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Left-tail risk and UK stock return predictability: Underreaction, overreaction, and arbitrage difficulties

Maher Khasawneh [a], David G. McMillan [b,*], Dimos Kambouroudis [b]

[a] Department of Banking and Financial sciences, Business School, The Hashemite University, Zarqa, Jordan
[b] Division of Accounting and Finance, University of Stirling, UK

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ABSTRACT

Recent studies challenge the standard model risk-return trade-off by showing inverse predictive power of firm-specific left-tail risk for future returns (i.e., left-tail momentum). In this work, we investigate the pricing of left-tail risk in UK stocks. Both the portfolio construction approach and Fama-MacBeth regressions reveal the underperformance of stocks with high left-tail risk. We examine alternative channels behind this pricing anomaly, namely, investor underreaction behaviour, continuous overreaction behaviour, and limits to arbitrage. Our findings suggest that the observed underperformance associated with high left-tail risk is largely a manifestation of investor underreaction to bad performance. However, the results also show that the predictable underperformance of high left-tail risk stocks is manifest in past winners. The empirical investigation reveals that, in addition to underreaction, limits to arbitrage interacts with investor high attention levels to explain part of the anomaly. The empirical findings provided here suggest several important implications for practitioners in the equity market.

1. Introduction

A fundamental view within finance is that investors trade-off risk for return – to tolerate more risk, a higher return is required. This rational view of the market dominates various decision-making approaches, including, most, if not all, the widely applied financial asset pricing models, which assume some form of a strict positive linear relation between risk and return. However, non-normally distributed financial markets experience crashes and the effect of these events encourages researchers to consider left-tail risk behaviour. As a result, several papers are devoted to testing the pricing of left-tail risk (see for example, Arditti, 1967; Atilgan, Bali, Demirtas, & Gunaydin, 2020; Bali, Demirtas, & Levy, 2009; Harvey & Siddique, 2000; Huang, Liu, Ghon, & Wu, 2012; Lee & Yang, 2022; Long, Jiang, & Zhu, 2018). Consistent with risk aversion, many studies find a positive relation between left-tail measures and stock returns (see for example, Bali, Gokcan, & Liang, 2007; Bali et al., 2009; Huang et al., 2012; Kelly & Jiang, 2014; Lee & Yang, 2022).

However, recent empirical evidence challenges this rational paradigm. Notably, Atilgan, Bali, et al. (2020) identifies a left-tail momentum anomaly for US stocks. They report a negative association between left-tail risk and future returns and thus, there is a premium for stocks with low left-tail risk. Moreover, they demonstrate the inability of available risk models to explain this puzzling behaviour. This result implies that investors prefer stocks with higher left-tail risk and accordingly, that the market behaves in a manner opposite to the rational model of risk aversion and informativeness efficiency. This pattern poses a challenge to the rational view of a positive risk-return trade-off and the many applications based on this theoretical approach. Atilgan, Bali, et al. (2020) argue that investors underestimate the persistence of large losses.

The nature of the anomalous left-tail momentum effect and the associated failure of available risk factors in accounting for it, opens the door for alternative explanations, with, arguably, the most plausible lying within behavioural finance that offers an alternative framework. Notably, the behavioural approach challenges two main assumptions of standard capital market theory. The behavioural approach allows investors to exhibit cognitive biases and thus, are vulnerable to producing erroneous decisions. This approach also suggests that there are potential limits to arbitrage. This can result in profitable opportunities appearing in the market as noise trading behaviour is not fully arbitraged.

Behavioural finance can be considered as a general framework with a

* Corresponding author at: Division of Accounting and Finance, University of Stirling, Stirling, FK9 4LA, UK.
E-mail address: david.mcmillan@stir.ac.uk (D.G. McMillan).

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diverse range of potential biases that drive mispricing in financial markets, such as overconfidence, representativeness, mental accounting, and conservatism (see, for example, De Bondt & Thaler, 1985; Odean, 1998; Hong, Lim, & Stein, 2000; Jegadeesh & Titman, 2001; George & Hwang, 2004; Grinblatt & Han, 2005; Hirshleifer, Lim, & Teoh, 2011).

In this paper, we contribute to the literature by providing evidence of left-tail momentum for UK stocks and testing the potential behavioural factors behind this anomaly. First, we investigate whether left-tail momentum exists in UK stocks, noting that Atilgan, Bali, et al. (2020) does provide evidence for an international sample that includes the UK. Nonetheless, with the majority of evidence based on US stocks, it is important to note UK market characteristics that differentiate it from the US market. For example, UK stocks are more illiquid, smaller in size, and are traded under different organisational rules compared to US stocks (see, Dimson, Nagel, & Quigley, 2003; Doukakis & Papanastasopoulos, 2014; Petrovic, Manson, & Coakley, 2016). The UK also has a high degree of institutional and overseas share ownership.[1] Second, and more importantly, we test the potential factors that may generate the poor performance associated with high left-tail risk stocks. Atilgan, Bali, et al. (2020) provide evidence on the ability of limits to arbitrage and attention to explain left-tail momentum. In this paper, we extend the analysis by considering the potential behavioural channels that may generate or amplify left-tail momentum. We re-examine the limited attention and limits to arbitrage channels by employing different proxies to those used by Atilgan, Bali, et al. (2020). Further, other, competing, pricing biases are tested, including, notably, the continuous overreaction channel.

A sample of UK stocks is analysed on a monthly basis from January 1996 to August 2021. Using both traditional portfolio construction and Fama and Macbeth (1973) cross-section regressions, we analyse limited attention proxies that are not considered by Atilgan, Bali, et al. (2020). Four different measures of attention are used, the 52-week high ratio, abnormal change in volume, continuous information index, and delayed response to market information. Each of these attention proxies measures a different aspect of the limited attention channel. To reduce the measurement error problem, we then combine these measures into one underreaction index by summarising their joint variation using principal component analysis (PCA). We believe that this common variation is a good proxy for investor limited attention. Thus, our underreaction proxies differ from that employed in the literature. For instance, Atilgan, Bali, et al. (2020) use the number of analysts following a stock, while DeLisle, Michael, Kassa, and Zaynutdinova (2021) add the Google search index to an analyst coverage proxy.

In addition to underreaction bias, investors in financial markets are found to overreact. Byun, Lim, and Yun (2016) demonstrate that investor overreaction may persist in the near future and represent a strong return predictor. Considering this evidence, we extend the analysis by examining whether the continuous overreaction index developed in Byun et al. (2016) generates, or at least impacts, the left-tail momentum anomaly.

The pricing literature also reports evidence on the importance of limits to arbitrage in generating many of the pricing discrepancies found in financial markets. In this paper, we proxy for the limits to arbitrage by five different variables, idiosyncratic volatility, bid-ask spread, size, age, and price synchronicity. As with the limited attention proxies, we combine these limits to arbitrage proxies into one index by performing PCA and extracting their first common component. We expect limits to arbitrage to play a vital role in the pricing of UK stocks and as a manifestation of anomalous pricing behaviour in stocks.

The results in this study show that left-tail risk is mispriced in UK stocks. The stocks with the highest left-tail risk (e.g., high expected shortfall) underperform stocks with lower left-tail risk. Thus, left-tail momentum exists in UK stocks, while adjusting for widely considered pricing factors fails to explain this underperformance. These findings suggest that investors in the UK market do not adhere to standard finance rational paradigms in the pricing of left-tail risk. Instead, behavioural explanations play an important role in generating left-tail momentum. Notably, both portfolio analysis and cross-section regressions reveal that underreaction-based bias plays a crucial role in generating the poor performance of the stocks with high left-tail risk.

In addition, introducing limits to arbitrage reveals an interesting pattern, which adds information to the underreaction bias in explaining left-tail momentum. The observed inverse relation between left-tail risk and subsequent returns is stronger when investors are confronted with arbitrage difficulties. Further, our results reveal that combining arbitrage difficulties with underreaction uncovers key trends that enhance our understanding of left-tail momentum. When investors are more inattentive (i.e., the attention index is low), the level of arbitrage limits provides little, if any, information regarding left-tail momentum. However, when investor attentiveness is high, we observe the disappearance of left-tail momentum only for stocks with low arbitrage difficulties. Left-tail momentum remains when investor attentiveness is combined with arbitrage difficulties. Considering the past performance of stocks, in contrast to the continuous poor performance of stocks with low attention, high limits to arbitrage and attentive behaviour induce a reversal in the performance of high left-tail risk stocks. Accordingly, these results provide evidence that left-tail momentum is also a manifestation of investor overreaction to speculative stocks. Irrational behaviour such as the lottery-like effect, documented in Bali, Cakici, and Whitelaw (2011), may provide at least a partial explanation for the anomalous poor performance of stocks with high left-tail risk. Therefore, the evidence in this study implies that stocks with high left-tail risk earn low returns, largely because of investor underreaction to bad news and partially as a result of investor overreaction to attention-grabbing fluctuations.

The rest of this article is organised as follows: Section 2 reviews the existing literature that is relevant to the left-tail momentum and the behavioural explanation of the return predictability observed in the stock market. Section 3 describes the sample, variables, and methodology. Section 4 presents the empirical results. Section 5 provides the robustness check and further analysis. The last section concludes the paper.

2. Literature review and hypothesises development

Empirical work reports numerous pricing anomalies that challenge the traditional theory of rational decision-making in financial markets. Moreover, the universe of behavioural explanations is diverse and there is no agreement on the nature of cognitive biases within markets. In this section, we review the literature that is most closely related to left-tail momentum, the associated payoffs and the potential behavioural biases that can explain it.

2.1. Pricing of the left-tail moment

The positive risk-return relation is a keystone of asset pricing theory, in which rational agents aim to maximise their quadratic utility (see, Markowitz, 1952). Accordingly, the behaviour of these rational investors is described by maximising their return to a specified level of risk or minimising the amount of risk for a targeted level of return. Early asset pricing models, like the CAPM of Sharpe (1964) are built on this view of investor behaviour, where the main ingredients are the mean and standard deviation of the asset returns distribution. At the same time, some researchers argue that investors may be concerned with distributional events beyond the first two moments. Roy (1952) argues that investors are principally concerned with extreme losses and avoiding disastrous events. Therefore, the left tail of the return

[1] While current institutional share ownership is roughly similar in the UK and US, overseas ownership is higher in the UK at 58%, compared to approximately 40% in the US.

distribution is crucial for an investment decision. Adopting this view, several models are developed to extend the standard two moments paradigm (i.e., return and standard deviation) and to incorporate information from the distribution's tail (for example, Arditti, 1967; Arditti & Levy, 1975; Harvey & Siddique, 2000; Kraus & Litzenberger, 1976).

Frequent financial crises, and notably after the Global Financial crisis, attracts the attention of market participants to the importance of a deeper understanding of investor response to such events, with the left tail of the return distribution longer and thicker than under the normal distribution (see Bali et al., 2009). Therefore, many works investigate the pricing of left-tail risk. However, they produce inconclusive results, either by the sign of the relation or its significance, on the pricing factors of left-tail risk.

One strand of the literature focuses on the systematic behaviour of left-tail risk. Harvey and Siddique (2000), Kelly and Jiang (2014), Van Oordt and Zhou (2016), Lee and Yang (2022), and Chabi-Yo, Ruenzi, and Weigert (2018) employ different approaches to measure left-tail risk and report evidence in favour of the standard risk-return relation i.e., the expected return is an increasing function of left tail systematic risk. Based on this, investors shun stocks that are more likely to generate worse outcomes conditional on the downstate of the market. Therefore, risky stocks, in terms of left-tail co-variation, should command higher expected returns for risk-averse investors. Ignoring the information content of the left-tail distribution would lead investors towards an inefficient position.

However, contrary evidence to this position exists. Bali, Cakici, and Whitelaw (2014) argue that the observed investor behaviour in holding an under-diversified position undermines the importance of a purely systematic risk measure. Building on this insight, they develop a hybrid tail measure, using US stocks, that accounts for both stock-specific and aggregate market events. In a comprehensive study, Long, Zhu, Chen, and Jiang (2019) contradict Bali et al. (2014) and demonstrate the failure of their hybrid left-tail measure in predicting returns across stocks in large-scale international samples. Atilgan, Demirtas, and Gunaydin (2020) re-examine the relation between systematic downside risk and returns in the US stock market and demonstrate the insignificance of systematic risk in predicting the future returns of the U.S. equities.

A recent trend in financial market studies notes that as opposed to optimal diversification behaviour, investors, especially individual ones, hold a highly concentrated position involving only a few stocks (see, Goetzmann & Kumar, 2008; Polkovnichenko, 2005). Accordingly, firm-specific characteristics and idiosyncratic information may have important content in the pricing of financial securities. Indeed, research in this vein questions the positive risk-return trade-off, suggesting that across stocks, risk is negatively related to future returns.

One of the recent contradiction to the rational paradigm of risk-return trade-off is introduced by Atilgan, Bali, et al. (2020), who investigate the ability of individual stock downside risk to predict future returns in both a US and international sample. Strikingly, their findings document an inverse association between left-tail risk and future returns. This result means that there is momentum in those returns associated with the level of realised losses. By its nature, this left-tail momentum is unexplainable by available risk models and is not subsumed by documented pricing anomalies.

To quantify downside risk, Atilgan, Bali, et al. (2020) utilise the information in the left-tail distribution of equity returns, specifically they employ value-at-risk (VaR) and expected shortfall (ES). These measures reflect the magnitude and the likelihood of extreme losses that could be generated by a specific asset.

The puzzling left-tail momentum documented by Atilgan, Bali, et al. (2020) is investigated and confirmed by several studies. Bi and Zhu (2020) confirm the anomaly in the US market. For the Chinese market, the second largest stock market in terms of market value, Zhen, Ruan, and Zhang (2020), Sun, Wang, and Zhu (2021), Yang and Ma (2021), Gui and Zhu (2021), and Wang, Xiong, and Shen (2022) report supportive evidence on the existence of left-tail momentum. Additionally, Kim, Park, and Truong (2023) and Eom, Eom, and Park (2023) investigate the Korean market and show significant evidence of the anomalous inverse association between the left-tail risk and future returns. Also, the anomaly appears to be a pervasive phenomenon. Using a large-scale sample of 26 developed markets, Atilgan, Bali, Demirtas, and Gunaydin (2019) confirm the negative association between firm-specific measures of left-tail risk (i.e., VaR and ES) and future returns. It is noteworthy that all the aforementioned studies apply a similar methodology to that in Atilgan, Bali, et al. (2020) and that they use VaR and ES to measure firm-specific tail-risk and the raw stock return distribution. Nonetheless, some other studies utilise different methodologies to measure firm-specific tail risk by isolating the idiosyncratic component of daily returns. DeLisle et al. (2021) extract the residual return using the Carhart (1997) 4-factor model and find a negative association between the minimum daily idiosyncratic return and future returns across US stocks. Applying a similar methodology to extract the idiosyncratic returns, Long et al. (2018) employ a parametric methodology of generalised extreme value to measure the idiosyncratic tail risk index and confirm the negative relation in the Chinese market. A similar parametric setting and results are obtained for an international sample in Long et al. (2019). The above discussion leads us to the first hypothesis in this study:

H1. There is an inverse relation between stock-specific left-tail risk and future returns across stocks in the UK market.

It is worth noting that the left-tail measure employed in Atilgan, Bali, et al. (2020) is distinctive from the systematic downside risk measures used in studies such as Kelly and Jiang (2014). The latter framework gauges tail risk by the co-variation between the left-tail distribution of a stock return and that of the aggregate market, i.e., the likelihood and magnitude of losses given there is a loss in the aggregate market. Theoretically, pricing of the systematic component assumes rational behaviour and a trade-off of higher expected return for the higher left-tail risk. Mostly, empirical results are mixed, some demonstrate a positive premium for high systematic downside risk (see, for example, Ang, Chen, & Xing, 2006; Kelly & Jiang, 2014), others report a discount (see, for example, Atilgan et al., 2019), while there is also evidence of no relation (see, for example, Atilgan, Demirtas, & Gunaydin, 2020). The firm-specific measures in Atilgan, Bali, et al. (2020) and Long et al. (2018) consider the whole stock return distribution without conditioning on market state, and therefore, includes idiosyncratic variations. As noted, the empirical evidence on these firm-specific measures mostly indicate a significant discount for stocks with high left-tail risk after controlling for systematic risk (e.g., beta and downside beta). This suggests irrational behaviour that may lead investors to overprice stocks with high left-tail risk, resulting in inverse predictability for stock returns. This leading to a violation of the rational pricing paradigm.

Failure of fundamental pricing explanations encourages investigation of irrational pricing behaviour and behavioural finance suggests many cognitive errors that could generate overpricing and underpricing of stock characteristics. One explanation is underreaction generated by investor limited attention. Atilgan, Bali, et al. (2020) demonstrate that limited investor attention to losses generated by stocks with high left-tail risk leads them to underestimate their persistence. DeLisle et al. (2021), Li, Yuan, Jin, Long, and Guan (2022), Sun, Wang, and Zhu (2022), and Wang et al. (2022) provide additional evidence on the validity of underreaction behaviour as a source of predictable poor performance associated with left-tail risk.[2] Overvaluation of stocks with high left-tail

[2] Several studies report evidence on the ability of underreaction behaviour to explain pricing anomalies, for example, Huang and Georg (2004), Barber and Odean (2008), Hou et al. (2009), Loh (2010), Da et al. (2014), Cheng et al. (2015), Ben-Rephael et al. (2017), Byun, Goh, and Kim (2020), Chen, He, Tao, and Yu (2023), and Khasawneh, McMillan, and Kambouroudis (2023).

risk is also supported by several authors (see, Bi & Zhu, 2020; Gui & Zhu, 2021; Kim et al., 2023; Yang & Ma, 2021). This leads to the second hypothesis:

H2. *The poor performance of a stock with high left-tail risk is stronger when investors are likely to be less attentive.*

Another possible explanation is overreaction combined with the self-attribution bias. Such an explanation is empirically introduced by Byun et al. (2016). Building on the theoretical model of Daniel, Hirshleifer, and Subrahmanyam (1998), Byun et al. (2016) develop an index of continuing overreaction and demonstrate that this continuing overreaction, triggered by investor overreaction and self-attribution towards their past performance, could generate momentum in short term performance. They provide empirical evidence suggesting continuing overreaction as an underlying mechanism behind the price momentum anomaly. Building on this, the suggested continuing overreaction channel would be proposed as an underlying driver of the persistent poor performance of the stocks with high left-tail risk. Byun et al. (2016) extract the continuous overreaction index from the monthly signed volume of an individual stock, a lower value of this index indicates the continuous overreaction associated with previous negative information. Thus, generating the third hypothesis:

H3. *the poor performance of a stock with high left tail-risk would be stronger when the stock has a lower value of the continuous overreaction index.*

Comparing limited attention behaviour with continuous overreaction behaviour generates a similar price trend over the short term but distinguishes longer term consequences. For underreacting generated by limited attention, investors miss price relevant information, such that it is reflected slowly until the price is fully updated. For overreaction, investors are attentive, perhaps, excessively, which can create a continuing trend in prices (i.e., momentum). However, as investors overreact to information, a reversal in the stock prices over the longer term is likely to emerge.

