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Applying time delay convergent cross mapping to Bitcoin time series

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ABSTRACT

This paper explores the applicability of Convergent Cross Mapping (CCM) and its extension, Time Delay Convergent Cross Mapping (TDCCM), to assess the causal relationships between Bitcoin, the S&P 500 index, and gold. Unlike conventional causality analysis methods, such as Granger causality or transfer entropy, CCM accounts for non-separable, weakly connected dynamic systems, and TDCCM explicitly incorporates time lags during cross-mapping, enabling the detection of complex causal relationships in systems with shared nonlinear behavior. This makes it particularly suitable for financial time series that often exhibit chaotic and nonlinear dynamics, particularly during periods of market instability. We integrate TDCCM with simplex projection and sequential locally weighted global linear map (S-map) algorithms, applying a sliding window approach to identify short time intervals characterized by high levels of nonlinearity and chaoticity. Using this approach, we uncovered a strong causal relationship between Bitcoin and the S&P 500 index during the onset of the COVID-19 pandemic. Our analysis reveals a bidirectional causal relationship between Bitcoin and the S&P 500 index, highlighting their interconnectedness during periods of heightened economic uncertainty. Furthermore, we find a unidirectional causal influence of Bitcoin on gold, reflecting Bitcoin's evolving role as a macroeconomic indicator and its growing relevance as an alternative store of value. These findings provide insight into the dynamics between cryptocurrencies and traditional financial markets, particularly during periods of global economic disruption.

1. Introduction

A time series is a set of observations taken sequentially over time (Shumway & Stoffer, 2017), for example, daily closing prices of cryptocurrencies or stock markets. Time series play an important role in financial data, for which temporal evolution is paramount to economic analysis and investment decisions. Time-series analysis helps us understand how the price of an asset evolves over time, providing insights into its patterns and trends. Analysts can obtain valuable information on how markets behave by analyzing patterns, trends, cycles, and potential chaos in these observations (Tsay, 2010). Beyond traditional analysis and forecasting methods, the study of relationships between univariate time series has attracted growing interest as both an explanatory and predictive mechanism (Granger, 1969). Uncovering the influence that one temporal variable exerts on another is often described as causal analysis.

Causality between time series is an important element, as it can help uncover influences that play an explanatory role or can inform various financial strategies, including investment or trading strategies. Studies such as Peia and Roszbach (2015) show that time series causal analysis is important to explore the bidirectional relationships between financial development and economic growth.

A key question in financial analysis is whether stock prices follow a random pattern or exhibit chaotic behavior. The Positive Feedback Trading Hypothesis (PFTH) suggests that stock price movements are chaotic and predictable to some extent (Antoniou, Koutmos, & Pericli, 2005; Sornette, 2009). Similarly, some studies such as Partida, Gerassis, Criado, Romance, Giráldez, and Taboada (2022) have shown that cryptocurrencies such as Bitcoin and Ethereum exhibit chaotic behavior. However, in our study, we want to understand the dynamics of Bitcoin in relation to other financial assets under conditions of chaos.

Traditional methods such as Granger causality (Granger, 1969) or transfer entropy (TE) (Schreiber, 2000) often assume linearity between variables, making them less effective when markets are highly volatile or show strong nonlinear interactions.

- The Granger causality's fundamental criterion is separability, which implies that causal elements can be distinguished from

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their effects. Separability reflects the view that systems can be understood a piece at a time rather than as a whole. This property is satisfied in linear systems and strongly coupled nonlinear systems. However, separability fails for general dynamic nonlinear systems. In dynamic systems, if X causes Y, complete information about X is encoded in Y, so X cannot be removed formally. Therefore, Granger is limited to cases where separability is met (Sugihara et al., 2012).
- Transfer entropy, on the other hand, is a model-free and an information theoretic generalization of Granger causality (Lindner, Vicente, Priesemann, & Wibral, 2011). TE is able to capture how much knowing the past of time series X decreases the unpredictability of the future of time series Y, beyond what is already known from the past of Y itself. The model-free nature of TE allows it to operate without being constrained by specific modeling assumptions, such as linearity. Although it avoids restrictive modeling assumptions, TE can be computationally expensive and often requires large datasets to estimate the conditional entropies reliably.

To overcome the limitations of these conventional approaches, we explore the effectiveness of convergent cross-mapping (CCM), a recent causality analysis method introduced by Sugihara et al. (2012), as well as its extension, time delay convergent cross mapping (TDCCM) (Ye, Deyle, Gilarranz, & Sugihara, 2015), which explicitly considers different lags in cross mapping to identify causal relationships from observational data.

CCM is important in studying financial time series because of its ability to detect causality in complex, nonlinear, potentially chaotic dynamics, which are conditions often found in modern financial markets. The CCM systematically reconstructs the underlying state space, allowing it to uncover hidden causal patterns even when the time series are short, noisy, or subject to rapid fluctuations. The ability to capture nonlinear causal effects not only enriches our understanding of market dynamics but also has direct applications in risk management, portfolio optimization, and strategic decision-making (Ma, Prosperino, Haluszczynski, & Räth, 2024; Ong & Herremans, 2023).

This versatility has driven its adoption across diverse fields, including ecology (Bonotto, Peterson, Fowler, & Western, 2022; Roy, Howes, Müller, Butail, & Abaid, 2019), neuroscience (De Castro Martins, Chaminade, & Cavazza, 2022), and finance (Azqueta-Gavaldon, 2020; Javarone, Di Antonio, Vinci, Cristodaro, Tessone, & Pietronero, 2023; Wu, Gao, An, & Liu, 2021).

Our objective was to utilize a suite of empirical dynamic modeling frameworks to evaluate the suitability of the CCM for financial time-series data, and assess how effectively the CCM identifies causal relationships between financial assets. Specifically, we applied this method to the Bitcoin, gold, and S&P 500 index datasets.

The main contributions of this study are as follows:

- We apply time delay convergent cross mapping to Bitcoin, the S&P 500 index, and gold.
- We present an approach to study nonlinear and chaotic short-term intervals by combining the S-Map and Prediction Decay algorithm with a sliding window technique.
- We identify and quantify lagged causal interactions between the time series of Bitcoin, gold, and the S&P 500 index.

The remainder of this paper is organized as follows. Section 2 discusses the related studies. Section 3 provides an overview of the general concepts required for the analysis in this study. Section 4 details the methodology used to infer causality and describes the experimental setup. The empirical findings are presented in Section 5. Finally, the 6 section summarizes the main findings and suggests directions for future research.

2. Related works

2.1. Causal analysis with transfer entropy and Granger causality

Dimpfl et al. used transfer entropy (Schreiber, 2000) to better understand financial markets. Initially, he applied it to study Bitcoin's nonlinear relationships (Dimpfl & Peter, 2013) and later expanded it in 2019 (Dimpfl & Peter, 2019) to explore causality between Bitcoin and other cryptocurrencies. However, transfer entropy is most effective for large datasets and excels in detecting long-term causality. This makes it unsuitable for short- or medium-term analyses.

