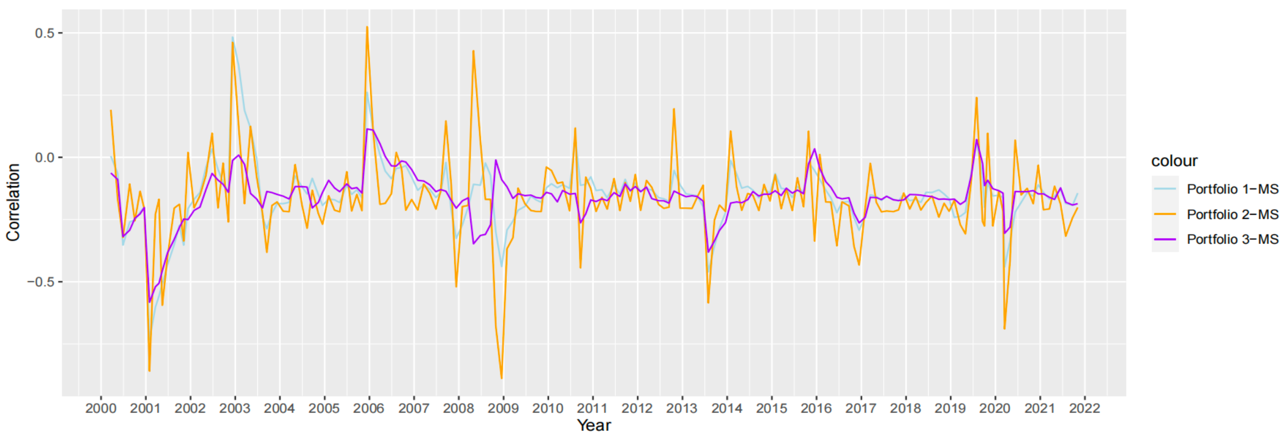


**Figure 4.7.** The plot of time-varying correlation between each asset index and the monetary policy shocks.


**Figure 4.8.** The plot of time-varying correlation between each portfolio and the monetary policy shocks.




To examine the robustness of the time-varying correlation, we get the fixed correlation matrix between 6 variables, 3 portfolios and monetary policy inside our dataset. The results of the fixed correlation are presented in Tables 4.5 and 4.6. As we can see from these two tables, all indexes and portfolios in our data set are negatively correlated with U.S. monetary policy surprise. In Table 4.5, we can obviously find that the three indexes in Portfolio 2 are positively correlated with each other. In Table 4.6, we can find that three indexes in Portfolio 3 are positively correlated with each other, except for 10-year treasury bonds. It is noteworthy that the negative correlation between the 10-year treasury bond and other time series in Portfolio 3 can be seen in Table 4.6. Our results show that the fixed correlation is in line with the time-varying correlation.

**Table 4.5.** The fixed correlations between the abnormal return of 3 stock indexes in Portfolio 2 and monetary policy surprise.

| | MSCI EAFE | MSCI EM | S&P 500 | Monetary Policy |
|---|---|---|---|---|
| MSCI EAFE | 1.000 | 0.814 | 0.635 | -0.253 |
| MSCI EM | 0.814 | 1.000 | 0.561 | -0.086 |
| S&P 500 | 0.635 | 0.561 | 1.000 | -0.241 |
| Monetary Policy Surprise | -0.253 | -0.361 | -0.241 | 1.000 |

The correlation here is the Pearson correlation. The data used to calculate the correlation for each stock index is the simple return. The data frequency here is daily.

**Table 4.6.** The fixed correlation between the abnormal return of 4 indexes in Portfolio 3 and U.S. monetary policy surprise.

| | Gold | Oil | S&P 500 | 10-YTN | Monetary |
|---|---|---|---|---|---|
| Gold | 1.000 | 0.232 | 0.029 | -0.170 | -0.097 |
| Oil | 0.232 | 1.000 | 0.261 | -0.158 | -0.081 |
| S&P 500 | 0.029 | 0.261 | 1.000 | -0.036 | -0.241 |
| 10-YTN | -0.170 | -0.158 | -0.036 | 1.000 | -0.080 |
| Monetary Policy Surprise | -0.097 | -0.081 | -0.241 | -0.080 | 1.000 |

The correlation here is the Pearson correlation. The data used to calculate the correlation for each asset index is the simple return. The data frequency here is daily.

## 4.5. The Effect of the Fed Monetary Policy Surprise on the Correlation

### *4.5.1 Response of Correlations between Assets and Fed Surprises to Fed Surprises*

In this section, we examine how the correlations between the abnormal return of each index we collected in this chapter and monetary policy surprises are affected by the monetary policy surprise (unexpected target rate change). To accomplish this, we first extract figures of the time-varying correlation between the abnormal return of each index and monetary policy surprises from the DCC-GARCH model in Section 4.4.1.

The next step entails determining how the correlations between abnormal returns of indexes change caused by the monetary policy surprise. we use the following regression specification to assess the impact of U.S. monetary policy surprises on the correlations between abnormal returns of the assets or indexes,

$$\Delta c_t^{\text{index } i \text{ and u}} = \alpha^{\text{index } i \text{ and u}} + \beta^{\text{index } i \text{ and u}} \Delta F_t^u + \epsilon_t^{\text{index } i \text{ and u}} \qquad (4.10)$$

where $\Delta c_t^{\text{index } i \text{ and u}}$ represents the correlation between the abnormal return of asset i and monetary surprise (*e.g.* MSCI EAFE and Monetary surprise), and $\Delta F_t^u$ is unanticipated Fed funds target rate change (monetary policy surprise). The coefficient $\beta^{\text{index } i \text{ and u}}$shows how much the correlation between the abnormal returns of assets *i* and monetary surprise responds to a 100-basis point unanticipated Fed funds target rate change.

We estimate the equation (4.10) and present the results in Table 4.7. First, we find that correlations of MSCI EAFE-MS, MSCI EM-MS and S&P 500-MS are highly affected by U.S. monetary surprise. From Section 4.4, we find that the correlations of the abnormal returns of the MSCI EAFE index, MSCI EM index and S&P 500 index are highly correlated with U.S. monetary surprise, which is probably the reason why the U.S. monetary surprise highly affects the correlation between the abnormal return of equity-related index and the U.S. monetary surprise. Second, the monetary policy has a little effect on the correlation between gold and U.S. monetary surprises, but the result is statistically insignificant. Third, noticeably, the correlation between the 10-year treasury bond and the U.S. monetary surprise is negatively affected by the U.S. monetary surprise. Fourth, the correlation between assets-diversified Portfolio 3 and U.S. monetary policy is less affected by U.S. monetary policy surprises than the correlations of Portfolio 1-MS and Portfolio 2-MS. Overall, Table 4.7 suggests that an unanticipated target rate rise causes all variables and U.S. monetary policy surprises to move together, with the exception of 10-year treasury notes.

**Table 4.7.** The time-varying correlation between abnormal returns of three indexes and the monetary surprise regressed against the monetary surprise.

| Correlations | Full sample | | | |
|---|---|---|---|---|
| | Estimate | R-Sq | Nobs | P-value |
| MSCI EAFE-MS | 0.396 | 0.021 | 178 | 0.031* |
| MSCI EM-MS | 0.634 | 0.059 | 178 | 0.001*** |
| Gold-MS | 0.082 | -0.001 | 178 | 0.383 |
| Oil-MS | 0.332 | 0.064 | 178 | 0.000 *** |
| S&P 500-MS | 0.530 | 0.140 | 178 | 0.000 *** |
| 10-YTN-MS | -0.254 | 0.038 | 178 | 0.005 ** |
| Portfolio 1-MS | 0.740 | 0.145 | 178 | 0.000 *** |
| Portfolio 2-MS | 0.808 | 0.099 | 178 | 0.000 *** |
| Portfolio 3-MS | 0.397 | 0.081 | 178 | 0.000 *** |

The table shows the estimates from the regression equation for the full sample (2000–2021) sample period. The estimated regression equation is:$\Delta c_t^{\text{index } i \text{ and u}} = \alpha^{\text{index } i \text{ and u}} + \beta^{\text{index } i \text{ and u}} \Delta F_t^u + \epsilon_t^{\text{index } i \text{ and u}}$, where $\Delta c_t^{\text{index } i \text{ and u}}$ represents correlation of the abnormal return of asset index or portfolio *i* at the event date t as defined earlier, and $\Delta F_t^u$ is monetary policy surprise. The coefficient $\beta^{\text{index } i \text{ and u}}$shows how much the correlation between the abnormal returns of asset index or portfolio *i* and monetary surprise responds to a 100-basis point unanticipated Fed funds target rate change. The P-value is reported under the estimates. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively. The data frequency here is daily.

### *4.5.2 Response of the Correlation between Assets to Fed Surprises*

In this section, we examine how the correlations between abnormal returns of the indexes will respond to U.S. monetary policy disruptions (unexpected target rate change). To accomplish this, we first extract figures of the time-varying correlation between abnormal returns of indexes from the DCC-GARCH model in Section 4.4.1.

The subsequent step entails determining how the correlations between abnormal returns of the assets or indexes change caused by the monetary policy surprise. we use the following regression specification to assess the impact of U.S. monetary policy surprises on the correlations between abnormal returns of the indexes.

$$\Delta c_t^{\text{index } i \text{ and j}} = \alpha^{\text{index } i \text{ and j}} + \theta^{\text{index } i \text{ and j}} \Delta F_t^u + \epsilon_t^{\text{index } i \text{ and j}} \quad (4.11)$$

where $\Delta c_t^{\text{indexes } i \text{ and j}}$ represents the correlation between the abnormal return of asset $i$ and j (*e.g.,* MSCI EAFE and MSCI EM), and $\Delta F_t^u$ is unanticipated Fed funds target rate change (monetary policy surprise). The coefficient $\theta^{\text{index } i \text{ and j}}$ shows how much the correlation between the abnormal returns of assets $i$ and j responds to a 100-basis point unanticipated Fed funds target rate change.

We estimate the equation (4.11) and present the results in Tables 4.8 and 4.9. Table 4.8 provides the effect of the Fed monetary surprise on the time-varying correlation between abnormal returns of three indexes among Portfolio 2 and the monetary surprise regressed against the monetary surprise. There is no significant relationship observed between the S&P 500 and MSCI EAFE and between MSCI EAFE and MSCI EM, by the effect of U.S. monetary policy, while the S&P 500 and MSCI EM (emerging markets) move apart with a statistically significant estimate. Table 4.9 provides the effect of the Fed monetary surprise on the time-varying correlation between abnormal returns of three indexes among Portfolio 3 and the monetary surprise regressed against the monetary surprise. The S&P 500 index and oil move apart significantly due to the effect of U.S. monetary policy. Interestingly, we find that the S&P 500 index and 10-year treasury bonds move together because of the effect of the U.S. monetary policy surprise.

**Table 4.8.** Effect of the fed surprise on the correlations between abnormal returns of indexes in Portfolio 2.

| Correlations | Full sample | | | |
|---|---|---|---|---|
| | Estimate | R-Sq | Obs | P-value |
| MSCI EAFE-MSCI EM | -0.126 | 0.001 | 178 | 0.289 |
| MSCI EAFE-S&P 500 | 0.145 | 0.003 | 178 | 0.206 |
| MSCI EM-S&P 500 | -0.352 | 0.020 | 178 | 0.031 ** |

The table above shows the estimates from the regression equation for the full sample (January 2000–December 2021) sample period. The estimated regression equation is:$\Delta c_t^{\text{index } i \text{ and j}} = \alpha^{\text{index } i \text{ and j}} + \theta^{\text{index } i \text{ and j}} \Delta F_t^u + \epsilon_t^{\text{index } i \text{ and j}}$, where $\Delta c_t^{\text{index } i \text{ and j}}$ represents the correlation between the abnormal return of asset *i* and j (e.g. MSCI EAFE index and MSCI EM index), and $\Delta F_t^u$ is monetary policy surprise. The coefficient $\beta^{\text{index } i \text{ and j}}$ shows how much the correlation between the abnormal returns of assets i and j responds to a 100-basis point unanticipated Fed funds target rate change. The P-value is reported under the estimates. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively. The data frequency here is daily.

**Table 4.9.** Effect of the fed surprise on the correlations between abnormal returns of indexes in Portfolio 3.

| Correlations | Full sample | | | |
|---|---|---|---|---|
| | Estimate | R-Sq | Obs | P-value |
| Gold-Oil | 0.004 | -0.006 | 178 | 0.968 |
| Gold-S&P 500 | -0.041 | -0.005 | 178 | 0.717 |
| Gold-10-YTN | -0.081 | -0.003 | 178 | 0.466 |
| Oil-S&P 500 | -0.294 | 0.05 | 178 | 0.002 *** |
| Oil-10-YTN | 0.172 | 0.011 | 178 | 0.091 * |
| S&P 500-10-YTN | 0.213 | 0.011 | 178 | 0.070 * |

The table above shows the estimates from the regression equation for the full sample (January 2000–December 2021) sample period. The estimated regression equation is:$\Delta c_t^{\text{index } i \text{ and j}} = \alpha^{\text{index } i \text{ and j}} + \theta^{\text{index } i \text{ and j}} \Delta F_t^u + \epsilon_t^{\text{index } i \text{ and j}}$, where $\Delta c_t^{\text{index } i \text{ and j}}$ represents the correlation between the abnormal return of asset *i* and j (e.g. MSCI EAFE index and MSCI EM index), and $\Delta F_t^u$ is monetary policy surprise. The coefficient $\beta^{\text{index } i \text{ and j}}$ shows how much the correlation between the abnormal returns of assets *i* and j responds to a 100-basis point unanticipated Fed funds target rate change. The P-value is reported under the estimates. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively. The data frequency here is daily.

### 4.6. Conclusion

In this chapter, we estimate the responses of four different portfolios, including a U.S.-only, a stock-bond portfolio, an international-diversified portfolio, and an asset-diversified portfolio, to U.S. monetary policy surprises derived from the Federal funds futures market. We also estimate the responses of six variables, including MSCI EAFE, MSCI EM, S&P 500, gold, oil and 10-year treasury notes, to U.S. monetary policy surprises derived from the Federal funds futures market. First, we find a surprise rate cut (negative surprise) of 25 basis points by the Fed triggers a jump in MSCI EAFE by 0.85 per cent, the MSCI EM index by 1.1 per cent, the gold index by 4.7 per cent, and the S&P 500 index by 0.6 per cent, while the results for both oil and 10YTN are not statistically significant. This finding is consistent with Roley and Sellon (1995) that find that the bond rate rose a statistically insignificant four basis points for each percentage point increase in the target funds rate. Second, we find that a target rate cut (negative surprise) of 25 basis points by the Fed triggers an increase in Portfolio 1 by 0.85 per cent, Portfolio 2 by 0.46 per cent, and Portfolio 3 by 0.4 per cent. Our results can prove that monetary policy shocks are a systemic risk factor, which is in line with Thorbecke (1997) that suggests that monetary policy might be a systematic factor that affects ex-ante returns. Third, we find that among these four portfolios, stock-bond Portfolio 1 and assets-diversified Portfolio 3 are relatively less affected by monetary policy shocks than the U.S.-only and the international-diversified Portfolio.

We note that a fixed-coefficient approach of estimating the equity return response to U.S. monetary policy surprise is unable to capture the gradual evolution of different indexes and portfolios to U.S. monetary policy changes. Third, our chapter estimate DCC-GARCH model and capture the time-varying correlation between all of our variables (MSCI EAFE, MSCI EM, S&P 500, gold, oil, 10-year treasury notes, Portfolio 1, Portfolio 2, Portfolio 3 and the monetary surprise) and Our results show significant time-varying co-movement in the response of global equity markets (the MSCI EAFE, MSCI EM and S&P 500 indexes) to U.S. monetary policy surprises. we also find that correlations between each portfolio and U.S. monetary policy surprises exhibit a mostly negative correlation with the U.S. monetary policy surprise throughout the study period, with the exception of a few outliers. Fourth, we also find that the S&P 500 index and the developed market index exhibit a high degree of co-movement in their response to the U.S. monetary policy unanticipated change, while the S&P 500 index and the emerging market index move apart in their response to the U.S. monetary policy unanticipated change.

Overall, from the analysis and the results we presented above, we conclude the following implications. First, our findings demonstrate that U.S. monetary policy poses a risk to global asset markets. Our regular regression analysis reveals a negative relationship between the monetary surprise and all six assets and four portfolios in this chapter. Second, we preliminarily suggest that investors can use gold, oil and U.S. 10-year treasury notes to hedge risks because we find that, 1) all stock-related indexes (the MSCI EAFE, the MSCI EM, and S&P 500 indexes) respond more to U.S. monetary policy surprises than the other three indexes (gold, oil, and 10-year treasury notes), 2) when we run the simple regression to measure the effects of monetary policy on the gold and 10-year treasury notes, the effects of monetary policy on the gold and 10-year treasury notes are very small and also statistically insignificant, 3) the monetary policy surprise has a little effect on the correlation between gold and U.S. monetary surprises although the result is statistically insignificant, 4) the time-varying correlation between the 10-year treasury bond and the U.S. monetary surprise is negatively affected by the U.S. monetary surprise, 5) the time-varying correlation between the S&P 500 index and gold is negatively affected by U.S. monetary policy, although the result is not statistically significant. Third, when we run the simple regression to investigate the effects of monetary policy on the four different portfolios, all four portfolios (U.S.-only, a stock-bond portfolio, an international-diversified portfolio, and an assets-diversified portfolio) are very negatively related to the monetary policy surprise; we contend that the U.S. monetary policy may be a systemic risk that cannot be fully diversified. Fourth, as the stock-bond diversified portfolio and the assets-diversified portfolio respond less to U.S. monetary policy surprises than the other two portfolios (U.S. only and international diversified portfolio), we suggest that the stock-bond diversified portfolio and the assets-diversified portfolio are more suitable for risk-averse investors.

# **Chapter Five:** Conclusion

## 5.1. Summary

This thesis contains three complete empirical studies, presented in chapters two, three, and four, respectively, each with different objectives. We summarise them as follows:

### *5.1.1 Chapter Two*

Chapter two investigates the diversification benefits of portfolio choices of U.S. investors, given the three major crisis periods and the apparent dominance of the U.S. market. More specifically, this chapter examines which of these four investment options, including three portfolio diversification options (a stock (60%)-bond (40%) portfolio, an international diversification portfolio, and an asset-class diversified portfolio) and a U.S.-only investment strategy, is more beneficial to U.S. investors over the sample period from January 1995 to December 2021.

The data frequency of this chapter is monthly. The whole period is divided into six sub-sample periods to estimate and compare the four investment options for the full sample periods and for the six sub-sample periods. We classify these sub-sample periods into two broad categories: three crisis periods and three non-crisis periods. The crisis periods include the Dot-com bursting period from April 2000 to December 2002, the Great Recession period from December 2007 to June 2009, and the COVID-19 health crisis period from December 2019 to December 2021. The non-crisis periods include the Dot-com booming period from January 1995 to March

2000, the 2003-2007 period, and the 2009-2019 period. We consider three types of diversification opportunities for U.S. investors, which are compared against a U.S.-only position that involves the S&P 500 index as the portfolio. The first one is a stock (60%)-bond (40%) portfolio, which is composed of the S&P 500 index and U.S. 10-year Treasury note using a 60/40 weighting (Markowitz, 1952). The second portfolio is an internationally diversified stock portfolio, comprising the S&P 500 index, the EAFE index, and the EM index. The third portfolio is an asset-class diversified portfolio, which is constructed across different asset classes and is constituted of the S&P 500 index, gold, oil, and the 10-year T-note. The currency of all series is the U.S. dollar. The data used in this chapter consists of six variables, including three stock indexes (S&P 500 index, MSCI EAFE index (developed market index), and MSCI EM index (emerging market index), three assets (gold (Gold Bullion), oil (Brent Oil), and bonds (U.S. 10-year Treasury note).

Our main findings are as follows. First, since 2009, compared with the MSCI EAFE index, and the MSCI EM index, the S&P 500 index has been the best performer, with a higher average monthly real return and a higher Sharpe ratio. Second, the ARMA (0,0)-DCC-GARCH (1,1) model shows that the S&P 500 index and the other five variables have an interaction relationship. Third, by combining the time-varying correlation and fixed correlation, we reinforce the existing argument that correlations between national stock markets have been increasing in recent years (Longin and Solnik, 1995; Forbes and Rigobon, 2002; Kim et al., 2005; Morana and Beltratti 2008; Christoffersen et al., 2014) and we also support the existing argument that the correlation between stock markets in crisis periods is higher than in non-crisis periods (Roll, 1988; Bertero and Mayer, 1990; King and Wadhwani, 1990; Solnik et al., 1996; Butler and Joaquin, 2002; Guidi and Ugur, 2014). Fourth, the asset-diversified portfolio consisting of the S&P 500 index, gold, oil, and U.S. 10-year Treasury Note can offer substantial diversification benefits for U.S investors for both long-term and short-term investments, no matter whether investors choose the equally weighted or optimised asset-diversified portfolio. Fifth, the cross-asset diversified portfolio outperformed the U.S-only, the stock (60%)-bond (40%) portfolio, and the international diversified portfolio, so the asset-diversified portfolio consisting of the S&P 500 index, gold, oil, and U.S. 10-year Treasury Note is the best choice for U.S investors. Sixth, before 2009, U.S. investors could benefit from the international-diversified portfolio consisting of the S&P 500 index and MSCI EM index. However, since 2009, the international-diversified portfolio is less likely to benefit U.S. investors. There are two possible reasons behind it. The first one is that since 2009, compared to the MSCI EAFE index, and the MSCI EM index, the S&P 500 index has been the best performer. Another reason might be that the correlation between international stock markets has been increasing recently, which may eliminate the benefits of international diversification. Seventh, compared with the Dot-com bursting crisis and the Great Recession, the COVID-19 health crisis did not have an evident impact on the return of the four portfolios, although it increased the volatility of each variable.

This chapter contributes to the literature from the following aspects. First, it compares the impact of the financial crisis and the health crisis on the benefits of investors' diversification portfolios. Financial markets are characterised by uncertainty and unpredictability. Financial crises are one of the main reasons that cause substantial volatility on the financial market, leading to a change in the connection between stock markets and between assets and a change in the risk characteristics of certain assets, and it in turn affects investors' investment allocation strategies and the performance of portfolio diversification. Some literature (*e.g.*, Holton, 2009; Ilmanen and Kizer, 2012; Miccolis and Goodman, 2012; Statman, 2013; Fabozzi et al., 2014) investigates the effect of the financial crisis on portfolio diversification, but there is no work comparing how different the impact of the financial crisis on portfolio diversification is from the impact of the health crisis on that. This chapter also examines which of these four investment options, including three portfolio diversification options (a stock (60%)-bond (40%) portfolio, an international diversification portfolio, and an asset-diversified portfolio), and a U.S.-only investing option, is more beneficial to U.S. investors, and to the authors' greatest knowledge, there is no other work conducting this examination in the literature.

### *5.1.2 Chapter Three*

Chapter three optimises portfolio selection for an investment universe of developed and emerging market stock indexes using the Parametric Portfolio Policy (PPP) approach of Brandt et al. (2009) for the period from December 2004 to December 2023, and compares the results to the performances of naïve diversified portfolios (1/N-rule), market capitalisation weighted, risk parity (equally weighted risk contribution), mean-variance (MV), and Black Litterman (BL) optimised portfolios.

To empirically test the in- and out-of-sample performance of our portfolio strategies, our modelled sample set comprises seven global indexes from developed economies (i.e., USA, Japan, UK, Italy, France, Germany, and Canada, known as the G7), and five global indexes from emerging economies (i.e., Brazil, Russia, India, China, and South Africa, known as the BRICS). In total, we model 12 global indexes with each monthly price series covering the period from December 2004 to December 2023.

