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Exploring new ways to connect

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Editorial

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This editorial of the Proceedings of the Eleventh International Mathematics Education and Society Conference explains the circumstances and organisation of the conference, presents the contents of this book and the review process that lies behind its production, and acknowledges the contributors who invested their resources to make this conference and this book a reality.

The conference: Exploring new ways to connect

These are the Proceedings of the Eleventh International Mathematics Education and Society (MES) Conference, to be held in September 2021. Although this is already the eleventh MES conference, this is the first one that is held entirely online, as so many conferences are at times of the COVID-19 pandemic. It is a huge challenge to organise a conference, which is traditionally held in a physical form, purely virtually – even more so in the case of an MES conference, which does not only feature presentations, which can easily be streamed, but includes intense discussions in various formats that cannot be easily copied to the digital realm. In this sense, the conference organisers and participants face the task of 'exploring new ways to connect'.

The conference organisers sticked as closely as possible to the traditional Principles and Guidelines of the MES community (https://www.mescommunity.info). This includes

- ensuring regional, ethical, and gender diversity concerning the choice of the four plenary speakers, of their two respondents each, and of the four plenary discussants,
- providing space for intense small-group and plenary feedback discussions on each plenary presentation,
- allowing for thematic specialisation through symposia, and
- facilitating lively discussions of individual research papers, project presentations, and posters.

A special challenge was the organisation of the times slots in which conference activities would be scheduled. Our experience from other online conferences, where the usual conference day was organised in eight consecutive hours, was that this solution suits people in a few time zones very well but makes participation for people in other time zones nearly impossible. We decided to distribute three three-hour time slots around each day, so that only one time slot would be at night at any time zone on Earth. Although this approach largly fragments the conference days and makes it very complicated to enjoy all programme points of the conference, we hope that it will stimulate participation from a large varierty of places.

The difficulty of scheduling meetings across time zones motivated us to introduce the rule that live meetings should be used for exchange, while the reception of information should be possible on an individual schedule. Thus, we do not only follow the traditional

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MES policy to publish the proceedings before the conference, so that the papers will be only for reading in advance. We also invited contributors to create short videos of their paper presentations and to host them for the conference participants, so that participants can come to the live sessions well prepared and use them mostely for feedback and discussion. We asked our participants to include the tag 'MES11' in the videos they host on video hosting platforms such as YouTube, and as long as authors followed this request and left their videos online, the reader might be able to still find some presentations searching for this tag.

Although switching to an online conference brought the obvious challenges, it also broad new possibilities, which we are eager to explore. Apart from technological innovations such as discussions on Padlet and informal meetings on SpatialChat, the online character of the conference allowed for a more inclusive participation from around the world. While physical conferences cause expenses for travelling, accommodation, eating out, and hosting, which not every scholar from any cultural and economic background can easily shoulder, our online conference only asked for a conference fee in three different income-dependent tariffs. Consequently, we could witness a wide variety of places from where contributions were submitted, and from where participants registered. At the time of the publication of these proceedings (7 September 2021), we had 46 registrations from the United States, 31 from Brazil, 21 from Sweden, 17 from India, 14 from Canada, 13 from the United Kingdom, 11 from Norway, 9 from Austria, 7 from Germany, 7 from New Zealand, 5 from Greece, 4 from Israel, 3 from Colombia, 3 from Japan, 2 from Nepal, 2 from Spain, 1 from Armenia, 1 from Australia, 1 from Chile, 1 from Egypt, 1 from Ghana, 1 from Indonesia, 1 from Rwanda, 1 from Saudi Arabia, 1 from South Africa, and 1 from Turkey.

The contents of this book

In this book, you find the manuscripts of nearly all contributors to the conference. Thereby, MES aims at facilitating a 'wider discussion of the social, ethical, and political dimensions of mathematics education for disseminating theoretical frameworks, discussing methodological issues, sharing and discussing research, planning for action and the development of a strong research network on mathematics education and society' ('Mathematics education and society', n.d.).

In the *first part*, titled 'Plenaries and Responses', you find the papers which accompany three of the four plenary presentations. We were not able to include the paper of plenary speaker Maisie L. Gholson, nor the papers of the respondents Ana Carolina Faustino and Luz Valoyes-Chávez, but we hope to be able to provide them later on the MES website. In all cases, both plenary speakers and respondents were chosen and invited by the MES11 organising team, and we are very happy and grateful that they agreed to invest their qualities in MES.

In the *second part* of this book, titled 'Symposia', you find the texts of the symposium proposals. These papers merely outline the contents of the symposia and were used to judge the quality of each proposal for its acceptance for the conference. The *third part* of this book, titled 'Project Presentations', contains the short papers in which envisaged or currently running research projects are being presented. The *fourth part* of this book, titled 'Poster Descriptions', comprises the descriptions of the posters that were submitted to the conference. The last and largest *fifth part*, titled 'Research Papers', includes the full-length manuscripts of the research papers submitted for presentation and discussion at the conference.

Symposia proposals, project presentations, poster descriptions, and research papers were each reviewed by two peers, who were already acquainted with the goals and policies of MES, and finally evaluated by a member of the International Committee of MES. Although the reviewing process of MES aims at productive feedback that allows manuscripts to reach the quality necessary for publication, it still checks the academic quality of each submission and its fit to the interests and aim of the MES community. In a few cases, where contributors challenged the boundaries of what would usually be accepted as academic work, we strived to tolerate and accept these contributions, also in the interest of further developing the forms of academic inquiry and exchange through experimental formats.

Nevertheless, in the reviewing process, some submissions had to be withdrawn or rejected because of a lack of fitting to the interests and aims of the MES community or because of a lack of academic quality. Other submissions were withdrawn by their authors for personal reasons or rejected because revisions were not sent even after a widely extended deadline. Eventually, this book contains 133 of 139 submitted research papers, 29 of 35 submitted project presentations, 7 of 8 submitted symposium proposals, and 4 of 5 submitted poser descriptions.

Acknowledgements

Organising a high-quality conference for more than 200 participants is a huge task that cannot be shouldered by a few. Consequently, there are many people to give thanks to.

First of all, I would like to thank the MES11 Organising Team, that is Yasmine Abtahi, Lisa Darragh, David Kollosche, Renato Marcone, Amanda Queiroz Moura, João Pedro Antunes de Paulo, Luz Valoyes-Chávez, and David Wagner, for our weekly online meetings, where countless issues were discussed and decisions were reached as some drank their morning coffee while others had their evening drink. It was a pleasure to learn and work with you, and your help is unvalueable.

Then, I would like to thank the International Committee of MES, that is Yasmine Abtahi, Jehad Alshwaikh, Arindam Bose, Anna Chronaki, MeganClune, Lisa Darragh, Peter Gates, Maisie Gholson, Beth Herbel-Eisenmann, David Kollosche, Gregory Larnell, Brian R. Lawler, Jasmine Ma, A Mani, Renato Marcone, Alexandre Pais, Julio César Paro, João Pedro Antunes de Paulo, Anita Rampal, Milton Rosa, Kate le Roux, Jayasree Subramanian, Luz Valoyes-Chávez, David Wagner, Paola Valero, Steven Watson, and Pete Wright, first, for their trust in my organisational skills, but then more specifically for providing advicing and supporting decisions, for monitoring the complex review process, and for organising the early bird language support.

Further, I owe a debt of gratitude to my local supporters, that is Amanda Queiroz Moura, Margit Pirker-Zedlacher, Gordana Gajić, and especially Martin Köfer, for shouldering much of the work that often goes unnoticed but is urgently necessary for the administration of such a big event. In this sense, I also thank our institution, the University of Klagenfurt, whose administration was always eager to help and meet our needs.

Eventually, I would like to thank all the participants for bringing this intellectual richness to the conference. Special thanks go to those 46 participants who donated money to support socio-economically unpriviledged scholars to participate at future physical MES conferences.

References

'Mathematics education and society'. (n.d.). [Website]. Retrieved September 1, 2021, from https://www.mescommunity.info

Plenaries and Responses

Innovative learning environments and the digital era: Finding space for mathematics identity

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The dichotomy of traditional versus reform mathematics classrooms has been of much research interest, including how students and teachers perform agentic identities in these contexts. Nowadays, however, mathematics learning in Aotearoa New Zealand plays out on a stage that may be a vast departure from either of these classrooms. So called "innovative learning environments" are characterised by fluid seating arrangements, multiple teachers, many digital devices; and they may be predominantly 'online'. In fact, we might argue that mathematics instruction today is 'device-centred' more than being teacher- or student-centred, a trend exacerbated by distance-learning during the pandemic. How then do teachers and students develop mathematics identities in this new era? A performative definition for identity allows us to see identity more easily as existing outside the individual; not only produced in and by social contexts, but also in wider societal narratives. In this paper I will discuss how 'innovative' learning environments are situated in neoliberal ideology and "twenty-first century" discourses, and I consider the production of teacher and learner identity scripts in these spaces.

Setting the scene

Let me start by painting a picture of a primary classroom in Aotearoa New Zealand. First you must delete your image of four walls, a desk for each child, and a forward-facing orientation. The floor plan of this classroom is hexagonal – an irregular polygon that would fit three or more traditional classroom squares. There are sectioned off spaces or smaller rooms with walls of glass. The furniture is varied and optional; children may sit on the floor, on cushions, on beanbags, at low tables, at higher tables, or they may stand or even lie down. Their belongings are in 'tote trays' so that they are mobile. The devices are mobile as well – children may bring their own laptops, Chromebooks, or tablets/I-pads; and the classroom also has its own collection of these for the children to use. There are multiple teachers but they may not be easy to spot, not being situated front and centre. I suspect for some it will be difficult to reconcile this scene with a more traditional image of the mathematics classroom that would otherwise automatically spring to mind. I invite you to click on the link below and watch the short video titled "Understanding pedagogy" (from: Te Kete Ipurangi (TKI): Ministry of Education, 2021).

https://www.inclusive.tki.org.nz/guides/planning-innovative-learning-environments-iles/

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The scene above does not describe *every* classroom in Aotearoa New Zealand, there is certainly a great deal of variety in classroom types, however it (and the embedded video) represent the direction taken in designing educational contexts over the past decade. Let us call these examples the "new" classroom, in order to differentiate and acknowledge that there are certainly other, more traditional classrooms to be found elsewhere throughout the country.

Of course, I have described a pre-pandemic classroom. Even after the return from lockdowns and emergency remote teaching it was certainly inappropriate to have such a high level of interaction between children and free movement through the space. On the other hand, the use of digital technologies that were already prevalent has increased. In Aotearoa New Zealand we are fortunate to have returned (at the time of writing) to 'normal' classroom interaction, but there remains the question of what is normal, or what might be the new normal? As Borba and colleagues suggest, the pandemic may entirely change the agenda for mathematics education (Borba, 2021; Engelbrecht et al., 2020). However, we have already seen how crises may be harnessed as a rationale for wide-scale educational change (Mutch, 2017; Williamson, Eynon & Potter, 2020), and I suspect it is safe to assume that these technology-rich, innovative learning spaces, such as seen in the link above, will become increasingly common, at least in Aotearoa New Zealand.

One might ask what *mathematics* teaching and learning look like in the 'new' classroom space. Whilst it seems clear that the 'new' classroom is a vast departure from the traditional classrooms of last century, my question is not whether the innovation constitutes an improvement to mathematics teaching and learning, rather, I am interested in how these environments (including online environments) produce identities as teachers and learners of mathematics. This is an important question given the value of identity research in understanding teacher change (e.g., Chronaki & Matos, 2013; Lutovac & Kaasila, 2017) and students' relationships with and participation in mathematics (e.g., Mendick, Moreau, & Epstein, 2009; Radovic, Black, Salas, & Williams, 2017).

In this paper I first outline a definition of identity as performative, following Judith Butler's work on gender identity. I extend her theatrical metaphor to propose a method by which we may more easily understand identity as beyond the individual, and produced by the wider socio-cultural and political context. Next, I will briefly give an account of the historical, educational context of Aotearoa New Zealand – the 'theatre'. Then I will discuss the 'stage' for learning mathematics (and producing identity), in this case innovative learning environments (ILEs) that include online programs for mathematics instruction, as I term the 'new' classroom. The discussion is based on my current research into the phenomenon of learning mathematics via online instructional programs and my personal observations of ILE classroom spaces. Additionally, in lieu of data, I invite you to consider the video linked above, and also the website for *Mathletics* (3P Learning, 2021, https://www.mathletics.com/nz), both of which I will refer to throughout.

Performative identity and extending the theatre metaphor

A few years ago, Darinka Radovic and I were invited to write a definition for mathematics learner identity for the *Encyclopedia of Mathematics Education* (Lerman, 2020). We were tasked to give a definition that *reflected* (rather than advanced) the work on identity in mathematics education currently. We finally settled on the following:

> A socially produced way of being, as enacted and recognized in relation to learning mathematics. It involves stories, discourses and actions, decisions, and affiliations that people use to construct who they are in relation to mathematics, but also in interaction with multiple other simultaneously lived identities. This incorporates how they are treated and seen by others, how the local practice is defined and what social discourses are drawn upon regarding mathematics and the self. (Darragh & Radovic, 2018, para. 1)

Although the above definition focuses on mathematics learner identity, a mathematics teacher identity may be thought of in a similar way. We aimed to write a definition that encompassed the various ways that mathematics learner identity is defined and operationalised in the discipline. Each aspect: (e.g., socially produced, enacted, recognized, multiple) is evident in the wider literature, but each of these aspects may be understood slightly differently depending on the perspective taken. There are certainly many ways to understand identity, and sometimes definitions are vague or absent altogether in literature published in the field. Elsewhere we have both argued that authors must be explicit in the way they define and operationalise identity for the purpose of their research (Darragh, 2016; Radovic, Black, Williams, & Salas, 2018), and so I will attempt to be explicit here. I take a *performative* view of mathematics identity, drawn from Butler (1988), that explains the social production of identity acts and highlights the role of recognition as part of identity. Others in mathematics education have seen value in Butler's work for identity (e.g., Chronaki, 2011; de Freitas, 2008; Gholson & Martin, 2019; Mendick, 2017); I find it useful myself because it allows an operationalisation of identity that incorporates more than interview narratives, and focuses the gaze beyond the individual to look at identity enactment and the production of identity within various layers of context. This enables a more thorough understanding of the relationships people form with mathematics learning and teaching and of the decisions they make regarding future participation in higher education or engagement with professional learning.

Judith Butler defines *gender* identity as performative, that is, a "stylised repetition of acts" (Butler, 1988, p. 519). Butler argues "a body becomes its gender through a series of acts which are renewed, revised, and consolidated through time"; thus identity is not predetermined but rather "the legacy of sedimented acts" (p. 523). I find it very useful to consider mathematics learner or teacher identity in the same way, that is, a series of acts which are renewed, revised, and consolidated over time. Butler wrote this particular definition in the '*Theatre Journal*', which may explain the theatrical emphasis on the idea of an 'act'; however, it is worth noting the theatre metaphor has generated some confusion as it tends to imply a separation between the actor and the act, as opposed to a poststructuralist understanding of

subjectivity where the act produces the actor (Jagger, 2008), which is key to performativity. Yet, I see much potential in exploiting the theatre metaphor in order to make explicit the wider social and political context in which identities are performed and performatively produced. We may consider the socio-political context to be the *theatre* and the immediate social context (e.g. the classroom) as the *stage*. We might ask ourselves what the typical *scripts* or normative ways of being are, and whether *improvisations* are possible. Finally, the *audience* takes up the important role of recognising (and validating) identity performances. I contend that understanding how identity is performatively produced requires a consideration of the wider temporal, ideological, and physical contexts.

I would like to unpack and operationalise this theatrical metaphor as I consider how the 'new' classroom context produces mathematics identity. I invite you to contemplate this metaphor alongside me as you view the linked video, the website from one popular online mathematics instructional program, *Mathletics* (3P Learning, 2021); or you may like to reflect on your own context. First of all, I situate us within the wider socio-political context of Aotearoa New Zealand's neoliberal educational system.

The 'theatre': Socio-political-historical context of education in Aotearoa New Zealand

In Aotearoa New Zealand, like many other countries around the world, neoliberal policies have dominated the political scene over the past three decades thus impacting the education system considerably (Ladd & Fiske, 2003; McMaster, 2013). Neoliberalism is an economic and political ideology that proposes individual, entrepreneurial freedom and is characterised by free markets (Harvey, 2005). Some of the ways we see neoliberalism at work in education is the devolution of control from central government to individual schools; 'free choice' for parents in where they send their children to school; voucher systems where the money follows the student; and outsourcing of educational provision to private providers (Thrupp, O'Neill, Powell, & Butler, 2020). Although an early adopter of neoliberal education, Aotearoa New Zealand has resisted some of the policies that have become entrenched in other nation's educational systems (McMaster, 2013). However, some neoliberal features that remain are: devolution of governance and curriculum to local schools (Lange, 1988; Ministry of Education, 2007; O'Neill, 2011), and private providers being allowed to make profit in public schools via educational provision of particular subject areas, including mathematics (Thrupp et al., 2020).

The beginning of neoliberalism within education for Aotearoa New Zealand was the policy document "Tomorrow's Schools" (Lange, 1988). *Tomorrow's Schools* devolved much responsibility for education to individual schools, managed by 'Boards of Trustees', which were made up of parents in the community. One intention of the policy was to give greater voice to parents and the community (McMaster, 2013). This allowed a great deal of autonomy to individual schools and, together with the 1991 abolition of school enrolment zones,

encouraged school choice and competition for students (Ladd & Fiske, 2003; McMaster, 2013). Curriculum reform swiftly followed, with new curricula in 1992, and again in 2007. The 2007 curriculum is remarkable for the small size of the document. All eight content areas and all 13 years of schooling are contained in a document of only 65 pages, of which mathematics has no more than 10 pages (Ministry of Education, 2007). In other words, the national curriculum contains only a list of achievement objectives without guidelines of how to teach them. The impetus was given to schools to create their own localised curriculum based on this document. On one hand this meant schools were able to cater educational experiences to the local community's wants and needs (see also McMurchy-Pilkington, Trinick, & Meaney, 2013). On the other hand, some schools struggled to develop a curriculum that was sufficient for the needs of their students. In either case, the variety of different practices in schools has increased under this policy.

Another prominent feature of the 2007 curriculum is the focus on producing "twenty-first century learners" (Benade, 2015; Ministry of Education, 2007). This is a global movement, driven in part by the Organisation for Economic Cooperation and Development (OECD), which has questioned "'outmoded' transmission models of teaching" and called for reform of educational systems (Benade, 2019, p. 58; see also OECD, 2013). In Aotearoa New Zealand this "transformation is increasingly evident in new technology-rich flexible learning environments, characterised by large open spaces, permeable boundaries and diverse furnishings emphasising student comfort, health and flexibility" (Benade, 2019, p. 58). Benade also notes that the design reflects the "neoliberal concern with ensuring that education is relevant to the realities of the twenty-first century workplace" (Benade, 2017, pp. 804–805), as shall be elaborated further later.

A more recent event that had a significant impact on schooling in Aotearoa New Zealand was the 2010 earthquake in Christchurch. The disaster was used as a justification for widespread permanent closure of some schools, to the devastation and surprise of their local communities (Mutch, 2017). With the post-earthquake re-building of schools, many were designed using "Innovative Learning Environment" (ILE) guidelines – with classroom spaces as described in the opening paragraphs. Subsequently, all school builds, expansions, and renovations in the entire country have been required to follow the ILE design in order to receive funding (Bradbeer et al., 2017). The ILE space certainly looks innovative and modern, but it has been criticised for not being based on research evidence (Bradbeer et al., 2017), motivated either by lower cost or neoliberal ideology, and evidence of the way a crisis can be harnessed by politics for educational change (Mutch, 2017). Over the past decade this has meant many schools have realigned their classrooms to the ILE model, although others remain traditionally 'single cell'.

Within this neoliberal and 'technology rich' context, mathematics education has increasingly drawn on digital technologies as a regular part of instruction. A recent OECD survey found that children in Aotearoa had the fastest growing rate of computer use in mathematics classes in the OECD with 89% of students using this technology as part of their

mathematics learning (Vincent-Lancrin, Urgel, Kar, & Jacotin, 2019). With such high availability of the technology, it is hardly surprising that the majority of schools have turned to commercial programs to provide instructional material using these computers and mobile devices. As many as 80% of primary schools in Aotearoa New Zealand subscribe to at least one online mathematics instructional program (OMIP), such as *Mathletics, Study Ladder, Mathsbuddy*, and *Sumdog* (Darragh & Franke, 2021). These online learning platforms are run by private corporations; they are typically of international origin but adapted to the national curriculum. Schools or parents may purchase an annual subscription and there is sufficient content on the platforms to provide mathematics learning for the full year. Although the COVID-19 pandemic exposed inequities of access to internet and digital devices at home (Riwai-Couch et al., 2020), children in Aotearoa New Zealand now have considerable access to devices and the internet at school (Vincent-Lancrin et al., 2019).

Aotearoa New Zealand was fortunate to avoid the health crisis experienced by many other nations during COVID-19, however the *educational* impact has been widely felt as we had repeated (albeit brief) lockdowns to halt community transmission and children were in and out of school depending on the health alert levels. Given that some parents were desperate to provide extra learning activities for their children during lockdown (RNZ, 2020, 28 July) those companies that provide online instructional programs may have had the opportunity to cement themselves further into the educational landscape; indeed, the marketing of such programs worldwide has ramped up considerably since the pandemic began (Williamson et al., 2020).

To summarise, the context of ILEs and online learning platforms for mathematics instruction can only be understood within the neoliberal policy agenda of education in Aotearoa New Zealand. This agenda includes: the necessity for schools to create their own local curriculum and decide whether to use private providers for part of the mathematics instruction, and the governmental requirement to create ILE spaces with new builds. In the next section I zoom in so that we may examine more closely this particular stage for the learning and teaching of mathematics. As we do this, I invite you to consider your own particular 'person of interest'. This may be the newly qualified mathematics teacher, the teacher who works in poverty-stricken areas, the 'out-of-field' teacher, or the white, middle-class teacher in diverse contexts. Or the person of interest may be the neuro-diverse child, the high achiever, the hearing impaired, the Black girl, the recently arrived immigrant, or the child who engages with schooling to revitalise their indigenous language.[1] As I present the 'new' classroom stage for mathematics teaching and learning in Aotearoa New Zealand, I invite you to ponder this question: Can you picture your person in this space?

[1] My apologies here – this list is clearly non-exhaustive and I am aware that I may have missed your particular 'person of interest' thus marginalising them further (literally) in this footnote.

The 'stage': Innovative learning environments and online mathematics learning

Whilst the 'theatre' attends to the broader socio-political context, the 'stage' may describe the context at the local level, i.e., the mathematics classroom. There are a number of studies on identity at this level of the 'zoom lens' (Lerman, 2001), but most are based in single-cell classrooms, whether they incorporate 'traditional' or 'reform' instruction (see for example Esmonde, 2009; Heyd-Metzuyanim & Shabtay, 2019; Ma & Singer-Gabella, 2011; Valoyes-Chávez, 2019). In this section I will first describe the 'new' stage for learning mathematics – the ILE space and online instructional programs. I follow with an explanation of how students may negotiate the space when learning mathematics and then discuss a couple of features of this 'new' stage that set it apart from classrooms of the past.

In a typical ILE environment, children learn in classroom spaces that are designed to fit 40-160 students together with 2-6 teachers (Everatt, Fletcher, & Fickel, 2019). Whilst the teachers are supposed to work collaboratively to teach the entire group, children are required to navigate the space independently. Benade gives an evocative description based on research within Aotearoa New Zealand schools:

> The placement of tables and chairs, often boardroom style, is a place where a 'workshop' can take place, facilitated by a teacher, or, more appropriately, a 'learning advisor' or 'coach'. A private space off to the side for a small group to work together is a 'breakout space'. Along with this neoliberal language of the business conference is the imagery of the future hunter-gatherers of the twenty-first century knowledge economy gathering at the 'campfire' (a circular formation of Ottomans). Redolent of captivating tales or fellowship, this is a space of gathering together before expedition, or debriefing after. Thirsting for knowledge, some young cubs work intently at a 'watering hole', a circular arrangement of seats and tables, where they plan their next project. For those who are required to work on complex tasks (such as numeracy) there are the high tables and chairs that provide a 'lookout', allowing these students to gaze intently into the long distance, as they solve challenging problems. (Benade, 2017, p. 802)

Where might we see the teacher in this space? There is little room for the traditional 'chalk and talk'. The teacher becomes a facilitator - assigning learning activities and managing the learning environment. And at times we may see the teacher engaging with a small group of children (at the "campfire") and teaching them some mathematics content.

The hunter-gatherer imagery employs a different metaphor to the idea of the stage within a theatre, yet it is scenery that is taken up by a number who write about ILE classrooms. The metaphor comes from Thornburg (e.g., 2004), who also describes the 'cave' as being a space for internal reflection, in addition to the more populated learning spaces of waterhole and campfire. In the above scene, the place given to mathematics appears isolated – children at high tables (rather than the low, collaborative spaces) deep in thought as they solve challenging problems. However, mathematics learning may be seen in the other spaces too. Children might be able to engage in small group, collaborative problem solving (at the watering hole) – but we also see many children seated either individually or in pairs and with some kind of digital device (laptop, chromebook, or tablet, for example). What

mathematics learning do these devices provide? In Aotearoa New Zealand it is likely the children will be engaged with commercial platforms that provide individualised learning online (Darragh & Franke, 2021).

Thornburg suggests that each of the neolithic scenes (the campfire, the watering hole and the cave) may be replicated in the online environment. Indeed, one thing missing from Benade's description above (though he refers to it elsewhere, e.g., Benade, 2015) is the presence of modern technologies in these spaces. As previously discussed, a key aspect of twenty first-century learning and ILE spaces are their "technology-rich" nature. Children and teachers in Aotearoa New Zealand have ready access to computers and hand-held devices and the internet at school (Vincent-Lancrin et al., 2019), and this makes the use of OMIPs during mathematics somewhat unsurprising. The most popular platform is *Mathletics* (3P Learning, 2021), originally designed in Australia, and subscribed by approximately 40% of the schools in Aotearoa New Zealand that use OMIPs (Darragh & Franke, 2021); there are dozens of other OMIPs to choose from.

The OMIP platforms draw on behaviourist techniques to motivate children to do maths (Jablonka, 2017); for example, completing a number of exercises is rewarded with a game, or the mathematics work is 'disguised' in a gaming format. Jablonka (2017) discussed this gamification in the context of OMIP "Sumdog" at a previous Mathematics Education in Society conference. Many of the platforms offer 'payment' for doing the mathematics in the form of 'tokens', 'coins', or 'points' that may be spent on designing avatars (or purchasing related paraphernalia) during the reward phase. This is the case for *Mathletics* (3P Learning, 2021): Figure 1 below shows the points, gold bars, and certificates achieved by the depicted (cartoon) 'learner'. Figure 2 depicts information directed to the teacher – learner analytics, with an example of the type of statistical information given about the learner's performance and progress.

In general, research within mathematics education is remarkably silent on the use of OMIPs. The plethora of studies into technology use in mathematics tend to focus on issues of teaching and learning (Young, 2017), and they tend to take an uncritical view of the technology itself; greater attention is given to the benefits to learning or the challenges of having teachers adopt the technology. By contrast, outside of mathematics education and particularly in the field of Media Studies we find much critique of the EdTech (educational technology) industry. The aspects of EdTech of concern centre on the personalisation of learning (Roberts-Mahoney, Means, & Garrison, 2016), and 'big data' collected via 'learner analytics' (Knox, Williamson, & Bayne, 2020). Personalising learning may be argued a valid goal for education but, as pointed out by McRae (2013), the 'hyper-individualisation' of the programmes is reductionist (mathematics becomes basic facts, for example) rather than providing personalised learning based on student interest or cultural background. In other words, learning is individualistic and individualised (Biesta, 2012), that is, centered on the *individual learner,* rather than being centred on students as a group. Such an approach does

not consider the backgrounds and interests of the children as a community of learners in the classroom and thus the teachers' situated knowledge of them is de-valued in this model.

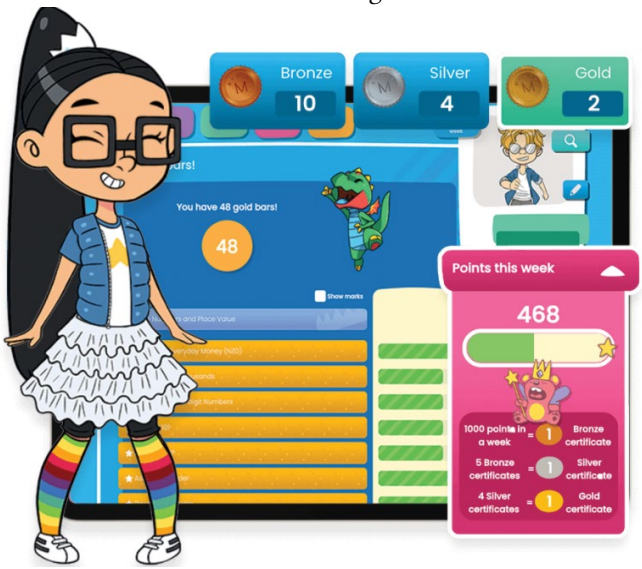

Figure 1: Rewards earned by the *Mathletics* student (3P Learning, 2021) see: https://www.mathletics.com/nz/features/

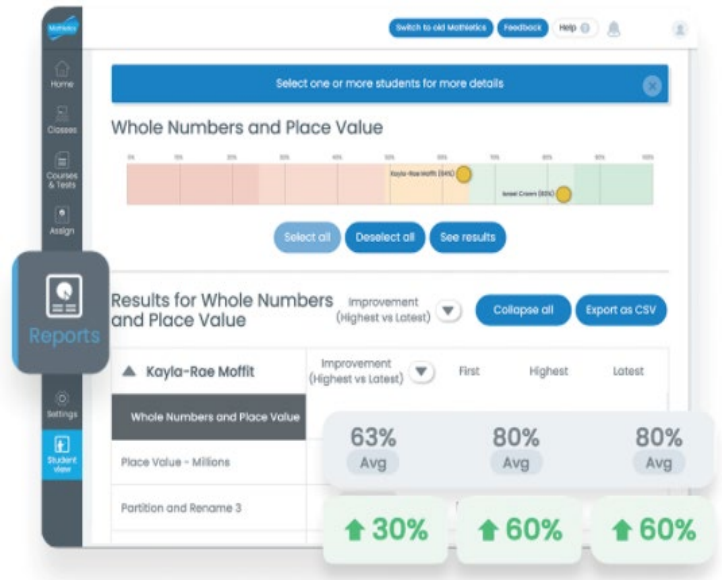

Figure 2: Learner analytics for one student
(3P Learning, 2021) see: https://www.mathletics.com/nz/features/

Teaching and learning mathematics on the 'new' classroom stage

It may be challenging to see clearly the teaching and learning of mathematics in ILE or OMIP spaces, and so it is perhaps useful to explore how a mathematics 'lesson' may proceed. In most Aotearoa New Zealand classrooms, a group rotation is used in mathematics whereby one group works with the teacher and the other groups are assigned other activities. In many schools the groups are ability based (Anthony & Hunter, 2017), despite research critiquing this practice. In some schools, however, groupings are instead fluid, and membership changes according to the needs of the child. In either case, because the class is not taught all together as a whole, communication as to what each student should be doing becomes complex. The illustration below (taken from the embedded video) depicts an "Action Stations" board (see Figure 3). The various activities on offer are represented by pictures and underneath each activity are the names of a group or individual children. The students find their allocated activity, locate the resources they need, and find a space in which to do their task, and often there is the option of 'free choice activities' included.

Figure 3: "*Action-stations" board: Children find their name and allocated* activity (Ministry of Education, 2021) see: https://www.tki.org.nz/

Typically, some students are directed to a digital device, and likely an activity on an OMIP such as *Mathletics*. When engaging with mathematics on the OMIP, students might have some choice as to which activity to complete, or they may have been assigned an activity - either by the teacher or by the OMIP's learning analytics.

(Co-)performing agentic mathematics teacher or learner identities in the 'new' mathematics classroom

Returning to the question of your particular person of interest, how do you imagine they co-perform mathematics teacher/learner alongside their other identities on this 'new' classroom stage? Might they be agentic in these performances, or would the context constrain how they may *be* a mathematics teacher or learner? To answer these questions, it may help to first look more closely at the way in which the 'new' classroom stage may constrain (or enable) particular identity performances, and secondly, consider what the *identity scripts* are that normalise performances on such a stage.

There are a couple of features of the 'new' classroom stage that set it apart from classrooms of the past. These may be seen in both the ILE space and the OMIP platforms; *free choice*, and the use of *surveillance*. These two features work to produce the mathematics teacher and learner in particular ways, as discussed below.

Free choice

One notable aspect of the 'new' classroom stage is the bodily freedom allowed to students as they perform the learner role. The children in the video certainly seemed in control of their own movement through the classroom space, bodies were not constrained in chairs facing in one direction. The space for being a mathematics learner is considerably expanded; in fact, it even extends beyond the classroom walls, as children may take a device elsewhere or log on to their learning portal at home. Agency for the learner is emphasised in both the ILE environment and the OMIP platforms with this notion of free choice. The school principal speaking in the video linked above mentioned students' freedom of choice and student agency. The online instructional programs also claim to allow student agency in selecting their own pathways through the available lessons (see *Mathletics* example in Figure 4 below). Within OMIPs, the students' freedom of choice is further evident in their choice of which game to play or what they might 'buy' for their avatar (using the credit points awarded for their mathematics work – see Figure 5).

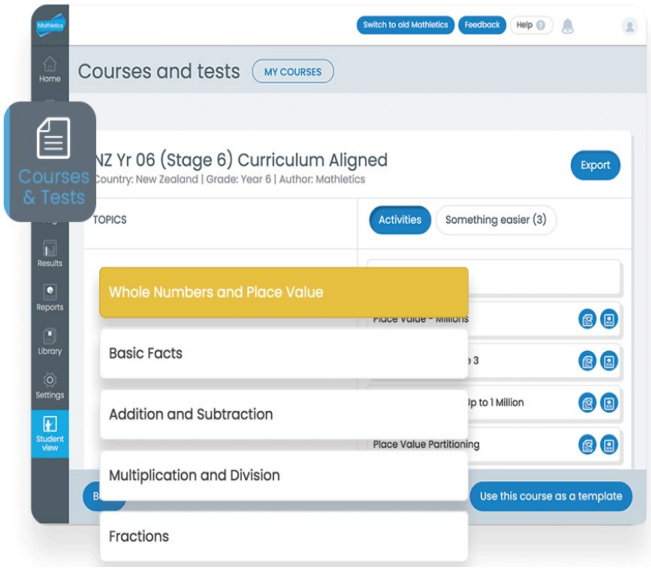

Figure 4: Choice of learning activities (3P Learning, 2021), see: https://www.mathletics.com/nz/

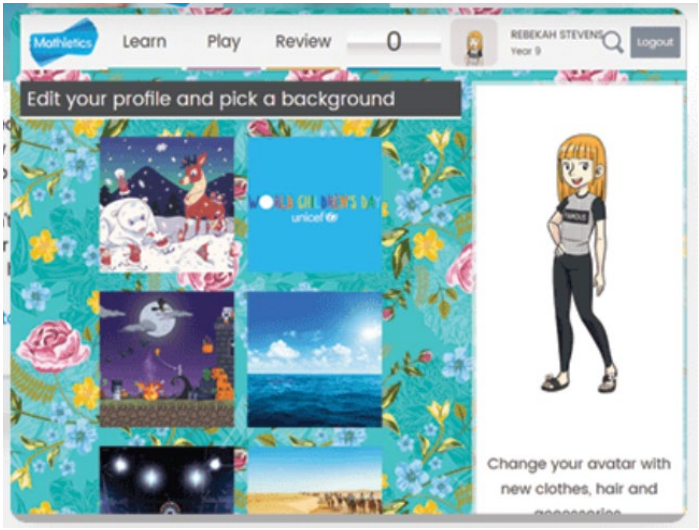

Figure 5: Choosing an avatar (3P Learning, 2021), see: https://www.mathletics.com/nz/

Such choice should be understood within neoliberal ideology, where free choice is a key feature of a market model of education. In Figure 5 we see how OMIPs further promote 'marketisation' by producing the mathematics learner as being a consumer in capitalist

society, what Jablonka (2017) calls a 'token economy', as students are invited to purchase items for their avatar based on their points 'earned' by doing the mathematics work.

Of course, we might question the level to which choices are in fact 'free'. Whilst children may be able to choose for themselves what to do and how/where to do it, their choices may affect the way in which they are recognised as a learner. Recall Walkerdine's (1990, 1998) research into mathematics of girls and boys in working-class schools: girls taking on the overt messages about good behaviour and following the rules were seen as hard-working (but not very bright), whereas the boys following the covert messages of exploration were attributed with having 'real understanding' and 'potential' despite lack of achievement (Walkerdine, 1990). I wonder what might be the overt and covert messages in these 'free choice' ILE spaces, and which children might be recognised as normal or pathological due to the messages they follow. Relatedly, Benade (2019) considered whether the ILE space is an inclusive design and identified the challenges faced by those with auditory, sensory and socio-cognitive issues; including the noise, the self-regulation required, anxiety, and getting 'lost' in the space. How might such children be recognised as mathematics learners in these cases? Where does your 'person of interest' fit here?

The notion of agency when using the OMIPs may certainly be problematised also. The complex learning analytics generate an assessment of students' next steps for their mathematics learning, and provide activity options based on this assessment (Knox et al., 2020). What appears to be freedom is in fact a tightly constrained choice based on the program's analysis of required next steps. As cautioned by Lupton & Williamson (2017):

> […] learning analytics platforms appear to displace the embodied expert judgement of the teacher to the disembodied pattern detection of data analytics algorithms [...] A significant risk that children's opportunities might be narrowed by the assumptions encoded in algorithmic processes is raised by such techniques (p. 787).

In other words, the assessment capability of the OMIPs means that decision-making may be taken out of the teachers' hands and this power placed with the OMIP instead. This fact constitutes a key area of criticism from the field of Media Studies, as mentioned earlier. Therefore, not only is the child constrained in their choice, but the teacher is de-professionalised (Roberts-Mahoney et al., 2016) through their reduced ability to select their own learning pathways for their students. The mythical free choice in the 'new' classroom that appeared to be on the agenda for students is even less evident for teachers. Here we see a further shift in who (or what) is centred on the stage of the mathematics classroom.

In short, the fiction of free choice, promoted by neoliberal ideology, appears to produce agentic performances of mathematics learner or teacher, but this agency is limited and it is controlled - as shall be seen further in the section to follow.

Surveilled performance

Another key feature of the 'new' stage of mathematics classrooms is *surveillance*. Researchers in the field have often used Foucault's (1977/1991) *Discipline and Punish*, applying the "panoptican" to the classroom setting (e.g., Hardy, 2004; Jablonka, 2017; Walshaw, 2010).

Typically, it is the student who is surveilled, but in the modern ILE, full of glass in place of walls, the teacher faces an increased surveillance, their teaching displayed to an extent not possible in the single cell classroom. In the linked video we could spy the teacher in a breakout room behind glass - her mathematics teaching easily visible. The principal in the video used the term "transparency" to describe the teaching practices, conveying a sense of openness both in terms of the physical and the pedagogical. Benade's (2017) study of teachers' work in ILE spaces juxtaposes the collaboration and transparency in teacher practice with the "stress of making collaboration work, the feelings of vulnerability, and a sense of always being on show" (p. 803).

The students face a very different kind of surveillance. On the one hand their physical selves may avoid surveillance – they might disappear into a dark corner or beneath a tent – but their learner selves are continually tracked through data collected. Lupton and Williamson (2017) call this "dataveillance", that is, digital surveillance. In the ILE space, students' learning and behaviour are firstly tracked via platforms such as Seesaw, ClassDojo, or Google classroom; both teachers and parents can view children's 'learning progress' on these platforms. The OMIPs go further with the sophisticated 'learning analytics' as seen earlier in Figure 2, (see also Jablonka, 2017). It is worth noting that this kind of data surveillance forms a violation of an unwritten rule of assessment: that students should know when an activity is being assessed. Pepin and colleagues call this "stealth assessment" (Pepin, Choppin, Ruthven, & Sinclair, 2017), when *everything* the child does is assessed, but they do not necessarily know it. This may be problematic as it denies them the opportunity to knowingly deliver their best effort for the assessed tasks.

What is also important to note here is that a huge amount of data may be collected from children engaging with the OMIP. For example, every keystroke, the length of every pause, the ratio of productive vs unproductive screen time, all may be recorded and used to build up a picture not only of the learner, but also to form a massive database of many thousands of learners.

> Performing such analysis depends not just on surveillance of the individual but also on massive dataveillance of millions of data subjects to generate the kinds of big databases from which accurate predictions are made by comparing individuals against norms derived algorithmically from the masses. (Lupton & Williamson, 2017, p. 787)

Ultimately this 'big data' (McRae, 2013; Roberts-Mahoney et al., 2016) is used to make the program more addictive, marketable, and profitable. This has been called "colonial design" technology, whereby the providers of the learning platform learn about the learners rather than the learners learning about themselves (Macgilchrist, 2018).

The emphasis on free choice from both the ILE and OMIP spaces also contributes to a form of surveillance. The students are required to be self-managing as they exercise their choices in selecting the activity likely to optimise their learning. For example, students often must manage their own learning to ensure they have completed a list of weekly tasks, and have made progress towards their own individual goals. This kind of self-management is a form of surveillance, whereby the child surveils themselves (see also Jablonka 2017).

"Learning personalization ensures, in this sense, the learner's individualized and responsibilized investment in their anticipated future self" (Macgilchrist, 2018, p. 242). Being a mathematics learner is about being future-focused via self-management.

Being a mathematics teacher and a mathematics learner therefore includes the performance of self-governance (Foucault, 1991) that results from surveillance. Being a mathematics learner additionally means being datafied, and continually subject to measurement against a norm. To summarise, the stage of the 'new' mathematics classroom in Aotearoa New Zealand produces particular identity acts that involve performances of agency, being surveilled, and self-governance. Such acts may be further understood by considering what the identity scripts are that describe normative ways of being the mathematics teacher or learner.

Dominant scripts for identity on the 'new' stage

The various identity performances as enabled and normalised on the stage and theatre constitute the *performance scripts*. These are expected or idealised ways of enacting identity that may be taken up by the individual, or ignored/altered via 'improvisation'. However, I wish to point out that neither the 'taking up' of a script nor rejecting it, is an easy (or even available) decision. Scripts, as normative models for identity performance, are powerful and impactful because they privilege a particular way of being. Scripts draw from societal narratives – in this case neoliberal ideology, twenty-first century narratives and the EdTech discourse of educational corporations – which all speak to how one should *be* a learner or a teacher of mathematics. Scripts, whilst produced in wider narratives, are made available in the local context – the classroom stage. On any stage there may be a variety of scripts available for individuals to take up (or there may be very few). Popkewitz's problem solving child is an example of a script; he shows how for some the desirable script is not available and these learners are "those left behind" (Popkewitz, 2004). In other words, the concept of 'scripts' answers the question of how one *should be* - in this case a mathematics teacher or learner in the 'new' mathematics classroom. In this section I present two dominant scripts produced in neoliberal and EdTech narratives, which are made available on the 'new' classroom stage. We might name these scripts: '*twenty-first century mathematics teacher*' and '*twenty-first century mathematics learner'*.

The twenty-first century mathematics teacher

Performing the twenty-first century mathematics teacher means being a coach/guide, being a collaborator, and having an audience (being surveilled). Whilst these performance aspects may be applied to teachers of any subject, they each have implications specifically for the mathematics teacher identity.

The production of the teacher as a coach or guide is evident in OECD texts about the ILE space, where the term "learning professional" (OECD, 2013) may equally be used. Here the teacher is made invisible, in a manner also evident in reform mathematics discourse (Valoyes-Chávez, 2019). Within EdTech discourse they even lose the name 'teacher', reduced

instead to 'coach', 'facilitator', or 'data-analyst' (Ideland, 2021). A student-centred emphasis means that the teacher is sidelined; their job is to guide students to their learning, but they are no longer required to teach. Similarly, when using the OMIPs, teachers need only direct students to the computer and then let the program do the teaching – including actual mathematical instruction (some OMIPs have instructional videos for this purpose), assessment, and assigning of tasks. In the case of primary school teachers – who are typically generalists (rather than having a speciality in mathematics teaching) and often described as having a problematic relationship with mathematics (e.g., Boylan & Povey, 2009; Hardy, 2009; Hodgen & Askew, 2011) – the OMIP space encourages them to sidle away from the responsibility for mathematics instruction.

The distancing of teachers from mathematics may be further exacerbated by the notion of teacher as collaborator (Benade, 2017). Collaborations enable teachers to divide out responsibility for various subject areas. In this situation just one of the teachers in the ILE space may take on the mathematics teaching, whilst the others may be responsible for other subject areas instead. Here the reduction of the teacher role goes further; for some, their mathematics teaching services are no longer required at all.

Yet a third aspect of the twenty-first century teacher is somewhat in contrast to the previous two. Whilst the mathematics teaching may be side-lined, the teachers themselves are not at all invisible. As discussed earlier, teachers are visible in a way not typically experienced in a single-cell classroom: their mathematics teaching performance may be observed and judged at any time. Given the literature about primary teacher anxiety regarding mathematics (Hodgen & Askew, 2011; Intawati & Abdurrahman, 2019; Jenßen et al., 2020) this visibility may indeed be intimidating as teachers are forced to put their mathematics teaching on display when they may prefer to avoid it altogether. In this *physical* aspect the contrast between the ILE and the OMIP spaces becomes apparent. In one the mathematics teacher is on show, in the other they are completely hidden. However, *pedagogically* the two spaces generate a similar effect.

The twenty-first century mathematics learner

Performance scripts for the twenty-first century mathematics learner are abundant. This type of learner is produced globally in OECD texts (e.g., OECD, 2013) and produced locally in Ministry of Education documents, for example:

> New Zealand needs an education system that provides its people with the skills and knowledge they require to be successful in life and in an increasingly global economy. An effective education system provides qualifications that open doors to future opportunities and the skills needed in today's society and the modern workplace. Equipping learners for a digitally enabled future is a key goal of our Four Year Plan. Demand for future-focused learning is increasing – the Ministry's ICT strategy and our twenty-first century practice in teaching and learning priority ensure we have the right focus to meet this need. (Ministry of Education, 2016, p. 10)

The discursive emphasis is very clear in this excerpt that prioritises "success" in a "global economy" and ties together "future-focused learning" with digital skills. The twenty-first

century learner is also produced in EdTech discourse, and marketed via the OMIP websites (Darragh, 2020). As Macgilchrist explains, to be successful in the twenty-first century, according to the mainstream argument, requires the "skills of creativity, collaboration, critique, and communication [...] This type of success is thoroughly entangled with the neoliberal, self-optimizing, 'entrepreneurial self'" (Macgilchrist, 2018, p. 242). Accordingly, I argue that performing the twenty-first century mathematics learner means to be entrepreneurial, self-managing, and individual.

Both the ILE space and OMIP platforms position the mathematics learner as a *self-managing entrepreneur*. If the 20th century classroom was designed to produce factory workers then the ILE space clearly prepares for a very different workplace, one that likely matches up with our mental images of the Google offices - themselves having become something of a trope. The entrepreneur is also produced in the OMIP platforms as children are encouraged to engage in the capitalist behaviour of buying products for their avatars. The twenty-first century mathematics learner is self-managing: they must meet their own learning goals and manage not only their own behaviour but also take responsibility for their own learning. It was notable that the Action Station task board in the video contained pictures to direct children to learning tasks, meaning that even children who cannot yet read are responsible for their own learning. The OMIPs encourage the children in this self-management, offering rewards for those who engage in large quantities of activities.

Finally, the twenty-first century mathematics learner is an *individual*, as made explicit in the hyper-personalised learning emphasised on both ILE and OMIP spaces. The ethos of twenty first-century education is very much student-centered (OECD, 2013), and we could certainly see this in the video linked earlier. The field of mathematics education has long been a proponent of 'reform' mathematics, with a de-emphasis on teacher-centred, traditional practices. However, there are some differences between the student-centred, problem-solving reform mathematics class (see also Lundin, 2012; Popkewitz & Lindblad, 2004) and the student-centred, individualised learning promoted by neoliberal ideology. The OMIPs and ILEs are firmly situated within the neoliberal version of 'personalised learning' (Darragh, 2020). Learning within this ideology entails receiving a separate and individualised learning plan – a task much more achievable by the artificial intelligence of learning analytics (and making the teacher ever more irrelevant). Further, there is a competitive aspect to this script; competition in games becomes competition for jobs in the future workplace.

To summarise, the twenty-first century teacher and learner are scripts produced by the neoliberal theatre, on the 'new' student- or device-centred classroom stage, and in the narratives of EdTech and educational policy documents. However, I wish to reiterate that performing mathematics teacher or learner is not about the exact following of a script. The script forms a notion of the ideal mathematics teacher or learner, against which an identity performance might be measured, recognised, or found lacking. Whilst any individual's performance of mathematics teacher or learner identity is formed at least in part by the available scripts, following the script is not always equally available for all people, and divergences from the script may be differently recognised depending on the person also. In

this co-performance of identity emerges issues of intersectionality (Bullock, 2017; Leyva, 2016, 2017) and where we may see the impact of the power of recognition (or 'recognition power'). Therefore, scripts are problematic in a number of ways: firstly, because they are not equally available to all, secondly, because improvisation from the script may be seen as pathological, and finally, scripts narrow the possibilities for ways of being.

Improvising the mathematics teacher or learner?

The scripts for mathematics teacher and learner seem inevitable given the neoliberal theatre of education, and the 'new' classroom stage that emulates the twenty-first century workplace. However, considering identity as performative, we might ask what divergence from these scripts are even possible. Butler's notion of the repetition of acts allows a certain agency here. Each time we perform mathematics teacher or mathematics learner we may renew the performance differently – we may improvise. It is precisely in the repetition of identity that an individual may revise the act, and any act may follow scripts closely or deviate. While performances are always constrained by the context, there is a whisper of agency here; the possibility of revision raises the question of what the alternative ways of being a mathematics teacher or learner are. Already we see alternatives to the mathematics teacher script; it may be sidestepped via collaboration with other teachers, or the use of online programs enable teachers to avoid the role completely. What of the mathematics learner? Is there a way to be a mathematics learner that denies the self-managing, independent entrepreneur? What sort of mathematics learner would this improvisation produce? And, finally, how might the improvisation be recognised?

To conclude, performative identity as a stylised repetition of acts means that we need to take seriously the physical, temporal, and ideological space in which these acts are made. A mathematics teacher or learner identity is made, renewed, and revised in every individual performance of 'teacher' or 'learner'. In this paper, I have shown how ILE classroom design and online mathematics platforms dictate certain behaviours for mathematics teaching and learning and make available scripts for the normal (twenty-first century) mathematics teacher/learner. A few questions remain, and I ask the reader to once again consider their person of interest. For whom are these scripts more available? How might improvisation away from the script be recognised? Finally, what alternative ways of being a mathematics teacher or learner are imaginable beyond these scripts?

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Identity as a significant concept in mobilising collective action in mathematics education and beyond: A response to Lisa Darragh

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It is my pleasure to be invited to offer a response to Lisa Darragh's plenary on 'Innovative learning environments and the digital era: Finding space for mathematics identity'. In doing so, I will argue that the concept of identity is not only useful in understanding students' and teachers' relationships with and participation in mathematics. Identities, like critical knowledge, are also potentially powerful tools in mobilising collective action that can be transformative of practice.

The myth of free choice and individualism

At first glance, Lisa provides an account of mathematics teaching and learning in Aotearoa that appears quite idyllic. This is especially so given my experience of an education system that is heavily regulated by central government (e.g. through standardised testing, a national curriculum etc.) and increasingly run in the interests of private businesses through the joining of schools into academy trusts (run by private trustees from business/charities), a predisposition towards consultocracy (Gunter, Hall, & Mills, 2015) and the outsourcing of curricula, resources and professional development to commercial enterprises. Whilst Lisa's account resonates with some of this, she also describes a localised curriculum that is devolved to the school level offering schools, teachers, parents (and students?) agency over what is learnt with potential to serve the needs of the community. With colleagues at Manchester, I have been involved in projects inspired by the Funds of Knowledge (FOK) approach (Moll, Amanti, Neff, & Gonzalez, 1992) and it's more recent adaptation Funds of Identity (FOI) (Esteban-Guitart & Moll, 2014). Both of these approaches involve teachers working with oppressed/minoritized groups to locate knowledge and identities in the home and community as a resource for developing curriculum projects in schools. The aim is to connect the school curriculum to the 'everyday' knowledge, practices and needs students experience in their communities. So in one sense Lisa's description of educational policy in New Zealand seems to align with the fundamental principles of a FOK approach and its overarching aim of developing curricula that challenge the privileging of elite forms of knowledge (and identification) in the academic curriculum.

Yet Lisa critiques educational policies in New Zealand as maintaining the fiction of 'free choice' in pursuit of a neo-liberal subject who is then held accountable for such choices. Indeed parental choice has been long identified as an 'essential circuit' of neoliberal policy

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in education (Ball, 1994), which assumes notions of self definition, self actualisation and thereby individual responsibility for the success and/or failure of the child. According to this kind of critique, the fiction lies in the myth that choice proffers agency when in reality it is restricted to the privileged few – e.g., in the mathematics classroom this may be those in the right position to accept or reject what Lisa refers to as 'identity scripts' associated with success.

The concept of identity here is pertinent since, as Holland et al. (1998) note, any moment of (or engagement in) social activity not only involves what one does 'in practice' but also a self-authoring[1] of the subject in ways that are culturally or socially recognised for and by others. Thus learners in the mathematics classroom are not only subjectively experiencing mathematical practices they are also self-authoring as mathematics learners - or not - as the case may be. Indeed, Lisa notes how this self-authoring is framed by power relations when she states that "following the script is not always equally available for all people, and divergences from the script may be differently recognised depending on the person also" (Darragh, 2021, p. 23, in this volume). So how or why do policies that aim to give greater voice to students/parents/teachers become, 'in practice', a learning environment that is potentially stratifying and exclusive?

In the aforementioned work on FOK, we have offered a critique (using Bourdieu) of so called 'domesticated' versions of the approach for their propensity to surface capital in students' homes in the interests of serving the needs of the school or the educational field (Black et al., 2019; Williams, 2016). Clearly those who have access to such capital are more able to offer the kinds of resources that the school might want or the kinds of identity scripts that are ascribed value in the mathematics classroom. However, the fundamental concern here must be with the structure of the educational field and the way its social, political and economic function is refracted through pedagogic practices. Such a critique suggests a more radical agenda is necessary whereby critical pedagogies are employed to challenge even transform (rather than serve) education as a process of reproduction.

The commodification of learning

Lisa's focus on the identity scripts produced through Innovative Learning Environments (ILE) and commercial Online Mathematical Instruction Programmes (OMIPs) links acutely to debates around privatisation in education and the economic commodification of learning and teaching which I refer to above. Initiatives such as the introduction of Mastery Mathematics in England, exemplify the complex fuzziness of the private/public distinction in relation to forms of curricula/pedagogy innovation. Notions of pedagogy, curricula and learners are constructed in the name of 'public value' whereby responsibility for reform is shifted away from the institution (education system) but towards complex relationships involving 'local publics'[2] (Newman, 2013) and private actors. Such initiatives involve the

[1] Note there are fundamental differences between self-authoring and neo-liberal concepts of self-actualisation as outlined by Holland et al (1998).

[2] Newman (2013) refers to new localism – where central functions of the state are devolved, fragmenting a unitary public in the name of flexibility, responsiveness and goals.

production/consumption of particular methods and resources which are in part public (i.e., openly stated as for the public good with public funds attached) but are also mobilized through both public (e.g., national organisations like NCETM) and private interests (e.g., paid consultants, academy trusts etc.). This complexity then produces particular relations of power between various stakeholders, and between organisations which permeate at every level. Quite literally this involves the commodification of students' educational labour for profit by private providers, which adds another layer to previous economic analyses of education whereby student labour is commodified in the form of qualifications that have value to be exchanged in the world of labour relations (Williams 2012).

However, Newman (2013) proposes a reconceptualization of 'public value' to incorporate a concept of 'public action', which involve new forms of alliance between activists, academics and policy actors. This arguably means such initiatives can potentially provide space for diverse collective forms of social agency and empowerment - that can lend themselves to more radical or progressive appropriations than we might otherwise see ventriloquated through policy discourse[3]. For instance: (i) alliance between teachers, academics and professional organisations which can produce collective agency for change (e.g., primary assessment reform); (ii) alliance between teachers within/across schools enabling some increase in professional autonomy locally; (iii) pedagogic practices which offer 'voice' to the mathematical learner and diverse forms of learner identity. Arguably, these forms of alliance engender ideas of voice and activism which are to be distinguished from the neo-liberal notions of 'free choice' and 'individualism' which Lisa alludes to. Public action engenders ideas of collective agency rather than the pursuit of private or individual gain.

Harnessing contradictions

Nevertheless, it is not my intention to dichotomise concepts here such as private/public, individual/collective and capital gain/human need. In Black et al (2021) we discussed how ideas associated with private capital gain (individual) and public good, collective agency and human need can be understood as a dialectic relation of exchange value – use value, drawing on Marx's concept of the commodity relation. In line with others in CHAT (Blunden, 2009), we argue for a unit of analysis which preserves the living dynamic unity of exchange value - use value and which, recognises this relation as fundamentally one of contradictory moments. Recognising and harnessing such contradictory moments can be developmental which can bring about transformations in practice. In the description Lisa offers, I suggest we might see such contradictory moments when the needs of a school community (run by parents) come into conflict or tension with the overarching 'need' of the school system to grow 'cultural capital' (exchange value) for those privileged enough to access it. For example, Lisa points out how some OMIPs promote a hyper individualisation of the learner (presumably to foster performances/actions/knowings that are transferable to standardised

[3] Newman (2013) points out that marginalised or politically less powerful publics are more likely to be mobilised through autonomous groups rather than via official consultation and participation in established government.

tests) that conflicts with the diverse needs and interests of a community of learners as a group rather than as a sum of individuals. This raises a question: at what point might this community collectively decide to challenge the system as it stands? In CHAT terms, this is where the harnessing of contradictions can be productive through the creation of a shared joint object that motivates a group towards collective action through the kinds of alliances Newman refers to above.

And so to identity…

With the above in mind, I argue that the concept of identity is not only useful for exploring students' relationships and participation in mathematics education, identity or identities are also potentially powerful in mobilising collective action. Think, for instance, of the identity work taken on by oppressed groups that is central to so many struggles for change (e.g., we recently analysed the case of the Mexican American Studies programme in Tucson, Arizona; Black et al., 2021). But this requires a concept of identity that not only emphasises individual repetitive acts that come to carry cultural significance (as identity scripts) - it also requires some concept of motive (collective-individual) 'to act' and a sense of reflection on how our subjective experiences make sense in terms of who we are and what we are becoming (Holland et al., 1998). Change occurs not only through stylised individual performances (improvisation) but also through collective action and I argue that is through latter that imagined alternative identity scripts can be realised.

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Innovation in school mathematics? Historical iterations and other enduring dangers: A response to Lisa Darragh

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In this response, I interrogate the limits of innovation in school mathematics within a historical context. I explore the continuities as well as shifts in the normalizing practices of school mathematics. I argue that the notions of "free choice" and "surveillance" are not only specific to neoliberal regimes but also are embedded in histories of modern schooling. The historical context enables us to explore the dangers of innovative learning environments such as ordering the differences on a hierarchy in addition to the production of particular identities.

Introduction

Seeking a change in teaching and learning practices has long been a concern for mathematics education. Innovative pedagogical methods and curricular ideas are always presented to ensure 'better' learning environments for all students. While these 'innovative' approaches are considered to improve teaching and learning mathematics, they do more as argued in Darragh's paper: Identities for students and teachers are produced, regulated, and normalized by the multiplicity of societal narratives such as neoliberalism, colonialism, racism, sexism and so on.

Darragh's paper revisits how the identities of mathematics learners and mathematics teachers are being produced and regulated in "technology-rich, innovative learning spaces". These learning environments are located in Aotearoa New Zealand; but she also situates the processes of identity formation of learners and teachers within neoliberal ideology, twenty-first-century narratives, and the EdTech discourse of educational corporations. Rather than positioning teachers or learners as fully agentic humans, her conceptualization of identity enables an analysis of the multiplicity of discourses that regulate the identities and normalize particular actions and participation in "ILE (innovative learning environment) spaces".

My response draws on the historical background of normalization practices, including "free choice" and "the use of surveillance" in learning spaces. Although Darragh notes that these two normalization practices are the features of "new" classrooms or online learning platforms in our digital era, I discuss how these practices historically have been part of the modern world, particularly they are embedded within the practices of schooling and school mathematics. In my response, first, I explicate the historical emergence of sciences of

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decision-making, which is beyond neoliberal ideology, that makes discourses of "free choice" possible and reasonable. Here, I also bring historical shifts in the practices that organize and regulate uncertain learning spaces that are presumed to be planned, stabilized, and secured. Following these, I consider the dangers of the common way of thinking about change and innovation, including the differentiating mechanisms in school mathematics.

Historical continuities of normalization in educational spaces and shifts in the practices of educational decision making

Educational spaces are complex, dynamic, and uncertain. Social actors of (mathematics) education experience several predicaments when they are asked to make choices among a range of options. While one decides different choices, the notion of uncertainty embedded in decision-making processes is not always subject to endless possibilities. Rather, decisions are produced in systems that include scenario planning, risk profiling, algorithmic modelling, and data analysis (Amoore, 2011). Are these emerging practices of data collection, analysis, and representation new to social and educational spaces? How might we historically think about these processes and their exacerbation with the increase of online education?

How to act and participate in the real world under uncertain conditions is not a new problem. Hacking (1990), for example, explored how statistics and probability became technologies to formulate complicated realities into stabilized entities to tame the chance in the modern world. These technologies of data collection and analysis have been concerned with "making up people" as administrable citizens of the state. With the avalanche of printed numbers, future society became designable through counting people and their habits. The enumerations resulted in populational categories that constitute human kinds (Hacking, 2007). The categories for humans such as effective housekeeper, intelligent adult, or democratic citizen have been placed into enclosed and disciplinary spaces to order, differentiate, classify and normalize proper and improper modes of actions and participations in the world (see Foucault, 1995).

One of the most familiar examples from schooling has been the wide circulation of intelligence tests in the late 19th century modern nation-states, a particular context that can be remembered as a major breakthrough in education with the industrialization, public education, and waves of migration. Schools were seen as an effective technology that prepared children for industrial work and average adult life (Danziger, 1997). While later these tests were to compare the 'national' IQ level of countries and classify the regions along a continuum of values (Valero, 2017), the widespread adoption of standardized tests were linked to eugenic projects that aimed to purify population as well as maintenance of a White supremacist society (Davis & Martin, 2018). Back then, ability groups were considered as an innovative strategy to plan effective learning environments. The societal hope of dividing students was not only about economic development and progress but also was concerned with race betterment and ensuring the well-being of population(s) (Yolcu & Popkewitz, 2019). Commitment to the knowledge produced through multiple data points, including scores of

standardized math tests and time on solving mathematics questions, instead of arbitrary decisions, was a tactic to rationalize the tracking of students.

The contemporary calculations of the future and uncertainty have shifted. They are less about spatial classifications but more related to the configurations of spaces of security and control (Foucault, 2007). These spaces are not enclosed in the disciplinary sense: Rather than spatial distribution of individuals in advance, there is a widespread installation of control technologies across spaces and possibilities (Deleuze, 1992). The tools like robots, smartphones, or networked machines enable perpetual training, frequent and faster surveillance, and continual monitoring of communities to maintain the safety and stability of the world. Within the data produced through these devices, practices of algorithms, data analytics, or risk profiling become "the authoritative knowledge of choice" to anticipate the future uncertainty (Amoore, 2011). Here, the notion of free choice would not simply be constrained by data, but data analytics is part of what we call 'free choice' or 'informed decisions' that we make under uncertain conditions. Despite the changes in the tools and technologies, the uncertainty of educational spaces was resolved through apparently precise, specific, and quantitative data networks in which reasonable and rational choices could be made. The explosive interest in data based decision-making can be framed as a historical reiteration of the hope for a safe and stable world (Heyck, 2015).

In contemporary educational research, while tracking and assessing students' IQ levels become unwanted, old-fashioned practices, we do still have standardized exams. However, today, standardized assessment items emphasize 21st-century skills such as problem solving, modelling, or systems thinking. That is, despite the changes, there is persistent trust in the data produced through the standardized tests. Nevertheless, contemporary educational choices could no longer rely only on the tests. There should be more to attend to the contextuality and uncertainty of learning environments.

In addition to contemporary modified testing practices, students and teachers are asked to produce their data in their contexts. For example, continual in-class tracking of children's mathematical learning trajectories is considered as active agents to close the "education gap" between ambitious goals of reform and actual student mathematical thinking (Daro, et al., 2011, p. 11). With the tools of the digital age, ongoing classroom assessment of mathematical trajectory becomes possible (e.g., Confrey & Maloney, 2012). Installation of these tools into the classrooms does not only provide rapid and frequent feedback for teachers who make instructional decisions but also contributes to the ongoing surveillance of learning environments.

The historical desire for stable and secure world orders the calculation of uncertain educational spaces. As I have briefly discussed, and as Darragh argues in her paper, these social processes have normalizing effects in educational settings. Nevertheless, the normalization has long occupied the landscape of school mathematics despite the changes in technologies and tools such as IQ tests, skill-based assessment items, or classroom trajectories. So, it is possible to refer to the process of normalization as a historical spiral,

moving from layer to layer, never stabilizing itself and the practices are always open to modification and adjustment with the changing conditions.

Dangers of normalization practices: Differentiating axes of school mathematics

Administration of the landscape of school mathematics with normalizing practices has been a way to make the children as a particular kind people. Darragh discusses this process as production of identities that are "scripted" by the contemporary neoliberal regime. Particularly, she talks about the 21st-century mathematics learner (and the teacher) who embodies capitalist behaviour in online platforms, takes responsibility for their own learning, and performs identities as an entrepreneurial. With the discourses of "free choice", she takes our attention toward the generation of agentic performances that are controlled through ongoing surveillance and data collection.

The network of school mathematics practices produces a normative and regulatory space for 21st-century mathematics learners and it simultaneously generates axes of differentiation. Children are no longer categorized as mathematically defective, disable, slow or remedial, but they are profiled as "at-risk" not only through the generalizations of national or international exam score but also through ongoing classroom assessment results. The children who are outside of the normative accounts of educational spaces are categorized as at risk, disadvantaged or underrepresented and become the objects of interventions, such as teaching, research, or reform to conserve the historically planned order and stability of the world.

While the normative accounts regulate and produce particular human kinds, they simultaneously generate the "others". The differentiated spaces for children are configured as the laboratories of experimenting the innovative or new ideas of school mathematics. In order to be prepared to the shifts in the educational spaces, novel psychological categories are generated in addition to the desired identities. This includes, for example, the interest and willingness of students to persist on mathematical tasks (Organization for Economic Co-operation and Development [OECD], 2018). While willingness to do mathematics is formulated as one of the desired distinctions of 21st-century mathematics learners in these accounts, the differentiated spaces are simultaneously generated for others who are seen and perceived as 'unwilling' to do mathematics.

At the end of the plenary paper, there is an important question that Darragh raises: "For whom are these [identity] scripts more available?" Taking into account the differentiated axes embedded in school mathematics, I want to take this question a step further. I wonder, what specific technological devices are available for whom? Are there any additional and modified pedagogical strategies for those who act outside of the boundaries of produced identities? What differentiated categories are designed for those who push against the boundaries of 'innovative learning environments'?

Conclusion

In mathematics education, everybody wants to make a change and innovation: Teachers, researchers, students, parents, policymakers, and curriculum reformers to name a few. These innovations are not only concerned with teaching and learning mathematics but also with producing identities, normalize particular subjectivities and also generate spaces for others. Exploring the history behind the reform and change offers ways to problematize what is given as natural, sensible, and necessary part of mathematics education including those rules and conventions that configure what we perceive as "change" within the boundaries of how we conventionally reason about school mathematics.

If we think of the normalization processes in the innovative learning spaces as historical, the identity "scripts" for 21st-century mathematical learners are also embedded in the numerical practices of testing, visualizing, or modelling the big data. As more teachers and learners get enumerated, the complicated realities of learning spaces are formulated into stabilized entities. Application and production of data are to render classrooms certain, secure, and stable with rational decisions. The stabilizations do not only make up people but also enable axes of differentiation. It is a simultaneous process of production of identities and their differential constitution.

Despite the shifts in the tools and practices of normalization and differentiation, the historical reiterations to secure the uncertainty in learning environments reveal that there is something sticky in the 'reason' of school mathematics. How we think about change in mathematics education is embedded in a style of reasoning that normalizes particular subjectivities while differentiates the others. Despite the shifts in the tools and technologies, mathematical learning environments have been occupied with the production of objects of teaching, research, and policy. Then, the snapshots of learning environments, which were narrated at the beginning of the plenary paper, are not a change in the premises that constitute objects in educational spaces. Rather, it is a historical iteration of 'reason' of school mathematics that makes, normalizes, and differentiates particular human kinds.

One might ask: Isn't there a possibility to perform any agentic identities in this digital era? Is there no space to be free in our choices? Is nothing changing at all? Are we going to give up inventing digital technologies or searching for possibilities of change in mathematics education? I would say no. "What is given up", as Popkewitz (2008) writes, "is the notion of planning people" that "stabilizes and fixes the boundaries of freedom" (p. 184). So, the change is never deadlocked. On the contrary, the spaces for performing freedom and other potentialities could be found in the very act of exploring historical shifts and iterations, where the resistance can become the continual interrogation of what is think-able and say-able within the boundaries of current practices.

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Rethinking exemplification in mathematics teacher education multilingual classrooms

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Examples that teachers choose and use are fundamental to what mathematics is taught and learned, and what opportunities for learning are created in mathematics classrooms. In this paper, I bring together three frameworks which have been used separately in mathematics education research – variation theory, meaning making as a dialogic process framework, and the notion of interacting/multifarious facets/dimensions within teacher education. The emergent framework consists of a triadic approach to understanding exemplifying practices within teacher education, and in particular, within multilingual teacher education classrooms. Lesson transcript data from an introductory class in probability in one teacher education multilingual classroom is used to illustrate how working with the amalgamated framework conduces to a powerful way of examining the choice and use of examples in mathematics teacher education multilingual classrooms, and how the three frameworks work together to attend to three critical layers involved in the complexity of teaching and learning in mathematics teacher education multilingual classrooms.

Introduction

As a teacher educator involved with both pre-service and in-service teacher education (TE), I have often tried to model how to teach using different practices in my class. These practices in themselves were never the object of attention in our discussions beyond their mere definitions. I became more sensitised to the importance of engaging my students on what makes for a good practice in multilingual classrooms in the course of my study (See Essien, 2014) using Wenger's (1998) communities of practice theory to engage with teacher preparedness for teaching mathematics in multilingual pre-service teacher education classrooms. Subsequently, how teacher educators in multilingual classrooms choose and use examples – that is, exemplifying as a practice – became an important focus of my research and practice. The importance of mathematical examples – that is, of tasks which are used to illustrate concepts in mathematics (Essien, 2021) – cannot be overemphasised. My focus on exemplifying as a mathematics practice was motivated by the centrality of examples in the teaching and learning of mathematics. Research has shown that the examples which teachers choose and how these examples are used play an important role in what mathematics is taught and how students learn and understand the mathematics that is taught in class

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(Arzarello, Ascari, & Sabena, 2011; Bills, Dreyfus, Mason, Tsamir, Watson, & Zaslavsky, 2006; Goldenberg & Mason, 2008; Zaslavsky, 2010; Zodik & Zaslavsky, 2008).

In my attempt to better understand the choice and use of examples, the following questions became important: what examples can teachers use in multilingual mathematics classrooms to help mediate knowledge in mathematics? And fundamentally, how can pre-service teachers be enculturated into how to choose and use examples that would help their future (multilingual) students better understand mathematics? What affordances do the selection and use of examples offer to pre-service teachers regarding the multiple dimensions of TE in mathematics? And what should a "good" (use of) example be in multilingual pre-service teacher education mathematics classrooms – given that these pre-service teachers themselves are most likely to teach in multilingual classrooms at the end of their qualifications? While it is not the intention of this paper to answer all the above questions, these questions were, for me, the driving force behind my quest for an all-encompassing framework which has the ability to not only provide a gaze into the types of examples that are chosen and used within TE, but also a gaze into how the choice and use of examples in teacher education can attend to the complexity involved in enculturating pre-service multilingual teachers into the intricacies of teaching in multilingual mathematics school classrooms. In doing this, the words of Lester (2010, p. 83) who argues that "rather than adhering to one particular theoretical perspective, [that] we act as bricoleurs by adapting ideas from a range of theoretical sources to suit our goals", came to mind. In an earlier paper for ZDM (Essien, 2021), I attempted to do this using a dyadic framework that accounted for both the examples chosen and used and the interactional pattern during the enactment of the examples. My contention is that using the dyadic framework did not account sufficiently for how pre-service teachers (PSTs) were enculturated into the multifaceted dimensions within teacher education.

In analysing the nature of examples used in mathematics classrooms, variation theory as a theory of learning has become ubiquitous in research about how the structure of an example space is not only an important mathematical process but also a critical didactical goal (Arzarello et al., 2011). This paper is premised on the notion that variation theory, which is commonplace in research involving the choice of examples in mathematics classrooms, may be insufficient to provide adequate perspectives on the quality of instructional examples (Zaslavsky, 2010) in teacher education classrooms of pre-service teachers who are themselves multilingual and who will teach in multilingual classrooms at the end of their qualification. Using a multifocal framework designed to unpack such complexities, I argue that in order to take into account the full extent of the multifaceted nature of teacher preparation for teaching in multilingual context, exemplifying as a mathematical practice in teacher education needs necessarily to also account for 1) how language is used to enable what some authors (e.g., Stein, Engle, Smith, & Hughes, 2008) have referred to as productive mathematical discussions in the class, and others (e.g., Engle & Conant, 2002) as productive disciplinary engagement, and 2) how the different facets involved in pre-service teachers education are (co-)constructed in multilingual context.

I start the next section by a brief exposition of variation theory subsequently bring in two 'bricoleurs', – meaning making as a dialogic process framework and the notion of interacting/ multifarious facets or dimensions within TE. Then I present the amalgamated framework. Although this paper is a conceptually rather than an empirically driven piece, using transcribed data of an introductory probability lesson, I show the applicability of the framework (so formed) and discuss the relevance of the framework both conceptually and empirically and how the interactional process in the enactment of examples bring into focus (or not) the multiple dimensions characteristic of teacher education in multilingual contexts.

Variation theory in mathematics teaching and learning

There have been several theoretical expositions on variation theory as a pedagogic theory in mathematics classrooms (see Kullberg, Runesson, Kempe, & Marton, 2017; Marton & Booth, 1997; Pang & Marton, 2005; Watson & Mason, 2006). For this paper, suffice it to indicate that variation theory holds that learning is a function of discernment, that is, of seeing or experiencing critical aspects of what is to be learned (Marton & Booth, 1997). Three core tenets of variation theory are important to my overall framework: Object of learning, Critical features, and Patterns of variation. The object of learning is what is to be learnt (Pang & Marton, 2005) – it is the focus of attention. In mathematics, this would be the mathematical object of learning. Pang and Marton (2003, 2005) and Runesson (2005) make a distinction between the intended, the enacted, and the lived objects of learning. The intended object of learning is the capabilities the teacher wants the learners to develop and the enacted object of learning is how these capabilities are realised in the classroom. As such, the enacted object of learning "is co-constituted in the interaction between learners and the teacher or between the learners themselves" (Runesson, 2005, p. 70). The lived object of learning is what is actually learned, that is, how the object of learning is experienced by the students. This brings to focus the important role the teacher plays in how his/her chosen examples are used in relation to the context in which the teaching is imbedded so that the intended object of learning aligns as much as possible to the lived object of learning.

Variation theory defines critical features are those aspects of a phenomenon that are necessary for the learner to discern in order for the learner to develop a particular understanding of the object of learning in focus. Variation theory holds that learners own experience of certain patterns of variation and invariance of novel situations and discernment or awareness thereof of the critical features of the object of learning is a *sine qua non* condition of learning. This means that discernment is not possible without the experience of difference between two of more different things/situations and without the experience of difference, it would not be possible to discern similarities. The kinds of awareness brought about by patterns of variation include contrast, separation, generalisation and fusion. Regarding contrast, variation theory holds that learners are more readily able to discern the critical features of an object if they are able to contrast it with other objects or another object. Similarity is what is kept invariant (Watson & Mason, 2006) in the mathematics structure within the sequence of mathematics examples. Drawing on Marton

and Tsui (2004), Olteanu and Olteanu (2013) assert that separation "refers to the other dimensions of variation that need to be kept invariant or varying at a different rate in order to discern a dimension of variation that can take on different values" (p. 515). Variation theorists argue that in order to fully understand a concept, it is necessary to also experience varying features of the concept so as to separate the features that are not critical (that is, that are not defining features of the concept). Fusion is when there is simultaneous variation of several critical aspects of the object of learning in an example space (Lo & Chik, 2016; Olteanu, 2018). The interrelationship between the different constructs of variation theory is represented in Figure 1 below:

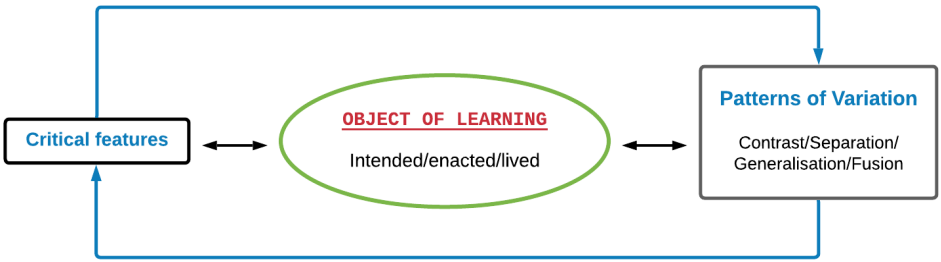

Figure 1: How the various concepts in variation theory interconnect

Perhaps, a good way to summarise variation theory is in the words of Lo and Chik (2016, p. 296) who assert that "necessary conditions for learning include focusing on the object of learning [hence the central position occupied by the object of learning in Figure 1], identifying which of its aspects or features are critical, and exposing learners to appropriate patterns of variation that help them discern these critical aspects or features".

'Bricolaging' variation theory

What does it mean to use variation theory in multilingual classrooms? More specifically, how can variation theory be used in multilingual teacher education classrooms so that it accounts for the complexity involved in preparing teachers for teaching in multilingual classrooms?

To better understand the choice and use of examples in teacher education, I draw on Mortimer and Scott's (2003) notion of meaning making as a dialogic process, and the notion of interacting identities within teacher education (Essien, 2014) and bring these to bear on variation theory. Lo (2012) argues that variation theory has two aspects: "the specific aspect, which refers to the subject matter, knowledge or skill that we wish students to learn (short-term goal), and the general aspect, which refers to the capabilities that can be developed through the learning of the specific aspect (long-term goal)" (p. 25). The long-term educational goal of pre-service teachers needs necessarily to go beyond knowledge acquisition and knowledge of subject matter. In any mathematics classroom, not only are examples used in the teaching and learning process important, but also important is the discourse that is

used to engage with the chosen examples and how language is used to negotiate meaning in the interactional process leading to the construction of the mathematical knowledge and the development of mathematical thought (Jung & Schütte, 2018; Scott, Mortimer, & Aguiar, 2006). Through such interaction, the multiple dimensions within teacher education are attended to (or not) as the class engages with the examples at hand. My use of the term 'discourse' resonates with Monaghan's (2009, p. 15; drawing on Morgan, 2007) understanding of discourse to mean the "patterned uses of language and other forms of communication whose deployment identifies the user as belonging to a particular community at a particular time in a particular setting". In what follows, I engage with meaning making as a dialogic process.

Meaning making as a dialogic process

Mortimer and Scott (2003) conceive of meaning making as fundamentally a dialogic process, where different ideas are expressed and acknowledged by the teacher, and worked upon. The framework focuses specifically on ways in which the teacher acts in order to guide meaning making interactions within the classroom. The framework comprises of five different, but interconnected aspects of interactions in the classroom 1) teaching purposes; 2) content; 3) communicative approach; 4) patterns of discourse; and 5) teacher interventions. Mortimer and Scott's framework positions dialogue as connecting participants to meaningful, purposeful and valuable processes of knowledge construction.

In the current framework, I have focused more on the aspects of the framework that deal with classroom interaction, namely, the communicative approach and patterns of discourse as it is my contention that variation theory, in its focus on the object of learning, already engages with the mathematics content within the classroom. It is also my contention that in the mathematics teacher education classroom, the framework on the multifarious dimensions of teacher education (which I engage with in the subsequent section) is more adequate for delineating the 'teaching purposes' aspect of Mortimer and Scott's framework.

Communicative approach focuses on "how the teacher works with students to develop ideas in the classroom" (Mortimer & Scott, 2003, p. 33). For Mortimer and Scott, talk can be dialogic or authoritative, but it can also be interactive or non-interactive. A dialogic talk allows for different points of view even if the talk is orchestrated by one person, while an authoritative talk focuses on one point of view – usually that of the teacher. Talk can also be interactive which means it is structured to allow for the participation of other people, or non-interactive when it excludes the participation of other people. Mortimer and Scott conceive of the meaning making process as two continuums in which in the first continuum, at one extreme there is the dialogic communication approach and at the other extreme, there is the authoritative communication approach (see Scott et al., 2006). In the second continuum, there is interactive talk at one end and non-interactive talk at the other.

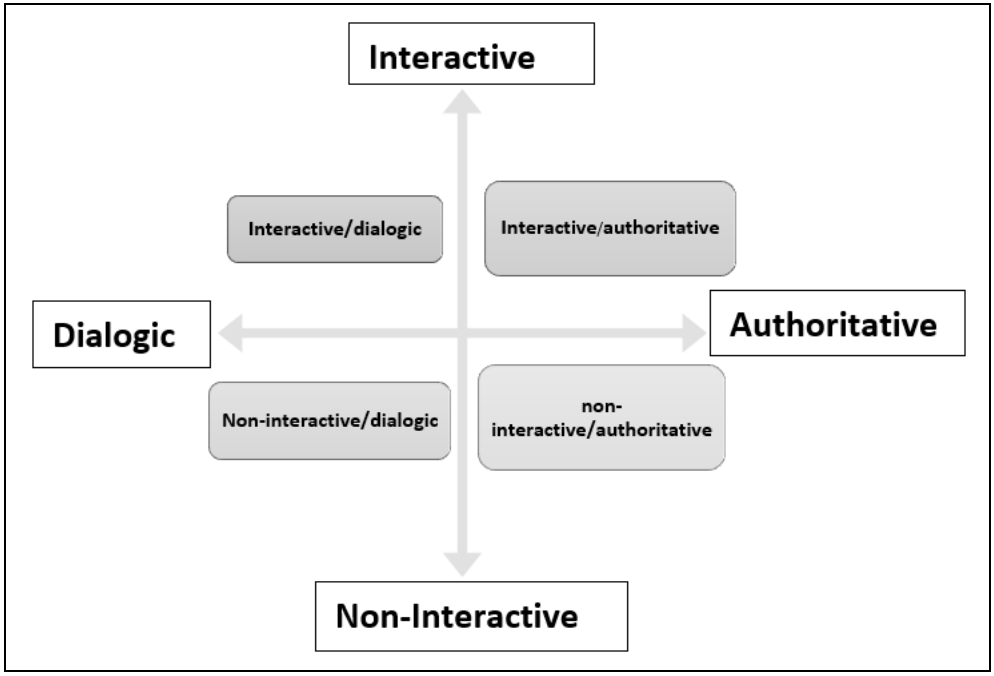

Figure 2: The dialogic-authoritative dimensions of discourse on an interactive–non-interactive continuum (adapted from Mortimer & Scott, 2003, p. 35).

The two dimensions of communicative approach (dialogic–authoritative and interactive–non-interactive) can be categorised into four classes of communicative approach with which discourse can be analysed (see Figure 2). The four classes are: 1) *Interactive/dialogic* where the teacher seeks to elicit and explore different ideas about a particular issue or concept and involves the students in the interactional process through, for example, questions which probe students' points of view; 2) *Non-interactive/dialogic* where the teacher is involved in presenting a specific (mathematics) point of view in a presentational mode (non-interactive), but at the same time, explicitly considering and drawing attention to different points of views (dialogic); 3) *Interactive/authoritative* where the focus is on one specific point of view that leads students through a question and answer routine with the aim of establishing and consolidating that point of view; 4) *Non-interactive/authoritative* which involves the teacher presenting a specific mathematics point of view or concept in a formal lecture mode (Scott et al., 2006).

Each communicative approach is put into action through specific *patterns of discourse* used by the teacher. Mortimer and Scott (2003) introduced the Initiation, Response, and Prompt (I-R-P-) pattern of discourse (Aguiar, Mortimer, & Scott, 2010; Mortimer & Scott, 2003; Scott et al., 2006). They argue that this pattern of discourse can also occur either in form of a closed chain or open chain in the interactional process. For closed chain, the pattern

takes the I-R-P-R-P-R-…E form, where the prompt (P) by the teacher is followed by a further response from the student (R), and so on until the chain is closed by an evaluation (E) by the teacher. In the open chain, there is no final evaluation by the teachers, and so the interactional process takes the I-R-P-R-P-R-P-R- form (Aguiar et al., 2010; Scott et al., 2006). This pattern may be different if students (rather than teachers) are the initiators of the question in the above chain. They pattern would then be in the form I-R_{s1}-R_{s2}-R_{s3}- (where S_1 would be student 1, S_2 student 2, etc). Mortimer and Scott argue that this pattern of discourse can be used to support dialogic interaction while most authoritative interactions are played out through the I-R-E pattern.

Multifaceted dimensions of teacher education

For teachers of mathematics intending to use variation theory in their classrooms, variation theory provides a way of structuring their lessons to maximise the chances of students' lived object of learning aligning as closely as possible to the teacher's intended object of learning. In teacher education multilingual classrooms, however, both the intended object of learning and the lived object of learning are more complex because they both necessarily need to go beyond the acquisition and/or the construction of disciplinary (content) knowledge. Hence, it is not simply about choosing examples that will achieve the target objectives as this could very easily place an emphasis solely on content. In pre-service teacher education mathematics classrooms, in addition to being knowledgeable about the mathematics content the Pre-service teachers (PSTs) will teach at the end of their qualification, PSTs need to develop an awareness of the context in which they will teach and have knowledge about instructional practices that are pertinent for this context. In this vein, research in multilingual classrooms has argued for the need to attend to linguistic aspects of mathematics teaching and learning and for attention to be paid to the language needs of multilingual learners (Barwell, 2020; Erath, Prediger, Quasthoff, & Heller, 2018; Schleppegrell, 2007; Smit & van Eerde, 2011). Moschkovich (2013) and Moschkovich and Zahner (2018) also argue that in mathematics classrooms, attention needs to be paid to enculturating students into participating in valued mathematical practices. In the specific context of teacher education for teaching mathematics in multilingual contexts, it can be argued that the multifacetedness of teacher education necessitates that PSTs need to at once be enculturated into becoming teachers of mathematics, becoming teachers of mathematics in multilingual classrooms, becoming learners of mathematics content, becoming learners of mathematical practices and becoming proficient LoLT Users for the purpose of teaching/learning mathematics (Essien, 2014). My elaboration of these multiple dimensions involved in TE draws from both the field of mathematics education (as indicated above) and from Wenger's (1998) notion of identity as a 'constant becoming', as trajectories which are not necessarily linear, and which has no fixed destination.

Becoming teachers of mathematics is about teaching, and the teacher educator sees herself/himself as developing this dimension within teacher education in the PSTs, while the PSTs see themselves as imbibing this identity. By the same token, in *becoming learners of*

mathematics content, the teacher educator sees herself/himself as responsible for the development of disciplinary knowledge in the pre-service teachers, and the pre-service teachers see themselves as learners of mathematics content. *Becoming learners of mathematical practices* relates to becoming knowledgeable about mathematical processes such as the processes of coming to define, exemplify, code switch, revoice, etc. If, for example, a teacher educator teaches a particular content using her/his chosen examples, and the focus is on the mathematics content, the teacher educator is enculturating the PSTs into becoming learners of mathematics content. But if the focus is also on the logic behind the examples that have been selected for use and what makes for a good set of examples in the topic at hand, then the teacher educator is enculturating the PSTs into becoming learners of mathematical practices/processes. In *becoming teachers of mathematics in multilingual classrooms*, there is something specific about teaching in multilingual contexts, and as such, attention is not only paid to the fact that the pre-service teachers would become teachers, but that they would become teachers in multilingual contexts. *Becoming proficient language users for the purpose of teaching/learning mathematics* describes a situation in which attention is paid to how the mathematics language and the language of learning and teaching (LoLT) in which it (mathematics language) is imbedded, are used in class.

A triadic framework for understanding examples in multilingual classrooms

How do these different frameworks come together to provide a gaze into examples and example spaces used in teacher education multilingual classrooms?

I contend that while variation theory provides perspective into the choice of examples by the teacher educator and the mathematics made possible to learn, Mortimer and Scott's (2003) framework illuminates how language is used to engage with these examples in practice, and finally the framework on the multifarious dimensions within teacher education provides perspective on how through the teacher educator's use of language, these dimensions are either attended to (or not) in teacher education multilingual classrooms. In Figure 3 below, I present the resultant triadic framework.

At the centre of the merger framework is the mathematical object of learning. This is because the object of learning is the focus of attention for the lesson. The teacher educator needs to first determine what the object of learning is for a class, and what critical features within an example set are best suited to achieve her/his object of learning. It is at this point that the patterns of variation become key (hence the question: "what examples would best bring out the critical features" is crucial). But also, it is essential for the teacher educator to engage with the kinds of discourse (patterns of discourse), communicative approach (collectively called interactional process), and teacher moves that are more adequate not only in the teaching process, but also in the discernment of pre-service teachers' understanding of the object of learning. The question as to which combination of interactional process and teacher moves will best enable pre-service teachers to develop a relevance structure on the topic at hand so that learning is made more meaningful to them becomes important.

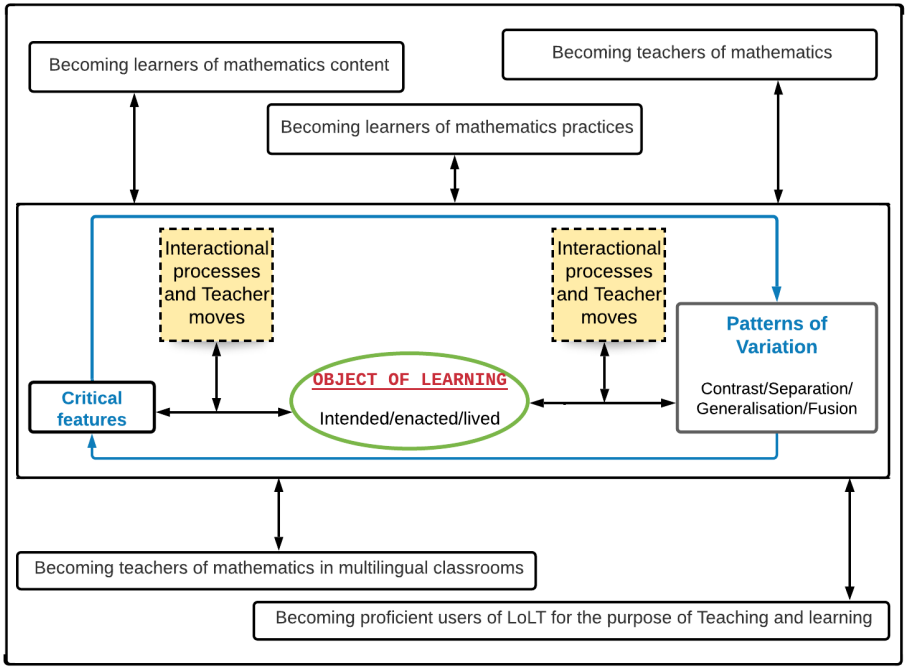

Figure 3: A triadic framework for understanding examples in mathematics teacher education multilingual classrooms

The bidirectional arrows between the 5 dimensions within TE and the core concepts of variation theory, and those from Mortimer and Scott framework are an indication that through the interactional process in engaging with the examples, multiple facets in TE are attended to (or not). But also, the awareness of the context of teaching and learning and the context the PSTs will teach at the end of their qualifications need to also inform the interactional process in class.

These three frameworks have been used separately by researchers to analyse data. I bring all three frameworks together and show an example (using empirical data) of how it can be used holistically to gain insight into not only the exemplifying practices within teacher education, but into how through the interactional process associated with the examples used in the classroom, opportunities for the enculturation of PSTs into the different facets within TE can be teased out. Zaslavsky (2010) argues that some examples have more explanatory power than others depending on the context and the classroom activities surrounding these examples. In this sense, the amalgamated framework provides tools for analysis of the instructional examples in classrooms which are both pre-service and multilingual in nature. It must be noted that in this framework, the examples that are chosen and used, the pattern of discourse that is enacted need not necessarily be fixed before the lesson.

Using the triadic framework to understand examples in mathematics teacher education multilingual classrooms

In the transcripts that follow, I bring the triadic framework (henceforth 'the framework') to bear on the example space used by a teacher educator in an introductory class on probability in order to show the value of the framework in analysing classroom data in multilingual teacher education classrooms. I engage with the classroom interactional process that occurred during the enactment of these examples and also engage with what facets of teacher education were attended to in the course of the enactment. For brief background, the teacher educator is a monolingual first language English speaker and does not share a common first language with most of her PSTs, most of whom are multilingual. It is important to note that as a product of the old South African high school curriculum, the PSTs in this class were encountering the topic probability for the first time, having not done it previously at high school. In the introductory class, focused on teaching the meaning of probability and its scale, the teacher educator provided the class with these four examples:

> **Example 1**: Chances of the teacher educator coming to class in the subsequent lesson
>
> **Example 2**: Tossing a coin. The game of football is used in which the referee tosses a coin after two captains have chosen their side of the coin
>
> **Example 3**: Throwing a dice: Throwing a dice and finding:
> 3.1 P(4)
> 3.2 P(Even number)
> 3.3 P(number less than 5)
>
> **Example 4**: Pack of cards and finding:
> 4.1 P(Jack)
> 4.2 P(10 Diamond)
> 4.3 P(Odd number)
> 4.4 P(Heart)
> 4.5 P(Black/Suit)

After this, in the next lesson, the teacher educator performed an experiment involving the law of large numbers and subsequently explained theoretical probability. Due to space limitations, in this paper, using classroom observation transcripts, I focus on the four examples above used in the introductory lesson. Using the merger framework, I start by analysing the example set based on variation theory as it concerns the object of learning, patterns of variation and critical features before engaging with the teacher moves and the interactional process in the enactment phase of the examples.

While this lesson was not theory-driven based on variation theory, a number of observations can be made on the teacher educator's choice of examples using variation theory as a lens, and in terms of what the examples make possible to learn. First, in considering the set of examples used in this introductory lesson on probability, it can be deduced that even though the initial object of learning was the definition and the meaning of probability at the start of the class, this object of learning shifted to the relevance of probability in everyday context. In terms of variance and invariance, all four examples are similar in the sense that

they relate to (and were taught in such a way that they related to) the PSTs' real-life context. Examples 1 and 2 are also invariant in as much as there are two possible outcomes. What is possible to discern in Examples 1 and 2 is that even though both have two possible outcomes (in terms of probability), the desired outcome can be an "either/or" situation. In Example 1, the teacher educator will either be in class or not be in class, and in Example 2, one captain either wins or loses. But beyond this, the two examples also allow the PSTs to discern the difference between the two scenarios in both examples in that while the probability of coming to class for the teacher educator is either 0 or 1 (for theoretical probability) or dependent on the relative frequency in terms of the number of times the teacher educator has been present/absent from class (relative probability), the probability of either winning the toss or not in Example 2 is $\frac{1}{2}$. The two examples make it possible for the PSTs to be able to discern and generalise that situations of "either/or" are not necessarily 0 or 1 in probability. The question that can be asked here is to what extent the teacher moves and interactional process enabled the PSTs to discern the above from the examples.

What contrasts Example 3 from the first two examples is the fact that there are six possible outcomes in Example 3. In Example 4, the structure of the question is invariant with Example 3 but can also be differentiated by the fact that Example 4 has 52 possible outcomes. Overall, four aspects or dimensions of variation are present across Examples 1 to 4.

Aspects	Examples of Critical Features
Events	Tossing a die, tossing a coin, or Obtaining a particular suit from a pack of cards
Sample Space	Coin: 2; Dice: 6; Cards: 52
Sample points	P(H), P(4) or P(10 diamonds)
Conditions	P(less than four) or P(Black or Suit)

Table 1: Critical features evident in the teacher educator's example set

These are events and their corresponding sample space and/or sample points and the stated conditions around the desired outcomes. The critical features or the values related to these aspects (see Table 1) are different and vary according to the nature of the presented event. So the example space can be described as simultaneously varying and thus Fusion.

So far, my analysis has focused on using variation theory to analyse the examples chosen for the introductory class on probability. In what follows, using transcripts of the interactional process in the class, I show a snippet of how the teacher educator enacted Examples 2 (see Essien, 2021, for analysis of Example 3).

Transcript 1: Tossing the coin

1	TE:	So for example *[writes:* e.g.*]* …has anyone got a coin here please? I didn't bring one in. *[gets a coin].* Right. We are now about to kick off with the Confederation Cup. I'm the captain of South Africa Bafana Bafana and you *[points to a student]* are the captain of Iraq *[everyone laughs].* And the referee comes along. Now before that you know that the coin is tossed. What do you call it in your language? How do you call that?

2	PSTs:	*(Students shout out answers)*
3	TE:	Anyway, so he [Referee] comes along *[hands the coin to a student]* You are the referee. Now before he does the activity, the referee is going to toss the coin. This is the event. Now whoever gets like a head or a tail, whoever gets it, what happens then?
4	PST1:	Reward
5	TE:	Ja, but what is the reward for getting the... you know, if I call 'heads' and she tosses heads
6	PST1:	Then you get to kick off
7	TE:	Then I get the kick-off, don't I? OK.
8	PST2:	Choose sides.
9	TE:	Now, oh hang on, before you toss the coin, listen to me, what are the chances that heads are going to come up?
10	PSTs:	*[in chorus]* Half-half
11	T:	Half-half
12	PST3:	2 to 1
13	TE:	2 to 1. What else?
14	PST2:	*[softly]* Unlikely
15	PST4:	Even odds
16	TE:	How do we speak?
17	PST5:	Equally likely
18	TE:	Equally likely chances that heads will come up. OK, do you see how I'm using the language with the number, with this? What's the number? You said it just now? Choose one of the numbers. [...]
19	TE:	So basically you can read this in mathematics just like this and immediately, instead of saying, 'Well it's half-half' or 'it's equally likely' what you can do is give me a number that goes with this event. What are the chances of getting heads? So please put equals *[next to* P (heads) *writes =].* And after that you said ... *[after = writes ½]* ...It's 1 out of 2 chances. Now I'd like you please to put the 2 in another colour. In fact let's just put the 1 in another colour *[goes over and writes the* 1 (numerator) *in pink and the* 2 (denominator) *in yellow)*
20	TE:	I'm doing this for a reason.
21	TE:	If you create a fraction like this, people, the numerator... here *[writes N]*, the numerator tells me something and the denominator tells me something. *[writes* D = *with a line pointing to the* 2*].* Now think of the coin, think of the ½, the 1 and the 2 and tell me what they tell me. *[TE cleans the scale off the board]*
22	TE:	You see a half in fraction work we teach the little ones it's 1 out of 2 equal pieces, isn't it? So if I cut an apple into 2, half-half, that's it. In Probability, this fraction is telling me more in English because it's linked to an activity, OK. So what's it telling you? Right, what's the numerator telling me?

23	PST:	I think the numerator is telling you that out of the 2 chances available of(?) which(?) the denominator, there's a probability of you getting only one. So of which it can be either the head or the tail.
24	TE:	OK, so the numerator, do you see that the numerator is linked to what I want? *[points to* N *and then to* heads*].* That's very very important you understand that. […]

In Transcript 1, the teacher educator taught the concept of tossing a coin (the object of learning) as something that is connected to the PSTs' everyday life by first using a physical coin but also by using soccer as an example. Using the triadic framework, it is possible to see how the pattern of discourse and the communicative approach enabled the teacher educator to bring into focus (or not) the different facets of teacher education. Interactive/authoritative approach is clearly the predominant communicative approach in the transcript. As indicated previously, using the interactive/authoritative communicative approach entails that even though the teacher educator welcomes PSTs' viewpoints, the interactional process is directed to one viewpoint – in this case, the viewpoint of the teacher educator. In Turn 9 where the teacher educator asks the PSTs what the chances are of obtaining a Head even before commencing with the experiment of tossing a coin, the teacher educator uses and I-R-P-R-P-R-P-E pattern of interaction to allow for different views to be offered (Turns 12, 14 and 15) on the probability of tossing a coin. The different viewpoints however converge to the teacher educator's viewpoint (hence interactive/authoritative as opposed to interactive/dialogic). It is important to note that through this interactional process, the teacher educator attempts to incorporate cognitive academic language opportunities after noting the PSTs' one- to two-word responses. She therefore asked in Turn 16, *how do we speak*?, and provides the answer in Turns 18 and 19 drawing the PSTs' attention to the correct way of expressing the probability of obtaining a Head when a coin is tossed. By so doing, attention is paid to the development of the PSTs into becoming proficient users of the LoLT.

A parallel is seen later in the transcript where the teacher educator instructor draws on language to explain the meaning of the answer (1/2) obtained in the probability question. Here, we see a focus on meaning rather than on procedures (for arriving at the correct answer) where the interactional process around the solution to the example attempts to give meaning to the mathematics symbol. Attention is paid to how the mathematical representation and the register around the mathematics symbol (1/2 in this case) are related. Such approach of interweaving content and language is well documented in recent literature (example, Wessel, 2019; Erath et al., 2018). But what the teacher educator does in addition to weaving the content and the mathematical language associated with the content is to draw on both content and the interactional context of the content to, in Turn 22, allude to how fraction is taught in earlier grades thus paying attention to the fact that the PSTs are becoming teachers of mathematics.

Discussions in relation to the triadic framework

Going back to the example space provided by the teacher educator, it is my contention that what the teacher educator could do further in enacting the examples is to focus on explanations that allow for the PSTs to see structure in the example space that makes generality possible. While this generality is evident using variation theory as a lens, in reality, it is mainly through the interactional processes and the teacher moves that this generality can become evident to PSTs. If the teacher educator had drawn attention to the pattern of variation and invariance in the questions she posed, she would have provided opportunity for the development of PSTs as learners of the mathematical practice of exemplifying. This is because in so doing, she would have drawn attention to exemplifying as a mathematical process.

While the framework allows for teasing out which dimensions of teacher preparation for teaching mathematics in multilingual context come into focus, at the same time, it also provides perspectives on which dimensions are backgrounded. In Transcript 1, through the interactional process around the examples at hand, it is easy to see attention paid to (and the PSTs enculturated into) 1) becoming learners of mathematics content, 2) becoming teachers of mathematics, and 3) becoming proficient users of the language of learning and teaching. Using the framework as a lens, it also becomes evident that in the enactment of the mathematical object of learning, becoming teachers of mathematics in multilingual context, and becoming learners of mathematical practices are not attended to in the presented transcript. A missed opportunity for the development of becoming teachers of mathematics in multilingual context is seen in Turn 1 of the transcript where the teacher educator asks the PSTs what they call "tossing a coin" in their home language. It is a missed opportunity because this question could have been used by the teacher educator to enculturate the PSTs into becoming teachers of mathematics in multilingual classrooms. The different ways of naming 'tossing a coin' in the different languages present in the class and their meanings in English could have been interrogated in class. This would have, no doubt, enriched their discussion around the meaning of tossing a coin in mathematics.

Variation theory provides conceptual learning opportunities but such opportunities in multilingual teacher education classrooms should go hand-in-hand with language learning opportunities deliberately tied into the interactional process that occur in class. The distinction between the two roles of language as a learning medium and language as a learning goal (Lampert & Cobb, 2003; Erath et al., 2018) comes to mind. Language learning opportunities are evident when using the amalgamated framework as opposed to using only variation theory either to teach or to analyse teaching.

Finally, the question could be asked as to how the class dynamics would have been affected if, for example, a different type of communication approach or a different type of pattern of discourse was used in enacting the examples. An I-Rs_1-Rs_2-Rs_3- pattern of discourse where PSTs are given the opportunity to engage or critique one another's solution to the problem would have enculturated the PSTs into practices that deal with judgments about what are mathematically legitimate claims, and practices such as providing justification, proving, critiquing conjectures, critiquing solutions, etc.

Concluding remarks

The triadic framework provides a multifocal conceptual lens for the exploration of three critical layers involved in the complexity of teaching and learning in mathematics teacher education multilingual classrooms. First, the mathematical object of learning needs to be central to the teaching and learning process. Regarding the object of learning, the choice of examples (using variance and invariance) needs to be such that it attends to the intended object of learning. Second, attention needs to be paid to the interactional process that takes place in the course of the enactment of the object of learning. Finally, in the context of teacher education, the framework provides for attention to be given to how the different dimensions involved in teacher preparation are (co-) constructed (or not).

The analysis in this paper has basically shown an inside-out approach (see Figure 3) in teacher preparation for teaching mathematics in multilingual classrooms where the starting point is the object of learning, culminating in the multiple facets of teacher education. My contention is that the choice of examples, and the accompanying interactional process need not necessarily gear towards developing the multifaceted dimensions involved in teacher education. The framework also provides for an outside-in approach where the teacher educator decides in advance what facets of teacher education to attend to at a particular point and then decides on which interactional process to use to give focus to these facets. This will in turn inform the type of examples that the teacher educator would choose to bring the object of learning into focus. In such a case, the framework suggested in this paper could be extended to professional development programmes involving multilingual mathematics teachers.

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Variation and dialogic communication in maths teacher education in a multilingual context: A response to Anthony Essien

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Anthony Essien, in his plenary session, presented a triadic Framework to analyse the choice of "exemplification" in teachers education mathematics classrooms in a multilingual context, with a focus on the role of the teacher educator, who is seldom the object of research in education. In my response, I will be reviewing his paper on two counts. A. I will be reflecting on the presentation of the triadic framework developed and B. reviewing how the selection of examples and the classroom interaction can be further extended to bring a greater multilingual and multicultural input into maths teacher education, with a greater focus on dialogue. I will end with a few suggestions as to how the framework may be extended and enriched to incorporate multilinguality in a stronger way.

The triadic framework for analysing choice of examples

As the elements of choice of examples in a multilingual mathematics classroom are complex, choice is determined by complex interrelated factors. A framework putting together these elements that can act as criteria of choice, is, indeed, important. It is, heartening that Anthony has taken upon himself this challenging task.

Another important aspect is that the site for the framework is teacher education classrooms and not school classrooms. This is a crucial element that is often neglected. Even though we're talking of Student Teachers here, the assumption that the student teacher has understood school mathematics concepts does not hold for developing countries like South Africa and India.

It is in the context of recognising its importance and its strengths that I now critique it. Maths is a subject that takes one through to an abstract realm. However, the human mind learns through concrete examples, from experience and reflections. Hence, exemplification is a necessary part of learning. It is interesting that Anthony tries to see "exemplification" through trying to make a triad of three different yet inter - related frameworks. This is an important exercise. He has chosen to do it theoretically rather than empirically. In the attempt to develop this framework theoretically, the elements of the framework have not been explicated enough and have become somewhat opaque. A little more elaboration with examples would have been more helpful.

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The triadic framework developed by merging variation theory, the dialogic interactive communicative process and the multifaceted aspects of multilingual contexts has captured a lot of important aspects and placed them in relation to each other. However, it seems to put multilingual aspects and contexts as an add on at the end, whereas this needs to be an integral and overarching part of the framework – like a canvas in a painting. Because language and multilinguality is what is the lifeblood of the children learning the mathematics.

The paper does not deal with the language aspects and its relation to mathematization at all. This is an important weakness. Different languages not only have different words, but different idioms and cultural contexts that need to be taken into account. A more centre-stage position to multilingualism would have brought out the immense potential of intertwining it with variation theory. This would map the examples from different languages and cultures onto the critical features of a mathematical concept and procedure.

Variation and dialogue: Two important elements for maths education in a multilingual context

The element of variation, in mathematics classrooms has two aspects to it – a) content variation and b) procedural variation. The latter is equally, if not more important in mathematization of concepts, but has not been dealt with at all. Procedural variation gives a lot of scope for multilingual and multi-cultural ways of solving problems, particularly oral ways – classical examples are using grouped addition from hundreds to units for multiplication – rather than the written algorithm of multiplying from the unit side and carrying over – this is procedural variation and is present in almost all algorithms. If connected with multilingual cultures, it would give a lot of mathematical power to deprived communities, whose procedural variations are excluded from mainstream mathematics.

Equally important are the communication and dialogue aspects, which have not been elaborated – the collective, reciprocal, supportive, cumulative and purposeful aspects of dialogue. It is not only language that is different in a multilingual context, but the cultural differences reflected by the language are much deeper and wider.

Those tribes and community groups that have developed with livelihoods which interact with numbers in certain ways, often give rise to quinary, quinary-decimal and vigesimal number systems in traditional societies. Even though other aspects of tribal living may decline – sometimes, when the livelihood remains the same, such number systems, ways of calculations, words, phrases, riddles continue. They make for a stronger procedure in the heads of such communities than the formal metric system introduced through school. Incorporating these in the examples and dialogues in the classroom has a number of advantages.

1. Most such groups are deprived communities and their language and ways of thinking have been excluded from the mainstream classroom. This invisibilises them, undermines their identity and demotivates them from continuing their studies. Including conceptual and procedural variation on mathemetisation from their lives, will enhance their visibility and motivate them to learn mathematics better and own that learning.

2. A diversity of variations will also help other students in class become familiar with a greater diversity of possibilities, other cultures and other words and meanings. This will develop cognitive robustness as well as empathy in all students.

Using the triadic framework to understand examples in multilingual mathematics classrooms

The concept chosen is an introduction to probability. While it is mentioned that the language background of the teacher educator and the student teachers is different, it is not clear which languages, tribes and cultures the student teachers hail from.

The choice of the four examples is mentioned, but not the bases of their choice. It is not clear whether they have been contributed by the student teachers, or the teacher educator decided on them by himself. Enough background is not available to say how they were chosen and any links to the cultures and languages of the student teachers are not mentioned. Football, dice games and cards all seem western contexts.

The aspect of probability being dependent on ambient conditions on a large scale is not mentioned nor incorporated in the activity or dialogue, or how this probability can be played around with – weighting the dice or coin. Just discussing one example does not bring out the conceptual and contextual features.

The teacher educator in the transcript excerpt also doesn't give ample space to discussing the student teachers' own conceptual, cultural ideas on probability and reveal their own language of probability. I don't know enough about South Africa, but in India the idea of probability is integrally linked with the idea of fate. Another area is gambling. We, in our alternative science curriculum have taken up the issue of odds in gambling, at the middle school level, and shown how it is impossible to have a winning situation in the end.

Secondly, this example did not have much dialogue or real interaction, somewhat vitiating the point of the dialogue part of the framework. It was presented by the TE in a relatively closed ended way – the coin was not tossed even once in the excerpt, whereas to get a hang of probability – you need to toss it a fairly large number of times, then go to the numerator denominator issue. Had it been followed up by a number of throws, the name for the throws, the use of coin throwing discussed, it may have brought forth a better dialogue necessary for better understanding. As presented both the activity and dialogue seem cosmetic rather than authentic.

A lot of mathematics – number, operations, time, weight, length, algebra can be dealt with in a multilingual way, using variation of examples and procedures from different cultures. This would benefit both the different language speakers as well as the dominant language speakers as through dialogue they could collectively bring consonance. This would require a framework for multilinguality which maps on to the variation theory framework.

In conclusion

In order to integrate a framework for multilinguality along with variation theory and communication, elements of language and culture, that could form criteria for selection of examples need to be laid out.

Some of these could be

- Words and phrases closest to the concept, that are available in the learners' context (e.g., number names in a quinary system, how number names for larger numbers are constructed in the language, or names for units of time, weight distance, etc., and for past, future, long short).
- Real life contexts where the concept is used (e.g., when teaching percentages look for examples of percentage in the community and how it is expressed – e.g., in poor communities and rural contexts when in the context of lending the phrase x rs. *prati saikda* (per hundred) is used not *pratishat* (percentage)).
- The local procedures for solving problems of that concept need to be looked for or extracted from students.

Secondly, a way of dialogue that brings out students' culture and language needs to be outlined. Elements of dialogue mentioned above also need to explicitly form part of the framework. These could then form a checklist and rubric for selection of examples to be incorporated when selecting examples according to variation theory. A revised framework could incorporate these aspects in a more authentic manner.

The ethical significance of exemplifying: A response to Anthony Essien

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In his plenary paper, Tony Essien wrote that "The importance of mathematical examples [...] cannot be overemphasised". Tony's paper opened my eyes to the significance of examples and of the complex practice of exemplifying in multilingual mathematics education in general, and in teacher education in particular. The triadic framework that Tony proposed captures exemplifying as illustrating concepts from the perspectives of (a) grasping the illustrated concept by discernment; (b) communicating about the illustration as a fundamental idea of teaching and learning; and (c) attending to the multilingual context (of pre-service teachers) in which the illustration is submerged.

Multilingualism in mathematics education

Before moving further, I would like to recognise that multilingual pre-service teachers and multilingual students in general are far from a homogeneous group either in, or across contexts (Barwell, 2016; Civil, 2012). However, multilingual students are often from marginalised groups whose opportunities to learn mathematics are diminished compared to their peers from majority/dominant groups whose first language is the language of power and hegemony and of learning and teaching (e.g., Chronaki, 2009; Källberg, 2018). I consider a multilingual classroom to be any classroom where more than one language is present. Hence, some students in a multilingual classroom may be multilingual, while others are not.

In multilingual classrooms some languages might be silenced due to monolingual norms and/or policies, but they are silently present in every instance of the interaction (García & Wei, 2014). Different languages in which different experiences, knowledges and worldviews are embedded (Knijnik, 2012; Radford, 2012) are brought into the learning spaces. This means that students may experience non-hegemonic and hegemonic cultural and language resources against each other at school, often finding some of their cultural, epistemological, and language resources and identities ignored or perhaps even rejected. Ultimately, this may influence multilingual students' future prospects. Therefore, multilingualism in mathematics classrooms is a matter of social, political, cultural, and ethical issues.

Ethical dimensions of a 'good' example

Initially, Tony shared some questions that inspired him to conduct research about the practice of exemplifying in multilingual mathematics teacher education. One such question

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was: "What should a "good" (use of) example be in multilingual pre-service teacher education mathematics classrooms?" This question invited me to consider value-laden or ethical dimensions of the practice of exemplifying in multilingual learning spaces. Indeed, the word example can be used to express conducts of behaviour; an example is one that/who serves as a desired pattern to be imitated – a role model. An example may also be a punishment inflicted on someone as a warning to others (Merriam-Webster Thesaurus Online, n.d.).

The philosophical branch of ethics is concerned with questions of right or wrong, of benevolence and harm, and about what is proper conduct. Ethics is not a monolithic concept. Rather, it covers a broad range of ideas that span from normative, utilitarian conceptions of virtue and duty, to postmodern ethics that embrace dimensions that may capture the ecological, the social and cultural, others, and self (Dubbs, 2020). Skovsmose (2020) suggested that adding an ethical dimension to mathematics opens up a space for reflection on the ways in which mathematical activity impacts upon society. He suggested that critical mathematics education offers the capacity to embrace such ethical reflection.

Opening up a space for ethical reflection

The triadic framework that Tony proposed helped me to identify one instance in the excerpt that he provided that has the potential to open up a space for ethical reflection on political effects of monolingual mathematics education in multilingual societies. In line 14 of the excerpt, a pre-service teacher softly replied "Unlikely" when the teacher asked what the chances of "winning" are when tossing a coin. It appears as if the coin-tossing example allowed the pre-service teacher who replied "Unlikely" to discern some critical aspects that relates to injustice. There ought to be equal opportunities that the heads or the tails appear when the coin is tossed, and the chances of winning should be 50 percent. The "unlikely" reply may be a reply that challenges the idea of equal opportunities. It could suggest a disadvantaged position where chances in life and opportunities, such as learning mathematics, are unequally distributed. In the interaction, the "Unlikely" reply seems to pass unnoticed by the teacher and by the other pre-service teachers because no one comments on it. The "Unlikely" reply could provide an opportunity to open up a space for reflection on the ways in which mathematical (school) activity impacts upon societies that are multilingual, which most societies are. In this space, reflections may capture and move forward a critique in multilingual mathematics education that revolve around discourses such as those discussed by Chronaki and Planas (2018) regarding representation and politics, language as resource, Eurocentrism, racism and epistemic violence, deficits, benevolence, dichotomies (formal-informal mathematics, first–second language, global-local, foregrounds-backgrounds) and othering, *et cetera*. These are discourses that pre-service teachers need to navigate as they are at once "enculturated into becoming teachers of mathematics, becoming teachers of mathematics in multilingual classrooms, becoming learners of mathematics content, becoming learners of mathematical practices and becoming proficient LoLT users for the purpose of teaching/learning mathematics" (Essien, 2021, in this volume, p. 45).

A relational perspective on ethics

What are good examples of how to navigate such spaces? How do we deal with the idea that, for instance, a 50–50 chance of "winning" might be true for some students in some situations, while winning might be very "unlikely" for other students, without reproducing stigmatising categorizations or deficit discourses (Norén & Boistrup, 2013)? Or without putting an extra burden on marginalised groups, demanding that they explain or share their experiences (Hand et al., 2021)?

The questions invite me to consider Swanson's (2017) writings:

> Perhaps it is time for us to remember what the intentions of mathematics education should be, to live well with mathematics education in order to live well with others; to live and research well with mathematics education in order to make possible futures of radical hope. (Swanson, 2017, p. 13)

The above quote shifts the attention from an ethical reflection on how mathematical activity may impact upon society to how we (for example educators and pre-service teachers) inhabit mathematics learning spaces in ways that allow us to live well with mathematics and with each other. In other words, the attention shifts from socio-political projects to a critique that centres considerations in multilingual mathematics activities from a relational perspective of ethics.

Swansson (2017) reminded me that for us to live well with mathematics, mathematics education and research, we may need to expand theories of ethics from non-Euro-centred perspectives. She called for a move towards, for example, indigenous, decolonial, posthuman ontologies, epistemologies, and theories to live better with each other as we engage mathematics. Drawing on indigenous knowledges (Ermine, 2007), Russell and Chernoff (2013) proposed ethics as a capacity and/or will to know what may harm or enhance ecological, social, and cultural well-being, and the well-being of others and self. While it may be challenging to grow a capacity to know what may harm or enhance the well-being of multilingual pre-service teachers or multilingual students in mathematics learning spaces, it is crucial.

Closing remarks

The triadic framework that Tony presented concurrently captures the significance of ensuring access to dominant mathematics for multilingual pre-service teachers *and* the significance of highlighting the presence of language and thereby also cultural and epistemological diversity, in exemplifying practices as part of language diverse learning spaces. In other words, the framework allows us to move away from dichotomising stands that revolves around either-or issues such as knowing and learning dominant vs non-dominant languages and/or mathematics. The capacity to recognise the socio-political significance of multilingual pre-service teachers and students getting access to dominant mathematics and dominant languages may be a matter of enhancing well-being. It could provide multilingual pre-service teachers with the language and mathematical resources

they need in order to be heard and listened to by others such as parents, students, and colleagues. However, without pausing for ethical reflection and critique (Hand et al., 2021), dominant languages and mathematics will simply be (re)produced, which could potentially cause harm (Le Roux & Rughubar-Reddy, 2021).

For example, ethical reflection could comprise illuminating the significance of the presence of non-dominant languages and mathematics in multilingual learning spaces. A both–and stand – that is, a stand that embraces both dominant languages and mathematics and non-dominant languages and mathematics, which I believe Tony's triadic framework does – is a good example that could help us inhabit mathematics learning spaces in ways that allow us to live well, or at least a little bit better with mathematics and with each other.

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Mathematics education, researchers and local communities: A critical encounter in times of pandemic, pareidolia and post-factualism

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Pandemic have increased the importance of social phenomenon involving mathematics and making crisis even more critical. This text studies the current global situation in order to enquire the role of mathematics education researchers in times of crises. A review of research partnerships with communities facing long-term critical situations is done, in order to reflect on the possibilities that mathematics education researchers have nowadays. Theoretical notions from theater anthropology and decolonial studies are borrowed to trigger insights on mathematics education as a field of practice and research.

If we keep quiet they kill us, and if we talk they do so anyways. So, we talk.
Cristina Bautista Taquinás, Indigenous Nasa leader, killed in 2019

In this plenary I want to invite the MES-community to revisit a recurring old theme: The relationship between communities and researchers in mathematics education research driven by social, cultural, and/or political interests. Researchers refer to diverse communities: local communities, cultural groups, teachers, or even students. In almost all previous conferences we can find a plenary talk in which this relationship is explicitly addressed or at least implied. The study of this relationship has almost become a genre within our community, in which issues such as the role of the researcher, coherence among methods, principles and ends, impact, validation, agency and reflexivity, just to mention a few, are deployed.

Knowing that the debates about these issues are far from being closed, I want to address some of them with two particularities that can bring new insights and concerns: 1) the contemporary situation assembled in 2020, and 2) features of certain experiences of long-term work with communities. These two particularities will allow me to unfold ways in which mathematics education and political endeavors can nurture each other. My strategy consists of sharing a series of images from research experiences that I am familiar with, in order to formulate questions that mobilize discussion.

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The haze dazes the gaze

Opening MES 8 in Portland, Ubiratan D'Ambrosio foresaw a new era for MES issues. He was almost prophetic: "*there may be no future. Our existence, as a species, is threatened.* Our objectives must be even more than social justice and dignity for the human species, must be the survival of our species that is threatened by a societal breakdown" (D'Ambrosio, 2015, p. 20, original emphasis).

Just six years after we are watching cruel examples of what Ubiratan called social and environmental degradation. MES 9 and MES 10 served to highlight the new threats of post-factualism and the naturalization of crisis. We were experiencing similar or analogous cases of proliferation of (mis)information at different times of the year and (economical) crisis emerged at national scale, as if a ghost visited countries one at a time. First Greece, then Portugal, and so on. Nowadays, with the COVID-19 pandemic those threats get amplified. Crisis gained a global scale and citizens worldwide face the same problems almost simultaneously. Now it is undeniable that we all are on the same boat (although not in the same seat) and more people are conscious of the interconnectedness of our decisions.

Inside this evolving critical situation, mathematics has also amplified its power and agency within the assemblage of things. Mathematics-heavy entities appear everywhere: Models for virus propagation, estimation of vaccines impact, geo-refencing of hot spots of contagious. The incidence of past, current and future public policies at national and global scale, discussions of what metrics to use to assess and compare mortality, lethality, testing, treatments, vaccines and vaccinations. Rates, probabilities, charts, epidemic curves, cost-effectiveness analysis, forecast, simulation models and many more mathematical objects have invaded the public sphere. Debates on objectivity, bias and political consequences of mathematical models exploded everywhere, connecting politicians, scientists, philosophers, groups of interests and layman citizens. All of those debates require having some proficiency in mathematics and, simultaneously, put in doubt the affordances and assumptions that any mathematical model of a pandemic conveys.

I could not imagine a more powerful example of Ole Skovsmose's ideas on the critical nature of mathematics in society than the COVID-19 situation. Risk, uncertainty, responsibility, mathemacy and several types of mathematical practices in a globalized world are vividly exposed. Knowledge and truth have gone beyond modernist conceptions (Skovsmose, 2009, 2011, 2021). All in one single explosive situation. Decision making processes need to be accomplished in real time at every level (national, regional, municipal, and even personal) with the public scrutiny and turning accountability as a major issue for decision makers. Citizens can be (in/ex)cluded, engaged or discouraged to intervene according to their capacity to understand the mathematical discourse unfolded.

Past and present mathematics education practices exhibit their power within the COVID-19 showcase. Charts with several indicators are used and misused to sustain and criticize public policies. Laypeople assess and question the validity of nation-State measurements of the situation. Personal experiences with the virus, the treatment and the vaccine are used to

mistrust the statistical populational tendencies pointed by experts. Possibility gets equated with certainty (probablity equal to 1). Sample results get immediately extrapolated for the whole population. Stochastic thinking is needed not only for citizenship, but for survival, and its absence is evident. Public health offices try to create awareness campaigns and fail miserably. Some people cannot understand the information provided in real-time. Others do not want to understand. Improvements in the propagation models due to new information and subsequent changes in the official policies are labelled as symptoms of improvisation. Hard times to explain rules of scientific method and Popperian falsifiability. Official and dubious information about lethality, infectivity and treatments exploded social networks several times. Conspiracy theories flourish and spread faster than the virus itself, and therefore there emerges the clamor to increase the mathematical-scientific literacy needed to act as a barrier against this tsunami of (mis)information.

Uncertainty in the facts lead to uncertainty in the interpretation of the facts. This is connatural to any state of crisis, and it creates a certain entropy to make the crisis even more critical. However, it is important to inquire the mechanisms that allow the constitution of this situation. Why are fake news so easily propagated? How post-factualism can lead the scene? This is not only because the technological possibilities of sharing information worldwide in real time. It is also because a very old impulse got unleashed: *Pareidolia*, that is the tendency to perceive a specific, often meaningful image, in a random or ambiguous visual pattern. Pareidolia is the basis for psychological exams such as the Rorschach inkblot test, which tries to infer a person's mental state by studying the thoughts or feelings that the person projects into some images (Wikipedia, n.d.).

Pareidolia is one type of *apophenia,* defined as a "tendency to perceive a connection or meaningful pattern between unrelated or random things (such as objects or ideas)." (Merriam-Webster.com, n.d.) There are other kinds of apophenia such as the Gambler's fallacy and the confirmation bias. They are related to situations like echo chambers, or cognitive bias such as the Dunning-Kruger effect. Although these tendencies have been deployed historically to segregate people by political or religious reasons, the internet and the fostering of digital social networks in the last decades have been functional to their unlimited growth and use in generating other type of divisions. One of the results of this dynamic is the political radicalization and polarization that escalate differences, justify violence and damage the social fabric that sustains a community. These worrying *apophanies*[1] seem to be omnipresent since the pandemic exploded and echo chambers seem to be shaping communication more than ever: virus deniers combined with anti-vaccine movements are radicalized by politicians attempting to promote biological passports.

I want to mention just two factors (among many) sustaining the proliferation of echo chambers and mistrust: the historic role of governments and the ways in which information widespreads. The first one is about the eroded confidence of many citizens in private and public institutions. At least in poor countries like mine, the government is historically

[1] An apophany refers to an instance of apophenia that produces the exact opposite of an epiphany.

characterized by the systemic and permanent denial of the most basic human rights of the population; and by the rampant corruption in the state institutions that deviates money from taxes to politicians and the private sector. How can a state like Colombia, that has kidnapped and killed more than 6400 civilians to present them as guerrilla soldiers, ask for credibility? How can the Peruvian government request confidence in medical procedures, after the forced sterilizations of more than 2000 indigenous women? When the official version of facts and events is in the hands of mythomaniacs, citizens are eager to look at other sources.

A second factor is the way in which information is being accessed and filtered. For instance, how search-engines operate in the Internet. It is well known that content creators obtain their profits by the amount of clicks, likes and visits. Algorithms create lists based on the features of the content, registered in meta-data and tags. In order to appear in the top ranks of a list, creators need to make-up their content to look more urgent, paradoxical and scandalous; so they are urged to produce and label content that appeals to the morbid curiosity of the audience, that is, to create clickbait. As a reaction, some educational youtubers have buckled under the pressure and decided to quit their channels because of the induced trivialization of their content. This means that the quality and diversity of contents are constrained and biased towards particular types of information.

Just to summarize this overview, I characterize the contemporary conjuncture as the mingle of new challenges emerging (the current pandemic), not so recent situations (post-factualism), and very old situations (pareidolia); all of these repowered. This explosive cocktail is clearly critical, as it is uncertain, risky and demands responsibility (Skovsmose, 2011). The COVID-19 cocktail operates as anabolic steroids for ongoing social, economic and ecological processes of oppression: Economic exploitation and massive unemployment, surveillance projects of fascist nature, social inequality, ecological deprivation, defunding of public schooling, and many others. All these threats to basic dignity got boosted in the last two years.

Turning on the fog lights

Nonetheless, the pandemic, postfactual and pareidolic (3P) situation also creates possibilities. For instance, Colombian public universities used the pandemics to negotiate with the national government until reaching agreements on full tuition waving for students due to the pandemic, and to enhance part of the digital infrastructure. Of course they did not obtain all resources that public universities deserve, and the conditions are still far from reaching a minimal decency. But certainly students forced the government to do things that were systematically denied for three decades of neoliberal policies of dismantling public education.

Indigenous Nasa people started a self-imposed lockdown in their territory, and gathered healers from different shelters to carry research on the COVID-19 virus. The team of healers experimented with several combination of plants and, after a couple of months, released to the whole community two different recipes: one for prevention and other for treating the symptoms. More recently, when the national government created official vaccination campaigns, Indigenous communities were fearing that the government would provide them

with low quality vaccines or even introduce other diseases among indigenous communities. As a result, indigenous organizations decided not to participate, arguing against the lack of clarity on the procedures, expressing concerns on vaccine effectivity, and defending their indigenous protocols of biosecurity.

During the extended lockdown, the Colombian government also tried to pass a regressive tax law, that would increase the taxation burden on the poorest. That provoked a national strike. In April 28th 2021, there started a social revolt unseen in 50 years. Anger and indignation expressed in unexpected ways. Statues of colonizers were pulled down, public places were renamed, young people made blockages and barricades in strategic points of several main cities. They get organized through social networks and broadcasted live the confrontation with police squads, revealing police brutality.[2] That was useful to contest the fake news that proliferated through stablished mainstream media. Television news showed only the riots and looting, prompting the narrative that cities needed to declare martial law to control the protesters.

Internet activism served to made visible other dimensions of the protest. People organized in the neighborhoods and created several lines of resistance. While some conformed the front line of the confrontation, a side line of mothers went to the streets to take care of their kids in the frontline; lines of musicians went to play and serve as incidental soundtrack of the riot! Collectives of lawyers worked, free of charge!, to represent people detained and guarantee their civil rights. Even an anonymous guy who dressed with the Colombian flag was named "Captain Colombia", in a sarcastic real emulation of the fictional U.S.A. hero. People organized a "communitarian pot" to provide food to front-line protesters. This "pot" consists in closing a street and turning it into a kitchen for whoever needs to be fed. Some of these initiatives became regular and generalized, while some other disappeared or became replaced by others side lines.

Outside the streets, "pedagogical" lines of resistance were also created. In Cali, collectives of lecturers at universities joined high school teachers, engineers and other professionals to create "Pre-Icfes pa'l barrio" [Pre high-stake-test training for the "hood"]. This initiative created a set of free online resources and personal tutoring in mathematics, chemistry, physics and biology, aiming to prepare last year high school students to obtain better grades in the national high-stake test. Variations of this type of initiative has emerged in other cities. In all, the initiative gathers 1.700 volunteers, dispersed around the country and even in other countries in the world, and reaches more than 14.000 youngsters, interested in gaining access to university. Probably the initiative will not succeed in improving the test scores. But

[2] During the period from April 28 until June 26, 2021 Colombian NGO's have reported violence in the hands of public force: 1,617 victims of physical violence, 44 homicides presumably perpetrated by the Public Force, 2005 arbitrary arrests against protesters, 748 violent interventions of peaceful protests, 82 victims of eye aggressions, 228 cases of firing of weapons, 28 victims of sexual violence and 9 victims of gender-based violence (Temblores ONG et al., 2021).

certainly it mobilizes an important amount of people that usually did not play a part in political actions.

Beyond assessing the efficiency, continuity or coherence of these samples of applied pataphysics, it is important to stress their success in activating spaces of encounter among diverse people who were not were engaged before or were disarticulated. Their success is to be found in the care for the social fabric that pareidolia targets to damage.

Love at third sight

When trying to relate mathematics education (ME) with the recent communitarian contestations mentioned before, some experiences came to my mind. Mônica Mesquita developed, together with groups of fishermen of the Costa de Caparica in Portugal, several projects of communitarian education, using critical ethnography and participatory action research. They first devoted their efforts to obtain unpolluted water to a semi-illegal settlement. More recently they have worked to improve sustainable and safe fishing practices. These experiences have created spaces of formal and informal education that empower communities around issues of collaborative governance, participatory ICT, and fishing legislation. A 20-year process of critical encounters with fisherman, academics, and government representatives has impacted positively on the social conflicts of the Caparica region, working for spatial justice through intellectual justice (Franco & Mesquita, 2019; Mesquita, 2014, 2016).

Since 1998, Adailton Alves da Silva, a Brazilian mathematics educator, has been working with A'uwê/Xavante communities. He began as an advisor in educational projects for this Indigenous group. He still collaborates with the production of didactic materials based on rituals and ceremonies, professional development workshops, and the design and advice on the secondary education curriculum for the Xavante people (da Silva, 2006; da Silva et al., 2021).

Since 2004, Gentil Wejxia, an indigenous educator from the Nasa people in Colombia, has led several educational processes in the sacred region of Tierradentro. In 2008 Gentil became the team leader of an indigenous research group on Nasa mathematics, which studied past and current vernacular practices, relating them with the six universal proto-mathematical activities enunciated by Alan Bishop (Caicedo et al., 2009). Nowadays he is part of the educational experience of *Kiwe Uma,* one of the many enactments of "educación propia" or own indigenous education (Parra & Valero, 2021). This initiative is carried out by a small but growing group of indigenous families that want to develop a culture-rooted education, nurturing their seed (the children) in order to safeguard the indigenous territory, land and worldview. They have structured several development stages for the kids, according to Nasa cultural values and practices. The equivalent to the content in a curriculum is strictly related with the set of skills and knowledge that an indigenous person needs to manage as adult in indigenous and non-indigenous contexts. Gentil has developed further his insights on the cultural dimensions of mathematics. The mathematical component of *Kiwe Uma* includes

both the spirituality behind weaving of some traditional handbags, hats and clothes, as well as the study of statistics and algebra, in order to understand the process of economic and ecologic deprivation that Nasa people have been suffering. This confirms that the *Kiwe Uma* collective decided to teach mathematics for life, but an indigenous life of cultural and political resistance. Kids were deliberately not registered to participate in the national educational system and thus, they would not sit the mandatory high-stake test.

I also acknowledge that MES conferences have been a rich space to meet inspiring experiences. I remember Munir Fasheh telling his story of healing from modern superstitions and the explorations on Mujaawarah in Palestine (Fasheh, 2015) ; the group on Crisis in MES shared Brazilian, Colombian, Indian and Palestinian strategies to deal with educational crises (Marcone et al., 2019; Parra et al., 2017). Gelsa Knijnik has also reported on the educational projects of the land-less movement (MST) in Brazil (Knijnik, 2004, 2006).