Granger causality has also been widely applied in financial markets and cryptocurrency research to uncover directional relationships. Ausloos, Zhang, and Dhesi (2020) detected bidirectional Granger causality between the CSI-300 index futures and spot markets.

de Oliveira Carosia, Coelho, and da Silva (2021) applied sentiment analysis and the Granger causality test to analyze the impact of financial news sentiment on the Brazilian stock market.

However, because of the separability criterion, which is satisfied in linear systems and strongly coupled nonlinear systems, their application to dynamic nonlinear systems may result in spurious correlations.

For such purposes, the CCM is often preferred because it is not limited to large data volumes or linear constraints.

2.2. Causal analysis with CCM

Clark, Ye, Isbell, Isbell, Deyle, Cowles, Tilman, Tilman, and Sugihara (2015), Sugihara et al. (2012) applied the CCM to ecology and environmental science. This seminal paper introduces the CCM and demonstrates its application in identifying causal relationships in ecological data, such as interactions between sardine and anchovy populations. Building on this foundational work, Hao Ye et al. in collaboration with Sugihara, later introduced the TDCCM applied to environmental science (Tsonis, Deyle, Ye, & Sugihara, 2018; Ye et al., 2015), which extends the CCM framework by explicitly incorporating different time lags in cross-mapping. TDCCM enhances the ability to detect and characterize causal relationships by accounting for potential temporal delays between interacting variables, making it particularly useful for systems in which interactions occur with inherent time lags. This advancement provides a more nuanced tool for understanding complex systems and broadens the applicability of causality analysis in various fields.

Lin, Guo, and Luo (2024) applied time-lagged CCM in the field of renewable energy to identify the causal relationships between meteorological factors and offshore wind power generation.

The CCM, combined with chaoticity estimation using the S-map technique and a sliding window approach, has also been applied in various other fields. In animal behavior science, the CCM has been employed to identify directional coupling between flying bat pairs (Roy et al., 2019).

CCM has been widely applied across diverse fields. In neuroscience, CCM is used to analyze brain activity interactions and their underlying dynamics (Avvaru & Parhi, 2023; De Castro Martins et al., 2022; Wismüller, Wang, DSouza, & Nagarajan, 2014). In social media analytics, CCM has been employed to study user behavior and influence patterns (Luo, Zheng, & Zeng, 2014; Manchanayake, Zaidi, Karunasekera, & Leckie, 2024).

CCM has found applications in energy systems (Li, Li, Juguang, Yang, Qiao, & Xiaobing, 2023; Liu, Lei, Zhang, & Du, 2019); in climate science, (Bonotto et al., 2022; Huang, Franzke, Yuan, & Fu, 2020; Wang et al., 2018); in physics, it has been applied to analyze resistor-inductor circuit systems (McCracken & Weigel, 2014) as well as in medicine and healthcare (Cobey & Baskerville, 2016; Gu, Lin, & Lin, 2023; Li & Convertino, 2021).

2.3. CCM analysis of financial time series

Ma et al. (2024) demonstrate that financial markets exhibit significant nonlinear causality, and that traditional correlation measures may underestimate these effects. Focusing on the German DAX and U.S. Dow–Jones stock indices, they find that nonlinear causality plays a critical role in understanding market dynamics, especially during major events such as Black Monday and the global financial crisis.

Javarone et al. (2023) employed CCM to identify the causality between human behavior, blockchain dynamics, and market trends in Bitcoin. Their findings suggest a significant causal link between transaction anomalies within blockchain and subsequent market behavior. Wu et al. (2021) applied the CCM theory combined with a sliding window approach to investigate the time-varying causal relationships in global stock markets.

Ge and Lin (2021) demonstrated that sliding windows can clarify the understanding of causal effects, which is valuable for producing a detailed description of causal dynamics in real systems. They used a cross-mapping coefficient to quantify the causal relationship between Chinese and American stock market time series. Additionally, they propose kernel change point detection based on CCM, applied to the financial system during the 2008 financial crisis to assess the linkage of Chinese and American stock markets (Ge & Lin, 2022). Du and Zhang (2023) used CCM combined with the S-Map algorithm to explore nonlinear bidirectional causality between U.S. stocks and bonds, showing that causal strength significantly impacts stock-bond correlations and increases prediction accuracy. Sun, Fang, Gao, An, Liu, and Wu (2021) explored the time-varying causal relationships among nickel prices in spot, futures, and stock markets using CCM. Their findings provide insights into nonlinear interactions and causal strengths in commodity markets through a novel combination of CCM and complex network theory.

Azqueta-Gavaldon (2020) applied the CCM to explore the causal relationship between narratives circulated by the media and crypto prices. He found strong bidirectional causal relationships between narratives and cryptocurrency prices. Using CCM, Tu, Fan, and Fan (2019) found that Bitcoin plays a leading role in cryptocurrencies.

Mønster, Fusaroli, Tylén, Roepstorff, and Sherson (2017) evaluated the performance of CCM for causal inference in noisy time-series data. They found that while the CCM can infer causal relationships, its accuracy diminishes with increasing noise levels and in systems with intermediate coupling.

3. Empirical dynamical modelling

In this section, we introduce Empirical Dynamical Modelling (EDM) and its key methods, including simplex projection, S-map, and convergent cross-mapping. These techniques are used to determine the embedding dimensions, analyze nonlinear dynamic systems, and infer causality.

The EDM is a non-parametric framework for modeling nonlinear dynamic systems and is based on the mathematical theory of reconstructing attractor manifolds from time-series data (Takens, 1981). EDMs are an alternative and highly flexible approach for using explicit equations because these equations can be impractical when the exact mechanisms are unknown or too complex to be characterized with existing datasets. This framework is applicable to any stationary or quasi-ergodic dynamic process, including chaotic systems.

Attractor reconstruction forms the foundation of the EDM, which aims to recreate the dynamics of a system from time-series data. In dynamical systems theory, time series are viewed as "observation functions" of a dynamical system, representing the projections of the system's behavior over time. The state of the system is depicted as a point in high-dimensional space with the axes corresponding to the fundamental state variables. The key assumption is that the system state

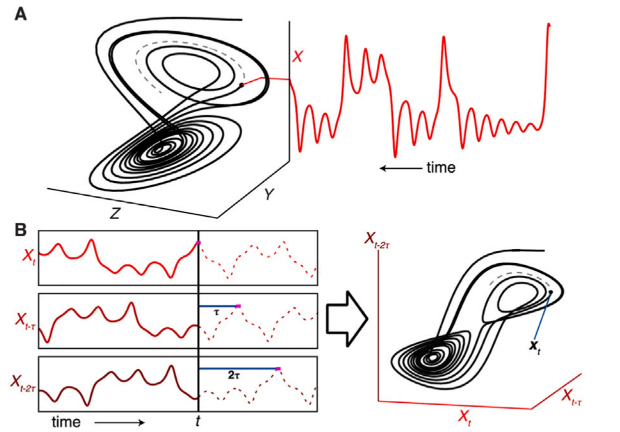


Fig. 1. In Panel A the states of the Lorenz Attractor are shown projected onto the x-axis, resulting in a time series for variable x. Panel B demonstrates the embedding process, where the original time series (X_t) is plotted alongside its time-delayed versions ($X_{t-\tau}$) and ($X_{t-2\tau}$), with (τ) being the time delay. These time-delayed components are then used to reconstruct the attractor in a new phase space (the rightmost plot in Panel B).
Source: Figure taken from Commons (2022).

evolves over time according to deterministic rules, meaning that the behavior is not entirely stochastic.[1]

Because time series are sequential observations of a system's behavior, information on the underlying dynamics is inherently encoded in their temporal ordering. Consequently, it is possible to reconstruct a "shadow" version of the original manifold using lagged versions of a single time series; if sufficient lags are used, the reconstructed states will map one-to-one to the actual system states.

