The relationship between concentration and realised volatility: an empirical investigation of the FTSE 100 Index

January 1984 through March 2003

Isaac T. Tabner

This thesis is provided in fulfilment of the requirements of the degree of Doctor of Philosophy at The University of Stirling

Acknowledgements

The author is grateful to the University of Stirling for providing financial support for this research. I would also like to thank all those in the department of Accounting, Finance and Law, and the Faculty of Management, who provided initial support and encouragement in my application for funding. Thanks are also due to all members of the Accounting, Finance and Law department for providing ongoing support throughout my time as a PhD student and more recently as a full time member of staff. In addition, Professor David Stopforth provided invaluable suggestions regarding the layout of the introduction and conclusions. Specific thanks are also due to Jim and Patricia Steeley for their initial feedback and later for honing my motivation to complete this project.

I would like to offer the greatest and most unreserved thanks to my principal supervisor, Dr Kevin Campbell, for his countless invaluable comments, support and patience, despite the volume of manuscript that I, and others, placed on his desk during his supervision.

Finally I would like to thank my family and friends for hearing and contributing to my ideas, encouraging me to think, to persevere, and to learn at every opportunity.

The author retains responsibility for any remaining errors in this document.

Dedication

To Noriko.

Thank you for continuing to be my company throughout this endeavour.

Abstract

Few studies have examined the impact of portfolio concentration upon the realised volatility of stock index portfolios, such as the FTSE 100. Instead, previous research has focused upon diversification across industries, across geographic regions and across different firms. The present study addresses this imbalance by calculating the daily time series of four concentration metrics for the FTSE 100 Index over the period from January 1984 through March 2003. In addition, the value weighted variance covariance matrix (VCM) of daily FTSE 100 Index constituent returns is decomposed into four sub-components: two from the diagonal elements and two from the off-diagonal elements of the VCM. These consist of the average variance of constituent returns, represented by the sum of diagonal elements in the VCM, and the average covariance represented by the sum of off-diagonal elements in the VCM. The value weighted average variance (VAV) and covariance (VAC) are each subdivided into the equally weighted average variance (EAV) the equally weighted average covariance (EAC) and incremental components that represent the difference between the respective value-weighted and equally weighted averages. These are referred to as the incremental average variance (IAV) and the incremental average covariance (IAC) respectively. The incremental average variance and the incremental average covariance are then combined, additively, to produce the incremental realised variance (IRV) of the FTSE 100 Index.

The incremental average covariance and the incremental realised variance are found to be negative during the 1987 crash and the 1992 ERM crisis. They are also negative for a substantial part of the study period, even when concentration was at its highest level. Hence the findings of the study are consistent with the notion that the value weighted, and hence concentrated, FTSE 100 Index portfolio is generally less risky than a hypothetical equally weighted portfolio of FTSE 100 Index constituents. Furthermore, increases in concentration tend to precede decreases in incremental realised volatility and increases in the equally weighted components of the realised VCM. The results have important implications for portfolio managers concerned with the effect of changing portfolio weights upon portfolio volatility. They are also relevant to passive investors concerned about the effects of increased concentration upon their benchmark indices, and to providers of stock market indices.

Table of Contents

List of Tables

List of Charts

Glossary of terms and abbreviations

Abbreviation	Term	Meaning
VCM	Variance covariance matrix	The sum of the VCM elements of portfolio constituent returns constitutes the portfolio variance
VWV	Value weighted portfolio variance	Equivalent to the sum of the value weighted VCM
EWV	Equally weighted portfolio variance	Equivalent to the sum of the equally weighted VCM
VAV	Value weighted average variance of portfolio constituent returns.	Equivalent to the summed diagonal elements in the value weighted VCM
VAC	Value weighted average covariance of portfolio constituent returns.	Equivalent to the summed off-diagonal elements in the value weighted VCM
EAV	Equally weighted average variance of portfolio constituent returns.	Equivalent to the summed diagonal elements in the equally weighted VCM
EAC	Equally weighted average covariance of portfolio constituent returns.	Equivalent to the summed off-diagonal elements in the equally weighted VCM
IAV	Incremental average variance of portfolio constituent returns.	Equivalent to the VAV minus the EAV
IAC	Incremental average covariance of portfolio constituent returns.	Equivalent to VAC minus EAC
IRV	Incremental realised volatility of portfolio returns.	Equivalent to the incremental average variance, IAV, plus the incremental average covariance, IAC
TVCM	Total variance covariance matrix	The sum of the absolute values of the EAV, EAC, IAV and IAC
SEAV	Standardised equally weighted average variance	The EAV divided by the TVCM
SIAV	Standardised incremental average variance	Equivalent to the IAV divided by the TVCM
SEAC	Standardised equally weighted average covariance	Equivalent to the EAV divided by the TVCM
SIAC	Standardised incremental average covariance	Equivalent to the IAC divided by the TVCM
SIRV	Standardised incremental realised variance	Equivalent to the IRV divided by the TVCM
RV	Realised variance of portfolio returns	Should be identical to the VWV
RS	Realised standard deviation of portfolio returns	Equivalent to the square root of RV
H	The Herfindahl Index of concentration	The sum of the squared constituent weights
DH	The differenced Herfindahl Index	H at time t minus H at time t - T
R	The reciprocal of Hannah and Kay's Index of concentration	One divided by the sum of the square root of constituent weights
DR	R differenced	R at time t minus R at time t - T
V	The variance of the logarithm of firm size metric of concentration	A concentration index like H and R that is relatively unbiased by either large, or small firms, when the distribution of firm size approximates to the log normal
DV	V differenced	V at time t minus V at time t - T
SK	The skew of firm weights metric of concentration	The sample skewness function in Excel applied to firm weights
DSK	SK differenced	SK at time t minus SK at time t - T

Suffixes are applied to the above abbreviations where appropriate to denote the number of days of returns used to calculate each individual data observation, or the differencing interval in days, in the case of the concentration metrics. For example, the abbreviation RV5 refers to the realised portfolio variance calculated as the sum of five consecutive daily returns squared, based on the procedure outlined in Chapter 5. A more comprehensive list of acronyms is provided in Table 4 on page 137 in Chapter 7.

Chapter 1 – Introduction

1.1 Context and background

In early 2000, Vodafone launched a takeover bid for Mannessmann, a German engineering and telecommunications conglomerate. This was in response to an earlier bid by Mannessmann for Orange, a key competitor to Vodafone in the UK mobile telephone market. On the 31st May 2000, barely two months after successfully completing the bid, Vodafone had a weight[1] in the FTSE 100 Index of 13% and, at that time, the ten largest firms in FTSE 100 Index accounted for more than 52% of its value.[2] Vodafone also accounted for 10.4% of the equity value of all firms listed on the London Stock Exchange (LSE), while the ten largest firms accounted for 41%.[3] Contemporary reports in the financial press, such as Riley (2000), argued that the concentration of the UK's leading market index into just a few sectors (namely banks, oils, telecommunications and pharmaceuticals) meant that investors tracking the index or attempting to hold the UK market portfolio were under-diversified. In addition, the foreword to a 2002 publication by the Association of Investment Management and Research suggested that during the inflation of the "technology bubble", in the latter part of the 1990s, investors in the US market seemed to completely ignore the principles of Modern Portfolio Theory (MPT) by concentrating the majority of their wealth into Technology, Media and Telecommunication (TMT) firms.[4] In fact, Hirschey (2001) reports that 87% of the Nasdaq 100 Index was accounted for by the TMT sectors on the 30th March 2001.

The concentration of the Nasdaq 100 equity market into technology firms, together with the rapid increase in UK equity market concentration during the latter part of the 1990s, is inconsistent with the strategy advised by Campbell et al (2001). One of the conclusions of their paper, based upon an empirical analysis of US stock returns over the period from 1962 through 1997, is that investors in the late 1990s would have achieved greater diversification benefits by holding the stocks of between fifty and one-hundred firms in their portfolios.

---

[1] Henceforth in this thesis portfolio weights are based on the equity market values in relation to the total equity market value of the index portfolio; therefore, there are two types of portfolio – value-weighted and equally weighted.

[2] Datastream

[3] LSE data-file

[4] "Analysis of Equity Investments" 2002 edition, Stowe et al (2002), in the foreword by George H. Troughton.

This is consistent with studies, such as the analysis of the UK market by Poon et al (1992), suggesting that more firms are required to achieve naive diversification than the eight to ten suggested by Evans and Archer (1968). This conclusion of Campbell et al (2001) is based on their findings that the idiosyncratic risk of individual firms listed in the US market had been increasing and that the average correlation of returns between firms had been decreasing over that period. The three years following the peak in concentration of the Nasdaq and the UK markets in March 2000 saw some of the most volatile and predominantly negative stock market returns in recent history; excessive concentration of investors' portfolios into a relatively few industries, or firms, may have exacerbated this.

Currently published research on the volatility of stock indices centres on time series analysis of historic index return data and time series models of option-implied volatility (OIV) against realised volatility. These methods are reviewed extensively by Poon and Granger (2003) and Knight & Satchell (1998). Other studies, such as Roll (1992), evaluate the relative importance of stock market volatility explained by industry specific factors and market volatility caused by country or market specific factors.[5] Results of the latter studies are often contradictory. Some authors find that industry factors are dominant in explaining overall market volatility, while others report a clear dominance of country specific factors. A key feature of Roll's 1992 study, and later works that cite him, such as Heston and Rouwenhorst (1994) and Isakov and Sonney (2002), that is relevant to this study, is their interest in decomposing stock market volatility into subcomponents. These include a country specific component, an industry specific component and, in the case of Campbell et al (2001), a firm specific component. The present study also decomposes market volatility into sub-components but, unlike the above studies, sub-components are derived from the variance covariance matrix (VCM) of FTSE 100 Index constituent returns. Elton and Gruber (1973), King and Wadhwani (1990), Brooks and Persand (2000), Andersen et al (2001a), Kearney and Poti (2003), Elton et al (1978), Malevergne and Sornette (2004), Poon et al (2004) and other studies examine the importance of the correlation between asset returns in determining the sub-components of the VCM and hence the aggregate volatility of a portfolio such as a stock market index. More specifically they investigate how the

[5] Roll (1992), Schwert (1989), Isakov and Sonney (2002), Griffin and Karolyi (1998), Heston and Rouwenhorst (1994)

correlation structure changes through time and how it behaves during extreme market events. These studies are reviewed in Chapter 3.

1.2 Purpose

This thesis explores the relationship between the distribution of investors' capital among the constituents of the FTSE 100 Index and the volatility of the FTSE 100 Index, in terms of how the capital is concentrated into the largest constituent firms. The fundamental questions asked are as follows:

1. Are changes in the level of concentration in the FTSE 100 Index associated with changes in the volatility of the Index?
2. If the answer to the first question is yes, are changes in volatility of the same sign or of an opposite sign to changes in concentration?
3. Finally, if there is evidence of a consistent association between changes in concentration and changes in volatility, can this association be used to produce forecasts of future realised volatility that are more effective than forecasts obtained by naive asymmetric autoregressive models?

Two preliminary issues related to the above three fundamental questions are:

a). How has the level of concentration and volatility evolved over the study period from January 1984 through March 2003?

b). Why might changes in concentration be associated with changes in volatility?

1.2.1 New data obtained to address the thesis questions

In order to answer the thesis questions, new data series have been constructed from the daily time series of the FTSE 100 Index constituent market values and price indices obtainable from Datastream. The FTSE 100 Index was recreated from January 1984 through March 2003. This allowed the calculation of daily values of four concentration metrics: the Hirschman-Herfindahl Index (H), the Sample Skewness of firm weights (SK), the Variance of the logarithm of firm size (V) and the reciprocal of Hannah and Kay's Index (R). Using squared daily returns, a time series of the realised standard deviation and the realised variance of FTSE 100 Index returns was also calculated for non-overlapping periods of five, ten, fifteen and twenty trading-days, over the entire study period. Time series charts of these variables, presented in Chapter 8, allow the evolution of concentration and volatility to be studied over the period from January 1984 through March 2003, as outlined by the first preliminary issue – a.

The null hypothesis of a unit root could not be rejected for any of the four concentration metrics. Therefore in order to model the association between changes in concentration and realised volatility, concentration metrics were differenced over the time interval used to estimate realised volatility, the dependent and autoregressive variable. This allowed the influence of concentration on realised volatility to be determined by comparing asymmetric autoregressive distributed lag (AARDL) models, in which differenced concentration is the distributed lag variable, with naive asymmetric autoregressive (AAR) models of realised volatility. In this study, such models are referred to as “direct models” in order to differentiate them from models in which the individual VCM sub-components are used as the dependent variables. Empirical results obtained from the direct models were ambiguous and did not provide a satisfactory answer to the three fundamental thesis questions. The reason for this unsatisfactory result can be explained by consideration of the second preliminary issue – b: why might changes in concentration be associated with changes in volatility?

For example, if investors concentrate their portfolio capital into the securities of firms whose returns have a high correlation with each other, and the market as a whole, portfolio volatility will increase as a result of the concentration. Likewise, if portfolio capital is concentrated into the securities of firms whose returns have an above average variance, portfolio volatility will increase as a result of the concentration. However, if capital is concentrated into firms with a below average covariance, or a below average variance, portfolio volatility will decrease. Furthermore, if the capital is concentrated into firms with a below average covariance but an above average variance, or vice versa, the two contradictory influences will confound one another and may cancel each other out completely. This makes it difficult to detect an association between changes in concentration and realised volatility using direct models that incorporate aggregate realised volatility as the dependent variable.

In order to explore the relative importance of the above possibilities, thus shedding light on how changes in concentration might influence changes in volatility, an additional group of new data series was derived from the recreated FTSE 100 Index portfolio. They were obtained by decomposing the variance covariance matrix (VCM) of FTSE 100 Index constituent returns into sub-components using a new methodology explained in Chapter 6. This enabled the effect of changes in concentration upon each individual sub-component to be examined in isolation. The new data series are as follows:

1. The equally weighted average variance of constituent returns, i.e. diagonal elements in the equally weighted VCM.
2. The incremental average variance of constituent returns that is conditional upon the portfolio concentration deviating from the lower limit defined by an equally weighted portfolio.
3. The equally weighted average covariance of constituent returns, i.e. off-diagonal elements in the equally weighted VCM.
4. The incremental average covariance of constituent returns that is conditional upon portfolio concentration deviating from the lower limit of an equally weighted portfolio.
5. A series consisting of the combined incremental average variance and incremental average covariance of constituent returns in the VCM, referred to as the incremental realised variance of the FTSE 100 Index.

The VCM sub-components consist of equally weighted average variance and covariance components and incremental average variance and covariance components. The incremental components represent the difference between the equally weighted and value weighted VCM sub-components. The combined incremental components are referred to as the incremental realised variances, which can be either positive or negative.[6] In order to determine the contribution of each of the four sub-components of the VCM, and hence sub-components of realised volatility, to the total realised volatility, a further standardisation procedure was devised and implemented for each sub-component, as detailed in Chapter 6.

The three fundamental thesis questions are then addressed, in part, by analysing the time series properties and descriptive statistics of the sub-components. However, in addition, time series model parameters are estimated for each sub-component and aggregate realised volatility series using the asymmetric autoregressive distributed lag (AARDL) models that are detailed in Chapter 7. The out-of-sample forecasting performance of the general AARDL models are compared to those of naive, asymmetric autoregressive (AAR) models.

1.3 Contribution to the body of knowledge

No published research investigates the relationship between concentration and volatility of a stock index using time series data. Only the study by Roll (1992), reviewed in Chapter 3, has examined the association between index concentration and volatility using a cross section of different indices. This thesis reviews and develops currently published work on volatility modelling and portfolio diversification. It adds a new dimension to existing

[6] Note that because the incremental realised variance is defined here as: the variance of an equally weighted portfolio of FTSE 100 Index constituent returns minus the variance of the market-value-of-equity weighted FTSE 100 Index returns, the incremental realised variance can be positive or negative.

research by focusing on the fundamental characteristics of index construction and index concentration at the company level rather than the industry level. Analysis of the evolution of sub-components of realised volatility extends the work of Campbell et al (2001) who found that the idiosyncratic risk of individual US stock returns increased over the latter part of the twentieth century. It also extends the work of Wei and Zhang (2003) and Kearney and Poti (2003). Wei and Zhang followed up Campbell et al (2001), demonstrating that the value weighted average stock return volatility is less than the equally weighted average stock return volatility, a result that is not inconsistent with the results of this thesis for the UK market. Wei and Zhang also demonstrate that the increase in the average stock return volatility can be explained by increases in the volatility of earnings and the return on equity. Kearney and Poti (2003) carried out similar research to Campbell et al on the Eurostoxx50 Index. These and other relevant studies are reviewed further in Chapter 3.

Considerable variation in volatility has been documented within major indices over time by studies such as Schwert (1989). In addition, markets appear to undergo periods of very high volatility, followed by periods of relatively low volatility, described in the literature as "volatility clustering". Furthermore, negative index returns have been found to lead increases in index volatility: a process described as the asymmetry, or leverage effect documented by Black (1976). The asymmetric autoregressive models employed in this thesis take into account the time varying nature of volatility, the clustering effect and the asymmetry effect in the FTSE 100 Index. The possible affects of contemporaneous and lagged changes in concentration are accounted for by additional distributed lag concentration variables incorporated in more general AARDL models.

The models evaluated in this study identify a number of issues relevant to portfolio managers and those engaged in developing equity market indices. Of particular significance is the behaviour of the incremental realised volatility during extreme market conditions. Poon and Granger (2003) highlight the behaviour of volatility series during extreme market conditions as a topic ripe for further research. The negative incremental average covariance and incremental realised volatility observable during past extreme events, as a result of this study, highlight the importance of portfolio weights in determining realised volatility during events that comprise the tails of the return distribution.

This thesis presents a new computationally efficient method for decomposing the VCM of portfolio returns. The findings will enable more efficient portfolio diversification strategies to be developed by providing a means by which the volatility of a market index can be attributed to the VCM sub-components and to concentration. A new method is proposed that will facilitate the interpretation of security betas calculated using regression methods. Indeed it is argued, here, that the calculation of the beta of a large security in a market value weighted portfolio is potentially misleading. This is because, if that security makes up a substantial proportion of the value of the market portfolio, much of the systematic risk of the market portfolio is in effect the idiosyncratic risk of that large dominant security. Furthermore, the beta of that security is largely attributable to its standardised covariance with itself.

The distribution parameters, and the degree to which the VCM sub-components can be forecast, are analysed in Chapter 8. The results are interpreted in the context of the "Overall Mean Model" of Elton and Gruber (1973).

1.4 Limitations

Research into the predictive power of option implied volatility (OIV) is an extensive and well-developed field in its own right. If the results of this study had provided decisive evidence that the forecasting performance of AARDL models incorporating distributed lags of differenced concentration was materially better than that of naive models, there would be grounds for comparing the out-of-sample performance of AARDL models of realised volatility with that of the various OIV forecasting models. As this was not the case further comparison with OIV models was not made. In any case, a consistent at-the-money OIV benchmark, such as the VIX calculated for the S&P 100 Index options, is not currently available for the FTSE 100 Index. The information contained in this study is likely to be more relevant to investors developing portfolio diversification strategies than to traders trying to estimate ex-ante realised volatility for pricing option contracts. Therefore, the literature review focuses on studies of portfolio diversification, leaving the review of OIV forecasting models to publications such as Poon and Granger (2003) and Knight and Satchell (1998).

1.5 Organisation

1.1.1.1.1 Chapter 2 – Literature review II: Studies of concentration

Various methods of measuring concentration are reviewed in this chapter. Earlier studies that have examined concentration and industrial structure, particularly in the UK market, are also reviewed, although none have considered the implications of concentration for the volatility of market returns. Some important properties of the different concentration metrics are discussed. Each numerical measure of concentration focuses on a different part of the Lorenz and concentration curves. The relative merits of the Hirschman-Herfindahl (H) index, the Simpson Index of Diversity, the Shannon Entropy Index, Hannah and Kay's Index (R), the variance of the logarithm of firm size (V^2) and other inequality measures are discussed in the context of this study. Consideration is given to some of the possible causes of changes in the level of concentration in a stock market portfolio.

1.1.1.1.2 Chapter 3 – Literature review II: Returns, volatility, theory and empirical studies

Different definitions and measures of stock return and stock return volatility are reviewed. Results of empirical studies on historical stock returns and stock return volatility in the UK market are also discussed. Empirical studies of co-movement are also reviewed and an extensive critique is provided of the study of concentration and volatility by Roll (1992). Subsequent literature citing Roll (1992) is also discussed. In addition, a detailed review is provided of the results and the conclusions presented by Campbell et al (2001), concerning the volatility of individual stocks in the US market up to the end of 1997. Finally, some key literature on time series models of realised volatility, the time series properties of realised volatility and the characteristics of the realised volatility distribution is also reviewed. The literature review provides the basis and justification for the methodology used in the empirical part of this study.

1.1.1.1.3 Chapter 4 – Literature review III: Equity market indices and their providers

A critical review of some major stock market indices and their providers is presented. The critique is based on the characteristics of an index that would make it suitable as a proxy for the market portfolio or a benchmark for evaluating the performance of active investment managers and a potentially investable passive portfolio. The problem of "Benchmark Error" identified by Roll (1977) and the paper by Bailey (1992b), concerning the characteristics of a suitable index, are reviewed. A table summarising the strengths and weaknesses of some well-known indices and their providers is presented. These are ranked in order to identify suitable indices for further research of the type reviewed earlier, as well as that implemented by this study.

1.1.1.1.4 Chapter 5 - Methodology I: Returns, concentration and volatility data

In this chapter a justification is provided for the choice of the FTSE 100 Index used in this study. Detailed information concerning the replication of the index using data sourced from Datastream is provided together with details of the return, volatility and concentration calculations applied to generate the data series.

1.1.1.1.5 Chapter 6 – Methodology II: Decomposing the VCM of portfolio returns

Justification is provided for the decomposition of the realised volatility of the FTSE 100 Index portfolio into those variance and covariance components that are conditional upon portfolio constituent weights and those that are not. A new method of achieving this by decomposing the VCM of constituent returns is presented. In addition, a method of standardising the sub-components is provided, so that their relative contribution to the total realised volatility of the index can be measured at any given time.

1.1.1.1.6 Chapter 7 - Methodology III: Sampling theory and models used

The rationale for breaking the data into sub-periods within the 1984 – 2003 data sample, and for focusing the analysis on the results obtained from January 1998 through March 2003, is explained. Specifications of the general asymmetric autoregressive distributed lag (AARDL) models, general autoregressive distributed lag (ARDL) models, naive asymmetric autoregressive (AAR) models, and naive autoregressive (AR) models, are also provided together with details of the out-of-sample forecast evaluation tests.

1.1.1.1.7 Chapter 8 - Results I: Characteristics of the data

Chapter 8 begins the empirical analysis. Tables are presented of the descriptive statistics for all the data series used in this study, both over the whole period from January 1984 through March 2003, and the sub-period from January 1998 through March 2003. Discussion of the descriptive statistics and comparison of the sub-periods with the whole period are provided. In addition, time series charts of selected data series plotted over the whole period provide a basis for discussion of the time series evolution of the selected data series. Extreme positive outliers are observed during the 1987 crash and the 1992 correction associated with the ERM crisis in the realised volatility data series and the three sub-components of volatility, namely: the equally weighted average variance, the equally weighted average covariance and the incremental average variance of constituent returns. In contrast to the positive outliers observed in the above series, extremely large negative outliers are observed, for the same events, in the incremental average covariance of constituent returns and the incremental realised variance of FTSE 100 Index returns.

1.1.1.1.8 Chapter 9 - Results II: Models of concentration and volatility

The results of direct models of the differenced concentration metrics and realised volatility in the FTSE 100 Index are presented. Some of the results suggest that when concentration is differenced in an attempt to make the time series stationary, it does not appear to influence either contemporaneous or future realised volatility in the FTSE 100 index. Models are estimated using realised volatility observations measured using five, ten, fifteen and twenty trading days. However, strong evidence is found in support of the asymmetry effect first documented by Black (1976) in addition to evidence suggesting that realised volatility can be forecast using a second order asymmetric autoregressive model when the series is estimated using five and ten trading days of returns to estimate each individual realised volatility observation, and a first order asymmetric autoregressive model when fifteen or twenty trading days are used.

1.1.1.1.9 Chapter 10 - Empirical Analysis III: Models of concentration and VCM sub-components

The results of AARDL models of differenced concentration and the individual sub-components of the VCM are reported and analysed. Evidence is found to suggest that the equally weighted average variance and the equally weighted average covariance of FTSE 100 Index constituent returns are positively related to lagged changes in differenced concentration. However, the incremental average covariance of the FTSE 100 Index constituent returns and the incremental realised variance of the FTSE 100 Index returns appear to have a negative relationship with lagged differences in concentration. This is evidenced by the significant negative model coefficients estimated. There is little evidence of a relationship between lagged changes in concentration and the incremental average variance of constituent returns, although some evidence is presented to suggest a positive association between contemporaneous changes in concentration and both the incremental average variance and the equally weighted average covariance of constituent returns. Once again the asymmetry or leverage effect is prevalent in all models, except those of the incremental average covariance and the incremental realised variance. However, despite statistically significant coefficients on many of the variables, there is little evidence to suggest that models of the sub-components are capable of producing better out-of-sample forecasts of realised volatility than naive AAR models of aggregate realised volatility.

1.1.1.1.10 Chapter 11 – Conclusions and contribution to current knowledge

The key empirical findings of this study are summarised and discussed. There does not appear to be a consistent link between lagged changes in stock index portfolio concentration and stock index portfolio volatility when the two variables are modelled directly. However, statistically significant relationships between differenced concentration and some of the sub-components of realised volatility are found. Of particular interest is the apparent inverse relationship between the lag of differenced concentration and both the incremental average covariance of constituent returns and the incremental realised variance. Of further note are the large negative outliers in these two sub-components of realised volatility around extreme market events, such as the 1987 crash, the 1992 ERM crisis and the volatility around the September 11th 2001 terrorist attacks. Positive outliers observed in the other sub-components – namely the incremental average variance, the equally weighted average variance and the equally weighted average covariance of constituent returns – mirror the negative outliers observed in the incremental average covariance.

The statistically significant coefficients for the differenced concentration data series contribute little, if any, economically significant out-of-sample forecasting ability to the VCM sub-component models. Furthermore, little evidence is found to suggest that either naive or general models of individual sub-components are better able to forecast the aggregate realised volatility than the direct models of aggregate realised volatility. Nonetheless, the models do aid the understanding of the time series dynamics of the VCM sub-components. For example, they illustrate the extent to which the various sub-components can be described by an autoregressive model, and the fact that the asymmetry effect is more pronounced in the equally weighted average variance and covariance components of the VCM than in the conditional components and, specifically, the incremental average covariance, where it is largely absent. In addition, the limitations of the autoregressive models for describing the evolution of VCM sub-components are also highlighted by reference to the extreme kurtosis and skewness of model residuals, despite the fact that autocorrelation can usually be eliminated from the residuals of AARDL and AAR models.

Chapter 2 – Literature Review I: Studies of concentration

2.1 Introduction to concentration

This chapter defines stock market concentration and describes various ways of measuring it. The Lorenz Curve and the Concentration Curve are introduced as graphical concentration metrics. The key attributes of a range of concentration indices are reviewed in section 2.2, including the Hirschman-Herfindahl (H) index, the Simpson Index of Diversity (D), the Shannon Entropy Index (E), Hannah and Kay's Index (R), and the variance of the logarithm of firm size (V^2). Studies that have examined industry concentration are reviewed in section 2.3. These include a paper by Roll (1992) that considers international diversification and relates concentration to stock market volatility using cross-sectional data. Section 2.3.1 provides an example of changing concentration in the FTSE 100 index over the period of the study and reviews earlier studies of concentration in the UK market including Hannah and Kay (1977).

The chapter then proceeds to consider possible causes of changes in the level of concentration in a stock market portfolio in section 2.3.2. A paper by Fernholz et al (1998) considers concentration and portfolio returns from a theoretical perspective, suggesting a theory as to why concentration might be expected to increase over time. In this chapter two theoretical proposals are considered: the first, known as the Gibrat effect, after Gibrat (1931), suggests that market concentration will increase over time due to differences in growth rates between firms; the second, proposed by Fernholz et al (1998), is that faster growing firms will eventually pay higher dividends, which will have the effect of countering the Gibrat effect. An additional practical explanation for changes in stock market concentration is also analysed, namely that changes in concentration are the exogenous result of corporate actions, such as take-overs, mergers, divestitures, de-listing and re-listing of existing firms, or the initial public offerings of previously unlisted large firms, including state owned monopolies and foreign firms. Section 2.4 concludes the chapter.

2.2 A review of diversity and concentration indices

2.2.1 Background to and definition of concentration

Diversity and concentration indices are analogous to one another, sharing some common origins in ecological science, engineering, mathematics, entropy and communication theory. The only difference between the terms diversity and concentration is that the former is used in conjunction with evenness and equitability when the principles have been applied in the ecological sciences, while the latter is used when referencing the same principles applied in the context of economics.[7] In the context of an investment portfolio, concentration would be high if the majority of capital was invested in just a few relatively homogenous securities, while concentration would be low if the capital were distributed evenly between many heterogeneous securities.

In this study, stock market concentration refers to the degree to which a few disproportionately large firms dominate the returns of value weighted stock market indices such as the FTSE 100. A notable example is the Finnish stock market and economy dominated by Nokia, the worlds largest mobile phone company. The opposite situation would be low concentration, or a fragmented market, in which numerous small firms each accounted for a relatively small share of the overall market. The research is based upon the idea that an index of concentration or diversity can be used to measure the distribution of constituent weights within a stock index portfolio. The relationship between changes in stock index concentration and changes in the volatility of stock index returns are examined in relation to the principles of Modern Portfolio Theory (MPT).

[7] The only references to concentration in finance research are a few papers published by Fernholz et al (1998) and a paper by Roll (1992), although these studies have a different emphasis to this study. However, the industrial economics literature contains more reference to concentration, which Clarke (1993) defines as the degree to which production for or in a particular market or industry is concentrated in the hands of a few large firms. Clark refers to aggregate concentration as the degree to which a few large firms control the production of the economy as whole, or at least broad sectors of it such as the financial or manufacturing sector.

2.2.2 Characteristics of concentration indices

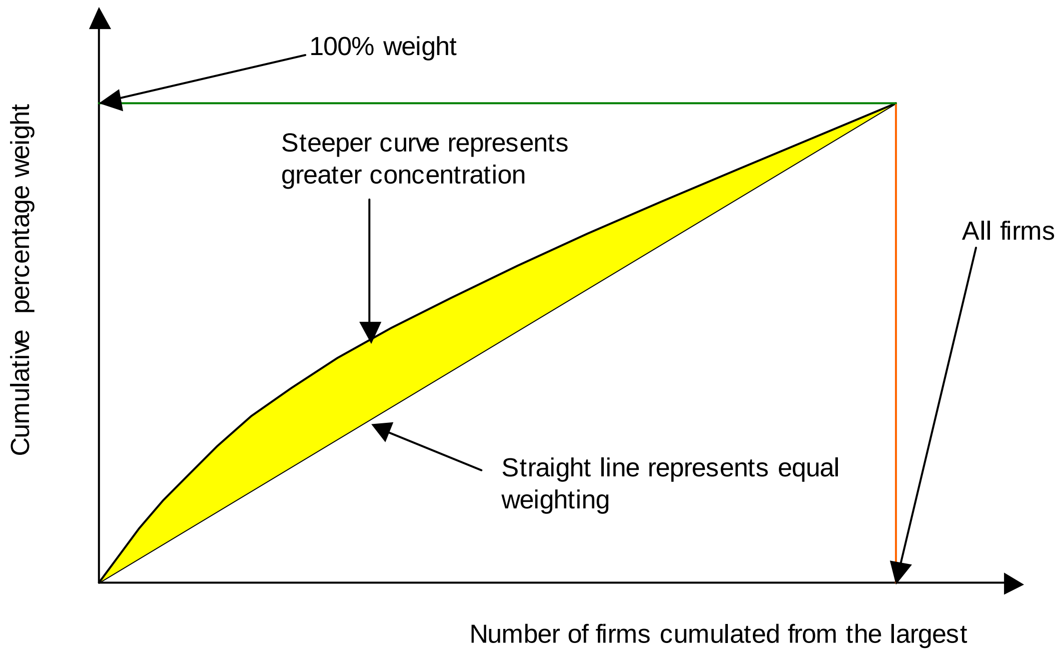
A number of different indices used for measuring concentration are reviewed extensively by Clarke (1993). As each index of concentration has its own merits and limitations, the optimal choice of index depends on its intended use. This section summarises some of the key points raised by Clarke (1993) and highlights issues that are relevant to this study.

At the generic level, absolute measures of concentration take into account both the number of different categories of units in a sample and the distribution of relative weights between these different categories. In an economic context, the units could be individual securities and the categories could define different firms or industries. Inequality measures of concentration only take into account the dispersion in the distribution of the weights between different categories and not the number of categories Clarke (1993). When comparing portfolios that have different numbers of constituents, or when the number of constituents in a portfolio or market is changing over time, there is an argument in favour of using absolute measures of concentration. As the FTSE 100 index has a fixed number of constituents, dispersion or inequality measures of concentration are adequate. An inequality measure of concentration applied to a portfolio with a constant number of constituents enables comparison of the same portfolio over different time periods and with other portfolios that have the same number of constituents.

2.2.2.1 Concentration Curve

The concentration curve is an absolute measure of concentration in which firm size inequality is represented by the convexity of the curve while firm numbers are indicated by the intersection of the curve at the 100% weight Clarke (1993), a hypothetical example is provided by Figure 1. The cumulative percentage weight is plotted on the y-axis against the number of firms starting from the largest on the x-axis. When concentration is at its lower limit, with weights equal for all firms, the line is straight. As concentration increases, the curve becomes more convex and moves further from the straight line, so that the shaded area in Figure 1 becomes larger. A curve that is very steep initially, and then flattens, represents a portfolio that is dominated by one or two large firms, but still contains many small firms that are, approximately, equal in size. On the other hand, if the concentration curve is smoother, large firms may still dominate, but the decrease in firm size, from largest to smallest is more continuous. Different concentration metrics place emphasis upon different parts of the concentration curve.

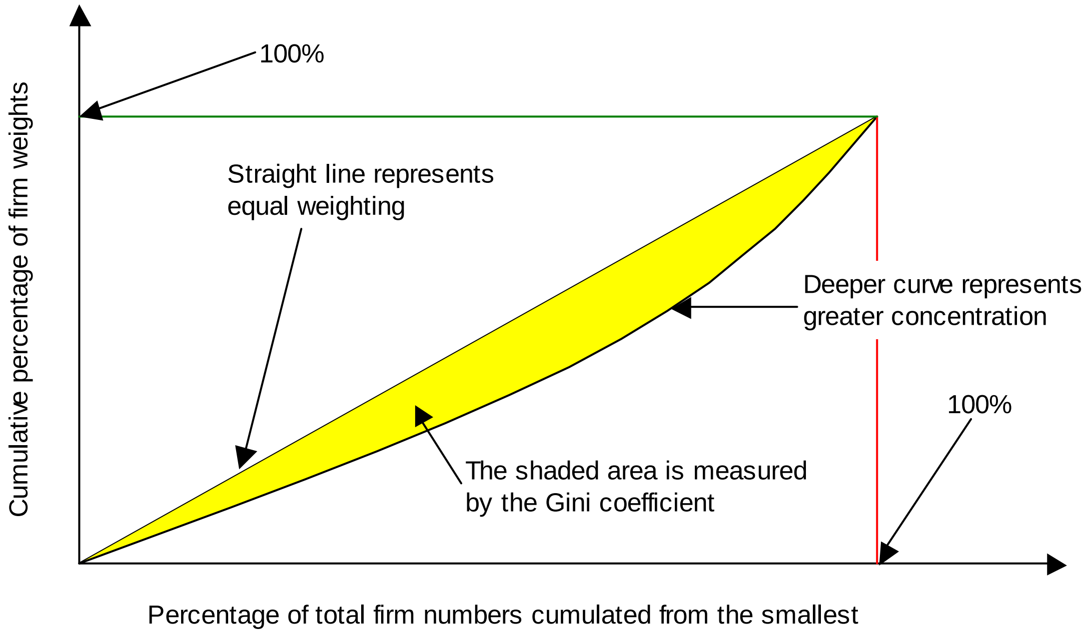
Figure 1 Concentration curve



2.2.2.2 Lorenz curve

Figure 2 illustrates a hypothetical example of a Lorenz curve. This differs from the concentration curve in that the cumulative percentage of the total number of firms are plotted from the smallest to the largest on the x-axis against the cumulative percentage of total market value on the y-axis. Because of the order reversal, a more concentrated market is indicated by greater concavity below the diagonal straight line rather than above it. Once again equal weighting, i.e. the lower limit of concentration is indicated by a straight line. Because the Lorenz curve is based on the percentages of total market value and the percentages of total firm numbers it is a unit-free measure of concentration. As with the concentration curve, the Lorenz curve depicts the degree of concentration as the area between the curve and the intersecting straight line Clarke (1993).

Figure 2 Lorenz curve of a hypothetical portfolio



2.2.3 Some absolute measures of concentration

2.2.3.1 Simpson Diversity Index

The Simpson's index is calculated as in Equation 1 and the value of *D* is an expression of the number of times one would have to take pairs of individuals at random from a population in order to select a pair of the same species Simpson (1949). In an economic context, D relates to the number of times one would have to select pairs of firms at random from the population universe in order to find a pair in the same industry.

Equation 1.
$$D = \frac{1}{\sum_{i=1}^{N} w_i^2} \quad \text{and} \quad E = \frac{1}{N} \times \frac{1}{\sum_{i=1}^{N} w_i^2}$$

Where: *E = Evenness of company distribution in the index or sample. E can have a maximum value of one in a situation where species, or industries, are equally weighted. Values decreasing below one indicate increasing dominance by relatively few species, or industries. N is the number of species, or industries, in the population, while w is the population weight of each species, or industry.*

In this study D and E are equivalent because the sample size N is the number of firms in the index and this is constant. Therefore $\frac{1}{N}$ can be deleted from the equation without loss of useful information. In some cases Simpson's index has been referred to as a dominance index as the squaring of the weights means that it is weighted towards the commonest species, or in this study, the largest firms.

2.2.3.2 Hirschman-Herfindahl Index H[8]

The Hirschman-Herfindahl Index (H) was initially attributed to Herfindahl (1950) in an unpublished dissertation for Columbia University. However, Hirschman (1964) later pointed out that an identical index, except for the presence of a square root, was in fact first published by himself in Hirschman (1945), although it has sometimes been incorrectly referred to as the Gini coefficient. The index he refers to as his own is represented by Equation 3 while the index usually referred to as the Herfindahl, or Hirschman-Herfindahl Index, is represented by Equation 2.

Equation 2.
$$H = \sum_{i=1}^{N} (x_i / x)^2 = \sum_{i=1}^{N} w_i^2$$

Where: *w_i i the weight of an individual company in a sample, x_i is the value of firm i and x is the total value of all firms i through N.*

---

[8]The Simpson Index can be traced to a paper published in Nature in 1949, by Simpson, titled "Measurement of Diversity". It has also been referred to as the Yule Index after the similar measure G Udney Yule devised to characterise the vocabulary used by different authors. Yule is cited by Simpson (1949) as a key reference for his index, which is a combination of the ideas of Yule (1944) with those of Fisher et al (1943) and Williams (1946) to form the basis of an index that is often cited alongside Shannon's Index in textbooks dealing with community ecology. It is almost identical to the Hirschman Herfindahl Index of concentration discussed above which is used in the empirical part of this study. The entropy index, first proposed by Shannon (1948), evolved from the concept of entropy as applied in the theory of communications engineering. Earlier researchers in this field include Nyquist (1924) and Hartley (1928). The objective of the research was to increase the efficiency of telegraphic information transmission. The Shannon Index is discussed further by Shannon & Weaver (1964) and later by Fernholz et al (1998). In ecological texts the index is often referred to as the Shannon-Wiener Index in deference to Wiener (1961) who arrived at a similar index independently, in 1948. In fact, in the 1961 edition of Wiener's book "Cybernetics", Wiener cites the statistician Fisher, who is the same Fisher, cited as a key reference in an article by Simpson (1949), which details the Simpson index of diversity. Wiener also cites Shannon (1948) as a key developer of the index. Shannon and Weaver (1963) in turn refer to Wiener in their book that updates and develops some of the ideas outlined in Shannon (1948). It therefore seems that a relatively small group of statisticians, mathematicians and engineers, exerted a common influence in the development of the Simpson, Shannon and market concentration indices. Without reference to Shannon, Hart (1971) discusses entropy and other measures of concentration in the context of economics and business concentration.

Equation 3. $$\sqrt{\sum_{i=1}^{N} w_i^2}$$

Given the similarity of this index to the Simpson's Diversity index, it is of interest to note that Hirschman's claim pre-dates the publication of the Simpson's paper by four years.

The H index takes account of all points on the concentration curve, as it is the sum of the squared weights of all firms in the industry. Therefore, if the total value of the market is defined as the sum of the market value of individual firms, then the share of the total market occupied by one firm is the market value of that firm divided by the value of the total market. This can be represented as follows: if *n* firms have market values x_i *(i = 1,……,n)* and the total value of the market is defined as *x* then $x = \sum_{i=1}^{n} x_i$ and hence the share of the total market accounted for by the firm (w_i) is $w_i = \frac{x_i}{x}$. The squaring process results in the greatest weight being given to the larger firms in the industry or market constituent sample as in Simpson's Index *D*. It can be seen that H is simply the reciprocal of Simpson's index of Diversity *D* by examination of Equation 4.

Equation 4. $$H = \sum_{i=1}^{N} \left(x_i / x\right)^2 = \sum_{i=1}^{N} w_1^2 \;\; and \;\; D = \frac{1}{\sum_{i=1}^{N} w_i^2}$$

In Equation 4, the weights reflect the contribution made by each firm, industry, or species to the population universe. Therefore, $\frac{1}{H} \equiv D$ while $\frac{1}{D} \equiv H$. Therefore, for practical purposes in the context of this study the Hirschman-Herfindahl *H* index and the Simpson's *E* are simply derivations of one another. In fact Clarke (1993) observed that the reciprocal of H is referred to as the numbers equivalent of H in that 1/H is equal to the number of firms in an equally weighted portfolio that would be required to produce the same value for H in the concentrated portfolio. Hence in the case of the FTSE 100 index, if H is equal to 0.05, then 1/0.05 = 20 and thus twenty firms in an equally weighted portfolio would have the same value of H as actually present in the 100 firm index portfolio. In other words if the total number of firms is 100 and as few as 20 firms in the equally weighted portfolio will give the same value of H, then the portfolio is much more concentrated than if an equally weighted portfolio of 50 firms were required.

If the average of firm size is taken together with the variance of firm size, H can be used to define a measure of inequality in the market values of different firms known as the coefficient of variation of firm size (*c*) represented by Equation 5 Clarke (1993).

Equation 5. $$c^2 = \frac{1}{N}\sum_{i=1}^{N} x_i^2 / \bar{x}^2 - 1 \quad and \quad H = \frac{c^2+1}{N}$$

Thus the H index depends upon the market share inequality as measured by c^2 and on the number of firms (*N*). H has an upper limit of unity in the hypothetical scenario in which the entire market is represented by just one firm. It has a minimum value of $1/N \rightarrow 0$ in the case of many small equally sized firms. The square of c can effectively be regarded as a unit free measure of inequality Clarke (1993).

2.2.3.3 Hannah and Kay's Index R

The index of Hannah & Kay (1977), represented by Equation 6, is very similar to the H index but it has the advantage of allowing the user to choose which part of the concentration curve to focus on by changing the value of α, an arbitrary elasticity parameter. This allows relatively greater weight to be given to large firms by increasing the value of α.

Equation 6. $$R = \sum_{i=1}^{N} w_i^{\alpha} \quad \text{Where } \alpha > 0$$

The Hirschman-Herfindahl index is equivalent to R when α = 2, while if α is sufficiently close to unity, the index will give the same ranking as that applied by the entropy index (E) discussed in the next section, Hannah & Kay (1977). The numbers equivalent to *R* is $R^{1/(1-\alpha)}$ or $HR = \left(\sum_{i=1}^{N} w_i^a\right)^{1/(1-\alpha)}$ and α > 0 but α ≠ 1. This means that if for example there are 100 firms in a portfolio and *R* = 5, and α = 0.5, then 1/(1-0.5) = 2 and the numbers equivalent is equal to R^2. Hence the numbers equivalent is 25, meaning that in an equally weighted portfolio of just 25 firms would give the same *R* of 5 that we found in the 100 stock concentrated portfolio. Note that if we set α < 1, for example 0.5, then we are allowing the smallest firms to exert a relatively greater influence on the calculation of *R* than the largest firms.

2.2.3.4 Shannon's Diversity index (H), sometimes referred to as the Entropy Index (E)

The Shannon Index (H_{sh}) is also often referred to as an entropy index (E). It is calculated using Equation 7.[9] Shannon and Weaver (1963) provide a general definition of entropy when they cite the statement of Boltzmann (1894), as follows:

"Entropy is related to the missing information inasmuch as it is related to the number of alternative outcomes which remain possible to a physical system after all the macroscopically observable information concerning it has been recorded."

Equation 7. $$E = -\sum_{i=1}^{N} w_i \ln w_i = E = \sum_{i=1}^{N} w_i \ln(1/w_i)$$

Equation 8. $$J = \frac{E}{E_{\max}} = \frac{-\sum_{i=1}^{N} w_i \ln w_i}{\ln N}$$ *E and E_{max} are diversity indices*

Where: *E_{max} is the maximum diversity that could be found in a situation where all species were equally abundant.*

The entropy index (E) is also related to Shannon's equitability index *J* as depicted in Equation 8. The equitability index J is equivalent to Simpson's E but in this case it is constrained between zero and unity, where unity represents the situation where all species are equally abundant. In addition, it will give a value identical to Hannah and Kay's *R* when α is set to equal unity Clarke (1993). The minus sign in the numerator of the expression, reflects the fact that any probability is a number less than or equal to unity, and the logarithms of numbers less than one are negative. Thus a minus sign is used here in order to ensure that Shannon's H_{sh} is positive. It is also worth noting that it is possible to use different logarithm bases for calculating *J*.

---

[9] In an ecological context the index simply questions how difficult it would be to predict correctly the species of the next individual sampled from a population of individuals. If uncertainty is low, the probability of a correct prediction increases. This implies that either the total number of species in the population is low or that the majority of individuals within the population are of the same species, i.e. diversity or evenness is low. On the other hand if the uncertainty number is high, the chance of a correct prediction is low implying that the total population is made up of a larger number of species and that they each represent a more evenly distributed share of the total biomass. In an economic context, this could represent the probability of a particular unit of output being derived from a particular firm, or a unit of capital in a portfolio being the security of a particular firm, when the units of the output or units of portfolio capital are sampled from the whole economy and the market portfolio respectively.

2.2.3.5 The Divergence Index of Kacperczyk, Sialm and Zheng (2003)

The above authors published a working paper titled *"On the industry concentration of actively managed equity mutual funds"* on SSRN in January 2003.[10] These authors report a correlation between concentration, measured using their divergence index (DI), and abnormal returns of actively managed mutual fund portfolios. Their index of divergence is related to the Hirschman-Herfindahl H index and is calculated as follows:

Equation 9. $$DI_t^F = \sum_{i=1}^{N} \left(w_{i,t}^F - w_{i,t}^M \right)^2$$

Where: $w^F{}_{i,t}$ = *The weight of the asset i in the fund portfolio.*

$w^M{}_{i,t}$ = *The weight of the asset i in the market portfolio.*

The basic function of the divergence index is to measure the degree to which the capital allocation of an individual, actively managed fund portfolio differs from the capital allocation that would result from a passive, market-tracking portfolio. Their ideas, results and conclusions are discussed in more detail in section 2.3.

2.2.4 Inequality measures of concentration

Inequality measures of concentration ignore the absolute numbers of firms in a sample and can thus be viewed as summary representations of the Lorenz curve in the same way that absolute measures summarise the concentration curve Clarke (1993).

2.2.4.1 Concentration ratio

Clarke (1993) defines the concentration ratio as the proportion of industry output accounted for by the r largest firms or, in the context of this study, the combined weight of the r largest firms in an index, when r is an arbitrary number. The index is calculated using Equation 10.

Equation 10. $$C_r = \sum_{i=1}^{r} \frac{x_i}{x} = \sum_{i=1}^{r} w_i$$

The concentration ratio is easy to calculate and easy to understand. However, it only focuses on a single part of the concentration curve and if the shape of the concentration

---

[10] According to information posted at: http://www-personal.umich.edu/~mkacpe/, a more recent version of this paper was under review at the Journal of Finance in October 2003. The updated version is available at the above url.

curve changes over time, comparison of the C_r values in a time series will provide an incomplete picture of the distribution of firm size in an index.

2.2.4.2 Coefficient of variation

The coefficient of variation has already been discussed in relation to the Hirschman-Herfindahl H index above. It is the ratio of the standard deviation of firm size to the mean of firm size, hence it is a unit free measure of dispersion and hence firm size inequality represented by Equation 5.

2.2.4.3 Variance of the logarithms of firm size V^2

The variance of the logarithms of firm size (V^2), defined by Equation 11, is particularly useful if the distribution of firm size in an index approximates to the log normal distribution, something that frequently occurs in practice and is loosely applicable to this study as discussed in the methodology and the results chapters.

Equation 11.
$$V^2 = \frac{1}{n}\sum_{i=1}^{n}\left[\log\left(x_i / \bar{x}_g\right)\right]^2$$

Where: $\bar{x}_g$ *is equal to the geometric mean of firm size.*

In the situation where firm values are approximately log normal and Lorenz curves are non-intersecting, V^2 provides an unambiguous ranking of firm size inequality Clarke (1993). V^2 is thus potentially useful for this study and will be discussed in more detail later when the empirical data is analysed in Chapter 8.

2.2.4.4 The Gini coefficient

The Gini coefficient, first proposed by Gini (1912),[11] (*G*) is a ratio of the shaded area between the Lorenz curve and the diagonal straight line, in Figure 2, and the area of the triangle falling below the diagonal straight line. The Gini coefficient is calculated as the relative mean difference i.e., the mean of the difference between every possible pair of individuals, x_i, x_j, in a matrix divided by the mean of the firm size, as illustrated by Equation 12. More details are provided by the online, encyclopaedia of mathematics, Mathworld.com edited by Weisstein (1999-2004), Gini's original reference is in Italian.

[11] http://mathworld.wolfram.com/GiniCoefficient.html and Gini, EAC. "Variabilitá e mutabilita." 1912. Reprinted in *Memorie di metodologia statistica* (Ed. AV. Pizetti and T. Salvemini.) Rome: Libreria Eredi Virgilio Veschi, 1955.

Equation 12.
$$G = \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} | x_i - x_j |}{2N^2 \mu}$$

Where: *N = The number of firms.*

μ *= The mean of the firm size.*

The Gini coefficient has a minimum value of zero, when all firms are equal in size and a theoretical maximum value of unity, when every firm except one has a weight of zero. G is not utilised in this study. Table 1 provides a summary of the concentration metrics discussed in this section.

Table 1 Summary of concentration metrics

Concentration metric	*Abbreviations commonly used (bold indicates abbreviation adopted in this study)*	*Equation cross reference*	*Author or authors.*	*Main uses*	*Classification*	*Used in this study*
Hirschman-Herfindahl	**H**	Equation 2	Hirschman (1945), Herfindahl (1950)	Used in economics, to describe the concentration curve that is influenced by the largest firms in the sample	Absolute	Yes, because it accounts for the whole of the concentration curve. It is influenced most by the largest firms in the sample.
Simpson's Index	E and **D**	Equation 1	Simpson (1949), Yule (1944), Fisher et al (1943), Williams (1946)	Used for describing the distribution of biomass, in studies of ecology.	Absolute	No, is the reciprocal transformation of **H**; therefore, there is little to be gained from including this index.
Hannah and Kay's Index	**R**	Equation 6	Hannah & Kay (1977)	Yes, can adjust emphasis on the smaller or larger firms in a sample, by adjusting the parameter α	Absolute	Yes, but α is set a 0.5 so that the smaller firms have a bigger influence than the larger firms to provide an alternative perspective to that provided by **H.**
Shannon's Entropy Index	H, J and **E**	Equation 8	Shannon (1948), Shannon & Weaver (1964), Nyquist (1924), Hartley (1928), Wiener (1961)	Used for describing the distribution of biomass, in studies of ecology. Is less influenced by the larger firms than the Simpson's E or the **H index.**	Absolute	No, the time series path is very similar to that of the Hannah and Kay's **R**; therefore, no reason to replicate **R.**
Gini Coefficient	**G**	Equation 12	Gini (1912)	Measures the area divergence between an equally weighted population and a non-equally weighted population using the Lorenz curve.	Inequality	No, does not add new information to that provided by the indices that are used.
Coefficient of Variation of firm size	$\mathbf{C^2}$	Equation 5	It is derived from the **H** index and discussion is provided by Clarke (1993).	Measures the inequality in the size of different firms.	Inequality	No because the result is very similar to that of the **H** Index.
Divergence Index	**DI**	Equation 9	Kacperczyk et al (2003)	Measures the degree to which the industry weights of an actively managed portfolio differ from the benchmark or the marked portfolio.	Absolute	No, because there is no theoretical basis for applying this metric in the context of this study.
Concentration ratio	$\mathbf{C_r}$	Equation 10	Not known, although Clarke (1993) state that it is one of the most widely used concentration metrics.	Measures the proportion of the total sample accounted for by the r largest individuals.	Inequality	No, r is an arbitrary figure and there is no theoretical basis, to adopt this measure for this study.
Variance of the logarithms of firm size	$\mathbf{V^2}$	Equation 11	Not clear, although it is discussed by Clarke (1993).	An unbiased measure of firm size dispersion if the distribution is approximately lognormal.	Inequality	Yes, because it adds additional information not provided by **H** and **R** and the distribution of firm size in the study sample is positively skewed, although not strictly lognormal.
Skewness of firm weights	**SK**	Standard formula for a sample skewness	This study	Measures the degree of asymmetry in the distribution of firm weights.	Inequality	Yes, because it is influenced by the largest firms in the sample to an even greater extent than **H.**

2.3 Review of some other studies involving concentration indices

A number of studies have examined concentration in British Industry as a whole and within different sectors of the economy. However, there are very few studies that have measured concentration using the equity value of firms listed on the London stock exchange, or any other stock exchange for that matter. There is only one published paper that links stock market concentration with stock market volatility, by Roll (1992).[12]

Kacperczyk et al (2003) have reported results of a study relating the concentration of actively managed US equity portfolios with the relative investment performance of those funds over the period December 1984 to December 1999. The authors use their divergence index, DI, to measure the degree to which the asset allocation of an actively managed mutual fund portfolio diverges from the asset allocation of the market portfolio. They suggest that active managers who diverge the furthest from the market portfolio generally, but not always, maintain portfolios that are more concentrated than the market portfolio. In addition, they propose that the greater the degree of divergence from the market portfolio, the more aggressive, and the more confident, is the active management. They argue that portfolio managers who adopt the strategy of diverging from the asset allocation of the market portfolio, do so, because they have a high degree of self confidence regarding their stock picking, asset allocation ability and information advantage in relation to the market as a whole. Kacperczyk et al use the divergence index of a portfolio as a measure of the manager's divergence from the market allocation. This is used as a proxy for the manager's self confidence and Kacperczyk et al attempt to see whether this self confidence is justified by superior risk adjusted returns attained by those managers who maintain the highest divergence from the market in their portfolios. They find that the added confidence implicit in the managers of more concentrated funds appears to be justified as concentrated funds produce better risk and style adjusted returns, on average.

Hannah & Kay (1977) in their book, "Concentration in Modern Industry[13]" mention some regression studies that have tried to relate concentration to profitability. Notable among these are Bain (1951) who studied a number of industries and observed that industries in which the largest eight firms held combined market shares in excess of 70% earned

---

[12] Roll's study is reviewed extensively in section 3.6.2 of Chapter 3.

[13] Pages 18 – 19

markedly higher returns. Kilpatrick (1967) finds that levels of profits and changes in profits relate to both the concentration ratio and a dummy variable reflecting whether or not a critical level of concentration is exceeded. Meehan and Duchesnau (1973) also found results consistent with those of Bain (1951). Hart & Clarke (1980) present results from an extensive study that they undertook of the UK market.

2.3.1 Current concentration in the UK FTSE 100 Index and previous studies

No academic research published to date has analysed concentration in the FTSE 100 Index of UK shares. However, Hannah & Kay (1977) studied concentration in British industry and cite earlier work of a similar vein. Given the absence of more recent work on this subject, it is useful to consider the following quote from Hannah and Kay.[14] This provides an insight into research opinion at that time, which can be compared with the current conditions in the UK equity market, and, more specifically, the FTSE 100 Index.

"If there is another merger wave comparable to that of the 1920s or 1960s, then apparently fanciful predictions such as those of Newbould & Jackson (1972) of a hundred firms controlling 70 or 80 percent of all manufacturing activity could be fulfilled within ten or fifteen years."

Hannah and Kay precede this statement with a suggestion that merger activity is a primary cause of increasing concentration and argue that small firms will only achieve sufficient growth to counter the trend towards increased concentration, if, for some reason future merger activity is very low. They follow on with the comment that at the time of writing, just prior to 1977, current merger activity was indeed at historically low levels and if it did not recover then the concentration level, predicted in the above quote, would not be reached until the next century. These ideas can now be viewed with the benefit of 20:20 hindsight in the light of actual concentration levels recorded for the FTSE 100 Index and the time series evolution of concentration since January 1984, reported in Chapter 8.[15]

The FTSE 100 Index is a value-weighted index that represented 80% of the total value of all the 2,428 UK firms listed on the London Stock Exchange (LSE) on the 31st of December

---

[14] Page 114

[15] The actual concentration in the UK equity market referred to in this study is measured using the distribution of equity market values, whereas the quote from Hannah and Kay (1977) refers to the distribution of UK manufacturing output. It is acknowledged that this is not a strictly like with like comparison, given that many of the firms currently listed on the LSE derive much of their revenue from operations outside of the UK.

2000.[16] Only ten firms accounted for 42% of the total £1.8 trillion market capitalisation of the UK stock market and the largest company, Vodafone, represented 9% of the total UK market capitalisation. This followed a series of mergers including: the formation of Glaxo-Welcome, which later became Glaxo-Smithkline, and BP-Amoco. However, the most notable of the mergers was the combination of Vodafone, the UK and international mobile telecommunications firm, and Mannessmann of Germany, to create the world's largest telephone company. Clearly the "apparently fanciful prediction" occurred within 25 or so years, although not within the ten or fifteen suggested by Newbould and Jackson (1972), largely due to renewed phases of intense merger activity, as predicted by Hannah and Kay.

On the 31st May 2000 Vodafone accounted for 13% of the FTSE 100 index while the ten largest firms accounted for 52% of the index. The FTSE 100 index in turn accounted for 80% of the value of all the UK listed shares traded on the LSE.[17]

Table 2 Percentage of FTSE 100 index and the total market accounted for by Vodafone and the ten largest firms on the 31st May 2000

	Vodafone	10 Largest firms	FTSE 100 Index
FTSE 100 Index	13%	52%	100%
Total Market (1,654 firms)	10%	41%	80%

Given the obvious inequality of the distribution of the equity market values across the firms listed on the London Stock Exchange and the constituents of the FTSE 100 index, it is fair to say that the LSE was characterised by a high level of concentration at that time, and it is still concentrated at the time of writing. Conversely it is also possible to say that the distribution displayed a low evenness.

2.3.2 Causes of changes in market concentration over time

A number of studies have examined concentration in British Industry from the 1950s through the 1970's. These studies have tended to focus on concentration within industries measured by the market share of firms and the proportion of total industry employment accounted for by the largest firms. What is clear from these studies is that concentration is not constant and appears to have increased over time. This is consistent with the time series data on various concentration measures reported in Chapter 8 of this study with respect to the market value of equity in the FTSE 100 index over the period from January 1984

[16] London Stock Exchange Data file 31st December 2000.

[17] London Stock Exchange Excel Data File of all listed firms at 31st May 2000.

through March 2003. There are a number of possible explanations for changes in concentration. Two notable examples are the so-called "Gibrat Effect" which is an endogenous result of differential stock market returns and the second is an exogenous result of corporate actions. The "Gibrat Effect" has received some attention in the literature and is discussed by (Hart & Clarke (1980); Hannah and Kay (1977) and by Clarke (1993)).[18] It is based on *"the law of proportionate effect"* which is based on the assumption that on average the distribution of growth rates is normal and not dependent upon firm size. The result is that although the proportionate growth rates and probability distributions are the same for all firms, the larger firms will grow by a larger absolute amount than the smaller firms with the result that, over time, firm size distribution becomes more and more positively skewed and may approximate to the lognormal.[19]

The second theoretical explanation is closely related to the Gibrat effect and is presented in a number of papers by Fernholz, the most recent being Fernholz (1999). Fernholz uses a model of stock prices based upon continuous semi martingales to achieve the same result as that demonstrated above, using Gibrat's law of proportionate effect. His conclusion is that a diverse distribution of capital throughout different firms and sectors of a security market is not a natural state. Hence in his conclusions, equilibrium can only be maintained if capital is redistributed from large firms as dividends and these dividends are then reinvested into small firms: an explanation that seems somewhat far fetched.[20]

Evidence in favour of the Gibrat effect has been found by Hannah & Kay (1977), although they argue that mergers account for the majority of the increase in concentration that has been observed in the UK market during the twentieth century. Although consistent with the

---

[18] Gibrat (1931) cited by Hart and Clarke (1980) p 4; Hannah and Kay (1977) p 99.

[19] The following simplistic example illustrates this process underlying the Gibrat effect. Consider a portfolio of sixteen firms each with a value of 100. For each firm the probability of zero growth is 0.5, the probability of growing by a factor of 0.909 is 0.25 and the probability of growing by a factor of 1.1 is 0.25. At the end of t_1 eight firms have a value of 100, four have a value of 91 and four have a value of 110. By the end of period t_2, six have a value of 100, four have a value of 91, four have a value of 110, while one has a value of 83 and one has a value of 121. The mode is still 100 but the maximum is 100+21 while the minimum is 100-17. Hence after only two periods the distribution is starting to gain a positive skew and concentration is increasing.

[20] Fernholz suggests that large firms must pay out larger dividends than smaller firms on average in order to maintain this equilibrium. He also lists a number of restricting assumptions, the first two of which are not realistic in practice and have severe implications for his theoretical argument. The assumptions are: 1. Firms do not merge or break up and the total number of shares of a company remains constant. 2. The number of firms in the market remains fixed. 3. Dividends are paid continuously rather than discretely. 4. There are no transaction costs, taxes, or problems with the indivisibility of shares.

idea of an upward trend in concentration, the results presented in this study also seem more readily explainable by merger activity than by stochastic processes such as the Gibrat effect. From a practical perspective, there are just two causes of change in concentration. The first is differential stock returns, stochastic or otherwise, that cause the distribution of capital throughout the market, or a portfolio, to change. The second is the result of exogenous events such as: takeovers, mergers, divestitures, new listings and de-listings. The importance of these issues in the context of this study are analysed in Chapter 8.

2.4 Summary

This chapter has defined the concept of concentration and reviewed a range of possible indices that have been used to measure concentration. The strengths, weakness and origins of the concentration indices have also been discussed and previous studies of concentration and application of concentration indices to the UK and other markets have been summarised. The relevance of these studies to this study of the UK equity market was illustrated by a brief commentary on the level of concentration in the UK equity market during the year 2000. Finally the possible causes of changes in concentration are discussed along with the implications of these causes for studies of stock market volatility, such as this one. The following chapter reviews some traditional finance theory such as MPT, and models such as the CAPM together with various studies of the correlation between securities in a market proxy portfolio. In addition, some important empirical studies of financial time series, including various methods of modelling volatility time series, are reviewed.

Chapter 3 – Literature review II: Returns, volatility, and capital market theory

3.1 Introduction: market volatility and portfolio risk

Portfolio risk has a variety of definitions but from the asset manager's perspective it usually relates to the probability of a loss occurring. An extensive literature on how to measure this probability, for a specified portfolio, already exists. Much of this is centred on value at risk (VAR) methodologies that are detailed in books such as Dowd (1998). Portfolio volatility encompasses the dispersion of returns around the expected value. However, as dispersion can include gains as well as losses, volatility is a more generic term than risk. This discussion focuses on the volatility of returns. Studies of volatility provide a fundamental basis for the development of more specialist risk measurement and control measures.

"Mathematical" models of prices and volatility attempt to describe the data generating process behind their respective time series. These models can be relatively simple, such as a random walk model, or they can be more complex, such as an autoregressive integrated moving average (ARIMA) model. Conversely, "equilibrium" models of asset prices, such as the capital asset pricing model (CAPM) and multi-factor models, such as the arbitrage pricing theory (APT), attempt to provide a theoretical explanation for the evolution of a data series.

Section 3.2 of the review begins with a discussion of the definition and measurement of returns. Measures of dispersion around an expected return, and how it may be measured, are then considered. Returns, expected returns and dispersion are then combined into the portfolio perspective by the introduction of co-movement, its measurement and its impact on aggregate portfolio volatility. Some important assumptions of MPT and the CAPM are then highlighted and discussed in section 3.3.

Section 3.4 provides an overview of the time series models used to describe the processes generating returns and the volatility of returns. Key terminology relating to time series models and characteristics specific to the time series of stock return volatility are also detailed in this section. The theoretical background is then followed in section 3.5 by a review of some key empirical studies on returns, expected returns, return volatility and co-movement of asset returns.

Studies that evaluate the benefits of applying MPT, consider aggregate portfolio or market volatility as well as individual firm volatility. They break aggregate volatility down into sub-components that can be attributed to market wide risk, country specific risk, industry specific risk and firm specific risk. These studies are reviewed in section 3.6 and their empirical findings are contrasted with the theoretical models. This is followed in section 3.7 by a review of the key literature on volatility modelling and forecasting with reference to how the various studies adapt the basic methods of return measurement and dispersion discussed at the beginning of the chapter to their real world data and modelling techniques. Chapter summary and conclusions are presented in section 3.8.

3.2 Measurement and definition of returns, volatility and co-movement

3.2.1 Measurement of returns

A simple definition of a return (R_i) is the percentage difference between the value of an investment today P_{it} and its value in the previous period, t_{-1}. This can be calculated as a discrete percentage return using Equation 13.

Equation 13. $R_{it} = (P_{it}/P_{it-1}) - 1$

Alternatively the continuously compounded return can be calculated using either Equation 14 or Equation 15, which both yield the same result.

Equation 14. $\text{Log}(P_{it}) - \text{Log}(P_{it-1}) = R_{it}$ or

Equation 15. $R_{it} = \text{Ln}(P_{it}/P_{it-1})$

Campbell et al (1997), recommend using the simple discrete return, as in Equation 13, when calculating the portfolio return from the returns of individual constituents. However, when studying a time series of returns data from a single stock, or portfolio, Campbell et al (1997) suggest using a continuously compounded return such as that represented by Equation 14 or Equation 15. This is because positive geometric returns of limited liability assets can be greater than 100% but are never less than minus 100%, therefore introducing positive skewness into the theoretical return distribution. Logarithmic differencing, on the other hand, produces returns that can, theoretically, have a normal distribution. Furthermore, a time series of logarithmic returns can also be added to give a multi-period return without an upward bias, unlike geometric returns, which can only be combined over successive time-periods by taking the product of price relatives. The additive property and assumption of a normal distribution make logarithmic returns more useful for the application of standard econometric modelling techniques for describing realised returns data and forecasting future returns.

3.2.2 Definitions of expected returns for volatility estimation

Expected returns and historic mean returns are key variables used for estimating the variance and covariance, because both estimates are based upon deviations of actual returns from a mean return. The definition of the mean return used in volatility and covariance metrics needs further justification. This is because the asset pricing models discussed in section 3.3 assume that the expected return is positively related to expected risk. However, given that historic volatility is measured using deviations from the historic mean return, and that the historic mean return has been used to provide forecasts of future expected returns; risk forecasts need to avoid problems of circularity in their estimation. Merton (1980) considers these issues in more detail; therefore, the following discussion provides a review of the assumptions concerning the expected return adopted in more recent studies.

There are three principle methods used to define the mean return when measuring realised volatility. The first is to take a simple average of realised returns over the period in which the variance or covariances are being estimated. This may be a fixed mean over the whole study period, or it may be a time varying mean of the returns realised within each discrete volatility-estimation period, i.e. the mean of daily returns used over the month in which an estimate of volatility is made. An alternative approach, adopted by Copeland et al (1994), is to assume that the expected return is equal to the contemporaneous risk-free rate of return, plus a risk premium, that may itself be an estimated figure based upon the long term average of equity market returns, after adjusting for the risk characteristics of individual securities. A third method suggested by Figlewski (1997) is to assume that the mean return is equal to zero. Deviations from the mean are simply the gross returns squared, when calculating the variance or covariance. Figlewski (1997) provides a justification for this method. Other studies such as Andersen et al (2000) and Areal and Taylor (2002) also assume a mean return of zero when measuring realised volatility. The three methods are analysed in more detail below.

3.2.2.1 Mean return; a simple average return over the sample estimation period

The standard definition of expected return in time series data is a sample average of returns measured over successive volatility estimation periods as shown by Equation 16.

Equation 16. Mean return, $\overline{R_i} = \frac{1}{T} \times \sum_{t=1}^{T} R_{i,t}$

Where: *t is the time interval, e.g. one trading day, over which a given return is measured and T is the number of time intervals t sampled in order to calculated the average, i.e. expected return.*

This method is appropriate when the population mean return is large in relation to the population variance. However, if the population mean is small in relation to the population variance, the sampling error from estimates of small samples obtained using Equation 16 will be large. Thus for discrete estimates, over periods of one month or less, it is prone to considerable bias due to the choice of sample period. This problem is sometimes referred to as "mean blur". Basically, a longer sample period should in theory yield a more reliable estimate of the mean. Nonetheless, it is likely that the sample mean will not be significantly different from zero when tested if the variance is large in relation to the mean. A problem with using a time varying mean to estimate volatility is that successive runs of returns with the same sign and similar magnitude may occur over a relatively short volatility estimation period. This results in a very unstable estimate of the mean and the tendency for dispersion estimates to be downwardly biased.[21]

3.2.2.2 Expected return as the risk-free rate plus an appropriate risk premium

An alternative approach is to assume that the true expected return is represented by the risk-free rate. However, this still leaves the question as to what is the appropriate risk free rate. Many studies such as Campbell et al (2001) use the one-month, two-month or three-month treasury-bill rate. It can be argued that this is only a valid proxy for the risk free benchmark of a stock portfolio, if the assumed investment horizon is the same as that of the treasury-bill maturity. Given that a stock portfolio is effectively an undated perpetuity it may be more appropriate to use the yield on undated consols so that the investment horizon of the risk-free benchmark is more compatible with that of the equity portfolio.

---

[21] A hypothetical example would be if the mean had been estimated during a run of successive returns of the same sign and a similar magnitude, e.g. 10% every day for an estimation period of one week. In this extreme case the mean return would be 10% and the dispersion from the mean would be zero even though the net change in portfolio value would be substantial. This is a problem of a volatility estimate being dependent upon the path of realised prices and the bias resulting from this could reasonably be termed "path dependent bias". Less extreme examples of such return runs are not unusual over periods of days, months and even years, in stationary return series that are mean reverting over long cycles.

The disadvantages with using the risk free return as a proxy for the expected return, is that it is harder to implement when using very high frequency intra-day returns, or when aggregating returns of many securities across a portfolio. In addition, comparability between markets requires adjustment for different exchange rates and inflation rates, as well as an assumption about the validity of the interest rate parity model.

One possibility is to estimate the mean return over the whole period in which discrete estimates of volatility are being studied, i.e. a fixed rather than a rolling estimate. However, a longer sample period may hide the existence of different populations of returns prevailing before or after major economic events. There is also the question of the length of a sample period that is adequate to estimate the true mean return. If an estimate had been made using data from October 1987 to February 2000 it would look very different to an estimate made between March 2000 and March 2003. Various papers published by Dimson and Marsh estimated the population parameters of UK stock market returns over the last half century. These provide useful benchmarks against which smaller more recent sample estimates of expected return can be compared, their results are discussed in section 3.5.2.

3.2.2.3 Assume that the mean return is equal to zero.

When the dispersion of returns is being studied over multiple discrete periods that are relatively short in length, i.e. one month or less, and the variance of the returns is large in relation to the mean, it is simplest to assume that the mean return is equal to zero. This eliminates the problems of mean blur and path dependent bias discussed above. It also simplifies the estimation of the realised variance, which can now be measured as the sum of the squared returns over the discrete sample-period divided by the number of return observations in the discrete sample. Unlike realised variance estimates that are based upon a time-varying mean, the assumption of a zero mean allows the denominator in the variance equation to be simply T rather than T-1, because a degree of freedom is not lost in the estimation of the sample mean Figlewski (1997). Realised sample standard deviation is then the square root of the realised variance as measured above.[22] This measure can be justified in a number of ways. For example, if the data-generating model that best describes security prices is a random walk with a zero intercept, then the best estimate of the future security price is the current price. If the expected future price is the same as the current

[22] Different authors refer to realised volatility as realised variance and realised standard deviation.

price, it follows that the expected return is zero. The process is more complicated if security prices are described as a random walk with drift. In this case, the intercept or drift parameter may be used as the expected return. An alternative is to estimate the sample mean over all of the available data. If it is not possible to reject the null hypothesis that the population mean is zero this provides support for the use of an expected return of zero**.**

3.2.2.4 Synopsis

Given the related problems discussed above, concerning mean blur and path dependent bias, it is likely that the assumption of a constant mean expected return of zero is likely to be less biased than an inappropriate sample estimate. At the very least a mean return of zero is consistent and provides a stable benchmark for potential bias that can then be evaluated against other empirical studies based on different markets, securities or asset classes. Several authors including Figlewski (1997) suggest using a mean return of zero and Andersen et al (2001a) follow this procedure when analysing the distribution of high frequency returns of the thirty Dow Jones Industrial Average constituents. Areal and Taylor (2002) also evaluate the distribution of high frequency FTSE 100 Index returns in relation to a mean of zero.[23]

The disadvantages of adopting the assumption of a mean return of zero are as follows. First, for the pricing of risky assets it is inconsistent with the assumption that rational investors demand a positive risk premium above the available risk free rate as a reward for holding risky assets. Second, if the true population mean return is positive, adopting an assumed mean return of zero will result in a positively skewed distribution of returns around the assumed mean when the true return distribution may be symmetrical. The estimated variance will also be upwardly biased in this case because the average of the squared deviations from the assumed mean will be greater. However, when the true mean is close to zero and the true population variance is large, the benefits of the zero mean assumption are likely to outweigh the disadvantages for discrete sample estimates over periods of one month or less.

[23] In fact, Areal and Taylor observe that the average intra-day FTSE 100 Index returns are negative. However, when returns are aggregated over separate trading-days they become positive. They conclude from this that most of the positive index returns occur while the market is closed, i.e. returns between close and open are on average more positive than intra-day returns to the extent that, although fewer in number, they reverse the sign of aggregate intra-day returns considered in isolation from overnight and weekend returns.