It is important to propose a thread that can bond all these experiences, and that sheds light on some of the ways that research and practice on social, cultural and political dimensions of mathematics education has been conducted. To do so, I want to introduce an insight coming from theatre theory, that can be used in many fields of human sciences, and that I find useful for mathematics education in particular. Eugenio Barba is the director of Odin Theatre, a group located in Holstebro, Denmark. He and his group are worldwide recognized for formulating the idea of theater anthropology. When reflecting about the several ways in which performative arts are developed around the world, Barba identifies a special kind of theatre, a "third theatre", that is:

> Almost unknown, it is rarely subject to reflection, it is not presented at festivals and critics do not write about it.
>
> It seems to constitute the anonymous extreme of the theatres recognised by the world of culture: on the one hand, the institutionalised theatre, protected and subsidised because of the cultural values that it seems to transmit, appearing as a living image of a creative confrontation with the texts of the past and the present, or even as a "noble" version of the entertainment business; on the other hand, the avant-garde theatre, experimenting, researching, arduous or iconoclastic, a theatre of changes, in search of a new originality, defended in the name of the necessity to transcend tradition, and open to novelty in the artistic field and within society. (Barba, 1986, p. 193)

From now on, I invite you to play a game of analogies. A "first ME" would be devoted to the ideology of improvement, trying to find and disseminate the "best practices" for mathematics classrooms, in line with official curricula. Large scale projects get funded. A "second ME" would be concerned with studying the complexities of the field, trying to comprehend the dynamics of classroom practices and public policies. Small scale projects receive some funds. Getting back to Barba:

> The Third Theatre lives on the fringe, often outside or on the outskirts of the centres and capitals of culture. It is a theatre created by people who define themselves as actors, directors, theatre workers, although they have seldom undergone a traditional theatrical education and therefore are not recognised as professionals.

> But they are not amateurs. Their entire day is filled with theatrical experience, sometimes by what they call training, or by the preparation of performances for which they must fight to find spectators. According to traditional theatre standards, the phenomenon might seem insignificant. But from a different point of view, the Third Theatre provides food for thought. (Barba, 1986, p. 193)

Could we think of a "third ME" as one that lives on the margins, one that has to face armed conflict, poverty, famines, cultural extermination, illegal migration? How does a ME that does not locate its strength in institutionalized funding or prestige look like? Although the ordinal third could refer to the "third world", I think that this kind of mathematics education is being conducted everywhere by teachers, scholars, activists or even enthusiasts, who do not to wait for approval or acknowledgement coming from funding agencies or fine academic circles to research and intervene reality. A "third ME" would conceive itself not as the education *for* mathematics, but an education *through* mathematics, just as Ubiratan D'Ambrosio proposed many times.

The third space of ME is what bonds Mônica, Gentil, Adailton, Munir and Gelsa. The third space is inhabited by them and many other people and doings that often are not considered as legitimate ME. They have in common an inner force to experience ME "as a bridge, constantly threatened, between the affirmation of their personal needs, and the necessity of extending them into the surrounding reality" (Barba, 1986, p. 194). A third ME encompasses the diversity of possibilities of adopting a conception of mathematics education research living at the margins. Many of their experiences are developed outside the school, but within politically organized groups; and others happen within the school system.

Third thoughts

In this section I engage in some further considerations (as remarks or extended aphorisms) that will elucidate the affordances of noticing this third region of ME. Some of them paraphrase what Eugenio Barba has proposed about the third theater in (Barba, 1992, 2002). I begin observing that the nomination of "third ME" could be new, but the concerns and purposes subsumed in the nomination are certainly not new. So, it is not aiming to propose a new pedagogic style, neither a new turn. Rather, it performs a sociology of the absences that "amplifies the present by adding to the existing reality what was subtracted from it" (de Sousa Santos, 2012, p. 56): A decolonial move to make visible knowledge, people and stories that were assumed as non-existent.

A third space of ME is not an educational style, nor an alliance of research groups, still less a movement or international association; nor is it a school, a paradigm, or set of techniques or methods. Instead, it points to a way of giving meaning to mathematics education. So, whereas many people wonder if school (and ME) make any sense nowadays (as if it were a thing capable to have or lack anything), a third space assumes that such problem belongs to the people who inhabits school (and ME). It prefers to deal with the question: "Are we able to give meaning to what we do?" This is far from a solipsistic, isolated approach to the problem. It is an awareness that answers need to be singular and embodied in actions.

A search for meaning signifies above all a singular discovery of a craft. It invites to a "patient building up of our own physical, mental, intellectual, and emotional relationship with [teachers, students and knowledge], without conforming to those balanced and proved relationships current at the centre of [mathematics education]" (Barba, 1992, p. 9, paraphrases in brackets). A craft does not fall entirely in techniques or routines, and what is crucial in a craft is the *ethos* that sustain it. An ethos is the ensemble of social, linguistic, political, existential, communitarian and ethical behavior, that expresses the ways in which we want to relate our local history with History. Our field is used to be thought in two dimensions, as if what mattered was only policies, ideological tendencies, institutions, learning outcomes, or different methodologies (Barba, 2002). Such framing falls short when inquiring for ethos and its concomitant craft. When mathematics education is addressed as solitude, craft, and revolt. Just like Paulus Gerdes did his work until his last day of life. I wonder if this paragraph meets with what Sonia Clareto and Roger Miarka have delineated as "minor mathematics education" (Clareto & Miarka, 2015).

The quest(ion) for a craft opens up an entire new terrain. Who does research? As far as everyone is in the race to create meaning for their own practice, the conventional colonial frontier among the one who thinks and the ones that are thought, gets blurred. It happens a displacement in the *locus of enunciation*[3]. Such displacement puts in doubt the very concept of research, and also its impact, pertinence and validation. For instance, in the indigenous regions of Cauca, Colombia, educational projects must be formulated and justified in regards of local community agendas of political and cultural resistance. They must involve members as active researchers (and not merely informants). This is to say, the academic outcomes are not the leading force, even as the research goals grow from the inside of community and reach the outside of legislative funding agencies. Mônica Mesquita has had similar experiences in Costa de Caparica. She has shared with me the need to have in mind the "hidden agenda", setting collectively, first, what would be the gain for the fishing community and, then, making adjustments to present projects to academic instances. The term "hidden" here point to the hybridity in the task. While the agenda is explicit, open, and co-constructed with the community, it is hidden because the bureaucratic eye of institutions is incapable to see it and grasp the vibrant desires for dignity of those involved. This creative role stresses the capacities of communities to generate knowledge for survival, enlarging the horizon of possibilities beyond what heterodox and disciplinary practices can do. These possibilities are part of a sociology of emergences (de Sousa Santos, 2012).

Once the geopolitics of knowledge is noticed, attempts to "include" by "giving voice", or by "acting on behalf of" should be reconsidered in research. The core idea of research as representation of the *other* is alien to a third ME, because una cosa es el indio, y otra cosa es la antropología. Instead, research becomes an experience of encounter among different entities (individual, groups and even non-human agents), that can be performative and transformative only as far as the experience nurtures the craft of parties involved.

[3] Mignolo (2000) introduced this concept first, but we take it here as "the geo-political and body-political location of the subject that speaks" (Grosfoguel, 2011, p. 4).

Just like the emulated third theater, a third ME circulates in a diversity of scenarios, audiences, procedures and outcomes. For instance, grandparents and healers have a central role in the *Kiwe Uma* activities. The simplest notion of school as fixed place operating at certain regular hours is completely detonated by the Nasa proposals of "educación propia". Participants travel across several *resguardos* (a type of indigenous reservation) and gather according to the moon phases and the healer' spiritual assessment that determines if the group of kids is spiritually clean and strong to embrace the work session. Mathematics educators visiting the third space embrace their accountability to instances far outside of the OECD-PISA domain.

When the first and second ME describe themselves as exploring a field, they really use the term "field" in a metaphorical way, because they are seldom located and immersed in a particular concrete space. Instead, incarnations of third ME explore a real, concrete field, a land in dispute, such as Palestine, the invaded *fazendas* near Porto Alegre, in Brazil, the illegal settlements of Costa de Caparica in Portugal, or the indigenous *resguardos* in Cauca, Colombia. This is not a pure coincidence. Fighting for the land means fighting for the material conditions of existence, a fight for survival with dignity. The complex articulation between materiality, knowledge and culture is expressed by the Colombian leader Francia Márquez:

> Territory for the black people is the real possibility of giving birth to freedom, autonomy, self-determination, it is our space for being. That is why we often harangue: territory is life and life is not for sale, it is loved and defended, likewise community wisdom shows us that *territory is life and life is not possible without territory.* (Marquez, 2020, italics added)

Solidarity with social processes and land, lead us to this enigmatic sentence: with mathematics we can inquire on/in time and space, with a mathematics education we can inquire on the rights to have a time and space.

At this point of the text, I aspire that this paragraph would be a truism: a third ME addresses the political nature of mathematics education in a particular way. Whereas conscious of the structural constraints that shapes school, education, mathematics and research, it decides to focus in the here-and-now of its particular political agency. Paraphrasing the anthropologist Joan Rappaport: "while some scholars engage in [political] description with an eye to analyzing it, [third ME researchers] study [politics] to act upon it" (Rappaport, 2008, pp. 20–21, paraphrases in brackets)

A last remark here could be the first to be said. The three types of regions in mathematics education research are not exhaustive nor exclusionary. There could be other unknown types of regions and many shared spaces among regions. It is important to stress that a third space is not conceived as superior or better than the others; it mainly indicates a different arrangement of priorities. Namely, whereas a first type is devoted to understand "how you do mathematics education", and the second will study "what mathematics education you do", a third will ask "why you do mathematics education". It is natural that each one of the three questions conveys part of the others, but depending on which one is prioritized, the other two questions are answered differently.

Close encounters of the third kind

This section explores the question of how we can address research and practices of mathematics education (ME) in the mingle of the 3Ps (pandemic, post-factualism and pareidolia). So far, I have focused on a particular stance about the field, called the third ME, in which collaborative practices of research with communities are central, and a permanent observance on ME research agency leads the practice.

I contend that the current context of threats to human survival and transcendence demands a reframing of the terms and purposes in which mathematics education researchers conceive their practice, especially when they encounter communities. The concept of *craft* can guide such reframing.

An appeal to consider the theoretical and methodological possibilities of including practitioners' agency into academic research projects has been raised by Renuka Vithal in MES 2 and by Orlando Fals-Borda in MES 3 as part of a paradigm shift in the social sciences. They discussed the dichotomy subject/object of study and explored the political relevance of participatory methods, ending with an invitation to seek possibilities to make more horizontal the research relation[4] (Fals-Borda, 2002; Vithal, 2000).

Almost 20 years later, the framework of decolonial studies (Castro-Gómez & Grosfoguel, 2007; de Sousa Santos, 2010; Smith, 2013; Zavala, 2013) lead us to go further in the paradigm shift. What would happen if not only local practitioners are welcomed on board to design and execute academic research projects on mathematics education, but also scholars try to join communitarian social processes that involve mathematics? What if we turn the inclusive action to ourselves? What if we do not assume to have the power to include, but rather have the aim of being included? What would happen if academic scholars decided not to have the main role in the play? My invitation, then, is to explore again (and still differently) the possibilities to make the research relation more horizontal.

A first thing that could happen is that scholars will need to consider the existence of mathematics education research outside the school, conducted by people interested in learning mathematics in action. Scopus-free researchers, working with PISA-free students! Many of them live on the margins, in the third ME. Some others live in between.

Consciousness on the responsibilities of contesting the matrix of hierarchizations that sustain modern rationality will help ME researchers to establish respectful relationships with communities. Collaborative and participatory approaches could be compatible with his/her decisions, unfolding exercises of co-theorization, mutual interrogation or endogenous research methods that build trust among different stakeholders involved. Instead of reducing accountability to heterodox criteria (such as consistency, scope or validity) or to the problems of reflexivity, academic researchers could also be aware of the problems of *symmetry*. The latter can be thought as "the study of how the participation of both researchers and practitioners has been conceived, unfolded, enacted, registered, and assessed in research"

[4] Concerns on practices of intellectual extractivism were raised by Setati in MES2.

(Parra, 2018, p. 150). Symmetry is also crossed by the complex power relationships around representativeness, legitimacy and intellectual property that surround any research. Integral part of mathematics education researcher's craft is the management of this complexity.

As Vithal and Fals-Borda noted, collaborative processes are inevitably critical. They demand intense interactions among different expectations, skills and backgrounds. Join communitarian processes attempt to congregate heterogenous agents. This is precisely the opposite of the dissociative power of pareidolia and echo chambers, which segregate homogenous agents. 3P does not like these kinds of barters!

In a very paradoxical way, this type of detours and escapes from the normal flux of academic practices could bring us to a renovated mathematics: One like the envisioned by Orlando Fals-Borda and Ubiratan D'Ambrosio; one that studies and expands the ways in which local communities have survived and transcended for many years (some of them for more than 5 centuries!). The third ME came to learn ethnomathematics in action. It is possible to conceive a kind of mathematics that turns its efforts to safeguard life, and that bring answers to the urgent needs of people; a mathematics that can support the struggle of people organized around the defense of a territory, like the experiences in Porto Alegre, Costa de Caparica, Cauca, or any other illegal[5] settlement.

Decoloniality has stated it clear and loud: An *ecology of knowledge(s)* rejects dichotomic thinking; it understands the relevance of disciplinary knowledge to contribute to the well-being. And for that reason, it does not endorse anti-intellectual gestures, like the monster currently in charge of Brazil, who has taken advantage of the COVID-19 cocktail as a biological weapon for exterminating indigenous people in the Brazilian Amazonia. Indigenous people kick back and share the developments of their struggle to their Brazilian brothers. The scientists of Instituto Butantan in São Paulo are doing the same. Every help counts now to stop the current genocide. A third ME researcher necessarily contests post-factualism.

In the time of the 3Ps, humanity needs to learn how to establish sustainable relations with non-human existences[6]. Capitalists and colonialists never understood how to make it (they only want to see measures, profits and borders in the land). The ones who understood that the Earth is not a resource to exploit but the very condition of our existence have been living and resisting for centuries. These are the communities of indigenous peoples, farmers, fishermen, among others. Decoloniality articulates diverse types of knowledge, trough cultural encounters that multiply experiences and share wisdom. A third ME researcher knows the importance of entering and committing to communitarian territories to listen, share, care and learn how to resist.

[5] Sometimes its illegality means that it is managed collectively and has not been measured (devoured) by capitalism.

[6] Paola Valero (2019) asked already what would be the mathematics education compatible with the imminent "new climatic regime", because that regime challenge many assumptions that sustain the current mathematics education.

Third time's the charm (or maybe not)

An important contribution that an ethical stance can bring to the field of ME is the chance to elaborate on certain insights that are gaining currency within the MES community. Colleagues (for instance, Andrade-Molina, Baldino, Pais, Valero) have raised warnings on the chances that ME research has to truly impact the school system in contemporary neoliberal societies. They have pointed out several constraints, like the capitalist system that needs to produce surplus-value at any time, or the subjectification processes that occur for students, teachers and researchers. Broadly speaking, colleagues refer to the *mechanisms of capture* (a.k.a Foucaultian dispositif), and point out that there are several mechanisms of capture jeopardizing the possibilities of "changing" the system.

> So we are always facing the danger of being deceived, as we think that we are struggling against oppression, when in fact we are being allowed by the dominant class to do so, just to cool down the rebellion. As I said before, the core things, like assessment and school, are here to stay. (Pais, 2008)

In that line of thought, when addressing the role of mathematics (and school) within the new normal state of global crisis due to the 3Ps (pandemia, postfactualism and pareidolia), ME researchers will need to be very careful not to get trapped by the mechanisms of capture, and not to end up subsuming their efforts to make a critical mathematics education instrumental to the increase of human exploitation. I could not agree more; that is definitely a possibility. Furthermore, I am sure that such a thing is happening already. However, if we have the analytical acumen to perceive how transnational forces of political and economic domination enter the school or enact themselves in mathematics education research, then we should also have the insight to notice how local forces of resistance also seek school and university spaces. Because education is just one more set-up in this antagonistic relationship, and since we inhabit that set-up, it is up to us to address the situation.

Previous sections showed us people around the world using the pandemic to create spaces and lines of cultural and political resistance. They appeal to an ethos of resistance, to activate *mechanisms of escape* (a.k.a Foucaultian counter-conduct or Deleuzian lines of escape or flight). My point is that ME researchers can indeed find resources to block the mechanisms of capture by engaging with sectors of society that are activating the mechanisms of escape. Playing with the words of Barba, one could ask:

> Why do they choose [mathematics education] in particular as a means of change, when we are well aware that other factors determine the reality in which we live? Is it a question of blindness, of self-delusion?
>
> Perhaps for them, [mathematics education] is a means to find their own way of being present and seeking more human relationships with the purpose of creating a social cell in which intentions, aspirations and personal needs begin to be transformed into actions. (Barba, 1986, p. 194, paraphrases in brackets)

A disenchanted but powerful idea of utopia is suggested here. There is no possible salvation, neither a hedonistic approach is attempted. I do not expect to "solve" the societal problems of post-factualism or pareidolia when I proposed to work more closely with

communities. I intend to do something different, closer to the concept of *hope* from Václav Havel: "It is not the conviction that something will turn out well, but the certainty that something makes sense, regardless of how it turns out" (Havel, 1990, p. 181).

Then, we should not wait for ME to make sense, but we should give it meaning. This in itself is a political act that is neither romantic nor useless. This is precisely the main task: to develop a craft, a care of the self:

> The hell of the living is not something that will be. If there is one, it is what is already here, the hell we live in every day, that we make by being together. There are two ways to escape suffering it. The first is easy for many: accept the hell, and become such a part of it that you can no longer see it. The second is risky and demands constant vigilance and apprehension: seek and learn to recognize who and what, in the midst of hell, are not hell, then make them endure, give them space (Calvino, 1978).

It is to develop a revolt, a rebellion, conceived as the stubbornness of doing something that is doomed to fail, of doing what is needed to do, or of doing what we are required to do in order to find/build meaning to our path.

Pataphysics is not romanticism, insofar as it boycotts the status of the thinkable. It is a corrosive force that denaturalizes normality.

With broken temples and arms beaten,
We sound the fanfare of those never surrendered
and of the always defeated!
(Leon de Greiff, Sarabanda, 1929)

O [assessment], where is your victory?
O [mandatory curriculum], where is your sting?
(1 Corinthians, 15: 55-57)

I look for life in death,
for health in sickness,
for freedom in prison,
a way out from the impasse,
and loyalty in the Judas.
But my destiny, from which I would
never expect anything good,
has decreed with the Gods
that, since I ask for the impossible,
they won't even give me the possible.
(Miguel de Cervantes Saavedra)

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Urgency and the shameful escape of privilege: We move differently when we refuse to set aside the weight: A response to Aldo Parra

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In his plenary paper, Parra adopts a tone that is simultaneously kindly and urgent, at once friendly and keenly critical, as he surfaces deeply visceral aspects of the world as it exists today and the untapped power Mathematics Education has to respond to seemingly omnipresent violences. His observations and urgency culminate in a call to action: That we should not let the world define Mathematics Education, but instead embrace our communal power to define it ourselves in ways that defy the white supremacist, cis-hetero patriarchal, abled, parochial, neoliberal, late Capitalist systems that currently confine us. Here, I aim to expand on and amplify what I perceive as the essence of Parra's message, with particular focus on unsettling anticipated neoliberal critiques/evasions of Parra's call to action.

> *The white conservatives aren't friends of the Negro either, but they at least don't try to hide it. They are like wolves; they show their teeth in a snarl that keeps the Negro always aware of where he stands with them. But the white liberals are foxes, who also show their teeth to the Negro but pretend that they are smiling. The white liberals are more dangerous than the conservatives; they lure the Negro, and as the Negro runs from the growling wolf, he flees into the open jaws of the "smiling" fox. One is the wolf, the other is a fox. No matter what, they'll both eat you. (Malcolm X)*

In his plenary address and manuscript, Parra (2021, in this volume) makes a series of thoughtful and keenly incisive observations regarding the state of the world as it exists today as well as regarding the often untapped power of communal/community organizing, culminating in a simple but extremely powerful call to action: "Then, we should not wait for [mathematics education] to make sense, but we should give it meaning [...] to develop a revolt, a rebellion, conceived as the stubbornness of doing something that is doomed to fail, of doing what is needed to do, or of doing what we are required to do in order to find/build meaning to our path." In this brief response paper, I aim to make use of my positionality as a member of the "global north," as well as my experience as an anarchist political activist in said global context (Bowers, 2021; Bowers & Lawler, 2021a, 2021b), in order to anticipate and subvert certain critiques and rhetorical evasions that I anticipate Parra's argument will prompt

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among many members of that broadly privileged community (and particularly among members with multitudinous intersectional privileged identities). None of this should be taken as a critique of persons, but instead as a surfacing of the tacit ethical weight that privilege allows us to choose either to consciously bear or set aside in our choices of response. This is not simple iconoclasm, but instead an invitation to join us in a "hyper- and pessimistic activism," (Foucault, 1983) an invitation to embrace "not the conviction that something will turn out well, but the certainty that something makes sense, regardless of how it turns out" (Havel, 1990, p. 181, as cited in Parra, 2021, in this volume).

To begin, I will briefly recapitulate the broad shape and sense of Parra's plenary address (or, rather, the key thread that I am addressing in this paper), in order to help ensure we (writer/speaker and readers/listeners) begin from a place of some shared understanding, even if my sense of Parra's words does not match your own. I will then outline some of the critiques and rhetorical evasions that I anticipate will comprise and confine the reactions of some, and especially many of those who have lived and been enculturated wholly or primarily into western neoliberal lines of thought (these lines of thought are also inextricably rooted in white supremacy, cis-hetero patriarchy, parochialism, abled supremacy, and so forth–these intransigent systems are part and parcel to western neoliberalism as surfaced in work such as Kendi, 2016). I will then respond to these anticipated reactions, not with an eye towards debunking or refuting per se, but instead with an eye towards consciously surfacing the ethical weight that they bypass or ignore. I conclude by restating and amplifying Parra's call to action: Let us leverage our communal power to give mathematics education, a rebellion of doing what is needed and "good" regardless or in spite of whether such efforts are doomed to failure.

Recapitulation of Parra: Urgency and ethics

Parra's text begins at the end, or rather an end: Apocalypse. Pandemic, postfactualism, and pareidolia, acting in concert, represent an existential threat, not just in the sense that the existence of many people/populations is literally threatened through violence (police brutality; Capitalists holding hostage water, housing, medical treatment, and other means of survival; etc.), but also in the insidious sense that the already distant possibility of a life (relatively) free of oppression is being crushed under the deleterious explosion of mounting exploitation, unemployment, inequality, deprivation, and so forth. We could add much more to this already horrifying list of existential threats, such as the accelerating climate catastrophe (IPCC, 2021) or the threat and risks associated with artificial intelligence, but even without expanding the list of threats it seems clear that there is an *ethical weight* tied to how we choose to respond (or not respond) to these threats, as well as an *urgency* to when we respond. It is also worth noting at this juncture that for many people, *apocalypse* is not a hypothetical future, but a reality of their present and past. "Dystopia" and "post-apocalypse" are common language used in privileged spaces to describe privileged people coming to experience what so many others across, for example, Indigenous and Black diasporic communities have already experienced (note that we could add to this list innumerable queer,

disabled/neurodivergent, and non-Christian communities). In other words, the ethical weight and urgency Parra describes already existed prior to the present moment, but the present moment has invited a growing group of people to consciously notice this weight and urgency.

From this apocalyptic beginning, Parra moves towards a more hopeful aspect of this line of inquiry: Extant resistance of omnipresent violence. Through a variety of examples, Parra discusses the potentially revolutionary power of redirecting our values and actions away from larger systems of oppression and towards something more local and communal. He describes examples of a vision of mathematics education that already exists in the margins, one rooted in resistance and emancipation. When the values of communities drive our work, when communities are able to determine the metrics of success, there is the potential to fundamentally change the nature of what we can accomplish. When communities are understood to include both human and non-human existences, a perspective which necessarily moves beyond viewing all symbolic and material assemblages as mere resources to be exploited, this revolutionary potential grows. Parra is careful to temper these claims with reference to work describing the innumerable ways oppressive systems capture and claim mechanisms intended as emancipatory (e.g., Cabral & Baldino, 2019; Pais, 2012). – Consider, for example, the way "strengths-based pedagogy" was/is intended to surface value in the margins of society, but is commonly enacted as "what can the center take from the margins to strengthen itself." This *capture* happens when the values ultimately being served are still the values of the oppressive systems (well symbolized by the absurd trend of rainbow-colored hostile architecture; the queer are disproportionately rendered homeless, so painting an anti-homeless bench in Pride colors represents a grimly fascinating material representation of this *capture*), and the value in the margin is simply a resource. Parra ends with a challenge: One piece of power that we inarguably have is the power of self- and communal-definition. What mathematics education *is*, broadly speaking, does not make ethical sense. If we want it to make sense, we have to leverage our power to make it make sense. We may still be doomed to fail against the overwhelming Authoritarian force we face, but even without the conviction that our efforts will turn out well, we can at least have "the certainty that something makes sense, regardless of how it turns out" (Havel, 1990, p. 181, as cited in Parra, 2021, in this volume). We can know that we are *doing good* even if we are doomed to fail.

Rhetorical evasion: Anticipating and responding

Based in my experiences as an Anarchist activist in the cultural context of the United States, there are a variety of common forms of rhetorical evasion or pushback I anticipate Parra's messaging prompting from the well-meaning privileged. I obviously can't name or respond to these exhaustively, but given the *ethical weight* and *urgency* of Parra's messaging, there are several that may warrant explicit naming and critical reflection. In particular, I will acknowledge and offer a reflective counter to these three responses, each of which have been extremely common roadblocks in efforts to organize with the intersectionally privileged: (1) I wish I could do more, but I can't; (2) Reform is a better path than revolution; and (3) I can

both satisfy what the system, as it exists, demands of me, *and* oppose the self-same system. In each case, my goal is not to offer an exhaustive response, but instead to invite critical self-reflection/discussion and advance more emancipatory futurities in the context of the *ethical weight* and *urgency* surfaced by Parra.

"I wish I could do more, but I can't…"

When I hear people voice this concern, what I actually hear them expressing is the tension they feel between their desire to *do good* and their anxiety or fear of the vulnerable position that *doing good* would ultimately demand of them. If you are white (as I am), then opposing white Supremacy ultimately demands that you let go of the safety net that privilege affords; if you are a cis-heterosexual man (as I am not), then opposing cis-hetero patriarchy demands that oppose the very ideologies and mechanisms that keep you safe; if you are neurotypical (as I am not), then opposing neurotypical supremacy demands that you let go of cultural norms that privilege your ways of being and thinking; etc. I do not blame you for feeling this tension. However, it is worth noting that this vulnerability that you can opt to ignore along your privileged identities is something that the marginalized can not opt out of. Thus, I ask: If you were to sit with this vulnerability, day in and day out, for days and months and years at a time, how would that affect your choices and the way you move through life?

"Reform is a better path than revolution…"

Is it possible to reform the system/institution to the point of emancipation without fundamentally changing it? If fundamental change is necessary, then what we are doing is revolution, and what you are distinguishing is a slow revolution from a faster one. When is it appropriate to allow a slow revolution? Certainly not in a moment of *urgency*. In contexts where your positionality is privileged, would you feel differently about how quickly we need change if you instead imagined yourself as one of the marginalized?

Consider police "reform." If you are relatively safe around police (my whiteness offers some safety, but my queerness and neurodivergence still put me in a great deal of danger when engaging with police), maybe a slow revolution sounds fine. However, if you or your children are at risk of extrajudicial execution, perhaps that is less likely to seem like a reasonable option.

"I can both satisfy the system and oppose it…"

I conclude at the same place conversations such as these so often seem to culminate: "Both/And." To be clear, it is not unreasonable to believe (or to advance the belief) that we can act in some ways at some times that empower the system, and in other ways at other times that oppose it – I certainly agree. However, "both/and" responses encounter an ethical problem when the two categories under consideration are not constructed as equal, as when one focus is more/less oppressive than the other. In such cases, this response advances equity only symbolically and not materially. In parallel to Dubbs' (2020) observations regarding Sfard's (1998) metaphors for learning, who diverges from the claim that "it is essential that

we try to live with both" (p. 8), this is a situation where it is uniquely trivial to observe that one focus can be said to be better than the other, with better being that which is more equitable. Uncritically suggesting that we can do "both/and" without a critical lens reifies oppression and marginalization, and it will continue to do so until such time as "We… normalize (and expect) the full taking up the philosophical and theoretical underpinnings of all of our work (even work that is not considered 'philosophical')" (Bakker, Cai, & Zenger, 2021, p. 12). In other words, if we do not consciously wrestle with the often tacit ways that our beliefs and consequent methods advance the interests of a violent system, we stand little chance of meaningfully opposing that violence. As it stands, "Both/And" is a product of late abled cis-hetero patriarchal white supremacist capitalism and neoliberalism, a signifier for the confrontation of postmodernity contra modernity, and as such it enables a disarmament of ideas that would otherwise be directed at critiquing late capitalism. In contrast to Hegelian synthesis, "Both/And" bypasses the negative moment of determination. Thus, "Both/And" isn't really "Both/And…" it's just an exercise in neoliberal thought (Bowers, 2021, p. 80).

Conclusion

I opened this paper with the words of Malcolm X, an evocative and deeply visceral attack on Centrism. That description of those words may sound strange to some who perceive Liberalism (e.g. the Democratic political party in the United States) as the "Left," but Liberalism is not and has never been the Left – instead, it is only the leftmost arm of the establishment, the leftmost position tolerated by the entangled authoritarian systems we occupy. In a word, it is the *captured* left. To be clear, if you currently identify as a Liberal or a Democrat, I do not think you are cruel or unreasonable; instead, I think only that you are *human*. In this culture of internalized white supremacy, cis-hetero patriarchy, abled supremacy, and neoliberal Capitalism, I would be deeply suprised by any relatively privileged member of society who did not identify in this way at some point. However, just because it is reasonable and human to find yourself here at some point in your journey of rhizomatic growth does not mean that it is an adequate place to stop. The fear, frustration, and rage we hear in Malcolm's words are real, and are not unique to him, nor unique to matters of race. The more marginalized the community I find myself learning with and from, the more likely I am to hear exactly Malcolm's sentiments. Now, in these final moments, I leave you with one final synthesis statement, one final call to action: Be not satisfied with centrism or the status quo, and force yourself to sit with the tensions that you have the privileged capacity to escape. We move differently when we force ourselves to carry the weight of the violences that privilege allows us to ignore (Bowers, Forthcoming).

> If you have come here to help me, you are wasting your time. But if you have come because your liberation is bound up with mine, then let us work together. (Lilla Watson in community with other Aboriginal Rights activists)

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Activism in mathematics education research: Stopping epistemicide by confronting and resisting modern forms of epistemic violence: A response to Aldo Parra

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In this paper, I will share my reaction and evoked thoughts inspired by Aldo's paper. I will argue about modern forms of epistemicide and how particular groups of people have historically lost the condition of humans to the eye of privileged groups. From a Postcolonial perspective, epistemicide problematizes how knowledge is, and has been used, to exert power according to national and global ideas for progress. Such ideas invalidate knowledge produced outside the boundaries of well-founded western forms of scientific practice. Violence against the Other in the path of organizing behaviour implies not only epistemological injustice but a systematic destruction—involving colonization, oppression and genocide. Here, the role of math education researchers should take a turn to give voice to those that have been silenced.

Have you watched the anime *The Promised Neverland*? This anime unravels the story of an orphanage in the year 2045. The orphanage was settled as part of "The promise" agreement made in 1045. Humans and demons set this agreement to keep both worlds—the human world and the demon world—separated. Demons eat humans, so the agreement was to create human breeding farms disguised as orphanages to provide food for the demons. In these orphanages, children believe they are orphans and have to stay in these houses until they get adopted. They are taught that they will be qualified for adoption if they acquire a certain level of intelligence and age. They learn all kinds of things, and, of course, they learn mathematics. When they get "adopted", they pack their belongings, have a farewell party, and leave the household thinking they will finally be free. Although you can probably see where this is going, they are murdered and sold as meat. They learn how to act and behave, how to eat and what to know, only to be profitable for the meat industry. And, well, this analogy could be as literal as your frame of reference makes you believe. But there is something I've been struggling with for quite some time now—and Aldo Parra's (2021) paper invites me to problematize—: how do we end up believing that mathematics is and should be universal at the cost of annihilating other forms of knowledge? How do we end up believing that epistemicide was the safest bet for economic progress? How do we end up using mathematically justified models to commit genocide—i.e., war math models to calculate how many people can be killed if a bomb is detonated in a certain location? How do we end up using mathematic

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illiteracy as an advantage to commit abuse—i.e., Aldo's example of tax increase in Colombia? And so, this is my reaction and evoked thoughts inspired by Aldo's paper.

One of the firsts things that come to my mind when trying to understand how we move in life through desire is the capitalist ideology that leads us to think we have to do what we do to be successful in the global world. One has to have a good job to have a good life. To have a good job, one has to have a promising career. To have a promising career, one has to have good grades and high levels of proficiency in particular school subjects, such as mathematics and science. Did we ever think as kids that the path we take was to become a productive citizen? Probably you didn't, me neither. But here we are, privileged by knowledge, socially validated as productive. Žižek talks about this with this example, people are more aware of the environmental impact of their choices. Hence, they decide to buy products that won't generate much waste (no plastic involved, ethically made, cruelty-free, and so on). The industry then starts marketing products that should respond to people's needs. If someone goes to the supermarket and buys a fair-trade chocolate bar with a paper wrap over a bar of plastic-wrapped chocolate, did this person do something to combat the environmental impact of food consumption? The industry with fancy labels—and so on—doesn't tell that, sometimes, paper wraps have a plastic coat to preserve some foods and to avoid moisture (who will buy a chocolate bar with a wet paper wrap?). This is why desire is intriguing. I watched the documentary *Seaspiracy* and saw how "dolphin-safe" labels are put into tuna cans when there is no certainty if dolphins were killed or not in the process. But people buy these products because of sustainable wildlife preservation. In this regard, Žižek says desire comes from fantasy. The fantasy keeps us in a constant state of desiring, probably feeling incomplete or an impulse of belonging.

We are constantly validated according to what we can give to society intellectually, financially, you name it! For instance, I don't see myself doing anything else than teaching, researching and being the best mom I can possibly be for my daughter. Do I talk to anyone about how epistemicide leads to perceive diversity as a threat to school mathematics? I write papers about it (and receive my monetary incentive for publications), go to conferences, give lectures without much result of being labelled as a pessimist by some mathematics educators or be well-received by my peers and mentors (probably reading this reaction). Aldo's paper presents experiences from other fellow researchers worldwide resisting and confronting epistemological violence and the use of mathematically justified forms to commit abuse to segregated and historically marginalized groups. This is done not behind a desk and a screen as I do, but in the front line. In academia, what we do is silenced, arguing what we do is not valid scientific research. They do not publish our work, even if we have enough data to make our claims clear and justified. What we do is not considered proper math education research. We live marginalized in the shadows of mathematics education. One of the main conclusions I got from Aldo's discussion is that it is not enough by recognizing forms of violence and the interplay they have with mathematics (either to help mathematically or to show how school mathematics alienates groups of people). It is not enough by doing research or problematizing the *status quo*. One should become a mathematics educator activist, as David Bowers (2021, in this volume, also commenting on Aldo's paper).

The second thing that came to mind is how power has been historically exerted through knowledge. I mean, it is not about how many things someone knows (for example, Nasa's knowledge in Aldo's discussion), but if people learn the right things to be validated in contemporaneity (if people qualify for "adoption" or not). Historically, a systematic epistemicide of particular groups of people have led them to lose the condition of humans to the eye of privileged groups, for example, indigenous people to the eye of colonizers. From a Postcolonial perspective, epistemicide problematizes how knowledge is, and has been used, to exert power according to national and global ideas for progress. Such ideas invalidate knowledge that is produced outside the boundaries of well-founded western forms of scientific practice: the taken as valid forms of knowledge. Here, the role of mathematics education researchers should take a turn to give voice to the abused, segregated, marginalized and invisibilized.

Globalization does not set equitable knowledge levels to all but to produce and naturalize forms of epistemic violence to communities that don't share validated capitalistic practices, conducts, and desires. Then, they must strike for survival only because they share other cosmologies than the "normal" and conceive mathematics in other ways. Here, a fine line between the right of remaining silent and the silencing of voices (either rhetorical or literal) decides people's future and attitude to engage with formal forms of schooling and towards learning school mathematics. I wonder how much mathematics a Mapuche kid could learn if their families and communities are portrayed as terrorists by the media and if their communities are constantly attacked and killed. Then, it is a bit obvious, at least for me, that they are not going to perform well in national standardized tests. (Please, I invite you to search the internet for "police brutality against Mapuche children" or "Mapuche children injured by police".) Their fight is not about equitable access to education but the right to live without life-threatening attacks. In such contexts, school mathematics is far away from saving Mapuche children lives. Is it enough by publishing about their cultural richness and mathematical knowledge they have? Should we partake in helping to solve the conflict between authorities taking over Mapuche lands and Mapuche families defending their territories? What is our role as mathematic researcher activists? Is it enough by researching multiculturalism, multilingualism, inclusion, equity, agency, social justice, and so on? Is it enough by giving voice to the voiceless (as Aldo once said when we were PhD students)? Should I share Mapuche experiences to make visible the lack of conditions and foregrounds, in Ole Skovsmose' terms, they have?

> En un allanamiento la agresividad es impactante, pocas veces hemos podido registrar imágenes donde Carabineros golpea a un niño, golpea a una mujer con una niña en brazos. Que tu casa esté llena de lacrimógenas al interior y tengas que salir producto del ahogo o ver a tu padre lleno de perdigones y sangrando por todos lados, son cosas que impactan mucho a un niño. Impactan a cualquier persona", dice Mijael Carbone. (Unrepresented Nations & Peoples Organization [UNPO], 2015)

> In a raid, the aggressiveness is shocking; we have rarely been able to record images where the Police hit a child, hits a woman with a girl in her arms. That your house is full of tear gas inside, and you must come out because you cannot breathe or seeing your father full of pellets and bleeding from all sides. These are things that greatly impact a child. They impact anyone," says Mijael Carbone. (UNPO, 2015, my translation)

In 2011, three ONGs demanded Chile for violence and child abuse, so the Supreme Court ordered not to apply the Antiterrorist Law to children. Although, everything remained the same.

> En el último informe anual de la institución, publicado a comienzos de 2015, se lee: "las violaciones de derechos –en particular de niños/as y adolescentes mapuche– producto del uso excesivo de la fuerza por parte de personal de Carabineros, se ha mantenido a lo largo del año sin mayores cambios en un patrón sobre el cual el INDH ha reclamado, y al cual se ha referido la justicia, en el sentido de cuestionar y sancionar el actuar de las fuerzas de seguridad, particularmente respecto de niños y niñas mapuche, exigiendo su apego irrestricto a las normas y reglamentos vigentes". El INDH ya expresó su preocupación al Comité de los Derechos del Niño de la ONU, que hizo público el recién pasado 2 de octubre [2015] un documento con observaciones y sugerencias respecto a la infancia al Estado de Chile. Allí declara que "El Comité sigue profundamente preocupado por la situación permanente de la desigualdad, la discriminación y la violencia contra los niños indígenas, en particular los niños mapuches". (UNPO, 2015)
>
> In the last institution's annual report, published at the beginning of 2015, it is written: "the violations of rights –particularly of Mapuche children and adolescents– as a result of the excessive use of force by Carabineros, have been maintained throughout the year without major changes in such a way that the NHRI has complained, and to which justice has referred, in the sense of questioning and sanctioning the actions of the police, particularly with regard to Mapuche children, demanding their unrestricted adherence to the current rules and regulations". The INDH has already expressed its concern to the UN Committee on the Rights of the Child, which published on October 2 [2015] a document with observations and suggestions regarding children to the State of Chile. There it states that "The Committee remains deeply concerned about the permanent situation of inequality, discrimination and violence against indigenous children, in particular Mapuche children". (UNPO, 2015, my translation)

Violence against the *Other* in the path of organizing behaviour and the quest for social order implies not only epistemological injustice but a systematic destruction—involving colonization, oppression and even genocide. So, what role can we play as mathematic education activists against epistemological violence? I still wonder.

Not only have indigenous groups experienced these practices, but women have also been alienated from mathematic practices, believing that women should not be educated. In Chile, the firsts school for women were not meant for them to actively participate in society but in learning how to be a good mother and wife. The mathematics they knew were not meant for them to pursue an academic life but to be able to help their kids with their homework. What contributions could women make if they were not authorized to engage in mathematician discussions? Women's mathematical knowledge was not allowed to exist. Although, when it comes to ethnic differences, other marginalized groups suffered brutal forms of violence (not only epistemic violence). Black slaves who came to Chile, not by choice (sold as products), were not even considered humans. They were reduced to serve the white men and lose the opportunity to access the spaces reserved only for the elite, such as schools. There were even discussions about "reproduction of Black populations" that proposed forced sterilization of

most Black people. In other parts of the globe, these white men supremacy included also forced sterilization of indigenous groups.

Losing humans' condition and being taken as a body (without agency or else) from which other people can decide (i.e., pro-choice versus pro-life debates about abortion) is much more than just school mathematics. But it should concern mathematics education activists. Mathematics is constantly used as a form of disseminating statistical results about everything, and people read it. More often, those numbers or graphs are manipulated for people to take directed decisions—this is when math illiteracy is used to commit abuse. Meanwhile, math is advertised as the key to escape from these unlivable spaces in educational policies and transnational reports. People should be empowered by mathematics; then, they will be able to read news and not be fooled by the media and take an informed decision. However, people wouldn't have to be empowered by math if the media didn't use maths to misinform and scandalized the public. What is our role as mathematics education activists here? Should we point to every time the press commits these abuses and explain why the numbers and graphs are adulterated? Does anyone care about charts at this point? As Aldo says, clickbait is created for the morbid audience to obtain profit from some random website: why informing people when you can make money from urgent, paradoxical and scandalous content, right? The wonders of school mathematics are another form of click bait. We have to ask who benefits from these classroom practices, is it children? I believe they don't.

Finally, I'm not comfortable with the idea of doing something if it makes sense for us, even if it doesn't turn out well, because that is the same train of thought that evangelization in Latin America was framed. They were saving us from our doom, from going to hell, whatever. We need to find a middle ground between not imposing our beliefs on *Others*, not alienating the *Other*. Probably Emmanuel Levinas could help in setting school mathematics from *Otherness* (*alteridad* in Spanish). We have so many possible paths to take from here, but I will start by challenging the assumption that all people need mathematics (and the same type of mathematics) for success. I mean, why mathematics? What mathematics? When mathematics? And for whom? We should challenge those political and economic decisions as mathematics education research activists to confront and resist modern forms of epistemic violence.

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Symposia

Mathematics teacher agency

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Anna Chronaki, University of Malmö and University of Thessaly
Herine Otieno, African Institute for Mathematical Sciences
Pete Wright, UCL Institute of Education

Neoliberal policies dominate in many parts of the world, setting a frame within which education practices are frequently constrained. In mathematics, perhaps more than other subjects, these constraints seem to be more keenly felt, not least because of the economic value placed on mathematics expertise and the related effects of performativity and accountability. In this symposium we explore potential sources of support that may enable mathematics teachers to challenge orthodox practices, facilitate creative responses to and/or rejections of policy constraints as they negotiate agency over their practice and their learning.

Introduction and aims

Neoliberal discourses of mathematics practice and of teachers' professional learning are often framed in terms of quality, with teachers viewed as deficient, their skills, knowledge and practice in need of improvement. Responses to these perceived deficits include large-scale, cascade models of professional development, albeit with increasing attempts to incorporate knowledge of what makes for effective teacher learning experiences. Such responses can constrain opportunities for teacher agency and contribute to teacher dissatisfaction, as teachers experience a lack of autonomy over their learning and their work.

To counter these discourses, in this symposium we share examples of and perspectives on the achievement of mathematics teacher agency, exploring implications for teacher learning, foregrounding questions of equity. Our interest lies in the creative ways that individuals and groups of mathematics teachers resist predominantly neoliberal discourses, determining their own learning goals and how they are supported to do this. We reflect on our own efforts to support teachers to transform practice, the challenges this raises and how teachers respond.

Exploring mathematics teacher agency

Agency is understood as a 'situated achievement' (Priestly et al. 2015, p. 29) – temporally embedded within a socio-cultural context. One approach to understanding agency focuses

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on three temporal dimensions, an 'iterational element', providing a stabilising influence from the past, a practical-evaluative element that focuses on an actor's capacity for making reasoned decisions, and a projective element where creative possibilities for future are imagined (Emirbayer & Mische, 1998). Threading through these three dimensions, we identify a variety of *teacher roles and practices* (e.g., practitioner researcher, 'champion' teacher, ...), *modes of collectivity* (e.g., research group, collaboration, peer learning), engagement with *external stimuli and support* (e.g., texts, networks, researchers, teacher educators…) and *tools* (e.g., resource design, pedagogic experimentation, video stimulated reflection, social media).

Symposium plan

Through sharing alternative perspectives and accounts of teachers' agency over their practice the session aims to stimulate discussion on our roles (as teacher educators/researchers) in supporting teachers' efforts towards transformative practices in mathematics. Each presentation will explore one or more of the themes of teachers' roles and practice, collectivity, external resources and tools in relation to agency. Presentations will be followed by small group discussions, inviting participants to reflect on their experiences in relation to these questions:

- How are possibilities for mathematics teacher agency supported or constrained in different contexts? Which groups/individuals are included/excluded?
- What responsibilities do we have (as teacher educators/researchers) in relation to supporting teachers' achievement of agency?
- What do we learn from teachers and students concerning 'agency'?

Introduction to the symposium and presenters (5 minutes)

Paper 1 Possibilities for mathematics teacher agency in England: Historical policy traces. Gill Adams & Mark Boylan (12 mins)

In this paper, we explore the state's changing role in shaping professional learning activities within the neo-liberal context, focussing on England. By examining ways that policy (and its absence) in relation to mathematics teacher learning influences broader socio-cultural conditions thereby offering shifting possibilities for teacher agency we outline the way that power relations, cultures and materialities operate across four time periods characterised by clear differences in policy and structures. These four are broadly: first, prior to the introduction of the National Curriculum (1970-1990); the second period from 1990 marked by national initiatives aimed at driving up standards in mathematics; the third centred on the introduction and early years of the National Centre for Excellence in Mathematics together with increased support for teacher led professional development and a fourth, marked by a reassertion of a central national agenda. We consider the roles and practices available to teachers in these times, drawing out possibilities for collaboration and examine how teachers (individually and collectively) interact with external influences.

Paper 2 AIMS Teacher Training Program: Working WITH Teachers and not ON teachers. Herine Otieno (12 mins)

In this paper, I reflect on the efforts I have made as a team lead for a Teacher Training Program for mathematics & science teachers in Rwanda, to transform a teacher training model originally shaped as a top-down cascade model to one which is largely hinged on teachers' individual and collective contributions. Citing specific examples, I reflect on the process of shifting the training program from using University lecturers as master trainers and a pre-defined, externally developed teacher training curriculum to promoting peer learning amongst teachers and drawing on individual teachers and teacher collectives referred to as champion teachers to organically identify key training content and interventions for improving quality of teaching & learning of mathematics in Rwanda secondary schools. Finally, drawing on observations and excerpts from two different threads of WhatsApp conversations with champion teachers and some of the participating teachers and employing the transformative professional learning framework (Jones & Charteris, 2017) I will explore the emerging 'impact' on the teachers' relationship with each other, teaching, and key stakeholders in their teaching 'environment'.

Paper 3 Participatory action research (PAR): A critical model for transforming classroom practice through developing collective agency. Pete Wright (12 mins)

PAR offers an alternative paradigm for research/ professional development in which teacher researchers (TRs) and academic researchers (ARs) collaborate in bringing about changes in classroom practice. Skovsmose and Borba (2004) outline a critical model of PAR which recognises the essential/ complementary roles played by both TRs (with their in-depth knowledge of the classroom situation) and ARs (with their expertise in research methods) in the research process. This model was adopted for the Teaching Maths for Social Justice (Wright, 2020) and Visible Maths Pedagogy (Wright, Carvalho, & Fejzo, 2020) research projects. Both projects involved the author as AR and sought to develop engaging and empowering practices in the mathematics classroom, a site that has historically proved highly resistant to change. The projects demonstrated how the mutual support and collective agency generated by a research group, or network of teachers, enables TRs to take risks and overcome constraints they face in developing their practice in line with a commitment to equity and social justice. The research groups provided opportunities for TRs to: engage with CME research literature; collaboratively plan, trial and evaluate classroom activities; design and implement their own data collection tools; and critique existing/new practices through video-stimulated reflection.

Paper 4 Teachers' relational agency: Affective bodying with children, materials, concepts and difference. Anna Chronaki (12 mins)

The purpose of this paper is to discuss teacher agency as a relational matter that grows through affective bodying with children, teachers, concepts and difference in the community revealing a process of minoritarian becoming(s) (Chronaki, 2019). It is based on the analysis

of recent experiences through collaborative work amongst children, teachers, student-teachers and researchers. In the project context, the author was involved in a process of creative design addressing mathematics in the context of 'the commons' of a specific community through radical pedagogic experimentations (i.e., playing and making mathematical games and crafts: spaces for coming together, global crises and local solidarity: debt vs money as common good and money, see http://www.citizenship-and-mathematics.eu). In a series of seminars and school-based work with participant teachers and children, these materials moved from the researcher's desk out to the public space of school classrooms, communal areas, the streets or the kafeneion. Here, we aim to denote aspects concerning this transformative move and to discuss teachers' agency as relational in multiple layers of research-creation with teachers, children and the community.

Group discussion & plenary (25 minutes)

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Parenting and educating in mathematics: Parental engagement during and beyond the COVID-19 pandemic

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Lisa Darragh, University of Auckland
Naomi Jessup, Georgia State University
Mary Candace Raygoza, Saint Mary's College of California
Tony Trinick, University of Auckland

In this symposium we engage participants in actively reflecting on our work with/as parents and caregivers in mathematics education and on their own experiences within their local contexts during the COVID-19 pandemic. Our goal is to reimagine parental engagement and plan for future collaborations to support new possibilities for and address exacerbated barriers to parental engagement in mathematics education.

Focus of the symposium

Parents' and caregivers' role in shaping children's mathematics education often goes overlooked, and the intersecting identities of mathematics teachers and teacher educators as parents is rarely considered. The COVID-19 pandemic, however, has placed parents in the spotlight. As nations aimed to slow the spread of the virus, 1.5 billion students worldwide abruptly transitioned to emergency remote mathematics instruction (UNESCO, 2020). A year later, the COVID-19 outbreak continues to cause significant disruptions to education. Parents have been thrust into unprecedented levels of engagement with school mathematics, and their responsibility for ensuring their child's mathematics learning has increased significantly. At the same time, as the work of educators has shifted into their homes, mathematics teachers and teacher educators are educating their students and parenting their children simultaneously.

Our intention for this symposium is to consider how COVID-19 might *enable* parental engagement by explicitly increasing expectations for involvement of parents in school mathematics and by drawing attention to the intersections of mathematics teaching and parenting. Symposium organizers will offer perspectives on parental engagement in mathematics during COVID-19 from two countries – the United States and New Zealand. By asking participants to reflect on our work with/as parents and on their own experiences in local contexts, we hope to build a global network of scholars committed to parental engagement and to plan for future collaborations.

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Parental engagement in mathematics education

Parents are uniquely positioned to support children's mathematics education across formal schooling. Unsurprisingly, parental involvement is strongly linked to children's mathematics achievement (Knapp et al., 2017). Yet, efforts to broaden mathematics learning opportunities largely ignore parents, which pits schools, teachers, and parents against each other rather than fostering partnerships for learning. Differences between parents' own mathematics education, which likely emphasized rules and procedures (Jackson & Remillard, 2005), and the curriculum and instruction experienced by their children present barriers to parental involvement, even in early grade levels (Muir, 2012). This challenge was intensified by the pandemic as parents struggled with or resisted the continuation of mathematical reasoning through child-centered strategies when learning shifted into the home. Despite these challenges, we see potential in blurring the line between parenting and educating in mathematics and seek to imagine possibilities for parental engagement. To that end, we ask questions such as:

3. What were the experiences of parents in diverse local contexts during the initial transition to emergency remote instruction? Presently?
4. How can we (re)envision parental engagement in mathematics education? How can we spark and sustain such efforts to engage parents in our local contexts?
5. How have parent experiences with mathematics education during the global pandemic created new barriers to engagement? How might we address those?

Symposium structure

After an overview (5 minutes), each organizer will present key ideas from their work with/as parents (20 minutes). Then, participants will engage in an interactive session to reflect on the experiences of parents with mathematics education in their local contexts in relation to the questions raised (40 minutes). We will conclude by building visions for parental engagement and planning for future collaborations (25 minutes).

Piata Allen and **Tony Trinick** both lecture at the University of Auckland where they focus on Māori-medium mathematics education. COVID-19 impacted differently on Māori-medium schools and whanau (extended families) than English-medium in Aotearoa New Zealand. This was in part due to the historic legacy of under resourcing of Māori-medium education by the State, in digital, human and print resources since the emergence of Māori-medium education 40 years ago. We discuss, our experiences as teacher educators, professional development facilitators and, in the case of Piata, a Māori-medium parent to support teachers and schools to teach mathematics in the medium of te reo Māori (the Māori language) during COVID-19 school closures. Our findings indicate that Māori-medium schools due to their legacy of being self-reliant, used their own agency to ensure a modicum of continuity in mathematics learning programmes. This was supported by wider community led initiatives that utilised social media and community networks to create, distribute and share resources quickly and efficiently in the face of inadequate state support.

Lisa Darragh lectures at the University of Auckland and her main research interest relates to learner and teacher identity in mathematics education. She will present results from a recent study she conducted with Dr. Nike Franke during the first couple of months of the nationwide lockdown in April 2020. The survey invitation was sent via Facebook community groups around Aotearoa New Zealand and received 634 responses over a three-week period. Parents were generally very engaged in the home learning of mathematics. They reported a range of opinions about quality of mathematics work and teacher support, and there was a correlation between general stress levels and negative opinions. To further support their child's mathematics learning, many parents turned to online mathematics programs, about which they were very positive, but the crisis brought to the fore a number of pre-existing issues. We argue that these findings have implications for all forms of mathematics home learning in the future, and suggest that schools need to listen to parental feedback regarding the quality, level and quantity of mathematics work. Additionally, schools could consider ways to deliver effective teacher support and to foster parental agency in helping their children with mathematics learning.

Frances K. Harper is an assistant professor at the University of Tennessee, Knoxville, where she focuses on how parent-teacher-community partnerships shift traditional power dynamics in mathematics education. She will present findings from a recent study conducted with Dr. Joshua Rosenberg, Sara Comperry, Kay Howell, and Sierra Womble during the initial transition to emergency remote instruction in 2020. Examining 100 survey responses and over 200 posts from Twitter, we saw a commitment among parents to engaging children with a range of mathematics topics from the elementary/primary school curriculum and a strong desire for collaborations among parents and teachers. We argue that these findings have implications for how we renew efforts to engage parents, such as placing greater value on the authentic ways families already engage in mathematics in the home and inviting parents into conversations about school mathematics on social media or in other accessible spaces. Harper will share how these implications inform the ongoing development of two networks of parent-teacher-community partners in her current projects, namely, networks aimed at fostering computational thinking among Black and Latinx pre-schoolers and at advancing racial justice in elementary/primary mathematics.

Naomi Jessup is an assistant professor at Georgia State University and one of her research interests examines the impact of COVID-19 on mathematics teaching and learning for Black communities as well as rehumanizing and culturally responsive approaches by parents. She will present preliminary findings of an ongoing two-year study that examined how Black parents in the United States responded to supporting their children's elementary mathematics learning with resources provided by schools throughout the middle of the 2019-2020 school year and subsequent school year. Initial findings indicate a range of approaches and strategies used by Black parents given that many of the strategies desired for learning mathematics were not common to them. In addition, parents voiced frustrations regarding the quality, rigor, and lack of cultural responsiveness of the mathematics tasks provided. Some parents modified and adapted the mathematics resources provided by the school to

better accommodate their child's learning and allow them to make cultural and real-work connections. Parents discussed increased agency in supporting their child's mathematics learning and due to their shift in roles and responsibilities from pre-pandemic to now. Implications from this study provide a counternarrative regarding Black parents' engagement in their children's mathematics learning and highlights their social and cultural capitol that is often not invited, noticed, or silenced in schools.

Mary Candace Raygoza is an assistant professor and STEMinist teacher educator at Saint Mary's College of California. This presentation examines her mathematical motherscholar praxis during the COVID-19 pandemic and present global reckoning for racial justice. In March 2020, her child's pre-school closed, as schools far and wide shuttered for in-person instruction. Alongside her paid work, she was also an informal pre-school teacher of one pupil, joining many parents, especially mothers, around the world whose roles in relation to schooling shifted suddenly and with little societal support. Informed by scholarship that highlights how much young children know and can do mathematically (Johnson et al., 2019), the mathematics that emerges from play (Wager & Parks, 2014), and the potential to connect (in)justice issues to early childhood mathematics (Ward, 2017), she will explore lessons from her three-year-old, namely how her child led the way in revealing mathematics connected to social-emotional growth and as part of learning about fairness and solidarity (e.g., the mathematics that emerged out of a community circle of stuffed animals). As the presenter interrogates her privilege as a white woman and a mathematics teacher educator mother, she also wonders: How can we support parents and caregivers to feel affirmed that meaningful mathematics lives within activity that is not intentionally mathematical (and is something accessible to them and their participation); and how can schools shape mathematics teaching and learning with that in mind?

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Interrogating common-sense assumptions toward a more just mathematics education

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In this symposium participants will interrogate common sense assumptions that serve to (re)produce long standing inequities in mathematics education. "Common sense" is historically and culturally constructed, rather than a natural reflection of "the way things are." Therefore, surfacing the historical roots of common-sense assumptions provides powerful opportunities to reframe problems of access, oppression, and erasure. We invite participants to consider common sense assumptions behind problems of equity in their own contexts, trace their histories, and offer alternative narratives to reframe these problems. We hope to better understand how common-sense assumptions have shaped mathematics education across the globe, and use these insights to reimagine mathematics education.

Aims of the symposium

In the context of a school year in which a horrific global pandemic and widespread racist violence have dominated the global discourse, we come together as a collective to question and reimagine a different world for mathematics education. In the U.S., at least, mathematics education, rooted in assimilationist labor, military-industrial, and capitalist agendas, has continued to play a significant role in perpetuating white supremacy in schooling practices, (Vossoughi & Vakil, 2018). Across Europe and around the world, research has argued that the simultaneous rise of neoliberalism and nationalism has resulted in educational policies in mathematics sustained by raced, gendered, and classed ideas of mastery, ability, and autonomous selfhood (e.g., Ineson & Povey, 2020). In this symposium, we explore how the practices of mathematics education rest on a set of assumptions, or a "common sense" that is historically and culturally, and politically constructed. Surfacing these common-sense assumptions, we argue, will provide opportunities to reframe the persistent problems of access, oppression, and erasure in mathematics education. The aim of this symposium is to invite participants to share and discuss common sense assumptions in their own contexts,

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then consider alternative narratives that may serve to reframe the problem space of mathematics education, thereby providing opportunities to ask new, more productive research questions and to reimagine how we design and take action for equitable teaching and learning.

In the following, we discuss what we mean by common sense assumptions and how they operate in our mathematics education scholarship and practice. We end with a description of proposed symposium activities.

Relevance of the symposium

Activists, mathematicians, and other scholars have highlighted the ways in which certain assumptions around mathematics have been deployed to support systems of oppression around the world (e.g., Wong, 2020). Mathematics education, which plays an exaggerated role in mediating the educational experiences and possibilities of students across the globe, has also been implicated in these conversations (e.g., Valero et al., 2012). For example, the discourse of "learning loss" presumes a particular set of knowledge must necessarily be acquired in each year of a child's life, and that any deviation from this progression will produce detrimental effects. In response to concerns around students' "learning loss" as a result of the COVID-19 pandemic, a host of responses have been proposed in the US, including the implementation of impractical and demonstrably inequitable standardized tests to measure "learning loss" while the pandemic continues to rage, and an increase of school hours and days in the next year (Ewing, 2020).

However, the very idea of "learning loss" hinges on common sense assumptions that include valuing a standardized yet arbitrary trajectory of achievement, as well as the insistence that the only learning of value occurs in school (McKinney de Royston & Vossoughi, 2021). We take up Schutz's (1953) argument that common sense, while critical for the accomplishment of everyday activity and social order, is, in fact, historically and culturally constructed (Garfinkel, 1967). What is considered common sense in one context (e.g., taking off one's shoes before entering homes is a common cultural practice in many communities) may seem absurd, or even offensive in another (e.g., guests from other communities may find it overly intimate to reveal their socked or bare feet). Common sense understandings also change across local contexts, including institutional spaces (e.g., one might expect to go to the bathroom when the need arises; in many school settings permission must be granted first) and disciplinary domains (e.g., some disciplinary conventions lead students to expect right and wrong answers, whereas others are presented as more flexible; Schutz, 1953).

Further, common sense understandings and assumptions serve to reproduce the cultures from which they originate. Garfinkel (1967) explained, "Not only does common sense knowledge portray a real society for members, but in the manner of a self-fulfilling prophecy the features of the real society are produced by persons' motivated compliance with these background expectancies…. Seen from the person's point of view, his commitments to motivated compliance consist of his grasp of and subscription to the 'natural facts of life in

society'" (p. 53). It follows that the common-sense understandings on which the inequitable conditions of mathematics education have been built serve to reproduce these conditions. In our example of "learning loss," we see how the common-sense assumptions about where valuable learning occurs and standardized achievement expectations bolsters the discourse of "learning loss," which in turn perpetuates inequitable practices like standardized testing and reifies the importance of normative, factory style schooling practices.

We argue that illuminating the historical origins and cultural development of these common-sense assumptions may allow us to understand anew the problem space of mathematics education. As Schutz (1953) notes, "all cultural objects--tools, symbols, language systems, works of art, social institutions, etc.--point back by their very origin and meaning to the activities of human subjects [...]. This historicity is capable of being examined in its reference to human activities of which it is the sediment" (Schutz, 1953, p. 3). There is a relationship between these histories and how they have sedimented in contemporary society. For our learning loss example, we can trace some of this discourse in the U.S. to the history of standardization of public education. Tyack and Tobin (1994) described how sorting classrooms into age-determined grade levels was an invention which, while commonplace today, did not become so until the twentieth century. Inspired by specialization and division of labor practices in factories, reformers from universities and state departments of education wanted to increase both efficiency of and control over public education. In a related effort, school content was divided into narrowly defined subject areas with required sequences, and tests were administered to determine whether students could advance to the next grade. While fears over learning loss is sensible within this context, examination of the context itself reveals a commitment to the needs of university reformers and politicians over concerns about, for example, the wellbeing of children, their teachers, and their communities. Considering this (simplified, for the purposes of this proposal) history of standardized, graded schooling in the U.S. allows us to reframe "the problem" of learning loss as many different problems, such as grade level standardization, whose knowledge counts in curricular sequences, and the role of efficiency in policy decisions in education.

Format of the symposium

This symposium is designed to support participant discussion and collective learning. The co-authors of the symposium will begin by introducing the concept of common-sense assumptions and provide the example of learning loss described above. The remainder of the symposium will support small group discussions of "a problem" related to mathematics education in participants' local contexts. In discussion, groups will unpack the common-sense assumptions required for defining "the problem" in a particular manner, and delineate the political and social premises from which these assumptions operate. As a consequence, these investigations will transform the problem space, allowing small groups to articulate different problems that may be addressed, expanding possibilities for reimagining mathematics education.

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Diversity and inclusion in mathematics teacher education: Lessons from Chile and Sweden

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Based on the examination of Chilean and Swedish research, the symposium addresses the possibilities and challenges for researching diversity and inclusion in mathematics pre- and in-service teacher education. Departing form concrete localized research and its contextual, theoretical and methodological stances, larger reflections and implications for the education of mathematics teachers that may lead to an increased sensitivity towards students' diversities and their impact in inclusion of students and change of educational experiences in mathematics are drawn.

Motivation

Transnational reports point to the correlation between students' diversities, including gender, socio-economic disadvantages, racial and ethnic differences, immigration background, etc., and low-performance in mathematics (e.g., OECD, 2014). The sharp inequities in results seem to undermine students' opportunities to access higher education and to break the poverty circle in which they live. Therefore, the connection between students' diverse position of disadvantage and the access to quality mathematics education is a problem to tackle by research in the field of mathematics education. This is an issue in many countries, among those Chile and Sweden, which have taken in substantial number of immigrants and have explicit policies for promoting equitable access to education.

The Chilean educational system is characterized by a deeply rooted inequity. Recent reform policies have taken steps towards improving quality and access to education. The "Law of inclusion" (MINEDUC, 2015) disallows schools to select students which is likely to

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help disadvantaged students; and the primary mathematics curriculum was re-written with a greater emphasis on problem solving (MINEDUC, 2012), as a way to improve the quality of education and students' results. However, little is known in about racial and ethnic minority students' mathematics experiences and performance, and about the challenges in educating teachers to implement reform-based mathematics instruction in diverse school contexts in Chile. Few studies have explored the teachers' learning process to teach mathematics to diverse student populations. This research sheds light on how teachers' views of students in positions of disadvantage play a critical role in successfully implementing reform-based mathematics instruction (Darragh & Valoyes-Chávez, 2019). Overall, the teacher is left to cope with other aspects of teaching, such as dealing with students' diversity.

In Sweden, educational policies have responded to the increase of diversity in student population by guaranteeing school attendance and thereby opening opportunities for social mobility and integration. The Swedish Agency of Education has launched reforms emphasizing students' attainment (Skolverket, 2015). Thus, success is equated to mathematics scores in a hierarchical system of evaluation. Even though gender differences in achievement have diminished, particular gender stereotypes are reproduced in mathematics teaching (Sumpter, 2016). Immigrant students' lower results in mathematics persists. Studies have focused on the impact of the language of instruction and students' home languages as a barrier or resources for learning mathematics (Caligari et al., 2021). The results show that teaching practices generate exclusion, and that teachers' views on students' abilities given their gender, socio-economic status or ethnicity are influential in students' opportunities for learning.

Furthermore, teacher education — initial and in-service — is recognized as a key for addressing diversity and generating inclusion of students in disadvantaged positions (Darling-Hammond, 2017). Researchers contend that a different type of understanding is needed to teach mathematics in diverse school settings (e.g., Anderson & Stillman, 2013). Thus, the question remains of how to educate teachers to support the mathematics learning of diverse student populations, so that quality and inclusion can go hand in hand; as well as of how research may support such education.

Aim

This symposium addresses the general problem of the challenges that students' diversity and its impact on more equitable school results pose for mathematics teacher education research. By taking the case of Chile and Sweden as a point of departure, the symposium aims at: (1) Discussing experiences that led to challenging teachers' education through distinctive research approaches to notions of inclusion and diversity. (2) Identifying theoretical and methodological ways of reasoning and operate with inclusion and diversity in mathematics teacher education research. (3) Rethinking the complexity of inclusion and diversity and its implications and possibilities for mathematics teacher education practices in different social contexts.

Organization

The symposium has a series of thematic conversations among researchers from Chile and Sweden. Each conversation involves a statement by each participant (10 m each), a time for discussion among them (5 m) and a time for conversation with the audience to draw connections with other contexts and problems (15 m). The symposium convener (Valero) will moderate the symposium. It is planned to take for 180 m.

Opening concerns: An introduction (10 m)

The symposium starts with a motivation by the convener for the topic and a series of questions about the relevance of the discussion for research and teacher education.

Challenging policies of inclusion and diversity (40 m)

This theme unfolds experiences that led to challenge inclusion and diversity in mathematics teacher education. The intention is to build an understanding of how diversity has entered the scene of mathematics teacher education practices and institutions in different settings. We draw on an experience with a group of embroiderers in Mexico to discuss how culture-centric views of mathematics can be questioned from cultural diversity perspectives to account for historically marginalized social groups' relationship to knowledge and school (Solares et al., forthcoming) Drawing on a teachers' experience in multilingual classrooms, we address the collision of expectations promoted—and imposed—by Swedish curricular guidelines, teacher training programs and parents (Caligari et al., 2021).

Dealing with teacher education's paradoxes (40 min)

This theme explores theoretical tools to trouble the current (and historical) state of mathematics teacher education. The toolboxes invite to examine the double gestures that make visible the paradoxes on mathematics teacher education. We explore how mathematics teacher education in Chile has been historically structured around discourses of a desired mathematics teacher in a constant state of becoming and quasi-Darwinism of mathematics teachers (Montecino, 2019). And, with this notion of the *image of a desired teacher* used for an analysis of policy and practice in teacher education, we show the operation of images as desire-constructs imposing inclusion/exclusion in teacher education in Sweden (Österling, 2021).

Experiences of diversity and inclusion in the mathematics classroom (40 min)

This theme focuses on how notions of diversity and inclusion are unfolded in particular research projects and educational settings. We revisit experiences collected in different school contexts and their implications for research. We discuss the limits of the Chilean law of inclusion and examine the challenges of implementing it in the case of Black immigrant students (Valoyes-Chávez & Darragh, under review). Then, we explore Swedish mathematics teacher's practices, based on research on multilingualism, to point call teacher educators for the need of elaborating different ways—both in practice and research—to cope with the diverse body of student teachers (Norén, 2015).

Challenges to and opportunities (40 min)

The symposium closes identifying key issues for further research and the implications for teacher education. We argue on how Chilean educational policies have been historically delineated to fabricate a particular citizen, currently attracted in making the global Homo-economicus that lead to invisibilize diversity and, therefore, negate inclusion (Andrade-Molina, in press). We also discusse whether inclusive methodologies that involve teacher students may be a path forward to generate new opportunities for inclusion for teacher students and teacher educators (Skog, 2014).

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Publish or perish: Power and bias in peer review processes in mathematics education journals

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The "publish or perish" slogan represents a constant pressure to survive in academia and to be considered a competent professional. This symposium will open conversation among researchers, editors and reviewers to address issues of diversity, ethics and politics in the publication process. We will facilitate discussions with colleagues around the world to explore biases in the scholarly publication process to uncover the mechanisms and practices responsible for the underrepresentation of particular groups of researchers. The symposium will be a meeting point, to find and explore ways to address biases in the processes of peer review. We hope to contribute to the efforts to make the publishing process more transparent and accessible to researchers.

Rationale

The publication of research in scholarly journals is a critical goal for researchers in mathematics education and in every academic field. This goal not only relates to scientific interests such as expanding the extant knowledge, disseminating novel theories and methods and engaging in academic conversations in our field. Getting papers published in prestigious mathematics education journals is almost the exclusive path towards academic recognition, promotion and job stability for novice and early career researchers. It becomes an accountability system that most universities use to measure academic productivity. The "publish or perish" slogan represents the Damocles' sword hanging over the researchers' neck and constitutes a constant pressure to survive in academia.

Researchers are expected not only to continuously publish their work but also to do so in high-impact journals. This is because, as Andrade-Molina, Montecino and Aguilar (2020) argue, publishing in well-known journals in the field adds value to both researchers and their institutions of affiliation. As different ranking systems consolidate in mathematics education, publishing in journals indexed in, for instance, the Web of Science and Scopus, constitutes an indicator of the researchers' productivity, a measure of the quality of their work and a criterion for allocating resources (Andrade–Molina, Montecino & Aguilar, 2020). A hierarchical system of universities and researchers is then introduced, validated and

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sustained; in this sense, scholarly publication transcends academic purposes and becomes a contested arena where different political and economic interests emerge shaping the entire process. Moreover, the publication process shapes ways of normalizing and conducting the researchers' work by delineating what is considered theoretically and methodologically valuable for academia and therefore publishable. In this vein, it draws an aesthetics according to what is established as "good/desired" research.

The phenomenon of exclusion in the mathematics education system of practices has been widely discussed (e.g., Louie, 2017). Nevertheless, little is known about inequity and marginalization in the processes of scholarly publication (Meaney, 2013). Until just recently and within the context of strong anti-Black racism protests and the consolidation of the feminist movement worldwide, calls for unpacking mechanisms of exclusion that lead to the underrepresentation of racially and ethnically minoritized scholars and female researchers in scholarly publications have emerged (Wagner et al., 2020). In particular, questions about how the review process is conducted and handled by editors have been voiced in different contexts. It is argued that, although peer review contributes to move the scientific field forward by awarding high quality research, far from being an objective and rational process, it is shaped by issues of power that end up rendering invisible particular voices and epistemes in academia. Although peer review is either a single-blind or double-blind process to ensure objectivity, transparency, impartiality and fairness, different studies evidence that these apparent goals are not realized (Lee et al., 2012).

Lee et al. (2012) point to research in diverse fields, which uncovers the existence of different biases during the review process ranging from errors in assessing a submission's "true quality" to the social characteristics of the authors. For instance, dominant representations about what high-quality research looks like may lead the reviewers to fail to assess the real qualities of the proposed work. In mathematics education, for example, prominence is given to cognitive investigations while sociopolitical and critical studies are delegitimized by the "where is the math" question (Martin et al., 2010). Also, in our field there are clear disparities among countries and regions in terms of which research gets published (Mesa & Wagner, 2019). Social characteristics of the authors also seem to play a critical role in peer review. These biases result in the differential evaluation of an author's submission as a result of her/his perceived membership in a particular social category. As Lee et al. (2012) argue, "social bias challenges the thesis of impartiality by suggesting that reviewers do not evaluate submissions–their content and relationship to the literature–independently of the author's (perceived) identity" (p. 11). Thus, national origin, language, gender, content, racial and ethnic biases seem to shape the review process.

With these ideas in mind, this symposium has the potential to generate discussions around the following areas, which we hope the participants will engage and contribute:

– What are the main obstacles that mathematics education researchers from underrepresented groups face when trying to get their research published?

- How do the researchers' social identities shape the chances of getting a paper published in a prestigious journal in the field?
- What is considered as high-quality research in mathematics education?
- What and whose knowledge are valued in mathematics education?
- What and whose knowledge are ignored in mathematics education?
- What can editors do to mediate or control biases in the peer review process?
- What is the impact of these values and practices on dominant views of what mathematics education looks like and what its concerns are?
- What are the ethical responsibilities of reviewers and editors?
- What can scholars in the field do to help develop the diversity of published research?

Aim

In this symposium we attend Mesa and Wagner's (2019) call for open conversations among researchers, editors and reviewers to address issues of diversity, ethics and politics involved in the publication process. We call on mathematics education researchers worldwide to fully engage in discussing their experiences in getting their papers published in high-impact mathematics education journals. The purpose of this symposium is twofold. First, it is aimed at facilitating discussions with colleagues around the world to explore biases in the publication process in order to uncover the mechanisms and practices responsible for the underrepresentation of particular groups of scholars in scholarly publications. Second, the symposium is thought as a meeting point to find and explore ways to address biases in the processes of peer review. We hope to contribute to efforts to make publishing processes more accessible to researchers who identify with groups that have been marginalized, as well as, to unpack mechanisms of power that normalize and control us.

Planned Structure

The introduction to the symposium (5 minutes) will be followed by brief input in which presenters will draw on their own experiences as researchers, reviewers, editors and editorial members. Melissa Andrade-Molina will problematize the ranking system of journals that govern the publishing practices in mathematics education (5 minutes). Alex Montecino will discuss a pseudo aesthetic shaped in mathematics education research (5 minutes). Luz Valoyes-Chávez will discuss research about gender, racial, language and national biases in peer review (5 minutes). David Wagner will outline recent discussions pursuing anti-racism amongst editors of mathematics education journals (5 minutes). Symposium participants will then be asked to bring their own histories to these thought-pieces and questions in small group discussion and plenary report-backs (45 minutes). The voices of participants with underrepresented identities will be encouraged and promoted in this time. Finally, participants will propose and discuss possible pathways for action to address the challenges identified in the symposium (20 minutes).

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Researching experiences of mathematics: Black/feminist and queer lenses

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This symposium will explore how critical methodologies in mathematics education shed light on the sociohistorical, political, and cultural role of mathematics in shaping experiences of learning mathematics. Drawing from Black/feminist and queer theories, panelists and participants will consider the subjective experiences of learners that tend to be obscured in research, and how critical shifts in methodology might bring such stories to light.

Aims of the symposium

In the last four decades, research in mathematics education has seen a shift toward an examination of the cultural practice of mathematics and of the identities and subjective experiences that unfold in and through this culture (Darragh, 2016). Decentering achievement, this research has sought instead to consider the role of mathematics—as a historical, political, and cultural institution—in engendering particular learner experiences in formal STEM spaces (e.g., Mendick, Berge, & Danielsson, 2017). As such work has been primarily concerned with surfacing as-yet untold stories of learners in mathematics, it has simultaneously fostered the development of a variety of critical methods in mathematics education research.

The Mathematics Education and Society conferences and the wider MES community have been at the heart of this shift. However, feminist, intersectional and queer perspectives have been less commonly taken up here than in the wider critical education research literature. This symposium will address this gap by offering three examples of these perspectives in action in order to open up discussion on what they offer for research into mathematics education.

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Relevance

Scholars within the MES community have championed the perspective that mathematics is a cultural and political practice and that mathematics education is governed by neoliberal and imperialistic interests (Gutiérrez, 2013; Pais & Valero, 2012). For example, Mendick, Berge, and Danielsson (2017) made sense of the identities of two students in a Swedish upper-secondary science program in terms of "active accomplishments, neither fixed nor singular but multiple and fractured, and as coming into being through talk, actions, and relationships" (p. 486). Through multi-modal micro-analysis we see how students positioned themselves in and are positioned by neoliberal discourses around equity in STEM. Sengupta-Irving and Vossoughi (2019) located their analysis in the view that equitable STEM educational policies are sustained by U.S. hegemonic agendas that shape the gendered and racialized experiences of minoritized girls. In spite of this, the authors found that participants were able to reclaim science with "ingenuity and humanity" (p. 495), refusing the prescription of science as primarily for U.S. hegemony.

Such studies surface important stories—both about learners and about STEM institutions—that have fallen by the wayside. Dotson (2014) argued that stories go untold due to our "inclination to understand oppression as we experience it and to extend our analysis of it beyond what we ourselves can see from our particular vantage point" (p. 57). Importantly, most "socioepistemic orientations towards oppression will illuminate as much as they obscure" (p. 45). Ahmed (2006) wrote similarly about orientation: "some things are relegated to the background in order to sustain a certain direction; in other words, in order to keep attention on what is faced. Perception involves such acts of relegation that are forgotten in the very preoccupation with what it is that is faced" (p. 31). That is, attending to something necessarily implies not attending to something else. Thus our work as researchers demands regular reflection and recalibration, which we hope to engage in at this symposium. We ask: what stories does our vantage as researchers render "theoretically invisible" (Dotson, 2014, p. 46)? And how might Black/feminist and queer perspectives bring these stories to light?

Format of the symposium

The symposium will begin with three provocations that present some possibilities of using Black/feminist and queer lenses on mathematics experiences.

Provocation 1

Maisie Gholson will offer new epistemological methods to resist deficit-perspectives on Black learners in mathematics education research. Expanding on earlier work in Gholson & Martin (2019), she centers performativity and pain to illuminate some of the ways in which mathematics education is "violent, painful, and dehumanizing" (p. 393) to Black learners. Through an examination of the repetitive and resistive identity-based performances and gestures of Black girls, Gholson is interested in methods that animate the oppressive

structures that operate within the mathematics classroom. Adopting Dotson's (2014) theory of oppression as a "multistable" phenomenon — which renders subjective experiences of oppression theoretically invisible in research — she argues that intersectional phenomenological lenses are crucial to telling such stories, while also helping demonstrate the broader politics of mathematics as raced, classed, and gendered.

Provocation 2

Sarah Radke's perspective seeks to disrupt dominant notions of learning and identity development as spatially and temporally linear. It builds on her collaborative work in Ma, Kelton, Radke, & Della Volpe (2020), in which a cross-setting analysis of participation illuminated how the "relational practices" (p. 259) of one youth were not merely incidental to a moment of statistical learning, but rather instrumental to it. Drawing on Butler's conception of performativity as contextual and structured by powered relations (1990), Radke's analytic lens is, in part, a response to Langer-Osuna and McKinney de Royston's (2017) call for tools to study how power mediates learning and positioning at multiple time-scales. Drawing from—and complicating—Saxe's (1991) focus on shifts in form-function relations across interactions, this view attends to the cultural, political, and social construction of a repertoire of forms across space and over time. Radke inspires us to reconsider where and how we look for learning, opening up the possibility that moments of learning and identity development are dispersed across experiences and interactions.

Provocation 3

Heather Mendick will speak about a research project with Eva Silfver, Maria Berge and Andreas Ottemo about contemporary geek identities. The data will be the opening sequences of the 2008 film *Iron Man*. These illustrate how mathematical, scientific and technological expertise is being repositioned. Previously the dominant image of the math/science/tech genius was a physically-weak socially-awkward geeky white man. Tony Stark / Iron Man exemplifies a new entrepreneurial configuration of the genius. While still geeky, white and male, he is both physically strong and socially confident. A queer poststructural reading of this geek entrepreneurial masculinity illuminates how it legitimates gender, economic and neo-colonial power relations, and represents a shift in hegemonic masculinity in which previously marginalised 'geeky' traits carry new status.

Symposium Structure

Participants and panelists will reflect on these provocations, sharing their own critical orientations to and analyses of the data. Following an initial discussion, panelists will describe the potential of their particular conceptual lenses for mathematics education research. Finally, the group will be encouraged to explore what intersectional, feminist, and queer lenses bring.

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Disrupting normativity in mathematics education: Meeting queer students at the intersection of their queer and mathematics identities

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In western mathematics and mathematics education, normative structures serve to reinforce hierarchies of oppression along lines of race, gender identity, class, dis/ability, and sexual orientation. This symposium aims to create a space in which participants can discuss and interrogate, from an intersectional perspective, the normative structures of mathematics and mathematics education, specifically those that reinforce heteronormativity and gender-normativity and to discuss ways of re/humanizing mathematics for LGBTQ+ people.

Aims of the symposium

This symposium aims to create a space in which participants can discuss and interrogate the hetero- and gender-normative structures of mathematics that serve to dehumanize mathematics for LGBTQ+ people. We will discuss research on queer identity in mathematics teaching and learning, queer theory as a means to re/humanize mathematics for LGBTQ+ students, and implications for mathematics pedagogy.

Rationale

With the dawn of the 21st century, the field of mathematics education began experiencing a shift in perspective—embracing the idea that the teaching and learning of mathematics, previously regarded as neutral, is influenced by political and sociocultural factors (Gutiérrez, 2013). This shift in perspective has led many scholars to investigate how race, sex assigned at birth, dis/ability and other identities impact mathematics teaching and learning. More often than not, the findings of such studies suggest that mathematics teaching and learning reproduce normative social structures that serve to sustain oppressive hierarchies (Leyva, 2017; Mendick, 2006). These normative structures often lead to dehumanizing mathematics experiences for students from traditionally marginalized groups (Goffney, Gutiérrez, & Boston, 2018; Tan et al., 2019).

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In Western schools, queer students are particularly vulnerable to dehumanizing experiences in educational spaces (Watson & Miller, 2012), given the politicization of their identities by religious groups and the experiences of harassment or assault that queer students report in K-12 and higher education institutions. Over the last decade, GLSEN has consistently reported that over 85% of LGBTQ+ students in U.S. grades K-12 have been harassed or assaulted at school (Kosciw et al., 2020). Such dehumanizing experiences continue for queer students in institutions of higher education, where they are significantly more likely than their non-queer peers to report experiences of harassment, discrimination, and feelings of being unsafe at their institution (Greathouse et al., 2018). Around the world, in the mathematics classroom, such experiences come in a variety of forms, from the complete erasure of LGBTQ+ people and experiences in textbook problems (Esmonde, 2011; Waid, 2020) to stereotypes pertaining to masculinity, femininity, and what it means to be "good at math" (Mendick, 2006). Such dehumanizing experiences may explain why, for example, queer students are less likely to complete Algebra II than those that do not identify as queer (Whipple, 2018).

At the time of writing this proposal (less than 70 days into the year), 2021 has seen the murder of least 10 transgender or gender nonconforming people (at least half of whom were Black transgender women) in the U.S. (Human Rights Campaign, 2021a) and there are 147 anti-LGBTQ+ measures (73 of which target transgender people specifically) being considered by U.S. state legislatures (Human Rights Campaign, 2021b). Considering the results of GLSEN's National Climate Surveys (e.g., Kosciw et al., 2020) and Greathouse et al.'s (2018) study of the experiences of queer people at institutions of higher education, it is clear to us that these hate-filled acts of violence against queer people are a symptom of a larger problem of homo- and transphobia that is reinforced by the normative structures of western society—normative structures that are reproduced in our mathematics classrooms. We believe, however, that this can change. Such change would require that mathematics educators begin to identify hetero- and gender-normative structures in mathematics education and develop tools to interrogate and disrupt those structures.

In this symposium we invite participants to envision a new, rehumanized form of mathematics, one that honors queer identity, knowledge, and experience. We will provide educators a lens through which to understand the intersection of queer and mathematical identity, namely through the use of the Queer Identity Intersection (QII) of Mathematics Education (Moore, 2020). The QII necessitates critiques of the ideologies of mathematics education and discussions of how they collide with students' subjectivities—gendered, sexualitied, and otherwise. Using this lens, we will then explore the following questions: What hetero- and gender-normative structures are present in mathematics education? What other normative structures exist in mathematics education? How do we interrogate and disrupt those normative structures in ways that honor and re/humanize mathematics (and the larger schooling environment) for queer students, educators, and students from queer families?

Symposium structure

The symposium will begin with a discussion of the mathematics classroom as a potentially dehumanizing space for queer students (building upon the literature presented earlier in this

proposal). This will be followed by an exploration of the intersection between mathematical identity and queer identity, using Moore's (2020) Queer Identity Intersection (QII) of Mathematics Education. To begin thinking about how one might navigate the "road" of Queer Identity, participants will be invited to identify (in small groups) hetero- and gender-normative structures present in mathematics education. Presenters will then discuss how they have worked to enact critical/queer pedagogies in their own teaching to disrupt those normative structures and re/humanize mathematics for queer students in K-12 public and independent school settings in the US, as well as with graduate and undergraduate students pursuing degrees in mathematics and mathematics education.

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Project Presentations

Teacher agency and professional learning: Narrative explorations

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There is considerable research on teachers' professional learning in mathematics, much of it written by those involved in leading 'development' activities and projects. In this paper, secondary mathematics teachers' accounts of learning provide a stimulus for an exploration of teacher agency and professional learning. Differing experiences and perspectives inform an evolving theoretical framing of mathematics teacher agency, an examination of how agency is achieved and restricted in professional learning.

Introduction

Much of the literature on teachers' professional learning in mathematics (and in other areas) is written by those involved in teacher professional 'development', with studies focussing on evaluations of specific professional development initiatives. Such studies contribute to knowledge of how these initiatives impact on teachers' beliefs and practices. However, such initiatives and programmes are frequently developed from a position that views teachers' skills, knowledge and practice as deficient in some way, with interventions taking the form of remediation. Where teachers' voices are heard, they are frequently restricted to their experience of a particular programme. What happens if we take a different starting point in a consideration of mathematics teacher learning, focussing on individuals' experiences and our own critical reflections?

In this paper I discuss an on-going exploration of teacher agency in relation to professional learning. I start from a fragmentary recollection of my own experience as a teacher, an account initially centred around learning through a one-day event. Other fragments are drawn from a life history study of mathematics teachers' experiences. These fragments provide snapshots of professional learning experiences from the early 1990s, when collective approaches to teacher learning and to curriculum development were beginning to be challenged by increasing state control, to a time of increasing national reform in the late 1990s, through to the early 2000s, when these reforms increasingly emphasised performativity.

Teacher agency

There is growing interest in teacher agency in relation to school reform (Pyhältö, Pietarinen, & Soini, 2014; Lasky, 2005) and teacher professional development (Insulander et al., 2019;

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Martinie et al., 2016). In England, continued challenges in recruiting and retaining secondary teachers in particular subjects, including mathematics, has prompted renewed focus on teachers' experience, particularly on their 'job satisfaction' (Worth & Van den Brande, 2020). In their study of retention and satisfaction in England, Worth & Van den Brande focus on autonomy, defined as a capacity to make informed decisions. The concepts of autonomy and agency are variously understood in the literature and often conflated. In this paper, I work towards a clarification of teacher agency through critical engagement with professional learning stories. I take as a starting point, Eteläpelto et al.'s (2013) definition of professional agency as 'exercised when professional subjects and/or communities influence, make choices and take stances on their work and professional identities' (p. 61). Their argument for a 'subject-centred socio-cultural and life-long learning perspective' (p. 60) facilitates a study of teachers' learning as they navigate their careers, changing roles and moving workplaces, engaging in identity work that encompasses professional and personal lives. An alternative framework advanced by Emirbayer and Mische (1998) enables the in-depth study of agency in action. They define human agency as 'a temporally embedded process of social engagement, informed by the past (in its habitual aspect), but also oriented toward the future (as a capacity to imagine alternative possibilities) and toward the present (as a capacity to contextualize past habits and future projects within the contingencies of the moment)' (Emirbayer & Mische 1998, p. 963).

Working with these temporal dimensions of agency, Priestley et al. (2015) develop a framework to aid enquiry into teacher agency. This encompasses teachers' life histories and their professional experiences (iterational element), social, cultural and structural aspects (practical-evaluative element) and long- and short-term aspirations (projective element) (Priestly et al., 2015, p. 30).

These conceptualisations of agency focus on the individual within a socio-cultural context. Relationships figure in these conceptualisations as workplace conditions. An alternative, perhaps complimentary theorisation of agency, relational agency, involves a focus on the recognition of others as a resource, acknowledging that 'work needs to be done to elicit, recognise and negotiate the use of that resource' (Edwards, 2005, p. 172). In shifting the focus from the individual, Edwards directs our attention to moral purposes of working together, highlighting possibilities of individual and collective benefits. Thus, relational agency is seen as an enhanced version of individual agency. Edwards draws attention to implications for professional learning, noting that the capacity for relational agency can be developed and that it may support teachers to recognise the value of working with others and negotiating meanings, rather than these actions being seen as evidence of a lack of skill or competence.

Building on the discussions above, I develop an analytic framework that enables exploration of the ways that policy might restrict or enable the achievement of teacher agency. Using vignettes drawn from life history studies, I consider policy effects across Emirbayer and Mische's temporal dimensions of agency (1998), focussing particularly on relational aspects.

Methodology

In this paper I revisit narratives co-constructed as part of my doctoral studies (Adams, 2013). These stories have remained significant to me, informing historical policy studies. Although they represent moments in time my understanding of them shifts as my knowledge, perspective and awareness shifts. Such revisiting of narrative research provides opportunities to 'explore the new and unfolding meanings' (Andrews, 2007, p. 5).

Vignettes or 'compact sketches' (Ely, Vinz, Downing, & Anzul, 1997, p. 70) are found in various forms in qualitative research; here they enable the introduction of characters, together with a glimpse of their experience. In addition to these vignettes, I begin with a reflexive account of my own experience as a secondary mathematics teacher, before working with documentary evidence to gain an alternative perspective on this experience. The account centres on recollections of one day in 1990, this day represents an impression of my experiences across several similar days across the first fifteen years of teaching. Much of my learning in that period was related to SMILE mathematics, a curriculum development project initiated by teachers in the 1970s (see Gibbons, 1975).

Vignettes drawn from the narrative study focus on two periods. The first, in the late 1990s, when changes initiated in the previous decade had gathered pace, the National Curriculum was undergoing its third revision and the focus on standards and accountability measures was growing. The second vignette is from the early 2000s, a time of global education reform, when the National Strategy was introduced in England.

Discussion

A focus on teachers' experiences of professional learning, albeit in the (short) form of vignettes drawn from a life history study, reveals how agency is achieved and constrained. The fragments provide starting points for an analysis of the complex factors influencing teachers' capacity for agency, broadening discussions of effective professional learning beyond specific opportunities provided to examine policy influences on individual and community. My reflections on learning through the SMILE project together with teachers accounts of learning in times of high accountability raise questions about scope for (and value of) collective learning activities, questions of learning what and why.

Accounts of complexity and variation are often missing from studies of mathematics teacher professional learning, yet such accounts can be productive, inviting us to reflect on our own experience. This initial work with selected vignettes will facilitate the development of an analytic framework to be utilized in further historic mathematics professional learning policy analyses.

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A critical gaze on new digital technology: Answers from mathematics education?

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An increasing concern have been expressed in both academic and public debate that new digital technology might undermine democratic values and practices. This paper explores how studies in the field of Mathematics Education could present different answers to ramifications of new digital phenomena for both individuals and society. A transdisciplinary approach under a post structural theoretical framework is suggested, and two tentative studies are presented. One study will carry out a critical text analysis of Swedish educational steering documents and one will be a classroom action research study, where discourses will be analysed.

New digital phenomena in the world of information

Digital technology is an integral part of our lives. Information flows from the Internet to us. However, scholars within different research fields have also drawn attention to the flow in the other direction, from us to the Internet (Kosinsky 2013; Zuboff 2019). Following this, the aim of this paper is to suggest possible studies in mathematics education as answers to ramifications of new digital phenomena for both individuals and society. For heuristic reasons, a distinction is made upon the direction of the information flow. Techniques that make use of the flow *from* us – our digital trace – are called *reading* techniques since they can predict attributes never explicitly stated. Techniques that make use of the flow *to* us are called *writing* techniques since the can change our behaviour, i.e. rewrite us. A full understanding of the new digital phenomena comes with the realization that they are formed by a symbiosis of reading and writing techniques, like lichen are formed by a symbiosis of algae and fungus.

Techniques reading more about us than we tell

We leave digital traces when we are online. They show our history of preferences, and are used to predict what videos, music or books we might prefer when continuing using such services. Thus, our behavior is constantly being read. The increasing size of this information flow have made it possible to see more general patterns in the data. This enables prediction of information that we might be unwilling to provide if asked. With methods from linear algebra, it is possible to predict attributes such as sexual orientation, ethnicity, religious and political views, and to some degree even personality. This is possible by merely having access

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to information of users' interaction with the 'like' function on Facebook (Kosinsky 2013). The accuracy of the predictions increases with more information, and companies that sell advertisement have access to much more information than just 'likes'.

The economic incentive to read humans through digital traces has been proposed by Zuboff (2019) to create a new form of capitalism, called surveillance capitalism, which operates with our traces as raw materials, mathematical methods as means of production and behavior predictions as products. She warns that this mostly unregulated form of capitalism is prone to exploit the lack of users' awareness for profit, no matter the human cost. In fact, she underlines that people need to be kept unaware, else the extraction of predictive value from digital traces would not work.

Techniques writing our future

Both online and offline behavior can be changed by what we see online, and we do not always select independently what will be shown. Techniques that affect the selection of content shown may therefore 'write' our behavior. One example was demonstrated by Bond et al (2010), who showed that voter turnout in the 2010 US congressional mid-term election was increased by planting a message on social media. Increasing voter turnout is beneficial for democracy, but the opposite has also been tried. In the United States 2016 presidential election, the Trump campaign used citizens' digital traces in an attempt to deter them from voting. Sponsored messages on social media are routinely distributed to people based on what is known about them. In this case though, the 3.5 million-person list destined for messages deterring them from voting, consisted *only* of individuals identified as not likely to vote for Trump (Sabbagh 2020). It is debated to what extent such measures can change the outcome of elections. Nevertheless, the fact that they are being used changes the democratic process of election campaigns to now incorporate mathematical modelling of Big Data.

Other writing techniques are the algorithms in search engines and on social media. They select what the user will be shown. However, it can easily be demonstrated that they show different content depending on historical digital traces. Reading and writing are thus intertwined here, both feeding the other. Limitation of the users' agency must therefore be understood in relation to this *interaction* of reading and writing, and not by them separately. This is also true for the attempted voter suppression.

Foucaultian theory and a transdisciplinary holistic approach

To be able to propose studies in mathematics education answering to the new roles of mathematics in society, a Foucaultian (Foucault 1995) inspired approach is one way. As an example, I appropriate the concepts *discourse, subjectification* and *dispositive* as tools to analyze the interaction between the new digital phenomena, the individual and society. *Discourse* will be used close to Foucault's (1995) own work, meaning not only language but also norms, habits, artifacts, institutional praxis etc. *Subjectification* as I use it, describes how the self is developed in interaction with the flow of information partly steered by algorithms.

The reading-writing algorithms would here be viewed as reinforcing the normalization process that produce subjects in relation to a milieu. By trying to resist this, the mechanics of power in the system may become confirmed and reproduced rather than refuted. One example is the usage of programs that generate random cookies to blur the digital traces. They confirm and propel the notion of a relation between the ability to read our digital traces and production of power. *Dispositive* in my adaptation, is used to envisage how the digital phenomena are linked and constitute an integrated knowledge structure that exercise power in society. This is exemplified by surveillance capitalism.

Research in mathematics education concerning the relation between mathematical techniques and democracy, should take into consideration the limits of what mathematics can achieve. Some mathematical problems are not solvable if all contemporary views on equity are to be respected. Thus, it would then be wrong to accuse algorithms that attempt the impossible to be unjust. It would rather be the expectations on what is solvable under certain combinations of societal principles of justice that would be unfair. One example is algorithms using criminological data to predict risk of recidivism. Such algorithms are used to select whom to keep in jail until trial. When optimized for public safety, they will either give more false positive in some groups (discrimination) or treat groups according to different standards (e.g., depend explicitly on ethnicity, which may be illegal) (Corbett-Davies et al. 2017).

Therefore, understanding of the general problem depends not only on contributions from several disciplines, but also on how these contributions interact. I suggest a transdisciplinary holistic approach including results from data science, sociology, mathematics and mathematics education.

Critical text analysis of steering documents

A starting point in the overall aim of exploring answers could be in the already present. In a critical text analysis it will be investigated what discourses can be construed in relation to democracy, in present steering documents for Swedish Upper Secondary School. Of special interest is mathematics role in subjectification of democratic citizenship and the dispositive of mathematical practices.

The text analysis will include both general steering documents and mathematics specific documents to see what differences exists. Inclusion of the mandatory national tests in mathematics will add the important aspect of assessment. Any discrepancies between what is to be taught and what is to be assessed is part of understanding the present. The critical aspect of the text analysis will be investigating what is present and what is *not* present in the steering documents, both in terms of current internal consistency and also when relating them to the new digital phenomena. This provides a point of departure when exploring revision of curricula.

Action research

An important aspect of the exploration of answers is what happens when teachers and students discuss the new digital phenomena in a classroom setting. For ecological validity, this will be investigated in classrooms in Swedish Upper Secondary schools together with teachers that already have an interest in teaching this content. Through negotiations with the teachers about details in objectives, methods, roles, etc, it will be attempted to achieve a milieu that augment learning for students, teachers and researchers under the framework of action research. This process will be unpredictable and the exact form of the study cannot be known beforehand. However, some general principles can still be sketched.

A concern is that the mathematical methods used from linear algebra are too different from the content of any pre-university curriculum, especially in their complexity. However, drawing on Skovsmoses (1990) three notions of knowledge, mathematical knowledge, technological knowledge, and reflective knowledge, I would suggest that the aim here is not for students to be able to use the methods themselves. The objective is rather reflective knowledge, i.e. knowledge about the methods, their limitations etc.

Data in form of observations, video, and interviews will be analysed though a discursive Foucaultian theoretical lens. Discourses can be construed from several viewpoints; (1) finding possible obstacles, what differences exist between teachers and students, before, under and after teaching (2) how is the complexity of the digital phenomena expressed, e.g., the reading-writing dialectic, (3) relating the classroom to research and public debate, (4) role of mathematics, (5) subjectification in the digital context, (6) dispositive of digital technology in a sociological context.

Which exact route the analysis will take depends on the nature of the discourses construed from the data. Nevertheless, the outcome will be relevant to the overarching aim of exploring how mathematics education can answer to new digital phenomena.

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Emerging teacher identities: Exploring the identity negotiation of early career teachers of mathematics

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In England, the recruitment and retention of secondary mathematics teachers is of continuing concern to governmental bodies. The teachers most likely to leave the profession are in their first five years of teaching, where their performance is subject to intense scrutiny, judgement, and measurement. Given this context, a better understanding of the identity negotiation of these teachers could improve teacher retention, as well as help prepare and support teachers at the start of their career. This presentation will focus on the early analysis of a pilot study which aims to understand how context and experiences with mathematics as a student can contribute to an emerging teacher identity.

Background

The manifestations of neoliberalism can be seen in all aspects of English education policy, including the way teachers' work is specified through detailed teacher standards and monitored and audited through performance management systems and mechanisms. The current context poses additional challenges for early career teachers (ECTs), defined as those with less than five years of experience, whose 'performance' is subject to even more intense scrutiny than that of other teachers (Hobson & Maxwell, 2017) and who are required to pass both their training course and first year of teaching against the same set of standards. In an attempt to assess the quality of teaching and learning, policy makers have often chosen lesson observations as the easiest way to capture the complexities of teachers' work. A result of such high-stakes assessment of teachers' practice is that many teachers discount pedagogical ideas and practices that do not work to their personal advantage in a performative system (Loh & Hu, 2014; Smagorinsky et al., 2004). ECTs in particular may find themselves having to make difficult choices between pleasing the cultural gatekeepers of the school and their own ideals and values.

This study focuses on the identity negotiation of ECTs of mathematics during the period of induction into their school. During socialisation into the school setting, the school culture coupled with workplace opportunities and constraints on pedagogical choices impact on an emerging professional identity. This research takes place at a time of significant policy change for the induction of ECTs with the launch of the Early Career Framework (ECF), which has been promoted as "the most significant reform to teaching in a generation"

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(Department for Education (DfE), 2019, p. 6). Introduced in a select number of areas from September 2020 and nationally from September 2021, the ECF entitles new teachers to a two-year induction, access to a trained mentor and a professional development programme delivered by one provider, chosen by the school from a small number of DfE approved providers. How each school chooses to interpret, enact and manage the framework means that the school context has never been of more importance when understanding the development of ECTs.

This research will provide an important insight into how the introduction of the ECF affects the identities of mathematics ECTs. This early stage in a teacher's career is crucial for researchers and government bodies to understand if the recruitment and retention of secondary school teachers is to improve. The recruitment of secondary mathematics teachers has been below target since 2014 (Worth & Van den Brande, 2019) and while ECTs are the teachers most likely to leave the profession, leaving rates for mathematics ECTs are amongst the highest of any subject (Worth & De Lazzari, 2017).

My ongoing research therefore seeks to answer the following questions: 1) what are the influences on ECTs of mathematics' developing identities over the two years of their induction to the school? 2) how does context shape the teacher identities of ECTs of mathematics? 3) how do teachers' experiences with mathematics influence the development of a teacher identity?

Theoretical framing

As ECTs are socialised into their schools, membership in communities of practice (Wenger, 1998) allow them to negotiate the meanings of their experiences and help build an identity. In these socially and culturally constructed realms, or figured worlds, ECTs negotiate their identities through participating in the "coproduction of activities, discourses, performances, and artifacts" (Holland et al., 1998, p. 51). Such situated theories, by focusing on the examples of participation that constitute learning, are useful for my study because they help gain insights in the emerging identities of teachers as ECTs are *becoming* a part of their community of practice.

Becoming a teacher demands significant personal investment, both of time and energy. It is also a period of intense identity negotiation. ECTs enter the profession with an identity which is strongly embedded (Flores & Day, 2006); they have a clear image and an ideal of what it means to be a teacher. This ideal, or designated identity, can influence ECTs' actions, and they may be disappointed if there is a perceived persistent gap between their current and ideal identities. This ideal may also be challenged by their workplace context, either positively or negatively, and ECTs therefore have to reconstruct their professional identities accordingly.

Establishing what impacts on a strongly embedded identity has prompted some studies to look explicitly at past experiences of school mathematics (de Freitas, 2008; Flores & Day, 2006) and conceptions of what it means to learn mathematics (Ma & Singer-Gabella, 2011). This recognises that mathematics teachers' experiences as learners and doers of mathematics

is vital to understanding their teacher identity. By paying attention to ECTs' past experiences and beliefs, it becomes possible to consider whether their start to teaching is as they had imagined, and how their experiences in school and classroom contexts have shaped or challenged their beliefs.

My initial literature review shows that the majority of studies utilise a sociocultural perspective, thereby responding to Lerman's (2000) call for a theory which equates learning with developing an identity in communities of practice (Lave & Wenger, 1991; Wenger, 1998). While this perspective is useful for my own study, as it focuses on the impact of social practices and context, I still appreciate that an individual's inner world is of importance. The individual emphasis alongside the social context would require a psychosocial approach and is used by far fewer researchers (for example Boylan & Woolsey, 2015), but offers a potentially more balanced approach. I consider the applicability of this approach through reflections on the pilot study.

Proposed methodology

In order to explore the identities of ECTs of mathematics, teachers' stories of their experiences will be collected to help develop a narrative study. In doing so, I equate these stories themselves with the identities (Sfard & Prusak, 2005). In sharing their experiences, I hope to allow participants the opportunity to reflect as they interact with the people and norms of their school context. Narratives allow me to explore the interaction of factors which contribute to a teacher's identity and look for any features in the teachers' narratives which demonstrate identity negotiation.

Final reflections

Identity in mathematics education research is useful as a way of understanding both individuals' experiences of teaching and learning mathematics and wider issues of context, social interactions and power dynamics (Darragh, 2016). My study will collect the stories of ECTs, which provide rich data about the process of *becoming* a mathematics teacher in a particular context. These stories can provide the means to effectively support ECTs to critically reflect on their learning and promote teacher agency throughout their early professional learning. By exploring the emerging identities of ECTs of mathematics, my study will make an important contribution to understandings of mathematics teachers' negotiations at a time of significant policy shifts.

In this presentation I will discuss my emerging theoretical framework, reflections on method(s) used in the pilot study and consequences for the main study and any further research.

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Assumptions, agency, and authority: Mathematical modelling and students' socio-critical reasoning

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Data from the pandemic has afforded the use of real-world modelling problems in mathematics classes to consider socio-critical mathematics. From an ongoing project, we examine grade 9 students' written work while solving a modelling task to consider their agency and authority as mathematicians. We use this analysis to posit that the "making assumptions" stage of the modelling cycle is a key moment in modelling for students' agentic decision-making via making and revising assumptions as they construct a model. We discuss our emerging conception of students' modelling authority and agency and conjecture about the ways that teachers' instruction may afford or constrain these acts. We contribute to the emerging research on modelling instruction and students' socio-critical reasoning.

Mathematical modelling from a socio-critical perspective focuses on how mathematics can be used to critique and inform decision making within society (Barbosa, 2006). Mathematical modelling is a non-neutral process in this perspective; the interpretations of a problem context and the assumptions that follow shape the development of the model and resultant solution (Anhalt et al., 2018).

The practices involved in socio-critical modelling activities ask students to reason throughout real-world tasks such that they author mathematical and situational-relevant knowledge to read and critique their world. They also propose solution paths and logically connect their interpretations and decisions with their model and solution (Blum & Leiß, 2005). The modelling cycle stages of identifying the problem and making assumptions can be seen as critical for achieving these goals, as they offer explicit opportunities for students to act agentically (Anhalt et al., 2018). While goals for student engagement in complex, critical modelling tasks have recently gained traction in the literature (Kaiser, 2017), understanding how they might be actualized in instruction needs further study. Additionally, there is minimal literature on instructional strategies to support students' agency in complex modelling tasks (Elliott et al., 2019; Kaiser, 2017). In this paper, we explore how U.S. grade nine students engaged in a modelling task, which explicitly attended to making and reflecting on assumptions, and developed agency and authority via socio-critical reasoning.

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Theoretical background

Socio-critical mathematical modelling is an opportunity for productive disciplinary engagement that examines the relationships between *problematizing* and *resources,* and *authority* and *accountability*, while recognizing how power shapes contexts for learning (Agarwal & Sengupta-Irving, 2019; Engle, 2012). Specifically, students' identities and histories shape their modelling experiences, in part through the assumptions they make and the reasoning they provide. Socio-critical modelling tasks provide opportunities for students to consider contexts in which they problematize situations, assert their authority, and leverage resources to solve the problem. These tasks, which are often ill structured and complex, require students to identify conditions and assumptions central to the task (Blum & Leiß, 2005). In this study, we consider the resources students accessed to support their problematizing and resultant reasoning. Resources for modelling include those students access from previous modelling experiences as well as new resources they seek as they consider the problem. We also consider the balance between *authority* and *accountability* as students develop socio-critical modelling practices while engaged in a task. *Agency* can be seen when students are authorized to take a stance on a task or method, and this can develop into *authority* as mathematicians and thinkers as they get recognized for their ideas in a more public space. With authority comes the need for *accountability*, or the ability to articulate how and why one's ideas make sense. As students build authorship, they must also build accountability to disciplinary practices, mathematical logic, and to their community of learners and stakeholders. Through a socio-critical modelling task which leveraged student assumptions, we seek to explore the relationship between agency, authority, and accountability evident in students' reasoning.

Study context and methodology

Data for this paper draw from broader study of teachers' instructional tools for modelling. The students in this study had abruptly shifted to remote learning in March 2020. In this transition, their teacher posed the following scenario, *Toilet Paper (TP) Task* (Figure 1).

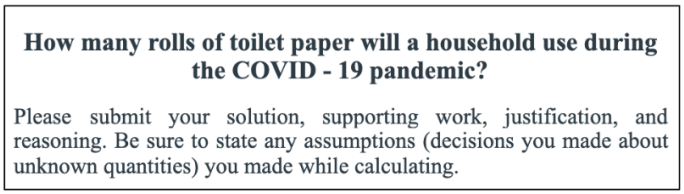

Figure 1: Modelling task for grade 9 students.

The grade 9 teacher and students had solved modelling tasks prior to remote learning using *modelling instructional routines*. These routines scaffolded students' learning of specific modelling practices, such as making assumptions and iteratively justifying models (Elliott et al., 2019). For the *TP Task*, the teacher asked students "to be thinking about those [modelling practices] … as they went through the task and answering like what assumptions they made and how did that affect the final solution." Students were asked to independently complete

the task as directed in Figure 1. The student work and the teacher's reflection on the use of the task were data for this paper.

We examined students' work for their agentic moves to author their own interpretations of the task, their assumptions about the scenario, and the solution paths they developed. We also looked for ways students held themselves accountable to different sources of authority in their socio-critical reasoning processes. After reading the student work independently, the research team met to discuss themes of *agency/authority* and *accountability* in student work. We coordinated these data with the teacher's reflections provided during an interview on modelling instruction. Looking at the themes and evidence across all of the data allowed us to notice patterns in the ways students acted agentically to construct solutions to a societal problem and supported their reasoning.

Findings and discussion

As students presented their models they attended to conditions and assumptions that would shape their models. Students' agentic moves were made apparent when they considered conditions such as students' own household size and the potential length of the pandemic. They used these conditions to make assumptions about variables and quantities needed for their models, including the average rate of TP use when quarantined at home, average household size, gendered composition of family, and length of the pandemic. Students' histories and identities shaped the nature of conditions and assumptions they considered. Several students weighed the benefits and limitations of using single- versus double-ply toilet paper; one student noted that they "refused to even think about 1-ply toilet paper" and supported their reasoning by citing average American incomes to justify that a typical budget would be able to sustain 2-ply paper. These assumptions capture students' underlying attention to socioeconomics and humor, while grounding their arguments using data.

Students' attention to sources of accountability to justify features of models varied. A quarter of the students provided websites or organizational references and most students drew upon their own authority and solved the task taking into account their own household data. Given our analysis occurred 12 months into the pandemic, we found students' attention to accountability and authority to assert the duration of the pandemic most interesting. One student held themselves accountable to the World Health Organization and the Center for Disease Control to articulate their model. Another referenced that fact that "vaccines are still being tested and thus the pandemic will likely be in effect 900 days." A third stated that "scientists" have suggested the pandemic will last 52 weeks. Others relied on their own authority to determine the duration of the pandemic, making explicit that they were assuming the particular time frame; some students claimed it could last anywhere from 25 days to seven months.

We found a majority of students made explicit their assumptions which, given the teacher's initial prompt, may seem unremarkable. However, modelling instructional research and the teacher's prior instructional experience would suggest that students' find these practices challenging (Anhalt et al., 2018; Blum & Leiß, 2005). The teacher shared that the students are "getting used to not being able to get all the same answers in the end … when they can figure out different ways they want to approach it and get into the problem, they

see it more as an exciting challenge." The TP task afforded the students to leverage their experiences relevant to constructing a model, thus asserting their agency while also considering different sources of accountability to productively engage. Given the unprecedented nature of the pandemic, students' attention to varying sources of accountability offers insights on how they were learning to read and write their world with mathematics through a lens of their experience and identity (Agarwal & Sengupta-Irving, 2019).

Conclusion

This ongoing study explores the possibilities for supporting students' development of agency, authority, and accountability through socio-critical modelling tasks that require attention to assumptions. Utilizing the structure for reflecting on assumptions provided via a routine for modelling and the problematization of a relevant, open-ended task, we analyzed student work for evidence of their agency, authority, and accountability. These reflections show a willingness to construct arguments based on student-made assumptions and interpretations of the scenario. Further, students referenced outside authority as well as their own experiences to defend their mathematical ideas, developing reasoning strategies in line with disciplinary expectations of the logical process of decision-making in modelling activities.

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Inclusion and social justice: Possibilities of mathematics education in the context with immigrants

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This article presents a doctoral research project that relates to the theme of inclusion and social justice from the perspective of critical mathematics education. The research aims at reflections beyond the school environment by researching immigrants and mathematics teachers from São Paulo State. In a qualitative methodological approach, interviews will be conducted, and comments will be made through a diary of notes. It is hoped that this research can contribute to the construction of critical perspectives on how mathematics education can be structured towards the inclusion of immigrant students and help to promote anti-racist mathematics education.

The theoretical frameworks

This doctoral research starts from concerns around issues related to immigration by dealing with themes of inclusion and social justice. According to the United Nations (UN) International Migration Inventory, in 2019, around 3.5% of the population on the planet were people who lived in countries other than the countries where they were born. Considering this amount, one in seven international migrants (about 38 million) are under the age of 20, which represents many school-age people.

International migrants are often faced with borders that can be material or immaterial. Material borders can be the concrete walls themselves, for example, which are placed as barriers, such as restrictions on mobility for immigrants. But also, the anti-immigration policies that are the subject of speeches by governments in different countries. Immaterial borders, on the other hand, are barriers that are often imperceptible or disregarded, such as those that appear during displacements in which people are subjected to precarious and dangerous modes of transport in crossings between countries. At the destination location, language barriers, customs, local laws, cultural differences, barriers to documentation, longing for the country of origin, barriers in the relationship with the local population, xenophobic speeches (whether recreational in jokes or more explicitly), also characterize immaterial boundaries.

In Brazil, the historical constitution of the population is also marked by international migrations, whether they are voluntary or forced. Amid the myth of racial democracy and

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migratory receptivity, research, and media show that migratory receptivity is often selective, that is, it may be related to the physical characteristics or the country of origin of immigrants. The myth of racial democracy distorts the real reality that racism in Brazil is structural and encompasses the reality of immigrants in the country today (Almeida, 2019).

In this text, racism is understood as a historical and social construction. As a systematic form of discrimination that gives a specific ethnic group a place of dominance over others. In other words, it is a systematic way of producing disadvantages and privileges depending on the group to which people belong (Almeida, 2019). In the context of immigrants, racism works more broadly and goes beyond the model of racism based on the black / white binary. It can be based on discrimination based on phenotype (visible characteristics of people's bodies), accent, geographic origin, religion, customs, and immigration situations.

In the face of diversified environments generated by migration, it is important that anti-racist practices are present inside and outside the school context. Therefore, the connection between tolerance and social and racial justice is fundamental. When talking about social and racial justice, it is necessary to recognize the existence of racism, to consider reparation for inequalities of opportunity, to question arguments based on meritocracy and to reflect policies of racial equality (Martin, 2013).

According to Gutstein (2006), mathematics education has the role of helping structurally marginalized groups to investigate and question injustices, such as racism and other social inequalities. Therefore, it is important to guide students to engage in the quest to challenge and transform structures of oppression. For the author, teaching mathematics for social justice is also teaching mathematics for racial justice. "This includes providing students with opportunities to analyze whether and how racism is implicated in social phenomena and to understand different forms of racism." (Gutstein, 2016, p. 490). In this context, it also includes rethinking the context of international migration and reflecting differences in the classroom.

For Skovsmose (2016), critical mathematics education has concerns with different groups of students and is relevant to teaching and learning about issues of social injustice and oppression. These concerns contribute to the organization of environments that favor inclusion and tolerance. Thinking about these environments with immigrant students requires considering social justice and racial justice, and the encounter between differences.

According to the UN (1995), education policies and programs must contribute to an understanding of solidarity, tolerance among ethnic, social, cultural, religious, and linguistic groups between nations. Tolerance, as defended by Freire (2017), is not a favor from the tolerant to the tolerated, in the sense of class, race, gender superiority. Nevertheless, tolerance is a virtue of human coexistence, of living with the different and not with the inferior.

Thus, the question of this research is: *How is mathematics education structured for the inclusion of immigrant students?* This research seeks to understand the context of immigration in Brazil and around the world, to reflect the connection between racism and xenophobia and to discuss possibilities for an inclusive mathematical education in the context of immigrants.

Research planning

This investigation perceives the complexity of the social, historical, and political context when working with social phenomena and the subjects involved in them, and in this way, this investigative process uses a qualitative approach to research.

It is also based on the assumptions of the Critical Race Theory (CRT) (Davis & Jett, 2019). For this reason, it gives special importance to the voice of people who are racially minorized through their reports and narratives about their lived experiences. It considers various forms of injustice and explores differences within and between groups when considering the historical and political context together with an awareness of racial inequalities.

In this sense, the data production of this research will be developed in the following moments: Interview with immigrants in which the following points will be considered: about the context of the immigrant family; about inequality / about exclusion / about social and racial justice; situation as a student / school context; about racism and xenophobia. Interviews with mathematics teachers and school managers in which the following points will be considered: context of immigrant students / context of schools with immigrant students/ about inequality, exclusion and social and racial justice/ about the role of mathematics in the context of immigration /about and racism and xenophobia/ about possibilities in mathematics classes in the context of immigrant students.

According to the data produced, we will point out research results in the light of Critical Mathematical Education (Skovsmose, 2019). The interviews will be recorded in audio and video. These moments will be transcribed and supplemented with notes made by the researcher that will contain memories of the context of the speeches, the atmosphere of the discussions, the episodes of silence, elements of the interactions between the researcher and the research participant, constituting information for the understanding and interpretation of the theme. There will be a reflection on the interaction of the materials and attention to the contents as official documents, responses to the production instruments, and to the content as elements of the social life of the participants, context of the countries of immigrants, context of Brazil, among others.

Comments

This research aims at reflections beyond classroom environments. It is hoped that the realization of this research can contribute to debates on the teaching and learning of mathematics and describe possibilities for actions that seek to promote contributions to an anti-racist and inclusive mathematical education in the context of immigrants. More broadly, favor conditions to combat social inequalities and injustices.

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Sustainable e-assessment in mathematics instruction

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This study aimed at moving beyond content mastery to help students develop sustainable, transferable skills such as self-regulated learning. Each Student conducted a self-inquiry on a selected topic in mathematics. They also formulated some multiple-choice questions, asked their peers to solve them and engaged in active discussions afterward. Collected data were analyzed in terms of Student-generated Questions' (SGQs) quality by two instructors independently. Findings showed while taking the responsibility of assessment is a promising strategy in developing self-regulated learning, it might not automatically lead to a higher-order learning (i.e., critical thinking). It is suggested that a combination of instructors' feedback and regular use of digital technologies can enhance students' questioning competence.[1]

Introduction

Assessment should meet both the specific goals of a course and equip students with necessary skills to undertake their own assessment activities, i.e., *judging the quality*, in the future professional workplace (Boud, 2000). However, the conventional assessment conducted in higher education mainly aims at measuring mastery of content knowledge at the end of a course (summative assessment) instead of developing sustainable competences in students (integrative or formative assessment). Mathematics is inherently an inquisitive discipline which evolves around questions and problems. In a typical classroom, teachers retain control of asking questions: the questions are initiated by teachers and students take their turn to answer. There are some opportunities when the teacher invites students to ask questions. However, when the teacher is the one who constructs the most interesting questions and problems, students become dependent upon the teacher to catalyze inquiry (Bowker, 2010). To facilitate development of mathematical competence, teachers should create effective learning environments and encourage students to ask relevant and scientifically sound questions (Foster, 2011).

[1] This paper is a short version of Caspari-Sadeghi, Forster-Heinlein, Maegdefrau, & Bachl (2021), where further explanations can be found.

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Objectives of the study

This study aimed at moving beyond content mastery to help students develop the sustainable, transferable skill of self-regulated learning or 'learning to learn' as one of the 'key competences for life-long learning' (European Commission, 2012). Generating well-crafted questions is a creative act, and at the heart of what doing science is all about (Chin and Osborn, 2008). SGQs open a window to the mind of the students: they indicate what counts as significant for the students, what they understood, misunderstood or missed altogether. In this empirical study, we tried to use SGQs as a technique to (a) foster a culture of inquisitiveness in mathematics learners, (b) involve students more critically and meaningfully with the content, and (c) make the students the owners of their own and peer assessment.

Context of the study

As higher education is forced to turn to online teaching and learning during the COVID-19 pandemic, the authors used this short-term crisis to reconceptualize what constitutes Student Learning Outcomes in an online mathematics instruction. This study was conducted in an applied mathematics course during COVID-19 pandemic (March–Sept. 2020). The course was run in online, synchronous mode (via ZOOM) with the further support of Learning Management System. The course was taught jointly by a professor and her teaching assistant who was recently graduated. The participants were Bachelor and Master students with no prior experience in online instruction.

Procedure

The students were required to select a related topic in mathematics, conduct an inquiry on it and present their summary and findings to the class. Self-directed learning was supported by the instructors, by recommending literature, answering questions, etc. The students also formulated some multiple-choice questions, which they asked their peers to solve and engaged in subsequent active discussions.

Results

Students' perception towards the value of the SGQ strategy was assessed through an online questionnaire. Findings revealed students' positive attitude towards the experience, with 88% of the participants reporting that it contributed greatly to their focused attention and engagement with content. The Quality of SGQs were analyzed based on a two-dimensional rubric, (a) the overall quality, i.e., content coverage, relevance/clarity, and plausibility of a question, and (b) cognitive demand involved in a question based on Bloom's Taxonomy (1956). Each dimension has several levels. The majority of questions generated by students (66%) were classified at the lowest category (remembering), 25% at level 2 (understanding), and less than 10% at level 3 (application) of Bloom's taxonomy. None of the SGQs was at the higher-order levels, such as analysis, synthesis, evaluation/creativity. No significant correlation

could be established between SGQ quality and students' academic attainment in the final exam (for full discussion, see Caspari-Sadeghi, et al., 2021).

Although Bloom's taxonomy received some criticisms (Moore, 1989), and there are alternative frameworks, i.e., Anderson & Krathwohl (2001) or Illeris (2002), Bloom's Taxonomy was preferred due to its clear categories as well as its widespread use in education, which facilitated comparing our results with other available studies.

Bottomley and Denny (2011) suggested such results are to be expected, since this was likely the first time these students were asked to write their own questions systematically. The development of appropriately aligned multiple-choice questions is not an easy or trivial task. The instructors decided to use both human support and digital technology solutions, i.e., *PeerWise,* to improve the process in the next course. Authoring questions for self and peer-assessment is an effective strategy to develop self-regulated learning which can facilitate future life-long learning beyond academia.

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Supplement: Sample of SGQs

Wandle die Dezimalzahl 67,25 in das Oktalsystem (b = 8) um. Welche Antwort ist die korrekte Ziffernschreibweise?

- ○ a.34
- ○ 103.2
- ○ 1000011.01
- ○ 1003.1

Was war nicht Teil der Kettenreaktion, die durch den ungeschützten Cast losgetreten wurde?

- ○ Die SRIs schalteten sich nacheinander ab.
- ○ Der OBC interpretierte nichts aussagende Bit-Pattern als korrekte Messdaten.
- ○ Das Haupttriebwerk wurde kurzzeitig abgeschaltet
- ○ Eine Hardwareexception wurde ausgelöst.

In welchem Fall ist das Nash-Gleichgewicht eindeutig bestimmt?

- ○ F ist quasikonvex
- ○ F ist pseudomonoton
- ○ F ist gleichmäßig monoton

Gibt es im Spiel „Schere-Stein-Papier“ ein oder mehrere Nash-Gleichgewichte?

- ○ Ja, in den Punkten {Schere,Schere}, {Stein,Stein}, {Papier,Papier}.
- ○ Nein, es existiert kein Nash-Gleichgewicht.
- ○ Ja, in den Punkten {Schere,Stein}, {Stein,Papier}, {Papier,Schere}.

Statistical literacy to empower coexistence within Brazilian semiarid region

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The Brazilian semiarid region is characterized by high temperatures and evaporation, in which during periodical crises, political actions to combating drought proved to be corrupt and ineffective, placing the population in vulnerability. Coexistence within the semiarid is a political approach that supports public policies to recover respect for the cultural diversity of peoples and territories, sustainability, agroecology, and food sovereignty. This project aims to investigate possibilities for the development of statistical literacy in a collaborative context of continuous teacher education with participants who teach in the semiarid region, intending to develop pedagogical activities that contribute to re-signify understandings of the paradigm of coexistence.

Introduction

This project intends to approach this statistical literacy perspective with elementary school teachers who live and teach in the Brazilian semiarid, which is a geographic region characterized by high annual averages of temperature (27° C) and evaporation (2,000 mm), with rainfall up to 800 mm per year (Lima, Cavalcante & Perez-Marin, 2011). The Brazilian semiarid is comprised of multicultural activities and territories, as well as a unique biome called *Caatinga* (which in Tupi-Guarani language means White Forest). The Brazilian semiarid comprises 969,589.4 km² which is equivalent to 11% of national territory.

The occupation of semiarid by Europeans began in the 16th century. The colonizers introduced unsuitable cultural practices, such as agriculture based on deforestation on the banks of water sources, burning, and exotic crop plantation. These activities are still current in several areas. However, due to the economical practices being unsuitable for the semiarid climate and soil, the social situation for most populations of the semiarid becomes a serious issue during longer drought periods. During these crises, it is common to develop ineffective temporary governmental projects, but this kind of projects it is just as a political instrument of domination and alienation, as well as a promoter of corruption. This problem has historically been used for political purposes.

From the 1990s, social movements and non-governmental organizations began to develop an alternative political project which we called *coexistence within the semiarid* to reduce historical problems, such as disputes over water resources and monopolization of arable land.

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The perspective of *coexistence within the semiarid* gives value to the processes of understanding these sociocultural aspects to plan, elaborate and carry out actions in search of the good living together. Therefore, the interventions must aim to rescue respect for the cultural diversity of the various existing peoples and territories, building the sustainability of water resources and a relationship between nature and agroecological practices, coexistence technologies and food sovereignty.

Silva (2006) identifies five meanings for the *coexistence within the semiarid:*

Meanings	Descriptions
1. Coexistence with the Environment	Management and sustainable use of natural resources in the ecosystem, without making their reproduction unfeasible, considering the balance of the common space experienced.
2. Coexistence Economy	Capacity for sustainable use of natural and cultural potential in productive activities that are appropriate to the environment.
3. Coexistence with the quality of life	To be able to identify the satisfaction of fundamental needs as a condition for expanding human capacities and improving the quality of life, conceived as a reduction in inequalities, poverty, and misery.
4. Coexistence Culture	Valuing and rebuilding local population knowledge about the environment in which they live, its specificities, weaknesses, and potential.
5. Political Dimension	Mobilization of civil society, through networks of social movements and organizations, which promote the dissemination of social values of coexistence within Semiarid and fight for the improvement of its economic and sociocultural conditions.

Table 1: The five meanings of *coexistence within semiarid*

Considering the context of Brazilian semiarid and the possible relationship with mathematics and statistics as discussed by Skovsmose (2021), we might ask the following questions: How can statistical literacy strengthen the political project of coexistence within the Brazilian semiarid from a critical and liberating perspective? What aspects were historically used to reinforce stigma and prejudice towards this region and its people? What is the role of mathematics in this process? As much in the possibilities of transformation and change as in its uses as a tool of oppression?

The general objective of this doctoral research project is to investigate how to utilize the theoretical-methodological assumptions of statistical literacy to promote the problematization of sociopolitical aspects and thereby empower the political project of coexistence with the semiarid.

The roles of mathematics and statistics to empower the coexistence with the semiarid

Skovsmose (2021) argues that the world faces more and more large-scale crises such as environmental ones, which might include those which happen in the Brazilian semiarid. The

author identifies at least three different types of relationships between mathematics and crises: mathematics *can picture a crisis*; *can constitute a crisis*; and *can format a crisis*. The political and economic interests play a decisive role to develop different crisis discourses about the reality, which can be based on mathematical and statistical arguments. For example, for over one century political actions to the semiarid emphasised *the combating of drought*, as it could benefit a hegemonic minority of wealthier groups dominating the semiarid. Then, in this case the statistical data is used to describe the extent of the drought, the resources people need, and somehow manipulate the reality. But also, Skovsmose considers that mathematics (and statistics) can play transformative roles in regards with crises if they support human-centred lines of arguing and actions. Statistics can support important social values, such as democracy and argument based on scientific evidence (Engel, 2019).

However, statistics can also be used to justify certain narratives by manipulating data representations that can emphasize or disguise some statistics in news (Monteiro & Ainley, 2010). In addition to that, there are disseminations of misinformation and malinformation associated with fabricated statistics, deliberately created to harm a people, social groups, organizations, or countries (Carmi, Yates, Lockley, & Pawluczuk, 2020).

Therefore, we understand that it is necessary to problematize the perceptions of how statistics can be used, either correctly and ethically, faithfully to the reality to be analysed, or in an unethical way, with the aim of pretending to validate distorted realities, favouring individual and antisocial interests, distant from the truths of the facts.

> The secret language of statistics, with so much appeal to our "fact-based" culture, is used to sensationalize, inflate, confuse, and oversimplify. Statistical methods and terms are needed to report data on social and economic trends, business conditions, "opinion", research, censuses. But without writers who use the words honestly and comprehensively, and without readers who know what they mean, the result can only be semantic absurdity. (Huff, 1993, p. 8)

In a panorama of contradictions and pitfalls, in which the biased uses of statistics can have strong implications in the social, economic, cultural, political, and historical contexts, we point out that statistical literacy is particularly important to achieve critical elements that can provide conditions to understand, transform, reflect and act up these controversial contexts.

Statistical literacy is "a stand-alone complex competency with many unique elements, well beyond knowing statistics per se. Further, I argue that statistical literacy, and its many building blocks, are seldom or insufficiently addressed in regular statistics or mathematics instruction" (Gal, 2021, p. 26). This competency is causally related to people's attitudes towards the countless statistical information that surrounds them daily and how they critically evaluate graphs, infographics, tables, charts, statistical data from journalistic, scientific, and informational texts. Gal (2002, p. 2) argues:

> the term "statistical literacy" refers broadly to two interrelated components, primarily (a) people's ability to interpret and critically evaluate statistical information, data-related arguments, or stochastic phenomena, which they may encounter in diverse contexts, and when relevant (b) their ability to discuss or communicate their reactions to such statistical information, such as their understanding of the meaning of the information, their opinions about the implications of this information, or their concerns regarding the acceptability of given conclusions.

Following the perspective of statistical literacy (Gal, 2002), our focus is on mobilizing among the participants the structural elements: elements of knowledge (literacy, statistics, mathematics, and context) and elements of disposition (beliefs and attitudes, and criticality).

In this context of ideological, political, cultural, economic dispute between political perspectives, narratives are constituted as background for the use of statistical elements for validation, confrontation, imposition, and refutation of the facts presented by each side. It is in this aspect that we set out to investigate possibilities to enhance the paradigm of living with the semiarid using the elements of statistical literacy.

Theoretical and methodological construction of research study

The theoretical-methodological construction process that we are proposing to carry out, seeks to articulate three interrelated perspectives, statistical literacy, *the paradigm of living with the semiarid* and the continuing teacher education.

We intend to propose a qualitative study based on a research-action type according to Tripp (2005). This type of research produces an analysis of reality while seeking to transform it through an intervention. Therefore, this project aims to contribute to the potentiation of *the coexistence within the semiarid*, using the elements of statistical literacy. For that reason, we decided to have an investigative structure as part of an activity of continuous teacher education which consisted of contextualized problem situations related to specificities of Brazilian semiarid. The referred continuing education will count on the participation of teachers and professors who teach statistics in basic education level in the semiarid territories.

The development of formative activities with teachers is based on three theoretical perspectives. Firstly, we dialogue with contextualized education for living with the semiarid region (ECSAB), a theoretical perspective that emerged from the context of the *paradigm of coexistence*, which carry out studies, pedagogical resources, and teacher education activities within the context of the Paradigm of Coexistence (Lima, 2008).

Another perspective was that of the investigative research cycle originally proposed by Wild and Pfannkuch (1999) and adapted by Guimarães and Gitirana (2013). These authors argue that an experience with engagement in statistical research, following the cycle and going through its stages in a critical and contextualized way, is a fundamental educational activity for the construction of statistical literacy.

A third perspective with which we dialog is proposed by Watson and Callingham (2003) who presents an analytical framework composed of six levels of positioning in relation to statistical literacy. It will help us to create the action research experiences, as well as to analyse whatever data produced through the project.

The participants will be six basic education teachers who teach statistics in public schools belonging to the semiarid territory. We called our initial approach with the participants as a *recognition step*, which consists of to know more about teachers' professional and academic backgrounds, as well as their knowledge and perspectives about semiarid socio-political aspects. This step will be developed by carrying out individual interviews.

A second stage is to propose to the teachers to participate in a cooperative group. The meetings will be based on a focus group approach in which will produce data about teachers'

perceptions, opinions, attitudes, critical interpretations, and beliefs. We plan to develop at least four meetings with an estimated duration of 2:30 hours.

During the focus group meetings, we will discuss the five meanings for the *coexistence within semiarid* associated with statistical literacy aspects. The meeting will be activities in which are involved in semiarid socio-political statistical contexts, which include statistical data. Table 2 presents examples of data which might be used to problematize.

Some social indicators of the Brazilian semiarid region
Division of semiarid lands suitable for agriculture About 1.5 million farming families (28.82% of all Brazilian farming families) occupy only 4.2% of agricultural land in the semiarid region. While 1.3% of rural establishments with more than 1,000 hectares (called *latifundios*) hold 38% of semiarid territory.
The economic situation of population The majority of Brazilians (59.1%) in extreme poverty are in the Northeast where the semiarid is located. Most of this population (52.5%) live in rural areas, and 4 out of 10 extremely poor people are between 0 and 14 years old (IBGE, 2010).
The vast majority of semiarid municipalities (60.09%) of the, with more than nine million inhabitants, the Human Development Index (HDI) varies from Very Low to Low. The HDI considers indicators of longevity, education and income. All towns in the Semiarid region had a lower Municipal human development index (HDI-M) than in Brazilian national rate, which is 0.727.