To estimate the impact of all six characteristics on the performance of the Parametric Portfolio Policy (PPP), we create three types of optimised portfolios: two-characteristic optimised portfolios (PPP-Two), three-characteristic optimised portfolios (PPP-Three), and all six-characteristic optimised portfolios (PPP-Six), all of which include the market capitalisation characteristic. Market capitalisation and the 12-month cumulative return optimise the two-characteristics portfolio; the market capitalisation, book-to-market ratio, and market capitalisation, following Brandt et al. (2009), optimise the three-characteristics portfolio; and all six characteristics, including the market capitalisation, return-to-equity ratio, book-to-market ratio, dividend yield, volume, and the 12-month cumulative return, optimise the six-characteristics portfolio.

The comparison between the mean-variance (MV) and Black-Litterman (BL) strategies, three benchmarks, and all three types of PPP strategies is complicated. We find the mean-variance (MV) and Black-Litterman (BL) strategies have more stable and better performance in terms of Sharpe ratio than the 1/N rule and risk parity, market capitalisation-weighted portfolios. In the in-sample simulation, the mean-variance (MV) and Black-Litterman (BL) strategies consistently beat all other strategies, no matter with or without short-selling constraints. In the out-of-sample simulation, all three types of PPP-optimised portfolios outperform the three benchmarks (equally weighted portfolio, risk parity portfolio, and market capitalisation-weighted portfolio) before controlling for short-selling. In addition, the six-characteristic optimised portfolio, without short-selling constraints, outperforms all other selected portfolio strategies before the deduction of transaction costs. Moreover, the six-characteristic-optimised (the market capitalisation, return-to-equity ratio, book-to-market ratio, dividend yield, volume, and the 12-month cumulative return) and the three-characteristic (the market capitalisation, book-to-market ratio, and market capitalisation) optimised portfolios seem to produce more stable and better performance than the two-characteristic (market capitalisation and the 12-month cumulative return) optimised portfolio. Furthermore, the market capitalisation-weighted portfolio performs better than the equally weighted portfolio and the risk parity portfolio within three benchmarks, both before and after the deduction of the transaction costs in our sample period.

### *5.1.3 Chapter Four*

Chapter four investigates the impact of changes in the U.S. monetary policy on portfolio diversification. The investing options in this chapter are consistent with Chapter two, including a stock (60%)-bond (40%) portfolio, an international diversification portfolio, an asset-class diversified portfolio) and a U.S.-only investment strategy.

Our results show that an unexpected Fed funds target rate cut (negative surprise) triggers an increase in all six variables and four portfolios, but the results for both oil and 10-year treasury notes are not statistically significant. Third, we estimate the effects of U.S. monetary surprises on the time-varying correlation between all six indexes, four portfolios, and monetary policy surprises by modelling heteroscedasticity. Our results suggest that all stock-related indexes (the MSCI EAFE index, the MSCI EM index, and S&P 500) respond more to U.S. monetary policy surprises than the other three indexes (gold, oil, and 10-year treasury notes), while the stock-bond diversified portfolio responds less to U.S. monetary policy surprises than the other three portfolios (U.S. only, international diversified portfolio, and assets-diversified portfolio). We also find that the S&P 500 index and the developed market index exhibit a high degree of co-movement in their response to the U.S. monetary policy unanticipated change, while the

S&P 500 index and the emerging market index move apart in their response to the U.S. monetary policy unanticipated change. Our results can prove that monetary policy shocks are a systemic risk factor, which is in line with Thorbecke (1997), who suggests that monetary policy might be a systematic factor that affects ex-ante returns.

Chapter four examines how different portfolios respond to surprises from FOMC announcements over the period January 2000 to December 2021. Our sample pool contains four main types of security portfolios. These portfolios consist of six different indexes. Our empirical analysis provides insight into how these four main types of portfolios are affected by surprises from FOMC announcements. To the author's best knowledge, this is also the first work that studies how different portfolios respond to monetary surprise and the most thorough analysis of how U.S. monetary policy shocks affect global asset markets. Several studies have examined how U.S. monetary policy affects global stock markets and asset prices (*e.g.,* Ehrmann and Fratzscher, 2004; Bernanke and Kuttner, 2005; Wongswan, 2006; Andersen et al., 2007; Ehrmann and Fratzscher, 2009; Wongswan, 2009; Hausman and Wongswan, 2011). However, these studies focus on a limited number of nations and a specific asset classification and there is no study examining how U.S. monetary policy affects portfolio diversification. This chapter examines the effects of changes in U.S. monetary policy on various portfolios by analysing their impact on bond prices, bullion prices, oil prices and equity markets. Compared to the existing literature, this should provide more exhaustive and reliable results.

Conclusively, this thesis provides new insights into how investors can diversify their portfolios by studying the benefits of different types of diversified portfolios, comparing the performance of various diversification strategies, and estimating the effect of U.S. monetary policy on portfolio diversification. Therefore, it provides new investment insights for investors to make their investment decisions, broadens the base of empirical research for policymakers to set regulations, and expands the empirical literature for academic researchers to explore the field of portfolio diversification.

### 5.2. Recommendations for Future Research

There are several ways to extend this thesis. First, examine how investors' psychology influences asset selection and allocation during portfolio construction. The prevailing shift in behavioural insights has a significant influence on investment decisions and portfolio construction (Barberis and Thaler, 2003). Behavioural finance demonstrates that psychological biases, such as herding behaviour, loss aversion, and home bias, profoundly influence investors' decisions, frequently resulting in suboptimal diversified portfolios. Second, new asset categories can be considered, such as crypto assets. The emergence of new asset classes, such as crypto-assets, and alternative investments necessitates ongoing research to understand their impact on portfolio diversification. Crypto assets, such as bitcoins, offer several potential benefits as innovative and efficient payment systems and portfolio diversification. Guesmi et al. (2019) find that hedging strategies incorporating gold, oil, equities, and Bitcoin significantly lower the portfolio's risk compared to a portfolio consisting solely of gold, oil, and equities. Third, studying how ESG investing criteria can be integrated into portfolio diversification. ESG investing emphasises the significance of sustainability in portfolio diversification, and integrating ESG investing criteria may offer the following advantages: 1) ESG factors help identify and mitigate risks related to climate change, social unrest, and governance failures, enhancing portfolio resilience. 2) Companies with strong ESG practices are often more sustainable and better positioned for long-term growth, contributing to stable returns. 3) ESG investing includes sectors like renewable energy and sustainable technologies, broadening diversification across emerging industries. 2) Investors can align their portfolios with personal or institutional values, promoting ethical and sustainable investment practices. 4) By integrating ESG criteria, investors can create diversified portfolios that are not only financially robust but also aligned with broader sustainability goals (Giese et al., 2019).

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# Appendix

**Table 3. 17.** Expanding window 1 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2018-12-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-01-31 |
| MV | 578.86% | 35.23% | -129.01% | -365.65% | -255.94% | 455.35% | -67.87% | -267.02% | 45.18% | -0.84% | -11.63% | 83.36% | -0.01567 |
| BL | 1364.63% | -158.64% | 183.08% | -740.30% | 246.91% | 883.84% | 8.41% | -1753.57% | -410.52% | -501.28% | -1212.79% | 2190.22% | 0.85277 |
| PPP-Two | 51.56% | 11.30% | 11.82% | 1.82% | 10.70% | 3.08% | 6.09% | 5.13% | -0.15% | -2.13% | 0.36% | 0.43% | 0.06995 |
| PPP-Three | 45.04% | 43.22% | 2.79% | -13.81% | 15.27% | -3.15% | -3.91% | 7.62% | 36.99% | -37.35% | 8.68% | -1.39% | 0.02343 |
| PPP-Six | -219.67% | 58.51% | 171.27% | 51.87% | 67.12% | 18.67% | 8.53% | 79.13% | -77.37% | -17.80% | -13.24% | -27.02% | 0.07890 |

**Table 3. 18.** Expanding window 2 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-01-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-02-28 |
| MV | 511.66% | 27.74% | -105.97% | -284.07% | -220.26% | 373.60% | -87.11% | -207.81% | 20.90% | -1.87% | -0.14% | 73.33% | -0.03156 |
| BL | 785.06% | -76.48% | 147.04% | -486.36% | 75.73% | 509.70% | 86.39% | -990.01% | -217.98% | -283.48% | -689.02% | 1239.41% | -0.50289 |
| PPP-Two | 54.87% | 12.60% | 10.52% | 1.25% | 10.68% | 2.58% | 5.83% | 4.88% | -0.92% | -1.85% | -0.21% | -0.24% | 0.04146 |
| PPP-Three | 47.86% | 48.24% | 2.67% | -15.27% | 11.38% | -2.08% | -2.26% | 4.15% | 38.77% | -34.49% | 7.58% | -6.54% | 0.07793 |
| PPP-Six | -179.30% | 99.97% | 192.38% | 37.47% | -45.88% | 31.68% | 41.00% | 37.49% | -47.75% | 20.28% | 7.10% | -94.43% | 0.17075 |

**Table 3.19.** Expanding window 3 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-02-28 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-03-29 |
| MV | 461.38% | 33.86% | -95.95% | -260.05% | -200.72% | 320.79% | -47.58% | -175.58% | 18.00% | -3.11% | -12.19% | 61.16% | 0.00865 |
| BL | 383.64% | 6.17% | -1.75% | -167.65% | -263.65% | 277.27% | -65.40% | -140.12% | -17.05% | -20.43% | -20.48% | 129.45% | -0.07292 |
| PPP-Two | 52.45% | 12.13% | 9.24% | 1.27% | 10.08% | 2.78% | 5.71% | 5.15% | -0.12% | -0.16% | 0.68% | 0.79% | 0.01463 |
| PPP-Three | 48.92% | 41.47% | 2.04% | -18.35% | 12.28% | -1.57% | -3.86% | 6.23% | 39.52% | -31.74% | 10.39% | -5.33% | 0.06218 |
| PPP-Six | -155.35% | 41.00% | 204.18% | 40.22% | -45.10% | 45.38% | 47.96% | 55.05% | -8.60% | -63.64% | 23.89% | -84.99% | -0.00122 |

**Table 3.20.** Expanding window 4 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-03-29 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-04-30 |
| MV | 448.55% | 35.25% | -93.45% | -242.86% | -185.35% | 279.23% | -35.21% | -172.73% | 29.56% | 1.52% | -17.41% | 52.90% | 0.14929 |
| BL | 355.10% | 11.89% | -11.06% | -144.97% | -280.62% | 255.68% | -73.67% | -86.56% | -1.79% | -2.69% | 21.46% | 57.24% | 0.15688 |
| PPP-Two | 55.97% | 10.47% | 9.19% | 0.20% | 9.69% | 2.01% | 4.56% | 5.12% | 0.21% | 0.47% | 0.57% | 1.55% | 0.03194 |
| PPP-Three | 48.05% | 37.86% | -2.43% | -14.69% | 12.03% | -1.83% | -3.93% | 6.66% | 40.08% | -29.65% | 11.72% | -3.87% | 0.00739 |
| PPP-Six | -147.63% | 41.48% | 143.82% | 72.45% | -26.47% | 28.02% | 60.69% | 38.25% | -23.87% | -30.88% | 24.39% | -80.25% | 0.06418 |

**Table 3.21.** Expanding window 5 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-04-30 | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-05-31 |
| MV | 439.73% | 32.98% | -89.45% | -239.60% | -186.16% | 282.67% | -35.92% | -167.66% | 25.50% | 1.55% | -13.59% | 49.96% | 0.05134 |
| BL | 334.73% | 12.04% | -8.59% | -134.54% | -257.16% | 233.12% | -67.48% | -80.99% | -0.73% | -2.64% | 18.37% | 53.85% | 0.02601 |
| PPP-Two | 62.18% | 9.50% | 9.98% | -3.05% | 9.10% | 0.75% | 2.21% | 5.66% | 0.25% | 0.33% | 0.82% | 2.26% | -0.06292 |
| PPP-Three | 45.34% | 31.90% | -4.03% | -13.05% | 10.72% | -2.52% | -2.32% | 5.65% | 49.93% | -27.96% | 9.63% | -3.28% | -0.05590 |
| PPP-Six | -154.63% | 25.15% | 138.46% | 79.32% | -37.96% | 19.82% | 55.37% | 26.01% | 12.82% | 4.11% | 12.86% | -81.33% | -0.11995 |

**Table 3.22.** Expanding window 6 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-05-31 | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-06-28 |
| MV | 492.74% | 36.31% | -105.41% | -296.38% | -227.37% | 346.72% | -35.16% | -193.04% | 39.35% | 8.91% | -29.03% | 62.37% | 0.09226 |
| BL | 394.24% | 13.11% | -16.80% | -161.23% | -319.21% | 287.55% | -84.44% | -98.22% | -1.52% | -3.43% | 25.07% | 64.87% | 0.13671 |
| PPP-Two | 56.28% | 8.22% | 10.71% | -1.49% | 9.31% | 1.77% | 3.19% | 5.68% | 1.37% | 1.41% | 1.04% | 2.50% | 0.05715 |
| PPP-Three | 42.51% | 43.40% | -3.84% | -15.62% | 7.93% | -4.68% | -4.66% | 5.28% | 47.14% | -33.72% | 14.99% | 1.27% | 0.01617 |
| PPP-Six | -160.46% | 70.08% | 135.14% | 80.63% | -43.79% | 16.46% | 57.14% | 31.57% | -0.43% | -58.06% | 33.85% | -62.13% | 0.04117 |

**Table 3.23.** Expanding window 7 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-06-28 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-07-31 |
| MV | 479.49% | 33.35% | -101.20% | -267.46% | -230.21% | 316.14% | -27.38% | -179.86% | 28.16% | 8.27% | -16.60% | 57.29% | 0.02419 |
| BL | 346.62% | 12.32% | -10.50% | -139.03% | -269.06% | 243.31% | -71.07% | -84.41% | -1.22% | -3.05% | 20.27% | 55.81% | 0.04539 |
| PPP-Two | 53.60% | 9.68% | 9.94% | -0.37% | 9.88% | 1.99% | 3.95% | 5.39% | 0.85% | 1.29% | 1.27% | 2.53% | 0.00196 |
| PPP-Three | 38.06% | 44.80% | -3.18% | -15.98% | 9.36% | -4.43% | -5.40% | 8.43% | 49.83% | -32.07% | 12.55% | -1.96% | -0.02426 |
| PPP-Six | -174.69% | 62.87% | 124.41% | 88.91% | -47.13% | 7.34% | 16.77% | 62.29% | 2.69% | -36.81% | 33.21% | -39.86% | -0.05707 |

**Table 3.24.** Expanding window 8 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-07-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-08-30 |
| MV | 477.64% | 31.16% | -89.66% | -246.98% | -224.96% | 288.45% | -32.25% | -174.10% | 23.45% | 8.31% | -21.48% | 60.42% | -0.05276 |
| BL | 345.85% | 12.46% | -9.98% | -141.06% | -267.81% | 245.38% | -71.95% | -84.83% | -0.13% | -3.11% | 19.09% | 56.09% | -0.03163 |
| PPP-Two | 53.63% | 10.88% | 10.54% | -1.26% | 9.89% | 1.54% | 3.21% | 5.61% | 1.08% | 1.27% | 1.07% | 2.52% | -0.02930 |
| PPP-Three | 39.81% | 29.46% | -1.27% | -10.28% | 9.73% | -2.79% | -1.20% | 5.44% | 52.10% | -33.92% | 14.68% | -1.77% | -0.04296 |
| PPP-Six | -147.73% | 8.11% | 123.23% | 100.86% | -39.09% | 14.18% | 31.14% | 51.67% | 10.91% | -42.39% | 36.87% | -47.78% | 0.02049 |

**Table 3.25.** Expanding window 9 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-08-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-09-30 |
| MV | 497.10% | 32.74% | -85.64% | -256.40% | -282.44% | 295.96% | -6.92% | -151.42% | 27.88% | 9.69% | -39.06% | 58.50% | -0.01735 |
| BL | 366.04% | 12.81% | -11.51% | -150.33% | -293.35% | 263.50% | -75.23% | -87.79% | 0.06% | -3.23% | 19.50% | 59.53% | -0.02953 |
| PPP-Two | 51.59% | 9.95% | 10.27% | -0.71% | 9.89% | 2.15% | 3.95% | 5.49% | 1.57% | 1.01% | 1.69% | 3.14% | 0.02106 |
| PPP-Three | 44.19% | 46.06% | -7.25% | -14.09% | 13.56% | -2.18% | -3.66% | 10.97% | 26.94% | -36.21% | 16.10% | 5.58% | 0.01197 |
| PPP-Six | -144.95% | 36.52% | 114.76% | 106.55% | -36.17% | 15.34% | 24.50% | 68.54% | 16.01% | -51.72% | -9.83% | -39.55% | 0.03678 |

**Table 3.26.** Expanding window 10 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-09-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-10-31 |
| MV | 452.36% | 29.65% | -71.05% | -243.30% | -238.80% | 274.72% | -4.83% | -143.38% | 29.41% | 9.21% | -51.19% | 57.20% | 0.06309 |
| BL | 352.13% | 12.43% | -9.22% | -145.34% | -277.96% | 252.92% | -71.89% | -84.75% | 0.60% | -3.07% | 16.46% | 57.69% | 0.07893 |
| PPP-Two | 50.63% | 11.42% | 10.08% | 0.54% | 9.89% | 2.45% | 4.69% | 5.16% | 0.76% | 0.77% | 0.96% | 2.66% | 0.02676 |
| PPP-Three | 42.14% | 47.47% | 0.27% | -17.76% | 15.90% | -1.13% | -4.55% | 9.58% | 25.70% | -32.18% | 18.92% | -4.36% | 0.00514 |
| PPP-Six | -168.45% | 34.53% | 136.96% | 112.60% | -26.97% | 5.77% | 24.20% | 54.06% | 10.60% | -58.83% | 12.73% | -37.20% | 0.05162 |

**Table 3.27.** Expanding window 11 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-10-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-11-29 |
| MV | 437.69% | 28.39% | -60.06% | -232.17% | -221.68% | 274.37% | -19.36% | -153.16% | 26.50% | 11.23% | -50.40% | 58.65% | 0.06024 |
| BL | 338.36% | 12.24% | -7.98% | -138.94% | -263.70% | 239.04% | -66.92% | -79.73% | 0.75% | -3.13% | 15.27% | 54.75% | 0.04545 |
| PPP-Two | 57.84% | 10.08% | 11.79% | -4.07% | 9.48% | 1.62% | 1.53% | 6.36% | 1.12% | 0.83% | 1.51% | 1.91% | 0.02013 |
| PPP-Three | 50.01% | 42.31% | -9.22% | -19.77% | 16.37% | -1.61% | -5.15% | 9.78% | 31.42% | -34.54% | 23.43% | -3.03% | 0.01091 |
| PPP-Six | -115.97% | 32.66% | 103.61% | 87.57% | -28.66% | 0.04% | 8.00% | 41.37% | 30.64% | -29.09% | 14.02% | -44.19% | -0.00508 |

**Table 3.28.** Expanding window 12 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-11-29 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-12-31 |
| MV | 433.84% | 26.12% | -62.86% | -222.30% | -217.29% | 262.39% | -19.85% | -143.30% | 25.53% | 11.48% | -46.13% | 52.36% | -0.00004 |
| BL | 324.88% | 12.03% | -6.23% | -132.54% | -249.40% | 226.40% | -63.19% | -75.94% | 0.76% | -3.03% | 14.08% | 52.18% | 0.00711 |
| PPP-Two | 57.10% | 8.57% | 9.18% | -3.79% | 9.65% | 1.88% | 1.78% | 6.94% | 1.78% | 1.82% | 2.99% | 2.10% | 0.03697 |
| PPP-Three | 43.84% | 36.08% | -8.71% | -15.35% | 12.72% | -2.17% | -3.91% | 6.77% | 50.23% | -31.16% | 13.54% | -1.87% | 0.03302 |
| PPP-Six | -98.47% | 62.19% | 89.82% | 76.16% | -20.64% | 7.25% | 10.25% | 36.20% | -26.90% | -14.13% | 9.58% | -31.30% | 0.01292 |

**Table 3.29.** Expanding window 13 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-12-31 | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-01-31 |
| MV | 415.89% | 26.17% | -61.31% | -210.88% | -183.47% | 236.04% | -19.20% | -151.74% | 20.66% | 11.93% | -37.90% | 53.81% | 0.04339 |
| BL | 311.95% | 11.94% | -5.03% | -126.28% | -233.61% | 213.14% | -59.65% | -73.74% | 0.46% | -2.79% | 13.56% | 50.04% | 0.02470 |
| PPP-Two | 57.46% | 10.58% | 8.89% | -0.41% | 9.90% | 1.87% | 4.07% | 5.28% | -0.20% | 0.41% | 0.26% | 1.88% | -0.01216 |
| PPP-Three | 46.61% | 40.26% | -2.69% | -11.79% | 13.16% | -10.75% | -3.63% | 2.66% | 43.10% | -27.57% | 12.12% | -1.47% | -0.01635 |
| PPP-Six | -103.76% | 75.98% | 100.29% | 76.38% | -20.94% | -15.95% | -23.05% | 35.95% | -20.88% | -4.63% | 22.69% | -22.08% | -0.01927 |

**Table 3.30.** Expanding window 14 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-01-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-02-28 |
| MV | 424.20% | 26.41% | -59.42% | -201.92% | -195.97% | 236.29% | -30.66% | -140.65% | 22.84% | 14.85% | -47.12% | 51.17% | -0.06510 |
| BL | 317.93% | 12.04% | -5.83% | -129.19% | -239.83% | 218.72% | -61.24% | -75.56% | 0.43% | -2.85% | 14.15% | 51.22% | -0.04960 |
| PPP-Two | 57.60% | 10.39% | 8.23% | -0.82% | 10.04% | 2.13% | 3.83% | 5.99% | -0.55% | 0.87% | 0.60% | 1.70% | -0.09027 |
| PPP-Three | 49.38% | 31.87% | -2.96% | -13.63% | 7.02% | -10.19% | -4.43% | 5.75% | 50.86% | -27.14% | 10.75% | 2.71% | -0.06267 |
| PPP-Six | -103.50% | 62.93% | 106.63% | 68.69% | -40.30% | -17.71% | -17.74% | 42.95% | 4.22% | -4.03% | 18.67% | -20.81% | -0.01795 |

**Table 3.31**. Expanding window 15 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-02-28 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-03-31 |
| MV | 526.66% | 35.89% | -80.84% | -236.14% | -315.06% | 309.24% | -38.71% | -149.72% | 33.02% | 16.92% | -62.97% | 61.72% | 0.12253 |
| BL | 391.29% | 13.47% | -14.88% | -163.17% | -319.19% | 286.75% | -81.25% | -95.51% | 0.43% | -3.59% | 20.35% | 65.30% | -0.03139 |
| PPP-Two | 54.63% | 10.49% | 8.24% | -0.31% | 9.52% | 2.36% | 4.00% | 6.11% | 0.34% | 1.37% | 1.02% | 2.23% | -0.14983 |
| PPP-Three | 58.22% | 42.60% | 0.71% | -18.47% | 13.50% | -5.72% | -5.23% | 11.14% | 17.68% | -36.54% | 21.68% | 0.41% | -0.10568 |
| PPP-Six | -56.81% | 80.24% | 115.98% | 64.04% | -15.60% | -10.10% | -15.43% | 50.03% | 6.87% | -59.50% | -26.34% | -33.38% | 0.05824 |