An alternative method of estimating mean returns is to use the prevailing risk free return, plus, a probability weighted expected risk premium based on the risk of the asset being evaluated. This method still does not avoid the problem of circularity because in order to establish an appropriate risk premium, it is first necessary to estimate the risk of the asset concerned, yet it is that risk premium that forms part of the expected return needed to estimate volatility in the first place.[24]

The choice of what mean return to use for realised volatility estimates ultimately depends upon the empirical characteristics of the data and the purpose for which the volatility estimate is being used.

3.2.3 Definition and measurement of volatility

Unlike many financial and economic variables that consist of primary factual data, such as stock prices, that are readily available, easy to define and easy to quantify at any given point in time, volatility is a ‘secondary’ or a ‘derived’ variable. This means that it is extracted from a series or sample of the underlying primary variable. Given that it is generally defined as the dispersion of possible data values around a mean value, the different definitions and ways of calculating the mean also introduce an additional layer of complexity, as evident from the discussion in the previous section. Stock market volatility may also be described as the stochastic deviation of short-term values around a long-term deterministic trend representing the expected return, as in the description of the evolution of stock prices as Brownian motion by Osborne (1959). As a result, the volatility of a data series cannot be measured for a particular point in time because the data value is known with certainty at that point. Therefore, it is only possible to either forecast the volatility that might be “realised” over a future time period, or to record the realised ex-post volatility.

The most common measure of volatility is the standard deviation, as it is the square root of the variance, this fits the definition of volatility as the dispersion of values around the mean. Because the length and number of time periods over which the average returns and average dispersion can be calculated are unbounded, realised volatility can be recorded in a variety of ways that may not always give compatible results. In addition, the variance and standard deviation are only one of a number of ways that stock market volatility can be recorded.

[24] A working paper by Graham and Harvey (2002) does aim to form an ex-ante estimate of the equity risk premium by surveying the opinions of Chief Financial Officers. This is reviewed in section 3.5.

Other metrics include the mean absolute deviation, option implied volatility, in addition to the extreme value metric suggested by Parkinson (1980). Parkinson's extreme value method is based on dividing the highest price by the lowest price realised in any given measurement period then taking the natural log of this figure as in Equation 17.

Equation 17. $$l = ln\ (H/L)$$

Where: *l is effectively a measure of the range*

H is the highest value attained in the measurement period

L is the lowest value attained in the measurement period

Parkinson demonstrates that the relationship between the variation in *l* and the population variance of the returns makes *l* a viable proxy for the variance that requires fewer observations and is more sensitive to changes in the variance over time than a conventional sample variance estimate. Garman and Klass (1980) include intra-day highs and intra-day lows in addition to overnight (close-to-open) returns in their volatility estimator. Wiggins (1992) use variations on both the Parkinson and the Garman and Klass method to estimate the volatility of the S&P 500 Futures prices. All three studies assume that the evolution of the logarithmic transformations of prices follow a random walk.

The definitions of volatility most relevant to this study are variations on the variance and standard deviation. Literature relevant to these is reviewed in more detail throughout the rest of this chapter with reference to the comprehensive analysis of the various measures of volatility provided by Figlewski (1997) and Poon and Granger (2003). In addition, skewness and kurtosis can be used to measure the shape of the return distribution. Recent studies that have done this are reviewed in section 3.7.5.

3.2.4 Definition and measurement of co-movement

Co-movement is the degree to which the returns of securities in a portfolio move in the same direction over a given measurement period. In an index or portfolio with more than one risky security, the portfolio variance is the sum of the VCM, which constitutes the weighted variances and the weighted covariances of returns within and between each security respectively. Empirical studies of co-movement are often based upon a VCM estimated using multiple time series observations of returns for each matrix. In this situation it is possible to estimate the conventional covariance matrix or correlation matrix for the assets in the portfolio. However, because the number of covariance elements in the

VCM is equal to the square of the number of securities (N), the size of the matrix rapidly becomes difficult to manage as N increases, so that calculating each element in the matrix becomes computationally intensive. This is even more complicated when each asset in the portfolio has a different weight and the weight changes over each period in which the return is being measured.[25] Chapter 6 provides a simplified procedure for estimating the average covariance in the VCM.

3.3 Equilibrium models of asset pricing and their assumptions

3.3.1 Overview

A number of models and concepts have come to dominate the finance literature over the last fifty years. Principle among these is MPT, first put forward by Markowitz (1952). This was followed by the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), Mossin (1966), and the Arbitrage Pricing Theory (APT) of Ross (1976). Single index models, the Market Model (MM) and the multi index models that form the basis of APT, attempt to provide a simpler model of asset pricing for the purpose of security selection and portfolio diversification. These models are discussed comprehensively in most finance text books, such as Elton et al (2003) and Bodie et al (1999). Therefore this discussion will focus only on some key principles and restricting assumptions behind the models.

3.3.2 Modern portfolio theory (MPT) and the capital asset pricing model (CAPM)

The most basic principle of MPT is that investors should attempt to optimise expected risk and return by changing portfolio constituent weights to create a “mean variance efficient” portfolio.[26] The CAPM makes the assumption that investors’ portfolios are mean variance efficient according to rational expectations based upon the best information available. The security weights that make up the risky portfolio depend upon, both, expected returns, variances and the covariance between security returns. Changes in any of these

---

[25] Various authors including Elton and Gruber (1973) and Campbell et al (2001), have achieved this, as will be detailed later in section 3.5.4. In addition, Elton et al (2003) detail methods that can be used to estimate the covariance between pairs of securities by using the beta of a security relative to an index. A more limited number of covariance estimates of each security with the market can then be used to form an optimal “ efficient” portfolio.

[26] MPT is described as a “normative theory” by Fabozzi et al (2002). This means that it describes how investors should behave, without making any assumptions about how they actually behave. In contrast, asset-pricing theory is described as a positive theory, in that it hypothesises about how investors actually behave, rather than how they should behave.

assumptions or expectations will result in changes in the optimal weights.[27] If the FTSE 100 Index is a valid proxy for the UK equity market portfolio and the CAPM holds, then the constituent weights in the index will proxy for those of the mean variance efficient market portfolio. Even if the assumption that investors are rational mean variance optimisers is relaxed, to allow for the existence of some naive or irrational investors, the potential for smart rational investors to arbitrage should maintain the equilibrium condition, so that on average portfolio weights reflect the collective attempts of investors to maximise expected return and minimise risk based upon an approach that is at least similar to that detailed by MPT.

In practice, few investors will hold a share of the real market portfolio because transaction and liquidity costs are too high. Instead they will aim to hold shares in a proxy for the market portfolio. Empirical studies, such as Evans and Archer (1968), Elton and Gruber (1977) and Poon et al (1992) have demonstrated that holding between twenty and fifty securities in a portfolio can reduce portfolio risk to a level that is close to the systematic risk of the entire market, although recent evidence presented by Campbell et al (2001) indicates that this figure may need to be increased due to a rising trend of idiosyncratic risk and declining average correlation between security returns. The problems associated with identifying and investing in a suitable proxy for the market portfolio are discussed further in Chapter 4.

Another key assumption of the CAPM, known as homogenous expectations (HE), states that all investors analyse securities in the same way and have the same economic view of the world. HE is restrictive in that it assumes investors form identical estimates of the probability distributions of future cash flows from investing in available securities. Hence for any set of security prices, they all derive the same inputs to feed into the Markowitz portfolio selection model. Given a set of security prices and the risk-free interest rate, all investors use the same expected returns and the same VCM of security returns to generate the efficient frontier and the unique optimal risky portfolio. If the entire market is the collectively optimised mean variance efficient portfolio of Markowitz, the weights of the market portfolio reflect investor’s collective expectations about the structure of the mean variance portfolio.

---

[27] Various methods of optimising portfolio weights, once the inputs are derived, are detailed by Elton and et al (2003).

In practice, investor's will have heterogeneous expectations. However, the market portfolio will be mean variance efficient according to the average of individual investors' expectations. Hence when the motivation behind the actions of individual investors forming their portfolios are considered collectively, the factors responsible for the resulting structure of the entire market portfolio are being considered. Therefore it is possible to regard the market capitalisation (market-value-of-equity) weights of constituents in the market portfolio, as the result of the weighted average of investor's expectations that leads them to optimise their portfolio weights in a certain way, based upon some kind of diversification strategy. For instance, if average investors expectations concerning the returns, variance and covariance of a particular stock in relation to others in their portfolio warrants a reduction in the weight of that stock in the optimal efficient portfolio, the equilibrium market price of that stock will fall and hence, its weight in the market portfolio will also fall, until a new equilibrium has been reached. This equilibrium weight will be determined by the new optimal weight of that stock in the efficient portfolio based upon expectations prevailing at the time. If these assumptions are accepted, it thus follows that the degree of concentration in the market portfolio arises as an outcome of investors' attempts to optimally weight their portfolios to minimise expected risk and maximise expected returns.

Average correlation models attempt to explain the correlation structure of security returns in an index. A model referred to by Elton and Gruber (1973) as the "Overall Mean Model" has direct relevance for this study. A review of this model and a subsequent study by Elton et al (1978) is provided in section 3.5.4, together with discussion of more recent studies of comovement in stock returns.

3.3.3 Limitations of modern portfolio theory

If investors are to create mean variance efficient portfolios they need to develop ex-ante estimates of security returns, risk and covariance. Obtaining these estimates is difficult and the following section reviews studies that have attempted to obtain satisfactory estimates, and the difficulties encountered. One problem identified by a number of empirical studies reviewed in section 3.5, is that during times of extreme volatility and market panic the correlation of most traded asset returns will approach unity, even though the same asset returns will appear to have a very low correlation during normal trading conditions. An additional problem, discussed extensively by Elton et al (2003) is that the models discussed above often explain historic data reasonably well but have poor out-of-sample performance in relation to naïve models. Therefore, there is still a need to identify a means of measuring and providing reliable ex-ante forecasts of the sub-components of the VCM, before investors are able to effectively create mean variance efficient portfolios that are suited to their current investment horizon. This problem is discussed further by Fabozzi et al (2002) when they compare the actual ex-post performance of a portfolio optimised using a global mean variance asset allocation model, based on the historic VCM, to the ex-ante expected performance implied by the historic data.

3.3.3.1 Behavioural finance

A further problem with equilibrium models, such as the CAPM, is their reliance on the assumption that investors exhibit rational behaviour with respect to risk and return. In fact, Statman (1999) challenges the assumption that investors are rational and cites studies, such as Kahneman and Tversky (1979), to indicate that investor behaviour is not consistent with the rational expectations assumption at the core of theories of economic equilibrium. In particular, investors are found to be loss averse rather than risk averse, whereby the pain experienced from realising a loss is greater than the satisfaction experienced by realising a profit of the same absolute magnitude. These and related issues, such as mental accounting, are examined more extensively in Shefrin (2002).[28]

Shiller (2003) highlights the problem of excess volatility in stock prices, pointing to the fact that stock returns have historically displayed considerably more volatility than the present value of the cash flows received by stockholders. If stocks are priced under rational expectations, and markets are in equilibrium, this suggests that the expectations of the values of future cash flows, i.e. analysts forecasts, are actually more volatile than the cash flows themselves – a finding that renders the forecasts meaningless and is difficult to explain using conventional economic theories.

An additional defence of equilibrium models is that, even if all investors are not rational, the existence of a few smart rational investors is sufficient to keep the markets in equilibrium as the smart investors take advantage of arbitrage opportunities presented by less sophisticated investors. However, Shiller (2003) cites work by De Long et al (1990b) indicating that rather than driving prices back to their intrinsic value, so called “smart money” investors

---

[28] Mental accounting is the tendency for investors to focus on separate “mental accounts” such as the “capital account” and “income account” when evaluating their own performance, rather than their overall portfolio gains and losses.

actually amplify irrational bubbles by anticipating demand from "dumb money" and buying into future bubble sectors in advance. Shiller goes on to give additional examples of disincentives and barriers to arbitrage activity with respect to short sales during the 1999 and 2000 technology bubble.

3.3.3.2 Empirical tests of the CAPM

Empirical tests of the CAPM and its derivations are numerous. Most finance textbooks summarise their outcome and provide references to the primary literature, so these studies are not reviewed here. Suffice to say that CAPM-type models are useful, but not perfect, and in many cases they are not entirely testable. However, various versions of the CAPM have been developed that take into account relaxations of a number of assumptions. For example, models that allow for differential borrowing and lending rates and asymmetric security returns have been developed. In Chapter 4, Roll's "benchmark problem" is reviewed.

3.4 Time series models

3.4.1 Overview

The equilibrium models discussed in the previous section attempt to explain the evolution of asset prices in an economic context by providing an intuitive reason for changes and imposing conditions such as the "no-arbitrage" condition. This section summarises some important time series models that are used to describe the evolution of asset prices. Unlike equilibrium models, time series models are descriptive only; their purpose is summarised well by Pindyck & Rubinfeld (1991):[29] *"A time series model provides a description of the random nature of the (stochastic) process that generated the sample of observations under study. The description is given not in terms of a cause-and effect relationship (as would be the case in a regression model) but in terms of how that randomness is embodied in the process."* Unlike a normal regression model, a pure time series model is a form of extrapolation whereby the future value of a variable is explained entirely by its past values without any additional variables. Thus the independent variables of pure time series forecasting models are derived from the dependent variable in some way, either by lags or some manipulation of the return distribution.

---

[29] 3rd edition page 440

Equation 18. $y_t = y_{t-1} + \varepsilon_t$

Time series models can be used to describe the deterministic component of a data series, the stochastic component of a data series, or both. The most basic definition of a stochastic time series is a random walk process described by Equation 18. The change in the series y is represented by the errors ε. These have a constant zero mean, are independent from all previous or future values of ε, have a constant finite variance and are drawn from the same probability distribution: characteristics sometimes referred to as "white noise". These conditions mean that the most likely change in the series at t_{+1} is the mean of the error, i.e. zero: hence the best possible prediction of future values of y is the current value y_t. Furthermore the forecast error equates to the variance of the ε. This is constant for any given lag length but it is proportional to the square root of the lag length.

A random walk model with an additional non-zero intercept term is described as a random walk with drift. The drift parameter represents an additional stochastic time trend, which can be positive or negative, depending upon the sign of the intercept term. Such models are often used to describe the evolution of asset prices as they are consistent with the "no arbitrage" assumption to the effect that it is impossible to make a reliable forecast of future security returns once the drift parameter, possibly representing the risk free rate plus the risk premium, has been accounted for.

A martingale process is similar to a random walk process but, although the mean is constant and zero like a random walk, the errors do not necessarily have a constant variance and are not always independent.[30] Hence a process that can be described using a similar model to Equation 18 but that has some predictable components in the errors can be described as a martingale.

The characteristics of the errors in deterministic trend models, stochastic trend models, unit roots and martingales are fundamental to the definition and application of the models. This is because the fundamental objective of any time series model is to obtain residuals that are independent Gaussian white noise and thus impossible to forecast using time series models. Having achieved this, it may then be possible to incorporate other non-time series variables into the model in order to further reduce the variance of ε_t that is conditional upon the other factors. Some of the key terms relating to volatility modelling in the above context are

[30] Page 229 of Watsham & Parramore (1997).

defined below. However, in order to effectively model the data series to achieve this ideal, the series must first be covariance stationary. Because of the importance of these conditions and their interpretation, the next section provides definitions of the terms "white noise", "Gaussian white noise" and "covariance stationary". These fundamental definitions are followed by some definitions of additional terms that describe the empirical characteristics of the volatility of financial time series that have been documented in the literature reviewed later in this chapter.

3.4.2 Some key terminology

3.4.2.1 A covariance stationary process

According to Diebold (2001), covariance stationary (weakly stationary) variables have a constant unconditional mean μ and a finite variance, σ^2.[31] The auto covariance structure must be stable over time. Therefore, the covariance of the variable y with former values of y must depend only on the lag length (displacement factor τ) and not upon the time factor, T, if a series is to be covariance stationary. However, covariance stationary variables are not necessarily independent and may be correlated with previous and future lagged values, provided the stability condition detailed above is met. The result is that the mean and variance of a covariance stationary series that is conditional upon previous realisations, i.e. the conditional mean and the conditional variance, need not be constant, unlike the unconditional mean and variance.[32] Therefore, forecasting in an autoregressive (AR) or moving average, (MA) process is possible so long as the autocorrelations are stable, i.e. "stationary", throughout the series. In addition, a covariance stationary series does not need to be normally distributed and it can display time varying skewness or kurtosis.

3.4.2.2 Strictly stationary or strict sense stationary series

Strictly stationary series must satisfy all the conditions of covariance stationary series but, in addition, there cannot be any autocorrelation with previous lags of the series. As a result, they cannot be forecasted using time series models. In this respect, the residuals of a random walk model are strictly stationary. Weakly stationary (covariance stationary) variables, by contrast, can display autocorrelation and partial autocorrelation with previous

---

[31] Pages 128-151. Note that other authors, such as Pindyck & Rubinfeld (1991) page 445 and Watsham & Parramore (1997) page 231 use slightly different definitions and notation.

[32] Pindyck and Rubinfeld use the term "wide sense stationary" as the equivalent to "covariance stationary" used here.

values, so long as the autocorrelations and partial autocorrelations are constant across all t for any given lag length (τ).

3.4.2.3 White noise

A white noise process is a form of strictly stationary process in that it has a constant mean, constant variance and no autocorrelation. If in addition to these conditions, a white noise process also has a zero mean, it is termed zero mean white noise. If the time series is independent across all lags in addition to the no autocorrelation condition, it is termed independent white noise. Finally if the series conforms to all the conditions of independent white noise and it is normally distributed, it is termed Gaussian white noise. A key assumption of ordinary least squares (OLS) regression models is that the disturbances are Gaussian white noise and the objective of all pure time series models is to achieve Gaussian white noise residuals. Once this condition is achieved it is impossible to improve the model forecasts by simply changing the time series specification of the model.[33]

3.4.3 Other time series models

If a time series is best described by a random walk model and hence can be defined as a random walk process, the errors are independent and hence impossible to forecast further using more elaborate time series models. However, if the series is a martingale or some other form of stochastic process, the innovations, may be covariance stationary, or difference stationary, enabling some degree of forecasting by more elaborate time series models. Some commonly used versions of these are defined below, although more detailed discussion and analysis is left to the many time series econometrics textbooks available.[34]

When a time series is covariance stationary and exhibits smoothly declining autocorrelation functions, as the number of lags increases, it may be described by an autoregressive (AR) model, in which the level of the process is a linear function of past levels. Chapter 7 details the specifications of AR models of the level of realised volatility used in this thesis.[35]

A moving average process describes a time series that is a linear function of past model errors and is specified by Equation 19. The errors could represent the differences in a

---

[33] Pages 123-124 of Diebold (2001) discuss the variations of white noise in more detail.

[34] Difference stationary refers to a series that is non-stationary in levels but stationary in differences.

[35] Note that in this study, the level of the realised volatility data is effectively the same as the non-overlapping sample standard deviations of the errors in a martingale process without drift.

martingale process, or the errors from an AR model. In the latter case the model would be termed an $ARMA_{(pq)}$ model. If the errors were the covariance stationary first differences of a non-stationary series, the series would be integrated with order one and defined as *I*(1); the process could then be described by an integrated moving average (IMA) model.

Equation 19. $E(y_t) = \alpha + \beta_1\varepsilon_{t-1} + ... + \beta_2\varepsilon_{t-q}$ *Where: q is the number of lagged errors needed to completely describe the series, hence the term* $MA_{(q)}$ *model.*

An integrated autoregressive moving average (ARIMA) model might be appropriate if, for example, a time series was non-stationary due to the presence of a stochastic trend but first differencing resulted in a covariance stationary series. Therefore, the series was *I*(1) so that levels of the differenced series could then be modelled using an AR model and the residuals of the AR model fitted an MA model.

Vector autoregressive models are vectors of two or more dependent variables, each evolving via autoregressive distributed lag models that contain lags of their own values and lags of the other dependent variables in the vector. They have been used to forecast portfolio returns and volatility based upon the evolution of constituent security returns in the VCM.

The autoregressive conditional heteroskedasticity (ARCH) models of Engle (1982) are based on the idea that the time series being modelled is a covariance stationary stochastic process, hence the unconditional population mean (μ) and the unconditional population variance (σ^2) are constant. However, the conditional mean (m_t) and the conditional variance (h_t^2) are time varying and conditional upon the information set available in the previous time period, for example previous realisations of the data series y. ARCH models can be estimated using either OLS or a maximum likelihood (ML) procedure, but the ML procedure is more efficient Engle (1982).

Bollerslev (1986) showed how to generalise the ARCH model by repeating the process over an additional stage in a process known as generalised ARCH (GARCH). When a GARCH model is applied the three stages in the estimation procedure are performed simultaneously using ML estimation.

The models described above depend upon the assumption that the time series is covariance stationary. That is, that the unconditional mean and variance are constant so that only the conditional mean and variance are allowed to be time varying. The time varying conditional variance allows ARCH and GARCH models to be used for forecasting realised volatility of asset returns. This is a valuable feature because empirical analysis of the time series of the realised volatility of stock returns suggests that realised volatility is in fact time varying. ARCH and GARCH estimation processes are described as a martingale differencing by Knight & Satchell (1998).[36] Like other time series models, the basic objective is to achieve Gaussian white noise residuals by modelling time varying conditional expected values of the underlying data, which is assumed to follow a martingale process with finite, but not necessarily constant variance.

Stochastic volatility models may incorporate features of the above models, but they start with the premise that the volatility data-series is a stochastic variable that is changing through time. An intuitive explanation of stochastic volatility models is provided in Chriss (1997) who describes volatility as a variable that changes in a similar random manner to stock prices in the theory of geometric Brownian motion.[37] Thus volatility may be a non-stationary variable that needs differencing in order to model its time varying behaviour. Alternatively, volatility may be a stochastic but covariance stationary process that can be modelled using an autoregressive model such as that of Poterba and Summers (1986) or Canina and Figlewski (1993). Some SV models make mean reversion of changes in volatility a basic assumption, in which the percentage change in volatility over short periods of time are normally distributed with a mean and standard deviation of their own, i.e. a volatility of a volatility.

Numerous articles defining complex variations of the above models are extensively reviewed by publications such as Poon and Granger (2003) and Knight and Satchell (1998). Rather than regurgitating these reviews, the following pages highlight some key characteristics of volatility time series and then proceed to examine a selection of studies, relevant to this thesis, that are either not covered by the existing reviews, or are not examined in the same degree of detail.

3.4.4 Characteristics of volatility time series

The volatility models summarised above describe the basic characteristics of a financial time series. However, other characteristics specific to the volatility of stock and other asset

---

[36] Pages 5 – 6.

[37] Pages 343 – 356.

price returns have been documented. These characteristics and the terms used to reference them are defined below.

3.4.4.1 Clustering

Clustering in volatility time series refers to the tendency for periods of high volatility to be followed by periods of high volatility and periods of low volatility to be followed by further periods of low volatility. This can be modelled using variations on the GARCH models, alternatively Poterba and Summers (1986) and Canina and Figlewski (1993) suggest that clustering of volatility can be described by an $AR_{(1)}$ process.

3.4.4.2 Long memory

Long memory, also referred to as persistence, is related to clustering in that the impact of large shocks tends to persist indefinitely. This is particularly apparent in higher frequency daily and intra-daily data. Poon and Granger (2003) attribute evidence that volatility is close to a unit root process to Perry (1982) and Pagan and Schwert (1990). Knight & Satchell (1998) cite Harvey (1993) as suggesting that volatility is fractionally integrated and thus stationary when the differencing interval used is less than 0.5.[38] Poon and Granger (2003) refer to Integrated GARCH (IGARCH), fractionally integrated GARCH (FIGARCH) and fractionally integrated exponential GARCH (FIEGARCH) models, as having been used to capture the persistence effect, although they observe that the drift term implicit in positively integrated processes is not observed in empirical volatility data.[39]

3.4.4.3 Asymmetry effect

The asymmetry effect is also referred to as the leverage effect, and like persistence, it relates to clustering. In effect the clustering effect outlined above is modified by the fact that although large shocks are followed by more large shocks, large negative shocks have a greater impact on future shocks than large positive shocks of the same absolute magnitude. The initial recognition of this effect is usually attributed to Black (1976). Studies that have incorporated the asymmetry effect into time series models, include Christie (1982), Schwert (1989), Nelson (1991), Engle and Ng (1993) and Glosten et al (1993). Hentschel (1995) and many others have carried out more recent work modelling the leverage effect. The term "leverage effect" arises from the suggestion that a partial explanation for asymmetry is

---

[38] See page 17 of Knight and Satchell (1998) and Hamilton (1994) for further details.

[39] Hwang and Satchell in Ch7 of Knight and Satchell (1998) and Granger (2001) are cited for this observation.

provided by increases in a firm's financial leverage, resulting from the rise in the debt/market value of equity ratio as stock prices fall. Bekaert and Wu (2000) argue that a process referred to as "volatility feedback" explains more of the observed asymmetry effect than the leverage effect.

3.4.4.4 Contagion and co-movement

Contagion is the tendency for a large volatility shock outside the normal distribution of the return time series in one financial market to occur simultaneously with, or in rapid succession to, similar shocks in different markets. This co-movement with different markets is also manifested between the securities within the same market, the classic example of such an event being the global market crash on the 19^{th} of October 1987. Various studies since 1987 such as King and Wadhwani (1990), Andersen et al (2000), Brooks and Persand (2000), Kearney and Poti (2003), Malevergne and Sornette (2004) and Poon et al (2004) have shown that the average covariance between securities and between markets tends to increase as the overall market volatility increases. Furthermore, an 'asymmetry in correlations' effect exists, whereby the increase in correlations is much larger in bear markets than in bull markets, thereby limiting the effects of portfolio diversification at the time when they are needed the most. This phenomenon is at the core of the empirical research in this study, so further discussion of this topic and reviews of prior studies in this area are discussed in detail in section 3.5.4.

3.4.4.5 Conditional heteroskedasticity

Conditional heteroskedasticity is the tendency for the magnitude of residuals in time series models of returns to be conditional upon the magnitude and, in some cases, the sign of the underlying return and, or, the residuals in the previous period. In other words, one of the basic assumptions of OLS models, that residuals should be independent of the explanatory variables, is violated. This characteristic forms the basis of the various forms of ARCH and GARCH models.

3.5 Empirical studies of returns, volatility and co-movement

3.5.1.1 Overview

The preceding pages have explained a number of important definitions in the study of asset returns and volatility. This section reviews some of the empirical data on the risk and return of the UK equity market presented by Dimson and Marsh (2001) and Poon and Taylor

(1992). A qualitative survey by Graham and Harvey (2002) of the ex-ante expectations of chief financial officers concerning the equity risk premium in the US market is also discussed. This is followed by a discussion of an empirical study of stock return volatility by Andersen et al (2001a) and an empirical study of the volatility of returns in the FTSE 100 Index futures contract by Areal and Taylor (2002). The section closes with a discussion of some empirical studies of co-movement between stock returns and stock index returns, dating from Elton and Gruber (1973) to Barberis et al (Forthcoming) and Malevergne and Sornette (2004). The empirical studies are discussed in the context of some assumptions concerning the behaviour and distribution of stock returns in equilibrium models such as the CAPM.

3.5.2 Empirical studies of returns, expected returns and expected risk premia

3.5.2.1 Poon and Taylor (1992)

Poon and Taylor (1992) study the weekly returns of the value weighted FTSE All Share Index from January 1965 to December 1989 and daily returns from 1969 to 1989. Data for dividends in the FTSE All Share Index are available from 1985, allowing Poon and Taylor to compare results for the data adjusted for dividends, and unadjusted data, over this period. They find no significant difference in the results and a return correlation between the two data series of 0.999. When they make a further adjustment for "settlement period effects" the correlation falls to 0.987.[40] Given the high correlation between the adjusted and unadjusted series, it seems unlikely that the omission of dividend data prior to 1985 would have adversely affected their results. Poon and Taylor (1992) find that distributions of daily, weekly and fortnightly returns are negatively skewed while monthly returns are slightly positively skewed. For all measurement intervals the kurtosis is considerably greater than three, indicating fat tails compared to the normal distribution. They also note significant autocorrelation at one lag, for daily and fortnightly time-series. Thin trading and settlement effects are suggested as a possible explanation. Squared returns have more significant autocorrelation coefficients than actual returns. This is indicative of volatility

[40] Settlement period effects relate to the cost of carry that arises from the delayed settlement of stock transactions. Prior to 1994, the UK market operated a system of delayed settlement on the second Monday, usually ten-working days after the end of each account period. Given that an account period lasted two and sometimes three weeks and that a trade could occur any time within an account period, this produced day of the week and seasonal anomalies on transaction prices. The effect was eliminated with the introduction of rolling settlement in 1994.

clustering and is more pronounced at higher measurement frequencies. Autocorrelations of absolute returns are also presented. This data is compared with our own FTSE 100 Index data for the period from January 1985 through March 2003 in Chapter 8.

3.5.2.2 Dimson and Marsh (2001)

Dimson and Marsh (2001) have compiled a number of monthly returns indices for the UK market. Their "All equities index" includes a total of 2,172 firms over the period 1955-1999, and on the 1st of January 2000 contained 1,308 firms. This has been subdivided into "high-cap equities", comprising 90% by market value of all equities, the "low-cap equities", which contains the next 9% by market value, and "micro-cap equities", which contains the smallest 1% of firms by cumulative market value. However, it is not clear how often these categories are re-balanced to take into account firms that move between categories. Dimson and Marsh also have separate indices for long maturity bonds, mid maturity bonds, index linked bonds, and treasury bills. They used the London Business School Share Price Database (LSPD), which starts in 1955. Their aim was to make indices that are as comprehensive and robust as those calculated by the Chicago Centre for Research in Security Prices (CRSP). After 1975, they have included all equities listed on the London Stock Exchange (LSE) with the exception of closed end funds, in order to avoid double counting. However, separate indices that include closed end funds are calculated. Dimson and Marsh estimate that closed end funds accounted for around 3.4% of the total capitalisation of the London market in January 2000. Prior to 1975 they used a random sampling methodology detailed in Dimson and Marsh (1983). The equity indices are value weighted in contrast to their bond indices, which are equally weighted; they also include dividends that are added on a precise ex-dividend basis. All returns are calculated assuming zero taxes and transaction costs and the results from a variety of calculation methods are compared, although the favoured method is the geometric differencing represented by Equation 20. This is used to calculate the premium over risk free treasury bills. It is argued that this has the advantage over arithmetic differencing between equity and treasury bills because it enables compounding of premia over any chosen differencing interval.

Equation 20. $$R_{gd} = [(1+r_a)/(1+r_b)]-1$$

Where: *R_{gd} equals the return premium on the asset, r_a equals the total return of the asset a and r_b equals the total return on a benchmark b, such as the risk-free rate.*

Descriptive statistics for annual nominal, real, arithmetic and geometric total returns, as distinct from risk premia, over the period 1995-99 are summarised for each asset class, in Table 5 on page 9 of Dimson and Marsh (2001). Annual equity returns are negatively skewed for all equities and the sub equity indices. Kurtosis is greater than three for the high-cap and the all equity index series but less than three for the low and micro-cap nominal equity index returns. High-cap equities have a lower standard deviation of returns than low or micro-cap equities. However, the kurtosis of high-cap equities is higher, suggesting that the returns of large dominant stocks in the index are more susceptible to extreme events. This could be due to their greater liquidity, allowing investors to buy and sell easily during times of panic or positive shocks, thereby forcing their returns to greater extremes. Intuitively this would seem to be more likely in the case of higher frequency returns than the annual returns described here. Dimson and Marsh also present descriptive statistics of return premia over the risk-free rate for both arithmetic and geometric differencing, in Table 7 on page 13 of Dimson and Marsh (2001). Skewness and kurtosis are lower when returns are calculated using geometric differencing.

3.5.2.3 Graham and Harvey (2002)

One method of estimating expected returns is to use the prevailing risk free return plus a probability weighted expected risk premium based on the risk of the asset being evaluated. This could be based upon a range of forecasts for key fundamental factors or the consensus of analyst's earnings forecasts. Graham and Harvey (2002) use a survey of chief financial officer's expectations, citing Welch (2000), as a study of views of financial economists. Fraser (2001) and Harris and Marston (2001) are cited as examples of studies that consider the views of financial analysts concerning the equity risk premium. However, Graham and Harvey argue that analysts have a tendency to be upwardly biased in their forecasts of future equity returns, while financial economists do not directly participate in asset allocation or capital structure decisions within the economy. Therefore, they propose that Chief Financial Officers (CFOs) are a better source of expectational data because they tend to be directly involved with their firms' capital structure decisions and asset allocation decisions.

They also argue that CFOs have less reason to be biased than stock analysts working for large investment banks. They cite work by Gerhardt et al (2001) and Fama and French (2002) who use fundamental data such as cash flow forecasts, cost of capital and stock price to determine an internal rate of return and risk premia.[41] In addition, Graham and Harvey (2002) argue that estimates of the equity risk premium based on historical data, such as that compiled by Dimson and Marsh, may be misleading and subject to sample period bias. They cite Fama and French (2002) who conclude that average realised equity returns may be higher than ex-ante expected returns over the last fifty years due to *"large unexpected capital gains"*. Graham and Harvey argue that if this is true, the use of historic average returns to estimate expected risk premia is misleading. Instead they propose using a survey of the expectations of CFOs, over one and ten year horizons. Although it is a qualitative survey, based upon the judgement of CFO's, it enables them to estimate a consensus forecast of the entire risk premium distribution, rather than just the mean return. Their study is still in progress and they intend to update their forecasts at quarterly intervals starting in the second quarter of 2000.[42] They anticipate that the data will enable them to relate changes in expected returns to recent historic and future actual volatility levels. In addition they are able to take into account asymmetric distributions of stock returns and differences between industry groups.

They find that the distribution of CFO expectations is much tighter than historic measures.[43] The results are interesting, although the relatively short time period over which data has been collected, to date, means that possibilities for comparison with more traditional estimation methods are limited. Furthermore, the potential for comparing CFO's ex-ante expectations with actual realised returns is limited at this stage, although this problem should be remedied in the future if Graham and Harvey are able to sustain their data collection effort for a significant period.

---

[41] Graham and Harvey (2002) also cite work by French and Poterba (1991) and by Graham and Harvey (1996), as examples of studies that estimated ex-ante expected returns by examining portfolio weights for different asset classes recommended by financial institutions. Based upon ex-ante assumptions about expected volatility and expected covariance between the assets, these authors gain an estimate of the ex-ante implied equity risk premium by re-arranging portfolio mean variance optimisation formulae employed in the implementation of modern portfolio theory. The following discussion reviews the study by Graham and Harvey (2002). The problem with all of these approaches is in the stability and validity of the restricting assumptions used.

[42] At the time of writing in February 2004 Graham and Harvey (2002) was still at the working paper stage.

[43] Between the second quarter of 2000 and the third quarter of 2001, their results indicate that the one year risk premium averages between 0.1 and 2.5% depending on the quarter surveyed while the ten-year premium is less variable and ranges between 3.6 and 4.7% per year. These figures are much lower than those of previous studies of the US market based on historical realised returns, which tend to imply a premium of around 9%.

When measuring realised volatility over a period, the return that is subtracted from discrete observed returns, such as in a typical formula for the sample variance, is not usually based upon ex-ante expected returns, such as those implemented by Graham and Harvey. Such methods are too subject to the vagaries of their specific estimation methodology to be readily compared across asset classes, across markets and across studies. Nonetheless, there maybe benefits from incorporating such estimates into models used for generating ex-ante volatility forecasts.

3.5.2.4 Areal and Taylor (2002)

Areal and Taylor (2002) examined five-minute interval returns of the FTSE 100 Index futures contract returns over a thirteen-year period from the 1st of January 1986 to the 29th of December 1998. Returns were calculated from the price of the nearest to delivery contract except on the days before expiry, after which the next contract was used. Data for the whole period was compared with data over a sub-period from 1990 through 1998, when trading volume was higher than at the beginning. For the sub-period the mean return was not significantly different from zero at the $\alpha < 20\%$ threshold, a substantially lower threshold than the conventional $\alpha < 5\%$ threshold.[44] Areal and Taylor also examine the distribution of standardised daily returns, where daily returns are defined using Equation 21 and the returns are standardised using Equation 22.

Equation 21. $$r_t = \sum_{j=0}^{n} r_j$$

Where: *r_t equals the daily return, j equals the intra day measurement period and j = zero refers to the market opening time.*

Equation 22. $$r_t^* = \frac{(r_t - \bar{r})}{\hat{\sigma}_t}$$

Where: *r_t^* equals the standardised daily return and $\bar{r}$ equals the mean daily return of 0.000483 or about 12% per annum over the measurement period.*

In the above equations Areal and Taylor note that the average five minute return is negative to the order of approximately 3% per annum; therefore, most of the positive returns

[44] This finding provides a justification for the assumption of a zero expected return, when estimating realised volatility with returns of this frequency, as in the discussion of expected returns in section 3.2.2.3.

in the Index futures must come from market closed periods, i.e. closing to opening price differences. The kurtosis of standardised daily returns is 2.77 when optimised weights are used to calculate returns compared to 4.81 for non-standardised returns. Nonetheless, both skewness and kurtosis values are significantly different from that of the normal distribution even for the standardised returns. The relative similarities of Areal and Taylor's results for returns and realised volatility for the FTSE 100 index and those of Andersen et al (2001a) for the DJIA are apparent.

3.5.2.5 Other related studies

Other empirical studies of returns include Schwert (1990a) who created indices of monthly US stock returns beginning as early as 1802 and daily returns beginning in 1885. Schwert (1990a) based his analysis on earlier indices created by Cole and Fricky (1928), Smith & Cole (1935) and Cowles (1939). Schwert concluded that the data exhibited a surprising degree of temporal homogeneity over the extended period, although this was punctuated by periods of high volatility during the depression of 1929 –1939 and later in 1973 – 1974. Furthermore, daily and monthly calendar anomalies documented in earlier studies of the CRSP data, from the post 1962 period, appear to be evident even in the earlier data examined by Schwert. Dimson et al (2002) extended their study of UK market returns to include an empirical analysis of market indices around the world. Since the publication of their book, Dimson et al have begun to combine their output with that of Ibbotson and Associates, who also publish regular data on the returns of both US and international stock and bond indices.

The study of realised volatility in the thirty constituents of the DJIA, by Andersen et al (2000) and Andersen et al (2001a), is reviewed in the next section.[45] However, Andersen et al (2000) also examine the unconditional distribution of returns in the thirty DJIA constituents. They find that they exhibit excess kurtosis and in the majority of cases are positively skewed. However, when the return series are standardised by dividing them by the realised standard deviation, the unconditional distribution of returns is approximately normal with sample kurtosis declining from 5.416 to 3.129.

[45] Andersen et al (2001a) is the published version of Andersen et al (2000); however, due to variations in the level of detail provided, citations from both papers are provided in this discussion.

3.5.3 Review of empirical studies of volatility

3.5.3.1 French et al (1987)

French et al use daily returns to estimate monthly volatility in the S&P Composite Index over the period from January 1928 through December 1984. In the spirit of Merton (1980) they examine the relationship between market risk and market expected returns, focussing specifically on the market risk premium component of returns. Because the S&P Composite portfolio contains the stocks of many small, thinly traded firms that exhibit non-synchronous trading patterns, the returns of the index are liable to exhibit spurious autocorrelation. Therefore, French et al correct for this and then decompose the monthly volatility into predictable and unpredictable components using univariate ARIMA models.[46] They find little evidence of a relationship between the excess holding period returns and the predictable components of volatility. However, they find strong evidence of a negative association between the monthly excess holding period returns and the unpredictable component of market volatility.

3.5.3.2 Poon and Taylor (1992)

As well the analysis of UK market returns reviewed in the previous section, Poon and Taylor (1992) also used Equation 23 to estimate monthly volatility for the FTSE All Share Index using daily returns.

Equation 23.
$$\sigma_t^2 = \frac{(1+2r_1)}{n_t - 1}\sum_{d=1}^{nt}(R_{d,t} - \overline{R}_t)^2$$

Where: *r_1 is the one period autocorrelation between successive returns and $\overline{R}$ is the time varying mean daily return in month t, so that monthly volatility is the sum of squared daily returns less the time varying mean adjusted for autocorrelation, nt is the number of trading days in a given month t.*

Equation 23 is an adaptation of Equation 24, applied by Merton (1980) to estimate monthly volatility when the moving average parameter is constant, the number of days in each month is constant and it is necessary to take account of possible autocorrelation between stock index returns.

---

[46] Following Merton (1980) and French et al (1987), Poon and Taylor (1992) develop this process, which is summarised in the next section.

Equation 24. $$\sigma_t^2 = (1+2r_1)\sum_{d=1}^{nt}(R_{d,t} - \overline{R}_t)^2$$

Autocorrelation can arise due to non-synchronous trading of the stocks of different constituent firms in an index. Poon and Taylor credit Merton (1980) for demonstrating that, if unaccounted for, it can lead to negatively biased estimates of variance.[47] Poon and Taylor point out that when σ^2_t is estimated using Equation 24, σ^2_t increases as *nt* increases. Given that *nt* varies from month to month, they argue that this makes it difficult to extract the predictable component using an auto regressive integrated moving average (ARIMA) model. They resolved this problem by scaling the variance estimate for the varying number of trading-days in each month using Equation 23.

Poon and Taylor adjusted for the positively skewed distribution of this estimate of variance by taking natural logarithms, although they note that some skewness still remains after the adjustment. These estimates are split into two components, an expected component and an unexpected component using both an autoregressive moving average model ($ARMA_{(1,1)}$) and an integrated moving average ($IMA_{(1,1)}$) model. The residuals observed in both models were close to white noise but the fitted values were skewed, with a high kurtosis. The R^2 is higher for the ARMA model, although Poon and Taylor conclude that the difference between the two models is small in terms of performance. Using a Dickey-Fuller test they are able to reject the null hypothesis of a unit root in the volatility series at the $\alpha < 1\%$ threshold. Much of their discussion is based on the results of the $ARMA_{(1,1)}$ model but they note that French et al (1987) chooses an $IMA_{(1,3)}$ model and that Poterba and Summers (1986) and Pindyck (1986) used an $AR_{(1)}$ model. As with the sample variance, Poon and Taylor report positive skewness and high kurtosis values for the conditional variance for both daily, weekly, fortnightly and monthly returns estimated using a standard ARCH model. Furthermore, the degree of skewness and kurtosis does not seem to be related to the length of estimation periods used. However, they report that the autocorrelation coefficients are higher, indicating greater persistence when estimates are made at higher frequencies. The results are robust to sample size effects as they appear even in smaller sub-samples of high frequency data, although the t statistics are lower than for the whole sample. They also

[47] It can be argued that this is likely to be a problem in variance estimates for indices containing the stocks of many small infrequently traded firms. However, in a large capitalisation index, such as the FTSE 100 Index used in this study, the liquidity of the constituent stocks is much greater and this effect seems unlikely to be a problem in daily returns.

find evidence to suggest that volatility data series become integrated as the sampling frequency increases, whereas they approach a stationary mean reverting process as the frequency of observations decreases.

Comparison can be made with the later results of Oomen (2001) and Areal and Taylor (2002) who use ultra high frequency data in the FTSE 100 Index returns and Index futures returns respectively. Poon and Taylor also test for relationships between returns, excess returns and each of the expected and unexpected volatility estimates. They find little evidence of significant cross correlations between the data series, although returns and unexpected volatility had a significant coefficient at the fourth lag.

3.5.3.3 Andersen et al (2001a)

Andersen et al (2001a) examine the distributions of daily-realised volatility for the returns of individual stocks in the Dow Jones Industrial Average (DJIA), using high-frequency intra day transaction prices. They find that the unconditional distributions of realised variances are positively skewed; however, the logarithms of the realised standard deviations are approximately normal. In addition, they find evidence that realised volatility exhibits the long memory characteristics described earlier in section 3.4.4.2, as well as limited evidence of the asymmetry effect, both for realised variances and realised covariances between returns of the thirty DJIA constituents. Despite the long memory characteristics observed in the logarithms of the realised standard deviations data series, the null hypothesis of a unit root can be rejected using an Augmented Dickey Fuller (ADF) test.

In the study by Andersen et al (2001a), realised volatility is the sum of squared returns over a given estimation period. Returns are measured over five minute intervals and the estimation period for realised volatility is one day. Estimates of realised daily variance are calculated from intra day returns using an $MA_{(1)}$ model to adjust for biases that are potentially induced into discrete returns by the spurious autocorrelation arising from non-synchronous trading. Andersen et al (2001a) argue that the highest possible frequency of sub-periods will theoretically result in the least biased estimate of volatility. However they acknowledge that, in practice, non-continuous trading and market microstructure effects, such as the bid-ask bounce, are likely to contaminate ultra high frequency returns data. Thus in a general context, the optimal frequency will depend upon the trading liquidity of the security, i.e. the size of the bid-ask spread and the frequency and volume in which it is typically traded. For example, Oomen (2001) presents a theoretical argument, backed up by high frequency empirical data, suggesting that the optimal sampling frequency to estimate realised volatility of FTSE 100 Index returns is around 30 to 35 minutes, compared to the five-minute intervals used by Andersen et al.

The limited evidence found by Andersen et al (2001a) for the asymmetry effect in individual firm volatilities contrasts with the strong evidence found by other authors for this effect in stock indices. Andersen et al argue that this may be evidence that the effect is primarily driven by a volatility feedback effect rather than a leverage effect. Weak evidence is also found of an asymmetry effect in individual firm correlations, although less than half of these are statistically significant.

3.5.3.4 Areal and Taylor (2002)

The study of FTSE 100 Index futures returns by Areal and Taylor (2002), and reviewed in the previous section, also examined the volatility of the Index futures returns. A modified version of the method adopted by Andersen et al (2001a), in their study of the thirty Dow stocks, was used in which the realised variance equalled the sum of the squared intra-day returns. Two temporal weighting schemes were used for the squared returns, equal weights and the "modification" which involved assigning closed market returns with different (smaller) "optimal" weights to intra-trading period returns. The adjusted weight was optimised to adjust for day of the week and weekend effects. Data for the whole period was compared with data just for the period 1990-1998, when trading volume was higher than at the beginning. They found that skewness and kurtosis were much higher for the whole period, which is not surprising since this included the 1987 crash. The distribution is approximately symmetrical with a skewness of 0.23 and a kurtosis of 25.2. Observed autocorrelations were very close to zero. Annualised estimates of daily realised-variance were obtained by multiplying daily variance by the root of 251. The average annualised estimate of standard deviation thus achieved was 14.2%, or 15.1%, depending on the weighting scheme adopted for the intra-day returns. The distribution of the logarithm of daily volatility had a skewness of 0.44, or 3.71, when optimised weights were used. Although these values are significantly different from the zero and three, that would represent a normal distribution, they are much closer to a normal distribution than the values obtained when equal weights were applied to both closed-to-open and intra-trading period returns. In the latter case, kurtosis of 5.96 fell to 4.22 when the overnight period was excluded. These results are not dissimilar to those of Andersen et al (2001a).

Unlike returns, the time series of the logarithm of estimated realised daily volatility Ln($\hat{\sigma}_t$), when calculated using optimised weights, shows positive temporal dependence, i.e. significant autocorrelations, for around 180 lags of trading day returns. However, the greatest autocorrelation of 0.65 occurs on the first lag. This is almost identical to the first lagged autocorrelation recorded by Andersen et al (2000) for the DJIA stocks. The augmented Dickey Fuller (ADF) test rejects the null hypothesis of a unit root in the series. However, Areal and Taylor present strong evidence suggesting the existence of a long memory process citing Baillie (1996) as earlier evidence for this effect.

3.5.4 Review of empirical studies of co-movement

Various studies have examined co-movement, correlation and covariance between securities and portfolios of securities. Elton and Gruber (1973) and Elton et al (1978) evaluate different methods of forecasting the correlation structure of portfolio constituent returns. King and Wadhwani (1990) examine co-movement during the 1987 stock market crash. King and other authors have found evidence to suggest that during times of market crisis, such as the 1987 crash, correlations across all markets have a tendency to approach unity.

A study, by Malevergne and Sornette (2004), focuses on the correlation between security returns during periods of extreme market volatility when stock returns are in the tails of the return distribution. They argue that the correlation structure of asset returns at the tails of the return distribution is different to the correlation structure recorded using returns that would fall within the standard normal distribution. They call the tendency of paired security returns to be highly correlated during extreme movements “tail dependence”. Their empirical study provides evidence that when portfolio weights are optimised with the objective of minimising tail dependence, portfolios exhibit more effective diversification characteristics without sacrificing returns than portfolios optimised using conventionally estimated correlations between paired assets.

Poon et al (2004) also examine the dependence structure of logarithmic returns using closing levels of the S&P 500, the FTSE 100, Dax 30, CAC 40 and the Nikkei 225 indices during extreme events. They refer to “left tail dependence” and “right tail dependence”, finding that left tail dependence associated with negative market returns is much stronger than the right tail dependence associated with positive market returns. Furthermore, they find that tail dependence appears to have increased in recent years, particularly among the markets of European countries.

Brooks and Persand (2000) also examine the relationship between the correlations and variance of daily logarithmic returns in five Southeast Asian stock market indices over the period from the 1st January 1985 through the 29th April 1999. Three separate market conditions are identified: "positive crashes", "negative crashes" and "no crashes". Positive crash periods are those in which one of the markets have returns greater than 1.645 standard deviations above the mean, negative crashes are the same distance below the mean and no crashes are when the returns of all markets are within 1.645 standard deviations of the mean. They present their analysis in the context of variance covariance value at risk methods, such as the J.P. Morgan RiskMetrics™ approach. Brooks and Persand find evidence that a positive association exists between the volatilities and correlations of the five market indices studied. They propose that value at risk models should incorporate a weighed average of VAR estimates derived using the correlation and variance structures observed in extreme trading conditions and those derived from normal trading conditions.

In addition to analysing the distributions and time series properties of returns and realised volatility of the thirty DJIA constituents, Andersen et al (2001a) also examine the four hundred and thirty-five covariance and correlation terms in the equally weighted VCM of the thirty DJIA constituents. They find that the average realised daily covariances between firms exhibit considerable temporal variation. Furthermore, the average realised covariance of a given firm, with the other constituent firms, exhibits considerable variation between firms. The distributions of the mean daily covariances are also unstable between firms and across time, with large variations in absolute skewness and excess kurtosis. However, distributions of the daily-realised correlations are more stable with lower absolute skewness and lower kurtosis. They find evidence of temporal dependence in the daily-realised correlations between individual firms but the null hypothesis of a unit root is rejected for the time series of daily-realised correlations. Andersen et al (2001a) also present evidence indicating that the average correlation between returns tends to increase as average volatility increases.

Elton and Gruber (1973) used the realised correlation matrix of a sample of seventy-six firms from seven industry groups to evaluate ten models for forecasting the ex-ante correlation matrix. The ten models included a "full historical correlation model" in which paired historic correlations were estimated between every pair of securities in the sample, a single index model, such as that of Sharpe (1963), and multi index models. The indices were selected by ranking them in order of the principle components. For instance the index for a single index model is the first principle component, or index that best explains the historical correlation matrix of the data sample. Elton and Gruber (1973) also tested their naive “Overall Mean Model”. This assumes that every correlation coefficient between paired security returns is equal to the average of all correlation coefficients. They found that the naive model, based on the average aggregate cross correlation of returns in the VCM, provided the best five-year out-of-sample forecast of the future correlation structure in the VCM, when compared to single and multi index models, and models that take into account the full historical VCM. Furthermore, the difference in the ranking between the overall mean model and the other models was both statistically and economically significant, allowing portfolios to be constructed with returns up to 50% higher for a given risk level. Thus the overall mean model is also better for optimising ex-ante portfolio weights than the other models tested.

The performance of the overall mean model for forecasting the ex-ante correlation matrix was tested again in relation to a full historical model and single index models by Elton et al (1978). Four variations of the single index models tested include an unadjusted beta model, in which betas are estimated using a regression model, a model in which the betas for all securities are assumed to equal one, a Blume adjusted beta model and a Vasicek adjusted beta model.[48] Once again the overall mean model continued to dominate the other five models tested. Elton et al (2003) provide a more extensive summary of these and other models that attempt to provide ex-ante forecasts and optimisations of the variance covariance matrix

Another study by Barberis et al (Forthcoming) defines three theories regarding co-movement: category based co-movement, habitat based co-movement and fundamentals based co-movement. The authors define *“category based co-movement”* as the idea that investors classify different securities into the same asset class and shift resources into and out of this class in correlated ways. They define *“habitat based co-movement”* as the co-movement that arises when a group of investors restrict their trading to a given set of securities and move in and out of that set in tandem. They define a *“fundamentals view of co-movement”* as, co-movement due to positive correlations in the rational determinants of asset values, such as cash flows or discount rates across co-moving assets. Their study

---

[48] Elton et al (1978) refer to Blume (1971) and Vasicek (1973) as the respective sources for the Blume and Vasicek adjusted Beta models.

provides evidence in favour of the category and habitat based theories of co-movement, but it does not support the concept of fundamentals based co-movement. Their study also provides a useful discussion of the economic theories behind various explanations of co-movement.

Bollerslev et al (1988) construct a tri-variate model of the quarterly percentage returns on twenty-year treasury bonds, three and six-month treasury bills and stocks from the value weighted NYSE composite index including dividends. Quarterly bond and stock returns are measured as an excess return over treasury bills from the first quarter of 1959 through the second quarter of 1982 to give a total of 102 observations. They find that the conditional covariance matrix of the three asset returns is autoregressive: therefore they reject the assumption that the matrix is constant over time. Use of a tri-variate GARCH model allows estimates of the conditional mean vector $\boldsymbol{\mu}$ and the covariance matrix $\mathbf{H_t}$ to vary over time. The covariance matrix of the three asset classes is value weighted by multiplying by the vector of asset class value weights $\mathbf{w_{t-1}}$. However, Bollerslev et al only consider three assets: an aggregate stock market index, a bond index and a risk free asset. Therefore they do not investigate the covariance matrix at the level of detail displayed by Campbell et al (2001), discussed in section 3.6.3, nor do they try to decompose the VCM of an entire stock index, as we do in this study. Their data provides evidence to reject the assumption that the covariance between the three asset classes studied is constant over time. They also suggest that the expected risk premium is significantly influenced by the conditional second moment of returns, i.e. the conditional variance. In addition they suggest that the risk premium of individual asset classes is more influenced by the conditional covariance of individual asset returns with the market returns than by the variance of individual asset returns. This is because the covariance represents the systematic non-diversifiable risk. Their study illustrates the time-varying nature of the covariance structure within the VCM. Campbell et al (2001) also demonstrate the time-varying characteristic when they find that average covariance has decreased over time, as does this study, which breaks the covariance down into incremental and equally weighted components that are also found to be time varying.

3.6 Studies related to concentration and portfolio diversification

3.6.1.1 Overview

Studies that have attempted to explain the time series behaviour of stock and stock index volatility using the principles of portfolio diversification are relevant to this study because portfolio concentration is related to portfolio diversification. Studies such as Elton and Gruber (1973) and Elton et al (1978), reviewed in the previous section, examined the correlation structure of portfolio constituent returns in the VCM. Roll (1992), and later works citing him, seek to explain whether or not international stock portfolio returns and return variances are driven more by country specific factors or by industry specific factors. Campbell et al (2001) focused on the variance of individual stock returns in relation to the variance of market returns, seeking to explain how much of the market variance is explained by the idiosyncratic risk of individual stocks and how much is explained by the covariance of individual stocks with one another and, hence, with the market as a whole. More recent studies have extended the work of Campbell et al in both the US and the European markets. Studies of country and industry sector diversification, such as Roll (1992), are reviewed below in section 3.6.2. Studies that examine the relationship between idiosyncratic and systematic risk, such as Campbell et al (2001) and later studies that develop their findings are reviewed in Section 3.6.3.

3.6.2 Studies of country and industry diversification

From an MPT perspective, studies such as that of Roll (1992), ask the question; which is more important, industry diversification or geographic diversification? One school of thought argues that globalisation and increasing market integration allows sector or industry specific factors to dominate portfolio returns and variances. The other school of thought, and a substantial body of empirical evidence, suggests that country specific factors still dominate sector or industry specific factors. A recent study by Isakov and Sonney (2002) indicates that the relative dominance of the two categories is shifting through time. Although, historically, country specific factors have dominated returns and variances in most markets, data over the last five years indicates that the balance has shifted towards industry specific factors as the pace of global market integration has accelerated, although Isakov and Sonney acknowledge that the bubble in technology stocks may have influenced their results.

Using a cross section of returns from the FT Actuaries/Goldman Sachs International Indices for twenty-four countries Roll (1992) investigated the link between country stock index returns, volatility and a number of different factors. The factors emphasised were: index industry composition, including index industry concentration, concentration of constituent company market values within the index and fluctuations in the value of local exchange rates against the US dollar. The methodology used, results obtained and conclusions drawn, are related to this study; hence the discussion that follows. The primary difference between Roll's study and this one is that Roll based his investigation on a cross sectional analysis of twenty-four national markets represented by the FT Actuaries/Goldman Sachs International Indices. He used daily returns for each index to calculate monthly volatilities over a three-year period from April 1988 through March 1991. In contrast, this study focuses upon just one index, the FTSE 100 for the purpose of developing the methodology. Whereas Roll's study uses a cross sectional approach, this study applies time series models of concentration, realised volatility and sub-components of realised volatility measured using daily FTSE 100 Index returns. A much longer time span is covered, compared to Roll's study and concentration is sampled at a much broader range of frequencies. The present study emphasises the impact of time varying concentration upon volatility, within a single market, whereas Roll's study examines the impact of cross sectional variations in concentration between twenty-four different markets.

Over the three-year period Roll collected data on monthly industry concentration and constituent concentration for each country index. For each of the thirty-six months of the study he performed the cross sectional regression represented by Equation 25.

Equation 25. $$\log_e(S_j) = b_0 + b_1 C_j$$

Where: *C_j equals a measure of concentration for country j at the beginning of the month, S_j equals the standard deviation of daily returns during the month, calculated using both local currency and dollar denominated price indices on each day during the month and j = 1,..24.*

Concentration of constituent firms for each index was measured by calculating the Hirschman-Herfindahl H index based on the market value of each constituent company in each index at the start of each month. In addition, the absolute number of constituent firms traded in each index for each month was also used as a measure of concentration.

Roll (1992) reports the thirty-six-month average of the monthly concentration slope coefficients, separately for each of the three concentration metrics. Each concentration metric has a thirty-six-month mean model slope coefficient significantly different from zero, when both local currency denominated returns and dollar denominated returns are used to explain monthly volatility in the regressions. However, despite being statistically significant Roll found the size of the coefficients to be quite small. The model R^2 values were not reported, making it difficult to determine the goodness of fit or the economic value of the model results. The slope coefficient for the H of the industry is 1.54×10^{-2}, and 8.76×10^{-3} for individual stocks, using dollar denominated returns. The mean monthly t-statistics of the concentration slope coefficients obtained for each of the thirty-six cross-sectional regressions are also reported, together with the standard deviation of the t-statistics. These results are revealing because they indicate that the mean monthly t-statistics are too small to reject the null hypothesis that an individual concentration slope coefficient is equal to zero, using a two-tailed test at the $\alpha < 5\%$ threshold. The only monthly mean t-statistic that would imply an individual monthly coefficient significantly different from zero, at the $\alpha < 10\%$ threshold, is that of the Hirschman-Herfindahl Index, and then only when returns are denominated in US dollars. However, Roll argues that the sample average of monthly-coefficient t-statistics is significantly different from zero, based on standard errors computed from the standard deviations of the reported t-statistics. Thus if the sample average t-statistic on the regression coefficient is significantly different from zero, he argues that we can infer, indirectly, that the sample means of the coefficients themselves are significantly different from zero.

Roll's analysis leaves opportunities to explore the possibility that the magnitude, statistical significance and sign of the slope coefficients for concentration may be small and inconsistent when studied over discrete periods. This provides a clear justification for the present study, which focuses upon the time series characteristics of concentration and its effect upon stock index volatility over a sustained period within a single stock market index.

Roll also estimated daily returns for each industry group based upon the weighting of each industry group within countries at the start of each month, while excluding the contribution of each country in turn. The motivation for this method of estimating daily returns for each industry group is twofold. First it allows the effect of returns for each global industry group on the returns of local stock market indices to be studied, when the estimated returns of the industry groups do not contain any component of the local index. In addition, it enables daily returns for each industry group to be estimated if they are not otherwise available, although it is not entirely clear from Roll's article whether this was in fact the case or not. Having obtained returns for each industry group Roll then estimated the time series model, represented by Equation 26, for each country index.

Equation 26. $$R_{jt} = b_{1,j} I_{j1t} + ... + b_{7j} I_{j7t} + b_{8j} D_{mt} + b_{9j} Z(j/\$)_t + e_{jt}$$

Where: *I_{jit} equals the industry index return for sector i, D_{Mt} equals the Monday seasonal dummy and Z(j/$) equals the relative change in the exchange rate.*

With the exception of the Monday dummy, one daily lead and one daily lag are included for each variable to correct for spurious autocorrelation induced by asynchronous trading between world markets. However, contemporaneous variable coefficients dominated the leading and lagged variables in the reported results. For each of the twenty-four countries most of the industry groups had slope coefficients that were significant. Unsurprisingly, the sign and magnitude of slope coefficients and t-statistics for a given industry in a given country could be related to the relative importance of the industry in the stock market of the country concerned. Essentially, US dollar (USD) denominated stock market returns, for a given country, were influenced extensively by the returns in the industries that make up the biggest portion of that index, even if that country's share of the global industry in question was excluded from the calculation of the overall global industry returns.

Overall, the article concludes that the volatility of a country's market index is inversely related to the number of firms in a country's index and positively related to the concentration index H. This is intuitively appealing and provides motivation for this study. However, as mentioned earlier, analysis of the Roll's data indicates that his results are not quite as clear cut as his conclusions imply, particularly when individual cross-sectional regression results obtained during the three-year study period are considered. Instability of the concentration slope coefficients, obtained from the cross-sectional regressions of concentration on realised monthly index return-volatility, provides further motivation for the present study that takes a more extensive look at the time series properties of concentration and its effect on volatility over a longer period within a single market index. Evidence in support of Roll's conclusions and in support of the idea that concentration may influence returns and hence volatility of returns is given by the results of Roll's time series regressions of industry index returns on country index returns, when those industry returns are computed strictly from returns in other countries. This, in turn, provides motivation for more detailed analysis of the internal weighted correlation structure within a stock index that displays time varying concentration, such as the FTSE 100 index. This study is partially motivated by the results and conclusions of Roll's study. Hence the emphasis of this study upon the time series characteristics of constituent concentration within indices and the sub-components of the VCM, all issues that are not addressed in Roll's article.

While it is possible to make a critique of Roll's 1992 study, the fact remains that it was almost certainly the first to look at industry concentration in this context and the findings and methodological shortcomings provide opportunities for the present study. Several more papers have since been published citing Roll (1992). The findings of some of these are consistent with Roll, to the effect that industry specific factors play a larger part in explaining stock returns than country specific factors, while others are contrary to Roll, arguing that country specific factors are more important than industry specific factors. The more prominent of these articles are reviewed below. The implications of their findings and methodology for this study, which focuses primarily upon the impact of concentration at the individual company level on ex-ante realised composite index volatility, are now discussed.

3.6.2.1 Other studies that examine industry and country diversification

Heston and Rouwenhorst (1994) studied monthly returns data for 829 firms contained in the MSCI indices of twelve European countries over ten years between 1978 and 1992. Like Roll they also assigned firms to indices based upon the FT Actuaries/Goldman Sachs indices using the same seven broad categories used by Roll (1992). When using a methodology based on that of Roll (1992) they obtain similar results to Roll with the effect that industry factors dominate country factors in influencing returns and variances. However, they are highly critical of Roll's methodology and, when using their own value weighted company, industry and country coefficients, their findings are contrary to Roll's in that the effect of industry factors on returns is considerably smaller than the country specific factors. Thus, Heston and Rouwenhorst conclude that country diversification is more important than industry diversification for portfolio optimisation in the twelve countries studied, with the exceptions of the Netherlands and Norway, which are highly dependent upon the energy sector.

Griffin and Karolyi (1998) broaden the scope of Roll (1992) and Heston and Rouwenhorst (1994) to include 66 industry classifications from twenty-five different countries over the period from January 1992 through April 1995. Using a dummy variable model, which they attribute to Solnik and de Freitas (1988), they decompose daily returns from the Dow Jones World Stock Index Database into industry and country specific components. They also look at nine aggregate industry sectors as well as the sixty-six sub classifications of the Dow Jones World Stock Index Series. Their findings, consistent with Heston and Rouwenhorst (1994) and contrary to those of Roll (1992), indicate that country specific factors dominate sector specific factors in achieving optimal portfolio diversification. When large portfolios are created by randomly combining firm securities across different industries within each country, portfolio variance is reduced to 21.9% of the variance of an average firm. Hence sector diversification without geographic diversification can reduce stock specific variance by 78.1% on average. However, diversification across countries but within the same broadly defined industries can reduce portfolio variance to just 8.4% of the average individual stock variance, i.e. a reduction of 91.6%.

Despite the above evidence indicating that country specific factors dominate returns, there are some anomalies. Griffin and Karolyi identify two categories of industry. Those that produce goods traded internationally, such as oil and specialist manufacturing or transportation services, and those that produce goods that are primarily consumed locally, such as food, aggregates, construction etc. The latter industries gain the greatest benefit from international diversification, as they seem less well integrated than the internationally traded goods and service industries, where the benefits or global diversification within industries are much less. The issue is further complicated by evidence of non-stationarity in the industrial structure of different countries. Furthermore, covariance between the returns of different industries appears non-stationary through time.

Isakov and Sonney (2002) note that while many academic studies appear to empirically demonstrate the dominance of country specific factors over industry specific factors in driving asset returns, many industry practitioners and financial institutions have begun to formulate asset allocation procedures based upon industrial sector selection rather than country or regional selection. This would seem unwise if the empirical studies are correct and country specific factors do dominate the return generating process. Such a condition would imply that global markets are fragmented, while the converse situation in which industry factors dominate would imply that global markets are on the whole integrated.

Isakov and Sonney (2002) perform their own empirical analysis using weekly price data on a sample of 4,359 firms from 20 countries over the period from June 1997 to December 2000. Returns denominated in local currency, and measured from Thursday to Thursday, are calculated to avoid the weekend effect. Data are obtained from MSCI and Datastream and they adopt the same methodology as Heston and Rouwenhorst (1994). Their results are consistent with earlier studies to the extent that country factors dominate industry factors. However there are large differences in the results depending upon which country or sector is analysed. Most countries are more important than most industries but the most important industries are more important than the less important countries for an investor wishing to achieve optimal portfolio diversification. This is consistent with Griffin and Karolyi (1998), who find that industries producing traded goods and service appear more integrated than industries producing non-traded goods or services.[49] Moreover, they also find that over the final 36 weeks of their data sample, the industry factor becomes more important than the country factor. They attribute this to telecom and information technology stocks, having a large influence on all the main market indices over this period. Baca et al (2000) are also cited as saying that the influence of the country factor was on average two to three times larger than the influence of the industrial factor before 1995, although this ratio has shrunk to just 1.23 during the last forty-eight months of their data set.

3.6.3 Studies of the relationship between systematic and idiosyncratic risk

In the current section, the focus shifts toward the review of studies of the relationship between systematic risk and the idiosyncratic risk of constituent securities in a market portfolio, and the implications for market concentration and volatility. Ever since the pioneering work of Markowitz (1952) portfolio diversification has been studied extensively. The pioneering work of Evans and Archer (1968) demonstrated the potential for naive diversification by simply spreading portfolio assets evenly between the stocks of a randomly selected sample of firms. They illustrated that as the number of firms in a portfolio increased the marginal reduction in portfolio risk for each additional firm declined rapidly. Most of the diversification benefits could be achieved by holding just ten firms according to Evans and Archer (1968). Later studies, such as Elton and Gruber (1997) and Poon et al (1992), revised that figure upwards and demonstrated that by sharing assets between the stocks of around thirty firms it was possible to reduce portfolio risk to just a fraction of that of a single firm and to little more than the systematic risk of the entire market portfolio. However a recent study by Campbell et al (2001) indicates that this

---

[49] "Traded goods" is referred to, in this paper, as goods exported abroad, as distinct from those consumed locally.

figure may have increased in the last thirty years to between fifty and one hundred firms. Their study has particular relevance for the present study because it analysis the time series of both the average variance and average correlation of firm returns over an extended period. It is reviewed in detail below.

Campbell et al (2001) decompose the total return and hence volatility of individual stocks into three components. A market return component ($R_{m,t}$), an industry specific residual ($\varepsilon^{*}_{i,t}$) and a firm specific residual ($\eta^{*}_{j,i,t}$). All stock returns and the aggregate market return are measured as an excess return over the treasury bill rate, thus enabling them to use the CAPM to explain the return generating process for individual firms without having to consider the risk free intercept. "Aggregate volatility" is defined as the volatility of the market as a whole and this is estimated using Equation 27. Value weighted excess market returns are calculated using Equation 28. They argue that this method does not require the specification of a parametric model to describe the evolution of aggregate volatility and their decomposition of firm volatility does not require the estimation of betas or covariances for individual firms.

Equation 27. $$Mkt_t = \hat{\sigma}^2_{m,t} = \sum_{s \in t} (R_{m,s} - \mu_m)^2$$

Where: *Mkt_t equals the volatility of the market, s is the interval at which returns are measured and t is the period over which volatility is measured, i.e. daily returns s, used to estimate monthly volatility σ^2_t and μ_m is the mean market return over the whole data-set.*[50]

Equation 28. $$R_{m,t} = \Sigma w_{i,t} R_{i,t}$$

Where: *$w_{i,t}$ equals the weight of industry i at time t.*

The excess return over the risk free rate of industry *i* at time *t* ($R_{i,t}$) is calculated using Equation 29. Industry returns are regressed on the market returns using the CAPM formula in Equation 30 so that the industry specific residual ($\varepsilon^{*}_{i,t}$) can be extracted from the industry returns.

---

[50] Campbell et al, also tried using a time varying mean but the results were similar, so they chose a mean estimated over the whole sample period to avoid potential sampling problems caused by instability in a time varying mean.

Equation 29. $R_{i,t} = \Sigma_{j,} w_{j,i,t} R_{j,i,t}$

Where: $w_{j,i,t}$ *equals the weight of firm j in industry i at time t.*

Equation 30. $R_{i,t} = \beta_{i,m} R_{m,t} + \varepsilon^*_{i,t}$

Where: $R_{i,t}$ *equals the industry return,* $\beta_{i,m}$ *is the beta for industry i with respect to the market return and* $\varepsilon^*_{i,t}$ *is the industry specific residual.*

The firm specific residual ($\eta^*_{j,i,t}$) is also estimated using a the regression model illustrated by Equation 31.

Equation 31. $R_{j,i,t} = \beta_{j,i} R_{i,t} + \eta^*_{j,i,t} = \beta_{j,i}\beta_{im} R_{m,t} + \beta_{j,i}\varepsilon^*_{i,t} + \eta^*_{j,i,t}$

Where: $R_{j,i,t}$ *equals the return of firm j in industry i at time t and* $\beta_{j,i}$ *is the beta of firm j with respect to industry i.*

They make the assumption that $\eta^*_{j,i,t}$, $R_{i,t}$, $R_{m,t}$ and $\varepsilon^*_{i,t}$ are orthogonal. Hence it is possible to ignore the covariances between these variables that arise as a result of the decomposition. Furthermore, they assume that $\beta_{j,m} = \beta_{j,i} \times \beta_{i,m}$ and that the weighted sums of the β_i equal to one. Because estimation of industry and firm level βs is difficult and they tend to be unstable over time, Campbell et al use the market adjusted return model shown in Equation 32 to provide a β-free estimate of industry and firm return variance.

Equation 32. Market adjusted return model $R_{i,t} = R_{m,t} + \varepsilon_{i,t}$

In the market adjusted return model $\varepsilon_{i,t}$ is the difference between the industry return and the market return. Although $R_{m,t}$ and $\varepsilon_{i,t}$ are not orthogonal, according to Campbell et al, the weighted average of the variances across industries are free of individual covariances in the specification described by Equation 33.

Equation 33.

$$\sum_i w_{i,t} Var(R_{i,t}) = Var(R_{m,t}) + \sum w_{i,t} Var(\varepsilon_{i,t})$$

$$= \sigma^2_{m,t} + \sigma^2_{\varepsilon,t}$$

Hence, $\varepsilon_{i,t}$ can be used to construct a measure of average industry level volatility without any estimation of β's. The weighted average industry volatility, expressed as the left hand side of Equation 33, can be interpreted as the expected volatility of a randomly drawn industry where the probability of drawing industry i is equal to it's weight $w_{i,t}$. This has a direct link with some interpretations of concentration. In fact the higher the concentration of the market portfolio with respect to industries, the closer the expected volatility of the portfolio will be to that of the largest industry or company by market value. Exactly the same principle applies to firm volatility in this context. In fact Campbell et al, use the same method for estimating the expected volatility of individual firm returns by modifying Equation 33 to Equation 34.

Equation 34.

$$\sum w_{i,t} \sum_{j \in i} w_{j,i,t} Var(R_{j,i,t}) = \sum_i w_{i,t} Var(R_{i,t}) + \sum_i w_{i,t} \sum w_{j,i,t} Var(\eta_{ij,i,t})$$

$$= Var(R_{m,t}) + \sum_i w_{i,t} \sum_i w_{i,t} Var(\varepsilon_{i,t}) + \sum_i w_{i,t} \sigma^2_{\eta,j,t}$$

$$= \sigma^2_{m,t} + \sigma^2_{\varepsilon,t} + \sigma^2_{\eta,t}$$

Using data spanning the period from 1926 through 1997, they concluded that there is no significant trend in market wide volatility or industry specific volatility over this period. However, firm level variance ($\eta^*_{j,i,t}$) displays a statistically significant positive trend, more than doubling between 1926 and 1997. They suggest that this implies that correlation between individual stocks has declined over the period and the predictive power of the market model for individual stocks has also declined. Their application of the Granger Causality test indicates that market volatility leads the other volatility series and that all volatility series tend to lead recessions.

As well as finding that firm level volatility is increasing, they also find that small firm volatility is much greater than large firm volatility and that overall firm level volatility is considerably greater in the equally weighted index. This reflects the fact that a small firm with higher volatility has the same influence in an equally weighted index as a larger firm with lower firm specific volatility. Hence large firms are those which have a greater weight in the index because they are perceived to offer lower volatility for a given level of risk; their attractiveness to investors trying to optimise their portfolio risk return profile enables them to issue more stock relative to less attractive firms offering a higher risk for a given level of return. This results in the low risk firms gaining size at the expense of the high-risk firms, resulting in increasing market concentration.

The data presented by Campbell et al is evidence in support of the suggestion that a more concentrated portfolio is not necessarily a more risky portfolio. However, in a more concentrated portfolio, it does become difficult to distinguish the idiosyncratic risk of the largest stocks from the systematic risk of the entire market of which the large stocks are a significant part. This is an issue that is not fully addressed by Campbell et al (2001).

One possible solution to the above problem would be to regress the returns of the largest dominant firms upon the returns of an equally weighted market portfolio. This would reduce the potential for estimated betas of the largest firms to be biased towards unity: a problem that is likely to occur if betas are estimated by regressing the dominant stock returns upon the returns of a value weighted index that is largely composed of that same stock. Campbell et al acknowledge this problem on page 22 of their article, where they observe that large industries by market value tend to have both low industry and firm level volatility but that this could be a result of the fact that shocks to large industries move the market as a whole. As a result market level volatility reflects shocks to these industries. They also note that the volatility of some industries exhibits a positive time trend over their study period. Notable among these are the telecommunications, computer and retail sectors, all of which have been growth industries over the latter part of the 20th century.