There is consensus within both theoretical and empirical work on the necessity of limits to arbitrage as an aspect of mispricing in financial markets. Mispricing, resulting from investor overreaction and underreaction, is considered within fundamental pricing theory, however, it is treated as noise and the existence of informed arbitragers is sufficient to eliminate its effect on prices. Therefore, the reported mispricing and appearance of pricing anomalies would be a manifestation of hindered arbitrage (Shleifer & Vishny, 1997). Empirically, limits to arbitrage as a source of mispricing are confirmed by many (see, for example, Ali & Trombley, 2006; Doukas, Kim, & Pantzalis, 2010; Stambaugh, Yu, & Yuan, 2015; Gu, Kang, & Xu, 2018). Atilgan, Bali, et al. (2020), Yang and Ma (2021), and DeLisle et al. (2021) show that the mispricing of left-tail risk across stocks is stronger when arbitrage is impeded. Therefore, combining limits to arbitrage with overreaction bias and/or underreaction bias is important to get a fuller understanding of the left-tail anomaly. This leads to both a fourth and fifth hypothesis where limited arbitrage and cognitive errors (e.g., limited attention and continuing overreaction) both play an important role in generating return momentum associated with the extreme left-tail:

H4. *The poor performance of a stock with high left-tail risk is stronger when stocks are costlier to arbitrage.*

H5. *: The interaction between limited-attention (continuing overreaction) and limits to arbitrage causes poorer performance for stocks with higher left tail risk.*

Empirical results support the theoretical behavioural models suggested by Barberis, Shleifer, and Vishny (1998), Daniel et al. (1998), and Hong and Stein (1999). Although with different ingredients, each one of these models offers a framework that could explain the observed predictable pattern in returns. The model of Barberis et al. (1998) relies more on cognitive biases such as representativeness introduced by Tversky and Kahneman (1974) and conservatism introduced by Edwards (1968). Daniel et al. (1998) suggest overconfidence and self-attribution biases as a driver of price anomalies. Hong and Stein (1999) rely less on cognitive biases and assume that investors are heterogeneous in their ability to process information, which diffuses slowly. Each one of these different cognitive biases slows information dissemination and impedes price discovery. Consequently, manifesting in the short run is price continuation.

3. Data and methodology

3.1. Data sample

To test the main relations in this study, a sample of all common stocks from the London stock exchange is used. The sample is selected to consist of both currently traded and delisted stocks in order to mitigate the well-documented survivorship bias found to affect cross-section asset pricing tests (e.g., see, Shumway, 1997). The pre-filtering sample includes 4515 stocks. The data includes daily and monthly prices and other trading data of the selected stocks and spans the period from January 1996 to August 2021. To mitigate the problem of non-synchronous trading, stocks with <120 trading days, over the past year, are excluded. We also rule out any stocks with a price less than £3 and any month with no available observations. The data source is Thomson Reuters DataStream. Following previous asset pricing literature in the UK, the sample only includes common equities (see, e.g., Florackis, Gregoriou, & Kostakis, 2011).

3.2. Variable definitions

As with many other financial concepts, left-tail risk and underreaction-related features have no explicit measure and are difficult to quantify. To represent them, various proxies are employed in the literature. In the following, we describe the employed proxies of these variables.

3.2.1. Left-tail risk (ES)

The literature adopts different metrics to proxy for left-tail risk (see, for example, Bali et al., 2009; Atilgan, Bali, et al., 2020; Huang et al., 2012; Bali et al., 2014). These measures can differ, for example, in capturing systematic rather than the total left-tail distribution and in parametric vs non-parametric approaches. In this study and following Bali et al. (2009) and Atilgan, Bali, et al. (2020), we measure total left-tail risk non-parametrically as represented by expected shortfall (hereafter, ES). Under ES, tail risk is the average amount of loss conditional on a given threshold (see, Artzner, Delbaen, Eber, & Heath, 1999). While there is no specific rule to select the threshold under which ES is calculated, we follow general convention by calculating left-tail risk/ES as the average of returns under the 5-percentile over the past one-year (250 days), as follows:

$$ES\alpha\% = 1\Big/N\sum R_i < \alpha\% \tag{1}$$

Where R_i is the daily returns of stock i over the past 250 trading days, α% is the selected threshold (5% in this study), and N is the number of observations less than the α% level. In the analysis we multiply ES by −1, therefore, a higher ES is associated with higher potential losses (i.e., higher left-tail risk). In addition to ES5%, to provide robustness, we measure left-tail risk by ES at the 1% threshold and by Value-at-Risk (Var5%) which is the observation that represents the 5-percentile of the distribution.

3.2.2. The underreaction-based cognitive biases

Limited attention and the associated underreaction behaviour, equally, have no direct and explicit measure. Therefore, we consider four proxies to capture underreaction behaviour. These measures are the delay to past market information, the abnormal change in volume, the information continuation index, and the 52-week high ratio. These proxies are motivated by existing empirical evidence and theoretical model building. Following Hou and Moskowitz (2005), the delayed response is measured as the sensitivity of stock returns to past returns of the wider market. A growing number of empirical studies employ trading volume as a reliable proxy of investor attention, examples include Barber and Odean (2008), Hou, Peng, and Xiong (2009), Loh (2010), Cheng, Yan, Zhao, and Gao (2015), Chang, Ko, Nakano, and Rhee (2018) and Chen, Tang, Yao, and Zhou (2019). Following these works, we employ the difference between trading volume in monetary terms over the past month and the 12-month average, standardised by its standard deviation.

A continuous information index is built following the method employed by Da, Gurun, and Walachia (2014). They argue that investors are more likely to miss the information that flows in as a small consistent piece (i.e., continuous information). The information is represented by the sign of daily returns over the past 12 months. If the cumulative return over the past 12 months is formed by daily returns of the same sign, then the stock is associated with continuous information. George and Hwang (2004) demonstrate the information content of the price to 52-week high (PH52). They argue that investors are more likely to underreact to price-relevant news when prices are closer to their 52-week high. They claim that this behaviour is a manifestation of anchoring bias, first theorised by Tversky and Kahneman (1974). Each one of these proxies is described in detail in the appendix.

All the aforementioned mispricing phenomena are potential manifestations of investor underreaction bias. However, none of them is a perfect measure. Thus, to reduce potential measurement error, a common component among these proxies is extracted to create an underreaction-based index. Notably, we apply PCA to the four proxies and extract the first common component, which is used as the index.

3.2.3. Continuous overreaction (CO)

Another potential source of price momentum is investor overconfidence and self-attribution (Daniel et al., 1998). Byun et al. (2016) propose a measure that captures the trend in investor overconfidence. They define this measure as follows:

$$CO_{i,t} = \frac{sum\left(w_j \times SV_{i,t-j}, \ldots\ldots\ldots\ldots\ldots\ldots, w_1 \times SV_{i,t-1}\right)}{mean\left(VOL_{i,t-j}, \ldots\ldots\ldots\ldots\ldots\ldots, VOL_{i,t-1}\right)} \tag{2}$$

where $SV_{i,t}$ is the signed volume for stock i in month t,

$$SV_t = \begin{cases} VOL_t if\ r_t > 0, \\ 0\ if\ r_t = 0, \\ -VOL_t\ if\ r_t < 0, \end{cases} \tag{3}$$

where VOL_t is the dollar volume in month t and r_t is the stock return in month t, J is the length of the formation period, and w_j is a weight that takes a value of $J\text{-}j + 1$ in month $t\text{-}j$ (i.e., $w_j = 1$ and $w_1 = J$). In this work, the continuous overreaction (CO) is measured using a 12-month formation period.

3.2.4. Limits-to-arbitrage index

Numerous works, theoretical and empirical, suggest costly arbitrage as an important ingredient of inefficient pricing (i.e., anomalies) and therefore, observed predictable returns. Limits to arbitrage prevent rational investors from eliminating any observed pricing deficiency and consequently, these deficiencies persist (see, for example, Pontiff, 2006; Stambaugh et al., 2015). As such, limits to arbitrage can affect asset pricing by providing a fertile environment for cognitive errors (e.g., higher information uncertainty) and/or preventing rational arbitrageurs from trading away mis-valuations. Like the other noted phenomenon, limits-to-arbitrage is unobservable and is, alternatively, measured by several reasonable proxies. We measure limits to arbitrage by five different proxies, idiosyncratic volatility, market capitalisation, firm age, return synchronicity, and the bid-ask spread. These variables are selected to represent both transaction costs and the costs of holding. Like the underreaction proxies, the limits to arbitrage proxies are combined via PCA to form a single index that is intended to mimic the variation of unobservable arbitrage difficulties.[3]

3.2.5. Control variables

The anomalous return predictive power of left-tail risk may represent previously published pricing anomalies and return predictors. To isolate the potential effect of such other predictors, we include a set of widely documented control variables. This includes, beta, downside beta, co-skewness, market value (size), book-to-market ratio (BM), price momentum, last month's return, long-term return, Amihud illiquidity ratio, accounting profitability, asset growth, dividend yield, idiosyncratic volatility, and the lottery-effect.[4]

Contradicting CAPM fundamentals, previous studies report poor performance for stocks with high beta (see, for example, Frazzini & Lamont, 2008). Bali, Brown, Murray, and Tang (2017) demonstrate that gambling-like behaviour by investors may account for the puzzling poor performance of high beta stocks. Adding systematic left-tail risk to the standard market factor may enhance explanatory power for the variation in stock returns. Harvey and Siddique (2000) add co-skewness as a determinant of stock returns, while Ang et al. (2006b) provide evidence in favour of downside beta. Stock size, proxied by market value, shows significant predictive power for future returns (Banz, 1981; Fama & French, 1992). One of the widely examined phenomena is the value premium, where stocks with high BM outperform stocks with low BM (Fama & French, 1992; Rosenberg, Reid, & Lanstein, 1985).

Research indicates that the performance of stocks exhibit predictable trends. This includes over the short-term (past month) and the long-term (past three to five years) where stock returns show predictable reversal (see, De Bondt & Thaler, 1985; Jegadeesh & Titman, 2001), while over the mid-term (the past 6 to 12 months), returns exhibit continuation performance, i.e., winners (losers) will be winners (losers) over the next period (see, for example, Jegadeesh & Titman, 1993). A further behaviour linked to the distribution of a stocks past return is the lottery-like effect. Bali et al. (2011) document an inverse relation between past extreme daily returns and future returns, i.e., stocks with high daily returns over the past month perform poorly over the next months. They suggest mispricing as a reason behind this persistent inverse relation. Coelho, John, Kumar, and Taffler (2014) suggest retail investors 'gambling' attitude would generate underreaction to bad news. Also, past studies report evidence that links illiquidity and idiosyncratic volatility to the emergence of mispricing in the stock market (see, Stambaugh et al., 2015; Wang et al., 2022). Au, Doukas, and Onayev (2009) show that the high cost of arbitraging (e.g., high idiosyncratic volatility) is likely to affect stock returns predictability in UK market. Hwang and Lu (2007) and Foran, Hutchinson, and O'Sullivan (2015) further report an anomalous poor performance associated with illiquid stocks in the UK.

Accounting information shows predictive power for stock returns. Fama and French (2015) show that ROA (return on assets) and growth in assets are pervasive predictors of returns across stocks. The return predictive power associated with ROA and the growth in assets may be a manifestation of mispricing behaviour (see, Hou et al., 2009; Jiang, Lee, & Zhang, 2005; Ma, Whidbee, & Zhang, 2023). Dividend-to-price ratio (dividend yield) also positively predicts future returns (see, for example,

[3] All the used variables are supported with evidence from the literature. See, For example, Kim and Verrecchia (1994), Zhang (2006), Jiang et al. (2005), Piotroski and Roulstone (2004), Chan and Hameed (2006), Gregoriou, Ioannidis, and Skerratt (2005), Kumar (2009). An and Zhang (2013), Godfrey and Brooks (2015), and Stambaugh et al. (2015).

[4] For more details about measuring variables, please refer to Appendix A.

Fama & French, 1988; Van Binsbergen & Koijen, 2010; McMillan, 2014).

The identified left-tail momentum may be a manifestation of one of these above-mentioned pricing deviations. For example, Aboura and Arisoy (2019) link the size effect, value premium, price momentum, and idiosyncratic volatility puzzle to left-tail risk. Therefore, it is important to include them in the statistical analysis to ensure robustness of our results.

3.3. Empirical methodology

Our analysis includes two commonly adopted approaches, the cross-section Fama and Macbeth (1973) regressions and a portfolio sorting technique.

We perform a single-sort approach to explore the performance of portfolios based on left-tail risk. Specifically, we create five ES (expected shortfall) portfolios by sorting individual stocks into quantiles according to their level of ES. The returns on these quantile portfolios are analysed over the subsequent three years. It is important to examine whether the left-tail anomaly is a result of any of the widely reported returns predictors, such as the stock size, beta, or any other well-known return-related attribute. Therefore, the independency of left-tail pricing behaviour from other returns predictors is examined. To do this, we perform a double-sort technique by first sorting on one of the selected predictors, and then, within each of these sorted portfolios, the stocks are sorted on ES.

In order to analyse a portfolio's alpha, we need a valid asset pricing model. To this end, we consider the widely used 4-factor model of Carhart (1997). Notwithstanding, despite the widespread use of this model, there are alternatives. Therefore, in the robustness analysis, we consider results using the 5-factor model of Fama and French (2015) and the 4-factor model of Hou, Xue, and Zhang (2015).

The above portfolio sorting techniques have both advantages and drawbacks. Sorting stocks is suitable as it mimics practical investment styles in the market. To illustrate, to generate a profitable trading strategy, investors would allocate their capital according to the different potential return predictors. Also, the non-parametric nature makes this approach free of any functional form and other requirements of parametric methods, such as a multivariate regression approach. However, portfolio sorts can only be undertaken on a limited set of factors as the quantity of stocks in each portfolio will shrink as the number of sorts increases. Therefore, in order to consider a larger number of factors for stock returns, the multivariate cross-sectional regression of Fama and Macbeth (1973) is used.

Under the first step, the following cross-section regression is estimated on a month-to-month basis:

$$\mathrm{R}_{\mathrm{i,t+2-t+13}} = \alpha_{\mathrm{it}} + \beta^{*}\mathrm{ES5\%}_{\mathrm{i,t}}\ (\text{or VaR}) + \Sigma\, \beta_{\mathrm{x}}^{*}\mathrm{X}_{\mathrm{i,t}} + \mathcal{E}_{\mathrm{i,t}}, \tag{4}$$

Where $R_{i,t+2\text{-}t+13}$ is the stock i risk premium over the next 2–13 months, $ES5\%_{i,t}$ is the expected shortfall, VaR is value-at-risk, and X represents the set of control variables. After estimating this model on a monthly basis, the averages of the estimated time-series coefficients are tested against the null hypothesis. The control variable list includes the attention proxy, information uncertainty proxy, illiquidity, and the other relevant variables as noted in the appendix. We estimate the regression both with and without controls. The purpose is to check the significance of the left-tail measures while controlling for a range of potential alternative mispricing sources. Each regression is estimated using Newey-West *t*-statistics to adjust for autocorrelation and heteroscedasticity.

Together both the non-parametric stocks sorting approach and the parametric Fama-MacBeth regression approach should provide sufficient empirical evidence on the nature of left-tail risk and pricing anomalies in UK stocks. Notwithstanding, regression results could be affected by extreme cases, for example, microcaps (small capitalisation stocks) or highly illiquid stocks. Therefore, we consider further robustness by re-examining the analysis for different capitalisation groups and after excluding low-priced and illiquid stocks. In application of the Fama-MacBeth regression, we use individual stocks. This is because, while some studies use portfolios as the base asset, Ang, Liu, and Schwarz (2020) highlight the potential problem of information loss by aggregating stocks into characteristic-based portfolios.

4. Empirical results

In this section, we present and discuss the analysis conducted to examine the relation between subsequent returns and left-tail risk, proxied using expected shortfall (ES). In considering the potential sources behind the effect, we include the effect of limited attention, continuous overreaction, and limits to arbitrage. Using UK stocks, we first examine the summary statistics and correlations between the main variables, then present univariate portfolio analysis to evaluate the performance of an investment strategy based on left-tail risk. Bivariate portfolio analysis and Fama-MacBeth regressions are then used to examine the forces that may generate or at least affect the payoff for the left-tail-based strategy.

4.1. Descriptive statistics

Panel A of Table 1 presents the descriptive statistics of the main variables. The table shows the time series average of the cross-sectional values. Our sample of stocks generates an average return of −0.35%. This negative return is not unexpected in light of a difficult time for the UK economy during the sample period.[5] The average expected loss beyond the 5% cut-offs is 5.53%. This average is comparable to that reported by Atilgan, Bali, et al. (2020) for US stocks.[6] The correlation values reported in Panel B show some notable observations. Our main measure of left-tail risk (ES) is negatively related to next month's returns (the correlation coefficient is −0.08). This suggests that, on average, the stocks with a riskier left-tail are more likely to be a loser next month. The left-tail measure also exhibits a noticeably high correlation with many of the previously reported return predictors. For instance, momentum (Mom), long-term past returns (LT), and market capitalisation (LNMV) all have a strong negative correlation with ES at −0.43, −0.50, and −0.51, respectively. Not surprisingly, left-tail risk is highly correlated with a right-tail measure (i.e., MAX) and a more general risk measure, idiosyncratic volatility (IVOL), with correlation coefficients at 0.71 and 0.90, respectively. These three measures (i.e., ES, MAX, and IVOL) represent different aspects of the returns distribution. Bali et al. (2011) employ the right-tail measure (MAX) as a proxy for the lottery-likeness, while idiosyncratic volatility is one of the widely reported asset pricing puzzles in recent times (see, Ang et al., 2006).[7] Therefore, this highlights the importance of controlling for these effects while analysing the effect of left-tail risk on subsequent returns of UK stocks.