**Table 3.32.** Expanding window 16 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-03-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-04-30 |
| MV | 594.23% | 47.80% | -89.52% | -282.81% | -360.87% | 358.67% | -31.20% | -185.36% | 32.29% | 27.19% | -68.15% | 57.73% | 0.49332 |
| BL | 641.98% | 16.92% | -48.59% | -282.11% | -581.73% | 521.45% | -152.11% | -168.50% | 0.20% | -6.14% | 43.14% | 115.48% | 0.61104 |
| PPP-Two | 53.70% | 10.28% | 8.39% | 0.09% | 9.91% | 2.77% | 4.40% | 5.80% | 0.30% | 1.14% | 1.11% | 2.11% | 0.09441 |
| PPP-Three | 51.32% | 29.59% | 4.26% | -21.32% | 8.95% | -13.81% | -8.78% | 12.45% | 49.68% | -28.74% | 12.93% | 3.45% | 0.12547 |
| PPP-Six | -71.15% | 54.32% | 125.78% | 48.19% | -13.90% | -35.45% | -27.79% | 61.46% | 42.24% | -34.46% | -21.31% | -27.93% | 0.06037 |

**Table 3.33.** Expanding window 17 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-04-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-05-29 |
| MV | 577.88% | 45.05% | -86.06% | -268.68% | -347.35% | 345.17% | -38.96% | -174.70% | 31.71% | 26.49% | -64.19% | 53.63% | 0.26203 |
| BL | 452.67% | 17.64% | -17.55% | -209.41% | -392.05% | 350.07% | -73.22% | -127.69% | -2.30% | -6.55% | 21.82% | 86.57% | 0.34881 |
| PPP-Two | 58.00% | 10.50% | 8.14% | -0.58% | 10.22% | 1.75% | 3.83% | 5.59% | 0.01% | 0.30% | 0.34% | 1.89% | 0.03785 |
| PPP-Three | 53.59% | 32.87% | 4.61% | -23.97% | 6.85% | -12.37% | -12.22% | 12.83% | 55.09% | -31.03% | 13.21% | 0.55% | -0.04075 |
| PPP-Six | -72.56% | 53.41% | 133.39% | 39.75% | -19.78% | -28.65% | -39.12% | 60.97% | 55.38% | -21.76% | -21.22% | -39.80% | -0.03397 |

**Table 3.34.** Expanding window 18 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-05-29 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-06-30 |
| MV | 570.78% | 42.39% | -75.07% | -260.42% | -355.93% | 349.73% | -45.46% | -170.76% | 22.89% | 26.74% | -60.62% | 55.75% | 0.01948 |
| BL | 405.17% | 18.09% | -18.35% | -188.72% | -331.48% | 297.71% | -57.16% | -113.45% | 4.33% | -6.64% | 16.20% | 74.30% | 0.08800 |
| PPP-TWO | 56.35% | 10.80% | 8.28% | -0.19% | 9.91% | 1.66% | 3.85% | 5.56% | 0.84% | 0.26% | 0.31% | 2.38% | 0.02789 |
| PPP-THREE | 51.70% | 29.15% | 4.12% | -23.15% | 9.56% | -13.48% | -15.48% | 12.91% | 59.47% | -28.76% | 12.50% | 1.46% | 0.05359 |
| PPP-SIX | -76.48% | 49.10% | 134.08% | 40.29% | -11.49% | -25.33% | -40.89% | 65.47% | 77.75% | -61.89% | -16.35% | -34.26% | 0.06674 |

**Table 3.35.** Expanding window 19 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-06-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-07-31 |
| MV | 587.68% | 45.24% | -85.15% | -270.75% | -397.67% | 380.27% | -40.24% | -172.14% | 24.45% | 19.61% | -49.30% | 58.02% | 0.39346 |
| BL | 386.51% | 17.55% | -15.55% | -178.79% | -310.75% | 279.54% | -53.56% | -107.51% | 4.12% | -5.93% | 14.14% | 70.23% | 0.29744 |
| PPP-Two | 57.70% | 10.89% | 7.90% | 0.04% | 10.19% | 1.47% | 3.77% | 5.35% | 0.10% | -0.01% | 0.23% | 2.36% | 0.05171 |
| PPP-Three | 50.04% | 29.31% | -6.15% | -12.82% | 7.35% | -17.11% | -7.62% | 12.85% | 50.58% | -32.19% | 19.47% | 6.29% | 0.10910 |
| PPP-Six | -73.46% | 40.50% | 113.95% | 71.33% | -52.54% | -29.57% | -25.56% | 50.93% | 59.40% | -59.33% | 10.83% | -6.47% | 0.06256 |

**Table 3.36.** Expanding window 20 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-07-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-08-31 |
| MV | 594.14% | 46.01% | -91.25% | -265.43% | -399.89% | 382.36% | -48.93% | -170.83% | 24.46% | 18.02% | -48.34% | 59.68% | 0.23915 |
| BL | 345.34% | 15.71% | -4.13% | -166.05% | -273.43% | 245.71% | -37.92% | -97.43% | 3.60% | -4.07% | 11.12% | 61.55% | 0.12998 |
| PPP-Two | 65.19% | 11.65% | 7.93% | -1.32% | 10.18% | 0.25% | 2.64% | 5.05% | -0.55% | -1.22% | -1.14% | 1.34% | 0.05890 |
| PPP-Three | 49.69% | 27.00% | -6.98% | -11.60% | 7.67% | -10.69% | -9.94% | 7.23% | 61.36% | -32.12% | 15.13% | 3.26% | 0.05145 |
| PPP-Six | -65.69% | 41.99% | 103.85% | 65.34% | -42.13% | -7.83% | -36.54% | 34.40% | 100.94% | -88.66% | 2.20% | -7.87% | 0.08887 |

**Table 3.37.** Expanding window 21 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-08-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-09-30 |
| MV | 545.47% | 42.63% | -76.07% | -241.03% | -370.00% | 345.34% | -41.29% | -146.28% | 23.66% | 16.99% | -47.91% | 48.50% | -0.01910 |
| BL | 313.59% | 14.96% | -0.74% | -148.66% | -237.92% | 215.25% | -32.16% | -88.33% | 3.00% | -3.92% | 9.14% | 55.78% | -0.00089 |
| PPP-Two | 75.36% | 11.68% | 8.77% | -3.06% | 10.13% | -1.10% | 1.62% | 3.87% | -1.95% | -3.02% | -2.46% | 0.15% | -0.03816 |
| PPP-Three | 56.61% | 16.47% | -16.59% | -17.62% | 5.71% | -9.26% | -7.14% | 7.75% | 63.12% | -28.43% | 20.02% | 9.36% | -0.02314 |
| PPP-Six | -63.03% | 20.21% | 73.01% | 46.35% | -47.19% | 3.39% | -16.68% | 42.33% | 73.41% | -52.60% | 2.42% | 18.37% | 0.01337 |

**Table 3.38.** Expanding window 22 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-09-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-10-30 |
| MV | 574.80% | 44.39% | -70.97% | -256.20% | -417.05% | 381.36% | -56.34% | -153.78% | 27.15% | 16.78% | -37.05% | 46.89% | -0.11941 |
| BL | 335.79% | 15.45% | -2.97% | -161.05% | -261.80% | 236.44% | -36.61% | -95.96% | 3.25% | -4.23% | 11.20% | 60.48% | -0.07199 |
| PPP-Two | 72.03% | 11.90% | 7.53% | -0.97% | 9.61% | -0.65% | 2.54% | 3.49% | -0.79% | -3.25% | -2.35% | 0.91% | -0.02078 |
| PPP-Three | 54.64% | 25.08% | -16.77% | -12.99% | 5.79% | -9.65% | -5.42% | 7.40% | 60.24% | -32.62% | 23.54% | 0.76% | 0.04745 |
| PPP-Six | -74.12% | 45.86% | 52.21% | 85.37% | -52.70% | -12.43% | -25.83% | 27.18% | 73.22% | -48.93% | 11.59% | 18.59% | 0.07515 |

**Table 3.39.** Expanding window 23 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-10-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-11-30 |
| MV | 545.92% | 46.37% | -45.16% | -243.57% | -414.61% | 301.61% | -7.47% | -144.36% | 36.26% | 12.84% | -32.23% | 44.41% | -0.24001 |
| BL | 358.10% | 16.58% | -2.61% | -173.64% | -288.95% | 248.77% | -33.84% | -103.50% | 5.11% | -4.94% | 13.53% | 65.40% | -0.08816 |
| PPP-Two | 80.78% | 10.95% | 10.21% | -4.79% | 10.43% | -1.31% | 0.56% | 4.12% | -3.73% | -2.94% | -2.85% | -1.42% | 0.09592 |
| PPP-Three | 71.00% | 20.31% | -4.45% | -24.38% | 7.38% | -11.35% | -11.28% | 9.62% | 54.74% | -30.68% | 19.08% | -0.01% | 0.04030 |
| PPP-Six | -9.54% | 32.17% | 66.40% | 31.86% | -42.64% | -20.35% | -41.50% | 26.23% | 63.37% | -17.55% | -1.93% | 13.47% | 0.09850 |

**Table 3.40.** Expanding window 24 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-11-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-12-31 |
| MV | 406.39% | 35.69% | -8.06% | -191.51% | -315.48% | 202.80% | 46.21% | -111.03% | 24.33% | 7.95% | -37.03% | 39.75% | 0.06739 |
| BL | 296.19% | 14.71% | 7.04% | -143.31% | -232.87% | 194.68% | -13.56% | -84.34% | 3.42% | -4.51% | 7.53% | 55.01% | 0.10538 |
| PPP-Two | 86.88% | 10.38% | 10.70% | -6.02% | 11.23% | -2.15% | -0.18% | 4.02% | -5.58% | -4.09% | -2.67% | -2.52% | 0.02960 |
| PPP-Three | 73.22% | 16.13% | -7.48% | -24.08% | 4.05% | -9.31% | -11.42% | 7.36% | 57.64% | -24.98% | 19.16% | -0.30% | 0.06443 |
| PPP-Six | -11.94% | 22.22% | 56.06% | 33.20% | -58.53% | -11.39% | -38.76% | 22.07% | 66.28% | 5.39% | -2.24% | 17.65% | 0.08462 |

**Table 3.41.** Expanding window 25 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-12-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-01-29 |
| MV | 397.83% | 33.29% | -7.35% | -184.47% | -291.58% | 204.12% | 25.79% | -119.30% | 24.34% | 8.01% | -32.06% | 41.38% | -0.06583 |
| BL | 282.14% | 14.62% | 8.45% | -135.52% | -219.27% | 180.64% | -9.34% | -77.77% | 3.16% | -4.37% | 5.91% | 51.35% | -0.05895 |
| PPP-Two | 79.49% | 10.39% | 8.57% | -2.49% | 9.81% | -1.13% | 1.40% | 3.80% | -3.35% | -2.63% | -2.80% | -1.07% | -0.00523 |
| PPP-Three | 69.92% | 20.31% | -6.25% | -11.94% | 3.56% | -6.98% | -5.52% | 7.94% | 49.30% | -25.00% | 11.53% | -6.85% | 0.00388 |
| PPP-Six | -15.84% | 31.32% | 74.05% | 50.72% | -54.83% | 3.52% | -13.35% | 17.15% | 36.29% | 6.16% | -27.82% | -7.37% | -0.02363 |

**Table 3.42.** Expanding window 26 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-01-29 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-02-26 |
| MV | 392.90% | 34.25% | -8.55% | -179.52% | -281.54% | 199.21% | 12.58% | -115.67% | 23.30% | 9.16% | -18.81% | 32.68% | -0.10257 |
| BL | 286.69% | 14.94% | 7.73% | -137.77% | -223.54% | 184.69% | -12.23% | -79.01% | 3.04% | -4.24% | 8.67% | 51.02% | -0.09402 |
| PPP-Two | 77.72% | 10.89% | 6.68% | -1.60% | 9.84% | -0.90% | 1.95% | 4.45% | -3.37% | -2.16% | -2.46% | -1.06% | 0.02376 |
| PPP-Three | 72.49% | 14.21% | -3.34% | -8.95% | 4.37% | -6.93% | -4.59% | 13.57% | 48.10% | -26.01% | 7.01% | -9.94% | 0.03990 |
| PPP-Six | -9.59% | 23.46% | 61.34% | 85.22% | -55.45% | -6.36% | -25.91% | 28.21% | 42.70% | 7.64% | -32.76% | -18.50% | 0.05734 |

**Table 3.43.** Expanding window 27 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-02-26 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-03-31 |
| MV | 372.20% | 32.82% | -6.57% | -168.41% | -284.16% | 173.67% | 28.02% | -95.57% | 25.42% | 10.04% | -6.50% | 19.05% | 0.07564 |
| BL | 276.27% | 14.58% | 8.48% | -131.76% | -218.59% | 172.93% | -7.32% | -71.86% | 3.73% | -3.79% | 11.42% | 45.91% | 0.07541 |
| PPP-Two | 83.05% | 10.01% | 6.43% | -3.27% | 9.58% | -1.43% | 0.99% | 3.83% | -3.15% | -2.54% | -2.40% | -1.10% | 0.02859 |
| PPP-Three | 79.88% | 21.19% | -12.39% | -16.87% | 5.44% | -9.96% | -10.00% | 14.20% | 46.11% | -26.70% | 14.06% | -4.95% | 0.01957 |
| PPP-Six | -3.37% | 44.24% | 36.05% | 69.75% | -53.90% | -12.70% | -37.33% | 29.03% | 74.60% | -14.60% | -22.99% | -8.79% | -0.01605 |

**Table 3.44.** Expanding window 28 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-03-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-04-30 |
| MV | 390.93% | 30.91% | -21.49% | -167.23% | -291.30% | 188.62% | 15.38% | -95.26% | 24.15% | 9.73% | -3.37% | 18.93% | 0.11659 |
| BL | 266.42% | 14.62% | 10.64% | -127.17% | -208.62% | 164.06% | -5.31% | -69.22% | 3.80% | -3.62% | 10.30% | 44.11% | 0.10143 |
| PPP-Two | 87.53% | 10.82% | 7.20% | -5.28% | 10.64% | -1.66% | -0.24% | 3.62% | -4.41% | -3.59% | -2.45% | -2.18% | 0.05000 |
| PPP-Three | 83.28% | 20.64% | -12.59% | -16.17% | 7.57% | -6.58% | -5.84% | 12.03% | 40.64% | -31.67% | 16.95% | -8.26% | 0.03435 |
| PPP-Six | 17.07% | 42.23% | 35.76% | 73.20% | -52.22% | -6.00% | -24.04% | 21.05% | 62.89% | -28.04% | -21.36% | -20.54% | -0.04815 |

**Table 3.45**. Expanding window 29 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-04-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-05-31 |
| MV | 381.52% | 29.65% | -25.11% | -171.67% | -275.92% | 176.37% | 34.90% | -92.09% | 20.29% | 7.07% | -6.21% | 21.20% | -0.08438 |
| BL | 254.34% | 14.38% | 12.91% | -118.37% | -195.69% | 154.28% | -8.34% | -65.31% | 4.18% | -3.07% | 9.63% | 41.06% | -0.04816 |
| PPP-Two | 99.76% | 13.60% | 10.07% | -9.96% | 10.76% | -3.40% | -2.80% | 3.05% | -6.52% | -6.65% | -3.11% | -4.78% | -0.00564 |
| PPP-Three | 94.59% | 12.48% | -9.85% | -19.97% | 9.83% | -8.79% | -8.46% | 10.43% | 44.21% | -30.90% | 15.75% | -9.32% | 0.01971 |
| PPP-Six | 37.22% | 16.23% | 48.96% | 74.94% | -50.20% | -13.22% | -26.97% | 16.93% | 72.08% | -20.35% | -25.31% | -30.32% | 0.03111 |

**Table 3.46.** Expanding window 30 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-05-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-06-30 |
| MV | 379.03% | 32.25% | -31.80% | -175.25% | -286.33% | 178.20% | 42.73% | -88.13% | 24.62% | 7.60% | -3.15% | 20.22% | 0.18110 |
| BL | 249.49% | 14.65% | 12.38% | -116.59% | -192.65% | 150.97% | -6.86% | -63.24% | 4.79% | -3.02% | 9.85% | 40.22% | 0.12079 |
| PPP-Two | 96.56% | 10.52% | 8.69% | -7.42% | 10.74% | -2.95% | -1.35% | 3.02% | -5.53% | -5.31% | -2.81% | -4.16% | 0.01680 |
| PPP-Three | 88.77% | 11.19% | -12.19% | -21.00% | 7.78% | -4.48% | -6.18% | 10.16% | 43.73% | -30.58% | 17.94% | -5.13% | -0.00573 |
| PPP-Six | 28.72% | 12.36% | 46.88% | 68.18% | -56.34% | -2.97% | -23.82% | 15.67% | 73.71% | -19.40% | -24.46% | -18.53% | -0.01871 |

**Table 3.47.** Expanding window 31 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-06-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-07-30 |
| MV | 384.88% | 31.78% | -31.73% | -174.33% | -285.64% | 172.56% | 45.13% | -90.23% | 24.44% | 9.72% | -9.92% | 23.33% | 0.03442 |
| BL | 242.43% | 14.66% | 13.05% | -114.52% | -187.90% | 149.81% | -7.91% | -60.92% | 4.66% | -4.03% | 12.56% | 38.12% | -0.00668 |
| PPP-Two | 97.89% | 9.13% | 8.22% | -7.85% | 11.05% | -2.63% | -1.21% | 3.40% | -5.75% | -5.57% | -2.57% | -4.11% | 0.01632 |
| PPP-Three | 88.12% | 11.31% | -8.45% | -19.38% | 8.60% | -5.97% | -10.01% | 9.53% | 44.74% | -28.70% | 18.66% | -8.45% | 0.02966 |
| PPP-Six | 34.39% | 12.57% | 43.97% | 66.31% | -51.36% | -20.82% | -28.90% | 14.96% | 73.26% | -10.43% | -26.30% | -7.64% | -0.01539 |

**Table 3.48.** Expanding window 32 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-07-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-08-31 |
| MV | 403.96% | 30.13% | -47.33% | -174.50% | -290.81% | 171.56% | 46.63% | -93.17% | 25.28% | 9.67% | -0.37% | 18.95% | 0.12512 |
| BL | 238.33% | 14.54% | 13.39% | -112.04% | -182.96% | 145.90% | -7.53% | -59.55% | 4.58% | -3.96% | 12.07% | 37.24% | 0.06276 |
| PPP-Two | 95.56% | 8.26% | 8.08% | -6.52% | 10.79% | -2.02% | -0.51% | 3.50% | -5.34% | -5.05% | -2.94% | -3.81% | 0.02536 |
| PPP-Three | 85.02% | 17.27% | -7.03% | -24.56% | 6.90% | -13.06% | -12.86% | 12.14% | 47.00% | -26.83% | 19.69% | -3.67% | 0.06157 |
| PPP-Six | 23.73% | 24.77% | 46.04% | 57.11% | -54.64% | -32.34% | -36.69% | 23.00% | 79.46% | -12.17% | -23.95% | 5.67% | 0.11917 |

**Table 3.49.** Expanding window 33 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-08-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-09-30 |
| MV | 396.72% | 30.21% | -42.81% | -166.60% | -284.00% | 161.72% | 43.21% | -92.41% | 29.66% | 12.07% | -4.61% | 16.84% | -0.16798 |
| BL | 235.82% | 14.18% | 12.29% | -110.07% | -178.45% | 142.56% | -6.89% | -58.34% | 4.03% | -4.22% | 12.83% | 36.24% | -0.14612 |
| PPP-Two | 93.35% | 8.41% | 8.37% | -6.26% | 10.50% | -1.84% | -0.57% | 3.67% | -4.73% | -4.61% | -2.88% | -3.41% | -0.03842 |
| PPP-Three | 84.13% | 21.90% | 1.66% | -21.21% | 10.10% | -15.08% | -11.39% | 11.49% | 48.95% | -35.86% | 16.11% | -10.80% | -0.03896 |
| PPP-Six | 21.48% | 36.79% | 65.68% | 69.67% | -50.52% | -36.09% | -33.25% | 18.05% | 69.95% | -26.17% | -16.35% | -19.23% | 0.06713 |

**Table 3.50.** Expanding window 34 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-09-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-10-29 |
| MV | 371.10% | 32.08% | -23.98% | -162.71% | -276.72% | 146.94% | 47.74% | -83.67% | 34.98% | 19.61% | -16.56% | 11.20% | 0.07314 |
| BL | 247.68% | 14.53% | 11.20% | -117.09% | -192.51% | 153.63% | -7.98% | -62.27% | 4.26% | -4.43% | 14.28% | 38.70% | -0.00549 |
| PPP-Two | 91.79% | 8.68% | 9.27% | -6.08% | 10.73% | -1.80% | -0.21% | 3.63% | -4.85% | -4.71% | -2.93% | -3.52% | 0.06202 |
| PPP-Three | 79.84% | 14.93% | -0.97% | -23.07% | 9.47% | -13.82% | -11.57% | 8.57% | 50.43% | -29.94% | 16.14% | 0.00% | 0.03684 |
| PPP-Six | 9.58% | 15.72% | 71.64% | 53.81% | -47.22% | -24.71% | -27.81% | 16.40% | 63.50% | -33.04% | -17.96% | 20.09% | -0.06868 |

**Table 3.51.** Expanding window 35 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-10-29 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-11-30 |
| MV | 365.97% | 31.85% | -29.86% | -152.76% | -270.12% | 132.03% | 47.12% | -70.00% | 35.46% | 23.73% | -14.43% | 0.98% | 0.12446 |
| BL | 230.21% | 15.02% | 16.87% | -109.04% | -179.21% | 137.82% | -5.34% | -54.07% | 6.23% | -0.77% | 9.37% | 32.91% | 0.08571 |
| PPP-Two | 91.74% | 7.27% | 9.82% | -6.09% | 11.06% | -1.12% | 0.31% | 3.53% | -4.28% | -5.04% | -2.79% | -4.41% | -0.00557 |
| PPP-Three | 83.06% | 21.38% | 0.08% | -27.87% | 9.93% | -13.34% | -15.18% | 10.80% | 40.01% | -28.49% | 20.35% | -0.74% | 0.02968 |
| PPP-Six | 11.05% | 50.87% | 78.84% | 45.32% | -51.36% | -24.27% | -36.09% | 19.54% | 36.12% | -37.70% | -11.12% | 18.80% | 0.02412 |

**Table 3.52.** Expanding window 36 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-11-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-12-31 |
| MV | 378.56% | 33.14% | -31.16% | -155.14% | -282.92% | 125.98% | 56.31% | -75.81% | 34.84% | 22.16% | -8.43% | 2.46% | -0.03759 |
| BL | 233.61% | 14.90% | 16.59% | -112.65% | -184.10% | 146.42% | -8.77% | -54.45% | 6.76% | -0.16% | 8.15% | 33.70% | -0.01301 |
| PPP-Two | 87.76% | 10.32% | 9.78% | -5.59% | 10.71% | -2.50% | 0.01% | 3.75% | -5.23% | -3.93% | -2.78% | -2.30% | 0.04274 |
| PPP-Three | 85.90% | 20.23% | 5.15% | -26.87% | 16.56% | -22.09% | -16.52% | 15.89% | 38.30% | -34.81% | 18.44% | -0.19% | 0.05153 |
| PPP-Six | 21.09% | 38.86% | 87.79% | 64.68% | -27.86% | -41.40% | -37.46% | 27.62% | 31.02% | -52.00% | -17.59% | 5.27% | 0.03437 |

**Table 3.53.** Expanding window 37 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-12-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-01-31 |
| MV | 356.96% | 30.78% | -33.21% | -151.92% | -251.42% | 116.38% | 63.23% | -72.44% | 32.29% | 15.03% | -9.41% | 3.75% | -0.23646 |
| BL | 225.25% | 14.50% | 16.19% | -108.56% | -172.66% | 138.92% | -6.64% | -52.12% | 6.39% | -1.05% | 7.35% | 32.43% | -0.11907 |
| PPP-Two | 87.97% | 10.00% | 10.43% | -5.96% | 10.28% | -2.09% | -0.02% | 3.18% | -4.69% | -3.87% | -2.85% | -2.39% | -0.05931 |
| PPP-Three | 85.29% | 14.22% | 4.84% | -34.87% | 14.09% | -18.71% | -17.44% | 14.78% | 41.99% | -31.82% | 23.58% | 4.04% | 0.00743 |
| PPP-Six | 17.88% | 17.88% | 82.13% | 41.94% | -29.38% | -33.81% | -29.36% | 28.34% | 41.50% | -47.05% | -5.03% | 14.95% | -0.00670 |