As shown in Fig. 1, the system state of the Lorenz Attractor can be represented using an embedding made up of E lags of X. While the behavior of a system is governed by a high-dimensional state space, unobserved variables can be replaced by the lags of a single time series. With sufficient lags, the reconstructed manifold retained the key mathematical properties of the original system. This ensures that the reconstructed states correspond one-to-one with the actual system states, and nearby points in the reconstruction reflect similar system behavior.

For a time series of length L, $X_t = \{X_t; t = 1, \ldots, L\}$, the reconstructed manifold is defined as a set of vectors.

$$X = \langle X_t, X_{t-\tau}, X_{t-2\tau}, \ldots, X_{t-(E-1)\tau} \rangle$$

where $t = 1 + (E-1)\tau$ to $t = L$, E is the embedding dimension (number of lags), and τ is the time lag between successive dimensions.

3.1. Embedding dimension using simplex projection

Simplex projection is a powerful method for identifying the optimal embedding dimension (E) and time delay (τ), both of which are essential for reconstructing the attractor manifold (M) and analyzing a system's deterministic behavior (Sugihara & May, 1990). Additionally, simplex projection can be used to distinguish between chaos and measurement errors by evaluating the forecasting skill, which is measured as the correlation coefficient between the predicted and observed trajectories. Embedding that yields the highest forecasting skill provides the best dynamic description of the data. If this model outperforms a comparable linear model, then the time series can be classified as nonlinear.

[1] The dissociation of determinism and predictability is characteristic of so-called "chaotic" nonlinear systems.

Earlier methods for delay coordinate embedding (Chan & Tong, 2001; Sauer, Yorke, & Casdagli, 1991) proposed selecting τ by minimizing the time-series autocorrelation or mutual information, followed by determining E through nearest-neighbor forecasting (Sugihara & May, 1990) or a similar false nearest-neighbor algorithm (Abarbanel & Kennel, 1993). Another method involves evaluating the prediction accuracy over a grid of E and τ values, and selecting the pair that produces the most accurate forecasts τ time steps ahead, with a preference for smaller values when the results are statistically indistinguishable (Sugihara, 1994).

To conclude, in simplex projection, the original data are embedded using the embedding dimension E and the time delay τ. The optimal embedding dimension corresponds to the highest forecast skill, as indicated by the correlation coefficient ρ between predictions and observations. For details on the implementation of simplex projection, refer to Sugihara and May (1990).

3.2. Prediction decay

In many natural systems, the key property is that nearby trajectories diverge over time. This phenomenon, known as deterministic chaos or the "butterfly effect" (Lorenz, 1963), implies that although short-term predictions may be feasible, their accuracy diminishes as the prediction horizon increases, making long-term forecasting progressively more challenging.

After identifying the best dynamic description of the data, we can use simplex projection to examine how prediction accuracy changes with time to prediction (T_p)—the number of future time steps for which forecasts are made (Sugihara & May, 1990). A decay in accuracy as T_p increases indicates that the system exhibits chaotic behavior, in which case the time series can be further classified as chaotic.

3.3. Sequential locally weighted global linear map

The simplex projection is limited to cases in which the observational noise is uncorrelated. Significant short-term autocorrelations in colored noise can cause the correlation (ρ) between the predicted and observed values to decay with an increasing prediction-time interval (T_p), potentially leading to a false classification of the time series as chaotic (Sugihara, 1994).

To distinguish between linear and nonlinear deterministic behaviors, Sugihara and May (1990) proposed that a time series may be regarded as nonlinear if the correlation obtained via a simplex projection is significantly better than that obtained using the best-fitting autoregressive linear model. However, this approach requires additional steps to identify the best-fitting autoregressive model and to perform a significance test (e.g., Fisher's z) to compare the correlations. As an alternative, the S-map method (Sugihara, 1994) offers a more efficient way to differentiate between linear and nonlinear dynamics, as well as between autocorrelated noise and deterministic chaotic behavior.

Unlike simplex projection, S-map constructs local linear maps to forecast from the reconstructed state space. By adjusting the parameter θ, the S-map can transition from a global linear model to a locally nonlinear model, making it a simple and effective test for nonlinearity in time-series data. For each prediction, the S-map constructs a unique exponentially weighted linear map with θ controlling the degree of local weighting. When $\theta = 0$, all the points are equally weighted, yielding a global linear solution equivalent to an autoregressive model. As $\theta > 0$, nearby points are weighted more heavily, allowing the map to vary locally and capture the nonlinear behavior.

The difference in the forecasting skill between the linear ($\theta = 0$) and nonlinear ($\theta > 0$) models can be used to assess the degree of nonlinearity in a stationary time series. If the time series is sampled from the autoregressive colored noise, the linear model ($\theta = 0$) is likely to yield the best forecasting skill. Conversely, an improvement in forecasting skill when $\theta > 0$ indicates nonlinear dynamics because the local adaptability of the model provides a better description of the system's behavior. For details on the implementation of S-map, please refer to Sugihara (1994).

3.4. Convergent cross mapping

CCM is a method designed to infer causality in dynamic systems. Unlike traditional causality methods, such as Granger causality, which rely on the predictive relationship between variables, CCM establishes causation based on the shared dynamics of interacting variables within the same underlying system (Sugihara et al., 2012). The fundamental principle of CCM is that if two variables X and Y are causally linked (i.e., they share a common attractor manifold M), then the states of one variable can be reconstructed using the historical record of the other.

CCM leverages the concept of manifold reconstruction, where the lags of a single time series are used to form a shadow manifold (M_x or M_y) that uniquely represents the system dynamics. By testing the extent to which one reconstructed manifold can map the other, CCM evaluates whether the two variables interact within the same dynamic system and are thus causally related. If Y influences X, then causality is established if the historical record of the affected variable (X) reliably estimates the state of the causal variable (Y). This is because M_x, reconstructed from X, must contain complete information about X which includes information about all the causes, including Y. The strength of this causal relationship is quantified using the correlation coefficient (ρ) between the predicted ($\hat{Y}|M_x$) and observed (Y) values (Sugihara et al., 2012).