Table 2: Social indicators of the Brazilian semiarid region

The information presented on Table 1 can be a starting point to interpret the semiarid reality based on statistical data. To begin the reflections, the researcher would ask questions, such as:

> What do you think about this information?
> Do you think it is reliable information?
> Do you think that this information can stereotype people who live in the semiarid region?
> What have you heard, during your life, about the semiarid region?
> Do you believe in these opinions?
> Have you ever used statistical data to describe the semiarid?
> What kind of information have you used to describe the people who live in semiarid?
> What did you want to argue?
> What other data can we use to demonstrate the potential of this region?
> Do you think the way this data is used by politicians leads to more inequality?
> How can aspects of statistical literacy help in building a dignified understanding of this region and these people?

The researcher will mediate the group proposing reflections, reviewing understandings and possible stereotypes, perceptions living, existing and resisting in the Brazilian semiarid.

Provisional considerations

The articulation between the theoretical perspectives of contextualized education for coexistence within the Brazilian semiarid and statistical literacy, suggests a promising path for the potentializing coexistence paradigm. In this sense, we expect that it is possible to transform and reframe the understandings and practices of teachers and professors of Brazilian semiarid so that they become mobilizing agents of this perspective.

We believe in the proposal under construction and in its effectiveness, so that we can disarm, disturb the hegemonic discourses, and introduce provocations into the debate, problematizing and denaturing exclusionary and silencing practices, so that we can dismantle the accepted and naturalized imagery about Brazilian semiarid.

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An Augustinian take: The loves of the mathematics education research community

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Psychoanalytic research in mathematics education focuses on the unconscious narratives in the field with a predominant focus on desire and its derivatives. Yet, over a millennium ago, Augustine of Hippo addressed similar themes with his concept of love and his definition of a community. In this project presentation, I argue that Augustine's notion of love as craving and his definition of a community can inform research efforts in mathematics education in an original way. Namely, I will use Augustine's work to help identify potential loves of the field and discuss how those loves subsequently shape the field and its research.

Introduction

There has been a recent uptake of psychoanalysis for research purposes in mathematics education (see the work of Alexandre Pais, Tony Brown, Tamara Bibby, etc.). In the introduction to the book *The Psychology of Mathematics Education: A Psychoanalytic Displacement*, Tony Brown, of Manchester Metropolitan University and the editor of the book, discusses how psychoanalysis entered the scene of mathematics education research. He describes how there was a shift away from traditional psychology towards "a psychology understood more through relations between people" (Brown, 2008, p. 1). Much of this initial work used Lacan's theories to promote "the shift from bio-scientific to narrative emphases in interpreting Freud's work" (p. 3) and is focused on what Lacan called "the truth of desire" (Lacan, 2019). Thus, psychoanalytic research in mathematics education attempts to identify subconscious or unconscious narratives in the field that often depend on desire.

Approximately 1,650 years ago, however, Augustine of Hippo discussed themes of desire and the role of desire in communities. Adjusting a definition from Cicero, Augustine (2009) wrote:

> […] for example, if one should say, 'A people is the association of a multitude of rational beings united by a common agreement on the objects of their love,' then it follows that to observe the character of a particular people we must examine the objects of its love. (§ 19.24)

For Augustine, "love is indeed nothing else than to crave something for its own sake" and "a kind of motion, and all motion is toward something" (as cited in Arendt et al., p.9). This elevation of love (as craving, or desire) shares similarities with some of the work in psychoanalysis. Thus, while I do not intend to diminish the work of psychoanalysts in

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mathematics education research, I'd like to offer an Augustinian "take" on mathematics education research and suggest that this ancient North African bishop can inform our work in an original and profound fashion—meaning, I'd like to use Augustine's conception of love as an opportunity to address the mathematics education research community's loves. This presentation reports the initial findings of an ongoing project where I attempt to answer two questions: first, what are the field's loves? And second, how do those loves shape and form the field and its research?

Before continuing, I think it is important to note that I am not suggesting a theological analysis of the field. As Hannah Arendt demonstrated in the field of political theory, Augustine's work extends beyond theology and Christian doctrine. Arendt thought that, regardless of culture, religion, or worldview, it was relevant that humans are first and foremost lovers. Arendt (1996) even suggested that, "Strictly speaking, he who does not love and desire at all is a nobody" (p. 18).

Arendt felt so strongly about Augustine's conception of love, she wrote her doctoral dissertation on the subject (entitled *Love and Saint Augustine*) and spent the remainder of her life refining and editing the work (Arendt et al., 1996). The lifelong work on her dissertation reflects her love of Augustine, demonstrates his influence on her work, and highlights how highly she thought of her original research question regarding "the relevance of the neighbor" (p. x). She explored Augustine's conception of love in order to study the phenomena of "neighbor" and, as Arendt explains, "to understand in what sense our neighbor is loved in adhering to the commandment of neighborly love" (p. 7).

Augustine's concept of love

Augustine distinguishes between two different types of love: *caritas* and *cupiditas*. For Augustine, *cupiditas* is love with a wrong object and *caritas* is love with a right object. Augustine's definitions of "wrong" and "right" are, of course, grounded in Christianity. But, as Arendt (1996) points out, "both right and wrong love (*caritas* and *cupiditas*) have this in common – craving desire, that is, *appetitus*. Hence, Augustine warns, 'Love, but be careful what you love.'" (p. 17).

Augustine offers this warning because he believes a person or communities' loves are central to who they are. Arendt writes,

> Desire mediates between subject and object, and it annihilates the distance between them by transforming the subject into a lover and the object into the beloved. For the lover is never isolated from what he loves; he belongs to it. (p. 18)

This close intimacy – this binding – of the lover and the beloved is defining for the lover. The stronger the love, the more intimate the union with the beloved. Augustine (as cited in Arendt et al., 1996, p. 18) writes, "Such is each as is his love." Arendt (1996) describes how Augustine uses the word *inhaerere*, which is translated as "clinging to," to denote the closeness of lover and beloved. Commenting on this theme of Augustine, she writes, "Happiness is achieved only when the beloved becomes a permanently inherent element of one's own being" (p. 19).

Arendt (1996) also highlights that, "A thing is sought for its own sake (in *caritas* or *cupiditas*) if its possession puts desire to rest. Thus, nothing can be said to be "loved" which is sought for the sake of something else" (p. 32). Which begs the question: what are the mathematics education research community's loves?

Science, our first love

In this presentation of my ongoing attempt to operationalize Augustine's concept of love, I'd like to suggest that the scientific paradigm is a core love of the mathematics education research community and describe how it can warp our field. Ernest (1998) describes scientific research in mathematics education as being founded on

> rationalism and the scientific method as employed in the physical sciences, experimental psychology, etc. It is concerned with objectivity, prediction, replicability, and the discovery of scientific generalizations or laws describing the phenomena in question. (p. 77)

Ernest goes on to describe two other major research paradigms: the interpretive, which for Ernest (1998) has to do with "understanding, interpretation, intersubjectivity, [and] lived truth" (p. 77), and the critical-theoretic, which aims for social and institutional change through social critique. He argues that each respective paradigm plays a role in the mathematics education research community, that no paradigm should be put on a pedestal, and that antagonistic attacks against entire paradigms should be discouraged.

Yet the scientific paradigm's strong grip on the field as a whole has ramifications for the field per Augustine's notion of love. Prominent researchers (e.g., Cobb, 2007; von Glasersfeld & Steffe, 1991) that work in the interpretive paradigm still frame the field as a science. Consequently, while many of these researchers assent to pragmatist notions of truth and meaning, they function within remnants of the scientific paradigm. This continued worship of the scientific paradigm has led to a scientism that looms large over the field. Thus, I argue that science is the object of the majority of the field's love and is sought for its own sake.

Per Augustine, I argue that this love is *cupiditas* and must be critiqued. While Ernest (2012) recognizes the consequences of adopting certain theories, he seemingly overlooks the consequences of having the scientific paradigm as a core love of the field. Yet the scientific paradigm distorts the field as a whole and skews the field's conception of knowledge. Anzaldúa (1987) identifies a major consequence of scientism. She writes,

> In trying to become "objective," Western culture made "objects" of things and people when it distanced itself from them, thereby losing "touch" with them. (p. 59)

Anzaldúa's description of the consequences of the West's pursuit of objectivity is precisely what is occurring within mathematics education research community today. In Augustinian fashion, Anzaldúa explains that the pursuit of objectivity ultimately leads to dehumanization. While researchers within the interpretive tradition reject the idea that knowledge reflects an objective, ontological reality (Ulrich et al., 2014), they also embrace obvious expressions of scientism. For example, Campbell (2020) suggests that soon we will be able to equip students to wear caps that transmit "high spatial and temporal resolution

EEG or MEG signals" (p. 99) to a central console that can then be analysed by their teacher in real time. Yet, per Anzaldúa, is there anything more distancing and dehumanizing? Where are the affections in such a cold, imagined future? For a field predominantly concerned with human beings, the scientific paradigm and scientism should not be objects of the mathematics education research community's love.

Concluding thoughts

In this project presentation, I argued that Augustine's notion of love can help diagnose the ills of the mathematics education research community. I then briefly excavated and examined the field's love of the scientific paradigm. In the future, further loves need to be explored as psychoanalytic research has already identified the field's love of capital (Pais, 2014). Ultimately, the mathematics education research community needs an imaginative overhaul. The field remains captive to unworthy loves and these objects of love continue to warp the field as a whole.

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Promoting early arithmetical skills by using part-whole thinking as a way to guide joint learning for students of all abilities

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This paper presents preliminary findings of the large-scale project Counting with all children from the very beginning, (Mit allen Kindern von Anfang an rechnen) a project that has the aim of developing and testing joint teaching-learning situations in primary schools, based on part-whole thinking ('Teile-Ganzes-Denken') from children's first maths lessons. Using design research, five mainstream teachers volunteered to participate in the Beta-cycle over a ten-week period in First Grade. Data analysis revealed that not all the teachers managed to define coherent aims for their students' mathematical learning, especially in the instances of children who have poor mathematical skills and of children with special educational needs (SEN).

Preliminary remarks

International organisations such as UNICEF, UNESCO or the European Union, define 'the right to education' as a common ideal regarding inclusion of all people. With the passage of time, the Austrian education system, like that of many other countries, has made great progress as far as the issue of inclusion is concerned. Since 1885, there have been special educational classes for children with learning difficulties, initially called 'auxiliary classes' and subsequently changed in 1956 to 'special schools'. A wide range of special schools were established as the proper place for children and young people with disabilities. The number of different special schools reached a peak in the 1980s with 10 different types available. Austrians' first attempts at integrative education in mainstream primary schools were in the 1980s (Buchner & Proyer, 2020). With the ratification of the UN Convention on the Rights of Persons with Disabilities in 2008, Austria set out on a path towards an inclusive education system. Even today, Austria, like many other countries in the world, still focuses on a so-called two-track approach: Parents are free to decide whether to send their child to a special school or a mainstream school. In 2018-19, of the 578,417 school-age children approximately 5% (29,000) had SEN. 63% of the children with SEN were included in mainstream schools, while 37% attended a special school (Statistik Austria, 2019). Currently the main focus of inclusive education is to promote the quality of teaching-learning situations (Florian, 2008). There seems to be uncertainty, and not only in German-speaking countries, about how to teach inclusively and how to create inclusive teaching-learning situations, mainly in early

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arithmetical education (Korff, 2016). Thus, this project seeks to design teaching-learning situations for First Grade based on part-whole thinking, in order to support teachers in their challenging task.

The TIGER concept of part-whole thinking

It seems to be accepted in the relevant German literature that at the end of First Grade children should learn that numbers (up to 20) are composed of other numbers. It might be helpful if teachers first focus on the consolidation of part-whole thinking and then, based merely on that, initiate children's understanding of addition and subtraction (see also, for example, Gaidoschik, 2019). These goals seem to be the norm in international mathematics education also, with for example 'Number framework' (New Zealand Ministry of Education, NZME, 2008) or 'Mathematics programme of study' (UK Department of Education, 2013). Part-whole thinking is incorporated as early as possible in the school programme in these guidelines for teachers' actions. The TIGER (Teile im Ganzen Erkennen und damit Rechnen) concept, developed by Gaidoschik (2017), focuses on solid number concepts in early arithmetical teaching. The concept consists of three fundamental aspects. Firstly, counting with an understanding of numbers. This includes children learning how to work out the 'counting principles' of Gelman and Gallistel (1987). Secondly, comparing quantities and numbers. On the basis of one-to-one matching up, children should learn and/or consolidate that no counting is needed for pairing the items or for number comparison such as identifying that there is one more left over. The third area is that of subitizing or perceptual subitizing without counting (see also, for example, Clements & Sarama, 2009). Teachers should work with students in all fields more or less concurrently.

The research project, research method and research question

The 13 teaching-learning situations of the current research project are based on the ideas of the TIGER concept, with a balance between individual and mutual learning. To be able to examine the effectiveness of the learning environments, collective case studies in the framework of design-based research (Euler, 2014) have been conducted. Within the framework of an Alpha-cycle, conducted in school year 2019-2020 with 45 First Grade students in two Tyrolean mainstream classes, video-recorded conclusions were drawn for the implementation, analysis and further development of the teaching-learning situations. The results were processed in an additional cycle. The Beta-cycle, which is relevant for data analysis, was carried out in five Tyrolean primary school classes in school year 2020-2021 with 95 pupils. To generate insights into students learning, each lesson was video recorded. Thus, in this paper, I write about six and seven-year-old children of the Beta-cycle over a 10-week period in First Grade in inclusive mainstream classes in Austria. The questions guiding the study are: Can teaching-learning situations based on part-whole thinking be designed to foster learning processes in early arithmetical teaching of all students in primary school? How should such teaching-learning situations be designed? A combination of 'Abstraction

in Context' (Dreyfus & Kidron, 2014) and 'Assessment of Teaching-Learning-Situations in Mathematics of the Early Grades' (Steinweg, 2010) is used for data analysis. AiC is a theoretical framework to analyze students' processes of constructing abstract mathematical knowledge. There are three epistemic actions: Recognizing (R), Building-with (B) and Constructing (C), hence the RBC-model. Steinweg's idea of dimensions focuses on the teachers' possibilities for action. The combination of both approaches gives indications for further development of teaching-learning situations.

Interim results

The following brief sequence of the teaching-learning situation Throwing Tiles focuses on six-year-old girls Anna and Rosa. They take turns throwing the 10 reversible tiles, which have a red side and a blue side. Then together they have to match one red with one blue tile, using one-to-one matching without counting (AiC - Recognizing). They create two rows to identify who has more tiles, and how many more there are of one colour than the other. Anna has six blue tiles, Rosa four red tiles. The teacher joins in:

Teacher:	Great! Both of you have already thrown tiles and matched them in two rows. Each blue tile in the top row has a red tile in the bottom row. Rosa, how many tiles do you have less than Anna?
Rosa:	I'd better count them again. *(Rosa takes her red tiles and starts again).*
Teacher:	How many tiles do you have, Rosa? Did you have to count them again?
Rosa:	I have four. *(Rosa is uncertain again).*
Teacher:	*(The teacher takes the blue tiles and arranges them in a row).* Can you match a red one of yours with each blue tile?
Rosa:	Yes. *(Rosa puts a red tile on each blue tile in a second row).*
Teacher:	Great. How many tiles more does Anna have than you? *(The teacher points to both rows with her index finger).*
Rosa:	Six? *(Rosa is unsure how to answer).*

Analyzing the transcript using AiC indicates that the ideal of learning processes is not achieved by all children. Regarding Steinweg's idea of dimensions, not all teachers were able to foster children's competence in one-to-one matching, as seen in Rosa's lack of understanding of the one-to-one matching up process. Particular attention should be paid to the mathematical thinking of underachieving children and to children with SEN, as in the case of Rosa mentioned above. In this sequence, the teacher did not realize that Rosa requires a solid understanding for one-to-one matching up (AiC - Recognizing) as a basis for further arithmetical strategies (AiC – Building with and Constructing). No further improvements need to be made to the teaching-learning situation Throwing Tiles. It is important to boost teachers' didactical knowledge and their knowledge of possibilities in the early arithmetical education of all children. In Austria, one third of children with SEN are integrated into mainstream primary school classes, but we can presume that their mathematical competence is not being continuously fostered, and neither are the abilities of children with poor

mathematical skills. Most authors, at least in the relevant German mathematics education literature, seem to agree that these children do not need completely different concepts, but instead need targeted support from highly qualified class teachers and teaching assistants.

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Disciplining bodies: Affect in mathematics education

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Identifying children's affective responses towards mathematics and putting efforts to change them are fairly common aspects of modern-day mathematics education. In this paper, I begin to explore how such efforts involve a disciplinary power that aims to maximize the economic utility of bodies and create a specific kind of citizen-worker.

Concerns about the consequences of how children feel about mathematics have often been expressed by researchers and educators alike (Boaler, 2009; McLeod, 1992). As a result, the identification and subsequent modification of affective responses such as beliefs, emotions, and attitudes have become integral to mathematics education. According to the OECD (2016), "teachers should be concerned about students' attitudes towards mathematics and should take steps to increase students' positive feelings, self-confidence and interest in mathematics when needed" (p. 77). Today, identifying and changing the way students feel about mathematics is aided by various resources – from teaching practices and strategies to even biosensors for monitoring children's emotional activity (Williamson, 2016). While these efforts often come across as 'innocent' pedagogical practices, "any attempt to promote subjectivity through governing thought is neither benign nor neutral" (Popkewitz, 2004, p. 27). In this paper, I argue that modulating affective responses involves a disciplinary power (Foucault, 1995) that aims to maximize the economic utility of bodies and create a certain kind of citizen-worker. I begin by examining a historically assumed motivation behind efforts to modulate the affective – the desire to increase test scores.

Affect and achievement

Affective responses include "a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition" (McLeod, 1992, p. 576). The association between affective responses and achievement became a naturalized belief due to considerable research in the latter half of the 20th century that posited attitudes and other affective responses as predictors of achievement. These statistically driven studies often established causal relationships between student achievement and a plethora of affective factors such as self-efficacy, confidence, and beliefs (McLeod, 1992). As these relationships became commonsensical, the belief that modulating affective responses will lead to higher test scores took hold.

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However, this belief has been challenged often – due to its lack of insights about the complex and unpredictable ways in which affect and achievement interact (McLeod, 1992), and the uncertainty of whether positive affective responses cause high test scores or the other way around. Still, the modulation of affective responses remains a significant aspect of 21st-century mathematics education. An additional impetus to the prevalence of this aspect has been the growing belief in a connection between mathematics competence and the economic steering of society (Valero, 2018). Thus, through achievement on tests, affective responses get linked to national economic progress. The usage of Likert-type questionnaires on student attitudes and beliefs in comparative global assessments like PISA validates this association.

However, increasing test scores does not seem to be the only motivation anymore for modulating children's affective responses towards mathematics. Several 'high-achieving' countries in the PISA mathematics test such as Korea and Singapore have continued to express concerns over low interest and self-confidence of students in mathematics (OECD, 2016), and made large-scale curricular revisions to improve student attitudes. It becomes imperative then to ask what else drives the efforts to change how students feel about mathematics, beyond mere desires for high test scores. In the next section, I explore the same.

Citizen-workers for modern-day economies

The scientific optimism that began following World War II intensified the popular belief that technological development was the solution to every problem (Skovsmose, 2005). Given the mathematical knowledge required to perform the supposedly essential functions of problem-solving and data-driven decision making in this technological landscape, mathematics has become indispensable. Consequently, mathematical knowledge – seen as a "human right in itself" (Valero, 2018, p. 108) – is now positioned as essential for any nation to advance its economy and technological competence. Anyone with a lack of this knowledge is perceived to be unprepared for 'an uncertain future' (Popkewitz, 2004) – a belief that has further amplified the 'STEM pipeline'.

Following WW II, a notion emerged – of human beings as citizen-workers who had to be trained and equipped with skills for the growth of a nation's economy (Valero, 2017). With the emergence of mathematical knowledge as vital during the same era, mathematics achievement became a way of assigning an economic value to a human being (Valero, 2018). This idea of human beings as bodies with economic exchange value placed the onus on school mathematics to produce the desired workforce for the technological future. In recent times, the economic potential of individuals is determined not just by their achievement, but through their "ability to apply knowledge to solve problems" and their propensity "to view mathematics as a useful tool that must constantly be sharpened" (National Research Council, 2001, p. 144). In other words, to be economically valuable in today's technological societies, one needs certain affective qualities too besides their cognitive skills in mathematics.

Due to such labor demands, it becomes necessary to socialize children to possess specific affective responses – turning mathematics classrooms into sites for inculcating these

attitudes and dispositions. With the emergence of cognitive capitalism – in which the production of wealth takes place through not just material production but also through developing the relational, the affective and the cognitive capacities of labor (Moulier Boutang, 2011) – achieving high scores in mathematics tests is not enough anymore. Children now also need to feel a certain way about themselves and mathematics in order to be 'productive' as a modern-day citizen-worker. Governing the child's affective responses, thus, becomes a focus within mathematics education.

Disciplining bodies

Considering such workforce requirements, I argue that the efforts to modulate children's affective responses are ways of governing their feelings and emotions towards a norm that is valued by modern-day jobs. I use the lenses of *discipline* and *docile bodies* (Foucault, 1995) to elaborate on this. Foucault defined docile bodies as ones "that may be subjected, used, transformed and improved" (1995, p. 136). He saw discipline as a mechanism that ensures that the increased utility of a human body makes it more obedient to commands, and vice versa through a constant subjection of its forces. Discipline produces docile bodies by increasing "the forces of the body (in economic terms of utility) and diminishes the same forces (in political terms of obedience)" (p. 138). Disciplinary techniques initiate "a subtle coercion, of obtaining holds upon it [the body] at the level of the mechanism itself – movements, gestures, attitudes, rapidity: an infinitesimal power over the active body" (p. 137).

To achieve a cognitively and affectively standardized labor force for modern-day technologized societies, mathematics education becomes a site for disciplining bodies. Specifically, with respect to the affective qualities of such bodies, the act of disciplining involves attempts to make all children feel *positively* about mathematics. Those who do not are identified and categorized for subsequent intervention – broadening the notion of what needs *fixing* in mathematics education beyond just the cognitive. A child's body language, pitch, stress, and other embodied aspects are made available for scrutiny by humans, or in some cases, by technologies such as student sensor bracelets (Williamson, 2016). Such scrutiny marks bodies that need disciplining – whose affective qualities must be standardized for enhancing their economic value. In the case of mathematics, this disciplinary power attempts to ensure specific mental habits, ways of thinking, and self-regulated behaviors that align with a modern-day desire for 'scientific rationality' (Popkewitz, 2004). *Negative* affective responses towards mathematics are seen as a threat to progress and development. For example, in the case of the United States, Boaler (2009) states:

> Far too many students in America hate math and for many it is a source of anxiety and fear. American students do not achieve well and they do not choose to study mathematics beyond basic courses, a situation that presents serious risks to the future medical, scientific, and technological advancement of society. (p.10)

Seen another way, not feeling a certain way about mathematics diminishes an individual's mathematical exchange value, and in turn their human capital value (Valero, 2018). Moving away from their earlier motivations of increasing test scores, efforts to

modulate children's affective responses towards mathematics have reorganized under a new goal – the creation of a productive technological workforce.

The aim of this paper is not to evaluate efforts to modulate affect as 'good' or 'bad'. My goal is to draw attention to the mechanisms of discipline entwined in these efforts which render a child's mind and thought governable towards a predetermined norm – one that emerges from the desires of neoliberal economies. Any attempt to standardize minds involves classifications based on idealized criteria and the subsequent exclusions of bodies. As a result, what I challenge through this paper is the standardization of the affective - the commonsensical idea that every child has to feel the same way about mathematics. In the current moment marked by the proliferation of STEM education and its associated exclusionary practices, any efforts that claim innocence in their work to change how students feel about mathematics require scrutiny and critical reflection.

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CiviMatics: Mathematical modelling meets civic education

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While mathematical modelling generally has an important position in mathematics education, discussions and approaches about normative modelling are underrepresented. Modelling can be seen as normative, when models are not only used to describe reality, but value or even generate reality, such as mathematical models used to assign legitimate carbon footprints within climate change discussion. Normative modelling for the political discourse requires reflecting about assumptions, simplifications and relevant consequences coming with the models. Starting from the well-known modelling cycle we integrate civic education perspectives to develop a didactical framework around normative modelling, suggesting an enhanced normative modelling cycle that explicitly refers to the judgement formation.

(Normative) mathematical modelling

Mathematical modelling is an important part of mathematics. A mathematical model is a simplified representation of reality that takes into account only certain aspects that can be sufficiently objectified and to which mathematical methods can be applied in order to obtain mathematical results. Models are not unambiguous, because it is possible to make simplifications in different ways. Thus, models should not be considered right or wrong, but more or less useful. The usefulness of a model can only be assessed with regard to the problem to be worked on (Greefrath et al., 2013).

Modelling may have different purposes, pending on the intendent use of the model. For example, descriptive modelling mainly obtains to describe reality, such as physical relations e.g. the planetary orbits. At the same time we may use modelling to "specify or design objects or structures that are meant to inhabit some extra-mathematical domain whilst possessing (if possible) certain required or desired properties" (Niss, 2015, p. 69). This is what we call normative modelling, when reality is not just described, but prescribed as well. Common questions that would indicate normative modelling could be: "Where should a new power plant or a huge shopping centre be located? In what way should seats be apportioned amongst parties in parliamentary elections?" (ibid.). It is clear that normative modelling requires taking a stance that values certain qualities of a solution differently. Unlike many modelling problems in mathematics education, normative modelling cannot be validated.

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A recent and significant example for the relevance of normative modelling is climate change. It is one of the most striking problems that people and governments across the world are facing and it heavily relies on mathematical models both to understand the problem and to evaluate possible solutions. This issue has the potential to change the social and political order of the world (Beck, 2015, p. 75). The introduction of a CO2 tax or global CO2 quotas, the orientation towards the 1.5-degree target or the calculation of ecological footprints are only a few examples of themes, that are widely discussed within politics and society involving mathematical models.

The political dimension of normative modelling

Normative modelling is not only a special kind of modelling but it takes on an important role for political decision-making processes. The application of mathematical models for political decisions is hence in need for reflection. Central, for example, is the understanding that political judgements can never be obtained by calculation alone and that political actors can deliberately influence outcomes by making different assumptions. This brings mathematics education into the focus of education processes for democratic competences, especially for critical analysis skills. It is therefore of democratic importance, both from an individual and a societal perspective, to provide citizens with the competences to understand the role of mathematics in complex situations, such as climate change. In this context, "the purpose of mathematics education should be to enable students to realise, understand, judge, utilise and also perform the application of mathematics in society" (Niss, 1983, p. 248). This includes the application of models for political decision-making, but also the reflection on the principles and assumptions that shape the transformation of societal issues into mathematic models. We build on the principles of civic education aimed at providing competences for the critical reflection of democratic processes as well as the principles of a critical mathematics education. Our goal is not only to focus on normative models as an example for conjunction of political and mathematical processes, but also provide approaches to enable learners to critically reflect on said assumptions (Reinhardt, 2013, p. 101; Skovsmose, 1994, p. 58).

Teaching (normative) mathematical modelling

Mathematical modelling has found its way into (German) schools due to different aspects – but not in terms of normative modelling. The curricula focus on descriptive modelling, promoting real world applications of mathematics in problem solving and modelling (Kultusministerkonferenz, 2015). A very popular model used for teaching mathematical modelling is the modelling cycle. It can be used to illustrate the concept of modelling, taken as an aid for learners in the process of modelling, as well as diagnostic tool and basis for empirical studies (Greefrath et al., 2013). The modelling cycle involves seven steps to model a situation, which are displayed in blue in Figure 1 (see Greefrath et al., 2013, for further explanations).

Taking a closer look on steps two and three, the differences between normative and descriptive modelling become clear: For all modelling processes, the simplifications made in these steps, determine the results. However, in normative modelling, the results may have a

political dimension that should be discussed. We can only critically reflect on the results, when we take the assumptions for the model into account. This suggests that the sub-processes of the modelling cycle may not be able to fully capture what happens in evaluating models arising from normative modelling (Niss, 2015, pp. 69–77). We thus aim for an enhanced modelling cycle starting with the following points: Normative modelling requires more, namely political analysis of the situation (in addition to mathematical analysis), discussion of political possibilities, and judgment formation as subsequent steps. At a minimum, the discussion of political possibilities requires that different possibilities emerge from the mathematical models or that they be considered from the very beginning. Political analysis also requires identifying interests of involved stakeholders.

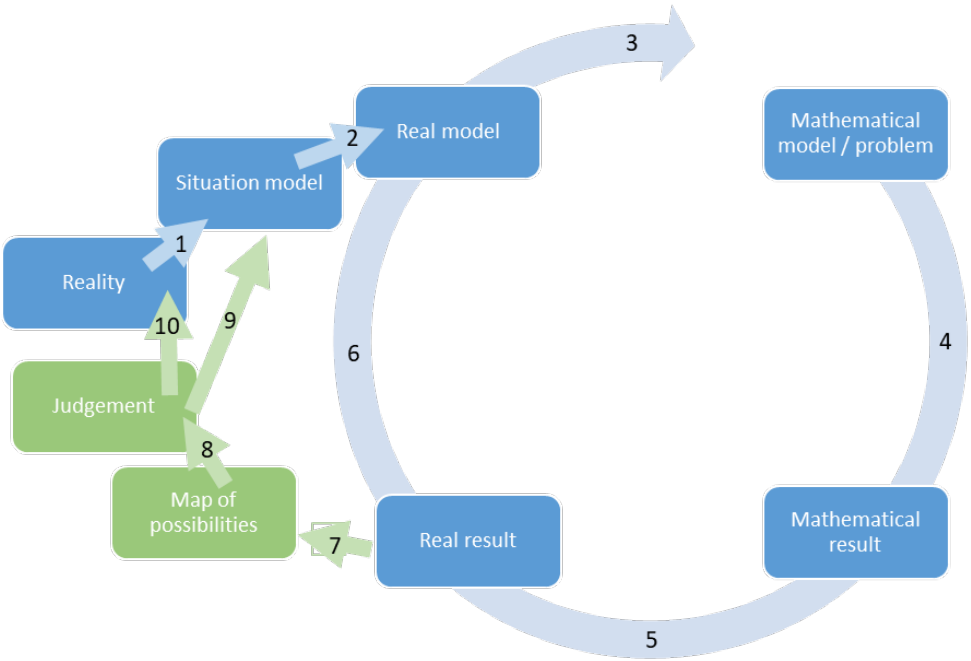

Figure 1: Our purpose of an adjusted normative modelling cycle

Developing an adjusted modelling

The modelling cycle provides indications of the steps in which alternatives and interests might be found. Understanding/Constructing (1.) at least does not involve conscious steps, but suggests that in normative modelling we may need to reconcile different conceptions of reality if we are to negotiate solutions in our societies. Simplifying (2.) is one of the most relevant steps. Which parts of the situation model are included at all and how interrelationships are simplified essentially determines the result. Here, alternatives have to be considered, their consequences for the model have to be estimated and they have to be classified with

regard to political interests. Mathematization (3.) is in itself a technical step, provided that the real model is specified precisely enough. In practice, however, the real model is specified more concretely in this step, so that simplifications similar to those in (2.) are to be expected here as well. The mathematical work (4.) will rarely provide starting points for the political discussion. Although alternative actions exist here (e.g., obtaining solutions algebraically or numerically), the differences, if any, should be irrelevant. Interpreting (5.) should also be more of a technical step because it initially involves only the translation of mathematical variables, functions, etc. into reality. However, generalizations could be made at this step, concerning e.g. model assumptions or restrictions of variable ranges, etc. Moreover, the presentation of the results will very often suggest actions, at least implicitly. Such (normative) statements can never be the result of a mathematical calculation and should therefore always be outsourced to the further steps. First, the different possibilities and the different implications related to the stakeholders' interests should be noted through the reflection and critique of the modelling just described. We named that to build a "map of possibilities". After that, everyone should form their own judgment (8.) by weighing the interests. Finally, we should acknowledge that our decision might have an impact on the world as we assume at this moment (relating to our situation model; 9.) and as it is (reality; 10.).

In relation to climate change, the normative modeling cycle could be used, for example, to teach about different ways of intervening to reduce CO2 emissions. One possibility discussed is that of the meat tax or, more generally, the question of the CO2 emission of various foods that are, for example, transported a long way or are expensive to grow or cultivate (1). Based on these individual situation models (2), a model could be set up in which different foods are each assigned a specific CO2 emission for the production process (3). For the transformation into a mathematical model, relevant categories have to be identified and defined to describe the CO2 emission of different foods. Among others, production, transport and the consequences of land use come into question here. In the simplest mathematical model, mean values per kilogram of the foodstuff are researched and summed up for these categories (4). The mathematical result is an overview of the total CO2 emission for one kilogram of the respective food. The result would show that beef on pure meat herds has a relatively high CO2 emission according to this model, as well as chocolate. In contrast, rice and tomatoes for example, have a significantly lower one (5). Scaling to 1000 kilocalories per food instead of kilograms would show a slightly different result: tomatoes would actually show higher CO2 emissions than chocolate (Ritchie & Roser, 2020; Poore & Nemecek, 2018). It could also be discussed whether the preparation and composition (e.g. sugar vs. protein) of the food should be considered in further models (6). The various results could then be the starting point for a discussion, for example, on a meat tax and the associated effects, for example, of an economic or social nature (7). Students may take up different roles and may discuss who would be using which mathematical model to build argumentations and why. Following this, the students may develop a reasoned judgment about the best possible solution to the problem from their perspective and to reflect on the associated effects on the original real model (8). The judgment and the associated decision can then serve as a starting

point for a new situation model (9) or guide the students' own political action outside the classroom (10).

Outlook

In July 2021, the first teaching materials on CO2 emissions from food as well as insect populations will be deployed and evaluated in a school context. First explorative results on student conceptions in the context of normative modelling will be collected. We hope to be able to discuss these with the scientific community, as well as our presented theoretical framework for normative modelling.

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Signs of power and dominance: Mathematics curricula in Indian boarding schools, 1879–1932

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This project examines an unexplored area of scholarship in the United States: mathematics education in Indian boarding schools, 1879–1932. The aims of this research include 1) contributing to historical analysis of the role of mathematics education in the U.S. colonial curricula and 2) increasing our understanding of the origins of contemporary mathematics education policy and practices. Theoretically, we argue that the systemic inequality and oppressive practices in mathematics education today are traced to U.S. assimilationist policies and thus rooted in the legacies of colonization and white supremacy. Using archival methods, we examine the history of mathematics instruction for Native youth in federal boarding schools. Finally, we discuss implications for contemporary mathematics education.

Description of the project

We recently launched a historiographic project that intersects mathematics education, educational history, and Native American studies. This interdisciplinary project examines an unexplored research area: mathematics education in Indian boarding schools (IBS) during 1879-1932 (assimilation era of federal Indian policies). U.S. historians have examined the assimilationist policies and practices of IBS; however, little work discusses the significance of mathematics education in these colonial institutions. Using archival methodology and interpretive approaches, our scholarship examines the history of mathematics instruction for Native youth in the federal boarding school system. We seek to develop a theoretical understanding of mathematics education as a colonial assimilatory project and its impact on the experiences of Native students attending IBS during that period. Exposing the powerful undercurrents of assimilation, colonization, and white supremacy running through the history of mathematics education in the U.S. and globally has implications for approaching future research on culturally sustaining pedagogies, specifically Native epistemologies and pedagogies focused on mathematics content.

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Research questions and significance of the project

- How did the mathematics curriculum in Indian boarding schools during 1879-1932 promote U.S federal assimilationist goals?
- How did the mathematics curriculum in Indian boarding schools institutionalize Eurocentric epistemologies and enforce U.S. citizenship and values?

In the United States, there is increased attention on Black and Latinx students in the mathematics-education literature. However, limited research exists for Native youth. As mathematics curricula "appear" unbiased, impartial, and apolitical (Bishop, 1990; d'Ambrosio, 1985), research on mathematics curricula as a colonial tool could illuminate how assimilationist discourses and practices continue for Native students and other underrepresented groups in the United States.

Whereas previous scholarship has examined other curricular areas within U.S. boarding schools, such as art education (Lentis, 2017), little research has been done on the history of mathematics curricula. For example, in the *Handbook on the History of Mathematics Education* (Karp & Schubring, 2014), none of the American chapters mention mathematics curricula in IBS. This omission in the literature is startling since mathematics was always included as a priority in the global colonial project (Bishop, 1990). In the United States, the Civilization Fund Act of 1819 provided funding to Christian missions to "civilize" Native children by "teaching [them] reading, writing, and arithmetic," and mathematics content also appears in the 1901 publication, *Course of Study for Indian Schools*, which was acclaimed for unifying the curricula across different federal Indian schools (see below). Our research begins to fill this gap in the literature while building on global Indigenous scholarship (e.g., Meany, 2020; Meyer & Aikenhead, 2021; Nutti, 2013; Parra & Trinick, 2018).

Conceptual framework: U.S. colonization, White supremacy, and mathematics

Horsman (1981) argues that the "American Anglo-Saxon" conceived of themselves as "a separate and innately superior people who were destined to bring good government, commercial prosperity, and Christianity to the American continents and the world" (p. 2). Our project is informed by scholars arguing that the dominant models of mathematics education function as racial projects within white supremacist structures (e.g., Gutiérrez, 2019; Martin, 2013). Monica Miles and her colleagues (2019) observe: "In mathematics in particular, Eurocentrism—in both epistemology and pedagogy—dominates, requiring students to conform to White ways of knowing and learning" (p. 105). The application of the IBS mathematics curriculum shows the distinct ways schooling is a form of white supremacist power and human dominance.

Utilizing TribalCrit (Brayboy, 2005) and Safety Zone Theory (Lomawaima & McCarty, 2006), we conceptualize contemporary mathematics education as the ongoing legacy of the U.S. colonial project of forced assimilation. During the federal Indian assimilation policy period and our period of study (1879–1932), the metaphoric safety zone of "allowable cultural

expression" (Lomawaima & McCarty, 2006, p. 5) had a diameter nearly at "0," in which "Indian-ness" was extracted and replaced with subservience to American citizenry. During the assimilation period, mathematics education at IBS did not include Indigenous knowledges (Holm et al., 2003) and instruction was for subservience in the manual labor market. The expectations for learning arithmetic are reflected in the following excerpt from the *Course of Study for Indian Schools*, published by the Office of Superintendent of Indian Schools in 1901:

> Let all problems be practical and so simple that the child has no difficulty in stating them before he performs the operation. Aim at only reasonable facility on the part of the child, but he must be accurate. All exercises in fractions and percent should be confined to small numbers and to subjects likely to come within the pupil's experience. Number work involving a labored [sic] process of reasoning as in "catch examples" should be discarded. (p. 42)

Policymakers and practitioners conceived of mathematics education imposed through the IBS to be innately superior as they attempted to extirpate "Indian-ness."

In the contemporary period, educational scholars have argued that liberal mainstream schooling for minoritized students functions as de facto assimilation (e.g., Paris & Alim, 2017), where the language and practices around academic learning employ English-only policy, reproduce cultural erasure, and inscribe dominant values and ideology rooted in whiteness and colonization. The de facto assimilation within contemporary schooling has not progressed far from the explicit assimilation policies enacted through the IBS. We hypothesize that both oppressive and "safe" policies and practices in contemporary mathematics education for Native students can be traced to early federal assimilationist policies. This period of mathematics education has implications also for math instruction for students of color as schooling and educational practices are also tied to this history.

History of boarding schools in the United States and their curriculum

In the late nineteenth century, the United States designed Indian boarding schools as a "civilization" project (Child, 1998). Their curricular model consisted of half a day of instruction in reading and mathematics and the remainder was manual labor. Previous research on IBS has focused on policies, sports and athleticism, and students' social and emotional experiences. The government developed a "Uniform Curriculum" in the early 1900s (Lomawaima & McCarty, 2006); however, research on the implemented mathematics curricula during 1879-1932 is non-existent. By the 1920s, teachers and school administrators developed local curricula while following federal guidelines. For example, Chilocco's curriculum differed from Haskell's because of their distinctive missions (agricultural vs. industrial training), but all integrated Christianity as part of their plan to "save" and "civilize" Native children. This project will contribute to the literature on Indian boarding schools by investigating the significance of mathematics in the assimilationist mission.

Archival methods: Analysis of IBS mathematics curricula

We have started a historiography of mathematics education in six Indian boarding schools, between 1879-1932: Carlisle Indian Industrial School, Pennsylvania (1879–1918); Chemawa Indian School, Oregon (1880–present); Chilocco Indian School, Oklahoma (1884–1980); Haskell Indian Industrial Training School, Kansas (1884–present); Phoenix Indian School, Arizona (1891–1990); and Sherman Institute, California (1892–present). This list was generated based on IBS historiography, geographical diversity, and well-documented secondary sources, which can enhance connections between our findings and previous scholarship. The U.S. Bureau of Indian Affairs operated dozens of off-reservation boarding schools between 1879–1932; we narrowed our project to six with the intent of expanding the list in the future. Specific analysis methods, examples of archival data (e.g., policy documents, photos of mathematics class, etc.), and preliminary findings will be provided in our presentation.

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Involving students' perspectives in multilingual mathematics learning spaces

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In a small on-going participatory research project, we collaborate with mathematics teachers. The project has reached a point where we, both the researchers and the teachers, have come to recognize the need to involve students in the design of their mathematics learning spaces because their knowledges and experiences may provide a crucial contribution. Therefore, we investigate in what way the students, teachers and researchers may collaborate to design mathematics learning spaces to facilitate multilingual students' mathematics learning. Involving the students in designing socially just mathematics learning spaces necessitates particular methodological decisions and identifies challenges in need of careful attention.

Focus and significance of the project

> *Elev:* *[…] jag menar vi pratar arabiska på lektionen. Vi säger så här för att vår lärare är arabisk så han låter oss skratta på arabiska mycket och vi skrattar och vi blandar arabiska och så.*
>
> *Student:* *[…] I mean we speak Arabic during the lesson. We say this because our [mathematics] teacher is Arabic, so he lets us laugh in Arabic a lot and we laugh and mix Arabic [and Swedish] and so.*
>
> *(Alhadi Alhasani & Zaki, 2021, p. 38)*

The quote above comes from an interview that two pre-service teachers conducted as part of writing their degree project. The student's mother-tongue is Arabic. She attends middle school in Sweden, where the language of instruction is Swedish. In the quote, she describes how her teacher promotes laughing in a mathematics class. That she could use her full range of language resources made us feel joy and sparked the hope for social justice in language-diverse mathematics classrooms. In an on-going small participatory research project with three in-service mathematics teachers from the south of Sweden, we have jointly decided to investigate in what way these teachers' students can be involved in the discussions on and design of their multilingual mathematics learning spaces.

Despite persevering efforts in Sweden to promote all students' first languages as a resource in mathematics learning, many students with other language backgrounds than Swedish cannot (or may not) use the full range of their language resources in school

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(Lundberg, 2019). Among other things, this influences the interaction in language-diverse mathematics classrooms (Ryan, 2019) and the multilingual students' foregrounds (Svensson, et.al., 2014). Previous studies have demonstrated in what ways teachers can design learning activities that involve the multilingual students' languages and cultures as resources for mathematics learning (Celedón-Pattichis, et.al., 2018; Planas & Setati-Phakeng, 2014), rather than viewing these languages and cultures as deficits, which often is the case (Gutiérrez, 2008; Källberg, 2018). Students are responsible, knowing, feeling, thinking and acting subjects. Students know about language diversity and mathematics learning. When taken seriously, their knowledges and experiences can make a crucial contribution to the establishment of learning spaces (Stith & Roth, 2006). This motivates us, both the researchers and the teachers, to involve the students in the design of their learning spaces. To develop the project along these lines, we aim to investigate how the students' perspectives can be incorporated in the design of learning spaces *by student participation* to enhance socially just mathematics learning opportunities for multilingual students. We recognize that external factors – for example, demands for summative assessment – position the students and teachers in ways that may limit the design of socially just mathematics learning spaces (Cabral & Baldino, 2019).

The participants in this project are the two authors, three mathematics teachers and their students. The teachers teach grade 4 and 8 (10- and 15-year-old students). Concept development and problem solving were identified by the participating teachers as areas in their teaching they wanted to develop, which, therefore, frames the project. The following two over-arching research questions guide the project:

1. How can the students, teachers and researchers collaborate to design mathematics learning spaces to facilitate multilingual students' opportunities to work with mathematical concepts and problem solving?
2. How do the participating students and teachers experience working together to design mathematics learning spaces to facilitate the students' work with mathematical concepts and problem solving?

Theoretical background

In the context of multilingualism, problem solving in mathematics class presents multidimensional challenges to students and teachers. For example, usually students need to read a text that mediates the problem to be solved. In other words, students usually approach problem solving as a matter of dealing with text-rich tasks. Word problems pose lexical and discursive challenges because of the use of concepts that range from subject-specific to everyday concepts. In addition, word problems in mathematics represent a very particular genre which students need to be familiar with to understand the problem posed. To add to this complexity, many mathematical problems are situated in the so-called everyday experiences and events, which suggests a cultural dimension to problem solving (Barwell, 2009). Language and culture are issues of, for example, communication, identity and epistemology because language and culture are matters of knowing and of knowledge (Ryan, 2019). Hence, to solve mathematical problems, multilingual students need to (re)produce

ways of knowing mathematics and mathematical concepts, and mobilize lexical, discursive and cultural resources across all their languages and cultures to do so. The notion of language-as-a-resource holds various epistemological potentials for multilingual mathematics activities as suggested in Ryan, Källberg and Boistrup's (2021) epistemological framework for multilingual mathematics activities. For example, the 'lever' potential when actualised, can move students from informal mathematics talk in their first language to formal mathematics talk in the language of instruction. Hence, this potential functions as a *lever.* The 'one new whole' potential draws on the theoretical construct of translanguaging and, therefore, constitutes prerequisites to produce new ways of languaging and knowing mathematics (Ryan, et al., 2021). Our assumption is that various epistemological potentials make different kinds of learning spaces available which influence students' and teachers' agencies and identities when they engage in multilingual mathematics activities.

Methodology

With the aim to develop innovative ways of facilitating learning spaces for multilingual students' work with mathematical concepts and problem solving, the project builds on a close cooperation between the teachers, their students and the researchers. We draw on ethno-mathematical knowledge to envision our cooperative meetings as 'barters', a way of perceiving

> elements that other participants do not produce. Sometimes these elements are either exchanged or given as a gift […] Every person involved in the barter returns home with something new gained in the barter. Roughly speaking, when people are engaged in a barter, tasks that only can be done with a joint effort are accomplished and the interdependence of the agents is emphasized, benefiting everyone. (Parra-Sanchez, 2017, p. 94)

To stage such 'barters', the students, teachers and researchers conduct workshops on themes that connect to the jointly identified problem solving in their multilingual classrooms. The workshops are designed to enable the participants to brainstorm new ideas and solutions in relation to the chosen theme (Alminde & Warming, 2019). In addition to the student-teacher-researcher workshops, the researchers and the teachers meet in seminars once a month. During the seminars, the researchers and the teachers discuss and reflect on the implementation of the outcomes of the barters in the classrooms.

We recognize that asymmetric power relations – for example, pertaining to authority, responsibilities and interests among students, teachers, and researchers – are embedded in our project. This necessitates being explicit about such asymmetries, making power relations visible and addressing them carefully. We look forward to learning from and discuss with the MES community how to handle asymmetric power relations ethically and responsibly in participatory research projects that involve students.

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Bridging mathematics students and the challenges of their learning disabilities

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Students without recognized levels of academic numeracy and literacy for university entrance are often diverted into a six-to-twelve month bridging or foundation programme where they must pass a mathematics course. However, a small number have struggled to learn mathematics from their earliest schooling and they again find themselves in an uncomfortably similar position. The students have known or unknown mathematical learning disabilities (MLDs) which may account for their own perceived lack of progress in mathematics, their frustrations and anxiety. An aim of the study is to find out how the mathematical learning needs of these students at a few New Zealand universities are being supported. In the early stages of this doctoral project, students with MLDs who failed at bridging mathematics share their experiences.

Introduction

Students entering degree programmes are expected to have sufficient levels of academic numeracy (and literacy) before being accepted. Most entrants arrive with Year 12 and 13 mathematics credits from the National Certificate of Educational Achievement (NCEA) as evidence of their readiness for university degree studies. Students without these levels and credits are often diverted into bridging, pre-degree or foundation programmes which may take from six months to a year to complete. Benseman and Russ (2003, p. 45) define bridging (or foundation) programmes as providing learners with "the requisite academic skills that will enable them to enrol in other tertiary programmes to which they would not otherwise have been able to gain entry." At one of the participant universities, each student on a bridging programme must pass at least one mathematics course to satisfy academic numeracy requirements. However, a small proportion of students on the programme have struggled to learn mathematics from their earliest educational experiences and often find themselves in an uncomfortably familiar situation when they must again face learning mathematics.

Some entrants have known or unknown mathematics learning disabilities (MLDs) which may account for their perceived lack of traction when learning mathematics, and the resultant frustrations and anxieties. This is exacerbated for those who fail their first semester bridging mathematics course, then find themselves once again having to take the same course in semester two. MLDs are described as "cognitive *deficits* in a student's processing of numerical information that lead to persistent and pervasive difficulties with mathematics"

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(Lewis, 2017, p. 321, original italics). However, labelling with a 'deficit' assumes that people without MLDs somehow occupy the high ground of normality while the diversity among human beings and their abilities is overlooked, and what mathematics they are actually being asked to do.

Positioning students with LDs is tenuous at best when describing what they contend with as some kind of medical condition and ignores their social identities (Lambert, 2018) and their working and learning environments. In terms of framing this study, Lambert's and Tan's (2017) disability studies in mathematics education (DSME) supports the privileging of student stories about learning and their reflections about the systemic trials they have encountered. Accordingly, the early part of this doctoral study documents some voices of students with MLDs who have shared their mathematical learning experiences, particularly those with MLDs who failed the bridging mathematics course in their first semester then went on to repeat the same course in their second semester to try to meet a challenging academic numeracy target.

Motivation for the study

The initial motivation for the study came from observing students' struggles in the years prior to the second semester repeat course being offered. Before 2015, even though students had failed the mathematics course in their first semester, they were still promoted into a more conceptually demanding second semester course to meet their academic numeracy requirements in the same calendar year. It is likely that these struggling mathematics students were given an even deeper hole within which to flounder and unsurprisingly several dropped out. In the 2011 cohort for instance, of the seventeen students who achieved D grades or less in the first semester course, and who then were invited to take the second semester course, seven recorded yet another D grade while the other ten finished with a Did Not Sit result.

With the repeat class being offered from 2015, although the same mathematical hurdles are still in place for failing students, the material is relatively recent. With there being a five-week mid-year break between the courses, these students have possibly maintained some learning momentum depending on what they understood in the first run. This understanding, however, is not always evident since several students with experiences of learning disabilities have inconsistencies in some fundamental mathematical ideas with which they were perhaps not supported through in their earlier learning. Their long histories of frustration with mathematics and the stories of their endurance provide a second motivation for this interpretative study.

Method

A small initial sample (n=7) of bridging mathematics students from a university bridging programme who failed their first semester maths course and then were asked to repeat it, were approached by a programme administrator to take part in an interview with the author. The group of student participants was spread across several cohort years from a large local

university. The bridging programme they were on typically had a year-long schedule of eight papers with at least one being mathematical. With the COVID-19 situation, three participants chose to be interviewed via Zoom while the others opted for a face-to-face setting while the city was between lockdowns. In a semi-structured interview setting, participants responded to a set of questions about their previous experiences when learning mathematics at school and their transition to university bridging mathematics. The author also took field notes during some of the conversations and two participants wrote to aid their explanations. The Zoom audio-recording function enabled transcribing, before each transcript was returned to the participant for checking. Some preliminary themes emerged from the interviews and several of these are discussed in the next section.

Preliminary findings

There is not room to include all of the preliminary themes so only a few key participant themes that recur have been discussed below. Most of the learners recounted struggles with learning mathematics from their primary schooling. Julia explained how her ADHD impacted on how she was treated by teachers.

> I was always labelled as the problem kid, and a little too difficult to deal with. And I think a lot of the time, especially in primary school, if I was disrupting, or I wasn't paying attention, they'd think that I ... I don't know, they'd just pop me off to something like they wouldn't bother with trying to teach me and actually sit me down.

Often other family members or tutors would be engaged for support, but this assistance with mathematics would sometimes backfire. Sarah's mother is a teacher and they would often work side by side after school, but Sarah's memories were far from fond.

> We always did our homework together and that's when you know we she has a very short attention. I have a very short attention span. She y ... she's very impatient. So, you know, we always ended up fighting and I always ended up crying. And it was just, yeah. I think that's probably how it started. Just associating maths words with crying and anxiety.

Another recurring theme was how just about every participant ended up in the lowest stream in secondary school. So not only was there the struggle to understand mathematical concepts, but there was what could also be described as the twin losses of face by being withdrawn from their peers and shifted into learning mathematics alongside a younger cohort.

After leaving school some went to work before entertaining the idea of returning to studies at university. They often worked in poorly paid jobs to support themselves during or between school and university. Gracie was one of the earlier work starters and describes how exhausted she would be in classes, which also perhaps contributed to her learning struggles:

> Even during school ... I started working when I was about 14. Um I was looking everywhere so I started working at Maccas ... and then in the end a lot of fast food places. And so even when I came to University I was still working. So it was pretty hard to keep up and I that's why I was like (someone would ask) did you go to that lesson? I was tired all the time ...

Drawing on fundamental mathematics remains a challenge for some. Two participants who work in hospitality separately described how the need for speed and accuracy when cashing up the register or balancing the electronic or credit card transactions at the end of their shifts made them feel inadequate beside other staff members who could be 'trusted' to complete the task. While there was no lack of either wanting to be able to manage those tasks by themselves, the amount of time they would need to deliberately check and recheck the money tallies was, after an unspecified period of time, either not offered them, or they ended up not volunteering for it altogether.

There is a lingering question about what actually happens when these learners are trying to understand mathematics. Julia describes a kind of 'brain fog' enveloping her when confronted with learning, and her exasperation with combatting this.

> So cognitively, since I already struggle to focus. I mean, obviously, with years of practice, I've become better at that but my brain fog is not great. It's just like your brain doesn't work properly. It almost feels like your brain half shuts down. You're very slow to think; things that should come quickly to you that should be basic, don't. Just legitimately feels like your brain's kind of shut down and given up and it's very frustrating because you are aware of it and you try to force yourself ...

Participants explained how they had neither the confidence nor the wherewithal to develop authority in mathematical learning, specifically the concepts presented in any mathematics courses. In earlier schooling they would often find themselves being compared to others who were adjudged to be more capable, perhaps reinforcing the negative mould that was never far away from them with regard to this subject.

Looking ahead

Two other universities are being invited into the study. Later interviews will employ focus groups with university bridging mathematics teachers and Disability Support Services staff. It is hoped that their perspectives will provide institutional data with what is being done to support these students.

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Abeng for multispecies' flourishing

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In this project/poster we present a collaborative poetic inquiry (Sameshima et al., 2017) that draws upon our overlapping sets of expertise as mathematics, science and quantitative literacy educators. We use as our prompt the metaphor of "breath in our bones" and start with the literal – how the atmosphere comes to life in our multispecies kin (Tsing et al., 2019). Shifting scales and working to bring light to the shadows that continue to be cast by plantation practices associated with the founding of the modern world economy—slavery, racism, genocide, and ecocide we attempt to signal that without multispecies' flourishing (Khan, 2020; Tran et al., 2020) the probability of widespread human flourishing is limited. We draw upon an analogy with the abeng as we present examples from our practice as teacher educators.

C de breath in dese bones

Who listens to the earth, to the species on it?
Who feels the shortness of their breath?
Whose job is it to give warning?
Most cultures have had elders or spiritualists to give the call,
to listen to the world,
to read and write the world and the word,
to blow the abeng,
to cause us to gather.
If not mathematicians, scientists, technologists and educators then who?

I think it needs to be more explicit.
It is the colonial cultures,
the 'more developed world',
the old and new Empires
that have contributed most of the carbon.
They have stolen the pathways
from "'lesser' developed" to "'more' developed."
Unjust as colonialist countries stole people, culture, resources,
they've stolen an 'easy' future,
they've stolen the breath in our bone

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by handcuffing those country's options for advancement.
All we have left are uneasy futures.

There is power in the Abeng
Gaia's call
The earth a breathing being, exhaling
inhaling...
Climate change as Earth's call...
the Abeng as humanity's call...?
Last call.

In what way might poetry inform the epistemic foundations of mathematics/science/technology?

Drawing upon the framing of ethnomathematics as mythopoetic curriculum (Khan, 2011) – a third approach in Curriculum Studies that establishes the imagination as central to curriculum work and brings together progressive and critical approaches in post-Industrialized contexts – we consciously and humbly draw upon an analogy with the *abeng*, a Ghanian word meaning an animal's 'horn.' The *abeng* is the archae[1]-texture anchoring our work. The blowing of the horn in the West Indies called slaves to the sugarcane plantations and allowed Maroon[2] armies to communicate among themselves (Cliff, 1984/1995). Today, 'New World' Africans blow the abeng symbolically as "a call to arm themselves…to stand up and defend their culture and traditions against extinction" (Abengcentral, n.d.).

As we ponder the breath in our bones, we wonder, what are changes in the composition of that atmosphere—the breath in our bones— due to rising anthropogenic carbon dioxide and human activity (e.g., logging/mining) doing to the bodies/bones of our multispecies kin? Our inquiry draws on recent scientific research into nutrient de-densification (Ebi & Loladze, 2019; Loladze et al., 2019; Zhang et al., 2020; Zhu et al., 2018) ocean acidification (Mekkes et al, 2021), changing seasonal patterns (Karrow, Khan & Fleener, 2018), biophonic 'desertification' (Krause, 2012), which we use as examples in our practice as teacher educators. Our poetic representations will be multimodal/synaesthetic, including poetic verse, visual data stories (graphs and infographics), photographs and soundscape. Some examples are included. Our work is a form of *symbiopoetics*, riffing off of Helmreich's (2009) *symbiopolitics*. In this sense, "A breath of our mouth becomes the portrait of the world" (Herder, quoted in Heidegger, 1971/1975, p. 139) and a poetic call to gather our marooned communities together. Our choice to use poetry is very much influenced by Feyerbend (2001).

Hearing the rhythm, pulse and pattern of data requires an attuning to its details – its silences and its rhythm - just as one listens to music. To many it would appear that the data

[1] Archaea are unicellular prokaryotes, obligate anaerobes, evolutionarily distinct from bacteria and other eukaryotes. The first species identified were known as extremophiles. They are important in oceans and in the microbiome of all species.

[2] Maroons were communities formed by slaves and Indigenous peoples who escaped plantations. These communities continue to exist today in the Caribbean.

representing climate change is akin to listening to Lou Reed's "Metal Machine Music" at full volume; incoherent, grating and without meaning or merit (to the casual listener the album sounds like a microphone buried in badly functioning industrial equipment occasionally being beaten with a sledgehammer). When one reads comments in news media posts about climate change one sees the raging voices of those to who the music of climate data seems to be just that (Bowen & Rodger, 2008). To a climate scientist that data is a rising orchestra of sound, an opera with more and more instruments and more and more singers joining in heading collectively towards a terrible climax, a crescendo followed by a sharp fall with a single keening, voice at the end slowly and plaintively fading away. Climate scientists don't want that opera, don't want that crescendo, don't want that outcome, do not want an opera in which everyone dies.

Quantitative literacies are important in rendering sensible (sense-able) the connections among multiple movements towards justice for example racial justice, ecological justice and economic justice. When combined with other multiliterate and multimodal practices, a focus on multispecies' flourishing and interdisciplinary practice decenters (but does not devalue) D'Ambrosio's (2010) ideas of mathematics as a means for human survival with dignity and as a means to more peaceful coexistence and extends it to our many multispecies partners on this planet and their impact on our mathematical attentions.

Figures

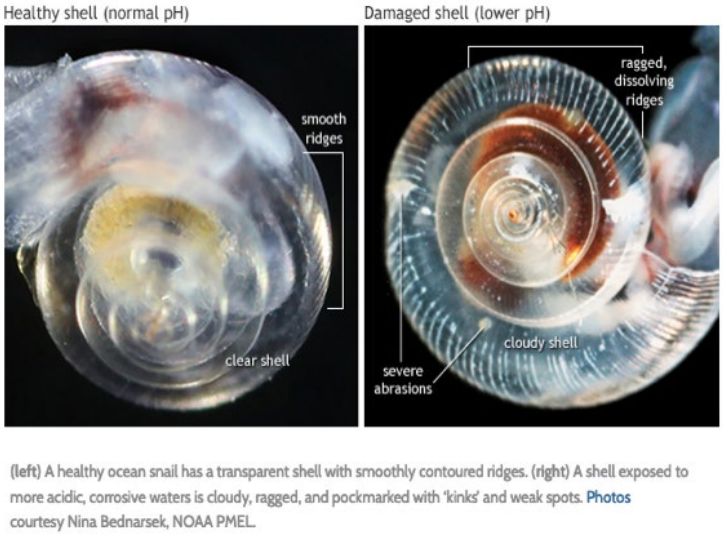

Figure 1: Scientific photograph of changes affecting shell integrity due to ocean acidification. (Image credit: National Oceanic and Atmospheric Administration)

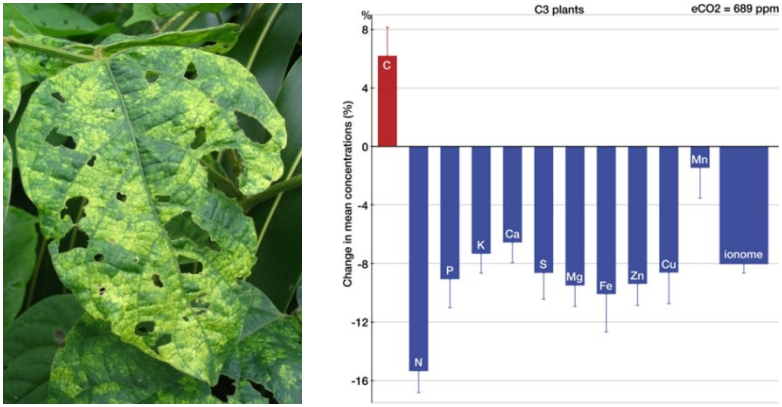

Figure 2: Representations of missing nutrients in plants under elevated CO2 levels. Left image S. Khan, Right image Irakli Lolzdze (2014)

Figure 3: Spring melt and tapping the sugar maples. (Images D. Karrow)

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Teaching middle school mathematics through global perspectives: An open online course

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Mathematical ideas are used across cultures to make sense of the surroundings, represent patterns, and predict future events. However, most students associate mathematics as a tool to manipulate numbers rather than a means to communicate complex ideas. To bridge this gap, an open online course was developed to introduce mathematics as a means to understand major social, political, and ecological issues in the world. The course is designed for middle school students (grades 6-8) and aims to build competence in statistics and data handling. The course content can be used to complement the existing mathematics curriculum and promote awareness about global issues. During the project presentation, we will showcase the open online course and the process of constructing the course.

Problem statement

We use mathematical ideas across cultures to make sense of our surroundings, represent patterns, and predict future events (Wagner, 2010). Some examples include (a) our representation of time through globally accepted 24 hour-days, or (b) our systems of evaluating the worth of goods and services using currency. Through the use of mathematics, we have often found ways to communicate our ideas and experiences with other people irrespective of geographical and linguistic boundaries (Orth, 2013). In fact, the scope of mathematics education to encourage active citizenship and help students understand global issues has been identified (Boylan & Coles, 2017; Renert, 2011).

However, contemporary school mathematics is designed to build a certain level of mathematical skill. Students are rarely exposed to the scope of applying their knowledge beyond classroom contexts. Even the application or story problems fit a specific template and students are taught to identify key words and perform relevant operations (Gerofsky, 2004). The emphasis on solving problems swiftly causes students to automate the process of calculation and desensitizes them to the context of the problem and meaning of the numbers (Wagner & Davis, 2010). As a result, most students associate mathematics as a tool to manipulate numbers (Davis, 2014) rather than a means to communicate complex ideas.

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Given this disconnect in the current system, there is a need to bridge the gap between school mathematics and the real-world. In response, we have developed an online course that introduces mathematics as a means to understand and communicate details about the major social, political, and ecological issues that our world faces today. This open online course aims to enable teachers and learners to complement the existing middle school mathematics curriculum with content that promotes awareness about global events and issues. Additionally, the project team will share the course development process which may be beneficial to other educators and researchers.

Significance

In 2015, the Sustainable Development Goals (SDGs)—a universal call to action to end poverty, protect the planet and ensure that all people enjoy peace and prosperity was adopted by the 193 member states of the United Nations. Of the 17 SDGs, SDG 4 focuses not only on providing 'quality, inclusive and equitable education' – but also an education that improves people's lives and seeks to build peaceful and sustainable societies. In order for SDG 4 to be achieved, the purpose of education needs to transform from a factory system of 'producing economically efficient beings', to 'learners empowered to question inequality, unsustainability, loss of common identity and violence'. In response to this need, the open online course is an initiative towards reframing the scope of mathematics education to promote awareness about critical global issues.

Outline of the course

The course[1] will build skills in statistics and data handling. Most curriculums (Common Core Standard, NCERT, Schola Europaea) around the world emphasize on teaching about data and data handling throughout middle school. Given that data handling is the process of data (a) collection, (b) organization, (c) analysis and (d) depiction of insights with the help of graphs or charts, the course consists of four modules targeting each step of the process. In each module, with the exception of the first module, the mathematical concepts are introduced in the context of a globally significant topic. A concise module-wise outline of the course is provided in Table 1.

Module	Context
1. Introduction	NA
2. Data Organization	Comparison across countries based on government system, levels of peace, and environmental sustainability
3. Data Analysis	Occurrence and impact of natural phenomena such as earthquakes, volcanoes, and lightning
4. Data Visualization	Water on earth and the impact of climate change on water related disasters.

Table 1: Module-wise outline of the open online course

[1] Link to the complete course: https://framerspace.com/course/BJaEOnDmI-sel-for-stem

Process of course building

Based on the mathematical focus of the module, suitable global data was chosen to highlight the key mathematical concepts. For example, module 3 focused on natural disasters, which were particularly relevant to the course due to their global ecological impacts. To discuss the concept of median, data related to deaths from volcanic explosions was used. Amidst the variety of data related to volcanic explosion, death rate could uniquely demonstrate the need for median due to the large variability within the data. Similarly, relevant data was selected from open sources for each module context (shown in Table 1).

The overall course construction process was based on the UDL planning process for instruction (Rao & Meo, 2016). Five pedagogical tools, namely, (i) storytelling; (ii) gamification; (iii) inquiry; (iv) reflection; and (v) dialogue were employed to design an engaging multisensory learning experience (Rautela & Singh, 2019). In line with the 7 design principles of UDL (Burgstahler, 2009), the following design considerations were used throughout the course to provide a conducive digital learning space.

Clear instructions and ease of Navigation

As the course was completely online, providing clear and simple instructions for students was key to sustain engagement. The course materials were colour coded to indicate the expected learner actions (as shown in Figure 1). Links to navigate through the course pages were made easily accessible through a course map.

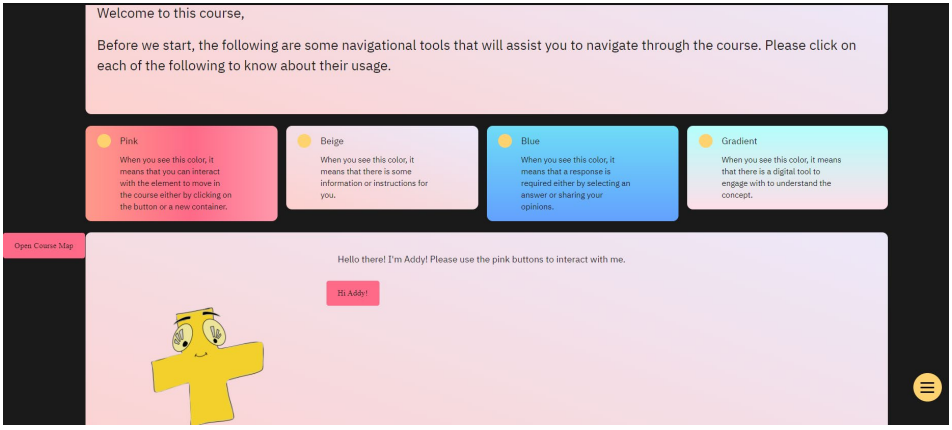

Figure 1: Example of a course page with instructions and a course map button.

Interactive and flexible delivery methods

The overall tone of the course was a dialogue with a peer rather than an instructor. The course had several mathematical characters, with the most prominent being Addy – a companion for students going through the course. The different modules within the course were designed as interactive conversations that the student could have with the various

characters, offering them the flexibility to engage with the concept. Elements of storytelling were used to introduce, explain, and explore ideas to creating an imaginative and emotional experience for the learners.

Self-paced content exploration

Following an inquiry-based approach, each lesson started with an observation or question that Addy brings up. Students were encouraged to explore the question through a variety of activities and digital tools. They were provided the opportunity to choose their learning trajectory with links to review older content, seek alternative explanations or engage with external sources for additional information.

Gamified feedback and assessments

Games were used to help students reinforce new concepts. Gamification or using elements of games within activities was to motivate progress through the learning material and provide the learners with opportunities for reflection. Based on their responses, learners received feedback and suggestions to engage with certain learning materials. The course not only tolerated error, but also used error as a learning opportunity rather than learning assessment.

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Ledor project: The role of the *ledor* (reader) of visually impaired candidates in job and universities admission exams

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Our research project intends to understand the role of the ledor (reader) of the visually impaired candidate in tests of job and universities admission exams. Such research would broaden the academy's understanding of the ledores's role in interacting with candidates during their tests, as well as helping to build mechanisms that make the participation of the visually impaired person in such processes more inclusive. This is an anti-deficiencialist perspective, which seeks to remove psychophysiological barriers from society, allowing people with disabilities to participate in selection/evaluation processes on a fair basis. The expected result from the work is the creation of an action protocol for ledores (readers), which assist visually impaired candidates in job and universities admission exams.

Introduction

Since the 2000s, there are laws that guarantee the right of people with disabilities to have priority service in several facilities in Brazil. In 2004, Decree No. 5.296/2004 regulated a previous legislation, specifying the services to which people with disabilities are entitled in various situations, among them the taking of job and universities admission examinations. Therefore, there is a look out for improvements in the task of meeting this right. One of the means that has been used involves the participation of a person, called *ledor* (reader), to read tests for the candidate who requests this service at the time of registration.

However, little is known about the interference that this actor exerts on the performance of the candidate's test. Therefore, this research will seek to understand in more depth the characteristics of this interaction, allowing a reflection on ways to improve this process and proposing actions to assist on this task.

To this end, a partnership was set between UNIFESP (Federal University of São Paulo) - Campus Diadema, UNESP (São Paulo State University) – Campus Rio Claro and VUNESP (Foundation for the Vestibular (Entrance Exam) of the São Paulo State University), a foundation that conducts job and universities admission exams, and assessments throughout Brazil, including the entrance exams for all UNESP and some UNIFESP courses, such as the medical course for example.

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The partnership

The beginning of this partnership took place in December 2017, when Renato Marcone was invited to VUNESP by Rodrigo Bortolucci to assist in a specific task: to discuss ways of adapting mathematical tests for a blind student on a specific exam.

After completing the task of thinking about mathematical tests adapted for candidates with visual impairment in general, Renato Marcone proposed that the collaborative experience did not end there, that it should become a research partnership between the two institutions they represented. The idea was well received by the board of VUNESP, as well as by the board of UNIFESP – Campus Diadema.

Theoretical basis

To build a theoretical basis to which we will focus to justify the research that we are proposing, we bring Renato Marcone's PhD research (2015), which was an effort to think about the dynamics of the relationship between the people with disabilities and the teaching and learning processes of mathematics. Marcone (2015) states that, since he started working with inclusion, he was always convinced that teaching mathematics to excluded people was something that would contribute to the autonomy of the society, and not only for the people with disability. As Paulo Freire (1996) taught, autonomy only makes sense when it is collective.

These arguments also apply to admission mechanisms, such as job and universities admission examinations, in the same way that they apply to assessment and evaluation mechanisms. They are almost invariably built by "normal" people for "normal" people. When *differences* arise seeking to participate in these selection and evaluation systems, the gap that exists between the desire to include and the ability to perform this inclusion in a satisfactory manner is evident.

Given this perspective, Marcone brought an analogy, at first, between the way the West sees the East, based on Edward Said's book, "Orientalism: the invention of the East by the West", first published in 1978, and the way "Normal" people see, often inventing, the "Abnormal", an analogy that gradually became theorizing, culminating in the creation of the concept: *deficiencialism*.

Deficiencialism would be the set of practices and networks of stereotypes that build a distorted view of people with disabilities as incapable of tasks, extrapolating the real limitation that the disability can cause in the person. For example, a blind person, who is unable to fly a plane with the technology currently available, is also stereotyped as a person who, consequently, will not be able to learn mathematics, as this is a very visual science (Marcone, 2010). The extrapolation of limitations is at the heart of what Marcone calls *deficiencialism* (for more, see Marcone (2015). The following statement by Vygotsky (our translation) reinforces what Marcone has been saying:

> The whole apparatus of human culture (of the external form of behaviour) is adapted to the person's normal psychophysiological organization. Our entire culture is calculated for the person with certain organs - hand, eye, ear - and certain brain functions. All our instruments, all the technique, all the signs and symbols are calculated for a normal type of person. And from here comes that illusion of convergence, of natural passage from natural to cultural forms, which, in fact, is not possible due to the very nature of things and which we try to reveal in their true content (Vygotsky, 2011, p. 5).

Still in his PhD dissertation, Marcone proposes an *anti-deficiencialist* attitude towards barriers arising from the psychophysiological organization of society in general. Then, this research presents itself as an *anti-deficiencialism* attitude, seeking to interfere in the job and universities admission examinations in Brazil, while proposing to build mechanisms that could overcome such barriers instead of only criticizing them.

Research Aim

This research aims to understand how the role of the *ledor* (reader) influences candidates with visual impairment and based on this understanding, to think of alternatives that can improve the relationship of the *ledor* with the candidate with visual impairment. As a secondary objective, this research intends, in the end, to propose an action protocol for *ledores* who work in tests assisting visually impaired candidates, as well as thinking about a digital solution, such as a software or a special device.

Methodology and evolution so far

Because of the new coronavirus pandemic, our research changed its course drastically. Instead of face-to-face interviews and pilot assessments with visual impaired volunteers, we are analysing audio recordings of the candidates taking their tests aided by a *ledor*. VUNESP records, in audio, all the tests where there is the aid of a *ledor*, as a standard, and it is these recordings that we are having access and analysing, to understand the interferences of the *ledor*.

All the participants of this research are 18 or more in age and have agreed to giving us access to their recordings, by signing an Informed Consent Form (ICF), created by us, which is provided by VUNESP, online, during the registration process for any admission examination, through their own website. Also, our research has been approved by an independent Ethics Committee in Brazil.

Preliminary data provided by VUNESP in 2018 showed that they received an average number of 150 requests per year for a *ledor*, including all the assessments and job and universities admission examinations they organized from 2012 to 2017. We got the permission to access all the authorised audio recordings of 2020. Our expectation was to receive, at least, 10 authorizations, to perform a relevant analysis. The average time of each audio is 7 hours, because the visually impaired students have the right of an extended time to perform their exams in Brazil. However, early January 2021, we have got the consolidated data: until October 2020, we had 321 authorizations from candidates who signed the ICF.

Then, our first decision was to classify those candidates in two groups: job admission candidates and university admission candidates and we started looking into the first group.

The authorizations referring to job admission examinations are a total of 167. Of that number, 58 registrations were not confirmed (some due to the suspension of the admission exam because of the COVID-19 pandemic restrictions, others because some lack of documents, etc.), 24 were absent and 84 showed to take their test. Among the 84, we have 72 audios available (sometimes the candidate releases the *ledor* at the moment of the test and audio is not recorded). Then, we have, potentially, 500 hours of audio recordings to analyse.

At this point, we are facing the challenge of: how to treat all this data to reach our research goal, without giving up the qualitative character of the research?

Last comments

Once we saw the huge amount of data, our first reaction was joy, because this is a clear sign that students with disability are fighting their way towards a public job or a seat in a university.

After the first feeling of rejoice, we felt very worried. We have in front of us a unique opportunity to make some change on the exclusionist processes that were consolidated for so many years in Brazil, helping to create a more inclusive environment on the job and universities admission examinations.

Hence, we are at a place where we are seeking how to use those data in the wisest way, and for that, we count on the MES colleagues' support to design our next step.

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Possibilities for mathematics education in a university-school partnership

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This ongoing project is about initial and continuing teacher education of science and mathematics, more precisely at the university-school interface. In this perspective, the relationship with the school which participates in the project, as well as the development of research, emerge from the processes of action research. In this text, we present the actions we have developed about mathematics education with all the participants of the project. Teachers and educators problematized ideas from an investigative activity on numbers, based on landscapes of investigation (Skovsmose). From this, the participants start to criticize the didactic textbook used in the school, as mandatory, contributing to their critical development teaching professional. The way the participants have faced teaching during COVID-19 pandemic is also discussed.

The research project

The present project is located in the field of research on initial and continuing education of science and mathematics teachers, more precisely at the university and school interface. Both institutions are interrelated in search of improving the quality of teaching and, for consequently, the professional development of teachers, teacher educators and managers involved, and the quality of learning of undergraduate students in science and mathematics, and students from the participating school. The project is financed by a Brazilian development agency (Grant #2018/16585-1, Sao Paulo Research Foundation – FAPESP).

The relationship with the school, as well as the development of research that are part of the project, emerged as a movement, an action research process (Thiollent, 2004), in which the school, the teachers and the management group appeared simultaneously as subjects and objects of investigation. Researchers from two Brazilian public universities, management participants and teachers from a public school in the State of Sao Paulo meet periodically in forums constituted at the school and at universities. In this process, the motives of the various participants, as well as the objectives of the group's collective work, have been refined and articulated in the direction of building the partnership developed in this project.

More specifically, we seek to discuss possible answers to the following questions: "Which changes in teaching, in school management and in the teacher education of prospective

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teachers and school teachers are being leveraged by the work developed in the collaborative partnership? What elements enhance such changes?".

The current group of researchers is made up of a very heterogeneous group, whether in their graduation and PhD (pedagogy, philosophy, specialists in teaching and training teachers in the areas of science and mathematics, and specialist in educational public policies), or in their professional experiences and school levels practices.

According to Foerste (2004), the partnerships between universities and schools can assume different objectives, levels of involvement and integration of participants. In this sense, Jones *et al* (2016) state that the types of partnerships are defined by three different levels of integration of the subjects: *connective partnerships* are those in which exchange relationships are established and each of the involved parties offers something to exchange with the partners; *generative partnerships* occur when subjects are able to help each other, through the development of collective projects, which leads to changes in the structure of existing activities; and *transformative partnerships* are those in which there is active and collaborative involvement of all participants in a reflexive-critical practice, guided by the articulation between theory and practice, in order to develop all participating members. It is through this last partnership that the project seeks to develop.

In this multiplicity, the relationship between university and elementary school is expressed in a heterogeneous and diversified way, generating networks in which acceptance/denial, rupture/continuity/discontinuity are threads whose intertwining produces interesting processes. Such processes "give meaning to partnership movements, teacher professional training and the insertion of research as a constituent of training, in which we have constituted ourselves as actors in the process" (Almeida et al., 2000, p. 53). In this sense, the authors point out that recognizing the various roles that the university and the school assume, modifying or producing them, is fundamental to identify the borders, the overlaps and the specific areas of work. According to the authors, the outcomes of these partnerships provide a "meaningful panorama of the teacher education, considering the teachers as critical producers of their pedagogical work" (Almeida et al., 2000, p. 53).

Based on Fiorentini (2009), Nacarato (2016) and Oliveira (2015), we established discussions regarding partnerships in mathematics education. The authors point out that the partnerships between the university and the school enable conditions for teachers and prospective teachers to reflect on trends in mathematics education, as well as to discuss the mathematics curriculum at school, in order to face the difficulties that often are raised in the teaching practice.

With such a perspective, we hope to bring contributions both to the improvement of the teaching at the partner school, as well as the learning of teaching, the professional teacher development and the respective areas of research in these fields. Thus, we understand the relationship between learning of teaching and professional development as processes that emerge from the needs created in their practice and from the internship activity. In this way, we hope that the actions shared between prospective teachers, school teachers and university professors can contribute to the dialogue between different types of knowledge.

In this present text, the focus is to present and the discus one of the training actions developed in the project in order to contribute to the two research questions, related to the changes in the mathematics teaching and in the teacher education of the participants of the project, and to the elements developed in the collaborative partnership that leverage such changes.

Teacher education meetings on mathematics education

In 2019, we held action research meetings of the group. In monthly meetings, at the universities the researchers met for reflect on relevant topics related to the project's issues. After some of them, on inquiry-based science teaching, there were two meetings on mathematics education in the early years of elementary school. The theme was chosen and organized by the group. Critical Mathematics Education (Skovsmose, 2011) was the theoretical perspective chosen by the universities researchers to base the meeting. The school teachers intended to discuss students' difficulties in learning mathematics and to know different ways of teaching. The teacher educators' intentions were to understand how school teachers used to teach mathematics, and to learn from them about students' difficulties.

In order to achieve these aims, previously, for the first meeting, we requested that participants to read the text "Landscapes of Investigation", by Ole Skovsmose (2000). We have chosen that text for three reasons: 1 – the text is based on critical mathematics education and by adopting this theoretical framework to reflect on the mathematics teaching is a possibility to question what is given as true; 2 – the proposal presented in this text (the landscapes of investigation) departs from a well known context to the school teacher and close to their practice (the paradigm of exercise); 3 – the author presents the mathematics as a tool to interpret situations and make decisions, and argues that mathematical discoveries made by the students can enable different qualities of learning. We expected these ideas raise in the study meetings.

Thus, at that meeting, an investigative activity on numbers, proposed in this text, was developed. An investigative activity aims to enable students to make discoveries, to have an active participation in solving the activity and to be responsible for their learning process. Such discoveries do not concern genuine knowledge of mathematics, but rather new knowledge for those who carry out the action. Dialogue is the form of communication that predominates in this type of activity.

The work with the landscape of investigation was developed in small groups and encouraged participants to explore a table to discover some regularities and numerical patterns. It was possible to notice that the participants shared their findings, doubts and hypotheses with their colleagues. Each idea raised in the small groups was presented and discussed with everyone. Views were shared. The action of understanding what the other was saying was present, and this is an essential element both for dialogue in the learning process and for the development of teacher education.

Some investigative questions were raised regarding the patterns and regularities of the referred numeric table. Surprise, restlessness and suspicious smiles were expressed by the participants when carrying out the activity. The invitation to investigate had been accepted by the group. The curiosity to try to find out why there were certain patterns persisted throughout the meeting. We realized that the discussion about generalizations, so important in the algebra of the early years, was put into action, because, during the activity, legitimate mathematical knowledge was raised for that group and essential for the development of the proposed task.

We reflected on the similarity between the activity carried out and those developed in previous meetings on inquiry-based science teaching. Raising hypotheses, evaluating them, arguing, making discoveries, dialoguing with colleagues are some of the common actions that we perceived among the proposals discussed for the teaching of mathematics and science. These actions are also important elements which enables changes in the mathematics teaching in the early years.

The second meeting on mathematics education began resuming the investigative activity carried out at the previous meeting. We emphasize the importance of teachers provide moments to the school students make discoveries in mathematics classes, playing an active role in the activity developed. Important aspects of Skovsmose's text (2000) were raised and discussed by the participants. We noticed that there is a huge difference between investigative classes, such as the activity we developed, and those based on the paradigm of exercise, in which the teacher explains the content, shows some examples and the students reproduce, in a list of exercises, what they heard from the teacher.

Because of the investigative activity we run in the previous meeting involved generalizations, we also discussed about Algebra in the early years. The search for patterns in numerical and graphical sequences and in tables, such as the one we explored in that meeting, is an aspect of algebra that can be worked on in the early years.

The second part of this meeting was intended for the presentation, by the school teachers, of the didactic material "Mathematics Education for the Early Years" (EMAI), used in the state schools of Sao Paulo, pointing out what the material is and how it is used. It should be noted that the teachers had the opportunity to dedicate themselves, in a group, to read and discuss the material. In their presentation, the teachers verbalized that this reading allowed them to demystify how to use it. They realized that the material is proposed as a support and not as a manual to be used daily and in a tied way, as it was presented to them in many moments of continuing education, provided by the management team of state schools. In this way, the teachers started to characterize it as a collaborative project of continuing education among the teachers of each school. Outbursts, complaints and solutions were verbalized.

Subsequently, we analyzed the summary of the material to see how it was organized. We identified that the language of the EMAI is very imperative ("do", "distribute", "ask"), which restricts the possibilities for the teacher to decide how best to develop the activity proposed

in the material. We also discussed the pressure that the author of these types of text experiences when producing didactic material.

The group started to question the possibilities of making the use of the didactic material more flexible. In order to connect the discussion on landscapes of investigation and the didactic material, one of the professors have raised the following question: Is it possible to create "landscapes of investigation" (Skovsmose, 2000) from the guidelines prescribed by the material? A deeply awareness and knowledge about the didatic material was one of the conditions considered by the group to be essential in order to proceed with the search for an answer to this question. The meeting ended at this time and with such a proposal. Considering the discussions played at these meeting we understand we were moving towards promoting changes both in the teaching of mathematics and in the teacher education.

The discussions and reflections, made by the school teachers about their teaching and didactic material, were important to their education, and also to highlight elements for thinking about and developing university-school partnership actions. For example, we realize that, in the beginning of the research project, the school teachers understood collaboration in an objective way to help them in some difficulty, and not as the possibility of a joint work that would contribute to thinking together about the development of a certain subject or a different approach to discuss with students. Thus, the discussions and verbalizations of the teachers show us how the partnership has potentialized the school's activities in relation to changes in the teaching.

The discussions were interrupted due to the end of the year and the advent of the COVID-19 pandemic. With regard to the specific issues of mathematics education, the year 2020 was intended for the collaboration between school teachers and educators in organizing the remote classes held with the few students who had technical, psychological and family conditions to follow on-line teaching. The classes in the State of Sao Paulo were held and made available by a media center, which did not count on the participation of the teachers themselves in its production. In this way, what was revealed was a loss of their didactic choices and, consequently, a challenge for the continuity of our project.

Thus, the two mathematics education meetings raised important questions regarding the professional development of the teachers. Throughout the year 2020, such meetings reverberated in their speech in the project remote meetings that followed and in the interviews provided by them. We intend, as one of the next steps of the project with regard to mathematics education, to return to the records related to these meetings (audios, interviews and written conversations on whatsApp), looking for elements that contribute to the potentiality of the collaborative partnership.

Finally, we understand that the project plays a political role, by offering, through the interaction between the participants, subsidies so that answers and solutions can be found in order to promote transformative actions in the professional development of all those involved.

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Borders, gender, and performative contradictions in active learning

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We present some ongoing (under review) work to conceptualize gender in mathematics education using a radically different approach: though an intersection of Nail's border theory, performative new materialism, and elements of Hegelian philosophy (as set forth by Marx and Žižek). Through this approach, gender is conceptualized as a border between the masculine and feminine, and for some, this is a border to be crossed. In response to researchers' call to elucidate the meaning of gender (e.g., Damarin & Erchick, 2010), the approach discussed here claims that much of gender's influence is lost on researchers due to overlooking the reflex-category nature of the masculine and feminine in performance. Metaphors of immigration/emigration, power differentials, and performativity are discussed apropos of participation in mathematics.

A gender problem in mathematics education and active learning

Active learning continues to spread its grasp on education fields as a "panacea" for mathematics and STEM education, with more and more literature being published that posit its beneficial universality for students. For instance, Freeman et al. (2014) found that the active learning environment (ALE) leads to "increases in achievement […] across all of the STEM disciplines and [these increases] occur in all class sizes, course types, and course levels" (p. 8412). In another study, Theobald et al. (2020) found that ALE is "disproportionately beneficial […] for underrepresented minority students" (p. 6478). Similarly, Laursen et al. (2014) found that ALE "levels the playing field" for men and women in terms of achievement. These large, widely-published studies clearly herald the trend in mathematics education towards seeing ALE as beneficial for all students, and even perhaps more beneficial for those who may need it the most.

However, the last decade has concurrently seen a confusing emergence of literature claiming that ALE impacts different groups of students in different ways, not always resulting in content gains or increased achievement. For example, Johnson et al. (2020) found that men outperformed women in a type of ALE called Inquiry-Oriented Instruction. In their study, ALE had no negative effect on the women, but rather it had no effect at all. Meanwhile, the men experienced content gains as a result of the ALE intervention. Compared to national representative samples that show no gender difference in student performance (with,

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presumably, data obtained from a traditional instructional environment), the results of Johnson and colleagues' study lead to a troubling contradiction. Their work mirrors the findings of Bando and colleagues' (2019) large-scale study, which found that a gender performance gap was exacerbated by ALE interventions, and, alarmingly, *increased* the longer that the intervention was being employed.

Gender-as-border: Towards a new theory of performative identity

These contradictory results have led an increasing number of scholars, including us, to ask "why," and to question the claim that ALE is indeed a universal panacea. While it is possible that the studies were not conducted in the same way, their overall claims apropos of gender are universal ones, and thus we approach our work from a position of troubling that universality. This paper reports on a larger, in-process theoretical development that attempts to re-examine these contradictions through a theoretical intersection of borders, gender performativity, and Hegel-Marx reflex categories.

Borders

Thomas Nail's (2016) border theory has emerged recently as a promising new contribution to the field of philosophy. Nail's work centres on motion and movement, and as such, his work on border theory is also highly dynamic. Borders, as mechanisms of social regulation, divide spaces and create their own unique ontological presence (cf. an ontological lack) as the divider itself. He describes two types of borders: extensive and intensive. *Extensive* borders create an "absolute break—producing… discontinuous entities" (Nail, 2016, p. 3); *intensive* borders create "new path[s] […] qualitative[ly] chang[ing] […] the whole continuous system" (p. 3). Consider that the masculine and feminine are historically constructed as an extensive border system. They are seen as quantitatively different, exemplified by the typical options on standard forms to check a box next to "male" or "female," as if they are mutually exclusive categories. We argue that, because of this extensive historical construction, "gender" leads to contradictions of many types—e.g., the current conflicts over transgender ontology—that could potentially be addressed via an intensive conception. Of particular interest here is the implications this impasse creates apropos of participation in mathematics classrooms.

Performance

Within the new materialist tradition can be found several variants, one of which is performative (Gamble et al., 2019). In this paradigm, the material real is a stage on which performances are made by actors: as such, ontology and epistemology are "inherently co-implicated and mutually constituting [of each other]" (p. 122), becoming one understanding of an *ontoepistemology* that, further, "problematize[s] anthropocentric binaries" (p. 111) such as gender. A key consequence of this perspective is that "humans can therefore never observe the universe as though from outside it […] [and thus, being bound by the] material configuration of the world […] [necessarily] leads to a thoroughly 'performative' and relational

materialism" (p. 123). This perspective allows gender to be seen as a performance of students as actors engaged in the performative role of producing themselves and their identities through the labour of their performance—a performance which, crucially, is political (cf. Butler, 1990; Moore, 2020). The politics of one's performance of gender is realized in the material real through the dynamics of Hegelian and Marxian power relations.

Hegel/Marx

Hegel (1807/1977), in one section of the *Phenomenology*, develops the notion of power dynamics between two self-consciousnesses in relation to each other by using the metaphor of lord and bondsman; the former is self-consciousness for itself and the latter is self-consciousness for another. The bondsman, in performing the lord's labour on the world of things (viz. the material world) reverses his subordinate position to become fully self-aware *through his labour*. The lord merely takes the enjoyment of the bondsman's labour *qua* the products of the world of things without becoming self-aware of his position as lord *vis-à-vis* recognition of the bondsman *as such*. The bondsman becomes ironically and symbolically free *vis-à-vis* the self-awareness he acquires in the act of labouring—labour imposed on him by the lord—that forces him to recognize the lord *as such*. Thus, Hegel shows that in being-for-another (viz. the bondsman), one transcends into a "more fully being-for-self" role. Hegel called this reflexive identity a reflex-category: the bondsman would not be a bondsman without the lord, and the lord would not be a lord without the bondsman, however neither is what the other sees himself as. This was later further developed by Marx: "[O]ne man is king only because other men stand in the relation of subjects to him. They, on the contrary, imagine that they are subjects because he is king" (Marx, 1867/1887, Footnote 22, p. 55, as cited in Žižek, 1989/2008, p. 20). Marx's symmetry here is homologous to Hegel's argument dialectically.

Consequences in the mathematics classroom

Combining all the above theory allows us to re-envision gender as a performative dimension in the classroom, a dimension which, crucially, conflicts with ALE. The Hegel-Marx axis gives us the power dynamics of masculine (lord) and feminine (bondsman), as co-constituted and reflexively determined, the source of our current gender impasse. Evidence of the extensive gender border, as the historical determination of gender's construction that we observe in modernity, can be found in such phenomena as women's suffrage, the gender pay gap, maternity leave and benefits, and the disproportionate ratio of women to men in professional roles. Nail's border theory describes how the division between them functions and describes the ontoepistemological "stage" over which students must perform their gender in the material real of the classroom. In both the active learning and lecture environments, the teacher takes the masculine position, and the students assume the feminine: "The teacher *knows for* the students, and the students—relegated to performing the teacher's labor—merely *work on it*" (Moore & Johnson, under review). However, in the ALE case, a split occurs: the teacher transfers some of her power onto the students in the form of

ALE tasks and expectations. When this occurs, the men and boys take the assertive voice, which is championed in ALE, whereas the women and girls are emburdened with the task of "emigrating"—leaving their feminine home and enacting a gender performance across the border to the masculine side—if they want to be successful. Crucially, the boys in lecture do not need to reciprocate this. Our goal with this project presentation is to discuss our theoretical development and its implications.

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Unpacking field trips: The role of a teacher educator in post-field mathematics teacher education courses

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This paper reports on an ongoing research project to study how mathematics teacher educators 'unpack' the field experiences of prospective teachers. By viewing post-field practices through the lens of disruptive pedagogies, we aim to better understand the roles of mathematics teacher educators and to reconceptualise post-field possibilities in teacher education.

Introduction

Research in the area of teacher education theory-practice transitions has been extensive (Gainsburg, 2012), including transitions from university (theory) to field experience (practice), as well as transitions from the process of *becoming* a teacher (university) to the first few years of *being* a teacher in schools. Another key transition in teacher education programs is the under-researched transition from field experience back to university. As noted by Eriksen and Bjerke (2019), "little is known about the way in which teacher educators integrate prospective teachers' actual experiences when they return to university after fieldwork" (p. 9).

The 'unpacking' of field-back-to-university transitions is relevant to the community of teacher educators since teacher education programs, and corresponding field experiences, are frequently critiqued for being steeped in technical-rational approaches (Nolan & Tupper, 2020). Mathematics teacher educators (MTEs) struggle with the tensions implicit in these transitions, as they seek to disrupt dominant 'technique-oriented' discourses of school mathematics and becoming a teacher.

Research theory and design

We first acknowledge the difficult, but necessary, task of moving away from using the language of theory and practice to describe the transitions between university teacher education courses and school-based field experiences. To counter this false binary and hierarchy, where expertise is seen to rest primarily with academics, Zeichner (2010) proposes teacher education hybrid or third spaces that "bring practitioner and academic knowledge

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together in less hierarchical ways to create new learning opportunities for prospective teachers" (p. 92). Similarly, Rust (2019) calls for teacher educators and teacher education programs to "be situated at the nexus between universities and schools—the place where theory and practice can come together" (p. 524).

In our study, we propose a hybrid space of research where our focus is on disrupting and reimagining knowledge constructed in the movement from university to field and back to university. Within this movement, it is the post-field context that we focus our attention. By viewing MTEs' post-field practices through the lens of *disruptive pedagogies*, we aim to better understand the roles of MTEs and to reconceptualise post-field possibilities in teacher education.

We draw on Anderson and Justice (2015) in describing a pedagogy as disruptive if it "requires students to challenge or change their epistemologies and participation in their learning" (p. 400). As Schulz (2005) reminds us, "[i]f teacher educators want to change prevailing practices … they must provide frameworks that encourage different ways of thinking about teaching and learning about teaching" (pp. 149-150). This applies to both pre- and post-periods of field experience, and hence, it underlines the importance of drawing on prospective teachers' (PTs') field experiences in post-field university courses, where different theoretical and pedagogical tools have the potential to better understand and unpack the field.

In the research design, we review literature on university to field transitions in mathematics teacher education to construct a list of the barriers/challenges in transitions as identified across the research. We are specifically interested in knowing whether the challenges in university-to-field transitions also carry weight in field-back-to-university transitions and how/if MTEs address them in post-field courses. From this list of barriers/challenges, we construct several questions to ask MTEs to understand their practices as post-field course instructors. All of these questions emerge from the central question of this research study: *What are mathematics teacher educators' roles in unpacking field experiences?*

With the questions constructed, the research study's data collection is divided into two parts. Part 1, the primary focus of this paper, includes conversations between the two authors—a dialogue made possible through our own self-study reflections on the questions. In part 2, which moves beyond the content of this paper, we use the questions to interview 20 MTEs from teacher education programs across Canada and Norway to gain broader perspectives on the practices of MTEs in disrupting the field-back-to-university transitions through post-field courses.

Barriers/challenges in theory-practice transitions: Review of literature

Given the self-study context of Part 1 of this study, here we focus our brief review of research in the area of theory-practice transitions primarily on our own findings; the two authors (Bjerke & Nolan) have written extensively on the barriers/challenges encountered in theory-practice transitions, revealing the following (abbreviated) list:

PTs as visitors: The visitor 'stamp' prevents PTs from trying out new ideas (Nolan, 2012), focusing on unquestioning alignment with existing norms and plans, deferring to the mentor teachers' accountability for their pupils' progress (Solomon et al., 2017).

The different roles of the involved parties: A lack of understanding of the roles of cooperating/mentor teacher, PT, and university supervisor (Nolan, 2015).

The theory–practice divide: A reported disconnect between university and school methods/theories, often resulting in PTs favouring school placement (Eriksen & Bjerke, 2019) and expressing a need to be armed with a 'toolbox' in order to be aligned more closely with the school and performing the role of a teacher (Solomon et al., 2017).

The demands of reform teaching: Reform, or inquiry, approaches not taken up by PTs during field experience, for several reasons: Inadequate modelling by MTEs; lack of 'recipes' for implementing inquiry; inquiry-based lessons reported as taking too much time to plan and implement; PTs' lack of conviction (Eriksen & Bjerke, 2019; Nolan, 2012, 2015).

Questions for MTEs about the field-back-to-university transitions

Based on the barriers/challenges outlined above, and with the lens of disruptive pedagogy informing our interest in unpacking the post-field context, we have constructed 8 conversation/interview questions. Given space restrictions, we present only 4 of these questions here as illustrations: (1) As a MTE and course instructor, what are the most significant challenges you experience in your work with PTs upon their return from a field experience? How do the challenges relate to the list of theory-practice barriers/challenges above? (2) What pedagogical strategies do you draw on in your post-field courses that you think might (a) intentionally or unintentionally, further re-affirm a university-school divide between theory and practice, and (b) challenge and/or disrupt the division between university/theory and field/practice classrooms, and instead portray them as being more in relationship with each other? (3) What theoretical tools do you draw on in your post-field courses to 'unpack' the field? How and to what end do you draw on these tools to understand, disrupt and/or support PTs' thinking and growth? Describe successes and failures in these efforts. (4) What do you view as your primary role(s) as a MTE in the post-field context?

Part 1: Dialogue between Authors

As an illustration of the research process, we present the following snapshot of the authors' dialogue around one of these questions (#3 above):

Kathy: I have drawn on Bourdieu's social field theory in my post-field courses, through a basic introduction of the concepts of habitus, field and cultural capital to PTs. Introducing PTs to these concepts in the context of discussing unchanging pedagogical practices in schools was meant to illustrate how a person feels comfortable in a field where their habitus is a good fit with the logic and operation of that field. I had hoped that these discussions, drawing on Bourdieu's concepts, would aid in disrupting technical-rationality in teacher education by building PTs critical capacities for thinking *with* and *through* theory.

Annette: You 'had hoped'. Does this mean that it did not happen? My latest effort has been to introduce Biesta's virtue-based approach to education, and hence to

mathematics teaching, discussing the PTs' experiences in relation to qualification, subjectification and socialisation. This has worked as a way to address both themselves as PTs, and as a way of talking about their experiences with different pupils.

Future directions and concluding thoughts

MTEs are called upon to make deliberate pedagogical choices toward "the disruption of practices which contribute to the reproduction of educational inequalities" (Beighton, 2017, p. 113). As this research focuses on disrupting and reimagining knowledge constructed in the movement from university to field and back to university, it is important ongoing work both for those teacher educators involved in our study (as a reflective self-study exercise) and for those reading about and relating to what we report. This brief introduction to our in-progress study highlights our approach to viewing MTEs' post-field practices through the lens of disruptive pedagogies.

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Mathematics education in a context of climate change

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The overall goal of the research project presented here is to investigate and produce knowledge about how students and teachers are learning mathematics when presented with mathematics in the context of climate change. In order to address this question, some ethical issues will be taken into account. First, there is a brief overview of different ethical issues concerning the questions, "Is there is an ethical responsibility to bring the concept of climate change into the mathematical learning situation?". Secondly, I will present the school-based-research. A number of lessons will be created, and an overview of the content of those will be presented here. They will contain, ethical considerations but also mathematical models and different perspectives on climate change, for the students to discuss and engage in.

Ethical responsibility to bring climate change into the mathematical classroom

For this research to be fruitful, some ethical issues will be taken into account. Why should climate change be part of the mathematical curriculum is one of those. Is there even a moral obligation to address these issues, and if the answer is yes, how would this affect the teaching and learning? Abtahi et al. (2017) address the question "how incorporating issues of climate change into the teaching and learning of mathematics can be understood as a moral and ethical act.". They conclude that although including climate change in mathematics classrooms can be viewed as an ethical or moral responsibility of mathematics teachers, in their day-today practice their decisions about issues are complex. These ethical challenges relate to, for example, the degree of involvement and interest of students in the issue of climate change, the possible discomfort of students, the uncertainty of how to respond, the unclear path of any possible contributions of their actions to the wider society, and finally a more general sense of dealing with the unknown. Teachers face practical obstacles in incorporating issues related to climate change in their mathematics classrooms, such as "lack of resources, lack of sources of data related to their immediate community, lack of curriculum mandates, and lack of time" (p. 11).

Karrow, Khan & Fleener (2017) also discuss mathematics education's ethical relation with the climate change. They argue that mathematics education is skewed towards economic interests instead of ethical relations. According to them mathematics education must move away from preoccupation with economics to one found by virtue ethics. In reviewing a

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number of papers, they conclude that the current mathematics education is formed through the prism of economy, instead they suggest a mathematics education that that concerns the development of the individual in relation with our Planets Ecosystem. To do this they suggest moving mathematics more into the realm of nature. For instance, recursive mathematics which can describe the reproduction of rabbits or formation of flower leaves. Fractals, complex systems and uncertainty are other areas that can be found in this mathematics of nature. This suggests a new epistemology, were learning is not an acquisition and accumulation of knowledge but rather a process of engagement with an understanding that all knowledge is uncertain and situated. This is also in line with UNESCOs slogan "changing minds not the climate" UNESCO (n.d.).

Perspectives on climate change

The teaching and learning in a context of climate change also involves a personal viewpoint towards the issue of climate change which is elaborated in several papers. How do we view ourselves in relation to the world, climate change and the dystopian future that is presented to us? Drawing on the notion of the *Anthropocene* which signals the shift from solving a problem to learning to live with a problem, Coles (2017) suggests the concept of *habit* that allow us to conceptualize ourselves in the world around us. Moving away from a savior narrative, *habit* will help people to connect individual and global perspectives. This issue is also discussed by Latour (2018) where he suggests a perspective of the *critical zone* to deal with the climate change. He argues that the scientific notion of us being ON the blue planet, leads us to take the role of savior instead we must face the reality of the critical zone in which we all are in some sense part of and trapped in. (Mikulan & Sinclair, 2017) draws on the notion of the *Anthropocene* claiming that we need to think in a more non-human way were the human is not the center of everything. They also show in their paper why mathematics is well suited for this endeavor. The above-mentioned examples are all ethical considerations for a teacher going into the mathematical learning situation, they all based on different ethical assumptions, and these will be investigated and analyzed in the research.

School-based-research in the mathematical learning situation

In my coming school-based-research, I will create several lessons that will touch on the different aspects of climate change. In the classroom there will be discussions about ethical considerations concerning climate change. Different ethical standpoints and perspectives will be presented to the students, illustrating the complexity of the problem. Articles dealing with the ethical implications concerning climate change are multiple in the research community. Gardiner (2011) argues climate change plays a fundamental role due to decision making, which effect animal and future generations. They also point out some reasons for why it is hard to be ethically sound. Local emissions have a global effect. A decision in one place may have implications in a totally different place in the world. Raymond (2004) argues for an ethics of commons. The atmosphere is a global common good and he proposes that

the emissions should be allocated between nations. It could be done by using the principle of equal burden. Meaning that nations should reduce their emissions based on the burden of this reduction. Another approach based on equal human rights would be to allow an emission level per capita. Shue (1999) concludes that "whatever needs to be done by wealthy industrialized states or by poor non-industrialized states about global environmental problems, the costs should initially be borne by the wealthy industrialized states" (p.111). He then describes the reasoning behind this conclusion and how proportional and progressive burden can be explained. All these suggestions, mentioned above, are based on different ethical assumptions and are examples of what can be presented and discussed in the classroom. They also deal with mathematical concepts, for instance proportionality and progression, that can be used as examples in the mathematical classroom.

When we say *climate change* what type of perspective and understanding are we referring to? Are there other models and perspectives that can be used to understand and explain the climate crisis and the world we live in? Huneman & Lemoine (2014) explore interesting distinctions of modeling. "Modeling can be seen in terms of representing a target system. Other epistemic functions, such as producing data or detecting phenomena, are at least as relevant" (p. 3). Other useful distinctions can be made, for instance between phenomenological and mechanistic models (p. 3). There are distinctions that can be applied to the different models of climate change as well. And relevant questions of which type of model to use in the classroom emerges. We may also ask ourselves if alternative models can bring light into the classroom concerning the climate crisis? Dutreuil (2014) investigates the epistemology of computational models that stem from an analysis of the Gaïa Hypothesis. The model has been criticized for being too abstract, describing fictive daisies on an imaginary planet, and trying to answer *what-if* questions, "how would a planet look like if life had no influence on it?". For these reasons the model has been considered not testable and therefore not legitimate in science, and in any case not very interesting since it explores non-actual issues. "This criticism implicitly assumes that science should only be involved in the making of models that are *actual,* by opposition to *what-if*, and *specific*, by opposition to *abstract*." Dutreuil (2014, p. 2). This research aims to challenge these criticisms and use the mathematical approach to illustrate how one can make a theoretical model first, without any empirical data and then see what understanding it will bring to the students. As an example, imaginary numbers were conceived long before there was any practical use for them. Several centuries later it was obvious that they were very practical tool to use in electrical engineering. A similar approach can be made to use an abstract mathematical model of the earth. And in this way remove a lot of variables, in doing so creating a new learning situation and a way to gain knowledge of how students learn mathematics in this context.

Analysis of data

In my analysis of the data from the classroom, I will use *actor-network-theory* (ANT). It is a methodological approach to study social phenomena where everything exits in a continuously changing network of relationships Latour (2005). As Latour (2005) describes,

everything that happens in a social situation takes place on the same level. So, for instance humans as well as objects have agency, and both play a role in creating a social situation. In ANT there are two main concepts, *medidators* and *intermedieries*. *Mediators* "transform, translate, distort, and modify the meaning or the elements they are supposed to carry". (Latour, 2005, p. 39). *Intermedieries* on the other hand, is what transports meaning without transformation: "defining its inputs is enough to define its outputs" (p. 39). But how do we distinguish between mediators and intermediaries? Latour mentions that to learn ANT is nothing more than to "become sensitive to the differences in the literary, scientific, moral, political, and empirical dimensions of the two types of accounts" (p. 109).

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Reading and writing the world with mathematics: Exploring possibilities with socially vulnerable Brazilian students

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This article presents a doctoral research project related to reading and writing the world with mathematics from a Critical Mathematics Education perspective. From a qualitative approach, the research aims to identify potentials landscapes for investigation so that students can read and write the world with mathematics, through generative themes based on Paulo Freire's theory. As methodological procedures, it's intended to produce field reports, interviews, and alternative resources and it's expected that great possibilities and potential scenarios for reading and writing the world with mathematics within the school will emerge.

The research project and the theoretical framework

The topic of interest is Teaching and Learning Mathematics for Social Justice, in the settings of a Brazilian public school, with the aim of identifying possibilities and potential for students, promoting reading and writing the world with mathematics from generative themes.

The main theorists who have supported this research so far are Eric Gutstein, Ole Skovsmose and Paulo Freire. These authors stated education must be intrinsically motivated by concerns related to social, democratic, economic, cultural and political issues, aiming the transformation of society through education for social justice.

Paulo Freire, probably the most important reference in critical education in Brazil, who developed part of his work in non-formal educational environments, introduces concerns and manifestations about the relations of power and social, political and economic inequality. According to Freire (2001), education is a political act and an act of knowledge, centered on the dialogic act. Freire stresses the importance of the subject learning to read the world, understanding the text and the context, saying his word.

> The reading of the world precedes the reading of the word, hence the subsequent reading of the word cannot do without the continuity of reading the word. Language and reality are dynamically linked (Freire, 2001, p. 11).

When a person learns to read the world and, consequently, to say his words, with the language and reality involved, he/she starts to reinterpret his own reality, while he/she learns

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to understand themselves in the world. The person is placed in the world as a person of a historic struggle against oppressions and inequalities.

Ole Skovsmose argues that mathematics education is not neutral, that it is necessary to be concerned about the interests behind the subjects, to question what or for whom it serves, what are the knowledge-forming interests that are connected to the content (Skovsmose, 2001). In this sense, Skovsmose (2010) emphasizes that the critical approach needs to rely more on uncertainties, than certainties and ready answers, considering that "any approach that can be characterized as critical is left open. And, with such uncertainty, a critical approach can be built" (Skovsmose, 2010, p. 13).

For Gutstein, a mathematical educator, reading and writing the world with mathematics is to investigate and criticize structures and situations of oppression that are present in everyday life, especially those that involve social injustices, such as racism, feminicide, social inequality, etc. It also highlights this action for education, so that students learn mathematics and, at the same time, use it to study their social reality, in the search for understandings and transformations of this world (Gutstein, 2016a). Gutstein believes that teaching mathematics for social justice is synonymous of reading and writing the world with mathematics or teaching critical mathematics, stating that Ole Skovsmose and Paulo Freire have a wide influence on his thoughts, practices and research.

It is necessary to highlight that those three authors, mentioned above, assume in their understandings the dialogue as the main element so that an education for social justice occurs. Understanding dialogue in the same sense Faustino (2018), as the encounter of different world views that build new world views.

In this perspective of dialogue, Skovsmose (2010) brings the notion of *landscape of investigation*, which seeks the active participation of students in the teaching and learning process, inviting teachers and students to dialogue and walk through different learning environments. According to the author, "landscape of investigation opens up new possibilities for reflection. And the notion of reflection is important for any type of critical mathematical education" (Skovsmose, 2010, p. 13).

It is in this literature that seek to expand the understanding of critical mathematical education and social justice, within the school environment, in which the research project is being built.

The context of research project and methods

The research will be carried out in a public school located on the suburbs of a city of the State of São Paulo, in Brazil, in a context of social vulnerability in basic sanitation, food, work, income, rights, affectivity, etc.

Based on this context, this project intends to list and negotiating with students, through dialogue, some possible themes of interest to them, which will become generative themes, from Paulo Freire's perspective. It is expected that themes related to the context of the school and its surroundings will emerge, with social dimensions, and from that start a reflective dialogue of students with the researcher teacher for reading and writing the world with

mathematics will begin. more details about the methodology and methodological procedures will be further defined, because the author is only in the first semester of the doctorate and the project is still in the initial phase.

In the book *Pedagogy of the Oppressed*, Paulo Freire brought generative themes as a teaching methodology in the context of literacy for peasants. Generative themes are concrete representations of the ideas, hopes, doubts, outlooks, fears, values, and challenges that arise from the thought-language of men and women in their relationship with the world (Freire, 2011). Noting "that the generative theme is not found in men isolated from reality, nor in reality separate from men. It can only be understood in human-world relations" (Freire, 2011, p. 136).

It is noticeable that he did not address ideas of mathematics in his work, but Guststein (2016a; 2016b) makes this connection between Paulo Freire's ideas and mathematics, bringing the concepts of reading and writing the world with mathematics. Gutstein (2016b) developed a work focused on mathematics education, that involved

> collaboration with students to discover their generative themes; creation of tasks based on these themes, so that students learn mathematics, at the same time that they prepare to read the world (Gutstein, 2016b, p. 462).

In this direction, in the research project, it is intended to use this methodology of generative themes in the context of mathematics, in a reality of periphery and social vulnerability in Brazil.

In the State of São Paulo, in Brazil, there is the Integral Education Program (PEI) for public schools, in which students spend most of their day at school. This program was created in 2012 for middle and high school and, since then, the number of schools that adhere to this proposal has been increasing. The author of this article teaches at one of these public schools, which is part of the PEI, and will conduct her research at her school, researching his own practice. The research will be developed with a group of students in an elective subject.

The elective subject was chosen for develop the research, since in this space the teacher is expected to promote the enrichment, expansion and diversification of the contents and themes. According to the Integral Education Program Guidelines (São Paulo, 2014), "elective subject occupies a central place with regard to the diversification of school experiences, offering a privileged space for experiment, interdisciplinary and further studies" (p. 29).

Thus, in the elective subject it is possible to work with mathematics covering different contexts, exploring interdisciplinary freely. Therefore, this project will not be developed in the mathematics subject which would have a limited space for the development of the proposal, due to the skills and programmatic content of the curriculum.

The elective subject also has a differential, which are the students who choose which elective they want to take in the semester. All teachers at the school offer subjects, disseminating the proposals, and students choose one that is close to their interest.

Considering the synergy of the ideas that have been explained so far, the reflections converge to the following guiding question: When working with generative themes in an

elective subject, what are the potentials landscape of investigation for reading and writing the world with mathematics?

It is expected that potential landscapes of investigation for reading and writing in the world with mathematics for social justice will emerge, in the context of public school.

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Interactions in mathematics classrooms over different timescales

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I here introduce a classroom study that takes a dialogical approach on mathematics teaching and learning to investigate in what ways interactions between the participants in the classroom are interconnected over different timescales when communicating mathematical reasoning. The study is part of my ongoing PhD-work with an aim to further understand the connections between in the moment interactions and longer patterns of interactions in mathematics classrooms in upper secondary school in Norway.

Context

To understand teaching, we need to think about how students take part in activities in the classroom and in education research look at what learning opportunities students are given and what systems they are part of (Staples, 2008). Franke, Kazemi and Battey (2007) show how teachers' choices of actions and activities in teaching influence the conditions for interactions in the classroom, creating different opportunities for students to engage with mathematics and with other students. Equally, students' reactions or expectations and attitudes towards mathematics influence interactions, how they engage and what meaning knowledge is given. In this flow of interactions between participants in the classroom there is also a negotiation for what is allowed or expected to be done in the mathematics classroom (Cobb, 1999). Depending on responses from students, the teacher makes new choices that have impact not only in the moment (Bishop, 2008) but affect what happens in the next lesson or several lessons to come, creating an ongoing change in conditions for teaching and learning.

Curriculum documents in Europe highlight developing a mathematical competence (OECD, 2018) as a central goal for teaching mathematics, but the translation of such competence differ between education systems and are hence used in different ways in research depending on the context. In the ongoing renewal of the curriculum in Norway (Kunnskapsdepartementet, 2020), the concept of competence is described in terms of basic communication skills (to speak, to read, to write, to count and digital skills) and core elements, such as reasoning and argumentation, that teaching should address in all school subjects and on all educational levels. In mathematics, such competence involves students' ability using mathematical concepts in different situations, their capability of engaging in posing and answering questions related to mathematics and their use of mathematical

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language. Such aspects of students becoming active members of the mathematical discourse in the classroom are also present in education research, for example how participating in conversations about mathematics promotes students understanding of mathematical concepts or development of a formal mathematical language (Schleppegrell, 2007; Barwell, 2016).

Changes in learning conditions influencing teaching in longer timescales than a lesson implies that research on teaching needs to include studies that tie together how students approach or understand mathematical content during parts of individual lessons with how a teacher organizes the classroom also for longer learning processes (Klette, 2007). Although there are several studies in education research that address how time can alter ones understanding of mathematics, there have been more studies on shorter social processes such as those taking place during classroom lessons than on processes that lasts days or weeks (Lemke, 2000). To look at a sequence of lessons in different timescales could affect how classroom activities and interactions are analysed and classified in research (Dalland et al., 2020) and to investigate connections between interactions from different parts of such a sequence could give a new contribution to the understanding of teaching and learning mathematics.

Purpose and aim

The project I present here is part of my ongoing PhD-work with a research interest in the interactions that take place in the classrooms of novice teachers when communicating mathematical reasoning. The study is to be conducted in upper secondary school in Norway and a novice teacher has in this context up to three years of experience teaching mathematics since graduating from teacher education. Three questions guide my research: 1) What kind of patterns of interactions are established in the classroom when the new teacher and students are communicating mathematical reasoning? 2) What are the relationships between the mathematical content and patterns of interactions in the classroom? 3) How do patterns in classroom interactions connect to, or influence each other, over different timescales?

Focus for this project presentation is the third research question, with the aim to further understand how individual events in the mathematics classroom can be interconnected with longer teaching and learning processes when looking at more than just a single lesson. My study proposes to examine the shifts in learning conditions and students' opportunities to participate in the classroom by following interactions over a sequence of lessons. I use 'patterns in classroom interactions' to address "identifiable types of exchanges" (Lemke, 2000, p. 276) that evolve or reoccur over time.

Theoretical aspects of the research

With an interest in classroom interactions and communication I choose a theoretical starting point looking at learning and teaching as social practice and that humans are interdependent of others (Linell, 2009). The meaning of actions and knowledge are created together by the

participants in a context that also influence the communication and negotiation of that meaning. In mathematics such meaning-making could be about a mathematical concept or to understand what counts as an acceptable mathematical explanation and justification in the classroom (Cobb, 1999).

One theoretical framework that highlights interactions, language and the influence of contexts is dialogism (Linell, 2009). In dialogism meaning-making is described as being multi-voiced and interactive and a dialogical research approach puts all the participants in the mathematics classroom as contributors to the dialogues and the meaning of mathematics (Barwell, 2016). Using the theoretical framework of dialogism as presented by Linell (2009) enables dialogues to be about all kind of human sense making and enables patterns in classroom discussions to be about social actions where participants interact with others depending on the situated linguistic practice. Participating in communication can then be a sign of learning and this classroom study seeks to develop understanding for what this participation looks like rather than for how or why interactions take place.

To address how interactions can be connected over time I will use the concept of different timescales presented by Lemke (2000). Timescales are then divided by powers of ten, for example utterance and the exchange of them being at the scale 1-10^2 seconds and a school day and units of them at the scale 10^5-10^6 seconds. According to Lemke it is especially interactions in the scale directly under and over the one in focus of an observation that is of interest to study. Lemke also uses semiotic artifacts to visualise how interactions on different timescales are connected in time and space through physical objects or utterances that reoccur in different situations. To follow dialogues over time, in a fairly limited context, could enable findings of interaction that form patterns both in language and other interactions with the environment that are not just accidental expressions and acts of routine but are baring some kind of meaning or reason (Linell, 2009).

Methodology

To address my research question about how events and interactions in mathematics classroom can be connected over time I plan to do classroom observations and visit three to four classrooms, and by that three to four different teachers. Observations and recorded video data from a sequence of lessons would enable studies of dialogues at micro and macro levels (Lemke, 2000) and by that identify interactions on different timescales. I intend to use Lemke's timescale of 10-12 days as a unit for the sequence of mathematics lessons to follow and then look at interactions within lessons, between lessons and over the entire period's timescale. Recordings make it possible to capture actions and things that can be analysed to connect different timescales, for example gestures, use of physical objects in different ways or words or utterances that reoccur over time. Patterns in interactions could also be connected to the level of responses given by students, the use of activities or the use of time and space in the classroom. Analysis of video recordings would make it possible to find thematic patterns in interactions, although there are challenges in finding those patterns and connecting interactions to a relevant timescale.

Challenges and final remarks

To conclude, previous research results have put into focus the importance of more detailed studies of how interactions affect the conditions for teaching and learning and I find the aspects of meaning-making, dialogues and timescales being highly relevant to further understand mathematics teaching and interactions in the classroom. Although video recordings make it possible to capture different interactions, finding those patterns in the recorded data pose a challenge not only in relation to the amount of data but in finding timescales and identifying relevant artifacts and aspects of dialogues to follow. I find this conference to be a good opportunity to meet with other colleagues of the community and discuss this study, its possibilities, and challenges.

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Developing mathematics education promoting equity and inclusion: Is it possible?

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This paper is a discussion of the possibility to develop an inclusive and equitable mathematics education in primary school based on success factors found in prior research. The overall goal is to develop a model for education and to develop an approach where sociopolitical and pedagogical issues are core. The study contributes to this important and challenging task by building on earlier research from different fields of relevance, generating a model for sustainable development of mathematics education, and at the same time, deriving from and anchoring the model in teachers and students' experiences of everyday life in the mathematics classroom.

Introduction

The Swedish school and mathematics education's ideal of a school for all has been heavily challenged in the last decade as segregation and inequalities have been enhanced at the same time as knowledge has decreased in the subject of mathematics. A struggle for both society, research and practice is to include an agenda of equity and inclusion in mathematics education (Roos, 2019; Bagger, 2017), even though these issues are central for every student learning in mathematics (Atweh, 2011). The struggle has been shown in research as complexity and multiple issues regarding equity and inclusion have been explained (Kollosche et al., 2019). Hence, the explanations to better understand inclusion and equity are found in very different fields. Examples are research on socioeconomics (e.g., Thien, 2016), gender (e.g., Leder & Forgasz, 2008), ethnicity (e.g., Martin, 2019) cultural background (e.g., Meaney, Edmonds-Wathen, McMurchy-Pilkington & Trinick, 2016), language (e.g., Planas, Morgan & Schütte, 2018), disability (e.g., Tan, Lambert, Padilla & Wieman, 2019), ability (e.g., Leikin, 2011), curriculum (e.g., Askew, 2015), educational approaches (e.g., Kolloshe et al., 2019), and assessment (e.g., Bagger, 2017). All these fields aim at creating a mathematics education for optimal opportunities to learn, though rather separate from each other. Out of this separate and multitudinous base of knowledge, success factors regarding sociopolitical and pedagogical issues can be retrieved. Thereafter these issues can be applied to develop an approach in mathematics education that promotes equity and inclusion. Following, this paper aims to investigate the possibility of a project focusing on promoting equity and inclusion in mathematics education in primary school in a Swedish context. This

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is done by presenting the core ideas behind a practice-based project with the aim to support the development of mathematics education which promotes equity and inclusion in the classroom. The result will be two-fold: to develop a model for education and to develop an approach where sociopolitical and pedagogical issues are core. The tentative research questions of the project are:

- What are already identified success factors on societal and classroom levels for equity and inclusion in education?
- How can this theoretically build a foundation for developing a model for inclusive mathematics education?
- How can criteria for equity and inclusion be developed in collaboration with teachers and students?
- How is the relation between this inclusive mathematics education in primary school and student's view of equity and inclusion?

Equity and inclusion in mathematics education

The notions of equity and inclusion are both complex and not always particularly defined in mathematics education research (Bagger, 2017; Roos, 2019). Although, they are frequently used in order to highlight the importance to consider every student's learning on both individual- and societal level (see for instance Askew, 2015). We define inclusion as processes of participation in learning and teaching in mathematics (Roos, 2019). We draw on Cobb and Hodge (2007) in our understanding of equity as something that "contributes to student empowerment, development, and in turn, their ability and agency to learn" (p. 71, Bagger, 2017). Consequently, we claim that inclusive and equitable mathematics education is an education that strives for every student's opportunity to participate in learning processes and develop the ability and agency to learn. Hence, education that considers equity and inclusion simultaneously. Research regarding educational quality and equity is complex and questions of who has access, or possibilities to get access are highlighted (e.g., Askew, 2015). Here, not only issues of what is happening within the mathematics classroom are at stake, but also issues of power and democracy in society (Halai, Mushaffar & Valero, 2016). This makes the learning of the individual student influenced by structures in society and continuous processes of in(ex)clusion is present (Halai, Mushaffar & Valero, 2016).

Bridging equity, inclusion and mathematics teaching – a reflection

In this future study both mathematics teaching and learning as well as students' own view of equity and inclusion needs to be studied at an individual and classroom level. Though, it also has to be understood and developed from a societal perspective foregrounding issues of power and democracy. To be able to, firstly develop a mathematics education based on success factors both on societal and classroom level, and secondly, investigate the relationship between success factors for equal and inclusive mathematics education and student's view of equity and inclusion, there needs to be a bridge between classroom and

societal issues. Also, there is a need to be attentive to power relations in the recontextualisation, acquisition and transmission of knowledge between research and practice (see Bernstein, 2003). Therefore, we build our framework on Bernstein's (1999) theories of the pedagogical device and vertical and horizontal discourses in education. Bernstein states that:

> The shift in equity from equality ('of opportunity') to recognition of diversity (of voice) may well be responsible for the colonisation of vertical discourse [knowledge retrieved from research] or the appropriation by vertical discourse of horizontal discourse [knowledge retrieved from practice] (Bernstein, 1999, p. 169).

Challenges for the future study

One challenge for this future study will be to explicitly find and investigate both research regarding societal issues on equity and inclusion, as well as classroom issues within different research paradigms of relevance. Another challenge is to be able to create a collaboration with teachers and students generating data consisting of observations and interviews with teachers and students as well as quantitative measures of knowledge development. In order to not colonise horizontal discourse (see Bernstein, 1999) while investigating prior research and researching with teachers and students important questions to ask are: What factor is important to start with? How (if possible) do different factors connect? How do the factors work in relation to specific cultures, contexts, and students? Is it possible to reconsider all factors identified when creating inclusive mathematics education?

The overall aim and hope for this future study is to contribute with a model for supporting the development of mathematics education that promotes equity and inclusion. The study will take both sociopolitical and pedagogical aspects into consideration and aims at shedding light not only on successful mathematics teaching but also on the students' part and perspective in such teaching.

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A socio-critical perspective in mathematics education: Doing interviews

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This paper aims to present a seminar proposal, to be developed during the summer semester of 2021 at Freie Universität Berlin, Germany, based on an on-going thesis. This seminar does reference to concepts like dreams, being more, background and foreground, from Critical Mathematics Education and Freire's Pedagogy, and aims to promote the study on the Interview research procedure from a socio-critical perspective in Mathematics Education. In this text, we briefly explore the thesis and seminar's theoretical framework, as well as present the implementation, highlighting the topics, the interview transcript, and some reflections that we can make about these works so far.

Introduction

This paper is about an ongoing seminar, given by the first author, based on interrelated concepts such as dreams, being more, background and foreground (Freire, 1983, 2000; Skovsmose, 2011, 2014).

For Paulo Freire (1983), human beings know themselves as unfinished, and therefore, they have hopes and dreams. They have hopes because of the human nature, that put themselves in a search movement, trying to be more. And human beings dream because they want to have experiences of humanization and freedom.

Inspired by this educator, we could say that dreams have a political perspective, related to the social, political, economic, and cultural contexts of human beings. They also have a subjective perspective, related to personal life experiences. It means that theses dreams are historical and represent an important connection with human beings' backgrounds. Although these dreams make reference to the past, they are not defined by them because, as a part of the humanity's search movement, they point to the future, and generate foregrounds.

Foreground is a terminology named by Ole Skovsmose (2014), which refers to future perspectives, such as dreams, desires, hopes, obstacles, fairs and frustrations. These foregrounds can be seen as landscapes of the future, those influenced by the past, but not determined by it.

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The seminar proposal was inspired by these concepts, as they are part of the first author's doctoral thesis (Soares, in progress). In her work, the research is being carried out based on Freire and Skovsmose's concepts, among others. The aim is to understand how teenagers from a social oppression context dream, what we can say about their life stories, and how they see the role of school and, specially, mathematics classes in helping them to develop their dreams. In the thesis, she works with teenagers from two schools: one in Sao Paulo, Brazil, and one in Bogota, Colombia. She refers to the case study and the life history (Goldemberg, 2004, Nogueira et al., 2017) as inspirations for and uses the interview as the most important data production procedure.

For the referred seminar, participants will be invited to produce data with students from public schools in Berlin, from diverse backgrounds. We will encourage them to be open to students from social oppression context, as was done in the thesis.

We understand that the inclusion of schools outside the Latin American context, in this case, from a German context, which is known as a more privileged region, will bring great gains for the scope of the research and for its impact. The inclusion of this new context will help to identify the dreams of young students from public schools in a global context, and expand the possibilities of mathematics classes in different classrooms around the world, about the understanding of these rooms as spaces that foster dreams and social transformation.

The seminar

The seminar has been prepared to be part of Freie Universität Berlin's course catalogue during the summer semester of 2021, as a result of the first author's work during her stay at this university, as part of a doctorate exchange period, and supervised by the second one. The seminar proposal was built pursuing the following general aim: to promote the study of an Interview research procedure, from a socio-critical perspective in Mathematics Education, based on the Case Study and Life History methodologies (Goldemberg, 2004, Nogueira et al., 2017). Another important approach for this seminar was the multicultural perspective in Mathematics Education (McLaren, 2000). The seminar will be arranged intending to develop a theoretical study in a dialogical way, and creating conditions for the participants to experience doing interviews in practice, going into the field to investigate the following guiding question: how Berlin teenagers dream, and what they think about school and Mathematics classes?

During the referred seminar, which will be held in English, participants (mainly bachelor's degree students) will be invited to produce data that will consist of semi-structured interviews, as in the thesis, with teenager students. At the end of the seminar, everyone will present their produced data and we, among the participants, will think about an initial interpretation of these data.

The interview script, that was also part of the first author' thesis, based on the theoretical framework and on the original one's (naturally, in Portuguese), is as follows:

Life:

1. Who are you?
2. How is your family?
3. How was your childhood?
4. And your adolescence so far, how was it?

Schools:

1. How were the schools that you have studied so far?
2. How do you relate to school, from childhood to today?
3. Who have been your math teachers since childhood?
   Tell me a little about them and your relationship with them.
4. What do you think about mathematics classes?

Dreams:

1. Do you identify injustices with you, or in your environment, or in the world?
   Which ones?
2. What makes you move on, or drives you?
3. What are your dreams? Why do you have these dreams?
4. How do you imagine yourself 4 years from now? And in 15 years?
5. If you were to choose a profession, what would it be?
   (Say the 1st, 2nd, and 3rd place, and justify).
6. Would you change something in your life or in the world? What and why?

All these questions seek to respond the aim's thesis. Briefly, we can say that the first four questions were inspired by the life history methodology and the concept of background; the next four questions were also inspired by the life history methodology and include concerns about mathematics education, and the last six questions were inspired by the concepts of foreground and dreams, including concerns about social and political aspects.

The goals to be achieved with this seminar proposal were categorized into ten seminars which include the following topics: socio-Critical perspective in mathematics education; multicultural perspective in mathematics education; case study and life history - understanding the methodology; "how to make an interview?" – theoretical framework and presenting the semi-structured model; transcription and textualization of interviews - understanding the differences; and data production presentation and interpreting results based on the theorical framework.

The classes will take place during the summer semester in 2021, between April and July. The topics, naturally, are related to the theoretical perspectives addressed in the thesis on which the seminar is based, except for the multicultural perspective. It was added due to the multiethnic and religious context in which many students in Berlin find themselves.

Last considerations

The thesis is at an advanced stage, and we could present in this text some of its results. For instance, we could say that families have a big impact on students' dreams, that their past experiences influence their foregrounds a lot, but not only 'negatively', and that many social oppressed students reveal, through their dreams, their desires for social justice. However, we think this is not the right place to share some of these initial conclusions because we tried here to focus on the seminar and, with this, we intend to look further, covering diverse backgrounds that go beyond.

Finally, in addition to the objectives linked to the thesis, we understand that the purpose of this referred seminar is that the participants learn in the field (in a practical way) how to conduct qualitative interviews in the socio-critical area, having a bibliographic basis in this regard, and start to learn how presenting research results and interpreting them.

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Timescales of transgressive teaching in social justice mathematics

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This project presentation reports on a conversation amongst in-service secondary mathematics teachers who had just participated in a social justice mathematics professional development session. As they discussed how they might incorporate the social justice activity into their classes, their conversation invoked a wide range of timescales. Timescales, as presented by Lemke, are categories of actions that are somewhat predictable, periodic and that either constrain or enact agency. Collectively, the teachers mentioned 20 timescales that imagine forms of agency that unfold over a range of time frames, from a moment to nearly a century. Timescale analysis of this conversation reveals the intensively hegemonic conditions which teachers must consider when preparing to teach transgressively.

Introduction

This paper analyses a conversation amongst several experienced, in-service secondary mathematics teachers sharing their thinking about how one should teach a social justice mathematics activity. The teachers had just participated in a professional development presentation of a mathematics activity on gender inclusivity that would require them to explain concepts of transgender identity and gender-neutral forms of speaking (Whipple, Staats, & Harrison, 2020). During a subsequent focus group interview on attitudes towards equity teaching, two teachers, one with a progressive and the other with a conservative political stance, commented that they had particularly enjoyed the presentation and wanted to present it in their class. However, their initial thinking about how to do this was divergent, and created a lively argument about how one should teach social justice mathematics topics. The conservative teacher described in detail techniques of maintaining mathematical neutrality. The progressive teacher emphasized alliance with a university professional development program as a technique for teaching as she wished to teach.

While imagining themselves teaching a topic that would be received as controversial by some school stakeholders, teachers mentioned an extreme range of timescales. Timescales are activities or processes that unfold over a relatively predictable period of time and that repeat periodically, such as the time it takes to complete a routine calculation, teaching over a class day or year, using a particular textbook or curriculum for several years, and so on

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(Lemke, 2000). Timescales are sometimes hierarchical. For example, a yearlong curriculum might influence the activities in class each day, and each day in class might be composed of multiple cycles of routine calculation, or components of inquiry learning, or other repeated activities specified by the curriculum. After some years, a new curriculum might be adopted with new nested cycles of activities.

Timescales can be seen as modes of action, but also, because they are somewhat predictable, speakers can also refer to them or invoke them in a conversation. Referring to a timescale can be a socially powerful act, a speaker's bid to frame the relevant scope or basis of interpretation for the ensuing conversation. This project presentation treats references to timescales as a way to understand teachers' imagination of the landscape of power that surrounds them should they embark upon teaching social justice mathematics.