**Table 3.54.** Expanding window 38 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-01-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-02-28 |
| MV | 390.21% | 27.97% | -57.54% | -173.09% | -258.08% | 153.95% | 45.37% | -83.67% | 36.70% | 1.84% | 2.72% | 13.62% | -0.14718 |
| BL | 236.63% | 13.67% | 13.22% | -117.15% | -181.78% | 155.08% | -13.25% | -56.65% | 6.62% | -4.25% | 10.78% | 37.08% | -0.07168 |
| PPP-Two | 92.32% | 10.09% | 9.45% | -6.29% | 9.90% | -2.63% | -0.52% | 2.77% | -5.21% | -4.12% | -3.25% | -2.52% | -0.00394 |
| PPP-Three | 87.24% | 6.01% | 1.86% | -29.32% | 13.83% | -15.13% | -16.32% | 10.76% | 43.14% | -28.81% | 23.31% | 3.43% | 0.11769 |
| PPP-Six | 26.98% | 4.54% | 79.11% | 14.40% | -24.25% | -26.08% | -28.86% | 18.47% | 41.01% | -46.88% | -11.27% | 52.83% | 0.20174 |

**Table 3.55.** Expanding window 39 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-02-28 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-03-31 |
| MV | 360.88% | 26.52% | -50.05% | -165.16% | -232.32% | 148.47% | 38.93% | -62.44% | 32.25% | -26.75% | 10.23% | 19.44% | 0.14066 |
| BL | 241.48% | 13.75% | 13.25% | -121.73% | -188.85% | 163.24% | -15.12% | -56.11% | 6.34% | -9.04% | 12.98% | 39.79% | 0.13504 |
| PPP-Two | 97.39% | 8.91% | 9.90% | -6.94% | 10.14% | -2.34% | -0.52% | 2.56% | -5.68% | -5.23% | -3.77% | -4.41% | 0.01395 |
| PPP-Three | 95.40% | 3.38% | 7.38% | -25.52% | 14.33% | -12.15% | -14.48% | 8.07% | 36.84% | -32.65% | 18.83% | 0.58% | 0.03030 |
| PPP-Six | 27.27% | -13.51% | 86.13% | 42.82% | -20.35% | -27.05% | -22.08% | 3.22% | 26.25% | -49.41% | 4.73% | 41.97% | 0.03115 |

**Table 3.56.** Expanding window 40 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-03-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-04-29 |
| MV | 372.43% | 24.36% | -51.97% | -165.08% | -241.43% | 144.12% | 38.40% | -62.06% | 32.21% | -24.84% | 11.79% | 22.08% | -0.22963 |
| BL | 231.89% | 14.53% | 14.40% | -118.65% | -179.16% | 160.06% | -14.25% | -54.77% | 6.22% | -9.64% | 11.76% | 37.61% | -0.21493 |
| PPP-Two | 96.73% | 8.06% | 10.00% | -6.28% | 10.31% | -1.93% | 0.28% | 2.17% | -5.28% | -5.65% | -3.83% | -4.58% | -0.09722 |
| PPP-Three | 97.30% | 12.93% | -0.42% | -35.51% | 18.23% | -7.25% | -16.51% | 10.92% | 32.20% | -31.38% | 27.76% | -8.27% | -0.11606 |
| PPP-Six | 22.04% | 13.23% | 75.12% | 18.13% | -19.95% | -25.26% | -32.77% | 15.04% | 29.77% | -54.12% | 24.24% | 34.54% | -0.20526 |

**Table 3.57.** Expanding window 41 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-04-29 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-05-31 |
| MV | 425.31% | 27.60% | -91.90% | -216.64% | -259.32% | 191.06% | 48.62% | -79.58% | 51.49% | -18.69% | 2.38% | 19.67% | -0.06732 |
| BL | 253.69% | 15.21% | 7.75% | -141.67% | -197.66% | 190.42% | -17.04% | -64.94% | 10.42% | -7.65% | 10.02% | 41.45% | 0.02951 |
| PPP-Two | 99.25% | 8.34% | 9.50% | -5.70% | 10.15% | -1.21% | 0.74% | 1.51% | -5.66% | -6.24% | -4.72% | -5.98% | -0.00165 |
| PPP-Three | 105.07% | 9.53% | 7.53% | -39.52% | 16.97% | -9.22% | -15.88% | 11.16% | 34.21% | -34.94% | 25.51% | -10.42% | -0.05333 |
| PPP-Six | 45.02% | -4.44% | 92.26% | -1.46% | -20.81% | -26.88% | -26.29% | 16.71% | 38.17% | -59.56% | 22.47% | 24.80% | -0.02415 |

**Table 3.58.** Expanding window 42 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-05-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-06-30 |
| MV | 419.70% | 28.58% | -83.40% | -211.41% | -257.56% | 208.31% | 27.59% | -81.22% | 43.58% | -16.14% | -2.58% | 24.55% | -0.01021 |
| BL | 255.47% | 15.27% | 7.60% | -142.88% | -199.78% | 192.44% | -17.37% | -65.53% | 10.46% | -7.70% | 10.17% | 41.85% | -0.09178 |
| PPP-Two | 105.32% | 5.51% | 13.10% | -6.88% | 9.85% | -0.33% | 0.98% | 1.02% | -6.59% | -7.45% | -5.59% | -8.95% | -0.07531 |
| PPP-Three | 110.22% | 4.70% | 4.46% | -32.93% | 17.40% | -8.93% | -13.14% | 9.14% | 34.29% | -36.63% | 23.12% | -11.71% | -0.09706 |
| PPP-Six | 56.02% | -13.36% | 82.87% | 14.84% | -30.09% | -30.22% | -22.28% | 7.54% | 42.42% | -46.47% | 17.37% | 21.35% | -0.19600 |

**Table 3.59.** Expanding window 43 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-06-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-07-29 |
| MV | 446.05% | 39.73% | -93.72% | -241.80% | -279.61% | 201.53% | 59.54% | -97.38% | 53.22% | -2.36% | -3.91% | 18.72% | 0.24311 |
| BL | 295.21% | 18.27% | 3.53% | -173.29% | -246.97% | 231.24% | -17.10% | -80.47% | 13.45% | -6.51% | 13.63% | 49.00% | 0.18079 |
| PPP-Two | 106.44% | 5.26% | 12.06% | -5.41% | 10.49% | -0.27% | 2.03% | 0.99% | -7.51% | -8.01% | -5.85% | -10.20% | 0.10102 |
| PPP-Three | 111.64% | 4.93% | 3.04% | -35.03% | 15.75% | -7.70% | -12.37% | 10.77% | 33.26% | -36.52% | 25.27% | -13.03% | 0.15990 |
| PPP-Six | 54.31% | -1.08% | 70.28% | -2.05% | -31.82% | -32.40% | -29.13% | 18.27% | 46.73% | -61.31% | 19.08% | 49.12% | 0.20830 |

**Table 3.60.** Expanding window 44 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-07-29 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-08-31 |
| MV | 422.41% | 34.88% | -80.99% | -229.09% | -260.51% | 176.38% | 70.20% | -89.23% | 51.39% | -6.51% | -8.67% | 19.75% | 0.02005 |
| BL | 268.17% | 18.55% | 4.99% | -153.86% | -214.39% | 207.91% | -21.29% | -72.94% | 11.16% | -3.81% | 12.94% | 42.55% | 0.02462 |
| PPP-Two | 105.74% | 8.00% | 13.00% | -6.15% | 10.80% | -0.81% | 1.56% | 1.00% | -9.32% | -7.93% | -5.69% | -10.19% | -0.07129 |
| PPP-Three | 109.70% | 2.03% | 5.46% | -31.77% | 13.38% | -5.70% | -10.29% | 8.72% | 33.52% | -34.90% | 22.72% | -12.87% | -0.07971 |
| PPP-Six | 50.45% | -15.78% | 65.95% | 8.08% | -34.75% | -25.38% | -21.26% | 15.17% | 50.71% | -58.24% | 14.46% | 50.60% | -0.01410 |

**Table 3.61.** Expanding window 45 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-08-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-09-30 |
| MV | 459.65% | 37.01% | -84.76% | -241.67% | -295.95% | 183.64% | 71.42% | -102.17% | 60.64% | 0.47% | -14.60% | 26.34% | -0.06665 |
| BL | 287.44% | 19.38% | 2.66% | -167.81% | -237.35% | 229.68% | -23.70% | -79.30% | 11.90% | -4.53% | 15.23% | 46.41% | -0.07836 |
| PPP-Two | 101.12% | 7.64% | 12.34% | -5.76% | 11.00% | -0.49% | 2.35% | 0.87% | -8.87% | -6.90% | -4.96% | -8.35% | -0.10376 |
| PPP-Three | 106.85% | -0.54% | 1.30% | -29.54% | 15.31% | -4.18% | -10.81% | 10.60% | 39.94% | -33.75% | 17.50% | -12.68% | -0.07118 |
| PPP-Six | 47.00% | -32.04% | 59.40% | 21.32% | -36.37% | -21.07% | -24.63% | 19.99% | 63.26% | -62.95% | 12.20% | 53.89% | 0.00024 |

**Table 3.62.** Expanding window 46 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-09-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-10-31 |
| MV | 566.17% | 44.34% | -126.81% | -298.93% | -387.72% | 249.26% | 87.95% | -126.78% | 80.30% | -6.86% | -21.04% | 40.12% | 0.13745 |
| BL | 344.34% | 21.77% | -3.47% | -208.05% | -305.20% | 292.20% | -31.11% | -98.48% | 14.52% | -5.92% | 21.25% | 58.15% | 0.12180 |
| PPP-Two | 100.52% | 6.30% | 10.47% | -4.74% | 10.77% | -0.05% | 2.35% | 0.88% | -7.48% | -6.36% | -4.81% | -7.84% | 0.06414 |
| PPP-Three | 105.59% | 2.26% | 3.70% | -32.04% | 14.99% | -3.39% | -11.25% | 11.59% | 37.58% | -35.46% | 17.84% | -11.41% | 0.03381 |
| PPP-Six | 55.84% | -12.56% | 49.23% | -23.06% | -41.72% | -21.66% | -32.36% | 9.87% | 78.48% | -49.22% | 10.92% | 76.24% | 0.04964 |

**Table 3.63.** Expanding window 47 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-10-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-11-30 |
| MV | 574.32% | 38.43% | -133.53% | -296.50% | -387.31% | 257.92% | 89.67% | -126.68% | 78.13% | -8.90% | -31.41% | 45.85% | -0.23170 |
| BL | 313.17% | 21.10% | -0.23% | -186.37% | -268.52% | 258.48% | -27.50% | -88.23% | 13.73% | -5.85% | 18.21% | 51.99% | -0.07268 |
| PPP-Two | 104.87% | 7.69% | 10.96% | -5.23% | 10.79% | 0.61% | 1.92% | 0.18% | -8.99% | -7.87% | -5.77% | -9.17% | 0.06710 |
| PPP-Three | 108.85% | -3.15% | 5.01% | -30.29% | 14.80% | -2.99% | -9.71% | 10.26% | 37.14% | -34.31% | 17.92% | -13.53% | 0.08163 |
| PPP-Six | 66.80% | -28.27% | 57.17% | -19.77% | -43.33% | -20.40% | -29.60% | 6.74% | 79.90% | -43.18% | 10.12% | 63.81% | -0.05337 |

**Table 3.64.** Expanding window 48 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-11-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-12-30 |
| MV | 507.40% | 38.36% | -125.27% | -265.67% | -342.36% | 248.29% | 68.28% | -118.83% | 65.67% | -13.36% | 15.93% | 21.57% | -0.23307 |
| BL | 285.01% | 20.41% | 1.41% | -168.09% | -238.55% | 234.66% | -26.84% | -80.42% | 11.55% | -6.10% | 23.16% | 43.80% | -0.12562 |
| PPP-TWO | 106.46% | 5.01% | 10.02% | -5.22% | 11.63% | 1.00% | 2.09% | 0.93% | -7.80% | -7.66% | -6.21% | -10.25% | -0.04258 |
| PPP-THREE | 108.81% | -0.59% | 2.39% | -28.12% | 15.55% | -2.25% | -9.37% | 11.01% | 32.85% | -36.13% | 20.32% | -14.48% | -0.02258 |
| PPP-SIX | 68.09% | -18.90% | 54.31% | -10.28% | -44.27% | -19.79% | -29.68% | 7.40% | 69.22% | -45.21% | 12.71% | 56.40% | -0.02105 |

**Table 3.65.** Expanding window 49 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-12-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-01-31 |
| MV | 499.78% | 44.22% | -128.79% | -315.02% | -376.59% | 301.38% | 99.83% | -112.73% | 76.10% | -23.08% | 10.86% | 24.02% | -0.07027 |
| BL | 277.93% | 20.81% | 3.63% | -170.61% | -234.39% | 239.00% | -24.85% | -80.53% | 11.09% | -8.70% | 21.76% | 44.87% | -0.00289 |
| PPP-Two | 108.45% | 4.79% | 11.03% | -5.00% | 11.98% | 1.16% | 2.00% | 1.26% | -8.28% | -9.31% | -6.41% | -11.66% | 0.06126 |
| PPP-Three | 110.26% | -0.66% | -0.63% | -26.02% | 16.55% | -0.54% | -7.44% | 10.30% | 32.25% | -34.70% | 20.32% | -19.69% | 0.01241 |
| PPP-Six | 49.13% | -23.81% | 69.22% | 50.69% | -44.54% | -6.21% | 7.47% | 36.05% | 40.47% | -55.77% | 13.31% | -36.00% | 0.05739 |

**Table 3.66.** Expanding window 50 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-01-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-02-28 |
| MV | 478.16% | 44.21% | -121.29% | -284.42% | -353.96% | 288.70% | 96.42% | -108.76% | 57.54% | -31.52% | 7.30% | 27.62% | -0.09421 |
| BL | 275.20% | 20.68% | 3.92% | -168.55% | -231.01% | 235.73% | -24.46% | -79.55% | 10.98% | -8.60% | 21.35% | 44.30% | -0.12162 |
| PPP-Two | 107.73% | 3.16% | 9.42% | -2.91% | 12.08% | 1.87% | 3.12% | 1.11% | -7.71% | -10.21% | -5.60% | -12.07% | -0.01072 |
| PPP-Three | 111.04% | -1.61% | 1.19% | -25.18% | 14.00% | -2.75% | -8.75% | 10.12% | 35.76% | -36.68% | 19.32% | -16.45% | -0.02240 |
| PPP-Six | 50.36% | -22.50% | 68.04% | 65.79% | -56.40% | -10.78% | 0.40% | 33.11% | 51.97% | -69.39% | 4.41% | -15.02% | -0.00370 |

**Table 3.67.** Expanding window 51 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-02-28 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-03-31 |
| MV | 485.29% | 47.11% | -135.01% | -295.76% | -351.00% | 301.87% | 113.08% | -115.70% | 62.26% | -33.42% | -5.15% | 26.44% | 0.26490 |
| BL | 285.97% | 21.78% | 1.30% | -177.85% | -245.15% | 251.68% | -23.02% | -83.58% | 11.19% | -9.35% | 20.50% | 46.52% | 0.20365 |
| PPP-Two | 105.08% | 2.94% | 8.89% | -1.31% | 10.17% | 1.30% | 2.81% | 0.92% | -6.15% | -10.22% | -4.90% | -9.53% | 0.03426 |
| PPP-Three | 112.17% | -3.48% | 1.97% | -28.53% | 17.29% | -2.15% | -8.14% | 13.55% | 32.97% | -37.66% | 18.52% | -16.51% | 0.01598 |
| PPP-Six | 46.67% | -21.95% | 71.01% | 64.73% | -48.80% | -9.37% | 4.68% | 40.49% | 42.43% | -67.82% | 1.32% | -23.41% | 0.02489 |

**Table 3.68.** Expanding window 52 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-03-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-04-28 |
| MV | 490.05% | 45.69% | -138.76% | -282.85% | -333.93% | 290.83% | 97.36% | -118.90% | 60.67% | -32.95% | -3.69% | 26.49% | -0.00557 |
| BL | 284.21% | 21.68% | -0.89% | -176.13% | -237.60% | 251.30% | -28.32% | -82.53% | 11.82% | -10.13% | 20.18% | 46.40% | -0.02606 |
| PPP-Two | 105.44% | 2.59% | 7.41% | -1.81% | 11.42% | 1.64% | 3.23% | 2.08% | -6.91% | -11.38% | -4.17% | -9.54% | 0.01275 |
| PPP-Three | 112.48% | -4.53% | 3.92% | -28.93% | 16.76% | -1.85% | -11.23% | 10.03% | 35.86% | -36.57% | 17.83% | -13.77% | 0.01210 |
| PPP-Six | 47.10% | -24.70% | 80.02% | 61.58% | -79.82% | -0.45% | -3.14% | 39.56% | 55.94% | -36.45% | 5.34% | -44.98% | -0.02397 |

**Table 3.69.** Expanding window 53 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-04-28 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-05-31 |
| MV | 450.86% | 44.48% | -125.74% | -300.69% | -316.83% | 287.96% | 134.47% | -114.13% | 60.96% | -29.96% | -19.70% | 28.32% | 0.20152 |
| BL | 271.66% | 21.03% | -0.10% | -169.57% | -221.37% | 238.44% | -24.98% | -78.83% | 11.70% | -9.63% | 17.19% | 44.47% | 0.20511 |
| PPP-Two | 104.72% | 0.70% | 7.18% | -2.71% | 10.82% | 0.89% | 2.31% | 1.80% | -4.47% | -8.79% | -3.79% | -8.67% | -0.00323 |
| PPP-Three | 111.57% | 0.55% | 4.31% | -30.42% | 14.83% | -3.62% | -12.73% | 11.70% | 35.50% | -37.41% | 18.02% | -12.29% | -0.00261 |
| PPP-Six | 52.89% | -15.75% | 80.27% | 62.27% | -77.80% | -2.72% | -12.68% | 44.35% | 46.33% | -36.36% | 7.81% | -48.61% | 0.02213 |

**Table 3.70.** Expanding window 54 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-05-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-06-30 |
| MV | 453.03% | 41.49% | -108.19% | -285.68% | -310.84% | 281.34% | 110.44% | -119.57% | 61.36% | -26.17% | -30.66% | 33.44% | 0.04925 |
| BL | 272.23% | 22.43% | -7.47% | -178.61% | -227.66% | 245.80% | -15.65% | -77.15% | 10.97% | -11.48% | 23.65% | 42.93% | 0.06175 |
| PPP-Two | 103.53% | 1.83% | 5.44% | -2.02% | 10.12% | -0.01% | 0.99% | 2.84% | -3.85% | -8.45% | -4.18% | -6.23% | 0.06279 |
| PPP-Three | 112.02% | 3.22% | 9.50% | -35.57% | 13.40% | -5.20% | -13.50% | 11.67% | 35.67% | -36.88% | 19.69% | -14.01% | 0.06569 |
| PPP-Six | 52.96% | -6.00% | 87.55% | 54.39% | -60.13% | -4.78% | 0.09% | 45.54% | 32.98% | -79.61% | 13.69% | -36.68% | 0.14786 |

**Table 3.71.** Expanding window 55 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-06-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-07-31 |
| MV | 453.03% | 36.89% | -104.15% | -262.25% | -311.40% | 266.51% | 101.84% | -114.46% | 56.50% | -32.66% | -28.28% | 38.44% | -0.07061 |
| BL | 256.79% | 21.60% | -5.02% | -166.48% | -208.58% | 226.73% | -13.99% | -71.82% | 10.37% | -10.57% | 21.20% | 39.77% | -0.00806 |
| PPP-Two | 104.56% | 6.08% | 6.85% | -4.92% | 9.72% | -2.08% | -1.31% | 2.65% | -2.95% | -10.03% | -4.07% | -4.50% | 0.02297 |
| PPP-Three | 113.58% | -4.02% | 4.26% | -33.53% | 14.91% | -2.78% | -7.03% | 11.26% | 31.17% | -31.72% | 17.36% | -13.45% | 0.01737 |
| PPP-Six | 60.58% | -18.25% | 88.25% | 59.67% | -67.47% | 5.64% | 19.56% | 34.35% | 23.69% | -76.98% | 10.37% | -39.40% | -0.00170 |

**Table 3.72.** Expanding window 56 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2018-07-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-08-31 |
| MV | 466.63% | 36.67% | -110.74% | -238.37% | -306.42% | 260.61% | 66.04% | -123.87% | 52.39% | -29.37% | -7.81% | 34.24% | 0.05816 |
| BL | 251.14% | 21.19% | -4.52% | -159.05% | -199.76% | 217.70% | -16.15% | -70.20% | 9.77% | -9.94% | 21.77% | 38.05% | -0.01562 |
| PPP-Two | 106.61% | 5.15% | 6.12% | -6.19% | 8.91% | -2.58% | -1.20% | 2.60% | -1.69% | -10.08% | -4.54% | -3.12% | -0.01869 |
| PPP-Three | 113.16% | 1.73% | 7.72% | -27.47% | 14.32% | -1.54% | -8.06% | 9.68% | 30.57% | -35.36% | 17.13% | -21.87% | -0.03117 |
| PPP-Six | 58.28% | 2.30% | 86.45% | 69.96% | -64.19% | 8.34% | 18.37% | 29.01% | 23.23% | -70.84% | 5.44% | -66.37% | -0.02671 |

**Table 3.73.** Expanding window 57 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-08-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-09-29 |
| MV | 489.56% | 36.90% | -117.74% | -246.27% | -323.07% | 266.58% | 74.48% | -127.37% | 56.88% | -26.68% | -18.37% | 35.10% | -0.16059 |
| BL | 261.89% | 21.74% | -6.24% | -167.15% | -212.54% | 230.18% | -17.06% | -73.79% | 10.31% | -10.28% | 22.90% | 40.06% | -0.13186 |
| PPP-Two | 109.95% | 8.68% | 6.77% | -6.33% | 8.59% | -2.98% | -1.19% | 1.86% | -2.97% | -10.04% | -5.50% | -6.85% | -0.04970 |
| PPP-Three | 114.19% | -3.51% | 7.15% | -30.81% | 12.75% | -2.34% | -10.24% | 6.96% | 33.62% | -31.67% | 15.28% | -11.39% | -0.03303 |
| PPP-Six | 57.01% | -6.12% | 91.59% | 61.09% | -71.56% | 6.50% | 11.59% | 22.42% | 30.26% | -45.35% | -3.03% | -54.40% | -0.08334 |

**Table 3.74.** Expanding window 58 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-09-20 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-10-31 |
| MV | 513.08% | 39.88% | -135.28% | -263.34% | -317.75% | 279.95% | 71.14% | -144.10% | 68.40% | -28.87% | -22.94% | 39.82% | 0.02467 |
| BL | 278.85% | 22.99% | -10.15% | -181.88% | -228.79% | 251.47% | -21.01% | -81.89% | 12.44% | -11.28% | 25.08% | 44.16% | -0.00745 |
| PPP-Two | 113.05% | 10.09% | 7.58% | -8.06% | 7.84% | -3.70% | -2.36% | 1.21% | -2.80% | -10.81% | -5.08% | -6.96% | -0.03379 |
| PPP-Three | 115.06% | -2.90% | 4.64% | -27.01% | 12.90% | -2.06% | -8.93% | 6.50% | 36.60% | -32.67% | 11.90% | -14.02% | -0.05654 |
| PPP-Six | 60.72% | -0.76% | 79.29% | 62.69% | -66.39% | 11.57% | 8.32% | 16.17% | 34.86% | -49.13% | -12.23% | -45.12% | -0.06984 |