4.2. Predictive power of the left-tail risk for the future returns: univariate portfolio analysis

Stocks are sorted into quantiles according to their ES level with both

[5] The sampling period includes some remarkable stock market crashes, most importantly, the bursting of the dot-com bubble of 2000, the global financial crises of 2009, and the then ongoing crisis of Covid-19.

[6] Considering the 5% percentile, they report 5% ES on average across the U.S. stocks.

[7] Strikingly, contrasting the standard rational paradigm of asset pricing and finance, Ang et al. (2006) document a negativeassociation between idiosyncratic volatility and the future returns among the stock in the U.S. market. In later work, Ang, Hodrick, Xing, and Zhang (2009) report pervasive phenomena in the international sample. Khasawneh et al. (2021) confirm this anomalous pricing behaviour in the UK stock market.

Table 1
Descriptive statistics and correlation matrix.

Panel A: summary statistics

stats	R	ES	Mom	Past	LT	LNMV	Beta	Dbeta	Cosk	Amih	IV	Max	Skew	BM	GA	ROA	DY
Mean	−0.35	5.53	5.03	0.02	14.06	5.79	1.67	1.82	0.03	0.28	2.55	5.13	0.27	0.68	0.15	0.06	2.43
SD	14.92	3.08	55.45	13.83	88.39	1.93	1.91	2.4	2.04	1.04	1.6	3.78	1.18	1.09	0.39	24.32	2.92
p5	−22.58	2.23	−93.43	−22.48	−152.36	2.76	−0.56	−1.24	−1.21	0	0.98	1.54	−1.58	0.05	−0.27	−41.5	0
p95	19.7	11.83	86.22	20.76	142.71	9.11	5.4	6.28	1.61	1.27	5.78	12.44	2.1	2.01	0.84	23.07	7.02

Panel B: Correlations Matrix

	R + 1	ES	Mom	Past	LT	LNMV	Beta	Dbeta	Cosk	Amih	IV	Max	Skew	BM	GA	ROA	DY
R	1																
ES	−0.08	1															
Mom	0.07	−0.43	1														
rev	0.06	−0.12	0.3	1													
LT	0.03	−0.5	0.55	0.17	1												
LNMV	0.05	−0.5	0.18	0.09	0.3	1											
beta	−0.02	0.22	−0.09	−0.03	−0.14	−0.11	1										
Dbeta	−0.01	0.18	−0.03	−0.01	−0.08	−0.1	0.76	1									
Cosk	0	−0.02	−0.02	0	−0.01	0.09	0.01	−0.17	1								
Amih	−0.007	0.32	−0.1	0.002	−0.23	−0.31	−0.04	−0.02	−0.01	1							
IV	−0.07	0.9	−0.2	−0.04	−0.43	−0.56	0.22	0.2	−0.03	0.34	1						
Max	−0.06	0.71	−0.19	0.1	−0.29	−0.41	0.03	0.04	−0.02	0.3	0.73	1					
Skew	0.03	−0.07	0.24	0.26	0.09	−0.07	0.01	0.03	−0.04	0.03	0.07	0.21	1				
BM	−0.01	0.35	−0.41	−0.14	−0.46	−0.24	0.08	0.04	0.01	0.18	0.28	0.24	−0.06	1			
GA	−0.05	0.09	−0.07	−0.05	0.16	−0.06	0.05	0.05	−0.01	−0.05	0.07	0.08	−0.02	−0.05	1		
ROA	0.06	−0.36	0.08	0.04	0.25	0.32	−0.12	−0.1	0.04	−0.13	−0.42	−0.31	−0.07	−0.05	−0.14	1	
DY	0.03	−0.1	−0.22	−0.08	−0.12	0.18	−0.07	−0.07	0.04	−0.05	−0.2	−0.11	−0.13	0.25	−0.11	0.23	1

This table reports the descriptive statistics (Panel A) and the correlation coefficients (Panel B) of the variables used in the empirical analysis. Ret is the monthly return, ES is the expected shortfall, VAR is the value at risk, Mom is the cumulative return over the past 12 months, Past is the previous month's return, LT is the return over the previous three years, LNMV is the logarithm of the market value, Beta is the market beta, Amih is the Amihud impact ratio, Dbeta is the down beta, IV is idiosyncratic volatility, Max is the average of the maximum 5 daily returns in the previous month, Coskew is the co-skewness, ROA is the return on assets, GA is the growth of the total assets, and DY is the dividend yield. Panel A reports the mean, standard deviation, 5%, and 95% percentiles. Panel B reports the cross-sectional Person correlation coefficient.

value and equal weighted portfolios constructed. The performance of each decile portfolio is evaluated by the next-month return. A zero-cost strategy is built by shorting the portfolio with the lowest ES and buying (long) the portfolio with the highest ES. These returns are then adjusted for risk using the pricing model of Carhart (1997).

Table 2 reports the ES-decile portfolio performance analysis. Panel A of this table shows the equal weight scheme and Panel B presents the value weight scheme. Contradicting the standard rational model and confirming left-tail momentum documented by Atilgan, Bali, et al. (2020), the results show an inverse association between the subsequent return and ES level. Stocks with high left-tail risk (i.e., highest ES-quantile) underperform stocks with low left-tail risk (i.e., low ES-quantile). This anomalous pattern holds regardless of the weighting scheme. For example, considering value-weighted performance, next month's return of the lowest ES-quantile portfolio is 0.5% (Newey-West t-statistic of 2.8), while for the highest ES-quantile portfolio, next month's return is −2% (Newey-West t-statistic of −2.6). Therefore, the zero-cost strategy generates a negative return of −2.5% (Newey-West t-statistic of −3.8). When adjusting for the Carhart (1997) four-factor model, this payoff falls to −1.8% but remains economically and statistically significant (Newey-West t-statistic of −4.0). This inverse behaviour is arguably more significant, both statistically and economically terms, for equally weighted returns.

The existence of the risk-adjusted negative returns for stocks with high ES (i.e., left-tail momentum) suggests, possible, overvaluation behaviour by investors. In other words, if investors underreact to stocks with high left-tail risk (high ES), their underperformance will persist in future, including relative to stocks with low left-tail risk (low ES). Alternatively, this negative return could be generated by continuous overreaction of investors (see, Byun et al., 2016). Under this channel, investors overreact to stock-specific news and, again, this behaviour persists in the future. We discuss these two possible explanations later in this work. Comparing the values in Table 2 to those reported by Atilgan, Bali, et al. (2020) for US stock, the effect appears stronger in UK stocks. This might be a result of the smaller market capitalisation and illiquid stocks that dominate the UK market.[8]

Table 2
Univariate portfolio analysis and the left-tail risk pricing.

Panel A: Equally-weight						
quantile	ES1	2	3	4	ES5	ES5-ES1
Ret_{+1}	0.8[a]	0.7[b]	0.4	−0.5	−2.4[a]	−3.2[a]
t-stat	3.2	2.1	1.1	−1.1	−3.6	−5.9
Alpha	0.8[a]	0.8[a]	0.6[a]	−0.1	−1.7[a]	−2.4[a]
t-stat	5.9	6.8	6.1	−0.8	−5.4	−6.3

Panel B: Value-weight						
quantile	ES1	2	3	4	ES5	ES5-ES1
Ret_{+1}	0.5[a]	0.3	0.4	−0.3	−2.0[b]	−2.5[a]
t-stat	2.8	1.2	0.7	−0.5	−2.6	−3.8
Alpha	0.2[c]	0.0	0.0	−0.5[c]	−1.6[a]	−1.8[a]
t-stat	1.7	0.2	0.0	−1.7	−4.3	−4.0

This table reports the monthly return of the ES-based quantile portfolios. Panel A reports the equally weighted return while in Panel B the return is weighted by the market value. Ret_{+1} is one month-ahead-formation return, Alpha is the Carhart 4-factor alpha, and t-stat is the Newey-West t-statistic. ES1 (ES5) includes the stocks with the lowest (highest) ES, and ES5-ES1 represents the spread between the returns of the highest and the lowest ES portfolios. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

4.3. Is the left-tail momentum persistent?

In Table 3, we extend the analysis from Table 2 by reporting the performance of ES-based portfolios over the next three years. The goal is to check whether the inverse ES-return pattern persists over this longer-run horizon. Again, performance is measured by the raw return and the Carhart (1997) risk-adjusted return. We only show the value-weighted returns. Although its significance diminishes over longer periods, the predictive power of ES for future returns persists. For example, looking two-months ahead, stocks with the highest ES, on average, earn a return of −1.92%. Therefore, considerably underperforming stocks with low ES, which earn an average return of 0.52%. The return to the zero-cost strategy is −2.44% and statistically significant. Over the next three years, these ES-based differential returns diminish gradually; however, they remain both economically and statistically significant. Therefore, we conjecture that underreaction bias is the more likely driving force behind left-tail momentum than the overreaction bias. For instance, the average monthly returns of the zero-cost strategy through the third year (T + 25–36) is −1.49% (Newey-West t-statistic of −3.77). This diminishing trend in differential returns comes from the partial reversal in the returns of stocks in the highest ES quantile.

Therefore, preliminary evidence suggests the existence of left-tail momentum in the UK market. Furthermore, this effect appears to be stronger and more persistent than that uncovered for the US. The ES retains its predictive power for future returns even after accounting for the Carhart (1997) risk model. This negative sign of abnormal return obviously violates the standard theory of rational investors.

4.4. Features of the ES-based portfolios and the bivariate-sort analysis

Above, we examine the association between left-tail risk, measured by ES, and subsequent returns in UK stocks. Here, we examine the relation between ES and a select set of control variables that includes the most important previously identified return predictors. We then consider the potential impact of these predictors on the performance of the ES-based strategy outlined in Tables 2 and 3. Our goal is to examine if the reported abnormal returns of the ES-based portfolio are a manifestation of any other predictors.

Table 4 presents the characteristics of the ES-quantile portfolios. Unsurprisingly, over the last month and the past three years, stocks with high ES are, on average, losers. This relation highlights the momentum nature in the performance of ES-based portfolios. For example, over three-years, high-ES stocks generate an average return of −8.34%. These stocks also have a smaller average market capitalisation, higher systematic risk (higher beta and downside beta), higher unsystematic risk (higher idiosyncratic volatility), and lower liquidity (higher Amihud ratio). Also, as shown in Panel B of Table 1, the high-ES stocks are more likely to be lottery-like, the difference in MAX between the highest and lowest quantile of ES is 4.9 and statistically significant. In terms of fundamentals, high-ES stocks are also unprofitable (ROA differential is −12.13% with Newey-West t-statistic of −12.47) but have high growth in assets (differential GA is 14% with Newey-West t-statistic of 5.55). In general, an investment strategy that sorts stocks according to their ES level also detects differentiation across return predictors.

It is worth noting that these relations are consistent with the predictive power ES and other selected returns predictors have for future

[8] As we will see later in this work, such characteristics of the limits to arbitrage argument would represent at least a partial explanation for the reported left-tail momentum.

Table 3
Long-run performance of the ES-based portfolios.

	Long-term performance												
	Raw return						Alphas						
	ES1	2	3	4	ES5	ES5-ES1	quantile	ES1	2	3	4	ES5	ES5-ES1
T_{+2}	0.52[b]	0.39	0.44	−0.49	−1.92[b]	−2.44a	T_{+2}	0.16	0.09	0.19	−0.68b	−1.45[a]	−1.62[a]
t-stat	2.52	1.40	1.16	−0.83	−2.53	−3.68	t-stat	1.49	0.65	1.11	−2.01	−3.49	−3.44
T_{+3}	0.51[b]	0.43	0.34	−0.39	−2.00[b]	−2.51[a]	T_{+3}	0.18	0.14	0.14	−0.47	−1.61[a]	−1.79[a]
t-stat	2.44	1.53	0.85	−0.68	−2.63	−3.75	t-stat	1.51	1.16	0.77	−1.57	−3.72	−3.55
T_{+4}	0.51[b]	0.36	0.20	−0.47	−1.77[b]	−2.29[a]	T_{+4}	0.19	0.10	0.00	−0.56[b]	−1.34[a]	−1.53[a]
t-stat	2.46	1.23	0.47	−0.87	−2.44	−3.54	t-stat	1.51	0.83	0.01	−2.06	−3.38	−3.19
T_{+5}	0.53[b]	0.37	0.04	−0.41	−1.76[b]	−2.29[a]	T_{+5}	0.24[c]	0.11	−0.13	−0.33	−1.36[a]	−1.60[a]
t-stat	2.55	1.23	0.11	−0.70	−2.27	−3.23	t-stat	1.81	0.88	−0.70	−1.16	−3.08	−3.05
T_{+6}	0.48[b]	0.38	0.05	−0.20	−1.44[c]	−1.92[a]	T_{+6}	0.16	0.17	−0.10	−0.13	−1.01b	−1.17b
t-stat	2.30	1.23	0.11	−0.38	−1.92	−2.80	t-stat	1.26	1.33	−0.52	−0.47	−2.55	−2.44
$T_{+7–12}$	0.49[b]	0.38	0.28	−0.24	−1.53[b]	−2.02[a]	$T_{+7–12}$	0.18	0.09	0.13	−0.28	−1.16[a]	−1.34[a]
t-stat	2.57	1.36	0.79	−0.51	−2.3	−3.35	t-stat	1.46	0.66	0.66	−1.18	−3.27	−3.11
$T_{+13–18}$	0.30	0.27	0.13	−0.34	−1.21[b]	−1.51[a]	$T_{+13–18}$	0.04	−0.03	0.01	−0.25	−0.68[a]	−0.72[c]
t-stat	1.27	0.97	0.34	−0.65	−2.00	−3.10	t-stat	0.25	−0.33	0.03	−0.99	−1.98	−1.64
$T_{+19–24}$	0.33	0.38	0.19	−0.18	−1.25[b]	−1.58[a]	$T_{+19–24}$	0.07	0.13	0.13	−0.12	−0.87[a]	−0.95[a]
t-stat	1.45	1.50	0.58	−0.38	−2.25	−3.58	t-stat	0.58	1.59	0.79	−0.56	−3.54	−2.84
$T_{+25–36}$	0.28	0.23	0.15	−0.27	−1.20[b]	−1.49[a]	$T_{+25–36}$	0.09	0.07	0.10	−0.29	−0.62[c]	−0.70[c]
t-stat	1.42	1.02	0.59	−0.85	−2.45	−3.77	t-stat	0.58	0.64	0.59	−1.35	−1.64	−1.73

This table reports the average monthly return of the ES-based quantile portfolios. T_{+n} is the average monthly return on n months ahead of portfolio formation, Alpha is the Carhart 4-factor alpha, and t-stat is the Newey-West t-statistic. ES1 (ES5) includes the stocks with the lowest (highest) ES, and ES5-ES1 represents the spread between the returns of the highest and the lowest ES portfolios. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

Table 4
Univariate portfolio characteristics.

quantile	ES	Mom	Past	LT	LNMV	Beta	Dbeta	Cosk	Amih	IV	Max	Sk	BM	GA	ROA	DY
ES1	2.80	15.30	1.29	36.76	9.22	0.93	1.01	0.06	0.00	1.17	2.66	0.28	0.43	9	7.27	3.55
2	3.82	12.06	0.82	34.26	9.01	1.32	1.38	0.21	0.01	1.49	3.43	0.14	0.51	10	6.89	3.19
3	4.72	8.56	0.64	29.50	8.91	1.69	1.72	0.28	0.03	1.78	4.13	0.06	0.53	14	6.23	3.02
4	5.97	4.60	0.07	21.22	8.28	2.08	2.20	0.24	0.03	2.27	5.06	0.02	0.59	17	5.31	2.50
ES5	9.00	−13.5	−1.26	−8.34	6.88	2.57	2.74	0.15	0.13	3.85	7.56	−0.08	0.72	23	−4.86	1.93
ES5-ES1	6.2[a]	−28.8[a]	−2.56[a]	−45.1[a]	−2.35[a]	1.63[a]	1.73[a]	0.09	0.13[a]	2.68[a]	4.9[a]	−0.36[a]	0.29[a]	14[a]	−12.13[a]	−1.62[a]
t-stat	26.2	−3.36	−4.55	−5.84	−12	10.2	8.4	0.97	6.43	22.9	16.3	−7.7	4.5	5.55	−12.47	−7.2

This table reports the average of the characteristics of the ES-based portfolios. ES is the expected shortfall, Mom is the cumulative return over the past 12 months, Past is the previous month`s return, LT is the return over the previous three years, LNMV is the logarithm of the market value, Beta is the market beta, Amih is the Amihud impact ratio, Dbeta is the down beta, IV is idiosyncratic volatility, Max is the average of the maximum 5 daily returns in the previous month, Coskew is the co-skewness, ROA is the return on assets, GA is the growth of the total assets, and DY is the dividend yield. ES1 (ES5) includes the stocks with the lowest (highest) ES, ES5-ES1 represents the spread between the highest and the lowest ES portfolios, and t-stat is the Newey-West t-statistic. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

returns. For instance, the negative relation between past performance (momentum) and left-tail risk (ES) is consistent with the poor performance of loser stocks with high ES stocks.[9] Therefore, one essential question that arises is whether left-tail momentum is a result of any previously widely documented return predictor.

To answer this question, we report bivariate portfolio results in Table 5. This analysis is performed in two steps. First, we sort the stocks into quantiles based on one of the selected characteristics. Second, within each of these quantiles, we re-sort stocks into another five quantiles based on ES. In performing these two steps, we obtain 25 portfolios. Our goal is to evaluate the behaviour of left-tail momentum when controlling for other potential effects.

Table 5 reveals that evidence for left-tail momentum is robust after accounting for a range of alternative stock return predictors, i.e., the ES level continues to inversely predict subsequent returns. In each case, the zero-cost ES-based strategy generates a significant abnormal return over the next month and in many cases is comparable to that reported within the univariate sort in Panel B of Table 2. Nonetheless, in some cases, accounting for predictive power from other stock characteristics tempers the return differential between high and low ES quantiles. For example, controlling for the effect of accounting profitability (ROA) and the dividend yield (DY) shrinks the abnormal returns of this strategy to −1.00% and − 0.93%, respectively, but they remain statistically significant.[10,11] Similarly, left-tail momentum remains significant when controlling for the IVOL (idiosyncratic volatility) effect, a well-known lottery-like characteristic, although with the return differential at −1.00%.