Timescales and agency

Discourse analysis using timescales requires attention to the ways in which forms of linguistic referencing specify elements of context — near or far in space or in time — and thereby establish the "scope of understandability" (Blommaert et al., 2015, p. 119). In mathematics education research, timescales have been used to clarify details of positioning theory (Herbel-Eisenmann, Wagner, Johnson, Suh, & Figueras, 2015); propose new, multilevel research methods (Noyes, 2013); and to understand processes of linguistically-mediated social stratification (Barwell, 2020).

Timescales invoke agency because they imply particular types of action carried out over a stretch of time, forms of agency that are somewhat predictable by virtue of being periodic, but never fully so. The hierarchical nature of timescales requires consideration of how different kinds of agencies relate to each other. Timescale analysis is interested in the interplay of agentic constraint, because longer duration timescales such as adopted curriculum tend to require particular kinds of actions at shorter timescales. However, actions at shorter timescales are the constituents of longer timescales, and through this, disruptive agency to change is always possible (Lemke, 2000). For example, what a teacher talks about in one class day might be partly constrained or formatted by the curriculum chosen by the school for use over several years. However, if the curriculum does not allow teachers to teach, and to speak, as they wish, these shorter timescales might provoke teachers to take action towards changing the curriculum. In this way, timescale analysis can give insight into teachers' experiences of power, and potentially, into their means of enacting power in their educational system.

Methods of timescale analysis

Although the timescale construct has been productively critiqued as being less-well-defined than Lemke's 2000 presentation might suggest (Blommaert et al., 2015), here, timescales usefully describe the political range and intensity of the teachers' conversation. For teachers from different schools to share teaching perspectives, they need to refer to situations that

are widely-experienced and that have partly-predictable responses. Teachers need to discuss the "scopes of understandability" of predictable events across school stakeholders. In this paper, we focus on identifying references to timescales in the conversation as a way to describe teachers' envisioning of the power dimensions of their work. A later paper will analyse the interactional processes of scale-jumping through which participants enacted power over each other during the emergent conversation (Barwell, 2020).

For this project, each sentence in each conversational turn was analysed for any reference to a social, physical or historical event that predictably repeats over some period of time. Shorter scale examples include how one should speak in class during a politically situated mathematics activity and how students, school administrators, and families might respond. Middle scale examples included references to political protests that were increasingly frequent at the time of the conversation in 2017, such as high-profile athletes' Black Lives Matter protests, the increasingly public presence of White supremacist racism and anti-Semitism, and student-led school activism towards gender diversity. Longer scale examples included references to the United States' periodic military actions so that a student's response to a social justice mathematics lesson might be conditioned by their relationships with multiple generations of military veterans in their families. We arranged these timescales in a roughly hierarchical manner in terms of the duration, that is, the time it takes to complete the process. Due to space constraints, only some of the 20 timescales are given in Table 1.

Timescale	Approximate duration
Multiple generations in a family: References to Brother, Dad, and Grandpa	Eight decades
The family life cycle: The 14-17 years it takes to raise a child to near-adulthood	1.5 decades
Passage into middle age	A decade
The cycle of the U.S. going to war periodically	Every few years
Types of contemporary political protest in the U.S.: Black Lives Matter; footballers protesting the National Anthem; right-wing marches	Monthly or weekly
The academic year shared by a teacher and a class of students	A year
Getting a pay check from your school	Every two weeks
Students' political expression during a day at school	A day
School administration, parental, community responses to student political expression	A day
Teaching a topic in a class	An hour
Introduce a conversational topic	A moment
Choose a response to a conversational topic	A moment

Table 1: Timescales relevant to social justice mathematics teaching.

Conclusion

Timescales mattered greatly to these teachers as they described how they might teach a social justice mathematics activity. As Table 1 indicates, teachers' discussion of whether mathematics should be, can be or must be taught neutrally was permeated with references to the periodic events from daily school routines, from the communities that the schools serve, and from contemporary and historical political actions. Analysis of this conversation suggests that as teachers consider transgressive teaching, they weigh matters of dramatically wide scale and scope: how to choose one's words, how hold one's face, how to maintain employment, how to respond to the spectrum of current political activism, how to address the displeasure of a family implicated in nearly a century of U.S. military action. The conservative teacher's vision of teaching in a neutral manner references timescales across a tightly regimented hegemonic system that strongly resists change. All the teachers in this conversation seemed to acknowledge that Table 1 lists a range of issues that they might need to address with local stakeholders when teaching social justice mathematics.

Future effective professional development towards social justice mathematics teaching may need to spend substantial time on uncovering these concerns, analysing with teachers which timescales afford them greater agency, and how to build discursive skill in re-framing discussions regarding the broader timescales. Beyond learning to teach a social justice mathematics lesson, with its associated techniques of engaging students' mathematical learning and socio-political consciousness, teachers may need explicit preparation to enact and protect their agency across a range of temporal events far beyond the classroom.

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Embodied and emplaced mathematical literacy: A refugee family's funds of knowledge toward regenerative farming

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In this paper, we present our preliminary findings from our ongoing ethnographic study on out-of-school mathematics learning for refugee families. Our paper provides a glimpse of embodied and emplaced mathematical literacy exercised by a Syrian refugee family engaging in intergenerational, small-scale farming practices, during the pandemic. Aligned with the funds of knowledge framework, we depicted a sketch of mathematical literacy that the family, including young learners, competently engaged. Our analyses call for the discussion on mathematical literacy that could challenge the hegemonic and normative relationships between body and place, and could lead us to the liberating interanimated relationships between body and place.

Conceptualizing embodied and emplaced mathematical literacy

Previous studies on non-dominant (im)migrant families' out-of-school practices have demonstrated funds of knowledge, which is "historically accumulated and culturally developed bodies of knowledge and skills essential for household or individual functioning and well-being" (Moll et al., 1992, p. 133). Funds of knowledge relevant to mathematical literacy include sophisticated geometric thinking in the practice of sewing (Gonzalez et al., 2001), the multiplicative thinking exhibited in gardening (Civil, 2007), and the proportional reasoning in calculating international currency conversions (Takeuchi, 2018). The funds of knowledge perspective have challenged the deficit views toward non-dominant, working-class families and demonstrated the possibility of transforming the school practice and curriculum, where teachers maximize the bodies of knowledge and skills that are embedded in family practices.

Such funds of knowledge are simultaneously embodied and emplaced. Interanimated relationships between a place and learners as agents are key in our inquiring into the family's knowing that is inextricable with the land that they are cultivating. In the recent scholarship (as seen in Krishnamoorthy & Ma, 2021; Marin et al., 2020; Takeuchi & Aquino Ishihara,

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2021), emplacement and embodiment have been synthetically analyzed. Learners are actively making places as they imagine new mobilities of bodies in the places salient to them (Marin et al., 2020). Embodied and emplaced mathematical literacy can thus be intertwined with social changes that the learners desire to envision (Takeuchi & Aquino Ishihara, 2021).

Methodology

Our ethnographic study focuses on a Syrian refugee family who has been engaging in small-scale farming after their resettlement to Canada in 2016, without the use of any pesticide or herbicide. The family used to engage in traditional farming in Deer al-Fardees village near the city of Hama in Syria. As the attacks on civilians in the city of Hama and Deer al-Fardees village escalated, the family evacuated to Lebanon and then moved to Canada as refugees in 2016. The family participants include three of the five siblings in early elementary years (age 6 years old to 9 years old at the time of the study in 2020), Aisha (9 years old), Rabih (8 years old), and Abir (6 years old), and the mother (Nahima) and father (Mohamed) of these children. The family lived in an inner city and commuted daily to a land located approximately 15 km away. Our ethnographic fieldwork was accompanied by video recordings to allow us to engage in the repeated and collective viewing of video data. We also used photographs and drawings that were produced with the children to understand the lived experiences on the farm. We conducted verbal interviews to understand the histories behind their farming practices. We also collected policy and media documents relevant to urban farming practices by this family. Emplacing our dialogues, together with the family participants, we engaged in a *shared* walk that "walkers have a particular way of being together that is more than just co-presence because it has sociability as the basis for bodily movement" (Lee & Ingold, 2006, p. 83). By walking together on the farm with each member of the family, we came to understand emplaced and embodied knowing of Science, Technology, Engineering, and Mathematics (STEM) enabled through their physical interactions with soil, plants, and animals on the farm.

Data and analysis

The data analyzed for this article includes video/audio recorded interactions collected over seven visits to the farm (each visit lasted 60 to 150 minutes) in the summer and fall of 2020. This ethnographic study is still ongoing for analysis of longitudinal development and program design for bridging informal and formal STEM epistemologies. Our analysis in this article focused on the embodied and emplaced mathematical literacy unveiled through the process of shared walks. For the analysis, we first created content logs of all the video/audio data and completed analytic memos for each data. For the parts of the interview conducted in Arabic, translations to English were completed by Author 2 (Raneem). Based on analytical memos, we inductively coded data (video/audio data) focusing on the participants' emplaced and embodied knowing of mathematics.

Findings

As we walked around on the land, the conversation went into the differences and similarities between the farm they had in Syria and the farm in Canada. Mohamed said they grew the same variety of plants that we saw on the farm: kouza, fava beans, parsley, chickpeas, beets, carrots, and so forth. Mohamed explained:

> The difference is… here is a short summer season while in Syria it is a longer summer season. The summer season is 4 months from when we plant till the end of the season. But we cannot forget that the daylight is twice as long here during the summer season (compared to Syria) and the sun is closer to us therefore the plants will grow faster. For example, Zucchini, we pick it every 3 days in Syria while here we have to pick it every day and sometimes twice a day because it is very quick to grow.

Then Mohamed grabbed soil from the ground and touched it with his hands to show us how much the soil can contain moisture. He added commentary about soil as follows:

> The land in Syria is a little hard to work with because there is no snow, therefore we are planting and planting every month of the year. We always need to add stabilizers to the soil in Syria. On the other hand, here we have about 6 months of snow/cold, which adds moisture to the soil which benefits us when we start planting in the summer. The soil in Canada generates around 200% more produces in the summer.

These excerpts from our dialogues demonstrate the proportional reasoning based on the relationships among the length of daylight, rate of plant growth, and length of a summer season between Syria and Canada. They came to notice that the same plant (kouza) produced "around 200% more produce" during the summer season in the Canadian city they were in, because of longer daylight. Based on such proportional reasoning, the family rationalized that shorter summer seasons in Canada would not be a disadvantage in the harvesting of produce.

During the pandemic, this urban farm attracted racialized families in an inner-city, especially those who live in the area of the city where many racialized immigrant and refugee people live. These communities are currently deprived of communal green spaces that served as a safer gathering space during the pandemic. This urban farm provided vegetables grown without the use of pesticides or herbicides with affordable prices or as donations for racialized immigrant and refugee families in need. Nahima explained, "I love the idea of how people come and pick vegetables by hand. Especially during the pandemic, we want people to be provided with fresh produce."

In the process of calculating the prices of vegetables, children, as early as 6 years old, were engaging in multiplicative thinking. As we walked on the farm together, we had conversations with Abir, Rabih, and Aisha about the quantity and weight of vegetables and their estimate of prices.

> Aisha said she'd take Author 1 and 2 to show the field of fava beans and we all walked together. Author 1 pointed at fava bean plants and asked how much it would be if you had a customer to sell. Aisha responded saying "1 kg is 5 dollars." Author 1 asked "okay, then what about if a customer takes 3 kg?" Aisha said, "20? No, 5, 10, 15."

A similar conversation happened when Author 1 asked Abir the price of 5kg of zucchini. Using skip counting, Abir said, "5, 10, 15, 20, 25. 25." Estimation of how much 1 kg of each vegetable would be and engaging in multiplicative thinking to calculate the price of vegetables were a layer of mathematical literacy that the children engaged in from the early years. Based on such experiences, Aisha surprisingly shared her observation on the affordability of fresh produces in the city, "do you know how much beets cost if you buy at a Superstore? So expensive!"

Discussion

Our paper provides a glimpse of embodied and emplaced mathematical literacy exercised by a Syrian refugee family engaging in regenerative farming practices. Aligned with the funds of knowledge framework, we depicted a sketch of mathematical literacy that the family, including young learners, competently engaged. Such embodied and emplaced mathematical literacy was "essential for household or individual functioning and well-being" (Moll et al., 1992, p. 133). However, the scope of this family's engagement in regenerative farming during the pandemic goes beyond the functioning and well-being of an individual household. The family was actively making a place for the collective good, by providing green spaces and making fresh produce free from pesticides and herbicides affordable to racialized refugee and immigrant communities in the city. Our preliminary analyses call for the discussion on mathematical literacy that could challenge the hegemonic and normative relationships between body and place in the discipline of mathematics.

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Exploring academic motherhood in mathematics education

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In this project presentation we describe an ongoing inquiry into the lived experiences of Academic Mothers in Mathematics Education (AM-ME) and the unique insights AM-ME gain while parenting, educating, and doing mathematics with their school-aged children. We focus on the rationale and significance for this inquiry and provide an overview of our project. During our session we will also discuss findings from a collaborative self-study, currently underway, that is part of our larger project on academic motherhood in mathematics education.

Rationale and significance

The 'academic gender gap' persists and disproportionately affects women with children (e.g., Huang et al., 2020; Thun, 2020) especially those who had children early in their career (e.g., Antecol et al., 2018; Mason et al., 2013). Women who are successful in combining full-time academic work with motherhood continue to face challenges in terms of working hours, stress levels, and work/family conflict, risking long-term health issues in the process (Ollianen, 2019). This reflects an historical bias against motherhood in academia, which has remained persistent despite more visibility of female faculty mothers (Mirick & Wladkowski, 2018), and has promoted a culture of silence around issues of academic motherhood (Pasque, 2015).

As we continue to live through a pandemic, during which inequities across the board are exacerbated, academic mothers may be the ones affected most in higher education institutions (Hermann & Neale-McFall, 2020) and risk suffering yet another 'motherhood penalty' (e.g., Baker 2012), as evident in journal submission data (e.g., Murdie, 2020; Staniskuaski, 2020), anecdotal reports from peers, and our own experiences as academic mothers of young children. Although academic fathers are not immune to the impacts of the pandemic, academic mothers have taken a greater hit (Langin, 2021). During the pandemic, academic mothers whose expertise lies in PK-12 education in particular became the default 'teacher' at home and continue to spend a significant amount of time supporting their children's' remote education, while focusing their academic work on serving their students and academic programs, rather than conducting research. In this way, the 'invisible work'

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that affects both the career advancement and overall well-being of academic mothers, has exponentially increased during the pandemic (Minello, 2020).

Working closely with their school-aged children during the pandemic, however, AM-ME in particular have been gaining unique insights into mathematics teaching and learning and are reimagining educational opportunities during this time of crisis (Vomvoridi-Ivanovic & Ward, 2021). This has positioned us and other AM-ME with school-aged children to become immersed in the teaching and learning of mathematics and draw expertise from mathematics education in novel ways. However, we find that it is these AM-ME who are less likely to find the time and energy to disseminate these insights to the broader mathematics education community. Further, AM-ME may not even consider these insights worthy of dissemination since doing mathematics with our children, and learning from this activity, is not typically considered publishable work. We are concerned that unique AM-ME insights that may contribute to advancing mathematics education will remain invisible, just like much of the 'invisible work' (Ahn et al., 2017) academic mothers do at home (Offer, 2014) and in the workplace (Guarino & Borden, 2017).

Project overview

Our project seeks to generate a content-specific counternarrative to the existing 'narrative of constraint' (Ward and Wolf-Wendel, 2012, p. 28) of what it means to be an academic mother in mathematics education by exploring AMs'-ME lived experiences during the pandemic and beyond, and identifying and naming the unique and multiple insights AM-ME bring to the professoriate while parenting, educating, and doing mathematics with their school-aged children. Our goal is to shift the discourse around academic motherhood to one that recognizes motherhood as an asset, rather than a deficit, through disseminating AMs'-ME unique contributions in mathematics (teacher) education scholarship.

Drawing on gendered views of work (e.g., Valian, 2005), post-structural feminism (Weedon, 1997), relational cultural theory (Miller & Stiver, 1997), and a funds of knowledge conceptual framework (e.g., Gonzalez & Moll, 2002), we seek to address the following overarching research questions: In what ways do AM-ME describe successes and challenges navigating their roles as academics, mothers, and mathematics (teacher) educators, during the COVID-19 pandemic and beyond? In what ways do these mothers perceive bi-directional influence between their roles? Subsections include:

1. What are the sources of these successes and challenges and how do AM-ME cope with the latter?
2. What are the funds of knowledge AM-ME with school-aged children gain while parenting, educating, and doing mathematics with their children, and in what ways do these funds of knowledge inform their work in the field of mathematics education and the teaching and learning of mathematics more broadly?

We, the three participant-authors in this study, are all academic mothers in mathematics education in the US. We have 7 children among us whose ages range from 4 to 14. In this

session we will discuss findings from a collaborative self-study, currently underway, whose purpose is to address the above questions as well as pilot data collection procedures for our larger project on academic motherhood in mathematics education. Our use of a collaborative self-study is purposeful in that we are aiming to focus on the nature of our work being self-initiated, improvement-aimed, interactive, inclusive of qualitative data, and trustworthy (LaBoskey, 2004). With our shared positions as AM-ME, we aim to guide each other towards sharing both our personal and professional growth and the ways in which we navigate our roles. It is from our collaboration that we begin to identify and illuminate our experiences in ways that we would not be able to individually (Baskerville & Goldblatt, 2009).

We engaged in narrative interviews (Jovchelovitch & Bauer, 2000), interviewed each other twice and offered probing questions until sufficient details were obtained, and transcribed each interview. Given the exploratory nature of the study, we used the principles of grounded theory to guide our coding and data analysis (Corbin & Strauss, 2015). We holistically read the interview transcripts and identified broad themes and categories. Then we used a combination of open coding and a-priori codes based on the lenses of our theoretical framework (Poststructural Feminism / Relational Cultural Theory / Funds of Knowledge). We engaged in multiple rounds of discussion about the codes. We used thematic analysis to identify, organize, and provide insight into the shared meanings and experiences of the participants (Braun & Clarke, 2012). Through the data analysis process, we named the specific funds of knowledge that we identified as leveraging across our roles as academics and mothers.

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Posters Descriptions

Intercultural dialogue in school mathematics: Ethics of school-free data collection

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Our project aims to promote intercultural dialogue in school mathematics, by exploring the experiences of migrant students and teachers in mathematics classrooms. Although data collection was initially designed to be conducted in person, some ethical issues have emerged as it moved online, due to the COVID-19 pandemic. This poster explores a series of 4 ethical considerations: a) the "shielding" role of school making some injustices invisible, b) the mediating role of school around the relationships that researchers need to create with the families, c) the consideration of the value of different mathematical knowledges, and d) some, practical issues.

We endeavour to study the impact of student migration on mathematics learning from the perspectives of students and teachers. There is an existing and growing body of research focusing on challenges in relation to migration in mathematics classrooms (Barwell, 2016). We aim to examine the potential richness afforded by students' multiple cultural, linguistic and educational experiences of mathematics, including different ways of doing, learning and thinking mathematics. This poster description is organised as follows: we first describe the project and its goals, we then elaborate on the data collection methods and how they changed due to the COVID-19 pandemic, and we finally discuss four ethical issues which emerged in relation to this transition.

A few words about the project: Migration in mathematics classrooms

More than 500,000 children in Canada come from migrant backgrounds (Statistics Canada, 2011) and all must study mathematics. For this study, we define migrant students as those who, as a result of a change of residence, experience differences of culture and language in

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school. Thus, migrant students include those whose families have moved for economic or employment reasons, those whose families are escaping war or other disasters as refugees and asylum-seekers, and indigenous students whose families have moved between a rural indigenous setting and an urban school. How does migration affect students' and teachers' experiences of school mathematics? Research suggests that many students from immigrant backgrounds face challenges in mathematics and underachieve. We conjecture that the mathematics teachers of migrant students often have little understanding of what their students are experiencing or what mathematics they already know. Migrant students can find the culture of teaching and learning mathematics quite alien, may bring novel ways of doing mathematics, and may encounter new ways of thinking about mathematics. Cultural differences in mathematics classrooms lead teachers to feel uncertain and unprepared. In an extention to the research on migration in mathematics classrooms, we aim to a. understand the experiences of students and teachers of learning and teaching mathematics in the context of migration b. promote dialogue between students and teachers in relation to these experiences and observe the impact of this dialogue on teachers' practice. The study inspires to stimulate multivocal, intercultural dialogue between students and teachers of mathematics, and helps us understand the impact of migration on the learning and teaching of mathematics. In the context of increasing mobility and superdiverse mathematics classrooms (Barwell, 2016), the study aims to create conversational relationship among families, children, teachers and school systems, in relation to learning and teaching of mathematics, as well as to highlight "unfamiliar" maths for teachers and for students.

Data collection: Pre-COVID and post-COVID

Originally, participants were being recruited from ten different schools and data collection to be in schools. After schools closed as a precautionary measure for COVID-19, this was no longer possible. Instead, we are recruiting participants through organisations which serve migrant populations and data collection is conducted online.

The initial pre-COVID design of the data collection

In the initial design of this study, there were two phases of data collection: the first phase would involve students and the second phase would involve teachers. In the first phase, our group would visit 10 schools and work with 5 students in each school. These students, would take the role of co-researchers, gathering accounts of their own prior and current experiences of mathematics, from their parents, and in their mathematics classes. Towards the end of our meetings, students would synthesize their collective experiences in the form of collages, in order to communicate their experiences with teachers. At the end of the group meetings, each student would be interviewed to gain their individual perspective. The second phase would focus on teachers' experiences. We would share each collage with teachers from the school, to understand teachers' responses to students' experiences of migration. Teachers, as co-researchers, would collect observations of their own practice, noting shifts arising from their new understanding of their migrant students' experiences. At the end of this phase,

teachers from the 10 participating classes would compare the collages created in their schools, their responses to them, and the resulting shifts in their practice in focus groups.

The post-COVID emerging data collection practice

As the study moved online due to the pandemic, the two data collection phases (one for students and one for teachers) are maintained but their character has changed. In the first phase, we hold ten virtual math clubs with six children in each club. Each group will meet virtually for five sesions and engage in both synchronous tasks as well as "take home" activities to work on with parents/siblings and to share at the next session. At the end of each math club, we hold interviews with each student and – if they want – a family member. The second phase, teachers' experiences, will include six virtual groups of mathematics teachers who will meet twice. We will share and discuss the student participant profiles with teachers as well as examples illustrating children's cultural-historical repertoires with respect to mathematics. Teachers will be asked to reflect on information shared, their reactions, and implications for practice.

Ethical considerations emerging

The transition to online data collection made visible to us a series of ethical considerations. As we are currently in the first phase of data collection, working with students, these ethical considerations are related to data collection with students.

The location of data collection changes from school to home

As the location of data collection changes from school to the virtual space (as students and researchers are physically at their home), aspects of the role of school are made visible to us. School conceals differences among students; the four walls of the school appear to be shielding students from these differences and to offer a sense of security. Inspired by Fasheh's (1998) question "Which is more fundamental? Outward peace or being true to our humanity?", we ask: What sort of an "outward peace" is created by schools and what becomes invisible by this portrayal of peace? What deeply rooted inequalities are given permission to be ignored in an "equal school"? For example, to identify potential participants, we visited richer and poorer parts of the city. This process made visible differences in the living spaces among students, which are concealed in school. We do not intend to evaluate this school function as "good" or "bad"; instead, we try to think about its complex social implications.

Relationships with children and their families are no longer mediated by school.

To conduct any research projects, relationships need to be built between researchers and participants, while in projects related to the concerns of the MES community, we hope that these relationships extend beyond whatever is necessary to obtain data from students or communities. The transition of data collection location made visible to us the school's role in facilitating the building of these relations. If we had been able to continue with research in schools – and given that at least one of the schools were participating based on established

relationships – the schools would mediate the relationship between researchers and students and their families. Due to the pandemic, we are attempting an online approach that reaches out to families directly using the connections we have in the group. When school stops mediating the relationship between researchers and children/families, questions arise: What is the nature of school's mediating role in building relationships between researchers and families? If the school is not there, for example at the time of COVID-19, how can/should researchers manage relationships with families and children?

Different mathematical knowledges are valued differently

Whose mathematical knowledge is highlighted? Our data collection methods entail doing mathematical activities with students with a migration background. While designing, conducting, and reflecting on these mathematical exchanges, a tension emerges about whose mathematics is highlighted. We aknowledge that these tensions cannot be resolved. We all bring our ideas and experiences of mathematics – our repertoires – to the project, and we 'see' participants' repertoires through our own. But we ask: whose knowledge counts as mathematical knowledge? (Abtahi, 2019). Extending the above line of thought, we are also thinking about what mathematical knowledge is valued. Walkerdine's "commodity" perspective draws our attention to how knowing the mathematics of the curriculum not only worths much more than other kinds of mathematics, but knowing "other kinds of mathematics" is often considered to be deficient and of lower level (Walkerdine, 1990).

Practicalities of online data collection

Finally, ethical concerns also emerge in relation to the practicalities of data collection. As many students' activities have moved online and students spend a lot of time in front of the screen, a question emerges about the ethical implications of asking students to spend more time online. Furthermore, issues emerge related to who has access to and who has comfort with technology, as well as the implications of video-recording students while they are physically at their homes.

Concluding thoughts

Through this poster, we present some ethical issues and dilemmas that have emerged in relation to our study's data collection transitioning from in-person to online. We hope that the conversations with the MES community will not only help us reflect on the ethical dilemmas that we are facing, but will also open up the discussion about ethical issues related to data collection in the cyber space and beyond.

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Teaching functions to 21st-century mathematics learners through a real-life problem

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Over a few decades now, educators all over the world are contemplating "How teaching-learning of mathematics should be?". In the 21st century, mathematics holds a special place due to the increased demands of skills that mathematics impart. Three major guiding ideas of 21st-century mathematics teaching are (a) Learning in a context, (b) Learning for the 21st century, and (c) Reducing the gap between school math and real life. This poster aims to provide a way to design a problem-based learning environment by posing a real-life problem in a high school classroom, specifically discussing the Vehicle routing problem designed around the day-to-day environment of students. This context of the problem will initiate the discussion among students and the facilitator on mathematical ideas around functions. This approach will provide a way for learners to understand the underlying mathematics of the real-life scenario under investigation.

Introduction

The Organization for Economic Co-operation and Development (OECD) in 2003 argued due to huge social changes and an explosion of knowledge in the 21st-century there is a dire need to change the fundamental units of education. The concept of formal education in the 21st century is characterized by the idea of the knowledge economy and globalization processes. In this century learners are expected to work towards generating new ideas and solve complex problems. According to OECD, these independent learners must acquire core competencies known as 21st-century skills to make them ready for future challenges of the 21st-century.

Mathematical competencies have been given a central position among the skills that are required by learners to thrive in the 21st century. A study (Gravemeijer et al., 2017) on future mathematical competencies required in a workplace explores what mathematics education prepares students for the future. The study suggests that modeling and applications should be one of the major goals for mathematics education, where learners can gain experience to devise solutions to an authentic real-life problem hence understanding the underlying mathematics.

Another research by the Advisory Committee on Mathematics Education (ACME) in 2011 shows how themes of mathematical modeling, costing, calculating risk, and quality control

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processes make up a huge portion of today's workplace. Through one on one interviews with employees (25 companies in the United Kingdom) across the sectors of employment, ACME has concluded that in a workplace more than the application of mathematics one should be able to solve problems within a context and communicate it effectively. They also recommend identified themes to be incorporated into the mathematics curriculum of the country.

This made us question:

1. What are the guiding ideas of 21st-century mathematics teaching? and
2. How can we design learning in a mathematical classroom that makes learners ready for the 21st century?

Taking a hint from aforementioned studies, we framed major guiding ideas for 21st-century mathematics teaching as (a) Learning for the 21st century, (b) Learning in a context, and (c) Making school mathematics workplace-ready i.e reducing the gap between school math and real life.

One of the possible solutions for our second question is provided by the pedagogy of problem-based learning (PBL) which has been gaining momentum lately. It is a blend of various past approaches (Hmelo-Silver et al., 2006) that had their influences on PBL during its development phases. Educators believe that problem-based learning has drawn inspiration majorly from (a) Constructivist Approach to Learning, (b) Critical Theory/ Critical Pedagogy, and (c) Pragmatism (Hirschman et al., 2018; National Council of Educational Research and Training, 2005; Savin-Baden et al., 2004).

PBL is a learner-centered approach that believes: a learner constructs knowledge while resolving a real-world ill-structured problem. PBL promotes the importance of peer-learning by enabling learners to work in collaboration. Throughout the process of resolving a problem a learner plays different roles (Savin-Baden et al., 2004, pp. 81–92) such as real-world problem solver, critical thinker, communicator and self-directed learners.

Keeping in mind the pedagogy of PBL and suggestions given in "The Role of Contexts in the Mathematics Classroom: Do They Make Mathematics More 'Real'?" (Boaler, 1993), the author has designed an activity. The objective of this project was to develop a lesson plan for the PBL environment to introduce the concept of functions to Indian high school students.

About the lesson plan

The lesson plan was developed to introduce the concept of functions to high school students. The topic skills that were taken into consideration before designing a problem statement are (a) Identifying independent and dependent variables, (b) Identifying the mathematical relationship between variables, (c) Designing a mathematical function from mathematical relation, (d) Domain and Range of a function, (e) Graph of the function, (f) Nature of graph of function and (g) Finding optimal solutions graphically.

The expected time to fulfill the objectives of the plan was kept 1.5 weeks. Understanding of the cartesian plane is the prerequisite knowledge required for this activity.

Choice of problem

Boaler, J., 1993 identifies contexts as general motivators to students which keep their interest going. But sometimes if a problem is an interpretation of a "real-life" scenario that expects students to enter a fantasy world they tend to think of it as some other textbook math problem. She also states "Using the real world, local community, and even individualized examples which students may analyze and interpret is thought to present mathematics as a means with which to understand reality. This allows students to become involved with mathematics and to break down their perceptions of a remote body of knowledge" (Boaler, 1993, p. 13).

On the other hand, Mauffette et al. (2004, pp. 11–25) explored the connection between the problem and the motivation of the students in a problem-based learning environment. Their findings suggest that initially for an introductory PBL level the facilitator should clearly identify and summarize the problem instead of putting it in a wider context which is the next level and is suitable for senior PBL learners. They also recommend that background information should be drawn from one source of data and information about the settings should be complete without omitting the details.

Optimization is one of the concepts we use often. In our day-to-day life from choosing a path to our destination or ordering food either we maximize or minimize certain quantities. Indian National Curriculum Framework 2005 put forward connecting knowledge to the outside world as one of the visions of mathematics education in the country. Choosing a problem that is deeply connected with the learners' life where the topic skills can be mapped was chosen, for the context Vehicle routing problem was finalized.

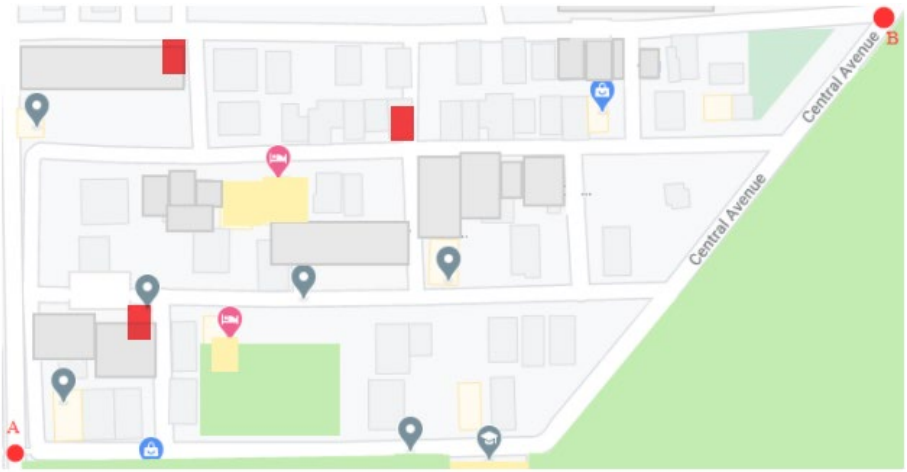

Figure 1: Map followed during the activity.
Note: The map was based on the locality students are familiar with.

Problem statement

"Using the map, find the best routes to reach point A to B in such a way you have to spend minimum time while covering all the highlighted places at least once."

In this, students are expected to answer: (a) Variables in the context, (b)Variables affecting the time, (c) Developing the understanding of their time function, (d) Representing their models on a cartesian plane, (e) Justifying their calculations, and (f) Finding accuracy in comparison with the estimated time given in Google Maps (see Fig. 1).

Critical discussions during the activity:

1. While finding variables both dependent and independent, the relationship between the mode of covering the distance (walking, running, cycling etc.), traffic encountered, and time spent will be discussed.
2. Interpreting mode of covering the distance in numerical terms.
3. Dependending upon students' approach/approaches different data sets will be obtained hence resulting in different functions.
4. Finding optimal solutions from the datasets obtained.

The rationale to the conference theme

Using problem-based learning for introducing concepts will help teachers to make mathematical learning relatable to learners and they will see how mathematics is a part of their life. Using PBL they will also help their learners to develop 21st-century skills. In the present time all over the world, there is a need for resources for mathematical teaching and learning which help learners to understand the underlying mathematics while interacting with real-life problems. When it comes to PBL, not many resources are available for teachers to implement in their classrooms. This presentation provides one way of designing a PBL environment in fulfilling the objective of mathematics education in the 21st century. It can be placed in the theme 'Sociology of Mathematics Education' of the conference as this presentation will talk about a new approach to teaching functions.

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Juxtaposing cases of delegating versus withholding authority of mathematical ideation in early algebra classrooms

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We will illustrate two cases of teachers implementing the same early algebra lesson exhibiting diametric forms of a teacher practice previously identified in the research literature as a means toward achieving equity—the delegation of mathematical authority. The teacher with greater student achievement gains on an early algebra assessment had greater occurrences of teacher moves that positioned students as having the power to rely on an intellectual authority situated internally or within the community of peers' "taken-as-shared" knowledge. On the contrary, the teacher with lower gains exhibited greater occurrences of teacher moves that, in effect, resulted in withholding authority from students to form mathematical justifications. Illustrative excerpts of both cases are shared.

Introduction

Non-dominate student populations are subjected to subtle detriments such as having teachers who assume deficits in students, have lower expectations, and hold biases that reinforce stereotypes about who belongs and excels at mathematics (Delprit, 1992; Flores, 2007; Oakes, 1990; Varelas, Martin, & Kane, 2013). Little is known about how day-to-day teacher instruction either conveys and reinforce or counteracts these messages. One way to counteract these messages is to position students as competent and capable of developing their own mathematical authority (Gresalfi & Cobb, 1996).

Research questions

We explore the ways in which elementary teachers of mathematics confer mathematical authority by asking the following:

How is the power to form and justify mathematical ideas either maintained by the teacher or delegated to students? How does the teacher's talk moves, uptake of student's ideas, participation structures, and general academic expectations of students constitute this delegation of mathematical authority? And, finally, how is the degree of delegation of mathematical authority associated with student academic outcome on an early algebra assessment?

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Methods

To study these questions, we use a comparative case study to offer a description of interactions in two third grade classrooms teaching an early algebra lesson in an urban, Title 1 school. Videos of two classroom observations are the in the process of analysis using open coding in two passes. The first pass we coded for classroom wide instances in which the power to form mathematical decisions is largely positioned within the teacher's power or students'. The second pass is ongoing and is an analysis of teacher's uptake of student's ideas, talk moves, and participation structures.

In order to measure each teacher's students' academic achievement, the gain or difference between a pre- and post-assessment scores of early algebra were used.

Preliminary findings

In order to illustrate the diametrically opposed implementation of delegation of mathematical authority, we will present two episodes extracted from two teachers teaching the same lesson within the same school. Teacher A had a student gain that was almost twice of that of Teacher B. Both excerpts are from the two classrooms as students consider the response to a warm up prompt: "If $a < b$ and $b < c$, how would you describe the relationship between a and c?"

Teacher A, episode of delegated mathematical authority

Teacher A reviews the problem after having students work on it within small groups.

Teacher: Who has something they want to present and defend? Sebastian, what were you and Jayden saying?
Student 1: If *a* is less than *b* and *c* is bigger than *b*, then it's bigger than all of them.
Student 2: What's bigger than all of them?
Teacher: Yes, which letter is the largest?
Student 1: The *c*-- the *c* is bigger than both the *b* and *a*.
Student 3: I don't get it.
Student 1: Look *(standing up, walking to the white board at the front of class)*, we can draw a picture to show it.

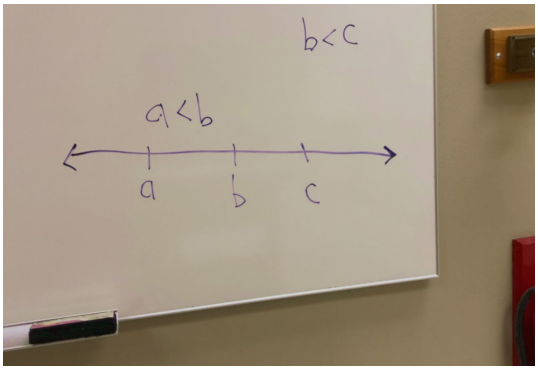

Figure 1: Student generated representation of the relationship between variables.

The two students presenting then draw a number line exhibiting the relationship $a < b < c$ for the class (see Figure 1). At the end of their presentation, the two students turn to their classmates and ask if others agree.

Teacher B, episode of withheld mathematical authority

Teacher B introduces the problem by having students read the problem out loud. There was no time given for students to consider the task within partners or small groups.

Teacher: Okay, so I'm going to write down two things… *(writes $a < c$ and $a > c$). (Turns to class)* Raise your hand if you agree with the first *(gesturing to $a < c$).* Okay, I count 5 votes. *(Writes 4 check marks).* And what about this one…a is more than c? *(gesturing to $a > c$)?* I see some maybes… one vote?

Student: I just guessed!

Teacher: Yeah? Well, it has to be the first one - *a* is less than *c*, because it is less than *b* and you know *b* is less than *c*.

The teacher moves on to the main activity of the lesson without further discussion.

Discussion and conclusion

The practices of Teacher A exhibit delegated mathematical authority in several ways. Firstly, students were given time to consider the task at hand suggesting that the teacher believes it is possible for the students to be successful with the problem and hold such authority. Secondly, the teacher asks students to present and defend their claims and justifications, thus situating mathematical ownership within her students. Lastly, it is evident that a classroom norm that students check for understanding and agreement with their peers. This evidences an established practice of the teacher delegating authority to the students as a whole.

The verbal and non-verbal actions of Teacher B largely withhold the authority of mathematical ideation from the students in that classroom. While the teacher does solicit votes from the students, there is a lack of opportunities for students to make sense of the relationship both in terms of time and peer-to-peer collaboration.

While these two interactions are brief, we maintain they are powerful in contributing to the ongoing conversations about equitable teaching practices in mathematics.

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‘There is no America without inequality’: Imagining social justice writing in a calculus class

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Swati Sircar, Azim Premji University

Social justice mathematics pedagogies envision students “writing the world” with mathematics, in ways that often involve literal rather than metaphorical writing. However, neither the textual form nor the pedagogical processes of developing these persuasive mathematical compositions are envisioned clearly in current mathematics education research. In this poster, we present several samples of social justice mathematics writing responding to an idealized model of the COVID-19 epidemic. One was written by an undergraduate calculus student and others are “creative writing” by professors who are answering the classroom task as if they were students. Through this poster, we hope to create conversation about how students can demonstrate both mathematical knowledge and social awareness through a written text.

Overview

Social justice mathematics pedagogies envision students “writing the world” with mathematics, that is, using mathematical learning to change their communities (Freire, 2005; Gutstein, 2016; Skovsmose, 2020). Quite often, this metaphorical writing involves literal writing, for example, when a curricular lesson concludes with students writing a reflection on their work or writing a letter to a powerful stakeholder (Berry III et al., 2020). This conceptualization, however, seems to be weakly envisioned in current mathematics education research, that students will learn mathematics in relation to issues of justice, compose a written statement that unites mathematics and their critical vision in some fashion, and deploy this writing into the world to some positive effect. Even a brief consideration will generate a complex constellation of composition decisions: How much mathematical knowledge must the writing demonstrate to the teacher? Will students explain how mathematics works in public statements, or merely present the outcomes of their analysis? How does one write persuasively across the epistemological divides of

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mathematical reasoning and moral critique? How do student groups negotiate the process of producing this collective statement?

This poster aims to create conversation among researchers on what they value in social justice mathematics writing. We present several samples of social justice mathematics writing in response to a calculus activity modelling the spread of COVID-19. One sample was written by an undergraduate student in a calculus class, Ijeoma, just as the pandemic began to spread through her city of Minneapolis, Minnesota, U.S.A. The other samples are "creative writing" provided by professor co-authors, written as if they were students in a similar calculus class. We do not present the professors' writing as ideal or perfect examples of social justice mathematics writing, but instead, as possibilities for how students might undertake the difficult task of calling attention to the appalling inequalities exacerbated by the COVID-19 pandemic while also demonstrating early-stage calculus knowledge. In effect, we have identified a lacuna in social justice mathematics research, and we take it as professors' responsibility to imagine a range of possible student responses.

Layout of the poster

The poster will contain the following sections: a statement of the research problem; a brief literature review; the focal epidemiology task with the writing prompt; four brief writing samples; and a collective reflection that outlines priorities for future research on social justice mathematics writing.

An idealized COVID-19 model

The focal task is a generalized Susceptible-Infected model of an epidemic which has many unrealistic features, but is useful pedagogically for discussing calculus topics such as derivative graphs, changing slopes, concavity and inflection points. Early-stage calculus students can use Euler's numerical method to generate graphs of monthly and of cumulative incidence of COVID-19 in an idealized community of 500 people:

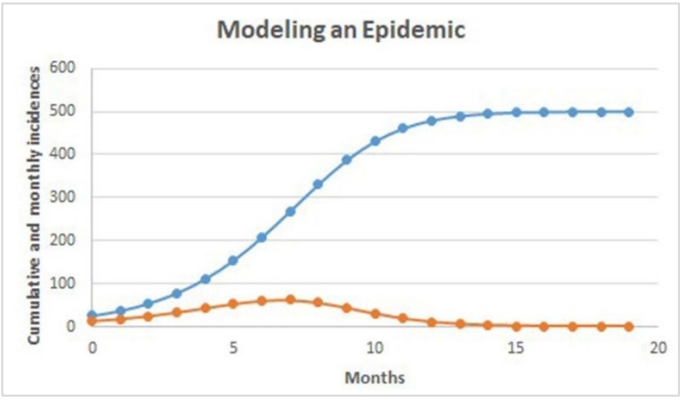

Figure 1: Modeling an epidemic

The poster will contain the following sections: a statement of the research problem; a brief literature review; the focal epidemiology task with the writing prompt; four brief writing samples; and a collective reflection that outlines priorities for future research on social justice mathematics writing. The task and the teaching process will be presented more fully on the poster.

The writing prompt, modified slightly for international professors' creative writing, is:

> Reflect on an issue of unfairness associated with the COVID-19 pandemic that affected any part of the world. Assume that eventually, the COVID-19 graphs in this place will look like the above graphs, except that they will have different scales on the axes and the "cumulative cases graph" will level out at a value or carrying capacity that is lower than the entire population.
>
> Explain carefully and in detail how COVID-19 raises issues of unfairness and social justice. Use the shapes of your graphs in your answer. For example, you could try to explain where the people you are writing about would show up in the graphs, or how they would be affected if governmental policies or changing social behaviours modified the various rates of change in the graphs.
>
> Your social justice writing should demonstrate your knowledge of calculus.

Envisioning social justice mathematics writing

In this proposal, due to space constraints, we present brief summaries and excerpts from a few of the social justice mathematics writing samples. The poster layout will present additional and longer texts. The layout will use graphic "callout" images to draw attention to writing techniques employed by each writer, particularly, demonstrating mathematical knowledge in order to criticize the dreadful impact of COVID-19 on exploited communities.

Ijeoma's real answer

> There is no America without inequality [...] The coronavirus is dangerously highlighting the inequality that lower class and working-class marginalized black and brown people face. [...] When referring back to the graph above, the people who are hit by this virus before the inflection point will consist of more working-class people of color. Where we see the graph leveling out will most likely be when people in privileged places of power take this situation seriously. It's heartbreaking to know that the numbers we calculated in class translate to real-life tragedies that lower-class black and brown communities will face.

Anna's fictional answer

Anna responded to the writing prompt as if she were an international student attending university in Minnesota, whose studies corresponded with both the experience of the pandemic there and the Black Lives Matter protests against the racist police killing of George Floyd, an African American man. Her concern was that the inflection point of the graph of cumulative cases is both an optimistic target to encourage social distancing but also, that it could also become a tool of governmental control against necessary political activism.

Swati's fictional answer

Swati's answer compared the classroom graph of "new cases per month" to public daily case data from India through the lens of national policies and seasonal events. She highlighted inequalities associated with social class. The coronavirus initially reached India through the air travel of middle- and upper-class people, whose ability to social distance caused unemployment among household service providers. She reads the growth and decline stages in the real data against governmental lockdown policies, seasonal movement for agricultural distribution, and festival gatherings. She highlighted continuities across the classroom and realistic graphs, but noted the significant difference that in the real data, the "new cases per day" approaches a positive, non-zero constant, so that cumulative cases will probably continue to rise.

The poster callout graphics on the poster will us notice writers' strategies for "writing the world" through mathematical knowledge. Some of these strategies include restoring a sense of racial and class-based difference to the mathematically uniform variable of "cases," interpreting growth and decline periods in terms of public policy and cultural activities, and reading the graphs in terms of their potential for public communication or manipulation.

Collective reflection

Our collective reflection will consider dilemmas in constructing texts that are intended to "write the world" with mathematics: How mathematically precise and explicit should this writing be? Must the student "teach" mathematics to the audience, or only present results of their mathematical investigation? What writing strategies manage to unify mathematical and political or moral positions? What kinds of writing prompts will encourage particular kinds of social justice mathematics writing?

We hope to point out varied strategies for writing that bridge mathematical insights and social, political or moral perspectives, and to notice different ways of valuing this kind of mathematical writing (Barwell, 2018). This conversation may heighten awareness of the complexity of social justice mathematics writing and the pressing need for wider pedagogical guidance towards it.

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Research Papers

“Communicate, argue, share your ideas”: Values in talking and values in silence

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Many mathematics curricula emphasise the importance of communication in mathematics classrooms, supported by an extensive body of research. In much of the world, this emphasis promotes specific communicational practices as necessary or desirable in order to support mathematical meaning-making and effective learning of mathematics. In this paper, we seek to question the implied universality of this approach: in Arendt’s terms, we seek to judge, in order to provoke questioning and rethinking. Our examination draws on the idea of social norms. We consider what happens when children in mathematics classrooms orient to different norms from those assumed to be beneficial for learning mathematics and how could the epistemological effect of such orientation be conceptualised.

Communication has come to occupy an important place in efforts to develop forms of teaching and learning mathematics that result in conceptual understanding, rather than an exclusive focus on procedural fluency. This emphasis is apparent in curriculum documents in many parts of the world. The recommendations or requirements of such documents often reflect what is known as ‘reform’ mathematics approaches, in which students are expected to play an active part in mathematics classroom activity, through posing and solving problems, often in small groups, as well as participating in whole-class interaction. Among other practices, students are expected to pose questions to their peers and respond to such questions, as well as explain their thinking to the class. These curricular guidelines are supported, often explicitly, by much research in mathematics education (e.g., Cobb & Bauersfeld, 1995).

While such work is valuable, and the goal of conceptual understanding is significant, our work with students from diverse cultural and linguistic backgrounds leads us to trouble its perhaps unintended and implied universality. In our experiences in mathematics classrooms with children from Indigenous backgrounds or immigrant backgrounds we have come to wonder if the reform approach to mathematics classroom interaction unjustifiably assumes or even imposes supposedly universal values. We also wonder and attempt to judge what the impact of these unstated assumptions might be.

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Who is to judge?

In her book of *Responsibility and Judgement*, Arendt (2003) analyzed the relation among thinking (and not thinking), responsibility, and the capacity for developing of moral judgement. As the foundation for thoughts and analysis, she uses events in the history (such as human interactions in the World War II) to relate judgment with human dignity. She explains human dignity could be claimed neither through valorising history nor through denying history's significance, but rather through judging. She views judging as an activity that recognises history but goes even further to deny the histories right to be the ultimate judge. That is, being historically "there", doesn't mean that it should be there! Arendt's view of judgment is particularly important because in different curricula, the endorsement of communication in mathematics classes is there, but the effects of is belonging for students with different epistemological roots (Abtahi, 2019) is not questioned. Here, we took the liberty to judge and we extend the invitation to others. While judging, Arendt explains, we are able to look back at, reflect upon, and begin to make sense of the affairs that are related to our communities, and to form individual standpoints. The more people's take stands and the more standpoints being present […] while we are ponder a given issue, "the better I [we] can imagine how I [we] would feel and think if I [we] were in their place, the stronger will be our capacity for representative thinking" (Arendt, 1993, p. 241).

Given how communication is endorsed in different curricula and given how values are therefore perpetuated or ignored, we seek to judge the significance of specific forms of commnunication as crucial foundations of the learning and teaching of mathematics. Communication is, of course, a part of human social life. So the issue is not whether there should be communication in mathematics classrooms (or in the curriculum) but how it is presented and practiced? And who decides? And how should we, as mathematics educators, judge its normalisation?

The origins of reform approaches: Sociomathematical norms

Ways of interacting in small groups, in large groups, among children or between children and adults vary in different societies or among different groups within societies. Heath's (1983) classic work on literacies, for example, showed clearly that children growing up in middle class White households, working class Black households and working class White households, in southern USA, all developed different repertoires of literacy practices, including different ways of talking and interacting around texts. For example, the uses of stories and ways of telling them varied, as did ways of interacting around a recently received letter.

The significance of Heath's study for education was that the repertoires of the middle class White children in her study aligned most closely with literacy practices in school. Those of the working class children did not align and were often seen as deficient. The example of Heath's work could be interpreted as an advantage of middle class White children, for whom

parents have time and resources to educate them in line with what they, themselves, have learnt as children – hence reproducing the same repertoires. Judging more closely, we see that the norms of literacy practices in school are also aligned with the reproduction of middle class White social norms. For children from the other two backgrounds, there was a misalignment. Such misalignments may not only be due to class: children of many cultural backgrounds whose parents have time and resources to help their children will develop different repertoires of literacy practices, would therefore experience a misalignment of norms and hence be perceived as having some kind of deficit. A study by Street, Baker and Tomlin (2005) focused on numeracy practices showed exactly this.

In a now classic paper, Yackel and Cobb (1996) proposed a social perspective on mathematics classroom interaction. This approach drew on social constructivism, social interactionism and ethnomethodological principles. This last influence was apparent in Yackel and Cobb's adoption of the idea of norms. They proposed two kinds of norm that were of significance in mathematics classrooms: social norms, and sociomathematical norms. It is important to note that from an ethnomethodological perspective, as Yackel and Cobb note, norms are not pre-ordained sets of rules governing human interaction. There is a reflexive relationship between behaviour and norms, through which interaction reflects prevailing norms, but also contributes to their interpretation, so that over time, norms will change. Here is how Yackel and Cobb explain social and sociomathematical norms:

> Our prior research has included analyzing the process by which teachers initiate and guide the development of social norms that sustain classroom microcultures characterized by explanation, justification, and argumentation. Norms of this type are, however, general classroom social norms that apply to any subject matter area and are not unique to mathematics. For example, ideally students should challenge others' thinking and justify their own interpretations in science or literature classes as well as in mathematics. In this paper we extend our previous work on general classroom norms by focusing on normative aspects of mathematics discussions specific to students' mathematical activity. To clarify this distinction, we will speak of *sociomathematical* norms rather than social norms. For example, normative understandings of what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant in a classroom are sociomathematical norms. Similarly, what counts as an acceptable mathematical explanation and justification is a sociomathematical norm. (pp. 460–461)

Yackel and Cobb acknowledge that social and sociomathematical norms will vary in different classrooms. Nevertheless, their paper clearly favours particular norms that they see as productive of mathematical meaning-making and learning, such as their focus on explanation, justification and argumentation. The paper appears to assume that certain practices are 'normal' in classrooms and that the teacher plays a strong socializing role in guiding the development of desirable norms. These ideas can be seen in many mathematics curriculum documents. To illustrate this point, in the next section we look at two mathematics curricula with which we are familiar.

Two mathematics curricula

Theories of social reproduction, such as those articulated by Bourdieu and Passeron (1977) and by Bernstein (1975, 1996), explain the fact that curricula play roles in reproducing social qualities and values by constructing and presenting particular types of knowledge as legitimate. Apple (1979) emphasises the idea that the ideologically-based contents of curricula legitimate some kinds of values and make them appear natural and consistent with common sense. That is, in mathematics curricula, some actions and interactions are valued and legitimatised as norms and natural. For example, in the new Norwegian mathematics curriculum, "fagfornyelsen" (Kunnskapsdepartementet, 2020), the broad concept of competence is translated into basic communication skills (to speak, to read, to write, to count and digital skills). All these competencies are also strongly linked to basic communication skills. There is a specific emphasis on dialogue and participation (Skolforskningsinstitutet, 2018). The curriculum includes the following:

> Dialogue is crucial in social learning, and the school must teach the value and importance of a listening dialogue to deal with opposition. When interacting with their pupils, the teachers must promote communication and collaboration that will give the pupils the confidence and courage to express their own opinions and to point out issues on the behalf of others. To learn to listen to others and also argue for one's own views will give the pupils the platform for dealing with disagreements and conflicts, and for seeking solutions together. Everyone must learn to cooperate, function together with others and develop the ability to participate and take responsibility. The pupils and their homes are also responsible for contributing to a good environment and sense of belonging. Just as each pupil contributes to the environment in school, so will this environment contribute to the individual's well-being, development and learning. (Ministry of Education of Norway, 2020)

The newly revised Ontario Mathematics Curriculum (OMC) for elementary schools also gives explicit attention to communication, and highlights "understanding local and global perspectives and societal and cultural contexts, and using a variety of media appropriately, responsibly, safely". The curriculum documents include the following:

> Communication is an essential process in learning mathematics. Students communicate for various purposes and for different audiences, such as the teacher, a peer, a group of students, the whole class, a community member, or their family. They may use oral, visual, written, or gestural communication. Communication also involves active and respectful listening. Teachers provide differentiated opportunities for all students to acquire the language of mathematics, developing their communication skills, which include expressing, understanding, and using appropriate mathematical terminology, symbols, conventions, and models.
>
> For example, teachers can ask students to:
>
> 1. share and clarify their ideas, understandings, and solutions;
> 2. create and defend mathematical arguments;
> 3. provide meaningful descriptive feedback to peers; and
> 4. pose and ask relevant questions.
>
> Effective classroom communication requires a supportive, safe, and respectful environment in which all members of the class feel comfortable and valued when they speak and when they question, react to, and elaborate on the statements of their peers and the teacher. (Ontario Ministry of Education, 2020)

These extracts raise many questions for us. What are the assumptions about communication (talking, interacting, social participation, social roles, etc.) revealed by these guidelines? One assumption seems to be that children should talk about their ideas. "Defending mathematical arguments" or "pointing out issues" might assume situations where someone has to convince others that they are right or criticise the ideas of classmates. Asking questions, active participation and communication seem to be idealised as signatures of a successful and engaging mathematics classroom.

We depart from Yackel and Cobb view of social norms and sociomathematical norms in our examination of the endorsement of communication in the mathematics classrooms, as highlighted in Norway (2020) and Ontario (2020) Mathematics Curricula. As mentioned by Yackel and Cobb, interaction reflects dominant norms, but at the same time, interactions contribute to the interpretation of the norms, making the norms change over time. We agree with such a view that norms and sociomathematical norms are reflexively produced and are changed in interactions and over time. We do not wish to be concerned with the soundness of this position but with what it means to a knower. Knowers are ones who grew up and were socialised into various norms (and sociomathematical norms) different from presented by the dominant classroom culture and of mathematics itself. We challenge Yackel and Cobb's perspective to examine epistemological assumptions such as: How do children come to know these norms? How are the epistemological effects of these norms conceptualised? And how do the norms of a group of people interact with and cause harm to the norms of another group of people? We think about these questions with a confined focus on the endorsement of communication. We attempt to judge the given value to 'communication' in mathematics classrooms. We judge to form a standpoint, and in the collection of many other standpoints, to contribute to a more just teaching and learning of mathematics.

These are mandates of the curricula, which by legitimising certain types of knowledge, and certain ways of acquiring that knowledge reproduce certain social qualities and values. Curricula, of course, are themselves embedded in and reflect particular societal norms and assumptions. Hence our fundamental questions become: how should cultural and, more importantly, epistemological sensitivity be conceptualised? And who is responsible to judge?

Speech, silence and communication in mathematics classrooms

Speech is silver and silence is gold – says a Filipino saying. Stay silent and reflect so that your thoughts become selected and strong speeches (or something like it) – says a Persian saying. One of the central instructions to many children worldwide is to practice quietness, listen, and speak only if one knows the full meaning of what one says.

Don't talk too often … Don't talk too long … Don't talk about those matters you know nothing about. Johnston (1990) explains that "Were a person to restrict his discourse, and measure his speech, and govern his talk by what he knew, he would earn the trust and respect of his [or her] listeners … people would want to hear the speaker again and by so doing would bestow upon the speaker the opportunity to speak, for ultimately it is the people who confer the right of speech by their audience." (p. 210). For many cultures, words are not

objects to be wasted. They represent the accumulated knowledge, cultural values, the vision of entire peoples. Armstrong, a Cree philosopher, explains:

> It is said that you cannot call your words back once they are uttered, and so you are responsible for all which results from your words. It is said that, for those reasons, it is best to prepare very seriously and carefully to make public contributions. (1999, p. 90)

Not once but many times both in Norway (2020) and in Ontario (2020), Mathematics Curricula, communication is encouraged for multiple reasons. For example, communication is valued as it gives "the pupils the confidence and courage to express their own opinions" (Ministry of Education of Norway, 2020). Pupils are inspired to "argue for one's own views" to create a "platform for dealing with disagreements and conflicts and seeking solutions together" (Ministry of Education of Norway, 2020). Similarly, in OMC, communication is highlighted as "an essential process in learning mathematics". In OMC, developing communication skills is directly related to "expressing" and "understanding mathematical terminology, symbols, conventions, and models" (OMC).

Following the line of arguments in Norway (2020) and in Ontario (2020) Mathematics Curricula, in emphasis on the benefit of having communications and expression of an idea in the mathematics classroom, specifically to better understand mathematics, raised concerns for us. Could such a view of communication imply that silence is the opposite of knowing? That a student who is silent and does not "express" or "argue" her position (because she has learnt the fundamental values of being silence) now is considered as not understanding "mathematical terminology, symbols, conventions, and models"? This consideration - of not understanding - is precisely the point in which epistemological sensitivity is vital. Without such sensitivity to the ways in which students with different social and sociomathematical norm become to be normalised in the dominant norms (i.e. those of the curricula), who thrives, who survives and who get disappeared? If according to Norway (2020) competence translates into basic communication then silence becomes a marginalised capacity, showing lack of competency.

Interpreting students' knowing and participation

The following extract is from an interview conducted in 2010 with a Canadian elementary school teacher. It was conducted at the end of the school year, in which she had taught a class composed for most of the year of Cree children (see Barwell, 2020, for information about the study). In this extract, she is reflecting on some of the children's participation.

> I would say [Curtis] is the strongest one in the whole class with regards to math skills and confidence (.) he is willing to take risks and doesn't care if he is wrong as he will learn from it very quickly (.)
> um the other kids […] um they are a lot less likely to take those risks (.) much more hesitant for a number of reasons I would say (.) one (.) they are not at school regularly (.) they are not here to hear everything like a concept or a lesson half way through and they have no idea what has been taught before or what is coming so (.) they have gaps of learning (.) they're not they're

> unsure and they just don't really want to participate (.) they don't see the importance in what they are doing (.)
> then you have someone kind of in the middle like Kevin (.) who wants to participate who wants to ask questions who wants to do well and try and things like that (.) but is so unsure (.) his skills like (.) he is so unsure of himself that he doesn't want to risk being embarrassed or things like that sometimes (.) sometimes he is okay but sometimes I know he is nervous about being wrong (.) and I think that is one of the biggest things that we have overcome.

The teacher's observations reflect the kinds of mathematics classroom norms we have been discussing. She refers positively to risk-taking, participating and asking questions. She refers to hesitancy, or being unsure as more problematic. But what if her interpretations, based on the implicitly adopted and reproduced norms, as found in many curriculum documents, are a misreading of behaviours of children from a cultural background quite different from her own and largely unfamiliar to her?

Abtahi (2019) warns us about the danger of an over-emphasis on cultural difference and neglecting the elements underlying the cultural differences, which are differences in ways of becoming to know, in mathematics classrooms. Similarly, for us, what is worrisome is beyond the cultural differences or even the diversity of social norms and sociomathematical norms. Instead, we are concerned with the epistemological differences and particularly are perturbed by the effect that such differences play in Curtis' or Kevin's mathematics learning. For example, in the above quote, the possibility of students' orientation to silence and humility as a cultural norm is ignored. This legitimate form of participation leads to the interpretation of Kevin's action as hesitancy, nervousness or a lack of willingness to take risks, leading to a (possibly harmful) epistemological assumption that Kavin "is someone kind of in the middle" and his sorts of behaviour "is one of the biggest things that we have overcome".

We are not attempting to force an interpretation on the data here. Instead, in our judgment of the content of the curriculum, concerning the significance of communication and the endorsement of silence in different cultures, we are highlighting possibilities of epistemological diversities that, if ignored, could be harmful. We note also that teachers judge their students' participation in mathematics – it can be difficult not to – and that sometimes these judgements will be harmful to the students.

Discussion

In many parts of the world's mathematical curricula, communicational practices are desirable to support better understanding and learning of mathematics. In many parts of the world, silence is desirable to support deeper understanding and reflection on any phenomena. Knowing the value of silence in many cultures around the world, knowing that human movement from places to places and knowing the significance of communication in mathematics classrooms, in this paper, we set out to judge the relationships among these three assumptions and highlighted possible epistemological implications. We examined the content of Canada (2020) and Norway (2020) mathematics curricula to better understand

their framing of the significance of communication in mathematics classrooms. We noticed not only that argumentation, expression and communication are highly endorsed in both curricula, but also that the lack of such is abilities is viewed as not understanding mathematical ideas. Following Arendt's view on judgment, we judged how the emphasis on communication, contradicts with social and sociomathematcial norms of some communities leading to not just cultural but epistemological ignorance. We finish this paper with an open question of how epistemological mindfulness could promote more just mathematics learning and teaching and how epistemological mindfulness could look to different teachers?

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Criticizing epistemic injustice: Rewarding effort to compensate for epistemic exclusion

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This is a theoretical paper prompted by reflection on practice. In the first part we criticize the notion of epistemic injustice in favor of epistemic exclusion and suggest rewarding effort to compensate for exclusion. In the second part, we briefly describe how we are rewarding effort to understand in our calculus courses. We ground the evaluation of this effort on firm theoretical basis, clearly distinguishing teaching, understanding, and learning. We frame the epistemology of twentieth century mathematics into an attempt to quilt signified and significant together through arbitrary language conventions called definitions. As teacher-researchers, we take our own classrooms as our object of study and ground theory on practice.

Epistemic injustice and mathematics education

In TWG-21 of Pre-CERME12, we reported our experience with calculus courses in the second semester of 2020. We received the suggestion to refer to "Epistemic injustice in mathematics education" (Tanswell & Rittberg, 2020). This we now do.

Epistemic injustice is a concept that made its *debut* in philosophy by the seminal work of Fricker (2007) who also coined the term.[1] Speaking of "injustice" unavoidably brings "victims" to mind. "Injustice" opens the issue of social victimization, discussed in Dolar (2017) and uttered by Žižek[2] about the METOO movement. Mladen Dolar remarks that the political correctness concern about social victimization is careful in defining a list of victimized minorities such as racial, sexual, religious, etc., but "curiously, the working class, the basis of capitalist exploitation, doesn't usually feature on this list" (p. 72). Indeed, he remarks, "the cultural and identity struggles inspire so much more passion and engagement than the political and the economic issues" (p. 72). However, this is not a mere curiosity. The apparent critical intentions of concerns about social victimization do not touch on the economic basis of the object of its critique. We go one step further and argue that this is the symptom of what we call *victimization ideology*.

[1] https://en.wikipedia.org/wiki/Epistemic_injustice

[2] https://youtu.be/ai_UAPaoEW4

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For the victimization ideology to be operational, the victim has to play its role. As Dolar (2017) remarks, a revolted Arab immigrant is no longer a victim, he becomes a terrorist. A conniving victim is no longer a victim, she becomes an opportunist. Therefore, repression and the Law have to be evoked to keep the victims attached to their roles, so that victimization ideology may display a humanist and charitable face. Here is our point: *victimization ideology functions to hide the fact that economic exclusion continuously produces "victims."* Consideration of economic exclusion would thwart the relief of culpability that victimization ideology is perceived to produce; it would reveal the true source of culpability. We will show how this cover-up of the economy happens in mathematics education.

Justice is not a natural law. Hegel (1991) shows that it is as old as humanity itself and stems from the exchange of commodities. For Marx (1962), Justice belongs to the ideological superstructure of society which, in its turn, depends on the economic infrastructure. Today, the economic infrastructure determines the dominant class almost directly as being the owners of capital. Finally, Althusser (1970) stresses that the dominant ideology is always the ideology of the dominant class. Along this line, we must conclude that an appeal for Justice and Right is an appeal for the reinforcement of the economic laws and the laws that ultimately ensure each of the myriad exclusion factors dealt with as epistemic injustice. In summary, *epistemic injustice is a call for more of what produces it.* This is the reason for its success.

The internalist view of mathematics

Epistemic injustice makes its entrance in sociology of mathematics via Rittberg, Tanswell and Bendegem (2020). Its fruits are surveyed in Kidd, Medina and Pohlhaus (2017). (How many trees had to be cut down to produce the cellulose for this edition?). It makes its entrance in mathematics education via Tanswell and Rittberg (2020), who endorse what they call the *apprenticeship model* of Dawkins and Weber (2017). Let us examine what this model brings into mathematics education.

Dawkins and Weber (2017) strongly advocate for teaching proofs in mathematics classrooms; they do not specify levels: "we maintain that proof instruction – as an apprenticeship in mathematical practice – should not shift away from proving practice itself" (p. 138). They address the recommendation that "enculturating students into classroom proving practice involves an enculturation into mathematicians' values" (p. 126). Clearly, their proposition amounts to aligning the mathematics classroom to the need of reproducing the mathematicians' qualified labor power (Baldino & Cabral, 2020), supported by the internalist view that mathematicians hold on their own practice.

Let us consider the epistemological rationale of Dawkins and Weber (2017). "By *proofs*, we are referring to the written artifacts that mathematicians call proofs" (p. 124). However, they do not say who mathematicians are. From the article, we infer that mathematicians are the members of a community of proof producers. Thus, they start from the circular definition: *proofs are the production of proof producers.* Hegel would say that there is nothing wrong with circular definitions; they may express the result of a dialectical process (Hegel, 1969).

As written artifacts, proofs are a form of speech. Cabral and Baldino (2021) show that the dialectics of history has generated a community of speech, together with a special form of speech of which proofs are a particular case. They call this community "mathematics" and its members, "mathematicians". In this sense, the circularity of the expression "mathematics is what mathematicians do", perfectly expresses the genesis of mathematics. We will see more of this below.

However, in the enunciation of that statement by Dawkins and Weber (2017), the dialectic character of the definition of proof is lost, in favor of a frozen image of proofs as construed by the internalist view of 20th-century mathematics. As spokesmen for the present state of their community, they seek to "conceptualize proof in terms of 'values and norms'" (p. 124). They need new definitions: "values represent a community's shared orientations that underlie shared activities" (p. 125) while the term norm "refers to the expectations on practice accepted by the scientific community to uphold a value" (p. 126).

Tanswell and Rittberg (2020) invoke Kant's philosophy to justify two of their values: "mathematical knowledge is justified by *a priori* arguments" and "mathematical knowledge and justifications should be a-contextual and specifically be independent of time and author" (p. 128). An effort in abstraction is necessary to make sense of these statements.

Let us consider "one plus one equal two", a mathematical knowledge which certainly the authors would "perceive as being held by the mathematical community" (p. 128). For indigenous populations of Brazil, one plus one may be one, as in the case of a piton and a duck. It may also be zero, if one misses two shots and loses two arrows; or it may be three, as in the case of a father and a mother (Ferreira, 1997). Is this knowledge independent of context? Can it be justified *a priori*? It can only seem so after a great effort of abstraction: we are not talking about ducks and boas. We could ask them: what are you talking about, precisely? They only know that it is something that has norms and values.

However, how did it happen that "one plus one equals two" has become an *a priori* knowledge to common sense? This is what Hegel's Logic is perceived to explain, but Hegel's criticism of Kant is precisely what Dawkins and Weber (2017) must ignore to uphold their internalist view of mathematics (Pais, 2016). They do not seem to realize that, ultimately, the adequacy of norms and values to conceptualize proofs must be subjected to the judgement of the same community of proof producers.

As spokesmen for the community, the authors express the difficulty of establishing pedagogic conditions that will put students in the mood of proving, but they do not regret castoffs. For their purpose, it does not matter that only a few students endure enculturation into proof writing, as they are careful not to fail too many. During summative assessment, they always put some easy questions to the majority and one or two difficult quizzes to the "good ones". This is what their internalist view calls mathematics education.

From injustice to exclusion

In Dawkins and Weber (2017) the words "justice" and "injustice" do not appear. This absence did not keep Tanswell and Rittberg (2020) from choosing this paper to support the entrance

of *epistemic injustice* into mathematics education. They open their case by stating two axioms: "mathematics practices are governed by norms and values" and "classroom mathematics should refer to professional mathematics practices" (p. 1199). They justify these statements with what they call the *apprenticeship model* of mathematics education, which, on page 1201, they erroneously attribute to Dawkins and Weber (2017).

Next, they observe that the norms and values of mathematics practices do not coincide with the cultural background values of students, a difficulty that they call the *research gap* (p. 1199). For ethical reasons, they say, it is necessary to "negotiate and adjust" (p. 1199) these values. For this negotiation, they "introduce *epistemic injustice* as a philosophical framework that helps reveal the ethical imperative to traverse the research gap" (p. 1199); they hope that "epistemic injustice in the mathematics classroom may be avoided" (p. 1200). They point out that the language used in written proofs that students should learn to reproduce may be a source of epistemic injustice, since it is "devoid of many features of natural language" (p. 1202). The authors conclude that "the apprenticeship model is in need of refinement" (p. 1209).

For Tanswell and Rittberg (2020), negotiation is also necessary due to the possible clash between ethical orders stemming from mathematics practice and ethical values present in students' cultural background. "The ethics we are subject to depends heavily on the role we are currently occupying" (p. 1205). The challenge to negotiating ethical values while seeking to traverse the research gap and avoid epistemic injustice is that "the classroom is a space in which multiple roles and their associated ethical orders are present and may come into conflict" (p. 1206).

Tanswell and Rittberg (2020) argue that "the framework of ethical orders also helps to identify the many roles that are inhabited in the mathematics classroom" (p. 1206). The reader expects that they will not miss two paramount teacher roles: their *tutoring* and their *summative assessment functions*. Summative assessment can put teachers into dramatic ethical dilemmas that Cabral and Baldino (2019a, 2019b) call the *splitting moment*. However, the reader's expectation is frustrated:

> Adopting the framework of ethical orders also helps to identify (…) the array of values, norms and responsibilities these entail. For instance, there are the distinct roles for the student and teacher, which come with very different perspectives, but beyond this they are both engaging with the ethical order of mathematics (Tanswell & Rittberg, p. 1206).

By looking for ethical values and responsibilities in the classroom, these authors are led back to the vague "ethical order of mathematics". This is astonishing: how is it possible to *look at* a classroom in search of roles that could lead to epistemic injustice and fail to *see* summative assessment?[3]. The mandatory universal summative assessment practice is the moment when a symbol is written beside the name of each student, meaning that a decision of passing/failing has been taken. This practice materializes epistemic exclusion in school, no matter its euphemistic disguises.

[3] "The officer paused with his glance on the space where Aureliano Segundo and Santa Sofía de la Piedad were still seeing José Arcadio Segundo and the latter also realized that the soldier was looking at him without seeing him" (Márquez, 317).

Summative assessment decides whether each student will get credit for the course, that is, whether she may or may not increase the value of her labor power by adding to it the work done by her, the teacher, and the staff during the course. Value is human labor crystalized in commodities, says Marx (1962). Upon graduating, the student will be the owner of a qualified labor power of higher value, which she expects to sell for a higher salary. Therefore, school is an economic enterprise and failing a course has economic implications (Pais, 2014; Baldino & Cabral, 2015, 2020).

As human work, value is substance and, as such, it does not disappear. In case the student fails, whither will the value already produced by the student go? It will be collected by her colleagues who passed, as a prize for having won a competition. *The work done by all is appropriated by those who pass.* This conclusion of Baldino (1998) is bound to raise objections. However, there is no success without failure. Even if it happens that everybody passes, the mere possibility of failure establishes the perspective of competition. Thus, the truth of epistemic injustice is *economic exclusion.* Epistemic injustice is a feature of the victimization ideology whose aim is to cover up and ensure the permanence of economic exclusion. This explains why Tanswell and Rittberg (2020) must avoid summative assessment when looking for epistemic injustice.

This point where the signified "exclusion" is attached to the signifier "injustice" is what Lacan (1971) calls a *quilting point*, borrowing the name from the mattress industry.[4] From this point on, Tanswell and Rittberg (2020) may be read back to reveal its political effect. The imposition of the "apprenticeship model" to enculturate students into the mathematicians' professional proving practice is actually intended to open the "research gap" that allowed the authors to present themselves as *defenders of their own victims.* The "apprenticeship model" assigns to teachers the impossible mission of teaching all students to "write proofs", under the sole argument that this is what mathematicians do. The qualified labor power of the students who succeed in this mission acquires a high sign value as the prize for winning a competition with so many losers. For these, it is necessary to "negotiate ethical orders" under the constraint of avoiding massive failure in the course. Thereby, *unspoken non-mathematical subsidiary criteria* are introduced into the summative assessment. This is the true ethical question that was kept in the shadows by the "ethics of mathematics", an abstraction ensuring that economic exclusion remain out of sight at adequate level.

Grounding theory on practice

To say it all at once: we too adopt non-mathematical, economically excluding, summative assessment criteria, but there is nothing in the shadows: *we reward effort to understand, instead of evaluating final learning.* To unfold this aphorism and justify our assessment criteria, we need three definitions, grounded on Lacan's psychoanalytic theory (Lacan, 1981).

[4] The quilting point is the stitch that transforms a sack into a cushion.

Evaluating learning or evaluate understanding?

Teaching is to ask questions and return the meaning of her answers to the student. The teacher assumes the position of the demanding Other: *what does he want?,* the student asks herself, an interrogation that Lacan writes in Italian *che vuoi?* to avoid the gender implication. In opposition to "conveying meaning," teaching is listening and showing understanding of what is heard.

Learning is to represent oneself as a subject by new signifiers. The student assumes the position of speaker and ventures a new representation into the field of the Other, where she must constitute herself as a desiring being. Learning is speaking, in the broad sense, including gestures, hesitations and silences. It is a restructuring of the subject's jouissance. It does not occur *in vitro*. It depends on a myriad of factors such as cultural background, gender, ethnicity, etc. These factors remain at work overnight.

Understanding is the student's ability to sustain her subjective representation by a signifier. In understanding, student and teacher are publicly interacting in the *agora*. Understanding is an objective process, it can be registered in video; it is a being-there (*Dasein*) and as such *has a measure* (Hegel, 1985, p. 330[5]); thus, at least in principle, understanding can provide a precise ground for summative assessment.

In summary, *one teaches by listening and learns by speaking*. Traditionally, one teaches by showing dominance over knowledge (*séance magistrale*. In this system, summative assessment is designed to give free course to the myriad of cultural biasing factors that work overnight to boost/hamper learning. One may say that this is a true epistemic exclusion of those who somehow have not incorporated (learned) the imponderable values of the ruling class present in the whole summative assessment situation.

To compensate for epistemic exclusion, we seek to develop a reliable way to evaluate the *effort to understand mathematics*. To unfold this statement, we must make precise what we mean by "effort" and by "mathematics".

What do we call mathematics?

It is useless to ask mathematicians. They do not know what mathematics is; they only say that mathematics is about this and that. They do not know what a proof is, either; they only say that it is "conceptualized in terms of values and norms". In Cabral and Baldino (2021), we describe the underlying historical process that started in Ancient Greece and has generated a special kind of speech, along with a *community of speech* that upholds it as epistemologically valid. These speeches we call *quilted speeches*.[6] Inside the community of speech, they came to be called proofs. The essence of a quilted speech is the *attempt to stop the sliding of the signified under the signifier*, that is, the possibility of justifying each inference from one

[5] *Alles, was da ist, hat ein Maaß*. Alles Daseyn hat ein Größe.

[6] Quilted speeches emerged in Ancient Greece, together with coinage; they were socially necessary to avoid intrafamily clashes between the economically broken landowner progenitor and his rich brothers who were free to go into commerce. These were indeed very special social circumstances.

statement to the next by an explicit consensual argument; these inferences are the *quilting points*. The community, with its statute of validity, is what can be properly called mathematics; its members are the mathematicians. An interesting description of the statute of this community, specifying what counts as a valid argument, may be found in Rittberg, Tanswell and Bendegem (2020).

To make clear what we mean by quilted speech, we present an episode of our current freshmen calculus course. The students had received a worksheet with the proof of the derivative of t^2 and had been required to justify each quilting point, represented by the numbered equality signs in the excerpt shown below. In the previous class, they had engaged in a lively discussion about "infinitesimals" and "limits," based on what they had found in books and online. Is the limit indeed reached? What is the threshold below which a number becomes infinitesimal? In the present class we were evaluating how they would justify the following quilting points.[7]

$$\boxed{f(t) \approx \frac{F(t)-F(t-dt)}{dt}} \overset{(0)}{\Rightarrow} \frac{t^2-(t-dt)^2}{\Delta t} \overset{(1)}{=} \frac{t^2-(t^2-2t\ dt+(dt)^2)}{dt} \overset{(2)}{=}$$
$$= \frac{2t\ dt-(dt)^2}{dt} \overset{(3)}{=} \frac{(2t-dt)dt}{dt} \overset{(4)}{=} 2t-dt \overset{(5)}{\approx} 2t \overset{(6)}{=} f(t) \overset{(7)}{=} \frac{dF}{dt} \overset{(8)}{=} F'(t)$$

At quilting point five we received three students' collaborations in a row (emphasis added):

Antônio: It's an *infinitesimal*, so it's only approximating 2t.

Cesar: You can divide by it and that's not wrong. It's not as if I were *dividing by zero*. But here, when I subtract, then it's like it were *really zero*.

Erick: It's like Antonio said, we have an infinitesimal number. So, we are practically *creating a limit* there.

We concluded:

Baldino: This whole discussion you've been struggling with since the previous class, and which mathematicians struggled with for 200 years, was resolved in 1824, when an important mathematician called Cauchy said this: "on dit que" – or "it is said that". It is said that the derivative is the name of 2t. The whole discussion you had was reduced to one name: "derivative" is the name given to 2t by *language convention*.[8]

[7] *F* and *f* respectively refer to the totalized and the daily deaths graphs due to COVID19 as popularized by the media; these graphs had been object of previous classes. We started with stereotyped graphs of a disease outbreak that lasted for 60 days. Once the students were able to express the *moving average of deaths based on one day*, we asked them to express this in the case of an endemic disease that lasted 60 years. Using the 4000-times zoom of CorelDraw, the students were naturally led to express the duration of one day as *dt*, "a little bit of", as in Thompson (1914).

[8] See the video in https://cabraldinos.mat.br/the-grounding-quilted-point-or-mathematics-of-the-20th-century/. The students authorized the publication of their names.

Point 5 is the grounding quilting point of twentieth century mathematics. It marks the moment when mathematics broke apart from philosophy and started determining its objects by arbitrary definitions solely aiming at the coherence of its global structure of quilting points. Hilbert hoped for absolute coherence of such language conventions. Gödel showed that this was impossible.

Quilted speeches make possible the historical development of the mathematics community and those quilting points make the *minimal units* of proofs. Therefore, *teaching proofs presupposes teaching the function of quilting points*. If students miss this point, they may continue looking for unquilted justifications, like the mathematicians of the past, and errors are bound to come up.

Evaluating effort

In our classroom practice, students are required to individually justify randomly assigned quilting points or sets of quilting points in previously assigned WS, thereby entering into a dialog with the teachers. *How much effort the student made to prepare herself for this moment becomes evident to all.* Close eye-to-eye conversation eliminates questions about the authenticity of answers. No written homework is required.

Studying a WS is an effort that can be demanded from any student of whatever cultural background. Contrary to what happens in evaluation of learning, this form of rewarding happens at the very moment and situation of understanding; there are no imponderable factors working overnight. Insufficient effort may lead to failure, and failure has an unavoidable economical implication. Thus, competition may still be present, but its criteria are explicit and measurable; thanks to systematic video records, grades may be contested by the students. If group heterogeneity becomes evident, it can be dealt with by establishing handicapped subgroups. Of course, we also evaluate learning for our own sake, not as a criterion for promotion. After a certain level of learning is achieved, it is not necessary to distinguish students with grades.

Does our strategy of rewarding effort ensure learning? It certainly ensures a certain network of learning, but this network must be evaluated by longitudinal and comparative mathematics education research. However, such research may reserve unpleasant surprises to those who support evaluating learning instead of effort, as Thompson (1994) showed.

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Critical mathematics education and social movements: Possibilities in an LGBT+ host house

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The LGBT+ community has historically been excluded from different spaces and has suffered with the prejudice and violence in Brazil and worldwide. The school has a fundamental role in the struggle for a less prejudiced society that respects and values differences. Through a study carried out in an LGBT+ host house, we will discuss in this article how spaces like this can become spaces for transformation and that bringing knowledge produced in these places to school and, specifically for mathematics classes, is a way to contribute to the struggle for a more just society.

Introduction

In the book *Pedagogia da Indignação*, more specifically in the chapter "Alfabetização e Miséria", Paulo Freire (2016) talks about an experience in Recife/PE, where he was walking with an educator friend in a slum in the city where many families lived from what they collected from a local garbage dump. The main question discussed was: "What to do in a context where many people are denied their humanity?" This is a recurring reflection of/in this book in which the author places/identifies indignation as essential for mobilizations to happen and changes to take effect.

In another chapter of the same book, entitled "Do assassinato de Galdino Jesus dos Santos – índio Pataxó", Freire reflects on what happened in the early hours of April 20, 1997, when a chief was murdered in Brasília. The indigenous leader, who was resting at a bus stop, was found by five young men who arrived by car and set him on fire. When they were accused of this inhumane act, they justified it by saying they believed he was a person on the street. Freire presents his indignation about this dehumanizing fact:

> What a strange thing, playing killing Indian, killing people. I keep thinking here, plunged into the abyss of a profound perplexity, amazed at the intolerable perversity of these young men disfiguring themselves, in the environment in which they have decreased instead of growing (Freire, 2016, p. 75).

This type of violence, suffered mainly by groups that we call underrepresented, is unfortunately common in our country. In this work, we will highlight the LGBT+ community (Lesbians, Gays, Bisexuals, Transsexuals, Transvestites, Transgenders and more) that has faced several struggles in our country. The reports of the Gay Group of Bahia carried out annually registered 329 deaths of LGBT+ people due to hate crimes in the year 2019 (Grupo

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Gay Bahia, 2019). In other words, several LGBT+ people are murdered in Brazil on account of their gender identity or sexual orientation.

These statistical data and the situations presented by Freire are examples of oppressions that underrepresented groups have faced in Brazil. When we look at the participation of these groups in decision-making spaces, such as political spaces, it has been very low, which can lead to the difficulty of public policies being designed and implemented in order to alleviate these situations of violence.

In Brazil, the National Congress is the national institution that exercises legislative power in Brazil and is divided into a chamber of deputies and a federal senate, where the chamber of deputies is understood as representatives of the people and has representatives in proportion to the number of voters in a given state and the senate has 3 representatives from each Brazilian state.

According to research by the National Congress, in the four-year term that began in 2019, of the 513 representatives of the people at the national congress, only 77 were female, 125 black and only one admitted to belonging to the LGBT+ community. These data do not portray the reality of the Brazilian population, because, for example, the Superior Electoral Court (TSE) points out that women constitute 51.3% of the total voters. As this data shows, we can think about representativity in the political space. However, when we discuss participation in formal or informal spaces, including the media, universities and the work market, we are also talking about representativity

As a result, underrepresented groups have been participating in social and collective movements in the search for a more just society. Butler (2018) highlights the importance of the struggle of social movements and emphasizes that not not all social movements are necessarily democratic, therefore, they need to be looked at carefully.

Keeping this argument in mind, we emphasize the importance of the role of social movements in the struggle for a more just society, promoting attitudes that favour the conquest of spaces by underrepresented groups to assist in the search to overcome stigmas and break barriers. The work developed in these spaces can also be understood as a promoter of learning, so the struggle is pedagogical (Gutstein, 2018). As an example, we can mention the work developed by some LGBT+ host houses in Brazil that are spaces that offer support to people who have been expelled from their homes on account of their sexual orientation or gender identity. The doctoral research (Barros, in progress), with which this article is linked, was conducted in one of these spaces. Information about the research will be presented below.

Conducting the research

The research reported in this article was carried out in a host house for LGBT+ people called Casassa in the city of Presidente Prudente/SP. The production of data took place in the months of August and September 2019 and the researcher sought to get involved in all the activities promoted, in addition to offering conversation circles.

Casassa is not for profit and everyone who works there is a volunteer. Its creation took place at the II Diversity Week that took place in the city of Presidente Prudente/SP in 2017. At this event there were several moments of reflection on gender and sexuality and in one of the activities the discussion arose about the need to have a permanent space so that welcoming work could be done to provide for LGBT+ people who were expelled from home and furthermore, that it was a space to hold discussions on relevant topics.

Casassa, not having any type of financing, carries out its work using money from donations and events that are promoted by volunteers. Every month there is a bazaar, called Bazarsasso, at which items donated to the house are sold and all the money goes to the local expenses. Educational and cultural events such as parties, English courses and workshops have already been held and promoted by the Casassa team.

Volunteers' work is organized by working groups. This work involves: reception, dissemination, donations, education, financial, legal and events. Casassa has two main purposes: welcoming LGBT+ people and carrying out cultural and educational activities. Thus, the word "welcoming" has two possible understandings, being a space for temporary housing of people who were expelled from home on account of their sexuality or gender identity and also a meeting place for discussions and interactions focused on the LGBT+ movement.

In August 2019, the researcher had his first face-to-face contact with the house, participating in meetings of the volunteer team, of which seven were interviewed throughout the month. The purpose of the interviews was to understand how Casassa originated, how work has been developed, relations with the community, the question of educational activities and the place of mathematics in this space. Participants were informed about the purpose and nature of the interviews and signed the Free and Informed Consent Form. The interviews were recorded in audio.

In September, two circles of thematic conversations were held. The first promoted discussions on representativity, focusing on the presence of LGBT+ people in politics. The second conversation circle dealt with the issue of visibility and stereotype thinking about how LGBT+ people are portrayed in diverse media. Casassa volunteers participated in the conversation circles, welcomed and also generated interested people from the community. In this stage, audio recordings were made during the activities and shortly after finishing the activity, the researcher made notes on important facts that happened.

The format of the conversation circles was based on Paulo Freire's (1987) cultural circles. These collective moments of dialogue sought to respect the knowledge of each participant and promote a democratic environment to reflect on reality inherent to the LGBT+ community. Critical Mathematical Education was used to help us think about the themes of representativity, visibility and stereotype thinking in the conversation circles. Looking at the mathematics that is involved in these processes helped us to compose the electoral system in the political space and to reflect on possibilities for social change. The theory from Critical Mathematics Education used to support the design of activities is the reading and writing of the world with Mathematics (Gutstein, 2006), where mathematics is used to promote

discussions that help in the understanding of situations in society and also helps in the realization of transformations.

Gutstein (2006) makes use of investigative activities that make a movement that starts from issues for the community and goes to the classroom. We sought to be inspired by this movement to bring issues that were relevant to the LGBT+ community to be topics of conversation circles. In addition, the construction of proposals and choice of mathematical tools and strategies had as protagonists the participants who reflect and put mathematics into action.

At this point in the research, the transcripts were having been completed, both of the interviews and of the conversation circles. The records have been organized and relationships are being established with the researcher's notes. This is a complex process that is not linear. The analysis has been carried out taking into account the research question, the objectives and the aspects that stood out in the data.

A close look has been started in order to understand what possibilities of reading and writing the world with mathematics can emerge from a social movement. Based on these reflections, I bring below some notes that help us to reflect on how Casassa (as a social movement) collaborates with the community in which it is inserted.

A host house changing a community

Casassa's proposal is to be a welcoming home, but this goes beyond the idea of housing. José brings this perspective by talking about how his friend Larissa, who is a transvestite, has experienced society as not safe for LGBT+ people*:*

> This whole neighbourhood is a place and having a transvestite living two blocks from here, there is Cláudio who will live two blocks from here too, because having Casassa makes these people more comfortable in that space. I remember when Larissa, this transvestite friend from the neighbourhood lived in another place, a neighbourhood far away and I went to visit her one day in the morning, we went from her house to the bakery which is two blocks away and just walking this way I came back tired, because there were a lot of whispered looks and comments with us passing by. They were simply looking, they didn't need to speak or shout anything, but I realized that Larissa was attentive all the time. That is why I hope that the whole community can be here at Casassa and that they get to know the LGBT + people to make the whole neighbourhood more welcoming, changing the structure of the place.

One of Casassa's goals is to create a host and support network that goes beyond the physical location restricted to the host home. In this quest, making a simple exercise like going to the bakery safer for LGBT+ people are a necessary exercise. However, for this to happen structural changes are necessary for society to understand the importance of respecting differences.

Another Casassa volunteer, Davi, sees the host house as a motivator for discussions on the reality of the LGBT+ community, after all it leads people to think about this, even though sometimes the discussions are initially marked by stereotypes:

> I think it influences in a positive and negative way. We have seen both things happen. Because it is an LGBT+ host house, it has the common sense that it is a place that is related to sex, so there are a lot of people who look here with a crooked eye and do not have the courage to come and ask what is happening and are judging. The positive impact is that people in need of care have a hope of knowing that there is a space like this.

This discomfort highlighted by Davi helps to get people out of their comfort zone and makes them think. In this reflection on the discussions on the reality of the LGBT+ community, it is possible that there will be an appreciation of Casassa and consequently of the LGBT+ community. Corroborating José's observations, Davi also reinforces the importance of the existence of host houses so that LGBT+ people can feel more secure in the process of developing their sexuality or gender identity.

Another interviewee, Olga, reports that the volunteers are beginning to understand the impacts of Casassa on society and that the host house represents hope for those who are expelled from home, in addition to being a space that promotes necessary discussions on LGBT+ reality:

> It has a significant impact in many ways. First the impact of the service provided, let's say that having this welcome is a refuge for these people, not only for those who are expelled, but those who were there without having a reference and did not understand their sexual orientation and gender identity and, suddenly, to see people thinking "Ah, I'm like them". It is an impact of visibility, because it started to be talked about this subject in a more open way. We do not know very well if it reached certain layers of society and if we arrived in many places, but we had contact with some businesses that offered support to us. Universities too and social movements themselves. I think the main thing was visibility as a whole. I see in my own family, when I say that I am working on this project, people start asking me, "What is Casassa?", "Do you accept it?", "Do you have families that expel children from home for this?". Have! So, it was this opening to a reality that sometimes these people did not know. We need to understand that there are realities different from ours and that something needs to be done.

A common response in the interviews was that the interviewees did not want a host house for LGBT+ people to be needed, after all, nobody should be expelled from their home on account of their sexuality or gender. However, we must emphasize that it is important to have host houses such as Casassa in the current society.

About the presence of mathematics in the daily life of Casassa, the volunteers had a little difficulty in saying about aspects that could be seen in their activities. It was evidenced by the interviewees in the financial issues of managing the house and the bazaar that is held to raise money. However, at no time was mathematics seen as a promoter of social and political discussions. The conversation circles sought to change this perspective.

When conducting the conversation circles, it was possible to put mathematics into practice, with mathematics providing a reading and writing of the world. In this sense, themes related to representativity, visibility and stereotypes were discussed. These are pertinent issues that reflect on how society views the LGBT+ community and the participation of this group in in a range of spaces. During the first conversation circle, for

example, we reflected on how underrepresented groups participate in the Chamber of Deputies, which is one of the houses of the Brazilian legislative power.

We used concepts such as percentage to compare the presence of underrepresented groups in these political spaces and in the community, and we find that there is a large discrepancy. We reflected on the reasons for this to happen from a critical perspective. These groups have historically been excluded from electoral processes, as many political parties have been concerned with issues that address the exclusive demands of a majority of the population.

In addition, we reflected on the electoral process, after all, the elections in Brazil are made by proportional votes by party. So, the votes destined for a certain candidate do not necessarily go to them, but these votes can help to elect another person from the same party. This proportional process could be a problem in the search for representativity.

Research participants showed themselves to be engaged during the discussions. The reality data on the representation of minorities surprised the participants in the conversation circle. Furthermore, mathematics helped to look to the future and think in an ideal context.

The need for greater participation by underrepresented groups in political and other spaces was evidenced so that, through public policies, transformations can be made showing a more inclusive society with more respect. Providing this type of debate in a space like Casassa proved to be necessary so that through struggle we can increasingly learn about human diversity. These were the conclusions of the participants in the conversation circles, who came to see mathematics as a possibility to discuss social and political issues.

At this point, we can see the importance of using mathematics from a critical perspective so that people who participated in that activity could reflect on the reality of the LGBT+ community and the participation of this group in politics.

These important discussions promoted in a space like Casassa can and should be taken to schools. However, this is not a simple task, since the advancement of conservatism has made it difficult for discussions about gender and sexuality to be held in Brazilian schools. However, teachers must take on this important task of promoting these discussions in their classrooms.

Conclusions

For change to happen, we cannot be unaware of the world's problems. We must be indignant about injustices, as proposed by Freire (2016) and in order to collaborate with the struggle of the oppressed, we must promote a pedagogy of resistance, where one learns to fight, fighting. Involvement with social movements in this sense is shown to be powerful, after all it is in these spaces that underrepresented groups organize themselves and carry out mobilizations seeking a more just society.

Through the realization of this research, we see that a space like Casassa is a pedagogical space. The engagement in the struggle of the LGBT+ movement can take place in several ways, but it is essential for us to overcome the prejudices that this community has suffered.

We as teachers are invited to be in these spaces participating in these discussions, but it is also important that we take these debates to our classes.

Mathematics showed its potential to collaborate with the reading and writing of the world when used from a critical perspective. The activities used in the circles helped us to think about ways to transform the reality of the LGBT+ community, which has still been largely excluded from society.

We fight for a less prejudiced society in which we will no longer have, for example, people being expelled from their homes on account on their sexuality or gender identity. In this sense, we believe that mathematics has the potential to support much needed discussions about diversity and difference.

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Engagement and resistance in an equity-focused professional development: Toward caring with awareness

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Teacher development toward equity should support teachers in seeing the ways in which their practice is part of larger social and political histories and structures so that they can then disrupt oppressive systems influencing students' opportunities to learn mathematics. We examine one teacher's engagement with an equity-focused professional development designed with this need in mind and explore what her experiences suggest about transitions from caring to caring with awareness, explicitly considering race, culture, gender, and power in relation to academic achievement.

Efforts in mathematics teacher education have long emphasized the need to prepare teachers for teaching diverse populations of students in ways that provide equitable opportunities to learn mathematics. However, mathematics education is positioned as part of a larger, neoliberal narrative disconnected from the experiences and realities of students from non-dominant communities (Martin, 2013). Neoliberalism reflects a shift in education from a social to a market concern and contradicts a focus on students' communities, beliefs, and points of view (Lipman, 2012). Teacher development toward equity, then, should support mathematics teachers in seeing the ways in which practice is part of larger social and political structures so that they can then disrupt oppressive systems influencing students' opportunities to learn mathematics.

The Access, Agency and Allies in Mathematical Systems (A3IMS) research project aimed to address these needs, designing and studying a professional development (PD) around five strands: mathematics, discourse, privilege & oppression, culture & community, and action research. Here, we examine one teacher's – Julie's – response to these PD efforts. On one hand, Julie was engaged in the PD and positioned herself as having good relationships with her students and putting students first, always. On the other hand, Julie's engagement was frequently marked with emotive language that suggested dislike and resistance to some PD

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efforts. In short, Julie was a complex participant who challenged us as PD facilitators, showing signs of being a caring teacher but not yet demonstrating caring with awareness (Bartell, 2011), or caring that includes explicit focus on racial, cultural, and political dimensions affecting relationships and mathematics education. We explore the question: *What does Julie's experience with the PD suggest about the transition from caring to caring with awareness in mathematics teaching?*

Background Literature

One long-accepted characteristic of an effective teacher is the ability to cultivate and maintain strong interpersonal relationships with students, often described as caring (Noddings, 1992). Caring is a process; it is something teachers do rather than something teachers feel. Caring relationships between teachers and students lead to higher levels of student engagement and achievement (Pianta, 1999). Mathematical Caring Relationships include teacher's choosing appropriate problems to pose based on students previously demonstrated mathematical reasoning, supporting students' mathematical learning (Hackenberg, 2005). Caring relationships in mathematics classrooms must also reflect *caring with awareness* (Bartell, 2011). Such caring relationships "acknowledge racial identity, culture, racism, and racial privilege as factors that shape and colour experience" (Thompson, 2004, p. 26). For teachers of mathematics, awareness includes an understanding of how mathematics has been used to socially partition individuals by identity markers such as race, gender, and ability. Caring mathematics teachers value and support Students of Colour' status, identity, and prior knowledge and centre issues of race and ethnicity in relationships with students (Rolon-Dow, 2005). Teachers demonstrating *caring with awareness* explicitly reject deficit perspectives of students and their communities. They do not attach mathematical (and other) "failure" to a student, but to themselves, searching within to find a more effective way to reach students (Collier, 2005). They explore students' mathematical thinking and in- and out-of-school experiences to inform changes in classroom practice. These relationships are also political, where mathematics teachers take up "active political stands in solidarity with students and their communities about issues that matter" (Gutstein, 2006, pp. 132-133) and engage students in using and learning mathematics while understanding the world. Teachers demonstrating *caring with awareness* work to ensure that the curriculum reflects the lived experiences of their students, recognize existing oppression in their students' lives, and seek to use their own status to encourage students (and themselves) to understand and undermine those oppressive conditions (Beauboeuf-Lafontant, 2005).

Teachers, and particularly White teachers, often resist discussions of race, culture, and power with respect to teaching and learning. Such resistance can take many forms. Hytten and Warren (2003) identified rhetorical strategies their study participants used in discussions of systemic oppression, including ignoring colour, focusing on progress, victim blaming, and focusing on culture rather than race. For example, people might make statements that suggest people of colour are racist, too, or complain in ways that position White people as

victims in interactions around race. Or people might argue that they "do not see colour," fuelled by a belief that by ignoring colour, racism is minimized. However, such colour-blind stances allow people to deny that "race, especially skin colour has consequences for a person's status and well-being" (Rosenberg, 2004, p. 257). It allows White people to dismiss their complicity in racial oppression. This dismissal of complicity is echoed in Annamma et al.'s (2017) expansion of colour-blind ideology where they argue that "colour-blind" is inadequate because it suggests passivity, hides the social construction of race and disability, and ignores the entanglement of these two in oppression. They instead define colour-evasiveness to call attention to the fact that avoidance of discussion about race is deliberate obliteration of the experiences of People of Colour.

It is important to note that many White women have been socialized toward resistance. Women tend to be socialized to avoid conflict (Gillespie et al., 2002). For White women, this avoidance of conflict exists within a social norm that talking about colour or difference is impolite (Mazzei, 2004). Thus, White female teachers [live within] ideologies that expect them to be complicit in the oppression of people of colour because of the expectation of gendered conflict-avoidance and deference. Instantiations of this ideology of the feminine interact with the ideology of colour-evasiveness in a manner that reifies the dominance of White male power and rests it in a space of academic achievement measured in abled ways. Teaching was historically viewed as women's moral obligation as mothers to raise a child as a productive member of the society (Grument, 1983). Relations of power and authority have historically been formed as patriarchal where teaching is considered both a feminine occupation and a kind of domestic service conducted by women, reifying a "caring teacher is a good teacher" narrative (Apple, 1983).

Method

Thirteen teachers from a small urban district in Granite City, located in the Midwestern U.S., signed up to participate in this PD. All the teachers were women; nine were White and four were African American. Julie is a White, middle-class, heterosexual woman who had taught in Granite City schools for 12 years. She grew up in Granite City but did not attend Granite City Community Schools (GCCS); rather, she attended schools also located in Granite City that she described as "straight down the street." Julie began teaching in GCCS as a graduate student; her undergraduate degree is in computer science and her graduate degree is in curriculum and secondary education. Most of her 12 years of teaching in GCCS has been at the high school level. At the time of this study, Julie was a math intervention teacher in a school that served grades 7-12.

The professional development

The PD was designed around five interrelated strands as noted above. The *mathematics* strand engaged teachers in mathematics tasks and in adapting curriculum to engage students in inquiry-based tasks and social justice activities. The *discourse* strand highlighted cultural assumptions embedded in expected mathematical discourse and built understanding related

to how these discourse practices contribute to students' positioning and developing identities (Herbel-Eisenmann et al., 2013). The *privilege & oppression* strand gave teachers an opportunity to explore their own racial, gender, class, etc. identities and consider the systemic influence of oppression on mathematics teaching and learning. The *culture & community* strand focused on developing teachers' recognition of the resources for mathematics learning that students bring from their community and cultural experiences. Finally, the *action research* strand emphasized the process of systematically examining, reflecting on, and making important changes to mathematics teaching practices that teachers identified that they wanted to improve. Each year of the PD began with a week-long institute followed by twice-monthly PD meetings throughout the academic year. The first four strands were woven together such that individual activities often reflected the goals of more than one strand (see https://olemiss.edu/a3ims). These four strands were the focus of the institute and Fall PD sessions to inform teachers' action research projects in the spring.

Data Collection & Analysis

Any activity that had goals related to the *privilege & oppression* strand or the *culture & community* strand in the first two years of the PD were selected for analysis, representing 81% of PD activities in those two years. All whole group interaction was video recorded and small group discussion was audio recorded. Interviews were conducted with Julie prior to the start of the PD, at the end of the first year, and at the end of the second year. The first interview collected demographic information, teaching history, and asked Julie what she thought equity meant and to describe successes and challenges specific to equity in the mathematics classroom. The final two interviews asked for Julie's reflections on aspects of the PD she found useful, her conceptions of equity and good mathematics teaching, and about tensions that she might be feeling with the dual focus on mathematics and equity in the PD. Finally, at two points during the first year, Julie's teaching was observed, and video recorded and a follow-up noticing interview took place where Julie watched a researcher-selected clip of teaching or student engagement and probed for Julie's thinking about the clip.

To analyse the data, three researchers read through the data set independently and identified three broad codes to look at more closely: Julie's positioning of self, students, and families and communities. We compared our coding and refined codes, resulting in these code descriptions: *Self* focused on Julie explicitly, consciously positioning herself in relation to mathematics, various communities and groups of people or other teachers. *Students* included Julie positioning herself in relation to a particular student as well as to students in her classroom, school, Granite City, and writ large. *Families and Communities* included references to the Granite City community, other cities and school districts, her personal family, and the PD community. The first author then analysed the text within these three broad codes, grouping similar concepts (e.g., Julie talking about an effective teaching practice) into themes. Transcripts were read and re-read for confirming and disconfirming evidence of each theme.

Findings

In our pre-PD interview with Julie, she noted that she enrolled in this PD largely for financial reasons, as she had just purchased a car. At the same time, she was oriented toward this PD as an opportunity to "get involved, to better my practice, to better myself, and to provide for my students." She had not attended much to the PD goals and aims, perhaps evidenced by her response that what equitable mathematics instruction meant to her was "like an investment, but I don't know what you mean by equitable." A moment later, when reflecting on challenges in supporting equitable teaching, Julie said that accountability was key:

> If a teacher makes a recommendation, it needs to be followed. If it's not followed, then the person that didn't make the change happen needs to be held accountable. Because we're hurting the student. It's not about us. It's not about me…it's about getting what's best for that child.

Julie's assertion that a key feature of her teaching is building strong relationships with students and prioritizing the needs of students was a consistent PD theme.

Strong Relationships, All About the Kids

Julie regularly shared statements across the PD that suggested that she believed she had strong relationships with her students. Prior to the start of the PD, when asked to elaborate on what she meant by getting to know her students, Julie described:

> It allows us to have empathy for our students. There may be a student that comes in and is sleeping in class. You don't know what they went through. You don't know if they were in the hospital all night. You don't know if they had to watch their baby brother because Mom had to go to work. When you have those kinds of things happening in your classroom and the administrator comes in and says, "Why is their head down?" you have an explanation because you're able to show that empathy and understand what the kid is going through. There are times when you may not be doing the curriculum. You may have to stop and talk about those recent shootings in those areas. What if it happens in Granite City?

During the first summer PD, Julie shared stories about students reciprocating care by calling her to see how her mom was doing when her mom was ill and she suggested that she had stronger relationships with her Black students than her White students:

> I feel like my Black students will ask more questions than my White students. I don't know if that is a cultural thing because they feel comfortable talking at home with other people. I mean, I relate better to Black people than I think I do with White people.

During Year 2, after attending a talk with Dr. Jeff Duncan-Andrade, Julie reflected, "I do that on a regular basis. I have that relationship with my students…I'm empathetic. I know that I feel what my students feel. If they're cryin', I'm cryin." At the end of Year 2, Julie positioned treating students with empathy in relation to culture and race:

> I understand that we have cultural issues. We have differences in our lives, and I get that. Every person is different, but just because you are a specific colour, you're a specific gender, that does not matter. It doesn't matter because I believe everybody can learn. I don't think our society believes that, and I think that's why we talk about social injustice so much, and for me it makes

> me angry because instead of talking about it, why aren't we doing something about it. That's what I do in my classroom. I do something about it. I don't look at them because they're Black. I don't look at you because you're Asian. You're a person. I treat you with decency…with humanity, compassion, empathy.

Julie's narrative about having strong relationships with her students was often voiced through her assertion that teaching is "all about the kids." During the first summer, teachers were asked to bring a cultural artifact with which to introduce themselves. Julie shared that she forgot to bring something, but that she knew her artifact was her students. She demonstrated her passionate stance and love for her students with tears:

> Since I was in 9th grade, I've struggled with depression…and it wasn't until a few years ago that I ended up going to a therapist…and it wasn't until that, those couple of years ago, that I realized what my purpose in life is. I have such a love for my students (begins to cry), they saved me. They have taught me so much.

Julie also demonstrated a passionate stance about teaching being all about the kids when she shared stories of speaking back to administrators, justifying her teaching and "standing up for what we believe in." During Year 1, for example, Julie expressed a tension between being "pleasers of the government because they pay our paycheck" and education needing to be all about the kids. She elaborated:

> It's about what the kids need, about what the kids want, and far too often we push kids on and push kids on and push them on, and they are not where they are supposed to be. It's a growing problem that I don't think our government recognizes. We're limited with you've gotta do this curriculum…you've gotta have this on the board. It's very difficult and frustrating because I just want to come and teach. I wanna be here for the kids. I wanna support the kids. If it takes me two days to go over something because they need that time, then I need to have that freedom, and I don't feel we have that.

In Year 2, Julie continued to stress the importance of instruction being all about the kids, even as she acknowledged that this could be exhausting and demanding: "we may be so burnt out, but we still have a responsibility to those kids…we're supposed to be a friend, confidant, mom, dad…the kids are what is supposed to be our main focus." Julie positioned this stance as standing up to those in power:

> We need to stand up for what we believe in. I understand that we are supposed to follow rules and do this and that. I get that, and I'm gonna do that. I'm gonna make sure I get there on time. I'm gonna make sure that I'm teaching my students…If [district mandated curriculum] is not providing my students with the best education, that's where I have to use my professional judgment to modify, to provide those kids with the education they want. I'm not going to provide them a crappy education…it's time we take back our schools, and we take back our classrooms, and we do what we know is best for our kids, no ifs, ands, or buts.

Throughout both years of the PD, Julie asserted teaching as her "mission field" or the place "where I am supposed to be." Julie seems to position her mission not so much about saving students, but about being the best teacher she can be:

> I have always felt that, since I've been involved in my teaching, that this is my mission field. I wanna be the best I can be for these kids, and I can't do that by myself. I need others to support me, to encourage me, to discipline me…to become a better teacher.

This notion of continually growing and working to know her students to "be the best teacher she can be" occurred throughout the PD. At the end of Year 1, Julie reflected:

> I don't feel that I know my students as well as I would like to know them. I would like to know what kinds of things they're going through…I feel like they're my children. I feel like I'm like a mom to them. I love them. I care about them. I don't want anything to happen to them. Far too often we don't know our students like we think we do… We had a student who died in a house fire. Did not know he had a sister. Did not know he had a stepbrother here in school. I didn't know that. Here I'm thinking I know the kid, but I really don't.

In Fall of Year 2, Julie again voiced that she doesn't "always know everything they're interested in" and in the Spring of Year 2 that she doesn't:

> Have the experiences that they have. I've not grown up in the same kind of culture they've grown up in, so what I think might be relevant may not be relevant for those students. I think there's that big cultural difference. I'm still learning.

Anger Toward and/or Dislike of PD Activities

In some instances, Julie seemed to use the defence of instruction being all about the kids as reason not to engage in politics or assertions of Whiteness. For example, during the first summer she stated:

> I'd say about 60 percent of the time, the parents, without finding out the facts, side with the child. Because it's automatic you're out to get them for some reason. I feel like, as a White person, I have that stereotype that I'm going to try to take advantage of a person's colour. That's hurtful to me. Because I love my kids…I will do anything for them.

Later in Year 1, Julie lamented that "people are so political about everything that they forget to think about the kids." She went on to say:

> Even though the district may require something, you can still make it about the kids. It's frustrating to hear how colleagues feel. They're worried about that student evaluation. That's not what you're here for. You're here for these kids. If you didn't meet the expectations…what are you gonna do to improve it? That's how I feel we have to look at it. I think I'm different from how most teachers feel and think, because I want what's best for the kids. It's not about me, it's not about the job, it's about the kids.

Julie also expressed frustration or dislike with PD activities throughout the two years, even if those activities, from our perspective as facilitators, put students first or had the potential to strengthen her relationship with students. For example, in the first summer when provided a handout that listed types of oppression (e.g., racism, elitism, ableism) in Column 1 and then non-target groups (e.g., White people formally educated, temporarily "able-bodied") and target groups (e.g., People of Colour, informally education, people with disabilities) in the next two columns, Julie immediately voiced:

> I can tell you right now I don't like this activity…What I feel when I look at this is division. I feel like, in our country right now, we are dividing ourselves so much already that –I don't look

> at these individual – I look at the person as a whole. I don't necessarily relate with everything on here. In racism. I don't necessarily relate better with White people…it makes me angry because I don't feel that we're looking at unifying as a whole.

Similarly, Julie's discussions of race were often points of resistance to the PD activities. For example, at the end of Year 2 Julie noted:

> I don't look at their colour, their gender…Yes, they're present, but I don't focus on them because they're a human being that is just trying to find their place in this world. Listening to them I think is the best thing that I can do for them, rather than trying to judge them, or tell them they can do this, or they can do that…[later] I understand that we have cultural issues. We have differences in our lives…but just because you are a specific colour, you're a specific gender, that doesn't matter. It does not matter because I believe everybody can learn. I don't think that our society believes that, and I think that's why we talk about social injustice so much, and for me it makes me angry because instead of talking about it, why aren't we doing something about it?

Julie's resistance was not limited to colour-evasiveness. For instance, in Year 2 after engaging in an activity adapting problem contexts to be more authentic to students' lived experiences, Julie expressed that it "gave me anxiety because our students, I don't focus on the same things our students focus on in culture." She went on to say:

> I do see the value in what we're trying to do, but I also see that it is a problem. … because if we look at how tests are presented to our students, and I'm not saying we need to teach to the test, but if they are not exposed [they'll have more trouble].

This desire of wanting the students to succeed academically as a source of resistance to the PD was inconsistent; while here, during Year 2, she expressed resistance to the activity, at the end of Year 1, Julie noted that although she became frustrated with some of the first-year activities, they were a source of growth/learning:

> What I've been learning about myself this year is some of those things that frustrate me the most are the things that are what is making me a better teacher. That frustration turns into memorable moments, powerful moments that help me become the individual that I am. Not necessarily just as a teacher, but as an individual as well.

Discussion

Julie's descriptions of her teaching indicated a caring for her students rooted in ethics and reflected some components of caring with awareness. She is working toward knowing her students culturally, acknowledging difference and race, and recognizing that knowing her students means knowing them beyond the school's walls. Her anger about demands on teachers that she saw as counter to teaching, such as district-mandated pacing, were expressed as care that considered the well-being of the students and could be interpreted as awareness of power structures; she was recognizing that in this realm she can exercise a type of specialized resistance (Collins, 1990). At the same time, in working to know her students beyond school walls, it is not clear that Julie has gotten past "dominant stock stories" (Rolon-Dow, 2005) or that she wanted to know her students well racially. Yet, she also was sometimes

reflective and glad for her growth related to learning about racialized experiences and students' cultures.

Applying a colour-evasive framework (Annamma et al., 2017), we see that the ideology Julie has taken up about race constructs race as an objective difference residing in the individual and does not include an understanding of how race has been socially constructed. Nor does it include the impact of the social and material consequences of racism on her students in their role of student or the educational structures that perpetuate racism. Julie's desire to ignore difference essentializes her students and neglects their complete humanity in favour of only specific parts of their humanity. Julie's belief of herself as a teacher who cares seemed to be a strong part of her identity as a teacher, and so it is possible that facilitating her toward growing the caring relationships to be more aware was felt by her as an attack on an identity she cherished, or even an attack on her students (and their opportunities to learn mathematics). Of note, Julie's assertion that she feels "like a mom" to her students reveals gendered assumptions in her role as teacher, and assumptions about what her students need: the unfit mother is a common stereotype of Black urban mothers in the United States (Gholson, 2016) and so Julie's assertion of taking up that role reifies this storyline. Julie's inconsistent colour-evasiveness made it difficult to know how to support her development from a caring teacher to one who cares with awareness.

Julie's responses in the PD align with documented acts of resistance when White people are confronted with conversations about systemic racism. But her inconsistent gender/colour-evasiveness also suggests that PD could meet her at her points of development and reflection and differentiate activities and experiences to target Julie's particular needs. In Julie's case, perhaps we would have leveraged differently her relationship with her students, having her listen to her students' stories of lived, racialized experiences and using her own students' voices as counter-narratives to her idea that talking about race is divisive and antithetical to caring. Or, instead of focusing on a multitude of intersectional identities, her experiences at some point should have been more narrowly focused on systemic racism, gendered uptake of roles, and their connection to students' opportunities to learn mathematics.

We wonder what it might mean to differentiate PD not only across participants and multiple intersectional identities, but also within groups of participants that share some characteristics (e.g., race and gender) but are at different places in their understanding of the role oppressive systems play in mathematics education. This differentiation likely requires PD facilitators to know their teachers, teachers' contexts, and even teachers' students to support each individual participant toward caring with awareness.

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Building agency in children through mathematics: Applying Conscious Full Spectrum Response for developing skills, competencies and inner capacities

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Sustainable, equitable and enduring solutions to the complex problems of the world require not only technical solutions, but also shifts in structural and social norms of society grounded in responsibility and interconnectedness. How do we as teachers look at these aspects? In this action research we develop a perspective through the framework of a Conscious Full Spectrum Response (CFSR) model to develop not only academic and technical skills in Mathematics, but also competencies – using skills to shift systems and culture and inner capacities - self-awareness, self-regulation, responsibility and courage to create. These together build agency – the ability to act and transform based on what I deeply care about. We review case studies of the work of children both academic as well as real life projects with this framework.

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Context

C3STREAM Land (C3 is *C*onscious for Self, *C*onscious for Others, *C*onscious for Environment, STREAM = STEAM + R (Research), henceforth referred to as C3SL) are rural STEAM centres in Tamil Nadu in India. STEM education can become "technology for the sake of technology" and miss out in addressing social, cultural and structural biases and disparities, it can also ignore the development of inner capacities of children. C3SL strives to address these as the deeper purpose of education.

Radical transformational leadership

Radical Transformational Leadership, the book, describes how we can generate equitable and enduring results using a unique response model based on extensive application world-wide in many sectors, themes and topics – the conscious full-spectrum response model. This model is designed for sourcing our inner capacities and wisdom to manifest change that embodies universal values of dignity, compassion and fairness, and simultaneously transform unworkable systems and norms in order to solve problems.

While each of us have an accountability of teaching Mathematics we are also trained through RTL workshops on the distinctions, design templates and tools. The distinctions allow each of us to work out of what I stand for (universal values I deeply care about) rather than out of socialized fear, the design templates allow for alignment of universal values, system and cultural shifts and actions when we design projects; the tools formalize processes that are cognitively coherent with the distinctions and templates.

Introduction to the Conscious Full Spectrum Response model

The Conscious Full Spectrum Response model is used to generate results at scale and addresses complex problems across domains while allowing for alignment in:
Technical Solutions to solve immediate problems, e.g., employment, education.
Shifting patterns and unworkable system and societal norms required for sustainability of the technical solutions, e.g., policies, casteism, race, gender.
Underlying factors of what we deeply care about – why we want these shifts and how we act when we embody these universal values, e.g., dignity, equity.

We give an example of C3SL in Figure 1 mapped to the CFSR model and also set the context of this work. The outer circle of universal values of C3SL are responsibility, equality and the courage to create. We want to see these values in the children we work with, in ourselves and in what we do.

The middle circle addresses systemic and cultural causes here we work in rural schools we address not only the digital divide, but also use the work on STEAM to interrupt the common ISMs like genderism, class and casteism in rural India. We work equally with both boys and girls in STEAM education. We also move from mediocracy to excellence with the older children taking responsibility of learning to plan and set their goals for the week. They choose to do this individually, in pairs, in groups and in consultation with facilitators. They are also provided RTL training for children to move from dependence on teachers to independence to interdependence and creating a learning community.

In the inner circle of technical solutions, the children have access to Mathematics materials, strategy games, puzzles that help them engage with Mathematics and play games. They have access to computers where they program in Scratch, Geogebra and Alice and also 3D modelling and printing. They also have access to electronics, Makey-Makey and other materials interacting with engineers who work in the industry. These help children not only address their curriculum, but also create projects that demonstrate their mastery on topics learned. With younger children we work on making Mathematics with real life (Education by Design) EBDs projects that they can do on themes they care about.

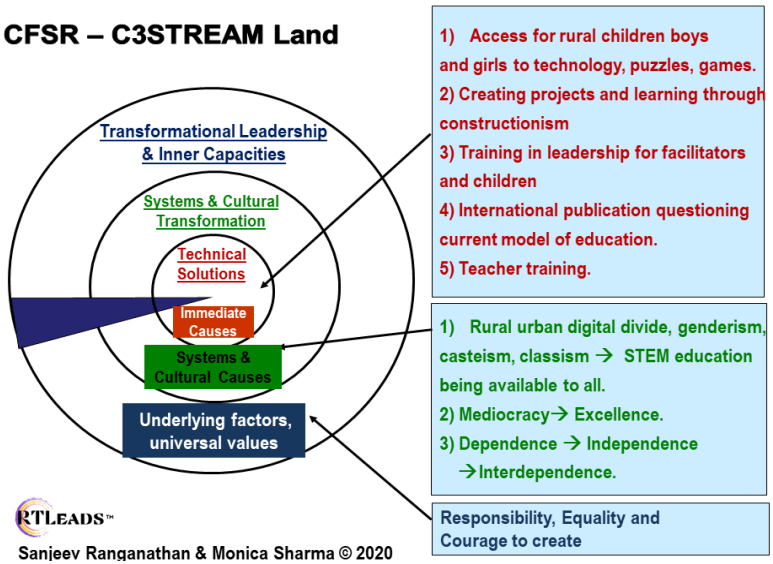

Figure 1: C3SL as an example of application of CFSR model.

All these move the children from mediocracy to excellence. The details of the activities of C3SL (formerly known as STEM Land) are documented in detail elsewhere (Ranganathan et al., 2018, pp. 294–302).

This research is conducted in two outreach schools of Auroville – Udavi School and Isai Ambalam School. The children attending come from villages surrounding Auroville. Udavi School follows the state board syllabus and we worked with 80 children from 6th to 10th grades intensively for 5 hrs/week for all their Mathematics classes. Isai Ambalam School follows the central board syllabus and we work with 71 children from 3rd to 8th grades intensively for 6 hrs/week as well as during Saturday activities and sleep overs for Mathematics as well as Environmental Sciences (EVS). In demographics, the primary occupation of parents in both schools is in unorganized labour, e.g., masons, painters, agricultural labours and schemes providing rural employment. The predominant community accessing Udavi School is MBC (Most Backward Caste) and that accessing Isai Ambalam School is SC (Scheduled Caste).

Philosophies underlying C3SL

The philosophy underlying the approach for C3SL is based on the principles of progressive and constructivist thinkers like Jerome Bruner, Seymour Papert in the United States, Sri Aurobindo in India. The philosophy of Sri Aurobindo of the integral development of the child (Aurobindo, 1921, pp. 1–8) emphasizes self-knowledge and assumes an important relevance in the recent National Education Policy (Government of India, 2020, p. 12) that is based on his work and states that "knowledge is a deep-seated treasure and education helps in its manifestation as the perfection which is already within an individual." The philosophy creates guiding principles as teachers and in how we engage with children. The three principles of true education by Sri Aurobindo are:

- Nothing can be taught
- The mind needs to be consulted in its own growth
- From near to far

The first principle can be linked to the constructivist theory that knowledge cannot be forced into the mind of a child. The role of a teacher is not to mould or hammer a child into the form desired by the adult. The teacher is a guide, or mentor that supports and encourages a child in the process of learning, enabling them to evolve towards perfection. Our engagement with children follows this principle.

The second principle indicates that the child needs to be consulted in his/her learning. This is done at C3SL as the elder children plan what they want to work on and how they want to organize themselves to do it with the broad ground rules of respecting themselves, others and the materials. With younger children this aspect was put in practice in the co-creation of challenges along with them.

The third principle is to work from near to far. To work from what is tangible and accessible to children to what is abstract to them. The children work on projects they care about in the environment they engage with and as they grow older move towards more abstract ideas. This paper will present projects both in the physical world and also in the abstract world.

Self-awareness and personal transformation are necessary, but not sufficient for social transformation. In this paper we take up a theoretical framework for social transformation that is aligned with values. This paper focuses on the application of the Conscious Full Spectrum Response capacity building framework that we use to review what we may be accomplishing through Mathematics and EVS.

Theoretical framework of CFSR for capacity development

We use the framework of a CFSR (Conscious Full Spectrum Response) capacity development (Monica, 2017, p. 236) as shown in Figure 2.

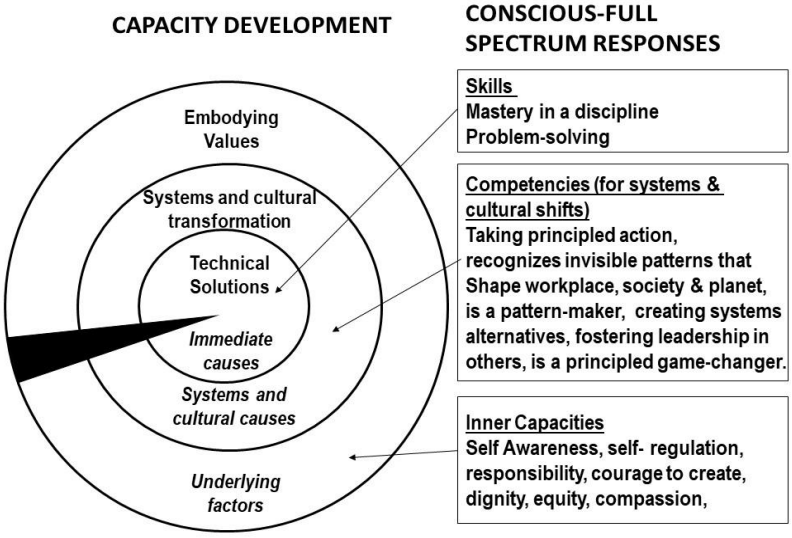

Figure 2: Capacity Development for Sustainable Results

A CSFR based capacity development simultaneously addresses:

1. Skills (inner circle) to generate technical solutions to address immediate causes. We look at mastery of the concepts as well as problem solving.
2. Competencies for systems cultural transformation (middle circle). We look at the ability of applying these skills in different contexts, pattern/system thinking, as well using skills to build healthy patterns in how children interact and learn.
3. Inner capacities to embody universal values (outer circle). We look what we noticed about children's responsibility, care and courage to create alternatives

Methodology

The topic/project and how the children went about creating them are described in each section. The skills are listed by analysis of the final product by the teachers. The competencies were observed by the teachers in the duration of the project and in conversations with the children. Inner capacities are not measured, but reflected. Opportunities were created for children to reflect on these and what were noted are derived through conversations on their reflections.

We will take a few case studies and deep dive into one of them to look at how these aspects are both supported and observed.

Case studies and observations

We first look at academic challenges and then at real world challenges. Can learning Set theory and algebraic identities be transformational?

Sets

A few children from 10th grade had built a physical game with a chart and materials with Sets. This physical game inspired Diva, a 9th grader the built a game in Scratch (a visual programming language) on sets shown in Figure 3. Each circle represents a hidden rule of either shape or colour. The player needs to determine the rules by guessing where the pieces fit. Programming helps children learn concepts because they need to break it down into simple instructions while improving problem solving, logical reasoning. The use of programming to develop mathematical thinking at C3SL has been documented before (Ranganathan, 2015, pp. 339–346). In this case, Diva realized that in order for the computer to understand which region of the Venn Diagram was being sensed by a new token (sprite) he needed to divide the Venn diagram into different regions (A-B, B-A, A∩B and U-AUB) helping him learn these better. He first made the game with a fixed rules and later generalized it for the computer to randomly pick the rules so it would be a challenge for him too. At C3SL we have sessions where children share their projects. Diva presented his project at one of these sessions. His presentation got the 8th graders, who were not expected to learn about sets, to learn about sets.

C3SL also organizes courses to learn programming to that gets children to make smaller projects and learn through interactions with peers and facilitators. After one such course the following project was created.

Algebraic identities

A few children from 8th grade created projects on algebraic identities. For example, Jan made a program that drew $(a + b + c)^2$ as three squares, i.e. a^2, b^2, c^2, and $2ab, 2bc, 2ca$ as areas of rectangles. Images such as these were also available in the text book, were static, but when children created them in scratch they were able modify their programs to use variable lengths for a, b, c and see for themselves that even with the different lengths the identity still held.

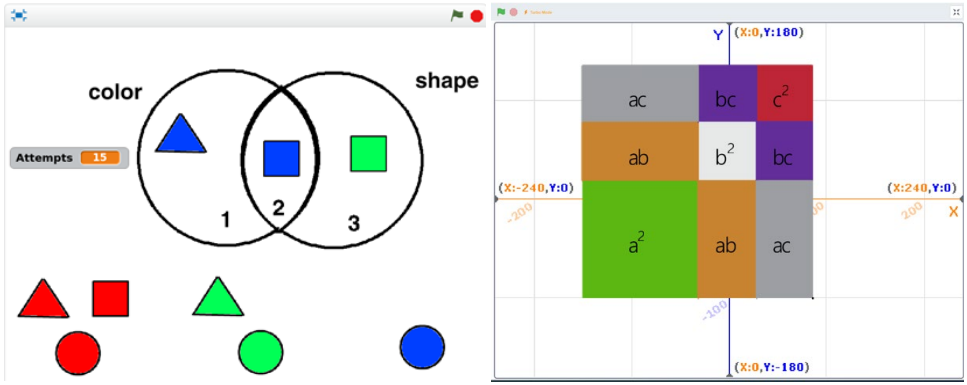

Figure 3 & 4: The two 2-sets game in scratch, algebraic identity in scratch

Observed skills, competencies and inner capacities

In the two virtual activities above through the CFSR framework for capacity development we see:

Skills: In academic skills they learned the different sections of overlap of Venn diagrams with 3 sets (A∩B∩C, A∩B-C, etc.), understanding that rules (descriptive form) can be used to define sets, deriving derive descriptive form from elements, algebraic identities. In programming, they learned interactive queries (sensing), drawing different shapes (pen), functions in scratch, variables, for loops, if, repeat.

Competencies: Ability to create projects to share their ideas, break down a problem into smaller components, move from specific implementation to generalization, shifting from dependency on teacher to independently working on projects to interdependent learning from peers and supporting peers learn through sharing projects.

Inner Capacities: Care – sharing knowledge & Courage to create.

Needful things co-operative (shop) project

We will now take an example of a real-life challenge and describe the methodology we follow in the guiding process as well as reviewing through CFSR model in practice.

In Isai Ambalam School the 7th and 8th graders had difficulty understanding profit and loss. Such skills (inner circle) could have been addressed by theoretical problems and even a mock market within the grade.

With most topics as teachers, we attempt to create opportunities for children to explore and understand the world around them and asked them to research what and where they buy the things they commonly use.

On looking at the prices of stationary in the shops they found that the price for the same product varied and the local shops in the village which were charging too much. The children began to wonder what is the 'real' price of a product. The children also noticed that it was not always easy for the young children to have access to shops for small items they needed like pencils, erasers, scales, notebooks that their parents were not always able to provide at the required time. Sometimes such explorations only support understanding, but the children felt a need to act and create a system that addressed this dependence on parents for time for purchases move towards independence of children and interdependence within the school. They decided to open a small makeshift store within their classroom at breaktime. This is the middle circle of looking at patterns and wanting to shift them.

Before starting the shop for a couple of weeks teachers organized group discussions on various topics, e.g., what are the needs of children, items that could practically be stored, investment required. The children surveyed and found preferred items that they would need to have in stock at the store. Practically, none of them had a background to fund the amount required. In conversation with their teachers the children felt that since it was a collective initiative it should not have distributed funding. They broke the amount down into 40 investors including the children themselves, volunteers and well-wishers of the school. For this they created a small kit for investors highlighting what they were attempting to do, the benefits it will give children, a period for which the funds would be locked and a small return that the investor could get.

Once the finances were raised and the items were purchased by the teachers in bulk from wholesale shops. The next set of discussions the teachers had with the children were on how things will be priced to meet all expenses including travel expenses, how it would be advertised, location of the shop, timings, roles and responsibilities.

Planning and Accountabilities: Children came up with several criteria for their shop including for investors, accounts, team work, being fair, following 5S system (Sort, Set in order, Shine, Standardise, Sustain) in their shop. They also came up with marketing strategies - cheap and best and rules of their shop - No borrowing, Fixed price and No bargaining.

Children divided their accountabilities among themselves – an accountant who collects all the cash and gives a bill, two helpers to sort the stationaries and arrange them in the appropriate place, a shopkeeper who gives the items a customer needs and one person to check the stocks at the end of the day. They exchanged their roles while maintaining the rigor of practice including those that included keeping the place clean. When keeping the shop space clean where they interrupted genderism when cleaners at the school initially objected to a boy doing a 'girl's job' the children stood for equality.

Addressing real life challenges also allowed them to demonstrate a variety of skills that academic classroom didn't and that we had not perceived in them in an academic classroom. The children ran the shop till the end of the schooling year and also realized that there are many other costs like electricity, rent, labour that they had waives for them to be able to make the products available significantly cheaper at the school.

Skills: Children learned to keep stock, write receipts, handle accounts and pricing, understand profit and loss, proportions, ratios and scaling, e.g., an individual item from a packet. Conversion from inches to milli meters, different angles, measurements, marketing strategies (by advertisements and attractive offers).

Competencies: The children noticed the patterns of how shops sell and noticed gaps in both the quality and pricing in local shops. They demonstrated the competencies to enrol others raise the investment for their initiative, to work as a team, allocate accountabilities among themselves and interrupt genderism. They moved from dependence on parents to have to find time to purchase stationary to interdependence and were able to handle real-life issues which helped us notice our own biases in children's capacity that was based on academic interactions.

Inner Capacity: We found that the children took responsibility and stood for well-being and care for children, demonstrated the courage to create an alternative. Further they found something each of them excelled at and felt more confident about themselves.

Pond Repair

The second case study is in Isai Ambalam School with real life projects. Taking responsibility for their school and surroundings, such as the water issue. The children had created a pond (Iyyanarappan et al., 2019, pp. 894–898). However, within a year the pond developed cracks due to roots from trees nearby. The children felt that they did not want all the work that they have done to go in vain so children wanted to create a stronger structure that would last.

Figures 5 & 6: Building mesh structure

The children supported by the facilitators built a frame in the shape of the pond and through this they learnt to bend metal rods (6mm and 12mm) at specific angles such as 90°, 45° etc. They also learnt unit conversion from inches to cm for buying the appropriate rods and to cut them in right dimensions. Once the frame was done, they mixed Reinforced Cement Concrete (ratio 1:2:3; cement : granite gypsum : sand) and poured into the structure filling all the rods and finally got some adult help to smoothen it.

Through this process they learnt angles and frames as well as ratios and proportions with more than two quantities. We observed children who are less engaged in academic classes are enthusiastic in building with their hands. In this example we have looked at building technology as a way for children to learn.

Skills: Children learnt conversion from inches to milli meters while building the mesh structure of the pond, angles such as 90°, 45° while bending the rods, they learned to measure length, calculated the circumference and how they wanted to mesh the pond. They learned to mix in the right proportions for the RCC mix and of course the practical skills of creating structure meshes and preparing the reinforced cement.

Competencies: The children took responsibility for what they have built, noticed the gap in what had been missed, worked as teams and shared learning and knowledge with each other, faced real-life problems and got the support they needed by enrolling partners.

Inner Capacities: Responsibility, self-awareness about what they cared about in the environment they wanted, Care – sharing knowledge, Courage - ability to create projects.

C3SL initiatives to support collaborative learning

As mentioned at C3SL we have sessions for children to share projects and conduct programming courses for children. We also work on initiatives across the schools we work with, e.g., a Rubik's cube tournament. The goal of the tournament was not so much to find the fastest solver, but to encourage people to learn to solve the cube. This included sessions at the tournament to learn the cube and teachers at the schools who were inspired by the children also learned to solve it from the children. This interrupted ageism where even teachers not part of C3SL were willing to learn from children.

We created open challenges for children to create videos for children to teach what they had learned visually with materials or drawings, e.g., integers. Children looked at different ways of demonstrating with materials integer addition, subtraction, multiplication and division. We used these videos across grades to encourage children to learn from each other.

Conclusions

In this paper we discuss the Conscious Full Spectrum Response model both in terms of a design template as well for capacity development that is needed for enduring and sustainable changes in the world in line with universal values. We give examples of the use of this model as a template of design for C3SL as well as how we used it observe what we are accomplishing with children beyond academic and technical skills.