Campbell et al (2001) calculated the full correlation matrix of the returns of all firms in their study. This enabled them to calculate the average correlations of the market constituents in the VCM at daily and monthly frequencies for annual and five-year periods in the NYSE, AMEX and NASDAQ. They acknowledge that complete data histories are not available for all the stocks over the whole period, but they do not say how they fill in the data gaps or maintain a consistent sample. Their results indicate a decline in the equally weighted average correlation of returns over the period of their study. This is consistent with their observation of increased idiosyncratic risk as a proportion of total risk in an average security's return. It is also consistent with the results of this study of the UK market. In addition, the decrease in average correlation could be consistent with the idea that increased concentration in the market portfolio does not necessarily imply increased volatility of the market portfolio if the correlation between dominant securities, and indeed all securities, is decreasing. However, Campbell et al suggest that more securities are required in order to form an optimally diversified portfolio if correlations between securities are decreasing. Furthermore, they find that during increases in market volatility, the average correlation between securities increases. This phenomenon is consistent with other studies suggesting that correlations between securities approach unity during times of market crashes and panics.[51]

Campbell et al (2001) do not find evidence of any ability for changes in average correlation to predict changes in aggregate volatility. However, their argument that more stocks are now needed in order to form an optimally diversified portfolio is supported by their separate comparison of the volatility of an equally weighted two stock portfolio, a five stock, twenty stock and fifty stock portfolio with that of an equally weighted index. The use of equally weighted rather than value weighted indices and portfolios is a focus on naive diversification and thus ignores the possibility that the market portfolio may have been subjected to more active diversification based on the principles of MPT.

By decomposing the total volatility of a firm into three components, they conclude that market volatility accounts for 16% of total volatility, industry volatility accounts for 12% and firm specific volatility accounts for 72% of total firm volatility. Most of the variation in volatility is accounted for by variation in market and firm specific volatility, while industry volatility is more stable over time. In fact the market volatility component, although accounting for a relatively small proportion of total volatility, displays the greatest variation in relation to its mean over time. Later in the report, Campbell et al try to link volatility with recessions and find that regressions of various measures of leading volatility against dummy variable models for recessions, non-recessions and models of GDP growth, indicate that volatility measures are useful for forecasting recessions. Furthermore, models that include leading stock market returns as explanatory variables for recession prediction are out-performed by models that include both leading market returns and leading volatility measures to the extent that coefficients on the leading returns are no longer significant when volatility series are included in the model.

Campbell et al (2001) also consider the predictive effect of different volatility components on industry output. They find that only firm specific volatility appears to have any significant forecasting power for industry output with an R^2 of just 1.2%.[52] They also discuss possible explanations for changes in idiosyncratic volatility. Two types of shocks are identified that affect stock specific returns and, hence, volatility: shocks to expected

[51] Andersen et al (2001a) and Kearney and Poti (2003) are examples of such studies.

[52] This would translate into an even lower R^2 value if an adjustment had been made for the loss of degrees of freedom due to the variables in the regression.

future cash flows discounted at a constant rate and shocks to the discount rate. They propose that unless "explosive rational bubbles" are assumed to exist, any model of stock returns should consider some consideration of the above two factors. It is not made clear why the bubbles should be "rational" and not irrational. However, the importance of these two factors for stock returns and stock return volatility is easily appreciated. Hence it is argued that, barring the above caveat, an increase in volatility must be brought about by an increase in the variance of one of the above factors or an increase in the covariance between the two factors.

Possible explanations for the increase in firm specific volatility suggested by Campbell et al (2001) are as follows. They argue that the trend towards focussed firms, as distinct from conglomerates, has resulted in an increase in average investment risk within firms. In addition, they argue that firms are increasingly reliant on external sources of finance, particularly in mature industries in which firms are pressurised by shareholders to return cash, rather than to diversify into new higher growth industries. Furthermore, Campbell et al argue that there was a trend, during the latter part of the 1990s bull market, for firms to issue stock earlier in their life cycle when there was greater uncertainty about their long-term business prospects. The latter trend is likely to have less of an effect on a market value-of-equity weighted portfolio than on an equally weighted portfolio, since new firms tend to be smaller. However, if the largest firms in a market are pure plays, rather than diversified conglomerates, and their idiosyncratic risk is greater than before as a result, then the increasing market concentration implies increasing market volatility, unless a commensurate decrease in the covariance of returns between the dominant firms can be demonstrated, an issue that is investigated later in this study. Other possible causes of increased firm specific volatility put forward by Campbell et al, include an increased use of stock options as compensation for executives providing an incentive to undertake riskier projects with a higher maximum payoff. They cite Cohen et al (2000) who find statistically detectable evidence for this effect, albeit at a small magnitude. Campbell et al also consider the leverage effect as a possible cause but argue that the bull market during the latter part of their study would have decreased corporate leverage as a proportion of the market value of equity at a time when firm specific volatility had been increasing, which suggests that the reverse should be the case.

In conclusion we can say that the study by Campbell et al (2001) raises as many questions as it answers and yet provides useful material for comparison with the results of this study focussing on the FTSE100 Index. The most relevant findings, as far as the present study is concerned, is that different components of total firm and, hence, total market volatility appear to be changing over time, in the same way that Isakov and Sonney (2002) find that the proportions of market volatility accounted for by industry and country specific components also appear to be changing over time. Therefore, it is not surprising that different components of market volatility, decomposed via the VCM, display time-varying characteristics as reported in the results chapters of this thesis. The comparisons made between volatility of equally weighted and value weighted portfolios are also relevant to the present study of concentration which effectively measures the difference in weighting between an equally weighted and market value weighted portfolio at any given time.

3.6.3.1 Other studies of idiosyncratic and systematic risk

Wei and Zhang (2003) extend the analysis of Campbell et al (2001) using Compustat and CRSP data from 1962 to 2000. In addition to extending the data sample, they link the increased volatility of individual stocks to increases in the volatility of firm earnings and return on equity, hence the title of their paper: *"Why did individual stocks become more volatile"*. However, of more relevance to this study is their comparison of the equally weighted average volatility of individual stocks and the value weighted average volatility. They find that the value weighted average volatility is usually less than the equally weighted average volatility. This implies that the returns of larger stocks are, on average, less volatile than those of smaller stocks, which are able to upwardly bias the equally weighted average volatility. This analysis is not dissimilar to that adopted in this study, although this study explores the issue in more detail as both the average variance and average covariance are decomposed into equally weighted components and incremental components conditional upon the weights.

Goyal and Santa-Clara (2003) use CRSP US stock returns data from July 1962 through December 1999 to decompose the variance of market returns into the average variance of individual stocks and the market wide risk. They find that the average stock risk is correlated with the total market risk. They compare the volatility of equally weighted and value weighted portfolios and find that the two are highly correlated. They use an equally weighted portfolio as a proxy for the market portfolio when estimating market risk to avoid potential bias caused by large stocks. Furthermore, they find a positive association between the lag of average stock variance and the return on the market, but no equivalent association is found between market risk and market returns. In addition they find evidence in support of the leverage effect of Black (1976). In line with the present study, they use a mean expected return of zero when calculating realised variances, unlike Campbell et al (2001) who use the risk free rate. They also find that the square roots and log transformations of variance estimates are closer to a normal distribution, as observed by Andersen et al (2001a) and also in this study. In an attempt to mitigate the effects of heteroskedasticity and possible autocorrelation in the residuals of their regression models they use Newey-West robust standard errors to estimate the p-values of model coefficients, as employed in this study.

Kearney and Poti (2003) follow the spirit of Campbell et al (2001) but use 42 constituents of the Dow Jones Eurostoxx 50 Index and six other stock market indices in the Euro zone. Their study period is also shorter than that of Campbell et al, but it includes more recent data, extending from 1993 to 2001. They calculate equally weighted and value weighted average firm variances and find a very high correlation between the two data series.

Malkiel and Xu (2003) also investigate the behaviour of idiosyncratic volatility, while Guo and Savickas (2004) explore the relationship between idiosyncratic volatility, market volatility and expected stock returns. Malkiel and Xu (2003) study the behaviour of the idiosyncratic volatility of US stock returns using CRSP data from 1952 through 1998. They find an increase in average idiosyncratic volatility that cannot be solely attributed to the rise in the prominence of riskier stocks traded on the Nasdaq market. Malkiel and Xu (2003) suggest that this could be due to the rise in the proportion of stocks owned by financial institutions, which receive information simultaneously from similar sources and are likely to interpret that information in a similar manner. In addition, Malkiel and Xu (2003) find evidence of a positive but non-linear relationship between earnings growth and idiosyncratic volatility. They find that the idiosyncratic volatility of returns for firms of similar absolute, and relative, size have increased over the time period of their study. On page 616 and in the appendix to their article, they provide details of their "indirect" method of extracting the market and idiosyncratic volatility from the weighted VCM of market constituent returns. This is based on subtracting the conditional volatility of market returns, based on the estimation method of French et al (1987), from the value weighted sum of the conditional volatility of individual firms' stock returns. Their method is dependent upon the assumption that individual firms' betas all equal unity and that the estimates of conditional volatility are appropriate. The present study also decomposes the value weighted VCM, but it differs from that of Malkiel & Xu (2003) in that it focuses on the importance of portfolio constituent weights in determining the aggregate structure of the VCM. In fact, in the present study, no assumptions are made about individual firm betas and the definition of idiosyncratic volatility is left open to debate. The only assumption inherent in the decomposition applied in this study is that the expected returns are identical across all firms and are equal to zero. However, the assumption of zero expected returns could easily be replaced by an identical risk free rate for all firms.

Guo and Savickas (2004) cite earlier versions of the paper by Malkiel and Xu (2003), and the published version, when they emphasise the importance of idiosyncratic volatility to private investors who may not have portfolios large enough to be optimally diversified. Guo and Savickas (2004) adopt a similar approach to Campbell et al (2001) and Goyal and Santa-Clara (2003). However, they found a negative association between value weighted idiosyncratic volatility and future excess stock returns whereas the association between market volatility and future excess stock returns was positive, contrary to the results of Goyal and Santa-Clara (2003). Guo and Savickas (2004) attribute the difference to the fact that they use a value weighted proxy for the market portfolio when estimating idiosyncratic and market volatility, compared to the equally weighted market proxy adopted by Goyal and Santa-Clara (2003). This provides further justification for the present study's investigation into the role of concentration in determining sub-components of portfolio volatility.

3.6.4 Synopsis

What none of the above studies do is evaluate the time series of concentration in the constituent weights of the markets studied. Nor do they examine the time series evolution of the differences between the equally weighted variance and the value-weighted variance, or the differences between the equally weighted covariance and the value-weighted covariance. When the realised variance of index returns is decomposed, it is either decomposed in relation to industry and country effects, or into idiosyncratic and systematic components. Even when equally weighted and value weighted components of the VCM are calculated separately and compared, as in Poti et al (2003) and Goyal and Santa-Clara (2003), the importance of the weighting is treated as a somewhat peripheral issue. Given that the actual market realised volatility is determined by the value weighted VCM, it seems surprising that the contribution of the value weights to the overall VCM is not examined. Hence this study fills the gaps in the above research, as detailed in subsequent chapters.

3.7 Modelling volatility

3.7.1.1 Background to volatility modelling

Methods that are used to measure, model and forecast volatility are reviewed extensively by Poon and Granger (2003). They compare the out-of-sample forecasting results of 93 papers published over the last twenty years. Studies that forecast volatility based upon past prices only, are evaluated in addition to, those that evaluate the forecasting potential of option-implied volatility (OIV). Papers that forecast correlations are excluded, although they acknowledge the potential value of such forecasts for portfolio risk management. Volatility forecasts based on non-parametric modelling techniques, neural networks and genetic programming models are also excluded from their discussion, together with studies that fail to provide out-of-sample forecast evaluations. The pure time series section includes models that aim to capture asymmetry, as well as volatility persistence and clustering, which are both salient features of volatility time-series. One very important issue identified by Poon and Granger, is the relative lack of attempts to separate the forecasting period into "normal" and "exceptional" periods. They highlight the possibility that different forecasting models might be better suited for different trading environments. It is further suggested that outliers might be drawn from a different distribution that occurs only during exceptional market conditions. Unfortunately, because outliers by definition are relatively few in number it is difficult to form an estimate of that distribution. They propose that the leptokurtic sample distribution represents two populations with distinct distributions, a "normal distribution" based on the return population generated when the market exhibits normal time series behaviour and an "outlier distribution" based on the observed data during extreme market behaviour. Poon and Granger (2003) conclude that OIV models provide the most effective forecasts, although suitable data is not always available. These models are closely matched by the performance of the ARCH and GARCH category of models. They suggest that future research should concentrate on combining forecasts from different models to improve forecast accuracy. Finally, the most useful, and to-date unattained goal, would be to achieve some success in forecasting extreme events.

3.7.2 Different approaches to TS volatility forecasting.

Volatility generally refers to the standard deviation (σ) or variance (σ^2) of returns. In the case where the lagged unconditional σ^2 or σ are independent variables in a time series model, care needs to be taken to avoid persistent but spurious autocorrelations of model residuals, and also multicollinearity when more than one lag is used. This is because the measurement period used to estimate the dependent variable may overlap the measurement period used to estimate the independent variable when the independent variables are simple lags of the dependent variable. This problem can be avoided if volatility is recorded in discrete non-overlapping time periods. Different studies have approached this problem in a variety of ways. For example, Officer (1973) used a simple rolling standard deviation based on the previous n months. Subsequent authors have attempted to mitigate the problem by using White's, Hansen's or Newey-West robust standard errors, when interpreting model results. Other studies, such as French et al (1987), Schwert (1989), Canina and Figlewski (1993), Andersen et al (2000) and Blair et al (2001), indicate that more reliable results can be obtained by using within period return data to estimate volatility for any given period. For example, monthly, weekly or daily data is used to estimate annual volatility, daily data is used for estimating monthly volatility while weekly volatility is estimated using either daily or intra-daily return data. In fact, Andersen et al (2000) use high frequency intra-day returns data taken at five-minute intervals, to estimate daily volatility, which can then be annualised. This method removes the problem of spurious serial correlation in the residuals and spuriously high R^2 values caused by modelling overlapping estimates of volatility. When this problem is eliminated model results are more powerful and robust. For example, Blair et al (2001), found that R^2 increased when intra-day rather than daily stock returns data was used to estimate daily realised-volatility for ex-ante forecasting.

Canina and Figlewski (1993)[53] and Poterba and Summers (1986) suggest that volatility follows a first order autoregressive (AR_1) process. Andersen et al (2003) propose that daily estimates of volatility derived from their five-minute interval, intra-day data set could be compounded to provide, weekly, monthly and annual forecasts of volatility. This is analogous to using exponentially declining moving average weights for TS volatility forecasts. For example, in the Andersen model, 100% weight is given to returns generated within the last day, while returns generated during preceding days of trading are given a zero weight.[54] Potential drawbacks of using high frequency intra-daily estimates of

[53] This study models OIV.

[54] Other models use a decay factor λ, with a weight such as the 0.94 optimised by Risk Metrics ™. Thus if there were ten successive values in a series used to calculate an exponentially declining average, the first would get a weight of 0.94, the second would get a weight of $0.94 \times (1-0.94)$ and so on. Boudoukh et al (1997) discuss this method in more detail.

volatility are increased noise, limited availability of data over extended time periods and market microstructure artefacts such as the bid-ask-bounce effect.[55] Andersen et al, used data sampled at five-minute intervals. Their data was taken from highly liquid foreign exchange markets in the Japanese Yen and the US dollar Andersen et al (2001b), Andersen et al (2000a), and major stock index price quotations Andersen et al (2003). Liquidity should be less of a problem in these markets than in the markets for individual stocks.

3.7.3 Time varying behaviour of stock price volatility

A key feature of stock return volatility that complicates the modelling procedure is that real world sample populations are continually evolving and display time varying parameters and time varying auto-covariance. For example, Peiro (1999) observes that different overlapping sub-periods of a volatility series display different inter-dependencies, variances, kurtosis and skewness. Variations on GARCH aiming to capture the leverage effect in volatility were developed by Engle et al (1987), Engle (1990), Engle and Ng (1993), Glosten et al (1993), Hentschel (1995) and others, some of which are reviewed later in this section.

Schwert (1989) asks why stock market volatility changes over time and, unlike, pure time series studies of volatility he studies the association between stock return volatility and a variety of exogenous variables. These include, economic activity and volatility in macroeconomic variables, such as growth in the money supply, inflation, industrial production and interest rates. He estimates a monthly variance of the Standard and Poor's Composite Index returns using Equation 35 over the period from 1928 through 1987 with daily data. He also estimates additional volatility data from 1885 through February 1927 using the Dow Jones composite portfolio.[56]

Equation 35. $${}^{n}\sigma_t^2 = \sum_{i=1}^{Nt} r_{it}^2$$

Where: r_{it} *represents the daily return less the mean daily return for the month and* N_t *is the number of trading days in a given month.*

---

[55] A tendency, observed in tick-by-tick data, for security prices to bounce randomly between the bid and the ask price.

[56] The validity of this section of the data may be open to question in the light of issues, raised in Chapter 4 of this thesis, concerning the suitability of indices, such as the Dow Jones composites, for empirical studies of this nature.

Squared daily excess returns were summed over the month to obtain non-overlapping estimates of the monthly variance with estimation errors that were uncorrelated through time. Monthly volatility was also calculated using a generalised version of the simple rolling method employed by Officer (1973). Schwert achieved this by a 12th order auto-regression of monthly excess returns including a dummy variable to allow for different monthly mean returns for the entire data series. Schwert then goes on to estimate a 12th order AR model for absolute values of errors from the former model including the dummy variables with the dependent variable $|\varepsilon_t|$ as an estimate of volatility for the one month stock market return. This uses one observation for each month, rather than the twenty-two used to calculate the non-overlapping variance. This method allows the conditional mean return to vary over time and allows for different weights of the lagged absolute unexpected returns.

Schwert (1989) concludes that volatility increases during recessions and that volatility can be used to forecast recessions. Furthermore, the causality is one-directional so that volatility in macroeconomic variables cannot be used to predict stock and bond return volatility: a conclusion that is consistent with the later findings of Campbell et al (2001). Schwert also finds evidence in favour of the leverage effect of Black (1976), but the results suggest that the leverage effect can explain only a small proportion of changes in stock return volatility over time. In addition, he finds evidence of a positive relationship between trading activity, measured by trading volume, and volatility. The number of trading days in a month was also found to be positively associated with volatility, an effect that Poon and Taylor (1992) correct for in their analysis. Schwert's overall conclusion is that many of the associations observed are insufficient, collectively or otherwise, to fully explain the variation in realised volatility over time. The results of his study are interesting, because, although Schwert does not try to find a theoretical economic model to explain the observed associations, he does step outside of the conventional time series-modelling framework, which is largely confined to mathematical models that attempt to describe the data-generating process.

3.7.4 Asymmetry and the leverage effect in volatility clustering

The asymmetry effect has been documented by Officer (1973), Black (1976), Christie (1982) and Duffee (1995). However, Chelley-Steeley and Steeley (1996a) also refer to "asymmetry" in the context of the transmission of volatility shocks in returns from large to small firms but not vice a versa. Essentially they find that large shocks in large firms are subsequently replicated in small firms, although large shocks in small firms appear to have no impact upon the returns of large firms, i.e. some evidence exists to suggest that volatility of small firms' returns may in part depend upon the volatility of large firms' returns but the volatility of large firms' returns appears to be independent from the volatility of small firms' returns, hence the asymmetry.

Black (1976)and Christie (1982) argue that much of the asymmetry between the impact of positive and negative returns upon future return volatility is due to the Leverage effect.[57] Nonetheless, Black (1976) acknowledges that the leverage effect alone is insufficient to entirely explain the asymmetry observed. He put forward the additional proposition that investors are able to anticipate future increases in firm risk; when this occurs without a commensurate increase in expected profits, firm values fall immediately, if, markets are in equilibrium, with the effect that falls in firm values precede increases in firm volatility. However, Black goes on to acknowledge that his empirical tests at that time were not entirely consistent with this proposition, later referred to by Bekaert and Wu (2000) as the feedback effect. Schwert (1989) confirmed that volatility is higher in economic downturns but he too was unable to demonstrate that this is entirely due to the leverage effect.

Bekaert and Wu (2000) argue that asymmetric volatility is consistent with the idea of time varying risk premia and the so-called "feedback effect". Therefore, rather than negative returns causing increases in volatility, as implied by the leverage effect, expected increases in volatility raise the risk premium required to compensate for the increased volatility; stock prices then fall in order to maintain the equilibrium. They point to the work of French et al (1987) and Campbell and Hentschel (1992) to support their argument. Their own study, Bekaert and Wu (2000), of Nikkei 225 Stock Index constituents, and portfolios made up of these, concludes that feedback effects are more important than the leverage effect in explaining asymmetry. They cite evidence of volatility contagion between securities to support their conclusion, at the portfolio level, noting that the leverage effect is prominent in the average covariance between securities, a finding consistent with the results of the present study.

---

[57] This refers to the theoretical argument that when stock prices fall the decline in the market value of the firm's equity is greater than any corresponding decline in the market value of the firm's debt. Hence the ratio of the market value of debt to the market value of equity is increased. Therefore, the increase in return volatility following negative stock returns is merely a reflection of the increased financial risk of the firms concerned due to their increased leverage. This same principle can also be applied to firms with no debt but some degree of operating leverage, according to Black (1976). This is because, as profits fall and hence the firm's share price falls, the firm's fixed costs are unchanged, hence operating leverage increases and the risk to the firm's free cash flows increases.

The results of Campbell et al (2001), discussed more extensively in section 3.6.3, are also consistent with the conclusion that volatility is greater in economic downturns for each of the three volatility components considered, namely systematic, industry specific and company specific volatility. They all increase substantially in economic downturns and tend to lead recessions. Campbell et al (2001) find that volatility can be used to forecast GDP growth, with an R^2 of 20%, but that GDP growth does not appear to be useful in forecasting market volatility.

Hentschel (1995) provides an extensive review of the development of symmetric and asymmetric GARCH models as well as a study of the distributions of stock returns. He credits initial documentation of the fat tails (excess kurtosis) characteristic of stock returns to Mandelbrot (1963), the leverage effect to Black (1976) and cites Black's acknowledgement that the leverage effect alone is insufficient to explain asymmetry. He points out that this latter point is consistent with Christie (1982) and Schwert (1989). Hentschel suggests that in the absence of a suitable theoretical model to explain asymmetry, the parametric ARCH models have been developed in the search for economical ways of describing the asymmetry.[58] He then cites the models of Taylor (1986) and Schwert (1989) as examples in which an ARCH model specifies the conditional standard deviation as a moving average of the lagged absolute residuals. As justification for these specifications, he cites "the demonstration by Davidian and Carroll (1987) to the effect that variance estimators based upon absolute residuals are robust to outliers in a regression framework".

Following his review of earlier models, Hentschel proposes a "unifying framework" based upon a family of nested GARCH models starting with an absolute value model. The absolute value GARCH model follows the general idea of univariate GARCH models in that it consists of two equations: a mean equation and a variance equation. The mean equation describes the observed data as a function of other variables plus an error term. The variance equation specifies the evolution of the conditional variance of the error from the mean equation as a function of past conditional variances and lagged errors. Hentschel provides detailed specifications of the adapted models in his article, although his version of the GARCH $_{(1,1)}$ model for the conditional standard deviation is repeated in Equation 36.

---

[58] For the convenience of the reader, the following sentence paraphrases Hentschel's summary of the development of these models. ARCH models are credited to Engle (1982), GARCH to Bollerslev (1986), exponential GARCH or EGARCH to Nelson (1991), Quadratic GARCH or QGARCH to Sentana (1991) and Engle (1990) then threshold GARCH or TGARCH to Zakoian (1991).

Equation 36. $$\sigma_t = \omega + \alpha\sigma_{t-1}\left|\varepsilon_t\right| + \beta\sigma_{t-1}$$

This differs from a conventional $GARCH_{(1,1)}$ in that the conditional variance terms σ_t^2 have been replaced by their square roots and the squared error has been replaced by its absolute value. Use of the absolute value of the error reduces the impact of large shocks on the incremental realised volatility compared to a conventional GARCH model. Hentschel attributes the development of this approach to (Taylor (1986),Schwert (1989) and Nelson (1991)). While Equation 36 is a symmetrical model of the incremental realised volatility, Hentschel also provides an asymmetric version of the absolute value GARCH model illustrated by Equation 37. Hentschel illustrates his asymmetric GARCH model using the news impact curves, thus described by Engle and Ng (1993) and introduced by Pagan and Schwert (1990).

Equation 37. $$\sigma_t = \omega + \alpha\sigma_{t-1} f\left(\varepsilon_t\right) + \beta\sigma_{t-1}$$

Where: $f\left(\varepsilon_t\right) = \left|\varepsilon_t - b\right| - c\left(\varepsilon_t - b\right)$; *b is a displacement parameter and c is a rotation parameter.*

Hentschel (1995) illustrates his use of "news impact curves" by demonstrating the impact of different ε_t on the absolute value function when the rotation parameter c and the displacement parameter b are assigned different values. Positive values of the displacement parameter b effectively apply a threshold below which positive shocks at time t do not have a positive impact on the incremental realised volatility at time t+1. The positive values of b also increase the impact of negative shocks on the incremental realised volatility by the amount of the threshold. This is achieved without changing the slope coefficient on either positive or negative values of ε in Equation 37. A positive value of the rotation parameter c, on the other hand, results in a damping of the slope coefficient applied to the absolute values when ε are positive. In addition, positive c values cause a relative increase in the slope coefficient on the absolute values when ε are negative, so that in effect, negative shocks increase the asymmetry function $f(\varepsilon_t)$ more than positive shocks of the same magnitude. Optimisation of *b* and *c* to obtain the best fitting model can be achieved by maximum likelihood estimation of restricted parameters, although one or other may be left free to be estimated empirically by the model according to Hentschel. Hentschel cites results obtained by authors of various other models where the parameters are both restricted and free to estimation. Hentschel himself uses daily CRSP data from the 2nd January 1926 to the 31st December 1990. The difference between the log of stock returns and the log of the risk-free rate forms the log excess returns $r_{t+1,}$ used in his analysis. He concludes that the symmetric GARCH of Bollerslev (1986) and the non linear GARCH of Bera and Higgins (1992) are rejected in favour of asymmetric GARCH models. In addition, models that incorporate both a shifting and a rotation parameter outperform models with just a rotation parameter such as the EGARCH of Nelson (1991). This is interpreted to mean that small asymmetric shocks are important, as well as large asymmetric shocks.

By combining the shifting and rotation parameters into his asymmetric absolute value GARCH model, Hentschel is effectively decomposing the information set into its component parts, so that the conflicting effects can be disentangled. For instance, positive shocks are generally deemed to have a different effect on realised volatility to negative shocks of the same magnitude, but very large shocks of either sign generally have a positive effect on the realised volatility. Thus the differing effects of large positive shocks, large negative shocks, small positive shocks and small negative shocks on the realised volatility, can be measured.

The paper by Hentschel (1995) provides a very useful review of reported applications of GARCH models for describing the importance of asymmetry for incremental realised volatility forecasts.[59] Good descriptions are provided of the various model specifications and comparison of their results over a long period. However, one critique of Hentschel's study is that the data are modelled over the whole sample period. It would be useful to break the data into repeated shorter periods for estimating each model. The relative temporal stability of model parameters and goodness of fit could then be tested.

3.7.5 Asymmetric distributions of stock returns and stock return volatility

Research into asymmetry in return volatility and return skewness is summarised by Premaratne and Bera (2002). Citing Hicks (1946), Mandelbrot (1963), Samuelson (1970), Kraus and Litzenberger (1976), Kane (1977), Peiro (1999), Premaratne and Bera (2002) use the non-parametric Pearson IV distribution of Pearson (1895) to simultaneously model the

[59] Another popular asymmetric volatility is the GJR – GARCH model of Glosten et al (1993). This is a form of GARCH model that includes an additional coefficient on a dummy variable equal to unity if the lagged return is negative.

time varying behaviour of the mean, variance, skewness and kurtosis of stock return data to explain risk.

Peiro (1999) makes an observation that draws attention to the need for greater understanding of asymmetry in stock returns. It is quoted verbatim as follows:

"CAPM assumes that only mean and variance of returns are important in asset pricing, therefore, higher order moments are unimportant, implying that upside and downside risk are considered equally by investors".

Peiro argues that numerous empirical studies indicate that this is not in fact the case and that negative shocks have a greater impact on future volatility and risk aversion than positive shocks of the same magnitude.

Evidence that investors prefer positively skewed returns has also been provided by Brennan (1979) and He and Leland (1993), while Chunhachinda et al (1997) found evidence to support the idea that investors trade expected returns in exchange for positive skewness. An analogy can be drawn to the popularity of lottery tickets, where the purchaser's probability weighted expected return is negative but the probability density function of possible returns has a very high positive skew.

3.8 Summary

The current chapter has outlined some key principles of capital market theory that are relevant to this study, with emphasis on the principle of mean variance optimisation put forward by Markowitz (1952) and the benefits of naive diversification observed by Evans and Archer (1968). Several expected daily return measures, from which unexpected innovations can be calculated, are also reviewed. The first is the average realised return over the realised volatility estimation period, which is allowed to change for each volatility estimation period. The second is the daily risk-free rate of return based upon contemporaneous one-month UK treasury bills. A third possibility is to use a mean or expected daily return of zero. The latter is consistent with the suggestion of Figlewski (2001), Poon and Taylor (1992), Andersen et al (2001a) and Areal and Taylor (2002). Although it is accepted that other authors have used considerably more complex methods of estimating expected daily returns, a simple estimate is likely to be more accessible and reliable over an extended period. The ultimate choice of expected return measure should be guided by analysis of the statistical properties of the different daily return measures over the sample period, as well as the descriptive statistics reported by Poon and Taylor (1992), Dimson and Marsh (2001) and Areal and Taylor (2002).

Empirical studies of co movement are then reviewed, including Elton et al (1973), Brooks and Persand (2000), Malevergne and Sornette (2004) and Poon et al (2004). The discussion of stock and stock index portfolio volatility is extended from the basic principles of MPT, the CAPM and the Market Model to include discussion of studies, such as Roll (1992), that examined portfolio diversification at the industry and country level. This is followed by a review of studies, such as Campbell et al (2001), that examine portfolio diversification in terms of the relationship between idiosyncratic risk, industry risk and market risk.

The focus of the review then shifts to outline some key articles detailing methods of defining, measuring and forecasting the realized volatility of stock market indices from a pure time series perspective. A number of core themes emerge as a result of the above survey that can be summarised as follows. Stock returns are not normally distributed; rather they display excess kurtosis and are often negatively skewed, although a few researchers have documented slight positive skewness when the large negative return for the 19th October 1987 is excluded. The sample variance of stock returns is not constant, normally distributed or symmetrical when repeated samples are estimated, i.e. there is time varying skewness and kurtosis in the underlying returns. Stock returns and the variance of stock returns change through time and exhibit serial correlation and clustering effects, with the result that large absolute returns tend to be followed by other large absolute returns and vice versa. In addition to the clustering effect, large negative returns have a greater impact on future absolute returns than positive returns of the same magnitude, resulting in an asymmetric autocorrelation effect. Within indices and markets, there is an additional asymmetric effect in the transmission of volatility between constituent stocks. Shocks in large firms are transmitted to small firms; however, shocks in small firms are not transmitted to large firms, or at least not in the same magnitude.

Estimates of volatility for a given period are more accurate if achieved using within period data, such as the model-free estimates of realised volatility of Andersen et al (2001a), than if they are generated using returns data from successive periods, or overlapping volatility estimates. This also helps to mitigate the problems associated with the moving average characteristics of multiple successive overlapping estimates of volatility used in early time series studies. The logarithms of daily-realised variance are approximately normal, as are daily-realised returns divided by daily-realised standard deviations and daily-realised correlations, according to Andersen et al (2001a). Naive estimates of volatility based upon simple rolling moving average estimates of variance have, over time, given way to exponentially declining average estimates. These can be related to estimates of conditional variance ε^2 derived from symmetric and asymmetric ARCH and GARCH models. More recently still, non parametric and "distribution free" estimates have been developed using high frequency data. Attempts to capture the asymmetry effect range from the news impact curves based upon shifting and rotation parameters described by Hentschel (1995) to the dummy variable model of Glosten et al (1993).

The conclusions reached are that the pure TS models used in this study should aim to capture the "asymmetry effect" and use within period returns data to obtain non-overlapping estimates of realised volatility for modelling purposes. In fact, given the approximation to normality and the reduction in estimation error discussed by Andersen et al (2001a), this study should measure realised volatility using the highest frequency returns data available. Possible models that may be used to capture the realised volatility forecasting potential of the asymmetry property include: absolute value asymmetric GARCH models discussed by Hentschel (1995) and variations of the dummy variable method used by Glosten et al (1993).

The next chapter reviews the choice of stock indices that are available as proxies for various market portfolios. The characteristics of a useful index are discussed and potential problems with unsuitable indices are discussed in the context of the benchmark problem identified by Roll (1978).

Chapter 4 – Literature Review III: Equity market indices and their providers

4.1 Introduction

In section 3.3 of Chapter 3 some key principles of the asset pricing models that are fundamental to traditional finance theory are outlined. A common theme among many of these models is the need for a measurable proxy for the market portfolio. In the strictest form of the CAPM, the market portfolio includes all tradable assets such as property, bonds, works of art, antiques, private businesses, and intellectual human capital as well as equities listed on the stock exchange. This definition of the market portfolio may be expanded to the global level, or alternatively confined within national boundaries. Calculating such an all inclusive index to measure the market portfolio in the strict sense of the above definition is difficult if not impossible to achieve without substantial measurement error and time delays due to the lack of transaction price data for the many assets that are infrequently traded, or traded privately without full price disclosure. This has led to the widespread adoption of national stock price indices as proxies for the equity market portfolio.[60] For example, the market model relies on this approach and recognises that the market portfolio per se is unmeasurable.

This thesis focuses on the effect of concentration upon different components of the VCM of the FTSE 100 Index. The ultimate objective of this approach is to attain greater understanding of stock market volatility. While the methodology is specifically applied to the FTSE 100 index, it could equally be applied to a variety of other stock market indices or to individual investors' portfolios. It is therefore appropriate to provide a review of the more important stock market indices, their purpose, and their suitability as a proxy for their respective market portfolios.

This chapter does not include bond indices, real estate indices and specialist equity indices built to reflect a particular sub-section of the market, such as growth or value stocks.

[60] The Dow Jones Transportation Index was the world's first stock market index, developed in the United States in 1884 by the journalist Charles Henry Dow. The index provided a means by which traders, investors and other interested parties could quickly gauge the condition of the market, which was dominated by stocks of transportation firms at that time. On the 26th of May 1896, Charles Dow began publishing a similar index that later became known as the Dow Jones Industrial Average (DJIA). Since Dow's indices were first published, there has been a huge proliferation of stock indices aiming to represent individual sectors, investment styles, local market conditions, national market conditions and international market conditions. Indeed some international market indices are in fact indices of indices.

Instead the discussion is restricted to equity market indices that are designed to reflect the average performance of their respective markets. Discussion of the necessary characteristics required in order to achieve the key index functions is initiated by a summary of the benchmark error problem, identified by Roll (1977). The requisite characteristics for a good benchmark identified by Bailey (1992b) are discussed in the context of these functions. Three principle index calculation methods are outlined and discussed. This is followed by discussion of the homogenous expectations assumption of CAPM and the importance of a market portfolio in determining the security market line (SML). A tabular summary is provided of some well-known indices, index providers and the extent to which they achieve their objectives. The section concludes by ranking the indices in the tabular summary and selecting those that best meet their stated objectives and those that are most suitable for the application of the methodology adopted in this study. Finally, the suitability of the FTSE 100 index in the context of this ranking is discussed.

4.1.1 Key functions of national and global market indices

National global and regional equity market indices have a variety of functions. Some prominent examples identified by the author, based on a review of the relevant academic and professional literature, are listed below.

1. Use as a popular indicator of a country or region's economic prosperity and status.
2. Often used as a proxy for the market portfolio of a given country or region against which the systematic and unsystematic risk of investors portfolios or stocks can be evaluated.
3. Used as a model or template upon which to base a "diversified" portfolio.
4. Used as a benchmark for measuring the performance of actively managed funds.
5. Frequently used as a model for creating passive "Market Tracker" portfolios.
6. Used as an underlying market for Index futures and options that are in turn used for hedging against major market moves, for speculation and for various forms of risk management.

Functions two, three and five above are particularly relevant to this study of stock market volatility because the FTSE 100 index provides a proxy for the UK market which can be examined in the context of portfolio diversification. The results and conclusions are relevant for both active managers aiming to create an optimally diversified portfolio and for investors in passive tracker funds who are trying to assess their VAR, as well as index providers who are trying to create an appropriate index for a given clientele. The characteristics required by an index in order to achieve the above functions successfully are discussed after the following summary of benchmark error.

4.1.2 Benchmark error

Benchmark error, identified by Roll (1977), arises when the benchmark portfolio or index used as a proxy for the market portfolio is materially different from the true market portfolio. Security and portfolio betas estimated using the incorrect benchmark portfolio differ from the true betas that would be estimated if the market portfolio were used. Furthermore, it is likely that the benchmark portfolio is not on the true efficient frontier. Hence the slope of the security market line will be incorrect. This means that actively managed portfolios that plot above the estimated SML may in fact be below the true SML or vice versa. Roll's findings are also a matter of concern to passive investors reliant upon index tracking, particularly if the passive strategy is motivated by a belief in the EMH. This rests upon the assumption that the level of a properly constructed national market index provides a reasonable approximation to the optimal efficient portfolio that is tangential to the SML.[61]

While finding a perfect proxy for the market portfolio may be an unrealistic goal, Roll's critique of the CAPM, nevertheless, serves to highlight the consequences of choosing an inadequate proxy for the market portfolio. Therefore when evaluating a potential market index in the context of the benchmark criteria discussed in section 4.2 and the extent to which the calculation methodology outlined in section 4.2.2 enable the criteria to be met, the objective of minimising benchmark error should be paramount.

4.2 Characteristics of a useful market index

Bailey et al (1990), Bailey (1992a) and Bailey (1992b) identified six characteristics of a useful benchmark index. These are relevant to market indices and apply to the functions of indices outlined in the previous section. The six characteristics, identified by Bailey et al (1992) and reproduced below, are discussed in relation to other important characteristics that are specifically relevant to academic studies.

1. An index must be unambiguous so that the names and the weights of constituent securities are clearly delineated.

[61] Subsequent papers by Roll (1978), Roll (1980), Roll (1981), Brown and Brown (1987), Grinblatt and Titman (1993), Titman and Wermers (1997) explore the issue of benchmark error in more detail and their findings are summarised in Reilly & Brown (2000).

2. It must be investable so that the option is available to forego active management and simply hold the benchmark.
3. It must be measurable so that it is possible to readily calculate the benchmark's own return on a reasonably frequent basis.
4. It must be appropriate in that it must be consistent with the investment manager's investment style or biases.
5. It must be reflective of current investor opinions, so that the benchmark is relevant in the light of current investment conditions. This should be reflected in the securities that make up the benchmark.
6. The methodology must be specified in advance so that the benchmark can be constructed prior to the start of the evaluation period.

The fourth criterion is fairly specific to the context of the Bailey (1992b) article. However, the principles inherent in the others can be transferred to the more general context of a market benchmark and the ability of an index to meet the functions of market indices, listed in section 4.1.1. Therefore, the above criteria are discussed in more detail below.

1. Unambiguous criterion: To be useful, any proxy for the market portfolio, or benchmark by which generalist investment managers are to be evaluated, must possess sufficient transparency to ensure that the constituent selection criteria and calculation methodology can be understood and, if necessary, replicated by its users. For example, the constituents of the FTSE 100 index comprise the 100 largest firms listed on the London Stock exchange based on a clearly specified quarterly review process. By contrast, the constituent selection process for the S&P 100 index is much more ambiguous and open to the discretion of the selection committee. The unambiguous criteria apply to all of the six market index functions listed in section 4.1.1.

2. *Investability criterion:* An index that claimed to represent the true market portfolio, or at least the equity market portfolio, would aim to contain every single stock traded on the index and would be market value weighted. The problem is that such an index, like the Wilshire 5000 index of US stocks, contains many small stocks held by just a few investors with the result that exact replication entails high transaction costs due to the limited liquidity available. By contrast a more limited index, such as the FTSE 100 index of UK stocks, may not reflect the market as accurately but it is much easier to replicate due to the greater size, greater liquidity and smaller number of the constituents. Thus the investability criterion is more important for functions 3, 4, 5, and 6 than for functions 1 and 2.

3. *Measurability criterion:* This is probably the easiest criterion for equity indices to satisfy given the relatively frequent and accurate transaction price data available and the usual availability of index prices at daily or higher frequencies. It is more of a problem for non-equity investments such as property or bonds where transactions are less frequent. It applies to all of the six functions of market indices listed above.

4. *Appropriate criterion:* For an equity market index to be appropriate, it should provide a reasonable representation of the underlying equity market that it claims to represent. Possible ways in which an index might fail to meet the appropriateness criteria with respect to achieving the goal of market proxy benchmark are as follows:

- Not containing a reasonable proportion of the total equity value of the respective market.
- Being price weighted rather than market value weighted.
- Being a geometric average rather than an arithmetic average.
- Being equally weighted rather than market value weighted, or vice a versa.

These are a little different to the definitions given in the Bailey (1992b) article where the emphasis was on selecting an index that reflects a particular manager's investment style, for example with respect to value and growth. The appropriateness criterion applies to all of the six functions of market indices listed above.

5. *Reflective of current investment opinion criterion:* A useful market index would reflect current investment opinion by being value weighted. This provides greater consistency with the homogenous expectations assumption of the CAPM and is thus consistent with the EMH. The EMH assumes that the price of a security is an unbiased, rational and appropriate evaluation of everything that is currently known or knowable about a given security. Homogenous expectations is summarised by Bodie et al (1999) on page 252 as follows:

"All investors have homogenous expectations, i.e. the same expected return, risk and covariance for and between each and every asset. If this holds, the optimum portfolio weight for each security in the MPT efficient portfolio will be the same for all investors. Thus if every investor's portfolio has the same security weightings, every investor will hold the market portfolio."

If the broad principles of the EMH and homogenous expectations are accepted, it follows that the constituent weights of a value weighted equity index reflect collective investors current capital market expectations concerning those constituents. Hence the index meets the criterion of being reflective of current investor expectations.[62] The requirement to be reflective of current investment opinions is implicitly relevant to all of the six functions of equity market indices listed above.

6. The necessity to be specified in advance criterion: While the constituent securities may not always be known in advance, the criteria for selecting the constituents and the index calculation methodology of a useful benchmark index must be specified in advance. In the case of a quantitative rule-based index such as the Topix 100 or the FTSE 100, this allows investors to anticipate constituent changes and constituent weighting changes in a timely manner. However, in the S&P 100 and the Nasdaq 100 this process is not transparent and, as such, these indices fail to meet this criterion. The requirement to be specified in advance is relevant to all of the six functions of equity market indices listed above. While it is not possible to specify the exact nature of the market portfolio in advance, the means by which the market portfolio proxy is defined should be specified in advance in order to determine appropriate risk evaluation, performance evaluation and research methodologies.

An additional criterion of a useful market index that is particularly relevant to studies such as this is the availability of sufficient data to be able to replicate the index retrospectively.

4.2.1.1 Retrospective replicability

This relates to the unambiguous criteria of Bailey et al, but it adds the specific requirement that a full history of original constituents, plus the dates of subsequent constituent additions and deletions should be available. In addition, sufficient data concerning constituent weights and prices should be available to allow a retrospective recreation of the index

---

[62] Some indices such as the DJIA, the S&P 100 and the Nasdaq 100 are ambiguous in this respect.

portfolio. Such a requirement is crucial for the present study concerning the FTSE 100 index, which meets the requirement well. It also allows recreation of the index without encountering the problem of survivorship bias, whereby data from deleted firms are missing resulting in an upward bias in the returns of the replica index. Although particularly relevant to the implementation of this study, the retrospective replicability criterion is relevant to all the six functions of an equity market index due to its impact on the unambiguous and measurability criteria.

4.2.2 Index calculation methodology

Index calculation involves several stages. The first is to identify the universe of eligible securities and define the criteria for determining whether to include or exclude securities in the index from the eligible universe. The next stage is to determine how frequently index constituents are reviewed and when changes may take place. The procedure by which constituents are added and deleted is also important. It is then necessary to determine how securities are weighted in the index, and hence, how the index prices are calculated. The three most common weighting schemes are: market value of equity weighting, equal weighting and price weighting. They all have their own strengths and weaknesses that need to be evaluated carefully when choosing a suitable proxy index for the market portfolio.

4.2.2.1 Constituent selection methodology

Constituent selection methods range from strictly rules based approaches such as, choosing the 100 largest securities by market value, and methods such as a random or stratified sampling. The approaches that best meet the suitability criteria of the previous section are unambiguous rules-based approaches that are clearly specified in advance.

4.2.2.2 Constituent review, addition and deletion procedure

Current and potentially eligible constituents for addition or deletion may be reviewed annually, quarterly or in real time. In the case of the FTSE 100, constituents are reviewed quarterly but when a significant corporate event such as a takeover, merger or de-listing occurs between quarterly review periods, the constituents are adjusted on, or close to the date that the event becomes effective on the stock exchange.

4.2.2.3 Weighting methodology

The returns of value-weighted indices are biased towards those of their largest constituents, while returns of smaller constituents have relatively little influence on aggregate index returns. This is a reflection of the fact that returns of the largest firms have a greater influence on the returns of the market portfolio than the returns of the smallest firms. A theoretical basis to justify the bias can be found in the CAPM, which assumes that investors have homogenous expectations concerning risk, return and covariance and also that the average investor holds the market portfolio. Taken in this context the large cap bias of value-weighted indices can be viewed as a reflection of investors' prevailing capital market expectations. Hence, value weighted indices meet the six criteria of a useful benchmark proposed by Bailey (1992b) better than either the equally weighted or the price-weighted indices detailed below.

Equally weighted index returns are not biased by large cap stocks or stocks with a high relative price. On the other hand, if the market portfolio is concentrated but the majority of constituent firms are small, it might be argued, that small firms have a disproportionately large influence on an equally weighted index when their collective value accounts for a relatively small proportion of the value of the market. Thus the returns of equally weighted indices are biased towards the returns of small firms. Furthermore, although equally weighted indices are easier to calculate than market value weighted indices, they do not pass the investability criterion so well. This is because in order to track an equally weighted index, continual portfolio re-balancing is required, therefore, it may be possible to only track an index that is calculated periodically, i.e. quarterly, otherwise transaction costs as a result of re-balancing will be excessive.

Price weighted index returns are biased so that stocks that have a high absolute share price, irrespective of their market value or weight in the market portfolio, have a disproportionately large influence compared to stocks with a relatively low absolute share price, irrespective of their market value or weight in the market portfolio. Given that absolute share prices convey little if any information about the overall success or position of a company in the market, the bias has no theoretical basis or justification. Hence, there is little justification for the use of, or the reporting of, price weighted indices. Despite this, price weighted indices such as the Dow and the Nikkei 225 are frequently reported in the media. Possible explanations for this seemingly undeserved attention may include their relative ease of calculation and their long history – the Dow averages, for example, are the oldest stock indices in the world. In addition, both the DJIA and the Nikkei might be considered key brands owned by the media firms who publish them and who have a vested interest in maintaining them to sustain their identity.

4.2.2.4 Accounting for corporate actions: dividends, stock splits and takeovers.

Dividend payments: Most market value weighted indices are based on share price changes without adjusting for dividend payments. However, in addition to the capital price index, some providers such as FTSE International also provide a total return index that is adjusted for gross dividend payments based on announced dividends. The adjustment takes place on the day the constituent company goes ex-dividend. The adjustment methodology used by FTSE international is detailed in their "calculation methodology handbook".

Stock splits: If unaccounted for, these distort the level of price-weighted indices. However, they do not affect market value or equally weighted indices. Price weighted indices adjust for stock splits by changing the index divisor so that the price index value is the same as if the stock split had never happened, bearing in mind that for an index without splits, the divisor would equal the number of stocks in the index. The method of adjustment is outlined in Reilly & Brown (2000) on page 156.

Capital changes and takeovers: These can take the form of rights issues and special dividends. Examples of how these are adjusted for in the calculation of the FTSE 100 Index divisor are provided in the FTSE guide to the calculation methods for UK indices, along with a procedure for dealing with constituent take-overs, mergers, initial public offerings and de-listings. Procedures for dealing with these corporate actions are explained in various levels of detail by different index providers.

4.2.3 Theoretical context

Although the validity of the EMH, and its assumptions, may sometimes be open to debate (as discussed in section 3.3.3 of Chapter 3) there is empirical evidence to suggest that it may be predominantly correct most of the time. Even if limited opportunities do exist for some portfolio managers to achieve average returns that are superior to the "*efficient*" market, the EMH still provides a valuable framework for analysis. Financial institutions adopting passive index tracking or enhanced indexing strategies make use of arguments such as this.[63] Furthermore it is possible to use this framework to argue that the importance to the market universe of an individual firm's stock should be represented accurately by the market value of that firm in proportion to the market values of other firms within the same

[63] Enhanced indexation strategies follow practices such as maintaining index sector weights but attempting to add value through stock selection.

universe, i.e. market constituent weights are a function of prevailing capital market expectations concerning returns, variances and covariances between constituent securities. Therefore, an index that claims to represent the market portfolio should be based upon strict rules to form a passive methodology that uses market value weights to identify constituents and calculate returns. The methodology should be consistently applied and transparent. This enables index users to develop a clear idea of what is being represented and to make fully informed risk assessments about future index behaviour.

It has been argued by certain index providers, such as Standard and Poor's, that a rule based passive index calculation methodology does not provide the best possible model portfolio for a tracker fund due to liquidity problems with some constituents, constituent turnover and diversification issues. Instead an actively managed index that applies a broad range of screening techniques and index manager discretion to determine constituents and weights may be more useful. While this use of discretion as well as pure quantitative factors may be understandable in some circumstances, such actively managed indices should be appreciated for what they are – namely, actively managed "model" or "normal" portfolios as distinct from passive proxies for the market portfolio. Such indices are effectively exposed to some manager specific risk as well as systematic risk.[64] Therefore they are inappropriate as proxies for the market portfolio. Furthermore, the ambiguity surrounding the proprietary and discretionary nature of the index construction process causes the indices to fall foul of the first criterion for a suitable benchmark as well being unsuitable for an academic study of this type, where transparency and replicability are crucial.

Based upon the above analysis of key index characteristics and the assumptions of capital market theory, the following list detailing appropriate methods for index constituent selection and calculation has been compiled by the author. The six items listed are consistent with the characteristics required for a suitable benchmark, a passive proxy for the market portfolio and the replicability criterion for academic studies.

1. The index size should be sufficient to represent the majority of the market-value-of-equity (MVE) of the market for which it is intended as a proxy. This should help to minimise benchmark error.

[64] Since the author completed this chapter, a similar point has been made by Haberle & Ranaldo (2003).

2. It should be MVE weighted so that prevailing capital market expectations concerning the optimal mean variance efficient portfolio are reflected as far as is possible given the limitations of the EMH.

3. Market value weights should be free float adjusted to take into account any cross-shareholdings between corporate entities that are not investable.

4. Adjustment should be made to avoid double counting of shares held by investment firms, which are themselves held by other investment firms.

5. Constituent firms should be chosen using clearly defined rules, based on their market value ranking in the country, market or stock exchange in which they are listed. This is consistent with the requirement that constituent selection and weighting criteria is specified in advance and it helps to avoid breaching the no ambiguity criteria of Bailey (1992b).

6. The index calculation methodology and constituent selection criteria should be transparent, so that users can understand precisely what it is that the respective index represents.

Two further practical characteristics of stock indices have been identified by the author, in addition to those listed above, they are as follows:

1. For the practical purpose of making a historical re-creation of the index, the index should not have more than a few hundred constituents.

2. It should be possible to obtain a list of the original base constituents together with the names and dates of subsequent additions and deletions from the index to enable accurate re-creation of the index for the purpose of avoiding survivorship bias in academic research.

The two additional characteristics are consistent with the investability criterion and relevant when implementing studies such as this one. However, the limit on the number of constituents does compromise the objective of achieving the most complete market proxy possible. It is argued here that this can be justified for practical reasons, when limitations arising from issues such as poor liquidity, unreliable data and limited resources outweigh the potential benefits of a more complete equity market index. In practice a number of different compilation and calculation methods are used to create indices, according to which of the functions listed above are to be emphasized by a given set of index users. Indeed, several of the worlds best known stock indices, including the S&P indices, the Nasdaq 100, the DJIA and the Nikkei 225, do not meet all of the criteria listed above. However, the providers of some of these indices, such as Dow Jones and Nikkei, do calculate a broad range of additional indices – many targeted at institutional clients, which do in fact meet most of the above criteria. The characteristics of some well-known indices are summarised on Table 3 in section 4.3. Those indices with characteristics that are most suitable for this study are identified.

4.3 Review of some well-known stock indices and their providers

During 2001 the author surveyed a large number of stock indices and their providers. The extent to which the indices measured up to the key criteria discussed in section 4.2 were evaluated. The aim of this exercise was to identify the most suitable stock index for developing and testing the methodology applied in this study. Other indices that may be suitable for similar research were also identified as well as indices that failed to meet the key criteria for this study, or other criteria detailed in the previous section 4.2. The extent to which major indices and their providers met the key criteria is summarised in Table 3.

The fourteen desirable characteristics of a stock index for the purpose of this study are listed. Eleven index providers, plus the FTSE 100 Index, are each given a score of one if they meet the key criteria, zero if they don't and not applicable if the criterion does not apply to a particular provider or index. The indices, or providers, with the highest scores are most suitable for this study and for use as market proxy portfolios. FTSE International attains a score of 14, the highest available. The FTSE 100 Index attains a slightly lower score of 13 because it is a national market index, rather than an index provider; therefore, one of the criteria is not applicable. The next highest score of 11 points is achieved by a number of different providers that may be useful for further research using the methodology developed in this study. This ranking is used in Chapter 5 to justify the choice of the FTSE 100 Index as a market proxy for this study.

Table 3 Extent to which major index providers achieve desirable characteristics in their indices

Information was collected to determine whether or not the most important stock market indices and their providers were able to meet the key index criteria discussed in section 4.2 and section 4.2.2 concerning calculation methodology, constituent selection methodology and the appropriateness of what they represent. The key criteria are listed in the left hand column. The name of the index or index provider is specified in the header row. If a particular index or provider meets a given criterion, it is awarded a score of one. If it fails to meet the criterion it is awarded a zero score and if the criterion is not applicable to the specific index this is recorded by the entry – "NA". If the information required to determine this was not readily available at the time of writing this is recorded by the entry "?" The total score is recorded in the final row.

Key Index Criteria	**Index or Index Provider**											
	FTSE 100 UK	**FTSE**	**MSCI**	**Topix**	**Russell**	**HSI**	**Dow Global**	**Wilshire**	**S and P**	**Nasdaq**	**Nikkei excluding the 225**	**Dow Averages**
MV Weighted	1	1	1	1	1	1	1	1	1	1	1	0
Free float adjustment	1	1	1	0	1	?	1	?	1	?	?	0
Indices that account for > 50% of value of total equity market	1	1	1	1	1	1	1	1	1	1	1	0
Constituent history	1	1	?	1	?	1	1	?	1	?	?	1
Rule based constituent selection	1	1	1	1	1	1	1	1	0	1	1	0
Indices which act as an underlying asset for a derivatives market	1	1	?	1	1	?	?	?	1	1	1	?
Transparent methodology	1	1	1	1	1	1	1	1	0	0	1	0
Provide indices that are easy to replicate	1	1	1	1	0	1	0	0	0	0	?	0
Regular review dates	1	1	1	1	1	1	1	1	0	1	?	0
Total return available	1	1	?	?	?	?	?	?	?	?	?	0
Daily or more frequent pricing	1	1	1	1	1	1	1	1	1	1	1	1
Indices with > ten years of history	1	1	1	1	1	1	1	1	1	1	1	1
Global and foreign market indices	NA	1	1	0	1	1	1	0	1	NA	0	1
Methodology specified in advance	1	1	1	1	1	1	1	1	0	1	?	0
Total score/14	**13**	**14**	**11**	**11**	**11**	**11**	**11**	**8**	**8**	**8**	**7**	**4**

4.4 Summary

The review of the major stock index providers and commentary on their various characteristics and calculation methodologies provides useful information for those involved in the interpretation of stock indices generally as part of the decision making process. In addition, new and existing index providers contemplating the development and introduction of new indices to cover various markets and economies as they develop will find the above discussion useful. One useful index that did not exist, at the time of this survey, is an index of the 100, or even fifty top US stocks by market capitalisation that meets all of the key criteria, as distinct from the S&P 100 and the Nasdaq 100 that fail to meet a number of the important criteria discussed. This may present a viable opportunity for a provider such as FTSE international, Frank Russell or Wilshire Associates to consider. While they are undoubtedly useful indices in themselves, at present the nearest equivalents provided by the S&P 100 and the Nasdaq 100 manifestly fail many of the criteria for a suitable market index outlined in section 4.2. In the context of this study this chapter also highlights the suitability of the FTSE 100 Index for the analysis employed. It provides a background for Chapter 5, which provides the justification for choosing this index, as the data source for the methodology applied in this research. Perhaps, more importantly, the critical review of major indices and their providers allows the reader to judge the relevance of the many academic and industry studies of stock market volatility and returns. There is little to be gained from developing a highly sophisticated forecast of the volatility of a stock index if the particular index is fundamentally flawed and meaningless from the perspective of an economist or an investment practitioner. Derivative products or so-called “passive” investments based on such questionable indices should be recognised as conceptually flawed and unsuitable to be marketed as a hedge, a diversification strategy or an investment portfolio. In fact, it could be argued that such indices, and their derivative products, are little more than complex gaming tools or investments in actively managed portfolios misrepresented as passive market portfolios. This concern is highlighted by the discussion of benchmark error and has particular importance when evaluating the performance of active managers against a benchmark, especially if that benchmark is represented as a proxy for the market portfolio.

Chapter 5 – Methodology I: Returns, concentration and volatility data

5.1 Introduction

This chapter begins by explaining the rationale and justification for the choice of the FTSE 100 Index as a data source for this study. The procedure used to replicate the index is then outlined. The concentration metrics used are selected from those reviewed in Chapter 2, and the rationale behind the choice is explained. The key assumption about the return generating process, namely that the expected return of the FTSE 100 Index and constituents is equal to zero, is justified. The implications of assuming that the expected return is equal to zero for the measurement of realised volatility are discussed, together with the method of calculating realised volatility that is used to generate the data-series investigated.

5.2 Justification for choosing the FTSE 100 Index

The FTSE 100 is a value-weighted index that represented 80% of the total value of all the 2,000 plus UK firms listed on the London Stock Exchange (LSE), on the 31st of December 2000.[65] The calculation methodology employed meets all of the key criteria listed in section 4.2 of Chapter 4. Furthermore, the index is one of the most transparent of all the major market indices. The precise calculation and constituent selection methodology, the list of base constituents and the names and dates of all subsequent additions and deletions are freely available.[66] This is particularly important for this study as it means that a historic recreation is possible, using information stored in Datastream. Table 3, in Chapter 4 of the literature review, presents a summary of characteristics displayed by suitable market proxies for the market portfolio and benchmark indices. The FTSE 100 Index attains the highest score of all the indices reviewed in this table.

There are a number of additional contemporary features that make the index suitable for a study of the interaction between index portfolio concentration and volatility. For example, 9% of the total equity market value of all 2,000 plus UK listed firms was concentrated in Vodafone as at the 31st December 2000. In fact, only ten firms accounted for 42% of the

---

[65] Source London Stock Exchange Data file 31st December 2000.

[66] "The FTSE International Guide to the Calculation Methods for UK Indices" January 1999, provides comprehensive set of guidelines regarding calculation methods. This is accessible at http://www.ftse.com. Base constituents and the names and dates of subsequent additions deletions is available by e-mail request at: info@ftse.com

total £1.8 trillion[67] market capitalisation of the UK stock market[68] at 31st December 2000. As the index accounts for such a large proportion of publicly traded UK firms, it could be argued that the index is a good representation of the UK market as a whole and a good indicator of UK economic performance. From the point of view of this research, the relatively limited number of constituents (100) means that the task of recreating this index over an extended period is manageable, particularly when the data is available, as in this case. All Index constituent firms had market values greater than 2.5 billion, on the 12th of January 2001.[69] Therefore, all constituents can be classified as large capitalisation firms, making like-with-like comparisons between constituents reasonable in terms of size. The Index constituents are also easily tradable compared to the many smaller firms listed on the London Stock Exchange. Thus the closing prices and market values shown in Datastream should be more reliable than those held for smaller, less liquid firms. FTSE International, manage an expanding range of indices and the principles of transparency and the market value weighted calculation methodology are becoming increasingly standardised across other global index providers. This has the benefit that a study such as this, developed using the FTSE 100 index data, can be replicated in other national and global markets.

5.3 Replicating the FTSE 100 index

5.3.1 Introduction

One of the most attractive attributes of the FTSE 100 index for a study of this type is the availability of a complete list of index base constituents and their subsequent additions and deletions. This list was compared with a similar list available from Datastream Windows using a programme called 99FTSE, which provides Datastream identification (DSCD) codes for all of the original firms and subsequent additions and deletions. During the study period, from January 1984 through March 2003, a number of firms have either ceased trading, been acquired or have changed their name. This, and additional errors in the 99FTSE programme means that the Datastream data is incomplete. However, using a combination of the definitive list of company names supplied by FTSE International, the data-base of dead firms and their DSCD codes in the Datastream Windows "Code?"

[67] Trillion defined as 10^{12} and not 10^{18}.

[68] The phenomenon is even more extreme in Finland with Nokia and in Holland with Royal Dutch Shell.

[69] Datastream – 12th January 2001

programme, and the Lexis-Nexis data-base of newspaper articles, it has been possible to recreate a complete and accurate replication of the index over the whole study period from January 1984 through March 2003.

The index was recreated using daily closing values for the total dividend adjusted returns data type (RI), market value of equity data type (MV) and price data type (P) extracted from the Datastream financial market data base for individual constituents and the index as a whole over the relevant periods. Full definitions of these datatypes are provided in the Datastream online help files.

5.4 Measuring concentration in the FTSE 100 Index

Chapter 2 provides a review of various concentration indices, including their historical background, and discussion of previous academic studies of concentration in the UK and other markets. This section details the methods by which concentration in the FTSE 100 Index has been measured. Four key concentration indices are identified as being suitable for the modelling of concentration and volatility using the methodology detailed in Chapter 7. Daily values of the following concentration-diversity indices were calculated for the FTSE 100 index over the study period. The formula for each measure of concentration (C) and their key characteristics are specified and discussed in Chapter 2 of the literature review, with the exception of the skewness of firm weights which is detailed below.

1. Hirschman-Herfindahl Index (H)
2. Variance of the portfolio weights (VW)
3. Coefficient of variation (C^2)
4. Shannon's Evenness index (E) which is equivalent to the Entropy index (E)
5. Hannah and Kay's Index with an alpha = 0.5 ($R^{\alpha = 0.5}$)
6. Variance of the logarithm of firm weights (V^2)
7. Skewness of portfolio weights (SK)

Apart from differences in the scaling factors, H, VW and C^2 all produce similar estimates of the general time series path of concentration. Likewise $R^{\alpha = 0.5}$ and Shannon's E provide similar results, apart from scaling effects.[70] However, the variance of the logarithm of firm size, V^2, provides radically different results that cannot be explained by scaling factors alone. To avoid duplicating the results, on account of the above mentioned similarities, the empirical analysis focuses on just four concentration metrics: H, $R^{\alpha = 0.5}$, V^2, and SK. These

[70] Time-series charts demonstrating these similarities between concentration measures in the FTSE 100 index have been produced. However, they are not reported due to space constraints.

four measures of concentration produce different results for the FTSE 100 index. This is due to the fact that they focus upon different parts of the Lorenz curve, as discussed in Chapter 2.

When comparing portfolios that have different numbers of constituents, or when the number of constituents in a portfolio or market is changing over time, there is an argument in favour of using absolute measures of concentration, as discussed in Chapter 2. However, as the FTSE 100 index has a fixed number of constituents, inequality measures of concentration can be used. Another key property of a concentration metric is that it should be unit free, i.e. the units of measurement should be constant through time and between markets. Hence a measure that is based upon weights will be more effective than one that is based upon absolute firm size. The four measures of concentration provide information about both ends of the concentration curve and the overall shape of constituent firm size distribution.

The skewness of firm weights is calculated using the sample skewness function in Microsoft Excel applied to the daily weights of FTSE 100 Index constituents. Like the Hirschman-Herfindahl H index it is heavily dependent upon the inequality between the largest firms in the portfolio and the majority of portfolio constituents. Skewness of constituent weights (SK) also provides a measure of firm size dispersion. A high positive skewness is likely to be associated with high dominance of just a few large firms, while a negative skewness implies that the majority of firms are of a similar size with just a few uncharacteristically small constituents. In other words, high positive skewness is associated with high levels of H and large negative values of SK are likely to be associated with relatively low values of H but not necessarily low levels of inequality.

5.4.1 Sampling and differencing of concentration

The daily values for the above four concentration indices are sampled at five, ten, fifteen and twenty-trading-day intervals. Time series of these samples for the whole study period from January 1984 through March 2003 are plotted in Chart 3 in Chapter 8. Analysis of the charts and diagnostic tests carried out on the time series data indicate that the level concentration series are not stationary, as the null hypothesis that each series has a unit root cannot be rejected. Therefore the four series were differenced at the four intervals listed above. The null hypothesis of a unit root could be rejected at the $\alpha < 5\%$ threshold using the Augmented Dickey Fuller (ADF) test and the Phillips-Perron (PP) tests for all differenced concentration series. Therefore, the differenced series were used in the modelling procedures outlined in Chapter 7.

5.5 Measurement of returns and realised volatility

5.5.1 Measurement of individual security returns

The daily total return index values extracted from Datastream were used to calculate daily percentage returns for each constituent of the FTSE 100 Index, over the period January 1984 through March 2003, using the formula for discrete percentage returns represented by Equation 13, in Chapter 3.[71] Individual daily percentage returns for each constituent were weighted by market value at the beginning of each measurement period in order to re-create the value weighted FTSE 100 Index VCM. Equally weighted percentage returns were used to calculate the customised equally weighted index of the FTSE 100 constituents.

5.5.2 Returns of the aggregate index

The Datastream total return index values for the aggregate FTSE 100 Index are used to calculate the continuously compounded returns using Equation 15 on page 32. The model results reported in Chapter 9 are estimated using continuously compounded daily FTSE 100 Index returns. This is because, when evaluating a single time series, continuously compounded returns have more convenient statistical properties, as discussed by Campbell et al (1997). However, the data used for the VCM decomposition results reported in Chapter 8 and Chapter 10 are derived from simple net daily percentage returns of individual constituents to allow for the aggregation of constituent returns in the portfolio, following the recommendation of Campbell et al (1997). When interpreting the time series data for the VCM sub-components in Chapters 8 and 10 and when comparing the results of Chapters 10 with those in Chapter 9, it is assumed that the difference between the two methods is immaterial. Campbell et al (1997) suggests that this is a reasonable assumption when returns are measured over the relatively short interval of a single trading day, as in this study.

---

[71] The justification for using discrete returns when aggregating individual security returns in a portfolio is provided by Campbell et al (1997) on pages 9 – 12.

5.5.3 Constituent weights and return weighting

Constituent weights are calculated by dividing the constituent daily closing market values by the total of all constituent daily closing market values. Closing market values in millions are obtained using the Datastream data type (MV), which is equal to the share price of a constituent multiplied by the number of shares outstanding for that constituent. The daily total market value of all index constituents, obtained by summing individual constituent weights, was checked with the total market value of the FTSE 100 Index. This technique of cross checking enabled identification and elimination of a number of errors in the constituent lists, from Datastream and FTSE International. The daily constituent returns are weighted using closing market values from the previous trading-day.

5.5.4 Calculation of realised volatility

Various methods of volatility estimation and measurement are reviewed in section 3.2 of Chapter 3. Based on the analysis of the various methods, this study uses the following procedure to record the volatility realised by the FTSE 100 Index and its constituents over the study period. It is assumed that a martingale difference equation, similar to the random walk represented by Equation 18 on page 44, describes the data generating process for the path of the FTSE 100 Index prices and the individual constituent prices. The assumed model has a zero intercept, such as that represented by Equation 18, and the error term (ε_t) is assumed to be covariance stationary. This means that it is assumed to have a constant mean of zero, a constant unconditional variance equal to the expected value of ε^2 and that the autocovariance function ρ of ε^2 is constant for any given lag length.[72]

Realised returns are thus represented by the error term ε_t, therefore their expected value, time series properties and distribution are that of the error term. In order to satisfy the assumption of covariance stationary behaviour, the error term is not required to have a normal distribution, although the mean and variance should be constant over time and identical for all constituent securities and the index. These constant distribution parameters are often referred to as the unconditional mean and variance, whereas the conditional mean and variance are allowed to vary according to the variations in the model variables that drive the return generating process, i.e. the variables detailed in Chapter 7.

[72] In this context, the lag length is sometimes referred to as the displacement factor τ.

Using the assumption that security prices are described by a martingale difference equation to justify an expected security return of zero, realised volatility can then be recorded as the square root of the sum of squared daily returns over a given measurement period. For example, the realised weekly variance of security returns in this study can be estimated using Equation 38.

Equation 38. $$\sigma_i^2 = \sum_{t=1}^{T} r_{i,t}^2$$

Where: *t equals 1-trading day,* r_i *equals the security or index return and T equals the number of trading-days used to estimate* σ^2*, which is the realised variance.*

Using Equation 38 the realised variance of FTSE 100 Index returns is estimated for T = 5, 10, 15 and 20-trading-days. The variance estimated, using 20 trading days, approximates to the realised monthly variance. In this thesis, the square root of the realised variance is referred to as the realised volatility of the returns. Subsequent references to five-day, ten-day, fifteen-day and twenty-day realised volatility, refer to the square root of σ^2 estimated using T = 5, 10, 15 and 20, where T is equal to the number of trading days of returns used for each discrete estimate of realised volatility. This contrasts with the approach used in some studies where realised monthly volatility might refer to the standard deviation of T monthly returns. Note also that, in this study, the realised variance is not an average of squared returns, or squared deviations from the mean; instead it is the sum of squared returns.[73]

5.6 Summary

The present chapter provides details of the data sources, the time period of the study, the FTSE 100 Index replication methodology, the concentration metrics and the realised volatility metrics applied in this study.

[73] In order to simplify the modelling process and discussion of the results, the variables examined in this study are abbreviated using the codes summarised in Table 4 on page 136.

Chapter 6 – Methodology II: Decomposing the variance covariance matrix (VCM)

6.1 Introduction

This chapter explains how the realised variance of the FTSE 100 Index can be decomposed into sub-components of the VCM. This enables the effect of changes in concentration upon realised volatility to be studied more precisely in a variety of different ways. The objective is to isolate the components of realised volatility that are a direct result of index concentration from those that are not influenced by concentration. It is then possible to analyse the effects of changing concentration on the individual sub-components of realised volatility, rather than realised volatility as a whole. This removes the ambiguity caused by inconsistency in the relationship between changes in concentration and the different sub-components, whereby the different influences cancel one another out, thus making it difficult to identify whether or not concentration and volatility are related. The following discussion specifies a method for decomposing portfolio variance into the sub-components of the VCM, so that the contemporaneous and lagged relationship between changes in concentration and the individual sub-components can be studied separately.

6.2 Method of decomposition

Equation 39 is the standard formula for the value-weighted variance (VWV) of a portfolio of N assets. The double summation represents the summing of all the $N \times N$ elements or covariance terms in the VCM. Equation 39 can be divided into the diagonal and off diagonal elements of the VCM. These are represented by the two terms on the right hand side of Equation 40.

Equation 39.

$$\text{VWV} = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{i,j} = \text{VCM}$$

Where: w_i *and* w_j *are the weights of securities i and j,* $\sigma_{i,j}$ *are the individual covariance terms for each pair of securities in the value-weighted VCM and* N *= the number of securities in the portfolio.*

Equation 40. $$VWV = \sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{i,j}$$

Where: *$i \neq j$ for any of the covariance terms $\sigma_{i,j}$. This is because the covariance of a security i with itself, represents the variance of an individual portfolio constituent σ^2_i.*

The weighted summation of the variance terms (σ^2_i) represent the diagonal elements in the VCM, while the off diagonals are represented by the double summation of the covariance elements ($\sigma_{i,j}$) for which $i \neq j$. These can be referred to as the value weighted average variance (VAV) and the value weighted average covariance (VAC), respectively, of constituent returns.

Estimating the variance and covariance for all $N \times N$ elements in the VCM, is laborious and computationally intensive. However, the average variance and average covariance of the VCM can be estimated easily if the expected returns in the variance and covariance calculations are assumed to be equal to zero for individual securities and for the FTSE 100 Index as a whole.[74] The procedure adopted by this study is computationally efficient and is detailed in the following steps.

6.2.1.1 Calculating the value weighted average variance of constituent returns (VAV)

The average variance of portfolio constituent returns, *VAV*, can then be estimated for any trading day (t), using Equation 41.

Equation 41. $$VAV = \sum_{i=1}^{N} w_i^2 r_i^2$$

For all i and j, where r = the security return over the time period t, w = the security weight in the portfolio at the beginning of period t and N equals the number of securities in the portfolio.

6.2.1.2 Calculating the value weighted average covariance of constituent returns (VAC)

The value-weighted average covariance of constituent returns for any trading day (t), VAC_t, can be calculated by subtracting the VAV_t from the squared portfolio return (R_p^2) as in Equation 42. R^2_p is simply the squared portfolio return, also equal to Equation 43.

[74] This is consistent with the random walk model discussed in section 5.5.4 of Chapter 5.

Equation 42. $$VAC = R_p^{\,2} - VAV = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j r_i r_j \quad For\ i \neq j\ .$$

Equation 43. $$R_p^{\,2} = \sigma_p^2 = \sum_{i=1}^{N} w_i^2 r_i^{\,2} + \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j r_i r_j$$

Where: portfolio variance estimated over T periods is defined as VWV and portfolio variance estimated over a single period t is defined as R^2_{p}.

Instead of calculating all the *N(N-1)* individual covariance terms in the portfolio and then taking an average, the same result can be achieved by simply subtracting the average of the *N* variance terms from the portfolio variance. This procedure is simplified further by using the squared constituent returns and the squared portfolio return for each t to proxy for the respective variance terms. The sampling bias that this simplification causes is easily corrected as follows.

6.2.1.3 Adjusting for sample bias

When Equation 41 and Equation 42 are combined as in Equation 43, the result begins to look very like Equation 40, which is the standard definition of portfolio variance σ_p^2 as detailed in Elton and Gruber (2003). Where the number of return measurement periods *(T)* over which σ_p^2 is estimated equals unity, or a single trading day as in this study, the two equations will give identical results. However, in order to eliminate small sample bias, it is better to estimate realised σ_p^2 as a sample adjusted average of T squared portfolio returns, where T is greater than unity. If a mean return of zero is assumed, as suggested by Figlewski (1997) this can be estimated using Equation 44.