[9] For evidence on the low beta anomaly (see, Black, 1972; Frazzini & Pedersen, 2014), idiosyncratic volatility puzzle (see, Ang et al., 2006), lottery-like effect (see, Bali et al., 2011), profitability (see, Fama & French, 2006), and asset growth (see, Cooper, Gulen, & Schill, 2008).

[10] We compare this with the performance of the univariate strategy In Panel B of Table 2, which is −2.35% with a Newey-West t-statistic of −3.74. Both are value-weighted strategies.

[11] The reported partial ability of the return on asset (ROA) and dividend yield (DY) to explain a large element of the ES-return relation could result from their ability to signal corresponding and future performance. To illustrate, extreme losses in the market are likely to be associated with poor accounting figures. Low ROA and DY are found to predict poor future returns (see, Hou et al., 2009; McMillan, 2014). Asem (2009) demonstrates that investor underreaction to dividend news may generate momentum in prices. Asem argues that investors underreact to dividend cuts by loser firms which lead to stronger momentum in prices. Dividends include information regarding future profitability (see, Ham, Kaplan, & Leary, 2020).

Table 5
Bivariate portfolios of the ES and the Control variables.

Control Variable		ES1	2	3	4	ES5	ES5-ES1
Past	Ret_{+1}	0.22[c]	0.04	−0.13	−0.34	−1.18[a]	−1.40[a]
t-stat	t-stat	1.71	0.29	−0.87	−1.54	−3.56	−3.47
MOM	Ret_{+1}	0.21[c]	0.06	0.06	−0.25	−1.45[a]	−1.66[a]
t-stat	t-stat	1.67	0.53	0.44	−1.41	−4.02	−3.89
LT	Ret_{+1}	0.42[a]	0.09	−0.15	−0.08	−1.38[a]	−1.80[a]
t-stat	t-stat	4.25	0.77	−0.84	−0.49	−4.90	−5.57
MV	Ret_{+1}	0.84[a]	0.62	0.23[b]	−0.39[b]	−1.03[a]	−1.87[a]
t-stat	t-stat	7.19	6.37	2.22	−2.40	−3.93	−5.76
Beta	Ret_{+1}	0.34[a]	0.12	−0.22	−0.43[b]	−1.13[a]	−1.47[a]
t-stat	t-stat	3.11	1.22	−1.30	−2.13	−3.82	−4.16
DBETA	Ret_{+1}	0.37[a]	0.27[a]	−0.14	−0.46[b]	−1.35[a]	−1.72[a]
t-stat	t-stat	3.30	2.69	−0.84	−2.15	−4.26	−4.57
COSK	Ret_{+1}	0.27[b]	0.08	0.01	−0.29	−1.55[a]	−1.82[a]
t-stat	t-stat	2.27	0.71	0.06	−1.22	−4.86	−4.60
IV	Ret_{+1}	0.21	−0.14	−0.36[b]	−0.75[a]	−0.79[a]	−1.00[a]
t-stat	t-stat	0.17	−1.02	−2.34	−3.95	−3.18	−3.79
MAX	Ret_{+1}	0.30[b]	−0.03	−0.46[a]	−0.52[b]	−1.05[a]	−1.36[a]
t-stat	t-stat	2.08	−0.22	−3.37	−2.61	−4.32	−4.52
ISK	Ret_{+2}	0.23[b]	0.14	0.25[c]	−0.35	−1.44[a]	−1.68[a]
t-stat	t-stat	2.18	1.06	1.75	−1.58	−4.30	−4.17
Amih	Ret_{+1}	0.79[a]	0.60[a]	0.26[b]	−0.47[b]	−1.42[a]	−2.21[a]
t-stat	t-stat	6.02	5.31	2.35	−2.56	−4.51	−5.88
BM	Ret_{+2}	0.20[b]	0.24[b]	0.04	−0.41[c]	−1.56[a]	−1.77[a]
t-stat	t-stat	2.04	2.05	0.25	−1.82	−5.17	−5.06
GA	Ret_{+1}	0.21[c]	0.09	0.00	−0.54[b]	−1.32[a]	−1.53[a]
t-stat	t-stat	1.87	0.76	−0.03	−2.01	−3.47	−3.37
ROA	Ret_{+1}	0.15	−0.03	−0.12	−0.69[b]	−1.11[a]	−1.00[a]
t-stat	t-stat	1.40	−0.23	−0.69	−2.50	−3.05	−2.97
DY	Ret_{+1}	0.32[a]	−0.01	−0.08	−0.43[c]	−0.61[b]	−0.92[a]
t-stat	t-stat	2.70	−0.08	−0.58	−1.81	−2.03	−2.66

This table reports the average monthly returns of the bivariate portfolios built based on the ES and one of the control variables. Firstly, the stocks are sorted into 5 quantiles based on one of the control variables, then, within each of these quantiles, the stocks resort to another 5 quantiles based on the ES. Ret_{+1} is the average of Carhart's alphas of the ES-based quantile over the control variable quantiles. Mom is the cumulative return over the past 12 months, Past is the previous month's return, LT is the return over the previous three years, MV is the market value, Beta is the market beta, Amih is the Amihud impact ratio, Dbeta is the down beta, IV is idiosyncratic volatility, Max is the average of the maximum 5 daily returns in the previous month, Coskew is the co-skewness, ROA is the return on assets, GA is the growth of the total assets, and DY is the dividend yield. ES1 (ES5) includes the stocks with the lowest (highest) ES, ES5-ES1 represents the spread between the highest and the lowest ES portfolios, and t-stat is the Newey-West t-statistic. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

The results in Table 5 suggest that left-tail momentum is a robust pricing anomaly and is not subsumed by other pricing anomalies reported in the literature.

4.5. Multivariate analysis: the Fama-MacBeth cross-sectional regressions

By its nature, bivariate portfolio analysis can only control for two variables at a time, while controlling for more variables becomes difficult. For instance, analysing three variables requires a large number of stocks, and four or more variables is practically inapplicable and hard to interpret. To control for the left-tail momentum in a multivariate setting, we perform the Fama-MacBeth two-step regression. The following regression model is estimated:

$$R_{i,t+1} = \alpha_{t+1} + \beta_{es}{}^{*}\left(ES_{i,t}\right) + \beta_{x}{}^{*}(Z) + \varepsilon_{i,t+1} \tag{5}$$

Where $R_{i,t+1}$ is the next month's return of stock *i*, $ES_{i,t}$ is the expected shortfall of the stock *i*, and *Z* is a set of control variables. α, β_{es}, and β_x are the estimated parameters. Table 6 reports the cross-sectional regressions analysis. The figures represent the average coefficients and their Newey-West t-statistics.

The empirical results confirm the left-tail momentum anomaly. The first column (C1) shows the results of the simple model regressing next month's return on ES. The average slope coefficient on ES is −0.46 (Newey-West t-statistic of −7.45). Accounting for other widely known pricing factors (Beta, LNMV, and BM) in column two (C2) has little effect on the ability of ES to predict future returns. The estimated average ES slope coefficient stays economically and statistically significant with a value of −0.45 (Newey-West t-statistic of −9.16). A further widely used pricing characteristic is momentum (MOM). The importance of this variable stems from its close resemblance with left-tail momentum.[12] In column three (C3), price momentum does have a more evident effect on ES predictive power, with the slope coefficient partially reduced to −0.36, although it remains statistically significant. Thus, left-tail momentum exhibits independent behaviour and is only partially subsumed by momentum.

The results continue to confirm the anomalous predictable ES-return pattern even when including other pricing factors, such as liquidity and accounting fundamentals. While the magnitude of the average slope coefficient may dimmish, it remains significant, both economically and statistically. One factor that it is of interest to note, is IVOL (included in C8 and C9), which explains a fair part of the left-tail momentum. Including IVOL in the model, reduces the average slope coefficients on ES to −0.118 (Newey-West t-statistic of −2.20). As noted, both ES and IVOL are two related features that describe the stock return distribution. An additional distributional feature that provides an ability to predict returns is the past maximum (MAX) daily returns (see, Bali et al., 2011). The inclusion further reduces the value of the left-tail momentum coefficient, which, nonetheless, remains statistically significant.

Both portfolio analysis and multivariate cross-section regressions reveal the ability of left-tail risk, proxied by expected shortfall, to inversely predict subsequent stock returns. This empirical evidence is consistent with pricing patterns detected in US stock by Atilgan, Bali, et al. (2020). Furthermore, by performing Fama and MacBeth regressions, we rule out the potential that previously identified factors explain the anomalous pricing of left-tail risk.

Notwithstanding, there is a notable partial ability for idiosyncratic volatility (IVOL) to explain left-tail performance. Previous studies demonstrate that the IVOL puzzle is a manifestation of noise trading and limits to arbitrage (see, for example, Pontiff, 2006; Stambaugh et al., 2015; Cao & Han, 2016). These results open the door for behavioural channels as a plausible source for left-tail momentum. Stocks with high left-tail risk (high ES) are likely to be mispriced by investors and combined with limits to arbitrage (due to arbitrage costs and information uncertainty) that are associated with such stocks.

4.6. Further analysis I: underreaction or overreaction

Here, we consider the role of underreaction or overreaction as the driving source behind the underperformance of stocks with high ES. The behavioural finance literature argues that pricing deficiencies can arise due to investor cognitive biases and the failure of arbitrageurs to exploit predictable profitable opportunities.

Two behavioural finance approaches suggested as plausible explanations for left-tail momentum are the underreaction (or limited attention) and the continuous overreaction biases. Underreaction-based behaviour is proxied by employing four different measures. The 52-week high ratio, abnormal trading volume, continuous information index, and

[12] A strand of previous work supports the underreaction origin of price momentum, for example, Hong et al. (2000), Hou et al. (2009), and Chen et al. (2023), among others. Byun et al. (2016) relate price momentum to the continuing overreaction bias. Accordingly, it could be that variations in left-tail momentum and the momentum factor are underlaid by the same behavioural biases. This makes them highly correlated and drives the ability of the momentum factor in pricing left-tail momentum. Even though they are distinguished anomalies, they exhibit close similarity.

Table 6
Firm-level Fama-MacBeth cross-sectional regressions.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
ES5	−0.46[a]	−0.45[a]	−0.36[a]	−0.345[a]	−0.36[a]	−0.354[a]	−0.29[a]	−0.118[b]	−0.105[b]
	(−7.45)	(−9.16)	(−7.6)	(−7.34)	(−7.42)	(−7.50)	(−6.51)	(−2.20)	(−1.99)
LNMV		−0.0003	0	0.0001	0.0002	0.0002	−0.0001	−0.0004	−0.0005
		(−0.78)	(0.09)	(0.23)	(0.411)	(0.53)	(−0.17)	(−0.90)	(−0.97)
Beta		0.0001	−0.0003	−0.0003	−0.0002	−0.0015	−0.0011	−0.0012	−0.001
		(0.09)	(−0.496)	(−0.42)	(−0.23)	(−1.28)	(−0.998)	(−1.03)	(−0.84)
BM		−0.0001	0.0034[b]	0.0032[b]	0.0031[b]	0.0031[b]	0.0025[b]	0.0025[b]	0.0025[b]
		(−0.082)	(2.18)	(2.32)	(2.24)	(2.23)	(2.25)	(2.27)	(2.31)
MOM12			0.0125[a]	0.014[a]	0.0144[a]	0.0144[a]	0.015[a]	0.017[a]	0.017[a]
			(5.43)	(6.291)	(6.59)	(6.61)	(6.82)	(7.93)	(7.97)
LT				−0.0008	−0.0009	−0.0009	−0.0008	−0.0011	−0.001
				(−0.791)	(−0.84)	(−0.87)	(−0.82)	(−1.15)	(−1.1)
Past				−0.013[b]	−0.015[a]	−0.017[a]	−0.019[a]	−0.018[a]	−0.015[a]
				(−2.44)	(−2.69)	(−3.06)	(−3.76)	(−3.61)	(−2.79)
Amih					−0.008	−0.0081	−0.0122	−0.0122	−0.012
					(−1.47)	(−1.36)	(−1.64)	(−1.64)	(−1.62)
Dbeta						0.0012	0.0011	0.001	0.001
						(1.55)	(1.39)	(1.30)	(1.28)
Coskew						−0.0045	−0.0043	−0.0043	−0.0041
						(−1.37)	(−1.23)	(−1.26)	(−1.21)
ROA							0.012[a]	0.010[a]	0.099[a]
							(4.11)	(3.58)	(3.4)
GA							−0.0046[a]	−0.0045[a]	−0.0045[a]
							(−3.51)	(−3.43)	(−3.52)
DY							0.056[c]	0.048	0.045
							(1.83)	(1.59)	(1.5)
IVOL								−0.365[a]	−0.254[b]
								(−3.50)	(−2.35)
MAX5									−0.091[a]
									(−2.89)
cons	0.022[a]	0.024[a]	0.0143[a]	0.0144[a]	0.014[a]	0.0137[a]	0.012[b]	0.0138[a]	0.0147[a]
	(7.69)	(5.68)	(3.32)	(3.31)	(2.96)	(2.87)	(2.42	(2.691)	(2.89)
Adj R^2	0.038	0.057	0.067	0.077	0.081	0.086	0.094	0.096	0.099

This table reports the results of the cross-sectional regressions of Fama and Macbeth (1973). ES is the expected shortfall, Mom is the cumulative return over the past 12 months, Past is the previous month's return, LT is the return over the previous three years, LNMV is the logarithm of the market value, BM is the book to market ratio, Beta is the market beta, Amih is the Amihud impact ratio, Dbeta is the down beta, IV is idiosyncratic volatility, Max is the average of the maximum 5 daily returns in the previous month, Coskew is the co-skewness, ROA is the return on assets, GA is the growth of the total assets, and DY is the dividend yield. In each column, the time-series averages of the cross-sectional regression slope coefficients and their associated Newey and West (1987) adjusted t-statistics (in brackets) are reported. Adj-R^2 is the adjusted coefficient of variation. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

the price delay measure. Each of these variables represents a specific aspect of investor underreaction behaviour that is observed in financial markets. To build an underreaction (attention) index we use principal components analysis (PCA) and extract the first common component of these variables. The alternative of continuous overreaction is measured by the index developed by Byun et al. (2016).[13]

Table 7 presents the analysis of the bivariate portfolios based on ES and the level of the attention (underreaction) index and continuous overreaction measure. Stocks are sorted into deciles based on the underreaction or continuous overreaction level and then, within each of these deciles, stocks are sorted into ES level deciles. The results reveal an evident trend in the magnitude of the left-tail momentum associated with the level of attention (underreaction). The profitability of the zero-cost ES strategy is most significant when attention is lowest (ATT1) where investors are likely to be inattentive. Here, the return to the ES strategy is −3.58% and statistically significant. However, for stocks within the highest decile of the underreaction index (ATT5), where investors are most attentive, this return spread is reduced in magnitude to an insignificant value of −0.16%.

Conditioning left-tail momentum on continuous overreaction (CO) produces some effect. Although the spread in returns between the highest and lowest ES deciles across different levels of CO remains statistically and economically significant, left-tail momentum is greater for stocks within the lowest CO-decile, with a return spread of −3.94%. When moving to the highest decile, the spread is reduced to −1.09%, but in all cases remains significant.[14] Thus, the bivariate portfolio analysis reveals that left-tail momentum is likely to be associated with, primarily, underreaction behaviour but also some effect from continuous overreaction.[15]

To further consider these results, we analyse both channels by using the Fama-MacBeth approach, with the results in Table 8. The analysis is similar to that in Table 6, but we add the underreaction and continuous overreaction behaviour both as individual regressors and also as interaction terms. This allows us to consider their effect on left-tail

[13] It should be noted that both behaviours can generate a continuous price trend. However, in the case of overreaction, the prices should experience a reversal in the future while the underreaction behaviour is more likely to generate a continuation of the prices, but, with no reversal. Therefore, according to the results reported before in this study, the underreaction more plausibly generates the left-tail momentum.

[14] Within the lowest level of CO, the investors are more likely to persistently overreact towards the bad performance (i.e., negative return) over the past midterm period.

[15] Both underreaction and overreaction are likely to lead to a similar short term price trend. Limited attention investors miss relevant information, which is then slowly reflected into prices, while the actions of attentive investors can lead to a price trend overreaction. However, over the longer term, while underreaction is marked by prices continuing to trend towards the new equilibrium, a reversal is observed when overreaction occurs. Looking at the empirical results revealed in Table 3 (and Table 11, below), they show that the anomalous trend in stock returns associated with left-tail risk is persistent and not reversed over the long run. Therefore, underreaction behaviour is a more plausible driver of this anomaly.

Table 7
Effect of the underreaction bias and the continuous overreaction bias on the left-tail momentum.