Such cognitively coherent framework allowed us to step beyond the comfort of our primary accountability as Mathematics teachers and assume the responsibility of global citizens and community leaders. It requires us to work on technical skills needed to solve immediate problems, competencies of using skills to shift culture and systems by noticing systems and patterns and learning how to work together towards interrupting disempowering ISMs, while being aligned with universal values such as responsibility, equality and courage to create.

Acknowledgements

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Mathematics in vocational education: An epistemic framework

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In this methodological paper, we present a framework, which was developed in an action research project where mathematics teachers and vocational teachers collaborated with a researcher. With this epistemic framework we challenge the view where mathematics is taken-for-granted as the theoretical knowing which is applied to the practical vocational knowing. The purpose of the framework is to aid teachers and researchers to capture theoretical and practical aspects of mathematical and *vocational knowing, both when the subject areas are separate, and when they are intertwined, in collaborative teaching. This way, power relations between different teaching contents may be reconstructed, which is helpful for collaborative teaching, and for students' learning in both mathematics and vocational teaching content areas.*

Ways of investigating mathematics in relation to vocational contexts

In the literature, mathematics in relation to vocational contexts in educational settings is conceptualised in various ways. We will describe examples of these here in the introduction, and then we will put the emphasis on presenting and elaborating on an epistemic framework addressing theoretical and practical aspects of mathematical *and* vocational knowing. The framework has been presented elsewhere with a focus on general design theoretical aspects (Boistrup & Hällback, in press), and here we focus mainly on epistemic aspects of mathematics in relation to vocational knowing. We discuss the framework in relation to how mathematics can be made relevant to students who, as a group, often experience obstacles in learning mathematics within vocational education.

One perspective adopted in the literature is Bernstein's theory of pedagogic practice (2000), which has informed research on, for example, how mathematics is recontextualised in different workplace activities. One example is FitzSimons (2015), who adopted the concept of recontextualisation by Bernstein when investigating the vocational mathematics taking place within the workers' own industry workplace. In this project, workers could identify unsuspected ways of the mathematics they already knew being transformed into authentic workplace activities, while at the same time appreciating their own roles within the overall setting of the workplace. Another theoretical perspective adopted in the literature is activity system theory by Engeström (e.g., 2001). A recent example is Frejd and Muhrman (2020) who

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investigated the learning space available for vocational mathematics education when carried out by teams with one mathematics teacher and one vocational teacher teaching collaboratively, as in this paper. The authors adopted the Engeström model in order to investigate notions like tools, norms, the division of labour and the community.

In critical mathematics education research with an interest in the characteristics of mathematical content in teaching, Chevallard's (e.g., 2006) Anthropological Theory of the Didactic (ATD), including the concept of praxeology, is quite often adopted (see, e.g., Straehler-Pohl & Gellert, 2013). Praxeology addresses two, albeit connected, dimensions of mathematical knowledge where praxis is *know-how* and logos is *know-why* (described below). Even though this model is mainly adopted in the literature of mathematics education, it is possible to adopt ATD also in other disciplines. This is described by Ladage, Achiam, and Marandino (2019), where one example is the didactic work in a museum. Another example is Quéré (2017), who investigated "mathematics in the workplace" in the context of engineering work in France. One outcome was that mathematics should not only be considered as a "tool" because engineers sometimes need to have an accurate understanding of the actual tools they use. Castela (2015) discussed the opportunities of adopting ATD in research on mathematics in connection to other knowledge areas, such as dress-making. The following quote highlights our rationale for choosing ATD as the framework we present:

> This anthropology of the mathematics should investigate social practices without too narrow restrictions on what is an interesting object. That is why I consider the praxeological model as previously presented as an interesting tool. It highlights dimensions of the institutional cognition that would be neglected otherwise, especially when the reference to acknowledged mathematics is too strong. (p. 18)

Background of the project

Boistrup, Bellander, and Blaesild (2018), in a study by the first author of this paper together with two teachers, drew on Bernstein's concept of recontextualisation to identify how mathematics was relocated and transformed (i.e., recontextualised) in vocational education (construction work). One conclusion, roughly speaking, was that there are at least two distinct ways to recontextualise mathematics in vocational activities. One is the explicit use of mathematics in, for example, problem solving, and another is mathematical concepts and methods being integrated –more implicitly – into the vocational activity. This project was helpful in finding ways to describe the interfaces between two teaching content areas, mathematics and construction work. Not surprisingly, in a survey with open questions, the students described that they found mathematics more accessible when the mathematics teacher and the construction work teacher taught together and discussed the different content areas' relationships. Since mathematics is the teaching subject which causes most problems for students in the Swedish upper secondary school to pass, this was of course good news. A surprise to the teachers and researcher in the project was that the students also expressed that the involvement of mathematics helped them in learning the vocational content.

In a subsequent action research project, in which the authors of this paper took part, new steps were taken in order to go into more detail on how epistemic aspects could be understood and interpreted when mathematics is taught in connection to vocational teaching content. The reason was that when the mathematics teacher and the vocational teacher taught collaboratively for one class per week, they wanted to move beyond a focus on practical matters and on which tasks to bring into the teaching. Instead they aimed at making mathematics relevant for the students in relation to the vocational teaching content, and to focus on explanations and reasoning in both teaching content areas. The researcher's choice then was to turn to Chevallard (2006) and his model of praxeology, both because praxeology consists of concepts which articulate epistemic aspects (described below), and because it is possible to adopt praxeology for analysis in a broad range of disciplines, including vocational knowing.

Why this epistemic framework?

The purpose of the framework is to aid teachers and researchers to capture theoretical and practical aspects of mathematical and vocational knowing, both when the subject areas are separate, but also when they are intertwined, as in collaborative teaching. This way, the power relations between the different teaching content areas may be reconstructed, which is helpful for collaborative teaching, and for students' learning in both mathematics and vocational teaching content.

We challenge the dichotomous conception where mathematics is taken as the theoretical knowing as opposed to the vocational knowing which is taken as purely practical. Rather, we view theoretical work as being developed by means of a variety of resources (such as body movements, artefacts, speech, and the like), maintained and changing over time in human practices (Selander, 2006) such as vocations and mathematics, as in the case of this paper. Rosvall, Hjelmér, and Lappalainen (2017) point to a connected tension between workplace and so-called academic knowing in vocational education in Sweden and Finland. This tension, as Rosvall et al. write, is exacerbated through the idea of a vocational learner as being practically oriented; using their hands instead of their heads and positioned as being in need. Such ideas constitute institutional norms, affecting the setting in which the teaching is designed. Through the framework presented in this paper it is possible, in research and teaching, to move beyond such ideas, and to strive to actively identify theoretical and practical aspects of different teaching content areas (in the case of this paper, mathematics and hair & makeup styling).

In Sweden, a large part of vocational education is included as study programs within upper secondary school, alongside study programs aiming for university studies. In these programs, the curriculum covers both the knowledge specified for the vocation in question, including periods of practicum, and general knowledge areas, for example English and Mathematics. In some schools, the vocational teacher and the mathematics teacher teach one lesson together each week, at least with the 1st year students. At one such school, a mathematics teacher (Hällback) was given the responsibility to take the lead in a process of

developing collaborative teaching between mathematics and vocational teachers, affording possibilities of learning for the students. As part of this, Hällback made contact with a researcher, Boistrup, and a joint action research project was initiated with four teachers at the school and one researcher.

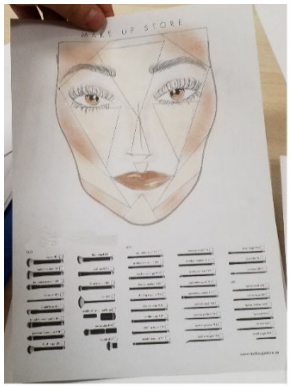

Figure 1: Student's face chart (left) and a student drawing triangles (right).

The framework of this paper was developed as part of the project and was helpful for the teachers in each team (one mathematics teacher and one vocational teacher). The examples in this paper derive from a lesson in the vocational knowledge area of styling, where the focus was on how to use triangles when doing facial makeup (see Figure 1). The lesson was video recorded, and documents and artefacts were photographed.

Theoretical underpinnings

In our action research project, we had a great interest in understanding vocational knowing aspects in relation to mathematics and, as mentioned above, we chose to draw on the model of praxeology. Praxeology is a model addressing the characteristics of knowing, and is part of Chevallard's (2006) ATD. Castela (2015) described ATD as being "interested in the processes and products of what we may consider as the institutional cognition, that is to say, in how institutions develop their socially acknowledged capitals of practices and knowing" (p. 8).

According to Chevallard (2006), praxeology is constituted by praxis and logos and offers us a foundation for addressing the practical and theoretical aspects in, and the connections between knowledge areas such as mathematics and vocational knowing. *Praxis* (know-how) concerns *tasks* (types of assignments) and *techniques* (procedures with which the task type can be carried out). An example of a vocational task in the project was curling hair with three different kinds of curls, while the mathematical task was to identify the angles of 45, 90, and 180 degrees, respectively, between the loop of hair and the surface of the skull. When curling the hair, aspects of the vocational technique concerned for example how to capture a loop of hair with the curling iron in a functional way. A mathematical technique could be to direct the loop of hair in the proper direction for the angle to be the intended one. *Logos*

(know-why) concerns *technologies* (why a procedure works in the way it does) and *theories* (overarching structures on a general level). An example of a technology connected mainly to styling was why the curling iron needs to be handled in a certain way in relation to how it affects the hair, while a mathematical technology was the explanation of why a direction of a hair loop creates a 90-degree angle and not 180 degrees. The main function of theory is then to provide a basis for the technology (Bosch & Gascón, 2014). This basis may be constituted by axioms, traditions, research findings, or theoretical assumptions. An example of a theory connected to the vocational knowing from the data was the overarching knowledge about hair styles, where curling all hair loops with the same angle creates for example a hair style similar to what Marilyn Monroe had. Examples of theoretical aspects more connected to mathematics were what constitutes the concept of an angle including the correct mathematical terminology.

We also drew on a theoretical perspective, based on social semiotics and institutional theory: Designs for Learning (DfL) (Selander, 2021). In DfL, the setting is always part of the analysis of teaching events, incorporating the resources available and the institutional norms which may restrict and/or provide opportunities for the teaching. Furthermore, in DfL the multimodal character of all communication is emphasized, with an interest in how knowing is transformed between and within modes (for example, speech, text, images, symbols) in all communication. These modes hold affordances for students' learning.

The framework

Through the project, we identified the praxis and logos of both vocational and mathematical knowing. The four Ts in the model — task, technique, technology, and theory — are intertwined and constituted by each other. Analytically, we identified them in the data from the collaborative teaching in styling and mathematics. For the analysis of knowing aspects, we developed a model (Figure 2), which reflects the epistemic framework.

	Styling				Mathematics
Task					
Technique					
Technology					
Theory					

Figure 2: A framework for capturing a continuum of aspects of knowing within styling and mathematics, and the intertwinement of both.

The framework in Figure 2 provides the opportunity to identify epistemic aspects that are a mixture of styling and mathematics, as well as aspects "belonging" more to either of the two knowledge areas, while at the same time identifying the praxis (task and technique) and logos (technology and theory) aspects.

Examples from collaborative teaching

When it comes to the setting of the makeup lesson, the location was the styling teaching rooms with the consequence that the learning resources connected to styling were there, with mirrors, hair and makeup materials, et cetera. The mathematics teacher added learning resources to the teaching as well, as will be shown below. The curriculum in the sense of teaching content derived from both styling and mathematics, with knowing aspects concerning carrying out makeup through highlighting and contouring, and handling triangles from a mathematical perspective. The institutional norm, that there is value in drawing on vocational knowing in the teaching of mathematics, is very much present. The overarching assignment of the lesson was a combination of these knowledge areas: to carry out facial makeup through the use of triangles. This overarching assignment consisted of task types, for example to know which parts of the face to highlight, and which parts to make darker through contouring. In order to make the modes and resources clear, the video episode is transcribed multimodally, with columns addressing various modes (see Excerpt 1).

In the first example, the styling teacher (D) discusses how the students should proceed to place triangles on the face (see Figure 1b). Normally, a stylist does not draw triangles on a person's face with clear lines, but during this lesson students would do this to emphasize the moment of seeing triangles in the face while doing makeup. Seeing the triangles facilitates, among other things, the work of making a face more symmetrical through make-up (which is a theoretical aspect, not highlighted by the teacher during this event). Before this event, D has asked some students where they can find relevant triangles on the face:

Time	Speech	Body movements and resources
20:07	D: You are really good; you know exactly where on the face the triangles should be.	Dips his makeup brush on the back of the hand, stands slightly forward leaning
20:08		Carries out make-up on a student under the eyebrows with light strokes
	[...]	
20:16	D: Here, here we should have light, right? And then we take all that	D puts a lot of makeup on the student's jawbone and tries to show a clear triangle

Excerpt 1. The styling teacher (D=Divo) discusses positions of triangles with a student (S).

The focus in Excerpt 2 is on the actual *technique* of doing makeup with the support of triangles. D articulates this in words ("…you know exactly where on the face the triangles

should be") in coordination with body movements and resources. The word "triangle" communicated through speech is transformed by D into body movements when he shows triangles as part of carrying out the technique of makeup. This transformation is part of making the technique clear to the students. This event received the following position in the framework (Figure 3):

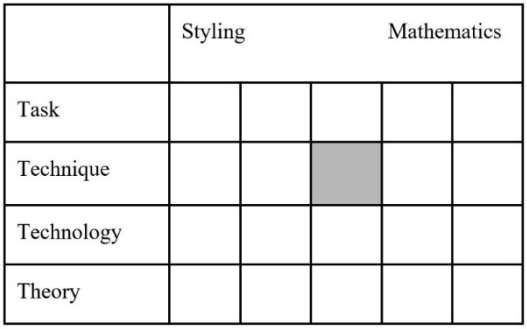

Figure 3. The interactions in Excerpt 1 interpreted as a technique with the focus equally on styling and mathematics.

The placement in the middle column in Figure 3 is due to the fact that D largely directs the students' attention to the triangles of the face (part of mathematical knowing), at the same time as he does this from a styling perspective.

The next example is that the mathematics teacher (M) and a student are looking for triangles on a face chart, as a basis for the makeup to be done (Excerpt 2). As mentioned previously, *theory*, in praxeology by Chevallard (2006), is about overall knowing and ideas which form the basis as to why *technologies* can explain certain *techniques*. The example in Excerpt 2 was interpreted to display knowing reflecting different areas of the model in the framework.

Time	**Speech**	**Body movements and resources**
23:01	M: Exactly mm… and then you can imagine that you have a triangle kind of like this… or?	Is squatting beside S. S looks at M, who makes a triangle in the forehead with her fingers
23:04	M: Or you want it so that it…	Shows with her fingers in the opposite direction.
	[...]	
23:24	M: Hey,look at you! You can really find many triangles.	Looks at the student.
	[...]	

23:32	M: Here we can make rather small triangles. Like this, right?	Shows with her thumb and index finger how thin the triangle may be.
23:34	M: Which has a small base.	Shows with both index fingers in the air, how small it may be.
23:26	M: But with higher sides, kind of.	Draws upwards with the index fingers. S nods.
23:29	M: So… there, right? Right under...	Points at the cheek of the face of the student's paper. S nods.

Excerpt 2: The mathematics teacher (M) and a student (S) are looking for triangles on a face chart, as a basis for the makeup to be done.

Excerpt 2 shows that M emphasizes that triangles can look very different from each other. The aspects of the interaction where they looked for triangles of the face, led to the middle box in the row of technology, since they were justifying how and why to use triangles when doing makeup. At the same time, they discussed the properties of the different triangles found, and this was inferred to concern theoretical aspects of mathematics:

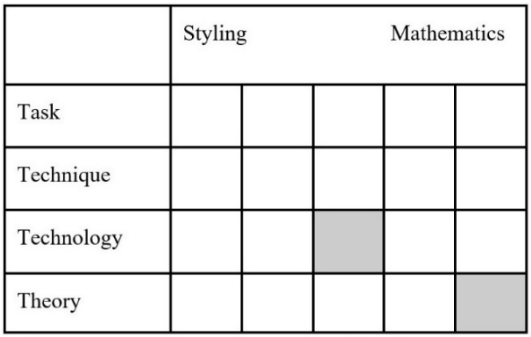

Figure 4. The interactions in Excerpt 2 interpreted as a technology with an equal focus on styling and mathematics, drawing on mathematical theory.

M also uses various terms that are relevant to descriptions of triangles, such as the word "base." This belongs to the overall mathematical knowing, which justifies the placement in the box in the lower right corner of Figure 4.

Discussion

In this paper, we have illuminated how it is possible to carry out a detailed analysis of data from collaborative teaching, with attention to aspects concerning for example transformations between modes/resources, and practical and theoretical aspects of knowing. With the attention to how theoretical and practical aspects of knowing can be understood as part of practices, we have contributed with a framework which challenges a dichotomous understanding of mathematics in relation to vocational knowing. The framework of this paper can

be viewed as a didactic model possible to adopt for teachers and researchers with a specific interest, such as collaborative teaching in mathematics and for example vocational subjects.

From a design theoretical perspective, this paper illuminates the setting of the teaching in terms of the institutional norms of the school, where it was promoted that mathematics should be integrated with vocational teaching for one lesson a week. Another setting aspect concerns resources where the paper describes how the styling classroom had many authentic artefacts which helped to strengthen aspects of knowing in styling, and also in mathematics. This is similar to the findings by Frejd and Muhrman (2020), although the framework of this paper extends the multimodal focus of the analysis, which also deepens the understanding of the meaning making that was afforded through the teaching. One example in the form of an artefact was the face chart which is normally used in styling practices, but was now also used with a focus on mathematical aspects such as symmetry and triangles. This focus, in turn, helped the styling content to be articulated more clearly. Through our attention to how knowing aspects were transformed between modes, we also identified the extent to which the knowing aspects reflected mathematics and styling, and also whether they were mainly about praxis or logos.

This paper aimed to take both mathematics and vocational knowing seriously, and to not take for granted that mathematics in vocational activities is not simply materialised as school mathematics. The framework of this paper adds ways of capturing the character of mathematical and vocational knowing in an educational setting, and how these knowledge areas can relate to each other in many different ways, and through a variety of modes. We argue that such a framework is helpful in making mathematics relevant for students attending vocational education programs, not least since it was shown in the project that a deliberate epistemic focus created opportunities for teaching and learning in both mathematics *and* styling.

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Anarchism as a methodological foundation in mathematics education: A portrait of resistance

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A large swath of research in mathematics education claims to serve an equity agenda. However, too often this research is conducted atheoretically, failing to disrupt the worldviews that produce injustice and oppression, for both the researcher and the reader. Responding to this commonplace divergence of intent and impact, we propose a methodological approach that anchors research and praxis in a sociopolitical foundation of anarchism. We seek cohesion of theory and practice by consciously demanding that values common to the equity agenda—cooperation, mutual aid, and freedom from hierarchy—provide explicit grounding for method and methodology.

> *Whenever we call something mathematics education research, we either reify existing lines along which something is included or excluded from the foam of mathematics education research, or we perturb them. We can blow additional air into bubbles that exist, we can reach in with our fingers and pop them, or we can blow—and hope—that a new bubble will emerge. The beauty of it all is that we cannot be sure what will happen. We can, however, be sure that things can change. Mathematics education research has not, and does not currently, have a fixed definition. Mathematics education research does not have a fixed, and proper, object of study. And it should not. (Dubbs, 2021, p. 165)*

Mathematics education is not a monolith with a singular identity. Instead, mathematics education is an amorphous collection of bubbles—conversations are raised or dropped; motivations expressed, rescinded, or revised; new conversants join a conversation while others leave, or perhaps just daydream for a time… Mathematics education is froth and foam, and we may at any moment shatter or breath new life into its constituent transience (Dubbs, 2021). At the same time, mathematics education research is commonly atheoretical, a term we use here to reference the commonplace disconnect (or lack of conscious interrogation) between researcher worldview and researcher methods (Stinson, 2020; Walter & Andersen, 2013). Framed in Dubbs' metaphor, researchers do not always know which bubbles they are growing, and which they are dissipating.

The purpose of this manuscript is thus twofold: To breathe new life into a bubble still not often explicitly explored in mathematics education (Bowers & Lawler, 2020), and to do so in

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a way that might give readers lost in the translucence an opportunity to notice and explore aspects of their own worldview that might previously have operated below the level of consciousness. Thus, in this text we explore the methodology—the combined worldview and method—of anarchist mathematics education. In so doing, we aim to contribute to the reflective awareness of ourselves and others regarding the bubble(s) that comprise anarchic methodology, as well as offer a counterpoint from which those of different perspectives might consciously notice aspects of their own worldview and its interrelationship or conflict with their methods (Wheatley, 2005).

Anarchism and the researcher: Conceptualizing anarchist methodology

> In the end, considerations of ontology, epistemology, ethics, values, subjective and ideological grounds, and so on—that is, the researcher's worldview—should precede not follow theoretical and methodological considerations. Explicitly and critically interrogating one's worldview should be the starting point of any research project. (Stinson, 2020, p. 13)

> We also noted that students and scholars (often of European descent), on hearing us present our work, would ask how our methodology differed from theirs. In response, we asked them to articulate exactly which aspects of standard quantitative methodologies they wanted us to contrast Indigenous methodology with […]. What intrigued us about such questioning was not that our audience wanted to know how an Indigenous quantitative methodology differed from other methodologies, but that they wanted us to provide a coherent picture of our methodologies when they could not provide a coherent picture of theirs. (Walter & Andersen, 2013, p. 43)

Anarchists and (mathematics) education researchers generally share convergent interests regarding social (in)equality—that is to say, both groups share a concern over noticing inequality and making efforts to pursue social justice. "Anarchists are principally and generally motivated by the presence of social inequality and domination to take action" (Williams, 2012, p. 10), and educational researchers have now spent decades producing such a volume of research noticing/analyzing inequality that printing the aggregate work (even of only the subset written exclusively by white cis-hetero men) might well blot out the sun. However, when looking beneath the surface, differences quickly materialize. Of particular note for our purposes, anarchism has clear theoretical (and, dare I say, methodological) underpinning, while substantial work in mathematics education is functionally atheoretical (Stinson, 2020) in the sense defined previously (i.e., commonplace disconnect, or lack of conscious interrogation, between researcher worldview and researcher methods). This atheoretical quality is, from our perspective, deeply concerning—even extremely well-intentioned equity work can actively operate against the goals of social justice when the researcher's worldview isn't consciously and critically interrogated, as for example in the case of work that fetishizes gap-gazing (Gutiérrez, 2008). Positivist framing and its analogues have been discarded but not replaced (Walter & Andersen, 2013), leaving static—white noise (pun intended). Thus, we seek in this section to utilize the strengths of anarchism to outline a broad but firmly grounded methodological approach for (mathematics) education research, one which we offer to those still lost in the static.

To frame this chiaroscuro sketch of anarchism as methodology, we will employ the framework of Walter and Andersen (2013), visually represented in Figure 1. Thus we begin with four key aspects of the researcher's worldview, which we will explore in this order: axiology (philosophy of ethical and aesthetic value), ontology (philosophy of being), epistemology (philosophy of knowing), and social position. These four are deeply interrelated and can't be meaningfully separated (in spite of our pragmatic choice for constructing this section), and we will surface a few of these connections as we go. Our decision to start with axiology rather than ontology might seem surprising, as the latter is suggested by the lingering positivist-shaped void many of us have experienced enculturation towards, but in truth the precedence of ethic is entailed in the selection of anarchism for our methodological framing (Bowers & Lawler, 2020)—anarchism is a methodology and lifestyle (Vellanki & Fendler, 2018) built upon a choice of ethical value. In short, anarchism is minimally built on the values of cooperation, mutual aid, and freedom from unjustified, coercive hierarchy (e.g., ablist cis-hetero patriarchal white-supremacist Capitalism).

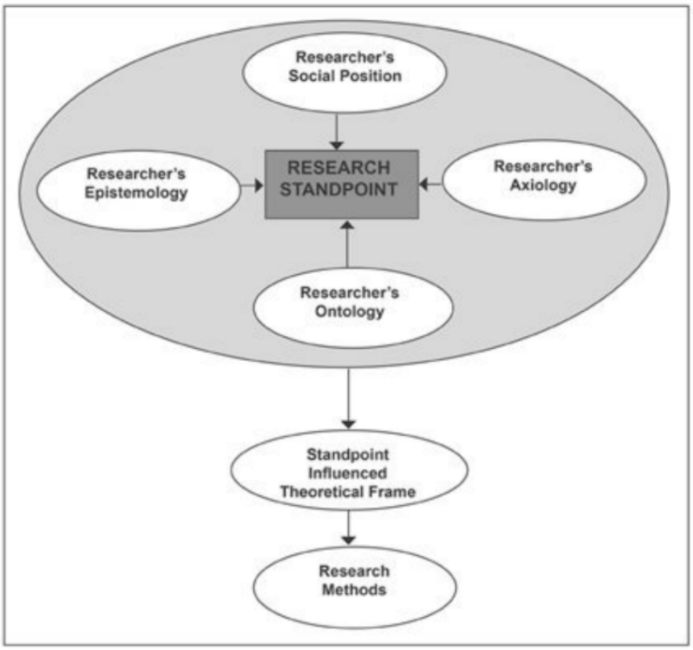

Figure 1: Conceptualization of Methodology (Walter & Andersen, 2013, p. 45)

Axiology

Axiology, the study of ethical and aesthetic value, deals with the intrinsic and extrinsic principles that shape our perception and practice. Axiology palpably shapes every aspect of research: our sense of how we do *good* while doing research, our sense of what questions are

interesting or *worthwhile*. Adding complexity, these values exist not just in the researcher, but also separately and rarely identically in the products and practices of research itself (Walter & Andersen, 2013).

As an articulation of ethics, anarchism is a mode of human organization with social self-determination, rooted in the experiencing of daily life. Anarchism, specifically social or communal anarchisms, holds a conceptual connection between freedoms of the individual and social equality, emphasizing cooperation, mutual aid, and rejection of hierarchy. "I am not myself free or human until or unless I recognize the freedom and humanity of all my fellow men" (Bakunin, in Suissa, 2010, p. 44). Anarchism values humanizing relationships that minimize if not eliminate coercive structures and interactions, taking seriously the hope for an equal and free society.

Ontology

Ontology, the philosophy of being, deals with how we perceive and operationalize a notion of reality. Far from concrete or immutable, our sense of what is *real* and how we respond to that *reality* can be fluid and even contradictory. Ontology establishes invisible boundaries around what is considered possible or meaningful, made visible in the ongoing clash between marginalized ontologies (e.g., indigenous, queered) and governing societal understandings (Walter & Andersen, 2013).

While the anarchic principles on which we build this methodological frame do not presuppose a particular ontology (or epistemology), they do foreground particular ontological (and epistemological) directions, substantively narrowing the array of ontologies (and epistemologies) we might consider reasonable. Phrased in the most concise possible terms, the ontological stance of an anarchist researcher is one of profound humility: that humans and nonhumans are complex, and that we are complex in ways that resist meaningful simplification (Smedslund, 2009; Weaver & Snaza, 2017). Efforts to simplify the human are, by definition, dehumanizing, and would thus conflict with our anarchic axiological foundation.

To efficiently convey a sense of the magnitude of this complexity within the confines of this manuscript, here we will surface four ontological characteristics of the human (note that the complexity amplifies when one extends beyond sole consideration of the human) and observe how they constrain one of the oft-touted goals of educational research—namely, the goal of surfacing absolute or general principles related to given measures based on the noticing of empirical regularity (Goddard & Wierzbicka, 1994; Smedslund, 2004; Wierzbicka, 1996). These principles have been associated with the ongoing disconnect between educational research and educational practice; they are principles that practitioners must take for granted, while research commonly tries to evade or methodologically exclude them (Smedslund, 2009; Weaver & Snaza, 2017).

Principle 1: Openness. The openness of the human means that, in principle, every single psychological/behavioral measure, and hence every composite measure, is open to an

indefinite number of possible influences, depending on how the situation is varied and how the task is understood.

Principle 2: Irreversibility. People are intentional (e.g., trying to pursue good outcomes or avoid bad outcomes), and do not completely unlearn or forget. Thus, observed regularities are conditional upon stable perception of outcomes, rendering absolute or general principles impossible. A valid general principle would entail a limit to intentionality, since it could not be modified by changing outcomes.

Principle 3: Shared Meaning Systems. People are forever part and parcel to innumerable overlapping shared meaning systems—the cultures of family, friends, workplaces, countries, regions, religions, ethnicities, and so forth. Regularities surfaced in research commonly reproduce what we already know (explicitly or tacitly) by virtue of their contingency on shared meaning systems.

Principle 4: Uniqueness. Chance plays a prodigious role in all aspects of our lived experiences, both inward and outward (e.g., Bandura, 1982). Serendipity and misfortune shape people in surreptitious ways, imposing a rigid barrier betwixt the ways the human is and the possibility of developing absolute or general principles.

Epistemology

Epistemology, the philosophy of knowledge and knowing, is central to the work of researchers ostensibly (per our positivist-shaped void) tasked with knowledge production. Whereas traditional Western philosophy constructed epistemology as outside of or prior to culture, the true span of epistemic consideration is wider: considerations of the (oft unwritten) rules of what counts as knowledge, who can be considered knowledgeable, and what knowledges are valorized or marginalized are key to epistemic consideration (Walter & Andersen, 2013).

Feyerabend (2010) argued for and outlines an anarchistic theory of knowledge (this was the subtitle of the first edition). Like us, Feyerabend's motivations in writing that text were built upon an axiological foundation:

> Anger at the wanton destruction of cultural achievements from which we all could have learned, at the conceited assurance with which some intellectuals interfere with the lives of people, and contempt for the treacly phrases they use to embellish their misdeeds. (p. 265)

The overarching thesis statement of Feyerabend's text is simply stated at the outset:

> Science is an essentially anarchic enterprise: theoretical anarchism is more humanitarian and more likely to encourage progress than its law-and-order alternatives… history generally, and the history of revolution in particular, is always richer in content, more varied, more many-sided, more lively and subtle than even the best historian and the best methodologist can imagine. (p. 1)

In essence, just as we describe the anarchist researcher as ontologically humble, Feyerabend describes an anarchist researcher as epistemologically humble. Indeed, we note that ontological humility almost demands epistemological humility, as we illustrated above in relating ontological principles to barriers to one of the commonly touted epistemic goals of hegemonic science.

To the anarchist researcher, anything goes; or rather, you are always more free than you realize you are. An anarchist researcher might use virtually any epistemic method to make sense of or shape the world, even methods that seem contrary to their perspective, as when anti-rationalist Feyerabend regularly made rationalist arguments to discomfit his rationalist opponents and friends (e.g., Lakatos). Viewing science (in our broad conceptualization) as a pedagogic project, one wherein we are forever learning with and from others, and imagining hegemonic science as institutionalized schooling, we find this reflection rooted in Freire and Rancière to be particularly meaningful:

> The unschooled world is only feared by those who have been thoroughly schooled. The emancipated world has no enemies among the truant. None among children and none among artists. None among those who would take equality as a point of departure. (Bingham & Biesta, 2010, p. 157).

Social Position

Our social position comprises much of who we are socially, economically, culturally, and racially. Social position is not just about the individual or individual choices—class, culture, race, gender, sexuality, (dis)ability/neurodivergence, and so forth deeply shape worldview, not least because so much is taken for granted. Social position is thus a verb rather than a noun. As researchers and as people, we do, live, and embody a social position (Walter & Andersen, 2013).

Obviously, we can make no particular observations about the specific social positioning of you, dear reader. However, the anarchist researcher does adopt a particular relationship with social positioning. In short, the anarchist researcher takes social positioning seriously, and may use it as a guide for noticing blind spots or disproportionate emotional/physical labor demands. Additionally, anarchist positioning is worthy of note in-of-itself, for it necessarily runs deep (per our axiological foundation), contrary to the disparate professional positionings that others may tend to write off as "just part of the job".

Anarchists take social positioning seriously (though people who don't have been known to attempt to co-opt the title). Per our axiological foundation, anarchists oppose unjustified, coercive hierarchies, including those of race, gender/sexuality, class, dis/ability, religion, nationality, and so forth. None of these hierarchies are individual monoliths—they silently embrace and dance a dance of violence, holding each other so close that their boundaries blur and disappear. Thus, anarchists are also deeply invested in intersectionality, variously in terms of: noticing and responding to the complex ways various intersections of identity shape lived experiences, accounting for constructed invisibility and cultural obfuscation of the multiply marginalized, and working to build solidarity across those of disparate backgrounds suffering at the hands of the same deathly waltz.

Furthermore, there are additional aspects of social positioning and hierarchy that are relevant to (mathematics) education researchers in particular. An anarchist researcher recognizes that they bear unique experience, knowledges, or tools that another might not have immediate access to, but will nonetheless express extreme skepticism of the many

coercive and unjustified aspects of the hierarchy which places researchers epistemically above others as knowers and learners. Relatedly, an anarchist researcher is deeply skeptical of the epistemic and social hierarchy that perceives established scholars as superior to emerging scholars, as well as the epistemic and social hierarchy that perceives "teachers" as superior to "students." An anarchist researcher further opposes hierarchies of discipline, such as the *Romance of Mathematics* (Lakoff & Núñez, 2000), the still commonplace mythology that contemporary disciplinary mathematics is epistemically superior to other disciplines or disciplinary perspectives.

As one final note that distinguishes this methodological approach from those typically constructed as existing at the center rather than the margins (work constructed on the margins, such as indigenous methodologies, is more likely to intentionally share this characteristic), anarchism as methodology is not limited to professional situations—it is a lifestyle. To borrow a metaphor from educational philosopher Lynn Fendler, imagine yourself as a chef, passionately dedicated to your craft. Can you dissociate the qualities of your ingredients from the qualities of the food they are used to create? By the same token, in our work as researchers, is it really possible to dissociate the qualities of the researcher from the qualities of the research they produce? To create the most delicious dish, we use ingredients that carry the qualities we wish to be present in that dish. To create the most ethical research, we must use ingredients that carry the axiological qualities we wish to be present in the research (Vellanki & Fendler, 2018).

Anarchism and research: Tracing the possibility space of anarchist method

> In [critically engaging with one's worldview], the frantic search that novice (and even seasoned) researchers experience in selecting theoretical frameworks and methodological approaches more times than not becomes self-evident and trivial. (Stinson, 2020, p. 13)

Having sketched the outlines of the worldview(s) associated with anarchism above, in this section we use that foundation to infer elements of the possibility space of anarchist research method. This space is vast, even as it excludes swathes of normative research methods (for example, psychometrics in its normative context—that is, the context of the worldviews and purposes that commonly underlay it—fall in steep conflict with an anarchist worldview). Thus, our goal is to be illustrative rather than exhaustive. In particular, we draw attention to three categories of method that have proved meaningful in our own work: collaborative action and design research, discourse-shaping and radicalizing research, and work that stands as iconoclasm of the oppressor within.

Collaborative action and design research

Despite ill-informed representations perpetuated by the media and other sources, the most likely places you might find anarchists in your community are at your local community garden, baby pantry, workers union, or co-op (we, the authors, have participated in each of these). These are loci of cooperation and mutual aid, places where people have noticed their community has a need that they can help to fill. As researchers with specialized knowledge(s)

that can be leveraged for the benefit of our communities, one notable analogue to these aforementioned spaces in research is collaborative action and design research. Action and design research represent

> an orientation to knowledge creation that arises in a context of practice and requires researchers to work with practitioners [and other stakeholders] […] its purpose is not primarily or solely to understand social arrangements, but also to effect desired change as a path to generating knowledge and empowering stakeholders. (Huang, 2010, p. 93)

Note that whereas more normative research might prioritize *knowledge* as a means towards *change and empowerment*, Huang instead describes pursuing desired *change* as a means to *knowledge and empowerment*. Along with relating to our axiological foundation, this also ties into the ontological and epistemological humility we referenced previously. In short, the goal of collaborative action and design research is to support communities, as for example through the collaborative development of "new theories, artifacts, and practices that can be generalized to other schools" (Barab, 2014, p. 151) or areas of praxis.

Discourse-shaping

Every publication, presentation, seminar, lesson, and conversation is necessarily a political act. Interaction either reifies or perturbs boundaries and beliefs, forever modifying the ideological and material translucence we occupy (Dubbs, 2021). Whenever we converse, the question is never "should I be political or not," for we are necessarily political in manners either hidebound or progressive. Instead, the question is, "in what ways should I be political?" or "in what ways should I shape this discourse?" Conscious of this, the anarchist researcher seeks to shape discourse in ways that advance our ethical goals of cooperation, mutual aid, and freedom from unjustified, coercive hierarchy. We might, for example, publish critical works in typically acritical spaces (Bowers & Küchle, 2020) in an effort to normalize critical perspectives therein, thus paving the way for further change or revolution. Discourse shaping is always a component of the work of any researcher (consciously or not), but for the anarchist researcher it can also serve as a goal in-of-itself, whether that means mobilizing/radicalizing potential future researcher-activists or simply offering a moment of critical introspection to an audience not often engaged in such.

Iconoclasm of the oppressor within

White supremecy, cis-heterosexual male supremecy, abled/neurotypical supremecy, capital supremecy… each of these (and other) oppressive paradigms/hierarchies surround us and can act through us.

> The true focus of revolutionary change is never merely the oppressive situations which we seek to escape, but that piece of the oppressor which is planted deep within each of us, and which knows only the oppressors' tactics, the oppressors' relationships (Lorde, 2007, p. 118)

The anarchist researcher has an ethical responsibility to always consider the ways oppressive systems may act through us, below the level of consciousness, notably (but not solely) along continua wherein our identity and/or positioning might place us among

oppressors. For example, a white researcher might be aware that whiteness is acting through a system in which they participate, such as mathematics doctoral coursework. It is common for disproportionate burden to be placed on BI-POC (Black, Indigenous, and People of Color) to surface white supremacy (with analogues applicable to each other system of oppression), so the white researcher might dedicate careful time, energy, and attention to analysing and thinking through how whiteness is acting in those spaces, then share what they surface with other researchers subject to similar positional blindness (e.g., Bowers, 2019). With this sort of persistent, critical reflection, we seek to free ourselves from such hierarchies of domination—by sharing this work with others, we engage in yet another type of cooperation and mutual aid. While we emphasize the ways positional blindnesses and disproportionate systemic expectations might make this work more important along the lines of our oppressive identities/positionings, we do wish to mention that such supremacies are commonly internalized along the lines of our marginalized identities as well, as when internalized neurotypical supremacy or cis-heterosexual male supremacy rears its ugly head in the work, thought, or action of this neurodivergent (autistic, ADHD), genderqueer author.

Conclusion

Anarchism is present in a significant portion of modern equity and justice research in mathematics education (Bowers & Lawler, 2020). Explicit identification of and attention to anarchist methodology provides researcher and reader the opportunity to more explicitly identify the manner in which knowledge production operates in harmony (or conflict) with stated aims of a just an equitable society: cooperation; mutual aid; and freedom from unjustified, coercive hierarchy. Phrased differently, building methodology from the foundation of anarchism allows for a cohesion of theory and praxis that would be beneficial in any work, but which carries particular weight when one's goals are emancipatory or anti-oppressive.

Critical theory is necessarily an inadequate force of change when not accompanied by critical praxis. The additional step of critical praxis has presented a hurdle to many, a fact at once shocking and wholly unsurprising—unsurprising, because critical and social justice work can't be built on a foundation of oppression such as that symbolically and materially reified in the norms and methods of much mathematics education research, but shocking, because a new reality lies just out of sight over the horizon. We hope this glimpse of one such reality, one such critical praxis developed through a cohesion of theory and practice, might offer a glimpse over such a horizon. While throwing off the reigns of oppression might seem at times an insurmountable challenge, we look forward, in solidarity, to basking under the warmth of new suns.

Acknowledgements

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Mapping conocimiento and desconocimiento in collaborative mathematics teacher professional development

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Using Case Study, this paper demonstrates the ways a group of teachers in Chile experienced collaborative knowing as they participated in Lesson Study for professional development. I use Anzaldúa's conception of Conocimiento, reimagined for math teacher learning by Gutiérrez, to analyse data and understand the ways this team of teachers co-created knowledge together. Data includes transcribed audio/video recordings of planning meetings, lessons, and post-lesson reflections as well as participant interviews and focus groups. This study contributes to the discussion on mathematics teacher professional development including the preparation of teachers for advocating for their students within systems of oppression.

Lesson Study in Chile

Lesson Study incorporates teachers working together to plan a lesson, enact and learn from that lesson and then participate in collective reflection on student learning observed in that lesson (Fernandez & Yoshida, 2004). It is a system of teacher learning that was developed in a grassroots way for over one-hundred years in Japan and prioritizes teachers' perspectives in their learning. Lesson Study can be done alone as its key feature is reflection on one's practice through attention on student learning. Collaboration with other teachers, coaches and administrators provides additional lenses for that reflection (Fernandez et al., 2003). Juxtaposed to more popular models where millions of dollars are spent on professional development, where many studies have shown that professional development for teachers is fragmented and superficial, largely ignoring what research has demonstrated about teacher learning (Ball & Cohen, 1999; Borko, 2004; Putnam & Borko, 1997). Gellert et al. (2013) found that with elementary math teachers in Chile nationwide professional development efforts had the opposite effect of what was intended. Due to Chile's neoliberal past, living in the shadow of the Pinochet dictatorship that produced the high-risk teacher evaluations coupled with job insecurity, researchers have found teachers will teach 'safe' lessons that refrain from activities that might demonstrate errors or misconceptions (Araya & Dartnell, 2008). It is likely these same precautions are demonstrated in Lesson Study.

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Methodology

This research sought to answer the following questions: *What cycles of Conocimiento do teachers of Mathematics experience while engaged in Lesson Study in Chile? and What do we as math teacher educators and math teacher education researchers learn about teacher learning by focusing on uncertainties as they arise when the group is trying to construct new knowledge?* Coming from a social constructivist perspective and using Anzaldua's framework of Nepantla/ Conocimiento and Gutierrez's reimagining of Nepantla for mathematics teacher learning in the Path of Conocimiento, I used Case Study with an ethnographic, qualitative approach to explore teacher learning through several cycles if Lesson Study as they were enacted over the course of one year. Data included analytic memos, audio and video transcribed recordings of collaborative lesson planning, lesson implementation, and collaborative lesson reflections. Data also includes teacher personal reflections, artifacts from the lessons and lesson planning. Lastly, teacher interviews and member checking sessions were also recorded and analyzed. Data was analyzed based on themes from the Path of Conocimiento as well as using inductive analysis. Nepantla was used to represent tensions or liminal spaces observed in the data. This is later represented by a 20-pointed star. When participants rejected interpretations in ways that shut down the exploration, they were coded as Desconocimiento. If comments or actions built on interpretations, clarified, or posited additional interpretations which respected both speakers, they were coded as Conocimiento, knowing. This longitudinal, situated Case Study design employed holistic single-case design outlined by Yin, (2018).

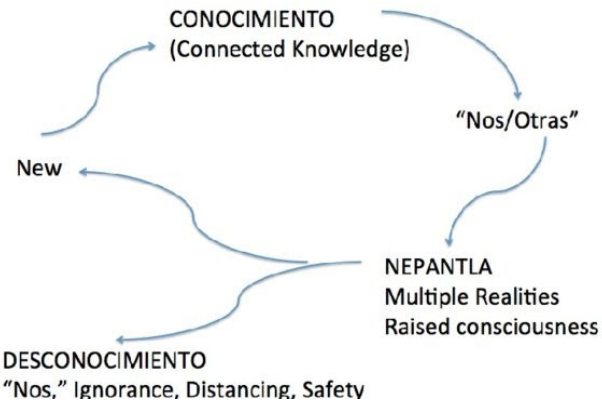

Figure 1: The Path of Conocimiento from Gutiérrez (2012)

Teacher	Grade/Subject	Experience (yrs.)	Yrs. at School	Gender
Valentina	Instructional Coach	30	30	F
Emilio	5th to 8th Grade Math	11	1	M
Violeta	2nd Grade	7	1	F
Martina	3rd Grade	6	0.5	F
Paloma	Classroom Aid	13	13	F
Elena	1st Grade	2	1	F

Table 1: Teacher/ Administrative Participants.

Context

This paper looks at a specific instance of conocimiento that emerged in the second of three cycles of Lesson Study carried out in one school. The objective of the lesson was to build understanding of the concept of tens and ones among a class of 1st grade students. As a secondary goal, the team sought to explore student collaboration through teacher-lead learning centers. The head teacher for the class, Elena, designed four centers which were introduced at the beginning of the class. Students were then sent to one of the four centers and then rotated as the teacher instructed. Video cameras were positioned around the room to capture the work in the classroom as a whole as well as at each of the centers. These videos then were edited to follow three focal students as they moved from center to center, so that the teachers could collaboratively reflect on one small set of students as the progressed through the centers. In the reflection meeting, teachers were given reflection sheets to record thoughts independently before they watched the edited video of the focal students.

Data analysis

After the lesson, students were dismissed to the school patio where they enjoyed a recess with the other classes. As we walked out of the classroom Martina and Violeta said that they felt the lesson had "gone well" and enjoyed "working together in the same room" but did not mention the lesson or their observations made in the lesson. Later, in Emilio's home, he began to process the events at his center. He explained that his student did not know her numbers yet, "She confused the ten and the one." He explained that generally the students performed poorly in his school and he cited poverty factors for why the school scores were so low on the state exams.

> And why do you see this? Why does this happen? It's because the parents have low income. I've checked those statistics and the parents sometimes have an eighth grade [education]... (Interview, June 28) [A]

Emilio's referencing poverty as an explanation of student performance was the beginning of the representations from the teaching group indicating that there were problems in

student performance. The second indication was found in the common language in the reflection, predominantly speaking about student engagement in general but positive terms. Specifically, teacher referred to the students as "happy" and "motived" or "happy and "excited to participate" (see Table 2.) Only three "suggestions" were given in all reflections. Two where on Emilio's reflection page, meaning that most did not offer suggestions.

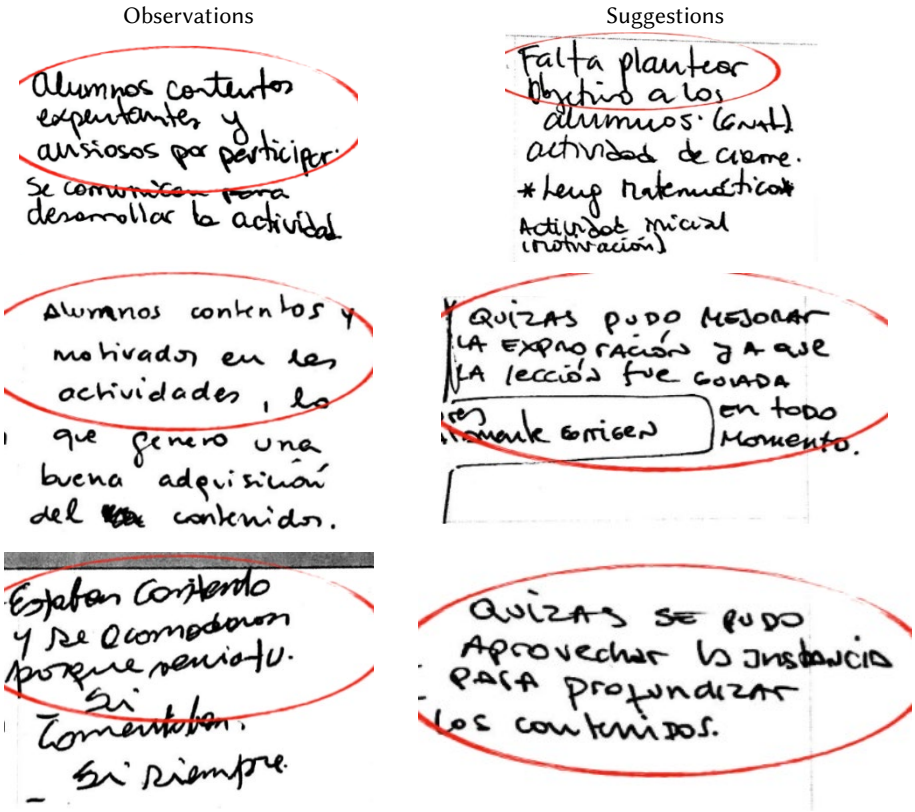

Table 2: Sample Comments from Teacher Reflection.

Despite predominantly positive comments from the independent reflections, there was a perceivable difference in mood among the teachers in the reflection compared to the planning meeting and the lesson. Where our planning time at the beginning of the week was energetic and full of ideas, the reflection was more reserved. One way to see this was in the coding: as the teachers were eager to share, they began to speak over one another, making transcribing difficult. These moments were coded as cross-speak. There were 3 cross-speak moments in this reflection.

Planning Meeting 1	Reflection 1	Planning Meeting 2	Reflection 2	Impromptu Reflection (3)	Reflection 3	Totals
24	0	9	3	12	2	50

Table 3: Cross-speech across lesson study meetings. Note that the first and second reflections had 2 and 4 participants, respectively.

We watched Emilio's center, as the focus student counted with tiles, "17, 18, 19, 30." To remediate, Emilio tried to get the student to count her tens and then to count on from the tens. He repeated the activity with different sets of tiles. When she answered correctly, she did so with an inflection at the end, indicating that she was not sure of her answer.

As the video played, it became evident that the teacher of the class, Elena, was uncomfortable. She shrunk down in her seat and kept her hands folded in front of her, partially obscuring her face, see Figures 2. (Note that the figures include a timestamp, this is only to communicate the passage of time.) As other teachers began mirroring her stance (see the third image in Figure 2), I was alerted to a tension happening in the group. As the video played, the mentor, Valentina quietly asked Elena (not the group) if the student knew the material. Elena responded, "Yes, she was just very nervous because of all the people and the cameras...".[B]

Figure 2: Teacher Tension

After watching one center, I paused the video and asked if the student had learned the objective (23:45.81). The teachers all said "yes".[C] These were the three moments leading up to this conversation that demonstrated the teachers were experiencing some conflict with what was observed in the lesson. A: Emilio explained that the student was mixing her tens and ones and then cited poverty. B: When asked whether the student knew the material, Elena said yes but that the situation in the room had made her nervous. C: When asked if the student had learned, everyone said yes, and then said nothing. These pre-indicators are representing by triangles in Figure 3.

2 (24:01.09) Valentina: This is a concept we teach through many grades. We have been working on it since kindergarten.

Comments A, B, C and 2 demonstrated that the teachers might have been expecting a fault to land, that I or others were looking for a person or factors that were to take the blame. This progression is represented in a straight path, opposed to the turn represented in conocimiento. In the straight path, we see desconocimiento, we see missed opportunities. The desconocimiento presents a refusal to see students learning as a place to hone practice, but as simply as a factor in grading teachers in summative ways. With new perspective/ interpretations, with challenges, we see the opportunity to begin making new knowledge collectively. A conocimiento move from 1 to 2 could have been *yes, and*: "Yes I noticed that she was using the numbers interchangeable, but I think it relates to her confusion of ones and tens." or "yes, but I wonder if it is a language thing… not a number thing?", Instead comment 2 re-centers the conversation and reiterates that this is something learned over time and that there is nothing to discuss concerning what was observed.

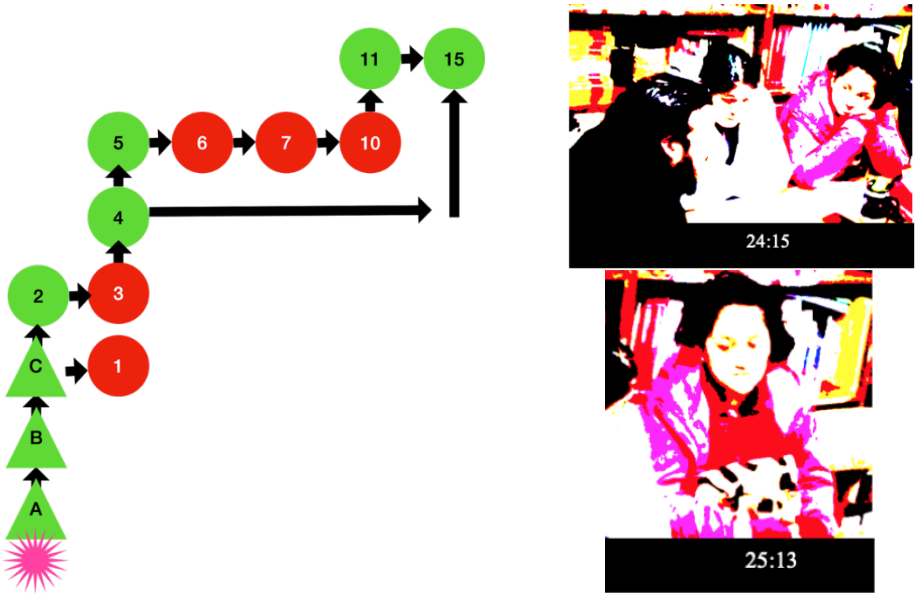

Figure 3: Building Conocimiento, A conversation diagram and a shift in body-language, observing Emilio (3) and agreeing (4).

3 (24:09:25) Emilio: I thought they understood that a ten was ten ones…. *maybe it was me. I think when I counted backward, going down and down and down, it confused her.* Also, all the people and the cameras could have made her nervous.

4 (25:07.31) Elena: Yes, because in fact, in class the kids do well. *But they struggle with complexity.* We committed a lot of time to it because it was hard for them to understand. They could understand that in tens there were 10 units representing 10 disks. Then for one group of tens they had a hard time because they had already said there were 18... for example, there are 18 already, but we have only one ten, 'How many were ones?' *[the children would respond] 'One!' What? No, a group of 10!*

Emilio presented himself as possibly making an error, an attention to discuss. Elena uses this moment to discuss an observation in the class, the confusion with the ten and the one. In this, she introduces an obstacle to understanding tens and ones well-documented in research (Fuson et al., 1997; Guerrero et al., 2020; Kevin Miller et al., 2000).

5 (25:53.11) Emilio: ...talking with another colleague... She did an activity where they gathered 10 balls of dough. *And those 10 balls of playdough, she combined... then perhaps the children will not be confused with a ones and the tens....* that they are not the same, because ten is bigger. Maybe that's what happened to the children with this activity. Because they confused one with a ten, when they are bigger... perhaps does not make sense, because it is obviously the ten largest. But for them as they were confused there.

6 (26:33.80) Elena: But first they were shown the loose quantity. There are 10 chips. Then I joined them and said, 'Look at these tens and there are ten. And this one, each of these pieces is called unit.' So, I show the units and they gather ten units...

Here Elena answers Emilio's suggestion with examples of what she did in class, and what she did at her center. Here we see Elena challenging Emilio's suggestion.

7 (27:06.61) Author: But maybe it was the way it was written? because it looks very similar... *Because normally we write 18 like this....* They might be guessing what we want.

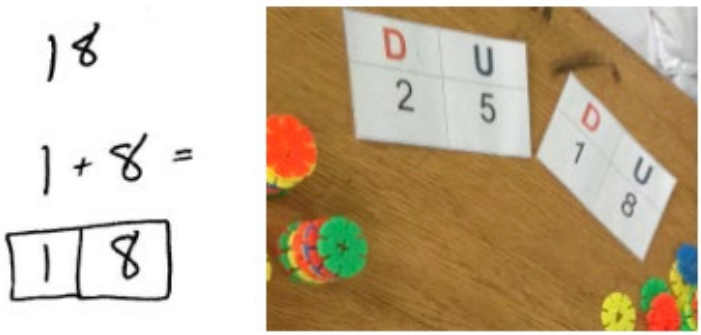

Figure 4: Artifacts from the reflection, My depiction in my notes and screen capture of cards at the observed center.

8 (27:19.83) Emilio:	Maybe they did not understand the instructions.
9 (27:25.34) Valentina:	Maybe it is the other way around…
10 (28:14.05) Author:	Judging by her facial expressions and how she was mixing 20 and 30… *[cross speak] …* I wonder if she was not hearing the difference between the sounds.

This marks our first cross-speech. We had entered the problem-solving stage; we had been building and considering each other's ideas/interpretations since Emilio presented that maybe he had confused her. Here we see increase excitement and engagement. We then received important information from the classroom aid, who had worked with the student across grades.

11 (28:43.93) Elena:	In fact, with her, you have to enunciate your words
12 (28:45.73) Paloma:	She *watches the mouth a lot.* [cross speak] I had her in prekindergarten. If she did not see your mouth, she did not understand.
13 (29:06.90) Emilio:	Was she evaluated?
14 (29:12.69) Paloma:	In kindergarten, she had hearing aids… These tests are done periodically; the mom controls that. [Cross talk] The mom says that it got better and then fell.
15 (29:24.46) Violeta:	… she must have gone many years without being diagnosed then *she learned to listen to people by the movement of their mouth.* Then it takes a few years with hearing aids… A year and a half? It is just processing the movement of the mouth with the sound coming out. *I also think the concept is difficult.*

Here we see the combination of observation culminating in identifying what will be specifically difficult for this student. In the post-reflection interview, it was revealed that one obstacle to a productive collaborative reflection was that criticizing felt like betrayal.

> It is my personal desire to support and not to betray my colleague, accusing her, accusing her that her work is not well done… It was Elena's class, her student. Elena said the student knew, but she was nervous… Then I also think that she was justifying herself… to not look bad, because that was her student. (Emilio, post-reflection interview)

Emilio's ability to have empathy for his colleagues while choosing to engage in difficult conversations helped to provide the brave space the group needed to learn from their practice. He could have simply agreed that the student was nervous in the reflection meeting, and the learning would have stopped there. Alternatively, he could have simply stated what he had told me, "The student doesn't know how to count." It would provide a missed opportunity to learn and to engage in professional development as the silence would likely have continued. It is clear for him, the tension included his interpretation, the care of his colleagues and their collaborative relationships and the interpretations being given from the group. Emilio prioritized the solidarity of the group to navigate the tension within the perspectives because he felt that an environment of solidarity is key to developing a productive learning space.

As a part of the member checking process, in zoom meetings with the teachers involved in this lesson and reflection, I showed the teachers the clip of this phenomena and asked them to remember what they were thinking. In the follow-up interview, Emilio stated, "I am a normal teacher and we all make mistakes. So, *I give the foot* or give the opportunity… I speak honestly and I think it drives others to see that we are all equal and they can tell their real experience without putting on a show." By modeling with self-criticism, he made space to engage with the differing perspectives in way that was candid and productive.

Discussion

Using this mapping tool, I was able to document how the group was able to turn into the tension for making knowledge together. This was represented by right turns, away from the straight path with the group's current conocimiento or desconocimiento. This tool then mapped the multiple ways the perspectives were combining to build rich, context-oriented complexity within the group. This tool made the combining of ideas as the group co-created knowledge explicit, visually representing teacher making sense together by expanding… verses a more linear path that often happened as teachers did not offer or incorporate each other's lenses.

Findings

By entering in a discussion, interpreting and making sense together, we developed a more complex interpretation of what was happening with the focal student in the lesson. In part, she may have been nervous. I and the other teachers had created a disruption and we were doing something very new. Yes, there is a well-documented struggle for children to understand the differences of tens and ones. Additionally, maybe the way we represented the tens and ones (with boxes) contributed to the confusion as to what were tens and what were ones and yes, in this center, we saw a student who had a history with hearing problems. This revelation about the hearing aid sparked an ongoing conversation at the school on student files so teachers could get a general report on each student's progress and understand abilities and student backgrounds that might impact teaching. Having overcome the pervasive high-risk nature of the neo-liberal education system in Chile, the teachers were able to address student thinking rich with context, combining student thinking, abilities, with research. The lessons learned in watching this focal student relied on open communication that was not initially happening. When they embraced the tension they paved the way for a more nuanced understanding among the group. That nuance resided in each person's privately held perspective of what was happening with that student, waiting to be held against the other perspectives. In this way we saw the collaborative making of new knowledge between the teachers, the research, through close observation of the student, and background knowledge of the classroom aid.

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About the useful uselessness and unimportant importance of mathematics

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In this presentation we explore the symbolic power of mathematics and mathematics education (ME), focusing on the trait of the socio-political perception of usefulness-importance of mathematics. We contribute excerpts from discussion groups with students from 2nd and 3rd year of secondary education, from a public school of Madrid. Then, we analyse and discuss those excerpts with the aim of understanding how the students live through school mathematics and its aforementioned "usefulness-importance".

Introduction

This chapter is an excerpt (and adaptation to the theme of this MES conference) of a chapter from a recently finished doctoral thesis (Bruno, 2020). Which in turns is heavily influenced by the young tradition of the socio-political perspectives in mathematics education (ME), as explained by Valero & Pais (2015), along many other authors. As we explain and elaborate throughout the mentioned Thesis, from a socio-political perspective in ME, the phenomena of the mathematics class or of the school cease (to a greater or lesser extent, as Valero & Pais, 2015, explain) to be the exclusive center of research attention. And they are understood as a fundamental, but not exclusive, node of an interweaving of social practices of different kinds: technologies of power and government, devices for evaluation and accreditation / social stratification, global agendas, dominant cultural discourses and evaluations, creation of subjectivities through schooling, "regimes of truth" (in Foucauldian language), and so on. All of them indeed closely related to mathematics, their schooling, and research in ME.

A relevant issue, both in the doctoral research and in the theoretical approach there adopted, is what authors like Skovsmose (1994) and Sáenz and García (2015) call the *symbolic power* of mathematics. We characterize this *symbolic power* (based on the developments of such authors) with the following fundamental features:

- Mathematics is considered an objective knowledge, aseptic of any ideological-political interest or struggles, neutral in values.
- Mathematics have, politically / symbolically, an almost unappealable truth value ("ideology of certainty", according to Skovsmose & Borba, 1997). "The numbers speak for themselves / they don't lie."
- Mathematics is very useful / important ("for everything", "they are everywhere", "in science and technology…but hey, also in the arts and… and in everyday life"). And

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therefore they are/should be a significant component of any educational curriculum in any country/community that claims to be up to date on any timeline of progress and innovation.

– Furthermore, considering the global aspirations of democratization, equity, social justice, "global citizenship" promoted from high levels of international organizations (i.e., the UN 2030 agenda; UN, 2015), teaching-schooling in Mathematics is promoted as a universal right / duty, as a key element to achieve such aspirations. Valero (2017) genealogically analyzes this ideal of "Mathematics for all" and how it has been developing in recent decades.

However, despite this broad socio-cultural legitimation, for a large part of the "ordinary people", mathematics is impenetrable or incomprehensible, if one does not have an important level of initiation and practice with them (i.e., university degree or profession that applies them in depth). Therefore, Mathematics has the remarkable political-cultural position of being a symbolic system that is both accepted and legitimized, but largely misunderstood and even feared-rejected by a large part of the population (Spanish, world). In our experience, both as mathematics teachers at different educational levels and as researchers in ME, this fear-rejection could be related to the systemic and generalized phenomenon of school failure in mathematics.

Now, considering the enormous embedding that mathematics has in decision-making and power spaces (in states, in international agencies, in economic and financial schemes, in electoral systems, in health systems, in modern ICT, even in the understanding/measurement of space-time...) the political consequences of this status of symbolic power of Mathematics are far-reaching. As expressed by Sáenz and García (2015, p. 26):

> By presenting itself so legitimized and at the same time so impenetrable for the uninitiated, it goes so far as to make the dominated assume the legitimacy of their submission [...]. Clouded in the shadow of this unattainable knowledge, anyone who ignores the language and mathematical methods by which social processes are usually expressed remains unarmed, without an answer, sometimes he must accept his bad luck, quietly waiting until the wisdom of who decided for him bears fruit.

As a part of a doctoral thesis project, field work has been developed in different Secondary Education schools in Madrid, Spain. The students involved are from 2nd and 3rd year of secondary education ("ESO", by its acronym in Spanish).

In the field experiences mentioned above, we seek to investigate the different aspects of the "symbolic power" of mathematics. In this presentation we will communicate some results referring to the socio-political and cultural *importance-usefulness* trait assigned to mathematics and its education.

For that purpose, we studied the perceptions and experiences of the participating students about mathematics and school math, considering different points of view: difficulty, liking or rejection, being a "person of sciences" or being a "person of letters", self-concept, etc. A significant part of the inquiries was around the question of the importance-usefulness of mathematics (in society, in daily life, in school, etc.).

Methodology of research

In her doctoral thesis, Suavita (2017) proposes an operational construction around the notion of *social imaginary*, that is, both notions and theoretical framework, as a methodology to study these socio-political, cultural and historical phenomena. His thesis is entitled "Imaginaries in mathematics teachers in training". Among others, Suavita collects the developments of the Ibero-American Research Network on Imaginaries and Representations (RIIR), which in turn bases its theoretical frameworks on the research of Castoriadis, Durand, Maffesoli, Baeza, Carretero, Silva and others.

In this chapter we follow her methodological approach in her study of social imaginaries. One of the main techniques she uses for obtaining data and evidence is the *focus group* or *discussion group*, and we thus will apply it for addressing our research concerns.

We organized the participant students in up to 7 focus groups, and in those we proposed to the students the discussion around topics that implied the trait of the use-importance of mathematics, like: why is mathematics taught in school? What are they used for in different areas of life? Why are they (or are they not) important-useful?

Mathematics are useful-useless, but not really important-unimportant

We will now transcribe excerpts from some of the discussion groups relevant to *use-importance* trait, and its perception by the participant students. We will follow each with analysis and commentaries considering the theoretical perspectives posited above.

The "R" abbreviation means "Researcher", the person who was conducting the discussion groups. The other abbreviations refer to the names of the participant students.

Group 1:

R: Let's see, one more question: if you're not good at math, can you be a sciences person?
(The students discuss, argue a bit before answering)

R: And if you are good at math, can you be a letters person?

A3: I'm bad at math, and I'm going for sciences.

A1 (to A3): And are you going for sciences? Well, not me, because I believe that everything is united, you know [...] That if you don't know mathematics, then, as you progress, you will not know physics and chemistry either, which is what is happening to me.

L (to A3): For example, Physics and Chemistry are not mathematics[sic], but there are formulas... and they are mathematics after all.

R: And if you were bad at math, it wouldn't be convenient for you to do science, or would it be convenient...?

A3: Let's see, for the career I want to do, yes. Because that also depends on what you want to do. I, for example, want to be a midwife.

A1: And what use is mathematics for you to get a child? One-two-three!

A2: Three minus one, two, minus one, one...

A3: If I want to go for that profession, then I must go for sciences. Then it [mathematics] will be useful to me. Let's see, math is useless to me, but it is necessary in sciences, so it's a no brainer.

We first observe that A3 classmates themselves question the decision to "follow a sciences degree", because science has/is a lot of math; or, without math there is no science, or without math you cannot do sciences. And they affirm it empirically, because at least in the current schooling-accreditation system, this is considered true. Whether or not math is used in one or another discipline, study, trade or criminal activity, to do "science" and anything that has been so labeled, you have to go through many and difficult math... and their assessments.

Of course, the issue of the usefulness of mathematics for an obstetrician is a sweet plate of non-sense, but we will return to it later.

Group 2:

C: Math is a bummer, but it is very important.

Q: That depends on what you study ...

R: And if you did not study engineering or "sciences", would they be useful to you? (Controversy, mixed voices).

B1: In certain things, yes. But like, above all, if you have a subject that has to do with mathematics. For example, if you go to "Sciences" ... also in "Letters" I think you must do mathematics. Or maybe it helps me to know the area of a certain something if I study Plastic Arts. But there are a lot of things that have nothing to do with what we are going to do.

S: But, for example, if I want to study psychology, mathematics will not help me to study psychology. For example, "basic math", as we said before, in real life does work for me, but ...

B1: In everything we have lived through, we haven't used a square root at all. Rule of three I have used, very few, but square root for nothing, knowing the apothem of whatever, either... I do not walk down the street calculating apothems.

R: And, for example, if any of you were a journalist ... Do you think math could help you to be a good journalist?

B: All knowledge helps you to come to more knowledge. But I think it could also do well without math. That you can do better, yes. But if you do not like it, why are you going to put in the effort? [sic!]

R: Do you think I could use math in a particular situation as a journalist?

E: For example, maybe you have to do a survey, and for that you do need math.

B2: You must calculate the schedule, what time do you have to be to do certain something...

B1: But, for example, with the survey, you can hire a worker to do it. But, to calculate the time, you need "basic" math, you don't need to know a rule ...

S: But, the basic thing, what we use in life, is addition, subtraction, multiplication, and division, and studying the hours. And the percentages, that is it.

L: Yes, things that are of no use to you. For example, equations of the second degree, me, not at all. I mean, and I am terrible at these.

E: Sure, imagine that you are a lifeguard, you are not going to calculate the distance you must jump to catch [someone drowning] ... it is already dead in the water.

The first answer of the students is an automatic "it's very important (despite its ugliness)", i.e., a conformity with the socio-political trait we mentioned at the beginning of this presentation. But shortly afterwards, the contradictions, questionings and even ironies emerge again, as in the first group. It seems that math is not so valuable in everyday life, but also in many professional activities; is useful mostly for those things labelled as "science", whatever that means. Only basic and elementary school math has proven to be useful to them.

Group 3:

R: Why do you think math is compulsorily taught in ESO?
G: It is something we have to know... at least a minimum.
A: For our future, I don't know, so that we can...
S: There is always the same excuse. What are you studying it for? To approve the test, but...
G: Let's see, until the ESO, it's fine. Then, more math, well ... whoever wants to aspire, well.
[...]
R: How do you see it?
S: Well, I say, math is going to be used and such, but to a certain extent. There are other things that you look at or whatever, but because you have to approve it. But that's the excuse you've always been given. You study it, whatever, because you have to pass. Because another thing's, what are you going to use them for?
G: I don't know, it's what they made us do. Study to pass, and the same with everything.
[...]
J: Man, people have to perfect themselves, and humanity also, if we stayed at a certain point, we could never move forward, we could not help other people.
G: But it's considered good [sic] to study just to pass. And not that they teach you to learn. Because, if you study it, you will forget it; If you learn it, it will stay with you. Someday you will forget, but ...

It is again difficult for students to justify the supposed importance-usefulness of school mathematics, at least from what they are studying in the ESO courses (2nd-3rd year at the time of the discussion groups). In general, the immediate or automatic response is "because we have to know it-it's very important-it's good for life". But then the doubts and questions appear...

¡No, wait! Math is important, you have to study it "to pass" (the test, the courses), to "be approved" ... to go on with more studies. To progress.

And beyond that, it's not so clear for the students.

Group 4:

R: What do you think mathematics is taught you for, and what do you think mathematics is used for? Please be sincere.
D: Let's see, mathematics is taught, really speaking, to teach us (sic!) ... to know how to unwind...
(J. laughs at the non-sense)
D: J. you're going to take one...

J: (ironic)... Mathematics is taught and used to teach...
D: Yes, they are taught because if you don't know mathematics, you will hardly know anything else...
J (sarcastic) But also, they are used to teach ...
D (annoyed) He won't let me finish the sentence... do you want to shut up? I didn't say that...
(Brief argument)
D: Anyway, we said... mathematics seems to me to be necessary, as J. said. It is "necessary" that they teach it. But sometimes I also thought that it is a lot to give #@*%*...
M: And that there are times when they are not understood ... Why do I have to learn this formula, or five hundred thousand formulas [...], if then maybe a mathematics teacher comes to give classes or is going to dedicate to some math, and he is going to go to the notebook and look at the formulas ... Why do you have to learn those formulas? Sometimes it doesn't make sense.
D: Because if they don't understand it then they won't know how to explain it ...
R: I mean, but you, think about it, why are Mathematics taught to you ...
(Discussion)
D: Let's see, let's be clear. Mathematics give a lot of #@*%*. And even more so when we are... consider that mathematics can make you repeat a whole course where you are doing well, and because you don't pass math, you repeat the course! I mean, that's really annoying!
R: If you don't pass math and language, you repeat the course. That is fashionable, it has become fashionable.
All: Yes, yes.

"They are important-useful because they teach us and they are useful-important." In all the discussion groups of the research experiences emerged some variant of what is shown in these excerpts: in a first automatic response, someone affirmed that mathematics "is very important-useful"; but shortly after the discussion began, the supposed importance-usefulness of math was either (not) justified with non-sensical expressions, or it was quickly questioned by the group. That is, there are contents of the school mathematics that are too abstract (difficult, without context?). And it is not at all obvious how they would be used in "daily life" (or in most careers, professions, vices, felonies); they may be used in specialized applications of certain professions, in "sciences" or for "engineers", not in "everyday life". Arguments of "common sense", but arguments after all...

All in all, in the discussion groups emerged a marked skepticism about the importance-usefulness of school mathematics, despite the first norm-conforming automatic responses.

However, another *sense-event* comes to light, perhaps more clearly in the last excerpt: the compulsory nature of school math, and its strong character of selection/exclusion through the assessment device. There are many expressions like "you study them because you have to pass", and D.'s elegant rhetoric in the last excerpt. Math is useful, but not really, and yet, they are tough and compulsory and can jeopardize the future perspective of students. What?

Let's go back now to the student who wanted to study obstetrics: school mathematics would hardly be important-useful in her intended profession, at the time of assisting in childbirth (we can't imagine how an obstetrician would need to calculate the sum of an infinite series at that given moment; perhaps we are wrong ...). But if she doesn't perform "well" on school math (that is, high enough grades), she won't be able to earn enough political credits to be a midwife. It is not difficult to extend this idea to the paradox of useless-usefulness of school mathematics: "you have to study them to approve them, because they will be useful to you, not in themselves (rather the opposite, they are applied little to nothing in most circumstances of "daily life" and many trades); but because if you do not pass them or do not have good enough grades, your personal possibilities in the future will be limited". Assessment, the sine-qua-non device of this stratification machinery, is implicit in all these disquisitions and *non-sense*...

Non-sense in a plate: math is useful and useless, essential but not really important...

Prudentia

In his own personal experience as students (teen and adults), we never had major problems with math and even enjoyed studying them... However, about tastes and colors... mathematics is a very particular discipline, and even when taught wonderfully, the students may or may not like it ... no, yes?

Well... we are also a bit fed up with seeing over and over the same disconnected, non-sensical, useless topics with our students; repetitive assessments, the suffering in our own flesh of the failures of our students and having to repeat again and again ... What we did not personally experienced in person of this selection/exclusion system, we are living it now through our present students. (And yet, somehow, we need the failure of the students... because in part we gain our bread out of it, and this chapter is and the participation in this conference is partially financed with that income...).

Furthermore, the students do not have such a naïve perspective, nor do they lack a critical view of the arbitrariness of ME, as a selection/exclusion mechanism; and of the doubtful importance of the "contents" of school mathematics, in daily life. We do not necessarily agree with the perceptions and evaluations of the students (rather, the opposite in many cases...). It cannot be denied that in their own way, the kids question some features of the symbolic power of mathematics and ME, as explained in the first part of this chapter.

So yes... there is something paradoxical, non-sensical, in the students' perception regarding the importance-utility of mathematics. Somehow, like in a Lewis Carroll novel or an El Chavo del Ocho episode, they consider mathematics useful and useless, important and unimportant, at the same time. What can the students, their teachers, we researchers, do about it?

Well, first of all acknowledge that, most likely, we also have a lot of paradoxes and internal, repressed non-senses about the sociopolitical place of mathematics, ME and the school as political technologies. So, we adults can, at least in principle, assume that this intrinsic tension is present.

As Cancino (2011) explains, in the individual psyche-social imaginary tension, "society imposes socialization on the psychē through its institutions." In contrast, "the psychē imposes an essential requirement on the social institution: the social institution must provide it with meaning" (Cancino, 2011, p. 73; Castoriadis, 2002, p. 268).

So, we also propose to interpret this non-sense as a way of experiencing the symbolic power of mathematics. More precisely, as a way of conforming their (our) own subjectivity in face of the symbolic power of mathematics, and especially of ME and school mathematics as technologies of selection/exclusion and/or accreditation/stratification in contemporary capitalist societies.

Valero and Pais, in an article from 2015, summarize:

> What if school mathematics is not important in society due to the exceptional and intrinsic characteristics of the academic field that gives this school subject its name, but rather due to the place it occupies within a particular social configuration of power? […] A political approach thus assumes that the teaching and learning of mathematics are not neutral practices, but that they insert people —be it children, youth, teachers, adults— in socially valued mathematical rationalities and forms of knowing. Such insertion is part of larger processes of selection of people that schooling operates in society. It results in differential positioning of inclusion or exclusion of learners in relation to access to socially privileged resources such as further education, labor market, cultural goods, etc. (Valero & Pais, 2015, p. 178).

As Gilles Deleuze elaborates in *La logique du Sens* (1969), *non-sense* and paradox are intrinsic to *sense*. Either in the proposition or in the (emerging) event and singularity, non-sense allows for the operation of *donation of sense*. By exploring (exploiting without mercy) these internal tensions of *non-sense* we attempt to reveal some chin in the armor of the symbolic power of mathematics (education).

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White intellectual alibies in use: A critical analysis of preservice teachers' rhetoric

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The demographic disconnect in the U.S. between the majority white, female teacher workforce and the diverse students they serve perpetuates white supremacy in various ways. These relationships can be especially problematic in mathematics settings, where race issues are often disguised behind discourses of neutrality, intelligence, and meritocracy. To further understand how white supremacy is enacted in educational spaces, we applied Leonardo's theory of "white intellectual alibis" to critically analyze interview data involving a pair of white-identifying preservice teachers engaging with novel hypothetical scenarios. Findings show that participants utilized various alibis that reinforced racist narratives and silenced possible antiracist conversations. Implications for teacher education are discussed.

Introduction

One way that white supremacy is perpetuated in U.S. classrooms is through disproportions between white teachers (79%) and students of color (50%) (Taie & Goldring, 2020; Yoon, 2012). Often, white teachers fall back on "hidden expressions of disgust for the Other" (Matias & Zembylas, 2014) and rely on the privilege of whiteness afforded them in existing systems. Within schools, mathematics spaces can be especially problematic as white privilege is further exacerbated by narratives of meritocracy, knowledge neutrality, normative intelligence discourses about who is capable and who is not, and white masculinity (Bullock, 2017; Martin, 2009; Warburton, 2015).

For the white teachers who wish to take part in mending this broken system, and would like to understand their role in perpetuating its preservation, the view from their classrooms can seem daunting. Perhaps white teachers are aware of centuries of inequality in the U.S. (Kendi, 2016; Ewing, 2018), which result in disparate, racialized, educational outcomes that significantly disadvantage students of color (e.g., academic outcomes, graduation rates, and college admissions). They might wonder if their daily interactions with students will have any impact on students' lives inside and outside of the classroom, or affect lasting change in helping reform inherently racist systems.

Although it is tempting to argue that white teachers are not the *problem*, but rather the *symptom* of these racist structures, we choose to focus on them because we contend that the

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actions of individuals can either rescript existing narratives or challenge them (Perry, 2011). Teachers who enter their professions with antiracist identities and the skills necessary to enact antiracist education are better equipped to be part of positive systemic change (e.g., Lewis, 2018; Tilson, Sandretto & Pratt, 2017). Thus, we seek to address the practices of white teachers as leverage points for potential change.

Teacher education programs are potential sites to address these issues. However, it has been argued that teacher education has itself been a site of whiteness's remaking (Daniels & Varghese, 2020, p. 57; Jupp, Leckie, Cabrera & Utt, 2019). Even within teacher education programs that are dedicated to social-justice issues, there continues to be disconnects between antiracist education and teacher practices (e.g., Agarwal, et al., 2010; White, Crespo, & Civil, 2016). This is often due to how white preservice teachers (PSTs) work to avoid and deflect interrogations into their own positionalities, consequences of whiteness, and how racism and white privilege are enacted (e.g., Buchanan, 2016; Lewis, 2018; Matias & Zembylas, 2014). A recent review of 39 peer-reviewed journal articles (all focused on the intersection of preservice teacher education and race) found that "teacher educators continue to face the problems of race talk evasion, colorblind racism, and even retaliation" that were evident in early white teacher identity studies (Hambacher & Ginn, 2020, p. 339).

Examining how PSTs affective responses and white subjectivities are enacted during "uncomfortable conversations devoted to naming the consequences of racism" has potential to illuminate various leverage-points and potential pitfalls for teachers and teacher educators to design and implement antiracist education (Buchanan, 2016; Lewis, 2018; Matias & Zembylas, 2014; Sharma & Lazar, 2014). In this study, we look carefully at the discourse of two preservice elementary teachers, Abby and Sarah, as a window into practices that evade race and racism. Data come from a pair interview; during which, the pair were given several scenarios and were asked to imagine that they were teachers in a fictitious classroom and would need to provide next pedagogical steps that would support students pertaining to content or issues of equity within a mathematics or science setting.

This paper focuses on one of these discussions in which participants were faced with hypothetical scenarios where a student shared a concern regarding how they perceived that "white kids" were being unfairly called on more during math class. Their discussion offers clues into ways which white teachers might unknowingly perpetuate racist status quos, and how they can often see defensive and race-evasive behaviors in another teacher's reactions (e.g., denial, avoidance, unfair blaming), though not in their own, similar, reactions.

Theoretical framework: White intellectual alibis

In this paper, we focus on how the participants used *white intellectual alibis* (Leonardo & Zembylas, 2013) to "prove" their innocence when faced with the possibility of their own racism. The alibi is a spatial metaphor: "The criminal is ruled out as a suspect once he furnishes a fail-proof accounting for his whereabouts […] he cannot be in two places at once" (p. 152). White intellectual alibis create a racist binary; whites positioning themselves as good nonracists, and "other" whites (or their former selves) as bad racists.

White intellectual alibis leave no room for the possibility that one could espouse antiracism yet maintain discriminatory practices (Leonardo & Zembylas, 2013; Perry, 2011). This suggests an extant nonracist, rather than keeping focus on possible antiracist narratives. Through avoidance and silences, racism proceeds unchecked and internalized messages about white superiority are perpetuated, rather than investigated and challenged (Souto-Manning, 2013). In this way, *acting* antiracist takes precedence over *advocating* for antiracism projects.

There are many possible alibis, such as: "my best friend is black"; "the n-word is just a word"; or claiming "I don't see race" (Nishi, Matias, and Montoya, 2015, p. 462). The findings presented here show how participants Abby and Sarah created white intellectual alibis for themselves through various discursive strategies. Here we pay special attention to three of these alibis—*I'm not like bad racist "others"; I'm a strategic problem-solving teacher;* and *Well, life isn't fair.* Through the lens of white intellectual alibis, we analyze how discourse can work to "prove racist innocence" and avoid "difficult" conversations; we suggest the growing list of white intellectual alibis in teacher education might be expanded to include our findings.

This framework, as well as the white intellectual alibies described in this paper, are informed by a rich tradition of CRT and whiteness theory. For example, Bonilla-Silva (2018) provides a useful guide to recognize rhetorical moves that signify "color blind racism": In this data corpus we see how, as Bonilla-Silva outlines, meritocratic-thinking functions as race evasion (seen in *the strategic problem-solving teacher* alibi). Additionally, in avoiding race talk by discussing the how life may not be fair, or focusing on the behaviour of others, participants were discussing "anything but race" in order to "dismiss the fact that race affects an aspect of [their] life" (Bonilla-Silva, 2018, p. 86). In focusing this analysis on white intellectual alibies, we do not intend to overstate the power of this theoretical lens. We use it to show one of the ways whiteness functions in action – as "evidence" for white innocence.

Participants and methodology

The qualitative data for this study come from a pair interview with two preservice elementary teachers (Abby and Sarah) enrolled in a university teacher training program. Both participants and the interviewer (Author 1) identified as white women. The interviewer approached the participants (and approaches this work) in a spirit of complicity–understanding that her work as a white teacher has been problematic at times, and she must confront and challenge her own white intellectual alibis.

The interview protocol was created as part of a larger project focused on improving math and science content and methods courses in a university teaching licensure program. As part of this project, a team of researchers gathered qualitative audio data of PSTs discussing various classroom case-study scenarios, which we call *hypothetical teaching scenarios* (HTSs). Using hypothetical scenarios in teacher education settings to approximate real classroom interactions is common practice (Shaughness & Boerst, 2018). However, what is less common is gathering and analyzing qualitative data of teachers *collaboratively* engaging with HTSs, in small groups or dyadic interviews, for example. The HTSs in this project were designed

to elicit rich discussion about issues that might arise in a mathematics or science classroom and covered a wide range of content and pedagogy (e.g., using representations in mathematics; evaluating arguments in science; addressing sexism, racism, or intelligence discourses in STEM settings; see Gutiérrez et al., 2019).

This paper focuses on two HTSs (Figures 1 & 2). Next, we describe our protocols.

Responding to a Student Grievance

Imagine you are teaching a 5th grade math class. One of your students asks to talk with you privately and you agree to meet with them. They start the conversation as following:

Student: "I don't know how to say this, but… it seems like you only call on the same three white students to show their work at the board when we're doing math. And I have my hand up too! But you never call on me… What's up with that?"

Figure 1: "Responding to a Student Grievance," a hypothetical teaching task.

After reading this scenario, small groups of PSTs were prompted to imagine how they might respond to the student, then write and discuss their responses. From analyzing these responses, the research team created a coding scheme that highlighted common ways PSTs used language to respond (e.g., apologies, explanations, or "solutions"; Gutiérrez et al., 2020). The research team proposed taking this HTS into an interview setting where the interviewer would ask follow-up questions and provide individualized prompts. We chose to utilize a pair interview format, where participants could interact back-and forth with a friend as well as the interviewer. We hypothesized this format would create an environment for participants to discuss difficult or uncomfortable topics, such as race, in ways that they might not otherwise. In this setting, at times, the interviewer prompted participants to discuss responses with one another and took an active observer role (Morgan, Ataie, Carder, & Hoffman, 2013). At other times, the interviewer jumped into the conversation, sharing relevant experiences and prior racial assumptions or beliefs.

For this pair interview study, we also sought to understand how PSTs viewed another teacher's response; to see if they might notice whiteness discourse outside of themselves. Thus, we added an HTS, *Student Grievance Conversation* (Figure 2), that included an imaginary conversation between the hypothetical teacher and student. The hypothetical teacher's responses in this HTS were crafted to closely follow common themes from the small-group discussion analysis (Gutiérrez et al., 2020). During the interview, Abby and Sarah were presented with paper versions of these scenarios, one at a time. For *Responding to a Student Grievance* they were asked to write how they would respond to the student, then share their responses with one another and discuss. They were also asked follow-up questions and prompted several times throughout. Next they were asked to read the *Student Grievance Conversation* and were given similar protocol prompts. However, in this case they were asked to discuss their thoughts about the teacher's response, rather than their own responses.

Student Grievance Conversation

Imagine the same student and a teacher (other than yourself) had the following conversation after the student shares the same concern:

Student: "I don't know how to say this, but… it seems like you only call on the same three white students to show their work at the board when we're doing math. And I have my hand up too! But you never call on me… What's up with that?"
Teacher: "I'm so sorry you feel this way. I never meant for this to happen. I will try and call on students more fairly in the future."
Student: "Thanks. That's cool. I don't think it's just that I feel this way, though. I notice the same thing happening with my friends or at the store. I just feel invisible sometimes. I just want white people to admit that they treat me this way."
Teacher: "Well, again, I'm sorry you feel this way. Not every white person treats you this way. I care for all my students. I can't control your friends or people at the store. But I can control what I do in my classroom."
Student: "OK. Thanks." [walks out of the room, but seems disappointed and has head down.]

Figure 2: "Student Grievance Conversation," a follow-up HTS.

The interview was video and audio recorded, and transcribed. We applied Critical Discourse Analysis methods to analyze, understand, and explain the data in order to "speak to and, perhaps, intervene in institutional, social, or political issues, problems, and controversies in the world" (Gee & Handford, 2013, p. 9).

Findings

Here, we highlight three of the white intellectual alibis consistently employed by Abby and Sarah. Much of the discourse seen here is discussed in whiteness literature to explain, for example, ways in which avoidance, deflection, image management, or cultures of caring work to shift conversations away from antiracist ones (e.g., Lewis, 2018; Orozco, 2019). We present these data under the lens of white intellectual alibis in order to further highlight ways that rhetorical moves are used as spatial dividers between "innocent" nonracists and "guilty" racists.

The strategic, problem-solving teacher alibi

Both Abby and Sarah wanted to "solve the problem" as evidenced in Abby's statement, "I would feel horrible if a kid felt that way, and I would want to fix it." Solving problems and fixing things is generally considered an important part of a teacher's job; however, using pedagogies and classroom practices as comprehensive solutions to racialized situations is, we submit, a type of white intellectual alibi: *I'm a strategic, problem-solving teacher*. The use of this alibi both exonerates white teachers from being racist, and from participating in further discussions of race. Notice how this alibi functions by considering Sarah's initial response to the first scenario:

> I would never call on the same three white kids […] I wouldn't teach like that […] [I would] pull sticks […] making sure every child can answer the question.

Although Sarah used the term "white" (a rare occurrence in the data corpus), there was no prior or further discussion about race or racism. She proceeded as though she had solved the problem by pulling sticks in order to randomize student participation (her solution for equity.) This illustrates how an alibi works to "solve the problem," and shut down possible conversations about race. Sarah could not be in two places at once: with strong pedagogies and classroom management skills, she precluded herself from committing a racist crime and interrogating potential racism or biases.

This alibi also worked to pacify the student and "prove to him that they were not doing it on purpose" (Sarah). Instead of engaging in critical self-reflection, Abby and Sarah addressed the student in order to "fix the problem," proposing several solutions. For example, at one point Sarah offered the following solution: "I will make sure to wait a little bit longer for people to put their hands up as well." And another time, Abby sought to involve the student in the solution: "Like, do you have a solution […] so how can I fix this for you? What I'm doing might not be working for you. What do you have in mind?" In focusing on pragmatic solutions for the issue at hand, and in working to "fix" things by getting the student's input, Abby distanced herself from the guilty racist charge. There was no further discussion about racism afterward; these conversations about classroom pedagogies were used to absolve Abby and Sarah and convince themselves (and others) that there was no possibility that a racist crime had been committed.

Well, life isn't fair alibi

Another alibi can be seen in the following excerpt that occurred after Abby scanned the second HTS and audibly sighed:

> I can see […] it talks about the white people, and stuff, and the sad thing is, you can't control every aspect of their lives […] I can only do so much […] Like, I can't go everywhere and make sure everybody treats them fairly.

This statement seemed to put-an-end-to, or take place of, possible discussions of racism. As with other alibis, on its face, this is a true statement: Abby cannot "control every aspect" of a student's life. However, the work is not done through the statement alone, but through the insinuation that this fact is not compatible with problematizing racist selves and practices.

We call this alibi *Well, life isn't fair*. One of the most notable incidents of this alibi was near the end of the interview when Sarah explicitly addressed the possibility of her own racism by saying:

> I feel like I'm coming off racist […] I'm not […] Like it's always a battle […] but you're not going to win every battle […] like there are going to be really hard things to deal with in life […] and there's some really crappy people in the world, but we need to, like, be above those people.

Sarah's contention that she was "not" racist was followed immediately with the *Well, life isn't fair* alibi (e.g., "Like it's always a battle […] but you're not going to win every battle"). The way that the nonracist comment was paired so closely with this alibi is evidence that Sarah saw the two as mutually exclusive. Her racism couldn't coexist with the fact that life was, in general, unjust and "hard." For Sarah, the two ideas could not occupy the same space at the same time.

I'm not like bad racist "others" alibi

The last part of the excerpt above, "there's some really crappy people in the world, but we need to, like, be above those people," is an example of a common theme in Abby and Sarah's speech: racism does exist, but only in others ("crappy people".) This alibi, *I'm not like bad racist "others,"* was especially apparent when Abby and Sarah, positioned as observers of another teacher, responded to the second HTS. They spoke as though the teacher's response was racially motivated, whereas, they projected their own motivations as unintentional—creating a nonracist alibi while distancing themselves from the racist "other" white in the scenario (Leonardo & Zembylas, 2013).

Especially revealing of this was when Sarah critiqued the teacher's apology:

> The *but* takes away the sincerity of it. It's like, 'blah, blah, blah, this, this, and this. *But* I can't really help you.' I hate the comment, 'I'm sorry you feel this way.' To me, that's like, it's almost like, 'Oh, yeah. I'm sorry you feel this way!' [sarcastic tone] That's not validating their feelings at all […] it's shutting it down and trying to make it a smaller problem.

Here, Abby's critique of the teacher's apology ("I'm so sorry you feel this way") was very similar to Abby's apology from the first scenario ("I'm so sorry if, like, that made you feel […] I'm so sorry if that made you think that I never call on you."). Despite the obvious similarities in the apologies, Sarah never mentioned this irony. In fact, she "hates" the teacher's apology. Looking at this phenomenon through a white intellectual alibi lens, we can see that it functions similar to a legal battle where the defendant's character is either defamed or commended. Again, we see how the spatial binary works: a defendant either has "good" behavior and, thus, is not capable of committing the crime of racism, or has "bad" behavior, and thus, is likely to be racist. Further, in pointing-out the bad behavior of others, the defendant separates themselves even more from the scene of the crime.

Discussion and future research

White intellectual alibis

This analysis shows how, as Abby and Sarah navigated a racialized discussion, they spent a significant amount of time creating white intellectual alibis that carefully managed their images as nonracist. Although both teachers seemed genuinely interested in addressing and rectifying the problem broached by the hypothetical student, they were rarely direct in addressing racism as a possibility. Abby and Sarah seemed, primarily, concerned with proving their innocence to themselves and the hypothetical student through the use of alibis

These findings validate the claim that "turning to whiteness in education means that the subjects who are least individually prepared and collectively underdeveloped for race dialogue occupy a central place at the table" (Leonardo & Zembylas, 2013, p. 155). This is a call to white educators like me (Author 1) to interrogate the alibis we use that allow us to excuse ourselves from racist practices. So doing, I hope to model for PSTs a process of critical self-reflection, attempting to expose *my* white intellectual alibis and engage in race-visible pedagogy (Jupp et al., 2019).

Hypothetical teaching scenarios

The novel use of HTSs in a pair interview has several implications for teacher education and critical education research. Our findings reveal that, although the participants were engaged in discourse, they carefully avoided discussing racism and possible antiracist pedagogy. This implies that teacher educators who wish to use HTSs as antiracist pedagogies will have to carefully consider design that nudges participants to explore issues of racism more directly. Otherwise, group discussions might be spaces where whiteness continues to have a primary role, instead of discussions where whiteness is questioned and PSTs are able to see their roles in problematic racist practices. In the case of Sarah and Abby, one possible tactic might be to circle back to their response to the first scenario and highlight the similarities in apologies. This might create a generative tension that can carry them through a crucial dialogue and confronting their potential racial biases.

Limitations and extensions

This HTS was couched in a mathematics-centric protocol; however, narratives about race in mathematics settings were backgrounded in this particular scenario. We were curious to see if participants would be primed (by the previous HTSs) or notice that the student referred to math in his complaint; however, none of the participants focused on this point. In future studies, we hope to find a way to include race issues while simultaneously highlighting the mathematics setting, in order to see how PSTs discuss these issues in tandem.

Through much feedback and discussion, we imagine there are many variations of this HTS that might extend our understanding of how whiteness functions in teacher education settings. For example, as suggested by an MES reviewer, the HTS might be structured to "reveal the kind of anti-racist response" that we might encourage in PST learners. As per his suggestion, perhaps the teacher would be directed to "ignore the dominating white male students in order to allow other voices to emerge." In this setting we could investigate how PSTs respond to the idea of "using discrimination to overcome discrimination?"

Finally, this work might be extended to other settings where whiteness is at work. It seems natural, for example, to use HTSs in professional trainings for university faculty and staff in STEM degree settings where, typically, student diversity is low, unexamined biases are high, and individual faculty members are engaged in practices that can either reinscribe or challenge this status quo (Killpack & Melón, 2016).

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Mathematics in action: an approach in civil engineering and ecology courses

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This paper presents part of the results of a doctoral research linked to the teaching and learning of mathematics in higher education. This study aims to reflect on the ways in which mathematics can be perceived in real life contexts – this conception is known as Mathematics in Action. The data were produced from activities developed based on the perspective of Problem-Based Learning, using a qualitative approach. They were developed in Civil Engineering and Ecology courses at two public higher education institutions in the state of São Paulo, Brazil. The results show evidence that learning contexts that present mathematical formulas based on reality can favor reflections on actions or decisions based on mathematics. They also contribute to the production of other studies involving the field of mathematics education in higher education.

Introduction

In recent years, concerns about the challenges and perspectives of higher education have been the subject of international level discussions. The aspects involving the teaching and learning processes, as well as the understanding and use of knowledge, in different courses, are linked to future actions of individuals in society, whether personal or professional. As a result, concerns also emerge regarding the ways in which the teaching and learning processes occur.

These concerns also extend to the field of mathematics education. After all, the notions linked to mathematical knowledge move in several directions. Reflecting on them can stimulate the development of critical perspectives. In addition, the aspects that revolve around the teaching of mathematics can be explored through an environment that encourages investigation processes.

The main objective of this paper is to propose discussions about the perceptions of mathematics being put into action in real life contexts, which is understood in the literature as the concept called mathematics in action, proposed by Skovsmose (2006, 2014a, 2014b, 2020). The inclinations of the respective work encompass perspectives based on Problem-Based Learning (PBL). The intention was to establish greater approximations between theoretical studies involving mathematical knowledge and its practical applications projected in real life contexts.

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In the academic world in general, there are already studies related to mathematics and learning based on problems and projects in the context of universities. Among them, it is possible to mention works such as: Vithal, Christiansen and Skovsmose (1995); Christensen (2008); Hernandez, Valero and Ravn (2015); Gouvêa (2016); Souza (2016); Valero and Ravn (2017). However, there are no empirical studies related to the concept of mathematics in action in Higher Education from PBL perspectives. This is a fundamental characteristic that demonstrates the originality of this research. Making investigations possible in this sense is something relevant to students, the academic community and our society. Thus, the purpose of this study is to reflect on some learning contexts that are based on reality and the use of mathematical concepts or formulas, in two bachelor degrees: Civil Engineering and Ecology.

Initially, this paper presents the methodological perspective and some procedures used in the referred research. In addition to describing the elaboration and development of the proposed activities, which were outlined based on a real situation, inspired by the PBL. Following, the theoretical assumptions related to the development of mathematics in action, which can be perceived in reality through five performance aspects are highlighted.

To conclude, are presented discussions about elements that were present in the construction of one of the three themes elaborated from the analysis and interpretation of the data, which was entitled "Learning contexts with formulas or mathematical concepts can tell us many things". In this stage, we highlighted three aspects of mathematics in action: technological imagination, hypothetical reasoning and realization, in addition to relevant considerations about the learning principles related to PBL.

The study proposes reflections on the different ways of perceiving the presence of mathematics in real life. Actions or decisions based on mathematics can be analyzed and questioned in universities through practices that lead to reflection, in addition to collaborating with the personal and professional training of students. Contributions such as these can favor studies in the field of mathematical education and concerns related to the challenges and perspectives of higher education.

The study design

The research molds were built from a qualitative approach, which seeks to interpret in a careful way the dynamics of the observed study.

The data production stage took place at two different times, in two public higher education institutions in the state of São Paulo, Brazil, during the second semester of 2018, through weekly meetings. In the first institution, named in the research as institution A, a group of first year students of Civil Engineering participated in the investigation, during extracurricular hours. In the case of the second institution, called institution B, the participants were second year students of the Ecology course and the whole process took place during some classes in the course of Differential and Integral Calculus II.

The study and analysis of a real problem mobilized the dynamics of meetings. There were four meetings at both institutions. At institution A, activities were carried out by the researcher herself. In the case of institution B, in addition to the students and the researcher,

there was the participation of the class teacher and a research assistant, who was part of the group of studies and research in mathematics education to which the author of this paper is linked. In this same institution, the students were divided into small groups and the discussions held took place through three meetings. Furthermore, the fourth meeting was destined for the final presentation. At this stage, the students could approach themes addressed throughout the process.

All data produced were recorded through audio recordings, and the researcher wrote comments in field notes, respecting the research's ethical principles.

Regarding the data presentation, there were concerns about demonstrating the potential of the work based on the discussions generated by a particular problem. The transcription of the data resulted in the first drafts, which were read and rewritten a few times. This process culminated in the final production of two texts, presented in two distinct chapters, one focused on data produced at institution A and the other, focused on institution B.

The data analysis process was inspired by the theoretical assumptions of Creswell (2007). For the author, the constitution of the analysis occurs through some steps that range from the organization and preparation of the data to the final interpretation of the results obtained.

After carrying out the analysis some categories emerged, which led, finally, to the stage of elaboration and description of the themes. According to Creswell (2007), these themes represent the main results in qualitative studies. Furthermore, they must be based on the theoretical contributions of the study and on specific evidence related to the research issue. In this thesis, three central themes emerged, which are directly or indirectly linked to the five aspects of mathematics in action. The three themes are:

1. Learning contexts with formulas or mathematical concepts can tell us many things
2. Elaborating legitimations and justifications through mathematics
3. Reflections on responsibility, ethics and valuation

In this paper, the discussions refer to the first theme, which discusses the use of certain concepts or mathematical formulas in different situations. The contexts analyzed by the participants mobilized different reflections about possible applications of a mathematical model in reality.

The problem that was used as the guiding thread of the investigation process is presented below.

From the organization to the understanding of the problem used

The material organization that led some of the face-to-face meetings as part of the master's research developed by the researcher (Souza, 2016). Its structuring followed theoretical assumptions related to the PBL's use. At that time, four problems were elaborated, which could be worked on in the exact, humanities and natural science areas, for example. These problems were structured by the researcher based on real or potentially real contexts and, among them, there was a suggestion of possibilities to assist in conducting the activities.

Soon, during the doctorate, there was a restructuring of one of the problems presented in the dissertation, based on new theoretical contributions. The intention was to use the chosen problem as a trigger for the discussions that would be held during the face-to-face meetings, seeking to ascertain whether the choice would open up possibilities to foster different reflections, in addition to the field of mathematical education.

"Environmental impacts caused by chemical pollutants" was the title of the problem. It consisted of two approaches: one involved a description of a fire that occurred in fuel tanks, in 2015, in the Alemoa neighborhood, in the city of Santos, state of São Paulo, Brazil, causing contamination of the region and several environmental impacts: social, political and economic. The other approach called "Support study" was elaborated from some adaptations of an activity found in a Calculation book. It addressed a fictitious situation, similar to the real case that occurred in Santos, and sought to contribute to discussions related to different mathematical knowledge worked in university.

The subsequent section includes theoretical references to the themes involved in the present study.

Understanding the concept called mathematics in action

The concept called mathematics in action was developed by Ole Skovsmose and is part of the concerns associated with the field of critical mathematics education. We understand that mathematics is associated with a variety of situations and practices, like social, economic and political aspects. Thus, it is possible to propose discussions about the operationalization of mathematics in different contexts.

When talking about mathematics in action we are referring to

> All those practices that include mathematics as a constituting part. It could be: technological innovation; forms of production; automation; management and decision making; financial transactions; risk estimation; cost-benefit analysis, etc. (Skovsmose, 2006, p. 323).

For this author, practices like these contain actions based on mathematics and, therefore, there is a need to promote reflections around them. In this way, this concept can be understood from five aspects: *technological imagination*, *hypothetical reasoning*, *legitimation or justification*, *realization* and *dissolution of responsibility*.

The technological imagination refers to the possibilities of exploring the construction and development of hypothetical situations in the form of technological alternatives, based on imagination. Every type of enterprise can be based on imagination, that is, on an imaginary scenario. Hypothetical situations can be raised and mathematics can provide materials for the construction of such situations.

Elaborate the hypothetical reasoning it concerns the analysis and evaluation of the possible consequences of an imaginary scenario. In other words, this aspect contributes to investigations about a given situation before an undertaking is executed or a decision is made. Mathematics can help in the approaches related to hypothetical reasoning because the hypotheses and investigations about the probable results of something not yet realized can be made through the use of mathematical models.

The construction of justifications and legitimations math based can be used for certain actions or decisions be taken. The aspect of realization takes action when the mathematics becomes part of reality in everyday life, but many effects may be hidden in these models and the categories and discourses produced from them are varied and their consequences can sometimes be pleasant and beneficial, sometimes they can present risks and disadvantages.

Finally, the dissolution of responsibility occurs when actions supported in mathematics include an exemption of liability. Normally, any action refers to an agent subject, but, in mathematics in action, that subject's performance does not seem to exist and the relevance of responsibility seems to be in charge of the mathematical model used. When addressing the question of responsibility, it is necessary to analyze whether the methods and tools used are reliable; whether the calculations performed are reasonable; whether (and which) aspects were ignored in the formulation of the model; etc.

Next, we presented some analyzes regarding the understanding of three aspects of mathematics in action constructed from the constitution of scenarios that encompass concepts or mathematical formulas.

What learning contexts involving reality and mathematical formulas can tell us

This section presents some previous results of the doctoral research. In different moments of data production, the use of some concepts or mathematical formulas in different situations mobilized many reflections. These contexts contributed to some aspects of mathematics in action that emerged during the discussions.

The construction of the theme "What learning contexts with mathematical formulas based on reality can tell us" contemplates three aspects of mathematics in action: technological imagination, hypothetical reasoning and realization. Thus, to contemplate them, another perspective is presented to approach the different mathematical content in universities.

A teacher putting different mathematical relationships on the board. A textbook or handout material. Students copying examples or solving exercises. Usually, this landscape prevails in many classes that contemplate mathematics in higher education.

The contexts presented below involve a blackboard, study materials, examples or exercises. However, everything was contemplated through the students' protagonism. The perceptions that emerged during the meetings were possible through the material delivered to the students about the case of the accident in Santos-SP. To start the discussions, the following a student's speech from institution A:

> Rogério: [...] we could work with some mathematical models thinking about the influence that [the fire in Alemoa] would have on the traffic of that region. They could be linked to a project that was done or that we want to do. You can see how much this will influence the fauna, you will have to move the animals that are around and you have to have all this planning part of how much this would change in the surroundings. For example, statistical data ... I think it would be interesting to include them in the analysis to quantify how much would change and to take the respective measures to mitigate the impacts, as much as possible.

According to Rogério's speech, we understand that mathematics is associated with the aspect of *technological imagination*. The examples he describes reveal the development of hypothetical situations and show that there are possibilities to think about different alternatives based on the analysis and understanding of mathematical models and the use of statistics. The notes made by the student concern the forms of organization, know-how and procedures related to the execution of projects and undertakings, as well as decision-making.

Another issue that emerged in the discussions was present in the "Support study", and that referred to the self-purification process. At institution A, the participants worked effectively with the material delivered at the beginning of the activities. When elaborating and developing it with the students, there were no specific explanations as to the meaning of the term self-purification. Interpretations related to that word were developed over the meetings and were made by the students themselves, based on the analysis of a bay's self-cleaning ability. The following dialogue was based on the following information and took place between researcher Débora and students Luara and Renata, both from institution A.

> After a careful analysis of the situation, environmental scientists have guaranteed that the bay has a capacity for self-cleaning at a rate of 20% per year.
>
> Débora: Here I would like to ask just one question: "Ability to self-purify at a rate of 20%. What is your understanding of this?"
>
> Luara: I don't know what self-cleaning is, but it must be related to something that is over.
>
> Renata: I think it must be the rate that contamination takes to decrease, for example, in one year, it will return to 20% of what it was before.

The situation presented contributed to the engineering students elaborated interpretations of what would be the representation of a scenario elaborated according to this information. And the raising of these hypotheses was associated with the following mathematical model covered in the support study:

> Based on this hypothesis, the specialists then established a model for the concentration of Oily Agent over time:
>
> $$f(1) = 10$$
> $$f(x+1) = 0{,}8f(x)$$

Soon, the students stated:

> Luara: So, it will always decrease by 20% and 20%; it will remove more and more oil and there will always be a little bit leftover.
>
> Renata: It means that the next [value] will always be 80% of the previous one.

It can be seen that although they did not have specific knowledge about self-purification, this was being built from the set of data analyzed. There was an opening for the *technological imagination* emerged. The students explored hypothetical situations based on imagination. The organization of the mathematical information constructed in this imaginary scenario allowed them to analyze and evaluate the consequences of that scenario in reality. These perceptions are in line with Skovsmose (2014b, p. 96) when he says that "one can reflect on

the nature of technological imagination supported by mathematics in view of specific issues [...]. This imagination can generate new guidelines [...] It can enable actions that otherwise would not be possible". With this, if the relations given were placed to investigate the decontamination processes in that bay, it would be necessary to put into action the *hypothetical reasoning* aspect because through it the hypothesis survey would be analyzed using the models considered.

Luara and Renata stated that self-purification was associated with the rates of decrease in the concentration of pollutants. A similar placement was also made by the ecology students, Katia and Carlos, from institution B. They highlighting other important elements:

> Katia: In this process, depending on the location, if it is in the sea, in the lake, it can have a difference in oxidation. As in the seas, the current is faster and larger, it occurs faster than in the river.
>
> Carlos: And self-purification also depends both on the speed of the watercourse and on the morphology of the riverbed or the quantity of the substance that is emptied, we can associate it with the transparency of the river.

This proved to be relevant as the analysis of the participants evolved, highlighting that as proposed by Skovsmose (2008), the analysis of an imaginary scenario is not only related to mathematics, after all, it involves other fields of knowledge. Katia and Carlos pointed out to evaluate the self-purification process, it would also be necessary to understand the location's oxidation, the speed of the watercourse, among other factors related to environmental issues. This placement of students reinforces that hypothetical reasoning should not be supported by mathematics alone. It is in line with Skovsmose (2008) when stating that in situations like these, the details of the investigation are only represented within a specific mathematical construction within a given alternative and, thus, there are limitations regarding real reasoning because the reasoning itself is founded on mathematics.

When advancing in the reading and analysis of the support study, the engineering students deepened the discussions. They hypothesized about the limit formula of a function: $\lim_{x\to\infty} f(x) = 0$. For example, Luara, in his first perception, deduced:

> Luara: I think there is a limit to how much the place can get rid of the pollutant. [...] It seems that it will never reach zero. We know that this is decreasing ...

As the readings progressed, in association with the tables and graphs of the support study, Luara and Renata elaborated more interpretations about this formula:

> Luara: It makes no sense! That's because, in the given expression, x tends to infinity! So, it will only return to the natural, to the infinite ... only that we will never reach the infinite because it is infinite.
>
> Renata: It may be a very small amount, but it will still be there.

The hypotheses raised by the students demonstrate reflections on the real impacts of applying a mathematical formula in reality, which is related to the aspect of *realization*. For them, the limit formula interpretation in the given context could be somewhat wrong. For them, the bay, would not be totally decontaminated, because even over the years there would

still be some pollutant in the place, from the spill of the oily agent. The students' statements reveal that certain effects may be hidden in a situation like this. In addition, the speeches produced from such an interpretation, could present risks and disadvantages.

Subsequently, in the context of the material used, the expression of the limit of a function was used as an argument by the defense lawyer of the company responsible for the accident. According to him, the formula supported the thesis that the cleaning of the bay would occur naturally over the years, which, therefore, would influence the value of the fine for the environmental impacts caused. Even before reaching the part of the material where this conclusion would be made, student Rogério emphasized:

> Rogério: As I understand it, the damage that was caused in that environment, over time, would gradually be cured. Over time, the more it would improve, until it reached the point where he would be completely cured, so the lawyer approached this for a time equal to infinite. So based on that he gave a natural response, as if not as if there was no real damage to nature because over time she can fix things herself [...]

The student imagined a scenario constituted by the understanding of several elements that emerged during the meetings: the limit formula, the tables and graphs of the support study, the deepening of research on self-purification and, such analyzes, would influence in reality, leading us to perceive the aspect of *realization* as well.

In these analyzes it was possible to verify that the knowledge used was applied in a particular situation and new possibilities of learning, of studies, emerged from the materials and the discussions provided. With that, different actions and decisions could be taken in face of the conclusions obtained. For example, the entire context involving these self-cleaning processes would contribute to the assessment of the analyzed aquatic environment and, consequently, this would allow different actions to be planned and executed, such as the release of fishing, the ways of compensating the damages caused, among other factors.

This entire process of discussion and reflection was supported by theoretical foundations by Skovsmose. The construction of a set of hypothetical situations is a powerful act. It is understood that opening possibilities for technological imagination, hypothetical reasoning and realization to appear in higher education mathematics classes is something powerful. Thus, this opening can transform learning contexts supported by the predominance of conventional classes of the explanation-exercises-correction type.

Concluding remarks

In this paper we presented some results of a PhD research in progress. In it, it is possible to contemplate ways of approaching some aspects of mathematics in action in higher education, inspired by a work perspective based on the PBL.

We tried to highlight the path of the investigative process, describing from the organization to the stages of development of the activities carried out. There was also a concern to explain the understanding of the five aspects of mathematics in action, even though only three of them were actually worked on in the presentation of the results.

Develop this theme in the thesis revealed some research potentialities. Inspired by Skovsmose it was possible to perceive, through learning around a problem, how the mathematics in action's aspects could reveal themselves, even in different university contexts.

Learning contexts that present mathematical formulas based on reality can promote different reflections. This theme emerged from moments when participants made interpretations regarding the use of formulas, equations, mathematical indexes or even concepts related to this field of knowledge. In different circumstances they sought to understand the ways in which mathematics operated in certain situations, whether implicitly or explicitly. In certain cases, the participants seemed to perceive the context described in each case addressed. They raised different hypotheses, sought to identify and assess the possible consequences of a project, undertaking or deciding, and also seemed to understand the real application of mathematical models in their fields of activity. It is understood that these reflections were mobilized by the work with the PBL. This teaching methodology contributed to the students' concerns being expressed. The problem adopted in the investigation was used to trigger the discussions. Several of them related to technological imagination, hypothetical reasoning, and achievement. However, the notes made by the participants did not only concern mathematics. They were associated with a real event and was structured to promote reflections on social, environmental and economic aspects, for example. Furthermore, the students worked in groups, discussed real issues and problems, there were perceptions about different fields of knowledge, they deepened research involving their fields of interest and presented criticisms regarding mathematics and their future area of expertise in this case, as engineers or ecologists, etc. Finally, the elaboration of this theme aimed to highlight these perceptions, in addition to proposing discussions that aim to contribute to the field of mathematics education in universities.

This was a way of presenting some reflections in this article. By linking the mathematical field to different learning situations, it is possible to perceive the ways in which mathematics can present itself in reality. Furthermore, this allows students to reflect on the knowledge learned and analyze how the presence of mathematical models in reality can actually happen. Being aware of such perceptions is something that cannot be lacking in the field of mathematical education and this can contribute to the adoption of different critical thoughts and attitudes in society.

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An informal mathematical education project aimed at contrasting early school leaving: Potential and criticality

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High student dropout rates are a common phenomenon in the suburbs of many metropolises, even in high-income countries such as Italy. Although the factors prompting students to take this course of action are numerous, there is a need for research in mathematics education to address this phenomenon as well. An attempt can be made through informal mathematics education, as explained by the design research project focusing on students in grades 4–7 described in this work. Herein, I reflect on the potential of informal mathematics education in preventing student attrition, before presenting a case study to identify the weaknesses of a teaching experiment in terms of lack of democratic dialogue needed for the creation of a meaningful and accessible educational environment.

Introduction

According to the latest official Italian report on school drop-out rates (ISTAT, 2019), which refers to the year 2018, preceding the recent COVID-19 pandemic, 22% of adults aged 18–24 in the city of Naples were Early Leavers from Education and Training[1], compared to 14.5% and 10.6% at the national and EU level, respectively. This is a very alarming figure, reflecting the urban marginality in the large metropolises of Southern Italy, often accompanied by semi-illiteracy and hostility toward institutions (Camera dei deputati, 2000). In this particular context, failure to graduate from secondary school also increases the likelihood that young people will be approached by gangs and organized crime units, with devastating social repercussions. More generally, early school leaving is a very complex phenomenon, motivated by a range of factors that can be broadly classified as exogenous and endogenous (Morgagni, 1998). Exogenous factors are social, economic and cultural in nature, and pertain to students' family and non-family environment. On the other hand, endogenous factors relate to educational policies, but also concern the particular educational process in which the individual learner participates. Little research has been conducted on the link between early school leaving and school mathematics education. However, in a very enlightening study, at least for the Italian

[1] According to Eurostat, the statistical office of the European Union, ISTAT measures school dropout rates by considering the percentage of adults aged 18–24 who have not completed secondary education and did not take part in education or training.

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context, it emerged that when a student drops-out from high secondary school, almost always in the same school year s/he failed in mathematics (Moscucci, Piccione, Rinaldi, Simoni, & Marchini, 2005). Thus, in order to focus on the prevention and contrast of early school leaving, it seems crucial to deepen and reflect on educational contexts not only from a more general pedagogical perspective, but also in the mathematics education research field.

The Proud of You project

Since 2018, my research group has been involved in projects aiming at preventing early school leaving. In particular, as a part of our Proud of You (PoY) project, we offer extra-curricular activities to primary and middle school students living in the suburbs of Naples. In 2021, the project scope was extended to students living in the city of Polistena, situated in another region of Southern Italy. PoY offers a further opportunity of learning to graders 4-7 through the school, but at the same time going beyond some of its usual practices. Indeed, PoY's strategy is the promotion of a fascinating, and thus often alternative, image of School. For this purpose, my research group and I developed a research-based design aimed at promoting a positive attitude towards mathematics (Di Martino & Zan, 2011). In some cases, we also had the opportunity to involve the teachers in professional development courses and to co-design with them the didactical activities (Carotenuto, Mellone, Sabena, & Lattaro, 2020). PoY is now in its third edition, allowing us to implement a cyclical design process of invention and revision, as is typical in design-based research (Cobb, Confrey, Di Sessa, Lehrer, & Schauble, 2003). My research group's overall research aim inside PoY is to explore how an informal mathematics education project could prevent early school leaving, hopefully with an effective and long-lasting action. As this is a highly innovative and ambitious project, some teaching experiments failed to yield the desired results, but nonetheless generated highly informative findings. Thus, in this work, focus is given to the importance of a meaningful and accessible educational environment. In this paper, I present a case study from which a lack of *dialogue* between teachers and students emerged (Alrø & Skovsmose, 2004), which runs counter to the PoY goals of democratic citizenship education.

Informal mathematics education's potential for preventing early school leaving

The research work related to PoY may be situated in the emerging field of Informal Mathematics Education (Nemirovsky, Kelton, & Civil, 2017). This study stream explores learning spaces that differ from the usual school settings and from everyday mathematics. The three main characteristics of informal mathematics education contexts are: the voluntariness of learner's participation, the fluidity of disciplinary boundaries, and the absence of traditional forms of assessment. As a result, informal mathematics education has "a unique potential to disseminate alternative images about the nature of mathematics and to realize the potential for everyone to engage with mathematics in creative and diverse ways". (Nemirovsky, Kelton, & Civil, 2017, p. 975). Taking care of students' vision of mathematics in *inclusive* environments is certainly a crucial issue in socially disadvantaged contexts such as those targeted by PoY. Here, the term "inclusive" is used in a general sense, as in the Italian educational system. By inclusive mathematics education I refer to the

participation in mathematical activities by individuals with claimed disabilities or with special educational needs. The latter group also includes students with very critical cultural disadvantage. In PoY, the cultural disadvantage often takes the form of limited Italian language (considered mother tongue at school) proficiency due to the exclusive use of the local dialect in everyday life outside of school, and the lack of cooperation, if not hostility towards the school, by the families.

Nevertheless, the only students' participation in informal educational contexts may not be sufficient to improve their attitude towards (school) mathematics and school in general. Indeed, projects as PoY are expensive offer brief educational experiences, while change in attitudes generally takes times. Moreover, the mathematical activities carried out within informal contexts constitute, by their nature, a significant departure from the manner in which students are taught in school. In this regard, the findings reported by Deborah Perry are highly informative, as they indicate that, even though 5–12 years old children can enjoy visits to a mathematical exhibition, they may not relate those experiences to school mathematics (Nemirovsky et al., 2017).

Thus, an effective struggle against early school leaving should aim to create the conditions for every student to develop and maintain a positive attitude towards mathematics *within the school*. Assuming a truly inclusive perspective, this means that focus should be given to tasks students perceive as *meaningful* and that makes them feel sufficiently mathematical competent, thus *accessible*. This is a very ambitious and challenging goal that requires teacher involvement. What potential does a project like PoY have for achieving this goal? Probably, one of the greatest potentialities of projects like PoY stems from the first feature of informal education spaces: the voluntary nature of the learner's participation. Indeed, being oriented to the design and creation of educational spaces in which the children participate voluntarily, an informal mathematics education project compels (more than curricular mathematical activity) teachers' educators-designers and teachers to reflect and search for significance and accessibility of the activities that they offer to learners. Furthermore, the organizational, scientific, and economic support generally offered by such projects allows teachers to experiment with new methodological approaches.

PoY teachers' training courses and educational paths

Due to the space constraints, only the main characteristics of PoY teachers' training courses and mathematics education paths are described here, but further details can be found in the article published by Carotenuto, Mellone, Sabena and Lattaro (2020). As soon as it was organizationally possible, PoY educational design work was shared with teachers. This was done in order to benefit from the teachers' professionalism and experience in their particular school, to involve and empower teachers, and to better spread the research group's vision of mathematical learning. Drawing upon embodied cognition (Radford, 2014), in PoY research-based design, movement, material, and sensory experiences were considered as central elements of the work of teachers and students in mathematical activities. These components are particularly relevant in contexts such as those targeted by PoY, given the linguistic difficulties of many learners. In particular, for the manipulative activities with artifacts we

were inspired by the pioneering work of Castelnuovo (1963). Moreover, in response to social backwardness and hostility toward institutions characterizing the communities in which PoY was conducted, mathematics education and *citizenship education* were intertwined in the program design. As a result, a pleasant epistolary meeting, even if only fictitious, was created between students and people belonging to the world of institutions and the world of culture. Moreover, many historical and naturalistic sites of the city were explored during all the project. Therefore, PoY was also a chance for the involved learners to come out from their suburbs, often lived as *ghettos*, helping them transcend their geographical and social borders.

A case study conducted to understand what went wrong

During the PoY execution, communication between teachers and students was not always consistent with the idea of meaningful and accessible educational environment. This phenomenon concerned almost exclusively few secondary school level classes, and fortunately was sporadic. Therefore, a case study of a particular conversation between a secondary teacher and a group of pupils was conducted to shed light on this critical issue, as described below.

Dialogue, absolutism, and authority in mathematics classrooms

Alrø and Skovsmose (2004) explored the role of communication in learning mathematics from a *critical mathematics education* point of view. Their perspective is particularly relevant for the PoY project, given its citizenship education component, and its goal of preventing early school leaving in socially disadvantages contexts. In fact, one of the issues of critical mathematics education is the promotion of individual empowerment through citizenship development.

Drawing upon the works of Freire (1972) and Rogers (1994), Alrø and Skovsmose (2004) described the educational *dialogue* in general terms as an "*inquiry process* which includes an exploration of participant *perspectives* as well as a willingness to suspend one's pre-understandings – at least for a moment" (p. 15). Participating in a dialogue implies verbalizing one's perspectives and listening and accepting those conveyed by others. However, the authors also noted that perspectives are rarely expressed in a conversation, but rather remain in the background, despite influencing interlocutors' interpretation of what is being said by others. They also observed that a dialogue requires participants to suspend their own perspectives in order to welcome those of others and participate in a process of collective construction of new perspectives. The authors noted that such a conceptualization of dialogue necessitates that participants are willing to *run a risk* and to *maintain equality*. In the educational context, a collective inquiry process requires the teacher to share power with all students in the classroom, which can be risky, as something unexpected can happen at any time. From the point of view of students, participating in a collective inquiry means taking the control of the activities, and thus assuming the *responsibility* for its development and of what can be learned from the dialogue. This dynamic is akin to Rogers's (1994) "person-centered" approach to learning, which not only facilitates learning and learning how to learn, but also develops student's responsibility and their democratic citizenship competencies. Conversely, in the "traditional mode", the teacher has the sole control of all interactions within the classroom, due to which democratic values are absent and students

are taught to obey those in power. As emphasized by both Freire (1972) and Rogers (1994), dialogue is a mode of analysis and a fundamental way of learning, but it is also a type of human interaction, which requires a lovely, respectful, and faithful relationship. Teachers and their students are certainly closed in an asymmetrical relationship, since they do not have the same role in the educational process. Nevertheless, in a dialogue, they are required to maintain equality, i.e., to deal with diversity and differences with *fairness*, while taking care of the emotional aspects and the diversity and differences in the content of the dialogue.

Extraneous to their idea of dialogue, but connected to the Roger's general conceptualization of traditional mode of teaching, Alrø and Skovsmose (2004) also introduced the notion of "bureaucratic absolutism" in mathematics classroom, which they defined as a system in which all mistakes, misconceptions, and alternative conceptions are to be eliminated. There, the process of eliminating errors is carried on by an "unified authority", composed by the teacher, the textbook and the answer book, which corrects errors without providing a transparent explanation to the students. In this dynamic, students could feel as clients facing a bureaucracy which refuses their application without providing any argumentation. A manifestation of classroom absolutism is the "sandwich" pattern of communication, described by Stubbs (1976) as "anything the pupil says is sandwiched in anything the teacher says" (p. 99). Conforming to this pattern, the teacher would ask a question and the student would provide a *minimal response*, which the teacher would evaluate. When adopting this approach, teachers are supposed to know in advance the answer to their questions and students are supposed to guess what the teacher expects to hear as a response. Thus, only the teacher knows the direction of communication and has full control over it, while students are concentrated in guessing, with no autonomy or responsibility for their learning process.

The phenomenon of classroom absolutism can be read in terms of *authority*; as said before, also Alrø and Skovsmose (2004) spoke about a unified authority. More recently, Wagner and Herbel-Eisenmann (2014) studied the different ways through which authority is at work in mathematics classroom discourse, focusing on pervasive speech patterns and more general indicators. They identified four authority structures—personal authority, discourse as authority, discursive inevitability, and personal latitude—which often co-occur in the same conversation. The personal latitude structure aligns with Rogers's person-centered teaching mode, as it allows all participants to make decisions (and be aware of this possibility), thus sharing authority in the classroom. Conversely, the first three categories depict different nuances of classroom absolutism, as they respectively distinguish whether authority is held by a single person (usually the teacher), whether it is attributed to the discipline of (school) mathematics, or whether it is hidden (and what is perceived in classroom is just a sense of predetermination). Teacher's personal authority can be the most pervasive structure in secondary mathematics classrooms, and the evidence that students are following the wishes of the teacher for no explicitly given reason is its general indicator (Wagner & Herbel-Eisenmann, 2014).

Context and methodology

The case study described here concerns a particular conversation between a highly motivated secondary teacher, Beatrice (pseudonym), and a group of pupils. This incident occurred during the PoY pilot phase conducted in a single school in the Naples suburbs of Scampia. The didactical paths of this pilot project were carried out from June 2018 to March 2019 and consisted of a summer camp away from school (with four morning activities related to mathematics) and an after-school educational program (comprising of eight mathematical activities organized on the school premises) for each student. The aforementioned conversation was selected because it was typical of the communication I observed between Beatrice and students.

During after-school educational program, Beatrice was involved in two didactical activities per week (involving grade 6 and grade 7 students), together with another teacher and two tutors. She conducted almost all whole-class activities, with the other teacher and the tutors taking the supporting roles. However, they shared the workload on small-group activities, which they conducted individually.

The data gathered during the PoY pilot phase consisted of initial and intermediary interviews with teachers, as well as videos and photos capturing the activities. Here, I present a fine-grained analysis of a brief transcript of a video-recording, from the point of view of the dialogical-absolutistic features and of the authority structures discussed in the previous subsection. The analysis also focused on non-verbal communication components, such as voice intonation and volume, facial expressions, gestures, and other contextual elements pertinent to participants' stances.

Analyzing a conversation between Beatrice and a group of pupils

The conversation between Beatrice and the group of students that is analyzed here took place during a didactical path aimed at exploring graphs. The students were first introduced to graphs as a phenomenon associated with body movement (by using a position sensor), after which they completed graphs and associated stories. Finally, the students interpreted choreographies conveyed through graphs and invented dances that they expressed through graphs, as shown in Figure 1.

Figure 1: From left to right, pupils observing, manipulating, and dancing graphs, respectively.

In the group task to which the analyzed conversation pertains, the main goal was to complete a graph from which some strokes had been erased, using particular pieces of cardboard, as in a jigsaw puzzle game. Students were required to choose the pieces in

accordance with a written narrative describing two mathematicians, Mary (Sommerville) and Hypatia, taking a particular walk. The overall atmosphere of the class appeared friendly. Below is the transcript of the selected short video excerpt, about 30 seconds long (the Italian language version is provided in the appendix). Beatrice and the three students in the group (Antonio, Brando, and Ciro) appeared very excited. From their facial expressions, postural attitudes, tones of voice, and the speed of communication, it is evident that all students (and Beatrice) were eager to complete the task.

1	Beatrice:	What happens to the graph if I quickly move away from the sensor?
2	Ciro:	It goes up, it goes up [he points with his right hand upwards, extending his arm as well]
3	Beatrice:	Ok . . .
4	Brando:	It goes above [nodding]
5	Antonio:	At the beginning . . . [teacher interrupts with a stop gesture]
6	Beatrice:	But if I go slowly, how does it go? [turning to Ciro]
7	Ciro:	It goes down [he points with his finger downwards]
8	Antonio:	Ah [he appears impatient and very eager to speak]
9	Beatrice:	Does it go down? [addressing Ciro again with a stern voice and look]
10	Ciro:	No, it goes straight [he makes a gesture with his hand to retract]
11	Beatrice:	[She continues to stare at Ciro with a stern facial expression]
12	Antonio:	Wait, wait, teacher! [speaking in dialect]
13	Brando:	If you go near the sensor it lowers [making a downward movement with his hands], if you move away . . . [making an upward movement with one hand] [teacher interrupts]
14	Beatrice:	Ok, then, if I go slowly . . . if I move away from the sensor slowly, as it is written here?
15	Ciro:	It comes down [pointing with a finger downwards]
16	Beatrice:	[She shakes her head]
17	Antonio:	Teacher, I understood that it goes like this . . . [speaking in dialect] [He traces with his finger a zigzag line on the paper, in the missing part of the graph] [teacher interrupts]
18	Beatrice:	No, it is a straight line, it is a straight line, it is a line [with an annoyed tone of voice and repeatedly moving the hand from left to right in a stop gesture]. It's not *zzz* [accompanies the sound with a gesture that recalls Antonio's zigzag line].

In this transcript, the total absence of argumentation is striking, especially considering students' incorrect answers and teachers' corrections. Moreover, although this phase of the activity was designed as group work, there was no exchange between the group members; instead, all exchanges were between the teacher and individual students, countering the PoY objective of promoting collective inquiry. Beatrice did not share power with the students in managing the task, but rather maintained full control of it, establishing a bureaucratic absolutism (Alrø & Skovsmose, 2004), in the particular form of teacher's personal authority (Wagner & Herbel-Eisenmann, 2014). This is also made evident by the fact that almost the

entire excerpt is characterized by the sandwich pattern of communication (Stubbs, 1976), which can be recognized (with small variations) in turns 1–4, in turns 6–14 (excluding turns 8 and 12), and in turns 14–16. All three line intervals begin with a (closed) question from Beatrice, which is followed by a response from one or two students, and end with Beatrice evaluating it (even with a simple head shake, as in turn 16). It can also be seen that almost all student responses to these questions were minimal. This is probably what the teacher expected, since the only longer response (turn 13) was interrupted, probably because it was anticipated by a 'correct' hand gesture. The fact that the students were busy guessing what the teacher was thinking, rather than taking responsibility for their learning process, is evident in turns 7–10. In turn 7, Ciro said "It goes down," but in turn 10, he changed his answer to "No, it goes straight" for no apparent reason, but probably because of Beatrice's negative evaluation in turn 9 ("Does it go down?" [addressing Ciro again with a stern voice and look]).

Another characteristic that made the observed communication far from being a dialogue—as conceived by Freire (1972) and Rogers (1994)—was the absence of fairness in the teacher–student relationship. From the point of view of managing difference and diversity, the lack of argumentation has already been pointed out. Thus, Beatrice did not take care to manage her students' unanticipated responses by investigating their origins (nor did the students request explanations for the corrections they received). Even on the emotional level, however, the conversation was unfair and Antonio, the student that struggled the most with the task (as evidenced also by his use of dialect), was the most penalized. First, from the beginning, he was not allowed to participate equally with his peers. This is evident in turn 5, when he was interrupted by the teacher with a hand gesture, and in turns 8 and 12, when he unsuccessfully requested to intervene. Moreover, in the final passages of the transcript, when Antonio finally spoke up, Beatrice responded with an intervention that can be defined as censoring. Indeed, Antonio was first interrupted (turn 17), and subsequently received a very stern correction (turn 18). The latter took place through a repetition of the same words ("No, it is a straight line, it is a straight line, it is a line"), pronounced with an annoyed tone of voice, accompanied by the same stop gesture. Moreover, his previous intervention was almost ridiculed through a sound ("*zzz*"). Although to a lesser extent, the conversation was not fair to Ciro and Brando either. Beatrice gave Ciro a stern tone and look (turns 9 and 11) and, as noted above, like Antonio, Brando was interrupted (turn 13).

Discussion and conclusions

At the request of the project funder, both completed editions of PoY were evaluated by an external party. Based on field observations, questionnaires, and focus groups with teachers and tutors, the overall project evaluation was positive. For example, increased school attendance on the days of extra-curricular activities was recorded, and mathematics activities were highly appreciated by both students and teachers. The weaknesses that emerged from the external evaluation were mostly organizational in nature. Beyond these evaluations, as noted in this paper, a didactical weakness emerged almost unexpectedly in some teaching experiments at the secondary level. Indeed, the elements of classroom absolutism and

personal authority exposed through this case study run counter to the PoY goals of democratic citizenship education. In addition, they testify to an incomplete realization of the project's potential, in terms of teacher training, aimed at the creation of meaningful and accessible learning contexts that could reduce school dropout rates. This is not to say that the dialogue between educators and teachers has totally failed. Indeed, they were able to share some approaches to teaching and implement different methodologies, valuing everyone's experiences and visions. Such dialogue, however, was not always effective in developing a shared democratic vision of teaching, and in preventing some non-dialogue practices typical of the school mathematics tradition. This outcome can be attributed to the teachers' educators lack of communication capacity in the first two cycles of experimentation. Paraphrasing Freire (1972), the teachers' educators did not yet have the words to share a dialogic vision of teaching:

> To exist, humanly, is to name the world, to change it. Once named, the world in its turn reappears to the namers as a problem and requires of them a new naming. Human beings are not built in silence, but in word, in work, in action-reflection. (Freire, 1972, p. 76)

Mathematics education, with the opportunity it offers to engage learners in explorative activities and problem solving, has great potential for supporting the goals of democratic education. Among the latter is education for democratic dialogue, which can only be pursued through classroom communication that is itself dialogic.

In light of the findings yielded by this study, it seems crucial that educators become more sensitive to democratic values in mathematics education and take time to reflect with teachers on the educational dialogue and to share explicitly, in words, their perspectives on the subject.