Equation 44. $$\sigma_{p,T}^{\,2} = \frac{1}{(T-1)} \sum_{t=1}^{T} R_p^2$$ *Where* R_p *is measured over each of T intervals t, where t = one trading day.*

Likewise it is more conventional and statistically robust to estimate *VAV* and *VAC* over *T* periods rather than over just a single period as indicated by Equation 41 and Equation 42. It is not too demanding to calculate the average of the *N* constituent variances estimated over *T* periods. However, calculating the average of $N \times (N-1)$ covariance terms estimated over *T* periods is more demanding unless the following procedure is followed.

The portfolio return for a single measurement period t is squared to give *(R^2_p)*. The single period *VAV* is then estimated using Equation 41. This is a relatively simple procedure using a spreadsheet. Given that R^2_p is calculated using Equation 43 the single period *VAC* can be found by subtracting *VAV* from R^2_p using Equation 45.

Equation 45. $$VAC = R^2_p - \left(\sum_{i=1}^{N} w_i^2 r_i^2 \right)$$

Having found the single period estimates of *VAV* and *VAC,* sample averages over T periods can be found by summing individual values over T periods and dividing the result by T-1, as represented by Equation 46 and Equation 47 respectively.

Equation 46. $$VAV_T = \sum_{t=1}^{T} [VAV] \times \left(\frac{1}{T-1} \right)$$

Equation 47. $$VAC_T = \sum_{t=1}^{T} [VAC] \times \left(\frac{1}{T-1} \right)$$

For multi-period estimates of VAC_T this is much more computationally efficient than estimating the $N \times (N-1)$ paired covariance terms individually and summing them as in the right hand side of Equation 40.

6.2.2 Incremental average variance (IAV) and covariance (IAC)

The *VAV* and *VAC,* of portfolio constituent returns, can each be further subdivided into the equally weighted and the incremental components. Equation 48 shows the modifications made to Equation 40 for calculating the variance of an equally weighted portfolio (EWV). The significance of this will become apparent shortly.

Equation 48. $$EWV = \left(\frac{1}{N} \right) \sum_{i=1}^{N} \frac{\sigma_i^2}{N} + \left(\frac{(N-1)}{N} \right) \sum_{i=1}^{N} \sum_{j=1}^{N} \left[\frac{\sigma_{i,j}}{N(N-1)} \right] \text{Where } i \neq j$$

The components of Equation 40 and Equation 48 can be used to calculate the influence of individual subcomponents of squared portfolio returns on realised volatility, over any given measurement period *(t)* or mean estimation period *($T \times t$).* The sub-components of squared portfolio returns are described below.

6.2.2.1 The equally weighted average variance of constituent returns (EAV)

The *EAV*, of constituent returns is equal to the sum of the equally weighted diagonal elements in the VCM and it is calculated using Equation 49.

Equation 49. $$EAV = \left(\frac{1}{N}\right)\sum_{i=1}^{N}\frac{\sigma_i^2}{N}$$

The lower limit of *EAV* is zero as the individual squared returns or σ_i^2 elements cannot be negative.[75]

6.2.2.2 The incremental average variance of constituent returns (IAV)

The *IAV* is the difference between the market value weighted and the equally weighted sum of the diagonal elements in the VCM as depicted in Equation 50.

Equation 50. $$IAV = \sum_{i=1}^{N} w_i^2\sigma_i^2 - \left[\left(\frac{1}{N}\right)\sum_{i=1}^{N}\frac{\sigma_i^2}{N}\right] = VAV - EAV$$

This can be either positive or negative because if the constituents that held the greatest weight in the portfolio also had the largest squared returns this would be positive. However if the constituents that had the greatest weighting also had smaller squared returns than the equally weighted average the difference, represented by *IAV*, would be negative.

6.2.2.3 The equally weighted average covariance of constituent returns (EAC)

The *EAC* of constituent returns is equal to the equally weighted sum of all the off diagonal elements in the equally weighted VCM and it is calculated using Equation 51, which simply subtracts the equally weighted average variance of constituent returns, EAV, from the variance of the equally weighted portfolio, EWV.

Equation 51. $$EAC = EWV - EAV \quad = \left(\frac{(N-1)}{N}\right)\sum_{i=1}^{N}\sum_{j=1}^{N}\left[\frac{\sigma_{i,j}}{N(N-1)}\right]$$

Where: *$i \neq j$, and $E\sigma_p^2$ equals the variance of the equally weighted portfolio.*

This can be positive or negative, depending upon whether returns of securities in the portfolio are generally moving together or in opposite directions. The "generally" is based

[75] Note that in these equations for the single period case the squared constituent returns r_i^2 and variance terms σ^2 are one and the same.

upon an equally weighted average, i.e. the sum of the equally weighted co-movement or off-diagonal elements in the matrix.[76]

6.2.2.4 The incremental average covariance of constituent returns (IAC)

This *IAC* represents the difference between the market value weighted and the equally weighted sum of all the off-diagonal elements in the VCM and it is calculated using Equation 52.

Equation 52.
$$IAC = VAC - EAC$$
$$= \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{i,j} - \left(\frac{(N-1)}{N}\right) \sum_{i=1}^{N} \sum_{j=1}^{N} \left[\frac{\sigma_{i,j}}{N(N-1)}\right]$$

Recall that VAC is obtained using Equation 45, and that EAC is estimated using Equation 51.

Like the incremental average variance, the incremental average covariance can be positive or negative, depending upon whether or not the greatest weights are found in the largest off diagonal elements in the VCM, or the smallest off diagonal elements in the VCM.

6.2.2.5 The incremental realised volatility of portfolio returns (IRV)

When the incremental average variance and incremental average covariance are combined using Equation 53, they can be referred to as the incremental realised volatility of the portfolio (IRV). As a measure of the difference between the variance of the equally weighted and value weighted portfolio, this is a measure of the total net effect of portfolio concentration upon realised portfolio volatility.

Equation 53.
$$IRV = IAV + IAC$$

If the IRV is positive, the concentration of the portfolio has increased the realised volatility of the portfolio and vice a versa if IRV is negative, at any given time t.

6.3 Standardising the VCM sub-components

The calculations in the previous section 6.2 enable the four sub-components of realised volatility at any given time t, EAV_t, IAV_t, EAC_t *and* IAC_t, to be derived from the summed diagonal and off diagonal elements in the VCM. Because of the nature of the

---

[76] The empirical data analysed in Chapter 8 indicate that only a small minority of observations are in fact negative and then only when they are estimated over relatively short periods, i.e. T is five or less.

decomposition process, if $R_p^2 = \sigma_p^2$, the following relationship represented by Equation 54 holds, by definition, over any measurement period t.

Equation 54. $$VWV = \sigma_p^2 = EAV_t + IAV_t + EAC_t + IAC_t$$

The above relationship has been established by simple decomposition of the standard portfolio variance equation used in MPT. The important issue, for those implementing portfolio diversification strategies and attempting to understand portfolio volatility, is the relative importance of the four sub-components for determining σ_p^2. Furthermore, it is of interest to know whether the relative importance of each factor is stable through time or whether it changes in relation to changes in portfolio variance, or to changes in concentration (ΔC).

Given that IAV_t, IAC_t, and EAC_t, can be either positive or negative and that they will change through time as σ^2_p changes, it is necessary to derive standardised measures of their influence on σ_p^2. These standardised measures can be either positive or negative according to the sign of the respective sub component. However, because negative values of respective sub-components tend to coincide with very low values of σ_p^2, simply dividing the respective sub component by σ_p^2 will give a ratio that is both very unstable and peculiar in its distribution[77]. An analogy would be the calculation of a price earnings (PE) ratio for a firm that had zero or slightly negative earnings and a very volatile share price, volatile earnings, or both. The problem is addressed by dividing the respective sub-component by the sum of the absolute values of all sub-components. This will be referred to as the gross, total variance covariance matrix (TVCM), which can be calculated simply by adding up absolute values of the four sub-components as represented by Equation 55.

---

[77] It is possible for the realised variance to be smaller than the absolute value of an individual sub-component on occasions when one or more sub-components are negative. In cases where a sub-component takes a large negative value, division by a small positive variance value results in an even larger negative value. This causes the time series properties to be very unstable and the distribution to be extremely left skewed, to the extent that the data is not very informative, if sub-components are standardised simply by dividing by the realised variance. This issue becomes less of a problem as T-values increase, due to the smoothing of extreme negative values. Nonetheless, the time series properties are still not as stable, and the distributions are not as symmetrical as when the sub-components are standardised using the method outlined in this thesis. As this is a new VCM decomposition method, no precedent has been identified for this standardisation procedure; it is simply an ad-hoc practical solution that seems to work. Further research, examining VCM sub-components estimated with T-values of ten-days or more, might explore the possibility of standardising the sub-components simply by dividing each by the portfolio variance, although the distribution properties are not as convenient.

Equation 55. $$\text{TVCM} = [|EAV| + |IAV| + |EAC| + |IAC|]$$

The four sub-components and the combined incremental components are then standardised by dividing each by the TVCM to give the following ratios.

6.3.1.1 Standardised equally weighted average variance (SEAV)

$$SEAV_T = EAV_T / TVCM_T$$

Where T is the number of daily returns used to estimate *SEAV*. Thus if, for example, T is equal to five trading-days, the acronym would be SEAV5. SEAV will always be positive and less than one because EAV is a squared term and the TVCM is the absolute sum of the four sub-components EAV, IAV, EAC and IAC.

6.3.1.2 The standardised incremental average variance (SIAV)

$$SIAV_T = IAV_T / TVCM_T$$

The SIAV may be positive or negative. If portfolio assets are concentrated in constituent securities that have absolute returns larger than the equally weighted average the weighted average variance of returns will increase and the IAV data-series will be positive. If, on the other hand, assets are concentrated in securities that have absolute returns smaller than the equally weighted average, the weighted average will decrease, IAV will be negative and hence SIAV will be negative. In the former case, raising portfolio concentration above the lower limit will have the effect of increasing portfolio risk. In the latter case, raising portfolio concentration above the lower limit will have the effect of reducing portfolio risk, if all other factors remain constant.

6.3.1.3 The standardised equally weighted average covariance (SEAC)

$$SEAC_T = EAC_T / \text{TVCM}_\text{T}$$

SEAC is most likely to be positive because in practice, it is not common to find many securities whose returns have a negative covariance over a long period, hence EAC will generally be positive. However, it is theoretically possible for EAC to be negative and, for short periods of time, negative values of EAC have been observed in the empirical data reported in Chapter 8. This means that, on occasion, SEAC will also be negative.

6.3.1.4 *The standardised incremental average covariance (SIAC)*

$$SIAC_T = IAC_T / TVCM_T$$

SIAC may be either positive or negative. For example, if portfolio assets are concentrated in pairs of securities that have a lower or more negative covariance with one another than the equally weighted average, the weighted average portfolio covariance elements will be lower. Therefore IAC and SIAC will be negative. Conversely, if assets are concentrated in pairs of securities that display a covariance with one another that is higher than the equally weighted average, the value weighted average covariance will be greater, hence IAC and SIAC will be positive. In the latter situation, increasing concentration will have had the effect of reducing portfolio diversification and will result in greater risk, ceteris paribus. In the former case, an increase in concentration will increase portfolio diversification and will reduce portfolio risk, ceteris paribus, i.e. the portfolio will be more efficient in the former case and less efficient in the latter. The sum of the absolute values of the four ratios listed above must always equal unity for any time t.

6.3.1.5 The standardised incremental realised portfolio volatility (SIRV)

$$SIRV_T = IRV_T/TVCM_T$$

The SIRV provides an indicator of the proportion of total realised volatility that is due to the concentration of the portfolio being greater than its lower limit of 1/N. It can be either positive or negative.

6.3.2 Acronyms

A relatively large number of data series have thus been derived each with their own notation. In order to simplify the following discussion and to provide convenient acronyms for programming software and discussing the results, the sub-components, standardised sub-components, the TVCM and realised volatility have been abbreviated as detailed in the footnote below.[78]

---

[78] This notation can be linked to the actual data series used in the empirical analysis by reference to Table 4 on page 136 in section 7.7 of Chapter 7.

EAV = Equally weighted average variance of constituent returns

IAV = Incremental average variance of constituent returns.

EAC = Equally weighted average covariance of constituent returns

IAC = Incremental average covariance of constituent return

IRV = IAV + IAC = The incremental realised volatility

TVCM = |EAV| + |IAV| + |EAC| + |IAC| = The sum of the absolute values of the sub-components of the VCM

RV = $\sigma^2_p = R^2_p$

6.3.2.1 Acronyms and the subscript T

In the results chapters, all of the acronyms defined above have a number suffix, indicating the value of T used to estimate them. Hence, EAV5 means that discrete observations of the equally weighted average covariance time series are each estimated with a T equal to five days. Model results are reported for T-values of five, ten, fifteen and twenty trading days for most of the above subcomponents of the VCM.

6.4 Summary

This chapter has provided a new method of decomposing the realised variance of portfolio returns into that which is conditional upon concentration being greater than the lower limit of an equally weighted portfolio, and that which is not. In addition, it has detailed a method for decomposing the realised variance of portfolio returns into the summed diagonal and summed off-diagonal elements of the VCM. The diagonal and off-diagonal components, representing the average variance and average covariance of constituent returns respectively, are each decomposed further into those parts which are conditional upon concentration being greater than the lower limit of an equally weighted portfolio and those parts which are not. This enables the generation of a number of data-series each representing different sub-components of the VCM. A method for standardising the sub-components of the VCM, so that the relative importance of each sub-component in determining the overall VCM can be evaluated, is then detailed. The next Chapter explains the modelling process applied to the data derived by implementing the methodology outlined in this chapter, as well as the differenced concentration and realised volatility data described in Chapter 5.

RS = The realised standard deviation of returns, i.e. the square root of S

Chapter 7 – Methodology III: Models and sampling theory

7.1 Introduction

The objective of any model building process is to be able to specify a model with parameters that are stable in any given time period. The residual errors should also be white noise. These conditions are often difficult to achieve in practice when modelling financial time series data, although they form the ideal objective of the modelling process. Furthermore, if the process generating a data series undergoes a fundamental regime shift, the population parameters of the series may change. In effect, two populations will have been generated, each with different parameters. In such circumstances it is not unreasonable to suppose that different model specifications are required in order to describe and explain a data series containing different populations of data. The models discussed in this chapter produce residuals that are not always consistent with the ideal of white noise. The implications of this for the interpretation of the results are discussed, where appropriate, in Chapters 8 and 9.

Model dependent variables are, realised volatility and the sub-components of realised volatility. Time series of the dependent variables consist of discrete non-overlapping observations, estimated using intervals of T daily returns as specified in the preceding two chapters. Each series is estimated with T equal to five, ten, fifteen and twenty trading-days. Forecasting models are used to predict future values of the dependent variables. They can be complex (general) models with many different contemporaneous and distributed lags of the independent variables, as well as autoregressive lags of the dependent variables. Alternatively, they can be simple (parsimonious) models with one or two lagged values of the dependent variable, an intercept and an error term. One of the simplest models of all is the random walk model discussed in section 5.5.4 of Chapter 5. This assumes that the best predictor of the future value of the independent variable at time t + 1 is the current value at time t. However, if a time series is covariance stationary, it may be possible to predict it using a relatively simple autoregressive (AR) model.[79]

General models may incorporate both simultaneous and lagged values of an independent variable in addition to autoregressive lags of the dependent variable. The residuals of general models usually have a lower variance indicating a better model fit within the

[79] The definition of covariance stationary is provided in the literature review, section 3.4.2.1.

estimation-period (in-sample) than the parsimonious models: unfortunately, when such models are used to generate actual forecasts outside of the model estimation period (out-of-sample forecasts) their superiority is often lost in favour of the more parsimonious models. However a variety of tests discussed in section 7.4.2 can be used to compare the fit and forecasting performance of general models with parsimonious models. These tests apply penalties of varying severity for each degree of freedom lost, or gained, as additional variables are added to or subtracted from the model specification. To avoid the danger of "omitted variable bias" general models are usually refined to more specific parsimonious models rather than the other way round. Finally, the models that appear to provide the best within sample fit according to these tests can then be used to generate out-of-sample forecasts that can also be compared to see if the out-of-sample comparison is consistent with the within-sample results. Methods of comparing out-of-sample forecasts are provided in section 7.4.3.

In this study general asymmetric autoregressive distributed lag (AARDL) models, discussed in section 7.3, use autoregressive lags of the dependent variable as well as contemporaneous and lagged differenced concentration as the independent variables. These general models are compared with the basic autoregressive (AR) and asymmetric autoregressive (AAR) models of each volatility series, discussed in section 7.2. Henceforth, the basic models are referred to as the naïve or benchmark models, against which the more complex models are compared.

All model parameters are estimated using OLS regression. Newey-West heteroskedasticity and autocorrelation robust standard errors are used to generate the t-statistics of model coefficients. This is because of the persistent autocorrelation and the excess kurtosis evident in the residuals of some of those models. The restrictive assumptions of the OLS methodology are well known and documented. However, there is a large volume of literature published on the subject of time series models of volatility and an ongoing debate as to the most appropriate models to use. As there is no precedent, in the literature, for modelling the relationship between changing concentration and realised volatility, or indeed the sub-components of realised volatility, it seems appropriate to start the analysis with a well-known approach such as OLS. In addition, it can be argued that residual analysis is one of the more productive aspects of model building, implementation and interpretation. This is because, even if some of the assumptions of the OLS models are not fully met, the results can still be usefully interpreted, so long as the residuals are also analysed to highlight areas of weakness and possible means of improving the specification. With this in mind, descriptive statistics and autocorrelation characteristics of model residuals are provided for all model results. Anomalies within these are discussed where appropriate.

Table 4 on page 136 lists and defines the data series that represent measures of FTSE 100 Index realised volatility, sub-components of the realised volatility and differenced concentration. The objective of this thesis is to model and describe the relationship between concentration and volatility in the FTSE 100 Index, and to determine whether or not it is possible to forecast realised volatility, or the sub-components of realised volatility, using models that incorporate the various measures of differenced concentration. Such forecasts are only useful if they are more accurate than naïve models such as random walk models, or simple autoregressive models of realised volatility.

7.2 Naïve benchmark forecasting models of realised volatility

Consideration of the correlogram for each volatility data series reveals the existence of autocorrelation that is significantly different from zero for a large number of lags. However, the greatest autocorrelation is usually found in the first two or three lags. Therefore, the naïve model used for each series is based on a second order autoregressive ($AR_{(2)}$) model for data estimated with a T equal to five or ten trading days and a first order $AR_{(1)}$ model when T is equal to fifteen or twenty trading days. The basic structure of an $AR_{(1)}$ model of realised volatility is represented by Equation 56.

Equation 56. $$\sigma_t = \alpha + \beta_1\sigma_{t-1} + \varepsilon_t$$

Where: σ_t *= the realised standard deviation of returns estimated for t= 1,...t = T*

These series are referred to as RS and are defined on page 136.

An ideal specification of an $AR_{(p)}$ model would result in the residuals ε becoming white noise. However, even sophisticated volatility models using advanced variations on ARCH and GARCH techniques often struggle to achieve this ideal. Therefore, for each model result reported, some information is also provided on the residual distribution and autocorrelations.

Given the extensive documentation of the asymmetry effect in the literature reviewed in Chapter 3, it is appropriate to include an asymmetric slope dummy coefficient (λ_1) in the basic naïve model. Hence Equation 57 represents the naive asymmetric autoregressive

(AAR_1) model. The dummy variable, D, is equal to unity if the average return over the period t = 1 to t = T is negative and zero if the return is positive.

Equation 57. $$\sigma_t = \alpha + \beta_1\sigma_{t-1} + \lambda_1 D_{t-1}\times\sigma_{t-1} + \varepsilon_t$$

Model coefficients on the dummy variable allow the model intercept to change when returns are negative, whereas model coefficients on the explanatory variable multiplied by the dummy variable enable the autoregressive slope coefficients to differ, depending upon whether corresponding FTSE 100 Index returns are positive or negative. Results of initial modelling demonstrated that coefficients on the intercept dummy variable were not significantly different from zero and were inconsistent in sign. In contrast, the slope dummy coefficients were, almost universally, significant and consistent in sign. Therefore, the results reported in Chapters 9 and 10 are for models that include slope dummy coefficients but not intercept dummy coefficients. For each data series modelled as a dependent variable, asymmetric autoregressive distributed lag (AARDL) models were compared with naïve AAR models and naïve AR models. Only one lagged asymmetric slope dummy coefficient was included in the asymmetric models, although models with both one and two autoregressive lags were estimated according to the criteria detailed above.

7.3 General asymmetric autoregressive distributed lag models (AARDL)

Initially AARDL models were estimated with up to five autoregressive lags, five asymmetric slope dummy variable lags, five asymmetric intercept dummy variable lags and five distributed lags of differenced concentration. However, following a general to specific modelling procedure, higher lags of all variables were found to have unstable coefficients that were inconsistent in sign and not significantly different from zero. Exclusion of the asymmetric intercept dummy variable coefficients and only including one lag of each of the asymmetric slope dummy variable, and the distributed lag of differenced concentration, resulted in estimated model coefficients on the remaining variables that were more stable between sample sub-periods. In addition, the coefficients were statistically significant at lower p-values, when evaluated using the diagnostic tests outlined in section 7.41 and overall model fit improved according to the metrics discussed in section 7.42. It is the results of these most parsimonious AARDL models that are reported in Chapters 9 and 10, for comparison with the corresponding naïve AAR and AR models. Equation 58 represents an example of a general AARDL model estimated for realised volatility is represented.

Equation 58. $$\sigma_t = \alpha + \beta_1\sigma_{t-1} + \lambda_1 D_{t-1}\times\sigma_{t-1} + \delta_1\times\Delta C_{-1} + \varepsilon_t$$

Where: ΔC *= differenced concentration*

The AARDL model specification for forecasting realised volatility represented by Equation 58 is a generic form that is applied to all forecasting models employed. When sub-components of the realised volatility are to be forecast, the respective sub-component dependent variable is substituted for σ_t in Equation 58.

Chapter 9 and Chapter 10 report model results estimated using the sub-period of the data sample from January 1998 through December 2003. During this sub-period, the most general models estimated for each dependent variable and differenced concentration metric also included a contemporaneous differenced concentration variable. The specification of these models was similar to that of Equation 58 except that an additional contemporaneous concentration variable was included, as displayed in Equation 59.

Equation 59. $$\sigma_t = \alpha + \beta_1\sigma_{t-1} + \lambda_1 D_{t-1}\times\sigma_{t-1} + \delta_1\times\Delta C_{-1} + \delta_2\times\Delta C + \varepsilon_t$$

General AARDL models for each dependent variable and differenced volatility series were estimated with and without the contemporaneous differenced concentration variable in this period. The results are reported in Chapter 9 and Chapter 10.

7.3.1 The 1987 Crash

The global stock market crash on the 19th October 1987 produced an extreme outlier in the dependent variable data series. This outlier was positive for realised volatility and all sub-components of realised volatility except for the incremental average covariance and incremental realised volatility, where it was negative. All time series models estimated over periods including this event that did not provide special treatment for the outlier exhibit extreme excess kurtosis and skewness in the residuals. In order to adjust the data for this outlier a dummy variable was included in the models. This was equal to one in the data estimation period corresponding with the crash and equal to zero at all other times. Coefficients on this dummy variable were significant in all models where it was included. The skewness and kurtosis of model residuals were considerably reduced and the adjusted R^2 values were considerably higher compared to corresponding models estimated without this dummy variable. However, significant autocorrelation was prevalent in the residuals of models that included the dummy, whereas it was generally absent in those that did not. The dummy variable method of treating this event is an alternative to the method of winsorizing of the data adopted by Campbell et al (2001) in which the largest outlier was replaced by the second largest value.

7.4 Diagnostic tests

7.4.1 Coefficient tests

The residuals of all the models estimated had means and medians very close to zero in relation to their standard deviations. However, the residuals of many of the models displayed evidence of autocorrelation in some lags. In addition, there was evidence that the residuals did not have a constant variance and were not normally distributed. This was evident in the excess kurtosis and skewness of model residuals, the Jarque-Bera normality tests of model residuals, the time series charts of model residuals and the Q-statistic for partial autocorrelation coefficients. The properties are all violations of the assumptions of OLS regressions, although they are common problems encountered in models of financial time-series. One adjustment that can be made to improve the reliability of the interpretation of model results, when the residuals display autocorrelation and heteroskedasticity, is to use Newey-West standard errors to calculate the t-statistics for model coefficients. The t-statistics for all model coefficients discussed in this study were estimated using Newey-West heteroskedasticity and autocorrelation robust standard errors.

7.4.2 Tests of model fit

The most basic test of the explanatory power of a model is the coefficient of determination, or R^2. However, as R^2 will increase automatically as the number of variables in the model increases, trying to find the model that maximises R^2 is not an appropriate way to choose the best model. There are a number of other measures of model fit, which are penalised by varying amounts, for each degree of freedom lost by the addition of a new variable to the model. The most common of these is the adjusted R^2. Other metrics include the standard error of the regression, the Akaike Information criterion (AIC) and the Schwartz Information criterion (SIC). Further review of these and other metrics is left to Diebold (2001).[80]

The E-views econometric software, used in this study, reports the natural log of the AIC and SIC, so reported values in Chapter 9 and Chapter 10 are negative because standard error

[80] Diebold (2001) pp 20-25 and 83-89

values are below one. Thus more negative values of the AIC and SIC indicate a better model fit. When comparing general AARDL models with the more parsimonious naïve AAR and AR models, the four metrics of model fit in a general model should all be superior to the naïve model, before it can be concluded that the general model is likely to have a better out-of-sample forecasting ability. If the SIC gives a conflicting result to the other metrics the distinction between the models is more ambiguous (Diebold, 2001). The SIC is more consistent than the AIC; however, the AIC is asymptotically efficient, whereas the SIC is not. If the goal is to find a simple forecasting model that is practically useful, the SIC will avoid inclusion of variables that add little to a models out-of-sample forecasting ability. This is because it is always biased in favour of the more parsimonious models. On the other hand, if the true model that describes the data generating process of the population is not among those considered for the sample estimate, a case that is usual in practice where models are at best a gross oversimplification of the data generating process, excessive reliance on the SIC may result in asymptotically useful variables being discarded. In other words, a variable that has real explanatory power in the data population as a whole may be erroneously discarded as irrelevant due to variations between the sample and population of the data if excessive reliance is placed on the SIC. This is specifically relevant to the interpretation of the results in Chapter 9 and Chapter 10.

For example, there is an issue of interpretation when the coefficient on contemporaneous or lagged differenced concentration is significantly different from zero with a very low p-value, the AIC and the adjusted R^2 in the AARDL model are marginally better than the naïve AAR model, while the SIC is worse than the naïve model. A reasonable explanation is that inclusion of the differenced concentration variable is, at best, likely to provide an inconsistent effect on the out-of-sample forecasting accuracy of the model. However, given the very high t-statistic and low p-value for the differenced concentration (ΔC) coefficient, it is also likely that ΔC is an important but perhaps small part of the data generating process. This means that for the immediate purpose of forecasting imminent changes in volatility ΔC may not be of great practical value. However, if the long-term aim is to construct a mean variance efficient portfolio, then taking into account the effect of changes in concentration upon the sub-components of the VCM may have much more practical significance. This is because, if this allows a relatively small reduction in volatility to be maintained over a long period without impairing returns, it will have a material and beneficial effect on risk-adjusted performance.

7.4.3 Tests of model out-of-sample forecasting ability

The previous section discusses some standard indicators of model fit that were evaluated for the each of the models estimated in this study. The main body of the results in Chapters 9 and 10 focuses on evaluation of models estimated for each dependent variable data series from January 1998 through December 2002. However, a total of fifteen observations were withheld from the model estimation sample for data series estimated with T-values equal to five trading days in order to evaluate the out-of-sample forecasting ability of models estimated for those data series. The fifteen out-of-sample observations included the first in January 2003 through to the last, ending on the 16th of April 2003. Eight observations were withheld for data series estimated with T-values equal to ten trading days of returns, starting from the first in 2003 to the observation ending on the 7th April 2003. Because forecasting models included lagged values of the dependent variables, static forecasts were estimated rather than dynamic forecasts. This means that each forecast was a one-step-ahead forecast with actual values of the dependent variable inputted into the AARDL model up to the last observation before the forecast.

The mean absolute percentage error (MAPE) and Theil Inequality Coefficient (TIC) are two unit free out-of-sample evaluation metrics provided by E-Views. The MAPE measures the mean of the absolute out-of-sample errors as a percentage of the corresponding realisations of the variable using Equation 60.

Equation 60.
$$MAPE = \sum_{t=T+1}^{T+h} \left| \frac{\hat{y}_t - y_t}{y_t} \right| \div (h+1)$$

Where: h = the number of out-of-sample observations used to evaluate the forecast.

This measure can give very unstable results when the number of out-of-sample forecasts h is small and the value of y_t can be zero or close to zero. This means that if the absolute value of the error is greater than the value of y_t at a time when y_t has a value of zero or very close to zero, the absolute percentage error can be infinitely large. The TIC is another unit free metric that compares the error of the model forecast with the error of a random walk model and is estimated using Equation 61.

Equation 61.

$$TIC = \frac{\sqrt{\sum_{t=T+1}^{T+h}\left(\hat{y}_t - y_t\right)^2 \div (h+1)}}{\sqrt{\sum_{t=T+1}^{T+h}\hat{y}_t^2 \div (h+1)} + \sqrt{\sum_{t=T+1}^{T+h}y_t^2 \div (h+1)}}$$

Where: y_t *is the realised change in the value of y that is equivalent to the error* ε_t *in a random walk model.* y_t *hat is the forecast change in the value of y at t.*

Thus the TIC is the ratio of the root mean squared error (RMSE) of the model forecast divided by the root of the mean of the squared forecast, plus the root of the mean of the actual squared realisations of the data. It takes a value between zero and unity where zero equates to a perfect forecast and unity means that the forecast is no better than that of a random walk model. Because the denominator is always at least as large as the numerator so that the upper limit of the TIC is unity, this measure of forecast accuracy does not suffer from the problem described for the MAPE.

In the case of an ideal forecasting model the out-of-sample forecast errors should be white noise with a mean of zero. However, if the sample mean of the forecast errors is significantly different from zero the forecast is biased and the implication is that the model could be better specified. Nonetheless, a biased forecast with a low forecast error variance is often more useful than an unbiased forecast with a large forecast error variance. Having evaluated the size of the forecast error using a metric such as the MSE, the MAE, the MAPE or the TIC, it is then possible to investigate the likely consistency of the forecast by decomposing the MSE into the bias proportion, the variance proportion and the covariance proportion, as indicated by Equation 62.[81] Although the MSE itself is not unit free, the three sub-components are and they sum to unity. This is because E-views reports each of the three components of the MSE as a proportion between one and zero by dividing them individually by the MSE as a whole.

The variance component of the MSE represents the contribution to the MSE made by the variance of the forecasts (y-hat values) from their mean, which will be different to the mean of the realised y values if the forecasts are biased. If the variance of the forecast y-hat values is different to the variance of the realised y values, the variance component will be

[81] These are discussed in more detail by Pindyck and Rubinfeld (1991), Chapter 12: also pp 316-317 Diebold (2001) and pp 337-338 of the “E-Views 4 User Guide”.

non-zero. However, the size of the variance proportion will depend upon the relative size of the variance component in relation to the bias component and the covariance component of the MSE. Ideally, the mean and variance of the forecast y values should not differ from the mean and variance of the actual y values, resulting in a variance and bias proportion of zero.

Equation 62. $MSE =$

$$\frac{\sum_{t=T+1}^{T+h}(\hat{y}_t - y_t)^2}{h} = \left[\frac{\sum_{t=T+1}^{T+h}(\hat{y}_t - y_t)}{h}\right]^2 + (\sigma_{\hat{y}} - \sigma_y)^2 + 2(1-\rho_{\hat{y},y})\sigma_{\hat{y}}\sigma_y$$

> *The three terms on the right hand side, beginning with the square brackets, represent the bias proportion, the variance proportion and the covariance proportion respectively.*

The covariance component of the MSE measures the covariance between the forecasted y hat values and the actual realised y values. Ideally the covariance proportion of the MSE should be close to unity while the bias and variance proportions should be close to zero. This is because a high covariance component signifies that a change in the forecast is of the same sign as a corresponding change in the realisation of y. Thus, even if the forecast over or under predicts the extent of the change in y, the direction is correct, so from a portfolio manager's perspective a predicted rise in volatility would be followed by an actual rise in volatility and vice versa. Therefore the decisions based on the forecast should lead to outcomes no less favourable than outcomes resulting from decisions based on no forecast or no decisions. The worst-case scenario implicit in a low covariance proportion of the MSE would be a forecast reduction in volatility leading to a decision to increase exposure to risky assets at a time when volatility is actually increasing.

7.4.4 Residual analysis

Ideally the residuals of a time series model should be white noise. Unfortunately it is very difficult to achieve this ideal with any models of financial time-series, least squares or otherwise, and a variety of complex procedures for modelling realised volatility are discussed in Chapter 3. In order to allow the reader to form their own opinion as to the robustness of the results, descriptive statistics of the residuals are reported, together with a note detailing the existence or otherwise of autocorrelation. Included in the descriptive statistics are the mean, median, standard deviation, skewness and kurtosis. In addition the

Jarque-Bera test statistic is reported and in the few cases where the null hypothesis of normally distributed residuals cannot be rejected the respective p-values are also reported. The mean, median standard deviation and skewness of the residuals allow the reader to assess the extent to which the model results might be biased. The skewness and kurtosis enables the reader to assess the influence of positive and negative outliers on the distribution of the residuals. Finally the existence or otherwise of residual autocorrelation provides an insight into the potential for improving the model specification by including more lags of the dependent variable, or by adopting some form of ARCH or GARCH specification.

7.5 Sampling theory and sample-period selection

7.5.1 Time period of study

The time period of this study is from the creation of the FTSE 100 index in January 1984 through March 2003. As this is a time series study, the time period studied represents the sample of the data. Therefore, any interpretation or breakdown of the whole period into sub-periods must give consideration to potential methodological problems relating to sampling theory and good practice for empirical research.[82] When a data sample consists of a time series over a given period, two approaches can be adopted. The first assumes that the data are all sampled from the same population. The second acknowledges the possibility of a regime shift occurring during the sample period, with the result that the sample period effectively spans two or more distinct populations of returns, each with different parametric characteristics. When such a regime shift is thought to have occurred, it makes sense to estimate model coefficients for the period before, and after, the regime shift, separately.

Time period bias relates to the fact that use of a relatively short sample time period for model parameter estimation, will only give results that are specific to that period. The use of a long sample period may give a more accurate picture of the true population characteristics, but it is vulnerable to the possibility that it contains different populations of data with different distributions arising from structural changes in the data generating process. For this reason it is necessary to construct time series models for the whole study period, in addition to shorter sub periods within the overall period. Results can then be compared between sub-periods, or between individual sub-periods and the overall period.

[82] These issues are discussed in more detail by DeFusco et al (2001) on page 285

7.5.2 Data mining

Data mining is the repeated "drilling" in the same sample of data until some interesting results are obtained[83]. These are then reported, while inconclusive results, or results inconsistent with the theoretical hypothesis put forward, are ignored. Although this study examines a number of different measures of differenced concentration, realised volatility and the sub-components of realised volatility, and tests various models over different time periods, the analysis and conclusions are robust to the criticism of data mining. This is because summaries of all results are reported, regardless of whether or not they are consistent with the a priori hypothesis. The primary objective of this study is to develop a method for explaining and forecasting realised volatility in the context of changes in concentration. The stability and persistence of the results obtained are evaluated by estimating model parameters over the whole study period and different sub-periods. In addition, out-of-sample forecasts are used to evaluate the results of models estimated in the sub-period from January 1998 through December 2002.

The existence of separate regimes in the data series, characterised by different variances and means, enable the data to be divided into separate five-year periods that can be modelled separately. These are from January 1988 to December 1992, January 1993 to December 1997 and January 1998 through March 2003. The additional four-year period from January 1984 through December 1987 can be used to isolate, from the rest of the data, the period leading up to and immediately following the 1987 crash. In addition, the data estimated with fifteen and twenty-trading-days worth of returns per observation was modelled separately for the periods from January 1984 through December 1990 and January 1991 through December 1999 as well as the whole period from January 1984 through March 2003. The sub-divisions in the data are identified in the time series charts, reported in Chapter 8 by vertical lines.

7.5.3 Focus of analysis

The modelling procedure specified in this chapter aims to take into account the characteristics of the data described in Chapter 8. Time series data involving concentration and realised volatility is modelled over both the whole study period 1984-2003 and separate

[83] See De Fusco p 298.

sub-periods within this. The choice of sub-periods is determined by the following practical and theoretical considerations:

- The need to identify stationary data for modelling.
- The need to identify a population of data that is consistent, i.e. data that does not include regime changes in the underlying data generating processes.
- The requirement to obtain a sample of data large enough to allow a reliable estimation of model parameters.
- The need to use data that is sufficiently recent to reflect current market conditions, while recognising that these conditions might change in the future.

Section 8.2 in Chapter 8 describes the evolution of the whole data sample from January 1984 through March 2003. The time series charts presented and discussed in Chapter 8 indicate that the time series of realised volatility, and of the level of concentration and differenced concentration, all undergo a regime shift beginning in the latter part of 1997, after the Asian financial crisis in October of that year. Despite the volatility induced by the Asian crisis, Western stock market indices rose rapidly until the late summer of 1998 when the Russian sovereign debt default and the collapse of Long Term Capital Management in September 1998 resulted in a sharp plunge in the major market indices. Markets rebounded rapidly in October 1998 and the rise associated with the inflation of technology and telecommunication stocks continued until the bubble burst in March 2000. The deflation of the bubble continued until the end of the data series.

The period from January 1998 through March 2003 was characterised by an exceptional period of volatility in all of the series studied. All measures of concentration increased rapidly during this period reaching a peak in March 2000. The variability of the concentration measures, as evidenced by the differenced concentration series discussed in sections 8.3 and 8.5 of Chapter 8, was also at levels unprecedented in the data sample as a whole, from January 1998 through to the end of March 2003. Based on this analysis it is reasonable to infer that the data series during this period resulted from a different data-generating regime to that of the study period as a whole from 1984 through 2003. Given that market conditions in 2003 appear to be no less unstable than those preceding the end of the data sample, due to the continuing "war on terror" and global economic uncertainty, it seems reasonable to assume that the characteristics of the data observed from January 1998 through March 2003 are likely to continue into the foreseeable future. Hence the discussion and analysis focuses on the results of the models of realised volatility and the sub-components of realised volatility that are obtained for this period. These results are then compared, in general terms, with those obtained by estimating model parameters over the whole study period, as well as earlier sub-periods.

7.6 Conclusion

This chapter has outlined the models used in the analysis and justified the sample period and sub-periods based on sampling theory. A summary of the various diagnostic tests used to evaluate the performance of the models applied is also provided.

7.7 Appendix: definitions of the variables used

7.7.1 Variable abbreviations

The four concentration metrics used are the Hirschman-Herfindahl Index (H), the reciprocal of Hannah and Kay's Index (R), the variance of the logarithm of firm size (V) and the skewness of firm weights (SK). These are differenced over intervals of five, ten, fifteen and twenty trading days. Thus the differenced data for the respective concentration metrics and differencing intervals are abbreviated using the acronyms DH5, DH10, DH15, DH20, DR5, DR10, DR15, DR20, DV5, DV10, DV15, DV20, DSK5, DSK10, DSK15 and DSK20 respectively. The realised variance of the FTSE 100 Index is measured as the sum of squared returns over a given number of trading days, T. The realised variance is abbreviated to the acronym RV, estimated with T-values of five, ten, fifteen and twenty trading days respectively. Hence for the four different T values the respective acronyms for realised variance are RV5, RV10, RV15 and RV20. Likewise, the realised volatility, which in this study refers to the square root of the realised variance, is abbreviated for different T-values using the respective acronyms, RS5, RS10, RS15 and RS20. A complete list of variable definitions and abbreviations is provided in Table 4 overleaf.

Table 4 Variable definitions and abbreviations

Panel A: Realised volatility variable definitions and abbreviations	
Abbreviation	**Variable definition**
RV1	Squared daily returns
RV5	Sum of the squared daily returns over 5 trading-days
RV10	Sum of the squared daily returns over 10 trading-days
RV15	Sum of the squared daily returns over 15 trading-days
RV20	Sum of the squared daily returns over 20 trading-days
RS1	Absolute daily returns estimated over 1 day, i.e. square root of S1
RS5	Square root of RV5
RS10	Square root of RV10
RS15	Square root of RV15
RS20	Square root of RV20
Panel B: Concentration variable definitions and abbreviations	
Abbreviation	**Variable definition**
H20	Level of the daily Hirschman-Herfindahl concentration index, sampled every twenty trading days
R20	Level of the daily Reciprocal of Hannah and Kay's Concentration Index R with alpha set at 0.5, every twenty trading days
V20	Level of the daily variance of the logarithm of firm size V^2, sampled every twenty trading days
SK20	Level of the daily skewness of constituent firm weights, sampled every twenty trading days
DH5	Difference in the Hirschman-Herfindahl concentration index over 5 days
DH10	Difference in the Hirschman-Herfindahl concentration index over 10 days
DH15	Difference in the Hirschman-Herfindahl concentration index over 15 days
DH20	Difference in the Hirschman-Herfindahl concentration index over 20 days
DR5	Difference in the Reciprocal of Hannah and Kay's Concentration Index R with alpha set at 0.5, over 5 days
DR10	Difference in the Reciprocal of Hannah and Kay's Concentration Index R with alpha set at 0.5, over 10 days
DR15	Difference in the Reciprocal of Hannah and Kay's Concentration Index R with alpha set at 0.5, over 15 days
DR20	Difference in the Reciprocal of Hannah and Kay's Concentration Index R with alpha set at 0.5, over 20 days
DV5	Difference in the variance of the log of firm size V over 5 days
DV10	Difference in the variance of the log of firm size V over 10 days
DV15	Difference in the variance of the log of firm size V over 15 days
DV20	Difference in the variance of the log of firm size V over 20 days
DSK1	Difference in the skewness of constituent firm weights, over 1 day
DSK5	Difference in the skewness of constituent firm weights, over 5 days
DSK10	Difference in the skewness of constituent firm weights, over 10 days
DSK15	Difference in the skewness of constituent firm weights, over 15 days
DSK20	Difference in the skewness of constituent firm weights, over 20 days
Panel C: Sub-components of the variance covariance matrix, variable definitions and abbreviations	
Abbreviation	**Variable definition**
EAV5	Equally weighted average variance of the FTSE 100 Index constituent returns estimated using 5-trading-days of returns
EAV10	Equally weighted average variance of the FTSE 100 Index constituent returns estimated using 10-trading-days of returns
EAV15	Equally weighted average variance of the FTSE 100 Index constituent returns estimated using 15-trading-days of returns
EAV20	Equally weighted average variance of the FTSE 100 Index constituent returns estimated using 20-trading-days of returns
IAV5	Incremental average variance of the FTSE 100 Index constituent returns estimated using 5-trading-days of returns
IAV10	Incremental average variance of the FTSE 100 Index constituent returns estimated using 10-trading-days of returns
IAV15	Incremental average variance of the FTSE 100 Index constituent returns estimated using 15-trading-days of returns
IAV20	Incremental average variance of the FTSE 100 Index constituent returns estimated using 20-trading-days of returns
EAC5	Equally weighted average covariance of the FTSE 100 Index constituent returns estimated using 5-trading-days of returns
EAC10	Equally weighted average covariance of the FTSE 100 Index constituent returns estimated using 10-trading-days of returns
EAC15	Equally weighted average covariance of the FTSE 100 Index constituent returns estimated using 15-trading-days of returns
EAC20	Equally weighted average covariance of the FTSE 100 Index constituent returns estimated using 20-trading-days of returns
IAC5	Incremental average covariance of the FTSE 100 Index constituent returns estimated using 5-trading-days of returns
IAC10	Incremental average covariance of the FTSE 100 Index constituent returns estimated using 10-trading-days of returns
IAC15	Incremental average covariance of the FTSE 100 Index constituent returns estimated using 15-trading-days of returns
IAC20	Incremental average covariance of the FTSE 100 Index constituent returns estimated using 20-trading-days of returns

Table 4 - continued

Panel D: Standardised sub-components of the variance covariance matrix, variable definitions and abbreviations	
Abbreviation	**Variable definition**
TVCM5	Sum of the absolute values of EAV5, IAV5, EAC5 and IAC5
TVCM10	Sum of the absolute values of EAV10, IAV10, EAC10 and IAC10
TVCM15	Sum of the absolute values of EAV15, IAV15, EAC15 and IAC15
TVCM20	Sum of the absolute values of EAV20, IAV20, EAC20 and IAC20
SEAV5	EAV5 divided by TVCM5
SEAV10	EAV10 divided by TVCM10
SEAV15	EAV15 divided by TVCM15
SEAV20	EAV20 divided by TVCM20
SIAV5	IAV5 divided by TVCM5
SIAV10	IAV10 divided by TVCM10
SIAV15	IAV15 divided by TVCM15
SIAV20	IAV20 divided by TVCM20
SEAC5	EAC5 divided by TVCM5
SEAC10	EAC10 divided by TVCM10
SEAC15	EAC15 divided by TVCM15
SEAC20	EAC20 divided by TVCM20
SIAC5	IAC5 divided by TVCM5
SIAC10	IAC10 divided by TVCM10
SIAC15	IAC15 divided by TVCM15
SIAC20	IAC20 divided by TVCM20
SIRV5	IAV5 plus IAC5 then divided by TVCM5, i.e. (IAV5+IAC5)/TVCM5
SIRV10	IAV10 plus IAC10 then divided by TVCM10, i.e. (IAV10+IAC10)/TVCM10
SIRV15	IAV15 plus IAC15 then divided by TVCM15, i.e. (IAV15+IAC15)/G150
SIRV20	IAV20 plus IAC20 then divided by TVCM20, i.e. (IAV20+IAC20)/TVCM20

7.7.2 Dummy variable definitions

The following table details the variable names used to abbreviate the asymmetric and 1987 crash dummy variables used in the E-views models described earlier in this chapter.

Table 5 Asymmetric dummy variable and 1987 crash dummy variable: definitions and abbreviations

Abbreviation	Variable definition
DD5	Dummy variable equal to unity if average 5 day return is negative, equal to zero otherwise
DD10	Dummy variable equal to unity if average 10 day return is negative, equal to zero otherwise
DD15	Dummy variable equal to unity if average 15 day return is negative, equal to zero otherwise
DD20	Dummy variable equal to unity if average 20 day return is negative, equal to zero otherwise
DD587	Dummy variable equal to unity on two RS5 values spanning the October 1987 crash, equal to zero all other days
DD1087	Dummy variable equal to unity on the RS10 value spanning the October 1987 crash, equal to zero all other days
DD1587	Dummy variable equal to unity on the RS15 value spanning the October 1987 crash, equal to zero all other days
DD2087	Dummy variable equal to unity on the RS20 value spanning the October 1987 crash, equal to zero all other days
DRS5	RS5 multiplied by DD5
DRS10	RS10 multiplied by DD10
DRS15	RS15 multiplied by DD15
DRS20	RS20 multiplied by DD20
DEAV5	EAV5 multiplied by DD5
DEAV10	EAV10 multiplied by DD10
DEAV15	EAV15 multiplied by DD15
DEAV20	EAV20 multiplied by DD20
DIAV5	IAV5 multiplied by DD5
DIAV10	IAV10 multiplied by DD10
DIAV15	IAV15 multiplied by DD15
DIAV20	IAV20 multiplied by DD20
DEAC5	EAC5 multiplied by DD5
DEAC10	EAC10 multiplied by DD10
DEAC15	EAC15 multiplied by DD15
DEAC20	EAC20 multiplied by DD20
DIAC5	IAC5 multiplied by DD5
DIAC10	IAC10 multiplied by DD10
DIAC15	IAC15 multiplied by DD15
DIAC20	IAC20 multiplied by DD20
DIRV5	IRV5 multiplied by DD5
DIRV10	IRV10 multiplied by DD10
DIRV15	IRV15 multiplied by DD15
DIRV20	IRV20 multiplied by DD20

Chapter 8 – Results I: Characteristics of the data

8.1 Introduction

In the preceding three chapters the complexity of the interaction between concentration and volatility in the FTSE 100 index is highlighted. Chapter 6 describes a process for decomposing the FTSE 100 Index variance into equally weighted and incremental average variance and covariance components of the VCM. The methodology detailed in the preceding chapters enables the generation of a large number of data series that are discussed in the current chapter. Acronyms and definitions of these data-series are provided in Table 4 on page 136 and in Table 5 on page 139. Although these acronyms are initially defined in more detail in Chapters 5 and 6, the reader will find this table a useful reference point.

Section 8.2 discusses the evolution of the FTSE 100 Index data sample in the context of various macro-economic and political events that took place over the study period. This is followed in section 8.3 by a discussion of the level and differenced concentration data series in terms of their descriptive statistics and time series properties, some of which are evident from the charts. The time series properties and descriptive statistics of the realised volatility data, the sub-components of the realised volatility and the standardised sub-components of the realised volatility, are then discussed in section 8.4. The time series properties and descriptive statistics observed for all the data series over the whole period are then compared with those observed over the sub-period from January 1998 through March 2003 in section 8.5. Section 8.6 comprises the chapter summary and conclusion.

8.2 Evolution of the data sample

The time series charts presented in this chapter illustrate the evolution of the data series, over the period from January 1984 through March 2003. A number of changes and events regarding the levels of the concentration, the differenced concentration, the realised volatility and the sub-components of the realised volatility of the FTSE 100 Index are reflected in the charts. These can be explained by the various financial and economic conditions prevailing over the respective time periods. In addition, some major events that may have constituted a regime shift in the structure and behaviour of financial markets in the UK are discussed below.

Chart 1 Monthly FTSE 100 Index level: January 1984 through January 2004

FTSE 100 Index excluding dividends, during and since the study period. Vertical lines indicate divisions between the sub-periods within the data sample.

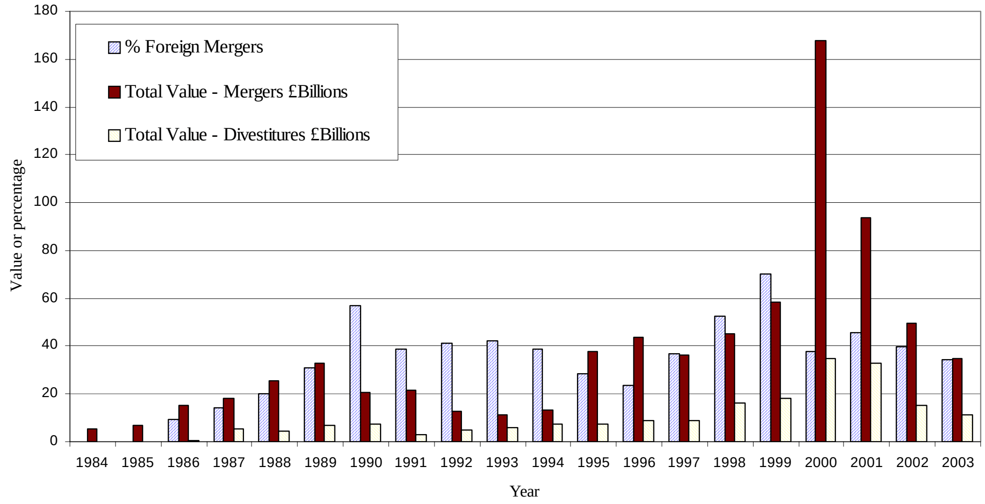
Chart 1 plots the value of the FTSE 100 Index over the entire study period, and beyond, to the beginning of 2004. The peak in the mid 1980s, and subsequent "Black Monday Crash" on the 19th of October 1987, is clearly visible, as is the recovery, punctuated by a few corrections, in the early 1990s. The corrections included volatility associated with the first Gulf War and "Black Wednesday" on the 16th of September 1992.[84] After Black Wednesday, there was a further mini bear market during 1994 and the first half of 1995. The UK stock market then embarked on a bull run that accelerated during the bubble from 1997 until its peak in March 2000. This bubble subsequently deflated, with a fall of more than 40% between March 2000 and the end of the study period in March 2003. Throughout the 1980s, both before and after the 1987 crash, the number and value of mergers between UK firms increased to a peak in 1989, as indicated by Chart 2.[85] However, Chart 2 also reveals that the recession of the early 1990s was characterised by a fall in the number of mergers and an increase in the number of divestitures, resulting in a trend towards more 'focussed' firms. This period corresponds with a decline in the levels of both H20 and R20 concentration observed in Chart 3 from the middle of 1985 through to the mid 1990s. In fact, this period was relatively stable compared to the post 1996 period.

---

[84] This event is also referred to as the "ERM crisis" as it is associated with Britain's exit from the European Exchange Rate Mechanism.

[85] ONS data file at: http://www.statistics.gov.uk/StatBase/tsdataset.asp?vlnk=993&More=N&All=Y

Chart 2 Value of mergers and divestitures: UK Office of National Statistics

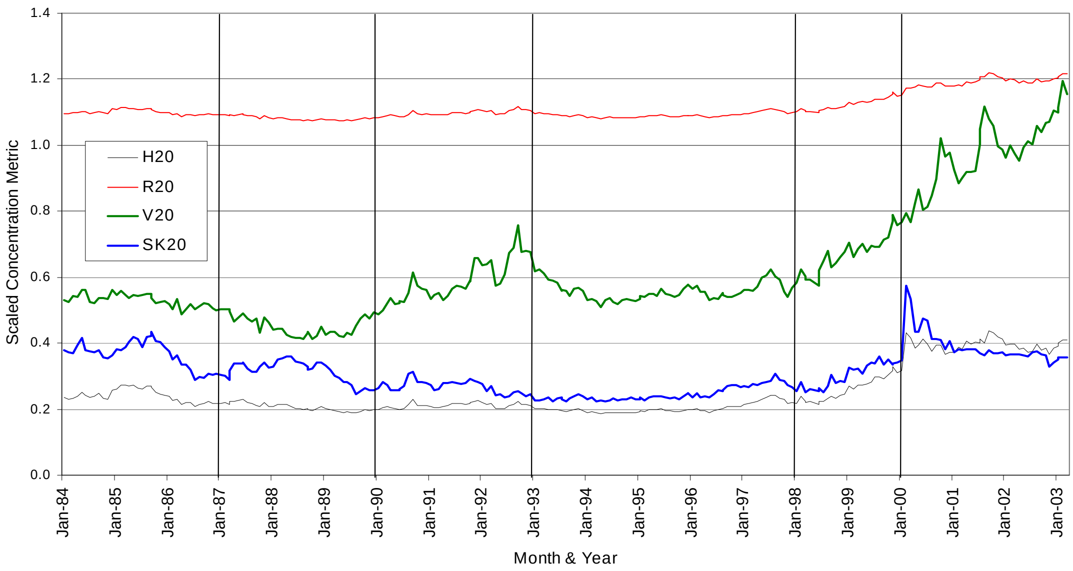


Office for National Statistics data file referenced in footnote 85. Total value includes mergers between UK and foreign and UK and UK firms. The % foreign refers to the proportion of UK firms acquired by, or merging with, foreign firms.

From 1995 the total value of mergers in the UK by domestic and foreign firms increased to a peak in 2000, while the percentage accounted for by foreign firms increased until 1999, as plotted by Chart 2. The value of divestitures also increased over this period, but by a lesser amount. Three of the four concentration metrics plotted in Chart 3 reached a peak in 2000, the same year that the value of mergers peaked, as plotted in Chart 2. However, V20 continued to increase until the end of the data sample period, in March 2003. The other three concentration metrics declined following the peak in merger activity.

From January 1998 onwards the levels of concentration increased rapidly and the variability of the concentration indices also increased. Increases in the levels are evident from Chart 3 while increases in the variability are evident from both Chart 5 and Chart 6, which plot the differenced concentration data.

Chart 3 Level concentration metrics (scaled): January 1984 – March 2003



The four level concentration metrics, sampled at twenty trading day intervals, are plotted over the entire study period. In order to fit the four series onto the same chart, H20 and R20 have been scaled up by a factor of ten, while SK20 has been scaled down by a factor of ten. V20 is unchanged.

Many of the mergers in the mid 1980s involved the formation of conglomerates, whereas the mergers of the 1990s tended to result in the creation of multinational firms, often leading specific industry sectors, such as pharmaceuticals, telecommunications, software development and oil production. The increased importance of international mergers is evident from the greater percentage of the total value of mergers accounted for by foreign firms through much of the 1990s, apart from the dip in 1995 and 1996, which is followed by a rise to the peak in 1999 as illustrated by Chart 2. Notable examples include, the merger of BP with Amoco Oil, Vodafone with Mannessmann, the German telecommunications conglomerate, and Glaxo-Welcome with SmithKline Beecham to form Glaxo-Smithkline. In addition, London's prominence as an international financial market attracted some large foreign multinational firms, such as HSBC, BHP Billiton and South African Breweries (SAB). These are listed on the London stock exchange (LSE) and are constituents of the FTSE 100 Index. These developments are the result of the increasing globalisation of world industries that is facilitated by reductions in foreign exchange controls and restrictions on cross border investment. The result is that many national stock markets are host to one or two global industry leaders, such as Nokia in Finland and Royal Dutch Shell in Amsterdam, as well as a much larger number of smaller domestic firms. Because the LSE is regarded as a major international exchange, it is also host to the global industry leaders mentioned above. In addition, a few firms, formerly identifiable as domestic firms, have become global industry leaders, such as BP Amoco, Glaxo-Smithkline and Vodafone. The latter have expanded overseas to such an extent that the majority of their revenues are generated outside of their home market. Thus in the UK market, the majority of firms may be relatively small, domestically-orientated firms, while the majority of the market value is accounted for by the relatively few large global industry leaders.

As concentration increases to the point where just a few firms account for a large percentage of the value of the entire market, changes in the relative size of the few largest firms can have a greater impact upon the overall measure of concentration. This is particularly true if a measure of concentration that is biased towards the largest firms in the index is used, such as the Hirschman-Herfindahl Index. This could help to explain why, as the levels of concentration increased, the volatility of concentration also increased.

The evolution of realised volatility of the FTSE 100 Index returns and the sub-components are discussed in section 8.4, where comparison is made between the most recent sub-period of data and the whole data set. Analysis of the time series data provides justification for estimating model parameters over both the whole data sample, from January 1984 through March 2003, and sub-periods within this. Because the period from January 1998 through March 2003 is the most recent, and arguably the most reflective of current market conditions, results from this period are analysed in more detail in Chapter 9 and Chapter 10 than the results from earlier sub-periods, or the period as a whole.

8.3 Concentration data

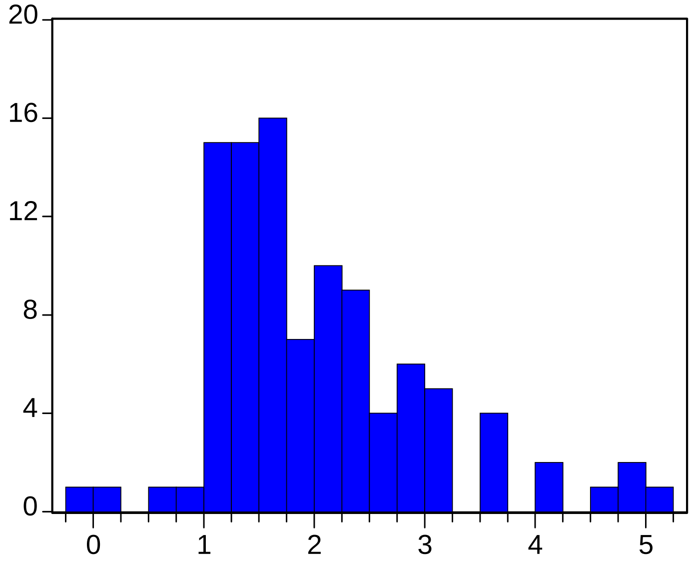
As outlined in Chapter 5, four different concentration indices are used to measure the daily distribution of constituent weights in the FTSE 100 Index. The Hirschman-Herfindahl index (H), the reciprocal of Hannah and Kay's Index (R), with an Alpha = 0.5, the variance of the logarithm of firm size (V^2) and the skewness of the firm weights (SK). The levels of the four different concentration indices recorded at intervals of twenty trading days (H20), R20, V20 and SK20 respectively, are plotted over the period from January 1984 to the end of March 2003 in Chart 3.

8.3.1 Distribution of firm size

The variance of the logarithm of firm size is arguably the most appropriate measure of concentration in the FTSE 100 index because, if the distribution of firm size is log normal it is not disproportionately influenced by either the smallest, or the largest, firms in the index.

The assumption that the distribution of firm size within the FTSE 100 index is approximately log normal provides justification for focussing upon the V^2 index as a key measure of concentration in the FTSE 100 Index portfolio. The Jarque-Bera (JB) statistic, for the natural log of FTSE 100 Index constituent firm size distribution is significantly different from zero at $\alpha < 10\%$ for the whole of the study period. Therefore, the assumption that firm size is log normally distributed in the FTSE 100 Index is not supported. However, as the distribution of the log of firm size illustrated in Chart 4 is reasonably symmetrical, although not strictly normal, V^2 is arguably a relatively unbiased estimate of firm size dispersion, when compared to the SK and H metrics that are influenced disproportionately by the largest firms.

Chart 4 Log of FTSE 100 constituent values (£ billions): 29th December 2000



Mean	2.040178
Median	1.759000
Maximum	5.066000
Minimum	-0.223000
Std. Dev.	0.974134
Skewness	0.978629
Kurtosis	4.063258
Jarque-Bera	20.87915
Probability	0.000029

Chart 5 Scaled DSK20 and DV20 data: January 1984 – March 2003

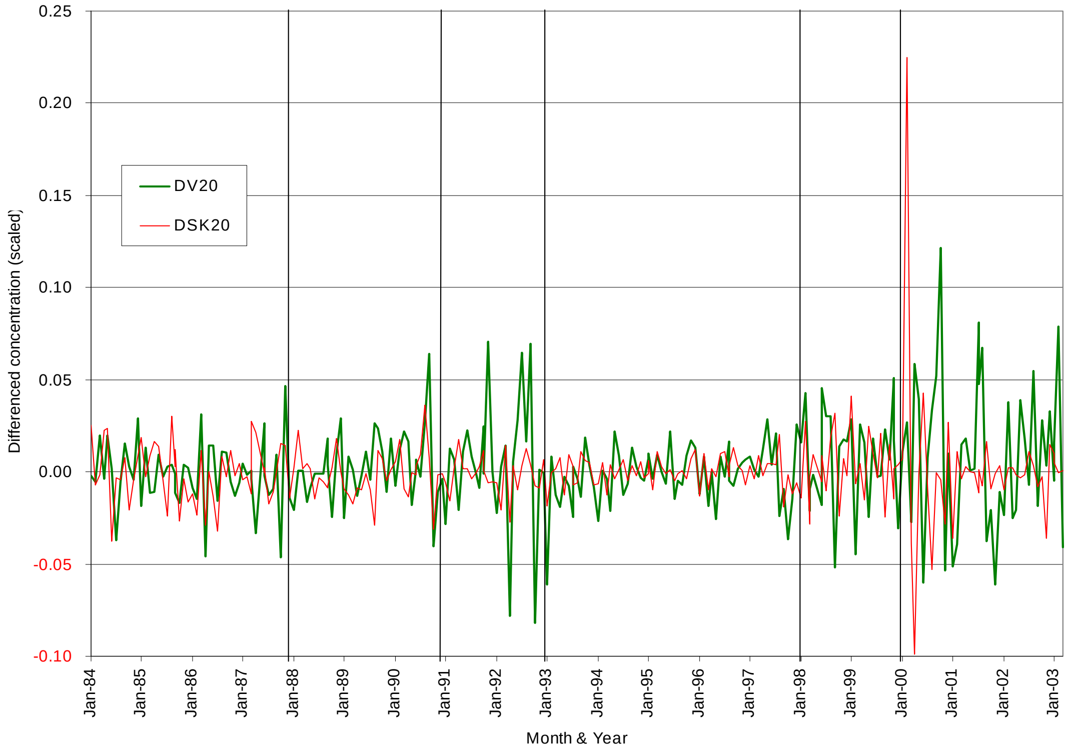


Chart 5 plots the time series of DV20 and DSK20 over the entire study period. In order to fit both series onto the same chart, DSK20 has been scaled down by a factor of ten. Vertical lines indicate the divisions between sub-periods of the data.

Chart 6 DR20 and DH20: January 1984 – March 2003

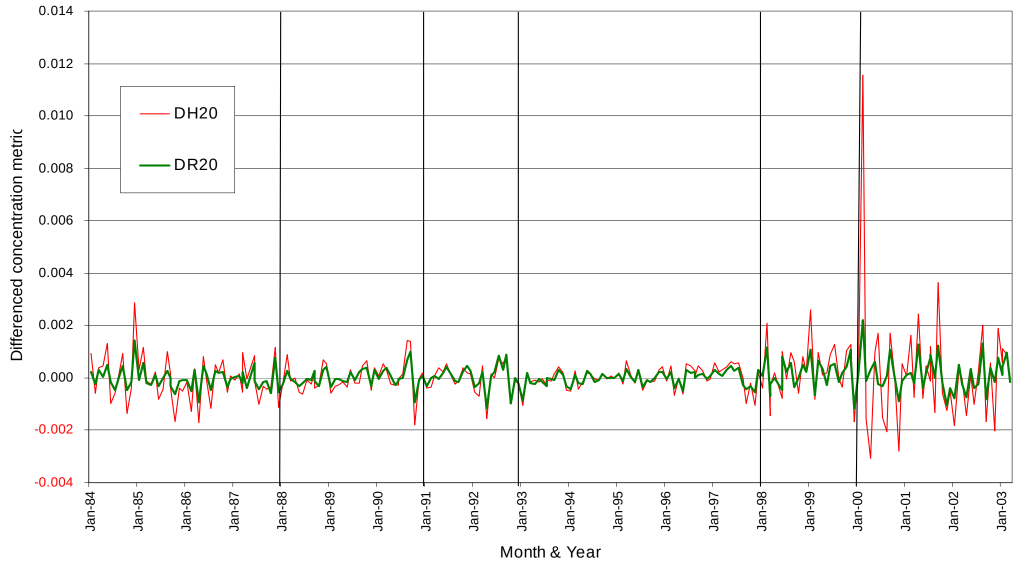


Chart 6 plots the DH20 and DSK20 over the entire study period. Vertical lines indicate the divisions between sub-periods of the data.

8.3.2 Time series characteristics of the concentration data

Chart 3 reveals that V20 exhibited an upward trend from 1996 through to the end of the data sample. SK20 jumped almost 80% as a result of the Vodafone-Mannessmann event but during the implosion of the technology bubble it subsequently declined to a little above the level before this event.[86] None of the level concentration series appear stationary over the period 1984 – 2003. H20 trends upwards from January 1994 until March 2000 when there is a structural break resulting from the Vodafone-Mannessmann event. It stabilises thereafter and resembles a mean reverting series, at the new higher mean level, for the remainder of the period. In addition, all-four concentration metrics appear to have undergone an earlier structural break during the latter part of 1997. From this point onwards, the upward trend becomes steeper and the level series become more variable. Many of the other data series, studied including realised volatility and the sub-components of realised volatility, also appear to undergo a structural break at this time, justifying the decision to focus the analysis on model results obtained for the sub-period beginning in January 1998. The non-stationary appearance of the four level concentration time series charts is supported by the results of the Augmented Dickey Fuller (ADF) and Phillips Perron (PP) tests for a unit root, reported in Table 6 and Table 7. The null hypothesis of a unit root cannot be rejected for any of the four level series but it can be rejected for the differenced series.

8.3.3 Descriptive statistics for the concentration data

The descriptive statistics for the concentration time series for the whole period are reported in Table 6, panel A. The significant JB statistics for each of the differenced series is a reflection of the excess kurtosis and the positive skewness exhibited by all differenced concentration metrics, except for the DV data, which is negatively skewed when differenced at intervals of less than twenty days. The mean is positive for all of the differenced series, except DSK5, although it is likely to be upwardly biased by positive outliers, such as the one resulting from the Vodafone-Mannessmann and other similar events. Such events account for the positive skew observed in most of the differenced series. The anomalous

[86] The take-over of Mannessmann by Vodafone resulted in Vodafone accounting for around 16% of the FTSE 100 Index MVE at this time.

DV data could be a result of the reduction in the influence of the Vodafone-Mannessmann event due to the absence of size bias in this measure.

8.4 FTSE 100 Index volatility and sub-components

8.4.1 Time series properties and descriptive statistics of the volatility estimates

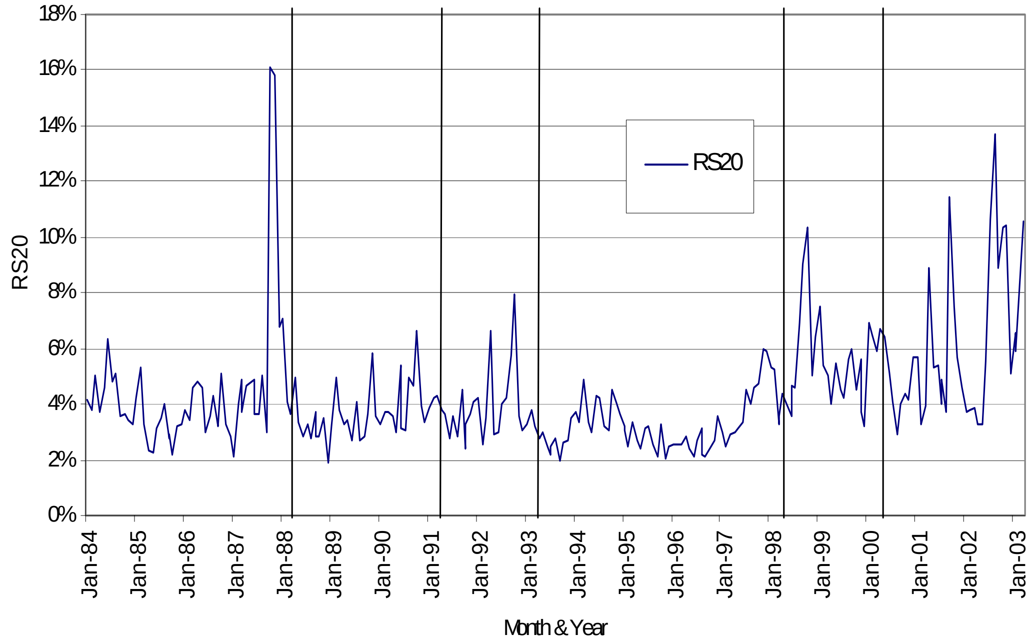
Descriptive statistics for each realised volatility variable RS5, RS10, RS15 and RS20 are reported in Table 6, panel B. These four data series represent time series for the realised standard deviation of FTSE 100 Index returns estimated with five, ten, fifteen and twenty trading days respectively, as detailed in Table 4, panel A.

The mean and the median of the monthly-realised volatility (RS20) series, over the whole study period are 4.31% and 3.72% respectively. These values can be annualised by squaring them multiplying by twelve and then taking the square root, giving 14.9% and 12.9% respectively. This is lower than the 22.7% standard deviation of nominal annual returns calculated by Dimson and Marsh (2001) for their “all UK equities” data series and 22.9% for their “high cap” equities data series, for the period from 1955 through 1999. The discrepancy could be due to the inclusion of the volatile 1972 – 1974 data in the Dimson and Marsh study. The difference could also be due to the fact that the standard deviation reported by Dimson and Marsh is defined as the standard deviation of annual returns, as distinct from an average annualised monthly standard deviation estimated using daily returns. However, this explanation is likely to explain a smaller proportion of the difference, given the established time-varying nature of stock market volatility, discussed in Chapter 3. In fact, the sensitivity of the mean realised volatility to the choice of sample period is illustrated by comparison of the mean monthly realised volatility recorded over the whole study period from 1984 through 2003 with that recorded from 1998 to 2003. The mean and median of RS20, recorded in panel B of Table 7, are 5.8% and 5.3% respectively. This translates into an annualised mean and median standard deviation of 20.1% and 18.35% respectively, levels that are much more compatible with those recorded by Dimson and Marsh (2001).

The realised volatility of FTSE 100 Index returns recorded in this study over the period January 1984 through March 2003, can also be compared with the standard deviation of monthly, fortnightly and weekly returns, recorded by Poon and Taylor (1992) for the UK FTSE Allshare Index over the period from January 1965 through December 1989. Descriptive statistics for daily returns are not reported in this thesis. However, reasonable comparison can be made between the realised volatility series, RS5, RS10, RS20 and the weekly, fortnightly and monthly series reported by Poon and Taylor. Data for the whole study period from 1984 through 2003 is reported in Table 6, panel B. Mean weekly volatility (RS5) of 2.1% is less than the 2.7% reported by Poon and Taylor, fortnightly volatility (RS10) of 3% compares to 4% recorded by Poon and Taylor, while monthly volatility (RS20) of 4.3% compares to 6.3% recorded by Poon and Taylor. When the realised volatility series observed in the sub-period, reported in panel B of Table 7, are compared with the results of Poon and Taylor, the values are more similar. Mean values of 2.8%, 4%, and 5.8% are observed for RS5, RS10 and RS20 respectively, compared to values of 2.7%, 3.9% and 6.3%. Thus the choice of sample period clearly has a substantial influence on the descriptive statistics of stock index returns. Again, it should be noted that these comparisons, although insightful are not, strictly, like-with-like as they relate to different stock indices, different time periods and different calculation methodologies. For example, Dimson and Marsh (2001) report the standard deviation of annual returns, while Poon and Taylor (1992) report standard deviations of monthly, fortnightly, weekly and daily returns. By contrast, this study reports the means and medians of successive realised standard deviations of daily returns estimated using twenty, fifteen, ten and five trading days, as distinct from calendar months, weeks etc. Furthermore, realised volatility series reported in this study are estimated using an assumed mean daily return of zero. The comparison provides a demonstration of the degree to which the realised volatility of UK stock market returns have varied through time and the degree to which different calculation methods and different return indices may influence the recorded results.

The mean and volatility of RS20, plotted in Chart 7, appears stable until the end of 1992, apart from the 1987 crash period. Both the mean and volatility of RS20 drop to a lower level over the period from January 1993 through January 1997. Then, from late 1997 onwards, both the mean and volatility of RS20 increase with some particularly large changes between January 2001 and March 2003, although none of these variations are as large as the 1987 crash event. This raises the possibility that different regimes exist in the FTSE 100 Index realised volatility time-series. These results are consistent with those identified from the time series charts of differenced concentration and the levels of concentration.

Chart 7 RS20: January 1984 – March 2003



Time series of non-overlapping estimates generated with T equal to twenty trading days. Vertical lines indicate the divisions between the sub-periods within the data sample.

8.4.2 Sub-components of FTSE 100 Index VCM

8.4.2.1 Descriptive statistics of the VCM sub-components

The null hypothesis of a normal distribution can be rejected, at the $\alpha < 1\%$ threshold, for all of the sub-components for which descriptive statistics are reported in panel B of Table 6. This is due to excess kurtosis and positive skewness present in all of the sub-components, except the incremental average covariance (IAC). The IAC data are negatively skewed for all T value estimation periods. In fact, the model results discussed in Chapter 10 demonstrate that the extreme negative values responsible for the negative skewness occurred in market crashes such as that of October 1987. The time series mean and medians of the equally weighted average variance (EAV), the equally weighted average covariance (EAC) and the incremental average variance (IAV) are positive for all T values. This suggests that, on average, the EAV, IAV, and EAC make a positive contribution to total realised volatility. Furthermore, the EAC accounts for the largest proportion of the realised variance, on average, and it has the highest maximum value. This is consistent with the principles of MPT to the effect that in a large diversified portfolio the VCM is dominated by the off-diagonal covariance terms.