Convergence is a critical property for inferring causality with CCM, as it reflects improved predictive skills with increasing time-series length or library size. This occurs because the attractor manifold becomes denser, and the nearest neighbors become closer, more accurately representing the system's dynamics (Tsonis et al., 2018). However, convergence is limited by factors such as observational errors, process noise, and time-series length. Failure to account for convergence can lead to spurious results because cross-mapping based solely on statistical associations between variables does not improve with the addition of data.

The significance of CCM is typically assessed using predictive skills with the largest possible library size. A more robust method involves a surrogate time series that preserves the linear characteristics of the data while removing any putative nonlinear structure. Null distributions can be generated by cross-mapping from surrogates, and a 95% quantile can be used to evaluate the significance of the observed cross-mapping skill ($\alpha = 0.05$) (Tsonis et al., 2018).

One of CCM's key advantages is its ability to detect causation in nonlinear and chaotic systems, for which traditional parametric approaches often fall short. Moreover, CCM does not assume linearity or require explicit models of the system, making it robust for analyzing complex real-world data. For a comprehensive introduction to CCM, its theory, and implementation, please refer to Tsonis et al. (2018).

3.5. Time delay convergent cross mapping

Time-delay convergent cross-mapping builds upon the traditional CCM to address its limitations in identifying delayed causal interactions and distinguishing between bidirectional causality and strong unidirectional causality that leads to synchrony (Ye et al., 2015). By introducing an additional lag parameter (l), TDCCM evaluates how the cross-mapping skill varies across different time lags, facilitating the identification of causal delays and clarifying ambiguities related to synchrony.

When using TDCCM, the causal interactions between two variables, X and Y, can be analyzed by varying l. When cross-mapping from X to Y, a negative lag ($l < 0$) indicates that the causal signal appears first in Y and later in X, which is consistent with Y causing X because causes must precede effects. Conversely, positive lags ($l > 0$) suggest that X may influence the future value of Y. By exploring this "asynchrony", TDCCM helps distinguish bidirectional causality from synchrony. In addition, TDCCM can be used to identify time delays in causation, understand the effects of stochastic drivers, and determine the sequence of variables in causal chains.

The implementation involves running CCM over a range of lags and constructing a ρ–l plot to identify the optimal lag (where the cross-mapping skill is maximized). The analysis proceeds by performing CCM with the identified lag and evaluating convergence to confirm causality under these optimized settings. For further discussions on the topic and details of the implementation, please refer to Ye et al. (2015).

4. Causal analysis methodology

The rapid evolution of global financial markets has introduced unconventional assets, such as Bitcoin, into mainstream investments. As Bitcoin's prominence grows, there is an increasing need to explore novel methodologies suitable for analyzing its complex dynamics, especially in relation to traditional assets such as gold and stock indices. This study employs time-delay convergent cross mapping to analyze the causal interactions between Bitcoin and gold and between Bitcoin and the S&P 500 index.

4.1. Data

Bitcoin is a decentralized digital currency introduced in 2009 by the pseudonymous entity Satoshi Nakamoto (Nakamoto, 2008). Operating on blockchain technology, Bitcoin enables peer-to-peer transactions without intermediaries. It has gathered attention as a speculative asset due to its significant price volatility and potential for high returns (Cheah & Fry, 2015).

The Bitcoin market is characterized by high volatility. Price swings are common and driven by market speculation, regulatory news, and technological developments (Corbet, Lucey, Urquhart, & Yarovaya, 2019). For example, Bitcoin's price can experience double-digit percentage changes within short periods, reflecting its speculative nature (Cheah & Fry, 2015; Corbet et al., 2019). This high volatility poses challenges for traditional linear causality methods, such as Granger causality, as it is often attributed to nonlinear chaotic dynamics and may not adequately capture the nonlinear dynamics present in Bitcoin's price movements (Diks & Wolski, 2016).

Recent research by Tong, Chen, and Zhu (2022) revealed that Bitcoin's price fluctuations exhibit nonlinear and chaotic behavior. Their analysis demonstrates that Bitcoin's volatility is time-varying and displays clustering effects, indicating that price movements are not random but follow a complex, nonlinear system. This chaotic nature signifies inherent long-term unpredictability, challenging the applicability of traditional linear causality methods such as Granger causality.

The S&P 500 index is a market capitalization-weighted index comprising the 500 largest publicly traded companies in the United States. It serves as a market proxy, representing approximately 80% of U.S. equity market capitalization, and is a benchmark for the overall performance of the U.S. stock market (S&P Dow Jones Indices LLC, 2024). Movements in the S&P 500 reflect investor sentiment regarding economic growth, corporate earnings, and market risk[2] (Fama & French, 2004).

The S&P 500 exhibited nonlinear dynamics in response to complex economic events. Studies find evidence of nonlinear behavior and chaotic dynamics in stock market indices (Hsieh, 1991; Scheinkman & LeBaron, 1989). These dynamics suggest that traditional linear models may not fully capture the complexities of market movements, particularly during periods of economic stress.

Gold has played a fundamental role in the global financial system for centuries owing to its scarcity and enduring value. It is widely recognized as a safe haven asset; investors prefer to invest in gold during periods of economic uncertainty or market volatility because it tends to retain or increase value when other assets decline (Baur & Lucey, 2010). Gold often preserves its purchasing power over time, making it a preferred asset during inflationary periods (Capie, Mills, & Wood, 2005). Its inverse correlation with the U.S. dollar allows gold to act as a hedge against currency fluctuations (Capie et al., 2005; Joy, 2011).

Although gold is less volatile than Bitcoin, it still experiences price fluctuations influenced by factors such as changes in real interest rates, which can affect gold's attractiveness (Tully & Lucey, 2007), and geopolitical events, as political instability often leads to an increased demand for gold. These dynamics may involve nonlinear relationships, which traditional causality methods may overlook (Bekiros, Boubaker, Nguyen, & Uddin, 2017).

The high volatility and complex dynamics of Bitcoin, along with the complex behaviors of the S&P 500 index and gold, necessitate advanced analytical tools. Traditional causality methods, such as Granger causality, assume linear relationships and may fail to detect causality in nonlinear systems. By employing CCM to analyze the causal relationships between Bitcoin and the S&P 500 and between Bitcoin and gold, this study aims to demonstrate the applicability and advantages of CCM in financial time series analysis—specifically for highly volatile assets such as Bitcoin.

Our dataset comprises daily closing prices ranging from 17/08/2017 to 19/04/2024. We utilize two data sources.

- For Bitcoin prices, we gather daily data from CoinMarketCap website.[3]
- We access historical financial market data for S&P 500 (SPX) and gold (GL) from NASDAQ.[4]

Bitcoin prices are available every day of the week, whereas gold and S&P 500 index prices are available only on US trading days. To ensure consistency, we removed weekend data from Bitcoin prices, resulting in a total of 1680 daily price points.

To apply CCM, which requires only one pair of time series, we focus on the closing prices of the assets under study. However, because closing prices are non-stationary, we analyze log returns instead of raw prices, following the approach of Ge and Lin (2021). Additionally, all data values were converted into integers to meet the requirements of the CCM analysis.