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Supplement: Italian language version of the transcript

1	Beatrice:	Cosa succede al grafico se mi allontano velocemente dal sensore?
2	Ciro:	Va sopra, va sopra [indica con la mano destra verso l'alto, stendendo anche il braccio]
3	Beatrice:	Ok…
4	Brando:	Va sopra [annuendo]
5	Antonio:	All'inizio… [l'insegnante interrompe con un gesto di stop]
6	Beatrice:	Ma se vado lentamente come va? [rivolgendosi a Ciro]
7	Ciro:	Scende [indica con un dito verso il basso]
8	Antonio:	Ah [si mostra impaziente e molto desideroso di intervenire]
9	Beatrice:	Scende? [con voce e sguardo severi, rivolgendosi ancora a Ciro]
10	Ciro:	No, va dritto [fa un gesto con la mano per ritrattare]
11	Beatrice:	[Continua a fissare Ciro con sguardo severo]
12	Antonio:	Aspetti, aspetti, professoressa! [in dialetto]
13	Brando:	Se vai vicino al sensore si abbassa [compiendo con le mani un movimento verso il basso], se vai che ti allontani… [compiendo con una mano un movimento verso l'alto] [l'insegnante interrompe]
14	Beatrice:	Ok, allora, se vado piano…mi allontano dal sensore lentamente, come sta scritto qua?
15	Ciro:	Scende [indicando con un dito verso il basso]
16	Beatrice:	[Fa cenno di no con il capo]
17	Antonio:	Professoressa, ho capito quello fa così… [parlando in dialetto] [Traccia con il dito una linea a zig-zag sul foglio, nella parte mancante del grafico] [l'insegnante interrompe]
18	Beatrice:	No, è una linea retta, è una linea retta, è una linea [con un tono di voce infastidito e muovendo ripetutamente la mano da sinistra a destra con un gesto di stop]. Non è zzz [accompagna il suono con un gesto che imita la linea a zig zag di Antonio].

The systemic dimension of financial numeracy education as a possibility to counter individualistic and neoliberal discourses

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Financial education has been criticized for reproducing neoliberalist and individualistic discourses, often ignoring structural issues related to inequality, power and social justice. Mathematics education can play a role in countering these discourses through what I have come to define as dimensions of financial numeracy education. In this presentation, I report on a project which investigated how financial situations are depicted in mathematics textbooks from Quebec, Canada. I focus on the systemic dimension to argue that mathematics classrooms can address some concerns of individualism and neoliberalism in financial education by presenting financial situations in which sense making is informed by mathematical thinking in connection to other systems of beliefs such as ethics, personal values, beliefs, politics, etc.

Introduction

Financial education has become increasingly important in schools. Despite governments' attempts to tackle successive global economic and financial crises, changes in pension funds, deregulation of housing markets, housing unaffordability, the spread of gig economy practices, and the deepening precarity in work conditions are just some of the challenges faced by individuals and communities in Canada and around the world (Zhang, 2019; McGregor, 2018; Grifoni & Messy, 2012). Most recently, the outbreak of the novel coronavirus and its associated COVID-19 pandemic indicate a severe economic recession potentially worse than what we experienced more than a decade ago.

However, this matter has yet to find mainstream attention in the field of mathematics education. Few researchers have actually developed research programs or lines of inquiry devoted to investigating financial education with a mathematics educational perspective (e.g., Lucey, 2002; Savard, 2008; Hamburg, 2009; Pournara, 2013; Sawatzki, 2014). Their research reveals the uniqueness of dealing with numerical information in financial situations in schools. The implication is that financial education cannot be a mere reproduction of financial knowledge in classroom settings.

Notwithstanding the evident need for students to understand how money circulates in communities and societies, financial education curricula has been heavily criticized. The first critique to financial education (or financial literacy) is that it promotes individualistic

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perceptions of money and personal finances. Critics point out that most curricula around the world strongly emphasize issues of personal finance at the expense of structural inequalities or the role of different actors and institutions in the promotion of well-being. In addition, the content seems to be heavily skewed toward middle class families and their needs: saving for college education, considering payment options, avoiding credit card debt, investing for retirement. Others have also critiqued mathematics education textbooks for constructing an image of middle-class life as the only possibility for happiness and well-being (Manoel, Silva & Valero, 2019; Silva & Valero, 2018).

The second critique is that financial education promotes neoliberalist discourses on the value of education in general, and mathematics in specific. With increasing deregulation of labour conditions, pension plans, health and education, the burden to support oneself and their family has also increased. Much of the arguments for the promotion of financial education acknowledge these realities without promoting critical perspectives (Arthur, 2012). According to critics of financial education, these arguments take economic reforms for granted and do not invite students to question and resist them.

The goal of this paper is to discuss these two critiques within mathematics teaching. In the Canadian province of Quebec, financial education is integrated to secondary mathematics in two ways: 1) as its own strand in grade 11 (when students explore simple and compound interest); 2) as a cross-curricular topic covered in all strands and secondary grades through the *Broad Areas of Learning* (Government of Quebec, n.d.). I have decided to focus on the latter because it represents a longer spam of time, more diverse, and yet implicit, content being portrayed to students.

Three dimensions of financial numeracy education

Mathematics has been pointed as an important aspect to successful financial education (Sole, 2014; Lusardi, 2012). However, the mathematics to which I make reference here does not mean its formal or decontextualized representation commonly taught in schools. Instead, I refer to the mathematics that is connected to everyday life. That is what researchers point to as critical to the development of financial education (Sawatzki, 2017; Lucey & Maxwell, 2011). In other words, I use the concept of *financial numeracy* to refer to the knowledge, confidence and ability to use numerical information in financial situations. Within this concept, I draw on the framework of numeracy as social practice (Yasukawa, Jackson, Kane, & Coben, 2018) to conceptualize three dimensions of financial numeracy education: contextual, conceptual and systemic.

The *contextual dimension* refers to the study of mathematics in financial contexts. Consequently, learning about financial concepts and practices is not necessarily part of the goals of the contextual dimension of financial numeracy. Because of that, financial measures (such as price) are only explored as a pretext to learn mathematical concepts (such as decimal numbers or percentages). Furthermore, the teaching of financial numeracy through this dimension tends to be limited to tasks that involve financial contexts. The context itself does not play a major role in justifying solutions of the task.

The *conceptual dimension* refers to the teaching and learning of financial concepts which requires multiple mathematical processes such as: modelling, representing, estimating, measuring, comparing, counting, predicting, rounding, etc. These processes are part of our everyday practices of financial numeracy; hence they reflect the role of mathematics in its development. Many ideas, such as profit, debt and investments, gain deeper layers of complexity once we incorporate the conceptual dimension. Hence, those mathematical processes are necessary to understand financial concepts.

The *systemic dimension* refers to the justification, or unpacking, of financial practices and concepts in relation to other epistemologies. The systemic dimension entails investigating certain financial measures and how they are calculated, defined, modelled and portrayed in society. For that reason, it is situated in pragmatic measurement. While money is used in daily life as a unit to measure goods and services, questioning these measures and unpacking what is behind their establishment is not as common in mathematics curricula. The reasoning behind these measures reveals social, cultural and political actors that use mathematics to convey values. The concept of inflation is a good example: it is common knowledge that inflation is the general increase of prices that happen over time in a given economy, a phenomenon that is real and recognized in our daily lives. However, the calculation of this index is a social and economic convention that requires selecting the basket of goods and services to represent the average increase. The weight with which the items are accounted for in the basket is also a matter of convention. How the final index rate will be used by different institutions, companies and individuals depends on the understanding of this measure as well as the values possessed by each actor. Mathematics plays an important role in empowering individuals to engage in critical debates about how financial concepts are mobilized and measured in daily financial practices, so I incorporate such a role in the systemic dimension of financial numeracy education.

These three dimensions engender a spectrum of financial numeracy. The question that remains to be answered is how these dimensions come together in moments of teaching and learning financial numeracy. In this paper, I focus particularly on the textbooks used by secondary mathematics teachers. I aim to investigate how financial situations are depicted and how those within the systemic dimension can address the issues of individualistic and neoliberalist discourses.

Methodology

I report on my doctoral research which investigated how financial numeracy is depicted in secondary mathematics in Quebec, Canada. The research sought to identify how each dimension of financial numeracy emerged in the textbooks, perceptions and practices of mathematics teachers. In this paper, I discuss the data from the textbooks used in the province's public schools. Three collections of Ministry-approved textbooks were analysed for all secondary grades (grades 7 to 11) with a focus on tasks (exercises, projects, activities, etc.) that incorporated financial situations. These collections covered all 6 grades of secondary school in Quebec, from grades 7 to 11. It is important to notice that in Quebec,

students in grades 10 and 11 are streamed into three mathematics sequences: Culture, Society and Technique (CST); Science (S); and Technique and Science (TS). The CST sequence is typically regarded as the lowest of the three and is targeted to students who aspire careers in the social sciences or trades.

A total of 40 books were analysed. Within these collections, a total of 1362 financial tasks were identified, for an average of 30.3 tasks per secondary grade per collection. Although the majority of tasks were situated within the contextual and conceptual dimensions (which seems to corroborate the notion of mathematics as a neutral space for financial education), 18% of them reflected the systemic dimension of financial numeracy education. The following section presents an overview of what these tasks look like and why they reflect the systemic dimension.

Findings: Systemic dimension in financial tasks

Financial tasks situated within the systemic dimension were often composed of open-ended problems that required students to make sense of their own personal experience in relation to the financial situation being portrayed (e.g., explain why some people go into debt, or justify their purchasing methods). In these tasks, students were being directed to give their personal perspective on the numerical results. Consequently, they could mobilize other forms of reasoning that endorse or challenge the strictly numerical solution. In other words, at the core of the systemic dimension of financial numeracy education lies the possibility of making sense of financial situations through multiple mathematical, cultural and socio-political lenses.

In terms of distribution, the lower grades of secondary school seemed to have a higher incidence of systemic financial tasks. This trend was particularly noticeable among the strands of arithmetic, algebra and, to a lower extent, statistics. The strand of probability, on the other hand, seemed inconclusive given the inconsistency across different collections. Table 1 summarizes the quantitative data collected from the textbooks.

Domain	Unified stream (grades 7, 8, 9)			Streamed curriculum (grades 10, 11)					
	Collection			Collection A			Collection B		
	A	B	C	CST	S	TS	CST	S	TS
Arithmetic	11	14	35	NA					
Algebra	12	11	22	9	5	11	7	4	7
Statistics	8	6	14	5	4	1	1	0	2
Probability	1	0	3	19	NA*	18	2	NA*	6
Geometry	2	1	1	0	0	0	0	0	0
TOTAL	34	32	75	33	9	30	10	4	14

* The Quebec curriculum does not allocate any probability content in the SN stream.

Table 1: Summary of financial tasks within the systemic dimension

In the following three subsections, I present a qualitative analysis of the mathematical domains with highest incidence among the textbooks collections (arithmetic, algebra and probability) and discuss how these tasks seemed to incorporate the systemic dimension of financial numeracy education.

Arithmetic

Arithmetic tasks mostly contained multi-step problems that included questions in which the student would provide their own idea (which can be based on mathematics but also on their personal experience). Figure 1, for example, shows a task with the price of gas which posed several questions that aim at understanding how such a financial measure is determined.

Translation:

Reading decimal numbers

a) How much does one litre of gas cost today?
b) How can you answer the previous question without using a comma or a dot?
c) What causes the increase of the price of gas? Ask your parents.
d) What do you think of Ariana's comment?

On the right, Ariana's comment: "My father doesn't care about the increase: he always purchases gas for $10."

Figure 1: Arithmetic task (Coupal, 2010, p. 56)

Question c asks the students why the price of gas fluctuates. The answer connects to a key feature of the systemic dimension: understanding that a financial measure (price of gas) is established based on a variety of factors. It involves not only costs associated with its production, but also elements such as weather, geopolitics and technical infrastructure. Accordingly, this line of questioning encourages students to engage in critical debates while unpacking these measures. In doing so, students can mobilize different epistemological systems to engage in the problem as opposed to simply carrying a calculation. Also, question d demands that students analyse a statement in which, instead of purchasing the same volume of gas, Ariane's father always purchases $10. This statement can be analysed in multiple ways that involve mathematical thinking, financial strategies, personal values and sociopolitical stances. Overall, tasks in this category also encouraged students to create their own questions based on rates, proportions and number representations.

Algebra

The systemic dimension emerged in algebra tasks whenever they asked students to give their own idea or make a judgement about a financial situation. They did so through multiple mathematical representations which supported the connection between mathematics and other epistemologies brought up by the students. In Figure 2, for example, different phone plans were presented with graphs in a Cartesian plane.

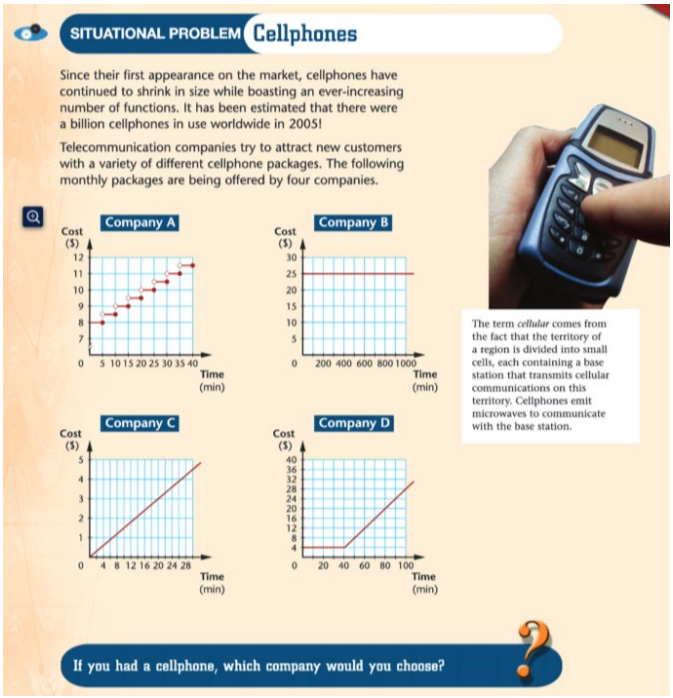
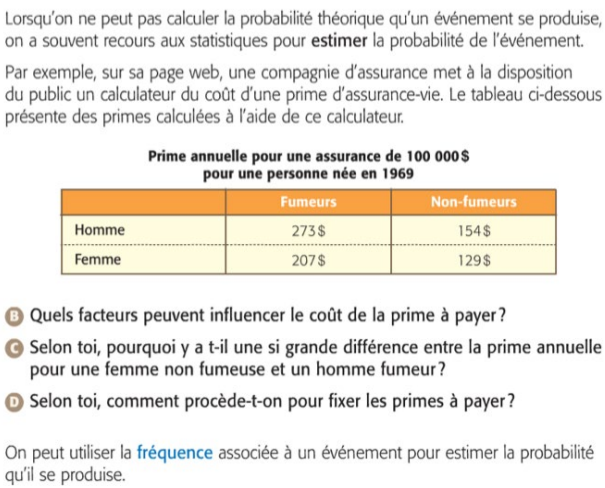


Figure 2: Algebra task (Boivin, 2009, p. 18)

Each plan was represented by a different function (either linear or stepwise). The question posed to the students was: "if you had a cell phone, which company would you choose?" There is no one single right answer to this kind of question. Instead, students should analyse each phone plan and come up with their own ways of measuring the best choice. This measurement, which is unpredictable, is fundamentally systemic: it will be based on students' assessments of their own needs, budget, habits, values, sense of fairness, etc. In this case, mathematics acts as a tool that allows them to make sense of their options, but their choice is unavoidably connected to other epistemological systems. In other words, the reasons behind their choice will be informed by multiple epistemologies that might or not include mathematics. This task, then, allows students to work on the systemic dimension financial numeracy education. Other possibilities of questions in this category include giving advice to characters in the task.

Probability

Probability tasks situated within the systemic dimension proposed that students use probabilistic reasoning as an argument to justify financial situations. Figure 3 exemplifies this category in the situation of insurance policies. The task lays out the different prices for life insurance according to the sex of a person and whether they smoke.



Lorsqu'on ne peut pas calculer la probabilité théorique qu'un événement se produise, on a souvent recours aux statistiques pour **estimer** la probabilité de l'événement.

Par exemple, sur sa page web, une compagnie d'assurance met à la disposition du public un calculateur du coût d'une prime d'assurance-vie. Le tableau ci-dessous présente des primes calculées à l'aide de ce calculateur.

Prime annuelle pour une assurance de 100 000$ pour une personne née en 1969

	Fumeurs	Non-fumeurs
Homme	273$	154$
Femme	207$	129$

B Quels facteurs peuvent influencer le coût de la prime à payer?

C Selon toi, pourquoi y a t-il une si grande différence entre la prime annuelle pour une femme non fumeuse et un homme fumeur?

D Selon toi, comment procède-t-on pour fixer les primes à payer?

On peut utiliser la fréquence associée à un événement pour estimer la probabilité qu'il se produise.

Figure 3: Probability task (Coupal, 2010, p. 29, our translation below)

Translation:

Since we cannot calculate the theoretical probability of an event, we can resort to statistics to estimate the probability of the event. For example, on its webpage, an insurance company publicizes a calculator for a life insurance premium. The following table presents premiums calculated with the calculator

(in the table: annual premium for a $100,000 insurance for a person born in 1969, columns: smoker/non-smoker, rows: male/female)
b) What factors can influence the cost of the premium?
c) In your opinion, why is there a big different between the annual premium for a non-smoking female and a smoking male?
d) In your opinion, how are the premiums established?

We can use the frequency associated with an event to estimate its probability.

Given that prices vary, question b inquires students about what influences the price of an insurance policy. Here, students can use the data provided in the task, but also build on their personal experiences to know that age can impact the price as much as gender, lifestyle, family composition, profession, etc. Furthermore, question c asks about the difference between prices for men and women. Again, justifications can vary, and students can use probabilistic thinking to justify higher prices (men are more likely to die at a younger age) coupled with

a range of other justifications and measures. The systemic dimension is evident here given the opportunity to engage in matters of gender inequalities, corporate policies, public regulation, etc.

Discussion and Conclusion

As it was evidenced by the quantitative data, systemic financial tasks are depicted more frequently in early grades of secondary mathematics in Quebec. In later grades, when students are positioned in streamed sequences, a heavier focus on university entrance is placed onto the curriculum and, consequently, the textbooks. In such cases, mathematics tends to become more abstract and resemble the traditional scholarly discipline. Unsurprisingly, only the CST sequence has introduced financial mathematics into its composition (students who pursue the S and TS sequences tend to explore pre-calculus courses).

Overall, results from the textbook collections show that the systemic dimension of financial numeracy education can be integrated in secondary school in multiples ways with a range of mathematical domains and concepts (particularly arithmetic, algebra, statistics and probability). Mathematics has been identified as a critical component of financial education given that money is essentially a measurement concept (Savard, Cavalcante, Turineck & Javaherpour, 2020). These also show that, if implemented properly, mathematics classrooms can generate perspectives to counter individualistic and neoliberal discourses in financial education.

Individualistic discourses can be addressed in financial tasks through a recognition that different groups and communities do not have access to the same resources (income, credit, confidence, infrastructure, etc.). When students are invited to share their own perspectives to financial situations, there is an opportunity to develop a sense of empathy in mathematics classrooms. They learn that financial situations do not entail one single numerical answer in every case. Mathematics operates then as a medium to communicate these differences, or even as an instrument to unpack and dismantle the myth of meritocracy.

Neoliberal discourses can be addressed in financial tasks when we recognize that economic value should not be the only measure to define our thinking. When students consider the impact of financial situations (portrayed in textbook tasks) to their communities and society, they have the opportunity to unpack the discourses of rational choice when it comes to matters of personal finance. They can also generate knowledge about the roles of government and other institutions in the protection and promotion of financial well-being.

In this paper, I have argued that some of the tasks presented in mathematics textbooks have the potential to construct a more empathetic and critical mathematics classroom environment. However, I also recognize that this is only possible if teachers are sufficiently prepared to identify and tackle sociopolitical matters in mathematics. That includes being aware of what financial education can look like in their classes, hence the importance of incorporating the three dimensions of financial numeracy education in teacher education and professional development. In essence this is about constructing a different epistemology of mathematics education, one in which mathematics is in continuous dialogue with other epistemological systems. The idea that mathematics is a neutral setting for financial

education does more harm than good in countering individualistic and neoliberal discourses, and this is why mathematics educators must be engaged in defining the directions of financial numeracy education.

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Students' difficulties and attitudes facing contextualized mathematical problems: A teacher perspective

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This research is carried after the implementation of the Mathematics Study Programs in Costa Rica, where one of the main curricular strands is contextualized problem solving. The aim is to find out the perceptions of teachers regarding the difficulties and attitudes of students when faced with mathematical problems in contexts close to everyday life. The survey of 67 teachers collected teachers' opinions on various statements. The results show that teachers mainly agree that students are actively involved and motivated in working with contextualized problems; but do not show a clear position about difficulty and complexity of contextualized problems as it is currently indicated in the literature. There are no significant differences between groups according to level of education, work experience or dedication to planning.

Introduction

The history of mathematics shows how knowledge has been constructed and reconstructed responding to the needs and demands of the social environment (Schwantes et al., 2019). Consequently, problem solving has accompanied the daily and academic work of humanity, and since the 1980s it has gained strength as a vitalizing strand in mathematics curricula (Blanco, 2015).

In the last thirty years, the presence of problems contextualized in the reality of the students in the curricula of several countries has increased (Puig, 2008), so as research in the contextualization (Albanese et al., 2017). In the case of Costa Rica, since the implementation of the "Mathematics Study Programs" (Ministerio de Educación Pública [MEP], 2012), the teacher has had to choose, create and adapt mathematical problems to increase the cognitive and motivational action of students. According to Baltodano (2018), this has been a very demanding task for the teachers, who have not had previous preparation for that. In the curricular documents available to teachers examples of problems provided are not always consistent with the stated theoretical references about contextualization, as we analysed in another publication (Chavarría & Albanese, 2021).

Moreover, as indicated by Mayela and Ballestero (2008), the teachers must not only select and adapt mathematical problems, but also be a guide and motivator of their students in the

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process of problem solving. Therefore, among others, it is essential that they know the difficulties their students face when solving these contextualized problems.

Despite it being a decade since the implementation of the mathematics study programs, we do not know the teachers' perceptions of students' difficulties and attitudes in solving mathematical problems contextualized in their sociocultural environment. This is precisely the objective of this paper.

Contextualized mathematical problems and attitudes

The Ministry of Public Education of Costa Rica indicates that working with contextualized problem solving not only allows activating higher order cognitive skills, but also promotes student motivation, as they perceive mathematics as being closer to their reality (MEP, 2012).

In this same line, a study by Chavarría (2014) indicated that contextualized problems (as compared to abstract mathematical problems) were better received by students, generating a feeling of being challenged and motivated to solve them. Similarly, Gómez-Chacón (2002) highlights that contextualized problems that emerge from real life can become motivating elements for students as they allow interaction with their experiences and close environments.

Difficulties in solving mathematical problems

Several studies have inquired about the difficulties that students face when solving problems and this includes, of course, those problems that are contextualized. In this regard, Chavarría (2014) explains that there are affective factors (lack of motivation, fear and doubts), poor relationships between mathematical concepts and previous knowledge, insecurity when facing new mathematical situations, poor relational comprehension and difficulties in reading comprehension. Regarding this last aspect, Blanco and Caballero (2015) reaffirm that among the most observed difficulties in their research, the following stand out:

> the lack of reading comprehension or lack of attention when reading the statement, the tendency to literally translate the problem statement into a mathematical expression, and the lack of knowledge of analysis elements of the situation posed and of specific heuristics. (p. 114, own translation)

Sánchez (2001) adds that the difficulty in solving mathematical problems lies not only in the student, but also in factors such as teaching methodology and teacher motivation. Buschiazzo et al. (1997) indicate that a problem has a difficulty in itself since it presents a novel situation for the student.

Methodology

This study is exploratory and descriptive, with a mixed quantitative and qualitative approach.

Participants were 67 in-service mathematics teachers from Costa Rica, selected through virtual sampling (González et al., 2018). The sample was characterized according to:

- Education: 43 teachers (64%) have studies beyond a Bachelor's degree in mathematics education, it means a Master or a PhD, while the remaining 24 (36%) have only a Bachelor's degree or a lower level diploma.
- Teaching experience: 29 teachers (43%) have less than 10 years of teaching experience, while 38 (57%) teachers have 10 or more years of teaching experience.
- Dedication to planning: 32 teachers (48%) report spending 3 or more hours per week choosing or developing problems, while 35 teachers (52%) spend two or less hours on this task.

A questionnaire was developed as data collection instrument, previously validated by the judgment of nine experts, specialists in different areas of research.

The data analysed in this communication correspond to items constructed in such a way that participants must provide a degree of agreement or disagreement according to a Likert Scale (from 1 to 5) with respect to statements elaborated by the researchers on various aspects related to the perception of students' difficulties and attitudes when solving mathematical problems. In each item of the Likert Scale, teachers were asked to indicate how much they agreed with certain statements, where 1 means strongly disagree and 5 means strongly agree.

Nonparametric hypothesis testing was performed due to the limited sample size.

Results

The contextualization proposed by the MEP (2012) in the Mathematics Programs of Study seeks to promote the active participation of students. In this regard, almost half of the teachers surveyed agree with this statement. In fact, 49% of them expressed they agree or strongly agree (score 4 and 5), while 39% express neither agreement nor disagreement (score 3) and only 12% express a disagreement (score 1 and 2). The box diagram in Figure 1 summarizes the information collected.

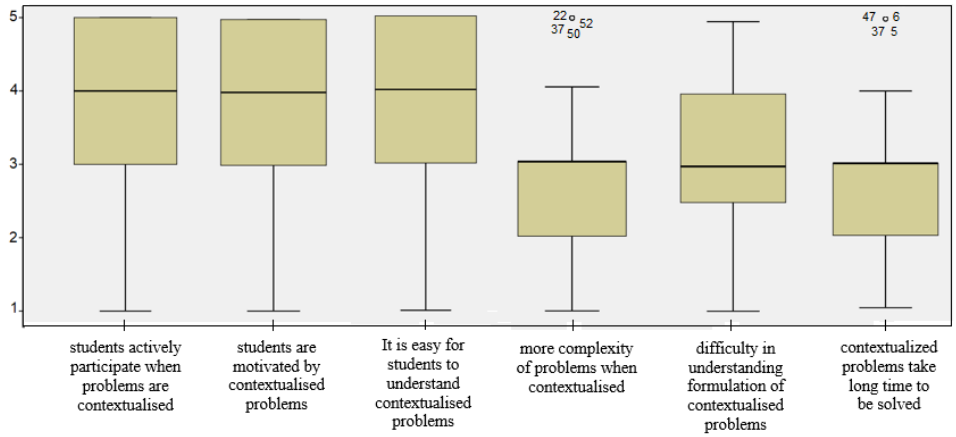

Figure 1: Box diagrams of teachers' level of agreement on the statements proposed.

In relation to a more emotional aspect, more than half of the teachers surveyed (58%) agree or totally agree that their students are motivated when mathematics problems contextualized in their reality are proposed to them, which is in line with what Gómez-Chacón (2002) stated. Only 11% of the teachers indicated that they disagreed or totally disagreed and the remaining 31% express neither agreement nor disagreement (score 3).

On the other hand, when teachers were asked whether they agree that it is easy for students to understand problems involving everyday situations, only 15% of them considered that they disagreed or strongly disagreed (score 1 or 2) with this statement, in contrast to 52% who agreed or strongly agreed (score 4 or 5). Teachers who neither agreed nor disagreed (score 3) were 33% (Figure 1).

On the same topic, teachers provided their opinion on whether a mathematical problem becomes more complex for the student when it is presented in a contextualized situation. In this regard, 48% indicated a degree of disagreement, 30% showed neither agreeing nor disagreeing, and the remaining 22% showed agreeing or strongly agreeing with this proposition (Figure 1).

Now, in order to deepen the teachers' perception of their students' reading comprehension, they were asked if, when they propose a mathematics problem with active contextualization, they consider that the student show difficulty in understanding the formulation. In this regard, the opinions were varied, 33% indicated agreeing or strongly agreeing, 42% neither agreeing nor disagreeing and the remaining 25% showed a degree of disagreement (Figure 1).

Statements	Mean	Median	Mode	Standard deviation
Students are actively involved in working with contextualized problems.	3,79	4	3	1,03
Students are motivated by contextualized problems.	3,82	4	5	1,07
It is easy for students to understand contextualized problems	3,36	4	3	1,11
Problems are more complex if they are contextualized	2,63	3	3	1,17
Student has difficulty understanding the formulation of a contextualized problem.	3,1	3	3	0,97
Contextualized problems take a long time to be solved.	2,87	3	3	1,04

Table 1: Descriptive statistics about the level of agreement on the statements proposed.

In addition, when the teachers were asked if the resolution of these problems requires a lot of time to be solved in class, 31% indicated disagreement or strong disagreement (score 1

and 2) and 24% showed a degree of agreement (score 4 and 5), while almost half of them indicated neither agreed nor disagreed (score 3). Figure 1 summarizes the information obtained on this aspect.

Finally, according to Blanco and Pino (2015), a mathematical problem presents a greater complexity in its resolution, compared to a traditional exercise, which implies a greater effort on the part of the students. Precisely, 58% of the participating teachers express to agree or totally agree in this aspect, 34% neither agree nor disagree and 75% are in disagreement.

In addition, when performing the nonparametric Mann Whitney test on the opinions of teachers regarding the items analysed, no statistically significant differences were identified between the degree of agreement and disagreement in all the statement according to the teachers' education. Nor were there statistically significant differences between the groups of teachers according to years of experience or the time they dedicate to planning problems per week. Table 1 summarizes the information with the respective statistical and significance values.

Conclusions

Teaching mathematics through contextualized problem solving is currently a very important curricular strand in countries such as Costa Rica. Its implementation and scope in terms of learning, motivation and skills achieved should be the subject of research.

This paper, addressing the teachers' perspective on this topic, showed that almost half of the participants agree or strongly agree that contextualized mathematical problems increase active participation in their students, while indicating that these problems allow the student to more easily understand the mathematical knowledge involved.

On the other hand, more than half of the respondents agree that the implementation of contextualized problems promotes students' motivation.

Regarding the teachers' perception of the complexity level and the difficulties that students may face, as well as the need for more time when solving contextualized problems, the data have shown a not so clear position. In fact, in the corresponding items the sample has been more evenly distributed, indicating that teachers do not clearly position themselves either in agreement or in disagreement of such statements. This has been surprising since contextualized problems are usually perceived as more demanding for the students, as previously highlighted in the literature (Sánchez, 2001; Buschiazzo et al., 1997; Blanco & Caballero, 2015).

Finally, it should be noted that no statistically significant differences were found in the opinion of the teachers about the statements described with respect to the groups of teachers divided according to education, teaching experience or dedication to planning.

We are aware of the limitations of studies based on a closed-items questionnaire. But this study is part of a broader research in which we propose to establish a more fruitful and dynamic dialogue with teachers to give answers to the some of the questions envisioned here.

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Children's literature, mathematics education and diversity

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This article aims to investigate the potential of connecting mathematics, children's literature, and ethnic-racial issues in the early years of elementary school. Based on qualitative methodology, the data were produced in a group of 1st-graders of a public school in Brazil and registered through videos, photos, and notes in the field diary. We analysed a didactic sequence developed over three days, working with magnitudes and measures based on the book Mama Panya's Pancakes: A Village Tale from Kenya. *The results indicate that students participated more when the mathematical concept was introduced through a children's literature story. The protagonism of the black characters, inserted in a positive context in the story, contributed to constructing a positive cultural identity, leading children to appreciate cultural diversity.*

Introduction

The book Reading and writing the world with mathematics: toward a pedagogy for social justice (Gutstein, 2006) emphasizes the importance of the political and social dimensions of the teaching of mathematics. Inspired by Freire (2014), the author emphasizes that mathematics education can also contribute for students to read the world and actively participate in the transformation of situations of oppression. Among the goals of mathematics education for social justice is the development of cultural positive and social identities (Gutstein, 2006). In the author's words, "by positive cultural identities, I mean that students are strongly rooted in their home languages, cultures, and communities, but at the same time, can appropriate what they need to survive and thrive in the dominant culture" (Gutstein, 2006, p. 28). In this sense, a critical educator should support the development of positive cultural identities through actions in the classroom that values the students' background, language, and culture, developing a pedagogy in which these aspects are contemplated, becoming a source for learning (Gutstein, 2006). In other words, a culturally relevant pedagogy (Ladson-Billings, 1994).

Gutstein (2006) developed his conception of mathematics education for social justice by working mainly with adolescents rather than with young children. However, his in-depth analysis of mathematics education for social justice inspires thinking about mathematical

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literacy. What does it mean to develop positive cultural identities when the mathematics teaching and learning process has children from the early years of elementary school as participants? Seeking to discuss possibilities inspired by this perspective, we will explore some potentialities of children's literature linked to mathematics and racial issues.

Children's literature and mathematics

Several mathematical educators (Montoito, 2019; Souza, Carneiro, 2015) have highlighted the importance of literature and children's literature in mathematics classes. According to Souza and Carneiro (2015, p. 393), "children's literature encompasses the so-called "classic" literature books and texts, such as the Brothers Grimm's stories and Andersen's tales, among others; contemporary texts, also called "realistic tales," and aimed at children, and paradidactic books."[1] Souza and Carneiro (2015) also point out that paradidactic books are developed to teach didactic content using literature playfully, with illustrative characters in real and hypothetical contexts, enabling dialogue through textual elements such as setting, characters, and conflicts.

Teaching mathematics connected to children's literature enables a pleasant approximation between children and mathematics, provides a context for mathematical terms, besides being based on the commitment of all disciplines to the children's literacy process. Montoito (2019) emphasises that all disciplines are committed to students' literacy, mathematics included. The author also points out how important it is to put on mathematical glasses when reading a story to analyse what it can represent in aspects different from merely literary ones, paying attention to the story's potential rather than the plot. This is the first step to start seeing literature from other aspects, as a literary tale can bring several teaching possibilities. Therefore, all teachers should encourage reading, regardless of the subject they teach. This posture is crucial for the development of a good reader. This perspective is productive not only for students but also for the teacher, who can help students develop creativity, can explore their imaginary and symbolic potentials.

In this way, mathematics classrooms have become a meeting place for children, characters, and mathematical concepts through the books *Problems of the Gorgonzola Family*[2] (Furnari, 2004), *The Duck Lollipop*[3] (Machado, 2003), *Who's Going to Keep the Peach?*[4] (Ah-Hae; Hyewon, 2010). Fairy tales, also offer rich situations for the elaboration of mathematical problems and investigative situations. However, looking at the mathematics classroom and children's literature, inspired by the perspective of mathematics education for social justice, means being concerned with developing positive cultural identities for children and going

[1] "a literatura infantil engloba os livros e os textos ditos "clássicos" da literatura, como as histórias dos irmãos Grimm e os contos de Andersen, entre outros; os textos contemporâneos, também denominados "contos realistas" e direcionados ao público infantil; e os livros paradidáticos" (Souza; Carneiro, 2015, p. 393).

[2] Os problemas da Família Gorgonzola (Furnari, 2004).

[3] O Pirulito do Pato (Machado, 2003).

[4] Quem vai ficar com o pêssego? (Ah-Hae; Hyewon, 2010).

beyond the "Once Upon a Time," in which most characters have a stereotyped beauty for example, linked to characteristics of white phenotypes and European culture.

Children's literature books have verbal and non-verbal elements that are not neutral and can contribute to constructing a positive cultural identity for children but can also make them disregard their culture and background. Therefore, it is essential that teachers connect children's literature, mathematics, and the positive representation of black characters from the beginning of schooling. Therefore, a set of essential questions emerged: What children's literature book should we take to the classroom? What does a book offer so that students enjoy it? How are the characters in this book? What are the settings? Will this book help children travel to other places? Does it bring the diversity and richness of different cultures and contribute to constructing a positive cultural identity? Does it contribute to mathematics being worked in an interdisciplinary way and potentiates the triggering of problem situations?

It is essential that books such as *Bia na África* (Dreguer, 2016), *Handa's Surprise* (Browne, 2001), *Mama Panya's Pancakes: A Village Tale from Kenya* (Chamberlin & Chamberlin, 2005), *The Color of Coraline* (Rampazo, 2017) and *The Little Black Prince* (França, 2020), whose characters are black children in positive contexts, also enter mathematics classes. In this way, this article aims to investigate the potential of connecting mathematics, children's literature, and ethnic-racial issues in the early years of elementary school.

Methodology

The development of this work was supported by a qualitative approach (Bogdan & Biklen, 1994) and used participant observation. This study was conducted in a class of 1st-graders of elementary school of a state public school in a city in the countryside of Mato Grosso do Sul (MS). The group was composed of 31 students and the teacher. The activities analyzed in this paper were developed in this class by the first author, as part of the activities of the Institutional Program for Teaching Initiation Scholarships (PIBID)[5], of the UFMS/CPNV subproject, coordinated by the second author.

The activities were developed in three meetings, held on Tuesdays, preceded by the subproject meetings at UFMS/CPNV, which aimed to deepen the theoretical knowledge about mathematical literacy and develop didactic sequences based on the discussions. The data were registered through videos, photos, and notes in the field diary, which we analysed later. We watched the video recordings several times to delve into the data and made notes to help describe the context. We also identified events that contributed to understanding the research object and confirming or refuting our hypotheses. Among those events, there were some of the students' speeches, which we transcribed, developing the episodes.

This article addresses a didactic sequence to work on mathematics along with children's literature and ethnic-racial issues. The didactic sequence is described in three episodes and analysed from its contributions to the teaching and learning of the mathematical concepts, appreciation of diversity and the fight against racism.

[5] Programa Institucional de Bolsas de Iniciação à Docência (PIBID).

A history in Kenya: Possibilities in mathematics classes

At the beginning of the class, it was agreed that students could participate and ask by raising their hands so that everyone could hear their arguments or their questions. With the support of a projector, we began a journey with them based on the story *Mama Panya's Pancakes: A Tale from a Village in Kenya* by Mary and Rich Chamberlin (Figure 1). It is about Mama Panya and a boy named Adika, who walk to a street market to buy ingredients for pancakes. On the way, the boy meets several friends and invites them all to eat pancakes at his home. During the development of the plot, the story also covers several aspects of Kenya and the African continent.

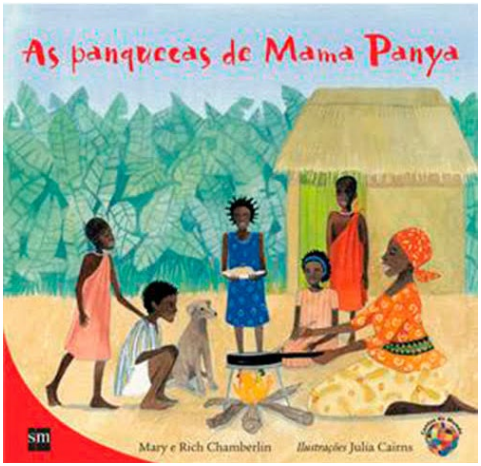

Figure 1: Book cover

Silence filled the classroom. The students were delighted and attentive. At the end of the story, we began the discussion by asking them whether they had noticed anything different in the history of that country. Some observed Adika and Mama Panya's clothes, while one student spontaneously and curiously exclaimed: "Wow, how well they dress, the clothes are so beautiful and colourful!" Others commented that the market was different from the ones they knew. They highlighted the characteristics of the animals, the trees, and the school. In the final pages of the book, there is information about the school, the village, and the animals. One student mentions the long distance between school and homes. After that, we started talking about the routes they had to take from their homes to school, describing it as long or short.

Some students said they lived very close; others emphasised that they lived far. To understand better the routes they had to take, they drew a picture. While they were drawing the routes, a student asked: "Teacher, do I need to draw the curves I make when I turn around a corner?" Seizing the moment, another sketched another question: "Do I need to draw the houses I see on the way home?" We replied that they could draw the way they thought the

best and bring details like houses, markets, and corners when they took other directions. Realising the students' doubts, we went from table to table, solving questions and discussing ideas of specific points, among other details that emerged during the drawings. Then, a dialogue was opened on the importance of using terms such as right, left, starting point, and it was collectively established that the starting point would be the school gate.

After finishing the drawing, each student shared their route with the rest of the class, explaining it orally. The first student found it a little challenging. We told them that they could walk around the room, showing the route. The students understood it. Some got up and walked around the room as they had described. Others explained their spatial location orally, without getting up. Subsequently, we read the recipe that Mama Panya used in the story and brought a text with curiosities about the origin of pancakes and how different they can be from country to country. Besides mathematics, we addressed the identification of textual genres. The students were encouraged to notice the difference between a recipe and other textual genres. For example, when asked if they knew how to identify a recipe, a student shared her answer with the class: "You can see it because the recipe the teacher gave is smaller and has numbers, and I don't see it in the other texts." Therefore, all learned the structure of a recipe: ingredients, preparation method, how to serve, and how many people it serves. Subsequently, we commented on the fractional numbers present in the recipe. Some students explained that these numbers represented divisions; others could not answer.

A cup, spoons, packages of wheat flour, and containers were brought into the classroom and used to contextualise the fractional numbers in the recipe. After that, the students read the recipe on the board and identified the mixed number referring to the measure to be placed, taking the amount in a cup straight from the wheat flour package and placing it in the container.

As some students did not understand the fractional part of the number, one child was willing to explain. Going to the front of the class, she read the recipe on the blackboard and said, "This recipe says to put two cups, you will take the cup and fill it twice and put the filling in the bowl. In front of the number two, there is another number, ½, which is the same as for the chocolate. You have a bar of chocolate with two pieces, don't you? If I eat one piece, how many are left?" The rest of the class replied that it was one. Next, she said, "Yes, but as the chocolate was composed of two pieces and I ate one, there was one piece left, which was half! We know it represents half; then, you will put half a cup more," showing everyone in the class with the wheat and the cup. After this explanation, no questions remained, and the students took turns to participate with different measures until everyone understood it. With this, the class was finished.

Pancakes

There were thirty students in class at the second meeting, which we began by asking if the students remembered the story read in the previous class. They explained some characteristics of the book *Mama Panya's Pancakes* orally. They mentioned the market, how

the characters dressed and spoke, the trees, the animals, the school, and the distance the characters had to walk to get there.

Then, we asked whether they remembered what Mama Panya was planning to cook when she decided she was going to the market to buy ingredients, to which they all replied very enthusiastically that it was the pancakes. Next, the conversation was about pancakes around the world, how each culture had its way and adaptations to prepare them: some usually use meat, ham, and mozzarella; others make them sweet, with chocolate, with condensed milk, and even only the dough with pepper, just like Mama Panya's. Then, we discussed who had already eaten pancakes and how they had been prepared. Everyone got involved and told how families used to prepare them. "Mine [referring to the cook] puts on minced meat, and we eat them for lunch," one student exclaimed. "Mine puts cheese on top and bakes in the oven!" stated another, and some reported that they had never tasted it.

Thus, a pancake recipe was written on the board to be prepared in the classroom. We organised some space, placing a table in front with all the ingredients to analyse the instructions and measure. We asked the class to read the whole recipe aloud so that they could get used to the structure of the genre. Then, students came in pairs to prepare the food. However, it was not that simple; they had to get into the mood of chefs de cuisine. To characterise the moment of preparing recipes, the students put on caps and aprons. One of them joked, "Now we're mini master chefs." The recipe was made step by step: each one went to the front, put on a cap, an apron and read out loud the ingredient to be added and its quantity, placing it in the container as shown in Figure 2.

Figure 2: Student adding milk.

After finishing adding all the ingredients, the next in line placed the mixture in another container. Then, they read the recipe, analysing what should be done next. The students pointed out that the recipe is for six servings and that this would not be enough for everyone.

They calculated how many times it would be necessary to repeat the recipe for the 30 students to eat. After talking to each other, they solved the issue together on the board.

The recipe was duplicated by adding six and six, and the teacher took notes on the board, and the students said: "12". Then, we analysed: "Is a recipe for twelve people the ideal?" They said: no. It would be necessary to add one more. The teacher wrote on the board: 12+6, and some students answered "18!" They were asked, "Is it fine now? Will all be served?" And they said: no, it would be necessary to add some more. Then, the teacher wrote: 18+6, and they replied, after thinking for a few minutes: "24!" "And now, is everything all right?" asked the teacher. And they answered: no, because there would be not enough for everybody. In this way, another addition was written on the board: 24+6. This sum was not calculated mentally as the previous ones. To solve it, the students used their notebooks and soon answered that the result was 30 and that now it was right, that it would be enough for all students. At this point, one of the students suggested that one more recipe be made so that the teachers could also taste it and, whatever was left, they would share with everyone else. Thus, it was done.

In this process, the students asked a question: "How many times have we repeated the number six in our calculations?" It was easier to conclude, as the sum was exposed on the board, and the students soon replied loud and clear: "Six times". They were asked if they all knew what it meant, and a student replied, "That means how many times we're going to repeat the recipe." The recipe was repeated six times, giving everyone the opportunity to execute a part of it. Finally, all the dough was placed in a larger container. The students then agreed that the last step, to cook the dough, should be carried out by an adult for safety reasons, so the pancakes were prepared by the first author and the class teacher, and all students tasted them. After that, teachers and students talked about the importance of mathematics for the recipe to end up perfect and the pancakes tasty.

Butterflies

In the third meeting of the discussion on the book *Mama Panya's Pancakes*, 30 students were present. We began a dialogue on animals, diversity, and richness of the fauna of each country. They were asked if they remembered the animals they had seen in the story. All answered they did and cited some: the giraffe, the zebra, and the tiger. Then again, they were asked, "And the butterfly, does anyone remember?" Students began answering all at once. When they calmed down, we all agreed that everyone could speak, one at a time. They characterised the butterfly as big and beautiful, according to the words of the class. During the discussion, we asked them to give the main characteristic of the Kenyan butterfly.

The class went silent, so we brought up the moment when the story reports the characteristics of the butterfly. Some students started shouting, "Teacher, the Kenyan butterfly is giant." Then, we looked for the measure of the butterfly in the text, and the students identified that it measured 20 cm from one wing edge to the other. The children sat in pairs, and each received two pieces of string, one measuring 20 cm and the other 7 cm. First, they were asked to analyse the strings and say which one they thought represented the

butterfly from Kenya and which one represented the butterfly from Brazil. They immediately noticed the difference and, to be sure, they all took the rulers. In pairs, they measured the strings to find out the exact size of each one of them. We told them that to use the ruler, they had to start counting from the number "0" onwards, and so they did. After they finished measuring, they chose two children to register the measure of the strings on the board.

Finally, they drew a 20 cm straight line in the notebook with the ruler and drew a butterfly underneath, following the rule that consisted of starting one wing on one end of the line and the other wing on the other end. The students draw beautiful Kenyan butterflies in their notebooks. With the colourful wings of beautiful butterflies on a paper page, the class was finished.

Data analysis

This study shows the potential of working on mathematics along with children's literature and ethnic-racial issues in the first years of elementary school. This connection enables students to learn mathematical knowledge such as quantities and measures addressed in the research while expanding their knowledge of ethnic-racial aspects. Students broadened their knowledge of magnitudes and measures in context, for example, the measures with natural numbers and the introduction to fractional numbers through the recipe brought from a children's literature book. Those numbers had a referent in the story, which helped children to understand them and identify that reference. The preparation of the recipe in the classroom allowed them to handle the ingredients and containers, reflecting on the measures specified in the instructional text and the relation to the equivalent amount of ingredients they should add.

The students also talked about displacement and reference points, reflecting on Adika and Mama Panya's route to the marketplace and described the routes they used to take to go from school to home. The students expanded their vocabulary during the descriptions and started to incorporate words such as turning right, turning left, near and far, and selected reference points they could use. We could show the children how to measure with a ruler from baseline, not using the space before the zero. This activity was developed in the context of the fauna of Kenya, represented by its butterflies, which favoured the children's engagement. At this moment, we also approached the concept of difference between measures by using the strings.

As for ethnic-racial issues, among all aspects, the study emphasised black characters in children's literature in the mathematical literacy process, which plays an extremely relevant role in bringing positive perspectives on black people to the classroom. According to Munanga (2005), the discourses about blacks throughout history, from a western and ethnocentric perspective, presented wild, poor, and inferior peoples, devoid of knowledge, history, and culture. Thus, ethnic-racial relations must be discussed from the beginning of schooling to change this stereotypical view. Educators and authorities must be committed to the educational, cultural, and political fields to present and discuss other approaches on the black population. To this end, Munanga (2005, p. 16) points out that:

> The rescue of collective memory and the history of the black community is not only of interest to students of black descent. It is also interesting to students of other ethnic backgrounds, especially white students, because upon receiving an education poisoned by prejudices, they also had their psychic structures affected.

Given this statement, regarding the learning provided on ethnic-racial issues, students could know a little about Kenya and learn that it is a country in the African continent, which allowed them to distance themselves from the Western ethnocentric gaze and recognise the importance of cultural diversity. Students also used positive adjectives to refer to the characters, for example, when they observed Adika and Mama Panya's clothes, and one student spontaneously and curiously argued: "Wow, how well they dress, the clothes are so beautiful and colourful!" According to Pereira and Dias (2020, p. 193), "[...] children deserve a literature that contemplates the multiplicity of stories and lives, recognising the various identities and valuing those that have long been disregarded."

Through this immersion in Kenya, they could value a visit to Africa and its riches without leaving the classroom, as Mary and Rich Chamberlin brought a piece of the country to them. Students were able to create meanings for a children's literature book that brings the representativeness of black characters in a positive context, contributing to the children's development of a positive cultural and social identity (Gutstein, 2006).

Considerations

Throughout this article, we emphasised the potential of combining mathematics, children's literature and ethnic-racial issues in the teaching and learning process, showing that the student-reader can build mathematical knowledge when it is worked in an interdisciplinary, playful, and contextualised way. Among the concepts approached, we highlight: units of measure involving natural and fractional numbers, displacements and spatial notions, use of the ruler to measure, and the concept of difference. Data analysis also allowed us to consider that education is a crucial tool to fight racism. The black characters in stories with positive contexts contribute to the construction of social and cultural identity, the learning and representativeness of the black population, and the construction of a democratic, human, and anti-racist society.

The connection between children's literature, mathematics, and ethnic-racial issues that value the history and culture of the black population is essential for the learning process of all children, regardless of their ethnic-racial belonging. This combination also enabled the teaching and learning of respect for the culture of the other, the identification of diversity and its appreciation, provided to students through the story *Mama Panya's Pancakes: A Tale from a Village in Kenya.*

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The experience of teaching mathematics in a multilingual classroom: The case of a bilingual teacher

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This study seeks to unfold the factors that contribute to the teaching experience of a bilingual mathematics teacher in multilingual classrooms in the British educational system in two countries, the United Kingdom and Gre ece. The data produced by the narratives of a case study were analysed following an inductive thematic analysis. The school setting, language matters, the multilingual students, and the personal motivations to act in multilingual classrooms proved to influence her experience of teaching. Her approach of language as a resource in teaching in this context was enhanced or suppressed by the schools' policy and its agents. Her personal motivation, her linguistic experiences, and the nature of mathematical symbols and representations come to counterweight the influence of external factors in her teaching.

Introduction

Language diversity characterises many mathematics classrooms, and researchers from different countries all over the world have shown attention to the specific context (Barwell, Wessel & Parra, 2019). Research in mathematics education has studied both learning and teaching mathematics in language diverse classrooms. Moschkovich (2002) supports a socio-situated approach for bilingual students learning mathematics, which focuses on different resources students use when communicating mathematically and may help teachers to build on those during instruction. At the micro-level of mathematics classrooms, the teachers' role in relation to multilingual students and classrooms is observed. Cases of bilingual teachers have been studied regarding the way they teach, and positive effects on bilingual students' participation have been observed (Adler, 2001; Norén, 2008; Delacour, 2020). Nevertheless, conflicting situations teachers face when teaching in multilingual classrooms have been noticed in different contexts. Specifically, bilingual teachers teaching native bilingual students in South Africa faced teaching dilemmas such as transparency, code-switching, and mediation dilemmas (Adler, 2001). In a different context of teaching bilingual immigrant students, the bilingual teachers were hesitant to use a language other than the official language of instruction, as pointed out by Norén (2008).

The appearance of the dilemmas is mainly evidence of teachers' intentions to include all students in the teaching practice, according to Barwell, Moschkovich & Setati (2017).

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Nevertheless, tensions and dilemmas teachers are facing, are not completely personal but are connected to opposing circuit of cultures (Chronaki & Planas, 2018). Thus, Chronaki & Planas (2018) mentioned that teachers are not the only ones creating the classroom reality, but also the policies and educational programs with different views on language diversity have a significant role in that. Additionally, municipal and school policy regarding students with different language backgrounds than the language of instruction affects the way that school masters and teachers provide support for those students (Valero, 2007).

In the micro-level of the multilingual classroom, it has been observed that teachers can not only draw on their own and the students' similar learning experiences (Norén, 2008) but also have a great understanding of parents' concerns regarding their children's success in society (Delacour, 2020). Moreover, the fact that teachers have similar backgrounds and common native languages with some students influences the way they help them familiarise themselves with and learn the language of instruction to promote mathematical knowledge or communication (e.g., Norén, 2008, Delacour, 2020). The official language of instruction seems to be a key component contributing to teachers' practice in multilingual classrooms. The excluding discourse of the official language of instruction use only might be a factor to teachers' fear of using their native language in the classroom (Norén, 2008). Moreover, the unwillingness of students to use the official language of instruction contributes to teachers' dilemmas in a multilingual context (Adler, 2001). Giving attention to bilingual teachers, especially immigrants, may face additional difficulties as they might not be prepared for the cultural differences occurring in the work environment (Seah, 2014).

Following the research of the complexity of teaching in language diverse classrooms, the present project aims to study the experience of teaching in multilingual mathematics classrooms of a bilingual mathematics teacher, through her narratives regarding teaching in the United Kingdom and Greece. We address the question of which factors influence a bilingual mathematics teacher's experience of teaching in multilingual classrooms?

Conceptual background

Language in the mathematics teaching and learning in the context of language diverse classroom follows two interconnected social and political approaches that are represented in different metaphors. Looking at a social approach, the sociolinguistic perspective on language stresses the social nature of language, starting from the assumption that language is not only a cognitive but also a cultural and social phenomenon. The current study draws on Valdes-Fallis's (1978) view on bilingualism through the participation of linguistic communities that have linguistic norms. In that way, bilingualism is described not only as an individual but also as a social and cultural phenomenon involving participation in language practices of communities (Barwell et al., 2017, p. 9). Moreover, the idea of *language-as-resource* for teaching and learning mathematics has been advocated for many years in research, as an idea to move away from and challenge the deficit model of the official language of instruction use only (Moschkovich, 2002; Norén, 2008; Planas & Civil, 2013). The idea of *language-as-resource*, advocates for the potential for thinking and doing specifically

learning and teaching mathematics (Planas & Civil, 2013, p. 363). Similarly, Planas & Setati-Phakeng (2014), describe the same metaphor as the view of language as a potential resource for gaining access to particular spaces and sorts of capital. Language as a resource in practice refers to a combination of strategies, norms and process that seek to promote balanced integration of language and mathematics (Planas & Setati-Phakeng, 2014). A particular practice promoting *language-as-resource* stemming from the sociolinguistics tradition is code-switching. Code-switching is defined, through the sociolinguistic perspective, as the complex linguistic practice of using more languages during a communication, specifically the alternating use of two languages on the word, phrase, clause, or sentence level (Valdés & Fallis, 1978). The value of code-switching, as a source of communication by teachers and students, during the teaching process has been highlighted (Barwell et al., 2017).

Moving on to the political role of language, which is considered to be highly embedded with multilingual issues, has been stated in the macro- level of society and micro-level of classroom interaction. This role of language is located in the socio-political turn that point out the taken for granted views of thinking and acting that privilege some people and exclude others (Gutiérrez, 2013). In the micro-level of mathematics classroom language-as-political relates to the processes that place languages and speakers at an inferior position (Planas & Civil, 2013). Thus, it can be viewed as a way of in or exclusion from the learning process, communication and decision-making (Setati, 2008; Planas & Civil, 2013).

Research methods

This study is an extreme case study, focusing on a bilingual secondary mathematics teacher, named Layla who immigrated in her adult life and since then she works as a bilingual at schools with high educational standards. Layla is a graduate of a Mathematics Department in Greece, with 16 years of teaching experience in secondary mathematics, 12 of which happened in multilingual classrooms. Her native language is Greek, and she has been fluent in English since a young age. During her undergraduate studies, she attended two courses about mathematics terminology in English. Layla immigrated to the United Kingdom and worked for two years in multilingual secondary education classrooms, teaching mathematics and mechanics. She had no previous experience or specialization in teaching in language diverse classrooms. Since her return to Greece, she has been teaching mathematics in multilingual classrooms.

The schools she has worked at are both private schools, with students of high economic and social status. The school in the UK is a boarding school while the school in Greece is an international school which follows the British educational system. Their students are characterized by linguistic diversity and come from a variety of cultural backgrounds. During her first year in the UK, Layla taught in classrooms with immigrant students exclusively whose native language was not English. In the second year, classrooms consisted of both immigrants and British students. Most students were from Germany, China, Japan, Bulgaria, and South Korea, while British students were in the minority. At the school in Greece, the classrooms consist mainly of Greek, Arab, Chinese, Russian, and British students with a

Greek or British cultural or/and educational background. Students are distributed among classrooms based on their performance in each course. Teachers at the school in the UK were mainly British. Teachers at the school in Greece had different cultural backgrounds and they are proficient in English or native English speakers, as are directly related to British culture either because of their nationality or their immigration experiences.

The selection of Layla was based on the researchers' prior knowledge about her teaching experience in multilingual contexts in different countries, and on the fact that she lives and works as a bilingual. Regarding ethical considerations, is noted that the teacher and the researchers knew each other and had collaborated in the past. Layla consented to be a part of this research and to be interviewed about her experience.

The data collection includes three interviews which were conducted in two phases. In the first phase, two semi-structured interviews were conducted, the duration of each was about one and a half hours. Interviews had a narrative approach as they followed the timeline of Layla's experience of teaching in the two countries. Specifically, the first interview focused on recording her language, educational, and teaching background as well as her immigration experience in the UK, in terms of her adaptation as a teacher in multilingual classrooms. The second interview focused on her experience of teaching in multilingual classrooms at the school in Greece and the relation to her experience from the UK. In the second phase, a third interview was conducted after the first analysis of the data. The results from the first two interviews were introduced to Layla and discussed from her point of view. All interviews were conducted by distance, via a video conferencing platform during the period of COVID-19 pandemic, were videotaped, and transcribed in full.

Data analysis was conducted in three phases through an inductive thematic analysis. In the first phase, through a process of coding and categorizing the generated codes, themes in the data were recognized and described (Braun & Clarke, 2006). The themes are derived from and connected with the data and not with a code framework (Braun & Clarke, 2006). The narratives were analysed interpreting the content of the text, "what" rather than "how" was told (Riesman, 2005). In the second phase, after distancing from the data, the preliminary results from the first phase were discussed again from a social and political perspective. In the third phase, the data were analysed again with a different perspective, after a peer-critique of the data analysis which was done by a group of colleagues. The last interview was included in the analysis as well. During the data analysis, the researchers were working separately, and their results interpretation was discussed to come to a common ground in the results' themes.

Results

In the following section, we address the factors that Layla views influencing her teaching experience in multilingual classrooms at the schools in the UK and Greece. The main themes identified are *the school setting, language matters, the multilingual students,* and *personal motivation to act in multilingual classrooms.*

The school setting

The setting of each school seems to play an important role in Layla's experience of teaching. In the UK, Layla appeared to face some *conflicts* with the head of the mathematics department (HoD) who was the main agent of the school's policy about teaching multilingual students. In particular, she claimed that the HoD had a strict attitude towards Layla's teaching method, which she brought from her teaching experiences in Greece.

> The HoD wanted to keep coming into the classroom, to see what I do, how I do it, […] if I apply some other method. Many times, afterward he told me to do it differently, without explaining to me.

Moreover, Layla highlighted that the HoD did not allow her to use the students' native languages while teaching ("They told me that we are in the UK and we have to do everything in English."). Furthermore, the HoD had complained about the use of the teacher's native language, when she practiced basic operations, such as multiplication.

In contrast, at the school in Greece, Layla seemed to *agree* with the school's policy about teaching multilingual students and to have gained the support of the HoD for her way of teaching. Specifically, she mentioned that the HoD did not discourage the use of her or the students' native languages during the lesson and at school in general.

> We take for granted that students who have the same native language will help each other, and one of them can be used as a translator. The school is open to this.

In addition, the *teachers' community* at each school had an important role in Layla's experience of teaching. On the one hand, at the school in the UK, the teachers' community did not support the use of students' native language in teaching. In that context, they claimed that students should use the language of instruction and learn in this language because they were on British territory. However, Layla disagreed with that approach for teaching multilingual students, resulting in her *opposition with the teachers' community*.

> My British colleagues did not do that and many times when I asked them if they were sure their students understood, I got answers like "it does not matter, and if they do not understand now. They will spend more time in the country and they will learn". […] I support that learning means that I am gaining knowledge and If I do not understand the language, I think that knowledge cannot be gained.

On the other hand, at the school in Greece, the teachers' community accepted language diversity and tried to establish a common communication code in order to include all the students in the learning process. This approach agrees with Layla's one, so she manages to *participate* in the community.

> In a very similar way to me. They want to find communication codes too. They do not say "It does not matter; students will get used to it". They are looking for more efficient ways to integrate children into their classroom.

Furthermore, each school's policy of providing students' educational background to teachers appeared as a significant element for Layla's experience of teaching and her preparation for it. In Greece, HoD informed teachers of students' backgrounds and their possible difficulties in learning and school life, contrary to school in the UK.

> At the beginning of the school year, we have a staff meeting and we have informed on a variety of issues about our students such as how many of them do not speak English and then we go to their class […]. So, the school has already prepared me for this, and I already know to whom I have to pay attention. […] So, we have prior knowledge about students, their previous performance, if they face difficulties in their understanding or if they face language issues.

Language matters

Language is recognised to be an important matter for Layla when teaching in multilingual classrooms. Specifically, the *native language* of students is being recognised as a *tool* or/and an *obstacle* for her teaching. In the UK, the native language of students was used by her as a *tool* for their participation in teaching processes.

> The Chinese students explained to each other and I allowed it. One of them said to me, "I understand what you are saying, can I translate it to them?"

In Greece, Layla had a common language with some of the students and found code-switching to be a way to help them with mathematics terminology. Although the *native language* played the role of a *tool,* it can also be an *obstacle.* Students who have a common native language with her may resist speaking the official language of instruction creating conflicting situations.

> The school has English as the language of instruction. We must speak English. There are students who for some reasons do not want to speak English but they want to use their native language. [...] The fact that I speak Greek is sometimes an obstacle.

Moreover, the role of language when teaching in multilingual classrooms became clear when she had to teach the *terminology of the mechanics* course, which was unknown to her. That brought to the surface the matter of teaching and learning in a different language and the need for a different way of representing the terminology which was accomplished by pictures.

> There are things we used in mechanics, like terminology, that I didn't even know. [...] If the student does not know what a metal bar is, how do I translate it? In which language; [...] I was making something like PowerPoint [...], every word I thought they would not know, I found it in a picture and wrote the relevant terminology.

The *way of teaching mathematics* influences Layla's experience of teaching in multilingual classrooms from the beginning of her teaching in the UK. Specifically, the *role of symbols* played an important role in finding or creating a common code with students when language seemed to be an obstacle.

> When there was a barrier, I said "Fine I will forget the language and I will go to symbols". I was trying to use only mathematical symbols.

Layla perceived teaching mathematics in multilingual classrooms the same as *teaching students with a common native language* since she began teaching in the UK. Layla appeared to practice labelling in both linguistic contexts, acting like she was introducing a new mathematical concept.

> Due to the terminology of mathematics [...], I showed them a symbol and told them that this symbol is called integral. There were no big differences. It is like teaching in the third class of a high school in Greece and telling them that I will call this symbol integral.

Multilingual students

Layla mentioned that her multilingual students were an important element in her experience of teaching in the UK and Greece. Both in the UK and Greece, she seems to adjust her teaching according to students' reactions, considering students' *confirmation of non-verbal teaching practices* (e.g., the use of symbols and pictures).

> My students said "It is easier when we use photos. We can understand what we are talking about. Can you use pictures every time"?

In addition, Layla noticed that the *students' opposition* to the British way of teaching encouraged her to reverse her alignment with the given teaching practice. That way, she was led to include elements in her teaching that appealed to the educational backgrounds of her multilingual students.

> A student complained to me that "I do not understand. Where we found this formula.", [...] and I proved to them the formula and we ended up with the theorem. The students said to me "Finally, we got it. In Germany, we did it like this, and in the UK I do not understand anything. Can we do lessons like this every day?"

At the same time, Layla noticed that the culture of multilingual students can act as an *obstacle* when trying to approach them.

> With Japanese students who did not express themselves, it was difficult to communicate. The fact that they cannot talk to their teacher because they respect his role was an obstacle.

The culture of multilingual students can act also as a *tool*, promoting cooperation between students, highlighting their common cultural elements and characteristics.

> I have allowed students from China to help each other. They do it very well. The Chinese are very hardworking, in general, as a people, and when they tell you that "we will help each other", they usually do it.

Personal motivation to act in multilingual classrooms

Layla presented the willingness to act in multilingual classrooms, to meet all students' needs and provide them with opportunities for learning. Her personal *experiences of learning (in) a different language* were a key point of her teaching approach in the UK. Terminology and everyday life language were distinguishable for her.

> It is different to speak a language in your daily life and use this language in teaching terminology. [...] I thought that I did not understand all the terminology even though I know English, so my students who did not know English well how they will understand it?

Learning basic mathematical skills were also recognised to be more accessible in her native language.

> When you are learning mathematical skills, you are learning them in one language. I have learned the times table in Greek. I cannot do it in another language, it would take me too long to do that.

Moreover, in Greece drawing on her *experience of teaching in a different language* she became more compassionate towards multilingual students' needs.

> [Ever since teaching in the UK,] I started to understand students' needs, because we were all foreigners, without a common language code, so I had to find a way to communicate. And I brought this to Greece as well.

Her *general intentions for* students to participate and understand her teaching were highly connected with her drive to provide *equal opportunities for multilingual students*. They were also connected to her need to find or create a *common communication code* with students, to her attempt to *distinguish mathematical and language issues* to support students appropriately, and finally to her feeling of *responsibility for teaching the terminology* to students.

> I thought that when I was learning a foreign language I didn't learn this terminology so the students would not know it either, so I had to do something to teach them the language, while my colleagues did not take this into consideration.

Concluding remarks

The results revealed that Layla's teaching experience was influenced by the school setting, language matters, the multilingual students, and the personal motivations to act in multilingual classrooms.

The issue of language is central to the bilingual teacher's experience. In both schools, English as the language of instruction has been established, pointing *"language-as-political"* (Planas & Civil, 2013), excluding students with a dominant language other than English from the learning process. On the contrary, Layla view on language follows a perspective of *"language-as-resource"* (Planas & Civil, 2013). Specifically, she practiced code-switching between English and Greek for students with Greek educational background and encouraged students to use their native language, resisting to the experience of *"language-as-political"* (Planas & Civil, 2013). Despite her efforts, she seemed to use native languages more as a tool for communication than a resource for her teaching, due to the schools' language policy restrictions.

Regarding the school setting, the school in the UK did not provide sufficient support and resources for teaching in multilingual classrooms, leading her to look for a variety of strategies that allowed her to act according to her personal motivations. However, school's policy in Greece differs in the flexibility of the use of her or the students' native languages. There is a stark difference between her levels of participation in the teachers' community in these schools. Bilingual teachers at the school in Greece adopting a similar approach to multilingual students learning with Layla and leading her to participate in the community and be an active member.

Following, the school's policy of teaching in multilingual classrooms and the lack of resources lead the teacher to act autonomously. In this context, she uses her personal experiences and intentions to establish inclusive mathematics teaching and to provide equal opportunities and create conditions that allow students to access mathematics knowledge.

Specifically, the teacher's experiences as a bilingual seem to encourage her to create a common code of communication and to establish a language that includes all students in the learning process. Moreover, immigration experiences (Norén, 2008; Seah, 2014) not only played a role in the teaching in the UK but also reinforced her empathy for her multilingual students which, in turn, is strengthening her approach to inclusive teaching until the present time.

The demands of the contents had a different role in her teaching. Specifically, the mechanics course, which Layla was not experienced with, lead her to reflect on her teaching strategies due to the unknown terminology for her and her students. The use of symbols, symbolic representations, and graphs as non-verbal teaching practices were encouraged by multilingual students' culture and educational background, so the teacher had adjusted her practices according to these. Thus, the relation of the school's policy on language diversity and practice is renegotiated by both students and teacher (Planas & Civil, 2013). In mathematics teaching, even though she recognised the need for additional teaching strategies in a multilingual context, her prior experience with the respective terminology and the nature of mathematics representations did not lead to a differentiation to the core organization of her teaching between multilingual and monolingual classrooms.

It is a challenge to understand the complex situations a teacher experiences and tries to handle in their teaching. These situations are created not only by the multilingual classroom setting but also by the school's policy and external factors (Chronaki & Planas, 2018).

Although this research is based exclusively on a teacher's narratives, her approach of language-as-resource is very stark and likely to have been further supported through classroom observations. The influence of institutional aspects on bilingual teachers' experiences and teacher's agency in teaching mathematics in those classrooms are issues that need to be further studied and problematised.

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Utility in context: A sociohistorical lens for examining students' conceptions of the usefulness of mathematics

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As the emphasis on mathematics in our society has increased, so too have discussions of the usefulness of mathematics. Research has highlighted many benefits of viewing mathematics as useful, from increased performance to enhanced interest. However, we argue that discussions of usefulness have largely been devoid of context, and that to understand current ideas about the usefulness of mathematics, we must consider them in relation to narratives of economic competition and global dominance. Here we propose applying a sociohistorical lens to students' discussions of the usefulness of mathematics, exploring the case of one student to consider what such a lens can offer. We conclude by raising questions about the risks of perpetuating and potential benefits of rethinking existing narratives about the usefulness of mathematics.

Introduction

Discussions of mathematics education in the United States are often filled with questions and claims about applications, or usefulness, of mathematics. Students are encouraged to pursue advanced courses and learning opportunities in order to secure careers that require mathematics, such as those in engineering and information technology. And for students who are not interested in math-related careers, we often try to convince students of the usefulness of mathematics by explaining how math can be used to split checks with friends at a restaurant, balance budgets, or calculate discounts when shopping.

In recent decades, there has been an increasing push to consider the sociopolitical context of mathematics education (e.g., Gutiérrez, 2013; Valero, 2004), as well how various aspects of the historical context have shaped our understanding and assumptions of processes related to learning and learners (e.g., Philip & Sengupta, 2020; Schoenfeld, 2004). When applying these lenses to consider discussions around the usefulness of mathematics, numerous questions arise: Why and how have specific narratives about the usefulness of mathematics developed? Who benefits from the framing of mathematics as useful in these ways? Whose voices and histories are not represented in current rhetoric regarding the usefulness of mathematics?

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In this work, we place the notion of usefulness in context, exploring some of the ways in which the history of racial capitalism and emphasis on global dominance in the United States have influenced ideas about usefulness. In particular, we explore the following question: What new understandings can be made visible by applying a sociohistorical lens to the construct of usefulness? We address this question by first highlighting the sociohistorical context of mathematics education, with particular attention to the United States. Then we overview research on utility value as related to mathematics education and propose the value of applying a sociohistorical lens to understand narratives about the usefulness of mathematics. We propose that this lens can be productive for understanding students' talk about usefulness, illustrating this approach with the case of 13-year-old Liliana. We conclude by considering what that approach allowed us to see—as well as what silences it made visible—and raising questions for collective consideration moving forward.

Sociohistorical influences on mathematics education

In order to consider current ideas around the usefulness of mathematics in context, we begin by briefly exploring the sociohistorical context of mathematics education, with particular attention to our local context, the United States. Around the globe, scholars have highlighted how mathematics as a field has been positioned in service of capitalism and global competition (e.g., Greer & Mukhopadhyay, 2003; Valero, 2017). In the United States, Gutstein (2009) describes how mathematics education "is framed as a way to serve capital and to continue U.S. global dominance" (p. 138). He explores a range of influential reports that emphasize the importance of strengthening our global competitiveness through an emphasis on STEM and the need to prepare our workforce to meet the needs of STEM-centric jobs. Gutstein unpacks how the American Competitiveness Initiative (Domestic Policy Council, 2006) was developed as a mechanism to achieve these aims, and how the primary beneficiary is not the majority of the U.S. but rather the wealthy and corporations.

Furthermore, many STEM initiatives have been touted for promoting equity and making STEM accessible for all; however, researchers have repeatedly shown how such initiatives continue to perpetuate inequities and actually serve capitalism and U.S. economic interests (e.g., Bullock, 2017; Morales-Doyle & Gutstein, 2019). For example, Morales-Doyle and Gutstein (2019) highlight how STEM schools in Chicago claim to promote equity but actually serve the interests of racial capitalism (Robinson, 2000) by seeking to attract white, middle class families; prioritizing the interests of corporations; and perpetuating racist tracking practices. Vossoughi and Vakil (2018) highlight the connections between STEM education and militarism in the U.S., making visible how narratives about the need for diversity and equity in STEM education are often rooted in concerns for national security and global competitiveness. These various examples highlight how increasing calls in the U.S. for more diversity and equity in STEM are often in the interest of the nation and not indicative of a true commitment to people (e.g., Martin, 2009).

Alongside this sociohistorical positioning of mathematics education, exclusionary norms have developed regarding what it means to do mathematics in the U.S. In particular,

mathematics and mathematics education have been constructed as white, patriarchal spaces (e.g., Battey & Leyva, 2016; Stinson, 2013) with associated white male values, such as speed, competition, and independence (e.g., Leyva, 2021). These norms have resulted in the exclusion of Black, Latinx, and Indigenous students (e.g., Leyva et al., 2021) and influenced who has had access mathematics and been viewed as belonging in STEM spaces. Before applying this sociohistorical lens to the construct of usefulness, we provide a brief overview of research on utility value in mathematics.

Perceived utility in mathematics

Currently, the concept of perceived usefulness—often referred to in terms of "utility value"—in mathematics has been examined from the field of educational psychology and has largely been investigated without much consideration of context. Research has described utility value as the degree to which a task is "useful and relevant for other aspects of [one's] life" (Harackiewicz et al., 2012, p. 1), and studies have focused largely on interventions to enhance utility value and to explore the effects of perceiving a subject or task as useful. Two primary types of interventions have been explored. One focuses on direct communication of utility value, where educators or researchers explain the usefulness of certain subjects or tasks to students and then explore what impact that has (e.g., Durik et al., 2014). In such studies, applications of mathematics tend to focus on usefulness for everyday life in general, specific daily activities (especially those focused on money), careers, and current or future schooling (Dobie, 2019). A second type of intervention focuses on self-generated ideas about utility, where participants come up with their own applications of various techniques or tasks (e.g., Hulleman et al., 2010). Some recent work in this area has begun to characterize the connections students make and has identified a focus on community and connection to others—previously absent from this work—among first-generation under-represented minority college students (Harackiewicz et al., 2016). Across these various studies, researchers have found that perceived usefulness can have numerous positive benefits including enhanced interest in a subject and improved course performance (Hulleman et al., 2010; Hulleman & Harackiewicz, 2009). However, some limitations of these studies include the fact that they typically involve white, middle-class college students, and they are often not responsive to students' sociocultural contexts or the larger history of mathematics education in the United States (Dobie, 2019).

In one study seeking to expand whose voices were represented in discussions of the usefulness, Dobie (2019) examined perspectives on the usefulness of mathematics among 84 seventh-grade students (mean age = 12.7 years), the majority of whom identified as Mexican, Mexican-American, or Chicanx (76%). Students' conceptions of usefulness (related to how mathematics content could be applied in their lives) were coded as focused on job/career, everyday life (general), daily activities, or current or future schooling. Across a range of survey and interview responses, nearly 70% of responses mentioned specific daily activities math could be useful for, and more than 50% of responses cited usefulness for everyday life in general and usefulness for one's job or career. Zooming in on the daily activities category

revealed an emphasis on money-related activities, with 45.8% of those responses focusing on managing money (e.g., paying bills, budgeting, paying taxes) and 41.7% of those responses focusing on using math when buying and selling. Within the job/career category, students primarily focused on applications of mathematics to future jobs in general (63% of responses), though 33% of the responses mentioned application to a specific job, such as being a cashier, teacher, business owner, architect, or banker. Relatively few responses focused on the usefulness of mathematics for getting a job (4%), rather than performing a job.

Applying a sociohistorical lens to perceived utility in mathematics

Building on these findings from Dobie (2019), the current research considers the affordances of applying a sociohistorical perspective to explore adolescents' perceptions of the usefulness of mathematics. In particular, we argue for an examination of young people's talk about the usefulness of mathematics through the lens of long-standing narratives emphasizing STEM in service of capital, global dominance, and economic competitiveness. We apply this lens to the discourse of middle school students in a large urban area in the midwestern United States who were surveyed and interviewed about their perceptions of, experiences with, and ideas about the usefulness of mathematics. Here we offer one example of what such a lens might elucidate by examining the case of Liliana. We selected Liliana's case to explore here because of the prevalence of many common student narratives in her survey and interview responses. By applying this lens, we can consider how the sociohistorical context of national narratives about mathematics can influence adolescents' ways of thinking about the usefulness of mathematics.

The case of Liliana

Liliana was a 13-year-old female who identified herself as Mexican/Chicana. In her survey, she reported that mathematics was the most useful subject to learn because, "Whether buying coffee or a car, basic principles of math are in play." When asked during her interview to explain what she meant by "basic principles of math," she said the following:

> Um, so Miss Martinez always tells us that you're gonna need math in everything you're doing. She was telling about us today [sic] as well because students are complaining, like, how does this help us in life? Um, when are we gonna need this? Well, like well you're gonna need, you know, addition, subtraction when you're buying things. You gotta know how much you're gonna receive or how much you're gonna give back, you know? Taxes and everything.

Here Liliana explained how her teacher, Miss Martinez, influenced her ideas about the usefulness of math. The specific examples she provided focus on money-related utility, specifically, taxes and purchasing ("buying" things, and knowing "how much you're gonna receive...or give back"). When asked what kinds of things she thought math would help her with, Liliana replied, "Just in my career or just anything that I'm—well, buying things, depending what I do I guess and later in the years. And even just teaching other people what it is." Here Liliana again echoed ideas about math as useful for "buying things," also noting usefulness for a potential future career (though she doesn't know what that will be yet) and

for helping to teach others. Initially she mentioned helping her brother in particular, and then later in the interview she noted that she could use math for "helping my cousins or my brother with homework." This theme of helping others reoccurred several times in the interview, such as when Liliana spoke about using mathematics to help her mom "when we would go, you know, just grocery shopping or anything" and to help her dad in his locksmith shop.

In another part of her interview, Liliana described whether she thought certain math concepts they had worked on during the year were useful. Liliana again emphasized usefulness for careers when she evaluated whether perimeter and area were useful concepts to learn: "depending on what's your job like." She also mentioned how that topic could be useful for school tests since she had seen it on tests previously. When discussing the topic of calculating commission and mark-up, she thought it could be useful for "going to the store." And for three of the topics (adding and multiplying fractions, writing equations, and finding equivalent ratios), she reported that she was "not sure" where you might use them. When shown a range of images of math classrooms and asked whether or not useful math was happening, Liliana often judged usefulness based on whether it was something she had seen in school. For example, when looking at a picture of students hanging graphs they created, Liliana shared, "This one was—I would say...it's useful. Mainly because I—I've seen something similar to this that we're doing."

During her interview, Liliana mentioned a range of sources that influenced her thinking about the usefulness of mathematics. When responding to the aforementioned images, Liliana explained why she viewed mathematics she saw in school as generally useful, pointing to her teacher, Miss Martinez. She commented, "Miss Martinez always says that what we learn in math is useful and we'll need it. So, I guess, that's it." At other times, she referenced her parents and media portrayals. For example, when grappling with the potential usefulness—or lack thereof—of graphing, Liliana initially noted, "I didn't know what we were gonna need it for. Um, and I told my mom and she wasn't really sure either." Later, however, she described how graphing was "kind of useful," suggesting the influence of media portrayals on her perceptions:

> I've seen, like, movies or TV shows where they're like, in a business room and they're showing, like, how much um, how—just like, you know, like, the weekdays and everything. Or how much money they received over the month. So, I guess that's kind of useful as well.

Liliana also mentioned looking to adults around her to figure out whether mathematics truly is useful. She noted, "We don't really see a lot of the things we do in—well I haven't seen a lot of the things we learn in school, what my parents, you know, do when we're buying things or in general." She continued, "The people that I talk to never heard of what we're learning, so I was like, do we really need it?" In this response, Liliana grappled with the narratives she had previously echoed about the usefulness of mathematics, admitting that she didn't often see the math she learned in school being used by people around her.

Applying a sociohistorical lens to key themes emerging in the case of Liliana

Four primary themes emerged in Liliana's responses. First, she most commonly conceptualized mathematics as useful for careers and money-related activities, such as doing taxes, purchasing, and budgeting. Second, Liliana expressed a desire to help others, especially her family members, and thought about how mathematics could be useful for doing so; those conceptions were focused primarily on helping others to learn math, and helping with career and money-related activities (her mom's grocery shopping and dad's work). Third, Liliana referenced a range of influences on her ideas about usefulness, including media portrayals (via TV or movies, for example) and significant adults in her life (e.g., her mother and teacher). Finally, Liliana expressed both a trust in significant adults in her life, as well as some scepticism, as she worked to make sense of whether math was truly useful in the ways she was hearing, given its visible absence on many occasions.

Applying a sociohistorical perspective to these four primary themes in Liliana's interview reveals several ways in which STEM narratives emphasizing capitalism and global competition influenced Liliana's thinking about the usefulness of mathematics. We are also able to see how she relied on these narratives although they were often in conflict with her experiences and values. To begin, in drawing attention to Liliana's most frequent narratives of mathematics utility—career and money-related activities—Liliana's perceptions serve as a mirror into the dominant narratives of capitalism and economic uses of mathematics (e.g., Harouni, 2015). Relatedly, we see that although Liliana valued helping her family, her ways of conceiving what that might look like were limited to career and money-related activities, as well as simply helping others to learn math. This is not to say that Liliana should not desire to help her mother shop for groceries, help her brother with his math homework, or help her dad in his job; rather, applying this lens highlights how Liliana's conceptions of how mathematics could be used to help others were constrained to conceptions that aligned with dominant sociohistorical narratives (e.g., individual self-interest, economic competition).

Next, when applying this sociohistorical lens, we see how these narratives may have trickled into Liliana's world from larger conversations. Narratives about U.S. global dominance through STEM and preparing a STEM-centric workforce were absorbed and recapitulated by authorities like teachers and popular media, providing Liliana with larger capital-serving narratives (e.g., buying and selling, taxes, or cliché boardroom scenes). Consequently, through this line of communication, Liliana trusted and absorbed these narratives and easily (and often) accessed these narratives when discussing mathematics utility.

Finally, this lens helps us make sense of Liliana's reluctance to wholeheartedly embrace these narratives (including her confusion when working to reconcile Miss Martinez's explanations of usefulness with her observations of the world and conversations with her mother). As discussed above, these dominant narratives about mathematics are inherently exclusionary, even though public messages about the usefulness of STEM often come in the form of blanket statements assumed to apply to all citizens. For example, students are frequently told that STEM learning will be beneficial for their future studies and careers;

however, we know that Black, Latinx, and Indigenous folks are underrepresented in STEM majors and careers (McGee & Bentley, 2017), and so many students lack visible role models who look like them in these areas. Furthermore, society does not truly expect all students to enter STEM careers. Rather, the continuation of racial capitalism actually requires a hierarchy of jobs and the economic exploitation of many racial groups (Morales-Doyle & Gutstein, 2019; Robinson, 2000). Thus, while Liliana heard blanket statements about the usefulness of math, it makes sense that she did not see a wide range of uses of mathematics around her and so questioned the narratives she was hearing.

Reflections on examining usefulness through a sociohistorical lens

Applying a sociohistorical lens to Liliana's talk about the usefulness of mathematics allowed us to see how her conceptions echoed dominant narratives, and helped us to understand her scepticism and questioning. Additionally, applying this lens can make visible narratives that were not voiced. Liliana clearly cared about others and wanted to help others, yet she was only able to see how she could help her cousins and brother to learn math, her mom with grocery shopping, and her dad in his job as a locksmith. She—understandably, given the sociohistorical context—offered no discussion of how math could be used to critically analyse local issues of relevance to her family and community (e.g., Morales-Doyle & Gutstein, 2019; Skovsmose & Valero, 2002). She never mentioned a teacher telling her how mathematics could be used to challenge the status quo and focus on people's needs and interests, rather than the economy's. Furthermore, the emphasis on application of specific techniques and skills at the expense of larger goals focused on cultural competence or deeper understanding of the world, for example, risks actually reducing the relevance of math education (Kollosche, 2017). When contemplating the responses we typically provide to students' questions about the utility of mathematics, Kazemi (2018) questions, "Do we teach mathematics in ways that help students make sense of themselves and their world, to imagine the world as they wish it to be not as it is?" (p. 3).

Applying this sociohistorical lens to students' discussions about the usefulness of mathematics can help to raise questions for both teachers trying to help students see the usefulness of mathematics and researchers investigating the construct of usefulness. As D'Ambrosio (1990) notes, "It is misleading to see mathematics education primarily as something that prepares for a job. Instead it should be looked upon as something that prepares for full citizenship, for the exercise of all the rights and the performance of all the duties associated with citizenship in a critical and conscious way" (p. 21). Applying this lens to our responses to the age-old question of "How is this useful?" suggests that we need to critically examine the words that come out of our mouths and the narratives we've been programmed to repeat. Who, or what, does it serve when our answer is "if you want to become an engineer"? What are potential affordances and constraints of responses focused on buying, selling, and paying bills? And what can be gained from exploring responses that encourage critical analysis of our society and the use of STEM in service of humanity, rather than capital? These questions align with Harouni's (2015) call to explore "the *why* questions

of mathematics" (p. 71). Additionally, researchers conducting utility value interventions should apply this sociohistorical lens to critically examine the examples provided to learners, as well as learners' own reflections. While this body of research is all about values of the learner, the discussion of values guiding researchers' and educators' assumptions related to the usefulness of mathematics has been noticeably absent. We must begin to critically examine whose interests are being served, as well as what is lost, by perpetuating dominant narratives about usefulness, and to take seriously Vossoughi and Vakil's (2018) question, "Toward what ends?"

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Student perspectives on group exams as a rehumanizing mechanism in college calculus

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College mathematics courses have historically embraced white, patriarchal norms that marginalize Black, Latin, and Indigenous students – and women, in particular (Leyva, 2021). We use Gutiérrez's (2018) framework for rehumanizing mathematics to consider how group exams might help to shift norms related to participation and positioning of students in college calculus courses. We report on one attempt to use group exams as a rehumanizing mechanism in Calculus 2, sharing the experiences of Black, Latin*, Native Hawaiian, and Pacific Islander students in the class. Given the racialized and gendered history of mathematics, we then zoom in to examine a tension that emerged for several Latina and mixed-race women. We conclude by discussing the importance of seeking out the experiences of Black, Latin*, and Indigenous women and critically examining rehumanizing efforts moving forward.*

Introduction

College mathematics courses in the United States have a long history of serving as racialized and gendered spaces that have been dominated by white patriarchal norms (Leyva, 2021; McGee & Martin, 2011; Oppland-Cordell, 2014). Furthermore, schooling structures have served to separate the person from the mathematics, often making students feel dehumanized in mathematics classrooms (Gutiérrez, 2018). Thus, Gutiérrez (2018) argues for a process of rehumanizing mathematics, which entails putting the person back into the mathematics and honoring the ways in which people have been doing mathematics for years. This process of rehumanizing is especially important for Black, Latin*, and Indigenous students, who have been marginalized in mathematics classrooms; and women in these groups, in particular, given their experiences of racialized and gendered oppression (Borum & Walker, 2012; Charleston et al., 2014; Leyva, 2021; Ong et al., 2011).

The second author took up this call to rehumanize mathematics in her university Calculus 2 class. As a key element of this approach, she implemented a group exam structure into the class. While differences in structures can exist, all forms of group exams entail students taking exams—or portions of exams—in collaboration with several classmates, rather than individually. Such collaborative exam structures have the potential to improve

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student learning, support positive engagement practices (such as debate and discussion), and increase student attitudes towards classes (e.g., Bloom, 2009; Eaton, 2009; Gilley & Clarkston, 2014; Wieman et al., 2014). We propose that these structures might also serve to rehumanize mathematics and counter the white patriarchal norms that currently dominate college mathematics courses. At the same time, we must critically examine the implementation of group exam structures and, in particular, attend to the experiences of Black, Indigenous, and Latin* women. Doing so will allow us to identify potential barriers to rehumanization and optimize the use of group exams moving forward.

In this paper, we explore students' experiences with the group exam structure, with particular attention to the views of Indigenous, Black, and Latin* students. We then zoom in to examine the experiences of four mixed-race and Latina women. In particular, we investigate the following questions:

1. In what ways did Black, Latin*, and Indigenous students view group exams as rehumanizing (or not) their mathematics experience in Calculus 2?
2. What barriers did mixed-race and Latina women experience that stood in the way of group exams serving a rehumanizing function in Calculus 2?

Conceptual grounding

There is widespread concern nationally about creating inclusive mathematics classrooms and addressing issues of access, equity, and diversity in the field (Abell et al., 2017; Bressoud et al., 2015; Hagman, 2021; National Council of Teachers of Mathematics, 2014). At the same time, there are limitations to discussions of equity, as the term is often used to convey a value without real action taken to effect change (e.g., Castagno, 2014). Additionally, Gutiérrez highlights how equity is often viewed as a destination, instead shifting focus to the action of rehumanizing mathematics, which "reflects an ongoing process and requires constant vigilance to maintain and to evolve with contexts" (Gutiérrez, 2018, p. 4). Mathematics classroom structures that are rehumanizing serve to disconnect students from simple compliance and reconnect them with the joy of doing mathematics, with a focus on Latin*, Indigenous, and Black students. Rehumanizing practices seek to transform the balance of power in the classroom and intentionally create a space where students feel a sense of belonging. Being guided by a rehumanizing approach entails acknowledging that human beings have always been doing mathematics in their daily lives in meaningful ways.

For educators who want to heed this call, Gutiérrez suggests eight possible domains of attention that constitute a sort of framework for ways in which we can rehumanize mathematics. These include "(1) participation/positioning, (2) cultures/histories, (3) windows/mirrors, (4) living practice, (5) creation, (6) broadening mathematics, (7) body/emotions and (8) ownership" (Gutiérrez, 2018, p. 4). Of particular interest for this paper is the first category, participation and positioning. This domain focuses on challenging the historical classroom power dynamic between teachers as authority figures and students as vessels into which teachers pour information. Rehumanizing efforts seek to upend that power dynamic and

replace it with more discussion-based classrooms where students view themselves and their peers as doers of mathematics, seeking evaluation and approval from each other as well as the teacher, rather than only from the teacher. Through the implementation of group exams, the second author sought to make exactly these kinds of shifts in her classroom to change how students viewed each other and their relation to mathematics.

Attempts to rehumanize mathematics must also take into consideration the long history of mathematics as a racialized and gendered space (e.g., Leyva, 2017; Leyva et al., 2021; Martin, 2006). Mathematics classrooms have been guided by white patriarchal norms that emphasize individualism and competition (e.g., Leyva, 2021). Given the way in which whiteness has served as an organizing construct not only in mathematics but in education overall and society at large (Leonardo, 2002, 2004), we focus on the influence of whiteness as a racial category, rather than delving into ethnic identities within whiteness. We align with Leonardo (2002) who highlights how "whiteness as a privileged signifier has become global" (p. 30), the world has been created according to white ideals, and white folks must acknowledge their racial privilege. Thus, in our own positionality statement below, we identify our whiteness, rather than our ethnicities, given the primary influence of our whiteness on our experiences in mathematics throughout our lives.

Finally, when we apply this lens of rehumanizing mathematics in a racialized, gendered context to exam structures, we must grapple with the role of exams in the political economy of schooling (Pais, 2013). As Pais notes, "schools are places of social selection and teachers are agents of exclusion" (p. 30), with exams being a key mechanism for carrying out that exclusion. As we seek to rehumanize exams, we are cognizant of their role in a system that stratifies and dehumanizes. At the same time, instructors face many constraints, and there are often limits to what can be changed in college courses. Thus, here we choose to explore ways to improve the exam structure, while continuing to think critically about the function of exams in the current system and potential structural changes as we move forward in this work.

Author positionality

Aligning with folks who urge mathematics education researchers to be explicit about how our identities influence our research (Aguirre et al., 2017), here we briefly describe our positionality in relation to this work. Both authors identify as white, cisgender women—one situated in an education department and the other in a mathematics department. Both of us have experienced mathematics as patriarchal and exclusionary in various spaces as a result of our gender; at the same time, we have never felt alienated or oppressed in mathematics as the result of our race. We come to this work seeking pathways for rehumanizing mathematics, recognizing both the limitations of our own experiences and the need to listen to students whose voices are not often centered in conversations about mathematics classroom practices.

Course context

Data for these analyses come from two sections of Calculus 2 taught by the second author at a predominantly white institution in Fall 2018. The course is taken by engineering, science, and math majors and taught from a calculus text focusing on computation and applications that has been used by the university for roughly 30 years. As part of an effort to implement more humane pedagogies and work to rehumanize calculus, the instructor incorporated several changes into the course over a five-year period including a class mission statement, active-learning strategies, small group discussions to work on math problems in every class period, and group exams. Additionally, the instructor provided guidelines and resources about productive and respectful group discussion strategies (e.g., giving others time to think, allowing group members to arrive at different conclusions). The exam structure entailed both group and solo portions of each exam, with the group portion typically occurring the class period prior to the solo portion. During the group portion, students could troubleshoot, collaborate, and debate with group members; however, each student turned in their own exam, allowing them to come to their own conclusions. Groups changed for each exam and were semi-randomly created (see Dobie & MacArthur, in press for additional details about group creation).

Data collection

Three surveys were administered throughout the semester to elicit students' perspectives on the course structure, group exams, and ideas about rehumanizing mathematics. Across both sections, 216 students (90%) submitted responses to at least one survey. Of those 216 students, the breakdown of reported identities included 71.3% white, 14% Asian, 9% Hispanic or Latin*, 2.7% mixed race, 1% Black, 1% Middle Eastern, and 0.5% Pacific Islander or Native Hawaiian students. Of those students who reported their race and/or ethnicity, 29% identified as female, 70% identified as male, and 1% didn't report their gender identity.

Following analysis of survey results, six women were invited to participate in follow-up interviews. These women were selected based on two criteria: 1) identification as Latin* or mixed race (there were no exclusively Black or Indigenous women in the class) and 2) current enrollment in university courses (so that we could easily contact the students). We chose to interview these women due to Gutiérrez's call to gather "evidence from those for whom we seek to rehumanize our practices that, in fact, the practices are felt in that way" (Gutiérrez, 2018, pp. 3-4). Since several of these women were among the few to express any negative sentiments towards group exams—and also the exact population of students for whom the instructor sought to rehumanize calculus—we felt it was important to learn more about their experiences. Four of the six women accepted the interview invitation: Denise and Sofia identified as Hispanic/Latina; Lilly identified as white, Hispanic, and Native Hawaiian/Pacific Islander; and Selena identified as white, Asian, and Pacific Islander (all names are pseudonyms). Individual interviews were conducted via Zoom by one of the two authors in Fall 2020 and lasted between 46 and 86 minutes. Questions focused on four

primary areas: overall experiences in their major and with university mathematics courses, experiences in Calculus 2, survey responses from Calculus 2 focused on group exams and rehumanizing mathematics, and the role of identity in STEM learning experiences.

Data analysis

Given our interest in the potential of group exams to rehumanize students' mathematics experiences, one survey question was selected for analysis for this study: "In your experience, describe how the group portion of the exams has made the classroom/learning environment more or less humane for students, compared to more traditionally structured exams." Each author coded all 196 responses to this question as either positive (if the student supported group exams and/or expressed a belief that they made the learning environment more humane), negative (if the student opposed the group exam structure and/or expressed a belief that it made the learning environment less humane), and neutral/mixed (if the student's response was neither positive nor negative, or if the student shared both positive and negative responses). All points of disagreement were discussed, and we agreed on one final code for each student. We then calculated frequencies and percentages of responses in each category for a) the class as a whole and b) all students who identified as Black, Latin*, Pacific Islander, or Native Hawaiian; or mixed race including one of those identities (23/196 students).

After the four interviews with mixed race and Latina women were conducted, each interview was transcribed, and both authors read through all transcripts, engaging in an open-coding process (Strauss, 1987). Emergent themes were discussed, and four coding categories were selected for the purposes of this analysis: interaction with and relation to peers in the course, reported challenges with group exams, expectations for group members, and expectations for oneself when working with others. A detailed coding scheme was developed and again both authors coded each interview, comparing points of disagreement until each was resolved.

Findings

Here we first explore students' reflections on whether group exams made the Calculus 2 learning environment more or less humane, drawing on survey responses. Then we dive into one theme that emerged from interviewees, unfulfilled group expectations.

Student reflections on the humane-ness of group exams

Overwhelmingly, students viewed group exams as positive and as contributing to a more humane learning environment than traditional solo exams. Out of the 196 students who responded to the aforementioned survey question, 81% provided responses indicating that the group portion of exams made the learning environment more humane, while only 1.5% of students suggested that the shift made the learning environment less humane. The remaining 17.5% of responses were either mixed, with students reporting both positive and negative impacts of group exams, or neutral.

For the 23 Black, Latin*, Native Hawaiian, and Pacific Islander students who responded to the survey question, 17 students (74%) provided positive responses, suggesting that the group portion of the exams made the learning environment more humane; five students (22%) provided neutral or mixed responses; and one student (4%) answered negatively. Below we explore the nature of some of these positive responses, and then we investigate some reasons for the neutral, mixed, and negative comments among this population of students.

Many of the positive survey responses from students in this subgroup centered on the community-oriented benefits of being able to work with and learn from each other. One Black male student wrote, "I think it makes the classroom more humane since you're adding a group collaboration aspect to the exam," while a male student who identified as a Native Hawaiian reported that the group exams felt "more 'humane' because the collaboration was fun and I was able to learn how others use the same math." Survey responses also highlighted perspective-expanding experiences with the group exams, as can be seen in one response from a Latino male student who wrote that he "get[s] to see how others might approach problems and try to understand them differently."

Students also articulated the two-way benefits of group exams since they are able to both help and be helped by others. One Latina woman commented that if students cannot "comprehend a certain detail of the problem," then they have other people in their group to "explain and help you out." Additionally, this same student noted that "you are also able to help others," showcasing the give-and-take of group exams. Another female student who identifies as white, Hispanic, and Native Hawaiian also mentioned that "if you're stuck someone else in your group might be able to explain it and visa [sic] versa," adding, "it's really a good teamwork experience."

While students' reported experiences were positive overall, it is critical that we closely examine the neutral, mixed, and negative responses, as we recognize the potential for further oppression and harm to Indigenous, Black, and Latin* students given the strong role of interaction with others during group exams. When examining the neutral, mixed, and negative comments from the subgroup of students more closely, we discovered that such comments came primarily from women (5 of 6 responses). There were nine Latina and mixed-race women who responded to the survey overall, meaning that five of nine (55.6%) provided neutral, mixed, or negative responses about the group exams, with only 44.4% providing positive responses—a drastically lower percentage than found among all other groups of students.

Upon examining these comments more closely, the primary theme that emerged was concern regarding the lack of preparation of one's groupmates. Four of the five women mentioned something akin to "some students are not prepared," with one Latina woman also expressing concern that she might be "not prepared enough" or might not "understand things as much as i [sic] should." Another Latina woman who provided a mixed response broadened this perspective on a lack of preparation to include group member attitudes, writing that "when group members actually try," her experience has been "very positive"; however, "when they are not prepared nor have a positive attitude, i [sic] feel it negatively impacts

me." This mention of the lack of a positive attitude suggests broader concern about group member relations and interactions.

A challenge to group exams as a rehumanizing mechanism: unfulfilled expectations of group members

Individual interviews with four of the aforementioned women allowed us to further investigate these reported issues with group members, particularly around perceived preparation. A more nuanced theme emerged from the coded interviews related to the women's expectations for themselves and others in group exams. In particular, three of the four women commented on experiences where they had to carry the weight of their team, and the burden they experienced in doing so. Denise captured this experience in the following excerpt:

> When a student isn't prepared or, like, when one of your group members isn't prepared, and then it impedes your conversation with other group members who attempt to be prepared but may also not be prepared at all, what you end up getting is nothing and what you end up giving is everything.

This sense of "getting…nothing" and "giving…everything" was echoed by Sofia when she shared about two teammates who would "stare at the questions but they weren't willing to help." She explained how she was "annoyed" because "I felt like I was writing the whole time trying to figure out the problems without getting a lot of help." Similarly, when Selena reflected on her group exam experiences, she recalled, "I was usually the one helping people instead of being the one helped."

As part of this discussion about group member roles, the three women also conveyed a sense of responsibility to their teams. Sofia captured this notion when describing how she spent more time studying for group exams than solo exams, noting, "If I had to, I would carry the group." For Denise and Selena, this sense of responsibility came through as they discussed their desire for teammates to challenge their ideas and their concern about failing the team when others don't challenge them. Selena shared,

> Every time there was some kind of discrepancy between our work they would just be like, "Oh, well you're probably right" instead of actually thinking about how they were wrong or how they could be right or I could be wrong… I'm not going to go in thinking that I did everything right, especially when it was my first time learning these things, and the fact that they kind of were so, like, deflated to the point where they didn't even want to question me made me think, well, I'm going to take down the rest of my team like this because I don't know if I'm right and no one is questioning anything.

Here Selena conveyed a concern of "tak[ing] down the rest of my team" due to the fact that she was the only one doing the work, and no one was challenging her. Denise also expressed frustration that her team members "wouldn't call you out," which was something she "really wished for." She added, "I feel bad cause in the sense of, I don't even know if that answer is right, which is why I was trying to ask questions to them."

Interestingly, despite the challenges experienced and the frustration caused by group members' lack of participation and commitment to the team, three of the four women reported positive feelings towards group exams now, and one (Denise) described her group experience on the exams as "relatively good" overall.

Discussion

Native Hawaiian, Pacific Islander, Latin*, and Black students' reported experiences on group exams in calculus reveal many perceived affordances that align with ideas in the Participation/Positioning category of the rehumanizing framework (Gutiérrez, 2018). In particular, students reported appreciating the opportunity to collaborate with peers, see others' perspectives, and both help and be helped by others. At the same time, some Latina and mixed-race women in the course reported frustration at having to carry the weight of their team due to team members not fulfilling their expectations. This frustration was accompanied by a sense of responsibility to the team, as well as a desire to be challenged by their team members. In contrast with some students whose frustrations were embedded in deficit-based language and judgements about their peers' abilities (Dobie & MacArthur, in press), it is notable that none of these women made assumptions about what their teammates were capable of. Sometimes these women even gave their classmates the benefit of the doubt, as when Denise spoke about students who "attempt to be prepared but may... not be prepared."

These findings highlight the ways in which group exams serve to restructure possibilities for participation and positioning, providing a potential rehumanizing mechanism in college mathematics courses. However, with opportunities for more student collaboration comes the possibility of racialized and gendered mechanisms (e.g., Leyva et al., 2021) playing out within student groups. While none of the women who were interviewed attributed their increased responsibility and lack of engagement from group members to their race or gender, these findings raise questions about potential extra burdens placed on mixed race and Latina women in group exam formats. We believe these questions should not cause us to abandon group exam formats, especially since most of the Latina and mixed-race women still supported group exams and identified many benefits of such structures. Rather, it is imperative that as we move forward in this work we continue to seek out the voices of Latina, Indigenous, and Black women; and especially to take an intersectional lens and explore the experiences of Black women, who are often erased in analyses (Crenshaw, 1989) and who were not represented in this study. Additionally, we must systematically examine who is served by group exam structures and work to identify and address any oppressive and hierarchical structures that emerge within groups. By doing this work, we can take next steps in considering mechanisms for improving group exam structures to further rehumanize mathematics for Black, Latin*, and Indigenous students, and women, in particular.

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A mapping of PhD dissertations inspired by critical mathematics education: Some voices from Brazil

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This article maps doctoral dissertations inspired by critical mathematics education that were produced by Épura- Research Group on Mathematics Education and Inclusion, from 2008 to 2020. We applied the inclusion criteria, using the descriptors "critical mathematics education", "Skovsmose", "landscapes of investigation", and "social justice", we selected 10 PhD dissertations to analyse. The results show an advance in research inspired by critical mathematics education in Brazil, which include concerns such as inclusion, dialogue, landscapes of investigation, work with projects, affirmative action policies for access to, and permanence, in higher education, and foregrounds, at different levels of education.

Introduction

Critical mathematics education is defined in terms of concerns that include political and social dimensions related to mathematics education. These concerns involve democracy, inclusion, exclusion, dialogue, social justice, equity, among others (Skovsmose, 2011). Brazil has experienced an increase in research that connects mathematics education and society and is inspired by critical mathematics education. This growth is expressed by the creation of research groups, publication of doctoral dissertations, and events based on this perspective, such as the Colloquium on Research in Critical Mathematics Education, held in Brazil in 2016, 2018 and 2019. This growth is followed by the need for studies that seek to map scientific production, critically analysing it and indicating possible trends.

This article maps doctoral dissertations inspired by critical mathematics education that were produced by the Épura Research Group on Mathematics Education and Inclusion, from 2008 to 2020. The group is based in the Post-Graduation Programme in Mathematics Education (PPGEM) of the São Paulo State University (UNESP), Campus of Rio Claro (SP, Brazil). The limitation of the database to this research group is justified for two reasons. The first, because the Épura Group is coordinated by Prof. Dr. Miriam Godoy Penteado and Prof.

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Dr. Ole Skovsmose, researchers recognised nationally and internationally for their contributions to critical mathematics education. The second reason is that researchers from the Épura group have dedicated themselves to developing research that contributes to the education of students from underrepresented groups. Thus, for theoretical and methodological support of what we propose to accomplish, the article is organised by this introduction, a methodology, presentation and analysis of the mapped PhD dissertations, and final considerations.

Methodology

The captions of research indicators: (1) Critical Mathematics Education, (2) Social Justice (3) Landscapes of Investigation (4) Skovsmose

Author	Title of the dissertation	1	2	3	4
Elielson R. de Sales (2013)	Visualisation in mathematics teaching: An experience with deaf students	-	x	-	x
Vanessa de Paula Cintra (2014)	Work with projects in the initial education of mathematics teachers from the perspective of inclusive education	-	x	-	x
Luciano F. Lima (2015)	Conversations on mathematics with seniors enabled by a university extension action	x	-	x	x
Renato Marcone Souza (2015)	Deficiencialism: the invention of disability by normality	-	x	-	x
Raquel Milani (2015)	The process prospective mathematics teachers' learning to dialogue with their students in the supervised teaching practice	x	-	x	x
Denival Biotto Filho (2015)	Who has never dreamt of being a football player? A work with projects to rework foregrounds	x	-	x	x
Guilherme H. Gomes Silva (2016)	Equity in access to and permanence in higher education: The role of mathematics education regarding affirmative action policies for underrepresented groups	x	x	-	x
João Luiz Muzinatti (2018)	The soothing "truth" in mathematics education: How middle-class students' arguments can reveal their view of social injustice	x	x	-	x
Ana Carolina Faustino (2018)	"How did you come to this result?": The dialogue in mathematics classes of the early years of elementary school)	x	x	x	x
Amanda Q. Moura (2020)	The encounter between the deaf and the hearing in landscapes of investigation: From uncertainties to possibilities in mathematics classes	x	x	x	x

Table 1: PhD dissertations of the Épura group that composes this study and descriptors present in the research.

We used a qualitative approach (Bogdan & Biklen, 1994) in a descriptive-analytical study. To achieve the objectives proposed, to map PhD dissertations in the field of critical mathematics education, we used state-of-the-art (Ferreira, 2002) or state-of-knowledge-type research, particularly demarcating investigations developed by the Épura research group, interrelated

as a web, from 2008, when the group was founded, through 2020. Therefore, the methodological approach chosen for the article is supports the need to know in more detail the contexts, objectives, main results, and conclusions of the localised production of the knowledge. We located 11 PhD dissertations on the website of the Épura group, in which we applied as inclusion criteria the descriptors "critical mathematics education", "Skovsmose", "landscapes of investigation", and "social justice", and the condition that at least two of these descriptors were present in the text of the PhD dissertation. Thus, we selected 10 titles (Table 1) for reading and analysis.

To present the research mapped, we prepared an authorial guideline specifying the data extracted during the critical analysis of the texts. Namely: 1) study problem; 2) objectives; 3) study participants and location; 4) methodological approach; 5) methodological procedures; 6) theoretical framework; 7) main references and 8) results.

What do the PhD dissertations say? Some voices from Brazil

In this section the dissertations located are presented in the chronological sequence of defences, as given in Table 1.

Sales' research (2013) was carried out in a public school in the city of Rio Claro/SP with eight 5th grade deaf students, users of the Brazilian Sign Language (Libras). The proposal was to analyse whether the resources that privilege visualisation contribute to teaching mathematics to deaf students through geometry activities. The researcher discusses mathematics education and its relations with inclusive education. He addresses the concept of empowerment in mathematics education, the construction of the concept of deafness, the visuality of the deaf person, and visualisation in mathematics education. He also analyses the importance of the sign language in this process.

The research is exploratory, descriptive and qualitative in nature. The tools used to produce data were the field notebook, filming and transcription, written records of the activities developed, documents, and semi-structured interviews with parents, students, teacher, and interpreter of sign language. The researcher developed a pedagogical intervention plan to create an environment for teaching and learning mathematics based on visual aspects for deaf students participating in the research. It is noteworthy that Libras was the language of instruction throughout the development of the intervention plan. The activities developed with the students were related to some geometric concepts and the identification of figures, to explore the visual-spatial thinking resulting from the manipulation and mental construction of relations between images, local and dynamic thinking. The researcher also conducted a survey with the students to gather information about some renowned paintings and artists who were inspired by mathematics. At the end of this stage, the students produced canvases. At the end of the intervention plan was a pedagogical excursion to an outstanding art museum and a cultural centre.

Sales (2013) emphasises that the movement made to develop the activities of the intervention plan meets the ideas of "empowerment" in mathematics education, i.e., students use mathematics to improve life chances, relating it to study, work, and more effective

participation in society through critical mathematical citizenship. Students, also, began to take a mathematical look at the world in which they were inserted, identifying points, lines, curves, maps, paths, for example. He identified four aspects of the students: curiosity, involvement, interaction, and interest in the activities. He also identified that "seeing, obtaining information to then perceive, visualise and understand"[1] (Sales, 2013, p. 160) is not something natural for anyone (whether deaf or not) and needs to be developed. In this sense, he verified the importance of the activities for the development of visuality, opportunities for the deaf students to expand their "way of looking". Sales (2013) concludes that the participation in the intervention plan "affected" the way each participant thinks and acts towards geometry and brings a new meaning to the understanding of this area of knowledge in their personal and school life. The researcher states that working with activities planned to teach the deaf, combined with sign language, produced the involvement and mathematical development of the group of deaf students.

Cintra's PhD dissertation (2014) is concerned about the initial formation of mathematics teachers for inclusive education. She mobilises concepts of critical mathematics education, such as work with projects and risk zones. The objective of the research was to investigate the involvement of 10 students from a mathematics teaching degree course when creating and executing research projects that addressed inclusive education. Based on a qualitative approach, data production was carried out in two mandatory classes, called "Project Development I" and "Project Development II", taught by the author of the dissertation, in the mathematics degree course of a public university in the state of Minas Gerais. During the first, students were divided into four groups and created projects that involved the following themes: the contributions of technology for students with hearing impairment to learn mathematics; the transcription process of mathematics books to Braille; the identification of deaf students' difficulties in logical reasoning activities in an urban school; the creation and application of geometric materials for students with visual impairment to learn mathematics.

In "Project Development II", the students visited schools that serve students with disabilities and an institute of pedagogical support for students with visual impairment and, after having developed the projects in schools in the city, prepared a final report, which was configured as part of the research data, along with the final version of the project and questionnaires with open questions each student in the study answered. The questionnaires aimed to know the students, their motivations for choosing the project theme, aspects of project preparation and execution, and preparation of the final report. In a complementary way, the author used the field diary. The data analysis was based on content analysis and two categories were constructed. In the first, the author discusses students' learning in the construction of projects and in the second, she addresses aspects of teacher education and mathematics education. The results of the research show that the preparation and execution of a project that connects mathematics education and inclusion contributes to the students

[1] "ver, obter informações para, então, perceber, visualizar e compreender" (Sales, 2013, p. 160).

in mathematics teaching degree course changing their conceptions regarding inclusion. Initially, students expressed feelings of pity and willingness to help people with disabilities., Through contact with the literature in the area of inclusion and with experiences in schools, students began to recognise people with disabilities as beings with rights and duties, rights that include learning mathematics, as they are able to overcome obstacles and learn in a school together with other students.

Lima (2015) investigated an extension action[2] that aimed to foster conversations about mathematics with senior people. The research question sought to answer: What do we reveal in a university extension action involving conversations on mathematics with senior people? To this end, the researcher engaged with theoretical references in the field of critical mathematics education, justifying the relevance of the study based on the low proportionality of extension activities with senior people in Brazil, specifically related to mathematics. The context of data production involved a group of seniors over 50 years of age participating in the activity fortnightly for one year. Each meeting lasted approximately one hour. The problematisation of mathematical dialogues had subjects such as loans, Moebius range, numerical regularities and symmetries, drawn from a variable set of resources. The methodological approach adopted was qualitative and the instruments used were the field diary and interview with eight members of the group participating in the mathematical conversations.

Particularly, with regard to mathematics education and the possibilities of educational projects for the seniors, the author states that activities of this nature can contribute to the strengthening of the formation and maintenance of affective ties and to intellectual habits through improving coexistence and interaction between peers (Lima, 2015). The methodological section of the dissertation describes the participants' productions, data from the researcher's field diary, way of analysing the results, as well as the respondents' characterisation.

Lima argues that the interactive possibility resulting from the analysis demonstrates that two themes are essential for the guiding question of the PhD dissertation, namely: 1) motivation to remain in action; and 2) ways of seniors' participation in mathematical conversations. The reasons relate to the fact that the tasks exposed, debated, and reflected seniors' cognition. There is extensive social interaction among all, and this leads to new learning. The group surveyed showed they liked and craved learning mathematics. The conclusions highlight elements that are strong indicators for future work with senior education, namely: Financial Education for Senior Citizens; Mathematics and Art; Games and Mathematics, among others.

[2] In Brazil, the epistemic basis of Universities is consolidated in activities that involve Teaching, Research and Extension. Extension actions aim to dialogue with social sectors in order to contribute to problems that emerge from the context of daily experience and work of people without a hierarchical relationship. We can understand the extension as being communication practices between University-Society in order to share plural knowledge (scientific-cultural) in which the participants can experience teaching and research processes so that the knowledge is complementary.

Souza (2015) investigated inclusion in the light of the concept of deficiencialism, which is elaborated by him in the dissertation. When referring to "deficiencialism", in the journey through theories, this term is analysed from postcolonial perspectives emphasizing the constitution of the colonised subject. For him, "The construction of the colonised subject is then an effect of stereotyping discourse, and I understand that the construction of other excluded subjects, other excluded groups, is done in a similar way" (Souza, 2015, p. 52). The dissertation advances the discussion of the stereotype in relation to disability, highlighting elements of colonial studies in which Fanon is an inspiration to think about the field of disability. In analogy to Fanon's statement in "Black skin, white masks" that "The black man wants to be like the white man. For the black man there is only one destiny. And it is White." (Fanon, 2008, cited on p. 53), the researcher understands that "in something similar, a kind of anti-deficiencialism, we could adapt this phrase saying that the disabled person wants to be normal and, for her, there is really only one destination: normality" (p. 53)[3].

In the dissertation, episodes of empirical data production in the context of Brazil and India are highlighted. One of the papers refers to a situation experienced by the author when he participated in a "Teacher Qualification Course in the Vision Impairment Area", offered via Instituto Benjamin Constant (IBC) in Rio de Janeiro (RJ), a school for the education of visually impaired people in Brazil. In addition, reflections are presented on the experience in which Souza (2015) interviewed professionals from the National Association for the Blind (NAB) in Mumbai, India. In summary, the researcher concludes that the post-colonial perspective contributed as a tactical element to resist the classifications of disability and its forms of understanding throughout history. According to the author, the central action of post-colonialism would be to rewrite colonial narratives from the perspective of the colonised, such as the case of Carlos, a student with disability. As the concept of "deficientialism" is understood in the work as "the networks of stereotypes used to define abnormalities in the face of normality, thinking of people with disabilities, it is natural that the notion of anti-deficientialism and post-deficientialism arises" (Souza, 2015, p. 149)[4].

Milani (2015) chose as a question of his qualitative research: how do prospective mathematics teachers and students learn to dialogue during their supervised teaching practice? To answer his question, Milani (2015) listed as an objective to propose and evaluate actions to promote dialogue learning in teacher education. The participants in the research were two prospective teachers the researcher followed during their teaching practice. The text focuses on the concept of dialogue in landscapes of investigation by Helle Alrø and Ole Skovsmose, characterising their theoretical and empirical aspects that permeate the entire PhD dissertation. Thus, Milani (2015) introduces the definition of dialogue according to these

[3] "em algo similar, uma espécie de anti-deficiencialismo, poderíamos adaptar tal frase dizendo que a pessoa com deficiência quer ser normal e, para ela, há realmente apenas um destino: a normalidade" (Souza, 2015, p. 53).

[4] "as redes de estereótipos utilizadas com o intuito de definir os anormais perante uma normalidade, pensando em pessoas com deficiência, é natural que surja a noção de anti-deficiencialismo e a de pós-deficiencialismo" (Souza, 2015, p. 149).

authors, i.e., a type of conversation with some special characteristics that aims at critical learning. The researcher reflects on the action of asking questions in mathematics classes and proposes to insert dialogic questions as another element of dialogue. The data was produced during the third set of classes of supervised teaching practice in mathematics, which includes in the syllabus the teaching performance in mathematics in high school classes involving diagnosis, planning, execution, and evaluation during the prospective teachers' classes, and the discussion between the supervisor teacher and the prospective teacher. The instruments were of two types: written and audio records. The practice that was developed in that discipline was a collective practice of the researcher and the teaching practice teacher. The activities were prepared for the development of the discipline and provided information on how the prospective teachers thought and practised the dialogue.

As a result, Milani (2015) lists some actions that can be part of learning to dialogue by future teachers with their students: experience dialogue in investigative activities; recognise themselves as people in dialogue; imagine themselves as teachers in dialogue; explain the concerns that emerge in this process of imagination, reflect on them and look for solutions; create imaginary dialogues, predicting speeches and actions of those involved; transform more closed communication patterns into more dialogic interactions; and engage in moments of guidance with the supervisor teacher. In the final considerations the researcher presents her definition of dialogue: "it is a form of interaction between teacher and students, engaged in a learning activity, in which speech and active listening are shared, ideas are discussed and the understanding of what the other says is fundamental" (Milani, 2015, p.202)[5]. She emphasises that the choice to dialogue in mathematics education is not neutral.

Biotto Filho's PhD dissertation (2015) investigated the re-elaboration of children's foregrounds, in a non-formal educational environment, based on the pedagogical approach of working with projects. According to the author, "foreground" means the future horizons of a person, or group of people, and can generate intentions and reasons for students to engage or not in school activities, such as those involving mathematics. In the theoretical framework, the author mobilises concepts of critical mathematics education such as foreground, work with projects, arranged situation, and border position. Based on a qualitative methodology, data production was carried out in a semi-boarding school social institution that children from six to twelve years of age attended in the school counter-shift. Thus, in his choice of the context for data production, the author considered not only the existing situation (traditionally organised school), or the ideal situation (school based on the perspective of work with projects), but an arranged situation, in which he developed activities during 11 meetings, based on the approach of work with projects in the semi-boarding school with 11 children aged around 11 years old, i.e., school age. These meetings were followed by interviews.

[5] "É uma forma de interação entre professor e alunos, engajados em uma atividade de aprendizagem, em que a fala e a escuta ativa são compartilhadas, ideias são discutidas e a compreensão do que o outro diz é fundamental" (Milani, 2015, p.202).

The project activities involved two phases: introducing the theming phase and the student group research phase. In the first, the participants chose to work with the football theme, related to the aspect of becoming a football player. The second phase involved dividing the students into three groups that researched the themes football, future, and professions. Afterwards, at the end of the project, each group presented its final product. Seeking to read the students' foregrounds, the author conducted interviews, inspired by the concept of InterView, which took the form of a structured conversation between interviewer and interviewee in which both look together at the subject of the conversation, negotiate meanings, and the interviewee participates in the interpretation of the data. Another important aspect of this process was the search to understand the elements present in the students' backgrounds, as well as the social context in which they live. The results of this study show the interpretative character of the foregrounds and point out the possibilities of investigating them by reading between views. It is also evidenced that the activities with projects contribute to the re-elaboration of foregrounds.

Silva's research (2016) sought to understand how mathematics education could contribute to the permanence and progress of students who benefited from affirmative action policies of higher education courses in the exact sciences. The theoretical framework used was mathematics education for social justice and critical mathematics education. The study participants were managers/professors of higher education and students who benefited from affirmative action policies in the exact sciences of two federal universities in Brazil. The investigation had a qualitative approach, was inspired by the case study, and the researcher conducted documentary analysis and semi-structured interviews, keeping a field notebook during the interviews. The data analysis was based on content analysis and critical survey. At the end of the research, Silva (2016) emphasises that even if it is not enough, the Quota Law is of paramount importance to combat social injustices and inequities present in society. Affirmative action contributes to students' access and permanence in public universities.

Silva (2016) argues that mathematics education can help students who intend to enter (access) and those who are already attending (permanence). For example, mathematics can be used as a "filter" to select people who meet the university's criteria through the selection process and, at the same time, the student who understands the "dominant" knowledge can be selected to win a scholarship, i.e., mathematical literacy can be a means to open doors. Aware of this, many of the students in the study who entered through the quotas, at the end of the course, returned to their cities to teach mathematics classes, voluntarily, in community courses, to help other students to enter higher education. Silva (2016) stresses that quota students reported having suffered microaggressions in the classroom from teachers and colleagues. Microaggressions can have consequences such as, for example, withdrawal from the course, anxiety, and depression. The researcher points out that affirmative action are more than mere welfare action; it aims to improve students' access to permanence and their social and academic integration, for a more just and equitable society, overcoming microaggressions and other forms of prejudice and discrimination they suffer.

Faustino's research (2018) sought to understand how teachers and students dialogue in mathematics classes in the early years of elementary school, thus seeking to identify elements that favour the construction of a dialogic mathematics class. The theoretical framework used was critical mathematics education, mathematics education for social justice, and critical pedagogy. The study participants were two teachers from the 3rd and 5th grades of elementary school, in interaction with their students in a public school in the state of São Paulo – Brazil. The research had a qualitative approach, based on participant observation and during data production, the researcher, together with the teachers, developed the project entitled "Environment and Mathematics". This project was developed with students from both classes during one semester. The project focused on the content of quantities and measures. The tasks included, for example, producing videos in which mathematics was used to interpret environmental issues such as the use of water in school and students' homes. The dialogues established between teachers and students during mathematics classes were registered in field diaries, through audio and video-recordings.

At the end of the research, Faustino (2018) advocates that the way the teacher organises the learning environment influences the pattern of communication that will emerge. Dialogue emerged during the project activities, in which students had to make decisions and share their perspectives. At this point, aspects such as investigating, presenting arguments, taking risks, and building equity were identified in the interactions. The author emphasises that interactions in classes are not free of conflicts (existence of non-dialogical acts). Finally, Faustino (2018) emphasises that teachers must listen actively, moving epistemologically to understand children's perspectives, to subsequently ask questions that help them understand the concepts.

Muzinatti (2018) described and identified specific particularities of the thinking and ideology of 7th-grade students (aged between 11 and 13 years) in a middle/upper class of a school in São Paulo, also constituted and confirmed in mathematics classes. In the theoretical framework, the researcher seeks to understand how the so-called "truth of numbers" may be reaching elementary school students, more precisely the students participating in the research. To understand the paths by which this absolute truth travels from ancient Athens to the classrooms of today, the researcher presents some pre-Socratic influences and aspects of Platonism that pervade antiquity, the medieval, the so-called modernity. He also shows the idea of Truth that has surpassed the centuries, poured into the religions, and philosophers' offices. Finally, the researcher presents Nietzsche's thought and the contextualisation of such questioning through critical mathematics education as the most fundamental critique.

The data was produced from activities planned and developed by the researcher which were recorded. Muzunatti (2018) chose to describe the classroom episodes as three short stories with researcher and students as the characters. To analyse the data, the author reflects on the students' arguments, their postures towards situations related to social injustices, and

the role of mathematics education in their construction of world representation. Coming from the entire historical process of reinforcing mathematics as absolute truth, the effect of Platonism that shapes the West, this perspective reproduced in academia is also perceived in the speech of the students participating in the research. It highlights the relationship of power and exclusion that mathematics exercises within the participating group, and even within the family, showing that in the classroom, distinction and respectability accompany those who excel in learning numbers. When the researcher, aiming for students to interpret the world via numbers, brings an overview of the reality, he imagined that mathematical "inexorability" would be a decisive factor in argumentation. However, students did not seem to be touched by the information: they treated it as something normal or, at worst, a fatality. The social problems revealed by the numbers do not seem to sensitise them. Contrary to what the researcher expected, the numbers did not help to sensitise students to the Brazilian social reality.

Moura (2020) raises the topic of inclusive education by emphasising the importance of dialogue in landscapes of investigation. The participants in her study were deaf and hearing students, the interpreter teacher of sign language, and the regular teacher in a class of the initial years of elementary school. Her research aimed to understand the interactions in mathematics classes in which they study deaf and hearing, in an approach of landscapes of investigation. This approach brings as its main characteristic the openness to different forms of learning through dialogue. In the theoretical framework, the researcher mobilises concepts arising from critical pedagogy such as tolerance and dialogue; critical mathematics education, such as landscapes of investigation, microexclusion, deficiencialism, and meetings amongst differences; and mathematics education for social justice, such as reading and writing the world with mathematics and culturally relevant pedagogy.

The research was based on a qualitative approach, used participant observation, and the data was produced in a public school in the state of São Paulo, in a regular classroom of the 5th grade. Three teachers, three interpreters of signal language, and the 17 students of the class, of which 12 were hearing and five were deaf, participated in the research. The classes were recorded with audio and video, and then episodes were transcribed and analysed. In these classes, students engaged in inclusive landscapes of investigation that were developed by the researcher and the teachers. The results of Moura's research (2020) show that dialogue was a fundamental aspect of the students' inclusion and that it involved elements such as decentralisation, active listening, and the presence of dialogic acts, which proved to be fundamental for cooperation between the deaf and the hearing. The author emphasises how important it is to understand that inclusive education is an encounter of differences and that people value and respect those differences in mathematics classes, recognising them as a valuable resource for teaching and learning. According to the researcher, landscapes of investigation created opportunities for cooperation and the construction of equity through dialogue, which is fundamental in the development of inclusive actions.

Considerations

The results show an advance in research inspired by critical mathematics education in Brazil, which include concerns such as inclusion, dialogue, landscapes of investigation, work with projects, affirmative action policies and foregrounds, at different levels of education. All studies analysed were based on qualitative methodologies and are practice-based methodologies. The data from the fieldwork of the studies by Sales (2013), Souza (2015), Muzinatti (2018), Faustino (2018), and Moura (2020) were produced in schools of basic education; by Cintra (2014), Milani (2015), and Silva (2016) in undergraduate courses in mathematics; by Biotto Filho (2015) in a semi-boarding school institution that served school-age children; while Lima's (2015) research was intended for senior citizens. Thus, the research studies are strongly committed to understanding and changing the Brazilian school reality at different levels of education, as well as in the different spaces where educational processes can take place. Such commitment is not restricted only to formal educational spaces, but also to non-formal spaces and seeks to contemplate different groups of students.

The experience of contact with the studies in this paper indicates that the Épura Group's research represents a collective effort to address issues related to mathematics education and society in the Brazilian context. The research studies present dialogue as central source of the promotion of communication, cooperation, and investigation in mathematics classes in different – formal and informal – educational spaces, and indicate the relevance of thinking of mathematics as a tool of social inclusion of children, young people and adults.

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The incident of the quadratic equations: Recognising exclusion

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In this paper we explore the hidden nature of girls' exclusion from mathematics in a Norwegian lower secondary school. Drawing on ethnographic data from a three-year longitudinal study, we focus on a particular incident of 'choosing' a challenging task. Viewing the incident through the lens of Figured Worlds we note how the researcher's perception of mundane acts in the classroom is expanded by witnessing a rupture within one student's narrative of the incident. As researcher and researched co-construct events, it becomes apparent that equal opportunity in the form of choice does not necessarily deliver inclusion.

> Miss A has done a bit of a stupid thing with that, the quadratic equations then, when she says that not everyone is skilled enough, then it's difficult to learn such things yourself. (Emilia, 10th grade)

Emilia's words, spoken in an interview in 10th grade, signalled an apparent critique of her mathematics teacher, Miss A. Although the first author had spent over two years in Emilia's mathematics class, she had not realised that Emilia, and potentially some of the other students, did not feel that they could access the more challenging mathematics on offer. This moment in the interview became a 'breakthrough moment' which provided an opportunity to for the researchers to re-reflect on the nature of mathematics teaching and learning, and enabled recognition of exclusion in terms of gender.

Girls' exclusion from mathematics is an established issue in the research literature in mathematics education, from Walkerdine's early work (1989/1998) to more recent discussion (Jaremus et al., 2020; Radovic et al., 2018). The research reported here took place in Norway, which is known for its strong claims about gender equality. In this paper we focus on girls' exclusion from the most advanced mathematics in a lower secondary classroom. Focusing on the habitual acts that underpin expected ways of being and acting in classroom mathematics – the rules of the 'game', we address the problem of becoming aware of moments of exclusion that are embedded within the mundanity of a classroom culture, where it may be disguised as routine and innocuous. We illustrate how, in becoming aware of such moments, they may work as seeds of change, for both the researcher and the researched.

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Being a girl in the game of mathematics learning

Much research on girls' and women's experience of mathematics classrooms reports that they are excluded by the everyday actions and mundane aspects of mathematics classroom cultures in which students 'play the game' of learning mathematics. Many have recognised that girls in mathematics need to play this game differently in order to survive. For example, Bartholomew's (2002) study of high attaining groups found that girls often expressed a stronger desire to understand mathematics in order to enjoy it, and were consequently more vulnerable to the fast-paced teaching style common in such groups along with a classroom culture which "tends to marginalise many of the girls" (2002, p. 6), by its 'laddish' style. Echoing Boaler's (1997) findings that boys turn school mathematics into a game, Bartholomew (2002) argues that girls need to play the game of mathematics learning differently in such classrooms, illustrating this complexity of being both feminine and successful in her case study of Tanya, who realises that she must focus on personal progress rather than competition with others.

The intersection of femininity and mathematics achievement in a high attaining classroom is further investigated by Foyn, Solomon and Braathe (2018). They found that in this classroom culture the girls both self-policed and policed each other in order to prevent themselves from visibly performing ways of being good at mathematics through the enactment of 'natural ability' and competitiveness, which were acceptable for boys but not girls. This is exemplified in the case of Anna; like the other girls, she is 'invisible' during lessons, but her visible enjoyment of mathematics challenges and her overt interest in achieving high grades, combined with a 'boyish' performance of carelessness means that she crosses the line of what is acceptable for girls in this classroom. She pays the price for this behaviour by being called a 'nerd', while the other girls are careful to position themselves as 'not Anna'. In contrast with Bartholomew's Tanya, the high achieving girls in this classroom balance on 'a knife edge' as they focus on maintaining their positionality as good in mathematics at the same time as not exposing themselves as behaving inappropriately.

However, there might be alternatives for high achieving girls to play the game differently within a classroom culture other than taking up the ascribed positions connected to traditional femininity. This is demonstrated by Radovic et al. (2018) who notice how three high achieving girls in three different peer groups in a classroom culture relate to mathematics in different ways. They developed and negotiated their mathematical identities in accordance with their peer cluster memberships of mature/ popular/hyper-feminine, 'Korean/weird', and normal/quiet girls/loud boys, taking up a variety of stances on mathematics - as effortless/effortful, as a natural ability, as different, as male. The groups also related to what they valued about doing mathematics – independent and collaborative, wider and complex, straightforward and procedural (p. 449). Thus, they combined being female and good at mathematics differently within peer groups that existed on the margins of the classroom culture. Radovic et al. (2018) conclude, by drawing on a nested model of

identity, that "peer relations had a central role in mediating each girl's [mathematical identity]" (p. 457).

In these studies, there is no indication that girls explicitly recognise their situation as one of potential exclusion, compromise or difference. Solomon's (2012) study of Roz, a female mathematics PhD student, explores the nature of reflexivity in her account of her experience of being a woman in mathematics, and how her resistance to ascribed positionality may lead to a change. Roz describes her struggles with mathematics and her challenges to the dominant discourse of male competition, storying herself as part of a collaborative group of female students who raise a counter-voice to traditional values in mathematics. However, a contradiction in Roz' story emerges when she explains her success in mathematics as due to having a 'male brain', despite her claim that you "don't have to give up being a woman to be a mathematician" (p. 180). Solomon argues that conflicting voices such as these are indicative of a struggle to find a space combining being a woman and a mathematician, and that "there is no easy link to be made between the reflexive accounts of gender and ability and a change in the (self)positioning of women in mathematics" (p. 181). However, she argues that recognising the inevitable existence of multiple voices indicates that there are possibilities for creating new meaning: "'figuring it otherwise' is still on the agenda" (p. 182).

The studies reviewed here raise the issue of the extent to which girls may become aware of their positioning in mathematics, and the potential for reflexivity as a result of such awareness. In this paper, therefore, we address the research question: How is exclusion brought to consciousness, for researcher and researched?

Theoretical framework: bringing the unconscious into awareness

In this paper, we draw on Holland, Lachicotte, Skinner and Cain's (1998) theory of Figured Worlds in order to understand classroom cultures. The concept of a figured world draws attention to the role of norms and the significance of certain behaviours; it is a

> socially and culturally constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others […] moved by a specific set of forces. (Holland et al., 1998, p. 52)

A figured world is constituted by the actors within it, over time. Newcomers to the world learn which acts give value over others, what are the significant markers of positionality, and how power and privilege are distributed. Everyday happenings in the figured world become the frame for how to act and how to construct the meaning of actions, embedded within the mundane happenings that routinely take place:

> A figured world, too, is played out; a frame becomes a world—a space and time established imaginatively—that one can come to sense after a process of experiencing, acting by virtue of its rules […] Players become ever more familiar with the happenings of a figured world (...) and learn to author their own and make them available to other participants. By means of such appropriation, objectification, and communication, the world itself is also reproduced, forming and reforming in the practices of its participants (Holland et al., 1998, p. 53).