**Table 3.75.** Expanding window 59 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-10-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-11-30 |
| MV | 580.23% | 43.56% | -159.60% | -274.95% | -357.08% | 301.86% | 65.05% | -177.16% | 73.45% | -23.56% | -15.04% | 43.24% | 0.20910 |
| BL | 294.88% | 23.90% | -12.92% | -193.45% | -247.15% | 269.86% | -23.36% | -88.08% | 13.09% | -11.78% | 27.82% | 47.18% | 0.22885 |
| PPP-Two | 114.78% | 12.63% | 7.42% | -8.45% | 7.12% | -3.64% | -2.50% | 0.75% | -2.02% | -13.39% | -5.24% | -7.46% | 0.08068 |
| PPP-Three | 119.21% | 9.47% | 7.66% | -29.67% | 12.04% | -3.22% | -11.43% | 4.53% | 34.56% | -35.06% | 11.58% | -19.68% | 0.06807 |
| PPP-Six | 56.91% | 23.11% | 80.67% | 60.57% | -54.04% | 13.98% | 11.89% | 18.55% | 26.13% | -78.12% | -7.01% | -52.63% | 0.12683 |

**Table 3.76.** Expanding window 60 without short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-11-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-12-29 |
| MV | 544.12% | 39.29% | -139.13% | -256.80% | -344.19% | 293.07% | 57.99% | -158.46% | 64.63% | -25.94% | -18.71% | 44.13% | 0.04432 |
| BL | 269.56% | 23.09% | -11.00% | -173.05% | -210.53% | 232.69% | -18.90% | -81.30% | 13.31% | -9.12% | 24.63% | 40.61% | 0.04725 |
| PPP-Two | 118.46% | 20.86% | 7.76% | -9.84% | 6.50% | -4.78% | -3.77% | 0.32% | -3.27% | -17.18% | -5.93% | -9.12% | 0.03936 |
| PPP-Three | 119.50% | 7.90% | 5.75% | -29.02% | 10.51% | -0.29% | -10.74% | 2.83% | 43.59% | -36.83% | 11.81% | -25.01% | 0.07413 |
| PPP-Six | 54.06% | 16.55% | 80.74% | 67.53% | -58.03% | 20.26% | 17.09% | 12.91% | 37.83% | -58.01% | -11.42% | -79.52% | 0.06538 |

**Table 3.77.** Expanding window 1 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2018-12-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-01-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.07769 |
| MC | 49.47% | 9.43% | 8.86% | 1.79% | 8.53% | 3.55% | 4.83% | 5.13% | 1.97% | 1.88% | 2.36% | 2.20% | 0.07342 |
| RP | 12.36% | 9.05% | 12.43% | 6.89% | 9.70% | 7.42% | 7.92% | 8.78% | 6.95% | 5.84% | 7.04% | 5.63% | 0.07426 |
| MV | 84.57% | 15.43% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.07361 |
| BL | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 100.00% | 0.16333 |
| PPP-Two | 8.41% | 8.34% | 8.35% | 8.33% | 8.34% | 8.33% | 8.33% | 8.33% | 8.31% | 8.30% | 8.32% | 8.31% | 0.07766 |
| PPP-Three | 8.43% | 8.42% | 8.31% | 8.27% | 8.35% | 8.30% | 8.30% | 8.33% | 8.41% | 8.23% | 8.34% | 8.32% | 0.07757 |
| PPP-Six | 13.71% | 12.55% | 13.23% | 4.63% | 7.14% | 6.64% | 6.16% | 7.66% | 10.51% | 5.20% | 7.58% | 4.99% | 0.07012 |

**Table 3.78.** Expanding window 2 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-01-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-02-28 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.01924 |
| MC | 49.52% | 9.43% | 8.85% | 1.79% | 8.51% | 3.55% | 4.82% | 5.12% | 1.97% | 1.87% | 2.36% | 2.20% | 0.03256 |
| RP | 12.30% | 9.06% | 12.42% | 6.90% | 9.69% | 7.45% | 7.95% | 8.71% | 7.05% | 5.83% | 7.03% | 5.60% | 0.02325 |
| MV | 85.49% | 14.51% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.04393 |
| BL | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 100.00% | -0.04617 |
| PPP-Two | 8.42% | 8.35% | 8.34% | 8.33% | 8.34% | 8.33% | 8.34% | 8.33% | 8.31% | 8.30% | 8.31% | 8.31% | 0.01931 |
| PPP-Three | 8.43% | 8.43% | 8.31% | 8.26% | 8.33% | 8.30% | 8.30% | 8.32% | 8.42% | 8.24% | 8.34% | 8.31% | 0.01936 |
| PPP-Six | 12.46% | 13.55% | 12.71% | 5.00% | 8.15% | 5.12% | 5.19% | 8.88% | 10.62% | 4.58% | 7.86% | 5.85% | 0.02593 |

**Table 3.79.** Expanding window 3 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-02-28 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-03-29 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.00946 |
| MC | 49.58% | 9.45% | 8.84% | 1.78% | 8.48% | 3.54% | 4.82% | 5.12% | 1.98% | 1.87% | 2.36% | 2.19% | 0.01397 |
| RP | 12.29% | 9.03% | 12.43% | 6.90% | 9.69% | 7.45% | 7.94% | 8.71% | 7.06% | 5.84% | 7.04% | 5.61% | 0.01025 |
| MV | 82.70% | 17.30% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.02281 |
| BL | 72.48% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 27.52% | 0.00127 |
| PPP-Two | 8.41% | 8.35% | 8.34% | 8.33% | 8.34% | 8.33% | 8.33% | 8.33% | 8.31% | 8.31% | 8.32% | 8.31% | 0.00947 |
| PPP-Three | 8.44% | 8.41% | 8.31% | 8.26% | 8.34% | 8.30% | 8.29% | 8.33% | 8.42% | 8.24% | 8.34% | 8.31% | 0.00959 |
| PPP-Six | 13.00% | 13.91% | 13.23% | 4.80% | 5.66% | 5.59% | 5.68% | 8.05% | 11.38% | 5.63% | 8.46% | 4.60% | 0.01652 |

**Table 3.80.** Expanding window 4 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-03-29 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-04-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.02896 |
| MC | 49.64% | 9.46% | 8.83% | 1.78% | 8.46% | 3.53% | 4.81% | 5.11% | 1.98% | 1.86% | 2.35% | 2.19% | 0.03213 |
| RP | 12.29% | 9.02% | 12.44% | 6.90% | 9.69% | 7.46% | 7.94% | 8.72% | 7.05% | 5.84% | 7.04% | 5.62% | 0.02975 |
| MV | 81.89% | 18.11% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.03024 |
| BL | 92.89% | 6.66% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.45% | 0.03533 |
| PPP-Two | 8.41% | 8.34% | 8.34% | 8.33% | 8.34% | 8.33% | 8.33% | 8.33% | 8.31% | 8.31% | 8.32% | 8.32% | 0.02897 |
| PPP-Three | 8.43% | 8.40% | 8.30% | 8.26% | 8.34% | 8.30% | 8.29% | 8.33% | 8.43% | 8.25% | 8.35% | 8.32% | 0.02890 |
| PPP-Six | 13.89% | 12.45% | 14.08% | 4.21% | 5.37% | 6.19% | 5.29% | 8.70% | 12.26% | 4.82% | 8.88% | 3.86% | 0.02770 |

## **Table 3.81.** Expanding window 5 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-04-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-05-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.04652 |
| MC | 49.70% | 9.48% | 8.81% | 1.77% | 8.44% | 3.53% | 4.81% | 5.10% | 1.98% | 1.86% | 2.35% | 2.18% | -0.06199 |
| RP | 12.28% | 9.04% | 12.42% | 6.90% | 9.69% | 7.45% | 7.94% | 8.72% | 7.06% | 5.84% | 7.04% | 5.63% | -0.05100 |
| MV | 84.20% | 15.80% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.07074 |
| BL | 92.69% | 6.85% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.46% | -0.06885 |
| PPP-Two | 8.41% | 8.34% | 8.34% | 8.32% | 8.34% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | -0.04655 |
| PPP-Three | 8.43% | 8.39% | 8.30% | 8.25% | 8.34% | 8.30% | 8.29% | 8.34% | 8.46% | 8.25% | 8.35% | 8.32% | -0.04650 |
| PPP-Six | 13.50% | 12.51% | 12.37% | 3.70% | 6.45% | 6.22% | 5.11% | 9.05% | 12.95% | 4.38% | 9.51% | 4.26% | -0.04783 |

## **Table 3.82.** Expanding window 6 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-05-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-06-28 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.05726 |
| MC | 49.75% | 9.49% | 8.80% | 1.77% | 8.41% | 3.52% | 4.80% | 5.09% | 1.98% | 1.85% | 2.35% | 2.18% | 0.05836 |
| RP | 12.22% | 9.01% | 12.40% | 6.88% | 9.67% | 7.45% | 7.93% | 8.73% | 7.12% | 5.89% | 7.04% | 5.67% | 0.05570 |
| MV | 83.03% | 16.36% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.61% | 0.00% | 0.00% | 0.00% | 0.06073 |
| BL | 92.53% | 6.38% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 1.09% | 0.06446 |
| PPP-Two | 8.40% | 8.34% | 8.34% | 8.32% | 8.34% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | 0.05726 |
| PPP-Three | 8.43% | 8.40% | 8.30% | 8.25% | 8.33% | 8.30% | 8.29% | 8.33% | 8.45% | 8.24% | 8.36% | 8.33% | 0.05715 |
| PPP-Six | 12.62% | 11.77% | 12.50% | 4.62% | 5.83% | 6.45% | 5.13% | 8.69% | 13.90% | 4.90% | 9.34% | 4.26% | 0.05073 |

**Table 3.83.** Expanding window 7 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-06-28 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-07-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.01326 |
| MC | 49.81% | 9.50% | 8.79% | 1.76% | 8.39% | 3.52% | 4.80% | 5.08% | 1.99% | 1.85% | 2.35% | 2.17% | -0.00040 |
| RP | 12.17% | 9.03% | 12.40% | 6.87% | 9.68% | 7.45% | 7.91% | 8.73% | 7.16% | 5.90% | 7.03% | 5.67% | -0.01175 |
| MV | 85.71% | 14.29% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00858 |
| BL | 92.33% | 6.61% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 1.06% | 0.01114 |
| PPP-Two | 8.40% | 8.34% | 8.34% | 8.32% | 8.34% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | -0.01323 |
| PPP-Three | 8.42% | 8.40% | 8.30% | 8.26% | 8.33% | 8.30% | 8.29% | 8.34% | 8.44% | 8.25% | 8.35% | 8.32% | -0.01328 |
| PPP-Six | 12.81% | 12.79% | 12.49% | 4.97% | 5.53% | 6.09% | 5.23% | 8.50% | 13.17% | 3.71% | 9.76% | 4.95% | -0.01386 |

**Table 3.84.** Expanding window 8 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-07-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-08-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.04311 |
| MC | 49.87% | 9.51% | 8.78% | 1.76% | 8.36% | 3.51% | 4.79% | 5.08% | 1.99% | 1.84% | 2.35% | 2.17% | -0.02965 |
| RP | 12.18% | 9.03% | 12.41% | 6.87% | 9.68% | 7.44% | 7.91% | 8.73% | 7.16% | 5.90% | 7.03% | 5.68% | -0.03935 |
| MV | 87.09% | 12.91% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.02296 |
| BL | 92.18% | 6.78% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 1.04% | -0.02161 |
| PPP-Two | 8.40% | 8.34% | 8.34% | 8.33% | 8.34% | 8.33% | 8.33% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | -0.04308 |
| PPP-Three | 8.42% | 8.38% | 8.30% | 8.26% | 8.33% | 8.30% | 8.29% | 8.33% | 8.45% | 8.25% | 8.36% | 8.32% | -0.04313 |
| PPP-Six | 12.25% | 12.76% | 11.55% | 4.73% | 5.80% | 5.80% | 4.20% | 9.45% | 13.89% | 4.53% | 9.66% | 5.36% | -0.04189 |

**Table 3.85.** Expanding window 9 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-08-30 | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-09-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.02518 |
| MC | 49.93% | 9.51% | 8.77% | 1.75% | 8.33% | 3.50% | 4.79% | 5.07% | 1.99% | 1.84% | 2.34% | 2.17% | 0.02109 |
| RP | 12.18% | 9.01% | 12.42% | 6.88% | 9.66% | 7.44% | 7.92% | 8.76% | 7.16% | 5.90% | 7.00% | 5.66% | 0.02499 |
| MV | 88.43% | 11.57% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.01596 |
| BL | 92.39% | 6.64% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.06% | 0.00% | 0.00% | 0.91% | 0.01654 |
| PPP-Two | 8.40% | 8.34% | 8.34% | 8.33% | 8.34% | 8.33% | 8.33% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | 0.02517 |
| PPP-Three | 8.43% | 8.42% | 8.29% | 8.25% | 8.35% | 8.30% | 8.29% | 8.35% | 8.39% | 8.23% | 8.37% | 8.34% | 0.02514 |
| PPP-Six | 12.53% | 11.72% | 11.59% | 4.96% | 5.98% | 6.01% | 4.75% | 9.21% | 14.53% | 3.80% | 9.75% | 5.17% | 0.02429 |

**Table 3.86.** Expanding window 10 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-09-30 | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-10-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.03721 |
| MC | 49.99% | 9.52% | 8.76% | 1.75% | 8.31% | 3.50% | 4.78% | 5.06% | 1.99% | 1.84% | 2.34% | 2.16% | 0.02749 |
| RP | 12.19% | 9.02% | 12.40% | 6.88% | 9.65% | 7.44% | 7.92% | 8.76% | 7.15% | 5.90% | 7.01% | 5.67% | 0.03548 |
| MV | 88.95% | 11.05% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.02060 |
| BL | 92.33% | 6.69% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.98% | 0.02081 |
| PPP-Two | 8.39% | 8.34% | 8.34% | 8.33% | 8.34% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | 0.03720 |
| PPP-Three | 8.43% | 8.41% | 8.30% | 8.24% | 8.35% | 8.31% | 8.29% | 8.35% | 8.39% | 8.24% | 8.37% | 8.31% | 0.03713 |
| PPP-Six | 13.13% | 13.26% | 11.02% | 5.03% | 6.40% | 6.44% | 4.14% | 9.85% | 12.66% | 3.48% | 8.98% | 5.61% | 0.03311 |

**Table 3.87.** Expanding window 12 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-10-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-11-29 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.00741 |
| MC | 50.05% | 9.52% | 8.75% | 1.74% | 8.28% | 3.49% | 4.78% | 5.06% | 1.99% | 1.83% | 2.34% | 2.16% | 0.01887 |
| RP | 12.20% | 9.03% | 12.37% | 6.88% | 9.65% | 7.44% | 7.92% | 8.78% | 7.16% | 5.90% | 7.01% | 5.66% | 0.00916 |
| MV | 88.99% | 11.01% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.02772 |
| BL | 92.31% | 6.75% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.94% | 0.02926 |
| PPP-Two | 8.40% | 8.34% | 8.34% | 8.32% | 8.34% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | 0.00742 |
| PPP-Three | 8.44% | 8.40% | 8.30% | 8.23% | 8.35% | 8.31% | 8.28% | 8.35% | 8.40% | 8.24% | 8.39% | 8.31% | 0.00742 |
| PPP-Six | 13.46% | 12.64% | 10.85% | 2.93% | 6.90% | 7.00% | 3.47% | 9.87% | 13.17% | 4.15% | 10.30% | 5.25% | 0.00804 |

**Table 3.88.** Expanding window 12 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-11-29 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2019-12-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.04682 |
| MC | 50.12% | 9.52% | 8.74% | 1.74% | 8.26% | 3.49% | 4.77% | 5.05% | 1.99% | 1.83% | 2.34% | 2.15% | 0.03680 |
| RP | 12.19% | 9.03% | 12.37% | 6.88% | 9.65% | 7.44% | 7.92% | 8.78% | 7.16% | 5.90% | 7.01% | 5.67% | 0.04308 |
| MV | 91.10% | 8.90% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.03192 |
| BL | 92.06% | 6.89% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 1.05% | 0.03203 |
| PPP-Two | 8.40% | 8.33% | 8.33% | 8.32% | 8.34% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.33% | 8.32% | 0.04681 |
| PPP-Three | 8.43% | 8.38% | 8.29% | 8.24% | 8.34% | 8.31% | 8.29% | 8.34% | 8.45% | 8.25% | 8.37% | 8.31% | 0.04679 |
| PPP-Six | 13.63% | 12.08% | 10.80% | 3.01% | 7.21% | 6.91% | 3.44% | 9.83% | 13.33% | 4.49% | 10.54% | 4.73% | 0.04348 |

**Table 3.89.** Expanding window 13 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2019-12-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-01-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.03152 |
| MC | 50.19% | 9.53% | 8.71% | 1.73% | 8.23% | 3.48% | 4.77% | 5.04% | 2.00% | 1.82% | 2.34% | 2.15% | -0.01515 |
| RP | 12.20% | 9.01% | 12.38% | 6.89% | 9.63% | 7.45% | 7.93% | 8.79% | 7.18% | 5.89% | 7.00% | 5.65% | -0.02807 |
| MV | 89.83% | 10.17% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.00312 |
| BL | 91.78% | 6.93% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.45% | 0.00% | 0.00% | 0.84% | -0.00338 |
| PPP-Two | 8.40% | 8.34% | 8.34% | 8.33% | 8.34% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | 8.32% | -0.03149 |
| PPP-Three | 8.42% | 8.39% | 8.30% | 8.26% | 8.34% | 8.30% | 8.29% | 8.34% | 8.42% | 8.26% | 8.36% | 8.31% | -0.03149 |
| PPP-Six | 13.38% | 12.47% | 11.42% | 5.86% | 6.81% | 6.69% | 4.74% | 9.13% | 10.97% | 5.65% | 8.66% | 4.21% | -0.02649 |

**Table 3.90.** Expanding window 14 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-01-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-02-28 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.09948 |
| MC | 50.26% | 9.54% | 8.69% | 1.73% | 8.21% | 3.47% | 4.76% | 5.03% | 2.00% | 1.82% | 2.33% | 2.15% | -0.09083 |
| RP | 12.21% | 9.01% | 12.39% | 6.89% | 9.62% | 7.45% | 7.92% | 8.81% | 7.18% | 5.90% | 6.98% | 5.64% | -0.09676 |
| MV | 90.36% | 9.64% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.08374 |
| BL | 91.83% | 6.91% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.42% | 0.00% | 0.00% | 0.84% | -0.08525 |
| PPP-Two | 8.40% | 8.34% | 8.33% | 8.32% | 8.34% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.32% | 8.33% | -0.09946 |
| PPP-Three | 8.42% | 8.37% | 8.30% | 8.27% | 8.33% | 8.30% | 8.30% | 8.34% | 8.43% | 8.27% | 8.36% | 8.31% | -0.09942 |
| PPP-Six | 13.15% | 13.10% | 11.69% | 5.76% | 6.49% | 5.95% | 3.69% | 9.41% | 10.68% | 5.94% | 9.09% | 5.04% | -0.09367 |

**Table 3.91**. Expanding window 15 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-02-28 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-03-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.22331 |
| MC | 50.32% | 9.55% | 8.68% | 1.72% | 8.18% | 3.47% | 4.76% | 5.03% | 2.00% | 1.82% | 2.33% | 2.14% | -0.15673 |
| RP | 12.13% | 9.09% | 12.30% | 6.97% | 9.52% | 7.47% | 7.94% | 8.84% | 7.23% | 5.89% | 6.95% | 5.65% | -0.20399 |
| MV | 87.33% | 12.67% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.12421 |
| BL | 92.29% | 6.88% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.83% | -0.13159 |
| PPP-Two | 8.39% | 8.34% | 8.34% | 8.33% | 8.34% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.32% | 8.33% | -0.22320 |
| PPP-Three | 8.45% | 8.40% | 8.30% | 8.25% | 8.34% | 8.31% | 8.29% | 8.36% | 8.36% | 8.25% | 8.38% | 8.31% | -0.22307 |
| PPP-Six | 13.42% | 12.42% | 10.62% | 5.08% | 5.79% | 5.90% | 3.89% | 9.51% | 12.80% | 6.60% | 8.72% | 5.23% | -0.20656 |

**Table 3.92.** Expanding window 16 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-03-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-04-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.07825 |
| MC | 50.38% | 9.56% | 8.68% | 1.72% | 8.16% | 3.46% | 4.75% | 5.02% | 2.00% | 1.81% | 2.33% | 2.14% | 0.09382 |
| RP | 12.12% | 9.50% | 12.48% | 6.93% | 9.51% | 7.57% | 7.99% | 8.63% | 7.06% | 5.94% | 6.91% | 5.37% | 0.07909 |
| MV | 79.92% | 20.08% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.10384 |
| BL | 93.10% | 6.90% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.11407 |
| PPP-Two | 8.39% | 8.34% | 8.34% | 8.32% | 8.33% | 8.32% | 8.33% | 8.33% | 8.33% | 8.32% | 8.32% | 8.33% | 0.07827 |
| PPP-Three | 8.42% | 8.37% | 8.30% | 8.26% | 8.34% | 8.30% | 8.30% | 8.35% | 8.43% | 8.26% | 8.36% | 8.31% | 0.07837 |
| PPP-Six | 15.45% | 13.24% | 10.71% | 4.61% | 7.14% | 6.45% | 3.90% | 10.68% | 11.04% | 3.49% | 8.89% | 4.40% | 0.08540 |

## **Table 3.93.** Expanding window 17 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-04-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-05-29 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.04122 |
| MC | 50.45% | 9.58% | 8.67% | 1.72% | 8.13% | 3.46% | 4.74% | 5.01% | 2.00% | 1.81% | 2.32% | 2.13% | 0.03901 |
| RP | 11.97% | 9.55% | 12.44% | 6.99% | 9.54% | 7.58% | 8.05% | 8.58% | 7.03% | 5.95% | 6.92% | 5.40% | 0.03968 |
| MV | 84.71% | 15.29% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.03553 |
| BL | 92.07% | 7.93% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.03974 |
| PPP-Two | 8.40% | 8.34% | 8.34% | 8.32% | 8.33% | 8.32% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.33% | 0.04120 |
| PPP-Three | 8.43% | 8.37% | 8.31% | 8.26% | 8.34% | 8.30% | 8.29% | 8.35% | 8.43% | 8.26% | 8.36% | 8.31% | 0.04104 |
| PPP-Six | 14.52% | 12.42% | 12.06% | 4.91% | 7.05% | 4.85% | 3.58% | 10.59% | 13.19% | 4.23% | 8.03% | 4.55% | 0.03145 |

## **Table 3.94.** Expanding window 18 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-05-29 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-06-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.04901 |
| MC | 50.52% | 9.58% | 8.66% | 1.71% | 8.10% | 3.45% | 4.73% | 5.00% | 2.00% | 1.80% | 2.32% | 2.12% | 0.03125 |
| RP | 11.96% | 9.59% | 12.38% | 6.99% | 9.56% | 7.56% | 8.05% | 8.58% | 7.07% | 5.94% | 6.92% | 5.40% | 0.04547 |
| MV | 87.53% | 12.47% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.02285 |
| BL | 91.81% | 8.19% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.02126 |
| PPP-Two | 8.40% | 8.34% | 8.34% | 8.32% | 8.33% | 8.32% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.33% | 0.04899 |
| PPP-Three | 8.42% | 8.37% | 8.31% | 8.26% | 8.34% | 8.30% | 8.29% | 8.35% | 8.43% | 8.26% | 8.36% | 8.31% | 0.04902 |
| PPP-Six | 14.19% | 12.11% | 12.30% | 4.38% | 6.97% | 5.41% | 3.31% | 10.32% | 13.78% | 4.91% | 8.02% | 4.31% | 0.04597 |