4.2. Library and window size

A common challenge in statistical analyses, particularly in state-space reconstruction methods such as the CCM, is the requirement of a sufficient sample size, which is closely tied to the attractor's dimensionality; the higher the dimension, the more data are needed for accurate reconstruction. In our analysis, this issue was compounded by using a windowing method to examine how the coupling between variables changed over time, effectively reducing the amount of data available in each window.

Thus, the concept of stationarity is crucial. Stationarity implies that the statistical properties of a system do not change over time,

[2] In their paper, Fama and French provide a comprehensive review of the capital asset pricing model (CAPM), which posits that the expected return of a security or a portfolio equals the rate on a risk-free security plus a risk premium. The risk premium is proportional to the systematic risk of the security or portfolio, which is measured by its beta relative to the market portfolio. The market portfolio, often proxied by a broad market index like the S&P 500, embodies the aggregate expectations and sentiments of investors regarding economic prospects.

[3] https://coinmarketcap.com/coins/

[4] https://www.nasdaq.com/market-activity/

thereby ensuring that the reconstructed attractor within each window accurately represents the system dynamics during that period. While ergodicity is a stronger condition that equates time averages with ensemble averages, state-space reconstruction methods such as CCM primarily require the system to be stationary or quasi-ergodic within the analysis window.

Sugihara (Sugihara et al., 2012) noted that Empirical Dynamic Modeling methods, including CCM, remain effective even with noisy and short time series of 35–40 points. Similarly, De Castro Martins (De Castro Martins et al., 2022) applied EDM to sample sizes of 200, resampled from the original time-series length of 50. Although increasing the sample size generally improves the analysis, our objective is to find a window length that is short enough to capture the evolving dynamics but long enough to ensure reliable attractor reconstruction within each window. By considering stationarity, we emphasize the importance of selecting window lengths that maintain consistent statistical properties of the system, balancing the need for temporal resolution with sufficient data for robust analysis.

4.3. Optimal embedding dimension

The optimal embedding dimension E determined during the analysis, is a good indicator of the use of sufficient data. High embedding dimensions suggest that system dynamics are complex and require more data for accurate reconstruction. By selecting window lengths that resulted in smaller optimal E values (less than the maximum tested value of 10), we ensured that each window contained sufficient data points to reliably reconstruct the system's attractor while maintaining stationarity within the window.

To determine the optimal embedding dimension E and the time to delay τ, and to select the embedding parameters, we performed a grid search by evaluating the prediction accuracy across a range of E from 1 to 10 and τ values from 1 to 3. We select the pair that produces the most accurate forecasts (that best unfolds the attractor) T_p = one time step ahead across the entire dataset through sliding windows (Sugihara, 1994; Sugihara & May, 1990).

This approach balances the need to capture temporal changes in coupling with the need for sufficient data for robust state–space reconstruction. Empirically, we found that a sample size of 100 per window with a 20-point step satisfied these conditions, capturing short-term dynamics while minimizing the influence of long-term trends.

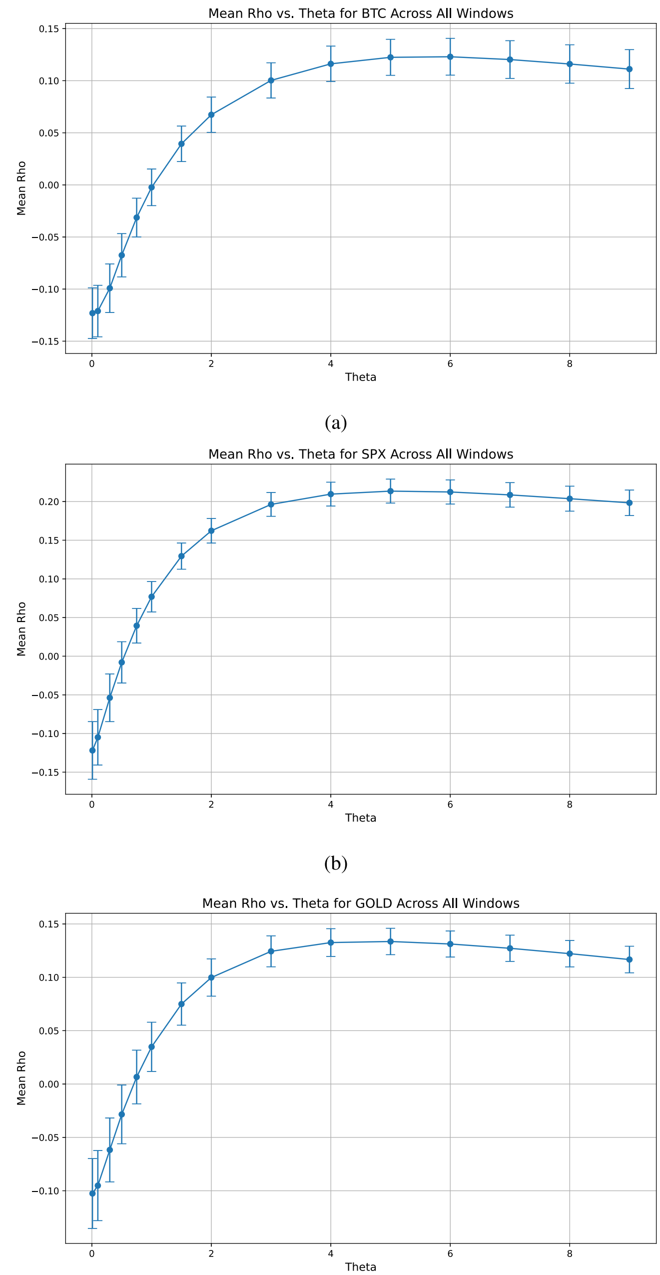
The optimal embedding dimension E and the corresponding τ are then applied to the Prediction Decay, S-map algorithm, and time-delay CCM analysis across all sliding windows to ensure consistency in our analyses.

4.4. Nonlinearity and chaos

After selecting the optimal embedding dimension and the corresponding lag of the time delay τ in each window, we used both the S-Map and Prediction Decay algorithms for the points in each window to measure the nonlinearity and chaoticity of the time series. Following Sugihara's work (Clark et al., 2015; Sugihara et al., 2012) it is important to understand that CCM is most effective in coupled nonlinear systems, meaning that both time series should exhibit nonlinear behavior.

Fig. 2 displays the mean and standard error of the prediction skill (ρ) of the S-Map algorithm, across all 53 sliding windows for the three assets: Bitcoin (Fig. 2(a)), the S&P 500 index (Fig. 2(b)), and gold (Fig. 2(c)). We see that as the weights (θ) increase, the prediction skill also increases, indicating nonlinear behavior for all variables.

Additionally, Fig. 3 shows the mean and the standard error of the prediction skill (ρ) of the Prediction Decay algorithm throughout all 53 windows. As prediction horizon (Tp) increases, we observe a sharp decrease in ρ, suggesting chaotic dynamics across all three assets (Sugihara & May, 1990).