The mean value of the IAC is negative, indicating that the actual market value weighted FTSE 100 Index portfolio has a lower incremental average covariance than an equally weighted proxy, on average. The median value for the IAC5 series is also negative, although the medians are positive in the IAC10, IAC15 and IAC20 series. Such an inconsistency of sign and persistent difference between the means and the medians suggest that the mean may be a biased measure of location that is heavily influenced by outliers. In fact, examination of the time series charts in the next section reveals large negative outliers associated with the 1987 and 1992 stock market events. Therefore, the median provides a better indication of the sign and magnitude of the IAC data for the majority of the time.

8.4.2.2 Time series charts of the VCM sub-components[87]

The time series of monthly realised-variance (RV20), the equally weighted average variance (EAV20) and covariance (EAC20) are plotted in Chart 8. From the chart, it is clear that EAC20 and RV20 are highly correlated, and that the majority of RV20 is accounted for by EAC20. Although EAV20 appears to follow the other two data series, its contribution to RV20 appears negligible. This is consistent with the MPT principle that the majority of the variation in a diversified portfolio is accounted for by the off-diagonal covariance terms in the VCM, a principle that is supported by the majority of the results in this study. However, it is also evident that there are times when the EAC is actually greater than RV20, as well as times when it is less, a difference that will be accounted for shortly. Noticeable peaks in EAV20, EAC20 and RV20 coincide with the 1987 crash, the ERM crisis and the technology bubble. The technology bubble appears to be associated with a regime shift in the data, beginning in late 1997 and continuing to the end of the data sample.

The RV20 data is also plotted with the two incremental VCM sub-components, IAV20 and IAC20, in Chart 9. Like EAV20, the contribution of IAV20 to RV20 appears relatively minor, although it increased during the peak of the technology bubble when concentration was at its highest in 2000. Although not as great as EAC20, the contribution of IAC20 is more noticeable than either EAV20 or IAV20. However, of greater interest is the fact that IAC20 appears to mirror the other three sub-components, and in particular the realised variance RV20. In fact, the largest negative values of IAC20 correspond with the highest positive values of the other three time series, including the 1987 crash and the ERM crisis. The only notable exception is during the recent bear market in 2002.

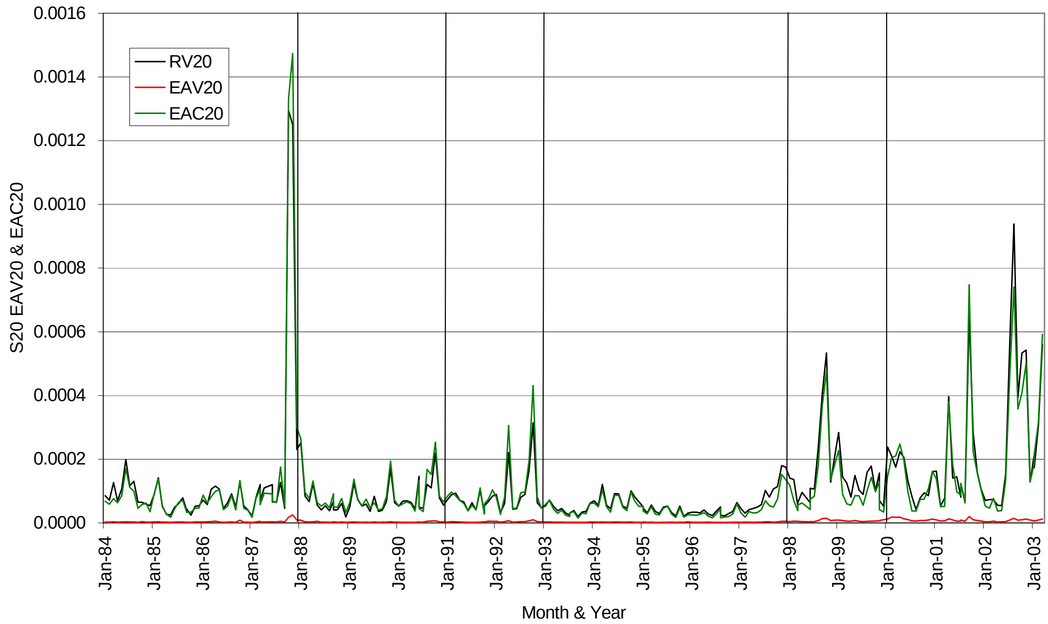
[87] The null hypothesis of a unit root can be rejected for all the sub-components of realised volatility when tested using both the ADF test and the PP test.

The net effect of the two incremental sub-components combined is illustrated by Chart 10, which plots the time series of the incremental realised volatility (IRV20). It is clear that the diversification benefits of the, often negative, IAC generally outweigh the disadvantage of the more frequently positive IAV because the combined IRV20 series follows an almost identical path to that of the IAC series. One easy way to characterise the effect of IRV20 on RV20 is to note that whenever IRV20 is negative, it has had the effect of reducing RV20 by that amount, or if it had not been negative RV20 would have been that much greater.[88]

Thus Chart 8, Chart 9 and Chart 10, provide further illustration of the historic benefits of a value-weighted portfolio versus an equally weighted portfolio in times of extreme market instability. In fact, during two of the largest volatility events in the data-series, concentration in the FTSE 100 Index had the effect of reducing volatility, although in the more recent period it has led to both increases and decreases in volatility.

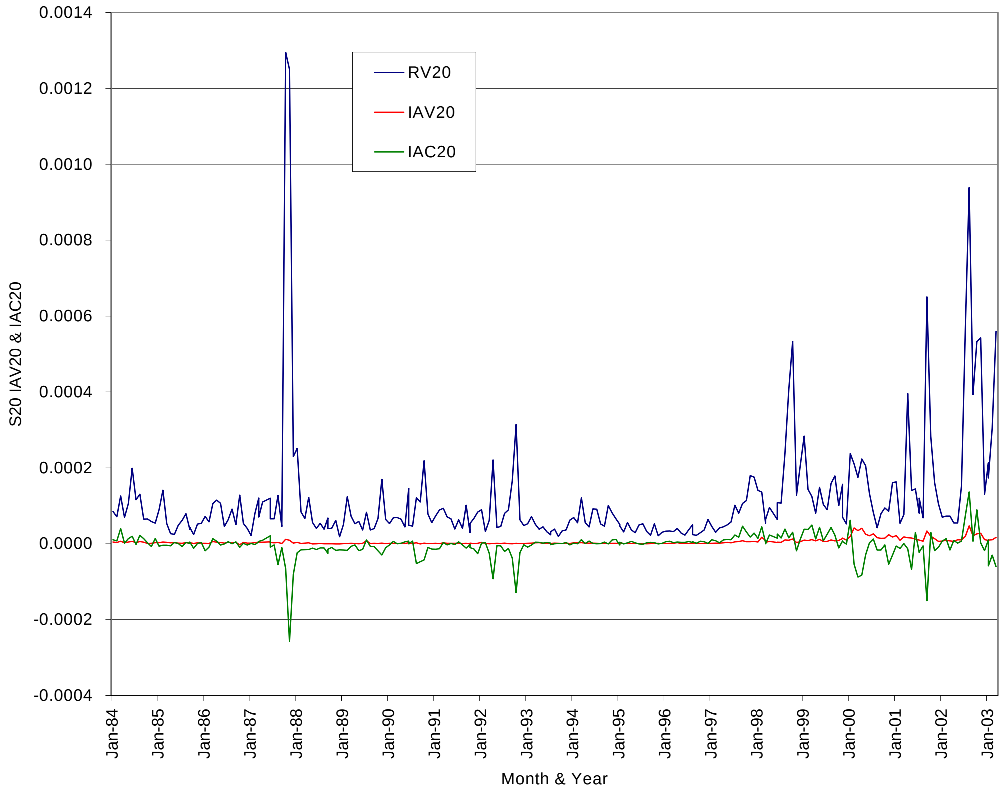
[88] The converse is not true for positive values of IRV20, because the RV20 plotted on the chart is the RV20 that has incorporated both positive and negative values of IRV20.

Chart 8 RV20 EAV20 and EAC20: January 1984 – March 2003



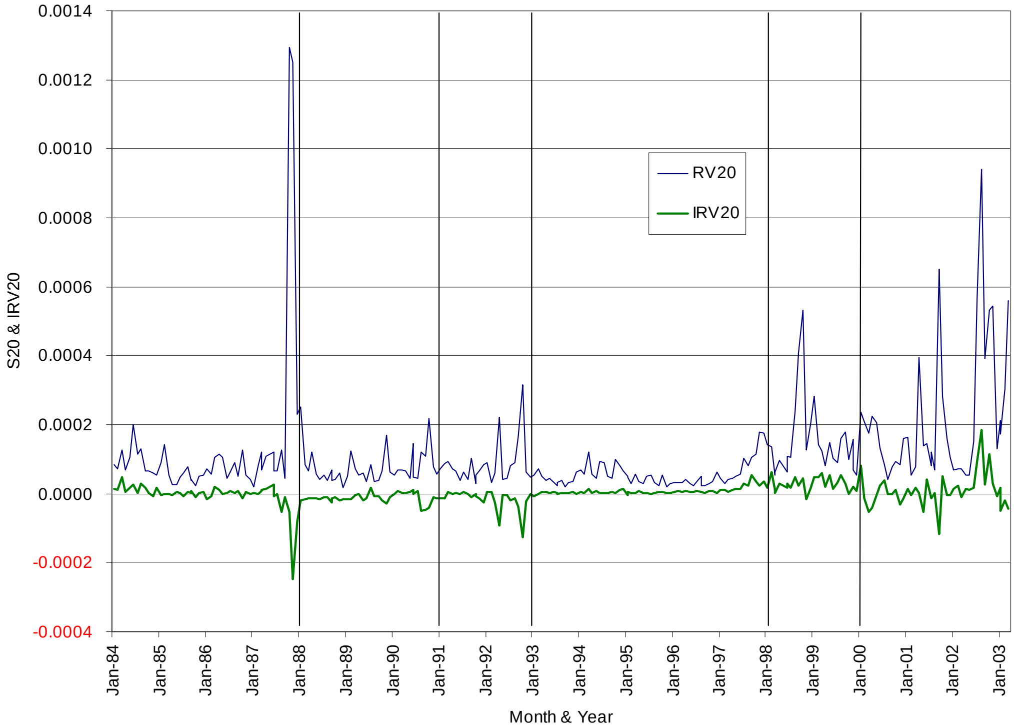
Time series of non-overlapping estimates generated with T equal to twenty trading days. Vertical lines indicate the divisions between the sub-periods within the data sample.

Chart 9 RV20 IAV20 and IAC20: January 1984 – March 2003



Time series of non-overlapping estimates generated with T equal to twenty trading days. Vertical lines indicate the divisions between the sub-periods within the data sample.

Chart 10 RV20 and IRV20: January 1984 – March 2003



Time series of non-overlapping estimates generated with T equal to twenty trading days. Vertical lines indicate the divisions between the sub-periods within the data sample.

8.4.2.3 Comparison with other studies

No other published studies have calculated incremental VCM sub-components in the manner adopted in this thesis.[89] However, Campbell et al (2001) report that the average variance increased and the average correlation decreased, with respect to individual stock returns in the US market, over the period of their study from 1962 through 1997. In a more recent study of the US market, Wei and Zhang (2003) made comparisons of the value weighted average variance and the equally weighted average variance of individual US stock returns in the US market, from 1976 through 2000. The patterns observed by Wei and Zhang (2003) are not dissimilar to those reported in this thesis for the FTSE 100 Index, particularly in respect of the large increases in both the equally and value weighted average variance of stock returns, post January 1998. Wei and Zhang (2003) find that the equally weighted average variance is generally higher than the value weighted average variance, another finding broadly consistent with these results. They suggest that the difference can be explained by the greater relative influence of smaller firms in the equally weighted

[89] Wei and Zhang (2003) calculate an "incremental variance of returns (IVR)" but this is different to the incremental realised volatility metric reported in this study.

average compared to the value-weighted average.[90] They attribute the increase in volatility, post 1997, to decreases in the average return on equity and increases in the volatility of the average return on equity. They also observe that firms entered the equity market earlier in their life cycle and had lower earnings quality during the inflation of the technology bubble than previously. They suggest that this is another factor that might explain increases in average firm volatility.

Kearney and Poti (2003) analyse market risk, idiosyncratic risk and average correlation patterns in the Dow Jones Eurostoxx50 Index over the period 1993 through 2001. They plot the time series of equally weighted and value weighted firm variance, market variance and total variance. Despite using different calculation methods and a different sample of firms to this study, their results are again similar to those of Campbell et al (2001), Wei and Zhang (2003) and to those reported in this thesis, in that both the equally weighted and value weighted average variance of firm returns increases dramatically post 1997. Kearney and Poti also estimate a measure of average correlation for their data sample. However, similarities between their time series of average correlation and the EAC, or IAC data reported in this study, are not readily apparent.

Goyal and Santa-Clara (2003) compare equally weighted and value weighted monthly standard deviations for their portfolio of US stock returns compiled from CRSP data over the period from 1962 through 1999. Unlike this study, they find that the value-weighted volatility is generally greater than the equally weighted volatility, even during the 1987 crash. Therefore their results for the US market are inconsistent with the results reported here for the FTSE 100 Index, in this respect. Nonetheless, in a manner consistent with this and all the other studies discussed here, they report an increase in overall portfolio volatility post 1997.

8.4.3 Standardised sub-components of the FTSE 100 Index VCM

Summary statistics for each of the 24 standardised VCM sub-components are presented in panel C of Table 6. The non-standardised or raw subcomponents of the VCM, discussed in the previous section, are useful for models that try to identify the relationship between changes in concentration and realised volatility by allowing specific subcomponents of the

[90] Smaller firms are generally regarded as more risky than larger firms hence the greater volatility in an index where smaller firms are given a greater relative weight.

VCM to be targeted in the modelling process. However, one needs to be cautious when interpreting the values of the raw subcomponents due to the fact that they are part of a matrix of security returns containing both squared variance elements and covariance elements. The standardisation procedure, outlined in section 6.3, resolves this interpretation problem and allows the relative importance of the individual subcomponents in the absolute total VCM, defined as the TVCM, to be evaluated.

Absolute values of a standardised sub-component that are close to zero indicate that the contribution to the realised volatility is relatively small over that estimation period. Large positive values indicate that the component is a major contributor to realised volatility, while large negative values indicate that it has an important role in limiting realised volatility. In other words, when large negative values of the standardised incremental average covariance (SIAC), or variance (SIAV) occur, portfolio diversification strategies that involve concentration of capital in fewer firms, have an important role in limiting the realised volatility of the FTSE 100 index.[91]

8.4.3.1 Descriptive statistics of the standardised sub-components

The JB test results for the standardised VCM sub-components reported in panel C of Table 6, indicate that the null hypothesis of a normal distribution can be rejected at the $\alpha < 1\%$ significance level for most of the series. However, it cannot be rejected at the $\alpha < 5\%$ threshold, for the SIAC15 or the SIAC20 series, suggesting that these series may in fact have a normal distribution. The time series mean and median values, for each standardised sub-component, are revealing for what they say about the relative importance of the VCM sub-components on average. According to MPT, the majority of the variance of a large diversified portfolio such as the FTSE 100 Index will be accounted for by the off-diagonal covariance elements in the VCM. The data in Table 6 supports this assertion as the mean and median value of the SEAC series are approximately 80% of the TVCM, for estimation periods of ten days and above. This is followed by the EAV and IAV series which each account for around 4% – 5% of the TVCM. Hence, the equally weighted average variance of constituent securities, and the incremental average variance, each account for approximately the same proportion of total realised volatility, on average. By contrast, the incremental average covariance accounts for a much lower proportion of the TVCM, on

[91] Large absolute values of the SIAC or SIAV, or, the SIRV that persist over time are inconsistent with the assumptions of the Overall Mean Model of Elton and Gruber (1973).

average. The mean value for each of the SIAC-series is less than 1%, on average, and the median is actually negative for the SIAC10 and SIAC5 series, suggesting that the incremental average covariance is negative for a substantial part of the time.

Given the very low mean values of the SIAC data, it might be tempting to conclude that the incremental average covariance is not useful in explaining realised FTSE 100 Index volatility. However, the large variance, the extreme negative and the extreme positive values present in the SIAC data, mean that such a conclusion would be unjustified. The maximum value for the SIAC data is nearly 71% of the TVCM and ranges between 43 and 50%, while the minimum values range between –45% and –30%. Hence, although the average contribution to total realised volatility of the incremental average covariance is close to zero it can, on occasion, account for a very large proportion of the TVCM. The time series charts discussed in the previous section, and the model results discussed in the next two chapters, indicate that these extreme contributions can occur during periods of extreme market volatility. Thus, although the impact of increased portfolio concentration on total portfolio volatility may be limited most of the time, it may mitigate or exacerbate market instability during periods of unusually high market volatility.

8.5 Comparison of the 1998 – 2003 sub-period with the whole period

The descriptive statistics and time series characteristics of all the data over the whole study period from January 1984 through March 2003 are discussed extensively in the previous section. Analysis of the time series charts in the previous section indicates that data sampled in the period post 1997 are from a different data-generating regime to samples from earlier sub-periods, and perhaps the period as a whole. Table 7 reports descriptive statistics from the shorter sub-period from January 1998 through March 2003 and follows the same format to that adopted in Table 6. This section compares the data in Table 7 with the data in Table 6.

High relative dispersion parameters, and in many cases non-standard distributions of the data, render formal testing of the difference between sub-periods difficult to implement and interpret. However, ad-hoc comparison of the descriptive statistics for the post 1997 data with data for the whole period is consistent with the data presented in the charts for all data types. For example, the level and the differenced concentration metrics are higher, both in means and in medians, in the later period. The means and medians of realised volatility are also generally higher in the sub-period than over the whole period, although skewness and kurtosis of the realised volatility sample is lower due to the absence of the 1987 crash data from the sub-period. The time series mean and median of the incremental average variance, IAV, are also higher in the sub-period, while skewness and kurtosis is lower. The time series average contributions of the incremental average variance of constituent returns to the TVCM is higher in the sub-period than it is in the whole period, as demonstrated by the higher mean and median of the SIAV data. The descriptive statistics, presented in Table 7, indicate that the time series average contribution of the IAV to the TVCM is between 7 and 9%, more than double the average over the whole period. The EAV data still make only a relatively minor contribution to realised volatility, although it is higher than in the pre-1998 periods, and the whole period. Therefore, higher levels of concentration have coincided with higher levels of incremental average variance of FTSE 100 Index constituent returns. The increase in the time series average SIAV data in the recent sub-period indicates that increasing concentration had the effect of increasing this sub-component of realised volatility, at this time, although this effect was cancelled out by the confounding influence of the predominantly negative incremental average covariance, IAC.

Like the other three sub-components, the time series means, medians and standard deviations of the IAC data were substantially higher in the 1998-2003-period than they were in the 1984 – 2003 period as a whole. This is due to the absence of some very large negative values, coinciding with Black Monday in 1987 and Black Wednesday in 1992. The absolute time series skewness of the IAC data was lower in the more recent period and the kurtosis values were lower, reflecting the absence of the aforementioned negative outliers. Unlike that of the other three sub-components, the contribution of the equally weighted average covariance of constituent returns (EAC) to the TVCM, reflected in the SEAC data, is substantially lower in the sub-period. As with the other sub-components, the EAC was higher in the recent sub-period than in the period as a whole, but the difference between the sub-period and the whole period was less for the EAC data than it was for all the other subcomponents. Thus the EAC accounts for a much smaller proportion of the total VCM in the 1998 – 2003 period than in earlier periods. This is partially offset by a higher average contribution to the TVCM from the incremental average covariance (IAC) of FTSE 100 Index returns.

8.5.1.1 Contribution of incremental realised volatility to the TVCM (SIRV)

Chart 11 reports the time series distribution for the (SIRV1) data. This is the contribution of daily incremental realised volatility to the daily total variance covariance matrix

(TVCM), over the entire study period. This distribution can be compared with that for the same data over the 1998 – 2003 sub-period, plotted in Chart 12. Neither distribution is normal, but both are positively skewed and both cover a similar range of values. The location parameters have shifted to the right in the sub-period, compared to the period as a whole, indicating that the IRV has been greater during the sub-period, when concentration was at its highest, than during the period as a whole. Comparison of Chart 13 and Chart 14, plotting monthly data, produces similar results backed up by the descriptive statistics. Therefore, higher levels of concentration have, on average, coincided with higher levels of incremental realised volatility. Nonetheless, this finding is still confounded by the greater dispersion of the SIRV data in the sub-period compared to the period as a whole. As observed earlier, negative outliers have often been observed in the IRV data during extreme volatility events, so although concentration may, on average, be shown to increase realised volatility, it may have diversification benefits during periods associated with the tails of the return distribution.

Note that negative values of the IAC imply that, on average, small firms have a lower covariance than large firms. The reverse is also true for positive values of the IAC. The same principle also holds for the IAV data. Therefore, if it can be established that either of the incremental sub-components are systematically non-negative, the assumptions of the Overall Mean Model of Elton and Gruber (1973) can be contested.[92] Likewise, if any of the incremental sub-components can be forecasted, it would imply a means of improving on the Overall Mean Model. Chart 13 and Chart 14 indicate that the incremental realised volatility may be systematically positive, although the bias is small in relation to the variance. Furthermore, IRV20 appears to be higher in the recent sub-period implying that it may not be constant through time. However, the relatively small change in relation to the variance, and the presence of outliers, makes such an interpretation subjective and prone to sampling error.

8.5.1.2 Explanations for the difference between the sub-period and the whole period

The latter part of the 1990s coincided with the end of a long bull-market in equities. This was associated with the ongoing process of globalisation, the increasing dependence of corporations on information technology and the widespread adoption of the Internet. This,

---

[92] As mentioned in the literature review, the Overall Mean Model assumes that covariances and variances are constant through time and equal for the returns of all firms.

and preparations for the so-called “millennium bug”, caused the values of telecommunications, media and technology (TMT) firms, to inflate, into a bubble that burst in the spring of 2000. The inflation of this bubble was punctuated by the Asian financial crisis in October 1997 and Russia’s sovereign debt default, together with the associated collapse of the hedge fund Long Term Capital Management, in August and September of 1998 respectively. The deflation of the technology bubble from March 2000 onwards was accelerated by the terrorist attacks on the World Trade Centre on the 11th of September 2001, the subsequent war in Afghanistan and preparations for the second Gulf war, which began at the very end of the data sample in March 2003.[93] These are all events and processes that had a major impact on the magnitude, direction and hence volatility of market returns. Therefore it is suggested that they may explain the increase in the level and volatility of the data series for realised volatility and the sub-components of the realised volatility. Whether or not these events are part of an ongoing structural change or just reflect temporary instability, resulting in a classic example of volatility clustering, remains to be seen. However, the issues prevalent in this period are still pertinent at the time of writing. Therefore, it makes sense to focus the analysis on the sub-period that is most relevant to volatility research going forward for the foreseeable future, although interesting events in the data sample as a whole are still accounted for, albeit in less detail.

[93] Between the end of the data sample and the time of writing in December 2003, the FTSE 100 Index has rallied more than 30% resulting in more market volatility, this time generated by positive stock returns.

Chart 11 SIRV1 distribution statistics: January 1984 – March 2003

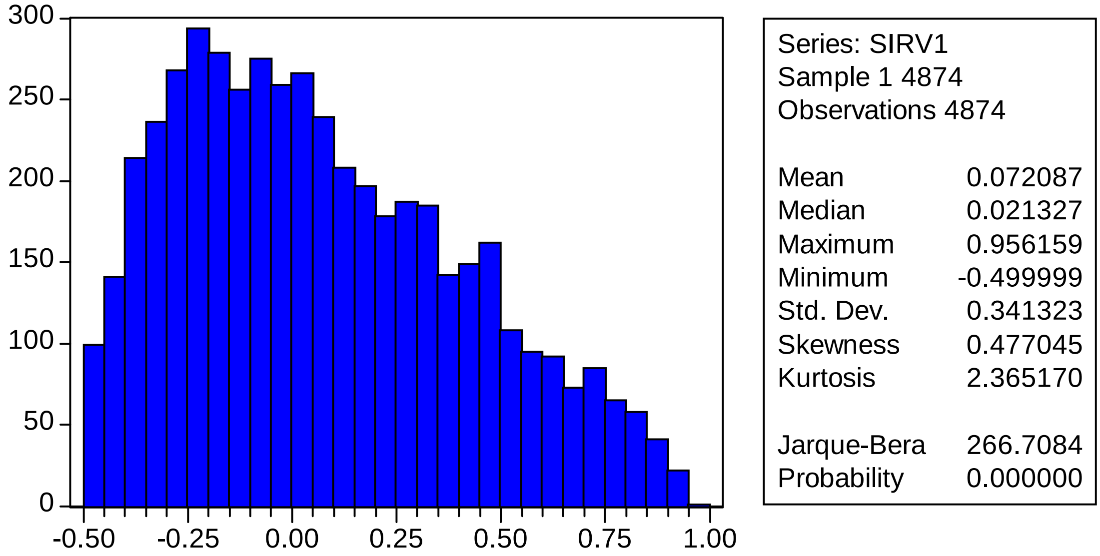


Chart 12 SIRV1 distribution statistics: January 1998 – March 2003

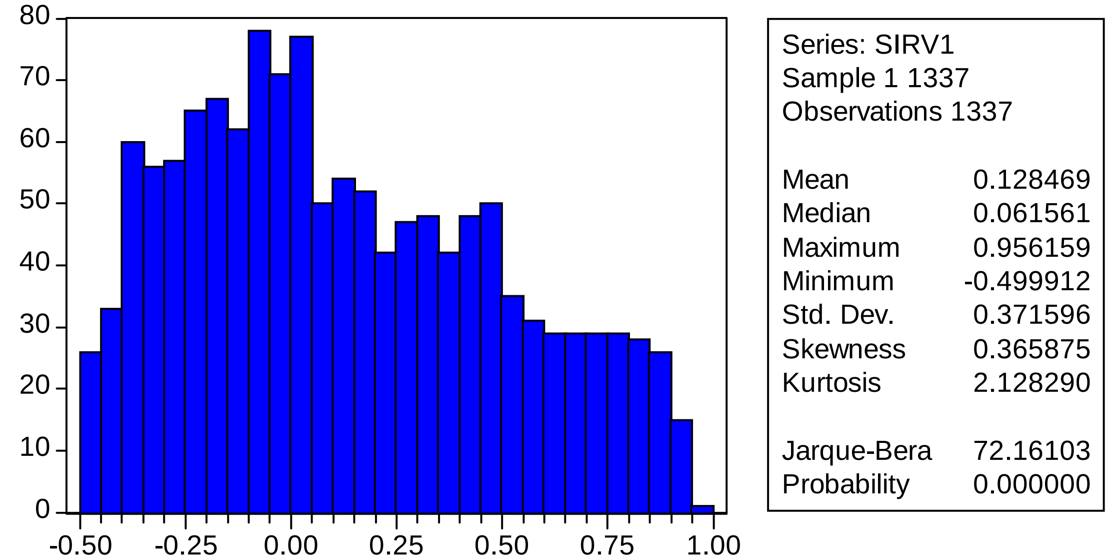
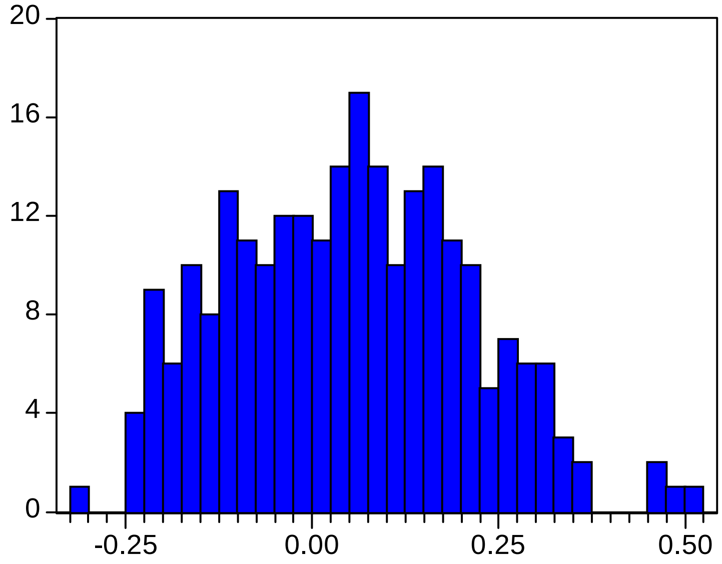
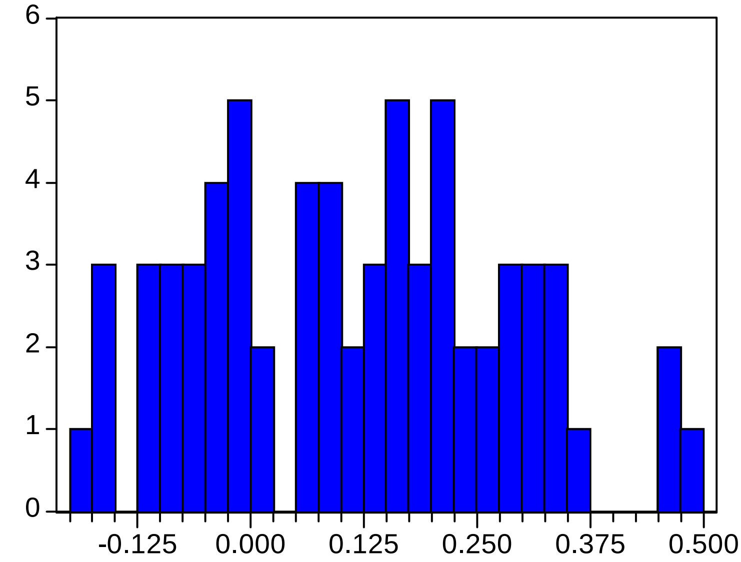


Chart 13 SIRV20 distribution statistics: January 1984 – March 2003



Series: SIRV20	
Sample 1 243	
Observations 243	
Mean	0.049813
Median	0.050279
Maximum	0.507792
Minimum	-0.318287
Std. Dev.	0.160130
Skewness	0.227802
Kurtosis	2.587376
Jarque-Bera	3.825562
Probability	0.147669

Chart 14 SIRV20 distribution statistics: January 1998 – March 2003



Series: SIRV20	
Sample 1 67	
Observations 67	
Mean	0.113611
Median	0.121616
Maximum	0.479255
Minimum	-0.191768
Std. Dev.	0.167713
Skewness	0.169687
Kurtosis	2.212129
Jarque-Bera	2.054427
Probability	0.358003

8.6 Summary

This chapter has analysed the evolution of the data sample from January 1984 through March 2003, relating anomalies, trends, and a suggested structural break in the data to key economic and political events. An increase in the number and value of domestic mergers is suggested as a cause for increases in concentration during the 1980s. An increase in the number of divestitures and a decrease in the number and value of mergers is suggested as a reason for the relatively stable levels of concentration in the early 1990s. The rapid increase in concentration during the late 1990s and early 2000s is attributed to an equally rapid increase in the number of both domestic and international mergers affecting the UK equity market. Changes in the time series properties of realised volatility and the sub-components of realised volatility in the latter part of the 1990s and early 2000s are related to the inflation of the technology bubble and commensurate increases in concentration.

Comparison is made between the sub-period from January 1998 through March 2003 and the period as a whole from January 1984 through March 2003. The analysis focuses upon the more recent sub-period as it is argued that it constitutes a separate regime within the data population as a whole. The key points to draw from the analysis are that both the level and variability of all four measures of concentration are generally higher in the 1998 – 2003 sub-period than they are in the period as a whole. This also applies to the realised volatility and the four sub-components of volatility.

The proportion of the VCM accounted for by three of the four sub-components is also higher, at the expense of the remaining equally weighted average covariance sub-component of constituent returns. Thus, while overall volatility has been higher over this period compared to the average volatility over the whole period, and all the sub-components of volatility have increased, the contribution to total volatility made by the equally weighted average covariance in the VCM has actually fallen. This is specifically due to the fall in the relative contribution, as distinct from the absolute values, of the equally weighted off diagonal elements in the VCM. By contrast the average contribution made by the incremental average covariance is greater in the 1998 – 2003 period. Furthermore, the incremental realised volatility and the average contribution of the incremental realised volatility to the TVCM, is higher in the 1998 – 2003 sub-period than in the period as a whole from 1984 – 2003.

On average, in the 1998 – 2003 period when concentration has been at its highest, the realised volatility of FTSE 100 Index portfolio volatility has also been at its highest and the net contribution to realised volatility made by the incremental realised volatility has been higher. This suggests that larger firms were, on average, more risky relative to small firms in this period and, hence, that increases in concentration were associated with increases in realised volatility, on average. However, the inference drawn from the time series averages is confounded by the behaviour of the incremental average covariance during the extreme tails of the distribution, particularly during the initial deflation of the year 2000 technology bubble, the 1987 crash and the Black Wednesday correction in 1992. During these tail events, the incremental average covariance was negative to the extent that the net incremental realised volatility was also negative. This implies that concentrated portfolios may offer diversification benefits to investors during tail events. The model results discussed in the next two chapters are consistent with the notion that the incremental average covariance falls following rises in concentration, and vice versa.

8.7 Appendix: tables of descriptive statistics

The following tables report the descriptive statistics discussed in the current chapter.

Table 6 Descriptive statistics for all variables: January 1984 through March 2003

Abbreviation	N	μ	Med	Max	Min	σ	Skew	Kurt	JB Stat	JB	ADF	PP
Panel A: Levels of concentration and differenced concentration												
H5	995	0.025	0.022	0.046	0.019	0.007	1.44	3.60	357	Yes	No	No
R5	995	0.111	0.110	0.123	0.107	0.004	1.47	3.80	386	Yes	No	No
V5	995	0.623	0.555	1.275	0.411	0.178	1.50	4.40	455	Yes	No	No
SK5	995	3.087	2.938	5.704	2.189	0.630	0.67	3.19	75	Yes	No	No
DH5	995	2.0E-05	-3.6E-06	6.9E-03	-3.3E-03	6.0E-04	3.74	42.29	66,330	Yes	Yes	Yes
DH10	486	3.9E-05	-1.7E-05	1.0E-02	-5.1E-03	9.1E-04	2.99	40.35	28,979	Yes	Yes	Yes
DH15	324	5.7E-05	1.7E-05	1.3E-02	-4.4E-03	1.1E-03	5.03	62.99	49,946	Yes	Yes	Yes
DH20	243	7.6E-05	9.5E-06	1.2E-02	-3.1E-03	1.1E-03	4.27	45.11	18,692	Yes	Yes	Yes
DR5	995	2.6E-03	1.0E-03	1.2E-01	-8.2E-02	2.6E-02	0.39	5.50	69	Yes	Yes	Yes
DR10	486	2.6E-05	1.7E-05	2.7E-03	-1.8E-03	3.8E-04	0.94	12.49	1,894	Yes	Yes	Yes
DR15	324	3.8E-05	1.8E-05	3.0E-03	-1.6E-03	4.5E-04	1.19	10.50	837	Yes	Yes	Yes
DR20	243	5.0E-05	1.9E-06	2.2E-03	-1.2E-03	4.6E-04	0.63	5.36	72	Yes	Yes	Yes
DSK5	995	2.6E-04	-7.9E-04	1.3E+00	-5.0E-01	9.4E-02	3.77	59.95	136,805	Yes	Yes	Yes
DSK10	486	1.0E-04	-6.1E-03	1.3E+00	-5.0E-01	1.3E-01	2.95	32.66	18,517	Yes	Yes	Yes
DSK15	324	1.0E-04	-1.6E-03	2.2E+00	-1.1E+00	1.8E-01	4.95	74.15	69,670	Yes	Yes	Yes
DSK20	243	1.4E-04	-4.9E-03	2.2E+00	-9.9E-01	2.1E-01	4.59	58.09	31,585	Yes	Yes	Yes
DV5	995	4.9E-04	8.0E-04	1.4E-01	-2.0E-01	1.6E-02	-1.49	37.77	50,482	Yes	Yes	Yes
DV10	486	1.3E-03	1.9E-03	1.2E-01	-2.0E-01	2.2E-02	-1.51	22.22	7,667	Yes	Yes	Yes
DV15	324	1.9E-03	2.4E-03	1.0E-01	-9.8E-02	2.5E-02	-0.10	6.32	149	Yes	Yes	Yes
DV20	243	2.6E-03	1.0E-03	1.2E-01	-8.2E-02	2.6E-02	0.39	5.50	69	Yes	Yes	Yes
Panel B: Realised volatility and the sub-components of realised volatility												
Abbreviation	**N**	**μ**	**Med**	**Max**	**Min**	**σ**	**Skew**	**Kurt**	**JB Stat**	**JB**	**ADF**	**PP**
RS5	995	2.1%	1.8%	17.7%	0.4%	1.2%	3.74	34.36	43,095	Yes	Yes	Yes
RS10	486	3.0%	2.6%	19.9%	1.1%	1.6%	3.81	30.81	16,839	Yes	Yes	Yes
RS15	324	3.7%	3.2%	15.9%	1.4%	1.9%	2.72	13.99	2,030	Yes	Yes	Yes
RS20	243	4.3%	3.7%	16.1%	1.9%	2.1%	2.64	12.49	1,195	Yes	Yes	Yes
EAV5	995	4.4E-06	2.9E-06	7.6E-05	1.0E-06	4.5E-06	5.87	73.05	209,166	Yes	Yes	Yes
EAV10	486	4.4E-06	3.0E-06	5.6E-05	1.1E-06	4.2E-06	4.91	47.75	42,515	Yes	Yes	Yes
EAV15	324	4.4E-06	3.0E-06	2.8E-05	1.2E-06	3.7E-06	2.48	10.76	1,145	Yes	Yes	Yes
EAV20	243	4.4E-06	3.0E-06	2.5E-05	1.3E-06	3.7E-06	2.46	10.34	791	Yes	Yes	Yes
IAV5	995	5.3E-06	2.1E-06	7.3E-05	-1.9E-05	9.1E-06	3.47	18.09	11,434	Yes	Yes	Yes
IAV10	486	5.3E-06	2.1E-06	6.4E-05	-8.0E-06	8.3E-06	3.19	16.06	4,279	Yes	Yes	Yes
IAV15	324	5.3E-06	2.1E-06	5.2E-05	-4.6E-06	7.8E-06	2.87	13.21	1,852	Yes	Yes	Yes
IAV20	243	5.3E-06	2.1E-06	4.7E-05	-2.6E-06	7.6E-06	2.71	11.62	1,049	Yes	No	Yes
EAC5	995	1.1E-04	5.7E-05	6.3E-03	1.3E-06	2.5E-04	16.31	376.19	5,817,971	Yes	Yes	Yes
EAC10	486	1.1E-04	6.0E-05	4.2E-03	1.2E-05	2.3E-04	12.90	222.31	987,467	Yes	Yes	Yes
EAC15	324	1.1E-04	6.4E-05	1.7E-03	1.2E-05	1.7E-04	6.06	51.41	33,618	Yes	Yes	Yes
EAC20	243	1.1E-04	6.3E-05	1.5E-03	1.5E-05	1.6E-04	5.39	39.80	14,884	Yes	Yes	Yes
IAC5	995	-3.5E-06	-2.6E-07	3.7E-04	-5.4E-04	4.7E-05	-2.85	40.36	59,204	Yes	Yes	Yes
IAC10	486	-3.6E-06	2.1E-07	2.2E-04	-3.7E-04	3.9E-05	-2.78	29.54	14,889	Yes	Yes	Yes
IAC15	324	-3.4E-06	9.8E-08	1.3E-04	-2.6E-04	3.3E-05	-2.41	20.28	4,347	Yes	Yes	Yes
IAC20	243	-3.4E-06	1.0E-06	1.4E-04	-2.6E-04	3.2E-05	-2.51	22.51	4,110	Yes	Yes	Yes
IRV5	995	1.8E-06	2.1E-06	4.2E-04	-5.3E-04	4.9E-05	-1.89	38.51	52869	Yes	Yes	Yes
IRV10	486	1.7E-06	2.5E-06	2.7E-04	-3.3E-04	4.0E-05	-1.64	26.71	11599	Yes	Yes	Yes
IRV15	324	1.9E-06	2.7E-06	1.8E-04	-2.5E-04	3.4E-05	-1.54	19.67	3881	Yes	Yes	Yes
IRV20	243	1.9E-06	2.6E-06	1.8E-04	-2.5E-04	3.3E-05	-1.40	21.59	3577	Yes	Yes	Yes

Table 6 continued

Panel C: Standardised sub-components of realised volatility												
Abbreviation	**N**	**μ**	**Med**	**Max**	**Min**	**σ**	**Skew**	**Kurt**	**JB Stat**	**JB**	**ADF**	**PP**
SEAV5	995	5.0%	4.1%	59.5%	1.1%	4.1%	5.14	49.49	93,972	Yes	Yes	Yes
SEAV10	486	4.5%	4.1%	25.9%	1.2%	2.4%	2.70	17.99	5,141	Yes	Yes	Yes
SEAV15	324	4.4%	4.0%	18.1%	1.2%	2.1%	1.84	9.45	744	Yes	Yes	Yes
SEAV20	243	4.3%	3.9%	18.3%	1.2%	1.9%	2.26	14.23	1,483	Yes	Yes	Yes
SIAV5	995	4.8%	3.0%	47.8%	-13.7%	6.0%	3.09	16.23	8,844	Yes	Yes	Yes
SIAV10	486	4.4%	3.3%	32.9%	-4.7%	4.7%	2.30	10.19	1,475	Yes	Yes	Yes
SIAV15	324	4.4%	3.1%	31.2%	-3.5%	4.5%	2.48	11.84	1,388	Yes	Yes	Yes
SIAV20	243	4.3%	3.3%	31.7%	-1.7%	4.3%	2.51	12.24	1,120	Yes	Yes	Yes
SEAC5	995	74.4%	77.7%	96.2%	5.7%	14.7%	-1.21	4.89	392	Yes	Yes	Yes
SEAC10	486	77.7%	79.9%	96.0%	27.3%	11.5%	-1.10	4.45	141	Yes	Yes	Yes
SEAC15	324	79.2%	81.5%	95.6%	40.3%	10.4%	-1.02	4.08	72	Yes	Yes	Yes
SEAC20	243	79.6%	80.7%	96.1%	45.4%	9.9%	-0.88	3.79	38	Yes	Yes	Yes
SIAC5	995	0.9%	-0.7%	70.8%	-45.4%	19.2%	0.35	2.71	24	Yes	Yes	Yes
SIAC10	486	0.6%	0.3%	49.8%	-38.5%	16.2%	0.32	2.73	10	Yes	Yes	Yes
SIAC15	324	0.7%	0.3%	46.9%	-30.7%	14.7%	0.24	2.60	5	No	Yes	Yes
SIAC20	243	0.7%	1.2%	43.3%	-31.3%	14.4%	0.17	2.51	4	No	Yes	Yes
SIRV5	995	5.6%	3.7%	86.3%	-37.0%	21.0%	0.45	2.86	35	Yes	Yes	Yes
SIRV10	486	5.0%	4.5%	67.0%	-35.0%	17.9%	0.38	2.93	12	Yes	Yes	Yes
SIRV15	324	5.0%	4.1%	55.9%	-30.9%	16.3%	0.31	2.74	6	Yes	Yes	Yes
SIRV20	243	5.0%	5.0%	50.8%	-31.8%	16.0%	0.23	2.59	4	No	Yes	Yes

Legend: Acronyms are defined in Table 4 on page 136. N = the number of observations, μ = the mean value, Med = median value, Max, = maximum value, Min = the minimum value, σ = the standard deviation, Skew = skew of the distribution, Kurt = kurtosis, JB stat = the Jarque-Bera statistic JB = whether or not the null hypothesis of a normal distribution can be rejected at the 5% level of significance using the Jarque-Bera normality test. ADF and PP = whether or not the null hypothesis of a unit root can be rejected at the 5% level of significance using the Augmented Dickey Fuller and Phillips Peron test respectively.

Table 7 Descriptive statistics: January 1998 through March 2003

Panel A: Levels of concentration and differenced concentration

Abbreviation	N	μ	Med	Max	Min	σ	SK	K	JB Stat	JB	ADF	PP
H5	274	0.034	0.038	0.046	0.022	0.007	-0.51	1.72	31	Yes	No	No
R5	274	0.117	0.118	0.123	0.110	0.004	-0.37	1.74	24	Yes	No	No
V5	274	0.856	0.859	1.275	0.577	0.177	0.12	1.77	18	Yes	No	No
SK5	274	3.555	3.606	5.704	2.519	0.597	0.61	4.40	39	Yes	No	No
DH5	274	7.4E-05	-1.5E-05	6.9E-03	-3.3E-03	1.0E-03	2.46	16.92	2490	Yes	Yes	Yes
DH10	134	1.5E-04	3.0E-05	1.0E-02	-5.1E-03	1.6E-03	1.81	15.39	930	Yes	Yes	Yes
DH15	88	2.3E-04	1.4E-04	1.2E-02	-4.7E-03	1.9E-03	2.33	17.59	859	Yes	Yes	Yes
DH20	67	2.8E-04	1.9E-04	1.2E-02	-3.1E-03	1.9E-03	2.96	19.06	818	Yes	Yes	Yes
DR5	274	4.0E-05	2.7E-05	2.0E-03	-1.3E-03	4.2E-04	0.77	6.17	142	Yes	Yes	Yes
DR10	134	8.9E-05	1.3E-04	2.7E-03	-1.8E-03	6.1E-04	0.43	6.03	55	Yes	Yes	Yes
DR15	88	1.4E-04	7.7E-05	2.7E-03	-1.9E-03	6.8E-04	0.51	5.05	19	Yes	Yes	Yes
DR20	67	1.7E-04	1.2E-04	2.2E-03	-1.2E-03	6.4E-04	0.37	3.30	2	No	Yes	Yes
DSK5	274	4.1E-03	-7.8E-04	1.3E+00	-5.0E-01	1.5E-01	3.51	35.26	12446	Yes	Yes	Yes
DSK10	134	6.3E-03	-8.0E-03	1.3E+00	-5.0E-01	2.1E-01	2.56	17.71	1354	Yes	Yes	Yes
DSK15	88	1.1E-02	1.0E-02	2.1E+00	-8.6E-01	2.8E-01	4.16	35.25	4068	Yes	Yes	Yes
DSK20	67	1.3E-02	-5.0E-03	2.2E+00	-9.9E-01	3.5E-01	3.58	27.25	1784	Yes	Yes	Yes
DV5	274	1.7E-03	2.2E-03	1.4E-01	-2.0E-01	2.5E-02	-1.13	19.63	3217	Yes	Yes	Yes
DV10	134	4.3E-03	7.5E-03	1.2E-01	-2.0E-01	3.5E-02	-1.39	12.10	506	Yes	Yes	Yes
DV15	88	6.6E-03	1.3E-02	1.1E-01	-1.1E-01	3.7E-02	-0.17	3.77	3	No	Yes	Yes
DV20	67	8.8E-03	1.4E-02	1.2E-01	-6.1E-02	3.7E-02	0.26	3.25	1	No	Yes	Yes

Panel B: Realised volatility and the sub-components of realised volatility

Abbreviation	N	μ	Med	Max	Min	σ	SK	K	JB Stat	JB	ADF	PP
RS5	274	2.8%	2.5%	8.1%	0.7%	1.4%	1.32	5.12	131	Yes	Yes	Yes
RS10	134	4.0%	3.6%	11.3%	1.5%	1.8%	1.52	5.66	91	Yes	Yes	Yes
RS15	88	1.3%	1.2%	3.3%	0.5%	0.5%	1.31	4.85	38	Yes	Yes	Yes
RS20	67	5.8%	5.3%	13.7%	2.9%	2.3%	1.33	4.33	25	Yes	Yes	Yes
EAV5	274	8.0E-06	6.7E-06	2.7E-05	2.2E-06	4.6E-06	1.56	5.73	196	Yes	Yes	Yes
EAV10	134	8.0E-06	7.0E-06	2.2E-05	2.5E-06	4.2E-06	1.34	4.51	53	Yes	Yes	Yes
EAV15	88	8.1E-06	7.1E-06	2.0E-05	2.7E-06	3.9E-06	1.07	3.68	19	Yes	Yes	Yes
EAV20	67	8.0E-06	7.5E-06	2.0E-05	2.9E-06	3.9E-06	1.23	4.13	20	Yes	Yes	Yes
IAV5	274	1.4E-05	1.0E-05	7.3E-05	-3.2E-06	1.3E-05	1.85	6.84	325	Yes	Yes	Yes
IAV10	134	1.4E-05	1.1E-05	6.4E-05	-4.7E-07	1.1E-05	1.87	7.21	177	Yes	No	Yes
IAV15	88	1.4E-05	1.2E-05	4.3E-05	2.6E-06	9.6E-06	1.32	4.42	33	Yes	Yes	Yes
IAV20	67	1.4E-05	1.0E-05	4.7E-05	3.0E-06	9.6E-06	1.51	5.12	38	Yes	Yes	Yes
EAC5	274	1.7E-04	9.8E-05	1.3E-03	2.0E-06	2.0E-04	2.77	12.23	1323	Yes	Yes	Yes
EAC10	134	1.7E-04	1.1E-04	1.0E-03	1.4E-05	1.9E-04	2.61	10.88	499	Yes	Yes	Yes
EAC15	88	1.7E-04	1.2E-04	9.0E-04	2.1E-05	1.7E-04	2.11	7.84	151	Yes	Yes	Yes
EAC20	67	1.7E-04	1.1E-04	7.5E-04	3.4E-05	1.6E-04	1.94	6.46	75	Yes	Yes	Yes
IAC5	274	5.8E-07	3.9E-07	3.7E-04	-2.9E-04	6.4E-05	0.39	10.91	721	Yes	Yes	Yes
IAC10	134	9.7E-07	2.5E-06	2.2E-04	-1.5E-04	5.0E-05	0.31	6.18	59	Yes	No	Yes
IAC15	88	1.3E-06	2.0E-06	1.4E-04	-1.3E-04	4.3E-05	-0.11	4.40	7	Yes	Yes	Yes
IAC20	67	1.6E-06	5.8E-06	1.4E-04	-1.5E-04	4.2E-05	-0.37	5.84	24	Yes	Yes	Yes
IRV5	274	1.5E-05	9.9E-06	4.2E-04	-2.5E-04	6.6E-05	1.02	11.62	895	Yes	Yes	Yes
IRV10	134	1.5E-05	1.2E-05	2.7E-04	-1.3E-04	5.1E-05	1.05	8.24	178	Yes	Yes	Yes
IRV15	88	1.6E-05	1.3E-05	1.8E-04	-1.1E-04	4.2E-05	0.62	5.58	30	Yes	Yes	Yes
IRV20	67	1.6E-05	1.7E-05	1.8E-04	-1.2E-04	4.1E-05	0.64	7.21	54	Yes	Yes	Yes

Table 7 continued: January 1998 – March 2003

Panel C: Standardised sub-components of realised volatility												
Abbreviation	**N**	**μ**	**Med**	**Max**	**Min**	**σ**	**SK**	**K**	**JB Stat**	**JB**	**ADF**	**PP**
SEAV5	274	5.6%	4.5%	34.9%	1.2%	4.3%	2.68	13.96	1698	Yes	Yes	Yes
SEAV10	134	5.1%	4.5%	25.9%	1.3%	3.2%	2.90	17.08	1294	Yes	Yes	Yes
SEAV15	88	4.9%	4.5%	16.6%	1.5%	2.5%	1.65	7.44	112	Yes	Yes	Yes
SEAV20	67	4.8%	4.5%	18.3%	1.5%	2.6%	2.39	12.74	329	Yes	Yes	Yes
SIAV5	274	9.2%	6.9%	47.8%	-1.6%	8.7%	2.03	7.38	406	Yes	Yes	Yes
SIAV10	134	8.6%	7.0%	32.9%	-0.2%	6.3%	1.34	4.61	55	Yes	No	Yes
SIAV15	88	8.6%	6.8%	31.9%	1.2%	5.9%	1.57	5.94	68	Yes	Yes	Yes
SIAV20	67	8.4%	6.9%	31.7%	1.8%	5.7%	1.65	6.22	59	Yes	Yes	Yes
SEAC5	274	67.0%	69.2%	95.6%	5.7%	17.2%	-0.88	3.79	43	Yes	Yes	Yes
SEAC10	134	70.6%	72.5%	93.7%	27.3%	13.7%	-0.64	3.15	9	Yes	Yes	Yes
SEAC15	88	71.8%	72.6%	92.1%	39.3%	12.2%	-0.54	2.77	4	No	Yes	Yes
SEAC20	67	73.1%	74.6%	92.3%	45.4%	11.3%	-0.39	2.56	2	No	Yes	Yes
SIAC5	274	1.4%	0.8%	70.8%	-45.4%	21.8%	0.27	2.48	6	Yes	Yes	Yes
SIAC10	134	2.6%	2.2%	49.8%	-38.5%	18.8%	0.26	2.38	4	No	Yes	Yes
SIAC15	88	2.5%	2.3%	47.0%	-27.3%	17.3%	0.20	2.21	3	No	Yes	Yes
SIAC20	67	2.9%	3.9%	39.2%	-28.0%	16.1%	0.07	2.17	2	No	Yes	Yes
SIRV5	274	10.7%	8.8%	86.3%	-35.5%	23.5%	0.36	2.61	8	Yes	Yes	Yes
SIRV10	134	11.2%	8.4%	67.0%	-35.0%	20.0%	0.29	2.72	2	No	Yes	Yes
SIRV15	88	11.1%	10.1%	56.7%	-21.6%	17.9%	0.24	2.39	2	No	Yes	Yes
SIRV20	67	11.4%	12.2%	47.9%	-19.2%	16.8%	0.17	2.21	2	No	Yes	Yes

Legend: N = the number of observations, μ = the mean value, Med = median value, Max, = maximum value, Min = the minimum value, σ = the standard deviation, Skew = skew of the distribution, Kurt = kurtosis, JB stat = the Jarque-Bera statistic JB = whether or not the null hypothesis of a normal distribution can be rejected at the 5% level of significance using the Jarque-Bera normality test. ADF and PP = whether or not the null hypothesis of a unit root can be rejected at the 5% level of significance using the Augmented Dickey Fuller and Phillips Peron test respectively.

Chapter 9 – Results II: Concentration and realised volatility models

9.1 Introduction

In this Chapter general asymmetric autoregressive distributed lag (AARDL) models, naive asymmetric autoregressive (AAR) models and naive autoregressive (AR) models are estimated for each of four dependent variables. The four dependent variables are the realised volatility data series RS5, RS10, RS15 and RS20, estimated with T values of five, ten, fifteen and twenty trading-days respectively.[94] AARDL models are estimated a minimum of four times for each dependent variable data series: once for each of the four differenced concentration indices discussed in section 5.4 of Chapter 5. The general to specific modelling procedure follows that specified in Chapter 7. The models applied in this chapter are referred to as "direct" models of realised volatility. In this context, use of the word "direct" refers to the fact that realised volatility as whole was modelled, rather than the separate sub-components of the VCM.

Section 9.2 in this chapter, reports the results for models of the relationship between realised volatility and changes in concentration over the sub-period from January 1998 through December 2002. Out-of-sample forecasts derived from these models are analysed and discussed in sections 9.3. Section 9.4 reports, analyses and evaluates a summary of key results from earlier sub-periods and the whole study period from 1984-2003. Section 9.5 concludes the chapter and highlights the connection between the results discussed in this chapter and those analysed in Chapter 10.

9.2 Model results for the sub-period: January 1998 – December 2002

This section reports the results of models estimated for the five years from January 1998 through December 2002. Table 8 through Table 11 report model results for the four realised volatility series RS5, RS10, RS15 and RS20 respectively. Each table provides the output of the general AARDL models, naive AAR models and the most basic AR models. The AARDL models are estimated both including and excluding contemporaneous differenced concentration variables. All AARDL models include one lag of the differenced concentration variable. Reported statistics include measures of model fit such as the

[94] Further details are provided in the methodology section 5.5 of Chapter 5 and in Table 4 on page 136.

adjusted R^2, the Akaike Information criterion (AIC) and the Schwartz Information Criterion (SIC). Model coefficients and p-values calculated using Newey-West heteroskedasticity and autocorrelation robust standard errors are also reported. Finally, descriptive statistics of model residuals are reported to allow evaluation of the possibility of model miss-specification or biased errors.

9.2.1 Coefficients on contemporaneous differenced concentration (DC) and the first lag of differenced concentration (DC – 1)

Because both the levels of concentration and the volatility of concentration are higher in the 1998 – 2002 sub-period, than the period as a whole, this would intuitively seem to be the period in which concentration is most likely to exhibit an observable effect on realised volatility. However, the results from the direct AARDL models of realised volatility indicate that there is little evidence of a relationship between changes in concentration and realised volatility. Only two models, out of a total of thirty-two AARDL models (the RS20-DR20 and the RS20-DH20 model), have significant coefficients on the lagged change in concentration (DC-1) variable – these coefficients are negative. Thus, the limited evidence for a link between changes in lagged concentration with changes in contemporaneous realised volatility suggests an inverse relationship between the two variables.

None of the contemporaneous or lagged coefficients of differenced concentration are significant at the $\alpha < 5\%$ threshold, except for the lagged DR20 coefficient. The lagged DH20 coefficient is significant at the $\alpha < 10\%$ threshold. Both of these coefficients are negative, suggesting that over this period increases in concentration resulted in decreases in realised volatility in the following period. DR20 is derived from the square root of firm weights and therefore is relatively more sensitive to changes in the distribution of the weights of smaller firms in the index than is DH20. Both measures exhibited large positive values when the merger of Vodafone and Mannessmann came into effect in March 2000, although the change was much less for DR20 for the reasons just discussed. However, it is unlikely that this effect alone could have resulted in the significant negative coefficients because it was even more pronounced in DSK20, where the coefficient is not significant at the $\alpha < 10\%$ threshold. The more symmetric DV20 measure of change in concentration has a coefficient of the opposite sign that is not significant.

Apart from the exception detailed above, the majority of the model results reported do not provide evidence of a significant relationship between either contemporaneous or lagged changes in concentration and realised volatility. Model coefficients on these variables are not significantly different from zero at either the $\alpha < 5\%$ or the $\alpha < 10\%$ threshold, as indicated by the p-values in each of the four tables. These results are robust for all four measures of differenced concentration and for all four differencing and volatility estimation intervals, apart from those discussed in the previous paragraph. In fact, including either differenced concentration or lags of differenced concentration does not improve the explanatory power of the models over naive asymmetric autoregressive models.

9.2.2 Coefficients on the asymmetric dummy variable (DRS –1)

The results discussed above do support the well-documented asymmetry effect whereby negative FTSE 100 Index returns appear to precede increases in index volatility. This effect is captured by the positive DRS coefficients, most of which are significant at the $\alpha < 1\%$ threshold. When model explanatory power is compared using adjusted R^2, the AIC or SIC criterion, the more parsimonious AAR models perform better than models that include differenced concentration. Even in the two exceptions noted in the previous paragraph, the improvement in adjusted R^2 is only about 6.5%, while the improvement in the SIC is small when the distributed lags are added to the model.[95] By contrast, when the simple AR models are compared with the AAR models, it is evident that the asymmetric variable is quite important in explaining future realised volatility. This is particularly evident when realised volatility is estimated using daily returns measured using 10 or more trading-days.

9.2.3 Residual analysis

Given that OLS regression model disturbances are assumed to be white noise, sample residual analysis can provide useful insights into the robustness of model results and help to identify model miss-specification. Each table of model results reports descriptive statistics for model residuals. For almost all models, residual means and medians are negative, indicating a potential bias in the realised volatility forecasts. However, both means and medians are very small in relation to the standard deviation of the residuals, suggesting that the true population mean may still be close to the assumed value of zero. Another OLS assumption is that the disturbances should be independent and exhibit zero autocorrelation. No residual autocorrelation, significant at the $\alpha < 5\%$ threshold, was present for any of the

[95] The Schwartz Information Criterion tends to favour more parsimonious models and is most sensitive to reductions in the number of degrees of freedom.

model results reported above, except for the simplest AR1 model estimated for RS5, RS10 and RS15. However, the assumption that residuals were normally distributed was tested using the Jarque-Bera normality test. The null hypothesis of a normal distribution could be rejected for all models except those estimated for the RS15 data. The RS15 data models also had the highest adjusted R^2 suggesting that a T-value of fifteen trading days is close to the optimal frequency for estimating realised volatility, at least during this five-year study period.

Table 8 Model results for RS5 and concentration differenced over five-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained from 259 observations, each estimated using five daily returns. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference RS5 DV5 indicates that five day realised standard deviation (RS5) is the dependent autoregressive variable and the variance of the logarithm of firm size differenced over five days (DV5) is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DRS – 1 denotes an asymmetric coefficient on the lagged DRS data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. No autocorrelation significant at 5% was evident in any lags of the model residuals except from the AR1 model where all lags had significant autocorrelation.

Model reference	**RS5 DV5**		**RS5 DR5**		**RS5 DH5**		**RS5 DSK5**		**AAR2**	**AR2**	**AR1**
Panel A: Tests of model fit	DC and DC - 1	DC - 1 only	DC and DC - 1	DC - 1 only	DC and DC - 1	DC - 1 only	DC and DC - 1	DC - 1 only			
R-squared	0.411	0.411	0.414	0.411	0.412	0.410	0.412	0.411	0.410	0.388	0.325
Adjusted R-squared	0.399	0.402	0.402	0.402	0.401	0.401	0.400	0.402	0.403	0.383	0.323
SE of the regression	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011	0.011
Sum of the squared resid	0.028	0.028	0.028	0.028	0.028	0.029	0.028	0.029	0.029	0.030	0.033
Mean of the dependent var	2.71%	2.71%	2.71%	2.71%	2.71%	2.71%	2.71%	2.71%	2.71%	2.71%	2.71%
SD of the dependent var	1.37%	1.37%	1.37%	1.37%	1.37%	1.37%	1.37%	1.37%	1.37%	1.37%	1.37%
Akaike info criterion	-6.23	-6.23	-6.23	-6.23	-6.23	-6.23	-6.23	-6.23	-6.24	-6.21	-6.12
Schwarz criterion	-6.14	-6.17	-6.15	-6.17	-6.15	-6.16	-6.15	-6.17	-6.19	-6.17	-6.10
Panel B: Coefficients											
Intercept	0.009	0.008	0.009	0.009	0.009	0.009	0.008	0.008	0.009	0.008	0.012
AR1	0.312	0.311	0.298	0.306	0.301	0.308	0.308	0.309	0.309	0.394	0.570
AR2	0.305	0.307	0.310	0.306	0.310	0.308	0.310	0.310	0.309	0.307	
DRS - 1	0.136	0.136	0.136	0.137	0.135	0.136	0.137	0.136	0.135		
DC - 1	0.015	0.016	1.256	1.125	0.186	0.170	-0.003	-0.003			
DC	-0.006		1.504		0.610		0.002				
Panel C: Coefficient p-values											
Intercept	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AR1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AR2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
DRS - 1	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01		
DC - 1	0.57	0.53	0.40	0.44	0.73	0.75	0.40	0.43			
DC	0.82		0.39		0.37		0.46				
Panel D: Residual descriptive statistics											
Mean	-7.8E-18	-5.5E-18	-7.4E-18	-5.6E-18	-8.1E-18	-7.7E-18	-7.7E-18	-4.2E-18	-7.8E-18	-8.4E-18	-9.2E-18
Median	-0.08%	-0.08%	-0.09%	-0.08%	-0.08%	-0.08%	-0.09%	-0.09%	-0.08%	-0.12%	-0.09%
Maximum	4.84%	4.81%	4.64%	4.82%	4.70%	4.84%	4.83%	4.86%	4.85%	5.01%	4.73%
Minimum	-2.30%	-2.31%	-2.35%	-2.31%	-2.34%	-2.32%	-2.33%	-2.33%	-2.33%	-2.54%	-2.92%
Std. Dev.	1.05%	1.05%	1.05%	1.05%	1.05%	1.05%	1.05%	1.05%	1.05%	1.07%	1.13%
Skewness	0.95	0.94	0.90	0.94	0.92	0.94	0.95	0.94	0.94	0.92	1.08
Kurtosis	5.40	5.36	5.17	5.38	5.25	5.37	5.40	5.38	5.37	5.38	5.89

Table 9 Model results for RS10 and concentration differenced over ten-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained from 126 observations, each estimated using ten trading days worth of data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference RS10 DV10 indicates that ten day realised standard deviation (RS10) is the dependent autoregressive variable and the variance of the logarithm of firm size differenced over ten days (DV10) is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DRS – 1 denotes an asymmetric coefficient on the lagged DRS data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. No autocorrelation significant at 10% was evident in any lags of the model residuals except from the AR1 model where all lags had significant autocorrelation.

Model reference	**RS10 DV10**		**RS10 DR10**		**RS10 DH10**		**RS10 DSK10**		**AAR2**	**AR2**	**AR1**
Panel A: Tests of model fit	DC and DC - 1	DC - 1 only	DC and DC - 1	DC - 1 only	DC and DC - 1	DC - 1 only	DC and DC - 1	DC - 1 only			
R-squared	0.482	0.464	0.486	0.460	0.478	0.458	0.467	0.460	0.457	0.407	0.401
Adjusted R-squared	0.461	0.446	0.464	0.441	0.456	0.440	0.445	0.442	0.443	0.397	0.396
SE of the regression	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.014	0.014
Sum squared resid	0.020	0.021	0.020	0.021	0.020	0.021	0.021	0.021	0.021	0.023	0.023
Mean dependent var	3.94%	3.94%	3.94%	3.94%	3.94%	3.94%	3.94%	3.94%	3.94%	3.94%	3.94%
SD of the dependent var	1.78%	1.78%	1.78%	1.78%	1.78%	1.78%	1.78%	1.78%	1.78%	1.78%	1.78%
Akaike info criterion	-5.788	-5.769	-5.795	-5.761	-5.780	-5.759	-5.759	-5.762	-5.772	-5.700	-5.714
Schwarz criterion	-5.652	-5.656	-5.659	-5.648	-5.643	-5.645	-5.623	-5.648	-5.681	-5.632	-5.668
Panel B: Coefficients											
Intercept	0.011	0.012	0.012	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.014
AR1	0.395	0.366	0.345	0.366	0.349	0.377	0.363	0.380	0.396	0.586	0.634
AR2	0.217	0.219	0.231	0.204	0.222	0.196	0.211	0.194	0.187	0.077	
DRS - 1	0.196	0.206	0.207	0.212	0.211	0.208	0.214	0.208	0.198		
DC - 1	0.050	0.044	2.310	1.594	0.470	0.440	0.004	0.005			
DC	0.070		4.706		1.539		0.007				
Panel C: Coefficient p-values											
Intercept	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AR1	0.000	0.000	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000
AR2	0.002	0.002	0.002	0.004	0.003	0.006	0.003	0.006	0.009	0.369	
DRS - 1	0.010	0.008	0.009	0.008	0.010	0.011	0.007	0.009	0.010		
DC - 1	0.173	0.188	0.273	0.355	0.515	0.461	0.380	0.277			
DC	0.083		0.091		0.069		0.081				
Panel D: Residual descriptive statistics											
Mean	-7.3E-18	-2.1E-18	-7.8E-18	-9.2E-18	-2.6E-18	-4.6E-18	-4.6E-18	-7.2E-18	-8.1E-18	-6.4E-18	5.2E-18
Median	-0.17%	-0.20%	-0.15%	-0.18%	-0.20%	-0.16%	-0.17%	-0.13%	-0.14%	-0.07%	-0.07%
Maximum	4.90%	5.49%	5.24%	5.66%	5.14%	5.69%	5.60%	5.69%	5.72%	6.01%	5.96%
Minimum	-2.82%	-2.92%	-2.74%	-2.97%	-2.78%	-2.97%	-2.90%	-2.96%	-2.96%	-2.66%	-2.81%
Std. Dev.	1.28%	1.30%	1.28%	1.31%	1.29%	1.31%	1.30%	1.31%	1.31%	1.37%	1.37%
Skewness	1.095	1.201	1.181	1.209	1.222	1.198	1.250	1.188	1.227	1.448	1.394
Kurtosis	5.430	6.062	5.922	6.308	6.201	6.337	6.530	6.338	6.439	7.199	7.115

Table 10 Model results for RS15 and concentration differenced over fifteen-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained from 82 observations, each estimated using data over fifteen trading days. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference RS15 DV15 indicates that fifteen day realised standard deviation (RS15) is the dependent autoregressive variable and the variance of the logarithm of firm size differenced over fifteen days (DV15) is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DRS – 1 denotes an asymmetric coefficient on the lagged DRS data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed could not be rejected at the 5% level of significance for any of the models except the AR1 model. No residual autocorrelation significant at 5% was evident in any of the models.

Model reference	**RS15 DV15**		**RS15 DR15**		**RS15 DH15**		**RS15 DSK15**		**AAR1**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.5191	0.5176	0.5221	0.5169	0.5197	0.5159	0.5230	0.5223	0.5158	0.4133
Adjusted R-squared	0.4938	0.4988	0.4970	0.4983	0.4948	0.4973	0.4983	0.5039	0.5036	0.4059
SE of the regression	0.0038	0.0038	0.0038	0.0038	0.0038	0.0038	0.0038	0.0037	0.0037	0.0041
Sum squared resid	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0013
Mean dependent var	1.26%	1.26%	1.26%	1.26%	1.26%	1.26%	1.26%	1.26%	1.26%	1.26%
SD of the dependent var	0.53%	0.53%	0.53%	0.53%	0.53%	0.53%	0.53%	0.53%	0.53%	0.53%
Akaike info criterion	-8.252	-8.273	-8.258	-8.285	-8.267	-8.283	-8.274	-8.296	-8.307	-8.140
Schwarz criterion	-8.104	-8.155	-8.110	-8.168	-8.120	-8.166	-8.127	-8.179	-8.219	-8.081
Panel B: Coefficients										
Intercept	0.0044	0.0045	0.0044	0.0045	0.0045	0.0045	0.0046	0.0046	0.0045	0.0045
AR1	0.5458	0.5395	0.5509	0.5460	0.5433	0.5406	0.5288	0.5277	0.5383	0.6424
DRS - 1	0.2410	0.2458	0.2293	0.2467	0.2347	0.2480	0.2588	0.2614	0.2496	
DC - 1	0.0054	0.0050	-0.2580	-0.2594	-0.0317	-0.0261	0.0015	0.0015		
DC	0.0056		0.5511		0.1735		0.0005			
Panel C: Coefficient p-values										
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DRS - 1	0.0102	0.0083	0.0238	0.0098	0.0205	0.0106	0.0076	0.0056	0.0073	
DC - 1	0.6514	0.6795	0.6119	0.6118	0.8392	0.8669	0.1580	0.1504		
DC	0.6436		0.3924		0.3537		0.5926			
Panel D: Residual descriptive statistics										
Mean	1.2E-18	-6.3E-19	1.3E-18	1.0E-18	1.5E-18	3.7E-19	4.1E-19	2.8E-19	1.1E-18	7.1E-19
Median	-0.049%	-0.049%	-0.042%	-0.033%	-0.041%	-0.033%	-0.048%	-0.037%	-0.034%	-0.063%
Maximum	0.950%	0.955%	0.958%	0.935%	0.953%	0.938%	0.919%	0.918%	0.935%	1.180%
Minimum	-0.875%	-0.893%	-0.858%	-0.892%	-0.868%	-0.891%	-0.887%	-0.888%	-0.890%	-0.786%
Std. Dev.	0.370%	0.370%	0.369%	0.368%	0.367%	0.369%	0.366%	0.366%	0.369%	0.406%
Skewness	0.27	0.29	0.23	0.23	0.26	0.25	0.25	0.23	0.25	0.82
Kurtosis	3.49	3.59	3.49	3.60	3.58	3.61	3.66	3.65	3.62	3.73
Jarque-Bera	No	No	No	No	No	No	No	No	No	Yes

Table 11 Model results for RS20 and concentration differenced over twenty-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained from 82 observations, each estimated using fifteen trading days worth of data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference RS20 DV20 indicates that twenty day realised standard deviation (RS20) is the dependent autoregressive variable and the variance of the logarithm of firm size differenced over twenty days (DV20) is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DRS – 1 denotes an asymmetric coefficient on the lagged DRS data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. No autocorrelation significant at 20% was evident in any lags of the model residuals except from the AR1 model where all lags had significant autocorrelation.

Model reference	**RS20 DV20**		**RS20 DR20**		**RS20 DH20**		**RS20 DSK20**		**AAR1**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.4150	0.4144	0.4867	0.4556	0.4591	0.4466	0.4156	0.4142	0.4133	0.2708
Adjusted R-squared	0.3747	0.3846	0.4514	0.4279	0.4218	0.4185	0.3753	0.3844	0.3938	0.2589
SE of the regression	0.0182	0.0180	0.0170	0.0174	0.0175	0.0175	0.0182	0.0180	0.0179	0.0198
Sum squared resid	0.0191	0.0192	0.0168	0.0178	0.0177	0.0181	0.0191	0.0192	0.0192	0.0238
Mean dependent var	5.68%	5.68%	5.68%	5.68%	5.68%	5.68%	5.68%	5.68%	5.68%	5.68%
SD of the dependent var	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%	2.30%
Akaike info criterion	-5.103	-5.134	-5.234	-5.207	-5.181	-5.190	-5.104	-5.133	-5.164	-4.978
Schwarz criterion	-4.933	-4.998	-5.064	-5.071	-5.011	-5.054	-4.934	-4.997	-5.062	-4.910
Panel B: Coefficients										
Intercept	0.0297	0.0300	0.0271	0.0284	0.0285	0.0288	0.0301	0.0302	0.0303	0.0273
AR1	0.3374	0.3345	0.3976	0.3833	0.3654	0.3598	0.3341	0.3328	0.3328	0.5209
DRS - 1	0.2823	0.2828	0.2654	0.2903	0.2866	0.3039	0.2848	0.2877	0.2842	
DC - 1	0.0218	0.0212	-6.4106	-7.4794	-1.8970	-2.1952	-0.0016	-0.0020		
DC	0.0155		6.3896		1.3506		0.0024			
Panel C: Coefficient p-values										
Intercept	0.0002	0.0000	0.0001	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
AR1	0.0085	0.0050	0.0029	0.0046	0.0042	0.0047	0.0079	0.0078	0.0071	0.0001
DRS - 1	0.0008	0.0007	0.0009	0.0001	0.0007	0.0001	0.0007	0.0005	0.0005	
DC - 1	0.7605	0.7689	0.0397	0.0223	0.0399	0.0512	0.6365	0.5147		
DC	0.8407		0.1054		0.4397		0.7065			
Panel D: Residual descriptive statistics										
Mean	-4.7E-18	-4.0E-18	-5.2E-18	-2.3E-18	-4.4E-19	-3.5E-18	-6.6E-18	-3.8E-18	-2.9E-18	-2.4E-18
Median	-0.424%	-0.385%	-0.330%	-0.197%	-0.388%	-0.357%	-0.439%	-0.458%	-0.452%	-0.517%
Maximum	6.057%	5.982%	5.439%	6.057%	5.407%	5.787%	6.053%	6.081%	6.102%	6.750%
Minimum	-3.224%	-3.229%	-3.276%	-3.331%	-3.485%	-3.359%	-3.347%	-3.346%	-3.262%	-3.060%
Std. Dev.	1.757%	1.758%	1.645%	1.695%	1.689%	1.708%	1.756%	1.758%	1.759%	1.961%
Skewness	1.01	0.99	0.77	0.92	0.73	0.85	0.99	0.99	1.01	1.36
Kurtosis	4.45	4.38	3.86	4.50	3.76	4.16	4.49	4.53	4.53	4.97

Table 12 Realised volatility forecasts: January 2003 – April 2003

Realised volatility final data points are as follows: RS5, 16th April, RS10, 7th April 2003. The difference is due to the fact that estimates are non-overlapping and hence not perfectly synchronised for different estimation periods over the whole sample period from 1984 through 2003.