	Attention Effect (ATT)					Continuous Overreaction (CO)				
Level	ATT1	ATT2	ATT3	ATT4	ATT5	CO1	CO2	CO3	CO4	CO5
ES1	−0.73	0.18	0.48[c]	0.81[a]	0.64[a]	0.44	0.44[c]	0.58[a]	0.60[a]	0.68[a]
t-stat	−1.38	0.45	1.64	4.26	3.44	1.61	1.78	2.91	3.24	2.97
ES2	−1.54[c]	0.07	0.45	0.52[c]	0.71[a]	0.06	0.25	0.57[c]	0.52[b]	0.58[c]
t-stat	−1.86	0.16	1.48	1.88	2.89	0.14	0.64	1.84	2.00	1.96
ES3	−2.40[a]	−0.31	0.22	0.51	0.62[b]	−0.35	0.13	−0.37	0.50	0.29
t-stat	−2.92	−0.55	0.51	1.52	2.20	−0.62	0.28	−0.75	1.41	0.68
ES4	−1.78[c]	−0.94	−0.15	0.43	0.64[c]	−1.39[c]	−0.75	−0.04	0.38	0.74
t-stat	−1.89	−1.49	−0.33	1.07	1.82	−1.75	−1.05	−0.08	0.95	1.54
ES5	−4.30[a]	−2.05[a]	−0.72	−0.15	0.48	−3.49[a]	−1.56[b]	−1.65[b]	−0.99	−0.41
t-stat	−4.37	−2.81	−1.12	−0.29	0.91	−3.57	−2.01	−2.28	−1.41	−0.68
ES5-ES1	−3.58[a]	−2.23[a]	−1.20[b]	−0.96[c]	−0.16	−3.94[a]	−2.00[a]	−2.23[a]	−1.59[b]	−1.09[b]
t-stat	−4.55	−3.36	−2.12	−1.91	−0.35	−4.32	−3.14	−3.28	−2.50	−2.03
α_{FF}	−3.15[a]	−1.80[a]	−0.89[c]	−0.60	−0.19	−3.29[a]	−1.39[a]	−1.61[a]	−1.27[a]	−0.91[c]
t-stat	−4.65	−3.15	−1.82	−1.57	−0.49	−4.28	−2.69	−3.31	−2.88	−1.89
α_{4F}	−3.02[a]	−1.90[a]	−1.13[c]	−0.96[b]	−0.52	−2.75[a]	−0.99[c]	−1.35[b]	−1.42[a]	−1.23[b]
t-stat	−4.22	−2.86	−1.91	−2.43	−1.41	−3.38	−1.74	−2.54	−3.20	−2.36

This table reports the effect of the underreaction-based biases and the continuous overreaction bias on the left-tail momentum. Firstly, the stocks are sorted into 5 quantiles according to the ATT or CO, then, within each of these quantiles, the stocks are resorted into another 5 quantiles according to the ES level.ES is the expected shortfall. ES1 (ES5) includes the stocks with the lowest (highest) ES, and ES5-ES1 represents the spread between the returns of the highest and the lowest ES portfolios, α_{FF} is the Fama-French 3-factor alpha, and α_{4F} is the Carhart 4-factor alpha. t-stat is the Newey-West t-statistic. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

momentum:

$$R_{i,t+1} = \alpha_{t+1} + \beta_1{}^{*}ES_{i,t} + \beta_3{}^{*}ATT\ (\text{or CO}) + \beta_2{}^{*}ATT_{i,t}{}^{*}ES_{i,t}\ (\text{or } CO_{i,t}{}^{*}ES_{i,t}) + \beta_4{}^{*}Z + \varepsilon_{i,t+1} \tag{6}$$

The results in Table 8 reveal several key observations. Confirming the bivariate portfolio analysis, higher ES predicts lower return, over the subsequent month for stocks with a lower attention index. To consider from an alternative perspective, when investors are attentive to information, they require higher returns on stocks with higher left-tail risk (higher ES). Therefore, left-tail momentum is stronger for stocks where investors are more likely to miss information. This result is evident under column C2, where the average slope coefficient on the ATTxES is 0.377 (Newey-West t-statistic of 4.31). As before, when controlling for additional return predictors, the nature of the result does not change. For example, the average slope coefficient of the ATT and ES interaction term is 0.223 and remains significant (Newey-West t-statistic of 2.82) in column C4.

Table 8 also shows that continuous overreaction influences the predictive power of left-tail risk, although much weaker than for attention. Column C6 shows that the interaction effect (COxES) is significant with an average slope coefficient of 0.127. However, when combining both the underreaction and continuous overreaction indexes in the regression, underreaction subsumes the effect of overreaction on the ES-return predictive relation. For example, under column C8, the regression includes ATT, CO, and their interaction with ES; the coefficient on COxES is indistinguishable from zero while that for ATTxES remains significant and close to the values under columns C2 and C3.

Table 8 reveals that underreaction and limited investor attention is the more plausible driver of left-tail momentum. Underreaction-related behaviours include anchoring to a reference point and the failure to recognise and update to new events that could lead investors to miss bad news. Therefore, this news is not immediately reflected in stock prices and thereby a continuous downward pattern in the prices of such overvalued stocks occurs. While there is some initial evidence towards continuous overreaction, this appears to be subsumed by the underreaction behaviour and has less impact on the ES coefficient.

4.7. Further analysis II: limits to arbitrage

The rational paradigm of pricing in financial economics does not exclude potential mispricing effects but assumes the existence of sufficient arbitrageurs to eliminate any such mispricing. Mispricing-related patterns, however, are more likely to persist when they are accompanied by frictions (see, Shleifer & Vishny, 1997) that limit arbitrage. For instance, various pricing anomalies are magnified by different deterrents to arbitrage (see, Cao & Han, 2016; Doukas et al., 2010). Kumar (2009) demonstrates that investors are more likely to make pricing errors when they face an informationally vague environment. Therefore, in this section, we examine whether the limits to arbitrage contribute to the identified left-tail momentum.

Limits to arbitrage are unobservable and there is no consensus on the appropriate measure, with many possible proxies suggested. To this end, we apply the same method as above and employ five different proxies to generate an index based on the first component of a PCA analysis. A higher PCA level corresponds to more costly arbitrage and, therefore, greater difficulty for informed arbitrageurs to trade and eliminate any mispricing. Limits to arbitrage may indicate higher information uncertainty, and thus it would be more likely for investors to make cognitive errors (see, Zhang, 2006).

Table 9 reports the bivariate portfolio analysis. Based on the limits to arbitrage index (LA), stocks are sorted into quantiles and then, re-sorted into ES-based quantiles. The LA5 portfolio contains the stocks with the highest limits to arbitrage. The results reveal evidence in favour of arbitrage frictions in generating left-tail momentum. Consistent with the limits to arbitrage argument, the underperformance of the highest ES-quantile (ES5) monotonically moves with the level of the LA index. The stronger the limits of arbitrage, the weaker the performance of the ES5 portfolio in the next month. The return of the zero-cost portfolio strategy is insignificant when stocks fall in the lowest quantile of the LA index. The spread is insignificant with a value of −0.31% for LA1, while for LA5, where the costs of arbitrage are most prohibitive, the next month's return for the zero-cost ES-based strategy is highly significant at −4.68% (Newey-West t-statistic of −6.00). Adjusting this performance for the Cahart risk factors confirm the pattern that emerges with raw returns. The four-factor model alpha (α_{4F}) is −0.26% and insignificant when arbitrage difficulties are low and − 3.84% and significant when arbitrage is difficult.

These results suggest that limited arbitrage is likely to be a notable force behind the mispricing pattern associated with high left-tail risk. Table 10 shows the Fama-MacBeth cross-section regressions to further confirm this. The same set of control variables is used as in Table 8. Confirming the results of Table 9, the Fama-MacBeth regressions show

Table 8
Cross-sectional regressions of Fama and Macbeth (1973): effect of the underreaction bias and the continuous overreaction bias on the left-tail momentum.

	C1	C2	C3	C4	C5	C6	C7	C8	C9[18]	C10[18]
ES5	−0.35^{a}	−0.51^{a}	−0.193^{a}	−0.317^{a}	−0.438^{a}	−0.508^{a}	−0.427^{a}	−0.49^{a}	−0.366^{a}	−0.315^{a}
	(−6.3)	(−9.22)	(−3.09)	(−4.73)	(−7.37)	(−7.11)	(−6.5)	(−7.62)	(−4.31)	(−4.1)
ATT5	0.018^{a}	0.0022	0.0082	0.0115^{b}			0.0172^{a}	0.0013		0.0104^{c}
	(4.38)	(0.38)	(1.526)	(2.3)			(4.50)	(0.197)		(1.81)
ATT5xES		0.377^{a}	0.105	0.223^{a}				0.385^{a}		0.251^{a}
		(4.31)	(1.32)	(2.82)				(3.69)		(2.631)
CO5					0.0089^{a}	0.0027	−0.005	0.0031	0.0033	0.0013
					(3.034)	(0.86)	(−1.49)	(0.813)	(1.145)	(0.358)
CO5xES						0.127^{b}	0.143^{b}	−0.039	0.0748	−0.0255
						(2.05)	(2.37)	(−0.525)	(1.134)	(−0.322)
Beta			−0.001	−0.0009					−0.0008	−0.0009
			(−0.87)	(−0.796)					(−0.678)	(−0.824)
LNMV			−0.0009^{b}	−0.0009^{b}					−0.0002	−0.0009^{b}
			(−2.15)	(−2.08)					(−0.42)	(−2.07)
BM			0.0025^{c}	0.0022^{c}					0.002	0.0023^{c}
			(1.899)	(1.677)					(1.43)	(1.689)
MOM12			0.012^{a}	0.009^{a}					0.0041^{c}	0.0087^{a}
			(5.75)	(4.32)					(1.723)	(4.00)
Past			−0.018^{a}	−0.016^{a}					−0.007	−0.016^{a}
			(−3.36)	(−2.94)					(−1.34)	(−2.91)
LT			−0.0014	−0.0007					0.0015	−0.0008
			(−1.363)	(−0.720)					(1.42)	(−0.762)
Amih			−0.0133^{c}	−0.0125^{c}					−0.0111	−0.0127^{c}
			(−1.785)	(−1.694)					(−1.51)	(−1.72)
ROA			0.0102^{a}	0.0103^{a}					0.0102^{a}	0.0103^{a}
			(3.43)	(3.49)					(3.438)	(3.482)
GA			−0.0044^{a}	−0.0048^{a}					−0.0056^{a}	−0.0048^{a}
			(−3.53)	(−3.76)					(−4.36)	(−3.77)
Dbeta			0.001	0.001					0.0011	0.0011
			(1.374)	(1.35)					(1.41)	(1.49)
Coske			−0.0035	−0.0036					−0.0041	−0.0036
			(−1.18)	(−1.22)					(−1.21)	(−1.18)
IVOL12M			0.1232	0.0262					0.0787	−0.034
			(1.10)	(0.211)					(0.567)	(−0.276)
MAX5			−0.101^{a}	−0.107^{a}					−0.11^{a}	−0.109^{a}
			(−3.21)	(−3.37)					(−3.40)	(−3.40)
Cons	0.0058	0.014^{b}	0.013^{c}	0.0135^{b}	0.0158^{a}	0.0193^{a}	0.0091^{c}	0.0124^{b}	0.0172^{a}	0.0134^{b}
	(1.139)	(2.377)	(1.88)	(2.042)	(4.21)	(4.87)	(1.823)	(2.236)	(3.237)	(2.071)
Adj R^2	0.048	0.051	0.10	0.10	0.044	0.047	0.053	0.056	0.097	0.102

This table reports the effect of the underreaction bias on the pricing of the left-tail risk. The reported results represent coefficients of the cross-sectional regressions of Fama and Macbeth (1973) and the corresponding Newey-West t-statistics. ES is the expected shortfall, ATT represents the attention index, CO represents the continuous overreaction index developed by Byun et al. (2016), Mom is the cumulative return over the past 12 months, Past is the previous month return, LT is the return over the previous three years, LNMV is the logarithm of the market value, BM is the book to market ratio, Beta is the market beta, Amih is the Amihud impact ratio, Dbeta is the down beta, IV is idiosyncratic volatility, Max is the average of the maximum 5 daily returns in the previous month, Coskew is the co-skewness, ROA is the return on assets, GA is the growth of the total assets, and DY is the dividend yield. In each column, the time-series averages of the cross-sectional regression slope coefficients and their associated Newey and West (1987) adjusted t-statistics (in brackets) are reported. Adj-R^2 is the adjusted coefficient of variation. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

In this regression, to account for the potential overlapping between the price momentum and the underreaction effect, we orthogonalize the momentum anomaly to the attention index.

that the association between limits to arbitrage and left-tail risk is significant, with more prohibitive arbitrage limits leading to poorer performance of high left-tail risk stocks. The average slope coefficient on the interaction term LA5xES is −0.257 and significant. Controlling for the set of previous return predictors does lead to this coefficient becoming insignificant, however, it should be noted that some of these control variables are themselves related to arbitrage limits.

The inclusion of the underreaction-based behaviour in these regressions reveals further interesting results. The limits to arbitrage index has a significant effect on the predictive power of ES through the interaction with underreaction behaviour (ATT). For example, Column 7 (C7) shows that when the triple interaction between ATT, LA, and ES is introduced, the direct effect of LA on ES predictive power of the future return (LAxES) is positive (0.215) but statistically insignificant. While its effect on ES through ATT (ATTxLAxES) shows an expected negative slope of −0.591 (and significant). However, when we control for the association with costly arbitrage, the interaction effect of underreaction behaviour on left-tail momentum stays significant.

This result suggest that the underreaction-induced mispricing is a significant determinant of left-tail predictive power for subsequent returns (left-tail momentum). Moreover, costly arbitrage appears to magnify this biased pricing behaviour. Such results represent empirical evidence in favour of the behavioural explanation of underreaction behaviour and costly arbitrage that lie behind the persistent underperformance of stocks with high potential losses (high ES).

Interestingly, the revealed interaction between the underreaction index and limits to arbitrage mean that costly arbitrage is associated with poor performance for all levels of underreaction behaviour. Therefore, considering costly arbitrage, poor performance of high left-tail risk stocks is equally evident when investors are likely to

Table 9
The limited arbitrage and the left-tail momentum.

Limits-to-Arbitrage (LA)					
Level	LA1	LA2	LA3	LA4	LA5
ES1	0.46[b]	0.94[a]	0.79[b]	1.01[a]	0.42
t-stat	2.13	3.77	2.62	2.87	0.94
ES2	0.49[b]	0.64[b]	0.74[b]	0.86[b]	−1.07[c]
t-stat	2.23	2.11	2.28	1.98	−1.68
ES3	0.48[c]	0.44	0.51	−0.10	−2.03[a]
t-stat	1.78	1.39	1.46	−0.20	−2.93
ES4	0.32	0.54	−0.20	−1.00[c]	−2.89[a]
t-stat	0.90	1.45	−0.45	−1.78	−3.77
ES5	0.15	−0.27	−1.29[c]	−2.32[a]	−4.26[a]
t-stat	0.30	−0.48	−1.95	−2.90	−4.25
ES5-ES1	−0.31	−1.21[a]	−2.08[a]	−3.33[a]	−4.68[a]
t-stat	−0.74	−2.64	−4.25	−5.32	−6.00
α_{FF}	−0.35	−1.29[a]	−1.90[a]	−3.07[a]	−4.13a
t-stat	−0.96	−3.76	−5.13	−6.26	−6.18
α_{4F}	−0.26	−1.06[a]	−1.78[a]	−2.86[a]	−3.84[a]
t-stat	−0.71	−2.86	−4.49	−5.59	−5.21

This table reports the effect of the arbitrage difficulties on the left-tail momentum. Firstly, the stocks are sorted into 5 quantiles according to the LA, then, within each of these quantiles, the stocks are resorted into another 5 quantiles according to the ES level.ES is the expected shortfall. LA1 (LA5) includes the stocks with the lowest (highest) arbitrage difficulties. The ES1 (ES5) includes the stocks with the lowest (highest) ES, and ES5-ES1 represents the spread between the returns of the highest and the lowest ES portfolios, α_{FF} is the Fama-French 3-factor alpha, and α_{4F} is the Carhart 4-factor alpha. t-stat is the Newey-West t-statistic. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

underreact to information as well as when they overreact.[16] That is, the negative slope coefficient of the interaction term ATTxLAxES may be a result of overreaction rather than underreaction. Thus, part of the left-tail momentum could be generated by investor overreaction. To distinguish between the two cases, we perform a triple-sort analysis, first on the underreaction index, second, on the limits to arbitrage index, and third, on expected shortfall. To ensure portfolios (as there will be 27) have sufficient stocks, at each level of this sorting, we group stocks into three levels with equal members. The performance of these portfolios and left-tail momentum are analysed over the past 12-months, past month, next month, and next 12-months.

Table 11 displays the results of the trivariate portfolio analysis. The revealed pattern confirms our argument. Conditioning on the likelihood of underreaction behaviour, interacting with the limits to arbitrage effect, uncovers different patterns in returns of stocks with high left-tail risk. When investors are more likely to underreact (i.e., L-ATT), regardless of the limits to arbitrage level, the spread between the high-ES and low-ES group of stocks (i.e., ES3-ES1) is negative over the past period (T-12) and this poor performance continues over the next year, although with declining magnitude. For example, with low limits to arbitrage, this spread, on average, is −1.68% over the past 12 months and − 0.81% over the next 12 months, both statistically significant. Interacting costly arbitrage with underreaction presents the same persistent underperformance. Therefore, underreacting-based pricing biases are associated with persistent poor performance regardless of the limits to arbitrage level. This suggests, for example, that low trading volume may indicate inattentive investors and thereby limited arbitraging capital. Abnormal low trading volume may indicate problems regarding information availability and difficulties in price discovery.

As shown in Table 7, the poor performance of stocks with high left-tail risk disappears when investors are attentive. The results in Table 11 reveal that this is only the case when limits to arbitrage are low. For high attention and low limits to arbitrage, the return of the ES-based zero-cost strategy is positive of 0.42% (Newey-West t-statistic of 3.43) over the past 12 months, while the return is insignificant at −0.11% (Newey-West t-statistic of −0.59) over the next 12 months. However, when combining costly arbitrage with high attention, the poor performance of high left-tail stocks returns. This poor performance is now a manifestation of price reversal rather than price momentum. To illustrate, over the past 12-month period, stocks in the high limits to arbitrage and high attention index are winners that generate, on average, a significant positive return of 2.12% but over the next 12 months these stocks underperform, with average monthly losses of −1.49%. This spread between the lowest and the highest tercile of ES is comparable to that for the stocks in the low attention group (i.e., underreaction-driven mispricing). Thus, limits to arbitrage reinforce and add information to the limited attention biases to explain left-tail momentum and show that left-tail momentum is significant even in the absence of underreaction-based mispricing.

In opposite to price momentum created by underreaction-based biases, the observed reversal in the past gains of stocks with high left-tail risk can be explained by investor overreaction to attention-grabbing stocks.[17] A high underreaction index signifies stocks that investors are attentive to, such as stocks with high trading volume and prices close to the 52-week high. Moreover, extreme left-tail risk is associated with many attention-grabbing features. For instance, Tables 1 and 2 demonstrate that high-ES stocks are lottery-like (i.e., experience extreme daily returns). This suggests that these stocks represent a potential target for individual investors who seek high fluctuations and gambling in the pursuit of rare but huge gains. Excessive trading is described as a symptom of overconfidence that would lead to poor performance (see, for example, Barber & Odean, 2000; Statman, Thorley, & Vorkink, 2006; Daniel & Hirshleifer, 2015; Liu, Peng, Xiong, & Xiong, 2022). Barber and Odean (2008) demonstrate that owing to limited cognitive ability, investors, especially individuals, tend to select their investments by relying on attention-grabbing features such as recent extreme returns and volume. Overweighting the importance of such a feature would lead to overvaluation and thereby to poor future performance. Combining these, with limits to arbitrage, causes this potential mispricing to persist creating a reversal in the returns of these attention-grabbing stocks.