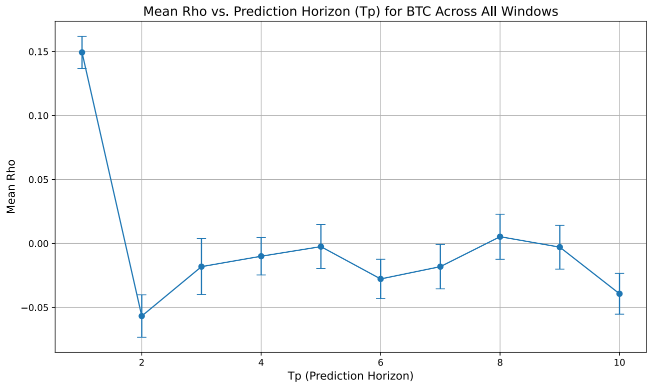
(c)

Fig. 2. The S-Map algorithm is combined with a sliding window approach to measure nonlinearity. For each value of θ, we calculated the mean and standard error of the predictive skill (ρ) across 53 windows. In all three assets – (a) BTC, (b) SPX, and (c) GOLD – the predictive skill increases as θ grows. This upward trend in ρ indicates nonlinear behavior in each of these markets.

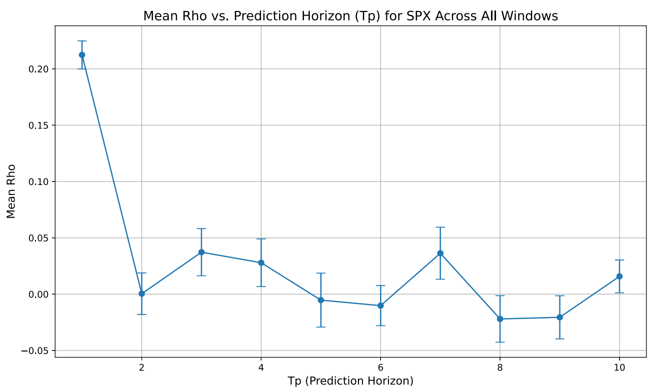
4.5. CCM analysis

Having confirmed that the variables exhibited nonlinear and chaotic behaviors across the sliding windows, our next objective was to determine the optimal lag between pairs of variables.

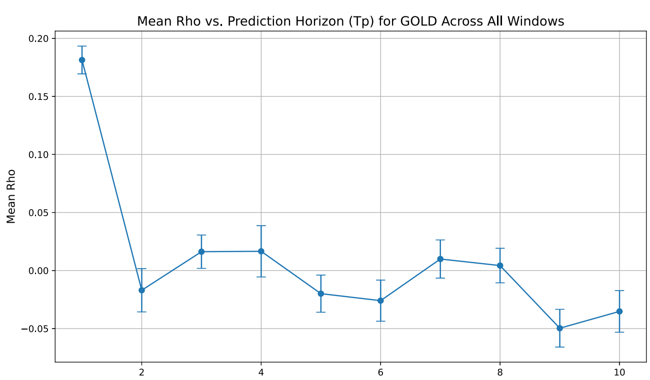
To do this, we ran TDCCM over a 10-day delay between variables to determine the duration of their effects on each other. By doing so, we identify the optimal lag at which a causal relationship appears, because true causation may be evident after several time steps and not immediately.



(a)



(b)



(c)

Fig. 3. The prediction decay algorithm is combined with the sliding window approach to measure chaoticity. For each value of the prediction horizon T_p, we calculated the mean and standard error of prediction skill (ρ) across 53 windows. In all three assets – (a) BTC, (b) SPX, and (c) GOLD – prediction skill has a sharp decrease with increasing prediction horizon (T_p), demonstrating chaotic behavior across all three assets.

We then used CCM to ensure convergence in certain windows because failing to account for convergence can produce misleading or spurious results.

To further validate our findings, we conducted a surrogate analysis. Surrogate testing (Theiler, Eubank, Longtin, Galdrikian, & Farmer, 1992) helps distinguish genuine interactions from random patterns. Specifically, we generated 100 surrogate time series from the original data and calculated the 95% quantiles of the resulting distribution of cross-mapping (ρ) values.

We then checked the convergence of each cross-mapping and compared the ρ values from these surrogate mappings to the actual cross-map ρ value obtained from the original data.

5. Experimental results

Our focus lies in exploring the relationships of Bitcoin paired with variables such as gold and the S&P 500 index. Here, we present and discuss the empirical results obtained from our analysis of CCM nonlinear and linear patterns of behavior. For simplicity, we present a few examples of the results.

5.1. Time delay CCM results

After identifying the time intervals at which both variables exhibit nonlinear or chaotic behavior, we determine the optimal delay (T_p) between the time series. Fig. 4(a), for instance, illustrates the effect duration between Bitcoin and the S&P 500 index. In this case, the optimal embedding dimension for Bitcoin is $E = 4$ with a corresponding $\tau = 3$.[5] In contrast, for SPX, $E = 3$ with $\tau = 1$. Time Delay CCM (TDCCM) shows that both variables share the same delay ($T_p = -3$) with a prediction skill of approximately 0.45 from SPX to Bitcoin—significantly higher than in the opposite direction. This suggests bidirectional coupling, although SPX appears to have a stronger influence on Bitcoin.

A similar analysis for Bitcoin and Gold is shown in Fig. 4(b), which shows the optimal embedding dimension of $E = 4$ for Bitcoin and $E = 1$ for gold, with $\tau = 2$ and $\tau = 3$ respectively. The highest CCM prediction skill was observed at $T_p = 9$ from Bitcoin to gold, and $T_p = -2$ from gold to Bitcoin. These results suggest that Bitcoin influences gold, as the dynamics of the Bitcoin time series best predict the future values of the gold time series.

5.1.1. Bitcoin & SPX result

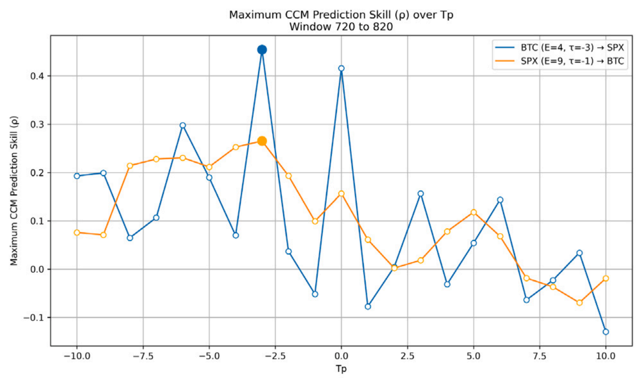
Fig. 5 illustrates the results of a CCM analysis between the S&P 500 index (SPX) and Bitcoin (BTC) over time. The presence of a solid marker indicates a statistically significant causation in a given time window. The blue line (BTC to SPX) indicates that the S&P 500 index influences Bitcoin, whereas the orange line (SPX to BTC) indicates that Bitcoin influences the S&P 500 index. The dashed lines mark the start (blue) and end (orange) of the COVID-19 pandemic period to contextualize the analysis.