Panel A: RS5 out-of-sample forecasts based on trading days from 19th December 2002 through 16th April 2003						
Forecasting model	**AR2**	**AAR2**	**AARDL2 DV5**	**AARDL2 DR5**	**AARDL2 DH5**	**AARDL2 DSK5**
Forecast variable	RS5	RS5	RS5	RS5	RS5	RS5
Forecast sample:	261 275	261 275	261 275	261 275	261 275	261 275
Included observations:	14	14	14	14	14	14
Root Mean Squared Error	1.55%	1.40%	1.39%	1.40%	1.40%	1.40%
Mean Absolute Error	1.20%	1.01%	1.00%	1.01%	1.01%	1.02%
Mean Abs. Percent Error	28.6%	24.2%	24.0%	24.2%	24.2%	24.4%
Theil Inequality Coefficient	19.9%	18.0%	17.9%	18.0%	18.0%	18.0%
Bias Proportion	6.2%	7.5%	7.5%	7.4%	7.5%	7.3%
Variance Proportion	13.3%	26.7%	27.9%	27.5%	27.0%	26.2%
Covariance Proportion	80.5%	65.8%	64.6%	65.1%	65.4%	66.5%
Panel B: RS10 out-of-sample forecasts based on trading days from 28th December 2002 through 7th April 2003						
Forecasting model	**AR2**	**AAR2**	**AARDL2 DV10**	**AARDL2 DR10**	**AARDL2 DH10**	**AARDL2 DSK10**
Forecast variable	RS10	RS10	RS10	RS10	RS10	RS10
Forecast sample:	128 134	128 134	128 134	128 134	128 134	128 134
Included observations:	7	7	7	7	7	7
Root Mean Squared Error	2.38%	2.11%	2.06%	2.08%	2.09%	2.10%
Mean Absolute Error	1.82%	1.50%	1.41%	1.44%	1.46%	1.47%
Mean Abs. Percent Error	30.0%	25.1%	23.1%	23.8%	24.2%	24.5%
Theil Inequality Coefficient	21.7%	19.8%	19.4%	19.5%	19.6%	19.7%
Bias Proportion	6.8%	14.9%	15.3%	15.7%	15.7%	15.6%
Variance Proportion	11.6%	34.0%	40.6%	39.5%	37.7%	36.4%
Covariance Proportion	81.6%	51.1%	44.1%	44.8%	46.6%	48.0%

9.3 Realised volatility forecasts: January 2003 – April 2003

The model results discussed in the previous section were estimated using differenced concentration and realised volatility data recorded over the period from January 1998 to December 2002. The remaining data in the series from January to April 2003 were used to test the out-of-sample forecasting ability of the models estimated using data from the preceding five years. Out-of-sample forecasts derived from AARDL models that include the first lag of the differenced concentration measures are compared with the naive AAR models for the realised volatility series RS5, RS10.[96]

Table 12 reports the results of forecast evaluation tests of static forecasts produced using E-views.[97] Static forecasts are successive one-step-ahead forecasts for each out-of-sample period in which the actual lagged data values from the out-of-sample period are inserted

[96] Further evaluation of out-of-sample forecasts using data estimated with T values equal to five, ten, fifteen and twenty-trading days, is currently in progress. The intention is to up-date the data sample on an ongoing basis for this purpose.

[97] Out-of-sample forecast evaluation metrics are discussed in section 7.4.3 of Chapter 7 and on pages 336-338 of the E-Views 4 user's guide.

into the models whose parameters were derived during the estimation period. The first row of the table details the nature of the model used to make the forecast: for instance AARDL2 DV10, indicates that the forecast used an asymmetric autoregressive distributed lag model in which the exogenous variable was the variance of the logarithm of firm size differenced over 10 trading-days. This model had two autoregressive coefficients, one asymmetric and one distributed lag coefficient. The forecast sample indicates the data range used in each out-of-sample forecast.

Evaluation of the forecast results in Table 12 indicates that comparison of model forecasts provides evidence consistent with the conclusions reached in the previous section 9.2, namely that differenced concentration has little explanatory power for overall realised volatility. The mean squared error (MSE), mean absolute error (MAE), the mean absolute percentage error (MAPE) and the Theil inequality coefficient (TIC) are almost indistinguishable between the general AARDL and the naive AAR1 models. The distribution of the MSE between the bias proportion, the variance proportion and the covariance proportion also makes it difficult to distinguish between the AARDL and the naive models as the RS5, and RS10 model forecasts give very inconsistent results in this respect. The MAPE in the RS10 models is marginally lower for the ARDL1 RS10 DV10 model than the AAR2 model, at 23.1% and 25.1% respectively. The ARDL1 model uses the first lag of DV10 but, as reported in Table 8, the positive coefficient on this variable was not significant at $\alpha < 10\%$, as it had a p-value of 18.8%. Also, unlike the RS15 and RS20 models, all coefficients on lagged changes in concentration are positive in the RS5 and RS10 models. This means that the small improvement in out-of-sample forecasting ability for models that include lagged change in concentration variables should be treated with caution. In fact, the covariance proportion of the MSE is higher in the simple AAR2 model than it is in the AARDL model for the RS10 models. The MAPE of the RS5 naive AAR2 model is 24.2% and the TIC is 18%.[98] This compares to an MAPE and TIC of 24% and 17.9% respectively for the AARDL DV5 model. The covariance proportion of the AARDL model is 64.6% compared to 65.8% in the naive model, while the bias proportions are the same.

---

[98] This result compares favourably with those of other published forecasts of realised volatility, when it is noted that that the covariance proportion of the RMSE dominates the bias and variance proportion at 66%, 7% and 27% respectively.

The results of models that describe the relationship between realised FTSE 100 Index volatility and differenced concentration over the period from January 1998 through March 2003 have provided little evidence of association between these variables. The next section summarises the model results estimated using preceding five-year sub-periods, and the period as a whole, to ascertain whether or not the results of the most recent five years are valid over an extended period of time, or merely reflect the most recent evolution of volatility and concentration in the FTSE 100 index.

9.4 Summary results from earlier sub-periods and the whole period

AARDL models were estimated for RS10, RS15 and RS20 using the four measures of differenced concentration as the distributed lag variables. Similar specifications to those that yielded the results reported in section 9.2 were applied. The only material difference was that for periods encompassing the 19th October 1987 a dummy variable was included. The periods over which model coefficients were estimated varied according to the dependent variable, as detailed in Table 13. The sub-period selection method adopted for the RS10 models breaks the data into identifiable regimes based on the time series charts reported in Chapter 8. The sub-period selection method, for the RS15 and RS20 models, involved a trade off between finding periods long enough to include a large number of data observations and separating the data into distinct regimes. In the end, a ten-year sub-period from January 1991 through December 2000 was selected. This had the benefit of excluding the 1987 crash while still including ten full years of data.

Table 13 Sub-periods for model estimation within the 1984 – 2003 study period

Seven periods were identified over which models where estimated for the three dependent variables, RS10, RS15 and RS20. The entry “Yes” indicates that model coefficients were estimated for that variable, the entry “No” indicates that model coefficients were not estimated for that variable, in the respective period[99].

Period	RS10	RS15	RS20
January 1998 through March 2003	Yes	Yes	Yes
January 1984 through March2003	Yes	Yes	Yes
January 1984 through December 1987	Yes	*No*	*No*
January 1988 through December 1992	Yes	*No*	*No*
January 1993 through December 1997	Yes	*No*	*No*
January 1984 through December 1990	*No*	Yes	Yes
January 1991 through December 2000	*No*	Yes	Yes

[99] The period January 1984 through March 2003 utilises the same data used for estimating the model coefficients in the previous section. However, no data is reserved for out-of-sample forecasts, so the coefficients are estimated using data up to and including the most recent available at the end of March and the middle of April 2003. The last RS15 estimated included data to the 31st March 2003, the last RS20 estimate included data to the 28th March 2003. However, the higher frequency RS5 and RS10 series included data to the 14th and 7th of April 2003 respectively.

A total of sixty AARDL models were estimated for the three realised volatility variables over the eight periods identified in Table 13. Panel A of Table 14 summarises the results of eight of these models that had coefficients on the first lag of differenced concentration that were significantly different from zero at the $\alpha < 10\%$ threshold, of which five had coefficients that were also significant at the $\alpha < 5\%$ threshold.

Table 14 Results summary for AARDL models with significant coefficients on lagged differenced concentration and contemporaneous differenced concentration

Results from the period 1998-2003 relate to models estimated using all the observations through to the end of March 2003. The reference codes identify the dependent variable and the distributed lag variable respectively. This is followed by the time period over which model parameters were estimated and whether or not models that spanned this time period included a 1987-crash dummy. The entry "yes and no" means that the relevant coefficient was significant both when the crash dummy was included and when it was omitted. The sign of the coefficient is a useful means of evaluating the consistency of the result over different estimation periods. The adjusted R^2 for the relevant AARDL model is also reported. This is followed by an indication of the presence, or otherwise, of residual autocorrelation. Here, the entry "yes and no" indicates that autocorrelation is present in models that include the 1987 crash dummy but not in models that exclude it. The null hypothesis that the model residuals are normally distributed can be rejected for all of the model results reported.

Panel A: Results summary for models in which coefficients are significantly different from zero on lagged differenced concentration

Model reference	RS10 DV10	RS10 DV10	RS10 DV10	RS15 DSK15	RS15 DH15	RS15 DSK15	RS20 DV20	RS20 DH20
Time period	1984-2003	1993-1997	1998-2003	1984-2003	1991-2000	1991-2000	1984-2003	1998-2003
1987 crash dummy included	Yes	NA	NA	Yes and No	NA	NA	Yes	NA
Coefficient significant at 5%	Yes	Yes	No	Yes	No	Yes	Yes	No
Coefficient significant at 10%	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sign of coefficient	+	+	+	+	+	+	+	-
AARDL model adjusted R^2	60%	40%	43%	46%	47%	49%	56%	37%
Residual autocorrelation	Yes	No	No	Yes	No	No	Yes	No

Panel B: Results summary for models in which coefficients are significantly different from zero on contemporaneous differenced concentration

Model reference	RS10 DH10	RS10 DR10	RS10 DSK10	RS10 DV10	RS10 DH10
Time period	1984-2003	1984-2003	1984-2003	1988-1992	1998-2003
1987 Crash dummy included	Yes and No	Yes and No	Yes and No	NA	No
Coefficient significant at 5%	Yes	No	No	No	No
Coefficient significant at 10%	Yes	Yes	Yes	Yes	Yes
Lagged coefficient	No	No	No	No	No
Sign of coefficient	+	+	+	-	+
AARDL model adjusted R^2	38%	38%	36%	13%	44%
Residual autocorrelation	Yes and No	Yes and No	Yes and No	No	No

All of the significant model coefficients reported in panel A of Table 14 were positive except for the RS20 DH20 model, which corresponds with the significant negative coefficients found in the RS20 DH20 model results reported over the same 1998 – 2003 time period in section 9.2. Overall, eight models with coefficients significant at the 10% threshold, out of a total of sixty, is only slightly more than would be expected due to chance. Therefore a conclusion that changes in concentration are significantly related to future realised volatility, based on the results reported here, would be presumptuous, particularly given the inconsistent sign of the coefficients.

Twenty-eight of the total of sixty models were estimated with both contemporaneous and lagged coefficients on differenced concentration. Panel B of Table 14 summarises the results of five of the twenty-eight models with coefficients on contemporaneous differenced concentration that were significantly different from zero at the $\alpha < 10\%$ threshold, one of which was also significant at the $\alpha < 5\%$ threshold. Once again, there is inconsistency in the coefficient signs in the small number of statistically significant results. Therefore, as with the results reported in Table 14, as well as in Table 8 through Table 11 presented section 9.2, it is not feasible to conclude from these model results that changes in concentration have a consistent effect on either current or future realised volatility.

9.5 Summary

This chapter has reported results from models of realised volatility on contemporaneous and lagged differences in concentration. It has presented a detailed analysis of model results for the period from 1998 – 2003, including out-of-sample forecasts for the period from January through March 2003. There is some rather limited evidence of an inverse relationship between changes in concentration and future realised volatility in the period January 1998 through December 2002, when volatility is measured using the Hirschman-Herfindahl concentration index and Hannah and Kay's R. However, there is also some evidence, particularly in earlier periods, of a positive relationship between changes in the variance of the logarithm of firm size, V^2, and future realised volatility. Unfortunately, the evidence is not sufficiently robust to allow either a positive or a negative association to be inferred. Overall, the relationship between changes in concentration and total realised volatility appears at best ambiguous and at worst non-existent according to the results presented in this chapter. In the light of these findings it is safest to conclude that changes in concentration are not a useful input for models that aim to forecast future realised volatility directly. By contrast, clear evidence is found in support of the well-documented asymmetry effect whereby negative market returns are followed by increases in future realised volatility. Furthermore, OLS models that account for this affect using a dummy variable appear to have improved forecasting ability.

A possible reason for the inability of the general AARDL models to provide better explanations and out-sample-forecasts of realised volatility than naive models was discussed earlier in the methodology Chapter 6. The explanation is based on the fact that total realised volatility of an index portfolio, such as the FTSE 100, is made up of the sum of the VCM of security returns. The effects of changes in concentration upon future realised volatility depend upon whether the change in the distribution of firm weights results in the concentration of index portfolio assets into securities which have a low covariance with the market as a whole, or into securities which have a high covariance with the market as a whole. All other things being equal, the former will have the effect of reducing future realised volatility, while the latter will increase it. Likewise, if the covariance is unchanged and assets are concentrated in securities that have a higher than average variance, portfolio volatility can be expected to increase and vice-a-versa. Hence the affect of changes in concentration on future realised volatility is ambiguous. It depends upon which of the above situations dominates, or to what extent the realisation of the conflicting scenarios cancel out each other’s effects. Chapter 6 of the methodology section outlines a method for decomposing the VCM of FTSE 100 Index volatility into sub-components, so that the effect of changes in concentration on the realisation of each of the above conflicting scenarios can be studied in isolation from their counterparts. These sub-components are modelled separately and the results of the models are discussed in the following Chapter.

Chapter 10 – Results III: Concentration and VCM sub-component models

10.1 Introduction

The previous chapter reported results of direct models of realised volatility and four measures of differenced concentration. For reasons outlined in Chapter 8 and Chapter 9 the focus of the discussion in Chapter 10 is upon models estimated in the five-years from January 1998 through December 2002. The remaining data from January through March 2003 is used to evaluate out-of-sample forecasts based on these models. Results of models estimated over the whole study period from January 1984 through March 2003, and sub-periods within this time frame, are summarised briefly in this chapter with the emphasis on highlighting inconsistencies with the period from January 1998 through March 2003.

The majority of model results reported in Chapter 9 did not provide any evidence of a direct relationship between differenced concentration and realised volatility. A possible reason, suggested in the conclusion to Chapter 9, is the potentially ambiguous affect of changes in concentration upon the VCM of index constituent returns. Chapter 6 details a method of decomposing the VCM into four sub-components. This allows changes in concentration to be modelled against individual sub-components, so that the influence of changing concentration upon one component of volatility is not hidden by a confounding influence on another component. It is the results of these models that are reported in the current chapter.

In addition to the introduction and conclusion, the current chapter is divided into five sections: one section for each of the four sub-components of the VCM and one section for the combined incremental components, referred to as the incremental realised volatility, IRV. Each of the five sections follows a similar format to that of the previous chapter. First, the results of models estimated over the sub-period from January 1998 through December 2002 are reported and discussed. This is followed by a report and discussion of the out-of-sample forecasts of the respective sub-component over the period from January 2003 through March 2003. Summary results are then provided and discussed for models estimated over the whole period from January 1984 through March 2003 and sub-periods within this time frame. Finally, a summary is provided to round up the results for each sub-component of realised volatility.

10.2 Differenced concentration and the equally weighted average variance (EAV)

10.2.1 Model results for the sub-period: January 1998 – December 2002

Each table discussed in this section reports model results for one of the four equally weighted average variance (EAV) estimates that comprise the dependent variables. These are EAV5, EAV10, EAV15 and EAV20 estimated with a T equal to five, ten, fifteen and twenty trading-days respectively. A total of thirty-two AARDL models are estimated: two for each of the four differenced concentration metrics on each of the four dependent variable data series. One AARDL model includes both a contemporaneous differenced concentration (DC) coefficient and a lagged differenced concentration (DC-1) coefficient. The other AARDL model only includes the DC-1 coefficient.

10.2.1.1 Coefficients on differenced concentration

Equally weighted averages are influenced more by the behaviour of smaller firms than value weighted averages. Therefore, if the equally weighted average variance is increasing, relative to the value-weighted average, it implies that the returns of smaller firms are becoming riskier. The results in Table 15 through Table 16 suggest that increases in contemporaneous and lagged concentration are associated with increases in the corresponding EAV values. Fifteen out of thirty-two AARDL models have positive coefficients that are significantly different from zero at the $\alpha < 5\%$ threshold for the first lag of differenced concentration (DC-1). A further three are significant at the $\alpha < 10\%$ threshold. Seven of the sixteen coefficients estimated in models with both DC-1 and contemporaneous differenced concentration (DC) data are positive and significant at the 5% level; another one is significant at the $\alpha < 10\%$ threshold. Thus, a positive association between differenced concentration and equally weighted average variance is indicated for both lagged and contemporaneous differenced concentration.

A possible explanation for these findings, suggested by the author, is that when investors anticipate that the market as a whole is becoming more risky they concentrate their capital into firms that they perceive as having either below average risk or below average covariance with the rest of the market. In other words, investors adopt a mean variance optimisation strategy consistent with their expectations concerning risk and return. Evidence presented later in the current chapter is consistent with the idea that investors concentrate their capital into firms that have a below average covariance, but not a below average variance.

10.2.1.2 Model comparisons

In addition to evaluating the statistical significance of individual coefficients on contemporaneous and lagged differenced concentration. The overall explanatory power of the models can be investigated by comparing values of the adjusted R^2, AIC and SIC. The highest adjusted R^2, at 35.5%, and the lowest AIC and SIC values for the models of EAV20, are obtained using the naive AAR1 model. This suggests that differenced concentration has no explanatory power for the EAV20 series over this time period, even when coefficients are significant at the α <5% threshold. In contrast, the EAV15 model results indicate that the inclusion of both contemporaneous and lagged values of DV15 results in the best fit, with an adjusted R^2 of 57.5% compared to 53.2% for the naive AAR1 model. AIC and SIC are also more negative for this model. The adjusted R^2 is around 55% in the EAV10 models that include DC-1 coefficients, 57% in models with both DC and DC-1 coefficients and only 52% in the naive AAR2 model. AIC and SIC are also generally more negative in all models that include DC and DC-1 coefficients than in the naive models. Among the EAV5 models, DV10 is the only concentration metric that improves model fit over the naive models when degrees of freedom are adjusted for, but no DC-1 or DC coefficients are significant at the α < 5% threshold.

10.2.1.3 Coefficients on the lagged asymmetric dummy variable (DEAV – 1)

Twenty-three of the thirty-two AARDL and four AAR models have positive asymmetric coefficients that are different from zero at the α < 10% threshold. Sixteen of these coefficients are also significant at the α < 5% threshold. Furthermore, the adjusted R^2, AIC and SIC values indicate that the models with an asymmetric coefficient fit better and thus have more explanatory power than models without the asymmetric coefficient. Thus the asymmetry effect, observed in the results discussed in Chapter 9, is also present in the equally weighted average variance of FTSE 100 constituent returns.

10.2.1.4 Residual analysis

Models of the EAV15 data have residuals that are closest to the OLS assumption of white noise as these are the only EAV-series models that have normally distributed residuals with no evidence of autocorrelation. The means and medians of the residuals for all models are very close to zero with absolute values that are several orders of magnitude less than the standard deviations. Residuals from the EAV20 models did not have any evidence of autocorrelation. Only the naive AR1 models of the EAV5 and EAV10-series had any evidence of autocorrelation in the residuals. The null hypothesis that the residuals are normally distributed can be rejected for all models of the EAV5, EAV10 and EAV20 series due to positive skewness and excess kurtosis. Therefore, the possibility that some of the estimated regression coefficients may be biased or inconsistent cannot be ruled out, although the consistency of the results across different models does counter this to some extent.

10.2.1.5 Synopsis

In summary, the results presented in Table 15 through Table 18 provide evidence of a positive association between both lagged and contemporaneous differenced concentration and the equally weighted average variance of FTSE 100 constituent returns. The AARDL models of the differenced concentration and EAV15 provide the best evidence of this association between concentration and the equally weighted average variance of constituent returns. In these models some of the coefficients on the DC-1 and DC variables are significant while the model residuals approximate white noise.

Table 15 Model results for EAV5 and concentration differenced over five-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 259 observations, each estimated using five trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference EAV5 DV5 indicates that the equally weighted average variance is the dependent autoregressive variable and the variance of the logarithm of firm size differenced over five days is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DEAV – 1 denotes an asymmetric coefficient on the lagged DEAV data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. No autocorrelation significant at 5% was evident in any lags of the model residuals except from the AR1 model where all lags had significant autocorrelation.

Model reference	**EAV5 DV5**		**EAV5 DR5**		**EAV5 DH5**		**EAV5 DSK5**		**AAR2**	**AR2**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**			
R-squared	0.536	0.535	0.531	0.531	0.521	0.521	0.519	0.519	0.519	0.504	0.447
Adjusted R-squared	0.527	0.528	0.522	0.524	0.512	0.514	0.510	0.512	0.513	0.500	0.445
SE of the regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum squared resid	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean dependent var	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06
SD of the dependent var	4.6E-06	4.6E-06	4.6E-06	4.6E-06	4.6E-06	4.6E-06	4.6E-06	4.6E-06	4.6E-06	4.6E-06	4.6E-06
Akaike info criterion	-22.48	-22.48	-22.47	-22.47	-22.44	-22.45	-22.44	-22.45	-22.45	-22.43	-22.33
Schwarz criterion	-22.39	-22.41	-22.38	-22.40	-22.36	-22.38	-22.36	-22.38	-22.40	-22.39	-22.30
Panel B: Coefficients											
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.4042	0.4006	0.3841	0.3841	0.3784	0.3782	0.3742	0.3758	0.3768	0.4496	0.6678
AR2	0.3190	0.3255	0.3247	0.3247	0.3317	0.3316	0.3438	0.3433	0.3396	0.3236	
DEAV - 1	0.1159	0.1148	0.1153	0.1153	0.1190	0.1190	0.1235	0.1231	0.1220		
DC - 1	0.0000	0.0000	0.0012	0.0012	0.0002	0.0002	-0.0000	-0.0000			
DC	-0.0000		-0.0000		-0.0000		0.0000				
Panel C: Coefficient p-values											
Intercept	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000
AR1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
DEAV - 1	0.0235	0.0259	0.0221	0.0224	0.0167	0.0165	0.0154	0.0157	0.0168		
DC - 1	0.0811	0.0763	0.0469	0.0444	0.3417	0.3340	0.5624	0.5625			
DC	0.5080		0.9971		0.9859		0.8889				
Panel D: Residual descriptive statistics											
Mean	-7.0E-22	-5.2E-22	-9.2E-23	-4.8E-22	-7.6E-22	-1.5E-21	-4.5E-22	-6.1E-22	-8.7E-22	-6.2E-22	8.6E-22
Median	-4.4E-07	-4.2E-07	-5.0E-07	-5.0E-07	-4.9E-07	-4.9E-07	-4.4E-07	-4.5E-07	-4.6E-07	-4.7E-07	-7.7E-07
Maximum	1.7E-05	1.7E-05	1.7E-05	1.7E-05	1.7E-05	1.7E-05	1.7E-05	1.7E-05	1.7E-05	1.8E-05	1.6E-05
Minimum	-7.4E-06	-7.5E-06	-8.9E-06	-8.9E-06	-1.0E-05	-1.0E-05	-1.1E-05	-1.1E-05	-1.1E-05	-9.0E-06	-1.0E-05
Std. Dev.	3.1E-06	3.1E-06	3.1E-06	3.1E-06	3.2E-06	3.2E-06	3.2E-06	3.2E-06	3.2E-06	3.2E-06	3.4E-06
Skewness	1.69	1.66	1.68	1.68	1.61	1.61	1.58	1.57	1.57	1.69	1.70
Kurtosis	8.58	8.39	8.61	8.61	8.52	8.53	8.49	8.45	8.44	8.60	8.19

Table 16 Model results for EAV10 and concentration differenced over ten-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 126 observations, each estimated using ten trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference EAV10 DV10 indicates that the equally weighted average variance is the dependent autoregressive variable and the variance of the logarithm of firm size differenced over ten trading days is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DEAV – 1 denotes an asymmetric coefficient on the lagged DEAV data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. No autocorrelation significant at 5% was evident in any lags of the model residuals except from the AR1 model where all lags had significant autocorrelation.

Model reference	**EAV10 DV10**		**EAV10 DR10**		**EAV10 DH10**		**EAV10 DSK10**		**AAR2**	**AR2**	**AR1**
Panel A: Tests of model fit	**DC & DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**			
R-squared	0.589	0.559	0.593	0.561	0.578	0.560	0.559	0.551	0.531	0.477	0.478
Adjusted R-squared	0.572	0.545	0.576	0.546	0.561	0.546	0.541	0.536	0.520	0.468	0.474
SE of the regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum squared resid	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean dependent var	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06
SD of the dependent var	4.3E-06	4.3E-06	4.3E-06	4.3E-06	4.3E-06	4.3E-06	4.3E-06	4.3E-06	4.3E-06	4.3E-06	4.3E-06
Akaike info criterion	-22.70	-22.64	-22.71	-22.65	-22.67	-22.65	-22.63	-22.63	-22.60	-22.51	-22.52
Schwarz criterion	-22.56	-22.53	-22.57	-22.53	-22.54	-22.53	-22.49	-22.51	-22.51	-22.44	-22.48
Panel B: Coefficients											
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.4980	0.4708	0.4201	0.4419	0.4224	0.4524	0.4595	0.4817	0.5258	0.6639	0.6913
AR2	0.1870	0.1736	0.1813	0.1534	0.1544	0.1304	0.1403	0.1187	0.1017	0.0357	
DEAV - 1	0.2242	0.2297	0.2587	0.2662	0.2772	0.2742	0.2671	0.2611	0.2241		
DC - 1	0.0000	0.0000	0.0014	0.0012	0.0005	0.0005	0.0000	0.0000			
DC	0.0000		0.0012		0.0004		0.0000				
Panel C: Coefficient p-values											
Intercept	0.0109	0.0042	0.0005	0.0020	0.0008	0.0012	0.0012	0.0012	0.0033	0.0022	0.0001
AR1	0.0000	0.0000	0.0001	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR2	0.0374	0.0625	0.0434	0.1323	0.1174	0.2120	0.2111	0.2745	0.3643	0.8054	
DEAV - 1	0.0023	0.0016	0.0002	0.0002	0.0001	0.0002	0.0005	0.0008	0.0034		
DC - 1	0.0040	0.0033	0.0001	0.0001	0.0001	0.0000	0.0069	0.0036			
DC	0.0183		0.0488		0.1272		0.2293				
Panel D: Residual descriptive statistics											
Mean	7.9E-22	7.1E-22	2.2E-22	-4.0E-22	4.0E-22	-7.9E-22	2.2E-22	-6.7E-22	1.6E-22	-6.1E-22	-3.0E-22
Median	-3.2E-07	-5.6E-07	-3.4E-07	-3.7E-07	-3.9E-07	-4.6E-07	-4.9E-07	-4.2E-07	-5.8E-07	-5.7E-07	-5.6E-07
Maximum	9.4E-06	1.2E-05	1.0E-05	1.3E-05	1.1E-05	1.3E-05	1.3E-05	1.3E-05	1.3E-05	1.4E-05	1.3E-05
Minimum	-7.5E-06	-7.3E-06	-6.0E-06	-6.9E-06	-5.6E-06	-6.0E-06	-5.9E-06	-6.3E-06	-6.8E-06	-7.3E-06	-7.5E-06
Std. Dev.	2.7E-06	2.8E-06	2.7E-06	2.8E-06	2.8E-06	2.8E-06	2.8E-06	2.9E-06	2.9E-06	3.1E-06	3.1E-06
Skewness	0.59	1.01	0.95	1.13	1.09	1.13	1.18	1.15	1.12	1.18	1.17
Kurtosis	4.64	5.85	4.70	6.11	5.28	6.00	5.87	5.99	5.90	6.40	6.36

Table 17 Model results for EAV15 and concentration differenced over fifteen-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 84 observations, each estimated using fifteen trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference EAV15 DV15 indicates that the equally weighted average variance is the dependent autoregressive variable and the variance of the logarithm of firm size differenced over fifteen trading days is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DEAV – 1 denotes an asymmetric coefficient on the lagged DEAV data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed could **not** be rejected at the 5% level of significance for any models except the EAV15 DH15, EAV15 DSK and the AR1 models. No residual autocorrelation significant at 5% was evident in any of the models.

Model reference	**EAV15 DV15**		**EAV15 DR15**		**EAV15 DH15**		**EAV15 DSK15**		**AAR1**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.595	0.585	0.587	0.568	0.577	0.557	0.576	0.558	0.543	0.514
Adjusted R-squared	0.575	0.570	0.566	0.552	0.555	0.541	0.555	0.541	0.532	0.508
SE of the regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum squared resid	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean dependent var	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06
SD of the dependent var	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06
Akaike info criterion	-22.84	-22.84	-22.82	-22.80	-22.79	-22.77	-22.79	-22.77	-22.77	-22.73
Schwarz criterion	-22.69	-22.72	-22.68	-22.68	-22.65	-22.66	-22.65	-22.66	-22.68	-22.67
Panel B: Coefficients										
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.6957	0.6720	0.6309	0.6117	0.6211	0.6094	0.6304	0.6213	0.6544	0.7153
DEAV - 1	0.1181	0.1292	0.1390	0.1866	0.1522	0.1942	0.1697	0.1880	0.1545	
DC - 1	0.0000	0.0000	0.0009	0.0010	0.0002	0.0003	0.0000	0.0000		
DC	0.0000		0.0009		0.0003		0.0000			
Panel C: Coefficient p-values										
Intercept	0.0057	0.0047	0.0011	0.0019	0.0015	0.0017	0.0023	0.0018	0.0046	0.0060
AR1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DEAV - 1	0.2162	0.1767	0.1788	0.0880	0.1867	0.0972	0.1586	0.1073	0.1799	
DC - 1	0.0524	0.0600	0.0384	0.0478	0.1061	0.1060	0.0287	0.0258		
DC	0.1416		0.0684		0.0233		0.0000			
Panel D: Residual descriptive statistics										
Mean	-1.1E-21	-2.0E-21	-2.3E-21	-1.5E-21	-2.1E-21	-2.0E-21	-2.3E-21	-1.9E-21	-1.4E-21	-1.3E-21
Median	-3.3E-07	-2.5E-07	-2.3E-07	-3.5E-07	-3.6E-07	-3.9E-07	-3.8E-07	-2.9E-07	-2.9E-07	-3.2E-07
Maximum	6.7E-06	7.5E-06	7.4E-06	8.3E-06	8.2E-06	8.6E-06	8.5E-06	8.5E-06	8.6E-06	9.0E-06
Minimum	-5.6E-06	-5.9E-06	-6.0E-06	-6.3E-06	-6.5E-06	-6.6E-06	-6.5E-06	-6.5E-06	-6.9E-06	-8.1E-06
Std. Dev.	2.5E-06	2.6E-06	2.5E-06	2.6E-06	2.6E-06	2.6E-06	2.6E-06	2.6E-06	2.7E-06	2.8E-06
Skewness	0.39	0.41	0.43	0.38	0.44	0.37	0.47	0.41	0.43	0.52
Kurtosis	3.04	3.44	3.56	4.01	4.18	4.13	4.37	4.00	3.91	4.34

Table 18 Model results for EAV20 and concentration differenced over twenty-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results obtained using 64 observations, each estimated using twenty trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference EAV20 DV20 indicates that the equally weighted average variance is the dependent autoregressive variable and the variance of the logarithm of firm size differenced over twenty trading days is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DEAV – 1 denotes an asymmetric coefficient on the lagged DEAV data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. No autocorrelation significant at 5% was evident in any of the model residuals.

Model reference	EAV20 DV20		EAV20 DR20		EAV20 DH20		EAV20 DSK20		AAR1	AR1
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.393	0.393	0.462	0.381	0.474	0.384	0.414	0.376	0.375	0.340
Adjusted R-squared	0.352	0.363	0.425	0.350	0.438	0.353	0.374	0.345	0.355	0.330
SE of the regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum squared resid	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean dependent var	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06	8.0E-06
SD of the dependent var	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06
Akaike info criterion	-22.39	-22.42	-22.51	-22.40	-22.53	-22.41	-22.43	-22.39	-22.42	-22.40
Schwarz criterion	-22.22	-22.29	-22.34	-22.27	-22.36	-22.27	-22.26	-22.26	-22.32	-22.33
Panel B: Coefficients										
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.4746	0.4748	0.5046	0.4587	0.5187	0.4606	0.4842	0.4446	0.4455	0.5804
DEAV - 1	0.1476	0.1476	0.1284	0.1850	0.1099	0.1928	0.1318	0.1660	0.1717	
DC - 1	0.0000	0.0000	-0.0002	-0.0005	-0.0000	-0.0002	0.0000	0.0000		
DC	-0.0000		0.0018		0.0006		0.0000			
Panel C: Coefficient p-values										
Intercept	0.0113	0.0048	0.0060	0.0074	0.0063	0.0049	0.0139	0.0071	0.0073	0.0097
AR1	0.0168	0.0126	0.0038	0.0179	0.0069	0.0130	0.0274	0.0261	0.0250	0.0005
DEAV - 1	0.2210	0.2169	0.2441	0.0636	0.3897	0.0700	0.2654	0.1064	0.0860	
DC - 1	0.2749	0.2804	0.7271	0.3103	0.8428	0.3794	0.3049	0.5350		
DC	0.9884		0.0212		0.0073		0.0438			
Panel D: Residual descriptive statistics										
Mean	-8.1E-22	-1.8E-21	-1.8E-21	-2.3E-21	-1.4E-21	-1.2E-21	-9.8E-22	-9.3E-22	-1.2E-21	-1.5E-21
Median	-6.9E-07	-6.9E-07	-5.4E-07	-6.0E-07	-6.8E-07	-7.9E-07	-6.5E-07	-8.3E-07	-8.0E-07	-8.0E-07
Maximum	1.3E-05	1.3E-05	1.2E-05	1.4E-05	1.2E-05	1.3E-05	1.3E-05	1.4E-05	1.4E-05	1.4E-05
Minimum	-5.8E-06	-5.8E-06	-5.7E-06	-6.3E-06	-5.7E-06	-6.3E-06	-6.2E-06	-6.5E-06	-6.5E-06	-5.4E-06
Std. Dev.	3.1E-06	3.1E-06	2.9E-06	3.1E-06	2.9E-06	3.1E-06	3.1E-06	3.1E-06	3.1E-06	3.2E-06
Skewness	1.55	1.55	1.23	1.54	1.35	1.49	1.68	1.61	1.58	1.70
Kurtosis	6.58	6.59	5.79	7.27	6.09	6.96	8.01	7.43	7.34	7.58

Table 19 Model forecasts January – April 2003

Final data points are as follows: EAV5, 16th April, EAV10, 7th April 2003. The difference is due to the fact that estimates are non-overlapping and hence not perfectly synchronised for different estimation periods over the whole sample from 1984 through 2003.

Panel A: EAV5 out-of-sample forecasts based on trading days from 19th December 2002 through 16th April 2003						
Forecasting model	**AR2**	**AAR2**	**AARDL2 DV5**	**AARDL2 DR5**	**AARDL2 DH5**	**AARDL2 DSK5**
Forecast variable	EAV5	EAV5	EAV5	EAV5	EAV5	EAV5
Forecast sample:	261 275	261 275	261 275	261 275	261 275	261 275
Included observations:	14	14	14	14	14	14
Root Mean Squared Error	4.1E-06	3.8E-06	3.6E-06	3.8E-06	3.8E-06	3.8E-06
Mean Absolute Error	3.4E-06	3.1E-06	3.0E-06	3.1E-06	3.1E-06	3.1E-06
Mean Abs. Percent Error	41.08	38.29	37.94	39.77	38.59	38.47
Theil Inequality Coefficient	0.223	0.207	0.198	0.206	0.206	0.208
Bias Proportion	0.000	0.000	0.000	0.000	0.000	0.000
Variance Proportion	0.137	0.235	0.283	0.248	0.247	0.230
Covariance Proportion	0.863	0.765	0.716	0.751	0.753	0.770
Panel B: EAV10 out-of-sample forecasts based on trading days from 28th December 2002 through 7th April 2003						
Forecasting model	**AR2**	**AAR2**	**AARDL2 DV10**	**AARDL2 DR10**	**AARDL2 DH10**	**AARDL2 DSK10**
Forecast variable	EAV10	EAV10	EAV10	EAV10	EAV10	EAV10
Forecast sample:	128 134	128 134	128 134	128 134	128 134	128 134
Included observations:	7	7	7	7	7	7
Root Mean Squared Error	3.9E-06	3.3E-06	3.2E-06	3.1E-06	3.1E-06	3.2E-06
Mean Absolute Error	2.7E-06	2.3E-06	2.1E-06	2.1E-06	2.1E-06	2.2E-06
Mean Abs. Percent Error	26.87	23.41	20.84	21.80	22.05	22.89
Theil Inequality Coefficient	0.218	0.190	0.182	0.180	0.181	0.186
Bias Proportion	0.018	0.065	0.063	0.063	0.073	0.078
Variance Proportion	0.084	0.279	0.348	0.415	0.432	0.364
Covariance Proportion	0.898	0.656	0.589	0.522	0.494	0.558

10.2.2 Forecasts for the period: January 2003 – April 2003

The model results reported in Table 15 through Table 18 were estimated using data recorded over the period from January 1998 through December 2002. The remaining data in the series, from January to April 2003, was used to test the out-of-sample forecasting ability of the models. Table 19 compares out-of-sample forecasts of EAV5 and EAV10 derived from general AARDL models with those derived from naive AAR and AR models. Forecast evaluation tests for static out-of-sample forecasts are presented, following the same format as that adopted in Table 12 in section 9.3 of Chapter 9, and the same explanation as that provided on page 177 is relevant here, the only difference being that the dependent variables in the model are the equally weighted average variance of constituent returns rather than the realised volatility.

The forecast results are mixed, although they provide some indication that the AARDL models incorporating lagged values of differenced concentration can provide better forecasts than the naive AAR models. The AARDL models that include the DV measure of differenced concentration do have lower MAPE and TIC values than the naive models, indicating that these models can produce more reliable out-of-sample forecasts. Analysis of the composition of the MSE reveals that the covariance proportion dominates the variance proportion, which in turn dominates the bias proportion of the forecasts of EAV10.

To conclude this sub-section, the models that produce the best forecasts for EAV10 are AARDL models that include the first lag of the respective DV series. There is limited evidence to suggest that including this measure of differenced concentration in forecasting models can improve the out-of-sample forecasts of the equally weighted average variance of constituent returns compared to naive AAR models. Furthermore, there is evidence to suggest that naïve AAR models have more forecasting power than the more parsimonious naïve AR models.

10.2.3 Summary results from earlier sub-periods and the whole period

In addition to estimating model parameters over the sub-period from January 1998 through March 2003, similar models were estimated for the three series EAV10, EAV15 and EAV20, over different sub-periods and the whole period from 1984 through 2003. These sub-periods are the same as those defined in Table 13 and the procedure followed is identical to that detailed on page 179, the only difference being that the dependent variables in the model are the equally weighted average variance of constituent returns rather than the realised volatility.

Table 20 summarises the results for the thirty-six models that have DC-1 coefficients significantly different from zero, at the $\alpha < 10\%$ threshold, from the total of ninety-six AARDL and ARDL models estimated for the EAV10, EAV15 and EAV20 data. Twenty-two out of the thirty-six models had DC-1 coefficients that were also significant at the $\alpha < 5\%$ threshold. Models are identified, in Table 20, using the model reference code described in the table legend. Panel A of Table 20 displays the results of twelve models out of a total of thirty-six estimated over the entire study period. Panel B displays the results of similar models estimated after the 1987 crash while panel C summarises the results of models estimated between January 1998 and March 2003.

The null hypothesis that the model residuals are normally distributed can be rejected in all models reported in Table 20. The significance levels of all model coefficients were determined using Newey-West heteroskedasticity and autocorrelation robust standard errors. The number of DC-1 coefficients that were significantly different from zero, at the $\alpha < 10\%$ threshold, was much greater than would have been expected if the first lag of differenced concentration had no explanatory power for forecasting the EAV data. In fact more than a third had coefficients significant at the $\alpha < 10\%$ threshold and more than a fifth had coefficients significant at the $\alpha < 5\%$ threshold. The general to specific modelling procedure allowed variables that did not have significant coefficients to be identified as redundant and, therefore, legitimately discarded, although their results are included in the above totals to avoid charges of data mining.

The results provide substantial evidence of a relationship between changes in concentration and future changes in the equally weighted average variance of constituent returns in the FTSE 100 Index. This relationship is robust across the four different measures of concentration used and over the different time periods. It is also consistent in the sign and the level of significance when the EAV data is estimated using a T value of five, ten and fifteen trading-days. The only inconsistency is in the models of the EAV20 data and all models estimated exclusively in the period 1984 – 1997 and 1984 – 1990. However, this period included the 1987 crash, which had a major distorting effect on the underlying data and on the behaviour of model residuals. Therefore it is reasonable to say that the results of the models estimated in this period are more difficult to evaluate even when the crash anomaly is mitigated using a dummy variable. The coefficients on the 1987 crash dummy variable were all positive and significantly different from zero in models that included this period. This demonstrates that the equally weighted average variance of FTSE 100 Index constituent returns made a positive contribution to the spike in realised volatility that defined this event.

Table 20 Summary results for models of EAV that have significant coefficients on lagged differenced concentration

Results reported for models estimated over the period 1998-2003 relate to models estimated using all the observations through to the end of March 2003. Therefore including the out-of-sample data that is used to evaluate the similar models reported in the previous section 10.2.1 where models are estimated using data up to the end of December 2002. The reference code identifies the dependent variable, the type of model, i.e. AARDL or ARDL and the distributed lag variable. For example, model reference EAV10 DH10 AARDL2 refers to an asymmetric autoregressive distributed lag model. This has two autoregressive lags of the dependant variable EAV10, one lagged asymmetric variable DEAV10 and one distributed lag variable DH10.

Panel A: Models estimated over the entire study period from January 1984 through March 2003

Model reference	EAV10 DH10 AARDL2	EAV10 DR10 AARDL2	EAV10 DV10 AARDL2	EAV15 DH15 AARDL2	EAV15 DR15 AARDL2	EAV15 DV15 AARDL2	EAV15 DSK15 AARDL2	EAV15 DSK15 AARDL2	EAV20 DV20 AARDL2	EAV20 DV20 AARDL2	EAV20 DV20 ARDL2	EAV20 DSK20 ARDL2
Time period	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003
1987 Crash dummy included	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No
Coefficient significant at 5%	Yes	Yes	Yes	No	Yes	Yes	Yes	No	Yes	No	No	No
Coefficient significant at 10%	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sign of coefficient	+	+	+	+	+	+	+	+	+	+	+	+
Adjusted R^2	72%	72%	73%	71%	72%	71%	71%	62%	68%	56%	56%	54%
Residual autocorrelation	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No

Panel B: EAV10 models estimated using data from the period from January 1988 through December 1992, plus EAV15 and EAV20 models estimated using data from the period from January 1991 through December 2000

Model reference	EAV10 DR10 ARDL2	EAV10 DV10 AARDL2	EAV10 DV10 AARDL2	EAV15 DH15 AARDL2	EAV15 DH15 ARDL2	EAV15 DR15 AARDL2	EAV15 DR15 ARDL2	EAV15 DV15 AARDL2	EAV15 DV15 ARDL2	EAV15 DSK15 AARDL2	EAV15 DSK15 ARDL2	EAV20 DR20 ARDL1	EAV20 DV20 ARDL1
Time period	1988 1992	1988 1992	1988 1992	1991 2000	1991 2000	1991 2000	1991 2000	1991 2000	1991 2000	1991 2000	1991 2000	1991 2000	1991 2000
Coefficient significant at 5%	No	No	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No	No
Coefficient significant at 10%	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sign of coefficient	+	+	+	+	+	+	+	+	+	+	+	+	+
Adjusted R^2	29%	31%	31%	66%	66%	65%	66%	65%	65%	66%	66%	78%	79%
Residual autocorrelation	No	No	No	No	No	No	No	No	No	No	No	No	No

Panel C: EAV10 and EAV15 models estimated using data from the period: January 1998 – March 2003

Model reference	EAV10 DH10 AARDL2	EAV10 DH10 ARDL2	EAV10 DR10 AARDL2	EAV10 DR10 ARDL2	EAV10 DV10 AARDL2	EAV10 DV10 ARDL2	EAV10 DSK10 AARDL2	EAV15 DR15 AARDL2	EAV15 DV15 AARDL2	EAV15 DV15 ARDL2	EAV15 DSK15 AARDL2
Time period	1998 2003	1998 2003	1998 2003	1998 2003	1998 2003	1998 2003	1998 2003	1998 2003	1998 2003	1998 2003	1998 2003
Coefficient significant at 5%	Yes	Yes	Yes	No	Yes	Yes	Yes	No	No	No	Yes
Coefficient significant at 10%	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sign of coefficient	+	+	+	+	+	+	+	+	+	+	+
Adjusted R^2	53%	45%	54%	45%	53%	47%	52%	52%	54%	52%	52%
Residual autocorrelation	No	No	No	No	No	No	No	No	No	No	No

10.2.4 Summary of all equally weighted average variance (EAV) model results

This chapter has reported results from models of the equally weighted average variance of the FTSE 100 Index constituent returns on contemporaneous and lagged differences in concentration, where concentration was measured using four different indices. It has presented a detailed analysis of model results for the period 1998-2003, including limited out-of-sample forecasts for the period January through March 2003. Evidence is presented to suggest that positive changes in concentration are associated with both contemporaneous and future positive changes in the equally weighted average variance. Limited evidence is also presented to indicate that this relationship can be used to improve out-of-sample forecasts of the equally weighted average variance. In addition to significant coefficients on distributed lags of differenced concentration, autoregressive and asymmetric coefficients are also significant for models of this data.

It should be emphasised that these results were not anticipated ex-ante. Possible ex-post explanations are as follows. For lagged coefficients, if investors expect the average volatility of index constituents to rise without an offsetting fall in the average covariance, the implication is that they expect total realised volatility to rise. In this case, if they wish to minimise future portfolio volatility, they may concentrate their assets into securities which they expect to have a lower than average variance. Alternatively, or commensurately, they may concentrate their assets into securities that have a below average covariance in an attempt to reduce the volatility of their portfolio. If their expectations are realised, increases in average constituent return variances will be preceded by increases in concentration, as evidenced by these results. If the former explanation is correct, increases in concentration will precede decreases in the incremental average variance of security returns, represented by the IAV series, a scenario that is not consistent with the results reported in the following section. However, if the latter explanation is correct, increases in concentration will precede decreases in the incremental average covariance, represented by the IAC data. The results for models of this series, discussed in section 10.5, are consistent with this explanation. This explanation may also apply to the positive contemporaneous relationship observed if it results from investors trying to reduce the volatility of their portfolio by concentrating portfolio assets into securities with a below average covariance or below average variance as the average volatility rises. The only difference between the two explanations is that the former is concerned with adaptations to changing ex-ante expectations while the latter is concerned with contemporaneous adaptations to changing realisations of volatility. Thus the idea that investors increase portfolio concentration prior to increases in the average variance of constituent returns is suggested as a possible explanation for the positive association between changes in concentration and changes in the four EAV data. Increases in portfolio concentration would result if investors allocated more of their assets into securities that had either a below average variance or a below average covariance with the index as a whole. Such strategies would be consistent with attempts to limit risk exposure in the face of increases in expected risk.

10.3 Models of the incremental average variance (IAV)

The presentation and discussion of results in this section follows the same format as that adopted for the models of realised volatility in Chapter 9 and the models of equally weighted average variance in the preceding section.

10.3.1 Model results for the period: January 1998 – December 2002

10.3.1.1 Coefficients on the first lag of differenced concentration (DC-1)

Of the thirty-two models for which results are reported in Table 21 through Table 24 only five have coefficients on the DC-1 variables that are significantly different from zero at either the $\alpha < 10\%$ or the $\alpha < 5\%$ threshold. Three of these are positive and significant at the 5% level, one is negative and significant at the 10% level and one is negative and significant at the 5% level. Therefore, slightly more than ten percent of the models have positive DC-1 coefficients significant, at the $\alpha < 10\%$ threshold, but the inference that can be drawn is limited by the inconsistent sign of coefficients in the other models. Residual analysis indicates that models estimated using the IAV15 data as the dependent variable have residuals that are closest to the OLS assumption of white noise. In other words, they exhibit no autocorrelation and a mean and median very close to zero in relation to the standard deviation. The skewness and kurtosis of model residuals is also lower than in the IAV5, IAV10 or IAV20 models.

10.3.1.2 Coefficients on contemporaneous differenced concentration (DC)

Of the sixteen models that include contemporaneous coefficients on the DC variable, twelve have positive coefficients that are significantly different from zero at the $\alpha < 10\%$ threshold, ten of these are also significant at $\alpha < 5\%$ threshold. The overall explanatory power of the models that have significant coefficients on the contemporaneous DC variable can be investigated by comparing values of the adjusted R^2, AIC and SIC with the naive AAR and AR models. The IAV5 models all have DC coefficients significant at $\alpha < 1\%$, apart from the IAV5 DSK5 model. The adjusted R^2 values range from 37.3% to 34.2% compared to 30.5% in the naive AAR2 model. Both AIC and SIC values are also more negative, indicating a better fit in the general models than naive models. All of the IAV10 models also have DC coefficients that are significant at $\alpha < 5\%$. The adjusted R^2 values range from 33% to 39% compared to 27% and 25% in the naive AAR2 and AR2 models respectively. The AIC and SIC values are all more negative in the general models than in the naive AAR and AR models. The IAV15 models all have DC coefficients significant at 10% or less, apart from the IAV15 DSK15 model. The adjusted R^2 on these models ranges from 54.5% to 57.2% compared to 54.2% in the naive AAR model and all AIC values are more negative in the general models. However, the SIC values are more ambiguous, and in some cases, slightly less negative in the general models than the naive models, indicating that the improvement in model fit resulting from the inclusion of the contemporaneous DC variable is less evident for the IAV15 models. The IAV20 DR20 and DH20 models both have DC coefficients significant at the 5% level with adjusted R^2 values of 47% and 43%, respectively, compared to 38% for the naive AAR model. Once again the AIC values indicate a better fit for the general models but the SIC value for the IAV20 DH20 model suggests that the naive model has as good a fit, or a better fit, when and adjustment is made for the difference in degrees of freedom.

10.3.1.3 Coefficients on the asymmetric dummy variable (DIAV - 1)

Out of the thirty-two AARDL models and four AAR models estimated in this section, thirty have positive asymmetric coefficients that are significantly different from zero at the α < 10% threshold, twenty-seven of these are also significant the α < 5% threshold. In addition, comparison of adjusted R^2, AIC and SIC values indicates that models containing an asymmetric coefficient fit better and, hence, have more explanatory power than those that do not. Thus, the asymmetry effect is prevalent in the incremental average variance component of realised volatility.

10.3.1.4 Synopsis

The results presented in this section provide little evidence to suggest that changes in concentration precede either positive or negative changes in the incremental average variance of FTSE 100 Index constituent returns. The few models that do have coefficients on the DC-1 variable that are significantly different from zero have inconsistent signs, making it impossible to unambiguously identify an association between lagged changes in concentration and contemporaneous changes in the incremental average variance. In contrast, many of the models do have positive coefficients that are significantly different from zero on contemporaneous differenced concentration. There is also evidence, provided by the many DIAV - 1 coefficients that are positive and significantly different from zero, to suggest that falls in the value of the FTSE 100 Index precede increases in the incremental average variance of constituent returns, i.e. the asymmetry effect.

Table 21 Model results for IAV5 and concentration differenced over five-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 259 observations, each estimated using five trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference IAV5 DV5 indicates that the incremental average variance is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DIAV – 1 denotes an asymmetric coefficient on the lagged DIAV data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. The final row indicates the number of autocorrelation lags out of a total of thirty-six that are significant at the 5% level.

Model reference	**IAV5 DV5**		**IAV5 DR5**		**IAV5 DH5**		**IAV5 DSK5**		**AAR2**	**AR2**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**			
R-squared	0.369	0.314	0.385	0.316	0.354	0.314	0.321	0.314	0.313	0.269	0.239
Adjusted R-squared	0.357	0.303	0.373	0.305	0.342	0.303	0.308	0.304	0.305	0.263	0.236
SE of the regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum squared resid	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean dependent var	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05
SD of the dependent var	1.3E-05	1.3E-05	1.3E-05	1.3E-05	1.3E-05	1.3E-05	1.3E-05	1.3E-05	1.3E-05	1.3E-05	1.3E-05
Akaike info criterion	-20.09	-20.02	-20.12	-20.02	-20.07	-20.02	-20.02	-20.02	-20.02	-19.97	-19.94
Schwarz criterion	-20.01	-19.95	-20.03	-19.95	-19.99	-19.95	-19.94	-19.95	-19.97	-19.93	-19.91
Panel B: Coefficients											
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.2817	0.2777	0.2452	0.2667	0.2656	0.2799	0.2871	0.2869	0.2837	0.3880	0.4886
AR2	0.2273	0.2064	0.2159	0.2072	0.1911	0.2014	0.1919	0.2003	0.2008	0.2029	
DIAV - 1	0.2996	0.2859	0.2911	0.2880	0.2849	0.2872	0.2915	0.2895	0.2872		
DC - 1	0.0000	0.0000	0.0023	0.0015	0.0003	0.0002	-0.0000	-0.0000			
DC	0.0001		0.0079		0.0025		0.0000				
Panel C: Coefficient p-values											
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.0000	0.0001	0.0002	0.0003	0.0002	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
AR2	0.0020	0.0080	0.0007	0.0057	0.0031	0.0088	0.0068	0.0091	0.0087	0.0073	
DIAV - 1	0.0000	0.0001	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001		
DC - 1	0.2802	0.6272	0.1883	0.3768	0.7476	0.8355	0.5750	0.6722			
DC	0.0061		0.0033		0.0087		0.1476				
Panel D: Residual descriptive statistics											
Mean	-1.4E-21	-1.3E-21	-7.5E-22	5.5E-22	-8.4E-22	-4.9E-22	-3.3E-21	-3.4E-22	-7.9E-23	-3.0E-22	3.7E-22
Median	-2.0E-06	-2.6E-06	-2.2E-06	-2.6E-06	-2.6E-06	-2.7E-06	-2.8E-06	-2.7E-06	-2.7E-06	-2.5E-06	-2.7E-06
Maximum	4.0E-05	4.7E-05	4.1E-05	4.7E-05	4.5E-05	4.7E-05	4.8E-05	4.8E-05	4.8E-05	5.3E-05	4.9E-05
Minimum	-2.6E-05	-2.7E-05	-2.4E-05	-2.7E-05	-2.4E-05	-2.6E-05	-2.6E-05	-2.6E-05	-2.6E-05	-2.2E-05	-2.6E-05
Std. Dev.	1.0E-05	1.1E-05	1.0E-05	1.1E-05	1.0E-05	1.1E-05	1.1E-05	1.1E-05	1.1E-05	1.1E-05	1.1E-05
Skewness	1.25	1.47	1.28	1.47	1.41	1.50	1.45	1.47	1.49	1.60	1.48
Kurtosis	5.72	6.72	5.79	6.68	6.22	6.79	6.57	6.65	6.77	6.89	6.23
Autocorrelation	0	2	27	5	32	5	12	2	5	0	35

Table 22 Model results for IAV10 and concentration differenced over ten-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 127 observations, each estimated using ten trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference IAV10 DV10 indicates that the incremental average variance is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DIAV – 1 denotes an asymmetric coefficient on the lagged DIAV data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. The final row indicates the number of autocorrelation lags out of a total of thirty-six that are significant at the 5% level.

Model reference	**IAV10 DV10**		**IAV10 DR10**		**IAV10 DH10**		**IAV10 DSK10**		**AAR2**	**AR2**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**			
R-squared	0.356	0.291	0.413	0.298	0.405	0.301	0.357	0.309	0.287	0.265	0.241
Adjusted R-squared	0.329	0.267	0.388	0.275	0.380	0.278	0.330	0.286	0.269	0.253	0.235
SE of the regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum squared resid	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean dependent var	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05
SD of the dependent var	1.1E-05	1.1E-05	1.1E-05	1.1E-05	1.1E-05	1.1E-05	1.1E-05	1.1E-05	1.1E-05	1.1E-05	1.1E-05
Akaike info criterion	-20.28	-20.20	-20.37	-20.21	-20.36	-20.21	-20.28	-20.23	-20.21	-20.20	-20.18
Schwarz criterion	-20.15	-20.09	-20.24	-20.10	-20.22	-20.10	-20.15	-20.11	-20.12	-20.13	-20.14
Panel B: Coefficients											
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.3037	0.2648	0.1948	0.2129	0.1875	0.1912	0.2105	0.1913	0.2893	0.3950	0.4900
AR2	0.2692	0.2690	0.3161	0.2873	0.3086	0.2930	0.3037	0.2987	0.2449	0.1870	
DIAV - 1	0.2169	0.2055	0.2375	0.2472	0.2507	0.2726	0.2601	0.2870	0.1984		
DC - 1	0.0000	0.0000	0.0032	0.0022	0.0010	0.0010	0.0000	0.0000			
DC	0.0001		0.0063		0.0023		0.0000				
Panel C: Coefficient p-values											
Intercept	0.0003	0.0002	0.0002	0.0003	0.0002	0.0003	0.0005	0.0004	0.0003	0.0001	0.0000
AR1	0.0043	0.0268	0.0806	0.0501	0.1089	0.0785	0.0564	0.1082	0.0047	0.0000	0.0000
AR2	0.0006	0.0012	0.0000	0.0002	0.0001	0.0002	0.0001	0.0005	0.0007	0.0024	
DIAV - 1	0.0442	0.0345	0.0462	0.0118	0.0355	0.0121	0.0300	0.0298	0.0389		
DC - 1	0.3774	0.5071	0.0418	0.1507	0.1396	0.1974	0.2218	0.2808			
DC	0.0206		0.0111		0.0026		0.0039				
Panel D: Residual descriptive statistics											
Mean	1.7E-21	2.3E-21	2.4E-21	1.8E-21	2.7E-21	2.5E-21	1.2E-21	1.8E-21	1.3E-21	2.1E-21	-1.0E-21
Median	-1.5E-06	-2.0E-06	-1.5E-06	-1.3E-06	-1.9E-06	-1.6E-06	-1.7E-06	-1.7E-06	-1.9E-06	-1.5E-06	-2.0E-06
Maximum	3.7E-05	4.7E-05	4.1E-05	4.7E-05	4.0E-05	4.7E-05	4.6E-05	4.8E-05	4.8E-05	4.8E-05	4.9E-05
Minimum	-1.9E-05	-2.1E-05	-1.6E-05	-2.1E-05	-1.6E-05	-2.0E-05	-1.8E-05	-1.8E-05	-2.0E-05	-2.0E-05	-2.1E-05
Std. Dev.	9.1E-06	9.6E-06	8.7E-06	9.5E-06	8.8E-06	9.5E-06	9.1E-06	9.5E-06	9.6E-06	9.8E-06	9.9E-06
Skewness	1.68	2.21	1.74	2.20	1.98	2.16	2.08	2.06	2.29	2.20	2.33
Kurtosis	7.93	10.63	7.92	10.56	9.05	10.18	10.03	9.57	11.04	10.50	10.62
Autocorrelation	1	3	4	2	5	2	4	3	2	0	8

Table 23 Model results for IAV15 and concentration differenced over fifteen-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 84 observations, each estimated using fifteen trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference IAV15 DV15 indicates that the incremental average variance is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DIAV – 1 denotes an asymmetric coefficient on the lagged DIAV data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals including the Jarque-Bera test statistic and p-values for rejection of the null hypothesis that the residuals were normally distributed. No residual autocorrelation significant at 5% was evident in any of the models.

Model reference	**IAV15 DV15**		**IAV15 DR15**		**IAV15 DH15**		**IAV15 DSK15**		**AAR1**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.570	0.544	0.593	0.548	0.567	0.543	0.572	0.566	0.542	0.447
Adjusted R-squared	0.548	0.527	0.572	0.531	0.545	0.526	0.551	0.550	0.531	0.440
SE of the regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum squared resid	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean dependent var	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05
SD of the dependent var	9.7E-06	9.7E-06	9.7E-06	9.7E-06	9.7E-06	9.7E-06	9.7E-06	9.7E-06	9.7E-06	9.7E-06
Akaike info criterion	-20.98	-20.95	-21.04	-20.96	-20.98	-20.95	-20.99	-21.00	-20.97	-20.81
Schwarz criterion	-20.84	-20.83	-20.89	-20.84	-20.83	-20.83	-20.85	-20.88	-20.88	-20.75
Panel B: Coefficients										
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.5899	0.5673	0.6271	0.5969	0.5748	0.5527	0.5253	0.5178	0.5635	0.6712
DIAV - 1	0.3371	0.3635	0.2267	0.3351	0.2896	0.3690	0.4059	0.4272	0.3579	
DC - 1	-0.0000	-0.0000	-0.0012	-0.0012	0.0000	0.0001	0.0000	0.0000		
DC	0.0000		0.0033		0.0008		0.0000			
Panel C: Coefficient p-values										
Intercept	0.0038	0.0006	0.0008	0.0009	0.0002	0.0002	0.0001	0.0001	0.0005	0.0000
AR1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DIAV - 1	0.0023	0.0018	0.0767	0.0052	0.0423	0.0055	0.0062	0.0021	0.0016	
DC - 1	0.7011	0.6657	0.3004	0.2903	0.9689	0.7653	0.0255	0.0220		
DC	0.0895		0.0152		0.0773		0.4927			
Panel D: Residual descriptive statistics										
Mean	2.3E-21	1.9E-21	1.8E-21	1.7E-21	1.3E-21	2.5E-21	2.1E-21	1.7E-21	1.3E-21	1.8E-21
Median	-9.0E-07	-1.5E-06	-8.8E-07	-1.5E-06	-1.3E-06	-1.6E-06	-1.3E-06	-1.5E-06	-1.6E-06	-1.0E-06
Maximum	1.8E-05	1.7E-05	1.8E-05	1.6E-05	2.2E-05	1.9E-05	2.2E-05	2.0E-05	1.8E-05	2.3E-05
Minimum	-1.6E-05	-1.9E-05	-1.5E-05	-1.8E-05	-1.8E-05	-1.9E-05	-2.0E-05	-2.0E-05	-1.8E-05	-1.3E-05
Std. Dev.	6.4E-06	6.6E-06	6.2E-06	6.5E-06	6.4E-06	6.6E-06	6.4E-06	6.4E-06	6.6E-06	7.2E-06
Skewness	0.51	0.55	0.52	0.59	0.70	0.59	0.61	0.51	0.60	1.04
Kurtosis	3.70	3.85	3.78	3.69	4.61	4.00	4.92	4.41	3.90	4.25
Jarque-Bera	5	7	6	7	16	8	18	11	8	21
Probability	0.07	0.03	0.05	0.04	0.00	0.02	0.00	0.00	0.02	0.00

Table 24 Model results for IAV20 and concentration differenced over twenty-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 64 observations, each estimated using twenty trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference IAV20 DV20 indicates that the incremental average variance is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DIAV – 1 denotes an asymmetric coefficient on the lagged DIAV data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed could be rejected at the 5% level of significance for all models. No residual autocorrelation significant at 5% was evident in any of the models.

Model reference	IAV20 DV20		IAV20 DR20		IAV20 DH20		IAV20 DSK20		AAR1	AR1
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.411	0.399	0.503	0.426	0.467	0.405	0.430	0.400	0.399	0.342
Adjusted R-squared	0.371	0.369	0.469	0.398	0.431	0.376	0.391	0.370	0.380	0.331
SE of the regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum squared resid	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean dependent var	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05	1.4E-05
SD of the dependent var	9.7E-06	9.7E-06	9.7E-06	9.7E-06	9.7E-06	9.7E-06	9.7E-06	9.7E-06	9.7E-06	9.7E-06
Akaike info criterion	-20.63	-20.65	-20.80	-20.69	-20.73	-20.66	-20.67	-20.65	-20.68	-20.62
Schwarz criterion	-20.47	-20.51	-20.63	-20.56	-20.57	-20.52	-20.50	-20.51	-20.58	-20.55
Panel B: Coefficients										
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.4332	0.4265	0.5195	0.4543	0.5094	0.4307	0.4808	0.4282	0.4265	0.5809
DIAV - 1	0.2749	0.2696	0.2250	0.3054	0.1990	0.2960	0.2199	0.2603	0.2694	
DC - 1	0.0000	-0.0000	-0.0018	-0.0026	-0.0001	-0.0004	0.0000	0.0000		
DC	0.0000		0.0043		0.0013		0.0000			
Panel C: Coefficient p-values										
Intercept	0.0001	0.0000	0.0003	0.0000	0.0003	0.0000	0.0004	0.0001	0.0000	0.0004
AR1	0.0011	0.0022	0.0001	0.0005	0.0021	0.0015	0.0050	0.0028	0.0025	0.0000
DIAV - 1	0.0916	0.1109	0.1668	0.0537	0.3507	0.1182	0.2623	0.1409	0.1065	
DC - 1	0.9926	0.9924	0.0915	0.0449	0.8525	0.4083	0.3875	0.6413		
DC	0.3985		0.0308		0.0301		0.1719			
Panel D: Residual descriptive statistics										
Mean	5.2E-21	4.0E-21	2.8E-21	4.4E-21	5.0E-21	6.2E-21	5.2E-21	5.0E-21	4.4E-21	5.2E-21
Median	-1.5E-06	-1.6E-06	-1.0E-06	-1.7E-06	-1.1E-06	-1.6E-06	-1.6E-06	-1.8E-06	-1.6E-06	-1.9E-06
Maximum	2.6E-05	2.8E-05	2.2E-05	2.6E-05	2.6E-05	2.7E-05	2.8E-05	2.8E-05	2.8E-05	3.0E-05
Minimum	-1.1E-05	-1.2E-05	-1.2E-05	-1.3E-05	-1.3E-05	-1.2E-05	-1.2E-05	-1.2E-05	-1.2E-05	-1.2E-05
Std. Dev.	7.5E-06	7.5E-06	6.9E-06	7.4E-06	7.1E-06	7.5E-06	7.3E-06	7.5E-06	7.5E-06	7.9E-06
Skewness	1.70	1.78	0.93	1.60	1.41	1.67	1.80	1.82	1.78	1.74
Kurtosis	6.24	6.56	4.33	6.27	5.91	6.36	6.97	6.62	6.56	6.83

Table 25 Model forecasts: January – April 2003

Final data points are as follows: IAV5, 16th April and IAV10, 7th April 2003. The difference is due to the fact that estimates are non-overlapping and hence not perfectly synchronised for different estimation periods over the whole sample 1984-2003.

Panel A: IAV5 out-of-sample forecasts based on trading days from 19th December 2002 through 16th April 2003						
Forecasting model	**AR2**	**AAR2**	**AARDL2 DV5**	**AARDL2 DR5**	**AARDL2 DH5**	**AARDL2 DSK5**
Forecast variable	IAV5	IAV5	IAV5	IAV5	IAV5	IAV5
Forecast sample:	261: 275	261: 275	261: 275	261: 275	261: 275	261: 275
Included observations:	14	14	14	14	14	14
Root Mean Squared Error	1.6E-05	1.6E-05	1.6E-05	1.6E-05	1.6E-05	1.6E-05
Mean Absolute Error	1.1E-05	1.1E-05	1.1E-05	1.1E-05	1.1E-05	1.1E-05
Mean Abs. Percent Error	233	229	229	231	230	229
Theil Inequality Coefficient	0.503	0.501	0.499	0.498	0.500	0.502
Bias Proportion	0.004	0.002	0.002	0.003	0.002	0.002
Variance Proportion	0.260	0.341	0.351	0.350	0.345	0.335
Covariance Proportion	0.736	0.657	0.647	0.647	0.653	0.663
Panel B: IAV10 out-of-sample forecasts based on trading days from 28th December 2002 through 7th April 2003						
Forecasting model	**AR2**	**AAR2**	**AARDL2 DV10**	**AARDL2 DR10**	**AARDL2 DH10**	**AARDL2 DSK10**
Forecast variable	IAV10	IAV10	IAV10	IAV10	IAV10	IAV10
Forecast sample:	128:134	128:134	128:134	128:134	128:134	128:134
Included observations:	7	7	7	7	7	7
Root Mean Squared Error	1.1E-05	1.0E-05	1.0E-05	1.0E-05	9.9E-06	9.7E-06
Mean Absolute Error	9.7E-06	8.7E-06	8.7E-06	8.6E-06	8.4E-06	8.1E-06
Mean Abs. Percent Error	108	95	98	100	96	91
Theil Inequality Coefficient	0.388	0.375	0.377	0.377	0.373	0.370
Bias Proportion	0.003	0.001	0.001	0.001	0.002	0.005
Variance Proportion	0.261	0.444	0.407	0.410	0.468	0.512
Covariance Proportion	0.736	0.555	0.592	0.589	0.530	0.483

10.3.2 Forecasts: January 2003 – April 2003

Results outlined in the previous section provide little evidence to suggest that lagged changes in concentration are useful for explaining the contemporaneous incremental average variance. This would imply that there is little difference between out-of-sample-forecasts obtained using the AARDL models and those obtained using the naive AAR model. The results reported in Table 25 indicate that this is in fact the case, when the MAPE and TIC are used to evaluate the forecasts. However, the AAR models perform a little better than the more parsimonious AR models.

10.3.3 Summary of results from earlier sub-periods and the whole period

A total of seventy-six AARDL and ARDL models were estimated for the three incremental average variance series IAV10, IAV15 and IAV20 over different sub-periods and the whole period from 1984 through 2003, in addition to the sub-period from 1998 through 2003.[100] Table 26, summarises the results for eight of the seventy-six models that have DC-1

[100] These sub-periods are the same as those defined in Table 13 and the procedure followed is identical to that described on page 179, the only difference being the dependent variables in the model.

coefficients significantly different from zero at the $\alpha < 10\%$ threshold. Seven these were also significant at the $\alpha < 5\%$ threshold. Hence, the number of models out of the total of seventy-six that had DC-1 coefficients significantly different from zero at $\alpha = 10\%$ or less is not that much greater than would have been expected due to random chance if the first lag of differenced concentration had no explanatory power for forecasting the IAV data. When the inconsistency in the sign of the coefficients is taken into consideration, it is even more difficult to conclude that lagged concentration is useful for forecasting the IAV. All of the coefficients on the 1987 crash dummy were positive and significantly different from zero at the 1% level. This suggests that the incremental average variance of FTSE 100 Index constituent returns made a positive contribution to this unusual volatility event.

Table 26 Summary results for IAV models with significant coefficients on lagged differenced concentration

Models are estimated over the periods within the sample from January 1984 through March 2003. The reference code identifies the dependent variable, the type of model, i.e. AARDL or ARDL, and the distributed lag variable. For example, model reference IAV15 DH15 AARDL2 refers to an asymmetric autoregressive distributed lag model. This has two autoregressive lags of the dependant variable IAV15, one lagged asymmetric variable DIAV15 and one distributed lag variable DH15. The null hypothesis that the model residuals are normally distributed can be rejected in all models. The significance levels of all model coefficients were determined using Newey-West heteroskedasticity and autocorrelation robust standard errors.

Model reference code	IAV15 DH15 AARDL2	IAV15 DH15 AARDL2	IAV15 DSK15 AARDL2	IAV15 DSK15 AARDL2	IAV10 DH10 AARDL2	IAV10 DR10 AARDL2	IAV10 DV10 AARDL2	IAV15 DSK15 AARDL2
Time period	1984-2003	1984-2003	1984-2003	1984-2003	1988 1992	1988 1992	1988 1992	1998 2003
1987 Crash dummy included	Yes	No	Yes	No	NA	NA	NA	NA
Coefficient significant at 5%	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes
Coefficient significant at 10%	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sign of coefficient	+	+	+	+	-	-	-	+
Adjusted R^2	68%	67%	69%	69%	0%	0%	0%	56%
Residual autocorrelation	Yes	Yes	Yes	Yes	No	No	No	No

10.3.4 Summary of all IAV model results

This sub-section has reported results from models of the incremental average variance, IAV, of the FTSE 100 Index constituent returns on contemporaneous and lagged differences in concentration, where concentration was measured using four different indices. It has presented a detailed analysis of model results for the period from January 1998 through December 2002, including out-of-sample forecasts for the period from January 2003 through March 2003. A summary is also provided of the results of an additional seventy-six AARDL models of the IAV data estimated over the whole study period from January 1984 to March 2003, and sub-periods within this. The analysis provides little evidence to suggest that lagged changes in concentration are useful for forecasting future values of the relevant IAV data. Such little evidence that exists is diminished by the inconsistent sign of the coefficients, which makes it unclear as to whether such a relationship, if it does exist, is positive or negative. The majority of the models provide no evidence of a relationship between lagged values of differenced concentration and changes in the realised IAV. However, the model results analysed in detail for the January 1998 – December 2002 period do provide consistent evidence to suggest that there is a positive association between the IAV and contemporaneous changes in concentration, DC. While this finding is of limited value for forecasting, due to the fact that DC is not known in advance of the forecast period, attempts to explain the observed association will further understanding about investor behaviour and portfolio theory. Hence, a possible intuitive explanation is suggested as follows.

As the equally weighted average variance, EAV, is influenced more by the variance of small firms than the value weighted average variance, VAV, the IAV represents the relative difference between the average variance of small firms and the average variance of large firms. If the IAV is positive, large firms contribute relatively more than small firms to total realised volatility. Increases in IAV that occur commensurately with increases in concentration imply that investors are concentrating more of their capital into the securities of larger firms in the index at a time when the returns of large firms are more volatile than those of relatively smaller firms in the large cap FTSE 100 Index. In order to rationalise this seemingly irrational behaviour it is necessary to assume that the greater risk of large firm returns is offset by, either greater expected returns or below average covariance, or both. The fact that these results were obtained during the technology bubble is consistent with the view that investors' optimism, during the first half of this sub-period, led them to expect greater returns from riskier firms.[101] In addition, the results presented in section 10.5 concerning the incremental average covariance, IAC, are consistent with the idea that large firms had below average covariance for substantial parts of this sub-period, and that concentration increased in advance of falls in the IAC.

Evidence is also presented suggesting that negative FTSE 100 Index returns precede increases in the IAV of constituent returns, i.e. the asymmetry effect.