**Table 3.95.** Expanding window 19 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-06-30 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-07-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.04991 |
| MC | 50.59% | 9.60% | 8.65% | 1.71% | 8.08% | 3.44% | 4.73% | 4.99% | 2.00% | 1.80% | 2.31% | 2.11% | 0.05003 |
| RP | 11.98% | 9.58% | 12.40% | 6.97% | 9.57% | 7.55% | 8.04% | 8.59% | 7.06% | 5.96% | 6.90% | 5.39% | 0.04657 |
| MV | 86.14% | 13.86% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.06232 |
| BL | 91.76% | 8.24% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.05880 |
| PPP-Two | 8.40% | 8.34% | 8.33% | 8.33% | 8.33% | 8.32% | 8.33% | 8.33% | 8.33% | 8.32% | 8.32% | 8.33% | 0.04992 |
| PPP-Three | 8.42% | 8.37% | 8.30% | 8.26% | 8.33% | 8.29% | 8.29% | 8.35% | 8.43% | 8.26% | 8.37% | 8.31% | 0.05002 |
| PPP-Six | 14.25% | 11.95% | 11.75% | 3.63% | 7.32% | 5.84% | 3.10% | 10.21% | 14.09% | 5.01% | 8.36% | 4.47% | 0.05368 |

**Table 3.96.** Expanding window 20 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-07-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-08-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.03209 |
| MC | 50.66% | 9.62% | 8.64% | 1.70% | 8.04% | 3.44% | 4.72% | 4.98% | 2.00% | 1.79% | 2.31% | 2.11% | 0.05335 |
| RP | 11.96% | 9.53% | 12.46% | 6.98% | 9.60% | 7.55% | 8.05% | 8.58% | 7.04% | 5.98% | 6.91% | 5.38% | 0.03847 |
| MV | 84.42% | 15.58% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.06401 |
| BL | 91.85% | 8.15% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.06578 |
| PPP-Two | 8.40% | 8.34% | 8.33% | 8.33% | 8.33% | 8.32% | 8.33% | 8.32% | 8.33% | 8.31% | 8.31% | 8.33% | 0.03212 |
| PPP-Three | 8.43% | 8.37% | 8.30% | 8.26% | 8.33% | 8.29% | 8.29% | 8.34% | 8.44% | 8.26% | 8.37% | 8.31% | 0.03214 |
| PPP-Six | 14.37% | 11.99% | 10.81% | 3.85% | 6.63% | 4.82% | 3.60% | 10.30% | 13.81% | 4.68% | 9.72% | 5.42% | 0.03807 |

**Table 3.97.** Expanding window 18 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-08-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-09-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.03940 |
| MC | 50.74% | 9.63% | 8.62% | 1.69% | 8.01% | 3.43% | 4.71% | 4.97% | 2.00% | 1.78% | 2.30% | 2.10% | -0.03791 |
| RP | 11.92% | 9.53% | 12.42% | 6.98% | 9.60% | 7.55% | 8.05% | 8.57% | 7.05% | 5.99% | 6.93% | 5.41% | -0.03670 |
| MV | 85.77% | 14.23% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.04077 |
| BL | 91.61% | 8.39% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.04046 |
| PPP-Two | 8.41% | 8.33% | 8.32% | 8.33% | 8.33% | 8.32% | 8.33% | 8.32% | 8.33% | 8.32% | 8.32% | 8.34% | -0.03940 |
| PPP-Three | 8.45% | 8.36% | 8.29% | 8.25% | 8.33% | 8.29% | 8.29% | 8.34% | 8.45% | 8.25% | 8.37% | 8.31% | -0.03935 |
| PPP-Six | 14.40% | 11.79% | 11.06% | 4.33% | 6.90% | 5.52% | 3.41% | 9.83% | 14.78% | 2.97% | 9.38% | 5.63% | -0.03354 |

**Table 3.98.** Expanding window 22 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-09-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-10-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.03689 |
| MC | 50.81% | 9.65% | 8.62% | 1.69% | 7.98% | 3.43% | 4.70% | 4.96% | 2.00% | 1.78% | 2.30% | 2.09% | -0.02783 |
| RP | 11.90% | 9.51% | 12.45% | 6.98% | 9.59% | 7.55% | 8.05% | 8.57% | 7.06% | 5.99% | 6.94% | 5.41% | -0.03342 |
| MV | 85.59% | 14.41% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.02161 |
| BL | 91.72% | 8.28% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.02435 |
| PPP-Two | 8.41% | 8.34% | 8.32% | 8.34% | 8.33% | 8.32% | 8.33% | 8.32% | 8.33% | 8.31% | 8.32% | 8.33% | -0.03686 |
| PPP-Three | 8.45% | 8.37% | 8.29% | 8.26% | 8.34% | 8.30% | 8.29% | 8.34% | 8.44% | 8.25% | 8.37% | 8.31% | -0.03670 |
| PPP-Six | 14.66% | 11.04% | 10.09% | 3.82% | 6.75% | 5.91% | 4.20% | 9.68% | 14.55% | 3.95% | 9.04% | 6.33% | -0.02397 |

**Table 3.99.** Expanding window 23 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-10-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-11-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.15509 |
| MC | 50.88% | 9.67% | 8.61% | 1.68% | 7.95% | 3.42% | 4.69% | 4.95% | 2.00% | 1.77% | 2.29% | 2.08% | 0.12179 |
| RP | 11.89% | 9.54% | 12.48% | 6.97% | 9.58% | 7.52% | 8.04% | 8.57% | 7.08% | 5.97% | 6.95% | 5.41% | 0.14835 |
| MV | 83.37% | 16.63% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.09641 |
| BL | 91.72% | 8.28% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.09929 |
| PPP-TWO | 8.42% | 8.34% | 8.34% | 8.32% | 8.33% | 8.32% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.33% | 0.15501 |
| PPP-THREE | 8.46% | 8.36% | 8.30% | 8.25% | 8.34% | 8.29% | 8.29% | 8.34% | 8.43% | 8.25% | 8.37% | 8.30% | 0.15482 |
| PPP-SIX | 15.65% | 11.19% | 9.81% | 3.51% | 6.84% | 5.78% | 3.79% | 9.88% | 13.86% | 4.16% | 9.70% | 5.83% | 0.13687 |

**Table 3.100.** Expanding window 24 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-11-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2020-12-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.05947 |
| MC | 50.95% | 9.69% | 8.60% | 1.68% | 7.91% | 3.41% | 4.68% | 4.94% | 2.00% | 1.76% | 2.29% | 2.08% | 0.04333 |
| RP | 11.93% | 9.71% | 12.29% | 6.89% | 9.53% | 7.53% | 7.94% | 8.61% | 7.18% | 6.00% | 6.99% | 5.42% | 0.05479 |
| MV | 85.20% | 14.80% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.03573 |
| BL | 91.25% | 8.75% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.03602 |
| PPP-Two | 8.43% | 8.34% | 8.33% | 8.32% | 8.33% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | 0.05944 |
| PPP-Three | 8.47% | 8.35% | 8.30% | 8.25% | 8.34% | 8.30% | 8.29% | 8.34% | 8.44% | 8.25% | 8.37% | 8.31% | 0.05948 |
| PPP-Six | 15.96% | 10.80% | 10.57% | 3.55% | 6.90% | 5.46% | 3.70% | 9.85% | 13.62% | 4.05% | 9.28% | 6.27% | 0.05861 |

**Table 3.101.** Expanding window 25 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2020-12-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-01-29 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.02078 |
| MC | 51.03% | 9.71% | 8.58% | 1.67% | 7.88% | 3.41% | 4.68% | 4.93% | 2.01% | 1.76% | 2.28% | 2.07% | -0.01122 |
| RP | 11.93% | 9.72% | 12.27% | 6.90% | 9.52% | 7.53% | 7.96% | 8.63% | 7.16% | 6.00% | 6.97% | 5.41% | -0.01695 |
| MV | 85.64% | 14.36% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.00699 |
| BL | 91.18% | 8.82% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.00861 |
| PPP-Two | 8.42% | 8.34% | 8.33% | 8.33% | 8.33% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.31% | 8.32% | -0.02077 |
| PPP-Three | 8.46% | 8.36% | 8.30% | 8.26% | 8.33% | 8.30% | 8.29% | 8.34% | 8.44% | 8.26% | 8.36% | 8.30% | -0.02072 |
| PPP-Six | 15.51% | 10.55% | 9.99% | 3.20% | 6.25% | 5.85% | 3.64% | 9.63% | 14.72% | 4.37% | 9.38% | 6.92% | -0.01615 |

**Table 3.102.** Expanding window 26 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-01-29 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-02-26 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.02775 |
| MC | 51.10% | 9.73% | 8.57% | 1.67% | 7.85% | 3.40% | 4.67% | 4.91% | 2.01% | 1.75% | 2.28% | 2.06% | 0.02577 |
| RP | 11.93% | 9.73% | 12.28% | 6.90% | 9.52% | 7.52% | 7.95% | 8.63% | 7.16% | 6.00% | 6.98% | 5.40% | 0.02882 |
| MV | 84.46% | 15.54% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.02182 |
| BL | 91.17% | 8.83% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.02352 |
| PPP-Two | 8.41% | 8.34% | 8.33% | 8.33% | 8.33% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.31% | 8.32% | 0.02775 |
| PPP-Three | 8.47% | 8.34% | 8.31% | 8.27% | 8.33% | 8.30% | 8.29% | 8.35% | 8.44% | 8.26% | 8.35% | 8.30% | 0.02776 |
| PPP-Six | 15.46% | 10.75% | 10.59% | 3.51% | 6.08% | 6.20% | 4.12% | 9.35% | 14.14% | 4.58% | 8.66% | 6.56% | 0.02749 |

**Table 3.103.** Expanding window 27 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-02-26 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-03-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.02720 |
| MC | 51.18% | 9.75% | 8.55% | 1.66% | 7.82% | 3.39% | 4.66% | 4.90% | 2.01% | 1.75% | 2.27% | 2.05% | 0.02691 |
| RP | 11.93% | 9.75% | 12.27% | 6.89% | 9.52% | 7.53% | 7.94% | 8.62% | 7.16% | 6.00% | 6.97% | 5.42% | 0.02336 |
| MV | 85.52% | 14.48% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.03098 |
| BL | 91.11% | 8.89% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.03506 |
| PPP-Two | 8.42% | 8.34% | 8.32% | 8.33% | 8.33% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | 0.02720 |
| PPP-Three | 8.48% | 8.36% | 8.30% | 8.26% | 8.34% | 8.30% | 8.29% | 8.35% | 8.42% | 8.25% | 8.36% | 8.30% | 0.02717 |
| PPP-Six | 16.41% | 9.93% | 10.60% | 4.42% | 6.20% | 6.35% | 4.03% | 9.61% | 14.15% | 4.20% | 7.97% | 6.13% | 0.02316 |

**Table 3.104.** Expanding window 28 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-03-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-04-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.02592 |
| MC | 51.26% | 9.76% | 8.53% | 1.66% | 7.79% | 3.39% | 4.65% | 4.90% | 2.01% | 1.74% | 2.27% | 2.04% | 0.03797 |
| RP | 11.92% | 9.76% | 12.30% | 6.89% | 9.52% | 7.52% | 7.94% | 8.61% | 7.16% | 5.99% | 6.97% | 5.42% | 0.02585 |
| MV | 88.08% | 11.92% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.04661 |
| BL | 90.92% | 9.08% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.04768 |
| PPP-Two | 8.43% | 8.34% | 8.33% | 8.32% | 8.33% | 8.32% | 8.33% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | 0.02595 |
| PPP-Three | 8.48% | 8.36% | 8.30% | 8.25% | 8.34% | 8.30% | 8.29% | 8.35% | 8.41% | 8.24% | 8.37% | 8.30% | 0.02593 |
| PPP-Six | 16.37% | 11.03% | 9.97% | 3.91% | 6.54% | 6.32% | 3.78% | 9.84% | 14.24% | 3.99% | 8.22% | 5.79% | 0.02350 |

**Table 3.105.** Expanding window 29 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-04-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-05-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.05051 |
| MC | 51.35% | 9.77% | 8.51% | 1.65% | 7.75% | 3.38% | 4.65% | 4.89% | 2.01% | 1.73% | 2.27% | 2.03% | 0.02468 |
| RP | 11.90% | 9.77% | 12.32% | 6.89% | 9.51% | 7.52% | 7.93% | 8.61% | 7.17% | 6.00% | 6.97% | 5.41% | 0.04460 |
| MV | 89.36% | 10.64% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.01171 |
| BL | 90.77% | 9.23% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.01088 |
| PPP-Two | 8.45% | 8.34% | 8.34% | 8.31% | 8.34% | 8.32% | 8.32% | 8.33% | 8.31% | 8.31% | 8.32% | 8.32% | 0.05044 |
| PPP-Three | 8.52% | 8.36% | 8.31% | 8.23% | 8.35% | 8.30% | 8.28% | 8.35% | 8.43% | 8.23% | 8.37% | 8.28% | 0.05044 |
| PPP-Six | 17.14% | 11.70% | 10.15% | 4.37% | 6.84% | 6.25% | 4.27% | 9.98% | 14.09% | 1.72% | 9.00% | 4.51% | 0.04555 |

**Table 3.106.** Expanding window 30 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-05-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-06-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.01019 |
| MC | 51.43% | 9.78% | 8.49% | 1.65% | 7.72% | 3.37% | 4.64% | 4.88% | 2.01% | 1.73% | 2.26% | 2.03% | 0.00300 |
| RP | 11.93% | 9.75% | 12.35% | 6.89% | 9.51% | 7.52% | 7.93% | 8.60% | 7.16% | 5.99% | 6.96% | 5.40% | -0.01117 |
| MV | 87.53% | 12.47% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.01670 |
| BL | 90.74% | 9.26% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.01805 |
| PPP-Two | 8.45% | 8.34% | 8.33% | 8.31% | 8.34% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | -0.01016 |
| PPP-Three | 8.51% | 8.34% | 8.30% | 8.24% | 8.35% | 8.30% | 8.29% | 8.35% | 8.42% | 8.23% | 8.38% | 8.29% | -0.01021 |
| PPP-Six | 17.21% | 10.37% | 10.53% | 5.80% | 6.57% | 5.98% | 4.65% | 9.60% | 14.84% | 1.97% | 8.55% | 3.94% | -0.01202 |

**Table 3.107.** Expanding window 31 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-06-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-07-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.00876 |
| MC | 51.52% | 9.80% | 8.47% | 1.64% | 7.69% | 3.37% | 4.64% | 4.87% | 2.02% | 1.72% | 2.26% | 2.02% | 0.00284 |
| RP | 11.94% | 9.75% | 12.35% | 6.89% | 9.50% | 7.52% | 7.93% | 8.60% | 7.16% | 6.00% | 6.96% | 5.41% | -0.00841 |
| MV | 88.91% | 11.09% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.01377 |
| BL | 90.68% | 9.32% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.01516 |
| PPP-Two | 8.45% | 8.33% | 8.33% | 8.31% | 8.34% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | -0.00873 |
| PPP-Three | 8.51% | 8.34% | 8.30% | 8.24% | 8.35% | 8.30% | 8.29% | 8.35% | 8.42% | 8.23% | 8.38% | 8.28% | -0.00866 |
| PPP-Six | 17.50% | 9.62% | 10.25% | 5.14% | 6.58% | 6.42% | 4.83% | 9.55% | 14.85% | 2.05% | 8.95% | 4.27% | -0.00503 |

**Table 3.108.** Expanding window 32 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-07-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-08-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.02070 |
| MC | 51.61% | 9.80% | 8.45% | 1.64% | 7.66% | 3.36% | 4.63% | 4.86% | 2.02% | 1.72% | 2.25% | 2.01% | 0.02453 |
| RP | 11.95% | 9.74% | 12.33% | 6.89% | 9.51% | 7.52% | 7.93% | 8.60% | 7.16% | 6.00% | 6.96% | 5.41% | 0.02182 |
| MV | 91.34% | 8.66% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.02977 |
| BL | 90.66% | 9.34% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.02987 |
| PPP-Two | 8.44% | 8.33% | 8.33% | 8.31% | 8.34% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | 0.02071 |
| PPP-Three | 8.50% | 8.33% | 8.30% | 8.25% | 8.35% | 8.31% | 8.29% | 8.35% | 8.43% | 8.24% | 8.37% | 8.29% | 0.02079 |
| PPP-Six | 17.87% | 9.03% | 10.31% | 5.44% | 6.52% | 6.17% | 4.71% | 9.78% | 15.23% | 1.13% | 9.12% | 4.68% | 0.02881 |

**Table 3.109.** Expanding window 33 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-08-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-09-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.02892 |
| MC | 51.71% | 9.81% | 8.42% | 1.63% | 7.62% | 3.35% | 4.62% | 4.85% | 2.02% | 1.71% | 2.25% | 2.00% | -0.03277 |
| RP | 11.95% | 9.73% | 12.33% | 6.89% | 9.51% | 7.53% | 7.94% | 8.61% | 7.14% | 6.00% | 6.97% | 5.41% | -0.02496 |
| MV | 90.98% | 9.02% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.04351 |
| BL | 90.59% | 9.41% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.04328 |
| PPP-Two | 8.44% | 8.33% | 8.33% | 8.32% | 8.34% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | -0.02893 |
| PPP-Three | 8.49% | 8.33% | 8.30% | 8.25% | 8.35% | 8.31% | 8.29% | 8.35% | 8.43% | 8.24% | 8.37% | 8.29% | -0.02897 |
| PPP-Six | 17.07% | 9.30% | 10.20% | 5.63% | 6.09% | 6.32% | 4.42% | 9.76% | 15.57% | 2.01% | 8.51% | 5.12% | -0.02731 |

**Table 3.110.** Expanding window 34 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-09-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-10-29 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.01701 |
| MC | 51.79% | 9.83% | 8.40% | 1.63% | 7.59% | 3.35% | 4.62% | 4.84% | 2.02% | 1.71% | 2.24% | 1.99% | 0.04001 |
| RP | 11.91% | 9.74% | 12.37% | 6.89% | 9.51% | 7.52% | 7.93% | 8.60% | 7.16% | 6.02% | 6.95% | 5.40% | 0.01911 |
| MV | 88.80% | 11.20% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.05940 |
| BL | 90.64% | 9.36% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.06062 |
| PPP-Two | 8.44% | 8.33% | 8.33% | 8.32% | 8.34% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | 0.01706 |
| PPP-Three | 8.48% | 8.33% | 8.31% | 8.25% | 8.35% | 8.31% | 8.29% | 8.34% | 8.44% | 8.24% | 8.37% | 8.29% | 0.01709 |
| PPP-Six | 16.76% | 9.27% | 10.59% | 6.28% | 6.14% | 6.38% | 4.77% | 9.76% | 15.35% | 1.56% | 8.85% | 4.30% | 0.02045 |

**Table 3.111.** Expanding window 35 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-10-29 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-11-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.03767 |
| MC | 51.88% | 9.84% | 8.38% | 1.62% | 7.56% | 3.34% | 4.61% | 4.83% | 2.03% | 1.70% | 2.23% | 1.98% | -0.01991 |
| RP | 11.89% | 9.75% | 12.40% | 6.89% | 9.50% | 7.52% | 7.93% | 8.59% | 7.16% | 6.02% | 6.95% | 5.41% | -0.03485 |
| MV | 90.61% | 9.39% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.00654 |
| BL | 90.14% | 9.86% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.00645 |
| PPP-TWO | 8.44% | 8.33% | 8.34% | 8.32% | 8.34% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | -0.03763 |
| PPP-THREE | 8.48% | 8.34% | 8.32% | 8.25% | 8.34% | 8.31% | 8.29% | 8.34% | 8.43% | 8.24% | 8.37% | 8.29% | -0.03753 |
| PPP-SIX | 16.32% | 8.39% | 11.31% | 6.04% | 6.49% | 6.70% | 5.16% | 9.49% | 15.42% | 0.69% | 8.96% | 5.04% | -0.02930 |

**Table 3.112.** Expanding window 36 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-11-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2021-12-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.03766 |
| MC | 51.98% | 9.85% | 8.35% | 1.61% | 7.52% | 3.34% | 4.60% | 4.82% | 2.03% | 1.70% | 2.23% | 1.97% | 0.04034 |
| RP | 11.91% | 9.77% | 12.39% | 6.88% | 9.49% | 7.51% | 7.93% | 8.58% | 7.16% | 6.00% | 6.97% | 5.42% | 0.03799 |
| MV | 89.99% | 10.01% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.04066 |
| BL | 90.16% | 9.84% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.04069 |
| PPP-Two | 8.43% | 8.34% | 8.34% | 8.32% | 8.34% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | 0.03767 |
| PPP-Three | 8.47% | 8.34% | 8.32% | 8.26% | 8.35% | 8.30% | 8.29% | 8.34% | 8.43% | 8.24% | 8.37% | 8.29% | 0.03770 |
| PPP-Six | 15.90% | 9.55% | 11.81% | 5.93% | 6.11% | 6.66% | 5.01% | 9.33% | 14.69% | 0.62% | 9.06% | 5.33% | 0.03946 |

**Table 3.113.** Expanding window 37 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2021-12-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-01-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.02159 |
| MC | 52.07% | 9.86% | 8.33% | 1.61% | 7.49% | 3.33% | 4.60% | 4.81% | 2.03% | 1.69% | 2.22% | 1.96% | -0.04359 |
| RP | 11.90% | 9.78% | 12.40% | 6.88% | 9.47% | 7.50% | 7.91% | 8.58% | 7.17% | 6.02% | 6.97% | 5.42% | -0.02779 |
| MV | 90.57% | 9.43% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.05650 |
| BL | 90.07% | 9.93% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.05663 |
| PPP-Two | 8.43% | 8.34% | 8.34% | 8.32% | 8.34% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | -0.02163 |
| PPP-Three | 8.48% | 8.34% | 8.32% | 8.25% | 8.35% | 8.30% | 8.29% | 8.34% | 8.42% | 8.24% | 8.37% | 8.29% | -0.02156 |
| PPP-Six | 16.05% | 9.64% | 12.05% | 7.24% | 6.87% | 6.29% | 5.01% | 9.16% | 14.23% | 0.53% | 8.79% | 4.16% | -0.02366 |

**Table 3.114.** Expanding window 38 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-01-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-02-28 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.05002 |
| MC | 52.16% | 9.87% | 8.30% | 1.60% | 7.46% | 3.33% | 4.59% | 4.81% | 2.03% | 1.68% | 2.22% | 1.95% | -0.02696 |
| RP | 11.87% | 9.75% | 12.36% | 6.88% | 9.49% | 7.51% | 7.92% | 8.59% | 7.18% | 6.01% | 6.99% | 5.45% | -0.04071 |
| MV | 91.48% | 8.52% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.02593 |
| BL | 90.27% | 9.73% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.02509 |
| PPP-Two | 8.43% | 8.34% | 8.33% | 8.32% | 8.33% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | -0.04997 |
| PPP-Three | 8.49% | 8.33% | 8.33% | 8.25% | 8.35% | 8.30% | 8.29% | 8.34% | 8.42% | 8.24% | 8.37% | 8.29% | -0.04964 |
| PPP-Six | 16.56% | 9.75% | 11.68% | 5.88% | 7.58% | 6.56% | 5.48% | 9.32% | 14.31% | 0.01% | 9.22% | 3.66% | -0.01740 |