The periods with a solid blue marker (BTC to SPX) demonstrate that the S&P 500 index significantly influences Bitcoin during these time windows. This implies that traditional financial market movements, represented by the S&P 500, have a predictive effect on Bitcoin's behavior during these specific periods. By contrast, periods with a solid orange marker (SPX to BTC) indicate a reverse relationship in which Bitcoin significantly influences the S&P 500. These findings suggest that, during certain times, Bitcoin serves as a leading indicator of market sentiment, possibly because of its growing role as an asset class with macroeconomic implications.

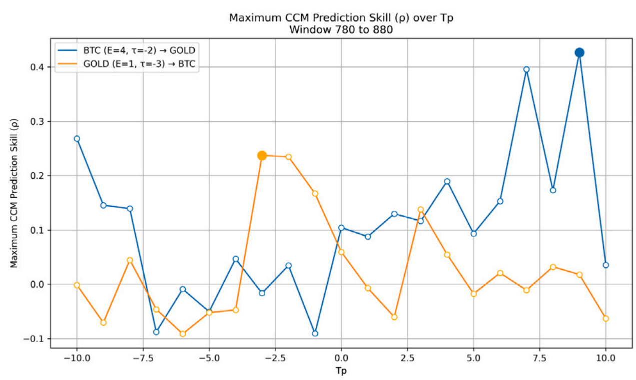
Time windows with blue and orange markers signify bidirectional causality, where the S&P 500 index and Bitcoin influence each other. These periods reflect stronger interdependence between the cryptocurrency and traditional financial markets. Bidirectional causality often occurs during periods of heightened financial stress or uncertainty when both markets react to shared global economic conditions or investor sentiment.

The absence of markers indicates a lack of significant causation in either direction, suggesting that neither the S&P 500 index nor Bitcoin has a measurable predictive influence on the other. This could reflect

[5] In the plots, correspondent negative τ values appear as a consequence of the Python implementation used in this analysis.



(a) Time Delay CCM between Bitcoin and S&P 500 index



(b) Time Delay CCM between Bitcoin and Gold

Fig. 4. In TDCCM, for $T_p \leq 0$, $X \rightarrow Y$ means Y is causing X; for $T_p > 0$, $X \rightarrow Y$ means X is causing Y because the dynamics of X best predicts future values of Y. If this is the case, then to confirm this, we use $Y \rightarrow X$, and for this, we should get values of $T_p < 0$. For cases where both $X \rightarrow Y$ and $Y \rightarrow X$ yields $T_p > 0$, then that is a sign that their interaction is being influenced by a third variable.

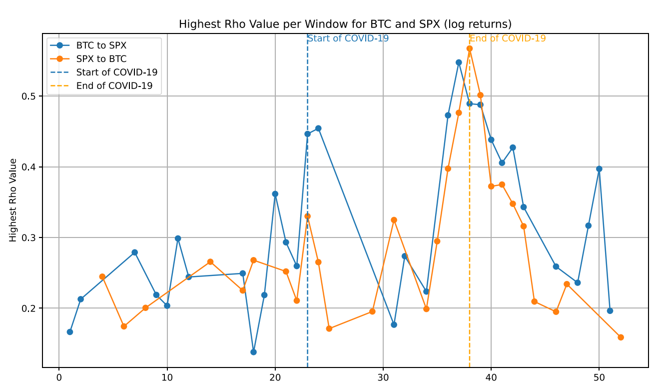


Fig. 5. CCM between Bitcoin and S&P 500 over time: Solid markers indicate significant causation in one or both directions, with dashed lines marking the start (blue) and end (orange) of the COVID-19 pandemic period.

periods of relative market independence or low volatility, in which each market follows its internal dynamics.

The dashed blue line represents the start of the COVID-19 pandemic, and the orange dashed line marks the end. At the onset of the pandemic, there was a clear spike in causation, with both unidirectional and bidirectional relationships emerging around that period. This finding indicates increased interdependence during this period, possibly driven by global uncertainty, synchronized market reactions, and heightened risk sentiment. Notably, SPX's influence on Bitcoin (blue markers) was

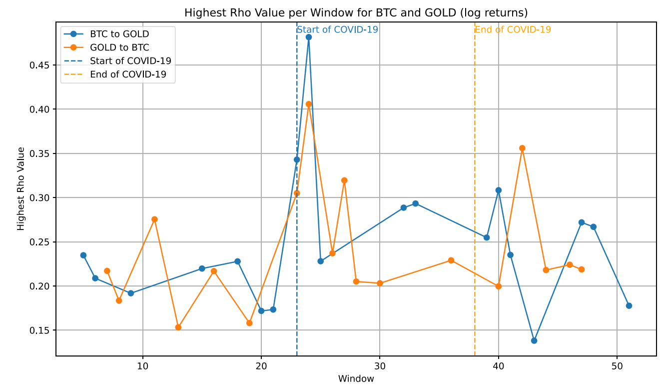


Fig. 6. CCM between Bitcoin and gold over time: Solid markers indicate significant causation in one or both directions, with dashed lines marking the start (blue) and end (orange) of the COVID-19 pandemic period.

more prominent during the initial stages of the pandemic, reflecting Bitcoin's sensitivity to movements in traditional markets.

Similarly, toward the end of the pandemic period, there was another peak in causation, with a notable bidirectional relationship. This likely corresponds to global economic recovery and shifting investor sentiment as markets adjust to post-pandemic conditions. The interdependence between SPX and Bitcoin during these periods underscores the influence of macroeconomic factors on both markets.

These findings suggest that the S&P 500 index, as a representation of traditional financial markets, often influences Bitcoin during times of economic stress or recovery. This highlights Bitcoin's integration into the broader financial ecosystem and its susceptibility to global market trends. Conversely, Bitcoin's influence on the S&P 500 index during certain periods could indicate its emerging role as an alternative asset class and an investor risk appetite indicator. The bidirectional causality observed during significant events, such as the COVID-19 pandemic, highlights the complex and evolving interplay between cryptocurrencies and traditional markets.

5.1.2. Bitcoin & gold result

Fig. 6 illustrates the results of a CCM analysis between Bitcoin and gold over time. Solid markers indicate statistically significant causation within a given time window. The blue line (BTC to gold) indicates that gold influences Bitcoin, whereas the orange line (gold to BTC) indicates that Bitcoin influences gold. The dashed blue and orange lines represent the start and end of the COVID-19 pandemic, respectively, providing context for the observed dynamics.

Periods with a solid blue marker (BTC to gold) demonstrate that gold significantly influences Bitcoin during these time windows. This suggests that movements in gold, as a traditional safe haven asset, have a predictive effect on Bitcoin's price dynamics. By contrast, periods with a solid orange marker (gold to BTC) indicate that Bitcoin significantly influences gold, which could reflect Bitcoin's growing relevance as a macroeconomic indicator or its competition with gold as an alternative store of value.