---

[101] At this time, investors around the world were pouring money into the stocks of technology, media and telecommunications (TMT) firms, as evidenced by the abundance investments companies specialising in those sectors that were launched at this time. In addition, data sources such as Datastream and the LSE data files at this time illustrate the weight of this sector in major stock market indices such as the FTSE 100, while Hirschey (2001) reports a similar phenomenon in the US market.

10.4 Models of the equally weighted average covariance (EAC)

This section reports the results of models of the equally weighted average covariance of FTSE 100 Index constituent returns, EAC, over the whole study period from 1984 through 2003 and sub-periods within this.

10.4.1 Model results for the period: January 1998 – December 2002

Model results obtained for each of the four time series of the equally weighted average covariance of FTSE 100 Index constituent returns: EAC5, EAC10, EAC15, and EAC20 are discussed in this section. AARDL models for each data-series are estimated over the period from January 1998 through December 2002, using four separate differenced concentration indices.[102]

10.4.1.1 Coefficients on contemporaneous differenced concentration (DC)

Out of a total of sixteen models estimated with both lagged and contemporaneous coefficients on the differenced concentration data, only two have positive DC coefficients that are significantly different from zero at the $\alpha < 10\%$ threshold. One of these, the EAC5 DSK5 model reported in Table 27, is also significant at the $\alpha < 5\%$ threshold, although there is no discernable improvement in the fit of this model over the AARDL model without the contemporaneous coefficient when adjusted R^2, AIC and SIC are compared. The EAC20 DR20 model, reported in Table 30, with a positive DC coefficient significantly different from zero at the $\alpha < 10\%$ threshold has slightly better adjusted R^2, AIC and SIC values than the equivalent model that excludes the DC coefficient. The significant coefficients should be viewed with caution, given that they are only two out of a total of sixteen, not a great deal more than would be expected by random chance with a threshold of $\alpha < 10\%$.

10.4.1.2 Coefficients on the first lag of differenced concentration (DC-1)

Of the thirty-two models containing DC-1 coefficients, for which results are reported in Table 27 through Table 30, twelve have coefficients on the DC-1 variables that are significantly different from zero at the $\alpha < 10\%$ threshold, eleven of these are also

[102] The EAC data and the differenced concentration indices are defined in Table 4 on page 136 of Chapter 7.

significant at the $\alpha < 5\%$ threshold. Of the eleven, nine are positive and two are negative. The remaining coefficient significant at the $\alpha < 10\%$ threshold is positive.

Slightly less than a third of all the models estimated for the 1998-2003 period have DC-1 coefficients that are significantly different from zero at the 5% level. The majority of these significant coefficients are positive and are obtained by modelling the EAC10 data. The adjusted R^2, AIC and SIC values of the AARDL models were compared with the naive AAR and AR models for the EAC10 and EAC15 models to see whether or not inclusion of the DC-1 coefficients improved the explanatory power of the models. For the EAC10 model results, reported in Table 28, AARDL models without contemporaneous DC coefficients all have adjusted R^2 values that are very similar to the naive AAR models at around 47 and 46% respectively. The AIC and SIC values are also very similar at around –15.1 and –15. This demonstrates that although, the coefficients on the DC-1 variables are significantly different from zero, inclusion of these variables in the model does not cause a detectable reduction in the variance of model residuals when the reduction in degrees of freedom is accounted for. Overall, the improvement in model fit, due to the inclusion of lagged differenced concentration data, is disappointingly low, given that the equally weighted average covariance accounts for a large proportion of the total FTSE 100 Index volatility.

10.4.1.3 Coefficients on the asymmetric dummy variable DEAC – 1

In contrast to the lack of improvement in explanatory power when distributed lags of differenced concentration are added to the naive AAR models, addition of the asymmetric slope dummy to the basic AR model, provides a discernable improvement in model explanatory power. This is evident in the lower adjusted R^2 and higher absolute AIC and SIC values in all of the more parsimonious AR models, except those estimated using the EAC5 data. Coefficients on the asymmetric variables are positive and significantly different from zero at the $\alpha < 5\%$ threshold in all models, except those estimated using the EAC5 data.

10.4.1.4 Residual analysis

The only model with evidence of residual autocorrelation is the AR1 model for the EAC5 data. Models estimated using the EAC15 data as the dependent variable have residuals that are closest to the OLS assumption of white noise with no autocorrelation and a mean and median very close to zero in relation to the standard deviation. The skewness and kurtosis of model residuals is also lower than in the EAC5, EAC10 or EAC20 models. However, the null hypothesis that the model residuals are normally distributed can be rejected for all models at the $\alpha < 1\%$ threshold due to the skewness and excess kurtosis of the residuals. This is most apparent in the EAC5 model results reported Table 27 where skewness and kurtosis are the highest. Although often unavoidable, high levels of skewness and kurtosis in the residuals of models estimated using OLS procedures imply that coefficients should be interpreted with caution.

10.4.1.5 Synopsis

The significant positive and significant negative model coefficients, presented in Table 27 through Table 30, are ambiguous in that both a positive and a negative association between lagged concentration and the equally weighted average covariance of constituent returns is indicated. Furthermore, inclusion of differenced concentration metrics adds very little, if any, explanatory power to the models as illustrated by the adjusted R^2, AIC and SIC values. Nonetheless, on balance, more coefficients on DC – 1 are significantly different from zero than would be expected by chance and the majority of these are positive, suggesting that the lag of differenced concentration may have some role to play in explaining the equally weighted average covariance. Evidence is much more conclusive in support of an asymmetry effect, whereby negative returns at the market level precede increases in the equally weighted average covariance of the VCM.

Table 27 Model results for EAC5 and concentration differenced over five-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 259 observations, each estimated using five trading-days worth of daily data. EAV full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference EAC5 DV5 indicates that the equally weighted average covariance is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DEAC – 1 denotes an asymmetric coefficient on the lagged DEAC data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. For each measure of concentration, results of two models are reported, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. Only the AR1 model had autocorrelation coefficients significantly different from zero at the 5% level.

Model reference	EAC5 DV5		EAC5 DR5		EAC5 DH5		EAC5 DSK5		AAR2	AR2	AR1
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**			
R-squared	0.423	0.421	0.420	0.418	0.419	0.414	0.418	0.414	0.414	0.405	0.284
Adjusted R-squared	0.412	0.412	0.408	0.409	0.408	0.405	0.407	0.405	0.407	0.400	0.281
SE of the regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum squared resid	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean dependent var	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04
SD of the dependent var	1.9E-04	1.9E-04	1.9E-04	1.9E-04	1.9E-04	1.9E-04	1.9E-04	1.9E-04	1.9E-04	1.9E-04	1.9E-04
Akaike info criterion	-14.81	-14.82	-14.81	-14.81	-14.81	-14.81	-14.81	-14.81	-14.81	-14.81	-14.63
Schwarz criterion	-14.73	-14.75	-14.72	-14.74	-14.72	-14.74	-14.72	-14.74	-14.76	-14.76	-14.60
Panel B: Coefficients											
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
AR1	0.2319	0.2231	0.1988	0.2033	0.2010	0.2033	0.2069	0.2081	0.2066	0.3133	0.5329
AR2	0.3909	0.4017	0.4134	0.4104	0.4159	0.4160	0.4175	0.4163	0.4168	0.4113	
DEAC - 1	0.1492	0.1494	0.1506	0.1532	0.1458	0.1511	0.1502	0.1504	0.1506		
DC - 1	0.0006	0.0007	0.0301	0.0286	0.0053	0.0050	-0.0000	-0.0000			
DC	-0.0003		0.0179		0.0127		0.0001				
Panel C: Coefficient p-values											
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.0549	0.0545	0.0933	0.0788	0.0750	0.0756	0.0673	0.0704	0.0685	0.0002	0.0000
AR2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
DEAC - 1	0.1276	0.1246	0.1097	0.1102	0.1159	0.1148	0.1118	0.1197	0.1164		
DC - 1	0.2335	0.2071	0.1766	0.1818	0.4251	0.4529	0.4193	0.5792			
DC	0.3674		0.5582		0.2288		0.0360				
Panel D: Residual descriptive statistics											
Mean	6.6E-21	9.5E-21	2.7E-20	7.4E-21	2.7E-21	2.4E-20	3.3E-21	1.3E-20	6.2E-21	-4.4E-21	4.7E-20
Median	-2.5E-05	-2.5E-05	-2.6E-05	-2.8E-05	-2.6E-05	-2.8E-05	-2.2E-05	-2.5E-05	-2.6E-05	-2.4E-05	-3.9E-05
Maximum	1.1E-03	1.1E-03	1.1E-03	1.1E-03	1.1E-03	1.1E-03	1.1E-03	1.2E-03	1.2E-03	1.2E-03	1.1E-03
Minimum	-3.1E-04	-3.2E-04	-3.4E-04	-3.4E-04	-3.8E-04	-3.8E-04	-3.8E-04	-3.8E-04	-3.8E-04	-3.3E-04	-3.6E-04
Std. Dev.	1.4E-04	1.4E-04	1.4E-04	1.4E-04	1.4E-04	1.5E-04	1.4E-04	1.5E-04	1.5E-04	1.5E-04	1.6E-04
Skewness	3.34	3.23	3.13	3.23	3.13	3.21	3.25	3.22	3.22	3.21	3.13
Kurtosis	22.85	21.94	21.20	22.19	21.26	22.21	22.50	22.33	22.29	22.17	18.70

Table 28 Model results for EAC10 and concentration differenced over ten-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 127 observations, each estimated using ten trading-days worth of daily data. EAV full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference EAC10 DV10 indicates that the equally weighted average covariance is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DEAC – 1 denotes an asymmetric coefficient on the lagged DEAC data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. For each measure of concentration, results of two models are reported, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. The final row indicates the number of autocorrelation lags out of a total of thirty-six that are significant at the 5% level.

Model reference	**EAC10 DV10**		**EAC10 DR10**		**EAC10 DH10**		**EAC10 DSK10**		**AAR2**	**AR2**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**			
R-squared	0.522	0.484	0.503	0.476	0.488	0.477	0.481	0.480	0.470	0.404	0.401
Adjusted R-squared	0.502	0.467	0.482	0.458	0.467	0.459	0.459	0.462	0.457	0.394	0.396
SE of the regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum squared resid	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean dependent var	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04
SD of the dependent var	1.7E-04	1.7E-04	1.7E-04	1.7E-04	1.7E-04	1.7E-04	1.7E-04	1.7E-04	1.7E-04	1.7E-04	1.7E-04
Akaike info criterion	-15.13	-15.07	-15.09	-15.05	-15.06	-15.05	-15.05	-15.06	-15.06	-14.96	-14.97
Schwarz criterion	-14.99	-14.95	-14.95	-14.94	-14.92	-14.94	-14.91	-14.95	-14.97	-14.89	-14.93
Panel B: Coefficients											
Intercept	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
AR1	0.2683	0.2280	0.2020	0.2376	0.2167	0.2427	0.2465	0.2534	0.2878	0.6743	0.6333
AR2	0.2022	0.1991	0.2079	0.1662	0.1859	0.1598	0.1654	0.1578	0.1383	-0.0646	
DEAC - 1	0.4619	0.4682	0.4816	0.4776	0.4866	0.4795	0.4828	0.4783	0.4447		
DC - 1	0.0007	0.0006	0.0299	0.0218	0.0091	0.0089	0.0001	0.0001			
DC	0.0010		0.0472		0.0117		0.0000				
Panel C: Coefficient p-values											
Intercept	0.0007	0.0007	0.0002	0.0017	0.0008	0.0016	0.0015	0.0015	0.0014	0.0005	0.0001
AR1	0.0792	0.1411	0.2085	0.1092	0.1856	0.1098	0.1145	0.0944	0.0500	0.0000	0.0000
AR2	0.0227	0.0134	0.0166	0.0525	0.0363	0.0726	0.0647	0.0760	0.1281	0.4700	
DEAC - 1	0.0021	0.0031	0.0023	0.0024	0.0025	0.0025	0.0022	0.0023	0.0031		
DC - 1	0.0404	0.0327	0.0240	0.0492	0.0591	0.0370	0.0233	0.0109			
DC	0.1696		0.2396		0.2728		0.4320				
Panel D: Residual descriptive statistics											
Mean	1.2E-20	2.0E-20	3.4E-20	3.4E-20	1.4E-20	3.9E-20	3.2E-20	1.9E-20	3.5E-20	4.2E-20	-2.2E-21
Median	-1.9E-05	-2.1E-05	-1.8E-05	-1.8E-05	-1.5E-05	-2.1E-05	-1.6E-05	-1.9E-05	-2.0E-05	-2.1E-05	-2.3E-05
Maximum	6.3E-04	7.6E-04	6.8E-04	7.8E-04	7.4E-04	7.8E-04	7.8E-04	7.8E-04	7.8E-04	8.0E-04	8.1E-04
Minimum	-2.5E-04	-2.9E-04	-2.5E-04	-2.7E-04	-2.4E-04	-2.7E-04	-2.5E-04	-2.5E-04	-2.4E-04	-2.6E-04	-2.6E-04
Std. Dev.	1.2E-04	1.3E-04	1.2E-04	1.3E-04	1.2E-04	1.3E-04	1.3E-04	1.3E-04	1.3E-04	1.3E-04	1.3E-04
Skewness	2.58	2.95	2.78	3.00	2.93	3.00	3.05	3.02	3.04	3.05	3.09
Kurtosis	13.38	16.78	15.34	17.64	16.82	17.72	17.94	17.88	17.69	17.29	17.66
Autocorrelation	1	0	0	0	0	0	4	5	0	0	0

Table 29 Model results for EAC15 and concentration differenced over fifteen-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 84 observations, each estimated using fifteen trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes indicate the dependent variable in the AR model and the independent distributed lag variable. For example, model reference EAC15 DV15 indicates that the equally weighted average covariance is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DEAC – 1 denotes an asymmetric coefficient on the lagged DEAC data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. The null hypothesis that the residuals were normally distributed was rejected at the 5% significance level for all models using the Jarque-Bera test. No residual autocorrelation significant at 5% was evident in any of the models.

Model reference	**EAC15 DV15**		**EAC15 DR15**		**EAC15 DH15**		**EAC15 DSK15**		**AAR1**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.587	0.580	0.583	0.579	0.581	0.580	0.594	0.594	0.579	0.384
Adjusted R-squared	0.566	0.564	0.562	0.563	0.560	0.564	0.574	0.579	0.568	0.376
SE of the regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum squared resid	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean dependent var	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04
SD of the dependent var	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04
Akaike info criterion	-15.46	-15.47	-15.45	-15.46	-15.44	-15.46	-15.48	-15.50	-15.49	-15.13
Schwarz criterion	-15.31	-15.35	-15.30	-15.35	-15.30	-15.35	-15.33	-15.38	-15.40	-15.07
Panel B: Coefficients										
Intercept	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
AR1	0.4094	0.3948	0.4012	0.3918	0.3867	0.3832	0.3783	0.3776	0.3912	0.6185
DEAC - 1	0.6083	0.6172	0.6017	0.6287	0.6253	0.6368	0.6580	0.6590	0.6289	
DC - 1	0.0002	0.0002	0.0002	-0.0007	0.0029	0.0030	0.0001	0.0001		
DC	0.0003		0.0152		0.0030		0.0000			
Panel C: Coefficient p-values										
Intercept	0.0001	0.0001	0.0001	0.0002	0.0001	0.0002	0.0002	0.0002	0.0001	0.0002
AR1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
DEAC - 1	0.0020	0.0021	0.0040	0.0021	0.0026	0.0017	0.0010	0.0009	0.0019	
DC - 1	0.6129	0.6585	0.9894	0.9505	0.4190	0.4138	0.0256	0.0238		
DC	0.3542		0.4481		0.5769		0.8195			
Panel D: Residual descriptive statistics										
Mean	-1.0E-20	-1.4E-20	-2.2E-20	-7.2E-21	-2.7E-20	-1.1E-20	1.8E-20	-2.3E-20	-2.0E-20	-2.4E-20
Median	-1.3E-05	-1.9E-05	-1.5E-05	-1.6E-05	-1.8E-05	-1.6E-05	-1.6E-05	-1.6E-05	-1.7E-05	-3.0E-05
Maximum	3.6E-04	3.9E-04	3.8E-04	3.9E-04	3.9E-04	4.0E-04	3.9E-04	3.9E-04	3.9E-04	4.9E-04
Minimum	-3.0E-04	-3.1E-04	-3.0E-04	-3.1E-04	-3.0E-04	-3.1E-04	-3.0E-04	-3.0E-04	-3.1E-04	-3.0E-04
Std. Dev.	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.0E-04	1.2E-04
Skewness	0.87	1.07	0.96	1.04	1.06	1.08	1.13	1.13	1.04	1.54
Kurtosis	6.09	6.81	6.45	6.84	6.81	6.89	6.91	6.90	6.84	7.14

Table 30 Model results for EAC20 and concentration differenced over twenty-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 64 observations, each estimated using twenty trading-days worth of return data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference EAC20 DV20 indicates that the equally weighted average covariance is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DEAC – 1 denotes an asymmetric coefficient on the lagged DEAC data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed could be rejected at the 5% level of significance for all models. No residual autocorrelation significant at 5% was evident in any of the models.

Model reference	**EAC20 DV20**		**EAC20 DR20**		**EAC20 DH20**		**EAC20 DSK20**		**AAR1**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.323	0.323	0.382	0.333	0.358	0.332	0.306	0.304	0.304	0.205
Adjusted R-squared	0.277	0.290	0.340	0.300	0.314	0.298	0.259	0.269	0.281	0.192
SE of the regression	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Sum squared resid	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mean dependent var	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04
SD of the dependent var	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04	1.6E-04
Akaike info criterion	-14.95	-14.98	-15.04	-14.99	-15.00	-14.99	-14.92	-14.95	-14.98	-14.88
Schwarz criterion	-14.78	-14.84	-14.87	-14.86	-14.83	-14.85	-14.75	-14.81	-14.88	-14.81
Panel B: Coefficients										
Intercept	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
AR1	0.2029	0.2030	0.2629	0.2385	0.2296	0.2111	0.2066	0.2025	0.2028	0.4528
DEAC - 1	0.4501	0.4501	0.4177	0.4541	0.4548	0.4828	0.4374	0.4401	0.4396	
DC - 1	0.0006	0.0006	-0.0338	-0.0425	-0.0111	-0.0137	0.0000	-0.0000		
DC	-0.0000		0.0538		0.0131		0.0000			
Panel C: Coefficient p-values										
Intercept	0.0067	0.0017	0.0033	0.0064	0.0031	0.0056	0.0061	0.0061	0.0057	0.0020
AR1	0.1127	0.0890	0.0916	0.1193	0.1060	0.1461	0.1275	0.1383	0.1335	0.0041
DEAC - 1	0.0234	0.0223	0.0293	0.0078	0.0189	0.0064	0.0242	0.0209	0.0206	
DC - 1	0.2882	0.2927	0.0463	0.0311	0.1468	0.1662	0.9153	0.9292		
DC	0.9929		0.0865		0.2872		0.5202			
Panel D: Residual descriptive statistics										
Mean	-4.3E-20	-3.7E-20	-4.0E-20	-5.1E-20	-4.7E-20	-4.1E-20	-4.4E-20	-5.1E-20	-5.4E-20	-3.1E-20
Median	-2.9E-05	-2.9E-05	-3.1E-05	-2.3E-05	-2.7E-05	-2.8E-05	-2.6E-05	-2.6E-05	-2.5E-05	-4.2E-05
Maximum	5.8E-04	5.8E-04	5.6E-04	6.1E-04	5.6E-04	6.0E-04	6.1E-04	6.2E-04	6.2E-04	6.3E-04
Minimum	-3.3E-04	-3.3E-04	-3.1E-04	-3.3E-04	-3.3E-04	-3.4E-04	-3.5E-04	-3.5E-04	-3.5E-04	-2.0E-04
Std. Dev.	1.3E-04	1.3E-04	1.2E-04	1.3E-04	1.3E-04	1.3E-04	1.3E-04	1.3E-04	1.3E-04	1.4E-04
Skewness	1.84	1.84	1.71	1.99	1.67	1.89	1.99	2.00	2.00	2.31
Kurtosis	9.28	9.29	8.91	10.74	8.62	9.99	10.52	10.61	10.61	9.60

Table 31 Model forecasts: January – April 2003

Final data points are as follows: EAC5, 16th April and EAC10, 7th April 2003. The difference is due to the fact that estimates are non-overlapping and hence not perfectly synchronised for different estimation periods over the whole sample period from 1984 – 2003.

Panel A: EAC5 out-of-sample forecasts based on trading days from 19th December 2002 through 16th April 2003						
Forecasting model	**AR2**	**AAR2**	**AARDL2 DV5**	**AARDL2 DR5**	**AARDL2 DH5**	**AARDL2 DSK5**
Forecast variable	EAC5	EAC5	EAC5	EAC5	EAC5	EAC5
Forecast sample:	261: 275	261: 275	261: 275	261: 275	261: 275	261: 275
Included observations:	14	14	14	14	14	14
Root Mean Squared Error	3.3E-04	3.1E-04	3.0E-04	3.1E-04	3.1E-04	3.1E-04
Mean Absolute Error	2.5E-04	2.2E-04	2.2E-04	2.2E-04	2.2E-04	2.3E-04
Mean Abs. Percent Error	72.12	67.02	64.74	67.36	67.13	67.21
Theil Inequality Coefficient	0.393	0.379	0.371	0.377	0.379	0.379
Bias Proportion	0.035	0.052	0.052	0.052	0.053	0.052
Variance Proportion	0.131	0.216	0.246	0.228	0.220	0.214
Covariance Proportion	0.834	0.732	0.702	0.720	0.727	0.734
Panel B: EAC10 out-of-sample forecasts based on trading days from 28th December 2002 through 7th April 2003						
Forecasting model	**AR2**	**AAR2**	**AARDL2 DV10**	**AARDL2 DR10**	**AARDL2 DH10**	**AARDL2 DSK10**
Forecast variable	EAC10	EAC10	EAC10	EAC10	EAC10	EAC10
Forecast sample:	128:134	128:134	128:134	128:134	128:134	128:134
Included observations:	7	7	7	7	7	7
Root Mean Squared Error	3.5E-04	3.0E-04	2.9E-04	2.9E-04	2.9E-04	2.9E-04
Mean Absolute Error	2.7E-04	1.9E-04	1.8E-04	1.8E-04	1.8E-04	1.8E-04
Mean Abs. Percent Error	67.0	43.6	37.7	39.8	40.0	41.1
Theil Inequality Coefficient	0.428	0.405	0.403	0.405	0.405	0.406
Bias Proportion	0.063	0.226	0.243	0.245	0.247	0.244
Variance Proportion	0.064	0.459	0.547	0.535	0.529	0.509
Covariance Proportion	0.873	0.315	0.210	0.220	0.223	0.247

10.4.2 Forecasts of equally weighted average covariance: January 2003 – April 2003

Positive coefficients significantly different from zero on the first lag of differenced concentration obtained from models of the EAC10 data are discussed in the previous section. Despite the low p-values obtained for the DC – 1 coefficients there was little evidence that inclusion of the DC – 1 variable improved model fit over the naive model, when the loss of degrees of freedom was adjusted for. Therefore it is difficult to ascertain whether or not differenced concentration is a useful variable for explaining the equally weighted average covariance. Out-of-sample forecast results reported in Table 31 provide evidence in support of a useful positive association, consistent with the coefficient p-values reported in Table 28. AARDL model forecasts reported in panel B of Table 31 have slightly lower MAPE values, of around 40%, than the naive AAR model, which has a MAPE of 43%, and the AR model, which has a MAPE of 67%. This indicates that concentration differenced over ten-days is useful for forecasting the equally weighted average covariance in the FTSE 100 VCM. It is also consistent with the finding that falls in the FTSE 100 Index value precede rises in the equally weighted average covariance, given the large difference in the MAPE between the AR and AAR models. However, the TIC forecast evaluation provides ambiguous results compared to the MAPE. Only the AR model has a noticeably higher TIC than the more general models, all of which have a TIC similar to the naive AAR model of around 40.5%, although the EAC10 DV10 model forecast has a marginally lower TIC, at 40.3%.

Further analysis of the MSE indicates that in fact the AR model estimated for the EAC10-series has the highest covariance-proportion, at 87%. The variance-proportion is about 50% of the MSE of the AAR and AARDL model forecasts for the EAC10-series. The covariance-proportion is slightly lower than the bias proportion in the AARDL models at around 22%, compared to 24%. In the AAR models the covariance proportion is 31% compared to a bias proportion of 22.6%. AARDL model forecasts for the EAC5-series have similar MAPE and TIC values to the AAR forecasts. However, the AAR forecasts appear better than the simplest AR forecast.

Overall, the relative performance of the out-of-sample forecasts reported in Table 31 are broadly consistent with the significant coefficients, identified during the within sample estimation period. There is clear evidence that including an asymmetric coefficient in the forecasting models can improve their out-of-sample forecasting performance. There is more limited evidence that inclusion of coefficients on the lagged differenced concentration variables can also improve out-of-sample forecasting performance of models that are used to explain the equally weighted average covariance.

10.4.3 Summary results from earlier sub-periods and the whole period

Table 32 summarises the results for fifteen out of the total of ninety-six AARDL and ARDL models estimated over the whole study period, and sub-periods within the whole period, for three data series EAC10, EAC15 and EAC20 that have DC-1 coefficients significantly different from zero at the $\alpha < 10\%$ threshold.[103] Nine of the fifteen model coefficients were also significant at the $\alpha < 5\%$ threshold. Hence the proportion of models that had DC-1 coefficients significantly different from zero is greater than would have been expected due to random chance, if the first lag of differenced concentration had no explanatory power for forecasting the EAC. All coefficients reported in Table 32 are positive, which suggests that, on average, increases in concentration lead increases in the equally weighted average covariance of FTSE 100 Index constituent returns. However, only two of the significant

[103] These sub-periods are the same as those defined in Table 13 and the procedure followed is identical to that detailed on page 179.

coefficients are estimated using a sample that does not include the latter part of the 1990s. These are derived from the EAC10 DR10 and EAC10 DV10 models estimated over the period 1988-1992. All other models include some, or all, of the most recent five years of data and Panel C of Table 32 reports results that were estimated over the period from 1998 through 2003. This suggests that the association between the equally weighted average covariance and differenced concentration may be a relatively new phenomenon, appearing towards the latter part of the study period.

An additional point is that all coefficients on the dummy variable for the 1987 crash are positive and significantly different from zero at the 1% level. This is consistent with the pattern displayed by the time series chart for the EAC10 series discussed in Chapter 8. It implies that the equally weighted average covariance of FTSE 100 Index constituent returns were a major contributor to this event.

Table 32 Summary results for EAC models with significant coefficients on lagged differenced concentration

Models estimated over the periods 1984-2003 and 1998-2003 used all the observations through to the end of March 2003. Therefore including the out-of-sample data that is used to evaluate the similar models reported in the previous where models are estimated using data up to the end of December 2002. The reference code identifies the dependent variable, the type of model, i.e. AARDL or ARDL and the distributed lag variable. For example, model reference IAV15 DH15 AARDL2 refers to an asymmetric autoregressive distributed lag model. This has two autoregressive lags of the dependant variable IAV15, one lagged asymmetric variable DB15 and one distributed lag variable DH15.

Panel A: Period from January 1984 through March 2003

Model reference	EAC10 DR10 AARDL2	EAC10 DV10 AARDL2	EAC15 DH15 AARDL2	EAC15 DSK15 AARDL2	EAC15 DSK15 AARDL2	EAC20 DV20 AARDL2
Time period	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003
1987 Crash dummy included	Y	Y	Y	Y	N	Y
Coefficient significant at 5%	N	Y	N	Y	N	Y
Coefficient significant at 10%	Y	Y	Y	Y	Y	Y
Sign of coefficient	+	+	+	+	+	+
Adjusted R squared	79%	79%	68%	68%	41%	67%
Residual autocorrelation	Y	Y	Y	Y	Y	Y

Panel B: Sub-periods

Model reference	EAC10 DR10 ARDL2	EAC10 DV10 ARDL2	EAC15 DH15 AARDL2	EAC15 DSK15 ARDL2
Time period	1988 1992	1988 1992	1991 2000	1991 2000
Coefficient significant at 5%	N	N	N	Y
Coefficient significant at 10%	Y	Y	Y	Y
Sign of coefficient	+	+	+	+
Adjusted R squared	10%	12%	32%	34%
Residual autocorrelation	N	N	N	N

Panel C: The sub-period from January 1998 through March 2003

Model reference	EAC10 DH10 AARDL2	EAC10 DR10 AARDL2	EAC10 DV10 AARDL2	EAC10 DSK10 AARDL2	EAC15 DSK15 AARDL2
Time period	1998 2003	1998 2003	1998 2003	1998 2003	1998 2003
Coefficient significant at 5%	Y	Y	Y	Y	Y
Coefficient significant at 10%	Y	Y	Y	Y	Y
Sign of coefficient	+	+	+	+	+
Adjusted R squared	43%	43%	44%	44%	56%
Residual autocorrelation	N	N	N	N	N

10.4.4 Summary of all EAC model results

This sub-section has reported results from models of the equally weighted average covariance of FTSE 100 Index constituent returns, EAC, on contemporaneous and lagged differences in concentration, where concentration was measured using four different indices. It has presented a detailed analysis of model results for the period from 1998 through 2003, including out-of-sample forecasts for the period January through March 2003. A summary is also provided of the results of an additional ninety-six AARDL models of the EAC data estimated over the whole study period from January 1984 through March 2003, and sub-periods within this time frame. Evidence is presented to suggest that lagged changes in concentration could have been used for forecasting future values of the relevant EAC data, over the latter part of this study period. Furthermore, the association between lagged changes in concentration and the EAC in the FTSE 100 Index is usually positive, although a few models indicate an inverse relationship. In contrast, there is very little evidence to suggest that there is a contemporaneous association between the EAC data and changes in concentration. There is evidence that falls in the value of the FTSE 100 Index precede rises in the level of the equally weighted average covariance of constituent returns in a manner consistent with the asymmetry effect that was observed in Chapter 9 and earlier sections of this chapter. This is consistent with the findings of Kearney and Poti (2003) who report that average stock correlations tend to spike up following negative returns. It is also consistent with the findings of Andersen et al (2000), who comment that the benefits of diversification are limited at the times when they are needed the most.

A possible explanation for the positive association between lagged DC and EAC is as follows. Equally weighted average covariance is influenced relatively more by the covariances between the returns of smaller firms than the value weighted average. Previous studies have documented a positive association between equally weighted average correlation and overall portfolio volatility, in various market indices. If investors anticipate increases in future volatility and increases in future equally weighted average correlation between security returns, a rational diversification strategy would be to concentrate capital into the securities of firms that have a below average covariance. If this strategy is adopted, the incremental average covariance will be negative during the periods of greatest volatility and it will tend to move inversely with portfolio volatility. The results of the next section are consistent with this explanation.

10.5 Models of the incremental average covariance (IAC)

This section reports results for models of the incremental average covariance and differenced concentration estimated over the whole study period, from January 1984 through March 2003 and sub-periods within this.

10.5.1 Model results for the period January 1998 through December 2002

The tables in this section report results for general AARDL models, general ARDL models, naive AAR models and naive AR models of the four incremental average covariance data series IAC5, IAC10, IAC15 and IAC20.[104]

10.5.1.1 Coefficients on the first lag of differenced concentration (DC – 1)

Of the thirty-six AARDL models for which results are reported in Table 33 through Table 36 twelve have negative DC-1 coefficients that are significantly different from zero at the $\alpha < 10\%$ threshold. Seven of these are also significant the $\alpha < 5\%$ threshold. Although models differ in terms of the concentration metric used and the number of returns used to estimate the IAC, when taken in the overall context of concentration and IAC the, number of significant coefficients is much higher than would be expected by random chance, if the null hypothesis that lagged changes in concentration had no association with contemporaneous incremental average covariance were to hold. Given that the signs of the coefficients are consistently negative, even in models where coefficients are not significantly different from zero, these results provide evidence that over the period from 1998 through 2002 changes in concentration, lagged over one period, were useful in explaining contemporaneous changes in the incremental average covariance. Furthermore, the negative sign of the coefficients would suggest an inverse relationship.

When the explanatory power of the AARDL models are compared with that of the naive AAR or AR models using the adjusted R^2, the AIC or the SIC, the adjusted R^2 of the AARDL model is higher than the naive model in every case where there DC-1 coefficient is significantly different from zero at the $\alpha < 10\%$ threshold. The AIC is, on the whole, more negative in the AARDL models than in the naive models, suggesting a better fit in the AARDL models. However, in the naive models the SIC is either more negative or identical to that of the AARDL models in every case. Thus, although the DC-1 coefficients are

[104] The four time-series are defined in on page 136 of Chapter 8 and in section 6.2.2.3 in Chapter 6.

significantly different from zero and do improve the adjusted R^2, their effect is not so great as to improve the explanatory power of the models when the most rigorous SIC metric is used to compare models. This suggests that the out-of-sample forecasting ability of AARDL models may not be consistently better than that of the naive models. This possibility is investigated further in section 10.5.2.

10.5.1.2 Coefficients on contemporaneous differenced concentration (DC)

Out of the twenty models estimated with DC coefficients on differenced concentration, only four IAC5 models, reported in Table 33, had negative coefficients that were significantly different from zero at the $\alpha < 5\%$ threshold. Two of these were IAC5 DSK5 models with negative coefficients, and two were IAC5 DV5 models with positive coefficients. The adjusted R^2, AIC and SIC all indicated a better model fit than the corresponding IAC5 models that did not include a contemporaneous DC coefficient. However, the inconsistent sign between the two differenced concentration metrics means that the significant coefficients should be interpreted with caution. While it may not be prudent to ignore the significant DC coefficients entirely, it seems reasonable to conclude that, for practical purposes, there is no material contemporaneous relationship between differenced concentration and incremental average covariance.

10.5.1.3 Coefficients on the asymmetric dummy variable (DIAC)

Eight out of a total of twenty-eight models estimated had asymmetric slope coefficients that were positive and significantly different from zero at the $\alpha < 10\%$ threshold. Five of these coefficients were also significant at the $\alpha < 5\%$ threshold. All significant coefficients were in the models of the IAC10 data reported in Table 34 and the adjusted R^2 and AIC of the AAR model was 15.3% and -17.12, compared to 12.7% and –17.06 in the basic AR2 model. However the SIC was identical for the AAR and AR model at –17.06 for the IAC10 data. Thus the asymmetry effect may not be as prevalent in the IAC data as it is in the other sub-components of the VCM.

10.5.1.4 Residual analysis

The only models that have evidence of autocorrelation in the residuals are the naive AR1 models estimated for the IAC5, IAC10 and IAC15 data-series. However the null hypothesis that the residuals have a normal distribution can be rejected using the Jarque-Bera normality test for all the models at the $\alpha < 5\%$ threshold. The lowest absolute skewness and kurtosis values occur in the residuals from models of the IAC15 and IAC20 data-series in which skewness is actually negative, although quite close to zero. Skewness becomes positive and kurtosis increases in models of the IAC5 and IAC10 data. The highest JB statistics occur in the models of the IAC5 data where the excess kurtosis and skewness of model residuals is highest.

10.5.1.5 Synopsis

Overall, there is consistent evidence that the lagged DV and DR differenced concentration metrics are useful for explaining contemporaneous realised incremental average covariance, due to the fact that model coefficients on these variables are significantly different from zero at the $\alpha < 10\%$ threshold. Furthermore, the consistently negative sign of all DC-1 coefficients indicates that the relationship is an inverse one. In other words, increases in differenced concentration are followed by decreases in the incremental average covariance. This result indicates that the distribution of capital invested in the FTSE 100 Index becomes concentrated into firms with below average covariance prior to increases in the equally weighted average covariance, or variance of returns. This may be interpreted to mean that investors have some ability to forecast increases in the equally weighted average covariance, a suggestion that is supported by the empirical results presented in section 10.4 for the equally weighted average covariance. However, it does not assume that this forecasting ability can lead to abnormal profits as it is a market wide ability and as such it is consistent with the EMH. The finding is consistent with that reported in section 10.2 where concentration increases prior to increases in the equally weighted average variance of returns, and also with that in section 10.4 where increases in concentration precede increases in the equally weighted average covariance of returns.

There is relatively little evidence to suggest that contemporaneous change in concentration is useful for explaining contemporaneous changes in the incremental average covariance. There is also relatively little evidence of the asymmetry effect in comparison to the EAV, IAV and EAC models.

Table 33 Model results for IAC5 and concentration differenced over five-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 259 observations, each estimated using five trading-days worth of daily data. A full list of acronym definitions is presented in on Table 4 page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference IAC5 DV5 indicates that the incremental average covariance is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DIAC – 1 denotes an asymmetric coefficient on the lagged DIAC data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. There was no evidence of autocorrelation coefficients significantly different from zero at the 5% level.

Model reference	**IAC5 DV5**			**IAC5 DR5**			**IAC5 DH5**			**IAC5 DSK5**			**AR2**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.138	0.138	0.115	0.113	0.112	0.110	0.107	0.107	0.097	0.128	0.128	0.092	0.092	0.014
Adjusted R-squared	0.121	0.124	0.105	0.095	0.098	0.099	0.090	0.093	0.087	0.111	0.114	0.082	0.085	0.010
SE of the regression	5.94E-05	5.93E-05	6.00E-05	6.03E-05	6.02E-05	6.01E-05	6.05E-05	6.04E-05	6.06E-05	5.98E-05	5.96E-05	6.07E-05	6.06E-05	6.29E-05
Sum squared resid	8.90E-07	8.90E-07	9.14E-07	9.16E-07	9.17E-07	9.19E-07	9.22E-07	9.22E-07	9.32E-07	9.00E-07	9.00E-07	9.37E-07	9.37E-07	1.02E-06
Mean dependent var	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06
SD of the dependent var	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05
Akaike info criterion	-16.60	-16.61	-16.59	-16.57	-16.58	-16.58	-16.57	-16.57	-16.57	-16.59	-16.60	-16.56	-16.57	-16.50
Schwarz criterion	-16.52	-16.54	-16.53	-16.49	-16.51	-16.53	-16.48	-16.50	-16.52	-16.51	-16.53	-16.51	-16.53	-16.47
Panel B: Coefficients														
Intercept	2.02E-06	2.02E-06	2.70E-06	2.61E-06	2.57E-06	2.84E-06	2.90E-06	2.89E-06	2.46E-06	2.46E-06	2.47E-06	2.18E-06	2.13E-06	2.88E-06
AR1	0.1275	0.1273	0.1144	0.1233	0.0947	0.0925	0.0958	0.0762	0.0761	0.0905	0.0809	0.0796	0.0831	0.1177
AR2	0.2490	0.2490	0.2648	0.2750	0.2731	0.2746	0.2854	0.2837	0.2837	0.3014	0.3005	0.2834	0.2829	
DC - 1	-0.0003	-0.0003	-0.0004	-0.0206	-0.0192	-0.0197	-0.0048	-0.0045	-0.0045	-0.0000	-0.0000	-0.0000		
DC	0.0004	0.0004		0.0060	0.0066		-0.0060	-0.0058		-0.0001	-0.0001			
DIAC - 1	-0.0006			-0.0717			-0.0508			-0.0244				
Panel C: Coefficient p-values														
Intercept	0.5852	0.5845	0.4629	0.4739	0.4816	0.4488	0.4476	0.4474	0.5227	0.5264	0.5224	0.5710	0.5779	0.5336
AR1	0.1141	0.0636	0.0900	0.1254	0.1661	0.1831	0.2032	0.3015	0.2893	0.2551	0.2765	0.2820	0.2357	0.2039
AR2	0.0024	0.0026	0.0025	0.0024	0.0026	0.0027	0.0027	0.0028	0.0026	0.0019	0.0019	0.0025	0.0024	
DC - 1	0.0287	0.0235	0.0167	0.0293	0.0264	0.0186	0.2085	0.2218	0.2481	0.9492	0.9468	0.8172		
DC	0.0310	0.0310		0.6195	0.5944		0.2313	0.2435		0.0003	0.0003			
DIAC - 1	0.9969			0.6974			0.7843			0.8896				
Panel D: Residual descriptive statistics														
Mean	-5.3E-22	3.7E-22	7.9E-22	-2.2E-21	-7.9E-22	-1.6E-21	-1.9E-21	2.6E-21	5.3E-22	1.1E-21	-1.5E-21	-2.5E-21	3.5E-21	-6.3E-22
Median	-1.9E-06	-1.9E-06	-3.2E-06	-2.4E-06	-2.4E-06	-2.7E-06	-8.3E-07	-7.0E-07	-2.4E-06	-8.7E-07	-5.8E-07	-1.7E-06	-1.6E-06	-3.5E-06
Maximum	3.5E-04	3.5E-04	3.6E-04	3.5E-04	3.5E-04	3.5E-04	3.6E-04	3.6E-04	3.6E-04	3.5E-04	3.5E-04	3.6E-04	3.6E-04	3.8E-04
Minimum	-2.8E-04	-2.8E-04	-2.7E-04	-2.8E-04	-2.8E-04	-2.7E-04	-2.6E-04	-2.6E-04	-2.8E-04	-2.7E-04	-2.7E-04	-2.8E-04	-2.8E-04	-2.8E-04
Std. Dev.	5.9E-05	5.9E-05	6.0E-05	6.0E-05	6.0E-05	6.0E-05	6.0E-05	6.0E-05	6.0E-05	5.9E-05	5.9E-05	6.0E-05	6.0E-05	6.3E-05
Skewness	0.68	0.68	0.69	0.61	0.62	0.68	0.76	0.78	0.63	0.75	0.76	0.59	0.57	0.47
Kurtosis	10.83	10.83	10.75	10.27	10.38	10.45	10.59	10.69	10.55	10.51	10.55	10.76	10.88	11.84

Table 34 Model results for IAC10 and concentration differenced over ten-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 127 observations, each estimated using ten trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference IAC10 DV10 indicates that the incremental average covariance is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DIAC – 1 denotes an asymmetric coefficient on the lagged DIAC data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. Only the AR1 model has autocorrelation that are significant at the 5% level.

Model reference	**IAC10 DV10**		**IAC10 DR10**		**IAC10 DH10**		**IAC10 DSK10**		**AAR2**	**AR2**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**			
R-squared	0.219	0.207	0.212	0.197	0.199	0.194	0.184	0.183	0.174	0.141	0.076
Adjusted R-squared	0.186	0.181	0.179	0.170	0.165	0.167	0.149	0.155	0.153	0.127	0.069
SE of the regression	4.42E-05	4.43E-05	4.44E-05	4.46E-05	4.47E-05	4.47E-05	4.52E-05	4.50E-05	4.51E-05	4.58E-05	4.71E-05
Sum squared resid	2.32E-07	2.36E-07	2.34E-07	2.39E-07	2.38E-07	2.40E-07	2.43E-07	2.43E-07	2.46E-07	2.55E-07	2.75E-07
Mean dependent var	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06	4.0E-06
SD of the dependent var	4.9E-05	4.9E-05	4.9E-05	4.9E-05	4.9E-05	4.9E-05	4.9E-05	4.9E-05	4.9E-05	4.9E-05	4.9E-05
Akaike info criterion	-17.17	-17.17	-17.16	-17.16	-17.15	-17.15	-17.13	-17.14	-17.15	-17.12	-17.07
Schwarz criterion	-17.03	-17.06	-17.03	-17.05	-17.01	-17.04	-16.99	-17.03	-17.06	-17.06	-17.03
Panel B: Coefficients											
Intercept	3.23E-06	2.67E-06	3.35E-06	2.53E-06	2.24E-06	1.99E-06	1.48E-06	1.44E-06	1.35E-06	2.26E-06	3.04E-06
AR1	0.0548	0.0531	0.0297	0.0317	0.0125	0.0132	0.0029	-0.0005	0.0094	0.2018	0.2776
AR2	0.3295	0.3167	0.3087	0.3021	0.3053	0.3026	0.3085	0.3075	0.2943	0.2674	
DIAC - 1	0.2906	0.2686	0.3145	0.3015	0.3538	0.3459	0.3828	0.3822	0.3695		
DC - 1	-0.0003	-0.0003	-0.0136	-0.0123	-0.0042	-0.0043	-0.0000	-0.0000			
DC	-0.0002		-0.0098		-0.0022		-0.0000				
Panel C: Coefficient p-values											
Intercept	0.3569	0.4509	0.3301	0.4641	0.5487	0.5885	0.7072	0.7109	0.7369	0.5815	0.5233
AR1	0.7301	0.7286	0.8365	0.8225	0.9306	0.9254	0.9837	0.9973	0.9511	0.0627	0.0434
AR2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	
DIAC - 1	0.0940	0.1034	0.0513	0.0562	0.0245	0.0258	0.0135	0.0129	0.0227		
DC - 1	0.0140	0.0167	0.0944	0.1470	0.1527	0.1756	0.4180	0.4037			
DC	0.1454		0.1767		0.3638		0.6316				
Panel D: Residual descriptive statistics											
Mean	-7.6E-22	-3.4E-21	-2.2E-21	-2.1E-21	-2.7E-21	-1.6E-21	-2.1E-21	-3.0E-21	2.2E-22	-3.7E-21	-2.4E-21
Median	-5.51E-07	-1.56E-06	-2.55E-06	-1.78E-06	-2.94E-06	-2.85E-06	-3.71E-06	-3.77E-06	-2.22E-06	-2.26E-06	-2.83E-06
Maximum	1.95E-04	1.99E-04	1.88E-04	1.97E-04	1.90E-04	1.94E-04	1.91E-04	1.92E-04	1.91E-04	2.01E-04	2.00E-04
Minimum	-1.37E-04	-1.35E-04	-1.42E-04	-1.37E-04	-1.44E-04	-1.42E-04	-1.47E-04	-1.47E-04	-1.48E-04	-1.40E-04	-1.35E-04
Std. Dev.	4.3E-05	4.4E-05	4.4E-05	4.4E-05	4.4E-05	4.4E-05	4.4E-05	4.4E-05	4.5E-05	4.5E-05	4.7E-05
Skewness	0.95	0.94	0.84	0.85	0.79	0.82	0.72	0.74	0.61	0.88	0.76
Kurtosis	7.39	7.60	6.90	7.17	6.84	6.99	6.86	6.92	7.02	7.61	6.78

Table 35 Model results for IAC15 and concentration differenced over fifteen-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 84 observations, each estimated using fifteen trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference IAC15 DV15 indicates that the incremental average covariance is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DIAC – 1 denotes an asymmetric coefficient on the lagged DIAC data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. The Jarque-Bera normality test results and p-values for rejection of the null hypothesis that the residuals were normally distributed. Finally the number of residual autocorrelation lags significant at 5% is reported out of the total of thirty-six.

Model reference	**IAC15 DV15**		**IAC15 DR15**		**IAC15 DH15**		**IAC15 DSK15**		**AAR1**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.265	0.251	0.251	0.243	0.236	0.234	0.231	0.230	0.225	0.223
Adjusted R-squared	0.228	0.223	0.214	0.214	0.197	0.205	0.192	0.201	0.206	0.213
SE of the regression	3.64E-05	3.65E-05	3.68E-05	3.67E-05	3.72E-05	3.70E-05	3.73E-05	3.70E-05	3.69E-05	3.68E-05
Sum squared resid	1.05E-07	1.07E-07	1.07E-07	1.08E-07	1.09E-07	1.09E-07	1.10E-07	1.10E-07	1.11E-07	1.11E-07
Mean dependent var	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06	3.3E-06
SD of the dependent var	4.2E-05	4.2E-05	4.2E-05	4.2E-05	4.2E-05	4.2E-05	4.2E-05	4.2E-05	4.2E-05	4.2E-05
Akaike info criterion	-17.54	-17.55	-17.53	-17.54	-17.51	-17.53	-17.50	-17.52	-17.54	-17.56
Schwarz criterion	-17.40	-17.43	-17.38	-17.42	-17.36	-17.41	-17.35	-17.41	-17.45	-17.50
Panel B: Coefficients										
Intercept	3.44E-06	2.69E-06	3.38E-06	2.67E-06	2.26E-06	2.08E-06	1.78E-06	1.74E-06	1.58E-06	1.47E-06
AR1	0.4300	0.4207	0.3994	0.3994	0.4029	0.4016	0.4131	0.4119	0.4267	0.4709
DIAC - 1	0.1171	0.0991	0.1394	0.1356	0.1389	0.1418	0.1301	0.1293	0.0962	
DC - 1	-0.0002	-0.0002	-0.0086	-0.0082	-0.0021	-0.0021	-0.0000	-0.0000		
DC	-0.0001		-0.0057		-0.0009		-0.0000			
Panel C: Coefficient p-values										
Intercept	0.3786	0.5025	0.3696	0.4996	0.5616	0.6005	0.6573	0.6630	0.7032	0.7111
AR1	0.0005	0.0003	0.0008	0.0004	0.0006	0.0004	0.0005	0.0004	0.0013	0.0000
DIAC - 1	0.5827	0.6205	0.5067	0.4888	0.5239	0.5019	0.5535	0.5522	0.6500	
DC - 1	0.0574	0.0668	0.1518	0.1612	0.3658	0.3716	0.5202	0.5195		
DC	0.3210		0.4191		0.7015		0.8298			
Panel D: Residual descriptive statistics										
Mean	-4.8E-22	-1.6E-21	6.9E-22	-8.1E-22	-8.5E-22	1.6E-22	-1.5E-21	2.4E-22	8.1E-22	6.5E-22
Median	1.1E-06	7.3E-07	-6.4E-08	-1.4E-07	8.8E-08	7.9E-07	3.1E-07	3.7E-07	1.0E-06	1.3E-06
Maximum	9.8E-05	9.3E-05	9.4E-05	8.8E-05	9.0E-05	8.8E-05	8.8E-05	8.8E-05	9.0E-05	9.5E-05
Minimum	-1.1E-04	-1.2E-04	-1.3E-04	-1.3E-04	-1.3E-04	-1.4E-04	-1.3E-04	-1.3E-04	-1.3E-04	-1.3E-04
Std. Dev.	3.6E-05	3.6E-05	3.6E-05	3.6E-05	3.6E-05	3.6E-05	3.6E-05	3.6E-05	3.7E-05	3.7E-05
Skewness	-0.03	-0.19	-0.17	-0.30	-0.27	-0.31	-0.26	-0.26	-0.27	-0.18
Kurtosis	4.13	4.40	4.43	4.69	4.71	4.79	4.69	4.71	4.72	4.80
Jarque-Bera	4	7	8	11	11	13	11	11	11	12
Probability	0.11	0.03	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Autocorrelation	1	4	1	2	2	5	6	6	5	17

Table 36 Model results for IAC20 and concentration differenced over twenty-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 64 observations, each estimated using twenty trading-days worth of return data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference IAC DV indicates that the incremental average covariance is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DIAC – 1 denotes an asymmetric coefficient on the lagged DIAC data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed could be rejected at the 5% level of significance for all models. No residual autocorrelation significant at 5% was evident in any of the models.

Model reference	**IAC20 DV20**		**IAC20 DR20**		**IAC20 DH20**		**IAC20 DSK20**		**AAR1**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.178	0.161	0.131	0.128	0.091	0.081	0.074	0.073	0.069	0.067
Adjusted R-squared	0.123	0.119	0.072	0.084	0.029	0.035	0.011	0.027	0.038	0.052
SE of the regression	3.93E-05	3.94E-05	4.04E-05	4.02E-05	4.13E-05	4.12E-05	4.17E-05	4.14E-05	4.12E-05	4.09E-05
Sum squared resid	9.11E-08	9.30E-08	9.64E-08	9.67E-08	1.01E-07	1.02E-07	1.03E-07	1.03E-07	1.03E-07	1.03E-07
Mean dependent var	2.8E-06	2.8E-06	2.8E-06	2.8E-06	2.8E-06	2.8E-06	2.8E-06	2.8E-06	2.8E-06	2.8E-06
SD of the dependent var	4.2E-05	4.2E-05	4.2E-05	4.2E-05	4.2E-05	4.2E-05	4.2E-05	4.2E-05	4.2E-05	4.2E-05
Akaike info criterion	-17.38	-17.39	-17.32	-17.35	-17.27	-17.30	-17.26	-17.29	-17.31	-17.34
Schwarz criterion	-17.21	-17.25	-17.15	-17.21	-17.11	-17.16	-17.09	-17.15	-17.21	-17.28
Panel B: Coefficients										
Intercept	2.99E-06	4.44E-06	4.85E-06	4.13E-06	3.17E-06	2.45E-06	2.11E-06	2.06E-06	2.02E-06	1.80E-06
AR1	0.3960	0.3610	0.3608	0.3732	0.2679	0.2681	0.2405	0.2359	0.2228	0.2636
DIAC - 1	-0.1581	-0.0993	-0.1811	-0.2071	0.0097	-0.0152	0.0598	0.0547	0.0740	
DC - 1	-0.0004	-0.0004	-0.0183	-0.0181	-0.0029	-0.0026	-0.0000	-0.0000		
DC	0.0002		-0.0037		-0.0022		-0.0000			
Panel C: Coefficient p-values										
Intercept	0.6072	0.4520	0.3770	0.4878	0.5876	0.6885	0.7347	0.7409	0.7455	0.7441
AR1	0.0838	0.0854	0.0869	0.0812	0.2620	0.2290	0.2781	0.2646	0.2401	0.0611
DIAC - 1	0.6735	0.7776	0.5910	0.5713	0.9791	0.9679	0.8694	0.8796	0.8263	
DC - 1	0.0111	0.0092	0.0752	0.0710	0.4123	0.4619	0.6142	0.6105		
DC	0.2434		0.6995		0.4786		0.8167			
Panel D: Residual descriptive statistics										
Mean	-8.5E-22	0.0E+00	-2.5E-21	1.3E-21	1.9E-21	-4.2E-22	-4.2E-22	2.5E-21	1.3E-21	2.1E-21
Median	2.7E-06	1.0E-06	9.3E-07	3.1E-06	1.1E-06	3.7E-06	1.2E-06	1.6E-06	9.9E-07	9.7E-07
Maximum	1.1E-04	1.1E-04	1.2E-04	1.2E-04	1.2E-04	1.2E-04	1.1E-04	1.1E-04	1.1E-04	1.2E-04
Minimum	-1.2E-04	-1.3E-04	-1.5E-04	-1.5E-04	-1.5E-04	-1.6E-04	-1.5E-04	-1.5E-04	-1.5E-04	-1.5E-04
Std. Dev.	3.8E-05	3.8E-05	3.9E-05	3.9E-05	4.0E-05	4.0E-05	4.0E-05	4.0E-05	4.1E-05	4.1E-05
Skewness	-0.24	-0.29	-0.39	-0.50	-0.30	-0.46	-0.41	-0.43	-0.48	-0.45
Kurtosis	4.20	4.60	5.92	6.12	5.56	5.94	5.55	5.63	5.59	5.62

Table 37 Out-of-sample forecasts: January – April 2003

The difference between end points for the IAC5 and the IAC10 static forecasts is due to the fact that non-overlapping data series estimated with different T values were not perfectly synchronised, between different T estimates.

Panel A: IAC5 out-of-sample forecasts based on trading days from 19th December 2002 through 16th April 2003						
Forecasting model	**AR2**	**AAR2**	**AARDL2 DV5**	**AARDL2 DR5**	**AARDL2 DH5**	**AARDL2 DSK5**
Forecast variable	IAC5	IAC5	IAC5	IAC5	IAC5	IAC5
Forecast sample:	261: 275	261: 275	261: 275	261: 275	261: 275	261: 275
Included observations:	14	14	14	14	14	14
Root Mean Squared Error	6.57E-05	6.28E-05	6.39E-05	6.52E-05	6.57E-05	6.57E-05
Mean Absolute Error	5.83E-05	5.66E-05	5.72E-05	5.79E-05	5.83E-05	5.83E-05
Mean Abs. Percent Error	265.8	274.2	276.9	269.8	265.7	265.8
Theil Inequality Coefficient	0.711	0.678	0.689	0.710	0.714	0.711
Bias Proportion	0.263	0.257	0.246	0.265	0.267	0.263
Variance Proportion	0.265	0.319	0.307	0.285	0.268	0.265
Covariance Proportion	0.471	0.424	0.446	0.450	0.465	0.471
Panel B: IAC10 out-of-sample forecasts based on trading days from 28th December 2002 through 7th April 2003						
Forecasting model	**AR2**	**AAR2**	**AARDL2 DV10**	**AARDL2 DR10**	**AARDL2 DH10**	**AARDL2 DSK10**
Forecast variable	IAC10	IAC10	IAC10	IAC10	IAC10	IAC10
Forecast sample:	128:134	128:134	128:134	128:134	128:134	128:134
Included observations:	7	7	7	7	7	7
Root Mean Squared Error	6.1E-05	5.7E-05	5.6E-05	5.7E-05	5.7E-05	5.7E-05
Mean Absolute Error	5.2E-05	5.1E-05	5.0E-05	5.1E-05	5.1E-05	5.1E-05
Mean Abs. Percent Error	131.2	140.5	152.8	139.9	137.0	141.4
Theil Inequality Coefficient	0.658	0.619	0.579	0.607	0.614	0.611
Bias Proportion	0.511	0.683	0.617	0.629	0.662	0.691
Variance Proportion	0.174	0.160	0.123	0.156	0.164	0.158
Covariance Proportion	0.315	0.157	0.259	0.215	0.174	0.150

10.5.2 Forecasts for the period: January 2003 – April 2003

Results outlined in the previous section indicate that the first lag of DR5, DV10 and DR10 may be useful for explaining the corresponding IAC data. Unfortunately, comparison of out-of-sample AARDL model forecasts with those of the naive AAR models, reported in Table 37, produces mixed results. The IAC5 forecast MAPE is lowest in the most parsimonious AR2 model while the AAR and AARDL models have higher, or similar, MAPE values. The lowest TIC occurs in the AAR2 model of the IAC5 data. The MAPE and TIC produce conflicting results when the IAC10 data are forecast. For example, the lowest TIC is obtained using the IAC10 DV10 model, but the lowest MAPE occurs in the AR2 model. The MSE of the IAC5 forecasts is dominated by the covariance-proportion. However, the bias-proportion accounts for a larger proportion of the MSE of the IAC10 forecasts, suggesting that the IAC5 forecasts may be more useful than those of the IAC10 data. In summary, the out-of-sample forecasts of the IAC data may be of only limited economic value, although the TIC, of the IAC10 forecasts, provides some evidence to suggest that inclusion of differenced concentration variables may improve out-of-sample forecasting ability. However, it should be emphasised that this evidence is by no means clear-cut.

10.5.3 Summary results from earlier sub-periods and the whole period

One-hundred-and-twelve AARDL and ARDL models were estimated for the three dependent variables IAC10, IAC15 and IAC20, over the whole study period and different sub-periods within this.[105] Table 38 summarises the results for the thirty-two of these that have DC-1 coefficients significantly different from zero at the $\alpha < 10\%$ threshold. Only one of these coefficients is positive. This was estimated using the IAC10 DV10 model over the period 1992-1997 and this is significant at $\alpha < 10\%$, but not at $\alpha < 5\%$. The remaining thirty-one coefficients are all negative and eighteen of these are also significant at $\alpha < 5\%$. The majority of the significant coefficients were estimated either over the whole period, or over periods that included the latter part of the 1990s when concentration in the FTSE 100 Index was at an historic high. However, three negative DC-1 coefficients, significant at the $\alpha < 10\%$ threshold, were also estimated using the IAC10 data in the period from 1988 through 1992. Nonetheless, it seems as if lagged differenced concentration had the greatest association with the contemporaneous incremental average covariance, during the latter part of the 1990s and the early 2000s, when levels of concentration were highest.

An additional point of note is that the coefficients on the 1987 crash dummy were negative and significant at the $\alpha < 1\%$ threshold for all relevant models. This means that, although the equally weighted average covariance and the average volatility of constituent returns spiked upwards during the 1987 crash, the incremental average covariance spiked downwards, as illustrated by Chart 9 in Chapter 8. Thus, if the FTSE 100 Index portfolio had been equally weighted at this time, i.e. not concentrated, the effect of the crash would have been much greater. A similar pattern is evident in Chart 9 for the 1992 ERM crisis, although this event is not explicitly modelled.

[105] The sub-periods are the same as those defined in Table 13 and the procedure followed is identical to that detailed on page 179, the only difference being the dependent variables.

Table 38 Summary results for IAC-series models with significant coefficients on lagged differenced concentration

Results obtained over the periods 1984-2003 and 1998-2003 relate to models estimated using all the observations through to the end of March 2003. Therefore including the out-of-sample data that is used to evaluate the similar models reported in the previous where models are estimated using data up to the end of December 2002. The reference code identifies the dependent variable, the type of model, i.e. AARDL or ARDL and the distributed lag variable. For example, model reference IAC15 DH15 AARDL2 refers to an asymmetric autoregressive distributed lag model. This has two autoregressive lags of the dependant variable IAV15, one lagged asymmetric variable DDD15 and one distributed lag variable DH15.

Panel A: Models estimated over the whole study period

Model reference	IAC10 DH10 AARDL2	IAC10 DR10 AARDL2	IAC10 DV10 AARDL2	IAC10 DV10 AARDL2	IAC15 DR15 AARDL2	IAC15 DV15 AARDL2	IAC15 DV15 ARDL2	IAC20 DR20 AARDL2	IAC20 DV20 AARDL2	IAC20 DV20 AARDL2	IAC20 DV20 ARDL2
Time period	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003
1987 Crash dummy included	Y	Y	Y	N	Y	Y	N	Y	Y	N	N
Coefficient significant at 5%	N	Y	Y	N	N	Y	N	Y	Y	N	Y
Coefficient significant at 10%	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Sign of coefficient	-	-	-	-	-	-	-	-	-	-	-
Adjusted R squared	36	37	37	18	39	40	26	37	39	21	21
Residual autocorrelation	Y	Y	Y	Y	Y	Y	Y	N	N	N	Y

Panel B: Models estimated over sub-periods

Model reference	IAC10 DH10 AARDL2	IAC10 DR10 AARDL2	IAC10 DSK10 AARDL2	IAC10 DV10 AARDL2	IAC15 DV15 AARDL2	IAC15 DV15 ARDL2	IAC20 DH20 AARDL2	IAC20 DH20 ARDL2	IAC20 DR20 AARDL2	IAC20 DR20 AARDL2	IAC20 DV20 AARDL2
Time period	1988 1992	1988 1992	1988 1992	1993 1997	1991 2000	1991 2000	1991 2000	1991 2000	1991 2000	1991 2000	1991 2000
Coefficient significant at 5%	N	N	N	N	Y	Y	N	Y	N	Y	Y
Coefficient significant at 10%	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Sign of coefficient	-	-	-	+	-	-	-	-	-	-	-
Adjusted R squared	5	10	1	23	17	17	28	29	32	32	35
Residual autocorrelation	N	N	N	Y	Y	Y	Y	Y	Y	Y	N

Panel C: Models estimated over sub-periods, including the most recent that includes data to the end of March 2003

Model reference	IAC20 DV20 ARDL2	IAC20 DSK20 AARDL2	IAC20 DSK20 ARDL2	IAC10 DV10 AARDL2	IAC15 DV15 AARDL2	IAC15 DV15 ARDL2	IAC20 DR20 AARDL2	IAC20 DR20 ARDL2	IAC20 DV20 AARDL2	IAC20 DV20 ARDL2
Time period	1991 2000	1991 2000	1991 2000	1998 2003	1998 2003	1998 2003	1998 2003	1998 2003	1998 2003	1998 2003
Coefficient significant at 5%	Y	N	Y	Y	Y	N	Y	N	Y	Y
Coefficient significant at 10%	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Sign of coefficient	-	-	-	-	-	-	-	-	-	-
Adjusted R squared	35	26	27	21	25	25	16	16	19	20
Residual autocorrelation	N	Y	Y	N	N	Y	N	N	N	N

10.5.4 Summary analysis of all IAC-series models and forecasts

The results presented in this section indicate that the incremental average covariance of FTSE 100 constituent returns tends to fall following increases in concentration, as evidenced by the negative coefficients on lagged changes in concentration. However, there is relatively little evidence to suggest that contemporaneous changes in differenced concentration are associated with contemporaneous changes in the incremental average covariance. In addition, unlike the model results for the other three sub-components of realised volatility, there is also relatively little evidence to suggest that falls in the value of the FTSE 100 Index precede changes in the incremental average covariance, i.e. the asymmetry effect. The negative sign of coefficients on the first lag of changes in concentration indicate that investors may concentrate their portfolios into assets with returns that have a lower than average covariance prior to increases in volatility. This may be an example of risk avoidance via active diversification efforts, prior to anticipated volatility, a behaviour that is consistent with the mean variance optimisation principles of MPT.

The out-of-sample forecast results indicate that, despite the significant coefficients estimated within the sample, the incremental average covariance is difficult to forecast. There is only very limited evidence to suggest that changes in concentration measured using the variance of the logarithm of firm size and Hannah and Kay's $R^{\alpha=0.5}$ may be able to improve the out-of-sample forecasting ability of the models. Overall, the MAPE and TIC of model-out-of-sample forecasts are high when compared to out-of-sample forecasts of the equally weighted average variance, the equally weighted average covariance and the incremental average variance.

The next section explores the relationship between concentration and realised volatility further by combining the incremental average variance and the incremental average covariance of FTSE 100 Index constituent returns to form a series referred to as the incremental realised volatility of the FTSE 100 Index returns (IRV).

10.6 Models of the incremental realised volatility (IRV)

This section reports results for the general AARDL models, the general ARDL models, the naive AAR and the naive AR models of the incremental realised volatility, IRV, over the whole study period and sub-periods within this.[106]

10.6.1 Model results: January 1998 through December 2002

This section reports results for models of the four incremental realised volatility series: IRV5, IRV10, IRV15 and IRV20 over the sub-period from January 1998 through December 2002. Two AARDL models are estimated for each of the four differenced concentration metrics with each of the four IRV series, giving a total of thirty-two. The reporting format adopted is the same as that applied in corresponding earlier sections. An additional four ARDL models are estimated for the IRV5 series, one for each differenced concentration index, because it was observed that the asymmetric coefficient was not significant in any of the IRV5 models.

10.6.1.1 Coefficients for the first lag of differenced concentration (DC-1)

Of the thirty-six AARDL models for which results are reported, in Table 39 through Table 42, eleven have negative coefficients on the DC-1 variables that are significantly different from zero at the α < 10% threshold. Eight of these are also significant at the α < 5% threshold. All DC-1 coefficients are negative in the IRV models, regardless of whether or not they are significantly different from zero. All models that use DV as a metric of differenced concentration have DC-1 coefficients that are significantly different from zero at the α < 10% threshold. Thus, these results present evidence consistent with the idea that the changes in the variance of the logarithm of firm size precede changes in the level of incremental realised volatility of the opposite sign. In other words, lagged changes in concentration have an inverse relationship with contemporaneous changes in the incremental realised volatility. Furthermore, the relationship is most evident when the differenced variance of the logarithm of firm size is used as the metric for changing concentration.

---

[106] As stated earlier in section 6.2.2.3, the IRV data represents the combined contribution of the incremental average variance and incremental average covariance of FTSE 100 Index constituent returns to the VCM. It is the difference between the sum of the market value weighted and the sum of the equally weighted diagonal elements in the FTSE 100 VCM, plus the difference between the sum of the market value weighted and the sum of the equally weighted off-diagonal elements in the VCM.

10.6.1.2 Coefficients on contemporaneous differenced concentration (DC)

Out of the thirty-six AARDL models discussed in this section, twenty were estimated with a contemporaneous DC coefficient to determine, whether or not, a contemporaneous association existed between changing concentration and changing incremental realised volatility. Four out of twenty had DC coefficients that were significantly different from zero at $\alpha < 5\%$. Two were observed in the IRV5 DV5 models with positive coefficients and two in the IRV5 DSK5 models with negative coefficients. Adjusted R^2, AIC and SIC values indicated that the IRV5 DV5 model with the contemporaneous DC coefficient had a better fit than the corresponding AARDL model without the DC coefficient. Only the adjusted R^2 and AIC indicated that the IRV5 DSK5 model with the DC coefficient had a better fit than the corresponding AARDL model without the DC coefficient. The inconsistent sign makes it difficult to ascertain whether or not an association does exist between these variables, or whether the result is a statistical artefact.

10.6.1.3 Coefficients of the asymmetric dummy variable (DIRV)

A total of forty models were estimated with asymmetric coefficients, including both AARDL and AR models. However, only five had asymmetric, DIRV, coefficients significantly different from zero at the $\alpha < 10\%$ threshold, two of these were also significant at the $\alpha < 5\%$ threshold. This seems to indicate that the asymmetry effect in the time series of incremental realised volatility is not clearly defined. It seems that the asymmetry effect is more prevalent in the equally weighted average variance and covariance of constituent returns in the VCM.

10.6.1.4 Model comparison

The adjusted R^2 values, reported in Table 39 through Table 42, are generally higher in the general AARDL and ARDL models than in the naive AAR and AR models while the AIC values are consistently more negative in the AARDL models. This indicates that the first lag of the differenced concentration variable improves the explanatory power of the forecasting models. However, the improvement in model explanatory power is not so great as to make the SIC more favourable in the AARDL models. This is arguably the most robust measure of model explanatory power because it applies the greatest penalty to models that have fewer degrees of freedom. Thus, comparison of the model fit between the general AARDL, ARDL and naive AAR or AR models, indicates that inclusion of the distributed lag DC-1 variable improves model specification in a manner consistent with the significance levels recorded on individual coefficients, although the improvement in model fit does not appear to be substantial. This implies that the general models are unlikely to have a better out-of-sample forecasting performance than the naive models. When the AAR models are compared with the more parsimonious AR models, the IRV10 AAR model has an adjusted R^2 and AIC values that indicate a better model fit than the more parsimonious AR model. Unfortunately, the SIC values are not consistent with the AIC or adjusted R^2 values.

10.6.1.5 Residual analysis

There is no evidence of residual autocorrelation in any of the model results reported in this sub-section. The mean and median values of the residuals in all models are close to zero in relation to the standard deviations. The highest skewness and kurtosis in the residuals exist in the models of IRV5 and IRV10. The IRV15 and IRV20 models have much lower skewness and kurtosis. However, the null hypothesis that the residuals are normally distributed can be rejected at the $\alpha < 1\%$ threshold in all models.

10.6.1.6 Synopsis

The model results provide evidence that changes in concentration tend to be followed by changes in incremental realised volatility of the opposite sign. However, evidence to suggest that contemporaneous changes in concentration are associated with contemporaneous changes in incremental realised volatility is more limited. Furthermore, there is little evidence to indicate that falls in the value of the FTSE 100 Index precede increases in the incremental realised volatility as a whole. This raises the possibility that the majority of the asymmetry effect documented in previous studies can be attributed to the equally weighted components of the VCM, such as the equally weighted average variance and covariance.

Table 39 Model results for IRV5 and concentration differenced over five-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 259 observations, each estimated using five trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference IRV5 DV5 indicates that the incremental realised volatility is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DIRV – 1 denotes an asymmetric coefficient on the lagged DIRV data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. There is no evidence of autocorrelation in any of the residuals of models reported in this table.

Model reference	**IRV5 DV5**			**IRV5 DR5**			**IRV5 DH5**			**IRV5 DSK5**			**AR2**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.120	0.119	0.083	0.088	0.087	0.078	0.073	0.072	0.070	0.094	0.094	0.068	0.067	0.006
Adjusted R-squared	0.103	0.105	0.073	0.069	0.073	0.067	0.054	0.058	0.059	0.076	0.080	0.057	0.060	0.002
SE of the regression	6.20E-05	6.20E-05	6.31E-05	6.32E-05	6.31E-05	6.33E-05	6.37E-05	6.36E-05	6.35E-05	6.29E-05	6.28E-05	6.36E-05	6.35E-05	6.53E-05
Sum squared resid	9.70E-07	9.72E-07	1.01E-06	1.01E-06	1.01E-06	1.02E-06	1.02E-06	1.02E-06	1.03E-06	9.99E-07	9.99E-07	1.03E-06	1.03E-06	1.10E-06
Mean dependent var	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05
SD of the dependent var	6.6E-05	6.6E-05	6.6E-05	6.6E-05	6.6E-05	6.6E-05	6.6E-05	6.6E-05	6.6E-05	6.6E-05	6.6E-05	6.6E-05	6.6E-05	6.5E-05
Akaike info criterion	-16.51	-16.52	-16.49	-16.48	-16.49	-16.48	-16.46	-16.47	-16.47	-16.49	-16.49	-16.47	-16.48	-16.43
Schwarz criterion	-16.43	-16.45	-16.43	-16.40	-16.42	-16.43	-16.38	-16.40	-16.42	-16.40	-16.42	-16.42	-16.44	-16.40
Panel B: Coefficients														
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.0702	0.1010	0.0835	0.0615	0.0700	0.0666	0.0432	0.0528	0.0519	0.0406	0.0539	0.0512	0.0542	0.0743
AR2	0.2171	0.2191	0.2336	0.2412	0.2417	0.2446	0.2513	0.2521	0.2513	0.2670	0.2680	0.2507	0.2502	
DC - 1	-0.0003	-0.0003	-0.0003	-0.0143	-0.0147	-0.0158	-0.0033	-0.0035	-0.0034	-0.0000	-0.0000	-0.0000		
DC	0.0005	0.0005		0.0150	0.0148		-0.0029	-0.0030		-0.0001	-0.0001			
DIRV - 1	0.0802			0.0218			0.0254			0.0348				
Panel C: Coefficient p-values														
Intercept	0.0045	0.0041	0.0016	0.0029	0.0027	0.0018	0.0025	0.0023	0.0027	0.0042	0.0041	0.0032	0.0029	0.0005
AR1	0.3122	0.1952	0.2984	0.3717	0.3916	0.4322	0.5097	0.5401	0.5417	0.5579	0.5296	0.5522	0.5205	0.4934
AR2	0.0077	0.0087	0.0102	0.0083	0.0085	0.0097	0.0098	0.0099	0.0095	0.0072	0.0072	0.0097	0.0096	
DC - 1	0.0927	0.0529	0.0308	0.1286	0.0725	0.0469	0.3692	0.3273	0.3412	0.8962	0.9010	0.7707		
DC	0.0116	0.0123		0.2313	0.2401		0.5524	0.5411		0.0007	0.0007			
DIRV - 1	0.6749			0.9230			0.9125			0.8747				
Panel D: Residual descriptive statistics														
Mean	-4.2E-21	-6.8E-21	-5.5E-21	-5.5E-21	-3.2E-21	-4.3E-21	-6.2E-21	-5.7E-21	-2.8E-21	-9.1E-21	-5.6E-21	-6.1E-21	-3.6E-21	-4.8E-21
Median	-5.3E-06	-5.7E-06	-4.8E-06	-6.1E-06	-6.3E-06	-5.5E-06	-4.8E-06	-4.5E-06	-4.6E-06	-5.5E-06	-6.0E-06	-4.4E-06	-4.2E-06	-6.4E-06
Maximum	3.8E-04	3.8E-04	3.9E-04	3.8E-04	3.8E-04	3.9E-04	3.9E-04	3.9E-04	3.9E-04	3.9E-04	3.9E-04	3.9E-04	3.9E-04	4.1E-04
Minimum	-2.7E-04	-2.7E-04	-2.5E-04	-2.7E-04	-2.7E-04	-2.5E-04	-2.5E-04	-2.5E-04	-2.6E-04	-2.5E-04	-2.5E-04	-2.6E-04	-2.6E-04	-2.7E-04
Std. Dev.	6.1E-05	6.2E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.3E-05	6.2E-05	6.2E-05	6.3E-05	6.3E-05	6.5E-05
Skewness	1.18	1.17	1.15	1.03	1.03	1.14	1.17	1.17	1.11	1.21	1.21	1.08	1.07	1.05
Kurtosis	11.34	11.23	11.21	10.76	10.72	11.03	11.28	11.24	11.12	11.18	11.12	11.21	11.27	12.24

Table 40 Model results for IRV10 and concentration differenced over ten-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 127 observations, each estimated using ten trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference IRV10 DV10 indicates that the incremental realised volatility is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DIRV – 1 denotes an asymmetric coefficient on the lagged DIRV data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed was rejected at the 1% level of significance for all models. There is no evidence of autocorrelation in any of the residuals of models reported in this table.