**Table 3.115.** Expanding window 39 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-02-28 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-03-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.02402 |
| MC | 52.24% | 9.88% | 8.27% | 1.60% | 7.43% | 3.32% | 4.58% | 4.80% | 2.04% | 1.67% | 2.21% | 1.95% | 0.01782 |
| RP | 11.85% | 9.80% | 12.37% | 6.88% | 9.51% | 7.49% | 7.91% | 8.62% | 7.17% | 5.89% | 7.02% | 5.48% | 0.01665 |
| MV | 89.04% | 10.96% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.02390 |
| BL | 90.19% | 9.81% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.02508 |
| PPP-TWO | 8.44% | 8.33% | 8.34% | 8.32% | 8.34% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | 0.02401 |
| PPP-THREE | 8.50% | 8.32% | 8.34% | 8.25% | 8.36% | 8.31% | 8.30% | 8.34% | 8.41% | 8.23% | 8.36% | 8.28% | 0.02396 |
| PPP-SIX | 16.52% | 8.51% | 12.07% | 5.18% | 7.68% | 7.03% | 5.77% | 9.06% | 13.92% | 0.00% | 9.00% | 5.26% | 0.01692 |

**Table 3.116.** Expanding window 40 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-03-31 | | | | | | | | | | | | Out-of-sample simulation |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-04-29 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.07595 |
| MC | 52.33% | 9.89% | 8.24% | 1.59% | 7.40% | 3.31% | 4.57% | 4.80% | 2.04% | 1.67% | 2.21% | 1.94% | -0.08689 |
| RP | 11.84% | 9.82% | 12.38% | 6.89% | 9.52% | 7.50% | 7.91% | 8.61% | 7.17% | 5.88% | 7.02% | 5.47% | -0.07854 |
| MV | 92.08% | 7.92% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.09322 |
| BL | 90.04% | 9.96% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.09352 |
| PPP-Two | 8.44% | 8.33% | 8.34% | 8.32% | 8.34% | 8.32% | 8.32% | 8.33% | 8.32% | 8.32% | 8.32% | 8.32% | -0.07597 |
| PPP-Three | 8.50% | 8.33% | 8.33% | 8.25% | 8.36% | 8.32% | 8.30% | 8.34% | 8.40% | 8.23% | 8.37% | 8.27% | -0.07603 |
| PPP-Six | 15.64% | 8.29% | 12.43% | 6.52% | 8.00% | 7.00% | 6.37% | 8.58% | 12.84% | 0.01% | 9.10% | 5.20% | -0.08428 |

**Table 3.117.** Expanding window 41 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-04-29 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-05-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.02377 |
| MC | 52.41% | 9.90% | 8.22% | 1.59% | 7.37% | 3.31% | 4.57% | 4.79% | 2.04% | 1.66% | 2.21% | 1.93% | 0.01189 |
| RP | 11.76% | 9.77% | 12.25% | 6.91% | 9.58% | 7.52% | 7.94% | 8.61% | 7.22% | 5.98% | 7.01% | 5.46% | 0.02146 |
| MV | 92.78% | 7.22% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00259 |
| BL | 90.29% | 9.71% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00346 |
| PPP-Two | 8.44% | 8.33% | 8.34% | 8.32% | 8.34% | 8.32% | 8.33% | 8.32% | 8.32% | 8.31% | 8.32% | 8.31% | 0.02374 |
| PPP-Three | 8.51% | 8.31% | 8.34% | 8.25% | 8.36% | 8.32% | 8.30% | 8.33% | 8.41% | 8.23% | 8.37% | 8.27% | 0.02358 |
| PPP-Six | 15.56% | 8.39% | 11.82% | 6.22% | 7.96% | 7.54% | 6.45% | 8.48% | 12.78% | 0.01% | 9.88% | 4.89% | 0.01320 |

**Table 3.118.** Expanding window 42 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-05-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-06-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.08923 |
| MC | 52.48% | 9.90% | 8.20% | 1.58% | 7.35% | 3.30% | 4.56% | 4.79% | 2.05% | 1.65% | 2.21% | 1.93% | -0.07894 |
| RP | 11.77% | 9.76% | 12.24% | 6.91% | 9.58% | 7.51% | 7.94% | 8.62% | 7.24% | 5.97% | 7.01% | 5.45% | -0.08555 |
| MV | 91.56% | 8.44% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.07516 |
| BL | 90.28% | 9.72% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.07326 |
| PPP-Two | 8.45% | 8.33% | 8.34% | 8.32% | 8.34% | 8.33% | 8.33% | 8.32% | 8.32% | 8.31% | 8.31% | 8.31% | -0.08922 |
| PPP-Three | 8.52% | 8.31% | 8.34% | 8.25% | 8.36% | 8.33% | 8.31% | 8.33% | 8.40% | 8.22% | 8.37% | 8.25% | -0.08928 |
| PPP-Six | 16.06% | 7.28% | 12.17% | 5.76% | 7.94% | 7.84% | 6.79% | 8.49% | 13.17% | 0.52% | 9.71% | 4.27% | -0.09514 |

**Table 3.119.** Expanding window 43 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-06-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-07-29 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.02573 |
| MC | 52.54% | 9.92% | 8.18% | 1.58% | 7.32% | 3.30% | 4.55% | 4.79% | 2.05% | 1.65% | 2.20% | 1.92% | 0.05587 |
| RP | 11.70% | 9.96% | 12.17% | 6.87% | 9.56% | 7.47% | 7.92% | 8.58% | 7.27% | 6.09% | 6.99% | 5.41% | 0.03067 |
| MV | 84.98% | 15.02% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.06653 |
| BL | 89.82% | 10.18% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.07318 |
| PPP-Two | 8.45% | 8.33% | 8.34% | 8.32% | 8.34% | 8.33% | 8.33% | 8.32% | 8.31% | 8.31% | 8.31% | 8.30% | 0.02582 |
| PPP-Three | 8.52% | 8.31% | 8.34% | 8.26% | 8.35% | 8.33% | 8.31% | 8.34% | 8.40% | 8.22% | 8.37% | 8.25% | 0.02604 |
| PPP-Six | 16.35% | 6.50% | 12.30% | 6.05% | 7.88% | 7.95% | 7.09% | 8.41% | 12.90% | 0.64% | 9.62% | 4.31% | 0.04646 |

**Table 3.120.** Expanding window 44 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-07-29 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-08-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.02146 |
| MC | 52.61% | 9.93% | 8.16% | 1.57% | 7.30% | 3.29% | 4.54% | 4.78% | 2.05% | 1.64% | 2.20% | 1.91% | -0.03917 |
| RP | 11.66% | 10.00% | 12.14% | 6.87% | 9.55% | 7.48% | 7.92% | 8.57% | 7.26% | 6.13% | 7.00% | 5.41% | -0.02754 |
| MV | 89.23% | 10.77% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.04271 |
| BL | 89.15% | 10.85% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.04270 |
| PPP-TWO | 8.45% | 8.33% | 8.34% | 8.32% | 8.34% | 8.33% | 8.33% | 8.32% | 8.31% | 8.31% | 8.32% | 8.30% | -0.02153 |
| PPP-THREE | 8.51% | 8.32% | 8.34% | 8.26% | 8.35% | 8.33% | 8.31% | 8.33% | 8.39% | 8.23% | 8.37% | 8.25% | -0.02165 |
| PPP-SIX | 16.15% | 6.84% | 11.82% | 6.28% | 7.55% | 7.66% | 7.05% | 8.82% | 12.66% | 0.05% | 9.91% | 5.19% | -0.03243 |

**Table 3.121.** Expanding window 45 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-08-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-09-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.09505 |
| MC | 52.68% | 9.95% | 8.14% | 1.57% | 7.27% | 3.28% | 4.54% | 4.78% | 2.06% | 1.64% | 2.20% | 1.91% | -0.09756 |
| RP | 11.65% | 10.00% | 12.13% | 6.87% | 9.54% | 7.47% | 7.91% | 8.57% | 7.28% | 6.16% | 7.00% | 5.43% | -0.09648 |
| MV | 88.96% | 11.04% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.09708 |
| BL | 89.19% | 10.81% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.09710 |
| PPP-Two | 8.44% | 8.33% | 8.34% | 8.32% | 8.34% | 8.33% | 8.33% | 8.32% | 8.31% | 8.31% | 8.32% | 8.31% | -0.09506 |
| PPP-Three | 8.51% | 8.31% | 8.34% | 8.27% | 8.35% | 8.33% | 8.32% | 8.33% | 8.40% | 8.23% | 8.36% | 8.26% | -0.09500 |
| PPP-Six | 15.98% | 7.10% | 12.12% | 6.50% | 7.08% | 7.64% | 7.07% | 8.57% | 12.54% | 0.01% | 9.77% | 5.62% | -0.08932 |

**Table 3.122.** Expanding window 46 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-09-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-10-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.05790 |
| MC | 52.73% | 9.96% | 8.13% | 1.56% | 7.25% | 3.28% | 4.53% | 4.77% | 2.06% | 1.63% | 2.19% | 1.90% | 0.05715 |
| RP | 11.57% | 9.99% | 12.00% | 6.93% | 9.53% | 7.50% | 7.94% | 8.57% | 7.34% | 6.13% | 7.01% | 5.50% | 0.05361 |
| MV | 88.42% | 11.58% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.05977 |
| BL | 89.36% | 10.64% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.06116 |
| PPP-Two | 8.44% | 8.33% | 8.34% | 8.32% | 8.34% | 8.33% | 8.33% | 8.32% | 8.31% | 8.31% | 8.32% | 8.31% | 0.05792 |
| PPP-Three | 8.51% | 8.31% | 8.34% | 8.27% | 8.35% | 8.33% | 8.31% | 8.33% | 8.40% | 8.23% | 8.36% | 8.26% | 0.05787 |
| PPP-Six | 15.60% | 6.78% | 11.54% | 7.13% | 7.45% | 8.03% | 7.32% | 8.54% | 12.75% | 0.00% | 9.36% | 5.48% | 0.05621 |

**Table 3.123.** Expanding window 47 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-10-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-11-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.08289 |
| MC | 52.80% | 9.96% | 8.12% | 1.56% | 7.22% | 3.27% | 4.52% | 4.76% | 2.07% | 1.63% | 2.19% | 1.90% | 0.07190 |
| RP | 11.53% | 10.09% | 12.00% | 6.91% | 9.51% | 7.48% | 7.91% | 8.57% | 7.35% | 6.13% | 7.03% | 5.50% | 0.08537 |
| MV | 93.36% | 6.64% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.05651 |
| BL | 88.76% | 11.24% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.05939 |
| PPP-Two | 8.45% | 8.33% | 8.34% | 8.32% | 8.34% | 8.33% | 8.33% | 8.32% | 8.31% | 8.31% | 8.31% | 8.30% | 0.08290 |
| PPP-Three | 8.51% | 8.31% | 8.35% | 8.27% | 8.35% | 8.34% | 8.31% | 8.33% | 8.39% | 8.23% | 8.36% | 8.26% | 0.08298 |
| PPP-Six | 15.96% | 7.28% | 11.62% | 6.49% | 7.26% | 8.38% | 7.38% | 8.25% | 12.88% | 0.46% | 9.04% | 5.01% | 0.08775 |

**Table 3.124.** Expanding window 48 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-11-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2022-12-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.03449 |
| MC | 52.86% | 9.96% | 8.11% | 1.55% | 7.20% | 3.26% | 4.52% | 4.76% | 2.07% | 1.62% | 2.19% | 1.89% | -0.04002 |
| RP | 11.57% | 10.04% | 11.97% | 6.89% | 9.47% | 7.46% | 7.89% | 8.60% | 7.39% | 6.19% | 6.95% | 5.58% | -0.03103 |
| MV | 91.17% | 8.83% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.05473 |
| BL | 88.85% | 11.15% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.05313 |
| PPP-Two | 8.45% | 8.33% | 8.34% | 8.32% | 8.34% | 8.33% | 8.33% | 8.32% | 8.31% | 8.31% | 8.31% | 8.30% | -0.03449 |
| PPP-Three | 8.51% | 8.30% | 8.34% | 8.27% | 8.36% | 8.34% | 8.31% | 8.34% | 8.39% | 8.22% | 8.36% | 8.25% | -0.03442 |
| PPP-Six | 16.20% | 6.94% | 11.88% | 6.23% | 7.26% | 8.44% | 7.12% | 8.23% | 12.87% | 0.69% | 9.13% | 4.99% | -0.02663 |

**Table 3.125.** Expanding window 49 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2022-12-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-01-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.07134 |
| MC | 52.91% | 9.98% | 8.10% | 1.55% | 7.18% | 3.26% | 4.51% | 4.75% | 2.07% | 1.61% | 2.19% | 1.89% | 0.06515 |
| RP | 11.53% | 10.05% | 11.98% | 6.90% | 9.49% | 7.47% | 7.91% | 8.59% | 7.38% | 6.15% | 6.96% | 5.58% | 0.06994 |
| MV | 88.31% | 11.69% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.06150 |
| BL | 88.60% | 11.40% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.06146 |
| PPP-Two | 8.45% | 8.33% | 8.34% | 8.32% | 8.34% | 8.33% | 8.33% | 8.33% | 8.31% | 8.31% | 8.31% | 8.30% | 0.07133 |
| PPP-Three | 8.51% | 8.30% | 8.33% | 8.28% | 8.36% | 8.34% | 8.32% | 8.34% | 8.39% | 8.22% | 8.36% | 8.24% | 0.07123 |
| PPP-Six | 16.20% | 6.67% | 11.50% | 6.76% | 7.47% | 8.57% | 7.19% | 8.65% | 12.76% | 0.56% | 9.24% | 4.45% | 0.06499 |

**Table 3.126.** Expanding window 50 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-01-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-02-28 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.03202 |
| MC | 52.97% | 9.99% | 8.08% | 1.54% | 7.15% | 3.26% | 4.51% | 4.75% | 2.08% | 1.61% | 2.19% | 1.88% | -0.02651 |
| RP | 11.52% | 10.04% | 11.98% | 6.87% | 9.49% | 7.46% | 7.88% | 8.57% | 7.45% | 6.16% | 6.97% | 5.60% | -0.02967 |
| MV | 87.91% | 12.09% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.02556 |
| BL | 88.59% | 11.41% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.02561 |
| PPP-Two | 8.45% | 8.32% | 8.34% | 8.33% | 8.34% | 8.33% | 8.33% | 8.32% | 8.31% | 8.30% | 8.32% | 8.30% | -0.03198 |
| PPP-Three | 8.51% | 8.29% | 8.33% | 8.29% | 8.36% | 8.34% | 8.32% | 8.34% | 8.40% | 8.22% | 8.36% | 8.24% | -0.03195 |
| PPP-Six | 15.56% | 6.52% | 11.25% | 7.92% | 7.57% | 8.93% | 8.00% | 9.30% | 12.17% | 0.40% | 9.59% | 2.78% | -0.02703 |

**Table 3.127.** Expanding window 51 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-02-28 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-03-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.01920 |
| MC | 53.01% | 10.00% | 8.07% | 1.54% | 7.13% | 3.25% | 4.51% | 4.74% | 2.08% | 1.60% | 2.18% | 1.87% | 0.02601 |
| RP | 11.52% | 10.04% | 11.96% | 6.89% | 9.50% | 7.47% | 7.90% | 8.56% | 7.45% | 6.16% | 6.96% | 5.59% | 0.01990 |
| MV | 87.58% | 12.42% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.03111 |
| BL | 88.65% | 11.35% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.03140 |
| PPP-Two | 8.44% | 8.32% | 8.33% | 8.33% | 8.34% | 8.33% | 8.33% | 8.32% | 8.32% | 8.30% | 8.32% | 8.30% | 0.01922 |
| PPP-Three | 8.51% | 8.29% | 8.32% | 8.30% | 8.36% | 8.34% | 8.32% | 8.35% | 8.40% | 8.21% | 8.36% | 8.25% | 0.01919 |
| PPP-Six | 15.16% | 6.54% | 10.76% | 8.73% | 7.06% | 8.60% | 7.86% | 9.16% | 12.75% | 0.00% | 9.31% | 4.08% | 0.01881 |

**Table 3.128.** Expanding window 52 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-03-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-04-28 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.02495 |
| MC | 53.07% | 10.01% | 8.06% | 1.54% | 7.11% | 3.25% | 4.51% | 4.74% | 2.08% | 1.59% | 2.18% | 1.87% | 0.01866 |
| RP | 11.51% | 10.05% | 11.95% | 6.89% | 9.51% | 7.47% | 7.89% | 8.57% | 7.46% | 6.16% | 6.96% | 5.59% | 0.02349 |
| MV | 88.53% | 11.47% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.01391 |
| BL | 88.61% | 11.39% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.01391 |
| PPP-Two | 8.44% | 8.32% | 8.33% | 8.33% | 8.34% | 8.33% | 8.33% | 8.33% | 8.32% | 8.30% | 8.32% | 8.30% | 0.02493 |
| PPP-Three | 8.51% | 8.29% | 8.31% | 8.29% | 8.36% | 8.34% | 8.31% | 8.34% | 8.40% | 8.21% | 8.37% | 8.26% | 0.02492 |
| PPP-Six | 14.99% | 6.39% | 10.33% | 9.03% | 7.44% | 8.69% | 8.12% | 9.36% | 12.34% | 0.01% | 9.19% | 4.13% | 0.02245 |

**Table 3.129.** Expanding window 53 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-04-28 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-05-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.03637 |
| MC | 53.11% | 10.02% | 8.05% | 1.53% | 7.09% | 3.25% | 4.51% | 4.73% | 2.08% | 1.59% | 2.18% | 1.86% | -0.01817 |
| RP | 11.51% | 10.05% | 11.95% | 6.89% | 9.50% | 7.47% | 7.89% | 8.57% | 7.45% | 6.16% | 6.97% | 5.59% | -0.03429 |
| MV | 88.73% | 11.27% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.00512 |
| BL | 88.57% | 11.43% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.00523 |
| PPP-Two | 8.44% | 8.31% | 8.33% | 8.33% | 8.34% | 8.33% | 8.33% | 8.33% | 8.32% | 8.31% | 8.32% | 8.31% | -0.03635 |
| PPP-Three | 8.51% | 8.28% | 8.32% | 8.29% | 8.35% | 8.33% | 8.31% | 8.35% | 8.41% | 8.22% | 8.37% | 8.27% | -0.03636 |
| PPP-Six | 15.36% | 6.35% | 10.35% | 8.30% | 6.92% | 8.93% | 7.45% | 9.27% | 12.79% | 1.19% | 9.34% | 3.74% | -0.03584 |

**Table 3.130.** Expanding window 54 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-05-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-06-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.04940 |
| MC | 53.17% | 10.03% | 8.04% | 1.53% | 7.07% | 3.24% | 4.50% | 4.72% | 2.09% | 1.58% | 2.17% | 1.85% | 0.05025 |
| RP | 11.53% | 10.02% | 12.02% | 6.88% | 9.47% | 7.46% | 7.87% | 8.55% | 7.47% | 6.17% | 6.94% | 5.60% | 0.04688 |
| MV | 91.25% | 8.75% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.05544 |
| BL | 88.60% | 11.40% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.05324 |
| PPP-Two | 8.44% | 8.32% | 8.32% | 8.33% | 8.34% | 8.33% | 8.33% | 8.33% | 8.32% | 8.31% | 8.32% | 8.31% | 0.04943 |
| PPP-Three | 8.50% | 8.29% | 8.32% | 8.29% | 8.35% | 8.33% | 8.30% | 8.35% | 8.42% | 8.22% | 8.37% | 8.28% | 0.04949 |
| PPP-Six | 15.38% | 5.91% | 10.52% | 8.36% | 6.77% | 8.73% | 6.94% | 9.53% | 13.08% | 1.49% | 9.56% | 3.72% | 0.05498 |

**Table 3.131.** Expanding window 55 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-06-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-07-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.03997 |
| MC | 53.24% | 10.03% | 8.02% | 1.52% | 7.04% | 3.24% | 4.50% | 4.72% | 2.09% | 1.57% | 2.17% | 1.85% | 0.03249 |
| RP | 11.51% | 10.07% | 12.01% | 6.86% | 9.48% | 7.46% | 7.86% | 8.55% | 7.48% | 6.21% | 6.94% | 5.58% | 0.03744 |
| MV | 93.68% | 6.32% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.03143 |
| BL | 88.35% | 11.65% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.03208 |
| PPP-Two | 8.43% | 8.33% | 8.33% | 8.32% | 8.33% | 8.32% | 8.32% | 8.33% | 8.33% | 8.30% | 8.32% | 8.32% | 0.03995 |
| PPP-Three | 8.50% | 8.30% | 8.32% | 8.28% | 8.35% | 8.32% | 8.29% | 8.35% | 8.43% | 8.21% | 8.36% | 8.30% | 0.03993 |
| PPP-Six | 14.73% | 6.48% | 10.35% | 8.32% | 7.16% | 8.46% | 7.14% | 9.75% | 12.77% | 0.00% | 9.73% | 5.11% | 0.03670 |

**Table 3.132.** Expanding window 56 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2018-07-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-08-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.04818 |
| MC | 53.31% | 10.04% | 8.01% | 1.52% | 7.02% | 3.23% | 4.50% | 4.71% | 2.09% | 1.57% | 2.17% | 1.84% | -0.03391 |
| RP | 11.51% | 10.07% | 12.02% | 6.86% | 9.47% | 7.47% | 7.86% | 8.55% | 7.48% | 6.21% | 6.92% | 5.58% | -0.04667 |
| MV | 93.17% | 6.83% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.02140 |
| BL | 88.34% | 11.65% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.02390 |
| PPP-Two | 8.44% | 8.33% | 8.33% | 8.32% | 8.33% | 8.32% | 8.32% | 8.33% | 8.33% | 8.31% | 8.32% | 8.33% | -0.04815 |
| PPP-Three | 8.50% | 8.30% | 8.32% | 8.27% | 8.34% | 8.32% | 8.30% | 8.34% | 8.44% | 8.21% | 8.36% | 8.29% | -0.04818 |
| PPP-Six | 14.55% | 6.51% | 10.27% | 7.93% | 7.09% | 8.27% | 7.19% | 9.55% | 12.88% | 0.01% | 9.47% | 6.28% | -0.04948 |

**Table 3.133.** Expanding window 57 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-08-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-09-29 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.03217 |
| MC | 53.38% | 10.04% | 7.99% | 1.52% | 6.99% | 3.23% | 4.50% | 4.70% | 2.10% | 1.56% | 2.16% | 1.83% | -0.04041 |
| RP | 11.52% | 10.04% | 12.01% | 6.86% | 9.47% | 7.47% | 7.87% | 8.55% | 7.49% | 6.23% | 6.90% | 5.58% | -0.03315 |
| MV | 95.23% | 4.77% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.04796 |
| BL | 88.44% | 11.56% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.04514 |
| PPP-Two | 8.44% | 8.33% | 8.33% | 8.32% | 8.33% | 8.32% | 8.32% | 8.33% | 8.33% | 8.31% | 8.32% | 8.31% | -0.03219 |
| PPP-Three | 8.51% | 8.31% | 8.33% | 8.27% | 8.34% | 8.32% | 8.29% | 8.34% | 8.44% | 8.22% | 8.35% | 8.29% | -0.03215 |
| PPP-Six | 14.69% | 6.94% | 10.51% | 7.88% | 6.86% | 8.33% | 7.34% | 9.29% | 13.26% | 0.57% | 9.08% | 5.26% | -0.03225 |