Time windows with both blue and orange markers signify bidirectional causality, where gold and Bitcoin influence each other. These periods reflect a strong interplay between the two assets, possibly driven by shared reactions to global economic conditions, shifts in risk sentiment, or changes in the perception of safe-haven assets.

The absence of markers in certain time windows indicates no significant causation in either direction. During these periods, Bitcoin and gold appeared to behave independently, likely reflecting times of low volatility or differing market drivers for these two assets.

The start of the COVID-19 pandemic (dashed blue line) corresponded to a significant increase in causation with notable bidirectional dynamics during this period. Gold's influence on Bitcoin (blue markers) is particularly prominent, highlighting Bitcoin's sensitivity to traditional safe-haven assets during times of heightened uncertainty. This suggests that Bitcoin reacted to movements in gold as investors sought to reposition their portfolios in response to economic shocks.

However, as the pandemic progressed, these dynamics shifted. Around the end of the COVID-19 period (dashed orange line), Bitcoin's influence on gold (orange markers) becomes more prominent, particularly in the early 40 s window. This suggests that Bitcoin was beginning to assert itself as a competing store of value and increasingly influencing gold's behavior.

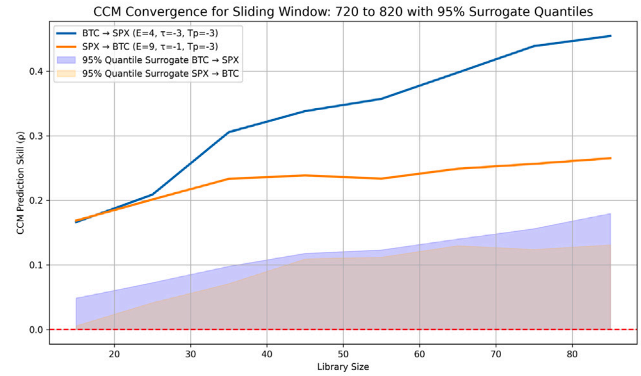
The results indicate that gold, as a traditional safe-haven asset, dominated the causality relationship with Bitcoin during the initial stages of the COVID-19 pandemic, underscoring Bitcoin's sensitivity to traditional macroeconomic indicators during periods of stress. However, the increasing influence of Bitcoin on gold toward the end of the pandemic signals a potential shift in investor sentiment, with Bitcoin emerging as a significant alternative asset.

5.2. CCM results

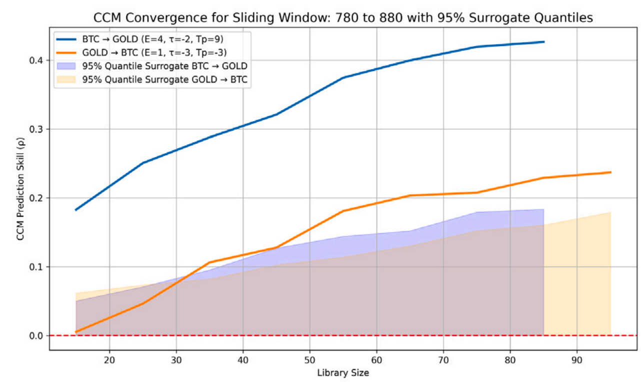
After determining the optimal delay (T_p) from the time-delay CCM, we analyzed the convergence of the prediction skill (ρ) over the same intervals. We show how prediction skills increase as the library size increases for each pair of assets. The results presented in Fig. 7(a) for Bitcoin-S&P 500 index and Fig. 7(b) for Bitcoin-gold, demonstrate that prediction skill (ρ) improves with larger library sizes, reinforcing the presence of a causal relationship between these variables.

5.3. Surrogate testing

To confirm the significance of the convergence of these causal effects, we conduct a surrogate time-series analysis. We applied the same parameters used for CCM, maintaining the optimal embedding dimensions of the variables across all windows. No significant causal links were observed between the variables, as shown in Fig. 7(a) for Bitcoin-S&P 500 index and Fig. 7(b) for Bitcoin-gold. By comparing the (ρ) values from the actual time series with those from the surrogate sets, we confirmed that the observed causal relationships – and their convergence – were statistically significant and were not simply artifacts of chance.



(a) CCM of Bitcoin and the S&P 500 index. The blue line represents the prediction skill from BTC to SPX, and the shaded blue area is the 95% quantiles of the corresponding surrogate analysis. The orange line represents the prediction skill from SPX to BTC, and the shaded orange area is the 95% quantiles of the corresponding surrogate analysis.



(b) CCM of BTC and GOLD. The blue line represents the prediction skill from BTC to GOLD, and the shaded blue area is the 95% quantiles of the corresponding surrogate analysis. The orange line represents the prediction skill from GOLD to BTC, and the shaded orange area is the 95% quantiles of the corresponding surrogate analysis.

Fig. 7. In both plots 7(a) and 7(b) we see an increase in the prediction skill as the library size increases, suggesting its convergence and resulting in the causal relationship. Furthermore, in both cases, we see from the surrogate testing that these values are significantly lower than the values from the real time series, confirming the causal relationship.

6. Conclusions

In this study, we applied the time delay convergent cross-mapping method to detect causality between Bitcoin and other financial assets such as the S&P 500 index and gold. Our goal is to explore how well CCM can identify the connections between Bitcoin prices and traditional financial assets over time. By combining CCM, simplex projection, and the S-Map algorithm with a sliding window approach, we aim to effectively identify and analyze nonlinear patterns in these time series over short periods.

To the best of our knowledge, there has been little research on the causal relationships between cryptocurrencies and financial markets using CCM. Existing studies focus on sentiment analysis to predict cryptocurrency prices and causality between the Chinese and American stock markets and global stock markets. We aim to explore the applicability of CCM in cryptocurrency and financial markets.

The effect of nonlinearity on the results is an important factor to consider. It is still uncertain whether chaoticity actually strengthens causal relationships or if it makes CCM more sensitive in detecting them. This distinction is important when applying this methodology because it could be a potential limitation in interpreting the results. We leave this investigation for future studies. Given the rapidly changing nature of cryptocurrencies and financial markets, further research is essential for a deeper understanding of these relationships. Future studies could benefit from incorporating more dynamic data, exploring the causality among multiple variables, and distinguishing between positive and negative causal interactions. Moreover, the use of the Lyapunov exponent to measure chaoticity can extend our approach. Further analysis is essential to fully understand how these causal relationships change over different periods.

CRediT authorship contribution statement

Albi Isufaj: Methodology, Software, Investigation, Data curation, Formal analysis, Writing – original draft. **Caio De Castro Martins:** Formal analysis, Writing – review & editing. **Marc Cavazza:** Conceptualization, Methodology, Resources, Writing – review & editing, Supervision. **Helmut Prendinger:** Conceptualization, Validation, Writing – review & editing, Visualization, Supervision.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the authors used ChatGPT from OpenAI in some sections for English editing and style-based reformulation, excluding ideation and the generation of supporting arguments.

After using ChatGPT, the authors reviewed and edited the content as needed and takes full responsibility for the content of the publication.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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