Model reference	**IRV10 DV10**		**IRV10 DR10**		**IRV10 DH10**		**IRV10 DSK10**		**AAR2**	**AR2**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**			
R-squared	0.157	0.155	0.144	0.142	0.138	0.138	0.131	0.131	0.129	0.104	0.049
Adjusted R-squared	0.121	0.126	0.108	0.114	0.102	0.110	0.094	0.102	0.107	0.089	0.042
SE of the regression	4.7E-05	4.6E-05	4.7E-05	4.7E-05	4.7E-05	4.7E-05	4.7E-05	4.7E-05	4.7E-05	4.7E-05	4.8E-05
Sum squared resid	2.6E-07	2.6E-07	2.6E-07	2.6E-07	2.6E-07	2.6E-07	2.7E-07	2.7E-07	2.7E-07	2.7E-07	2.9E-07
Mean dependent var	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05
SD of the dependent var	5.0E-05	5.0E-05	5.0E-05	5.0E-05	5.0E-05	5.0E-05	5.0E-05	5.0E-05	5.0E-05	5.0E-05	5.0E-05
Akaike info criterion	-17.06	-17.08	-17.05	-17.06	-17.04	-17.06	-17.03	-17.05	-17.06	-17.05	-17.02
Schwarz criterion	-16.93	-16.96	-16.91	-16.95	-16.91	-16.95	-16.90	-16.94	-16.97	-16.98	-16.97
Panel B: Coefficients											
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.0475	0.0474	0.0388	0.0395	0.0262	0.0261	0.0046	0.0061	0.0021	0.1680	0.2234
AR2	0.2772	0.2723	0.2650	0.2635	0.2643	0.2645	0.2655	0.2660	0.2623	0.2408	
DIRV - 1	0.2326	0.2257	0.2412	0.2365	0.2646	0.2657	0.2985	0.2999	0.3052		
DC - 1	-0.0002	-0.0002	-0.0101	-0.0097	-0.0030	-0.0030	-0.0000	-0.0000			
DC	-0.0001		-0.0031		0.0002		0.0000				
Panel C: Coefficient p-values											
Intercept	0.0035	0.0034	0.0023	0.0023	0.0068	0.0057	0.0118	0.0125	0.0116	0.0109	0.0034
AR1	0.7212	0.7160	0.7642	0.7559	0.8430	0.8433	0.9721	0.9631	0.9868	0.1293	0.1057
AR2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
DIRV - 1	0.1565	0.1479	0.1474	0.1330	0.0948	0.0815	0.0433	0.0384	0.0310		
DC - 1	0.0287	0.0323	0.2261	0.2589	0.3242	0.3164	0.6919	0.6890			
DC	0.5099		0.6519		0.9198		0.7656				
Panel D: Residual descriptive statistics											
Mean	5.0E-21	3.6E-21	4.0E-21	4.8E-21	2.0E-21	5.5E-21	6.0E-21	4.6E-21	5.6E-21	4.8E-21	1.1E-21
Median	-4.3E-06	-3.6E-06	-4.6E-06	-3.2E-06	-3.2E-06	-3.1E-06	-3.0E-06	-1.8E-06	-2.2E-06	-5.7E-06	-6.0E-06
Maximum	2.4E-04	2.4E-04	2.3E-04	2.4E-04	2.3E-04	2.3E-04	2.3E-04	2.3E-04	2.3E-04	2.4E-04	2.4E-04
Minimum	-1.4E-04	-1.4E-04	-1.4E-04	-1.4E-04	-1.5E-04	-1.5E-04	-1.5E-04	-1.5E-04	-1.5E-04	-1.5E-04	-1.4E-04
Std. Dev.	4.6E-05	4.6E-05	4.6E-05	4.6E-05	4.6E-05	4.6E-05	4.6E-05	4.6E-05	4.6E-05	4.7E-05	4.8E-05
Skewness	1.44	1.46	1.34	1.38	1.33	1.32	1.24	1.21	1.17	1.42	1.33
Kurtosis	9.71	9.84	9.13	9.30	9.13	9.09	9.01	8.92	8.90	9.80	8.86

Table 41 Model results for IRV15 and concentration differenced over fifteen-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results are obtained using 84 observations, each estimated using fifteen trading-days worth of daily data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference IRV15 DV15 indicates that the incremental realised volatility is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DIRV – 1 denotes an asymmetric coefficient on the lagged DIRV data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. The null hypothesis that the residuals were normally distributed was rejected at the 5% significance level for all models using the Jarque-Bera test. No residual autocorrelation significant at 5% was evident in any of the models.

Model reference	**IRV15 DV15**		**IRV15 DR15**		**IRV15 DH15**		**IRV15 DSK15**		**AAR1**	**AR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.223	0.217	0.213	0.212	0.200	0.200	0.194	0.194	0.191	0.184
Adjusted R-squared	0.184	0.188	0.174	0.183	0.159	0.170	0.153	0.164	0.171	0.174
SE of the regression	3.7E-05	3.7E-05	3.8E-05	3.7E-05	3.8E-05	3.8E-05	3.8E-05	3.8E-05	3.8E-05	3.8E-05
Sum squared resid	1.1E-07	1.1E-07	1.1E-07	1.1E-07	1.1E-07	1.1E-07	1.1E-07	1.1E-07	1.2E-07	1.2E-07
Mean dependent var	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05	1.8E-05
SD of the dependent var	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05
Akaike info criterion	-17.50	-17.51	-17.48	-17.51	-17.47	-17.49	-17.46	-17.48	-17.50	-17.52
Schwarz criterion	-17.35	-17.40	-17.34	-17.39	-17.32	-17.37	-17.31	-17.37	-17.42	-17.46
Panel B: Coefficients										
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.3601	0.3586	0.3586	0.3619	0.3620	0.3615	0.3634	0.3634	0.3638	0.4299
DIRV - 1	0.1855	0.1656	0.1690	0.1593	0.1617	0.1632	0.1575	0.1584	0.1527	
DC - 1	-0.0002	-0.0002	-0.0090	-0.0088	-0.0020	-0.0020	-0.0000	-0.0000		
DC	-0.0001		-0.0020		0.0002		0.0000			
Panel C: Coefficient p-values										
Intercept	0.0025	0.0027	0.0017	0.0019	0.0045	0.0043	0.0080	0.0076	0.0094	0.0087
AR1	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002
DIRV - 1	0.4920	0.5338	0.5188	0.5452	0.5507	0.5470	0.5760	0.5656	0.5762	
DC - 1	0.0348	0.0346	0.1006	0.0973	0.3254	0.3181	0.6300	0.6257		
DC	0.5185		0.7526		0.9347		0.9513			
Panel D: Residual descriptive statistics										
Mean	-3.3E-21	-2.3E-21	-4.0E-21	-6.5E-22	-3.4E-21	-2.9E-21	-1.1E-21	-1.5E-21	-3.2E-21	-3.5E-21
Median	6.1E-08	-1.7E-06	-1.3E-06	-1.7E-06	-2.9E-06	-2.8E-06	-3.6E-06	-3.7E-06	-3.4E-06	-2.2E-06
Maximum	1.1E-04	1.1E-04	1.1E-04	1.1E-04	1.1E-04	1.1E-04	1.1E-04	1.1E-04	1.1E-04	1.2E-04
Minimum	-1.0E-04	-1.1E-04	-1.2E-04	-1.2E-04	-1.2E-04	-1.2E-04	-1.2E-04	-1.2E-04	-1.2E-04	-1.2E-04
Std. Dev.	3.6E-05	3.7E-05	3.7E-05	3.7E-05	3.7E-05	3.7E-05	3.7E-05	3.7E-05	3.7E-05	3.7E-05
Skewness	0.33	0.27	0.21	0.18	0.16	0.16	0.19	0.18	0.18	0.38
Kurtosis	4.54	4.55	4.70	4.75	4.85	4.85	4.79	4.79	4.77	5.10

Table 42 Model results for IRV20 and concentration differenced over twenty-trading days: January 1998 – December 2002

Asymmetric autoregressive distributed lag (AARDL) model results estimated using 64 observations, each estimated using twenty trading-days worth of return data. A full list of acronym definitions is presented in Table 4 on page 136. Model reference codes identify the dependent variable in the AR model and the independent distributed lag variable. For example, model reference IRV20 DV20 indicates that the incremental realised volatility is the dependent autoregressive variable and the differenced variance of the logarithm of firm size is the independent distributed lag variable. The table is divided into four panels. Panel A provides metrics of model fit for model comparison. Panel B reports coefficients on the model intercept. First and second autoregressive lag coefficients are represented by AR1 and AR2, respectively, while DIRV – 1 denotes an asymmetric coefficient on the lagged DIRV data. DC – 1 represents the coefficient on the first lag of differenced concentration and DC represents the contemporaneous coefficient. Results of two models are reported for each measure of concentration, the first (DC and DC - 1) includes contemporaneous change in concentration and the second (DC - 1 only) omits this variable. Panel C reports the p-values for the respective coefficients for the rejection of the null hypothesis that the coefficient is equal to zero. T-statistics used to calculate the p-values are derived from Newey-West Heteroskedasticity and autocorrelation robust standard errors. Panel D reports descriptive statistics for the model residuals. Using the Jarque-Bera test, the null hypothesis that the residuals were normally distributed could be rejected at the 5% level of significance for all models. No residual autocorrelation significant at 5% was evident in any of the models.

Model reference	**IRV20 DV20**		**IRV20 DR20**		**IRV20 DH20**		**IRV20 DSK20**		**AARDL1**	**AAR1**
Panel A: Tests of model fit	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**	**DC and DC - 1**	**DC - 1 only**		
R-squared	0.154	0.139	0.105	0.105	0.075	0.072	0.068	0.068	0.066	0.046
Adjusted R-squared	0.096	0.096	0.045	0.060	0.013	0.025	0.005	0.021	0.036	0.030
SE of the regression	3.9E-05	3.9E-05	4.0E-05	4.0E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05
Sum squared resid	9.1E-08	9.3E-08	9.7E-08	9.7E-08	1.0E-07	1.0E-07	1.0E-07	1.0E-07	1.0E-07	1.0E-07
Log likelihood	561	560	559	559	558	558	558	558	558	557
Mean dependent var	1.7E-05	1.7E-05	1.7E-05	1.7E-05	1.7E-05	1.7E-05	1.7E-05	1.7E-05	1.7E-05	1.7E-05
SD of the dependent var	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05	4.1E-05
Akaike info criterion	-17.37	-17.39	-17.32	-17.35	-17.29	-17.31	-17.28	-17.31	-17.34	-17.35
Schwarz criterion	-17.21	-17.25	-17.15	-17.21	-17.12	-17.18	-17.11	-17.17	-17.24	-17.28
Panel B: Coefficients										
Intercept	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR1	0.1933	0.1706	0.1671	0.1717	0.1010	0.1076	0.0951	0.0948	0.0915	0.2179
DIRV - 1	0.1496	0.1882	0.1324	0.1224	0.2693	0.2403	0.2724	0.2690	0.2709	
DC - 1	-0.0003	-0.0003	-0.0136	-0.0136	-0.0018	-0.0016	-0.0000	-0.0000		
DC	0.0001		-0.0012		-0.0014		-0.0000			
Panel C: Coefficient p-values										
Intercept	0.0061	0.0040	0.0023	0.0034	0.0077	0.0088	0.0132	0.0123	0.0119	0.0092
AR1	0.0999	0.1139	0.1849	0.1698	0.4227	0.3703	0.3991	0.3929	0.3727	0.0776
DIRV - 1	0.6599	0.5763	0.6748	0.7223	0.4347	0.4995	0.4516	0.4460	0.4348	
DC - 1	0.0203	0.0173	0.1536	0.1473	0.6010	0.6340	0.7707	0.7653		
DC	0.2342		0.8905		0.5902		0.9305			
Panel D: Residual descriptive statistics										
Mean	-1.7E-21	2.8E-21	3.4E-21	8.5E-22	1.7E-21	2.5E-21	2.3E-21	2.1E-21	2.3E-21	1.7E-21
Median	-9.6E-07	-6.9E-07	-9.5E-07	-4.8E-07	-1.7E-06	5.5E-07	-2.4E-07	-1.2E-07	-1.1E-07	-7.3E-07
Maximum	1.3E-04	1.3E-04	1.4E-04	1.4E-04	1.4E-04	1.4E-04	1.4E-04	1.4E-04	1.4E-04	1.5E-04
Minimum	-1.1E-04	-1.1E-04	-1.3E-04	-1.3E-04	-1.3E-04	-1.3E-04	-1.3E-04	-1.3E-04	-1.3E-04	-1.3E-04
Std. Dev.	3.8E-05	3.8E-05	3.9E-05	3.9E-05	4.0E-05	4.0E-05	4.0E-05	4.0E-05	4.0E-05	4.0E-05
Skewness	0.38	0.37	0.31	0.28	0.37	0.29	0.29	0.29	0.27	0.50
Kurtosis	4.63	5.00	5.93	5.94	5.65	5.80	5.62	5.65	5.63	6.26

Table 43 IRV Model forecasts: January – April 2003

The difference between end points for the IRV5 and the IRV10 static forecasts is due to the fact that non-overlapping data series estimated with different T values were not perfectly synchronised, between different T estimates.

Panel A: IAC5 out-of-sample forecasts based on trading days from 19th December 2002 through 16th April 2003						
Forecasting model	**AR2**	**AAR2**	**ARDL2 DV5**	**ARDL2 DR5**	**ARDL2 DH5**	**ARDL2 DSK5**
Forecast variable	IRV5	IRV5	IRV5	IRV5	IRV5	IRV5
Forecast sample:	261: 275	261: 275	261: 275	261: 275	261: 275	261: 275
Included observations:	14	14	14	14	14	14
Root Mean Squared Error	6.61E-05	6.53E-05	6.39E-05	6.44E-05	6.56E-05	6.61E-05
Mean Absolute Error	5.75E-05	5.64E-05	5.59E-05	5.65E-05	5.72E-05	5.75E-05
Mean Abs. Percent Error	150	145	145	146	149	150
Theil Inequality Coefficient	0.885	0.876	0.869	0.878	0.886	0.886
Bias Proportion	0.342	0.343	0.333	0.324	0.341	0.345
Variance Proportion	0.269	0.279	0.307	0.306	0.284	0.272
Covariance Proportion	0.389	0.378	0.360	0.370	0.375	0.383
Panel B: IAC10 out-of-sample forecasts based on trading days from 28th December 2002 through 7th April 2003						
Forecasting model	**AR2**	**AAR2**	**AARDL2 DV10**	**AARDL2 DR10**	**AARDL2 DH10**	**AARDL2 DSK10**
Forecast variable	IRV10	IRV10	IRV10	IRV10	IRV10	IRV10
Forecast sample:	128:134	128:134	128:134	128:134	128:134	128:134
Included observations:	7	7	7	7	7	7
Root Mean Squared Error	5.95E-05	5.79E-05	5.60E-05	5.70E-05	5.74E-05	5.77E-05
Mean Absolute Error	5.21E-05	5.19E-05	5.12E-05	5.16E-05	5.17E-05	5.18E-05
Mean Abs. Percent Error	579	732	749	661	670	729
Theil Inequality Coefficient	0.811	0.759	0.716	0.751	0.759	0.757
Bias Proportion	0.606	0.715	0.678	0.675	0.693	0.713
Variance Proportion	0.165	0.143	0.117	0.144	0.149	0.144
Covariance Proportion	0.229	0.142	0.205	0.181	0.158	0.143

10.6.2 IRV forecasts: January 2003 – April 2003

Although the previous section reports evidence of a negative association between lagged changes in concentration and incremental realised volatility, the MAPE and TIC of the AARDL model forecasts, reported in panel A of Table 43, are little better than those of the naive AAR model forecasts, and in some cases not as good. They are marginally better for the AARDL model that incorporates DV5 as the differenced concentration metric, while naive AAR model forecasts have slightly better MAPE and TIC values than the AR model forecasts, despite the lack of significant asymmetric DIRV coefficients on the within sample model results. The same pattern is evident in panel B of Table 43 in which the forecasts generated from the model incorporating DV10 produce slightly lower MAPE and TIC values than the naive AAR model forecasts. The naive AAR model forecasts, in turn, out-perform the AR model forecasts, providing more evidence in favour of the asymmetry effect than the within sample results reported in the previous section. Furthermore, the decomposition of the MSE does not provide a favourable evaluation of the out-of-sample forecasts because the MSE values are all dominated by the bias proportion, although the lowest bias proportion and highest covariance proportion are found in the models that include the lagged DV10 data. Therefore, improvement in out-of-sample forecast performance of the models incorporating lagged DV data, although slight, is consistent across all forecast evaluation metrics, providing some encouragement for further research into the modelling potential of this concentration metric.

10.6.3 Summary results of models from earlier sub-periods and the whole period

Table 44 reports results for models similar to those discussed in the previous two sections. However, models were estimated for the three series, IRV10, IRV15 and IRV20, over different sub-periods and the whole period from 1984 through 2003.[107] Sixty-eight AARDL and ARDL models were estimated for the three IRV data series IRV10, IRV15 and IRV20. Table 44 summarises the results for the twenty-two of these that have DC-1 coefficients significantly different from zero at the α < 10% threshold. Nine of these were also significant at the α < 5% threshold. Hence, the proportion of models that had DC-1 coefficients significantly different from zero is much greater than would have been expected due to random chance if the first lag of differenced concentration had no explanatory power for forecasting the IRV data. All except one of the significant coefficients were negative.

An additional point of note is that the coefficients on the dummy variable for the 1987 crash were all significant and all negative. In addition, they made a substantial difference to the explanatory power of the model. This is particularly interesting because it demonstrates that the volatility associated with the 1987 crash would have been much greater, had the Index portfolio been equally weighted at that time. Thus, concentration in the FTSE 100 Index portfolio had the effect of reducing the index portfolio volatility during the 1987 crash.

[107] Sub-periods are the same as those defined in Table 13 in Chapter 10 and the procedure followed is identical to that detailed on page 179, the only differences being the dependent variables.

Table 44 Summary results for IRV models with significant coefficients on lagged differenced concentration

Results for models estimated over the periods from 1984 through 2003 and 1998 through 2003 relate to models estimated using all the observations through to the end of March 2003. Therefore including the out-of-sample data that is used to evaluate the similar models reported in the previous where models are estimated using data up to the end of December 2002. The reference code identifies the dependent variable, the type of model, i.e. AARDL or ARDL and the distributed lag variable. For example, model reference IRV15 DH15 AARDL2 refers to an asymmetric autoregressive distributed lag model. This has two autoregressive lags of the dependant variable IRV15, one lagged asymmetric variable DBD15 and one distributed lag variable DH15. The null hypothesis that the model residuals are normally distributed can be rejected in all model results reported. The significance levels of all model coefficients were determined using Newey-West heteroskedasticity and autocorrelation robust standard errors.

Panel A: Summary of results from model estimated over the whole period

Model reference	IRV10 DR10 AARDL2	IRV10 DV10 AARDL2	IRV10 DV10 AARDL2	IRV15 DV15 AARDL2	IRV20 DR20 AARDL1	IRV20 DV20 AARDL1	IRV20 DV20 AARDL1
Time period	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003	1984-2003
1987 Crash dummy included	Y	Y	N	Y	Y	Y	N
Coefficient significant at 5%	N	N	N	Y	N	Y	N
Coefficient significant at 10%	Y	Y	Y	Y	Y	Y	Y
Sign of coefficient	-	-	-	-	-	-	-
Adjusted R squared	34	35	19	39	34	37	21
Residual autocorrelation	Y	Y	Y	Y	Y	Y	Y

Panel B: Summary of results from models estimated over sub-periods

Model reference	IRV10 DH10 AARDL2	IRV10 DR10 AARDL2	IRV10 DSK10 AARDL2	IRV10 DV10 AARDL2	IRV10 DV10 AARDL2	IRV15 DV15 AARDL2	IRV20 DH20 AARDL1
Time period	1988 1992	1988 1992	1988 1992	1988 1992	1993 1997	1991 2000	1991 2000
Coefficient significant at 5%	N	N	N	N	N	Y	Y
Coefficient significant at 10%	Y	Y	Y	Y	Y	Y	Y
Sign of coefficient	-	-	-	-	+	-	-
Adjusted R squared	5	10	0	10	28	21	24
Residual autocorrelation	Y	Y	Y	Y	Y	Y	Y

Panel C: Summary of results of models estimated over sub-periods

Model reference	IRV20 DR20 AARDL1	IRV20 DV20 AARDL1	IRV20 DSK20 AARDL1	IRV10 DV10 AARDL2	IRV15 DR15 AARDL2	IRV15 DV15 AARDL2	IRV20 DR20 AARDL2	IRV20 DV20 AARDL2
Time period	1991 2000	1991 2000	1991 2000	1998 2003	1998 2003	1998 2003	1998 2003	1998 2003
Coefficient significant at 5%	N	Y	Y	Y	N	Y	N	Y
Coefficient significant at 10%	Y	Y	Y	Y	Y	Y	Y	Y
Sign of coefficient	-	-	-	-	-	-	-	-
Adjusted R squared	28	31	23	31	21	22	9	13
Residual autocorrelation	Y	Y	Y	Y	N	N	N	N

10.6.4 Summary of all IRV model results

Section 10.6 has reported results from models of the incremental realised volatility of the FTSE 100 Index on contemporaneous and lagged differenced concentration using four different concentration metrics. It has presented a detailed analysis of model results for the period from 1998 through 2003, including out-of-sample forecasts for the period January to April 2003. A summary is also provided of the results of an additional sixty-eight AARDL and ARDL models of the IRV-series estimated over the whole study period from January 1984 to March 2003 and sub-periods within this time frame. The data provides substantial evidence to suggest that lagged changes in concentration can improve the explanatory power of models of the incremental realised volatility of the index. Lagged concentration also appears to have limited out-of-sample forecasting potential, based on the results of the forecast evaluation period from January through March 2003. Furthermore, the consistently negative coefficients on the lagged change in concentration, in the AARDL models of differenced concentration and the incremental realised volatility, suggest that the relationship between the incremental realised volatility and changes in concentration is an inverse one. Rather more limited evidence exists to suggest that falls in the value of the FTSE 100 Index precede increases in the incremental realised volatility, in a manner analogous to the well-documented asymmetry effect. This contrasts with the strong evidence in support of the asymmetry effect reported for the equally weighted sub-components of the VCM.

10.7 Summary

Chapter 9 reported and discussed the results of direct models of differenced concentration and realised volatility. However, in recognition that changes in concentration may have a different effect on the various sub-components of realised volatility and that these confounding effects may cancel each other out, Chapter 10 has addressed this issue by modelling the effect of changes in concentration upon the sub-components of realised volatility separately. The results presented here justify this approach in that they do demonstrate that changes in lagged concentration are associated with contemporaneous changes in the sub-components of realised volatility. Furthermore, while it is evident that three of the four sub-components of realised volatility have a positive association with changes in lagged concentration, one sub-component, namely the incremental average covariance, does, in fact, have an inverse relationship with lagged changes in concentration. In addition, this inverse relationship between lagged changes in concentration and contemporaneous changes in the incremental average covariance more than compensates for the positive relationship between changes in concentration and the incremental average variance of FTSE 100 Index constituent returns. This is illustrated by the results obtained from models of changes in concentration with changes in the incremental realised volatility. The models have negative coefficients on lagged concentration that are significantly different from zero more frequently than would be expected if the result was a spurious artefact of data mining.

Overall, it appears from these results that increases in concentration precede decreases in incremental realised volatility and vice a versa. This is contrary to the view often put forward by market practitioners that increasing concentration in the market portfolio results in an increase in risk. In addition, the results presented here, for the 1987 crash dummy variable coefficients, demonstrate that during the 1987 stock market crash, the incremental realised volatility was negative. In other words, the market value weighted FTSE 100 Index portfolio had a lower volatility than a hypothetical equally weighted portfolio of FTSE 100 Index constituents in that period.

In addition to analysing the effect of concentration on the sub-components of realised volatility, this chapter has also reported substantial evidence in favour of the asymmetry effect. This is evident in all of the sub-components of realised volatility, although it appears to exist to a lesser extent in the incremental average covariance and the incremental average variance of constituent returns, and less still when these two series are combined to form the incremental realised volatility. The latter observation suggests that the majority of the asymmetry effect can be attributed to the equally weighted sub-components of realised volatility. It also appears that, while increases in portfolio concentration may, on average, be associated with increases in portfolio volatility, the effect may be reversed during extreme events.

Chapter 11 – Conclusions and achievements

11.1 Introduction

This final chapter outlines the extent to which the three fundamental thesis questions and preliminary issues identified in Chapter 1 have been addressed. It also highlights the significance of the results for investment practitioners and identifies areas for further research.

11.2 Observed changes in the data series

The first of the preliminary issues raised in the introduction to this thesis is: how does concentration and the realised volatility of the FTSE 100 Index evolve over the study period? This section provides answers to these questions concerning concentration, realised volatility and also the decomposed sub-components of realised volatility. The analysis of the incremental sub-components of realised volatility also sheds some light on the second preliminary issue, b, posed in the introduction: namely, how does concentration influence realised volatility?

11.2.1 Concentration

Despite the different attributes of the four concentration metrics, they all unambiguously demonstrate that the distribution of equity market value has become more concentrated in the largest constituents of the FTSE 100 Index over the study period. Furthermore the null hypothesis of a unit-root cannot be rejected in any of the four concentration indices, in levels. The variability of the differenced concentration indices has also increased over the study period with higher standard deviations observed in the period from January 1998 through March 2003, compared to the study period as a whole.

Possible reasons that have been suggested for the increase in the levels of concentration indices over the period are the cross-border mergers that have occurred between some of the largest firms listed in London and their overseas competitors and the listing on the London stock Exchange of multinational firms such as HSBC, BHP Billiton and SAB. The increase in the variability of concentration may be due to the fact that once concentration increases beyond a certain critical point, changes in the value of the few largest firms have a disproportionate effect upon the overall level of concentration.

11.2.2 Aggregate realised volatility

The time series of the monthly-realised volatility over the entire study period is plotted on Chart 7 in section 8.4. With the exception of the 1987 crash and the ERM crisis of 1992, realised volatility is relatively stable until the end of 1997. However, from January 1998 until the end of the data sample, it appears higher, on average, and displays greater instability. This corresponds with the inflation and subsequent collapse of the technology bubble, the terrorist attacks in September 2001, the war in Afghanistan and the build up to the second Gulf War.

11.2.3 Decomposed sub-components of realised volatility

Over the period 1962 – 1997, Campbell et al (2001) report an increase in firm level variance. Chart 8, in section 8.4, plots the total realised value weighted variance of the FTSE 100 Index together with the equally weighted average variance and the equally weighted average covariance of Index constituent returns. The three series have positive outliers associated with the 1987 crash and subsequent high volatility events, such as the ERM crisis in 1992. However, while the equally weighted average covariance closely tracks and, in some cases, exceeds the total realised variance, the equally weighted average variance is almost negligible in terms of its contribution to the total realised variance.

The incremental average covariance and the incremental average variance are plotted with the total realised variance, RV20, of the Index in Chart 9. The contribution to RV20 made by the incremental average variance is almost negligible, like its equally weighted counterpart, although it increased during the early 2000s when concentration in the Index was highest. However, the incremental average covariance makes a more substantial, and often negative, contribution to RV20. It is especially negative during the periods when RV20 is highest, with the exception of mid 2002 when it is positive for a few months, before turning negative again in March 2003.

During the latter part of their study of the US market, Campbell et al (2001) observe a fall in the average paired correlations between individual stock returns. They use these findings together with the finding that the average volatility of individual stock returns has increased, in recent years, to justify their argument that the diversification benefits obtainable from holding more than the traditional fifteen to thirty stocks in a portfolio has increased. The data reported in this study for the UK market is only partially consistent with the findings of Campbell et al (2001). Due to the added information provided by the decomposition into the equally weighted and incremental average covariance components, in addition to the standardising of the components, the results of this study are more revealing. In summary:

- The equally weighted average covariance of constituent returns increases post 1997 and becomes more variable. It also has very high positive outliers during the 1987 crash. A smaller positive outlier occurs at the time of the 1992 ERM crisis.
- The incremental average covariance of constituent returns becomes more volatile following 1997 and for a significant period after January 2000 it is actually negative. Furthermore, very large negative outliers coincide with the 1987 crash and the ERM crisis in 1992. This represents new, previously unpublished, data relating to the average covariance terms in the VCM.
- The time series of the standardised equally weighted average covariance of constituent returns falls post 1995, although it exhibits mean reverting characteristics. As the standardised covariance is in some ways analogous to the average correlation of security returns this finding in the UK market is consistent with the findings of Campbell et al (2001) for the US market pre - 1997.
- The standardised incremental average covariance of constituent returns shows some resemblance to a covariance stationary process, although it exhibits persistent autocorrelation.[108]

Thus, while Campbell et al (2001) find that the average co-movement, measured by the average correlation, had fallen in the US market, this study finds that the equally weighted average covariance rose in the UK market, after the end of the US study by Campbell et al. However, the contribution of the equally weighted average covariance to the aggregate realised volatility of the market proxy fell. This is consistent with the conclusions of Campbell et al. In addition to the findings of Campbell et al, this study finds that co-movement conditional upon concentration departing from its lower limit, as represented by the incremental average covariance, is increasingly variable. However, it is often negative during extreme market movements, such as the 1987 crash. This can be interpreted to mean that investors attempt to concentrate their assets into securities that have a below average covariance in an attempt to improve portfolio diversification.

When the combined effects of the incremental average variance and the incremental average covariance of constituent returns are considered using the incremental realised variance series, IRV, it is evident that the incremental average covariance dominates to the extent that any confounding effects of the incremental average variance are overwhelmed. This has the result that the incremental realised variance of the aggregate index portfolio is often negative, as illustrated by the time series of IRV20 plotted with RV20 in Chart 10.

---

[108] The mean is close to zero and the series appears to revert to the mean over an extended time period; however, significant autocorrelation coefficients persist up to 36 lags.

11.3 Models of concentration and aggregate realised volatility

The direct models of concentration and volatility provide a preliminary investigation into the three fundamental questions posed in the introduction to this thesis, notwithstanding their shortcomings identified there. Like earlier studies of the US market, such as Black (1976), Schwert (1989), Glosten et al (1993), Hentschel (1995) and Bekaert and Wu (2000), the direct models in this study also provide evidence of the asymmetry effect. In fact most models, estimated over the period from January 1998 through December 2002, have coefficients on the lagged asymmetric slope dummy that are positive and significantly different from zero at the $\alpha < 1\%$ threshold.[109] Furthermore, there is evidence that out-of-sample forecasts of realised volatility are improved by the inclusion of an asymmetric slope dummy coefficient. Using the asymmetric autoregressive (AAR) models it is possible to eliminate autocorrelation in the residuals of models estimated over the period from January 1998 through December 2002, although most of the model residuals exhibited positive skewness and excess kurtosis.

There is very little evidence of a direct relationship between either lagged or contemporaneous changes in concentration and realised volatility. Very few models had coefficients on these variables that were significantly different from zero, and those that did were inconsistent in sign and gave little improvement in model fit, after adjusting for the lost degrees of freedom in the more general models. Out-of-sample forecasts were marginally better in the autoregressive distributed lag model that included one lag of differenced concentration, measured using the variance of the logarithm of firm size.

11.4 Models of concentration and the sub-components of realised volatility

The models of concentration and the realised sub-components of the VCM provide more substantive answers to the three fundamental thesis questions than the direct models. In addition, they identify the VCM sub-components in which the asymmetry effect is most prominent. For instance, in addition to finding strong evidence in support of the asymmetry effect for models of aggregate realised volatility, this effect is found to exist in the sub-components of realised volatility. Furthermore, it is more apparent in some sub-components than in others. It is hoped that identifying the source of the asymmetry more precisely than in previous studies may assist in the search for a full explanation of the

[109] The only two exceptions have positive coefficients significantly different from zero at $\alpha = 0.05$

phenomenon. For instance, asymmetric coefficients are positive and significant, at the $\alpha < 5\%$ threshold, for most models of the equally weighted average variance, incremental average variance and equally weighted average covariance of constituent returns. However, in the models of the incremental average covariance the only asymmetric coefficients significant at the $\alpha < 5\%$ threshold are found in models of the data series estimated with T equal to 10 trading days. Therefore, the so-called “leverage” or “asymmetry” effect is evident in both the equally weighted average variance and covariance of constituent returns but it is not so prevalent in the incremental average variance and covariance.

11.4.1 Changes in concentration

Lagged changes in concentration measured using the variance of the logarithm of firm size are associated with decreases in the incremental average covariance of constituent returns and the incremental realised variance of the FTSE 100 Index returns, as evidenced by negative coefficients on this variable, significant, at the $\alpha < 5\%$ threshold. All other lagged differenced concentration metrics have negative coefficients for models of the incremental average covariance and incremental realised variance, some of which are significantly different from zero at the $\alpha < 5\%$ threshold. This indicates that the relationship between changes in concentration and the incremental realised variance of FTSE 100 Index returns is an inverse one. In other words, increases in concentration precede decreases in the future incremental realised variance of the FTSE 100 Index. Very little evidence exists of any association between lagged changes in concentration and changes in the incremental average variance of constituent returns. Limited evidence is presented to suggest a positive association between changes in concentration and the equally weighted average variance and the equally weighted average covariance.

In answer to the first and second fundamental questions of this thesis, the results indicate that lagged changes in concentration are associated with changes in realised volatility but that the relationship is different in sign and importance for different sub-components of the VCM. Furthermore, concentration increases prior to decreases in the incremental average covariance, while increases in concentration precede increases in the equally weighted average variance and covariance. One possible explanation for these findings is that investors in the FTSE 100 Index constituents concentrate their portfolio capital into firms with a below average covariance prior to increases in the average volatility of constituent returns and increases in the volatility of the overall index. If this is a reflection of investor behaviour, it is consistent with the CAPM assumption that investors are mean variance optimisers. Given the attention that applications of portfolio theory have received in the professional community and the integration of portfolio theory with value at risk methodologies, as outlined in books such as Dowd (1998), and articles such as Barry et al (1997), Brooks and Persand (2000), Putnam (2003) and Waring (2003), this is not entirely surprising.

The third fundamental question addressed by this thesis is: can changes in concentration be used to forecast future realised volatility? In fact, the model results provide little evidence to suggest that including lags of differenced concentration in the models improves out-of-sample forecasting capability with regards to the incremental average covariance and the combined incremental realised variance of the FTSE 100 Index returns. The same applies to out-of-sample forecasts of the equally weighted average covariance, the incremental average variance and the equally weighted average variance. In the few cases where out-of-sample forecasts are marginally better in models using lagged changes in concentration than in naive models, the concentration metric used is the variance of the logarithm of firm size.

11.4.2 The 1987 crash

All models estimated over the whole study period, and earlier sub-periods that spanned the 1987 crash period, incorporated a dummy variable for this event. Coefficients on the contemporaneous dummy variable were positive and significant at an α equal to 0.000 for all models of the equally weighted average variance, the incremental average variance and the equally weighted average covariance of constituent returns. However, coefficients on the contemporaneous dummy variable for the 1987 crash, in models of the incremental average covariance and the combined incremental realised variance of FTSE 100 Index returns, were also significant with α equal to 0.000. However, unlike the models of the other VCM sub-components, the coefficients were negative. The implication of this is that the actual value weighted FTSE 100 Index portfolio, at that time, displayed a higher degree of mean variance efficiency than a hypothetical equally weighted portfolio containing the same constituents. This is evidence in favour of the CAPM assumption that investors attempt to construct mean variance efficient portfolios based on some kind of Markowitz optimisation model, and that proxies for the market portfolio, such as the FTSE 100 Index are more efficient than a hypothetical equally weighted portfolio with the same constituents. In short, this study finds that incremental realised variance is often negative during extreme volatility events, due to large negative values of the incremental average covariance.

11.5 Contribution to the body of knowledge

The counterpart to this section in the introduction identifies how this thesis aims to contribute to the current body of knowledge. The extent to which these aims have been achieved is now reported.

Roll (1992) reached the conclusion that levels of concentration in national market indices are positively associated with the volatility of returns in those indices. However, in Chapter 3 a number of methodological limitations are identified in the cross sectional analysis that Roll applied in reaching his conclusion. This thesis investigates time series concentration data in the FTSE 100 Index and demonstrates that, historically, concentration has, at different times, had the effect of both increasing and decreasing the realised volatility of that index. The varying effects are explained using the principles of modern portfolio theory. Emphasis is placed on the importance of the average variance and average covariance components of the VCM for determining the aggregate realised volatility of any portfolio.

Isakov and Sonney (2002) extended the work of Roll (1992) and reached the conclusion that while country specific effects had historically been of greater importance than industry effects in determining the volatility of a globally diversified portfolio, during the latter part of their study period, industry effects had come to dominate country effects. They attributed this to the increasing importance of the TMT industries in explaining the returns of global equity portfolios, i.e. greater concentration in one volatile sector. This thesis demonstrates that, during the period in question, both concentration and volatility in the FTSE 100 Index were at a historically high level.

The results of Campbell et al (2001), based upon a study of US data, indicate that idiosyncratic risk increased and the correlation between securities decreased over the period from 1926 through 1997. They did not find that the market specific component of individual firm risk had increased over the study period, nor did they find that total market risk had increased. Rather the proportion of total firm risk accounted for by firm specific risk had increased. They argue that this result implies that investors should increase the number of securities in their portfolio in order to achieve optimal diversification. This thesis is restricted to the constituents of the FTSE 100 Index in the UK market between January 1984 and March 2003, while the analysis focuses on the period following that studied by Campbell et al, i.e. from January 1998 through March 2003. Furthermore the method of decomposing market volatility is different to that adopted by Campbell et al.

Notwithstanding the differences, it is useful to discuss the results of this study in the context of those of Campbell et al, and subsequent studies that develop their findings.

For instance, in the five years following the end of the period studied by Campbell et al, a large increase in the level of concentration in the FTSE 100 Index coincided with some of the most volatile stock market returns in recent history. It may be argued that the increase in the concentration in the UK market reflects collective investor behaviour that is opposite to that regarded as optimal by Campbell et al, i.e. concentration of market portfolio assets into the stocks of fewer firms, not more. If true, this could reflect irrational behaviour by investors ignoring the principles of modern portfolio theory and getting carried away by the "Irrational Exuberance" of the technology bubble. Alternatively, it could be explained by the rational behaviour of investors attempting to construct mean variance efficient portfolios in a manner consistent with the EMH. Unfortunately, any attempt to resolve this issue is frustrated by the number of input variables involved. Therefore in order to simplify the analysis, this discussion leaves aside the question of whether or not investors' expectations concerning risk, return and covariance were based upon rational or irrational analysis over this period. Instead it is assumed that the expectations were based upon the best information and decision-making tools available at the time. MPT involves optimising both risk and returns; however, if returns are left aside for the time being, it is then possible to focus on whether or not the concentration of the market portfolio resulted in greater risk or less risk for investors over the study period.

Rather than decomposing risk into industry specific, firm specific and market specific components in the manner of Campbell et al, (2001), this study returns to the basics of Markowitz portfolio diversification in the spirit of Elton and Gruber (1973) and Elton et al (1978). It examines the variances and covariances in the VCM of the FTSE 100 Index portfolio. It specifically identifies the variance and covariance that is conditional upon portfolio concentration deviating from its lower limit of unity. In this context, the "Overall Mean" model developed and tested by Elton and Gruber (1973) and later re-tested by Elton et al (1978) is useful. This is because the Overall Mean Model is a naive model in that it assumes that covariances between all paired security returns in the market portfolio are identical and hence equal to the average correlation. Furthermore, it assumes that covariances are constant over time, at least in the unconditional distribution. If this holds and all securities are also expected to have the same expected return, they also have the same variance and the same beta, of unity. Under these assumptions, the VCM and the realised volatility of an equally weighted portfolio would be the same as that of a value weighted portfolio. The methodology applied in this study, in effect, provides a means of evaluating those assumptions by isolating the incremental average variance and the incremental average covariance of constituent security returns and hence the incremental realised variance of the FTSE 100 Index portfolio.

For example, under the assumptions of the Overall Mean Model, the incremental sub-components and standardised incremental sub-components are zero and cannot be forecast. Using the standardised incremental realised volatility estimated with a T equal to 20, which has an approximately normal distribution plotted in Chart 13 through Chart 14, this thesis demonstrates that the incremental sub-components have positive location parameters over the study period. However, dispersion parameters indicate that the incremental components can make both large positive and large negative contributions to realised volatility. Furthermore, the large negative values of the incremental average covariance seem to be associated with extreme market shocks, a finding which is inconsistent with the assumptions of the Overall Mean Model.

In the spirit of Elton et al, the thesis modelled the sub-components of the VCM. However, forecasting the sub-components individually has not, as yet, led to forecasts of realised volatility that are superior to those that could be obtained using naive asymmetric autoregressive models, a finding consistent with those of Elton and Gruber (1973) and Elton et al (1978). Nonetheless, the study has presented evidence to suggest that increases in concentration precede falls in the incremental average covariance of constituent returns, something that does not seem consistent with the idea that a concentrated portfolio is less efficient than an equally weighted one.

Goyal and Santa-Clara (2003) have re-examined and updated the data used by Campbell et al (2001) for the US market. In addition, they found that the standard deviation of monthly returns in their value weighted US market proxy portfolio is greater than the monthly standard deviation in their equally weighted portfolio, a finding that is inconsistent with the results of this study of the FTSE 100 Index. Furthermore, no data is provided concerning the concentration of their US market portfolio and they do not separate out the incremental realised volatility, unlike this study. Wei and Zhang (2003), also re-examine the issues raised by Campbell et al (2001). They find, contrary to Goyal and Santa-Clara, that the equally weighted average variance of raw returns in the period from 1996 through 2000 is substantially larger than that of the value weighted portfolio. Although they calculate an incremental variance that is used for comparison between time periods, their metric is not compatible with the incremental average variance calculated in this thesis. Kearney and Poti (2003) examined the Eurostoxx50 index constituents, decomposing the VCM into average correlation and average variance components. In addition, they compared value weighted and equally weighted market variance, firm level variance and aggregate variance. Once again they do not separate out the incremental average variance, covariance or the incremental realised volatility. Although their comparison of the value weighted and equally weighted aggregate market variance is consistent with that in this study (for the FTSE 100 Index VCM), their findings are inconsistent in that the value-weighted averages are lower than the equally weighted averages. Therefore, comparison of their data sample with the FTSE 100 Index data, using the method adopted in this study provides an opportunity for further research.

The additional scope of the VCM decomposition methodology applied to the data analysed in this study, provides further evidence concerning the dynamics of the VCM and the interrelationship between idiosyncratic risk and systematic risk. In addition, it examines a market index not studied by the cited articles.

Many previous authors, such as Kearney and Poti (2003), Andersen et al (2000) and earlier articles, cited in Chapter 3, have observed that correlations between firm returns spike upwards during periods of extreme volatility, such as the 1987 crash. Such observations led Andersen et al (2000) to comment that the benefits of portfolio diversification are limited at the time when they are needed the most. The results of this study add a new perspective to these observations because they demonstrate that although the conclusions of the cited studies hold, for the equally weighted average covariance, they do not hold for the incremental average covariance of constituent returns. It is suggested that the benefits of a negative incremental average covariance and incremental realised volatility may be the result of successful mean variance optimisation strategies implemented by investors.

With respect to the asymmetry effect, the relatively stronger evidence of this effect in the equally weighted average variance and covariance, in contrast to the weaker evidence in the incremental averages, may provide a lead to researchers following in the footsteps of Bekaert and Wu (2000), and others, who are attempting to attribute and explain this phenomenon with respect to the different components of realised stock index volatility.

Poon and Granger (2003) suggest that the separation of volatility forecasting periods into those that are normal and those that are exceptional could be a fruitful target for further research. Poon et al (2004), as well as Brooks and Persand (2000), examine correlations between the returns of different stock markets during exceptional trading conditions. The evidence presented in this thesis highlights the difference in the behaviour of the VCM within a single market index during normal trading conditions when the incremental realised variance is generally positive, and during extreme events when it has, in most cases, been substantially negative.

11.6 Implications of the findings for investors in the UK market

This study presents unambiguous results demonstrating that investors in the FTSE 100 Index are concentrating more of their assets into fewer firms than they were twenty years ago. These large firms are global in scope deriving the majority of their revenues from outside the UK economy.[110] Hence, investors in the FTSE 100 Index can create an internationally diversified portfolio without investing in shares listed on foreign markets. By concentrating their assets into just a few relatively large firms, investors are not necessarily increasing the risk of their portfolios. This is evidenced by the fact that the incremental realised variance of the FTSE 100 Index is frequently negative and was very negative during the times of greatest market volatility, namely the 1987 crash, the 1992 ERM crisis and for a significant period following the collapse of the TMT bubble in 2000, 2001 and 2002. However, during the latter period, from 1997 through to March 2003, the incremental realised variance has also been positive, as well as negative, and it is difficult to forecast. Nonetheless, decreases in the incremental realised volatility appear to be preceded by increases in concentration, as evidenced by the negative coefficients, significant at the $\alpha < 5\%$ threshold, on lagged changes in concentration in models of the incremental average covariance and the incremental realised variance of FTSE 100 Index returns.

While this statistically significant relationship does not seem to confer any economically significant forecasting information, it is still relevant to investors' asset allocation decisions, because it highlights the importance of the covariance between the returns of the dominant securities. For example, a semi passive investor wishing to hold a proxy of the UK market

[110] This is evident from the segmental analysis data disclosed in the annual reports of firms, such as Vodafone that derived 80% of its revenue from overseas operations in 2000.

portfolio, such as the FTSE 100 Index, might question whether or not the UK market portfolio is more risky as a result of increases in concentration. The results of this study indicate that it is not, as well as providing a simple methodology to enable evaluation of the incremental components of the VCM in real time. Likewise, an active investor might want to know if the risk of their portfolio could be reduced, by down-weighting the dominant FTSE 100 Index constituents. In fact, the results suggest that such a strategy may increase risk if the incremental components of realised volatility are negative in the FTSE 100 Index. Therefore, only the possession of superior knowledge concerning the average covariance of the dominant securities in relation to the average covariance of all the securities in the investment universe could justify this decision. The studies by Elton et al suggest that superior information concerning covariances is just as difficult to acquire as superior information concerning returns. The results and methodology presented will also be of interest to stock index providers aiming to provide appropriate model portfolios for passive investors or active manager benchmarks.

11.7 Limitations and potential for further research

The decomposition of the VCM and volatility measurement was based on the assumption that daily returns of the FTSE 100 Index and constituents have an expected value of zero. Some theorists might argue that an expectation based on the contemporaneous risk free rate, or the risk free rate plus a risk premium, is more appropriate. Although the validity of this assertion is debatable, future researchers might consider applying the same methodology but based on excess returns over a non-zero expected return.

The use of daily squared returns to proxy realised volatility over periods of five, ten, fifteen and twenty trading days is an attempt to resolve the issues identified by Merton (1980) and cited by Andersen and Bollerslev (1998). Namely that, the variance of a “sample variance” of stock index returns is likely to be inversely related to the sampling frequency. In other words, if the sampling frequency of returns used to estimate return volatility decreases to weekly or monthly frequency, the volatility estimates become increasingly noisy due to the missing information that would be available in daily, intra-daily, or in the theoretical ideal of the Andersen and Bollerslev world, continuous time path of stock index returns. Hence, the twenty-day volatility estimates used in this study will be less noisy than fifteen-day estimates, that in turn will be less noisy than ten and five day estimates. Hence estimates of daily volatility are discussed in Chapter 8 would have been affected by this sampling error problem had they been used for modelling. Therefore, if future studies intend to focus on five day, or daily estimates of volatility, the use of high frequency intra-day returns would be appropriate. Furthermore, model coefficients were estimated using the rather restrictive OLS procedure, which may be less efficient than other estimation procedures available, such as maximising the likelihood functions. On the other hand, this basic method was sufficient to eliminate autocorrelation in model residuals in the majority of cases. The failure to achieve a Gaussian distribution in the residuals, although not ideal, is not unusual in more sophisticated models of financial time series. The use of robust standard errors will have mitigated the impact of these issues.

A further limitation of this study, in relation to published research on volatility modelling, is the relatively small number of out-of-sample forecasts estimated, particularly for data series estimated with T equal to 10 trading days or more. This is partly due to the identification of a new regime in the data series, from late 1997 onwards, thereby limiting the sample of suitable data available. However, implementing recursive out-of-sample forecasts over that period and updating the data sample from March 2003 to the present, so that further out-of-sample forecasts can be evaluated, provides an opportunity for further research.

This study has provided a relatively simple and easy to implement method of decomposing the VCM of portfolio constituent returns. The simplicity is due to the fact that, in order to obtain the average covariance, it is not necessary to estimate all of the paired covariance terms in the VCM. Furthermore, by making a comparison between the equally weighted and value weighted VCM it is also possible to further decompose the VCM into components that are conditional upon concentration and those that are not. Having established the methodology and applied it in the FTSE 100 Index, it may now be applied to further studies using other market proxy portfolios such as those reviewed in Chapter 4.

It is also suggested that further studies applying concentration metrics in modelling should focus on the variance of the logarithm of firm size as this provides the least biased measure of firm size dispersion in situations where the distribution of firm size approximates to the lognormal. Another issue that merits further investigation is the teasing apart of the ambiguous contemporaneous relationships between changes in concentration and changes in volatility. This study finds limited empirical evidence to suggest that such a relationship may exist in models of differenced concentration and the average variance of constituent returns. There is little evidence of such a relationship between concentration and the average covariance of constituent returns.

References

Andersen, T.G., and Bollerslev, T., (1998). Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts. International Economic Review. 39:4 pp 885-905.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Ebens, H. (2000). The Distribution of Stock Return Volatility. Earlier Version of Andersen Et Al (2001a)

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Ebens, H. (2001a). The Distribution of Realised Stock Return Volatility. Journal of Financial Economics. 61:1 pp 43-76.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Labys, P. (2000a). Exchange Rate Returns Standardised by Realised Volatility Are Nearly Gaussian. Multinational Finance Journal. 4:3 and 4 pp 159-179.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Labys, P. (2001b). The Distribution of Realised Exchange Rate Volatility. Journal of the American Statistical Association. 96:453 pp 42-55.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Labys, P. (2003). Modelling and Forecasting Realised Volatility. Econometrica. 71:2 pp 579-626.

Areal, N.M.P.C. and Taylor, S.J. (2002). The Realised Volatility of FTSE-100 Futures Prices. Journal of Futures Markets. 22:7 pp 627-648.

Baca, S.P., Garbe, B.L. and Weiss, R.A. (2000). The Rise of Sector Effects in Major Equity Markets. Financial Analysts Journal. 56:5 pp 34-40.

Bailey, J.V. (1992a). Evaluating Benchmark Quality. Financial Analysts Journal. 48:3 pp 33-39.

Bailey, J.V. (1992b). Are Manager Universes Acceptable Performance Benchmarks? Journal of Portfolio Management. 18:3 pp 9-13.

Bailey J.V., Richards, T. M. and Tierney, D. E. (1990). Benchmark Portfolios and the Manager/Plan Sponsor Relationship, New York: Harper Collins.

Baillie, R.T. (1996). Long Memory Processes and Fractional Integration in Econometrics. Journal of Econometrics. 73:1 pp 5-59.

Bain, J.S. (1951). Relation of Profit Rate to Industry Concentration: American Manufacturing, 1936-40 . Quarterly Journal of Economics. 65:3 pp 293-324.

Barberis, N., Shleifer, A. and Wurgler, J. (Forthcoming). Comovement. Journal of Financial Economics.

Barry C.B., Peavy, J. W. and Rodriguez, M. (1997). Emerging Stock Markets: Risk, Return, and Performance, Charlottesville, U.S.A. The Research Foundation of the Institute of Chartered Financial Analysts.

Bekaert, G. and Wu, G. (2000). Asymmetric Volatility and Risk in Equity Markets. The Review of Financial Studies. 13:1 pp 1-42.

Bera, A.K. and Higgins, M.L. (1992). A Class of Nonlinear ARCH Models. International Economic Review. 33:1 pp 137-158.

Black, F. (1976). Studies of Stock Price Volatility Changes. Proceedings of the 1976 Meeting of the American Statistical Association. Business and Economics Section: pp 177-181.

Blair, B.J., Poon, S.-H.H. and Taylor, S.J. (2001). Forecasting S&P 100 Volatility: the Incremental Information Content of Implied Volatilities and High Frequency Index Returns. Journal of Econometrics. 105:1 pp 5-26.

Blume, M. (1971). On the Assessment of Risk. Journal of Finance. 26:1 pp 1-10.

Bodie Z., Kane, A. and Marcus, A. J. (1999). Investments., USA: Irwin/McGraw-Hill.

Bollerslev, T. (1986). Generalised Autoregressive Conditional Heteroskedasticity. Journal of Econometrics. 31:3 pp 307-327.

Bollerslev, T., Engle, R.F. and Wooldridge, J.M. (1988). A Capital Asset Pricing Model With Time Varying Covariances. Journal of Political Economy. 96:1 pp 116-131.

Boudoukh, J., Richardson, M. and Whitelaw, R.F. (1997). Investigation of a Class of Volatility Estimators. Journal of Derivatives. 4:3 pp 63-71.

Brennan, M. (1979). The Pricing of Contingent Claims in Discrete Time Models. Journal of Finance. 34:1 pp 53-68.

Brooks, C. and Persand, G. (2000). Value at Risk and Market Crashes. Discussion Papers in Finance, University of Reading.

Brown, K.C. and Brown, G.D. (1987). Does the Composition of the Market Portfolio Really Matter? Journal of Portfolio Management. 13:2 pp 26-32.

Campbell, J.Y. and Hentschel, L. (1992). No News Is Good News. An Asymmetric Model of Changing Volatility in Stock Returns. Journal of Financial Economics. 31:3 pp 281-318.

Campbell, J.Y., Lettau, M., Malkiel, B.G. and Xu, Y. (2001). Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk. Journal of Finance. 56:1 pp 1-43.

Campbell J.Y., Lo, A. W. and Mackinlay, A. C. (1997). The Econometrics of Financial Markets, Princeton, New Jersey: Princeton University Press.

Canina, L. and Figlewski, S. (1993). The Information Content of Implied Volatility. Review of Financial Studies. 6:3 pp 659-681.

Chelley-Steeley, P.L. and Steeley, J.M. (1996a). Volatility, Leverage and Firm Size: the UK Evidence. The Manchester School Supplement. pp 83-103.

Chriss N.A. (1997). Black-Scholes and Beyond: Option Pricing Models, Chicago: Irwin.

Christie, A.A. (1982). The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects. Journal of Financial Economics. 10:4 pp 407-432.

Chunhachinda, P., Dandapani, K., Hamid, S. and Prakash, A.J. (1997). Portfolio Selection and Skewness: Evidence From International Stock Markets. Journal of Banking and Finance. 21:2 pp 143-167.

Clarke R. (1993). Industrial Economics, Cambridge Massachusetts: Blackwell.

Cohen, R.B., Hall, B.J. and Viceira, L.M. (2000). Do Executive Stock Options Encourage Risk Taking? Working Paper, Harvard Business School

Cole, A.H. and Fricky, E. (1928). The Course of Stock Prices, 1825 - 66. The Review of Economic Statistics. 10:3 pp 117-139.

Copeland, Koller and Murrin (1994). Valuation: Measuring and Managing the Value of Companies., New York: John Wiley and Sons.

Cowles A.I.A. (1939). Common Stock Indices, Bloomington, Indiana: Principia press.

Davidian, M. and Carroll, R.J. (1987). Variance Function Estimation. Journal of the

American Statistical Association. 82:400 pp 1079-1091.

De Long, B.J., Summers, L.H. and Waldmann, R.J. (1990b). Positive Feedback Investment Strategies and Destabilising Rational Speculation. The Journal of Finance. 45:2 pp 379-395.

DeFusco R.A., McLeavey, D. W., Pinto, J. E. and Runkle, D. E. (2001). Quantitative Methods for Investment Analysis., Charlottesville, U.S.A.: Association for investment management and research.

Diebold F.X. (2001). Elements of Forecasting, Second Edition., Cincinnati, Ohio: South-Western College Publishing.

Dimson, E. and Marsh, P. (2001). UK Financial Market Returns, 1955-2000. Journal of Business. 74:1 pp 1-31.

Dimson E., Marsh, P. and Staunton, M. (2002). Triumph of the Optimists: 101 Years of Global Investment Returns, Princeton, New Jersey: Princeton University Press.

Dimson, E. and Marsh, P.R. (1983). The Stability of UK Risk Measures and the Problem of Thin Trading. The Journal of Finance. 38:3 pp 753-783.

Dowd K. (1998). Beyond Value at Risk, Chichester, UK: Wiley.

Duffee, G.R. (1995). Stock Returns and Volatility: a Firm Level Analysis. Journal of Financial Economics. 37:3 pp 399-420.

Elton, E.J. and Gruber, M.J. (1973). Estimating the Dependence Structure of Share Prices. Implications for Portfolio Selection. Journal of Finance. 28:5 pp 1203-1232.

Elton, E.J. and Gruber, M.J. (1977). Risk Reduction and Portfolio Size: an Analytical Solution. Journal of Business. 50:4 pp 415-437.

Elton, E.J., and Gruber, M.J., (1997). Modern Portfolio Theory, 1950 to Date. Journal of Banking and Finance. 21:11-12 pp 1743-1759.

Elton, E.J., Gruber, M.J. and Ulrich, T.J. (1978). Are Betas Best. Journal of Finance. 33:5 pp 1375-1384.

Elton E.J., Gruber, M. J., Brown, S. J. and Goetzmann, W. N. (2003). Modern Portfolio Theory and Investment Analysis, USA: John Wiley and Son's Inc.

Engle, R.E. (1982). Autoregressive Conditional Heteroskedasticity With Estimates of the

Variances of United Kingdom Inflation. Econometrica. 50:4 pp 987-1008.

Engle, R.F. (1990). Discussion: Stock Market Volatility and the Crash of 87. Review of Financial Studies. 3:1 pp 103-106.

Engle, R.F., David, M.L. and Russell, P.R. (1987). Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model. Econometrica. 55:2 pp 391-407.

Engle, R.F. and Ng, V.K. (1993). Measuring and Testing the Impact of News on Volatility. Journal of Finance. 48:5 pp 1749-1778.

Evans, J.L. and Archer, S.H. (1968). Diversification and the Reduction of Dispersion: an Empirical Analysis. Journal of Finance. 23:5 pp 761-767.

Fabozzi, F.J., Gupta, F. and Markowitz, H.M. (2002). The Legacy of Modern Portfolio Theory. The Journal of Investing. 11:3 pp 7-22.

Fama, E.F. and French, K.R. (2002). Testing Tradeoff and Pecking Order Predictions About Dividends and Debt. Review of Financial Studies. 15:1 pp 1-35.

Fernholz, R. (1999). On the Diversity of Equity Markets. Journal of Mathematical Economics. 31:3 pp 393-417.

Fernholz, R., Garvy, R. and Hannon, J. (1998). Diversity-Weighted Indexing. A New Approach to Passive Investing. The Journal of Portfolio Management. 24:2 pp 74-82.

Figlewski, S. (1997). Forecasting Volatility. Financial Markets, Institutions and Instruments, New York University Salomon Centre. 6:1 pp 1-88.

Figlewski, S. (2001). Volatility: Forecasting, Interpretation and Option Pricing. SIRIF Presentation: The University of Strathclyde.

Fisher, R.A., Corbet, S.A. and Williams, C.B. (1943). The Relation Between the Number of Species and the Number of Individuals in a Random Sample of an Animal Population. Journal of Animal Ecology. 12:1 pp 42-58.

Fraser, P. (2001). How Do U.S. and Japanese Investors Process Information and How Do They Form Their Expectations of the Future? Evidence From Quantitative Survey Based Data. Working Paper, University of Aberdeen

French, K.R. and Poterba, J.M. (1991). Investor Diversification and International Equity Markets. The American Economic Review. 81:2 pp 222-226.

French, K.R., Schwert, W.G. and Stambaugh, R.F. (1987). Expected Stock Returns and Volatility. Journal of Financial Economics. 19:1 pp 3-29.

Garman, M.B. and Klass, M.J. (1980). On the Estimation of Security Price Volatilities From Historical Data. Journal of Business. 53:1 pp 67-78.

Gerhardt, W.R., Lee, C.M.C. and Swaminathan, B. (2001). Toward an Implied Cost of Capital. Journal of Accounting Research. 39:1 pp 135-176.

Gibrat R. (1931). Les Inegalites Economiques, Partly Translated and Reprinted As "On Economic Inequalities", Paris: Reprint "International Economic Papers (1957)".

Gini, C. (1912). Variabilita' e Mutabilata. Cited by Weisstein, E. W. (1999-2004)

Glosten, L.R., Jaganathan, R. and Runkle, D.E. (1993). On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. Journal of Finance. 48:5 pp 1779-1801.

Goyal, A. and Santa-Clara, P. (2003). Idiosyncratic Risk Matters! The Journal of Finance. 58:3 pp 975-1007.

Graham, J.R. and Harvey, C.R. (1996). Market Timing and Volatility Implied in Investment Newsletter's Asset Allocation Recommendations. Journal of Financial Economics. 42:3 pp 397-421.

Graham, J.R. and Harvey, C.R. (2002). Expectations of Equity Risk Premia, Volatility and Asymmetry From a Corporate Finance Perspective. Working Paper Downloaded From SSRN Electronic Journals.

Granger, C. (2001). Long Memory Processes - an Economist's Viewpoint. Working Paper.

Griffin, J.M. and Karolyi, G.A. (1998). Another Look at the Role of the Industrial Structure of Markets for International Diversification Strategies. Journal of Financial Economics. 50:3 pp 351-373.

Grinblatt, M. and Titman, S. (1993). Performance Measurement Without Benchmarks: an Examination of Mutual Fund Returns. Journal of Business. 66:1 pp 47-68.

Guo, H. and Savickas, R. (2004). Idiosyncratic Volatility, Stock Market Volatility, and

Expected Stock Returns. SSRN Working Papers

Haberle, R. and Ranaldo, A. (2003). Wolf in Sheep's Clothing: the Active Investment Strategies Behind Index Performance. SSRN Working Paper

Hamilton J.D. (1994). Time Series Analysis, Princeton, NJ: Princeton University Press.

Hannah L., and Kay, J. A. (1977). Concentration in Modern Industry. Theory, Measurement and the U.K. Experience, London and Basingstoke: The Macmillan Press Ltd.

Harris, R.S. and Marston, F.C. (2001). The Market Risk Premium: Expectational Estimates Using Analyst's Forecasts. Journal of Applied Finance. 11:1 pp 6-16.

Hart, P.E. (1971). Entropy and Other Measures of Concentration. Journal of the Royal Statistical Society. Series A (General) 134:1 pp 73-85.

Hart P.E. and Clarke, R. (1980). Concentration in British Industry 1935-75, Cambridge: Cambridge University Press.

Hartley, R.V.L. (1928). Transmission of Information. Bell System Technical Journal. 7:4 pp 535-563.

Harvey, A. (1993). Long Memory in Stochastic Volatility. Unpublished Manuscript Cited by Knight and Satchell (1998)

He, H. and Leland, H. (1993). On Equilibrium Asset Price Processes. Review of Financial Studies. 6:3 pp 593-617.

Hentschel, L. (1995). All in the Family, Nesting Symmetric and Asymmetric GARCH Models. Journal of Financial Economics. 39:1 pp 71-104.

Herfindahl, O.C. (1950). Concentration in the US Steel Industry. Doctoral Dissertation Unpublished, Columbia University

Heston, S.L. and Rouwenhorst, K.G. (1994). Does Industrial-Structure Explain the Benefits of International Diversification. Journal of Financial Economics. 36:1 pp 3-27.

Hicks J.R. (1946). Value and Capital: an Inquiry into Some Fundamental Principles of Economic Theory, Oxford: Clarenden Press.

Hirschey, M. (2001). Cisco and the Kids. Financial Analyst's Journal. 57:4 pp 48-59.

Hirschman A.O. (1945). National Power and the Structure of Foreign Trade, Berkeley:

Hirschman, A.O. (1964). The Paternity of an Index. The American Economic Review. 54:3 pp 761-2.

Isakov, D. and Sonney, F. (2002). Are Practitioners Right? On the Relative Importance of Industrial Factors in International Stock Returns. SRRN Electronic Journals, Working Paper.

Kacperczyk, M., Sialm, C. and Zheng, L. (2003). On the Industry Concentration of Mutual Funds. SSRN

Kahneman, D. and Tversky, A. (1979). Prospect Theory: an Analysis of Decision Under Risk. Econometrica. 47:2 pp 263-292.

Kane (1977). Skewness Preference and Portfolio Choice. Journal of Financial and Quantitative Analysis. 17:1 pp 15-25.

Kearney, C. and Poti, V. (2003). Idiosyncratic Risk, Market Risk and Correlation Dynamics in European Equity Markets. SSRN Working Paper June:

Khoury S. and Ghosh, A. (1988). Recent Developments in International Banking and Finance, New York, US: D.C. Heath.

Kilpatrick, R.W. (1967). The Choice Among Alternative Measures of Industrial Concentration. Review of Economics and Statistics. 49:2 pp 258-260.

King, M.A. and Wadhwani, S. (1990). Transmission of Volatility Between Stock Markets. The Review of Financial Studies. 3:1 pp 5-33.

Knight J. and Satchell, S. (1998). Forecasting Volatility in the Financial Markets., UK: Butterworth Heinemann.

Kraus, A. and Litzenberger, R.H. (1976). Skewness Preference and the Valuation of Risky Assets. The Journal of Finance. 31:4 pp 1085-1100.

Lintner, J. (1965). The Valuation of Risky Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. Review of Economics and Statistics. 47:1 pp

13-37.

Malevergne, Y. and Sornette, D. (2004). How to Account for Extreme Comovements Between Individual Stocks and the Market. The Journal of Risk 6:3

Malkiel, B.G. and Xu, Y. (2003). Investigating the Behavior of Idiosyncratic Volatility. Journal of Business. 76:4 pp 613-644.

Mandelbrot, B. (1963). The Variation of Certain Speculative Prices. Journal of Business. 36:4 pp 394-419.

Markowitz, H.M. (1952). Portfolio Selection. Journal of Finance. 7:1 pp 77-91.

Meehan, J.W. and Duchesnau, T.D. (1973). The Critical Level of Concentration. Journal of Industrial Economics. 22:1 pp 21-36.

Merton, R.C. (1980). On Estimating the Expected Returns of the Market: an Exploratory Investigation. Journal of Financial Economics. 8:4 pp 323-361.

Mossin, J. (1966). Equilibrium in a Capital Asset Market. Econometrica. 34:4 pp 768-783.

Nelson, D.B. (1991). Conditional Heteroskedasticity in Asset Returns: a New Approach. Econometrica. 59:2 pp 347-370.

Newbould G.D. and Jackson, A. S. (1972). The Receding Ideal, Liverpool: Guthstead Ltd.

Nyquist, H. (1924). Certain Factors Affecting Telegraph Speed. Bell System Technical Journal. 3:4 pp 324-346.

Officer, R.R. (1973). The Variability of the Market Factor of the New York Stock Exchange. Journal of Business. 46:3 pp 434-453.

Oomen, R.C.A. (2001). Using High Frequency Stock Market Index Data to Calculate, Model and Forecast Realised Return Variance. SSRN Working Paper

Osborne, M.F.M. (1959). Brownian Motion in the Stock Market. Operations Research. 7:2 pp 145-173.

Pagan, A.R. and Schwert, W.G. (1990). Alternative Models for Conditional Stock Volatility. Journal of Econometrics. 45:1-2 pp 267-290.

Parkinson, M. (1980). The Extreme Value Method of Estimating the Variance of the Rate of Return. Journal of Business. 53:1 pp 61-65.

Pearson, K. (1895). Contribution to the Mathematical Theory of Evolution-II: Skewed Variation in Homogenous Material. Philosophical Transaction of the Royal Society. A:186 pp 343-414.

Peiro, A. (1999). Skewness in Financial Returns. Journal of Banking and Finance. 23: pp 847-862.

Perry, P.R. (1982). The Time-Variance Relationship of Security Returns: Implications for the Return-Generating Stochastic Process. Journal of Finance. 37:6 pp 857-870.

Pindyck, R.S. (1986). Risk, Inflation and the Stock Market. American Economic Review 74:3 pp 335-351.

Pindyck R.S. and Rubinfeld, D. L. (1991). Econometric Models and Economic Forecasts, United States: McGraw-Hill.

Poon, S.-H. Rockinger, M. and Tawn, J. (2004). Extreme Value Dependence in Financial Markets: Diagnostics, Models, and Financial Implications. The Review of Financial Studies 17:2 pp 581-610.

Poon, S.-H. and Granger, C.W.J. (2003). Forecasting Volatility in Financial Markets: a Review. Journal of Economic Literature. 41:2 pp 478-539.

Poon, S.-H. and Taylor, S.J. (1992). Stock Returns and Volatility: An Empirical Study of the UK Stock Market. Journal of Banking and Finance. 16:1 pp 37-59.

Poon, S.-H. Taylor, S.J. and Ward, C.W. (1992). Portfolio Diversification: a Pictorial Analysis of the UK Stock Market. Journal of Business Finance and Accounting. 19:1 pp 87-101.

Poterba, J.M. and Summers, L.H. (1986). The Persistence of Volatility and Stock Market Fluctuations. American Economic Review. 76:5 pp 1142-1151.

Premaratne, G. and Bera, A.K. (2002). Modelling Asymmetry and Excess Kurtosis in Stock Return Data. Dept Economics, University of Illinois, Working Paper.

Putnam, B.H. (2003). Disaster Risk and Investment Guidelines. AIMR Conference Proceedings. Improving the Investment Process Through Risk Management. 2003:4 pp 4-16.

Reilly F.K. and Brown, K. C. (2000). Investment Analysis and Portfolio Management, 6th Edition., Fort Worth: South Western and Thomson Learning.

Riley, B. (Mar 11, 2000) The long view. Financial Times Finance week edn, Financial Times. Finance week.

Roll, R. (1977). A Critique of the Asset Pricing Theory's Tests. Journal of Financial Economics. 4:4 pp 129-176.

Roll, R. (1978). Ambiguity When Performance Is Measured Using the Security Market Line. The Journal of Finance. 33:4 pp 1051-1069.

Roll, R. (1980). Performance Evaluation and Benchmark Error I. Journal of Portfolio Management. 6:4 pp 5-12.

Roll, R. (1981). Performance Evaluation and Benchmark Error II. Journal of Portfolio Management 7:2 pp 17-22.

Roll, R. (1992). Industrial Structure and the Comparative Behaviour of International Stock Market Indices. Journal of Finance. 47:1 pp 3-41.

Ross, S.A. (1976). The Arbitrage Theory of Capital Asset Pricing. Journal of Economic Theory 13:Dec pp 341-360.

Samuelson, P.A. (1970). The Fundamental Approximation Theorem of Portfolio Analysis in Terms of Means, Variances, and Higher Moments. Review of Economic Studies. 37:4 pp 537-542.

Schwert, G.W. (1990a). Indexes of U.S. Stock Prices From 1802 to 1987. Journal of Business. 63:3 pp 399-426.

Schwert, W.G. (1989). Why Does Stock Market Volatility Change Over Time? Journal of Finance. 44:5 pp 1115-1153.

Sentana, E. (1991). Quadratic ARCH Models: a Potential Reinterpretation of ARCH Models As Second-Order Taylor Approximations. Unpublished Paper, London School of Economics and Political Science, London.

Shannon, C.E. (1948). A Mathematical Theory of Communication. Bell System Technical Journal. 27:3 pp 379-423.

Shannon C.E. and Weaver, W. (1964). The Mathematical Theory of Communication, Urbana and Chicago: University of Illinois Press.

Sharpe, W.F. (1963). A Simplified Model for Portfolio Analysis. Management Science. 9:2 pp 277-293.

Sharpe, W.F. (1964). Capital Asset Prices: a Theory of Market Equilibrium Under Conditions of Risk. Journal of Finance. 19:3 pp 425-442.

Shefrin H. (2002). Beyond Greed and Fear - Understanding Behavioral Finance and the Psychology of Investing, Oxford, UK: Oxford University Press.

Shiller, R.J. (2003). From Efficient Market Theory to Behavioral Finance. Journal of Economic Perspectives. 17:1 pp 83.

Simpson, E.H. (1949). Measurement of Diversity. Nature. 163:13 pp 688.

Smith W.B. and Cole, A. H. (1935). Fluctuations in American Business, Cambridge, Massachusetts: Harvard University press.

Solnik B. and de Freitas, A. (1988). Recent Developments in International Banking and Finance, New York, US: D.C. Heath.

Statman, M. (1999). Behavioral Finance: Past Battles and Future Engagements. Financial Analyst's Journal. 55:6 pp 18-27.

Stowe J.D., Robinson, T. R., Pinto, J. E. and McLeavey, D. W. (2002). Analysis of Equity Investments: Valuation, Charlottesville, V.A, U.S.A: Association for Investment Management and Research®.

Taylor S. (1986). Modelling Financial Time Series., New York: Wiley.

Titman, S. and Wermers, R. (1997). Measuring Mutual Fund Performance With Characteristic-Based Portfolios. Journal of Finance. 52:3 pp 1035-1058.

Vasicek, O. (1973). A Note on Using Cross-Sectional Information on Bayesian Estimation of Security Betas. Journal of Finance. 28:5 pp 1233-1239.

Waring, M.B. (2003). The Dimensions of Active Management. AIMR Conference Proceedings. Improving the Investment Process Through Risk Management. 2003:4 pp 22-31.

Watsham T.J. and Parramore, K. (1997). Quantitative Methods in Finance, London U.K.: Thomson.

Wei, S.X. and Zhang, C. (2003). Why Did Individual Stocks Become More Volatile? SSRN Working Papers, Forthcoming, Journal of Business.

Weisstein, E.W. (1999-2004). Gini Coefficient. Mathworld

Welch, I. (2000). Views of Financial Economists on the Equity Premium and Other Issues. Journal of Business. 73:4 pp 501-537.

Wiener N. (1961). Cybernetics, or Control and Communication in the Animal and the Machine., New York and London: M.I.T. Press and John Wiley and Sons, Inc.

Wiggins, J.B. (1992). Estimating the Volatility of the S&P 500 Futures Prices Using the Extreme-Value Method. The Journal of Futures Markets. 12:3 pp 265-273.

Williams, C.B. (1946). Yule's Characteristic and the Index of Diversity. Nature 157:13 pp 482.

Yule G.U. (1944). Statistical Study of Literary Vocabulary, Cambridge: Cambridge University Press.

Zakoian, J.-M. (1991). Threshold Heteroskedastic Models. Unpublished Paper (Institut National De La Statistique Et Des Economiques, Paris).