The rules and norms of positionality in a figured world are played out unconsciously for much of the time. However, sometimes, actors experience events – ruptures - that lead them to become conscious of the position they have developed, and which has been so far out of awareness. Holland et al. (1998) notice that this moment of realisation may happen in different ways: people may become aware of their positionality within the figured world, through a process of realisation of their entitlement or lack of entitlement, which have become habitual within the mundanity of the figured world. Or it can happen through more dramatic incidents, as sudden activities or happenings that radically affect their entitlement in accordance with social position. Either way, new consciousness of one's positionality may lead the actor to reconsider their access to acts, leading to a possibility of affecting their own behaviour, including active resistance to such positioning. Thus, positionality within the figured world is

> more or less conscious, more or less habitual, moving sometimes out of awareness, toward fossilization, and at other times toward consciousness and susceptibility to manipulation (Holland et al., 1998, p. 237).

Since a figured world is reproduced by the actors in that world, there is always the possibility of changing the rules of the game, but this is dependent on becoming conscious of those rules. Holland et al. note that

> Ruptures of the taken-for granted can remove these aspects of positional identities from automatic performance and recognition to commentary and re-cognition. […] Some signs of relational identity become objectified, and thus available to reflection and comment […]. Alternative figurings may be available for interpreting the everyday, and alternative ways of figuring systems of privilege may be developed in contestations over social arrangements. (Holland et al., 1998, p. 141–142).

Awareness that we occupy an unfavourable position which derives from the mundanity of the figured world is of crucial importance for 'figuring it otherwise' and moving towards a new figured world. These new worlds may just be images of how things should not be, a counter-world that is not necessarily played out. But this may be the seed of a new reflexivity in assessing how things are. In this paper we consider how rupture may not only function as a potential catalyst for one girl's reflection on her position in a mathematics class, but may also provide an insight for the researcher into the what lies beneath the mundanity of classroom events.

Methodology

This paper draws on a three-year ethnographic study of a mathematics classroom in a lower secondary school just outside Oslo, with students drawn from mid to high socio-economic backgrounds, conducted by the first author. Employing the lens of Figured Worlds provided a means of capturing the values, norms and frames of 'Class A' as the students and their teacher moved together through the years from grades 8 to 10. Conducting an ethnography in these circumstances means that the researcher herself is at least partly an actor in the figured world, providing various opportunities but also constraints. In this paper, we see that

everyday happenings may come to be what is expected, meaning that it is possible to overlook moments of exclusion.

A variety of data were collected in this study. Fieldnotes in every lesson aimed to record habitual acts and mundane activities and map students' relationships with the teacher and each other. They also aimed to capture the students' enactment of their relationships with mathematics as they moved around the room. Focus group interviews with the students while they were in grades 8 and 9 provided a broad information base about their shared beliefs about mathematics and what it was like to be a mathematics student in Class A. These accounts of Class A were supplemented by interviews throughout the 3 years with their teacher, Miss A, who played a significant role in this figured world as the bridge between the students' classroom lives and the world of education policy and assessment. Copies of Miss A's records throughout the period provided insights into her view of students' performance and progress, along with twice-yearly test results. Students themselves provided further data in written reflections after the end of year tests in 8th grade and 9th grade.

Individual interviews with the students in grade 10 aimed to capture their personal histories and individual accounts of Class A. As Braathe and Solomon (2015) argue, agency is enacted within the interview through the co-construction of a dialogue; this has particular significance in this paper in terms of its role in creating a space for recognition. In this paper, we focus on the interview with one student, Emilia, set against the backdrop of the figured world of Class A and a particular incident recorded in the fieldnotes. Analysis focuses on the role of rupture and recognition, and on reflexivity for both Emilia and the first author.

The incident of the quadratic equations: How exclusion is hidden in plain sight

The incident of the quadratic equations occurred in a 10th grade lesson on algebra and reduction of fractions. During this school year, and perhaps in response to pressure to ensure good grades in the national tests at the end of the year, Miss A had begun to bring extra problems which extended beyond the level targeted by the textbook the class was using. On this occasion, she presented a worksheet on quadratic equations, telling the students that these problems were for those who wanted to challenge their understanding of simplifying algebraic fractions by factorisation and who were confident they were able to cope with the most difficult questions in the textbook. The lesson continued with business as usual. The students chose freely what task level they would work on; most worked in their usual places, while some asked permission to move in order to work with students who were different from their neighbour.

From her vantage point in the classroom, the first author noticed that the only students who chose to work with the quadratic equations were a group of boys. She had learned from conversations with Miss A that these boys were called the 'smart boys' due to their high grades and their obvious (and sometimes noisy) interest in mathematics. Several students had mentioned this group of boys in their interviews as being 'quite a lot better than the

Ross and Alexander ('Smart boys')

Ross: works concentratedly on his own and raises his hand to ask me for assistance. He works on factorizing algebraic expressions using knowledge of the quadratic equations to reduce fractions. He is focused and thoughtful in our talk, and our conversation is about how to know when to use quadratic equations. He gives an impression of both enjoying and understanding the conversation

Alexander: works on his own on factorizing algebraic expressions using knowledge of quadratic equations to reduce fractions.

I 'look over his shoulder' to see how his work is going. What he has done in his workbook seems to be correct, and he gives me the impression that he has no need for assistance. I

Figure 1: Fieldnotes on two 'smart boys'

Emilia, Eva and Sarah

Sitting next to each other, work on tasks in the textbook – on level 2 tasks out of 3 levels. Do task after task checking the 'fasit' [solutions] and seem happy when it's right and carry on. I ask why they choose level 2 tasks, Sarah and Emilia reply that it is to be sure they get it right.

Figure 2: Fieldnotes on three girls

others' and 'at a higher level'. The rest of the students chose to work on problems in the textbook. The first author took field notes, as in Figures 1 and 2. At the time, the events recorded in these field notes were not seen as anything special; rather, they seemed to be just examples of everyday happenings in Class A, descriptions of habitual acts rather than of an incident as such. At most, they were seen as reflections of a pattern of test results which showed that the 'smart boys' achieved consistently high grades across the years; although two of the girls in Figure 2, Emilia and Sarah, were high achievers with scores of 5 on a 6-point scale, they did not score the 6's which were seen to define the 'smart boys'. At this point in time, the field notes had no singularity and no apparent significance among the other data. However, something happened to change this situation, and they became significant as part of our understanding of the impact of norms in the figured world of Class A.

Emilia's story

Emilia's words create a rupture in this perception of events as inconsequential. They are themselves a rupture in her otherwise positive narrative, which tells of her long-term love and enjoyment of mathematics, and indeed she is good at mathematics according to Miss A's account of her grades. Her story prompts a new view of the habitual acts in the figured world of Class A, drawing attention to how these field notes were describing more than the enactment of apparently innocuous choices. Her words describe a moment of exclusion from

the most advanced mathematics, situating the field notes and revealing the significance of what lies behind them, enabling the researcher to see through her eyes to what is hidden in plain sight.

Emilia stories herself as the main actor in her narrative, generally responsible for her own actions in Class A. Her story circulates around how she manoeuvres within the dynamics of this figured world to find a way to do what she needs to do in order to understand, which is important for her if she is to enjoy doing mathematics: "I need to understand what and why I'm doing it somehow, […] get it [the work] done and for it to be fun". Her need for understanding makes it important that she can ask questions, but she hesitates to do so in some whole class situations:

> if we're at the start, a startup, in a new topic, then it might be useful [to ask questions], but not if we're in the middle of a new topic and I don't understand it, because then, I don't know why, then I just don't understand it, because then it goes very quickly on the board, because then they go through it so fast.

It seems that there are others in the class who set the pace of teaching in these plenary sessions, described here by the unknown and distant 'they', and the pace means that she cannot or does not choose to ask questions. She adds that she prefers to ask Miss A questions afterwards instead, and this is a first indication that her situation in Class A is not as straightforward as it may seem.

Emilia says that she enjoys the freedom in Class A that means that she can choose for herself who to work with: "I like it the way we do it, because we know best who we like to collaborate with, and then we can choose more ourselves." This also applies to choosing the level of problems to work on, but tensions in her rationale for this emerge. She says that at school she works on easier problems than those she does for homework: "then I choose a bit more advanced". It turns out that, at school, working with the boys means that she can challenge herself, because they are more competent than the girls – "some of the boys are very smart, and sometimes I like to work with them occasionally because some of them have a lot more skill than many of the girls" - but the pace of work that she knows from the plenary holds her back. As she says, "I need to ask my questions during the explanations", and she prefers to work with her friends (the girls), even though "it might not be that smart, because I don't get to challenge myself that much". Despite these 'ifs and buts', Emilia doesn't blame anyone or express dissatisfaction with her situation in the classroom. This is in keeping with how she has acted since 8th grade, matching the first author's impression of her as a student who easily adapts to situations and rarely speaks out to express a negative opinion. But at the end of the interview, when asked if there is anything she would like to change about mathematics in Class A, she says there is:

> Miss A has done a bit of a stupid thing with that, the quadratic equations then, when she says that not everyone is skilled enough, then it's difficult to learn such things yourself.

Her critique of Miss A is unexpected, and creates a rupture in the genre of her talk so far. Suddenly, Emilia starts talking about grouping according to attainment in the classroom, and why she thinks this is a good idea, because it might provide opportunities for people like

her; groups would enable the teacher to work with "those who struggle with one thing to go through it with them. And those who are at a high level might learn something new". Furthermore, even though just some minutes ago she had emphasised that she enjoyed the freedom of choice that Miss A provides, she says that she should take control over this. This unexpected outburst and its associated contradiction strikes the first author the moment it is articulated, revealing that Emilia doesn't see herself as having access to the same mathematics as the smart boys. When she is asked if she has learned about quadratic equations, she laughs and says: "No, I never understood it, but then we haven't had any tasks with it, or we've had it, but I found another way to solve it." It seems that she has some level of awareness that she is missing out on the more advanced mathematics, but she plays its importance down:

> I'd love to learn how to understand maths better, but that's not one of the most important things I'm thinking of now, because I know we're going to learn it in high school.

Discussion: Gender breaks out of mundanity

Emilia's unexpected critique of Miss A captured Emilia and the first author jointly in a moment of co-construction of awareness, but in different ways and with different outcomes. For Emilia, this incident and its telling may have signified just a beginning of realisation. Without knowing what happened next, it is difficult to argue that this is a moment of realisation for Emilia, but it is possible to claim that it had the potential to lead her towards a new consciousness of her lack of access to the most challenging mathematics in the figured world of Class A. As Holland et al. (1998) point out, actors may experience events – ruptures – that lead them to become conscious of the position they have developed and which have been so far out of awareness. In this case, Emilia's articulation of the incident of the quadratic equations may play a part in a process of realisation of her lack of entitlement, until now embedded within habitual acts but now revealed in a first step to 'figuring it otherwise'. Perhaps Emilia is a younger version of Solomon's Roz, noticing injustice and articulating it, even if that articulation is contradictory and incomplete. For now, perhaps, the event is perhaps just "available to reflection and comment" (Holland et al., 1998 p. 141), and she improvises a response in terms of future opportunities for learning rather than addressing the present.

For the first author, although (or perhaps because) she had been a partial actor in the figured world of Class A for some time, and as such had not recognised the presence of differential access to more advanced mathematics, this incident was much more dramatic; it ignited a rethink of what underlay these mundane acts in Class A. Reflecting on this moment in the interview, we realised that Emilia's comment that she enjoys the freedom to choose who she works with and what they work on, while at the same time saying that she thinks the teacher should decide, is not the contradiction it appears to be. Her words suggest that she wants the opportunity to choose, but her story shows that she pays a price for the choice. If she chooses to work with the group of 'smart boys', she has to work in their way; consequently, she chooses to work with the girls on less demanding problems, even though

she would like to challenge herself more. Emilia's manoeuvring within this frame in order to keep hold of her enjoyment of mathematics is reminiscent of Bartholomew's Tanja: Emilia improvises so that she can play the game of mathematics differently from the hegemonic frame which supports her male peers at the same level.

Exposure to Emilia's view of the incident of the quadratic equations forced a re-reflection on what we had understood so far about the norms of positionality and access to what were deemed significant acts in Class A. It also provides a context for a possible interpretation of the gender differences in the attainment scores in favour of the boys in this class. As the field notes show, the first author had noticed that Emilia and her girlfriends did not choose to work with the quadratic equations, choosing to work instead from the textbook, even though they were high achievers. Emilia's story gives more colour to the picture of Class A, in particular its norms of positionality and their effect on her sense of entitlement to work on the more advanced mathematics – it can and must wait until she is in upper secondary school. What this reveals about an apparently simple difference in results is that there may be exclusion in Class A which goes beyond what appears to be just *self*-exclusion. The incident of the quadratic equations, seen from Emilia's complex perspective, makes it possible to understand how gender positions the students in this classroom differently. It provides a glimpse into how equal opportunity defined as freedom of choice does not mean equity for male and female students in Class A.

The incident of the quadratic equations illustrates how the norms of a figured world may be unconsciously reproduced and out of awareness. As Holland et al. (1998) emphasise, awareness is the first step towards change. For the researcher, recognising exclusion that is embedded within the mundanity of a mathematics classroom may be difficult when one does not experience the classroom culture from the perspective of the disadvantaged. But Emilia's participation in the co-constructive space of the interview enabled us to recognise what was hidden in plain sight. Exposing and opposing exclusion, raising one's voice and rejecting the rules, is difficult from a position of disadvantage, as in the case of Emilia. This makes it even more important for researchers in mathematics teaching and learning to be conscious of how to capture aspects of exclusion that may be embedded within the mundanity of classrooms, as hidden rules of the game. Researchers employ a space that enables us to use our voice, unlike those students who experience exclusion, and we are ethically bound to do so, especially in contexts where, like Norway, equal opportunities with respect to gender are seen to be no longer a problem.

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Mathematics education in middle schools of Ethiopia: Culture and language in the textbook

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In this paper I present a study of mathematics education in middle schools of Ethiopia, focusing on the textbook. Particularly, I examine how textbook is connecting mathematics education with the local culture. I also set out about the use of the local language in mathematics education. The study draws on cultural historical activity theory. The analysis reveals that the textbook has limitations in connecting mathematics with the local culture. Their limitations appear to be consequences of the education policy of the country. The implications for cultural and linguistic issues in mathematics education in Ethiopia and beyond are set out.

Introduction

This paper reports an examination of mathematics education in primary schools of Ethiopia with particular focus on how textbook is connecting mathematics education with the local culture and the use of the local language in mathematics education.

There have been studies, which expose about mathematics education and making cultural connection (e.g., Fyhn, et al., 2017; Jannok Nutti, 2013; Lipka & Adams, 2004). Jannok Nutti explored and exposed the mathematical activities in handcrafters and reindeer herders of Sámi people in Sweden, where she analyzed their stories based on the six activities proposed by Bishop (1988). Jannok Nutti expressed concerns about the in-availability of resources for designing and developing a mathematics education which is based on indigenous culture (e.g., Jannok Nutti, 2013). Sámi mathematics education has long been struggling to move from boarding schools, where children were isolated from their culture, to establishing a school situation where children are encouraged to build on their cultural heritage (ibid).

There are many factors for not getting the opportunity to learn mathematics of students' own culture. Jannok Nutti (2013) mentions that some of the factors are the dialect used in the textbook; the teachers' opinions about what works best for succeeding in the national examination, and that the textbooks are mere translations of the national textbooks with hardly any values of the communities. Moreover, time constraint and constraints in teachers' knowledge of teaching mathematics, which is based on the indigenous culture, can contribute to limiting the opportunity.

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There have been efforts to produce teaching materials through a collaboration of teachers and researchers (e.g., Fyhn et al., 2017). Fyhn et al. (2017) reported results of investigation of Sámi braiding where they explored ways of using it in teaching discrete mathematics at Sámi middle schools. They participated in exploring cultural artifacts and designed tasks based on their exploration (Fyhn et al., 2017). The Fyhn group stresses that the starting point is not the standard mathematics but, the Sámi culture. The issue of tension between Western traditions as reflected in the standard mathematics and Sámi traditions is also revealed. Fyhn et al. (2017) interpreted the Sámi descriptions used in braiding corresponding to words down/up, below/above and right/left as 3-dimensional space. Their results show that investigations of Sámi braiding can be utilized in connection with students' learning of variables based on numbers. It can also be used in the teaching of combinatorics.

Lipka and Adams (2004) reported their study about mathematics curriculum, which was based on the culture and implemented in Indigenous people of Alaska, USA. They examined the mathematics of Yup'ik Eskimo in Alaska. The authors mention the uniqueness of this Indigenous community's counting and quantifying ways and systems. Their study indicate that it is vital to involve the local community particularly elders in the development of mathematics curriculum for Indigenous people (e.g., Lipka & Adams, 2004). Lipka and Adams (2004) suggest starting with students own experiences in Indigenous community and challenging students to come up with their own conjectures and their own evidence for the conjectures they have made.

There are studies, exploring and exposing mathematics used in Africa, which are important for providing culturally connected mathematics education (e.g., Gerdes, 2014). According to Gerdes, diverse mathematics concepts and themes can be taught in African schools using African cultural activities and artifacts as a starting point. Gerdes provides an example where the use of geometry by Mozambican artisans in weaving of traditional basket can be utilized in teaching geometry.

On the other hand, there are studies examining the problem in learning mathematics in a foreign language (e.g., Setati, 2005). In her research project about learning mathematics in primary schools in South Africa, Setati (2005) examines the complex relationship language and mathematics learning. She stresses that mastery of mathematics' language is part of its learning. According to Setati (2005), language as a tool is both cultural and political. It not only is used to communicate thoughts but also creates aspects of who we are (ibid).

In a report about students learning by participating in practical activities, Miettinen (1999) stresses that better learning can occur when students participate in the actual out-of-school activities than through obtaining information about those activities in the classroom. He makes distinction between interest in a topic and motivation for the sake of succeeding. Contrary to the practice in the schooling activity, in enhancing better learning, interest in the topic is more important than the motivation to succeed (ibid). Consistent with Miettinen, the current study draws on cultural historical activity theory, and attempts to expose the activities which the Ethiopian textbook focuses on as well as to expose the potential influence of the textbook on the school mathematics.

Some studies of mathematics education in Ethiopia over the past decades at least mentioned issues of connection of mathematics with Ethiopian situation (e.g., Dutton, 1968). Traditional games, and other activities in Ethiopian culture involving mathematics are also documented in earlier studies (ibid), which have survived to this day. In another study, upper secondary students named several activities in Ethiopian culture, which involve mathematics (Gebremichael et al., 2011). In the current study, I attempt to answer the research question: How does textbook attempt to connect mathematics education with the local culture and utilize the local language.

This paper is structured as follows. This introduction section is followed by the remaining sections, which are presented in this order: activity theory, methods, and the data presentation and analysis. Then, I set out the discussion and conclusion.

Activity theory

I adopt cultural historical activity theory (CHAT) as a theoretical perspective to analyze the data. I examine the textbook in search of suggestion of activities, which are culturally and historically situated in Ethiopia. I also attempt to expose the other activities in Ethiopia.

Following Leont'ev (1979), I understand activity as being vital to human life. Activity has various components primarily the acting subject interacts with the object of the activity (ibid). This interaction is mediated by mediating instruments. The remaining components are division of labor, rules, and community (ibid). Human beings participate in activities and activities are historically situated (ibid). The Ethiopian traditional activities are historically situated in the culture even before the mathematics textbook and the schooling activity, as we know it now, were introduced in the country (Wagaw, 1979). In the current study, schooling is considered as an activity and learning mathematics is a goal directed action mediated by instruments as part of schooling.

When I examine the textbook, I search for suggestions of activities. The textbook is an instrument for the learning of mathematics. It mediates the interaction between the student and the goal of mathematics learning and the motive of schooling as well as between the student and the teacher. The teacher's guide suggests that teachers should use the textbook for preparing the lesson and in teaching. The teacher gives exercises to the student from the textbook. The exercises in the textbook help the student in acquiring mathematics knowledge and prepares the student to succeed in the examinations. The textbook also describes the division of labor by providing what the student should do as part of the learning process. It also describes the rules/norms of the mathematics classroom by letting the student do the exercises as homework before coming to class and as classwork while obtaining help from the teacher. The textbook also provides the student with suggestions of solutions for the exercises. The textbook may also determine the community by providing a suggestion of working in groups.

The school has rules, division of labor and tools. Teachers and administration are parts of the school communities. In the mathematics classroom, the object of the activity includes the goals and motives of the school. The goals of engaging the child in the process is

transferring mathematical knowledge. The school has a motive of producing students who can succeed in moving over to the next level by succeeding in the examinations. The outcomes of the activity are children with mathematics knowledge, skill and competence, possibly enabling the child to succeed in examinations.

Methods

I employ document analysis because it fits my purpose of examining and exposing the cultural issues as described in the textbook and the teacher's guide. This is consistent with the purpose method as set out by Bowen (2009). According to Bowen one purpose of document analysis is extracting information from a relevant resource. The data in this study is the texts in the textbook and the teacher's guide as well as in the policy document. The purpose in the current study is to expose how the textbook connects mathematics education with the cultural heritages of the Ethiopian society. The research method which fits to this investigation is document analysis method.

In examining the textbook, I first, explore the structure of the textbook as well as the chapters. Then, I examine the textbook in search of texts, which mention the available activities in Ethiopian culture and society including the schooling activity, and I code them. In the third step I summarize the available activities into groups. In the fourth step, I examine those activities to expose the components of the activities such as the instruments, communities, rules, and the division of labor, which I set out in the theoretical perspective section. I examine the level of detail about the exposition of the activities and the approach suggested with respect to their favorability to learning. I follow the same steps in examining the teacher's guide. Then, I merge my results from the two sources in the presentation of results. I also re-examine them, and cross-check when required. Some text in the policy document is also examined and extracted.

The data presentation and analysis

In this section, I set out the data and the analysis. the data in this study is the texts in the textbook and teacher's guide. I start with presenting the topics in the textbook 6-grade textbook. The section has four subsections, which are presented in this order: policy issues; the textbook and language issues; making connection with out-of-school activities and a further look at activities.

Policy issues

The ministry of education is the responsible organ for implementing the education policy (MoE, 1994). The policy document, MoE (1994) hardly has direction about the local culture. In its introduction, it mentions *culture* in connection with building culture of problem solving. The policy document has general and specific objectives. The general objectives, among others, focus on problem solving skills and creativity. It also states culture in connection with democracy, namely, about building culture of democracy. Ethiopia has diverse cultures and languages. The policy document states that one of the specific objectives

is ensuring that people will use their languages in education. On the other hand, there are no such statements about their culture.

As part of its strategy, the policy document has general statements about curriculum. It states that it intends ensuring development of curriculum and preparation of textbook, which maintain international standards while paying proper consideration to regional (local) circumstances and gender-related matters. However, it is not clear what these regional circumstances are, and apart from the attention given to the regional languages, the cultural issues are not specifically mentioned in the policy document. The education policy document mentions that one of its objectives is providing education which is secular. The Ethiopian people are largely religious. The country is home for Orthodox Christians and Muslims, which are the two largest religious groups, respectively. The apparent consequence for the textbook will be considered later in this section.

The textbook and language issues

The curriculum is designed centrally by the ministry of education. The ministry delegates the regional education bureaus to prepare textbook. The grade 6 textbook, which I examine, is written in local vernacular, Amharic (Ministry of Education of the Federal Democratic Republic of Ethiopia, 2011a). The grade 6 teacher's guide is also written in Amharic (Ministry of Education of the Federal Democratic Republic of Ethiopia, 2011b). Amharic is the official language of the country and has its own alphabets, which it shares with a very old Ethiopian language, Ge'ez. The textbook uses these alphabets as mathematical symbols such as variables and in naming sets.

At 6th grade, there is only one textbook. In general, the chapters of the textbook are structured in the same way, with little variation. The chapters start with stating the learning goals and contents of the chapter. This is followed by introductory words, which attempt to relate the topic with experiences, which the students are familiar with, or with their prior knowledge. Then, it provides the contents before, presenting proofs for some of the mathematical statements. The chapters end by providing a summary and key words followed by review exercises. The 6-grade textbook has six chapters. The following are the topics of the first four chapters, in this order: sets, composite numbers, fractions and whole numbers. Chapter 5 is a combination of topics. Its title is equations, inequalities, and proportionality. This chapter includes coordinate system and graphs. The final chapter is about geometry.

As mentioned earlier in this section, the textbook is written in the local language, Amharic. The mathematics terms are usually in Amharic. Some of the terms may be difficult for the student. The 6th grade textbook also provides description or definition of many terms. For example, in one of the three sections of Chapter 5, which is about coordinate system, the section starts with activity, and it provides description of terms later. The following text is an English translation of an excerpt from the first paragraph of the section:

> Under this heading you learn about coordinate system. You also learn about how you can describe points as ordered pairs on the coordinate plane (Ministry of Education of the Federal

> Democratic Republic of Ethiopia, 2011a, p. 123, author's translation, see Supplement 1 for original version in Amharic).

Following this excerpt, the textbook provides activity, where it provides a set of instructions to students, which includes asking to draw a coordinate system. Finally, it provides descriptions of most of the terms based on what the students would have done in the activity. However, there are some limitations in this regard. For example, the first instruction states, "draw two straight lines on your notebook." (Ministry of Education of the Federal Democratic Republic of Ethiopia, 2011a, p. 123, author's translation, see Supplement 2 for original version in Amharic). The author had to struggle to find out what the Amharic phrase which corresponds to "straight line" means. This key term may not be familiar to the students. Absence of description of the term might affect their learning of this topic.

Making connections with out-of-school activities

The textbook gives examples and exercises, which make connections between mathematics and the out-of-school activities. For example, in the sixth-grade textbook, Chapter 5 provides an exercise in the topic *proportionality*, which asks students a series of questions such as comparing the amount and prices if the student buys something. Then, it provides answers to some of these questions. In the first chapter of the sixth-grade textbook, which is about sets, mentions of wildlife, a set of traditional dishes in a restaurant, etc. the traditional dishes are parts of the Ethiopian culture. Apart from such mentions, the textbook hardly provides examples or tasks of mathematics' connections with the local culture. There are hardly any mention of traditional activities, institutions, communities, or instruments or artifacts in the Ethiopian culture.

The teacher's guide provides detailed guidance about what the teacher should do in teaching every topic. It also encourages making connection with the out-of-school activities. For example, in the first chapter, which is about sets, it states:

> Before providing definition of a set, explain to them those relating to the various examples given in the introduction of the chapter. Furthermore, these will help them to relate the sets they experience in their everyday life with those, which are available in mathematics. You can start your explanation with those which are locally available. You can start with the following: family […] a class of students, a football team. […] The example you give, and the concrete things should be clearly understandable to every student. Encourage them so that they can participate. Encourage them to give their own examples of sets which they find in their everyday life and in mathematics (Ministry of Education of the Federal Democratic Republic of Ethiopia, 2011b, p. 3, author's translation, see Supplement 3 for original version in Amharic).

However, the list does not include peculiar activities, artifacts, communities in the cultures of Ethiopian society. It suggests such examples for the teaching of empty set, "a set of cats with 8 legs" (Ministry of Education of the Federal Democratic Republic of Ethiopia, 2011b, p. 6, author's translation, see Supplement 4 for original version in Amharic). The teacher's guide provides further guidance. It states:

> From the textbook let students discuss about […] then, ask students to work on […] from exercise 1.2 let students discuss about differences. (Ministry of Education of the Federal

Democratic Republic of Ethiopia, 2011b, p. 8, author's translation, see Supplement 5 for original version in Amharic)

Then it provides answers to the exercises: The answers to the exercises include: "group of students, group of hens, people who play football" (Ministry of Education of the Federal Democratic Republic of Ethiopia, 2011b, p. 8, author's translation, see Supplement 5 for original version in Amharic). Moreover, the teacher's guide provides guidance about several issues such as students' motivation, building on students' answers, students' active participation and providing feedback. However, there is no such guidance about making cultural connections.

Like the chapter on sets, the geometry and measurement chapter of the teacher's guide hardly provides guidance about how the teaching would relate to the Ethiopian culture. The chapter provides a list of suggestions for teaching aids. The list does not contain any of the traditional instruments or artifacts in the Ethiopian culture. In its section about measurement there is no mention of the traditional ways of measuring.

A further look at activities

Here I attempt to investigate some of the activities further. In the shopping task, the activity of shopping is external to the schooling activity. In the schooling activity, a story about another activity is presented to engage students in mathematics task. The students do not have access to the shopping activity while engaging in the task. Nevertheless, it is likely that students already have the experience in participating in the activity of shopping. A reiteration of this external activity is used as a mediating artefact in the schooling activity. The textbook does not suggest providing an opportunity of some form of participation in the activity (cf. Miettinen, 1999). According to Miettinen (1999), actual participation in the activity than being presented about the activity results in better learning.

In a similar way, the textbook presents about the activity of restaurant. This activity involves the production and service of Ethiopian dishes, a cultural product. There are division of labor, rules and instruments of the production. However, they are not the focus of the task. The focus is only on the list of dishes. As mentioned earlier in this section, there are few out-of-school activities mentioned in the textbook and the teacher's guide, some of which have cultural connection of mathematics. However, these activities are not fully exploited in a way that gives the students the opportunity to participate and have a role in the activities. They are just presented with the story of the activities. In the schooling activity, these external activities are not presented in an engaging way. The teacher's guide hardly gives guidance on the issue of cultural connection. Suggestions of possible cultural communities to which the student belongs are scarce. It states that the teacher is encouraged to use out-of-school activities. However, it rarely mentions the out-of-school activities, which are embedded in the Ethiopian culture.

The textbook and the teacher's guide expose about the schooling activity in many ways. They determine the school rule and the classroom norms since they inform what is the right way to behave in the mathematics classroom and what rules to follow in the teaching and

learning of mathematics. They determine the division of labor by informing what the mathematics teacher and learner should do. The student uses the textbook in learning mathematics and the teacher gives homework from the textbook. Thus, the textbook is an instrument, which mediates learning mathematics. The textbook and the teacher's guide inform the teacher's decision of forming working groups in the classroom (school communities) or conceptions of school communities. It can also influence the teacher's perceptions of out-of-school activities. Another instrument, which mediates learning mathematics, is language. As mentioned earlier, the textbook presents mathematics in the students own languages. On the other hand, clarity of the use of terms in the language can promote or hinder their learning of mathematics.

Discussion

The results expose the limitations of the mathematics textbook and the teacher's guide in fostering learning mathematics with a cultural connection. The available literature suggests the possibilities as well as the advantages of making cultural connections (Jannok Nutti, 2013, Fyhn et al., 2017, Lipka & Adams, 2004). Besides earlier studies undertaken in Ethiopia also expose the potential cultural resources, which could avail themselves for the teaching of mathematics (Dutton, 1968; Gebremichael et al., 2011).

It was set out in introduction that studies undertaken about indigenous cultures show that the cultural activities and artefacts provide learning opportunities for the child. For example, in the production of braiding, which is a cultural and historically situated activity in the Sámi culture, children and elders take different roles. There are rules of the process as well as for the interaction, children have to listen and follow what parents/elders tell and do. The object of the activity includes the goals and motives. The goals for engaging the child in the process is transferring cultural knowledge and the motive for producing such artifacts is preserving the Sámi culture. The outcomes of the activity are the braiding and the child with a skill of making a braiding. As set out earlier in the introduction, earlier studies show that the Ethiopian context is also favorable for making cultural connection. It provides similar opportunities of diverse activities, which are parts of the various cultures in Ethiopia. These activities have own communities and involve rules, division of labor, instruments, and artefacts.

The textbook and teacher's guide do not suggest providing an opportunity of some form of participation in the activities (cf. Miettinen, 1999). Contrary to the suggestion by Miettinen the teaching materials stress motivation for the sake of mastery and succeeding in examinations instead of interest in engaging in culturally connected mathematics. I stated in the data presentation and analysis that the policy document objectively claims that it will provide education which is secular. It appears that the teaching materials almost ignored local contexts partly in the name of secularism.

Earlier studies in Ethiopia revealed that students of upper secondary were able to name several traditional activities which are inherent in the Ethiopian society where they tell about the utilization of elementary mathematics (Gebremichael et al., 2011). Given that the

teacher's guide provides detailed guidance about what the teacher should do in every topic, the absence of guidance on the issue of cultural connection is likely to hamper the teacher's actions in this respect. Another issue is language. Though students learn mathematics, using textbook, which is in their own language, description of some of the mathematics terms are not provided, which can hinder their learning (cf. Setati, 2005). For Setati part of learning mathematics is mastery of its language.

Concluding remarks

The educational policy document appears to influence textbook and the teacher's guide as the guiding documents in isolating the mathematics classroom from the local culture. Children need not be disconnected from their cultural heritages so that they can succeed at school. Lipka and Adams (2004) suggest otherwise. According to Lipka and Adams, mathematics education of children of indigenous communities need to be connected to their cultural heritages so that the children can succeed at school. As the studies show it, the issue of cultural connection in mathematics should not be seen as a dichotomy between developing children's knowledge of mathematics based on the local culture or developing knowledge of the standard mathematics. As Jannok Nutti (2013) puts it, it should rather be seen as uniting the two, with a due regard for the importance of both.

There should be more focus on cultural connection in mathematics, which is based on the diverse culture in Ethiopia. It is important that teaching resources with cultural connections are available for the teachers including guidance about how they can use these materials. The textbook is in students' own language. This is important for their learning. However, there are limitations in the textbook in that some terms are not described, which may affect their learning of topics.

The data analysis shows that the textbook and the teacher's guide would tell us most of what would happen in the mathematics classroom. However, it remains to undertake a study into the implementation of the mathematics curriculum using the textbook.

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Supplements

Supplement 1

5.2 ውቅሮች

በዚህ ርዕስ ስር ስለውቅሮች ትማራለህ/ሽ:: ከዚህም ሌላ ነጥቦችን እንዴት የውቅር ወለሎች ላይ በስርጥ ጥንድ ቅደም ተከተል መግለፅ እንደምትችል/ዪ ትማራለህ/ሽ::

Supplement 2

ሀ. በደብተርህ/ሽ ላይ ሁለት ቀጠ-ዘዌ መስመር የሆኑትን ስራ/ሪ::

Supplement 3

የስብስብን ትርጉም ከመስጠትህ/ሽ በፊት የተለያዩ ምሳሌዎችን በተማሪው መጽሐፍ መግቢያ ላይ ከተሰጡት ጋር የሚመሳሰሉትን ግለጽላቸው:: ይህም ተማሪዎቹ የስብስብ ፅነሰ ሀሳብ እንዲረዱ እና እንዲማሩ ይረዳቸዋል:: ከዚህም ሌላ ይህ በዕለታዊ ኑሮአቸው ውስጥ የሚገኙ የነገሮችን ስብስብ እና በሒሳብ ትምህርት ውስጥ የሚገኝ ስብስብ አንድ ላይ እንዲያጣምሩ ይረዳቸዋል:: ማብራሪያህን በአከባቢ የሚገኙትን ነገሮች ስብስብ የጋራ ስም ካላቸው ከታች ከተሰጡት መጀመር ትችላለህ/ሽ::

- «ቤተሰብ» የሰዎች ስብስብ «ልጆች እና ቤተሰቦቻቸው»::
- «ክፍል» በክፍልህ ውስጥ ያሉ የተማሪዎች ስብስብ::
- «ቡድን» የእግር ኳስ ተጫዋቾች ስብስብ እና የመሳሰሉት::

ለዚህ ጉዳይ ትግበራ 1.1 ምሳሌ 1 እና በተማሪው መማሪያ ውስጥ የተሰጡትን የቡድን ሥራ 1.1 ጥያቄዎችን መጠቀም ትችላለህ/ሺ:: የምትጠቀምባቸው ምሳሌዎች እና ተጨባጭ ነገሮች ለሁሉም ተማሪዎች ግልፅ(ሁሉም ተማሪዎች የሚረዱት) መሆን አለበት::ተማሪዎቹ በሚደረገው ውይይት ውስጥ እንዲሳተፉ አበረታታቸው:: ተማሪዎቹ የስብስብን ምንነት በዕለት ኑሮዋቸው ውስጥ የሚገኝ እና በሒሳብ ትምህርት ውስጥ የራሳቸውን ምሳሌ እንዲሰጡ አበረታታቸው::

Supplement 4

8 እግሮች ያሉዋቸው ድመቶች ስብስብ::

Supplement 5

ከተማሪው መጽሐፍ ምሳሌ 4ሀ እና መ ተመሪዎቹ እንዲወያዩበት አድርግ/ጊ:: በመቀጠል ተማሪዎቹ የአላቅ ስብስብ ምሳሌ ያልሆነውን እንዲሰሩ ጠይቅ/ቂ:: ከመልመጃ 1.2 ውስጥ በ5ኛ ጥያቄ ላይ ተማሪዎች እንዲወያዩበት በማድረግ አላቅ ስብስብ እና አልቆ-ቢሰ ስብስብ እንዲለዩ እርዳቸው/ጂ.ያቸው::

ግምገማ

ከመልመጃ 1.2 ውስጥ የቀሩትን ጥያቄዎች ለተማሪዎቹ የቤት ሥራ በመስጠት ስራቸውን ገምግሙ::

የቡድን ሥራ 1.2 መልስ

ሀ እና ሐ ባዶ ስብስብ ናቸው::

የትግበራ 1.1 መልስ

1. የተማሪዎች ቡድን፣ የእግር ኳስ ተጨዋቾች ቡድን፣ የዶሮዎች ስብስብ፣ ሁለት ሰልፍ የተማሪዎች ስብስብ::
2. እግር ኳስ የሚጫወቱ ሰዎች::
3. የ 9 ዶሮዎች ስብስብ::

Adults watching numbers: Numerical information about COVID-19 presented in Greek TV news

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Since March 2020 a state of emergency has been imposed on the people in Greece, under which restrictions in their movements and lockdowns in the economic, social and cultural activities are from time to time imposed or removed, limited or extended by governmental decisions. These decisions are justified almost exclusively by numerical data: numbers of infections, people tested daily, deaths, hospitalized patients and patients in intensive care units and at the same time numbers of daily individual violations of restrictions. This paper reports on an analysis of numerical information about COVID-19 communicated by the prime-time news bulletins of two national TV channels on a daily basis over the previous year and we are claiming that once again numbers were used to manipulate citizens' views and attitudes.

Governing by numbers

Nowadays numbers constitute a crucial element of every political decision-making process and are used to proclaim the "rationality" of choices and "objectivity" of options, while at the same time conceal the political expediencies and the power relations involved.

The daily TV news bulletins and the front pages of newspapers are typical examples of the multiple uses of numbers in public discourse aiming to present an existing "reality", on the basis of which governmental decisions or political actions seem to be "in fact inevitable". On the other hand, however, it may be claimed that numbers and more broadly numerical expressions do not simply describe an existing reality, but also, they co-create a reality which then is portrayed as existing reality. The numerical expressions of particular aspects of political, social or economic facts involve implied choices which overvalue some of their features and devalue some others. In portraying this reality, the media frequently employs reasoning which may not be considered as neutral. In this way, the numerical expressions used create a "topos", a mathematical term, which restricts our views of the respective aspects of political, social or economic life and determines our thinking and actions concerning issues related to these aspects. As was succinctly put by Rose (1991), "Numbers here delineate 'fictive spaces' for the operation of government, and establish a "plane of reality", marked out by a grid of norms, on which government can operate" (p. 676).

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How do numerical expressions delineate "fictive spaces" in political activities? The answer, of course, is not, simple and exceeds the limits of this paper. However, we may outline in a few words some points of an answer inspired by Rose (1991).

First of all, the use of numbers is a key element in any process of transforming an economic, social or political issue into a problem that requires a solution by quantifying and expressing it numerically. In other words, the numerical description of an issue allows its formulation as being an economic, social or political problem and correspondingly the tackling of such a problem presupposes its statement in terms of numerical expressions between its selected characteristics.

Secondly, numerical expressions restrict any evaluation of political actions and particularly of governmental decisions. Hence, the success or failure of such decisions are dependentable and expressed in quantitative changes and numerical correlations, which specify the achievement of goals, the fulfillment of declarations or the solution of problems, e.g., tackling unemployment.

Finally, numbers legitimize the exercise of power by certifying its representation to the people. For instance, the numerical expressions of public opinion polls express the correlations between governmental actions and orders of the people whose name power is exercised in.

The effectiveness of the above functions of numbers in politics is mostly grounded on the hegemony of numerical discourse in modern societies which provides apparent objectivity and legitimacy in numerical expressions as has been analyzed by Sfard (2009).

There is no doubt that the relationships between numerical expressions and political actions are much more complex than the ones outlined in a simplistic way above and they have been the subject of analyses from various viewpoints and approaches as we may see by the relevant literature (e.g., Bartl al. 2019; Chassapis & Giannakopoulou, 2015; Demortain, 2019; Fioramonti, 2014; Gilles 2016; Shore & Wright, 2015; Skovsmose 2010).

In this paper, the issue of using numbers by governments to manipulate citizens' views and attitudes is put as a preliminary note related to the politics concerning the pandemic, an aspect of which will be analyzed below.

The numbers of COVID-19 pandemic in Greek TV channels

The first cases of COVID-19 were confirmed in Greece at the end of February 2020, and since then a state of emergency has been imposed on the people. Under this state of emergency, restrictions on movements and lockdowns in the economic, social and cultural activities of the people were from time to time imposed or removed, limited or extended by government. All these restrictions were justified almost exclusively by numerical data. A daily briefing was introduced by government during which health officials and experts presented the current situation of the pandemic using, almost exclusively, numerical data. Such data consists of numbers of new cases of infection, of people tested and those found positive, as well as of deaths, numbers of people hospitalized, and of patients in intensive care units. In addition, various numerical indices also are included, such as the virus reproduction index

or virus risk degree. All these numbers are daily calculated and cumulative over various time periods. At the same time government announced numbers of daily violations of individual movements and activity restrictions. These briefings lasted about an hour and were being broadcasted live on all national television networks. The numerical information announced by officials and experts, was enriched with visual representations (graphics, diagrams, tables etc.) and supplemented with journalistic comments reproduced by TV channels in their main evening newscasts, where they usually have their maximum audience.

After a short lifting of restrictions during the summer all restrictions on people and activities were re-applied in October 2020. However, in this second wave of the pandemic, the role of informing the public was entirely assigned by the government to the TV channels. Since then, the prime-time TV news has started with a bombardment of numerical information about COVID-19 followed by a variety of interpretations provided by journalists, health experts and politicians, most of them supporting relevant governmental decisions which were sometimes hardly justifiable. Under the conditions created in the country by the restrictions on individual movements and the suspension of all labour, economic and social activities, the TV broadcasts became, for a majority of people, the main source of information about the pandemic. In this way, TV channels turned into a powerful instrument influencing the perceptions and shaping the social reality of the viewers by selecting, framing and transmitting numerical information about COVID-19, thus creating a meaning about the pandemic.

The governmental management of the pandemic, as evidenced by its decisions and proclamations, was based on two political strategies. The first one was the production and dissemination of a storm of numerical information which day by day shaped the image of citizens about pandemic and its dangers emphasizing above all their individual responsibilities in preventing virus spread. The second strategy was the mobilization and assignment of a primary role on TV news bulletins and broadcasts in tackling pandemic. On this ground, the government funded TV networks with special grants on the pretext of informing citizens. It is almost certain that no other event in recent Greek history has been covered to such an extent and duration by the media with such a flood of numerical information.

This paper reports on findings of an analysis of numerical information about COVID-19 which was communicated by the prime-time news of two national TV channels on a daily basis over the previous year.

The guiding theoretical perspective of our analysis stems from framing theory (Goffman, 1974). The basis of framing theory is that the media attracts attention on certain events and then places them within a field of meaning. In essence, framing theory suggests that the way in which something is presented to the audience (called “the frame”) influences the choices people make. Framing can support the investigation of media influence on the audience, by seeking to outline different patterns of presentation of the news broadcasted by them. According to this sociological approach, frames stand as the mechanism for perceiving complexity of the surrounding world in a simplified way and numerical depiction seems to

be a very effective instrument for highlighting significant information and putting aside unnecessary details. As put by Entman (1993, p. 53),

> to frame is to select some aspects of a perceived reality and make them more salient in a communicating text, in such a way as to promote a particular problem definition, causal interpretation, moral evaluation, and/or treatment recommendation for the item described.

To sum up, the structures of TV news bulletins (media frames) offer to their audience aspects of perceived reality so as specific interpretations, attributions or evaluations are suggested.

The numerical fluency of Greek adult TV audience

The numerical storming of TV channels was being addressed to an adult audience ignoring the only available nationwide research, the "Survey of Adult Skills". It was, carried out during 2014–2015 by the PIAAC-Programme for the International Assessment of Adult Competencies (OECD, 2016) which identified the low level of numeracy in the populace. In this survey numeracy is defined as the ability of adults to use numerical and mathematical concepts. The PIAAC survey found that the share of adults in Greece who score at the highest levels of proficiency in numeracy is considerably smaller than the OECD average (252 in Greece while 263 is the OECD average on a scale of 500 points). More specifically, about 6% of adults in Greece attain the higher levels in numeracy (OECD average 11%) and about 25% of adults attain the middle level (OECD average 32%). At the middle level, adults have a good sense of number and space, can recognize and work with mathematical relationships, patterns and proportions expressed in verbal or numerical form, and can interpret and perform basic analyses of data and statistics in texts, tables and graphs. On the other hand, the proportion of adults with poor numeracy skills is much larger in Greece than OECD average. Almost 28.5% of Greek adults score at the low level of numeracy (OECD average is about 23%). Some 27% of 16–24-years-olds are low performers in numeracy (OECD average 19%). Low proficiency is also prevalent among 55–65-year-olds in Greece: about one in three adults at this age group score at the low level in numeracy. Adults at the low numeracy level can perform basic mathematical processes in common, concrete contexts, for example, one-step or simple processes involving counting, sorting, basic arithmetic operations and understanding simple percentages. In relation to the educational level of adults it is interesting to note the finding that tertiary-educated adults in Greece have relatively low proficiency in numeracy (OECD 2016). Although we have to be skeptical when reading international survey reports (Evans, 2014), this is a picture of the adult numeracy situation in Greece as outlined by PIAAC findings and addressing such an audience TV channels broadcast every day a storm of numerical information.

Research methodology

This paper reports on an analysis of numerical information about COVID-19 communicated by selected news bulletins of two national TV channels. The two channels were selected on

the basis of their character (a public ERT1 and a private Skai) and their nationwide spectatorship. From all their everyday evening news bulletins were broadcasted from February 17, 2020 (when the first cases of COVID-19 confirmed) until October 4, 2020 (when a strict lockdown was imposed once again in the country), a convenience sample was selected. The main selection criterion was to have been broadcasted on dates when restrictions were imposed or removed, limited or extended by government decisions. So, it was analyzed 112 evening news broadcasted by the two TV channels in which the pandemic was their first issue covered mostly by numerical information and its interpretation lasting from 15 to 35 minutes. The TV news were analyzed by applying a variation of the content analysis (Neuendorf, 2017). As it is known, content analysis is a research tool used to determine the presence of certain concepts, within any occurrence of communicative language. Under the scope of our research, it is numbers and numerical expressions that are the organizing concept of our content analysis.

The main findings of this analysis are briefly presented in the following sections.

The choice of numbers

The TV news programs organize their reports around numbers. Although the information provided may sound real to the audience, its meaning is not clear for many reasons. According to our evidence, the news broadcasts are focused mostly on raw numbers or percentages without reference to unit populations. Graphics as in the following two examples were typical at the start of the news. Figure 1 shows the original TV screen while the next one is translated by the author. The presenters comment on the new confirmed cases (1678 cases) and the numbers of cases in one city and three regions of the country in which large numbers of infected people were detected. The caption "informs" the audience that "positivity is over 10%" without any explanation of the meaning of "positivity".

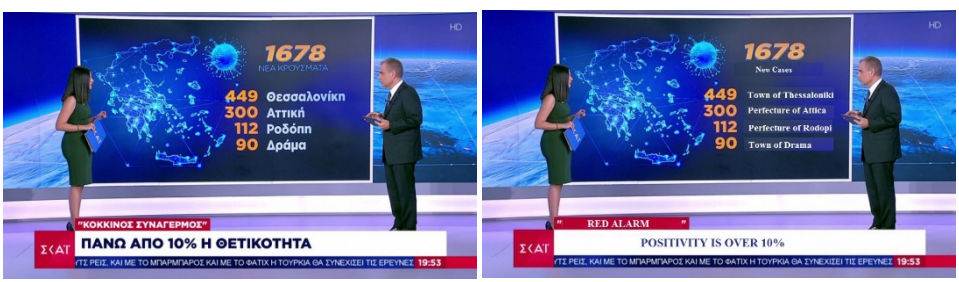

Figure 1: Raw numbers introducing pandemic situation one day on Skai TV (Skai TV, News bulletin on November 1, 2020, 19:50 o'clock local time).

Figure 2 presents a sequence of raw numbers reporting daily infected cases over the first month of the pandemic as well as their total (892 cases).

None of the numbers shown in these particular screenshots, and nor on all the TV news are standardized. Thus, without any reference to the population sizes, it is completely unclear

to what extent the numbers of cases depend on the relative sizes of the populations referred to. Nonetheless, the use of numbers conveys the information and comments on a sense of objectivity. As Porter (1995) puts it, the language of mathematics is well suited to embody objective judgments and it is adopted when claims to knowledge are needed to gain trust and credibility beyond the bounds of locality and society.

Figure 2: A sequence of raw numbers reporting the evolution of pandemic during the first month of pandemic on ERT1 TV (ERT1 TV, News bulletin on March 26, 2020, 21:00 o'clock local time).

Although seemingly extreme, it may be claimed that raw numbers displayed on TV screens highlight the individuality of cases by removing from the counts their frames of reference which are their social context, thus isolating the cases of infected, hospitalized or deaths and delete any sense of belonging. Everyone is alone in the pandemic and this is one of the "lessons" promoted by TV channels. Therefore, individual responsibility and not social policies is the weapon against the pandemic.

Beyond integers the second prevalent numerical expression in communications about COVID-19 are percentages denoting either indices, for instance the virus reproduction index, or parts of a whole which was usually not explicit. Whilst percentages are a seemingly simple numerical concept, it is known that they confuse and disorientate people. Here its use implies more than it displays, since it does not denote a quantity but a relation between two quantities; actually, it is a fraction whose denominator is 100 and thus the expression of increase or decrease of a quantity is not always easily conceived. Expressing a relation between two quantities, the value of a percentage is dependent on the change of the one or the other quantity or both. At the same time, when it is displayed and commented on, even when compared the one percentage or the another as a single numeral, these quantities are not referred to, at all. Since percentages are numerical expressions representing a ratio, not just a quantity, being displayed as single numbers, they become suitable for numerical alchemies and appropriate for manipulations projecting a false or a distorted picture of a reality.

In Figure 3 percentages and decimals are employed to display the hazards of the pandemic on a particular day using terms which may be unknown to the majority of the audience: positivity index 8.3%, alarm limit 4%, Rt (reproduction rate) 0,68 in Athens, 0,85 in Thessaloniki.

Finally, the third type of numerical expression which was mostly included in TV describes various aspects of the pandemic based decimal numbers. Decimals are rational numbers but

their writing refers, at first sight, to counting numbers and since each decimal number displays the "part" obscuring the "whole" to which it refers (its denominator), its comprehension often difficult.

Figure 3: Percentages and decimals describing hazards of pandemic on Skai TV (Skai TV, News bulletin on November 1, 2020, 19:50 o'clock local time).

As portrayed by our analysis, the decimal numbers broadcasted on TV news are rather perplexing than informative. As shown in Figure 3, the decimal numbers express rather incomprehensible concepts to the majority of audience, as is virus reproduction rate (Rt=0.68). In Figure 4, decimal numbers present discrete quantities as if they were continuous, as is the number of 0.625 people infected by one person. In the same screen the merits of individual self-restraint in dealing with the pandemic are exposed with the following implicit numerical argument: if self-restraint by a person is 75% (sic), then that person in 5 days will infect 0.625 persons and in 30 days 2.5 persons. A personal self-restraint by 75%, as explained by presenter, means that a person has to reduce his/her social contacts by that percentage. How could one do that? In any case, the question of how this percentage is calculated remains without an answer.

Figure 4: Decimals describing the merits of individual self-restraint on ERT1 TV (ERT1 TV, News bulletin on March 19, 2020, 21:00 o'clock local time).

Numbers embedded in charts and diagrams

In many broadcasts, numbers as numerical sequences of infections, deaths, hospitalizations or recovering were used to depict the development of the pandemic in comparison to restriction measures imposed by government. Figure 5 is a typical example.

Figure 5: Number of deaths in total since the onset of the pandemic on ERT1 TV (ERT1 TV, News bulletin on November 5, 2020, 12:00 o'clock local time).

Numerical sequences are almost impossible to be comprehended and interpreted by the majority of the audience, given the situation of adult numeracy in Greece outlined previously. Thus, numbers concerning development of numerically expressed aspects of the pandemic were, in my view, included in charts and diagrams of TV shows in order to assign validity to comments of the presenters or justifications to governmental decisions rather than inform the audience.

Framing the interpretations of numbers

The comments of TV presenters and the interpretations offered by health experts, were made with references to the displayed on-screen numbers. These numbers, however, being row or relational numbers, as percentages or decimals, acquired their specific meaning by their, not explicit, but implied referents. Mostly however, the numbers acquired particular conceptions of their values by the quantitative expressions used by the presenters and commentators. For example, "numbers show an alarming exponential growth of infections", "numbers reveal a significant, rapid and dangerous transmission rate", "numbers may not seem big but they have great importance" etc.

At the same time, a variety of visual displays added an elaborate visual dimension to the numerical expressions which, without a doubt, influenced the interpretation of the audience. It is commonly assumed that visually presented numerical data, known nowadays, as infographics, convey information which can quickly be consumed and easily understood by an adult audience and for this reason they were extensively used by TV news in broadcasts about the COVID-19. However, the visual displays that the TV news bulletins utilized highlighting the numbers, create a referential field which, as Potter et al (1991, p. 343) has put, function as "parallel commentaries" which reinforce numerical expressions.

Concluding comments

As outlined in our theoretical framework, numbers serve political projects and government decisions. Firstly, by defining problems and delimiting their solutions by quantifying and expressing it numerically. Secondly, as a consequence, restricting the evaluations of related political actions and particularly of governmental decisions, and thirdly, legitimizing the

power exercised of by political authorities. The data presented in previous sections showed that TV news about COVID-19 was structured, throughout the period reviewed, around numerical expressions, enriched with visual representations and all framed by journalistic comments and health experts' interpretations. The particular numerical expressions used in these communications created particular images of the pandemic and its risks thus indirectly specifying the proper measures to deal with it or in other words defining the problems created by pandemic and predicting their solutions. The use of some specific types of numbers rather than others by TV news, the utilisation of certain mathematical ideas and the discourse surrounding them were value-laden supporting political purposes.

Given the low levels of adult numeracy in the country, as shown by the mentioned PIAAC survey, we can reasonably assume that for sizeable shares of TV audience the numbers as presented and commented on TV, may generate confusion or misunderstandings about many aspects of pandemic rather than informing on the actual risks and the appropriate actions, both individual and collective.

On the ground of such confusions, the numerical discourse which frames the numbers presented on TV created to the audience feelings of fear and through an individuation of numbers and facts promoted a sense of individual blaming for the current picture of the pandemic and its evolution concealing at the same time any governmental responsibilities. From the first days of the pandemic onwards the governmental officials, on their TV appearances, made individuals jointly liable for the spread of pandemic and for the failure of measures to tackle it. After all, the numerical communications employed by TV restricted the evaluations of related political actions and legitimized the governmental decisions.

Public opinion polls of those days reflect many aspects of the above conclusions. People were confused about crucial aspects of pandemic, possessed by feelings of fear and insecurity and the management of pandemic by numbers seemed rather successful, since the majority of people questioned evaluated the government decisions as effective (e.g., Eurobarometer, 2020; diaNEOsis, 2020).

Beyond conclusions, our findings raise many questions whose answers, however, require further research. Questions related to the impact that the particular use of numbers about the pandemic had on people's attitudes, legitimating the choices made by political authorities and finally, on enabling the tackling of the pandemic.

So, what may be a strategic response from the viewpoint of critical mathematics education? I would suggest promoting adult critical numeracy and the "reading" of COVID-19 numbers from such a perspective and of course, the publicity of this "reading". It is accepted that critical numeracy incorporates the abundant ways by which number concepts and facts are used by various agents for promoting their agendas. In addition, the social practices involved in each of these uses, the effects on power relationships generated through processes of number uses and overall aims to an understanding their consequences for individuals and societies (Geiger et al., 2015).

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University mathematics students' use of resources: Strategies, purposes, and consequences

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This contribution provides a qualitative view of mathematics students' use of resources in their first year at a German university on the basis of problem-centred interviews. Results show that students use many different material resources, such as exercises, lecture notes, books, internet, etc., as well as the help of other persons to organise their learning of mathematics. The use of the respective resources differs in various phases of the study and serves different purposes. In addition, consequences of the use of resources on exam success and the experience of success when completing exercises are examined. These results have possible implications for the conception of mathematics courses, which are discussed.

Introduction & theory

The transition from school to university, especially in mathematics, is associated with various difficulties, which are due, for example, to differences in mathematical content, or in the way mathematics is communicated, thought and taught (e.g., de Guzmán et al., 1998; Thomas & Klymchuk, 2012). These differences are also reflected in the resources that accompany mathematics learning: "teachers 'change into' lecturers; lessons into lectures; homework into coursework; textbooks into course materials and lecture notes; tests into examinations; and school mathematics into university mathematics" (Gueudet & Pepin, 2017, p. 469).

The term "resources" is used in many ways. Adler (2000) distinguishes human resources (e.g., other students, lecturers, their knowledge base, etc.), material resources (e.g., chalkboard, textbooks, mathematical proofs), as well as social and cultural resources (language, time, etc.). In Schoenfeld's (2011) theory of goal-oriented decision making, resources are understood as people's knowledge in the context of available materials. Together with their goals and orientations (beliefs, values, dispositions, etc.) they are expected to explain persons' actions. Accordingly, an adequate use of different resources can be regarded as an important factor for successful learning.

In quantitative studies, the use of resources is often conceptualized and operationalized within self-regulated learning theories (e.g., Panadero, 2017, for an overview) as a special class of learning strategies (e.g., Credé & Phillips, 2011). Such studies provide empirical evidence that across study programs managing one's time and study environment as well as

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using literature is positively correlated with academic performance, whereas peer learning and help-seeking are not or negatively correlated with academic performance (Boerner et al., 2005; Credé & Phillips, 2011). In mathematics courses, these correlation patterns are rather not observed, with sometimes even negative effects of the use of literature (Griese, 2017).

This paper focuses on two classes of resources: mathematics learning materials, i.e., materials such as books, lecture notes, exercises, etc., and human resources, which are understood here as other people that students can draw on for peer learning or help-seeking.

In line with Schoenfeld's (2011) theory students' goals are considered a key factor for the selection and use of the respective resources, whereby students' goals are understood as "the conscious or unconscious aims they are trying to achieve" (Schoenfeld, 2011, p. iv). The different resources of school and university entail different goals as well as different expected and actually implemented strategies of students. Rezat (2013) found that secondary school students use textbooks mainly to practice similar tasks or worked examples. At university, lecturers expect students to use lecture notes to learn and understand mathematical concepts, and as a model for certain mathematical practices, e.g., for mathematical proof construction (Gueudet & Pepin, 2018). Students use resources provided by their institution most often, but they also use many other resources (Anastasakis et al., 2017). The use of resources changes during the course of study (Stadler et al., 2013), is strongly oriented towards exam-related goals (Anastasakis et al., 2017), and depends on the institutional framework (Gueudet & Pepin, 2018; Kock & Pepin, 2018). Therefore, the structure of typical mathematics courses for first-year mathematics students at German universities will be briefly described here.

Mathematics modules of the first semesters at German universities normally consist of lectures and related exercises. The lectures introduce mathematical theory, i.e., definitions, examples, theorems and their proofs are presented. The exercises are handed out weekly and worked on by students in self-study. Students' solutions are submitted, corrected and discussed in a separate lesson. In order to pass such a module, usually a certain number of exercises (often 50% of all exercises) has to be solved correctly and a written exam has to be passed. According to the study regulations of German universities, the self-study time is about two thirds of the scheduled learning time for the respective lectures of the first academic year.

This paper examines the use of materials and learning with others by mathematics students in this institutional context. The research questions are as follows:

- RQ1 Which resources do students use when learning mathematics at university?
- RQ2 For which purposes are the respective resources used? Which goals regulate the use of resources?
- RQ3 How are the different resources used?
- RQ4 How is the use of the different resources related to the learning success of students?

Method

In order to investigate these questions, problem-centred interviews (Witzel, 2000) were conducted and analysed in two consecutive academic years with a total of 18 students (13 of whom were female) at up to four interview times in their first year of study (see Göller, 2020, for a detailed description of the study design). To recruit participants for the study, the research project was briefly presented in a mathematics lecture attended by students from various (mathematics-related) study programs. The 18 interviewees volunteered to participate in the study. Participation in the study was not compensated. Gender and age of the participants were not specified by the interviewees. The gender reported here was assigned to the interviewees by the interviewer and is intended only to provide a rough description of the sample.

Ten interviewees (9 female) were enrolled in a degree program for mathematics teachers at upper secondary level, six (3 female) in the degree program Mathematics B.Sc. and two (1 female) in the degree program Business Education. The respective interviews had a duration of about 45 minutes, were audio-recorded, completely transcribed and analysed using methods from Grounded Theory Methodology (Strauss & Corbin, 1990). The interview questions addressed key topics of self-regulated learning and Schoenfeld's theory of goal-oriented decision making, such as students' strategies, goals, beliefs, and evaluations with respect to the mathematics lectures in their first study year. The use of resources, such as mathematics learning materials and human resources, was coded as a subcategory of strategies (see Göller, 2020).

All interviewed students attended mathematics courses in the form described above in which an exam was written at the end of the semester and at least 50 % of the points of the weekly exercises had to be completed correctly during the semester in order to achieve admission to the exam.

Results

Category	Quote example
Learning with other students	
- Work on exercises	And there we always meet and try to solve the algebra exercises.
- Work on lecture contents	And then on the very last day [before the exam] we met in a group and discussed last questions.
Help-seeking	
- Other students	... sometimes I asked other students if I had not understood something.
- Lecturers	Well, sometimes we ask the lecturer herself.
- Relatives and friends	And if I get stuck, I ask my uncle.

Table 1: Categories of learning with others

All interviewees reported learning with other people. This learning with other people has different forms, which are listed in Table 1. Most of the learning with others is spent working on the exercises, with smooth transitions between working together on the exercises and seeking help from other students.

The categories of materials used by the students interviewed are listed in Table 2.

Category	Examples	Quote example
Exercises	on the attended lectures	I try to work through the exercises.
Lecture notes	of the attended lectures	Tuesday, I worked on the lecture notes.
Books	Paper books, e-books, textbooks, etc.	There are also workbooks with solutions and where you just have to look if there is something in them, some kind of trick.
Websites	Google, Wikipedia, etc.	If I didn't understand definitions or such things, I often read something on Wikipedia.
Social Networks	Facebook, WhatsApp, Phone, etc.	We have a WhatsApp group, so I photograph the exercises and put them in the group.
Videos	YouTube etc.	YouTube has also helped me a lot.
Solutions of others	Solutions of peers, worked examples, etc.	if people already worked on this [exercise] then I look at it. And then try to understand it for myself and write it down again.
Previous exams	Exams of previous years	I also worked on some old exams.

Table 2: The different materials used by the interviewees

The institutional framework of the study implies two central performance goals, which were expressed by all interviewees: Passing the exams and achieving 50 % of the points for the weekly exercises to achieve admission to the exam. Some interviewees also formulated other goals, e.g., learning goals, but overall, these two performance goals were so dominant that strategies to achieve them often superseded strategies to achieve learning goals. Moreover, these two performance goals are consecutive, in the sense that admission to the exams had to be achieved first before passing the exams was addressed. As a result, these two performance goals define two different phases in which the use of resources and associated strategies differ: During the semester students' primary goal was to find solutions for exercises in order to achieve admission to the written exams. If admission was achieved at the end of the semester, the focus shifted to the preparation for passing the exams. In the following, these two phases will be considered separately.

The use of resources during the semester

During the semester, the students' mathematical work was dominated by the exercises. All other materials were primarily used to support the work on the exercises. A follow-up of the lecture contents after the lectures beyond working on exercises was rarely reported, and in many cases explicitly denied. The most frequent strategies were working together with others on the exercises, searching for solutions or worked examples on the internet or in books, and working with solutions of others.

The use of the various resources also depends on how autonomously the interviewees were able to solve the respective exercises. Generally, the interviewees first tried to work on the exercises on their own and then, depending on their progress, drew on more and more other resources. This process is shown in Figure 1.

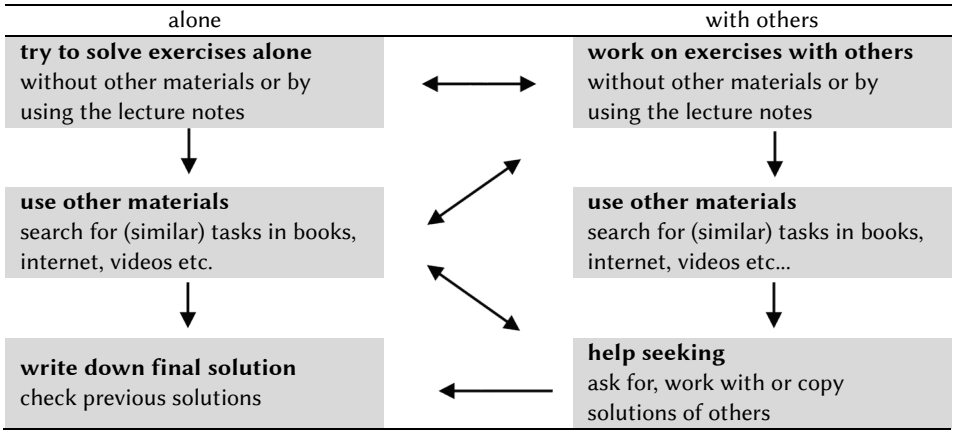

Figure 1: A typical sequence of students' use of resources when working on exercises

Figure 1 describes different possible and in the interviews reported steps to generate a final solution for exercises, which can be used to illustrate the approaches of different students. For example, the following interview excerpt illustrates the steps taken by a student (S1) in the fifth week of her first semester to solve exercises:

> S1: First of all, I read an exercise. At the beginning I usually think, oh my God, what is that? Then I check the lecture notes to see what we have done. Then it usually becomes a bit clearer what somebody wants from me on this exercise sheet. And then I try to write down my own thoughts about it. And if I can't get any further, I read in the book or on the internet and try to find a solution. And if I still can't get anywhere, then I put it aside and do the next one. Maybe I think about it again later. And if I still don't get ahead, then I just ask someone.

This interview excerpt shows that she first and foremost tries to solve the exercises alone, using the lecture notes (top left), then she uses other materials (middle left) and only as a last

step asks other students for help (bottom right). In the middle of the second semester, the same student reports an approach that focuses much more on working with others to solve exercises (right side):

> S1: Tuesday at noon we have two and a half hours off. All from my math crew. And then we always sit together and try to solve the algebra exercises. After the two and a half hours we usually realize that we have solved maybe one and a half problems and have no idea about the rest. Since the submission deadline for the exercises is on Thursday [...], the panic breaks out at the latest on Thursday morning, when we sit together for another two and a half hours. Because still no one has a clue about two tasks. And then we talk or write to other math people, where most of them have just as little idea as we do, and then at some point we just send the solutions back and forth, copy them and hand them in.

Overall, it can be observed that collaborative working on exercises with others increases during the first study year. However, regarding Figure 1, the interviewees differ in how much time and effort they invested in the respective steps, which steps they could possibly skip, because they were able to solve the exercises on their own with the lecture notes and then only compared solutions with others,

> S2: So, like I said, I'm reviewing the lecture notes first. First of all, I have to understand the exercise. I have to read it several times. And I find, especially when you look at the documents again, the lecture notes, that you always find something that you need for the exercise. [...] This is of course not the case with every problem, but that sometimes there were parts that have cross connections to the exercises, i.e., were quite similar to the exercises. I also bought literature to consult literature where it is written in more detail. And then I make my own sketches. My own approaches. Write them down. And then, as I said, we meet with the peer group and then just complement the things.

or which step they started with. For example, there are interviewees who started almost immediately to look for solutions on the internet or in books,

> S3: In the beginning I really tried to solve the exercises with the lecture notes. With some exercises it still works. But in the meantime, I immediately enter keywords into Google to see if I find anything similar. Then I try to understand it. And apply it to the exercise. But it always takes too much time to first look at the lecture notes.

who almost exclusively worked on exercises with others,

> S4: The main part, well, almost everything I do, I do in the group.

or even started straight away to copy solutions of others:

> S5: We have a study group now, and somebody always has the solution. And most of the time I just copy them to get my admission, because there is simply no time for it.

However, the exact path of the problem-solving process always depends on the concrete exercises and in particular on how far the individual students got with which strategies. Experience of success in completing the exercises was reported especially in the context of follow-up the lecture notes.

The use of resources for exam preparation

To prepare for the exams, the same materials were used by the interviewees as for solving the exercises during the semester. However, a different focus can often be observed here: While the work with mathematical contents focused on the exercises during the semester, some interviewees (also) focused on the lecture notes during the exam preparation. In most cases, this focus on the lecture notes consisted of extracting definitions and theorems from the lecture notes and then trying to understand and remember them. The proofs of the theorems of the lecture notes were only rarely followed up by the interviewees, and by many not at all. In addition, the students prepared for the exams mainly by themselves and usually only met once with peers to discuss questions.

Table 3 shows the relationship of the interviewees' exam grades to their use of resources in the exam preparation phase.

Grade	Preparation time	Focus
12	> 2 weeks	Lecture notes, all exercises
11	3 days	Previous exams
10	> 2 weeks	Lecture notes, (procedural) exercises
9.5	3 weeks	Lecture notes, all exercises
9	> 2 weeks	Lecture notes, books
7.5	9 days	Lecture notes, all exercises
7	15 days	Lecture notes
6	2 weeks	Exercises
5.5	3 days	Lecture notes
5	2 weeks	Videos of procedures
5	1 week	Lecture notes
4	1 week	Lecture notes (selected proofs)
3	3 days	Selected (procedural) exercises

Table 4: The relationship between exam grades (15 best, 0 poorest), exam preparation time, and the focus of the use of materials during exam preparation

Interviewees who invested more time in exam preparation tended to do better in the exams. Furthermore, it can be observed that those who did better in the exams often focused on both the lecture notes and the exercises, whereas those who did poorer in the exams focused rather on specific materials. This can also often be explained by the time invested, since students usually planned to repeat both materials, starting with one of them and then often had no more time for the other.

Discussion

The results show how diverse the resources used by students are and that these resources serve different purposes in different phases of a semester. During the semester the focus is on the exercises, and all other materials as well as the support of other people are mainly used to work on them. Typically, students first try to work on the exercises on their own and then, depending on their progress, draw on more and more other resources (see Figure 1). Accordingly, the use of such additional resources can be seen as an indicator of difficulties with solving exercises autonomously, which may explain negative correlations of academic performance with literature use and peer learning (e.g., Griese, 2017).

In the exam preparation phase, several students (also) focused on the lecture notes. While cooperation with others played a major role for working on exercises during the semester, all interviewees prepared for the exams mainly on their own. The results also indicate that the follow-up of the lecture notes is the most promising way to experiences of success while completing exercises, while a longer preparation time and a repetition of both the lecture notes and the exercises is the most promising way to succeed in the exams.

The results also show a considerable impact of the institutional settings on the goals and resource use of the students, which results here in a strong focus of the students on the exercises. Gueudet & Pepin (2018) also observed a focus on exercises at a French university, while at the examined UK university the focus of the students was on the lecture notes. This focus on the exercises led in the French and the German sample examined here to the situation that several students started to look for solutions or worked examples mainly on the internet and in books. This procedure does not meet the lecturers' expectations that students use the lecture notes to learn and understand the concepts, and as a model for certain mathematical practices, e.g., for mathematical proof construction (Gueudet & Pepin, 2018).

Overall, it seems that under the pressure of constantly having to produce solutions to exercises during the semester, some students use strategies that do not necessarily seem beneficial for learning (such as copying solutions to exercises from others) and postpone strategies for achieving their learning goals and for following up lecture notes. This applies in particular to students' work with proofs: Students try to construct proofs when the exercises require them to do so, but most of the students interviewed here almost never followed up the proofs of the lectures - neither in the semester nor in the exam preparation phase.

On the other hand, the focus on the exercises provides an important institutional tool to regulate students' strategies. Accordingly, strategies and goals considered essential should be represented as explicitly as possible in the exercises. For example, special task formats can be considered which support the following up of lecture notes, in particular the reading of proofs. Particular attention must also be paid to the difficulty of the exercises in order to enable as many students as possible to work on the exercises on their own while at the same time challenge all students.

When interpreting the results, the small number of students surveyed here must be taken into account and generalizations should be treated with caution. Nevertheless, the results of this and the other studies cited here show that a detailed examination of students' use of resources can provide an important contribution to research on students' learning of mathematics and the design of mathematics courses.

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Learning from history: Jens Høyrup on mathematics, education, and society

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I summarize just a few themes from the writings of the historian/anthropologist of mathematics, Jens Høyrup, that relate to the work of MES. His sheer hard work in striving to understand and explain the mathematical practices of others across millennia and multiple cultures, always in relation to sociopolitical contexts, underpins his ability to convey, to those prepared to make the effort, the relevance of the history of pre- and early modern mathematical practices to perennial issues in mathematics education.

A confession and an enigma

To begin with a confession, I have only recently become aware of how important Jens Høyrup is to our field. My interest was aroused in the last couple of years because of searching for writings on mathematics and war, which took me to Booß-Bavnbek and Høyrup (2003). Since then, I have begun to appreciate the measure, number, and weight of his writings, almost invariably available online (and mostly in English, which raises an important point about the distorting effect of the hegemony of English in our field, that space restrictions preclude me addressing) – notably, Høyrup (2020). Høyrup (2019) contains a small selection of his essays (a mere 31). Now that I have become aware of the importance of his work, I find it an enigma that it is so little recognized in our field, in particular the MES corner of it. Using the search specification "word: Høyrup, author: X" within Google Scholar, with X ranging over many obvious names, yields very little. In what follows, be assured that I am only scratching the surface.

History of mathematics is demanding work

Historians in general, and historians of mathematics in very specific ways, have to struggle with attempting to understand "the Other" from within the worldview of their own cultural matrix and personal history. It is a problem shared, in varying ways, by child psychologists, anthropologists, therapists, translators, and others, one that has been extensively debated within Ethnomathematics in particular.

Writing in the context of Chinese culture, but with general relevance, Cullen (2009, p. 592) cautioned against:

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> [...] the idea that there is *a priori* a universal ahistorical cross-cultural "natural kind" called "mathematics" that can simply be located and studied once one can penetrate the linguistic barrier to see what it is called in Chinese, and on which one can simply impose all the structures and expectations that a modern person finds in the subject called "mathematics" in twenty-first-century English.

Reading Høyrup, it is obvious how painstakingly he applied principles of close reading and structural analysis informed by his study of literary theory. The following quotation is offered as simply one illustration, related to an interest of mine, namely multiplication and division as models for situations, emphasizing distinctions such as the effects of the kinds of numbers and quantities involved, and the differences between "symmetric" and "asymmetric" cases (Greer, 1992; and see Hofstadter & Sander, 2013, Chapter 7 for an insightful discussion):

The four operations originally interpreted as multiplications are:

- steps of [...] a multiplication of pure number by pure number;
- to raise [...] originally used in volume calculation, where the base is "raised" to the default height of one cubit to the real height, and then transferred metaphorically to other calculations of concrete magnitudes by multiplication;
- to make [two segments] hold, name "hold" a rectangle [...] This is just no genuine multiplication but a construction, mostly implying, however, the determination of the resulting area;
- to repeat or repeat until n [...] a concrete doubling (e.g., of a right triangle into a rectangle) or n -doubling (n being sufficiently small to be intuitively graspable, $a \leq n \leq 9$).

(Høyrup, 2010, p. 6)

(His analysis of the diversity of usages for the four arithmetical operations strikingly echoes that of Urton, 1997, for Quechua mathematics). Note that technical details of transliteration are omitted in the above; the quotation is followed by interpretations of terms used and suggested inferences as to the cognitive operations involved. The complexities of translation, such as the principle of "conformal translation" that he pioneered can only be hinted at here; the following quotation makes a clear point:

> The technical meaning of the Babylonian terms has to be learned from their use, not imported from a different conceptual structure – who would get the idea to translate technical texts from the early eighteenth century using terms like oxygen, hydrogen, etc.? (Høyrup, 2010, p. 6, footnote).

His precision and the care that he takes with his analyses puts Høyrup in a strong position to critique less conscientious historians, particularly those relying on dated secondary sources, and he pulls no punches in this regard. Any connoisseur of long-drawn-out complex exegetical disputes would enjoy Høyrup (2017a).

On Ethnomathematics

In the preface to Høyrup (1994, p. xi) (and earlier in Høyrup, 1980) he explains his choice of the term "anthropology of mathematics" to characterize his approach:

> [...] a term which suggested neither crushing of the socially and historically particular nor the oblivion of the search for possible more general structures: a term which neither implied that the history of mathematics was nothing but the gradual but unilinear discovery of ever-existing Platonic truths nor ... a random walk [among] an infinity of possible systems of belief. A term, finally, which involved the importance of cross-cultural comparisons.

Like anthropology, cognitive psychology, and other fields, history of mathematics (in the European and extended-European context) is striving to emerge from its colonial and racist roots. In particular, one focus of Ethnomathematics is creating a counter-narrative to the Eurocentric myth (Høyrup, 1992). Morris Kline provided the easiest of targets when he wrote:

> Mathematics is a living plant [that] finally secured a firm grip on life in the highly congenial soil of Greece and waxed strong for a brief period. In this period it produced one perfect flower, Euclidean geometry. The buds of other flowers opened slightly [...] but these flowers withered with the decline of Greek civilization, and the plant remained dormant for one thousand years. Such was the state of mathematics when the plant was transported to Europe proper and once more embedded in fertile soil. (Kline, 1953, p. 27)

(See also Chemla, 2012). Commenting on the writing of Kline and two others, Høyrup (2010, p. 4) concluded that the other two interpreted Babylonian texts by "translating" them into what they took to be the contemporary equivalents, while Kline found in them something that he did not consider counted as mathematics.

> The implied message (probably resulting because it is the implicit starting point for all three) is the same: there is only one kind of mathematics: *ours*.

He located the counter-narrative within the context of imperialism and white supremacy in the unforgettably succinct and incisive phrase "the ideological shroud assigning the right to conquer and kill in the name of moral superiority" (Høyrup, 2020, p. 8). Just as powerfully, throughout his work, he draws attention to the cultural violence done in the name of intellectual superiority. At the same time, let me note, in passing, that his unflinching gaze distances him from any oversentimental notions about absence of brutality and injustice in other cultures (e.g., Chapter 24 in Høyrup, 2019, pp. 661–688).

With his penchant for terminological exactitude, he (very necessarily) problematizes terms such as "Western" and "non-Western" (2020, p. 8), and how about this for a footnote? (Høyrup, 2018, p. 7):

> *The ancient Greeks were not Europeans*. They would certainly have been no more pleased by being lumped together with Illyrian, Italic, Iberian, Gallic or Germanic barbarians than a Japanese teacher of mine when the Apartheid regime in South Africa decided in the late 1960s to consider Japanese "white by honour".

He aligns with d'Ambrosio's definition of Ethnomathematics as the study of the mathematical practices of cultural groups of all kinds (including academic mathematicians). And he pointed out that "Ethnomathematics", no less than "mathematics" is "*our* concept" (Høyrup, 1994, p. 67).

Of particular interest is his discussion of the relationships between aspects of "decoration, art, and structural inquiry" (Høyrup, 2000) that he approaches within the framework of a

"hermeneutics of non-verbal expression" (p. 160 in Høyrup, 2019), relating to the problem of "the Other" mentioned above. He concludes:

> The moral of the present tale is, firstly, that we should be careful not to extrapolate from *every* piece of geometrical decoration to such extensive symmetries which may be superimposed on its pattern but which are not needed to explain it... secondly, that no necessity leads from an *aesthetics of forms* to *formal investigation of forms*, nor from *formal investigation of forms* to *integration with mensurational geometry* or *into mathematics as a broader endeavour* (Høyrup, 2000, pp. 202–203 in Høyrup, 2019).

Here I take it that he is arguing that rather than a casual assumption that mathematics may be read into *any* aesthetic effort, a close study of each case is required. Thus, in his review of Gerdes (1994), he states that:

> All the examples explored by Gerdes (and sub-Saharan geometrical decoration in broad average as far as the reviewer is aware) belong to the [...] type [which] bears witness of deliberate explorations of symmetries and other formalizeable properties of figures (Høyrup, 1996, p. 203 in Høyrup, 2019).

(A fascinating modern example that I merely mention is that of Escher, how he related his art to mathematics, and his interactions with mathematicians, including Polya and Penrose; with a little online research, the interested reader will be able to find recent documentaries on the subject.)

Mathematics and the state

Høyrup (2009) asserts that mathematics is not generally implicated in the formation of "pristine" states (which I take to mean states in the process of initial formation), an exception being Mesopotamia of the late fourth millennium, when a system of accounting was instrumental in the emergence of the state and, conversely, the emergent system of mathematics was intimately bound up with its administrative role. Urton (2009, pp. 28–29) discusses similar issues in three contexts relating to: "mathematical philosophies and concepts of authority in the West" prior to invasion of the New World, including the rise of double entry bookkeeping; khipu record-keeping in the Inka empire; the uneven encounter between the two systems.

In 1800, Napoleon wrote that:

> The advancement and perfection of mathematics are immediately connected with the prosperity of the state.

and, generally:

> The functioning of the modern state presupposes a variety of mathematical technologies – accounting, statistics, and much more. Mathematics, on its part needs the institutions of the state (schools, universities, research institutions, etc.) to secure financing, recruitment and the rearing of competence. (Høyrup, 2009, p. 635 in Høyrup, 2019)

Hacking (1990) has documented, in Høyrupian detail, the ways in which the formal mathematics of probability and statistics developed within socio-political contexts, in close

relationship to changing views of the nature of humans, and in the service of states. In a rare overtly political statement, he trenchantly observed that:

> We obtain data about a governed class whose deportment is offensive, and then attempt to alter what we guess are relevant conditions of that class in order to change the laws of statistics that the class obeys (Hacking, 1990, p. 119)

Generally, the mathematical establishment in the United States – in particular, but by no means solely – is complicit with the political and military establishments in failing to address the several crises facing humanity. As Davis and Hersh (1986, p. 267) put it, "nuclear weapons mathematics is an accepted component of American mathematics life". Writing with Booß-Bavnbek in 1984 (reprinted in Høyrup, 1994, p. 225), Høyrup put involvement of the sciences with war next only to "the responsibility of the sciences for engendering the ecological crisis". The COVID-19 pandemic continues. And we may add global manifestations of resurgent white supremacy, accelerating wealth inequality, and the effects of antisocial media in destroying any viable notion of truth.

Supra-utilitarian, "recreational", and useless mathematics

Høyrup also makes abundantly clear that as people developed mathematical practices, essentially contemporaneously with the rise of civilization, such practices soon transcended the practical demands for surviving and then thriving. He also describes how pervasive is "recreational mathematics" and clarifies the uses for which such puzzles were developed. The mass and consistency of such "riddles" and "puzzles" across millennia and much of the globe is apparent in the enormous collection of sources compiled by David Singmaster. Of immediate relevance to contemporary education, in my view, is the class of problems that could be described as "only to be found within school mathematics", including problems using unnaturally "nice" numbers or absurdly high numbers. Indeed, he comments that examples which serve to show that a method "may be used for *any* problem of a similar structure, not just for trite commercial calculation" and that such problems pass "imperceptibly into general school mathematics" (Høyrup, 1994, p. 28).

As one who has questioned the need for "algebra for all" (Greer, 2008), and, in particular, wondered why quadratic equations enjoy such prominence in mathematical curricula, I was intrigued to read the following (Høyrup, 2013, p. 7):

> Many students will certainly be astonished to discover that even their teachers do not know why second-degree equations are solved. Students as well as teachers will be no less surprised that such equations have been taught since 1800 BCE without any possible external reference point for the students […].

Styles of mathematical reasoning

While Hacking (2016) refers to six styles of scientific reasoning that have characterized science in Europe, namely mathematical, experimental exploration, hypothetical modelling, probabilistic, taxonomic, and historico-genetic, it is equally possible to distinguish different styles of reasoning within mathematics. At a very general level, Raju (2007, p. 413) argues

that within European mathematics there are two main streams. From Greece and Egypt came a mathematics that was spiritual, anti-empirical, proof-oriented, and explicitly religious; from India, via Arabs, a mathematics that was pro-empirical and calculation-oriented, with practical objectives. For a different take on "style" that emphasizes "ways of writing" rather than shared epistemologies see Rabouin (2017).

A great deal of Høyrup's analysis of Old Babylonian mathematics is concerned with establishing the styles of reasoning that underlay their algebra, its relationship to geometry, and the relation of their "geometrical algebra" to that of the Greeks (Høyrup, 2005). He declares unequivocally that:

> It is therefore to be expected that mathematics teaching in any mathematical culture which went beyond mere routine [...] did include appeals to reason – whether naive or critical, and whether in Greek style (or that dubious reading of the Greek style in which we project ourselves) is a different matter. If we cannot find traces of this reasoning in extant sources we may safely conclude that this is due, *either* to failing understanding of the sources on our part, *or* to the insufficiency of extant sources as mirrors of educational practice. *Tertium non datur*. (Høyrup, 2019, p. 560).

What is mathematics? (Høyrup, 2017b)

I suggest a fundamental shift from asking about mathematics as some kind of entity to asking about the uses of the word and about the families of practices in which it may be said to be implicated (clearly that is closely related to the definition of Ethnomathematics as the study of the mathematics of cultural groups, including academics). In similar spirit, Harouni (2015) referred to categories of mathematics that he labels commercial-administrative, philosophical, artisanal, and social-analytic.

The title of Høyrup's (2019) selected essays refers to "mathematical practice". A book could be written just on the usages of singular and plural nouns in discussions of "mathematics", starting with that word itself in many languages (for example, Bourbaki used "mathématique" instead of the usual French plural form to proclaim its belief in the unity of mathematics). However, elsewhere he wrote that (1994, p. 67):

> [...] we have a cluster of indubitably mathematical practices, disciplines, and techniques, cohering through shared use or investigation of abstract, more or less generalized number or space or of other abstract structures.

A closely related question is: "Who is a mathematician?" (Høyrup, 2003). Again, Cullen (2009, p. 593) clarifies with the questions:

> [...] was there a self-conscious and publicly recognized group of people in ancient China with a family resemblance to what would be called nowadays "mathematicians"? What did these people call themselves? What did they consider their defining skill-set, or their common obsession to be?

(and similar warnings certainly apply to "schools" and "teachers").

Dialectical processes

I have developed an aversion to binary questions such as "Is mathematics discovered or invented?". It is safest to explore the terrain with a dialectical mine detector. For example, Høyrup (2002, p. 404) states:

> In mathematics as elsewhere, the expansion of knowledge is a dialectical process where neither naive creativity nor critical consolidation can function alone, each of them establishing that the other may become operative,

which resonates with the declaration by Hacking (2016, p. 13) that "the history of mathematics is one of diversity and unification". Further, he emphasizes that the historian can describe developments internal to a discipline (such as mathematics) and also external drivers but that such descriptions lack explanatory capability "as long as no dialectical synthesis takes place" (Høyrup 1994, p. xiii). For a mathematician's take on the internal/external interplay on developments within mathematics and mathematics education in the 20th century, see Mandelbrot (1994).

To make my final point that history also includes the future, I refer to Freudenthal on Piaget's supposed discovery of a direct correspondence between his hypothetical roots of mathematical cognition and Bourbaki's "mother structures":

> Piaget is not a mathematician, so he could not know how unreliable mathematical system builders are [...] Mathematics is never finished – anyone who worships a certain system of mathematics should take heed of this advice. (Freudenthal, 1973, p. 46)

Or, as Høyrup (1995, p. 5, footnote 4) put it:

> No critique is ever definitive. What seemed at one moment to be an absolute underpinning [...] turns out with historical insight to make other "naïve" presuppositions which in their turn can be "criticized".

Final comments

Philosophers, like most other people who think about it at all, tend to take "mathematics" for granted. (Hacking, 2014, p. 41)

The study of history of mathematics offers an extremely effective remedy. And, in relation to education, many have made the essential point that

> school is seen as a magical shortcut that allows ideas arduously developed by humanity over thousands of years to be transmitted in a few years to a random human being (Hofstadter & Sander, 2013, p. 391)

Further, Høyrup so clearly demonstrates the complexity of conceptual change and illustrates how analysing the epistemological crises of the past and how they were addressed could inform contemporary teaching.

The characterization of much mathematics as supra-utilitarian or useless reflects the imbalance (in my opinion) pervasive in mathematics education whereby curricula, pedagogy,

and assessment cater more for the preparation and selection of future mathematics students, rather than being guided by considerations of usefulness for the general population. This imbalance is, to a considerable extent, influenced by the desire of mathematicians to perpetuate their species, enabled by what could be termed the unreasonable political effectiveness of "mathematics" as a term of propaganda.

His work bears heavily, in my judgment, on the current ideological fault line in global mathematics education between homogenization and diversity in all its forms, including styles of reasoning. In the spirit of Ethnomathematics, he shows how diversity is manifest in all families of practices involving mathematics, with the notable, and arguably unfortunate, exception of school mathematics.

A partial explanation for the enigma to which I refer at the beginning is that Høyrup presents so much in the form of meticulous detail ("the socially and historically particular" in the quotation above about anthropology of mathematics) that a little work is necessary to find the "more general structures" (same quotation). I assert that such work is richly awarded; for me, the points made above have been greatly reinforced by reading him. As someone who needs to eke out his remaining intellectual resources, "discovering" Høyrup at my age is a blessing and a curse. To younger colleagues, I urge you to start serious study now of this remarkable scholar's work.

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Who plans mathematics teaching?

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Teachers plan and conduct mathematics teaching, and hence, are often seen as the key to change mathematics teaching towards a more equal and just activity. However, there are signs that other actors are also influencing the planning and thereby the mathematics teaching. To explore the influence of others, a focus group study was conducted and results from one of the group discussions are presented in this paper. The results show that there are individuals who are actors in the process of planning, but also organizational and material actors. There are also direct links from the actors that the teachers express to others, which means that these also influence the planning and the mathematics teaching. Hence, changing mathematics teaching is not just a matter of teachers.

It would be uncontroversial to say that all students should be given equal opportunities to learn mathematics in school. However, is not always the case. For example, in Sweden, although the Educational Act state students' equal rights to learn, results from TIMSS 2019 indicate that students' background and home conditions play a role in their learning in mathematics (Skolverket, 2020). In addition, the differences between different groups of students increase (Skolverket, 2020), which should be seen in the light of the fact that there have been initiatives, such as new national curriculum and a national educational initiative for in-service teachers, aiming at a more equal and research-based mathematics teaching.

Often curriculum materials are designed to promote reform in mathematics teaching and there are those who see texts such as the national curriculum as a way to govern teachers (Remillard, 2005). However, studies show that even if teachers use the same curriculum material their teaching is not the same (e.g., Remillard, 2005), which has been interpreted as a teacher's aims are the result of interaction between the teacher and the curriculum material, and the aims are influenced by the material and the teacher and her characteristics (Remillard, 2005). However, mathematics teaching and teachers' aims for teaching seems to be even more complex. When mathematics teachers plan, they are influenced by both formal and informal actors (Grundén, 2020) which means that what Remillard (2005) sees as an interplay between curriculum material and the teacher seems to be an interplay between the material, the teacher, and other actors.

In this paper, results from one focus group interview are used to exemplify how actors and networks of actors influence the way teachers plan for mathematics teaching. The results are used for a discussion about social and political dimensions of mathematics teaching that might have consequences for the possibilities of realizing the vision of an equal mathematics education.

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Background

The idea of governing teaching and thereby implementing new ideas often builds on the assumption that what is stated in formal curriculum also is what is enacted in the classroom. Instead, Remillard (2005) suggests that teachers interact with formal curriculum, plan their teaching based on this interaction, and in the classroom situation transform the planned curriculum into enacted curriculum. This means that there is not a direct link between what is in formal governing documents and what happens in mathematics classrooms – the teacher and her decisions are in between.

Teachers have to balance between obligations and need to attend to many different tasks – all with different motives (Skott, 2004). There are demands imposed on teachers and at the same time teachers are responsible for the enactment of the curriculum and thereby function as the link between ideas about mathematics described in curriculum and research and the context and social surroundings (Skott, 2004). This mix of demands and responsibilities is referred to by Skott (2004) as "forced autonomy".

Skott (2004) argues that when reform ideas are not implemented as intended this may be due to the teacher's different motives for the activities in the classroom "that force him [the teacher] to pursue one of these at the expense of the others" (p. 253). Hence, teachers do not always act in the way they think benefit their students' learning in mathematics the most. In the concept of forced autonomy lies a view of the teacher being at the center of curriculum enactment. However, in my research (Grundén, 2020) the complexity in the planning process is highlighted and teachers' autonomy is called into question. Instead of *forced autonomy*, I used the term *false autonomy* which means that "the teacher has the task of planning, but not the full mandate to do so" (p. 83).

Planning for mathematics teaching can be seen as different things including choosing activities, producing manipulatives, and thinking about ideas for teaching (Grundén, 2020). In this paper, planning is seen as:

> A situated process, hard to distinguish in time and place, that involves mathematics teachers' socially embedded considerations, decisions, and reflections on and about future teaching (Grundén, 2020, p. 35).

Theoretical standpoints

As stated above, planning can be seen as an interplay between the teacher, material, and other actors. This view of planning as socially embedded implies that planning can be seen as a social practice. There are different ways of describing practices (e.g., Boaler, 2003; Fairclough, 2003; 2015; Grootenboer & Edwards-Groves, 2013), but what they have in common is that people are involved in interactions and activities. Fairclough (2015) also includes language and the material world, and in Fairclough (2003) a social practice is described as mediating the relation between potential events defined by structures and actual events. Hence, the social practice of planning with its people acting and interacting, using language and materials, mediates the potential events, i.e., the ideas promoted by structures, and the actual events, i.e., what happens in the mathematics classroom.

The people involved in a practice can be seen as actors which can be defined as "a participant in an action or process" (Oxford Dictionary, 2021) who is linked to capacity and space for actions and to the ability to act differently (Giddens in Johnson, 2001). An actor is, according to Enserik, Hermans, Kwakkel, Thissen, Koppenjan, & Bots (2010), "a social entity, a person or an organization, able to act on or exert influence on a decision" (p. 79) and the influence can be directly or indirectly. Actors "have a certain interest in the system and/or that have some ability to influence that system, either directly or indirectly" (Enserik et al., p. 80). In this paper, the description of an actor as someone who "exert[s] influence on a decision" (Enserik et al., 2010, p. 79) and the view of practices as including material world (Fairclough, 2015) implies that physical objects can be actors as well.

Method

The material in this paper comes from a focus group study with mathematics teachers. Focus group discussions is useful for complex issues that can be explored with richer material than individual interviews since the participants interact and react to each other's statements (Carey & Asbury, 2012). Focus group material needs to be understood within the context in which it is produced as well as in relation to the larger social context.

In the focus groups, I wanted the participants – based on their view and experiences of planning – to reflect on and react to aspects that in a prior study (Grundén, 2020) were identified as related to planning for mathematics teaching as well as aspects and reflections that came up in the discussion. The aspects that were used as stimuli for the discussion were *students, school management, national tests, template/forms, parents*, and *textbook*. The study involved six groups with a total of 27 mathematics teachers. All groups got the same stimuli. In this paper results from one of the groups – consisting of six teachers working in school year 1–3 – is presented and discussed.

Each focus group discussion started with me putting notes in the middle of the table. On some of the notes I had written an aspect, others were blank. I told the participants that teachers in a previous study had identified the aspects as related to planning in one way or another. I asked the participants to consider whether any of the aspects were related to planning for them, and I also told them that they could add aspects they thought were missing or remove aspects that did not relate to planning for them.

When the teachers started to talk, my role was to ask clarifying questions, follow-up questions, invite all participants to the conversation, and through gestures and small words confirm that I was listening. The group presented here did not remove or add any aspects on the empty pieces of paper.

Analysis

The aim in this paper is to describe in what ways actors and relations between actors might have consequences for changing mathematics teaching, and hence, in the analysis I needed to identify actors, see what they do in the planning process and see in what ways they are connected to other actors.

Enserik et al. (2010) describe different techniques to identify actors, one of which being the reputational approach, in which key actors are asked to identify important actors.

However, it is only possible to talk about the actors you are aware of which means that there is a risk that actors' implicit influence on planning for mathematics teaching might not be identified. Instead of asking key actors – in this study the mathematics teachers – for actors influencing planning the teachers were asked to talk about planning with the help of stimuli, i.e., aspects earlier identified as related to planning and the identification of actors was done by me in the process of analysis.

To identify actors in this study the definition of actors as *participants that influence a process* – in this study the process of planning – was used. This means that just because the teachers talked about one of the aspects that were used as stimuli, the aspect was not identified as an actor. It became an actor if the teachers talked about what it did in the process of planning. For example, the group started to talk about students – which was one of the words on the notes – and their differences but it was only when they said that students' differences led to a situation where the teachers could not plan, for example, problem solving when they were teaching the whole class, that students were considered an actor. An example of an aspect that was not considered as an actor was parents. The group talked about parents as having opinions about the mathematics teaching but did not explicitly say that decisions in the planning process were influenced by the opinions.

Inspired by Enserik et al.'s (2010) description of actor analysis, guiding questions for the analysis procedure were developed. The questions were:

1. Who/what is doing something in the process of planning?
2. In what way(s) does the "who/what" participate in the planning process?
3. What relations between actors are expressed by the teachers?

The results are based on the first three steps but for the purpose of the discussion another question was used.

4. What known and obvious relations of importance for the process of planning exists between actors?

In the analysis, the transcript was read with the first two questions in mind. Sections where the teachers talked about something that somehow had consequences for decisions in the process of planning were marked. The actor was highlighted in one color, and the participation in another, and expressed relations with other actors were marked in a third color. In the process of identifying known and obvious relations, yet undetected actors emerged. In Table 1, examples from the analysis are presented with the actors in bold.

Action	Relations - expressed	Relations – known and obvious
The **textbook** gives the structure of the teaching, [e.g., how the mathematical content is distributed over the school year]	**Students** love the textbook **Skolverket** (the National Agency of Education) has approved the textbook. Some **parents** want us to work faster in the textbook	**Authors** has/have written the textbook A **publisher** has published the textbook

Table 1: Examples from the analysis.

Results

This section starts with a presentation of the actors that were identified in the focus group discussion and a description of the actions that these actors do in the process of planning. The section ends with the relations between actors that the teachers expressed.

Actors

There were several actors identified in the analysis, and an overview of them all can be found in Figure 1. In this section, the most prominent by virtue of how often it showed up in data are presented in more detail.

In the group discussion, the teachers seem to agree that students are important actors in the planning process. However, students are actors in different ways, and in the material, two main themes were found; students' differences require adjustments and students' needs guide the planning. Adjustments made in the planning process are, for example, based on different tasks being suitable for different students, that students' knowledge of Swedish affects their ability to understand written tasks, and that some students need challenges. What students need to achieve the stated curricula goals is something that according to the teachers highly affects the planning, and what they need is based on their prior knowledge. The teachers also said that students have the right to be prepared for what comes on the national tests.

The textbook emerges as an ever-present actor in the planning of mathematics teaching. According to the teachers, the textbook governs the structure of the teaching, what content is to be included, and when to work with that content. In the planning process, the textbook also acts as a source for tasks. The book is even described as a savior, although it also appears that the book has shortcomings that make the teacher in the planning process look for material elsewhere. When teachers want to supplement the textbook, they turn to different media – including websites and TV-shows from public service – and colleagues for inspiration and activities.

The national tests and assessment support material issued and regulated by Skolverket (the National Agency of Education) are actors in the planning process partly because the teachers are planning to "teach to the test", i.e., preparing students for the standardized tests. The tests and the support material are templates for what content is to be covered in the teaching. The presence of the tests also makes the teachers plan for repetition before the tests are carried out. The tests are actors also in organizational aspects. Carrying them out takes several lessons, and the teachers emphasize that time they could have spent on teaching and developing students' knowledge is wasted. Parts of the tests should be conducted with individual students or smaller groups of students, which means that the teachers have to plan for the other students. On the one hand the teachers emphasize this as problematic – they have to rely on a resource person to take care of the students while they conduct the test – and on the other they appreciate being "forced" by the regulations to plan for meetings with individual students or small groups of students.

An actor also mentioned in the discussion is mathematics as a school subject. The subject leads to that teachers plan for a teaching where the students work more independently than in other subjects. The planning also varies depending on which parts of the mathematical content are to be dealt with. For example, some content requires that tasks in the book are supplemented with other tasks. The content also might change which activities are planned for, and how much time is spent.

Although the teachers describe that an in-service development program called Matematiklyftet did not lead to any lasting changes, they state the program as an actor that has really changed not only the teaching they planned for, but also the process of planning. In the discussion, the teachers emphasized that during the program they did not rely on the textbook as much as they usually do, and they also talked about the introduction of a reflection phase in the process of planning. The teachers also talked about one page in a learning management system as an actor. Structure and goals from the textbook were available and used by some as a base for planning.

Relations between actors

When it comes to teachers' expressed relations between actors, the most prominent ones are related to the textbook. Firstly, students and the textbook with its teacher guide are related to each other in that tasks in the book need to be considered in relation to students and that different students approach the book in different ways. A statement in the discussion, namely "Students love the textbook", witness a relationship in which the students appreciate working with the textbook. Not only students, but also parents, are highlighted by the teachers as actors who appreciate the textbook and thus can be said to have a relationship with it. According to the teachers', there is also a relation between the textbook, the national curriculum, and Skolverket, so that textbooks are in line with what is stated in the national curriculum. The teachers also emphasize that Skolverket evaluates and approves textbooks[1]. Another actor is a learning management system in which there is a matrix with goals related to the textbook. The matrix is not published by the textbook publisher but made by "someone".

"Someone" emerges as an actor related to a national assessment support material because "someone" has decided that it must be conducted with all students. There is also an expressed relation between Skolverket and the support material. Skolverket should, according to the teachers understand that the guidelines for implementation of the assessment support material, pose concerns for teachers in planning. When conducting the assessment support material, there is according to the teachers a need for an extra teacher or a resource person to take care of the rest of the class.

In Figure 1, there is an overview of the actors and relations that were identified in the discussion in the focus group presented in this paper. The actors that have a direct impact on the teachers planning are on the periphery of the circle, actors mentioned by the teachers being related to the peripheral actors are outside the circle. Relations between actors are represented by lines between the actors.

[1] In Sweden, there is no review of textbooks and thus no guarantees that they meet the national curriculum.

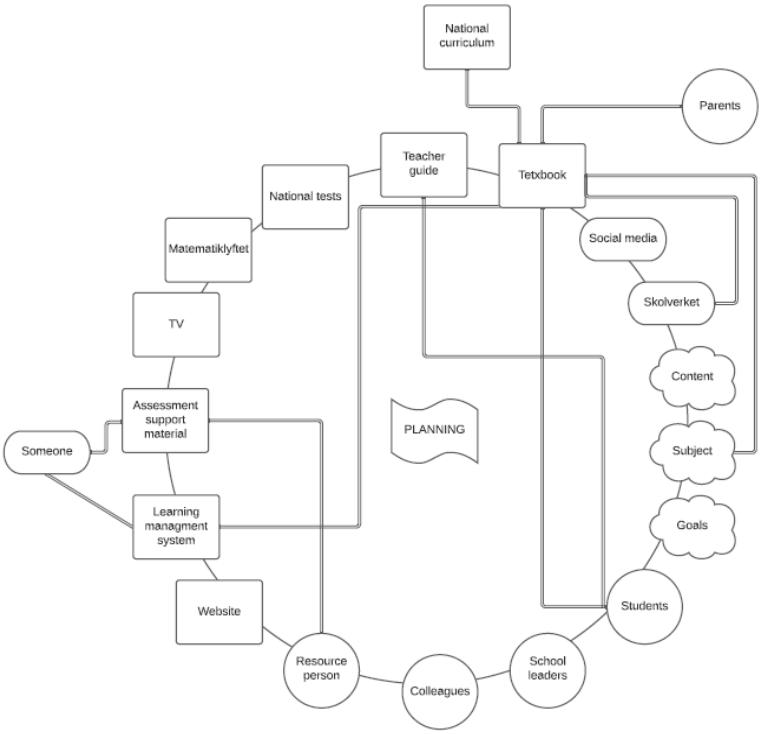

Figure 1: Actors in the process of planning and expressed relations between them.

Discussion

The discussion starts with a description of known and obvious relations between actors that appear in the teachers' stories and other actors, followed by a discussion about the influence of actors and relationships on planning in relation to the vision to develop mathematics teaching towards a more equal and just phenomenon.

Known relationships between actors

In addition to expressed relations, there are relations of importance for the planning between actors that are known and obvious (see Figure 2). These relationships are important to emphasize in order to get a more comprehensive picture of the influence. The textbook is emphasized as an actor influencing the decisions teachers make when planning. The textbook is related to students, parents, national curriculum, and Skolverket. Known and obvious relations to the textbook are that authors write the textbook and a company publishes it, teachers or others decide to buy the textbook. Other prominent actors are the mandatory assessment support material and mandatory national tests, which are related to

politicians who have decided that they are mandatory, and to Skolverket[2] because they design and administer them. The tests are also related to national curriculum. There is also a relation between the assessment support material, national tests and school leaders who are the ones that appointing funds for the resource persons the teachers say are required to implement the tests.

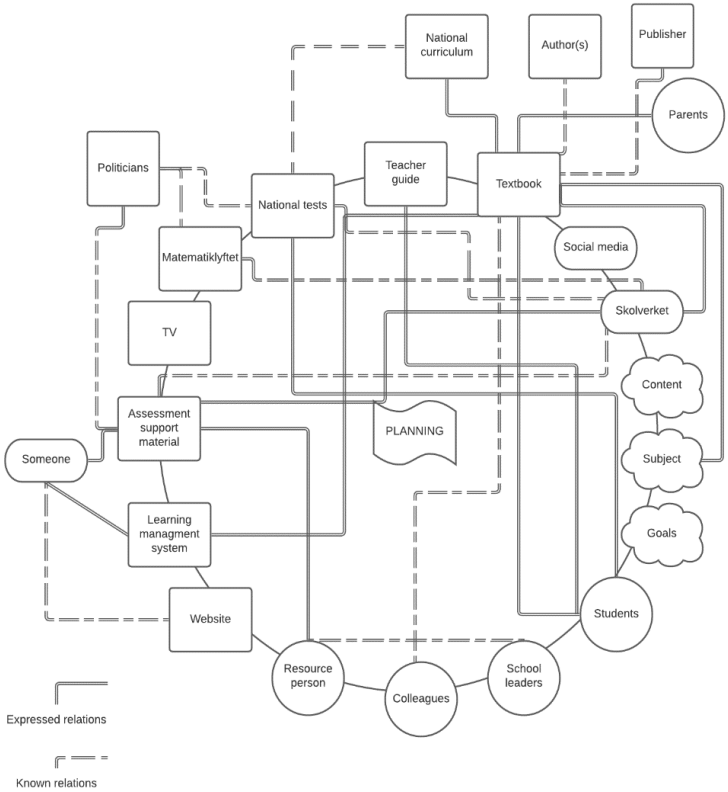

Figure 2: Actors in the process of planning and expressed and known and obvious relations between them.

In Figure 2 it becomes visible that decisions teachers make in the process of planning are influenced by many actors. What also becomes visible is that the actors who have relationships with most other actors are the textbook, assessment support and national tests, and Skolverket.

The network of actors that emerges in the analysis of the teachers' discussion provides a reason to question the view of the teacher as an autonomous implementer. Not only does a

[2] Here Skolverket is regarded as the actor although there are individuals employed by Skolverket that are acting.

teacher interact with curriculum materials as Remillard (2005) states, but there are also several actors involved in the decision-making process.

The textbook is an actor present in many of the decisions about planning. Important to bear in mind is that the teachers express relations between the textbook and Skolverket and between the textbook and the national curriculum that do not exist. It seems like the teachers trust the textbook more than themselves, which may seem logical given that the teachers think the textbooks are approved by Skolverket. Instead, in the long run, the teachers put their trust in authors and publishers. Even if there is no review and control of textbooks in Sweden, it seems reasonable to believe that these actors have an interest in contributing to students learning in best possible ways. However, these actors also have financial interests in selling as many textbooks as possible, which might mean that publishers are more willing to grasp trends than to profoundly change mathematics teaching.

On a couple of occasions in the discussion, *someone* appears as an actor that the teachers put their trust in. For example, when the Learning Management System appears as an actor and the teachers describe how they use a matrix in the system. The teachers use the matrix for planning but do not know where the matrix comes from. However, they "trust it fully" and "it follows the book". Another example of when *someone* is an actor is when the teachers search for material and activities on social media or on a website.

Teachers rely on actors with formal power – for example when they mention Skolverket – but also on actors such as authors, publishers, and "someone", that do not have formal power and might have an agenda not in line with what is best for students' learning based on research and national curriculum. The results give the impression that the teachers seem to have more trust in others than in themselves, even in "someone" they do not know who it is. "We fully trust that it is OK! It is well thought out!" The results speak against the image of the teacher as an implementer of ready-made ideas from formal decision-makers and suggest a broader understanding of Remillard's (2005) idea of the teacher as interacting with curriculum material. It is not enough to see the teacher as a key to change in mathematics teaching. Rather, the result supports the idea of *false autonomy* (Grundén, 2020) and to promote changes in mathematics teaching focusing on teachers is not enough. Those who want to contribute to a more equal and just mathematics teaching need to work to ensure that actors with knowledge of mathematics teaching are the ones who make decisions, and we as researchers need to address our research also to other actors who influence decisions in the process of planning for mathematics teaching.

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Political conocimiento in teaching mathematics: Intersectional identities as springboards and roadblocks

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Today's mathematics teachers must navigate the politics of teaching and learning mathematics in various contexts. Yet, many teacher preparation programs continue to focus on content and pedagogical knowledge, leaving teachers unprepared for these challenges. This study investigates the role of intersectional identities on the ways 49 teacher candidates (TCs) in the United States engaged with a tool designed to develop their abilities to analyze school politics to develop their propensities to intervene on behalf of historically marginalized students. Findings indicate TCs' racialized identities, personalities, ethical positions, and grade levels impacted the ways they were (or were not) compelled to act. Implications for future research are discussed.

Introduction

All teachers face politics in their work lives. Yet, teachers of mathematics are uniquely challenged by a subject that is the focus of high stakes tests and which many believe to be neutral, culture-free, and universal. Rather than believing they will develop this political knowledge "on-the-job," teacher candidates (TCs) need opportunities to critically analyze school policies (e.g., placement of students into courses, state/district mandates) and administrator and colleague actions (e.g., adopting textbooks that are difficult for multilingual students; enforcing pacing guides) to recognize injustices and advocate for students. This aligns with professional standards in the field, such as the Association of Mathematics Teacher Educators' Standards for Teaching Mathematics (AMTE, 2017).

Teacher identities are deeply connected to the kinds of actions they carry out in teaching (Holland et al., 2003) and are shaped by professional experiences and demands (Aguirre, Mayfield-Ingram, & Martin, 2013), as well as the communities in which they participate (Gonzalez, 2009). As such, we have argued that developing political conocimiento with TCs is not a universal activity that can be inserted into a methods course without careful consideration of other work in that program/course, the teacher educator's intersectional identity and what their goals are, who the teachers are, and the nature of their work contexts (Myers, Kokka, & Gutiérrez, 2021). In this paper, we offer empirical results from the point of view of three womxn scholars of Color and our teacher candidates who range in intersectional identities, but who are primarily white womxn. In particular, we focus on our use

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of one tool, The Difficulty Sort, which asks teachers to categorize how difficult a particular political scenario might be for them to respond to. We highlight the ways in which TC's intersectional identities influence the meanings they make and the actions they envision. We argue for the continued need to focus on TC identities and contexts, which helps counter the tendency of a methods fetish in education.

Political conocimiento in teaching mathematics

Political Conocimiento in Teaching Mathematics (PCTM) is a theoretical lens that highlights the political knowledge mathematics teachers need to navigate their practice (Gutiérrez, 2012). To date, research on the politics of mathematics teaching has primarily focused on calls for programs to develop teachers' political knowledge (AMTE, 2017; Gutiérrez, 2013) or the strategies that teachers have used to enact ethical practice (Brown, 2017). The field still knows little about the specific meanings that TCs make while they are developing such political knowledge, though noted exceptions include Gutiérrez, (2015) and Gutiérrez & Gregson (2014). Several questions arise, such as: Given the salience of race in the US, what is the meaning that teachers of Color versus teachers who are white make of these activities? Do teachers who focus on early grades (e.g., elementary) engage with these activities differently than teachers who teach middle/high school students? And, to what extent should the activities be designed to reflect the backgrounds and teaching contexts of TCs? Another limitation of PCTM research has been the focus on "rehearsals" (Gutiérrez, Gerardo, Vargas, 2017) that require significant time and the ability to facilitate a theater-like performance on-the-spot. More nimble and transitional tools are required if we expect to expand efforts. Our work seeks to fill this void in the field and answer some of the aforementioned questions by presenting empirical results from such activities with TCs in different contexts and over a range of grade levels.

Context/data

This study is a cross-case analysis of three teacher education sites in the United States (East coast, Southeast, and Midwest) that differ in several ways (e.g., grade level focus, intersectional identities of teacher candidates, program structure and focus on anti-racism/justice, types of school placements for observation and student teaching, etc.). In the larger project, we seek to understand how TCs make sense of activities that support them to develop political conocimiento, so as to further theorize PCTM and offer the field a set of tools to use with other TCs.

During the 2020-2021 academic year, after obtaining signed consent forms from participants, each of us carried out several activities with our teacher candidates in mathematics methods courses we taught via Zoom. We used four Political Conocimiento Development Tools we created to support them to develop the knowledge and ability to advocate for historically minoritized/oppressed students. These tools are: How Would You Classify It?, Difficulty Sort, Similarity Sort, and Mapping/Rehearsal (see Figure 1). All sessions using these tools were video recorded then transcribed by research team members who utilized Otter.ai transcription software. TCs' intersectional identities reflect the ways they described themselves in interviews as well as the grade level they will teach.

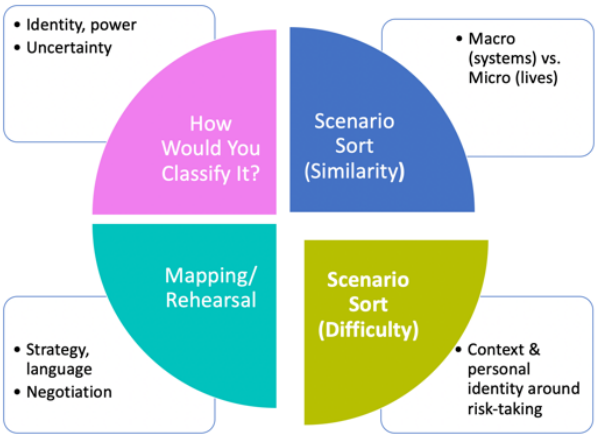

Figure 1: Political Conocimiento Development Tools

We focus here on the second activity in our suite of tools, the Difficulty Sort, which attends to their comfort with taking risks in teaching to advocate for students. TCs were asked to individually sort into three categories how difficult seven different politically-charged scenarios would be for them to respond to. An example of a scenario is: "The teacher walks past a pair of boys working at a table and overhears one say to the other: 'Turn on your Asian brain and help me do this problem.' It's said loud enough for the entire class to hear." After sorting their scenarios, the TCs were invited into breakout rooms via Zoom to discuss the ways they sorted the scenarios, noticing similarities and differences, reflecting upon their strengths/missed opportunities in advocating, and considering how their intersectional identities might trigger particular responses from others in the scenario. Then, TCs were asked to reflect on what they learned ("Something that is new for me from this activity is…") and what questions they have ("Something I'm now wondering about is…"). Finally, TCs were interviewed individually (1–1.5 hours each) about their backgrounds, teaching contexts, and experiences in the activity. We focus on the 49 TCs across our settings.

Following Boyatzis (1998), we created analytic memos on our sessions, giving attention to individual participants and coding general findings for our individual sites. We then met to discuss similarities and differences across our sites and to reflect on the variety of responses from our TCs. We video recorded these research meetings to capture our reflections and deliberations. After our initial meeting analyzing data from the Difficulty Sort, we returned to our data to refine our coding, based upon themes that arose in a given site but were less pronounced across all three sites. For example, in one site, TCs of Color versus white TCs differed in how they made sense of the activity and reflected on how their identities might trigger responses from others. But, upon further exploration, that was not the case in the other two sites. As such, we surfaced the influence of contexts and our TCs identities in the development of PCTM. We report our findings here.

The role of teacher candidates' identities

Most of the teacher candidates found the activity useful for bringing up issues they had not considered but that they will face in their teaching. They also appreciated hearing the views of other people (e.g., to help them note their strengths/challenges and to think about the different ways their peers might approach responding to the scenario). Even so, their identities influenced how they engaged in the session and how they might respond to the scenarios if they were faced with those situations.

Racialized identities

An important aspect of living in the United States is the way people are racialized, and several of the scenarios we crafted included either explicit references to, or coded language about, students of Color. Thus, it is not surprising that we saw concrete differences in the ways that whiteness played out in the activities. For instance, although we offered a specific prompt for breakout room groups to discuss how their identities may have influenced their responses, some white TCs responded that they had not considered their identities before and/or were not sure how the activity helped them further understand their identities. They said things such as, "I am not fully sure, I've never really thought about [my identity]" [white woman, secondary]. This was true even when some of these white TCs were in breakout rooms with TCs of Color who were explicitly discussing their racialized identities, commenting on the ways other people ascribe racial or ethnic identities to them. In this sense, these white TCs' sentiments were race evasive, centering their own white identities as normative. When whiteness was recognized by white TCs, it usually arose as a form of positioning themselves in contrast to or distanced from the students of Color in the scenarios, as if they could not imagine what it would be like to be them. They said such things as:

> Who am I to tell them what to think?? I am not a minoritized student. [white woman, secondary]
>
> Sometimes it's tough to speak with students when you're completely, completely different backgrounds. And, I always want to let you know, remind them that I'm here for them […] I'd love to understand their culture and their, [that's] my thought process. It can be tough to try to connect with those students. [white woman, secondary]

Whiteness also arose as a form of acknowledged authority for responding in the scenarios because they believed others will defer to white people when they take action. In this respect, several of the white teachers were fully aware of their privileged status as it would play out in these scenarios.

> I feel that my identity as a cisgender white female will make people listen to me. I am part of a majority and that people hold my identity higher than others. I do not think that it should be that way, but it is. I need to stick up for those that do not have the voice to. [white female TC, elementary]

In these cases, TCs recognized they were positioned in society with white privilege and saw their responsibility to use that privilege to intervene.

At other times, whiteness arose as uneasiness regarding "doing the right thing," which tended to refer less to ensuring justice and more to "not wanting to make a mistake or be seen as racist." They expressed such sentiments as:

> Anything with race is difficult as a white person. [white man, secondary]
>
> I'm worried that my good intentions will not be received as such. [white man, secondary]
>
> As a white teacher, I don't want to give anyone the wrong impression." [white woman, elementary]

One scenario depicted a teacher who is giving back student papers and passes by a student with their hand raised and the student exclaims, "Did you see that? She's not helping me because I'm Black!" The white elementary TCs in our study felt more compelled to point out how difficult the situation would be to attend to because of their whiteness, as if they were the victim, not the students in the scenarios. Some worried that they would/could not do the right thing simply because of their white racialized identities.

> As a white male I can 'be called anything under the sun' and people would justify it […] I wouldn't ever meaningfully do that to a student. But once the story gets told […] and if I don't see the student's hand […] and I tell them that, but they tell their parents and then they tell someone else […] I could lose my job. It's just very difficult for me to defend my position because I'm the only teacher in there. It's not like the students can defend me. So […] it's just difficult for me." [white man, elementary]

In this instance, "anything under the sun" means "being called a racist." Yet, even in attending to his whiteness, he is not able to name race/racism explicitly. He sees his whiteness as leading to an extreme situation – potentially losing his job, which is an unlikely scenario given that most teachers and administrators are white and support other white people, and students hold little status in schools. In feeling this unease, their approach seemed less reflective about what they could do to stand up for students and, instead, TC's expected someone to hand them the correct protocol to follow. This theme was more prominent within the elementary TCs.

> Is there a protocol to follow when we realize these educators may cause more harm to children just by being in their classroom? [white woman, elementary]
>
> How do I approach a situation with someone who is in charge of me without being rude or stepping on toes? I normally stand up against people but when I am working for the person it seems like a touchy subject. I know I need to stand up to them but how do I go about that in a nice way? [white man, elementary]

As such, although advocating for students is an explicit goal of the Political Conocimiento Development Tools, getting white teacher candidates to recognize their racialized identities does not guarantee they will acknowledge their white privilege or use it as a springboard to intervene. In fact, as noted here, it could serve as a roadblock to responding.

In contrast to themes that arose among white TCs, most of the secondary TCs of Color were more forthcoming about how their identities could influence how they process the scenarios and how their identity might trigger responses in others. In general, they saw their racialized identities as possible challenges with respect to other adults they might need to confront and as possible strengths with respect to students in the scenarios. Identifying possible negative responses they might get, some TCs expressed:

> Identity can trigger some prejudiced thoughts [about me] because I am a minority." [Asian American man, secondary]

> […] if I'm talking to someone who maybe I'm talking to them about […] bilingual students, if I was talking to them about that textbook thing [a scenario where the school wants to adopt a new textbook that might be harder for multilingual students]. Maybe they would feel like I was attacking how they viewed something just because they're like white, or just because they don't have that experience. Maybe they would think that because I relate to it more, I have like a higher, like I hold myself to a higher ideal than them because I relate to it […] that could be something that's running through their head that maybe they feel like they're more on the defense, because they don't relate to that as much. [Latina female TC, secondary]

In the first quote, the TC reflects on ways they have experienced prejudice simply because of their racialized identity, a recurring theme for this TC in our sessions. In the second quote, the TC is catering to the comfort of her white colleagues by suggesting that simply being Latina could trigger a defense by others.

On the other hand, in response to a scenario where a teacher overhears two Latinx students discussing how they would have to use their race (as opposed to their grades) to get into college, one TC sees her connection to the students as an asset.

> I would be able to maybe share my own experience and how […] in my college career, and like, I'm here teaching you. So it's possible for you to succeed […] And I would imagine that, because I have that connection, that we're both Latinx, that they would maybe respect that or like, like that they would understand that perspective that I'm coming from. [Latina female TC, secondary]

For that same scenario, another TC of Color also recognized her identity as a possible strength, but then cautioned against essentializing students.

> I did consider trying to connect with them. And the fact that I am a minority as well, but then that conflicts with 'you and I' statements. Just because I'm a minority doesn't mean that I'm going through the exact same thing as another minority." [Palestinian, female TC, secondary]

Because minoritized backgrounds can be seen in multiple ways (a strength or a challenge), simply recognizing one's racialized identity does not necessarily lead to feeling empowered to act. It can serve as a springboard or a roadblock.

The theme of wanting to "Do the right" thing also arose among TCs of Color, but tended to relate to one's ethical identity (e.g., propelling TCs to want to advocate for their students), something we discuss later.

Personality

In addition to racialized identities, self-described personality traits played a role for several of the TCs when considering how they would respond. Although personality can include more self-monitoring aspects than traditional definitions of identity, we include it here because it was explicitly raised by our participants. That is, when asked about their identities, they commented on how being "shy/unsure," a "peace maker," or "the kind of person who speaks their mind" would influence how they respond. Sometimes these personality traits served as a springboard for action, and other times they seemed to serve as roadblocks.

> If you ask my friends, they'll say […] I'm a very opinionated person […] when I believe in an opinion, it's hard to dissuade me off of it. [Asian American male TC, secondary]

> I think it would be hard to first, stand up by yourself because I'm shy, and second, to try to convince a room full of people to agree with an unpopular opinion. [Latina woman, elementary]

The first quote illustrates how identifying as someone who is "opinionated" may serve as a springboard to action, whereas the second quote suggests that being "shy" may serve as a roadblock to advocating for one's students. Below, we see a TC who begins with the idea that her personality will lead her to address the situation head-on, but then she quickly takes a more cautious tone when considering whether, as a white person, she will be able to do the right thing and intervene on behalf of her students.

> I like to be direct and address situations head-on in most cases. If it is something to do with an identity that I don't personally identify as, however, I struggle a bit with knowing what to do or what would be best because I don't want to make any mistakes in how I handle the situation. [white woman, secondary]

In this case, it is unclear if her fear of making a mistake will prevent her from acting, even though she identifies as someone who is "direct," but it raises the question: How do issues of personality intersect with racialized identities?

We consider that question by considering the following two TCs who share a personality trait but differ in racialized identities.

> Maybe it comes from the fact that I'm a middle child in a large family. I often was the peacemaker [...] the person who would come in between two people when they're fighting and try to get them to understand other perspectives. So, I don't know how good I am at that, but I do know that that often is my default in certain scenarios. [Palestinian female, secondary]

> I do just try to take the stance of like, I'm a very neutral, like, I'm not like, even in all of this, like election stuff, I don't talk about anything, because I like I just like, I like to make peace not create chaos or anything. [white female TC, secondary]

In these cases, we see that the Palestinian TC who describes herself as a "peace maker" sees that trait as an asset in intervening, whereas the white TC who says she likes to "make peace" seems to use that trait as an excuse not to "create chaos or anything."

Overall, while self-identified personality traits arose as an expressed variable in both TCs of Color and white TCs, we noted that TCs who, when asked about identity, commented only on the role of their personality and not race/gender (in terms of affecting how they made sense of, or might respond to, the scenarios) tended to be white. Those teacher candidates seemed to use personality in lieu of racialized identities. This interaction between racialized identities and personality traits has piqued our interest and is something we intend to consider further in future analyses.

Grade level

Less prominent were effects of grade level identities on the ways TCs engaged in the activities. We saw some evidence that elementary TCs were less likely to talk about their racialized identities and tended to project a value for Color-evasive teaching. For example, in one site, teacher candidates' racialized identities were mentioned explicitly in only one of eight breakout groups. When discussing how to respond to a scenario where the Black student accused the teacher of not helping them because of their race, several TCs expressed an interest in pointing out to students that race does not play a factor in the situation. This framing arises in the following group discussion.

> You could talk to the student(s) about how maybe you just didn't see the student's hand up and is willing to hear him/her out because everyone's voice is important in her class regardless of race, age, gender, color, hair color, etc [...] you could make it a group discussion to show how things are not about race, age, color, etc in the classroom. [Group, elementary TCs]

As mentioned, several of the TCs talked about how reading and engaging with the scenarios triggered strong feelings. Yet, the elementary TCs, more so than the secondary TCs, tended to focus on the need to regulate those feelings or remain "level headed." However, this distinction was also difficult to parse from identities related to their specific programs and teaching contexts where cooperating teachers might be conveying the preference for this Color-evasive stance. We suspect that because most of the scenarios felt difficult, there was less room for a distinction between the kinds of responses that elementary and secondary TCs might exhibit. Our use with other tools, such as the Similarity Sort (where TCs explore the stories that get told in mathematics and teaching) might show more significant differences between elementary TCs (who tend to focus on loving and caring for young children) versus secondary TCs (who tend to focus on conveying mathematical content).

Mirror test

Elsewhere, Gutiérrez (2015) conceptualized a form of ethical identity for teachers called The Mirror Test, wherein teachers need to be able to look themselves in the mirror and assess whether their actions reflect what they say they stand for. An important part of this stance is identifying to what/whom they hold themselves accountable. Different teachers will have different identities they are trying to uphold, or different Mirror Tests they are trying to pass. Some teachers might focus on fidelity to school mandates/reforms to be seen as a "professional," whereas others might focus on serving their most vulnerable students to be seen as "ethical," or to being seen as a "team player" by colleagues. We saw the Mirror Test as distinguishing between the kinds of reactions that TCs gave. Part of the Mirror Test for many TCs was being student-centered (responding to students' needs, empathizing with them in the scenarios). And, in these cases, TCs felt compelled to advocate. We saw evidence of this attention to students playing out across racialized identities and grade levels.

Teacher candidates who felt the scenarios violated their mirror tests did not always know what to do. But, they felt compelled to *do something*.

> I think a lot of the scenarios, in the 'I can do this now' [category], is, I don't know exactly what I would do. But I feel like I wouldn't be able to just listen to that statement and not do anything in the moment. But for this [scenario where a teacher suggests she can rely on her Asian American girls to tutor other students in math], I think I would just, I would just politely phrase to the teacher that, "You know, as teachers, we shouldn't be putting our students in boxes or labeling them [Palestinian female TC, secondary]

One TC described "pulling a Dead Poet's society," referring to a movie about a white teacher in an elite boarding school who gets fired for encouraging his white students to live their lives on their own terms. He clarifies the phrase, saying, "Dead Poet's society is an analogy about engaging students and then getting fired. It's a good movie for teachers to watch." [white man, secondary] This TC seems to be projecting himself as the kind of person

accountable to his students and not the school or administration, so much so that he would get fired for being so student-centered.

Other Mirror Test examples arose as several TCs articulated how they would need to respond because of the kind of teacher they believed themselves to be.

> In response to Scenario A [where a student called you out for not helping them because they are Black], I feel like it would just be kind of like really heartbreaking to hear a student say that […] to think that I'm not presenting myself or the way that I'm teaching is coming off in a way that like I have a prejudice, prejudice against certain students would be really difficult to hear, because then I think I would just have to do a lot of reflection, like personally […] it's definitely necessary to address that. [Latina female TC, secondary]

> […] we should always word things in a way that we are always on the side of our student, even if the student does something wrong or isn't flourishing as much as we'd want them to. We want students to know that we're on their side and that we're, we're here to support them. Even if the student isn't right in front of us, I feel like that should be our mindset. So I'd want to respond with something that would be in support of our students." [Palestinian female TC, secondary]

Each of the aforementioned TCs were commenting upon how they would hold themselves accountable to students of Color above other things, such as getting along with colleagues or "going with the flow." This sense of solidarity was extended to students who did not share their racialized background (e.g., TC of Color and a white student or a white TC and student of Color). Even so, being student-centered did not necessarily lead to TCs engaging in racial/gender analysis of the scenarios.

> […] it really is so important to stick up for the students in your classroom. I always knew it was the right thing to do, but with the dilemmas that we were given, I realized how important it actually is. [white woman, elementary]

In these cases, TCs saw themselves as accountable to students in a generic category ("all students") versus recognizing that certain students (e.g., students of Color, girls) were being stereotyped or harmed in the scenarios.

Overall, our findings indicate that for the teacher candidates with whom we worked, being highly conscious of their racialized identities, personality traits, grade band focus, and the kind of teacher they want to be served to elicit a range of responses, sometimes leading to a greater inclination to advocate for students (i.e., a springboard for action) and, other times, a hesitation around doing the "correct" thing (i.e., a roadblock to action). We also witnessed interaction effects (e.g., between personality, racialized identities) that demonstrated how a single attribute (e.g., personality) was not the primary predictor of advocating for historically minoritized students. Given the highly context-dependent nature of this work, there is no universal policy we can offer about making TCs aware of specific identities in order to promote advocacy for students. As such, we surfaced tensions in recommendations for future tool design. Should the tools help TCs acknowledge their racialized identities? Should the tools elicit a strong emotional response from TCs? Should the tools be reformatted to include different contexts that might correlate better with grade level focus? These are questions we will continue to grapple with as we move forward.

In this paper, we focused on race/ethnicity and gender identity in relation to TCs' Political Conocimiento in Teaching Mathematics. We found that for TCs who could empathize with students in the scenarios, prompting them to consider their identities might have allowed them to understand why, even if they were not sure of *what* to do, they felt compelled to act in order to reflect an identity they could be proud of. And, TCs who could not easily relate to the students in the scenarios might have used their identities as rationales to avoid intervening. These findings are especially important, as much of the literature on "best practices" in teacher education in the US is normed on white teachers. Future research should continue to investigate the role of these TCs' intersectional identities and also expand to include sexual orientation, religion, linguistic backgrounds, living with a dis/ability, etc. Moreover, further investigation of differences by grade level (elementary or secondary) and contexts (e.g., states that ban Critical Race Theory, teacher education programs with an explicit anti-racist or justice focus) will help the field better understand factors that influence TCs' commitment to take action. Programmatic features may also influence TCs' development of Political Conocimiento in Teaching Mathematics. That is, teacher education programs with an explicit anti-racist or justice focus, including cooperating teachers who share such commitments, may better support TCs to develop PCTM. Finally, TCs should be followed to see how they engage in the suite of tools over time and to document any possible lasting effects of such professional development on their beginning years of teaching.

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Secondary mathematics teachers in England who think outside the hegemonic discourse of "ability": Situation and horizon

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A neo-liberal discourse has been hegemonic in England for several decades and includes "ability" as a fundamental plank leading to and legitimising divisive and unjust grouping practices in schools. Some teachers, however, resist this discourse and believe and succeed in enacting all-attainment teaching in secondary mathematics. This paper is concerned with such teachers and, using Gadamer's concepts of situation *and* horizon*, offers a model for how they experience the world.*

Introduction

A neo-liberal discourse of education has been hegemonic in England for several decades sweeping aside all previous assumptions (Ball, 2017) and reframing education as a consumer good rather than a moral enterprise (Povey & Adams, 2018). The discourse promulgates the idea that the unequal way that society – and schooling - is structured is natural, simply the result of free individuals and their free choices within a market. Thus, it seeks to legitimate inequality, a process that leads those at both the top and bottom of society "to see themselves as largely responsible for their own places in it" (Oakes, 1982, p. 197). The associated discourse of "ability" forms a central plank in this neo-liberal discourse, in turn legitimating divisive and unjust grouping practices in schools.

The research[1] from which this paper is drawn focused on those who challenge the practice (almost universal in secondary mathematics classrooms in England) of grouping students according to their supposed "ability". The research participants were a small number of teachers who do not accept this "common-sense" thinking but who instead believe that all students should have access to all of the curriculum and that all students are capable of learning without limits (Hart, 2004). Using both a narrative and a thematic approach, the

[1] Much of the text here is drawn directly from Jackson, 2019. The research data and the models developed are the work of the first author; the second and subsidiary author contextualised the research and developed the paper itself.

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research attempted to identify aspects of who these teachers are and how they find a way to resist the hegemonic education discourse. As anticipated, the teachers were committed to social justice in education; but they also shared attitudes and practices related to pedagogy. Beyond this, however, they also shared a different world view, one in which they understood their current *situation* (Gadamer, 1960/1975) as having an *horizon* (ibid.) and therefore not the only way things could be.