Therefore, we can extend our conclusion and suggest that the predictable poor performance of stocks with higher left-tail risk is largely created by the underreaction to news but can also be attributed to investor overreaction to ‘glittering’ stocks. Such results are consistent with those in Chan (2003) who report drift after bad news and reversal following extreme price shocks.

5. Robustness

The above results report a significant left-tail momentum anomaly in UK stocks. In this section, we provide a battery of robustness tests to examine whether these results are confirmed or whether they are sensitive to the changes in sample and methodology. We consider alternative left-tail risk measures, alternative pricing models and different market states.

[16] By construction, the highest level of underreaction index (e.g., ATT5), used above in this study, could indicate an absence of the underreaction behaviour but more likely the overreaction behaviour. To clarify, stocks with a high underreaction index are likely to experience abnormal trading volume, an upward trend in the prices, and successive positive daily returns over the past year. Association of these features with a prohibitive costly arbitrage would likely lead to overreaction.

[17] Note that we must distinguish between the continuous overreaction, tested above in this work and the overreaction behaviour suggested here, the first one is tested to explain the subsequent drift in the return, following the bad news, while the latter is accused to generate a reversal in the future return after grabbing attention events.

Table 10
Interaction of the limits-to-arbitrage and the underreaction bias with the left-tail momentum.

	C1	C2	C3	C4	C5	C6	C7	C9
ES5	−0.46^{a}	−0.245^{b}	−0.287^{a}	−0.014	−0.269^{b}	−0.175	−0.507b	−0.443^{b}
t-stat	(−9.73)	(−2.11)	(−2.68)	(−0.13)	(−2.05)	(−1.44)	(−2.55)	(−2.41)
LA5	0	0.012^{c}	0.005	0.022^{a}	0.015^{b}	0.008	−0.04^{a}	−0.043^{a}
t-stat	(−0.103)	(1.90)	(0.866)	4.173	2.527	−1.457	(−3.55)	(−4)
LA5xES		−0.257^{b}	−0.122	−0.413^{a}	−0.248^{b}	−0.159	0.215	0.335^{c}
t-stat		(−2.38)	(−1.13)	(−4.14)	(−2.16)	(−1.38)	(1.07)	(1.66)
ATT5				0.021^{a}	0.008^{c}	0.016^{a}	−0.032^{a}	−0.022^{b}
t-stat				(5.51)	(1.81)	(3.5)	(−3.40)	(−2.42)
ATT5xES					0.275^{a}	0.154^{c}	0.471^{c}	0.381^{c}
t-stat					(3.14)	(1.89)	(1.883)	(1.67)
ATT5xLA5							0.08^{a}	0.076^{a}
t-stat							(6.012)	(5.92)
ATT5xLA5xES							−0.591^{b}	−0.592^{b}
t-stat							(−2.13)	(−2.14)
Betaw			−0.001			−0.001		−0.001
t-stat			(−1.20)			(−1.11)		(−1.061)
LNMV			0			−0.001^{c}		−0.001
t-stat			(−0.53)			(−1.76)		(−1.14)
BM			0.002^{c}			0.002^{b}		0.002^{b}
t-stat			(1.78)			(2.01)		(2.042)
MOM			0.005^{a}			0.009^{a}		0.009^{a}
t-stat			(2.68)			(4.35)		(4.32)
Past			−0.004			−0.017^{a}		−0.018^{a}
t-stat			(−0.75)			(−2.95)		(−3.03)
LT			0.002^{b}			−0.001		−0.001
t-stat			(1.98)			(−0.65)		(−0.67)
Amih			−0.009			−0.012^{c}		−0.012^{b}
t-stat			(−1.45)			(−1.892)		(−2.27)
ROA			0.01^{a}			0.009^{a}		0.009^{a}
t-stat			(3.54)			(3.46)		(3.53)
GA			−0.005^{a}			−0.004^{a}		−0.004^{a}
t-stat			(−4.66)			(−3.78)		(−3.87)
DY			0.034			0.049^{b}		0.04^{c}
t-stat			(1.39)			(2.09)		(1.89)
Dbeta			0.001^{c}			0.001		0.001^{c}
t-stat			(1.79)			(1.59)		(1.68)
Cosk			−0.004			−0.003		−0.003
t-stat			(−1.06)			(−0.85)		(−1.05)
IVOL			0.179			0.019		0.004
t-stat			(1.48)			(0.17)		(0.034)
MAX5			−0.092^{a}			−0.096^{a}		−0.107^{a}
t-stat			(−3.12)			(−3.32)		(−3.75)
Cons	0.022^{a}	0.013^{a}	0.015^{a}	−0.012^{b}	0	0.003	0.031^{a}	0.03^{a}
t-stat	(7.41)	(3.21)	(2.84)	(−2.536)	(0.018)	(0.399)	(3.82)	(3.63)
Obs	194,775	194,775	185,386	194,775	194,775	185,386	194,775	185,386
Adj R^2	0.046	0.05	0.1	0.06	0.062	0.106	0.065	0.11

This table reports the effect of the interaction between the limits-to-arbitrage and the underreaction bias on the pricing of the left-tail risk. The reported results represent coefficients of the cross-sectional regressions of Fama and Macbeth (1973) and the corresponding newey-west t-statistics. ES is the expected shortfall, ATT represents the attention index, LA represents the limits-to-arbitrage index, Mom is the cumulative return over the past 12 months, Past is the previous month's return, LT is the return over the previous three years, LNMV is the logarithm of the market value, BM is the book to market ratio, Beta is the market beta, Amih is the Amihud impact ratio, Dbeta is the down beta, IV is idiosyncratic volatility, Max is the average of the maximum 5 daily returns in the previous month, Coskew is the co-skewness, ROA is the return on assets, GA is the growth of the total assets, and DY is the dividend yield. In each column, the time-series averages of the cross-sectional regression slope coefficients and their associated Newey and West (1987) adjusted t-statistics (in brackets) are reported. Adj-R^2 is the adjusted coefficient of variation. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

5.1. An alternative measure of left-tail risk

We reproduce the main analysis for left-tail momentum but alter the measure of left-tail risk. We calculate expected shortfall at the 1-percentile of the distribution rather than the 5-percentile (ES1%). In addition, we also consider a further measure of left-tail risk by using value-at-risk (VaR). The VaR measure, like expected shortfall, also utilizes the left tail of the return distribution. However, VaR represents left-tail risk by the 5-percentile return (VAR5%) rather than the average of the returns lower than the 5-percentile.

Table 12 presents the results of the portfolio analysis where stocks are sorted into quantiles based on ES1% or VAR5%, with value-weighted returns of these portfolios measured over the next month. These results confirm those reported for ES5%. Sorting on either ES1% or VAR5% reveals that stocks with high left-tail risk underperform stocks with lower left-tail risk. Considering VAR5%, stocks with the highest left-tail risk (P5) underperform stocks with the lowest left-tail risk (P1) by −2.61% (Newey-West t-statistic of −3.73). This spread is −1.89% for the ES1%-based portfolios (Newey-West t-statistic of −3.51). Adjusting these differential returns for the four-factor model of Carhart (1997) reduces their magnitude but does not alter the inference. Therefore, changing the measures of the left-tail risk does not affect the results regarding left-tail risk.

Table 13 reports the Fama-MacBeth regressions to analyse the predictive power of the VAR5% for the next month's returns. The analysis is comparable to that in Tables 8 and 10. These results confirm the main findings. Primarily, next month's return is inversely predicted by VAR5%, i.e., the higher the value-at-risk, the lower next month's return. Moreover, the inverse VAR5%-return predictive relation is stronger for stocks with a low degree of investor attention (i.e., low ATT5). In

Table 11
Interaction of the limted attention with the limited arbitrage on the prcing of the left-tail risk.

		Underreaction							
LA		L-ATT				H-ATT			
		T − 12	T-1	T + 1	T + 12	T-12	T-1	T + 1	T + 12
L-LIMIT	ES1	-1.79[a]	−2.98[a]	−0.34	0.16	1.74[b]	2.09[a]	0.6[a]	0.62[a]
	t-stat	−7.37	−7.38	−0.74	0.66	14.68	9.61	3.45	4.69
	ES3	−3.47[a]	−3.9[a]	−1.1	−0.81	2.16[a]	2.91[a]	0.42	0.51[b]
	t-stat	−7.75	−4.57	−1.26	−1.42	11	8.19	1.12	1.99
	ES3-ES1	−1.68[a]	−0.92	−0.76	−0.81[b]	0.42[a]	0.83[a]	−0.18	−0.11
	t-stat	−6.88	−1.49	−1.29	−2.34	3.43	3.2	−0.58	−0.59
H-LIMIT	ES1	−1.12[a]	−3.00[a]	−1.13[b]	−0.95[b]	4.46[a]	7.4[a]	1.85[a]	0.92[a]
	t-stat	−3.1	−6.47	−2.12	−2.51	19.23	15.58	4.72	3.19
	ES3	−4.99[a]	−4.77[a]	−3.19[a]	−2.45[a]	6.58	12.74[a]	0.27	−0.57
	t-stat	−7	−5.01	−3.58	−4.62	0.04	15.72	0.46	−1.15
	ES3-ES1	−3.87[a]	−1.78[a]	−2.06[a]	−1.5[a]	2.12[a]	5.33[a]	−1.58[a]	−1.49[a]
	t-stat	−9.09	−2.74	−3.63	−5.22	6.32	8.61	−3.33	−4.02

This table reports the effect of the arbitrage difficulties on the left-tail momentum. Firstly, the stocks are sorted into 3 groups according to the LA and ATT, independointly, then, dependently, within each of these group, the stocks are resorted into another 3 groups according to the ES level.ES is the expected shortfall. L-LIMIT (H-LIMIT) includes the stocks with the lowest (highest) arbitrage difficulties. L-ATT (H-ATT) includes the stocks with the lowest (highest) attention level. The ES1 (ES3) includes the stocks with the lowest (highest) ES, and ES5-ES1 represents the spread between the returns of the highest and the lowest ES portfolios, α_{FF} is the Fama-French 3-factor alpha, and α_{4F} is the Carhart 4-factor alpha. t-stat is the Newey-West t-statistic. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

Table 12
Alternative measures of the left-tail risk: Univariate portfolio analysis.

Alternative Left-tail measure						
ES1%						
	P1	P2	P3	P4	P5	P5-P1
Ret_{+1}	0.53[a]	0.25	0.34	−0.37	−1.36[b]	−1.89[a]
t-stat	2.71	0.91	0.90	−0.67	−2.18	−3.51
Alpha	0.15	−0.10	0.02	−0.53[c]	−1.01[a]	−1.16[a]
t-stat	1.37	−0.71	0.12	−1.93	−3.13	−2.92
Var5%						
	P1	P2	P3	P4	P5	P5-P1
Ret_{+1}	0.59[a]	0.48[c]	0.29	−0.14	−2.02[b]	−2.61[a]
t-stat	3.05	1.87	0.84	−0.29	−2.56	−3.73
Alpha	0.26[b]	0.16	0.07	−0.43[c]	−1.81[a]	−2.07[a]
t-stat	1.98	1.25	0.42	−1.81	−4.04	−3.94

This table reports the value-weighted monthly return of the quantile portfolios built based on the alternative left-tail risk measures. ES1% is the expected shortfall measured at 1% percentile, Var5% is the value at risk measured at 5% percentile, Ret_{+1} is one month-ahead-formation return, Alpha is the Carhart 4-factor alpha, and t-stat is the Newey-West t-statistic. P1 (P5) includes the stocks with the lowest (highest) left-tail risk, and P5-P1 represents the spread between the returns of the highest and the lowest ES portfolios. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

addition, the interaction of VAR5% with ATT5 is always positive and significant. This indicates that the more attentive the investor, the less likely they are to misprice the stocks with higher left-tail risk. The effect of costly arbitrage on this potential mispricing behaviour is also confirmed with VAR5%.

5.2. Alternative risk factors

Recent asset pricing literature proposes additional factors that, empirically, reveal an ability to absorb many pricing anomalies. The q-theory of investment of Hou et al. (2015) and Fama and French (2015) propose two additional factors, the investment factor and the profitability factor, although the two papers suggest different methods to mimic these factors. Therefore, we test whether these factors could contribute to explaining left-tail momentum. Despite their empirical success in explaining return predictability, there is no consensus on the reason. Notably, there exists a debate between a rational risk and a behavioural explanation. The former includes Hou et al. (2015) and Cooper and Maio (2019), while the latter includes Li and Sullivan (2011) and DeLisle et al. (2021). Furthermore, some (e.g., Lin, 2021) argue that the suggested factors, especially profitability, are not consistent with covariance risk. Hou et al. (2009) attribute the positive profitability-return association to investor underreaction.

Table 14 presents the results of considering alternative risk models, i.e., whether the additional factors from the models of Hou et al. (2015) and Fama and French (2015) can better explain predictable returns associated with left-tail risk. To ease comparison, both raw returns and alpha of the Carhart (1997) model are again reported. Under the equal weighting option, adding the profitability and investment factors fails to explain the predictable underperformance of stocks with high left-tail risk. Regardless of the pricing model, left-tail momentum is significant. The adjusted alpha of the hedged portfolio is −1.64% for the q-factor model of Hou et al. (2015) and − 1.83% for the 5-factor model of Fama-French, with both statistically significant. Weighting returns by market value, the alternative pricing models do outperform the pricing model of Carhart (1997) and explain a larger portion of the left-tail momentum. However, the underperformance of stocks with high left-tail risk is still significant. The pricing models of Hou et al. (2015) and Fama and French (2015) produce an adjusted alpha of −0.92 and − 1.08 respectively.

5.3. Filtering out micro and illiquid stocks

Empirical asset pricing studies have shown that many pricing anomalies appear only in small and illiquid stocks (see, for example, Bali and Cakici, 2008). Drawing on this, we examine the robustness of our results to the exclusion of such stocks.

Table 15 presents the univariate portfolio analysis of left tail risk but after screening out stocks in the lowest market-value quantile (i.e., small stocks) and the highest Amihud ratio quantile (i.e., illiquidity stocks). The results from Table 15 show that small capitalisation and illiquid stocks are not the basis for the left-tail momentum anomaly. After omitting these stocks, the return differential, in raw and risk-adjusted terms, between the low-ES and high-ES stocks remains significant.

5.4. Sub-period analysis

We consider left-tail momentum over two sub-periods to ensure that it is not a temporary phenomenon, appearing only within a specific period. For this purpose, the full sample period (1996–2021) is divided into equal two sub-periods 1996–2007 and 2008–2021.

Table 13
Alternative measure of the left-tail risk: Fama and MacBeth regressions.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
VAR	−0.74^{a}	−0.39^{a}	−0.85^{a}	−0.39^{a}	−0.86^{a}	−0.83^{a}	−0.38^{a}	−0.27	−0.44^{b}	−0.77^{b}	−0.48^{c}	−0.56^{c}
	(−7.16)	(−3.8)	(−8.28)	(−3.15)	(−6.75)	(−6.96)	(−2.66)	(−1.4)	(−2.12)	(−2.08)	(−1.64)	(−1.86)
ATT			0.0002	0.0126^{b}		−0.001	0.0104^{c}		0.0061	−0.035^{a}	−0.022^{b}	−0.023^{b}
			(0.033)	(2.55)		(−0.166)	(1.94)		(1.1)	(−2.86)	(−2.26)	(−2.21)
ATTxVAR			0.672^{a}	0.335^{b}		0.69^{a}	0.431^{a}		0.527^{a}	0.79^{c}	0.587^{c}	0.64^{c}
			(5.2)	(2.5)		(4.81)	(3.02)		(3.65)	(1.78)	(1.7)	(1.76)
CO					0.0008	0.003	0.0025					
					(0.214)	(0.76)	(0.67)					
COxVAR					0.267b	−0.0544	−0.1134					
					(2.27)	(−0.429)	(−0.871)					
LA								0.014^{b}	0.016^{a}	−0.039^{a}	−0.042^{a}	−0.042^{a}
								(2.13)	(2.61)	(−2.77)	(−3.51)	(−3.01)
LAxVAR								−0.53^{a}	−0.42^{b}	0.222	0.358	0.343
								(−3.17)	(−2.42)	(0.597)	(1.14)	(1.013)
ATTxLA										0.0784^{a}	0.0751^{a}	0.0753^{a}
										(4.71)	(5.17)	(4.69)
LAxATTxVAR										−0.77^{c}	−0.87^{c}	−0.88^{b}
										(−1.75)	(−1.93)	(−2.027)
Beta		−0.001		−0.0011			−0.0011				−0.0014	−0.0013
		(−0.84)		(−0.91)			(−0.931)				(−1.39)	(−1.07)
LNMV		0.0002		−0.0008^{c}			−0.0008^{c}				−0.0007	−0.0007
		−0.47		(−1.77)			(−1.77)				(−1.25)	(−1.22)
BM		0.0018		0.002^{c}			0.0019^{c}				0.0018^{c}	0.0019
		(1.47)		(1.71)			(1.67)				(1.651)	(1.61)
MOM		0.009^{a}		0.0117^{a}			0.0118^{a}				0.0113^{a}	0.0103^{a}
		−4.54		−6.13			−5.87				−5.68	−5.47
Past		0.0014		−0.013b			−0.013b				−0.015^{b}	−0.0154^{a}
		(0.26)		(−2.32)			(−2.22)				(−2.45)	(−2.73)
LT		0.0024^{b}		−0.0005			−0.0006				−0.0007	−0.0006
		(2.27)		(−0.59)			(−0.659)				(−0.693)	(−0.647)
Amih		−0.0099		−0.0139			−0.0136				−0.014^{c}	−0.0126
		(−1.16)		(−1.6)			(−1.62)				(−1.9)	(−1.36)
ROA		0.008^{a}		0.0085^{a}			0.0084^{a}				0.0084^{a}	0.0086^{a}
		(2.69)		(2.77)			(2.71)				(2.84)	(2.83)
GA		−0.007^{a}		−0.0054^{a}			−0.0054^{a}				−0.0052^{a}	−0.0052^{a}
		(−4.9)		(−3.99)			(−4.05)				(−4.17)	(−3.92)
DY		0.0174		0.035			0.036				0.0253	0.027
		(0.54)		(1.15)			(1.19)				(1.04)	(0.88)
Dbeta		0.0011		0.0011			0.0011				0.0013^{c}	0.0012
		(1.28)		(1.33)			(1.43)				(1.71)	(1.5)
Coskew		−0.004		−0.0031			−0.003				−0.0029	−0.0031
		(−1.1)		(−1.01)			(−0.96)				(−0.776)	(−0.99)
IVOL		−0.19		−0.22^{c}			−0.23^{b}				−0.151	
		(−1.6)		(−1.9)			(−1.99)				(−1.31)	
MAX5		−0.09^{a}		−0.095^{a}			−0.097^{a}				−0.101^{a}	−0.11^{a}
		(−2.9)		(−3.18)			(−3.19)				(−3.462)	(−4.22)
Cons	0.024^{a}	0.0175^{a}	0.016^{a}	0.0117^{c}	0.022^{a}	0.0145^{a}	0.0114	0.0116^{b}	0.001	0.0326^{a}	0.0306^{a}	0.0311^{a}
	(7.98)	(3.43)	(2.8)	(1.67)	(5.20)	(2.61)	(1.63)	(2.44)	(0.126)	(2.97)	(3.28)	(2.83)
Adj R^2	0.037	0.095	0.05	0.1	0.046	0.055	0.104	0.05	0.061	0.065	0.11	0.11