**Table 3.134.** Expanding window 58 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-09-20 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-10-31 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | -0.02769 |
| MC | 53.44% | 10.04% | 7.99% | 1.51% | 6.97% | 3.22% | 4.49% | 4.69% | 2.10% | 1.56% | 2.15% | 1.83% | -0.02872 |
| RP | 11.50% | 10.05% | 11.99% | 6.87% | 9.48% | 7.46% | 7.87% | 8.55% | 7.51% | 6.24% | 6.90% | 5.59% | -0.03006 |
| MV | 93.88% | 6.12% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | -0.02285 |
| BL | 88.31% | 11.61% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.08% | 0.00% | 0.00% | 0.00% | -0.02342 |
| PPP-Two | 8.45% | 8.34% | 8.33% | 8.31% | 8.33% | 8.32% | 8.32% | 8.33% | 8.33% | 8.30% | 8.32% | 8.31% | -0.02771 |
| PPP-Three | 8.51% | 8.33% | 8.33% | 8.26% | 8.34% | 8.31% | 8.29% | 8.34% | 8.44% | 8.21% | 8.35% | 8.28% | -0.02779 |
| PPP-Six | 14.99% | 7.37% | 10.64% | 7.44% | 6.77% | 8.18% | 7.02% | 9.00% | 13.70% | 1.40% | 8.74% | 4.77% | -0.03358 |

**Table 3.135.** Expanding window 59 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-10-31 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-11-30 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.07968 |
| MC | 53.50% | 10.05% | 7.98% | 1.51% | 6.95% | 3.21% | 4.49% | 4.69% | 2.10% | 1.55% | 2.15% | 1.82% | 0.07984 |
| RP | 11.50% | 10.05% | 11.97% | 6.87% | 9.48% | 7.46% | 7.87% | 8.54% | 7.51% | 6.26% | 6.91% | 5.59% | 0.07878 |
| MV | 94.46% | 5.54% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.08228 |
| BL | 88.43% | 11.57% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.07886 |
| PPP-Two | 8.45% | 8.34% | 8.33% | 8.31% | 8.33% | 8.32% | 8.32% | 8.33% | 8.33% | 8.30% | 8.32% | 8.31% | 0.07968 |
| PPP-Three | 8.52% | 8.36% | 8.33% | 8.26% | 8.34% | 8.31% | 8.29% | 8.33% | 8.44% | 8.20% | 8.35% | 8.28% | 0.07965 |
| PPP-Six | 14.75% | 8.14% | 10.67% | 7.17% | 6.81% | 8.25% | 6.94% | 8.77% | 13.70% | 1.16% | 8.59% | 5.05% | 0.08090 |

**Table 3.136.** Expanding window 60 with short-selling constraint.

| | Weight allocation from in-sample modelling starting on 2006-01-31 and ending on 2023-11-30 | | | | | | | | | | | | Out-of-sample |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Methodology | SPX | SHANGHAI | NKY | FTSEMIB | FTSE | DAX | CAC | TSX | SENSEX | IMOEX | JALSH | IBOV | 2023-12-29 |
| 1/N | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 8.33% | 0.04173 |
| MC | 53.57% | 10.04% | 7.98% | 1.51% | 6.92% | 3.21% | 4.49% | 4.68% | 2.11% | 1.54% | 2.14% | 1.82% | 0.04001 |
| RP | 11.46% | 10.10% | 11.88% | 6.87% | 9.49% | 7.44% | 7.87% | 8.53% | 7.55% | 6.30% | 6.93% | 5.58% | 0.04133 |
| MV | 95.75% | 4.25% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.04087 |
| BL | 87.95% | 11.81% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.00% | 0.23% | 0.00% | 0.00% | 0.00% | 0.03666 |
| PPP-Two | 8.45% | 8.36% | 8.33% | 8.31% | 8.33% | 8.32% | 8.32% | 8.33% | 8.33% | 8.28% | 8.32% | 8.31% | 0.04173 |
| PPP-Three | 8.52% | 8.37% | 8.33% | 8.25% | 8.33% | 8.32% | 8.29% | 8.33% | 8.45% | 8.19% | 8.36% | 8.26% | 0.04182 |
| PPP-Six | 14.50% | 9.48% | 10.31% | 6.91% | 7.24% | 8.30% | 7.06% | 8.76% | 13.29% | 0.33% | 8.87% | 4.97% | 0.04707 |

**Figure 3.27.** The plot of the time-varying correlation between S&P 500 and SHANGHAI.

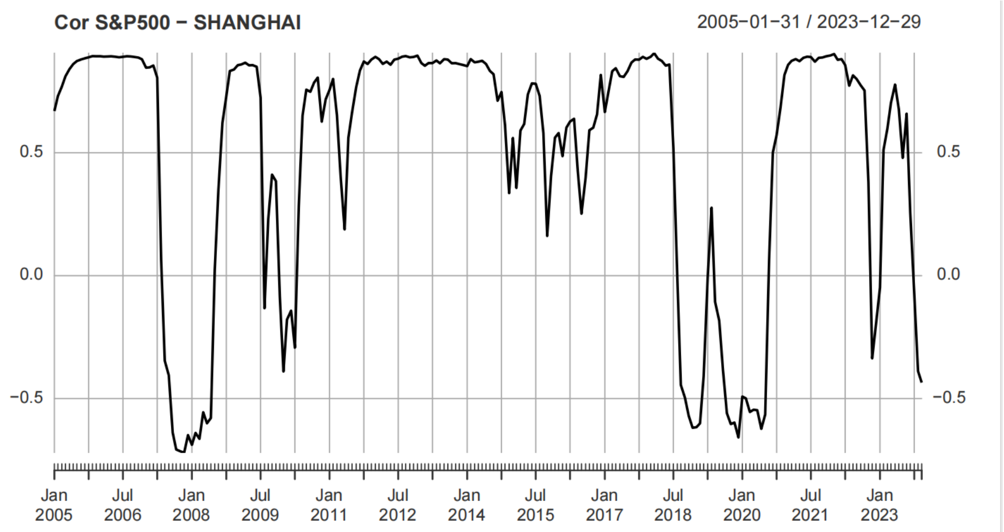


**Figure 3.28.** The plot of the time-varying correlation between S&P 500 and Nikkei 225.

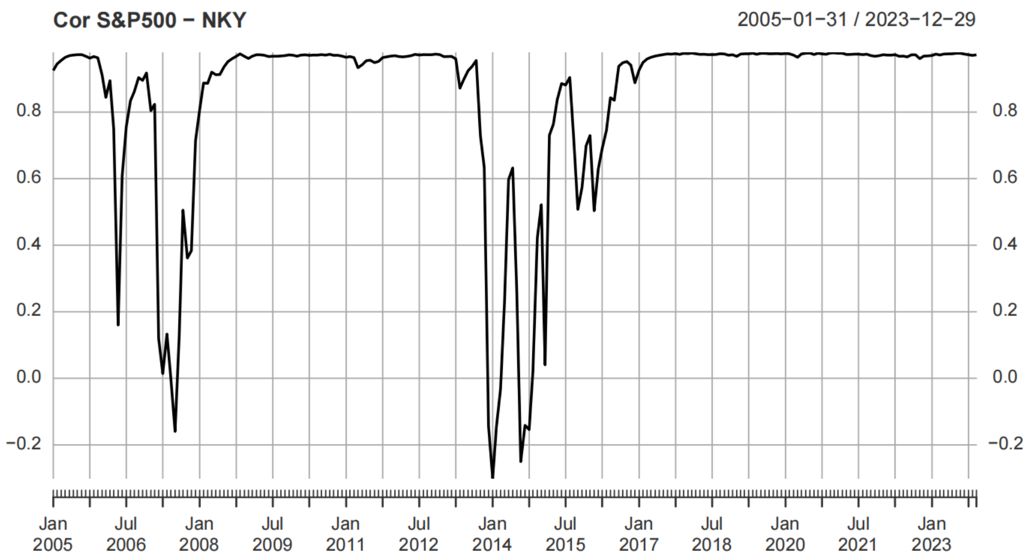

**Figure 3.29.** The plot of the time-varying correlation between S&P 500 and FTSE MIB.

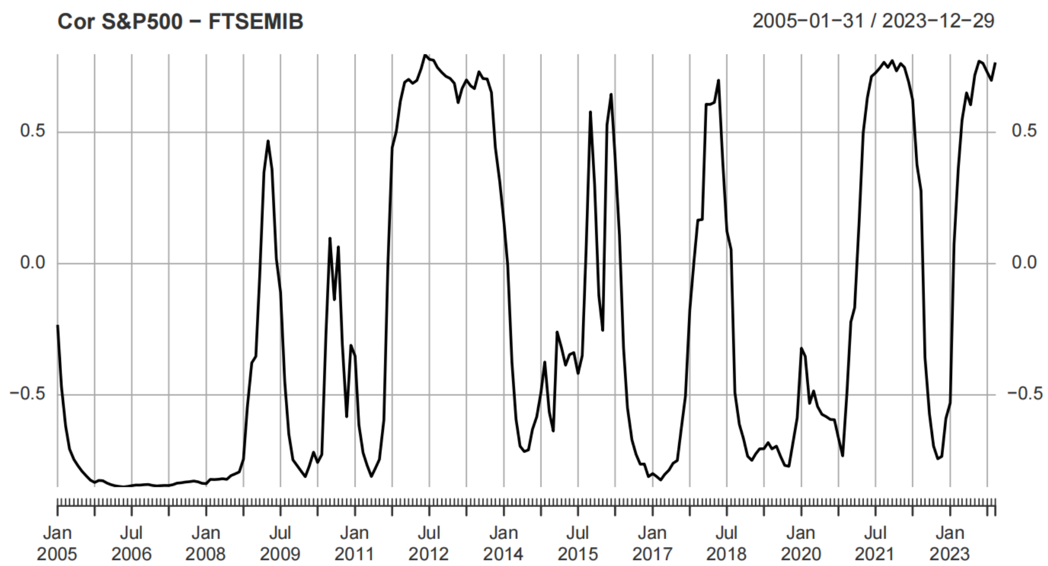


**Figure 3.30.** The plot of the time-varying correlation between S&P 500 and FTSE.

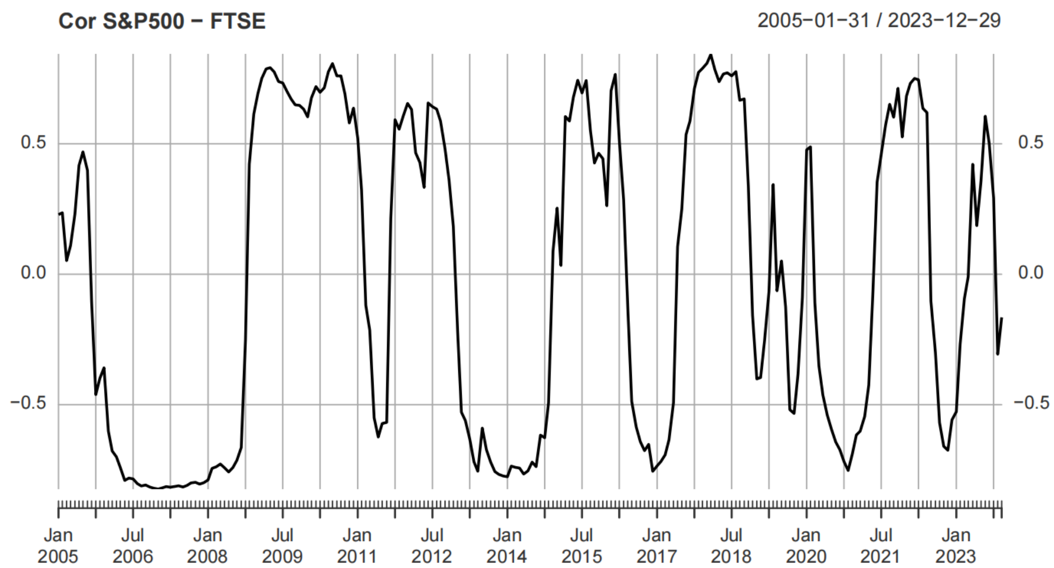

**Figure 3.31.** The plot of the time-varying correlation between S&P 500 and DAX.

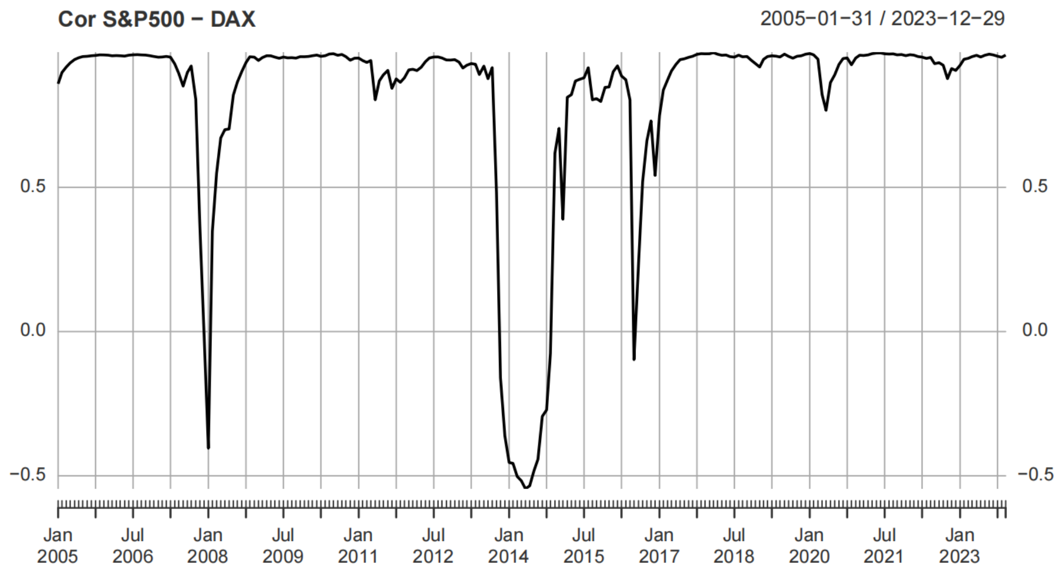


**Figure 3.32.** The plot of the time-varying correlation between S&P 500 and CAC 40.

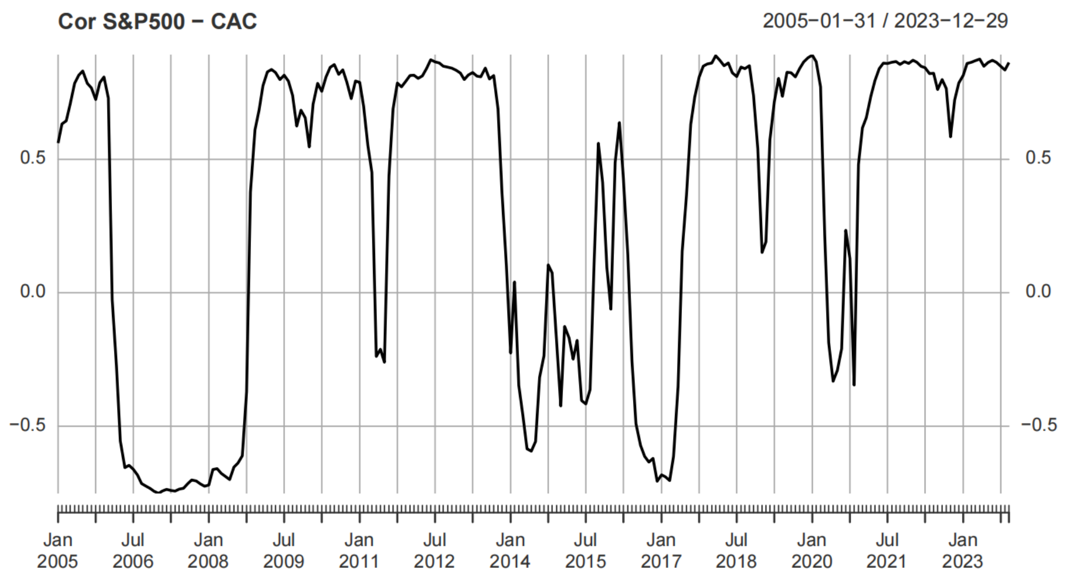

**Figure 3.33.** The plot of the time-varying correlation between S&P 500 and TSX.

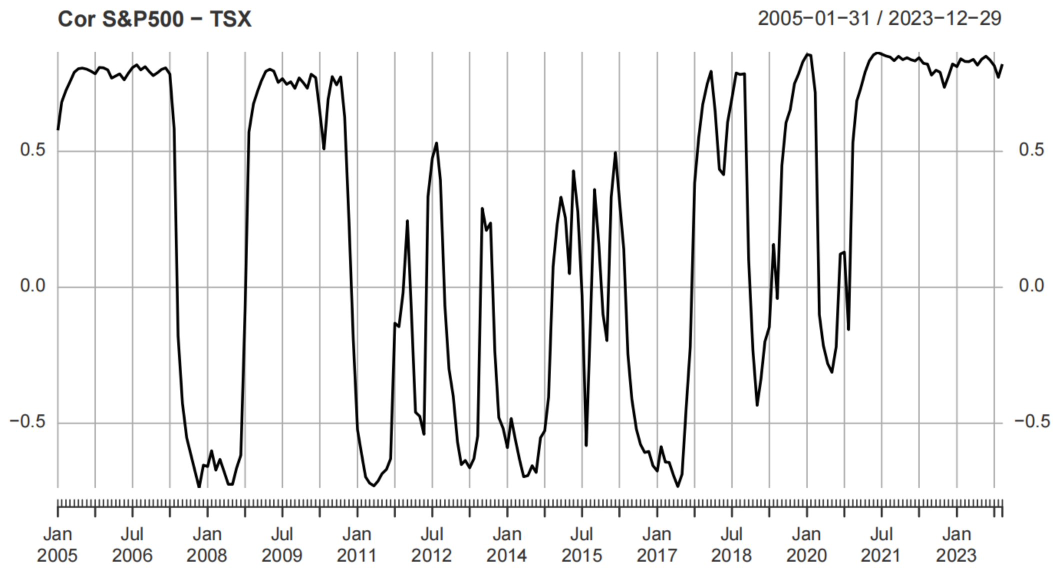


**Figure 3.34.** The plot of the time-varying correlation between S&P 500 and SENSEX.

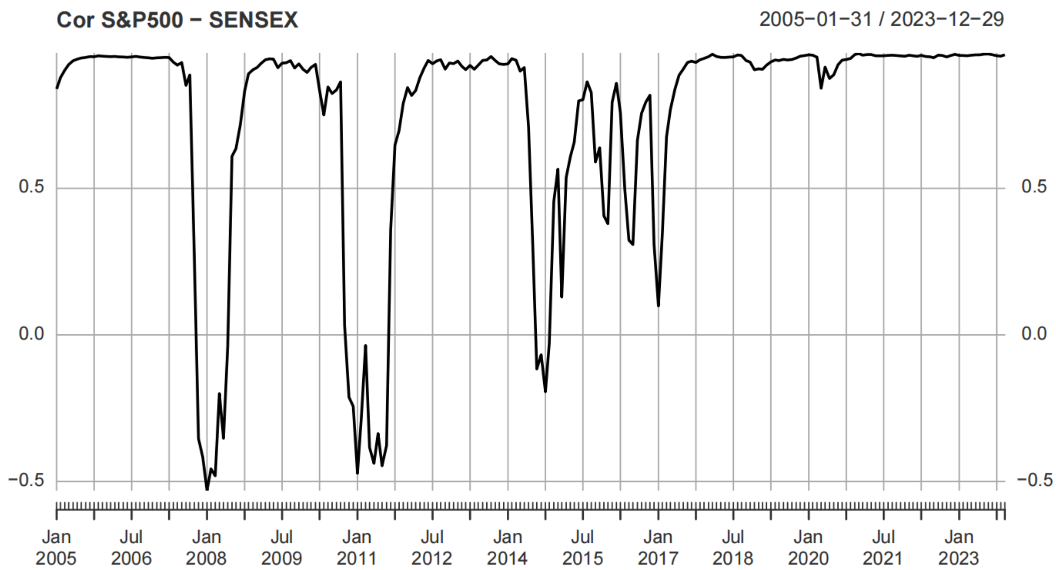

**Figure 3.35.** The plot of the time-varying correlation between S&P 500 and IMOEX.

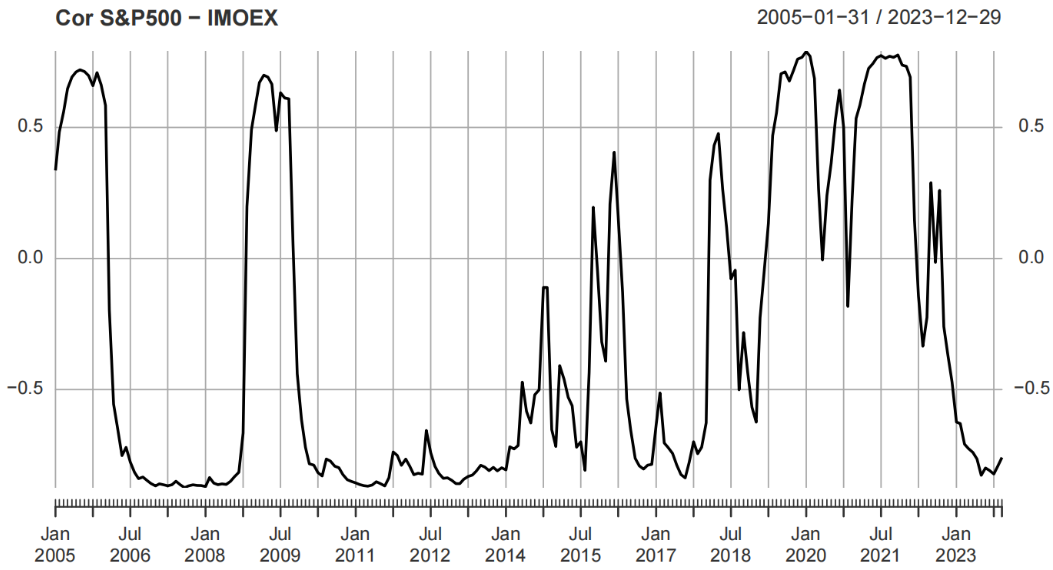


**Figure 3.36.** The plot of the time-varying correlation between S&P 500 and JALSH.

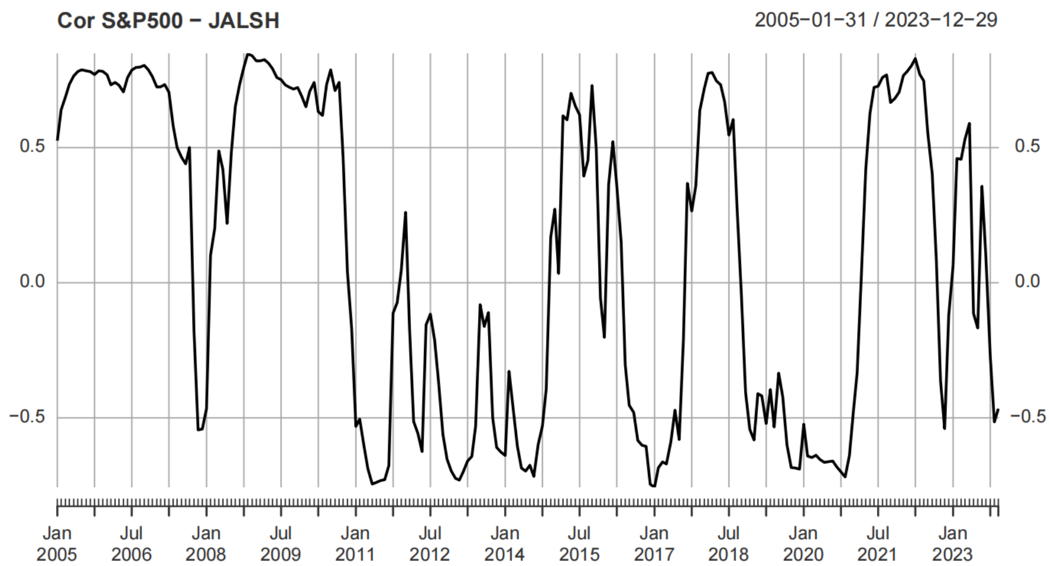

**Figure 3.37.** The plot of the time-varying correlation between S&P 500 and IBOV.