This table reports the effect of the interaction between the limits-to-arbitrage and the underreaction bias on the pricing of the left-tail risk. The reported results represent coefficients of the cross-sectional regressions of Fama and Macbeth (1973) and the corresponding Newey-West t-statistics. Var5% is the value at risk measured at a cut-off point of 5%, ATT represents the attention index, LA represents the limits to arbitrage index, Mom is the cumulative return over the past 12 months, Past is the previous month's return, LT is the return over the previous three years, LNMV is the logarithm of the market value, BM is the book to market ratio, Beta is the market beta, Amih is the Amihud impact ratio, Dbeta is the down beta, IV is idiosyncratic volatility, Max is the average of the maximum 5 daily returns in the previous month, Coskew is the co-skewness, ROA is the return on assets, GA is the growth of the total assets, and DY is the dividend yield. In each column, the time-series averages of the cross-sectional regression slope coefficients and their associated Newey and West (1987) adjusted t-statistics (in brackets) are reported. Adj-R^2 is the adjusted coefficient of variation. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

The results in Table 16 show consistency of the left-tail anomaly over time. Regardless of the weighting schemes, the spread in raw returns between the extreme ES quantiles is significant for both sub-periods, ranging from −2.12% to −3.45% and all statistically significant. For the risk-adjusted returns, again, the results hold in each case, with the Cahart alpha economically and statistically significant.

5.5. Sentiment effect

Sentiment plays an important role in shaping the pricing behaviour of financial securities. A growing set of empirical evidence notes that investors are more prone to committing a pricing error when they exhibit excessive positive sentiments (euphoric).

Emotional-driven pricing has long been recognised by researchers, theoretically and empirically. De Long, Shleifer, Summers, and Waldmann (1990) demonstrate that uninformed noise trading may put higher limits on arbitraging activities. This results in stock prices persistently deviating from their fundamental values, and such that noise-trading can explain several financial anomalies. Barberis et al. (1998) and Daniel et al. (1998) develop behavioural pricing models that motivate investor sentiment as being behind observed stock return predictability and rejection of the efficient market hypothesis. These models represent investor sentiment by a small number of cognitive biases, especially conservatism, representativeness, overconfidence, and self-attribution. Rather than updating their beliefs according to a rational Bayesian approach, sentiment-driven investors develop a distorted belief

Table 14
Alternative pricing models.

	Equally weighted					
Portfolio	ES1	ES2	ES3	ES4	ES5	ES5-ES1
Ret$_{t+1}$	0.79^{a}	0.68^{b}	0.43	−0.50	−2.42^{a}	−3.21^{a}
t-stat	3.23	2.14	1.14	−1.05	−3.62	−5.88
Car4F	0.75^{a}	0.78^{a}	0.64^{a}	−0.13	−1.67^{a}	−2.42^{a}
t-stat	5.89	6.75	6.12	−0.77	−5.44	−6.27
Q4F	0.74^{a}	0.73^{a}	0.67^{a}	0.24^{c}	−0.90^{a}	−1.64^{a}
t-stat	6.21	6.31	5.45	1.85	−3.83	−5.34
FF5	0.73^{a}	0.76^{a}	0.68^{a}	0.15	−1.10^{a}	−1.83^{a}
t-stat	6.16	7.25	6.53	1.26	−4.78	−6.15
	Value weighted					
Portfolio	ES1	ES2	ES3	ES4	ES5	ES5-ES1
Ret$_{t+1}$	0.54^{a}	0.34	0.39	−0.26	−1.97^{b}	−2.51^{a}
t-stat	2.75	1.23	0.70	−0.50	−2.64	−3.78
Car4F	0.20^{c}	0.03	−0.01	−0.50^{c}	−1.61^{a}	−1.81^{a}
t-stat	1.69	0.23	−0.03	−1.69	−4.33	−4.02
Q4F	0.14	−0.15	0.06	−0.07	−0.78^{b}	−0.92^{b}
t-stat	1.24	−1.07	0.33	−0.29	−2.20	−2.26
FF5	0.12	−0.03	0.10	−0.16	−0.96^{b}	−1.08^{b}
t-stat	1.10	−0.25	0.63	−0.67	−2.65	−2.57

This table reports the average monthly return of the ES-based portfolios priced by the alternative pricing models. Panel A reports the equally weighted return while in Panel B the return is weighted by the market value. Ret$_{+1}$ is one month-ahead-formation return, Car4F is the Carhart 4-factor alpha, Q4F is the Hou et al. (2015) f-factor model alpha, and FF5 is the Fama and French (2015) 5-factor model alpha. t-stat is the Newey-West t-statistic. ES1 (ES) includes the stocks with the lowest (highest) ES, and ES5-ES1 represents the spread between the returns of the highest and the lowest ES portfolios. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

Table 15
Filtering out the Micro stocks and the illiquid stocks.

Micro stocks omitted						
Portfolio	ES1	ES2	ES3	ES4	ES5	ES5-ES1
Ret$_{t+1}$	0.54^{a}	0.34	0.277	−0.257	−1.921^{b}	−2.46^{a}
t-stat	2.74	1.23	0.7	−0.49	−2.54	−3.66
Alpha	0.196^{c}	0.028	−0.008	−0.50^{c}	−1.59^{a}	−1.79^{a}
t-stat	1.68	0.21	−0.05	−1.68	−4.11	−3.86
Illiquid stocks omitted						
Portfolio	ES1	ES2	ES3	ES4	ES5	ES5-ES1
Ret$_{+1}$	0.537^{a}	0.333	0.274	−0.265	−1.76^{b}	−2.30^{a}
t-stat	2.73	1.21	0.69	−0.5	−2.3	−3.38
Alpha	0.193^{c}	0.020	−0.015	−0.51^{c}	−1.43^{a}	−1.62^{a}
t-stat	1.66	0.15	−0.08	−1.7	−3.68	−3.49

This table reports the average monthly return of the ES-based portfolios after filtering out the micro stocks and the illiquid stocks. Ret$_{+1}$ is one month-ahead-formation return, Alpha is the Carhart 4-factor alpha, and t-stat is the Newey-West t-statistic. Micro stocks are the stocks that make up the bottom quantile of the cross-sectional distribution of the market value, and Illiquid stocks are the stocks that make up the upper quantile of the cross-sectional distribution of the Amihud impact ratio. ES1 (ES) includes the stocks with the lowest (highest) ES, and ES5-ES1 represents the spread between the returns of the highest and the lowest ES portfolios. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

regarding the distribution of expected cash flows (e.g., the probability of an event). Rational investors exist but face difficulties and deterrent costs. Therefore, stock prices may fluctuate due to the common movement of such sentiment-driven investors who can either over- or under-react to value-relevant information, i.e., sentiment can lead investors to over- or under-estimate the probability of an event, resulting in mis-valuation of a financial asset.

Empirically, Baker and Wurgler (2006, 2007) build an aggregate measure of investor sentiment and document inverse predictability with

Table 16
Sub-period analysis.

Panel A	1996–2007					
	EW			VW		
Portfolio	ES1	ES5	ES5-ES1	ES1	ES5	ES5-ES1
Ret$_{+1}$	0.96^{a}	−2.5^{b}	−3.45^{a}	0.60^{b}	−2.36^{b}	−2.95^{a}
t-stat	2.95	−2.4	−3.82	2.00	−2.15	−3.00
Alpha	1.07^{a}	−0.91^{b}	−1.98^{a}	0.26	−1.32^{a}	−1.57^{b}
t-stat	6.05	−2.3	−3.78	1.38	−2.74	−2.62
Panel B	**2008–2021**					
	EW			VW		
Portfolio	ES1	ES5	ES5-ES1	ES1	ES5	ES5-ES1
Ret$_{+1}$	0.65^{c}	−2.35^{a}	−3.00^{a}	0.48^{c}	−1.64	−2.12^{b}
t-stat	1.83	−2.75	−4.64	1.89	−1.61	−2.42
Alpha	0.49^{a}	−2.30^{a}	−2.79^{a}	0.15	−1.86^{a}	−2.01^{a}
t-stat	3.82	−5.84	−5.62	1.14	−3.06	−2.93

This table reports the average monthly return of the ES-based quantile portfolios over the different periods. Panel A reports the results of the 1996–2006 period while in Panel B the results for the 2007–2017 period are reported. Ret$_{+1}$ is one month-ahead-formation return, Alpha is the Carhart 4-factor alpha, and t-stat is the Newey-West t-statistic. ES1 (ES) includes the stocks with the lowest (highest) ES, and ES5-ES1 represents the spread between the returns of the highest and the lowest ES portfolios. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

future stock returns, especially for stocks that are highly subject to speculative demand and arbitrage difficulties (e.g., small, young, high volatility stocks, unprofitable stocks, non-dividend-paying stocks, extreme growth stocks, and distressed stocks). Subsequent empirical analysis provides ample evidence of the role of sentiment in stock returns predictability (i.e., anomalies). Post-earnings announcement drift (Frazzini & Lamont, 2008), initial public offerings (IPOs; Cornelli, Goldreich, & Ljungqvist, 2006), momentum (Antoniou, Doukas, & Subrahmanyam, 2013), beta puzzle (Antoniou, Doukas, & Subrahmanyam, 2016), idiosyncratic volatility puzzle (Stambaugh et al., 2015), MAX effect (Fong & Toh, 2014), size premium (Qadan & Aharon, 2019), value-at-risk (Bi & Zhu, 2020), and earnings expectations (Riedl, Sun, & Wang, 2021), among others.

This research suggests that investors can be cognitively biased and the market subject to common waves of euphoric (i.e., high investor sentiment) trading. These waves bring investor overconfidence and other decision-making biases that create speculative episodes and adds to the costs of arbitrage. Consequently, stock prices show predictable patterns (e.g., momentum and reversal).

Defining investor sentiment is a complex task. To achieve this, we follow the prior literature that employs survey measures for consumer satisfaction and economic expectations (see, for example, Lemmon & Portniaguina, 2006). Here, we employ the Economic Sentiment Indicator, published monthly by the European Commission, as a proxy for consumer and business mood. To extract the irrational component of investor sentiment, this indicator is regressed against six economic variables, the market return, the change in industrial production, the unemployment rate, the change in the consumer price index, the difference between the yields of 10-year Treasury bonds and 3-month Treasury bills (i.e., term premium), and the OECD-based Recession Indicator. The residuals of this regression are employed as the proxy for irrational sentiment. This is then classified into three states, optimistic, middle, and pessimistic. Under the optimistic (pessimistic) state, the average of the sentiment index over the past three months is in the top (bottom) 40% of a 3-month rolling average.

Table 17 presents the results of considering left-tail risk and investor sentiment. The results demonstrate a significant effect of investor mood on the magnitude of the spread between high and low left-tail risk stocks. Consistent with the mispricing argument, investor optimism

Table 17
left-tail momentum and sentiment effect.

Panel A	EW						
	Pessimistic			Optimistic			
Portfolio	ES1	ES5	ES5-ES1	ES1	ES5	ES5-ES1	Opt-Pess
Ret_{+1}	0.84[c]	−1.45	−2.29[a]	0.81[a]	−3.94[a]	−4.75[a]	−2.46[b]
t-stat	1.84	−1.28	−2.75	3.37	−4.03	−5.25	−2.06
Alpha	0.51[a]	−1.59[a]	−2.10[a]	0.99[a]	−2.27[a]	−3.26[a]	−1.17[c]
t-stat	3.46	−3.66	−4.00	5.23	−4.54	−5.44	−1.66

Panel B	VW						
	Pessimistic			Optimistic			
Portfolio	ES1	ES5	ES5-ES1	ES1	ES5	ES5-ES1	Opt-Pess
Ret_{+1}	0.69[c]	−0.96	−1.64[c]	0.49b	−3.10[b]	−3.59[a]	−1.95
t-stat	1.92	−0.80	−1.65	2.24	−2.53	−3.04	−1.31
Alpha	0.13	−1.51[a]	−1.64[b]	0.24	−1.82[b]	−2.07[b]	−0.43
t-stat	0.72	−2.98	−2.63	1.52	−2.48	−2.52	−0.46

This table reports the effect of the market sentiment on the performance of the left-tail momentum. A pessimistic (optimistic) market is defined when the sentiment index is at the bottom (upper) of the 40% percentile of its distribution. Ret_{+1} is one month-ahead-formation return, Alpha is the Carhart 4-factor alpha, and t-stat is the Newey-West t-statistic. ES1 (ES) includes the stocks with the lowest (highest) ES, ES5-ES1 represents the spread between the returns of the highest and the lowest ES portfolios, and Opt-Pess is the average of differences between the ES5-ES1 between the optimistic periods and the pessimistic periods. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

leads to stronger left-tail momentum. The difference in this spread between the optimistic state and pessimistic state is −2.46% (Newey-West t-statistic of −2.06) and − 1.95% (Newey-West t-statistic of −1.31) for equal- and value-weighted portfolios, respectively. Adjusting for the Cahart risk factors does substantially reduce the sentiment effect on the magnitude of left-tail momentum, especially for the value-weighted scheme. The difference in the alpha of the ES-based strategy between the two sentimental states is only −0.43%. This might be a result of a relation with some of the risk factors and the sentiment effect.[18]

5.6. Market state

Investor behaviour changes with market state. Cooper, Gutierrez Jr., and Hameed (2005) point out that investor overreaction is more likely in a good market state. Therefore, we repeat the analysis decomposing the sample period into an Up-market state and a Down-market state. Following Cooper et al. (2005), the state is defined using market returns over the past 3-years. The market is in an Up-state if the whole market return over the past 36 months is positive, while the market is in a Down-state if the return for this period is negative.

Table 18 shows the results of this analysis. Weighting the returns equally, left-tail momentum is stronger following an Up-state than Down-state, which supports an investor mispricing explanation. However, the value-weighted portfolios do not confirm any effect of market state on left-tail pricing. This difference in results may be due to the differing contributions that small-size stocks make to the two weighting schemes.[19] Arguably, within an optimistic atmosphere and attracted by positive market returns, investors are more likely to act on negative news. Consequently, left-tail momentum is stronger with this market state.

6. Summary and conclusion

Contradicting the rational risk-averse view that prevails in finance theory. Recent evidence by Atilgan, Bali, et al. (2020) reveals an inverse relation between left-tail risk and subsequent returns across individual stocks in the US. In this paper, we seek not only to investigate this anomaly for UK stocks but also to model the driving forces behind it. Notably, we examine behavioural explanations such as investor underreaction and overreaction and consider the role played by limits to arbitrage. To do so, we utilise a sample of stocks for the period from January 1996 to August 2021.

Left-tail risk is measured using observations on the extreme left-tail of the empirical distribution of past daily returns. For each individual stock, we employ the expected shortfall (ES) and value-at-risk (VaR). Considering many potential proxies for factors that can determine the left-tail anomaly, indices are built via principal component analysis (PCA) to represent underreaction-based biases and limits to arbitrage. Overreaction is represented by the Continuous Overreaction (CO) index developed by Byun et al. (2016).

We can summarise our results as follows:

First, we establish that UK stocks with high left-tail risk earn abnormally low returns in comparison to those with low left-tail risk. This result remains robust when considering standard asset pricing risk factors. Moreover, the abnormal poor performance associated with investing in high left-tail stocks persists over the next three years.

Second, in seeking to understand this effect, we consider a behavioural explanation. In analysing both underreaction and overreaction biases, we find that the left-tail momentum is more likely to be associated with underreaction behaviour. Both portfolio-based and Fama-MacBeth regressions show that the magnitude of left-tail momentum is significantly stronger when investors are less attentive. These results support those of Atilgan, Bali, et al. (2020). The results also confirm related work on behavioural biases. For example, slow diffusion of bad news (see, Hong et al., 2000; Hong & Stein, 1999) and anchoring bias could explain the failure of investors to update their valuations (see, Tversky & Kahneman, 1974).

Third, in building upon these results, we introduce limits to arbitrage, which exerts a substantial effect on the underreaction channel.

[18] Qadan and Aharon (2019) demonstrate that investor sentiment and the size premium are highly related. Also, Antoniou et al. (2013) highlight the link between sentiment and the price momentum. Therefore, introducing the size factor and the momentum factor may subsume the sentiment effect.

[19] Technically, in moving from an equal- to a value-weight scheme, big stocks will dominate portfolio returns, while the contribution of small stocks is reduced (and becomes negligible for the smaller stocks). The size of stocks is positively related to sentiment and arbitrage difficulties. Behavioural studies demonstrate that small stocks are more subject to speculative demand and more difficult to value and arbitrage (see, for example, Baker & Wurgler, 2006; Han & Kumar, 2013; Qadan & Aharon, 2019). The results from Table 2 show that left-tail momentum is more prominent using the equal weighing scheme. Therefore, difference between the up- and down-states noted here, is diminished under a value-weight scheme.

Table 18
The left-tail momentum and the market state.

	Down-State			Up-State			
Panel A	EW						
Portfolio	ES1	ES5	ES5-ES1	ES1	ES5	ES5-ES1	Up-Down
Ret_{+1}	1.24[b]	−0.18	−1.42	0.68[b]	−2.96[a]	−3.65[a]	−2.23[c]
t-stat	2.27	−0.10	−0.93	2.56	−4.56	−6.82	−1.67
Alpha	0.68[a]	−0.71	−1.38[b]	0.77[a]	−1.92[a]	−2.69[a]	−1.30[c]
t-stat	5.51	−1.20	−1.99	5.51	−5.95	−6.54	−1.68
Panel B	VW						
Portfolio	ES1	ES5	ES5-ES1	ES1	ES5	ES5-ES1	Up-Down
Ret_{+1}	0.84	−1.05	−1.89	0.47[b]	−2.19[a]	−2.66[a]	−0.77
t-stat	1.58	−0.53	−1.18	2.20	−2.90	−3.83	−0.48
Alpha	0.16	−1.95[a]	−2.11[a]	0.21	−1.53[a]	−1.74[a]	0.37
t-stat	0.80	−3.08	−3.10	1.59	−3.37	−3.20	0.35

This table reports the average monthly return of the ES-based quantile portfolios over the different market states. Panel A reports the equally weighted return while in Panel B the return is weighted by the market value. UP-State (Down-State) is defined when the cumulative return over the previous 36 months is positive (negative). Ret_{+1} is one month-ahead-formation return, Alpha is the Carhart 4-factor alpha, and t-stat is the Newey-West t-statistic. ES1 (ES) includes the stocks with the lowest (highest) ES, and ES5-ES1 represents the spread between the returns of the highest and the lowest ES portfolios. a, b, and c indicate statistical significance at 1%, 5%, and 10%, respectively.

Notably, for stocks where investors are attentive and limits to arbitrage are weak, the left-tail momentum effect disappears. However, considering high-attention stocks, an inverse relation between left-tail risk and subsequent returns reappears when stocks are difficult to arbitrage.

Fourth, the reappearance of the left-tail risk effect in the absence of underreaction biases indicates the importance of limits to arbitrage as an additional factor and presents an interesting explanation for the left-tail effect. That is, although the left-tail momentum anomaly is largely explained by the underreaction channel, this does not explain the full picture. Even with high attention, left-tail momentum exists where limits to arbitrage are present, which suggests investor overreaction. Barber and Odean (2008) demonstrate that investors are net buyers of stocks with attention-grabbing features (e.g., high abnormal volume) and subsequently these stocks underperform. Also, difficult to arbitrage stocks (e.g., stocks with high idiosyncratic volatility) are more likely to experience extreme returns and thus are attention-grabbing for investors (see, Han & Kumar, 2013). As shown in Table 4, high left-tail risk is more likely to coincide with the extreme right tail (i.e., lottery-like feature). Our analysis demonstrates that difficult to arbitrage stocks with high attention index are past winners with a subsequent loss.

In short, the evidence presented here implies that: stocks with high left-tail risk earn anomalously low returns; this is linked to investor underreaction to negative information; where underreaction is not present but limits to arbitrage are, the effect remains.

Accordingly, the UK stock market exhibits elements of informational inefficiency. Heyman, Lescrauwaet, and Stieperaere (2019) show that investor attention reverts winner stocks to losers in the subsequent periods. Hong et al. (2000) and Coelho (2015) postulate that investors are more likely to underreact to bad news. Hur and Singh (2016) demonstrate that underreaction plays a more crucial role in explaining the price momentum anomaly, although overreaction also provides important information. In the light of evidence revealed by this work, we suggest that combining the information content of investor underreaction and overreaction provides us with a better understanding of the left-tail momentum.

The results presented above suggest some important implications for market participants:

First, the UK stock market exhibits predictability such that an abnormal return can be generated. On average, going short on stocks with the highest left-tail risk and going long on stocks with low left-tail risk earns a positive return. Conditioning this strategy on the attention index and the limits to arbitrage index reveals that the strategy is profitable when applied to stocks with low attention index and high limits to arbitrage index.

Second, consideration of the revealed relations may be useful in improving the performance of a pricing momentum strategy. Aside from the potential profitability, the returns of the conventional momentum pricing anomaly of Jegadeesh and Titman (2001) exhibit a fat left-tail distribution, i.e., rare but large long-lasting losses (see, Daniel & Moskowitz, 2016). Barroso and Santa-Clara (2015) and Yang and Ma (2021) suggest strategies to enhance this conventional price momentum strategy. Building on these insights, the results here imply that removing winners with high left-tail risk may improve this conventional momentum strategy, as the past good performance of these stocks is likely to revert in the future rather than continue.

Third, the evidence of inefficiency in the UK market is of interest to market regulators. The observed mis-valuation may lead to excessive volatility and destabilisation of the financial system. To mitigate this effect, policymakers could apply well-designed policies to tackle such undesired mis-valuation behaviour. For example, the controversial securities transaction tax may offer one potential policy to reduce speculative-induced trading. A good example of such a policy is the capital gain taxes introduced to UK stocks in 1998 and the turnover tax proposed by Stiglitz (1989). Underreaction of investors to bad information may require the intervention of regulators to improve the informational role of the market.

The results also provide some directions for future research. As noted, the UK (and US) have high levels of institutional share ownership.[20,21] The usual view is that dominance by institutional investors is key in arbitraging any deviations in the market price from fundamental value. Accordingly, institutional ownership should affect (eliminate) the presence of pricing anomalies. However, our results suggest that this is not the case.

This could arise for two reasons. First, the failure of institutional ownership could be traced to the costs of arbitrage associated with high left-tail stocks. High arbitraging costs could lead institutional investors to shun high ES stocks and preventing exploitation of the mispricing opportunities (e.g., Au et al., 2009). Moreover, governance and regulation of some institutional investors, such as banks, leads to caution when dealing with high-risk securities. Therefore, it could be the case that such institutional investors avoid stocks with high VaR and ES. This

[20] See, for example, https://www.oecd-ilibrary.org/governance/corporate-ownership-and-concentration_bc3adca3-en

[21] A further area of future research for UK stocks is the changing nature of institutional ownership, including recent (2020) pension rule changes that results in a lower stock holding by pension companies, see: https://www.ons.gov.uk/economy/investmentspensionsandtrusts/bulletins/ownershipofukquotedshares/2022

issue requires further investigation, for example, to distinguish between limited attention by institutional and retail investors (see, Ben-Rephael, Da, & Israelsen, 2017). Second, institutional investors may possess the same cognitive biases that shape the decisions of retailer investors. For example, Ben-Rephael et al. (2017) demonstrate that institutional investors underreact to performance-related news and contribute to post-announcement drift in stock prices.[22]

As a final point, the explanatory power revealed by the investor attention level and limited arbitrage may also generalise to other anomalies, which could be considered in future research.

Data availability

No

Appendix A. Variable definitions

A.1. Left-tail risk proxies

The expected shortfall at q% (ESq%): ES is the average of losses beyond the lowest 5-percentile of the distribution of the daily returns over the past 12 months. We multiply the ESq% by −1, thus the higher value implies higher left-tail risk.

Value-at-risk at q% (VARq): Value-at-risk is estimated from the left-tail of the empirical distribution of one-year daily returns and equals the negative value of the original q% percentile.

A.2. *Continuous overreaction (CO):* Following Byun et al. (2016), we define this measure as the following

$$CO_{i,t} = \frac{sum\left(w_j \times SV_{i,t-j}, \ldots\ldots\ldots\ldots\ldots\ldots, w_1 \times SV_{i,t-1}\right)}{mean\left(VOL_{i,t-j}, \ldots\ldots\ldots\ldots\ldots\ldots, VOL_{i,t-1}\right).}$$

where $SV_{i,t}$ is the signed volume for stock i in month t,

$$SV_t = \begin{cases} VOL_t if\ r_t > 0, \\ 0\ if\ r_t = 0, \\ -VOL_t\ if\ r_t < 0, \end{cases}$$

where VOL_t is the dollar volume in month t and r_t is the stock return in month t, J is the length of the formation period, and w_j is a weight that takes a value of J-j + 1 in month t-j (i.e., $w_j = 1$ and $w_1 = J$). In this work, the continuous overreaction (CO) is measured using a 12-month formation period.

A.3. Underreaction-based biases

Delayed response: Following Hou and Moskowitz (2005), for each individual stock we run the following market models:

$$R_{i,t} = \alpha_i + \beta_{i,t}{}^*R_{m,t} + \sum_{n=1}^{4} \beta_{i,t-n}{}^*R_{m,t-n} + \varepsilon_{i,t}, \text{(unrestricted)}$$

$$R_{i,t} = \alpha_i + \beta_{i,t}{}^*R_{m,t} + \varepsilon_{i,t}, \text{(restricted)}$$

Where R_{it} is the return on stock i and $R_{m,t}$ is the return on the general market index at time t. estimating the above model parameters, the main delay response measure is the fraction of the contemporaneous individual stock returns explained by the lagged market returns,

$$\text{Delay}_i = 1 - \frac{R^2_{restriscted}}{R^2_{unrestricted}},$$

where R^2 is the fraction of return explained by the corresponding model.

To keep aligned with the main goal of this study, we follow Boehmer and Wu (2013) and focus our interest on the left side of the information distribution. Thus, we will apply the above measure conditioning on the past negative market returns. We estimate the Delay_i using daily returns over the past 12 months.

Abnormal volume: the amount of trading volume over the past 12-month average volume, and measured

$$\text{ABnVol}_i = \left(Volume_{it} - \frac{\sum_{n=1}^{12} \text{Volume}_{it-n}}{observationshs} \right) \Big/ \text{Std}\left(\text{Volume}_i\right)$$

Where Volume is the pound trading volume in month t, Std is the standard deviation of the volume over the past 12 months. According to this equation, low ABnVOL indicates that the investors pay little attention to stocks i, and therefore they start to trade this stock less frequently.

Continuous information: Da et al. (2014) suggest the following measure to classify continuous from discrete information,

$$\text{ID}_i = sgn\ (\text{PRET})^*\ [\%\text{neg} - \%\text{pos}]$$

[22] Also see, Cen, Hilary, and Wei (2013), Puetz and Ruenzi (2011), Edelen, Ince, and Kadlec (2016), Wulfmeyer (2016), and Hudson, Yan, and Zhang (2020).

where sgn (PRET) is the sign of the cumulative return over the formation period, %neg is the percentage of negative-return days over the formation period, and %pos is the percentage of days with a positive return. In this study, we used a decomposed measure of the above ID measure. In particular, we classify the sample into the following three groups,

$$\mathrm{ID_i} = \begin{cases} \%pos - \%neg \; if \; sng(PRET) = sgn(pos \quad \% - \%neg) \\ 0 \; otherwise \end{cases}$$

The above classification aims to give separate values for the negative and positive continuous news. To illustrate, if the stock i return over the formation period is negative and generated by the frequent negative daily returns, then there is continuous negative information, and the measure will take a negative value (%neg > %pos). The same logic applies to the positive return. Notice that the stocks with discrete information will fall in the middle of the distribution with a value of 0.

Price to 52-week high (PH52): the ratio of the current price to the highest price over the past 52 weeks,

$$\mathrm{PH52} = \mathrm{P_{i,t}}/52 - \mathrm{week\ high\ price}$$

where $P_{i,t}$ is the current closing price for stock i and the denominator is the highest price over the past 52 weeks. According to the anchoring bias and the revealed empirical evidence, investors are more likely to miss out on the news of the stocks with a closing price far from or near to the past 52-week high price are more likely mispriced.

A.4. Attention index

To tackle the noisy measure problem associated with a single proxy, the underreacting behaviour and the limits to arbitrage will be integrated to an index via the so called principal component analysis (PCA). Using the above four proxies of underreaction behaviour, we employ the so-called principal component analysis (PCA) to weight the various measures of attention and extract the common attention index. Particularly, each month, we apply the PCA to measure the common variation across the stocks, decompose the variation into a linear uncorrelated components that contain most of the variance. To proxy the underreaction behaviour (i.e., the limited attention) the first principal component is employed. This component will be used as the attention index. In this step, we conjecture that the extracted first component measures the joint variation in the four proxies, which is assumed to represent the investors' attention level. Indeed, this index shows high correlation with the all individual proxies. Similar processes are used to gauge the limits to arbitrage index.

A.5. Limits to arbitrage proxies

Firm size: measured by the market capitalization of the firm which equals the stock price multiplied by the number of shares outstanding.

*A.6. **Age:** measured by the number of months since the firm's initial appearance in the DataStream database*

Return Synchronicity: To measure return synchronicity, on a monthly basis, for each stock, we first estimate the R^2 of the following market model over the past 3 months of daily data,

$$R_{i,t} = \alpha_i + \beta_{i,t} {}^{*} R_{m,t} + \varepsilon_{i,t},$$

where $R_{i,t}$ and $R_{m,t}$ are the return for the firm i and the market, respectively, on time t. The return synchronicity is the logistic transformation of the R^2.

$$\mathrm{Synch} = \mathrm{Log}\left(\frac{R^2}{1 - R^2}\right).$$

Idiosyncratic Volatility: To measure idiosyncratic volatility, we follow Ang et al. (2006). In specific, over the past 12 months, we run the following Carhart's model,

$$\mathrm{R_{it}} - \mathrm{rf_t} = \alpha_i + \beta_m{}^{*} (\mathrm{R_{mt}} - \mathrm{rf_t}) + \beta_{smb}{}^{*}\, \mathrm{SMB_t} + \beta_{hml}{}^{*}\, \mathrm{HML_t} + \beta_{umd}{}^{*}\, \mathrm{UMD_t} + \varepsilon_{it}, \quad (2)$$

Where R_{it} is the return of stock i on day t, R_{mt} is the market return on day t, rf_t is the risk-free rate, SMB_t is the small market capitalisation minus big market capitalisation factor, HML_t is the high minus low factor, UMD_t is the winner minus loser factor, and ε_{it} is the unexplained component of returns of stock i. Also, α_i, β_m, β_{smb}, β_{hml}, and β_{umd} are the estimated parameters. After estimating the model, the idiosyncratic volatility is calculated as the standard deviation of the residuals. To mitigate the effect of nonsynchronous trading we require a minimum of 120 observations to estimate Carhart's four-factor model. These procedures are re-estimated on a monthly basis therefore monthly series of idiosyncratic volatility is estimated for each individual stock.

Bid-Ask spread: this spread is represented by the average of the daily bid-ask spread over the past 12 months. The daily bid-ask spread is the difference between the closing bid and ask prices divided by the average of their mid-point, as follows,

$$\mathrm{Bid - Ask\ spread} = \left(\mathrm{Bid_{i,d}} - \mathrm{Ask_{i,d}}\right)/\left(\left(\mathrm{Bid_{i,d}} + \mathrm{Ask_{i,d}}\right)/2\right).$$

Where Bid and Ask are the closing bid and ask prices. We take the average daily bid-ask spread over the past 12 months.

Limits-to-arbitrage index (LA):

Similar to the Attention index, the LA is built by employing the PCA to extract the first principal component of the five proxies of the limits-to-arbitrage.

A.7. Other variables

To isolate the potential effect of other return predictors, we control for a set of return predictors that are widely documented in the financial literature. This set of control variables is as follows:

Maximum return (MAX5): represents the lottery likeness. Following Bali et al. (2011), we measure the lottery likeness by the average of the maximum 5 daily returns over the past month.

Amihud (2002)'s**Illiquidity measure (Amih):** Following Amihud (2002), we measure the price impact of illiquidity as the average of the daily absolute stock return to dollar trading volume ratio over the past 12 months,

$$\mathrm{Amih}_{i,t} = \Sigma\left\{|\mathrm{R}_{i,d}|/\mathrm{VOLD}_{i,d}\right\},$$

Where $R_{i,d}$ is the daily return of stock i, $VOLD_{i,d}$ is the daily trading volume in dollars for the stock i. This liquidity measure serves as a proxy for the impact ratio and the effect of the order flow on the prices which is inspired by Kyle (1985).

Market Beta (Beta): To mitigate the impact of nonsynchronous trading, we follow Lewellen and Nagel (2006) and Cederburg and O'Doherty (2016) by adding four lags of the market premium to the regression, as a following,

$$Rp_{i,t} = \alpha_i + \beta_{i,t} {}^* Rp_{m,t} + \sum_{n=1}^{4} \beta_{i,t-n} {}^* Rp_{m,t-n} + \varepsilon_{i,t},$$

$$\beta_i = \beta_{i,t} + \sum_{n=1}^{4} \beta_{i,t-n}$$

Where Rp_i and Rp_m are the daily risk premium for the stocks i and the market portfolio, respectively, and β_i is the estimated beta. The beta will be re-estimated on a monthly basis using the daily returns over the past three months.

Downside Beta (Dbeta): is a systematic left-tail risk measure, measured in a similar way to the market beta but the stock return is regressed only on the negative market returns rather than the total market returns series. Therefore, the downside beta is designed to measure the association between the stock and the market conditioning on the market downstate.

Co-skewness: Another measure of the association between the stock and the market conditioning on the extreme fluctuations in the market return. Following Harvey and Siddique (2000), to measure the co-skewness, the following quadratic form of the market model will be fitted:

$$\mathrm{R}_{i,d} = \alpha_i + \beta_{i,d}{}^* \mathrm{R}_{m,d} + C_{i,d}{}^* \mathrm{R}^2{}_{m,d} + \varepsilon_{i,d},$$

where $C_{i,d}$ represents the co-skewness measure. Positive co-skewness indicates that the stock has a lower tail risk, and it is more likely to generate a positive return conditioning on the tail of the market returns distribution.

Midterm Momentum (Mom): following Jegadeesh and Titman (1993), med-term momentum is defined as the cumulative return over the past 12 months after skipping a month between the portfolio formation period and the holding period, i.e., cumulative return over month t–12 to month t–1.

Short-term reversal (Rev): measured using the stock return over the past month (1-month return).

Long-term return: the stock's return over the past three years

ROA: is the return on asset and is measured by the ratio of earnings to the total asset.

Asset growth: measured by the growth in total assets

BMV: is the ratio of the book value to the market value.

DY: is the dividend yield, the ratio of dividend to the current closing price.

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