The neo-liberal education discourse in England

In England, the hegemonic neo-liberal discourse of education has swept aside all previous assumptions (Ball, 2017) about the purposes of education, the professionalism of teachers and what constitute moral and fitting ways of working with children and young people. A previous discourse that at least recognised the effects of systemic disadvantage has been replaced by one of blaming "failing" children, "failing" teachers and "failing" schools. Individuals are constructed as making free choices within a market, with the neo-liberal rhetoric, as always, disguising the coercion (Mattei & Salour, 2019).

An epidemic of testing beginning at age four has contributed to making children in the United Kingdom among the lowest in Europe for their declared educational well-being (UNICEF, 2013) as does the narrowness of the curriculum and teaching to the test. The explicit aim of our national compulsory curriculum is "knowing more, remembering more and being able to do more". Ofsted is the government service which polices the education system, in particular schools' performance in national tests (Perryman, Maguire, Braun, & Ball, 2018). As we have noted elsewhere (Povey & Angier, in press), in its *School Inspection Handbook* (Ofsted, 2021) there are 89 references to *knowledge* and 58 to *skills* but only 2 references to *thinking*, 2 to *creativity* and only 1 to *curiosity*.

Teachers too are subject to a relentless system of auditing and measuring with both teachers (Black, 2019) and students (Reay & Wiliam, 1999) being known by their "scores". They find themselves drawn into practices which they find morally dubious (Goodley, 2019) and a performative culture pervades all their experiences in school. As one early career teacher said:

> Performativity is known to all teachers whether by name or not. Its influence taints all the day-to-day activities of teachers and subconsciously affects the way they view their role. (Povey & Adams, 2017, p. 58)

The effect is to erode their sense of independence and moral authority and to challenge their collective professional identities (Povey & Adams, 2017). Many teachers[2] are attempting to "restory" themselves (Stronach et al., 2002, p. 130); there is indeed a battle over the teacher's soul (Ball, 2003).

[2] Personal experience and anecdotes from a wide range of sources suggest that this restorying has become much more prevalent during the COVID-19 pandemic, laying bare as it has both the inequalities and injustices of the current situation and the capabilities and professionalism of education workers.

Social justice, "ability" thinking and grouping practices

An essential plank of the neo-liberal educational discourse in England is the notion of "ability". The term is very seldom interrogated; it is simply accepted as unchallengeable "common-sense" that children come to school with different "abilities" which are measurable and more or less fixed. Thus school-based discussions of "ability" often take for granted:

> a series of assumptions that, while exerting a considerable influence on the life in school, are rarely voiced in an explicit or systematic way. (Gillborn & Youdell, 2000, p. 52)

In England it is now rare to find a secondary school where the pupils are not in "ability" groupings for mathematics (Francis et al., 2017). In addition, there is evidence that children in primary schools are increasingly being put into "ability" groupings (Hallam, Ireson and Davies, 2004; Drummond and Yarker, 2013). This is despite the very substantial research evidence showing that this is undesirable for students and unfair and unjust in its outcomes.

Allocating students to "ability" groups is a social justice issue, always disadvantaging somebody, and in England, amongst others, disadvantaging working-class children (Cahan, Linchevski, Ygra, and Danziger, 1996, p. 30) and some groups of black and Asian heritage students (Gillborn & Youdell, 2000). Research on "ability" grouping indicates that it has little, if any effect, on attainment averaged overall but has long term detrimental effects in terms of personal and social outcomes (Nunes, Bryant, Sylva, & Barros, 2009; Boaler, 2005). Indeed the social effects of "ability" grouping are found to be overwhelmingly negative (Boaler, 1997; Cahan et al., 1996; Marks, 2013). The practice is damaging to children's development with the long-term effect, continuing into adulthood, of resulting in more limited horizons, stunted intellectual growth and a lower self-evaluation of one's worth and the possibilities available (Boaler, 2005; Cahan et al., 1996).

Methodology, methods and analysis

The research project was framed within critical theory, actively acknowledging that knowledge and discourse are not neutral but are socially constructed, and with the intention of being "deliberately political" (Cohen, Manion, & Morrison, 2007, p. 26). It was conceived as an attempt "to free academic work from capitalistic domination and to help schools and other institutions to become places where people might be socially empowered rather than subjugated" (Tedlock, 2007, p. 153), to act as an emancipatory force in the world. Both authors are committed to pursuing social justice aims in and through mathematics education and the research was conducted from that declared standpoint.

Lather (1991) believes that for critical theory research to be taken seriously it must be trustworthy and needs to display catalytic validity: catalytic validity enables the participants to understand the reality of their lived existence and empowers them to do something about it. There is evidence that, for at least some of the participants, enhanced understanding and commitment to (continuing) action resulted from participation. In addition, we wanted the research to impact on the world more widely. So the key questions for us are: is the research credible? and does it have the power to act on the world?

Adopting critical theory had implications for the research design: "a critical epistemology assumes that valid knowledge is obtained in part through shared understandings, reflexivity, sensitivity to insiders" (Jungck, 1996, p. 624). Thus, the participants identified were six teachers committed to teaching their students in all-attainment groups. The intention was to uncover their attributes, intentions, outlooks and perceptions of all-attainment teaching.

Semi-structured in-depth interviews were conducted (with some ethnographic details collected to support these). Interviews as a research method fit well with critical research as they are adaptable; there are opportunities for personalisation and construction of knowledge through shared meaning. The interview transcripts and individual portraits constructed from them were returned to the participants for their comments. Of the six participants, five replied. Three of those suggested minor alterations, the other two accepting the portrait without asking for any changes. Indeed, one of the participants replied:

> I've just read your interview notes and they are extremely accurate. I had no idea I'd revealed so much! (Sarah, email correspondence)

All of these participants reported that the transcripts and the portraits were an accurate reflection of what they had said. These responses suggest that the data are reliable and offer (internal) participant validity (Cohen, Manion, & Morrison, 2007).

Constructing the narratives was the initial approach to data analysis. Some common patterns of response emerged and so an inductive thematic analysis was also undertaken. The analysis was extended, thorough and systematic (see Jackson, 2019, pp. 74–79 for details). The findings from this analysis produced three overarching categories which were:

- the teachers – what sustains them
- introducing, developing and maintaining all-attainment teaching while convincing others
- how the teachers make all-attainment work in the classroom.

Each of these had a number of themes and subthemes (Jackson, 2019, pp. 74–79). A final stage in the analysis involved stepping back from the details of these analyses and, drawing at this stage on Gadamer (1960/1975), coming to an understanding of how these teachers see themselves in relation to the world. It is this which is the subject of this paper.

Situation and *horizon* and English secondary mathematics teachers

After the thematic analysis was complete, it became apparent that, cross-cutting the themes, there was something else: there was a sense that how these teachers *were in the world*, their sense of themselves in relation to the world, what they took as being self-evident or given, was somewhat different from the current lifeworld of most secondary mathematics teachers in England. Drawing on understandings from Habermas (1987), we see that lifeworlds – those "panoramic constellation of contexts, realities and understandings" (Dufays et al., 2020, p. 3) – can become colonised by economic and political interference, creating "a growing sense of powerlessness felt by individuals in relation to their ability to self-determine their lives" (p. 2), including their working lives, and the erosion of shared experience within a community.

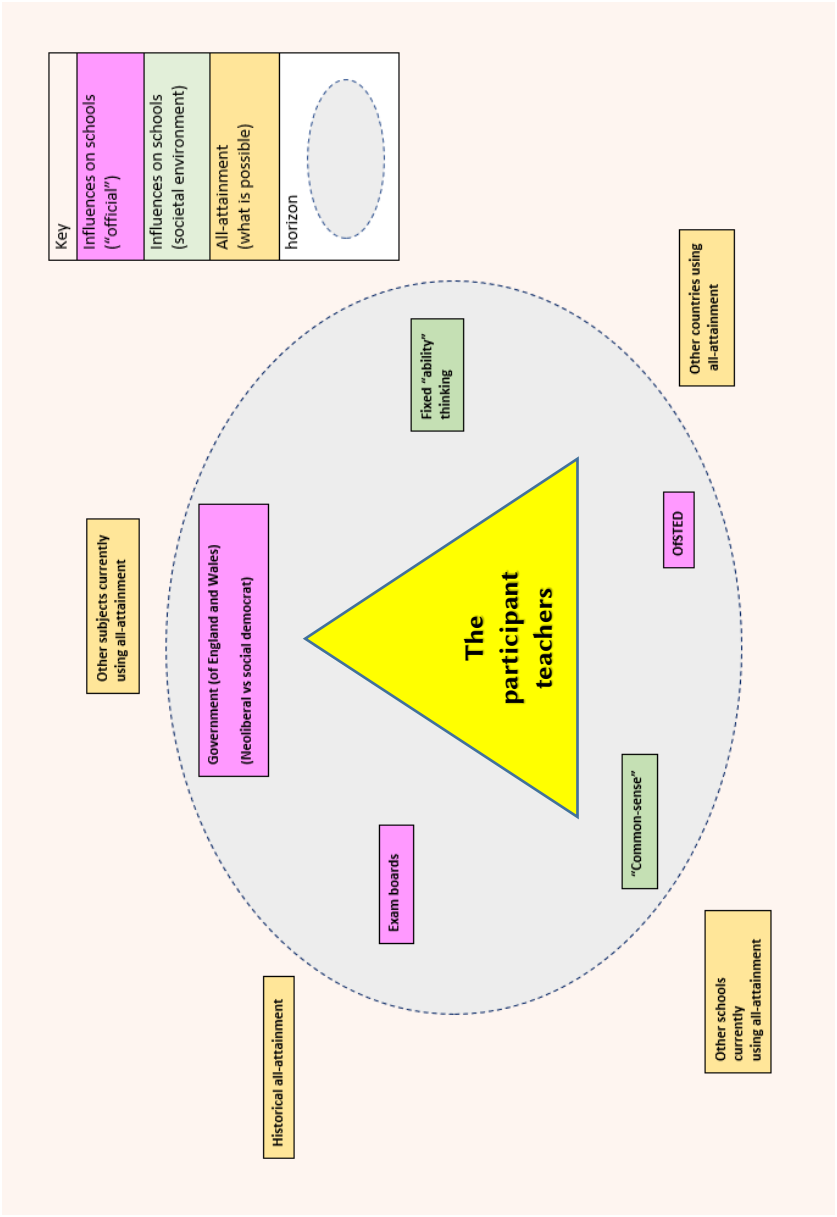

Figure 1: The teachers, the situation and beyond the horizon. (Jackson, 2019, p. 143)

In trying to understand this difference better, it has been useful to draw on the work of Hans-Georg Gadamer (1960/1975), in particular his two interlinked concepts of *situation* and *horizon*:

> We define the concept of "situation" by saying that it represents a standpoint that limits the possibility of vision. Hence an essential part of the concept of situation is the concept of "horizon". The horizon is the range of vision that includes everything that can be seen from a particular vantage point ... A person who has no horizon is a man (*sic*) who does not see far enough and hence overvalues what is nearest to him. (1960/1975, p. 269).

For Gadamer our understandings always take place in a particular situation, one in which we are entangled, one dependent on our history. This situation is the standpoint from which we experience the world, our vantage point. Our vision, then, is inevitably limited and has an horizon. However, our relationship with the horizon is not predetermined and varies between different people in the same situation. It is possible to experience our situation as *all that there is and all that there could be*. The way things are is inevitable and how things must be: there is no possible beyond.

In the context which we are considering, the current dominant educational discourse and the educational world that it produces are the situation for all teachers. Included in this is the ideological commitment to "ability" thinking and we see that, for many secondary mathematics teachers, the putative differences in something innate and relatively unchangeable – the ability to succeed in mathematics – is an unquestioned "truth" (Marks, 2013; Gillborn & Youdell, 2000). The landscape is horizonless and what is currently inhabited is all that there is and all that there could be.

But for the participants in the research study, in the same situation, they can see an horizon to the current landscape.

> To have an horizon means not to be limited to what is nearest, but to be able to see beyond it. A person who has an horizon knows the relative significance of everything within this horizon, as near or far, great or small. (Gadamer, 1960/1975, p. 269).

So they both see the situation they are in more clearly and can also envisage that things can be different from how they are proposed in and produced by the hegemonic educational discourse.

Figure 1 represents how the participating teachers see and understand the framework within which they are operating. They engage critically with and question the world within which they are situated. They reject the limitations on their students imposed by the hegemonic common-sense of fixed "ability" thinking (Gillborn and Youdell, 2000; Archer et al., 2017). Because they see the horizon to this thinking, they can see the opportunities beyond it.

In doing so they are open to thinking about the possibilities from both historical and current day perspectives. They are able to use these resources for communicative action (Habermas, 1987) to resist colonisation of their lifeworlds and to redefine the institutional arrangements imposed upon them (Dufays et al., 2020; Stronach et al., 2002). They are able to consider possibilities that have arisen in the recent past in England; for example, in the

late seventies/early eighties when all-attainment existed in around a fifth of secondary mathematics lessons (Ruthven, 1987). They can examine the practices in other curriculum areas in schools in England where all-attainment still exists today to a much greater extent than in mathematics (Archer et al., 2017). They can look for schools where all-attainment in mathematics is still practised – there are still a few. They can look at other countries and see that all-attainment may not just be a practice but a legal requirement (Gates & Noyes, 2021, p. 48); for example, it is illegal to put students into "ability" groups in Sweden due to issues of equality (Boaler, 2005). These teachers question the *status quo*, knowing that there are alternatives that serve all students better; they imagine what could be and attempt to enact it in their teaching, so things are better for all of their students, not just a select few.

In addition, all-attainment for these teachers is not only about the students' experience of learning mathematics important as that is for them: they are also concerned with the students' future life chances. Unsurprisingly, for most of these teachers the field of social justice impinges on the field of mathematics education informing their decisions. Bob expresses this quite succinctly:

> My reasons for mixed-attainment are the philosophical reasons of social justice and giving everyone a chance and putting the responsibility on the students, giving them the opportunity to show what they can do rather than me say 'This is what you can do and bad luck whether you think you can do anything else, you're not going to get the chance'. [Bob, transcript]

These teachers try to build an interdependent community (Healy, 2019) of independent learners and to enable the students to develop as critical thinkers so that they are not limited by their situation and they too develop an horizon, so they too can see beyond their current situation.

> The more you get people to (question the world), the less likely they are to then simply accept things that are handed down from above. They feel an equality. Because they have this richness and ability to [think] if I don't know it yet, I'm open to learning and I want to learn it, and I'm capable of thinking, capable of learning it, and therefore I won't simply have to bow down and accept and [not] question authority or expertise, I can challenge, question. You might tell me something, you might know more [than me] about this but I'm perfectly capable of thinking about that, if necessary going away and learning some more about it, and questioning. (Pete, interview transcript)

They believe that unless the students develop their critical thinking capacity, they cannot fully exercise their right to engage critically with and question the reasons why society and, indeed, schools are the way they are and to contribute to a more socially just world.

Conclusion

In this paper we have argued that teachers in England who are able to resist having their educational thinking colonised by the hegemonic neo-liberal discourse reject "ability" thinking and the associated practices of grouping students. Further, we have argued that one way of understanding these teachers is by considering their relationship to the *situation* in

which they find themselves. Specifically, we have drawn on Gadamer's notion of *horizon* to model how they are in the world and to draw attention to how what lies beyond the horizon can support more radical thinking. In turn this supports the teachers in working towards a more just mathematics education in schools.

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Counter-storytelling of La Raza at the Borderlands of race, language, and mathematics

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Through composite counter-storytelling, we share the collective experiences of Raza learners that highlight the relentlessness of racism and linguicism within white institutional spaces. We highlight the experiences of 11 fourth grade Raza learners who find themselves within the Borderlands of Raza and Anglo cultures. We use Borderlands, a space of inbetweenness, to foreground the mathematical experiences, contradictions, and ways of resisting in relation to race and language for Raza learners. We argue Raza learners should be able to embrace themselves as inhabiting Borderlands and be guided towards a path of conocimiento, where a deeper understanding of themselves and their place in the world produces knowledge from their unique perspectives.

Introduction

We have been gifted the opportunity to learn with 46 Raza[1] students in grades 3–5 and recognize the privilege and urgency to share their experiential knowledge with others in order to create more equitable spaces. This collaboration has pushed us to think more deeply about the experiences of Raza learners inhabiting predominantly white spaces, both numerically and ideologically. Therefore, we feel a responsibility to not only acknowledge their contributions to this work, but also share their experiential knowledge within a larger community working towards social justice. In this paper, we share the development of a composite counter-story of 11 Raza fourth grade students and their experiences learning mathematics while navigating white institutional spaces.

White institutional spaces refer to the "ideologies, discourses, and practices in [spaces which] serve to privilege white perspectives, white ideological frames, white power, and white dominance all the while purporting to represent [the spaces] as neutral and objective"

[1] We use Raza and La Raza as a political tool to disrupt the oppressive nature of Latinx and Hispanic being imposed on Raza communities and perpetuating Eurocentric storylines (see Anzaldúa, 1987; Martínez, 2017; Gutiérrez, 2001).

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(Martin, 2015, p. 443). One way our society creates white institutional spaces is through the use of storytelling. Anglos have centred their experiences and ways of knowing as Truth and passed down their stories through academic and non-academic spaces such as school, history classes, books, media, and news outlets. They have silenced the stories, experiences, and ways of knowing of People of Color and positioned our knowledge as un-American, unpatriotic, reverse racism, untrue, and inferior. Furthermore, storytelling from an Anglo-only perspective perpetuates white saviour narratives, protects white culture, and encourages the healing of genocide, war, and slavery by forgetting, avoiding, and providing inaccurate information.

Some cultures have been able to assimilate into the dominant culture and become white (see Yancey, 2004; Ignatiev, 2009). Yancey (2004) argued that in order for a culture (or person) to become white, they must repudiate their ancestral cultural values, and then adopt the beliefs, language, customs, and narratives of white culture. For example, the Irish are now recognized as white and part of the dominant culture due to their skin colour, adoption of dominant ideologies of a superior white race by perpetuating racial hierarchies, and abandoning their ancestral Irish culture (Ignatiev, 2009). When the United States claims all are welcome, it is on the condition that those from other cultures will strive to become white and take up the white culture dominant within the United States.

However, when cultures do not adopt the values of the dominant culture and instead choose to maintain their own cultural values and strive for equitable spaces, the dominant culture responds with *white rage* (Anderson, 2017) and keeps these communities on the margins of society. A recent example of white rage is the insurrection on the United States' capitol building on January 6, 2021 where white supremacists attacked with intentions to hinder the official counting of votes for a new president. White rage can also be seen when slavery was banned in the American south and African Americans tried to move to the northern US for more opportunities, but "white Southerners saw black advancement and independence as a threat to their culture, and, indeed, their economy" (Anderson, 2017, p. 46) and created laws preventing the recruitment of African Americans for employment in the north.

The process of schooling in the United States is a project to ensure students are provided the necessary skills to assimilate into white institutional spaces (Martin, 2015) or become more white (Carbado & Gulati, 2014). Students who are not part of the dominant culture are expected to assimilate into the dominant culture and forego any aspects of other cultures (Orozco, 2012). Students who learn to assimilate are considered successful and good students; whereas students who preserve an ancestral culture and values are seen as uneducable and burdens on society (Martinez, 2020). Furthermore, students from non-dominant cultures must traverse through multiple cultures, making sense of the tensions, contradictions, and experiences of not being fully part of one or the other, but located in between cultures.

Raza communities, however, must navigate discrimination based on race and language and are dehumanized in various ways despite our degree of assimilation into Anglo cultural

values and language. Flores and Rosa (2015) describe raciolinguistic ideologies as "conflat[ing] certain racialized bodies with linguistic deficiency unrelated to any objective linguistic practices" (p. 150). Therefore, placing Raza learners in contradictory spaces, where schools ask Raza students to assimilate, while also preventing them from ever fully reaching whiteness due to "looking like a language and sounding like a race" (Rosa, 2019, p. 2). Raza students are habitually othered due to their skin colour determining their linguistic repertoire as well as their linguistic repertoire positioning them as Other.

We share the development of a composite counter-story (Solórzano & Yosso, 2002; Yosso, 2006) of the tensions, contradictions, and ways of resisting while learning and doing mathematics as described by a few Raza learners. We aim to emphasize how white institutional spaces are not made with Raza communities in mind and therefore need to be revealed, questioned, and disrupted. We use Latinx critical theory to frame our exploration into how Raza students are racialized as well as provide our methodology of composite counter-storytelling (Solórzano & Yosso, 2002; Yosso, 2006). We couple Latinx critical theory with Borderlands theory (Anzaldúa, 1987) to explore how white supremacy and racism develop tensions and contradictions at the borderlands. These frameworks complement one another to centre the voices and unique experiences of Raza, challenge dominant narratives of how race and language play a role in the learning and doing of mathematics, and work for more socially just spaces for Raza students.

Theoretical framings

Latinx critical theory framed our exploration of Raza students' experiences in predominantly white spaces, allowing us to reveal how race and language are used to position Raza as inferior in a racial hierarchy. We overlap Anzaldúa's (1987) work on Borderlands theory with Latinx critical theory to dig deeper into how Raza students are navigating white spaces as a hybrid of Raza and Anglo cultures. In the following sections we describe each of these theoretical frameworks in more detail.

Latinx critical theory

Latinx critical theory guided our exploration to understand the lived experiences of Raza students in predominantly white spaces (see Solórzano & Yosso, 1998; 2001; 2002). Latinx critical theory is an extension of critical race theory to focus on how Raza communities are racialized due to phenotype, surname, language, and citizenship (Flores, 2018; Yosso, 2006). The tenets of critical race theory vary; however, the following tenets guide our work: 1) race is a social construct and racism is endemic in our society; 2) challenging dominant ideology; 3) centring experiential knowledge of People of Color; 4) progress towards change happens through interest convergence with the dominant culture; 5) commitment to social justice; and 6) the use of interdisciplinary studies (Delgado & Stefancic, 2017; Solórzano et al., 1998). Next, we briefly describe each tenet and how they inform our work.

Critical race theory is conceptualized through ethnic studies, women studies, and anthropology in order to highlight how racism impacts the experiences of People of Color.

Anglos positioned themselves as superior through their words and actions that dehumanize indigenous people, Africans and African Americans, and Raza communities through genocide, slavery, and war (Dunbar-Ortiz, 1994). Race is socially constructed in order to establish a dominant culture (middle- and upper-class Anglo males) in all aspects of our lives: education, labour, society, and politics. Furthermore, Solórzano and Yosso (2002) described critical race methodology that puts into action critical race theory in education by challenging dominant narratives, centring the voices of People of Color through intersectional lenses, and working towards social justice. Critical race scholars caution, however, that the dominant culture will not willingly give up control, but progress is only made when there is benefit to the dominant culture as well, or an interest convergence (see Bell, 2018). Latinx critical theory provides a specific lens to understand how the previously mentioned tenets are experienced within Raza communities. We use Borderlands theory in conjunction with Latinx critical theory to think more critically about the conflicting spaces Raza students must navigate in a white supremacist society and predominantly white schools.

Borderlands theory

Anzaldúa (1987) conceptualized Borderlands theory through her lived experiences growing up at the Mexico-U.S. borderland which forced her to traverse multiple cultures where she was neither fully immersed in a Mexican culture or an Anglo culture, but a hybrid of the two. Anzaldúa used borderlands to represent the physical borderland separating land either naturally (e.g., oceans, rivers) or unnaturally (e.g., walls). On the other hand, Borderland with an uppercase "B" is described as the emotional state of inhabiting a space of inbetweenness, or nepantla, which refers to the multiple cultures she inhabited. Anzaldúa highlights the unique perspective and lived experiences of those navigating Borderlands where knowledge can be gained through the complications of being both/and as opposed to one or the other. She argued "borders are set up to define the places that are safe and unsafe, to distinguish *us* from *them*…a borderland is a vague and undetermined place created by the emotional residue of an unnatural boundary" (p. 3). It is a space of the unknown, created and determined by each person living within multiple cultures "where one does not know whether to assimilate, separate, or isolate from the demands of the dominant culture" (Aguilar-Valdez et al., 2013, p. 829). Inhabiting a Borderland means one is not fully accepted within either culture but must navigate both cultures simultaneously, creating a new culture from knowledges and experiences within this space of inbetweenness. Anzaldúa (1987) described this duality in terms of Mexican-Americans living at the Mexico-U.S. border:

> Nosotros los Chicanos straddle the Borderlands. On one side of us, we are constantly exposed to the Spanish of the Mexicans, on the other side we hear the Anglo's incessant clamouring so that we forget our language…This voluntary (yet forced) alienation makes for physiological conflict, a kind of dual identity—we don't identify with the Anglo-American cultural values and we don't totally identify with the Mexican cultural values. We are a synergy of cultures with various degrees of Mexicanness or Angloness (p. 63).

Raza in the U.S. are neither one nor the other, but a hybrid of the two cultures, both Anglo and Mexican cultures influence ways of being, knowing, and understanding the world. Anzaldúa argued, because of this hybridity and influence of multiple cultures, experiences, and knowledges, nepantleras (those inhabiting Borderlands) are uniquely positioned to create and provide new knowledge.

Borderlands theory is a process one encounters when they are perpetually othered:

> Social hierarchies presuppose that the world is composed by rigid and definite categories of superiority and inferiority, where superiority is commonly associated with folks who are white, Christian, middle or upper classes while inferiority is related to those who fall outside of the previous category and are considered different or defective (Orozco-Mendoza, 2008, p. 42).

Inhabiting a Borderlands means one goes through a process beginning with an arrebato, or a trauma such as being othered due to skin colour or language. Once an arrebato is experienced, Anzaldúa described a person enters a state of nepantla where they can either follow a path of desconocimiento or conocimiento. Desconocimiento results in assimilating to the dominant culture whereas conocimiento is a path "to reverse the colonization that has been passed on…through cultural practices and the construction of myths that operate in everyday discourse" (Orozco-Mendoza, 2008, p. 42). Taking the path of conocimiento allows one to imagine a reality beyond that of the dominant culture and provide a unique perspective of ways of being and knowing. Once on the path to conocimiento, however, arrebatos continuously occur, making the journey everlasting, where the path of conocimiento is only possible through the continuous healing from arrebatos.

Methods

46 Raza learners in third, fourth, and fifth grades across three predominantly white schools—more than 50% of the student population identifies as white while less than 20% of the student population identifies as Raza—in the south-eastern U.S. took part in a research collaboration. For this study, we focused on the 11 fourth grade learners who participated in one semi-structured interview ranging from 12 to 45 minutes. The interviews focused on the participants telling stories about their experiences regarding race, language, and mathematics. For example, participants were asked: Has there ever been a time you didn't feel like you fit in because you identify as [nationality] (e.g., Mexican, Chilean, Argentine)? How does knowing Spanish help you in learning math? If your teacher could teach math in Spanish, would that be helpful to you? Why or why not? Following Latinx critical theory methodologies, we used a grounded theory approach to develop a composite counter-story based on Raza learners' experiences.

There was a conscious effort to not adultify the Raza children, their discourse, and experiences; keeping in mind their understanding of race is different from adults (Quintana, 2007). Quintana (2007) argued most elementary age children either have a literal perspective of race where physical characteristics are the main criteria for one's race, or a social perspective of race where "children begin integrating their own observations of their social world into their verbal reasoning about race" (Quintana, 2007, p. 26). The range of understanding of race needed to be considered as we conducted and analysed their interviews.

We use composite counter-storytelling (Solórzano & Yosso, 2001, 2002) to describe the experiences of the 11 fourth grade Raza learners with the Borderlands they inhabit, which allow them to learn and grow within multiple cultures. Composite counter-storytelling is a method used in critical race methodology to centre the experiences of People of Color and recognize individual narratives as experienced by a collective (Solórzano & Yosso, 2002). Therefore, composite counter-storytelling allows for a more complete understanding of the experiences of the Raza learners. Furthermore, race and language are vastly complex and unique experiences, however, composite counter-storytelling allows the overlay of individual experiences to provide a more complete understanding of systemic racism.

Following Solórzano and Yosso (2002), we incorporated a grounded theory approach (Glaser & Strauss, 1967) to understand how varying degrees of Latinidad and Anglo culture play a role in navigating predominantly white spaces. Through initial readings of the transcripts, we noted any utterances about context or language. We used data that was both similar across participants as well as unique experiences that stood out. These experiences became themes we used to explore for evidence in each of the other participants. After multiple iterations, these themes were combined or expanded upon to become codes that explain how language played a role in learning and doing mathematics for the Raza students. Following Solórzano and Yosso (2002) we paired the data from the interviews, various literature on Raza experiences, and our own anecdotal evidence to create a composite counter-story.

Composite counter-storytelling and Borderlands

The data from interviews with Raza learners guided our construction of a composite counter-story. Each of the themes were developed using the voices of the young Raza learners which became conversations with the research team relating our own anecdotal evidence to the data and literature. The utterances from the Raza learners were just a snapshot of larger stories, grounded within systems of power due to race and language (in this case). We could understand the experiences of the young Raza learners, because we too had similar experiences. Furthermore, we recognized larger systems of power in play when Mauricio said “there’s like not that much things to be talking about in Spanish during class”. This is an utterance, a small view into Mauricio’s experiences in learning math. However, it is grounded in systems of power around race and language, which we felt should be told through a composite counter-story for several reasons. As stated previously, composite counter-storytelling allows the voice of the Raza learner to be centred and access to various communities outside of academia. Furthermore, the composite aspect allows these utterances to be understood as embedded within other data, such as literature. In the example of Mauricio, his utterance provides a way to understand the effects of white superiority and how racism and linguicism play out in everyday life. In this section, we continue to highlight students’ utterances as moments of nepantla that guided our construction of a composite counter-story: a) Navigating location and audience; b) Value of the Spanish language; and c) Tensions of language in learning mathematics.

Navigating location and audience

Students often felt they needed to leave Spanish outside academic conversations while at school, however, they still identified with and used Spanish at lunch and recess. Therefore, navigating location and audience, where students must recognize safe spaces for using Spanish versus English, became an instance of nepantla. For example, Mauricio described not wanting to use Spanish in the classroom because he was afraid of getting caught, which guided his decision of when to use, and not use each language. Mauricio provides insight into how language is used against him, as a way to other and criminalize him for speaking Spanish. Raza students' entire linguistic repertoires, English, Spanish, and a mixture of the two languages, were used in other spaces such as their home, cultural events (i.e., fiestas), some stores (i.e., mercados), and with Spanish-speaking friends. We also acknowledge the depth of students' linguistic repertoire of being able to recognize their audience and make adjustments accordingly; however, they were also tasked with having to cross language borders of English, Spanish, and a hybrid of the two. For example, Carmen explained how she speaks Spanish with her parents, but English with her brother. Therefore, inhabiting a Borderland where she uses both Spanish and English depending on her audience.

Valuing of Spanish language

Raza students described another instance of nepantla as valuing the Spanish language. Students received messages from their parents that it is important to continue using Spanish because it is part of their culture. For example, Carmen discussed her father's desire for her to speak more Spanish, stating, "[my father] was saying let's just talk more Spanish because I'm like Spanish." The dominant narratives in schools, however, perpetuate English as the spoken language in the U.S. and required for academic purposes. Carmen and her father are finding ways to resist assimilation into the dominant culture and disrupt white supremacy. Furthermore, Mauricio had been led to believe Spanish is not an important language for academic purposes, stating, "there's like not that much things to be talking about in Spanish during class." Mauricio shows how white supremacist ideals of English-only spaces prevents him from using his entire linguistic repertoire. We see these two conflicting narratives told by Carmen and Mauricio to be a space where students are experiencing nepantla; they must make choices in each space about which parts of their linguistic identities to foreground.

Tensions of language in doing mathematics

Raza students also discussed navigating Borderlands of language in regard to the use of language in the mathematics classroom. Because mathematics was positioned as an English-only act in their schools, Spanish-speaking students negotiated their linguistic identities in order to navigate the mathematics classroom. Therefore, positioning Raza students as traversing the Spanish language along with the English-only norms of mathematics. Several students said they would like to learn mathematics in Spanish because it would help them learn more Spanish (Angelica, Estifan, Rocio). For example, when asked if learning math in Spanish would be helpful, Estifan stated, "Yeah… because then I would be able to speak Spanish anytime I wanted." Estifan desires to disrupt white supremacy and dominant

narratives of English-only norms in mathematics and valuing Spanish, often seen as deficit by educational institutions. Therefore, entering a state of nepantla where they must decide to either assimilate and use English-only, or push back and take a path of conocimiento where Spanish can be used as a resource in learning mathematics as well. For example, several students described their parents helping them learn mathematics in both English and Spanish. Specifically, Mauricio explained how he navigates these contradictions of language and mathematics by doing his mathematics homework in both Spanish and English. He provides his thinking in Spanish for his dad, but also does the work in English to prepare for class the next day. These narratives explain how Raza students value Spanish but are navigating an English-only mathematics classroom.

Conclusion

Using Borderlands theory to frame the composite counter-story of the 4th grade Raza students in predominantly white schools amplifies the unique experiences of navigating multiple cultures. The 11 Raza students shared multiple narratives regarding the negotiation of their linguistic repertoire. Within their Borderlands schooling pushes English-only academic spaces, while the home encourages the use of both languages to preserve their Spanish proficiency. Raza students must decide when, where, and how to make these negotiations in each space. Anzaldúa (1987) argues students in a place of nepantla choose to assimilate (desconocimiento) or disrupt dominant narratives and exist in multiple spaces and ways of knowing (conocimiento).

Asking students to assimilate into the dominant culture positions one culture superior to the other, forcing Raza students to make a decision to forego one culture over the other. In making that decision, however, one culture must be silenced. We wonder what it would mean for students to not have to decide, to not have to choose between cultures. We argue for the whole child to be accepted, seen as a resource, and able to exist as a hybrid of the cultures impacting their lives and learning of mathematics.

Finally, we can no longer ask 4th grade Raza students to navigate these spaces on their own; Raza students need support in learning about the cultures impacting their lives. This is not as simple as adding more Raza teachers, or other Teachers of Color to the workforce; the sole solution is not a mathematical one (Tate et al., 1993). Furthermore, this work goes beyond our schools; schools are one medium where racism and white supremacy are perpetuated, but these issues are systemic. While it is imperative for teachers and pre-service teachers to learn and be held accountable for the cultures of the students they teach, this cannot be the only way to address racism and white supremacy. There is also a necessity for the system and narratives in place which perpetuate white supremacy to be revealed, questioned, and dismantled. The Raza students have provided the knowledge of how they must navigate a system put in place beyond their discretion; thus, it is not their job to grapple with it, navigate inbetweenness, assimilate to one over the other, or to resolve these conflicts. Their work has been done, they have gifted us this knowledge, a glimpse into challenges, conflicts, and contradictions persistent in their lives. While we will keep fighting for and

providing counterstories of these injustices, it is time for those who benefit most from these systems to learn how to resolve and dismantle the system of exclusion of Raza and other Students of Color.

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Practices in teacher education for supporting pre-service teachers in language-responsive teaching of modelling

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In this paper, I investigate practices in teacher education for language-responsive teaching of mathematics. I use action-research in a mathematics education course for primary school (grades 1–7), to systematically investigate my practices to support pre-service teachers in identifying language demands of modelling activities. Two sets of practices were identified regarding teaching about supporting communication and supporting multimodality. These practices were associated with pre-service teacher actions of talking, noticing, planning, and applying language-responsiveness. The paper concludes with recommendations for further changes and improvements in these practices. The study contributes to insights on improving teacher education practice for preparing language-responsive mathematics teachers.

Στο παρόν κείμενο διερευνώνται οι πρακτικές εκπαίδευσης εκπαιδευτικών για την γλωσσικά-ανταποκρινόμενη διδασκαλία των μαθηματικών. Αξιοποιώντας την έρευνα-δράση διερευνώ τις πρακτικές που χρησιμοποιώ στο πλαίσιο ενός μαθήματος μαθηματικών για την υποστήριξη των μελλοντικών εκπαιδευτικών (τάξεις 1–7) στην αναγνώριση των γλωσσικών απαιτήσεων δραστηριοτήτων μοντελοποίησης. Αναδεικνύονται δύο θεματικοί άξονες πρακτικών, ως προς την υποστήριξη της επικοινωνίας και της πολυτροπικότητας, καθώς και εμπλοκή των μελλοντικών εκπαιδευτικών σε γλωσσικά-ανταποκρινόμενες δράσεις συζήτησης, παρατήρησης, σχεδιασμού και εφαρμογής. Η παρούσα μελέτη δύναται να συνδράμει στη βελτίωση της πρακτικής των εκπαιδευτών για την προετοιμασία γλωσσικά-ανταποκρινόμενων εκπαιδευτικών των μαθηματικών.

Introduction

As the number of multilingual students in school classrooms increases over the years, teachers need to be prepared to meet their needs in subject areas, like mathematics, in language-responsive ways (Prediger, 2019). Language-responsiveness that is specific to mathematics means that students' needs are to be met through teaching arrangements that support students' languages and diverse backgrounds, with the parallel development of the content language required. In this paper, I investigate a teacher educator's (TE) practices in the topic of mathematical modelling, for preparing pre-service teachers (PTs) for language-

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responsive teaching in their future school classrooms. The research question is: how does a teacher educator support pre-service teachers in identifying classroom language demands in a mathematical modelling activity?

Earlier studies show that many PTs lack formal preparedness for responding to the complexities of teaching mathematics in multilingual classrooms (e.g., Essien, Chitera & Planas, 2016). This is problematic, as it leaves PTs relying on personal experiences and natural inclinations, which are usually insufficient to support students both mathematically and linguistically (de Araujo et al., 2015). Consequently, teacher education programs need to reconsider how they prepare teachers for language-responsive mathematics teaching (Prediger, 2019) and how TEs provide explicit experiences for PTs, as part of the entire mathematics curriculum (Essien et al., 2016).

A reason for the lack of preparation can be that TEs have not developed appropriate practices to support PTs to reflect on language-responsive teaching in systematic ways, or are constrained by contextual and other challenges. For example, Eikset and Meaney (2018) identified situations where the TE prioritised mathematics content over language-responsive practices, despite having awareness of the need for such practices. In addition, Thomassen and Munthe (2020) suggested that teacher education programs might not have yet integrated recommendations from relevant research so that their programs were adapted to raise the responsibility of TEs to prepare PTs.

More knowledge about what practices TEs could use to prepare PTs for language-responsive mathematics teaching is therefore necessary. In this paper, I aim to explore the practices used to support PTs in identifying language demands of a mathematical modelling activity. The paper is part of a wider ongoing action-research project, which aims, first to describe initial practices that I, as a mathematics TE, use to support PTs to consider in issues of language-responsive mathematics teaching; and second, to identify how such practices are adapted to improve PTs' understandings of these issues.

To conduct this project, I draw on Lucas and Villegas' (2013) framework as a stimulus for preparing language-responsive teachers, which includes the element of identifying language demands of disciplines, such as mathematics. The framework describes orientations and types of pedagogical knowledge and skills that PTs could use for teaching language learners responsively, and has been utilised by researchers in mathematics education, such as Prediger (2019) in professional development.

Theoretical background: Language-responsive (mathematics) teaching

Lucas and Villegas' (2013) language-responsive teaching is concerned with taking into account learners' diverse backgrounds and languages as resources for academic learning. Responsiveness to student diversity is a critical equity concern in mathematics education regarding how students understand and participate in classroom communication (Vogler & Prediger, 2017).

Language-responsiveness can be considered related to Moschkovich's (2013) "equitable mathematics teaching practices", which highlights the importance of the role of language in

mathematics by extending the use of content-specific vocabulary to the use of a range of mathematical discourse practices for communicating ideas. Similarly, Barwell (2020) identified that when explicit attention was given to features of mathematical discourse, such as formal and informal language, linked to gestures, representations and body language, the school classrooms tended to be more language-positive for supporting students' learning. Therefore, students should be supported to use the resources they have in their first, second or additional languages in mathematical discussions. Based on Essien et al. (2016), considering students' languages as resources is one of the major challenges for mathematics in multilingual classrooms and, hence, TEs' practices should be structured with systematically supporting PTs to respond to these challenges.

Research by Prediger (2019) on language-responsive teaching in mathematics found that practices drawing on students' linguistic background also need to highlight the content language demands. This is important for teacher education, as many PTs can find it challenging to not equate language-responsive teaching to reducing academic standards for linguistically diverse students (Gay & Kirkland, 2003). It is TEs' responsibility to support PTs to change such beliefs. Developing an ability to identify language demands of particular disciplines is an essential element of teacher expertise in Lucas and Villegas' (2013) framework. Also, it is one the five teacher jobs that Prediger (2019) has described, including noticing, demanding, supporting, developing and identifying language, as part of requirements for classroom teachers to sufficiently support students' language and mathematics learning. For these reasons, this paper focuses on identifying mathematics-specific language demands.

Lucas and Villegas (2013) additionally suggest relevant "tasks for learning" about teaching language learners. For example, they explain that teachers need to be able to analyse language features of the communication and activities that play out in the classroom and would likely challenge multilingual students. In doing so, teachers could support multilingual students' participation and active engagement, in ways that approach what Moschkovich (2013) and Barwell (2020) emphasise for communication in mathematics. The framework with the "tasks for learning" can be used by TEs to inform practices within the teacher education curriculum to prepare PTs.

Methodology

To address the research question, I use the first cycle of my ongoing action-research project, which concerns teaching about modelling. This cycle, like all others in an action-research spiral (Kemmis et al., 2013), involves multiple sub-cycles, from which I use individual examples to identify and investigate my practices. As an action-researcher, I aim to understand my teacher education practices and change them for improvement (Kemmis et al., 2013). The examples described in this paper are about the practices I used in my teacher education work which would contribute to PTs being able to identify the language demands of a mathematical modelling activity.

Mathematical modelling is the focus of the teacher education course under investigation. In the mathematics course for teachers of grades 1–7, mathematics content and didactics are combined. The course included an assignment about the PTs' implementing mathematical modelling into their teaching while on practicum at local schools. The assignment was based on Barbosa's (2006) criteria for a mathematical modelling activity, in that it should be a problem (not an exercise); and it should be taken from everyday or other sciences that are not pure mathematics (p. 294).

The data collection in the first cycle comes from a workshop, at the beginning of the autumn semester of 2020, on modelling with three different groups of PTs, which was physical (first session) or digital (second and third sessions)[1]. Between the workshops, I discussed my practices about language-responsive mathematics with colleagues to consider how I used or could change them. Each workshop was about three hours long and was audio-recorded with the consent of participants, and then transcribed. Interactions between the TE and PTs were primarily in English, as their native languages were not shared, while interactions among PTs were in Norwegian. The TE used power-point slides in Norwegian and/or English as a visual aid and displayed a video of modelling in a school classroom with linguistically and culturally diverse students in which the students spoke English. Video screenshots were used as springboards for discussions about different aspects of language-responsive teaching to do with mathematical modelling into other parts of the sessions.

The analysis of my teacher education practices was done to identify which practices to change and in what ways. Therefore, I adapted Pierson's (2008) model of analysing teacher's follow-up interactions in mathematics classrooms, based on the constructs of "responsiveness" and "intellectual work". In the adapted version, responsiveness refers to how the TE takes-up PTs' ideas, and is not to be related to language-responsiveness. Intellectual work refers to how the TE engages PTs with cognitive work, by giving or demanding it, which draws on Lucas and Villegas' (2013) framework of language-responsive teaching within the context of mathematical modelling.

To do this analysis, I created tables where I categorised my interactions with the PTs in levels of responsiveness and/or intellectual work. Interactions categorised as low responsiveness were those in which I as the TE did not take up the PT's idea. Those classified as medium showed the TE's idea being the focus of the TE's response. High responsiveness was split into two levels: high-I when the TE's reasoning was on display when elaborating on PT's idea, while high-II when the PT's reasoning was displayed. Intellectual work was split into giving and demanding. Low level of giving intellectual work were interactions in which the TE gave information without integrating language aspects in the mathematics content, medium level were interactions in which the TE gave information about students, and high level interactions were when the TE gave information about implications for teachers. About demanding intellectual work, low level interactions were those in which the

[1] The regulations in Norway in autumn 2020 required 50% digital teaching due to the COVID-19 safety measures, unless other force majeure reasons eventuated.

TE did not request the PTs to use their own ideas, medium level were interactions where the request was implicit, and high level when the request was explicit.

A second analysis, in alignment with Pierson's (2008) coding of content/activity, provided deeper insights on the TE's practices about the element of "identifying classroom language demands". This analysis supported me as the TE action-researcher to decide whether my practices were effective for achieving my pedagogical goals, and consequently understand how to improve them. For instance, if the TE's support for "identifying classroom language demands" was in interactions with little or no uptake of PTs' idea (low responsiveness), then there is a need to determine alternative practices that would utilise PTs' own reasoning in future interactions.

As a result of the two analyses, two sets of practices were identified as important foci in regard to improving my work as a TE. The first set was identified as practices to do with "supporting communication rather than single language use", and a second set was about "supporting multimodality related to content-specific mathematical ideas". These results are described in the next section.

Results

In this section, I present representative examples of the two sets of practices related to "identifying classroom language demands" in modelling contexts. The first set of practices were aimed at developing PTs' understandings about the value of teachers moving past a focus on students' imprecise language when discussing a modelling activity. The second set of practices were aimed at developing PTs' understandings of how a teacher could encourage the use of multiple linguistic and non-linguistic modes to make the modelling content accessible to all students. In the examples, I use brackets to indicate the level of {responsiveness} or {intellectual work}.

First set of practices: "Supporting communication rather than single language use"

– Actions of talking about language-responsiveness

The most common practice was connected to PTs' actions of talking about language-responsiveness. For example, in the last part of the second modelling session, the TE asked PTs about what they thought the teacher in the video did to support multilingual students' engagement with modelling ideas. One PT wrote a response in the digital platform chat, shown in quotation marks in the excerpt below. The TE read it and further elaborated with her own reasoning {high-I responsiveness}.

> TE: [PT's name] says, "the teacher focuses on what they (the children) are trying to communicate rather than the words they are using. They are able to get across what they mean, and the teacher accepts it". Yes, the teacher actually does stand by the kids and […] encourages them to move forward. She does not insist on precise language to explain how they solve the problem, […] she does not lower the content that they are working with academically. So, everyone is working on the same content demand. […] Learning math does not need to wait for learning the language of instruction in the classroom. [It] can happen in parallel by learning the language through using the language.

After the TE affirmed PT's idea of accepting students' language when communicating meanings, she continued with how language and content were not reduced for multilingual students in that video. In this sense, the PTs interacted with the TE to explicitly talk about language-responsiveness in a modelling activity. In this action of talking, they shared opinions of maintaining the modelling content demands, without situating language as a problem in mathematics communication.

Another example from that session was after the TE asked PTs to work in groups and plan for supporting multilingual students in a modelling activity using Barwell's (2020) suggestions for language-positive classrooms. In one group, a multilingual PT shared personal stories about the opportunities of mathematics education received in a mainstream school classroom, compared to classrooms where mathematics content was reduced on account of communication skills in Norwegian being a problem. The TE followed up by asking smaller questions to facilitate a discussion focused on planning the modelling activity {low responsiveness}. The PTs continued with discussing the use of symbols, and the ways modelling can support working "across languages". Here, the PTs talked about language-responsiveness in a modelling activity, in a slightly different way than the previous example, as they reflected upon personal experiences and the role of languages other than Norwegian in mathematics education.

– *Actions of noticing language-responsiveness*

Teacher education practices in this theme were also identified in connection to PTs' actions of noticing language-responsiveness in a modelling activity. For example, in the second and third modelling session, the TE showed a video screen shot of a student who explained in a plenary session someone else's modelling strategy as "He minused until he got to the answer of how much are left". The TE elaborated on the creativity of the language involved and compared the expression in English (to minus) and Norwegian (å minusere). She then indirectly requested PTs to rethink their ideas by pointing out that the teacher again did not insist on precision, and that "learning another language provides more resources to think with, to understand the world and to understand the mathematics" {medium demand}. Thus, the TE engaged PTs in noticing language-responsiveness by paying attention to students' language use in a modelling activity, and to the valuable role of multilingualism in mathematics.

– *Actions of planning for language-responsiveness*

Another practice of supporting the identification of language demands was during PTs' planning a modelling activity using Barwell's (2020) suggestions for language-positive classrooms {high demand}. For example, one group of PTs planned a small-group activity, with the aim of encouraging multilingual students' participation. The TE briefly gave feedback by elaborating on the importance of encouraging students to use their existing language skills when working on a modelling activity in groups {high-I responsiveness}. In this sense, the PTs explicitly planned for language-responsiveness in a modelling activity.

Second set of practices: Supporting multimodality related to content-specific mathematical ideas – Actions of noticing language-responsiveness

The most common practice that I used in regard to raising PTs' awareness of the value of multimodality was connected to PTs' actions of noticing language-responsiveness, and in particular using gestures and representations, in students' contributions to interactions with their peers or the teacher. For example, in the third session, the TE showed a video screenshot of a student sharing with a learning partner what was noticed in a short film that the students watched in the school classroom for working with their modelling activity.

> TE: So this kid was using body language, kroppsspråk (body language), to explain, to show how she understands the film that she saw about [film content]. There was the concept of taking away, that something happened, and some [products] were taken away. That is a very central concept in subtraction. And that is the main idea that they were working with, […] and how modelling was used as a tool for them to learn about subtraction. And also her learning partner understood her.

The TE explained {medium giving}, first, that the student's body language was a means to show her understanding of "taking away" in the short film, which was integrated into the students' modelling activity, and second, that her learning partner understood her body language. In this case, the TE engaged PTs in noticing language-responsiveness related to using non-verbal modes to discuss modelling content.

A similar example was when the TE explained (second and third session) another video situation, where a student explained in the whole class the "decomposing" strategy he used in the modelling activity. The TE's explanation was about how the student used signs and symbols that represent how he "decomposed" the numbers and supported his classmates' understanding {medium giving}. In this manner, the TE's practice engaged PTs in noticing language-responsiveness related to using representations to understand content and emerging features of content language.

An example of a student and teacher interaction is when the TE showed a screenshot of the video of a teacher talking with a student about her strategies in doing the activity, at the second and third modelling sessions. The student had written on her worksheet numbers and symbols, represented relations of subtraction, and drawn a number line. Both the student and the teacher were pointing at the representations and supported the language features of the activity while talking about it. The TE gave intellectual work to PTs by explaining this and by stressing that the use of multimodal means to communicate provides students with more opportunities to participate and learn {high giving}. In this sense, the TE engaged PTs in noticing language-responsiveness related to how multimodality can be encouraged and allow students and teachers to discuss and make sense of each other's ideas about the content of the modelling activity.

– *Actions of applying language-responsiveness*
Applying language-responsiveness was identified in PTs' discussions in a multilingual setting. For example, in the first modelling session, the TE asked PTs to share ideas from the modelling activity they planned in groups {high demand}. The PTs, individually and with the help of their peers, switched between Norwegian and English, pointed at pictures they had taken for the activity (e.g., from the cafeteria) or at their written text describing the activity, and used body language about the activity's content, such as comparing (sammenligning) statistical data and finding the average (gjennomsnitt) height of students, or repeating these phrases and using synonyms to ensure understanding. Therefore, content language demands of each activity were met through interactions {high-II responsiveness}. The PTs interacted with the TE and applied language-responsiveness to support their ideas in a multilingual mathematics setting, and their efforts to reinforce others' understanding of the activity encouraged them to identify and utilise extra-linguistic supports.

Discussion and concluding remarks

In this study, I have investigated my own teacher education practices for preparing PTs to work with language-responsive teaching within the context of a mathematical modelling activity. I used Lucas and Villegas' (2013) framework of language-responsive teaching, focussing on the element of "identifying classroom language demands of particular disciplines", to gain insights on how does a TE support PTs identify classroom language demands in a mathematical modelling activity.

Two sets of teaching practices in teacher education emerged as being of interest: teaching about supporting communication rather than single language use, and teaching about supporting multimodality related to content-specific mathematical ideas. Supporting PTs to identify classroom language demands in a modelling activity was to do with preparing PTs to identify, first, when and why letting go of students' imprecise language can be appropriate and, second, to identify what and how multimodalities can be used in mathematics classroom communication.

The TE's practices for this preparation involved engaging PTs in actions of language-responsiveness: talking-about, noticing, planning, and applying. Noticing was the most common action, while planning was the least common action in relation to identifying language demands.

Noticing language-responsiveness was about being aware of language and language forms used by students and relates to Lucas and Villegas' (2013) task for learning through cultivating awareness of language as a focus of analysis. Making the action of noticing language relevant to specific mathematics content in this study aligns with what Prediger (2019) described about noticing being a necessary teacher job that supports the identification of language demands. However, in my teacher education practices, I did not involve PTs in noticing students' language resources and needs, like Prediger (2019) had emphasised as needing to be done. My practices for noticing language-responsiveness were identified either as giving intellectual work, where I gave information about students' language, or as implicitly demanding intellectual work from the PTs. As a result of this analysis, it is

important for me to consider how to change my practices to do with noticing so that I request explicit intellectual work from the PTs to notice language-responsiveness themselves within the context of a modelling activity.

Talking about language-responsiveness was about having PTs reflect on their experiences and share their understandings of teaching and learning in multilingual mathematics classrooms. This PT action is relevant to the first task for learning within Lucas and Villegas' (2013) framework. Talking about language-responsiveness occurred within multilingual interactions with multilinguals, between the TE and PTs as part of considering aspects of language-responsive teaching. However, talking about language-responsiveness should not be considered sufficient in teacher education (Prediger, 2019). In the analysis, my TE practices for engaging PTs in talking about language-responsiveness were identified as low or high-I responsiveness. Therefore, I need to consider how to provide explicit intellectual work so that the PTs will talk about language-responsiveness in a modelling activity and to address questions to the whole classroom so that all the PTs engage in discussion. In addition, I, as the TE, could look for meaningful opportunities to associate talking about language-responsiveness with other actions, such as applying language-responsiveness.

Applying language-responsiveness was about the PTs using the resources that were available in the environment when sharing ideas about a modelling activity in multilingual classrooms. This action was connected to Lucas and Villegas' (2013) task for learning through applying practices and tools in linguistically diverse settings. Thomassen and Munthe (2020) also highlighted that all PTs, especially at the beginning of their education, should be given practical opportunities to participate in multilingual classroom settings, and to then reflect on their experiences. However, as the TE, I did not explicitly provide opportunities for this. Therefore, I need to consider improving practices for applying, so that they contribute to high levels of responsiveness and demand for intellectual work, by making connections to actions of reflecting on language-responsiveness.

Finally, planning for language-responsiveness was about creating teaching arrangements that can support multilingual students and is related to Lucas and Villegas' (2013) task for learning through understanding and applying practices and tools to plan classroom instruction. In regard to mathematics education, this kind of planning seems similar to Prediger's (2019) description of teacher's job to demand language from multilingual students by activating teaching and learning situations that promote student communication. In the data, there was one example of demanding high level of intellectual work. Thus, I, as the TE, could consider how to improve my use of this practice by interacting more with the PTs and engaging with their ideas.

The analysis of the data showed that I, as the TE, used multiple practices for supporting PTs to identify language demands in a mathematical modelling activity. Also, I was able to identify those practices that needed to improve in order to increase the possibilities for the PTs to develop their language-responsive mathematics teaching. However, there remain issues to investigate as a TE to ensure that I continue to improve my own practices in the future.

The findings from this paper can provide information to other TEs, engaging in action-research, about how to attend systematically to preparing PTs for language-responsive teaching of mathematics. Becoming a language-responsive teacher is a process that begins in initial teacher education programs and continues throughout a teacher's professional career development (Lucas & Villegas, 2013). Therefore, it is important to better understand the challenges faced in preparing language-responsive mathematics teachers.

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Just mathematics? Fostering empowering and inclusive mathematics classrooms with Realistic Mathematics Education

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This paper considers how Realistic Mathematics Education (RME) can serve as a platform for developing socially just and inclusive mathematics classrooms, and examines how teachers new to RME interpret and enact this potential. Drawing on exit interviews from a large RME trial in England, we explore teachers' interpretations of RME's potential for transforming students' access to mathematics, and their understanding of the role of the "realistic" element of RME in inclusion. We also study one teacher's classroom practice and reflections to investigate how he endeavours to build on his students' "real life" starting points. Our analysis of how he appropriates RME to achieve his aims underlines the need to identify inclusion as an object of reflection in continuing professional development.

In this paper, we consider the potential of Realistic Mathematics Education (RME) to support a mathematics education that enables us "to do more than tinker with the arrangements in school that contribute to the production of inequities in the lived experiences of learners and educators" (Gutiérrez, 2013, p. 62). Recognising the debate around the concept of inclusion as often assimilative in nature (Ahmed, 2012; Martin, 2019), we examine how RME, as a pedagogical practice, supports teachers in enacting and reflecting on this mathematical mode of inclusivity, in which learners are invited to draw on their own experience to generate mathematics in participative classrooms. By exploring how teachers adopting RME described their interests in inclusion, and using a case study to examine how these interests may not play out in practice, we identify ways in which RME materials and strategies can serve as a catalyst for thinking about the nature of inclusion, and consider how CPD can highlight this potential.

RME as a platform for inclusive mathematics education

RME lends itself to the pursuit of inclusive classrooms in terms of instructional design features and their related pedagogy. A central design principle of RME is that mathematical

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strategies and models are generated from students' understandings of everyday situations, and that mathematics emerges from these informal models (Gravemeijer & Stephan, 2002). This grounding in students' own understandings of a realisable world emphasises their participation in mathematics at its core, supporting a 'bottom-up' pathway to formal mathematics, which means that students always have a route back to contexts in which they have constructed meaningful models.

RME's design features are supported by a pedagogy of *guided reinvention* (Stephan, Underwood-Gregg & Yackel, 2014). Shifting authority away from the teacher, *guided reinvention* promotes student agency in mathematical discussions where teachers serve primarily as facilitators. Rather than leading conversation, teachers aim to support students' mathematising of context by recognising and working from what is 'everyday' for their particular class, and encouraging students to state, restate and reflect on solutions and strategies in the room (Solomon, Hough & Gough, 2021). In addition to being meaningful to the speaker, students' explanations must be acceptable to other students, in the sense that they too can access and understand them - otherwise an explanation cannot be 'taken-as-shared', so as to become "objects of reflection" for the class (Yackel & Cobb, 1996, pp. 470–471).

In deploying RME, teachers need to be able to work with the hypothetical learning trajectories that underpin RME materials, and to anticipate, analyse and build on students' own informal models of everyday situations. This is a challenging re-direction of their pedagogic practices for many teachers, as illustrated in Wubbels, Korthagen & Broekman's (1997) study of an RME teacher education programme in the Netherlands. Wubbels et al. (1997) found many teacher trainees misunderstood RME as merely a one-way street – simply an innovative avenue toward formal mathematical concepts. Few teachers came to conceptualise mathematics as a more dynamic system of exchange, where mathematical concepts adhere to and live inside of the rhythms of our daily lives and fantasy worlds. Additionally, it was a challenge for teachers to reappropriate the perspectives of mathematical novices, for whom the reality of mathematical concepts can seem doubtful and arbitrary.

What is real? RME teaching, student empowerment and social justice

This struggle to stitch together the different mathematical realities of teachers and students is of particular interest in this paper, as it cues critical questions about *whose* reality matters in mathematics classrooms. The nature of reality is, of course, what is at stake in social justice efforts that seek to foreground the brilliance of black and brown, female, differently-abled, and poor students. As Delpit (2006) reminded us:

> We all carry worlds in our heads, and those worlds are decidedly different. We educators set out to teach, but how can we reach the worlds of others when we don't even know they exist? Indeed, many of us don't even realize that our own worlds exist only in our heads and in the cultural institutions we have built to support them. (p. xxiv)

US projects, like the Algebra Project (Moses & Cobb, 2001) or Civil's Bridge Project (Gonzalez et al., 2001), have set out to bridge the violent erasures enacted by white

supremacy and patriarchy, while also strategically navigating the way in which a white, male, middle-class mathematics serves as a gatekeeper for students' access to university education or other ambitions.

Working to tackle the complex cohesion between multiple real world*s* and mathematical concepts, RME's focus on students' understanding of 'realistic' contexts seems poised to take on similar issues. However, Gutstein (2006) argues that the 'realisable' contexts of RME were sometimes in tension with his vision of teaching for social justice. Working with *Mathematics in Context* (MiC), a US middle school implementation of RME (NCRMSE & FI, 1997–1998), Gutstein (2006) recognised the features of RME that "support teaching mathematics for social justice" (p. 103). He praises the MiC materials for presenting mathematics "as a sensemaking activity" and positioning students "as arbitrators of knowledge" (p. 103). However, while MiC helped his students to develop the necessary "mathematical power" (p. 107), "MiC by itself … does not challenge students to analyze injustice or see themselves as potential social change agents" (p. 104). While 'realisable' contexts of white middle class pursuits – a school camping and canoe trip for example – served to generate an emergent mathematics of ratio and ratio tables, students reported that this context had no experiential resonance for them. Nor were such contexts relevant to the political and social critique that Gutstein felt was most important for his students' agency.

For Gutstein therefore, there is a tension between teaching for social justice and teaching for mathematical power. In the Algebra and Bridge projects, there is also a tension between full ownership of mathematics and meeting the gatekeeping practices of our educational systems, where test scores are the main tokens of exchange value. How does one balance the need for an equitable and inclusive classroom and meaningful discussions which are 'real' for the students, against pressures to take shortcuts to mathematical models and debased but 'functional' ends? How teachers navigate these tensions as they interpret and implement RME is a core concern of this paper. Given these issues concerning the potential of RME pedagogy to serve as a vehicle for empowering or inclusive mathematical activity, we ask the following research questions: How do RME teachers describe their aims for inclusion? How do they describe and enact RME's 'realistic' component?

Methodology

The wider context of this study is our role as researchers, material designers and trainers in a randomised controlled trial of RME in England between 2018 and 2020, with Year 7 and 8 students (ages 11–13), involving over 120 teachers located in 60 intervention schools. Ten modules on number, proportion, geometry, algebra and data were developed by the team, building on our long-term development work with RME, including adaptation of MiC materials for use in the UK (Dickinson, Eade, Gough & Hough, 2010). Teachers attended 7 face-to-face training days, with on-line training sessions provided to replace a planned 8th training day during the COVID-19 school closures in England. The CPD programme exposed teachers to RME pedagogy, introducing the mathematical landscapes underpinning our

materials, and emphasising guided-reinvention strategies for orchestrating classroom discussion that sought to build on students' informal models. It is worth noting that we did not make inclusion an explicit part of the training, as we return to this issue below.

We collected a variety of data in the CPD sessions, making video, audio or photographic records of teachers' discussions and work. Six schools were asked to participate in the trial as Design Schools, in order to enable the team to gain some insights into how teachers were incorporating the RME materials and pedagogic strategies in the classroom. We visited these schools throughout the project, observing lessons and talking to teachers afterwards. We also interviewed 30 teachers in exit interviews at the end of their Year 8 teaching in 2020 about their aims in mathematics teaching and in joining the trial, their interpretation of RME and their experience of working with the materials. We did not explicitly ask teachers to comment on inclusion. Full ethics approval was sought and gained for this study from Manchester Metropolitan University. Participation in data collection was voluntary and all participants are anonymised.

For this paper, we selected a teacher from one Design School as a case study. Peter was responsible for a class of the lowest-attaining students in a large, suburban, co-educational, non-selective school in central England with a diverse intake (over 1500 students). Although the school's special educational needs and free school meals pupil numbers were below the national average, these students were over-represented in Peter's class. We visited Peter and his RME teaching partner three times over the course of the trial when the students were in Year 7 (March and June 2019) and then Year 8 (October 2019). On each visit, we observed one lesson with each teacher and we talked to them together at the end of the school day, with a focus on the lessons we had observed. Peter also participated in an exit interview after the trial had ended.

We analysed interviews thematically, focusing on teachers' accounts of the connection between features of RME and their global goals as mathematics teachers, paying particular attention to references to socially just or inclusive mathematics. We inspected the classroom data for the use of RME pedagogic strategies, and analysed Peter's post-lesson reflections on these events in terms of the relationships between his aims as a mathematics teacher and his appropriation and interpretations of RME.

Findings

Teachers' aims in developing inclusive classrooms

Before we turn to our discussion of Peter, we present a brief analysis of teachers' reasons for engaging with RME as expressed in the exit interviews. Of particular interest to us here are teachers' spontaneous comments on their inclusive aims. A majority described their aim in mathematics teaching in terms of emotional or nurturing themes. Many talked about building resilience and confidence, for example, "I try to create in my classroom ... a safe space to fail" (T1), or instilling their own love of mathematics into what they know is an unpopular subject: "wanting them to get to ... a curiosity ... to actually challenge think, and

wonder" (T2). There were also teachers who recognised their own privileges in life and wanted to "give something back": "I'd like a purpose … I thought well I can do maths ... and then I can just kind of make a difference that way" (T3); "everyone gets the same opportunity but it's not always the same based on kind of what you've got going on at home ... that made me just really want to help" (T4).

Some teachers extended these ideas to broader claims about the importance of access to mathematics:

> Maths is a worthwhile subject... it helps you with everything–it helps you with your life... And I don't care what you're going to be – you're going to be a hairdresser, you're going to need some maths (T5).

These aims fed into many teachers' interest in RME in delivering a reality-based curriculum and pedagogy, which their students needed because it related to their world: "some of them, they need something tangible to look at, don't they?" (T1); "trying to put the learning in a context that the kids care about and that makes sense to them" (T6). RME provided "that element that I think was missing from my maths learning when I was at school, this ownership of why we use the formulas that we do" (T7).

There were a few more forensic responses about how RME worked to build knowledge, as in T8's comment that his previous starting point of presenting a bar model to students had missed the point of emergent mathematics:

> I love bar models ... but I never got why kids don't like it. ... [But with RME] we didn't actually start with bar modelling, we started with a sandwich being shared... how we would simplify it and how that model was the bar model. I always thought the bar model is where you start ... but there was a step beforehand that helped pupils get there.

Thus, we see a spectrum of concerns about inclusion and the role of context in achieving this. These issues are also reflected in Peter's exit interview. To explore how these connections play out in the classroom, we next take a closer look at Peter's appropriation of RME.

Case study: Peter's aims as an RME teacher

In his exit interview, Peter emphasises his desire to nurture the "weakest" students, "doing the right thing by them". The role of relationships is critical in this:

> In the training year, I realised that teaching was not just solely about your subject, but… the whole child element of it... If you can get that relationships element working, then they will be far more perceptive to what you want to do.

Peter is quite conscious about the markers of academic success (in school and beyond) but he underscores the importance of the "real world" for his "weaker" students: "it's not about a GCSE grade. It's not about making them superb mathematicians. It's getting them to a point where they can function in the real world." Thus, despite it pushing him out of his "comfort zone", Peter was keen to "get realistic elements into maths education because" it is a means of "making sure that with every student you're doing right by them" and to "get the weakest to come along on the journey." How do these aims play out in practice?

Achieving inclusion? Peter's practice and reflections on working with RME

To delve into the interplay of Peter's inclusion-related goals and see how these play out in his classroom, we zoom in on part of an algebra lesson, observed during our second visit to Peter's class. The lesson started with two questions, introduced the day before, on symbolic representations of brick patterns (S = standing or short, L = lying or long). Peter deployed a number of RME strategies – drawing students into discussion, remaining neutral (i.e. not responding either positively or negatively to contributions), probing explanations to elicit justification (e.g., asking "because?"). We pick up the thread five minutes into the lesson, as the class moves from one question to the next. Peter flags up his assessment that the students will find the ideas challenging:

Teacher: Okay. So, this is where it can get a little more confusing. [Reading out question on the board.] Mike [a fictional character in the question] also says
Gina: [softly but audibly] Oh, not Mike!
Teacher: "SLSLSLSL could be written as: length of row = 4(S+L)". Thirty seconds with your partner, can you explain this?

[Students embark on 2 minutes of discussion, Gina can be heard saying "BODMAS, that's BODMAS." BODMAS is an acronym used in the UK for order of operations in arithmetic: first Brackets, then Order (exponentiation), then Division, ...]

Teacher: Gina, you like talking, what are you thinking? What's Mike thinking?
Gina: I don't know ...
Teacher: Why ... can you try and explain?
Gina: Why don't you ask Mike?
Teacher: Well, I can't. Mike's not here. That's why I'm asking you.
Gina: Does he go to a different school? [other student starts giggling]
Teacher: Yes.
Gina: What school does he go to? [breaking into giggle]
Teacher: School in Manchester. [a few students cry out "MAN CHES TUH!"]
Gina: How do you know?
Teacher: So ... because I do. What's he trying to do?
Gina: He's trying to do BODMAS.
Teacher: Why do you say that?
Gina: BODMAS? Because it looks similar to BODMAS. Because he has to use BODMAS. [in the background, a student asks, "is that true?"]
Teacher: Okay, yeah. You are true. The ... the ... we will have to do ... brackets and ... so there is an element of that. Can you ask someone else to carry on for me?
Gina: [tentatively] Ask someone else ...
Teacher: In the room. Pick someone else in the room. Don't have to have their hand up. [Gina nominates Anna ... giggling ...]
Teacher: Anna. Surprise, surprise ... Anna, can you explain what Mike is thinking?
Anna: No, I can't. ... I don't understand it.

[The lesson opens into a wider discussion with a number of students, including one working at the board who takes it back to the context of the bricks, but Peter isn't satisfied with the responses. We rejoin the lesson as Peter writes expressions on the board.]

Gina: [excited] They are all patterns. A pattern with brackets, a pattern with numbers, a pattern with letters.

Teacher: I know you are excited and loving maths but we need to stop shouting and raise your hand.

From the start, we hear that Gina is unsympathetic to the presentation of this question, perhaps critical of the use of the unknown but all-knowing "Mike" ("Oh, not Mike!"). We hear her talking about BODMAS, in response to the fictitious Mike's suggestion that the letters could be written as 4(S+L). Peter calls on her to share her thinking, but undermines any inclusive aspect of this move by positioning her as inappropriately talkative ("You like talking"). Gina appears to resist the doubled-edged message here (talk/be silent) by suggesting that Peter should ask Mike what he is thinking, forcing him to admit Mike's fictional nature, and to comment on his teaching strategy ("That's why I'm asking you"). As Peter pursues his request that Gina should tell the class what Mike is thinking, she neatly uses this as an opening to reintroduce her initial idea, now co-opting Mike as her mouthpiece: "He's trying to do BODMAS". Peter's ultimate dismissal of Gina's contribution is protracted, in his reluctant acceptance that BODMAS is indeed involved ("there is an element of that") and his invitation to pass on to someone else, which leads to some sarcasm about Gina's choice ("Anna. Surprise surprise"), to his later explicit request to "stop shouting and raise your hand".

Recalling this episode in the post-lesson interview, and in response to a question about whether the student's introduction of formal mathematics 'got in the way', Peter says:

> So – I pre-empted it. I knew that – I was surprised that it was the girl in question, because actually she's one of my weakest. So, for her to be shouting that out – and she always does – was actually good. I didn't ignore it but I sort of brushed it aside, and said "Oh, it's BODMAS, you did the brackets first. Oh, brilliant! OK, so how does that work in this context then?"

Peter would like his students to see beyond the procedural bracket operations and use the context of the bricks to notice that SLSLSLSL and 4(S+L) both represent "4 lots of S and L". He sees RME as delivering on making algebra meaningful: "What I think is really good in the RME is it ... brings in the concepts ... where the model is going to come from". But, we also see here that Peter's representation of events suggests praise for Gina, as a girl marked as "one of my weakest" who is "always" "shouting out", but this time has contributed something good - BODMAS. But there is a chasm to be bridged and, after working with several students, Peter invokes a mythical student, George, who 'provides' the bridging explanation. Peter explains this strategy:

> Sometimes, I have to use George and Finn and different ways and it's mainly when the students aren't bringing them up themselves. And I know ... it will start a different discussion and hopefully unlock a few more doors for certain students.

While the student discussion may not have generated the desired mathematics, we see Peter persisting in his endeavour to achieve his "inclusive" purposes: raising student engagement and enabling at least some students to understand the material. As the lesson progresses, students are able to connect with the initial context, with varying degrees of success, and have opportunities to exercise choice – which strategies they employ, which problems they work on – and to relate to the open-ended nature of the problems. Given the nature of some of the RME contexts (fair sharing, air travel, working in a fish and chip shop, national elections, ...), there were opportunities for discussions related to social justice but these fledgling discussions, when they did arise, were not connected to the mathematics. Ultimately, Peter's focus was on providing students with access to methods and knowledge by "giving them that stepping stone of realism" rather than building on what is real for them in order to make sense of both mathematics and the 'real world'.

What does RME achieve for Peter?

For Peter, RME provides a mechanism for students to participate and become more confident in mathematics, and his own role as 'facilitator':

> I really love teaching RME because, actually, it's given me a way into them, to show that they can be enthusiastic and try different things. ... I think the more I get them talking, the less I have to talk. So, I become sort of that facilitator that I want to be for RME.

Peter says that RME has "massively changed my approach to teaching maths ... opening my eyes to lots of realistic informal models, which I could then go in and use." He prioritises providing an accessible entry point to formal mathematics and developing functional mathematical skills over following through on his students' responses. Yet, Peter does not simply use RME for its "realistic informal models". He deploys a number of RME strategies to elicit student explanations and foster discussion, and recognises the value of giving time to discussion - to strengthen both his relationship with the students and their relationship with mathematics. Indeed, the students take these opportunities to engage with contexts, exercise their creative/resistant agency, and try to connect their various mathematical understandings to the materials. Given this mixed and nuanced picture, and the partially tapped potential of RME, how could we have better supported our teachers' reading, appropriation and enactment of RME? We pick this up in our concluding remarks.

Discussion: Just mathematics?

Inclusion is a complex concept and its relationship to ostensibly wider aims of mathematics teaching for social justice remains up for debate. In their exit interviews, teachers connected to the inclusion components of RME on a broad spectrum. Some approached inclusion from a nurturance stand-point, while others invoked the importance of finding ways to inspire. Some, however, were very precise in naming the twin force of emergent mathematics, identifying the way in which students' experiences can reverberate across mathematical thinking.

As we see in the example of Peter, putting these good intentions into practice is not straightforward. Peter appropriates and deploys certain elements of RME (e.g., informal models and strategies for fostering discussion), but, in doing so, he voices potentially damaging microaggressions (Ball, 2021) in the name of inclusive discussion. This points to the need for inclusion to become an "object of reflection" in future RME CPD projects. More explicit discussions about the social construction of mathematical ability and the way that mathematics *lives inside* of problems and contexts might have gone some way in nuancing Peter's rhetorical style. If possible, CPD should also support teachers in reworking RME materials to harness contexts most salient to their students' lives. These solutions, however, do not address Gutstein's (2006) rightful complaint that students should understand mathematics as a tool for "reading and writing the world" – reframing and uncovering injustice. As in Peter's lessons, there were opportunities during our training sessions for deepening discussion around the social reality of certain contexts. For example, one activity – ranking British Prime Ministers by age – elicited active discussion on elitism and colonial governance. But, we didn't always build on this. Just as RME teachers responded to pressures in their classrooms, or missed building on some student responses, so did we.

This brings us to the question of what counts in and as mathematics, what counts as worthy of mathematical measurement: Whose everyday activities take precedence in mathematising our shared world? While Peter may not have implemented an 'ideal' RME, he has evidence that overall his students are doing better – on assessments, in making sense of mathematics, and at 'having a go'. For him, this is a vindication of investing in RME. Mathematical empowerment today could open up mathematics for social justice tomorrow. In which case, is just focusing on 'the mathematics' a viable starting point? As Martin, Gholson and Leonard (2010) point out, one cannot separate mathematical practice from social justice, equity and inclusion. Our experience of supporting teachers to become adept at using RME materials in their desire to enhance students' access to mathematics has highlighted the need to address inclusion more explicitly as an object of reflection with respect to both curriculum and pedagogy.

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Mathematics for multispecies' flourishing

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Why teach mathematics? We introduce an ethical framework for teaching elementary mathematics, "Mathematics for Multispecies' Flourishing" (M4MSF). We draw upon Su, who has promoted the idea of doing mathematics as a route to human flourishing, and Seligman's PERMA theory of well-being. We frame M4MSF as connecting content to needs for survival, transcendence, belonging, dignity and challenge through considerations of land, language, lore (story), living, logic and learning. This framing extends ethnomathematical and culturally responsive approaches to mathematics and grounds our understanding of multispecies' flourishing as a right of all species. We present examples from a completed project with pre-service elementary teachers in Canada.

Conceptions of human flourishing are emerging as a focus in research on human well-being (eg., Seligman, 2011), teacher education (eg., Cherkowski & Walker, 2018) and mathematics education (eg., Su, 2020). In our opinion these frameworks while useful continue to privilege a specific anthropo/eco/ego/homo-cidal genre of being human (Wynter, 2015) as a bio-cultural exceptional species (Haraway in Mitman, 2019) and place human interest — including the interest of ahuman capital (MacCormack, 2020) — above all else. We recognise however that there is not and has never been human flourishing at community and population levels without — or that is independent of — multispecies' flourishing (Brink, 2008), and, at present, in many places, the multispecies world is far from flourishing.

Our thesis then is that current conceptualizations of a "Mathematics for Human Flourishing" (Su, 2020), or "Rehumanizing mathematics" (e.g., Goffney & Gutiérrez, 2018; Tan et al., 2019) while welcome, necessary, and important contributions to ongoing conversations in our fields and for practice, continue in a tradition of an anthropocentric mathematics education that fails to attend to the evidence that there has never existed widespread human flourishing anywhere without simultaneous attention to multispecies flourishing (the flourishing of a multiplicity of biological and other forms) and multispecies' flourishing (the right of other species to flourish). Our shift in thinking and argument in this paper is to invite readers into a mythopoetic re-framing of what would traditionally be rendered within ethnomathematical or culturally relevant frames.

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From human flourishing to multispecies flourishing to multispecies' flourishing

Kirksey and Helmreich (2010) describe the emergence of multispecies ethnography describing it as centering on, "how a multitude of organisms' livelihoods shape and are shaped by political, economic and cultural forces" (p. 545). van Dooren, Kirksey and Munster (2016) use the concept of "passionate immersion" as a frame for multispecies studies that unsettles notions of species, opening up ways of knowing and understanding others and has implications for epistemology, political philosophy and ethics around questions of, "liv[ing] with others in entangled worlds of contingency and uncertainty […] [and] inhabiting and co-constituting worlds *well*" (p. 1). This "species turn" in anthropology has spilled over into other social sciences (such as education, especially early years education (see for example the common worlds project, n.d.; Taylor & Pacini-Ketchabaw, 2018) (and has a much longer history in the humanities and arts) as well as beginning to influence Scientific practice.

According to van Dooren et al (2016)

> a multispecies approach focuses on the multitudes of lively agents that bring one another into being through entangled relations that include, but always also exceed, dynamics of predator and prey, parasite and host, researcher and researched, symbiotic partner, or indifferent neighbor. But these larger contexts are not mere environments in the sense of a homogeneous, static background for a focal subject. Rather, they are *complex "ecologies of selves," dynamic milieus that are continually shaped and reshaped, actively — even if not always knowingly — crafted through the sharing of "meanings, interests and affects," as well as flesh, minerals, fluids, genetic materials, and much more*...this multiplicity, this multiplying of perspectives and influences, is key to what multispecies studies is all about (pp. 3-4, italics added).

In short, multispecies work is a study in complexity of living, learning and becoming with, alongside and through other planetary beings and cosmological phenomena. Khan (2020) begins to develop the first aspects of the framework of a mathematics for multispecies' flourishing. Framed mythopoetically it puts into conversation the complex assemblage of Sylvia Wynter's formulation of Man and the ceremony found or second emergence of an autopoiesis of the human with that of multispecies' scholars such as Donna Harraway and her notions of making kin. Speculatively, Khan (2020) offers that a mathematics education for multispecies' flourishing might find a first analogy, "in enactivism's focus on structural coupling among individuals and their environment and extend this to include indigenous wisdom sensibilities/ spiritualities, which do not view the multispecies world through plantation logics and economics as 'resources' to be exploited and profited from but as 'kin' and 'nations' to be partnered with in teaching, learning, living, and dying well."

We offer that a Mathematics For Multispecies' Flourishing acknowledges and seeks to address the needs of some specific member(s) of the multispecies world for survival, transcendence, belonging, dignity and challenge. We attempted to illustrate as well that our notion is of a set of reciprocal relations among these elements, i.e., none is more important, all are necessary though in certain periods one may become a priority, e.g., survival. We note some similarity to Maslow's hierarchy/pyramid of needs but think our non-hierarchical

relational framing offers something different in its visual metaphor and the commitments entailed. The second part of the image identifies what we believe one must attend to in doing mathematics for multispecies' flourishing. In this first iteration we identify relationships with land, the languages that are spoken by humans and other species, the stories (lore) told about that land through those languages that enable a continuity of living forms over time, the logics (grammars and biosemiotic codes) that structure relationships of meaning from which learning emerges across different embodied complex systems ranging from the subcellular to the mytho-cosmological.

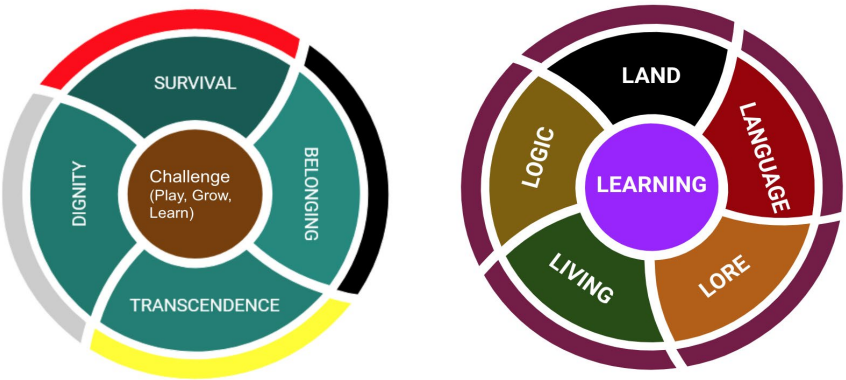

Figure 1: The frame of multispecies flourishing as a connection with course content.

In our meetings together we also discussed framing aspects as flourishing promoting, flourishing limiting, and anti-flourishing. Without getting into a full discussion here, we point to poverty, educational inequality, colonialism, racism, traumas, and pandemics as examples of flourishing-limiting situations in humans. We place, during the period of their expression, genocide (including species extinction), fascism, the coloniality of power and murder as anti-flourishing. We also found that across cultures, geographies and times, communal activities such as games, group hunting/agriculture/shelter construction, ceremonies and rituals, together with art/craft/making practices contributed to survival, transcendence, belonging, dignity and challenge while always being situated in certain places, with multiple overlapping multispecies languages, telling polysemous stories and allowing plural logics and ways of sense-making. These communal activities we often found framed within a respect and deep care for the multispecies world and they became multiple sources of inspiration and exemplification. We also discussed the model and logic and ethics of the plantation as a powerful contributor to flourishing-limiting practices and anti-flourishing events.

It is our belief that pursuing a mathematics for flourishing or a mathematics for multispecies flourishing requires attending to the life-histories of individuals, not as social workers, but as compassionate human beings interested in the flourishing of all learners and not just those of human beings. We suggest that a mathematics for multispecies' flourishing

steps further than the concerns of the mathematics proficiency framework and the anthropocentrism of most forms of ethnomathematics research by keeping foregrounded the 5 inter-related needs of survival, transcendence, dignity, belonging and challenge that necessarily require developing meaningful relationships with lands, languages, lores, living, logics through which one curates one's learning alongside multispecies others to whom one has responsibilities.

Research context

In the context of our work with pre-service elementary teachers we have used the provocation offered by Cherkowski and Walker (2018) below with teachers and ourselves,

> What if you were to imagine that your primary role as an educator is to learn how to thrive in your role, and, in so doing, *to continually co-explore and to enthusiastically facilitate all means by which each person in your learning community flourishes along with you* most of the time? (Cherkowski & Walker, 2018, italics added)

The provocation provided the impetus for the design of EDEL 415, Issues in Elementary Mathematics Education: Mathematics Education for Multispecies Flourishing which I (Steven) taught in the Winter term of 2020 (Jan.–April). The specific focus of the Section was identified as "Mathematics Education for Multispecies Flourishing" and offered participants opportunities to examine issues that affect mathematics education whilst continuing to develop their knowledge and skills in mathematics content knowledge and knowledge for teaching. In my outline I noted,

> We shall inquire into necessary conditions for promoting multispecies flourishing through mathematics education during the elementary grades. Consequently, some aspects of the content and discussions in this course may be unsettling, controversial, or emotionally and intellectually challenging. The instructor will do his best to make the learning spaces places of loving kindness where the students are able to critically, compassionately and thoughtfully engage with difficult content.

The course had an enrolment of 32 students and meet weekly for 3 hours. We were forced to move to emergency remote teaching in mid-March as a result of COVID-19 lockdowns and ended the course in this mode of engagement. This made the concerns of the course especially those of survival, transcendence, dignity, belonging and challenge and the places and paces of learning even more salient.

During our in-person meetings we explored a variety of mathematical activities. Hang opened one session with a presentation about one aspect related to the Lunar New Year celebration in Vietnam - a story of making Bánh Chưng. The presentation over time provided one of the concrete examples of a mathematics for multispecies' flourishing and lived alongside others such as understanding communal buffalo hunting on the North American plains, building paper fractal snowflakes, solids and shadows, appreciating Indigenous games and methods for measuring, and making kolam sand drawings. Importantly, it provided an early hub to which to refer back to as students' and our own understandings of what a mathematics for multispecies' flourishing might be about.

The data comes from weekly student mathcuration journals and reflexive conversations of the instructional/research team throughout the planning, delivery and post-course analysis.

Student responses to sessions

Through the presentation and discussion students became aware of several mathematical ideas related to the preparation of the cakes and the relevance of working to connect subject matter to learners' cultures and contexts. Students related the content of Hang's shared story to specific mathematical content in the programme of studies including estimation, symmetry and measurement. Some students made the connection that although this was a singular example it could be generalised to other cultural practices, i.e. that one could connect mathematics meaningfully and respectfully to aspects of learners' cultural backgrounds or bring one's own culture into the classroom. We find evidence for this for example in the following excerpts from several students:

> Something I understood today, was that *we can take situations that are not explicitly related to math and connect it to the subject*...When taking in [the guest speaker's] presentation from the stance of a teacher, I was able to view the relationships to math and it made math more meaningful. *She used math concepts like symmetry, estimating and measurement without explicitly teaching about them.* This presentation mattered, because in my future teaching I can try and use this to connect different students' cultures into my lessons.
>
> While I myself celebrate this holiday, *I have never considered it from a mathematical standpoint before.* It was interesting to learn about the ways mathematics is embedded into our everyday lives, especially within our culture. I think teachers have always struggled in finding out how to make math meaningful to their students, and I think this is a good strategy for doing so. As [R] said in class, a great extension task would be to have students find how math is present in their own cultures and practices.
>
> Not only would students be able to learn about another culture in a hands-on way, but they'd also be *able to incorporate mathematics (such as through finding perimeter, area, etc.).* It could be used as a nice Social Studies connection as well...making these Lunar cakes involves a variety of mathematical concepts. It is *something that requires actual understanding of math, not memorization.* I do think that education needs to move in this direction on this topic, and I believe we are slowly starting to see this trend.

Deep connections and relationships were conceptualized between the different perspectives thus showcasing the underpinning philosophies of making kin and multispecies flourishing *with* and *in* mathematics. The above quotation also showcases the dynamic and continuous shifting and developing of pedagogical beliefs that enables students to curate their mathematical and professional identities.

Students began to see the teaching and learning of mathematics as a plurality of cultural contexts and cross-curricular framings. In other words, students began to see the teaching and learning of mathematics, and school mathematics, as a culturally situated phenomena that spans multiple subjects and lifeworlds. This shift inherently provides a more rich and complex mathematical learning story to occur.

Bringing the 'real' world, life-worlds of students into mathematics was one aspect that pre-service teachers identified and valued prior to the course. On more than one occasion, students expressed their developing beliefs/values about the importance of connecting content to the life-worlds of students. This activity explicitly framed within a perspective of multispecies' flourishing, enabled students to 'see' and 'open' a route to engaging students beyond eurocentric/western curricular content by strengthening this connection between familial curriculum making and the Programme of Studies' learning outcomes.

In the quote below from another pre-service teacher we observe a valuing of language and aspects of culture. They also articulate clear connections to the ideas of flourishing and survival as well as explicit naming of "belonging" appropriate to this point in the course. In the excerpt we also see a possible value of making mathematics a place that is like a home for learners.

> Passing down language, celebrations, values, religion, and also games is an integral part of cultural flourishing and survival. Creating belonging, especially in a new homeland, is extremely important. Without belonging, there would be no "home".

In envisioning such a home - through mathematics education classroom practice - there is a recognition of the value of community formation and collective if not communal activity. Another writes,

> *Collaborating with parents and community members in classroom activities* would be a benefit to everyone as well. It would be interesting to see *how students could connect with each other and the community*, as well as think deeper about how their own *traditions can be connected to subjects like mathematics* (as in Tran's precision in creating her Chung cakes).

This looking forward (in time) and outward (to the community) is not framed within the typical discourse of the community as a resource or containing resources to be exploited for learning but the community as a true *partner*, a collaborator, in teaching and learning (mathematics). This is emerging as a critical theme as schools re-open.

What conceptions of mathematics for multispecies flourishing are developed?

Earlier we presented our explicit framework for thinking about a mathematics for multispecies' flourishing. This is more intentionally and fully developed than the one initially presented to the pre-service teachers. We acknowledge that the students' reflections on the nascent concept have shaped and contributed to this developing idea.

Through the experience of Hang's presentation, students not only began to attend to the cultural and language aspects of students' lifeworlds, they also began to interpret and develop their own conceptualizations of Mathematics for Multispecies Flourishing. In fact, students learned about a variety of different aspects of multispecies flourishing (and related philosophies) which act to open opportunities for possible futures, and strengthen their existing teacher identities (Garner & Kaplan, 2019).

Students appeared to have pre-existing teaching philosophies that value connecting mathematics to everyday life and practice. Through this activity they were invited and welcomed to see more concretely how that philosophy could be enacted. We find evidence for this in the following student notes:

> Although on the surface it may seem as if they are just "something you always do", there is always a rich meaning and sentiment behind every ritual. In this case, this Lunar New Year ritual is a way of connecting people to both multiple places and multiple people.

In these last sentences for example, we see students as expressing a value for "respect" and "deep understanding" (dignity), connecting it to "tradition" (belonging) through story (language and lore) within the Canadian context of ongoing multiculturalism where many parents are "raising children in a new home country" (land). We also note in the reflection (and from in-class conversations) that collaboration with parents and communities is expressed as part of their imagined future practice (and identity) as teachers of mathematics in the elementary grades (dignity and belonging).

The presentation was described as "memorable" and featured in the curated reflection on the session of many students. One student noted in their curations journal after this third session the difficulty or rather challenge of making sense of this concept of Multispecies Flourishing. However, a specific example allowed for connections to be made to the elements of the framework. They wrote,

> Trying to grasp *Mathematics Education for Multispecies Flourishing* has been difficult. It is so far from anything that I have ever known about math that the journey has been a bit rocky. It was enlightening in today's class to see the separate ways that math can allow a species to flourish. Looking at the example of the New Years cake from Vietnam, we can see multispecies flourishing *through survival, transcendence, dignity, belonging and challenge.*

Though this student did not elaborate on these connections, others did, for example another wrote,

> Making the cake brings up several connections [for me] for example: Survival- *of the culture*, food, water. Dignity- *sense of identity* related to accomplishment, the idea of *belonging* to a culture and family… Making a connection to *where all the materials are from*...Extend and challenge students to *see how math is present in their own lives. Thinking long term, timing, situation of practice, when will this be appropriate*, New year? Or could you build it off traditions. You have to think *across discipline.* (italics added)

Others more quickly made connections to the framework on flourishing and multispecies flourishing which had been presented in the previous session one week earlier.

> This tradition incorporates many elements, including survival (food is needed for human survival), transcendence (cake links to ancestors, the homeland, and any absent members of the culture), and dignity (learning to care for the leaves used to make the cake, and discovering a sense of identity [belonging] and accomplishment [challenge] after successfully making them).

For other students the connection was not to "multispecies flourishing" but to "multicultural flourishing" which is in itself a small but noteworthy elaboration of the

frameworks of culturally responsive mathematics (Nicols et al., 2020), ethnomathematics (Stathopoulou & Appelbaum, 2016) and mathematics for human flourishing (Su, 2020). Several noted that such activities could not only contribute to deeper awareness but deeper conceptual understanding of mathematics,

> My goal when I get into the classroom is to have students understand math in a way that not only makes them feel comfortable with the subject, but also so that they can use mathematics in the real world to solve problems. For example, making these Lunar cakes involves a variety of mathematical concepts [...] It is something that requires actual understanding of math, not memorization.

Repeatedly the pre-service teachers told us during the course that the concept of multispecies flourishing was new to them and found it difficult to make sense of it. This difficulty with a novel concept and framework was anticipated. We offered over the duration of the course multiple experiences and instantiations, with intentional and strategic variation, to make sense of the concept. For example, within the first half of the course I heard, "I'm struggling to understand how I can specifically incorporate multispecies flourishing in my classroom and lessons. I think because it is such a vague concept, it is hard to concretely apply it." In this I read the familiar press for immediate application prior to deep, connected understanding and the need for concrete examples. A specific learning strategy employed in the course however was the inter-leaving (reference) and recursive elaboration (Davis & Simmt, 2003) of examples across sessions. Hang's presentation represented an early example that was serendipitous but allowed for the concrete connections to begin to be made.

As the course instructor when I initially proposed the idea of mathematics for multispecies flourishing (M4MSF) as the theme I had a not quite fully formed idea of what it could mean (but an intuition that there was something there of value to teachers and myself). The elements of survival, transcendence, dignity, belonging and challenge were already present from my reading of Brink's (2008) study of Head Smashed In buffalo jump and other scholarship (see Khan, 2020). At the start of the course I was definitely thinking in terms not of multispecies' flourishing but of multispecies flourishing. Distinguishing between these two has been important for me in recognising the difference that each signals. The latter is about plurality, i.e. the ability of all species to flourish. The former is about *the right* of all species to be able pursue flourishing as an end. The pre-service teachers' ongoing struggles with the concept and providing suitable exemplification helped me to refine this distinction and by the end of the course I had shifted to using multispecies' flourishing in my thinking and writing.

Another learning from this example is the clear demonstration of the relationship and need for deep engagement, "passionate immersion" with the multispecies world. As we talk about what might be needed to do this work I find the traditional framings in mathematics education lesson plans as "materials" or "necessary *resources*" to be problematic and potentially exploitative. The bamboo, banana leaves, rice, beans, meat, the stories, even the plastic used to wrap the cakes are more lovingly framed as 'partners' in our learning to live well and not

just 'resources' to be consumed. There is a necessary recognition of the significance of relationships of interdependence and inter-vulnerability, of responsibility, care, compassion and ethics in this work.

One worry for me is just how easy it is for this still gestating concept of multispecies' flourishing to be consumed/subsumed (the mental image is phagocytosis) by other, already existing and related discourses such as ethnomathematics, culturally relevant pedagogy, rehumanizing mathematics education, mathematics for sustainability and mathematics for social justice as members of our field jostle to exploit the possibilities of their niche by colonizing other emerging discourses rather than starting with the pursuit of connection, collaboration and communal responsibilities. A mathematics for multispecies' flourishing acknowledges the debt owed to these important familial 'relatives' in providing a history and tradition of looking towards more meaningful ends for mathematics than mathematical proficiency and a less anthropocentric focus than ethnomathematics. Its specific contribution though is to attempt to de-center the privileged position of the human (and human economies and logics) as the sole benefactor in our considerations of *who* mathematics is for, with whom mathematics is done in the world, and what mathematics does *to* the world through human action and inaction. Beyond this paper what a mathematics for multispecies flourishing might look, sound, smell, taste, feel like and yet become is left to all of us, as an exercise of communal study, respectful partnerships, research, and praxis.

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Responding to a manufactured crisis: Discourses shaping mathematics-related challenges

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The U.S. has been subjected to a "manufactured crisis" that has promoted myths about education, and reform efforts have been generated to address fictitious problems. But to what extent have those reform efforts infiltrated districts and communities? This discourse analysis examined if and how district leaders' articulations of challenges were shaped by discourses that often remain unnamed. Standards and accountability discourses were consistently present across interviews with district leaders both resisting and succumbing to the discourses. While less dominant, school choice discourses were also evident. The presence of these discourses and the tensions that accompanied them suggest that district leaders are working within and negotiating the confines set by broader school reform policies.

> *This blanket statement has been used by certain political forces that in the United States, they're, "failing schools" and it's been said so much … that most people, especially outside of the education realm just tend to accept it. But I don't know who they're talking about … to just blanket say that is an absolute farce that has been created.*
> *Devin, superintendent, rural Missouri school district*

This quote embodies the motivation behind the analysis that we present in this paper. We were surprised when we came across Devin's candid articulation that "failing schools" and the narratives that accompany them are a "farce," created to drive educational reform efforts. we expected to find evidence of these narratives in leaders' talk but were surprised to hear district leaders resist them in such explicit ways. These narratives, or *discourses,* have infiltrated communities and schools to define the challenges district leaders articulate and take up.

The present paper emerged from a larger study that investigated the mathematics-related challenges identified by mathematics leaders across the U.S. state of Missouri. In that study, we sampled 50 districts in a variety of contexts and invited them to describe their biggest challenge related to mathematics instruction. District leaders overwhelmingly articulated challenges related to standardized test scores. Sometimes test scores were an explicit focus; they *were* the challenge (Munter et al., 2020). Other times they served as a mechanism

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measuring another challenge, such as the district's progress toward equity. Even still, test scores were present.

For us, the overwhelming presence of standardized test scores – both as an explicit challenge and appropriated to measure other challenges – invokes a broader set of discourses rooted in the neoliberal agenda (Croft et al., 2016). We have come to wonder if district leaders' articulations of challenges and subsequently the problems that are taken up within districts are shaped by discourses that are often unnamed—the "political forces" Devin referenced at the start of the paper. This analysis examines whether that is the case and, if so, identifies and describes how neoliberal discourses shape district leaders' articulations of problems. Two questions guide the study: 1) What are the neoliberal discourses shaping district leaders' articulations of challenges, their sources, and their efforts related to mathematics education? and 2) How do those neoliberal discourses shape district leaders' talk?

A manufactured crisis

Credible sources, "political forces" in Devin's words, have led well-meaning Americans to believe that the country's public schools are "failing." In 1983, the U.S. Department of Education released a critical report on the state of American schools titled *A Nation at Risk*. Authored under the direction of the Secretary of Education and promoted by President Ronald Raegan, the report asserted a decline of American education, the blame for which it assigned largely to teachers. This attack on American education continued with presidents and secretaries of education telling Americans about the shortcomings of their public schools. Biddle and Berliner referred to this "campaign of criticism" as the "Manufactured Crisis" (Biddle & Berliner, 1995, p. 4).

Because America has been subjected to a Manufactured Crisis that has promoted myths about education, reform efforts have been generated to address fictitious problems. Some end up creating problems for educators and students. Biddle and Berliner identified six key reform ideas put forth by critics responsible for manufacturing the crisis (Biddle & Berliner, 1995). One key effort is funding education through vouchers given to parents to use with schools of their choice in the private sector. These efforts suggest that more choice among schools would subject schools to market forces, which would promote efficiency and lead to higher levels of achievement. The second effort is to intensify, rather than replace, curriculum. These efforts often call for a return to "the basics" and increased instructional time. A third effort rests on the belief that students, teachers, and other educators lack the ability or will to improve education, leading to suggestions of increased state and federal controls. A fourth effort ties school funding and educator salaries to performance indicators. A fifth effort forces immigrant students into immersion programs in which all instruction is in English. The sixth effort is based on the notion that "gifted" and "talented" students should be given enriched instructional experiences, which leads to an "ability"-tracking system.

These arguments use market rationality to organize and reform education and schools. For example, funding education through vouchers and the privatizing of schools relies on the rationale that subjecting schools to market forces will promote efficiency – and that

efficiency is a worthwhile pursuit. The reform efforts identified by Biddle and Berliner (1995) align with what others have identified as neoliberalism. We turn our attention there next.

Neoliberalism

Croft, Roberts, and Stenhouse (2016) suggested that education reform efforts are part of a coordinated effort to erode public education and that neoliberal policies are the mechanism of the erosion. Neoliberal rationality is expressly amoral in its means and ends (Brown, 2006). Three features broadly capture a neoliberal political rationality: 1) Free markets are depicted as achieved, rather than just happening by nature; 2) Market rationality organizes social and political spheres; and 3) Governance criteria follow market-based rationality and become about productivity and profitability (Brown, 2006). The neoliberal agenda dominates by changing the "common sense" (Apple, 2017). According to neoliberal policies, education reform is necessary, will lead to improved achievement, and will close the achievement gap between students of color and white students (Hursh, 2007). Standardized testing and accountability reform efforts are "weapons" to privatize and increase social engineering through tracking of student data (McDermott, 2013).

Mathematics education is implicated in this system. It is seen as a driver of the economy (Darragh et al., 2017) and links personal progress to national progress (Llewellyn & Mendick, 2011). It plays a key component in testing regimes, which contribute to international competition and works to make reform policies seem inevitable (Darragh et al., 2017). If mathematics testing and accountability efforts are part of a larger, coordinated effort in which neoliberal policies are "eroding" public education (Croft et al., 2016) and testing and accountability efforts are the "weapons" (McDermott, 2013) of the neoliberal agenda, then districts and communities are likely experiencing and appropriating aspects of this agenda.

Data

For the present study, we examined the extent to which the reform efforts put forth by those responsible for manufacturing the crisis had been taken up by district leaders, and the extent to which neoliberal discourses had shaped their articulations of problems and responses. To examine those discourses, we transcribed and analyzed three semi-structured interviews with the assistant superintendent of Midwest City (Teresa), the assistant superintendent at Midwest Suburb (Brooke), and the superintendent of Rural Town (Devin). The interviews were about 40 minutes long and covered leaders' roles, perspectives on challenges in the districts related to and beyond mathematics and the districts' efforts to respond to those challenges. Table 1 describes the interviewees, their position, and the primary responsibilities they described.

We sampled in hopes of uncovering a range of discourses that might shape the ways district leaders identified and talked about problems of practice in mathematics. We selected one district from three categories in the original study (Munter et al., 2020): urban, large metropolitan, and rural. The districts offered a range of racial and socioeconomic contexts.

Table 2 describes the contexts and demographics of the schools including the percent white and free and reduce priced lunch (FRL).

District Leader	District	Position	Primary Responsibilities
Teresa	Midwest City	Assistant Superintendent for Curriculum and Instruction	Put in place a guaranteed, viable curriculum; formative and summative assessments
Brooke	Midwest Suburb	Assistant Superintendent for Curriculum and Instruction	Curriculum, instruction, professional development, oversees middle schools
Devin	Rural Town	Superintendent	Education of the children of the district, budget, oversee instructional leaders, vision of district, upholding all the policies

Table 1: Interviewees Positions and Primary Responsibilities

We also sampled for variation in a) the problems district leaders reported and b) the ways in which they talked about those problems (i.e., framed their problems). Standardized Missouri Assessment Program (MAP) test data suggested that the districts might be experiencing accountability pressures in different ways. In 2018, Midwest City and Rural Town scored 21 and six points below the state average, respectively. Midwest Suburb scored six points above the state average. Our initial analysis suggested that district leaders from each of the three districts identified different problem types (Munter et al., 2020). Midwest City described problems related to equity. Midwest Suburb and Rural Town described problems related to test scores.

	Student Population	Characterization	Percent White	Percent FRL
Midwest City	Over 14K	Urban	9.53%	99.9%
Midwest Suburb	Over 14K	Large Metropolitan	56.59%	71.3%
Rural Town	Under 2K	Rural	94.07%	32.4%

Table 2: Demographic and Characterization Summaries

Data analysis

For this analysis, we conducted a two-part qualitative analysis that consisted of qualitative coding and discourse analysis. The first part of analysis served to answer the first research question: What are the neoliberal discourses shaping district leaders' articulations of challenges, their sources, and their efforts related to mathematics education? To do so, we transcribed the interview data and coded them with Biddle and Berliner's (1995) reform efforts to identify places in which neoliberal discourses emerged. Table 3 describes those discourses.

Then, we employed Critical Discourse Analysis (CDA) to answer the second research question: How do those neoliberal discourses shape district leaders' talk? We attended to what Fairclough (2011) calls discourses (ways of representing) and style (ways of being) to analyze how neoliberal discourses shaped district leaders' articulation of challenges and responses to those challenges. To examine discourse, we asked: What challenges are foregrounded? Which discourses run throughout? How are the challenges represented? Which challenges are excluded? To analyze style, we asked: How does the district leader position the district in relation to the challenges and efforts to address those challenges? How does the district leader express a stance toward the problems and efforts? To whom or what does the district leader attribute the source of the problem?

Vouchers and Private Schools	Promotes choice that will subject schools to market forces. This will, in turn, promote efficiency through competition and lead to high achievement and equalize educational opportunities.
Intensification	Focuses attention on "the basics;" intensify existing efforts
Carrots and Sticks	Suggests a need for state and federal controls; specifies standards for students, teachers, and schools
Performance Indicators and Accountability	Ties funding to "objective" indicators.
Immersion Programs for Language Minority Students	Forces immigrant students into immersion programs in which all instruction is in English.
The Elect and the Damned	Sets up an "ability"-tracking system in which groups of students are exposed to different curricula.

Table 3: Coding Scheme for Neoliberal Discourses based on Biddle and Berliner's (1995) Identified Reform Efforts

We used these questions to structure memos that served as the basis for subsequent analysis in which we looked for patterns in how the identified discourses shaped district leaders' articulations of challenges. These patterns emerged as we considered the ways in which district leaders either explicitly identified neoliberal discourses and their response to them or seemingly unknowingly evoked neoliberal discourses in their identification of challenges and responses to those challenges.

Findings

Three of Biddle and Berliner's (1995) reforms were evident within and across interviews: vouchers and private schools, carrots and sticks, and performance indicators and accountability discourses. There was some evidence of intensification discourses with Devin, the superintendent of Rural Town, but only briefly and they were not evident in the other interviews. The other discourses—immersion programs for language minority students and the elect and the damned—did not emerge. Because there were not sufficient data around

these other reforms, they are not included in the subsections that follow. In the discussion section, however, we do offer some conjectures for their absence in the data.

The following subsections are organized by the discourse. The first attends to voucher and private schools discourses. The second – standards and accountability discourses – combines two of Biddle and Berliner's (1995) reform efforts: carrots and sticks and performance indicators and accountability discourses. Within these two subsections, we describe each discourse (research question 1) and describe the ways in which the discourses shape district leaders talk (research question 2).

School choice discourse

Vouchers and private schools discourses emerged in the interviews with Devin, the superintendent of Rural Town, and Teresa, the assistant superintendent of Midwest City. Both Devin and Teresa were aware of how those discourses influenced their districts. The discourses were visible and named.

Devin articulated the challenges around the threat that charter schools posed to his district and community. While this discourse was making its way into Devin's articulation of challenges, it was explicitly named, and Devin actively opposed it.

> That [Senate Bill 313] is going to take money right out of our pockets and it's going to hand money over to entities that don't necessarily have to accept some of the students that struggle that I just told you about, and most states, see that that money goes to upper-middle, the wealthy class individuals who take their voucher and help pay for their private school while it continues, you know to starve us out.

Teresa, assistant superintendent of Midwest City, described the context of her district's community, "It's like you have so many districts, you have so many schools and honestly, it looks different across the whole [Midwest City] area. You have charter. You have you have private. You have public, you have, we have signature schools." Teresa described the reality of the funding challenges that Devin described as threatening his district. In her context, these challenges were related to equity.

> So the equity issue is very different, also I think with what happens is you know, you have an area of the city that's considered poor, its impoverished it's highly really, African American, the resources are very slim because we share my budget, I have to share my budget with the charters, because we're all within the [Midwest City] area.

Like Devin, Teresa was aware of how school choice discourses affected her district. For both districts, school choice discourses created challenges related to funding. Devin was confident that community members in Rural Town would pass a local tax levy to fund the public-school system. According to Teresa, Midwest City lost students and funding to charter, private, and signature schools.

Both Rural Town and Midwest City experienced school choice and voucher challenges. In both cases, losing students to other schools meant losing much needed funding for federal and state mandates Those funds were (or would be) diverted to charter and private schools not held to the same accountability standards. While these efforts were in the name of equity

– to help the "poor kid" according to Devin – they ultimately lead to less funding for students attending public schools, "poor kids" in Rural Town and poor and African American kids in Midwest City.

Standards + accountability discourses

Carrots and sticks (discourses suggesting a need for state and federal controls and specifying standards for students, teachers, and schools) and performance indicators and accountability (discourses tying funding to "objective" indicators) emerged in all three interviews, evident in some capacity in almost every stanza. Often, standards and accountability *were* the challenge. For example, when asked about current challenges in mathematics, Brooke, the assistant superintendent of Midwest Suburb, immediately noted their district's concern with "lack of progress" according to achievement data. Based on these data, the district had explicitly identified ninth grade Algebra I as their focus. Brooke explained that grade level and department teams met with instructional coaches about once a month to discuss "what kids are telling us in that data."

Brooke articulated the specific focus, ninth grade Algebra I end-of-course (EOC) exam scores, and attributed the lack of progress to an incoherent progression in mathematics from the early grades. In the early grades, ninth grade Algebra I EOC scores were less of an explicit challenge but standards and accountability discourses still served as a mechanism for describing the challenge of incoherence. This manifested as a "progression of skills" to be mastered so that students do well on the ninth-grade test.

> There always, really, kind of has been a progression of skills that were supposed to have been taught. Right? And mastered at every grade level. And if those skills are missed, I didn't get to whatever that might be, addition, subtraction of mixed numbers or I didn't get to, you know, those sorts of things. [If] that doesn't happen then ultimately the impact is seen by the time kids get to high school, so I don't ever say to third grade, 'our ninth grade scores are not good; our ninth grade algebra one scores are not good,' but we have to look at it in such a way that sequentially, we have, we are each responsible for a set of skills and for deepening mathematical thinking so that by the time they get to ninth grade the assessment at ninth grade is not *that* difficult.

In Midwest Suburb, accountability discourses shaped the identification of the challenge (Algebra I EOC scores), the cause (incoherence), and the solution (delineating algebraic thinking). Both performance indicators and accountability discourses (the identification of Algebra I EOC scores as an explicit focus) and carrots and sticks (tasking grade levels with skills and standards) work together in Brooke's articulation of Midwest Suburb's challenge and response.

Devin, the superintendent of Rural Town, described challenges of acquiring curriculum materials aligned with the shifting of accountability standards from Common Core to the Missouri Learning Standards.

> We had been waiting Missouri has, of course, new Missouri standards that have come out, and we were heading head on toward the Common Core and were preparing for that. And then the brakes kind of got hit on all of us…Our district in particular had been waiting on buying and purchasing resources, and when I say resources, everything from curriculum [of] course because we didn't know…what the Missouri standards would say…So I would say the biggest challenge for us right now is, at least from the superintendent standpoint, is financial to meet all the curriculum needs because it's going to be extremely expensive to set everyone up and then you've got science and you've got all the other areas as well. So we may well be looking at some sort of local initiative to raise our tax levy in our operating funds in order to meet the demands and we're facing right now with the Missouri standards coming on at all the curricular needs that we have.

His challenge (curriculum acquisition), the source of the challenge (shifting standards), and his solution (wait until the shift happens and pass a tax levy) were all shaped by standards and accountability discourse. However, it was not clear if the challenge and the way he talked about the challenge were shaped by the need for federal and state control (carrots and sticks); that funding is tied to "objective" indicators (performance indicators and accountability); or most likely, both. It seemed that federal and state control created the need for new curriculum materials and determined the timeline of acquiring those materials across subject areas. However, the reason for aligning to standards remained unclear. While Devin did not explicitly say, we wonder if performance indicators and accountability – preparing students for state mandated tests – is silently implicated and so commonplace that it remains unsaid. In this way, it seemed that acknowledging one discourse necessarily evoked the other.

District leaders also resisted the imposition of standards and accountability discourses. In response to "the uncertainty of assessments, statewide assessments, constant changes and curriculum," Rural Town decided to articulate and focus on "what we thought was important." Devin, the superintendent of Rural Town, identified five aspects for their focus: kindergarten readiness, third grade reading, middle school and high school transition, college and career readiness, and ACT (a standardized assessment for college admission) scores. Even in this initiative, an effort born out of frustration with the uncertainty of state-mandated assessments, the district's initiative was framed with scores and readiness measures.

Discussion

School choice discourses and standards and accountability discourses – which included Biddle and Berliner's (1995) carrots and sticks and performance indicators and accountability reform efforts – were evident across the interviews. These discourses shaped district leaders' identification of challenges. Standards and accountability discourses found their way into the articulation of the problem, perceived cause of the problem, and initiatives to address the problem in all three interviews.

As noted above, some of Biddle and Berliner's reform efforts were absent from the data. It is possible that those discourses were not appropriated or experienced by the district leaders in our sample. Alternatively, perhaps they have become so dominate they are now "common sense" (Apple, 2017). Given the default organization of mathematics into tracks (Oakes, 2005), it seems likely.

Teresa and Devin explicitly recognized and resisted school choice discourses. In both cases, losing students to charter and private schools meant losing much needed funding for the students in their schools. Both Devin and Teresa recognized that while these discourses were often under the guise of helping poor and African American students, they worked against equity efforts. While the district leaders recognized and resisted school choice discourse consistently, all three district leaders appropriated standards and accountability discourses to identify or describe their problem. However, as McDermott described, testing, school reform, and neoliberalism are closely related:

> High-stakes testing is the thread that ties together a larger picture of reform that includes privatization of public education, replacing public schools with charter schools, enforcing a curriculum that "force feeds" meaningless data to already disempowered and disenfranchised communities and uses "accountability" to turn data into big profits" (pp. 78–79).

While there is hope that district leaders resisted neoliberal discourses of school choice, their appropriation of standards and accountability discourses to identify their challenges and describe their efforts further promotes the neoliberal agenda they fight.

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How post-factualism creates new needs for the epistemology of mathematics

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Discussions in mathematics education research have lately turned towards the phenomenon of post-factual discourses and reflected on the role of mathematics education in facing this challenge. While contributions so far have focussed on fact checking, source criticism, and a critical attitude towards mathematical models, this essay establishes that such scepticism lies at the very heart of post-factualism, and that its use in education bears the danger of producing and fortifying post-factual attitudes instead of overcoming them. It is shown that instead we require to acknowledge the roots of such scepticism in relativist epistemologies and to find answers to the question how facts are possible if all knowledge is relative. This essay refers to Hacking's framework of styles of reasoning as a possible solution to this dilemma.

Why bother?

The growing impact of post-factual discourses and the declining trust in science indicate that the rational society and liberal democracy are facing a crisis. Mathematics education, which may be regarded as part of an appropriate strategy to address post-factualism, might just as well bear responsibility in producing anti-rationalist sentiments, presuming it failed to introduce a wide selection of students to rational argumentation, and instead promoted dependence on teachers as authorities of knowledge, the uncritical processing of pre-determined rules, and an unjustified trust in the omnipotence of mathematical solutions in real-world applications. This alarming possibility should direct our attention at the epistemological question how mathematics is effective in creating factuality.

In the next section, I will discuss how post-factual attitudes have been approached in mathematics education research so far. After that, a closer look at post-factual attitudes and at epistemology more generally will recontextualise the above problem in a field of more general epistemological problems: Following the argument that it is unreasonable to conceive of truth and factuality as an attribute that is independent from humans, and embracing the relativist assumption that truth and factuality are products of human understanding, the urging question will be how truth and factuality can be conceptualised in the first place. I will end this essay will a brief outlook on a possible solution and its implications for mathematics education research.

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Mathematics education research on post-factualism

Published discussions on post-factual discourses are scarce in mathematics education. I searched the lists of contents and, if accessible in digital form, the whole text corpus of the articles published in eleven different Anglophone and six different Germanophone mathematics education journals for articles containing the terms 'fake news', 'post-factual', 'postfactual' or 'post-truth', or their German equivalents.[1] The search yielded five results (Heitwerth, 2020; Oldenburg, 2020; Orey & Rosa, 2018; Pais, 2018; Trakulphadetkrai, 2018), but in all cases post-factualism is only addressed causally. A search for these terms in connection with 'mathematics education' on Google and Google Scholar yielded three more results (Hauge, 2019; Hauge et al., 2019; Marcone et al., 2019), which feature post-factualism as a main theme. I will not discuss Pais's (2018) summary of Borovik's (2017) concern about the diffusion of fake news through automated big-data processing, nor will I address Trakulphadetkrai's (2018) report that two plenary speakers at the 9th British Congress of Mathematics Education reminded the participants 'that data needed to be interpreted with care, particularly in the age of fake news' (p. 50). The former is not of central importance for the question discussed in this essay, while the brevity of the latter makes it difficult to interpret the statement.

In their discussion of a mathematical modelling course in a virtual learning environment (VLE), Orey and Rosa (2018) argued:

> This particular form of pedagogical action on the VLE web site directed students towards the development of sound arguments based on data that they collected and towards observing, generalizing, and applying mathematical models to solve future problems faced by their own communities. Also, in a context where information is often unreliable, it helps individuals to move away from emotional manipulation (via social media and *fake news*) and to learn to search for deeper explanations and distinct ways of reflecting and dealing with societal issues based on the scientific method and factual data. (p. 183, original emphasis)

Heitwerth (2020) reflected on mathematical models that were used in public communication in Germany to legitimise the social lockdown during the SARS-CoV-2 pandemic in 2020. He argued that such reflections are necessary so that students can detect 'fake news' about the corona crisis and understand the importance of 'evidence-based politics': 'Following this idea, a pro-scientific perspective can be supported, so that the individual epistemic beliefs of the learners may be based on facts and not fakes' (p. 199, my translation). Heitwerth discussed how an understanding of graphs of functions, in this case relating time to the number of registered SARS-CoV-2 infections, can help to evaluate the

[1] The Anglophone journals were Canadian Journal of Science, Mathematics and Technology Education, Educational Studies in Mathematics, For the Learning of Mathematics, International Journal of Science and Mathematics Education, Journal for Research in Mathematics Education, Mathematics in School, Mathematics Teaching, Mathematics Teaching in Middle School, Research in Mathematics Education and ZDM Mathematics Education. The Germanophone journals were Der Mathematikunterricht, Journal für Mathematik-Didaktik, Mathematica Didactica, Mathematik Lehren, Mitteilungen der Gesellschaft für Didaktik der Mathematik and MNU-Journal.

legitimacy of graphs shown in media and the claims that can follow from their interpretation. Both studies illustrate how mathematics can be used to question the epistemic status of claims and to legitimise claims mathematically. Thereby, they suggest that an awareness of this role of mathematics is suitable to counter post-factual discourses. In both cases, the activities question the legitimacy of mathematical representations (numbers and graphs) which are derived from modelling activities. However, it remains opaque what 'the scientific method', 'factual data' or 'facts' might mean and in which way mathematics can contribute or not to reaching this standard. This leads to the epistemological question how mathematics is effective in establishing or refuting the fact status of claims.

The most elaborated work on post-factualism in mathematics education was developed by Hauge and colleagues. Her approach is more specific as it focusses on the role of mathematics in manipulation through fake news. Hauge (2019) discussed problems one faces when discussing the trustworthiness of numbers in news in mathematics education, including the difficulty of distinguishing uncertain, wrong or deceptive uses of mathematics, the arbitrariness of the social production of numbers in applications of mathematics, and the declining trust in science. Hauge et al. (2019) argued that '[s]uch situations require a critical investigation of data, simplifications, assumptions, graphic presentations, validity of conclusions and how limitations in mathematics-based argumentation can be linked to an underlying agenda' (p. 3). The breadth of these requirements indicates that epistemological awareness is required on top of mathematical knowledge. Based on her contemplations, Hauge (2019) suggested among other things that students should learn mathematical concepts, how to argue based on applications of mathematics, how argumentation is structured in general, and that mathematical methods are neither correct nor incorrect but arbitrary.

Hauge et al. (2019) discussed an example of fake news, where the choice of statistical data and a not fully justified application of mathematical concepts resulted in politically biased statements. This discussion, just as the studies by Heitwerth (2020) and Orey and Rosa (2018), faced applications of mathematics on an exemplary basis. Considering aspects of mathematics such as deductive proof, it appears necessary to expand the focus beyond mathematical modelling when evaluating how mathematics is productive in establishing facts. The merely exemplary character of the present discussions suggests that there is a lack of solid epistemological theory of how mathematics establishes facts.

The two remaining publications indeed turn towards epistemological problems, although in a very fundamental way. In his critique of the assumption held in radical constructivism that truth is relative, Oldenburg (2020) argued that

> an enlightened pedagogy must take a clear stand against such views. But it will not succeed in doing so if it relies on constructivism, which encourages such relativism and considers 'alternative facts', as the current US president [Donald Trump] likes to use them, to be perfectly normal. (p. 80, my translation)

The concern that post-modern tendencies in mathematics education may go hand-in-hand or even brisk up post-factual attitudes is shared by Marcone et al. (2019). The authors

referred to the increasing impact of fake news and asked how to relate to that problem in mathematics education. They suggested that the 'timeless reasoning of mathematics' is losing relevance as discourses become more 'appealing of the emotional side' (p. 186). Reflecting on the role of mathematics education research, they wrote:

> It is paradoxical that critical scholars have fought against the uncritical faith in mathematics and now the problem is that there is a uncritical suspicion in mathematics. We wonder if the Mathematics Education community played some part in this battle. Part of the Mathematics Education community always criticizes the ideology of certainty, but now that it is gone from some places, some of us are afraid. Now the dominant discourse is to bypass and ignore scientific facts. (pp. 186–187)

Marcone et al. (2019) concluded:

> Our contention is that we are not being successful at promoting a critical distance towards mathematics, and that many of our arguments against universality and neutrality have been trivialized and turned back against its original intention. […] We need to understand and operate within this mechanism of de-mathematising public discussion. We need to begin to spin in a different direction to rescue the critical perspective on mathematics. (p. 187)

For that purpose, this essay will look further into the phenomenon of post-factual discourses, before it will face the problem how an epistemology of mathematics could conceptualise truth without falling back into essentialism.

From post-structuralism to post-factualism

MacMullen (2020) asked 'what would it mean for discourses to be post-factual' (p. 97), only that he, working in political science, wrote 'politics' where I now write 'discourses'. He differentiated four ideal types of post-factual attitudes. This distinction will prove essential for my line of argument.

The first attitude is *unconscious* post-factualism. Here, someone is not even aware of her involvement in post-factual discourses. MacMullen (2020) highlighted that psychological mechanisms facilitate the acceptance of post-factual information. Irrespective of the factual status of a claim, we are more likely to accept it as true if it fits our values and our personal experience, if we hear it oftentimes, and if we trust the people holding it. Needless to say that many professional political campaigners have specialised in using such psychological mechanisms for the benefit of their party. These effects are further supported by the tendency of digital social networking to connect people with similar mindsets and not people with conflicting mindsets. People with unconscious post-factual attitudes might usually prefer factual discourses and might find it disturbing to realise that they have participated in post-factual discourses. An obvious educative approach to this attitude would be to teach people to be sceptical about truth claims and to question the interests and values that lie behind them. This appears to be the position of the studies in mathematics education discussed above, especially that of Hauge (2019), whose focus on fake news led her to assume a dichotomy between ruthless, interest-driven producers and passive, helpless victims of post-factual discourses. In contrast we should also consider in how far post-factual

discourses are actively produced and shared by a wide range of people, often with awareness of their problematic epistemic status, as in the following three attitudes.

The second attitude is *metaphysical* post-factualism. Here, post-modern critique of truth and objectivity is used to argue that truth and objectivity are nothing but interest-driven constructions, which blurs the distinction between factual and post-factual claims altogether. MacMullen (2020) remarked that this attitude is a rare intellectual variety of post-factualism. Nevertheless, this attitude highlights central epistemological problems, which I will address later in this essay.

The third attitude is *motivational* post-factualism. Here, not denying the existence of facts, people choose to rather trust in a discourse that 'expresses their values and makes them feel good, often by providing affirmation, a sense of community and identity, and perhaps some degree of entertainment' (MacMullen, 2020, p. 105). They acknowledge the problematic epistemic status of post-factual claims but refuse the effort that it would take to evaluate such claims and to counter the psychological mechanisms that lead people to unconscious post-factualism. They widely dismiss concerns for the effects of their reliance on post-factual information. An obvious educative approach to this attitude would be to raise awareness for the individual and societal effects of this epistemological idleness. Reflections on the effects of the reliance on post-factual claims on the policy making of some countries during the SARS-CoV-2 pandemic in 2020 might already teach such a lesson.

The fourth attitude is *epistemic* post-factualism, whose impact MacMullen (2020) held to be underestimated. Here, not denying the existence of facts, someone assumes to lack the skills or resources to evaluate claims but does not trust experts either: 'Seeing no way to be guided by the facts, such epistemically post-factual citizens consciously make their political decisions in a different way' (p. 109). MacMullen established that there is no one-dimensional approach to post-factualism, for the educative approach recommended for unconscious post-factualism, that is being sceptical of truth claims and questioning the interests and values they are based on, appear only to solidify the attitude of epistemic post-factualism. Instead, he argued that these people 'need a better understanding of how communities of experts and legitimate news organizations operate and the associated (defeasible) reasons to trust their empirical claims' (p. 116).

It becomes clear that it is insufficient and might be counter-productive to face post-factual discourses merely by establishing their post-factual status and by learning to question the interests and values of the participants in this discourse as done in the examples from mathematics education presented by Hauge et al. (2019), Heitwerth (2020), and Orey and Rosa (2018). People with attitudes of conscious post-factualism do not see the post-factual status of claims as a problem and already mistrust the representatives of factual claims. Considering the metaphysical, motivational and epistemic types of conscious post-factualism, what is needed to rebuild trust in factual discourses is an epistemological legitimisation that factual discourses are possible, that it is worth the effort to prefer facts over sentiment, and that laypersons have the ability to judge whether a claim is based on facts or not. Thus, the scene is set for the discussion of genuinely epistemological questions

in the mathematics classroom – at least if mathematics education is supposed to play a part in addressing post-factual attitudes.

Epistemological education in the mathematics classroom

Oldenburg (2020) hoped for a new kind of realism to rescue the idea of truth and insight which he saw represented in mathematics. Chakravartty (2017) proposed to call an epistemology 'realist' if it follows

- the metaphysical assumption that the objects of reality exist independently of human beings,
- the linguistic assumption that language allows for a true description of reality, and
- the epistemological assumption that knowledge resembles reality.

Especially mathematicians are said to often share realist positions (Davis & Hersh, 1980). This becomes obvious when we presume that the law of the excluded middle (assuming that a claim is true or false and not beyond or in between that) is an uncircumventable law of thought for any sane person, or that the decimal expansion of π, defined as the ratio of a circle's circumference to its diameter, stays the same anywhere, at any time, for anybody, also for aliens whom we send this decimal expansion to illustrate the intellectual achievements of us Earthlings.

However, closer analysis reveals some fundamental problems of realism. Although some of these problems were already debated in ancient anti-Eleatic philosophy and re-entered the philosophical stage in the eighteenth century by philosophers such as Berkeley, Hume and Kant (Liston, 2020), they were hardly taken into account in the wider field of science and arts until the emergence of post-modern and constructivist paradigms in the second half of the twentieth century. The following line of argument is an abridged selection from a philosophical discussion which easily fill books (e.g., Audi, 1997/2010; Landesman, 1997):

- Epistemologically, it has been established that we, limited by our sensual perceptions, cannot know in how far our theories accurately resemble an independent existence or if they leave blind spots (e.g., from a biological perspective, Maturana & Varela, 1984). The epistemological assumption had been called into question from within mathematics as the discovery of non-Euclidean geometries raised serious doubts whether geometric theories were indeed simply representations of an independent world (Davis & Hersh, 1980).
- Metaphysically, this means that neither can we determine if a world independent of human beings exists, nor does it matter epistemologically. The metaphysical assumption is therefore scientifically undecidable and reduced to a matter of belief. Yet, Horkheimer and Adorno (2002) noted that a belief in truths independent of humans can be frightening and dangerous, for it degrades the role of humans from active and ethically responsible constructors of their intellectual world to passive discoverers of already decided facts (p. 8).

– Linguistically, post-structuralist studies suggest that academic discourses, for example on insanity, delinquency, sexuality and truth itself, cannot be understood as a true description of reality, but constitute a historically contingent product which depends on values, interests and power relations (e.g., Foucault, 1961). Elsewhere, I presented such a post-structuralist argumentation for logical thinking (Kollosche, 2014b, or, if you can read German, preferably Kollosche, 2014a, Ch. 5). The ineluctable discrepancy between discourses and that which escapes articulation constitutes a central pillar of Lacan's psychoanalysis (Žižek, 1994). Subsequent studies following the post-structural paradigm have focussed on a deconstruction of ever more hegemonic discourses. This included changing perspectives on mathematics, which has come to be understood as an at least partly arbitrary product of history and social struggle (Davis & Hersh, 1980), as a weapon of colonising non-Western epistemologies (D'Ambrósio, 2007) and as a gender-biased style of thinking (Burton, 1995).

I referred to Marcone et al.'s (2019) and Oldenburg's (2020) fear that the growing acceptance of post-factual discourses coincided or has even been nourished by the refutation of realism and the orientation towards relativist epistemologies. Baghramian and Carter (2019) explain relativism as 'the view that truth and falsity, right and wrong, standards of reasoning, and procedures of justification are products of differing conventions and frameworks of assessment and that their authority is confined to the context giving rise to them', which would include constructivist and post-structural positions. Relativist epistemologies do not face the fundamental problems of realism discussed above, but, as Koschorke (2019) pointed out, they face the problem that they do not explain what should be regarded as truth, as there is no human-independent authority left to judge the suitability of any discourse for producing facts.

At least in the case of post-structuralism, this theoretical limitation has to be understood historically. Post-structuralism developed in a Europe reflecting on the horrors of the Second World War and experiencing the process of political decolonisation. Left protest movements were concerned with challenging authorities, refusing hegemonic world views, opening the public discourse for marginalised positions and advocating for a pluralism of lifestyles. The objective was to counter hegemonic discourses and to create spaces for alternative positions, not to (re)establish a basis for intellectual consensus.

Koschorke (2019) and MacMullen (2020) supported Marcone et al.'s (2019) and Oldenburg's (2020) fear that relativism has prepared the stage for post-factualism. McIntyre (2018) documented in great detail how political and economic agents learned to deploy post-structural lines of argumentation in order to change the rules of a debate to their advantage. Thus, relativist positions have become common among non-researchers, including the assumptions that truth is relative to arbitrary frameworks of reference, that experts are biased and therefore untrustable, and that truth claims are mere deceptions waiting to be

deconstructed. This is how the refutation of essentialism has facilitated the conscious acceptance of post-factual attitudes.

Koschorke (2019) and MacMullen (2020) assumed that the awareness of epistemological problems in the wider public cannot easily be erased in order to return to factual public discourses. This calls for more elaborated answers on how a consensus on knowledge can be achieved in contemporary societies. However, Koschorke (2015) demonstrated that contemporary attempts to establish a new form of realism do not solve but merely ignore the fundamental problems of realism. As of now, there appears to be no epistemological position which could reinstate the above-mentioned assumptions of realism while overcoming the problems they invoke. Any epistemological solution will therefore have to be sensitive to anti-essentialist arguments. Just like Koschorke (2019) argued for cultural studies, we face the open question how we can find our way

> out of the dead zone into which the discipline has maneuvered itself, following decades of deessentialization, doubt about norms, and institutional critique—without giving up the emancipatory potential and cognitive achievements of poststructuralist theories and their global advancement. (p. 1156)

Styles of reasoning: A solution!?

Elsewhere, I discuss Ian Hacking's epistemological framework as a possible solution (Kollosche, 2021). Here, I can only sketch the basic idea of the framework: Based on the study of the historical development of styles of scientific thinking by Crombie (1994), Hacking (1982) proposed that different styles of reasoning define which statements are candidates for truth-or-falsity, how their truth value can be determined, and what the objects of their statements mean. For example, the *postulation style of reasoning* creates deductively structured theories which include presupposed objects with presupposed properties, and in which the legitimacy of an assertion must be deduced. Thereby, its objects are defined by their suitability for a deductive theory. Hacking mentions six other styles, including the *taxonomic* style, the *statistical* style, and the *algorismic* style. All these styles add to our understanding by employing different ways of saying and determining 'what is', without claiming to describe any reality that would exist independently of them.

It is intriguing that most of the styles described by Crombie (1994) and Hacking (1982) have connections to mathematics. This suggests that mathematics education might be a priviledged if not even an imperative place to discuss in school settings how scientific reasoning is possible, even if it does not reveal the properties of any mind-independent reality. It is yet an open question how to approach specific styles in mathematics education and if traditional ways, for example for discussing proof and mathematical modelling in the classroom, are appropriate. However, the rich repertoire of relevant episodes from the history of mathematics and science provided by Crombie (1994), Hacking (1982), and others provides a powerful basis for conceptualising epistemological education through mathematics to face post-factual attitudes.

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Curriculum reconceptualization and rhizomatic thinking: Introducing Venn diagrams with Roma students

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This paper through examines how the mathematics curriculum may be reconceptualized through rhizomatic thinking. Students through this process co-shape the mathematics curriculum and create the conditions to acquire meaning through their personal history and their lived experience in a process which is characterized by unpredictable and uncertain events. Mathematics emerges as a useful and practical 'vehicle' helping students to reconsider common sense in elements of their daily lives, to soften power relations, to act towards the restoration of social justice and to lift conditions that are problematic for their personal lives.

Introduction

The internationalization of curriculum studies (Pinar, 2013) encourages post-colonial networks which ignore and challenge dichotomies such as formal/informal knowledge, theory/practice and epistemology/ontology as they engage each other in complicated conversations about mathematics curriculum reconceptualization. A non-hierarchical dialogue among the students and the teacher-researcher may lead to a commitment to forms of knowledge that are not linear but instead have multiple entry points and multitude of pathways which concludes to the need for a rhizomatic approach in research process.

To enact curriculum conceived as a subjectively oriented and historically attuned conversation (Aitken & Radford, 2018; Pinar, 2004) means associating academic knowledge with people involved, teaching not only what is mathematical knowledge, but also suggesting its possible consequences for the individual's self-formation in the historical present, allowing new knowledge to shape the individual's life. In that sense mathematical knowledge in schools should be framed neither with predetermined skills and tasks nor with standardized tests. A mathematics rhizocurrere is thus proposed to create new conditions for successful and meaningful mathematics learning for Roma students.

Mathematics usually act as a key subject which is of great importance in the school curriculum. Many students find mathematics difficult, and some scholars have argued that mathematics serves as a form of gatekeeper by determining social mobility. Relevant research reveals a strong relationship between mathematics performance and the socioeconomic status of the family. (Douglas & Attewell, 2017). D'Ambrosio (1990, p. 23)

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refers to this as the "social terrorism" of mathematics education and links it to the production of negative self-esteem among minority students. Regarding Roma students, they have poor interaction with the mathematics curriculum, and it is very common for them to complete the six grades of primary school knowing only the four basic arithmetic operations.

This paper is part of a doctoral dissertation proposing a reconceptualization of the mathematics curriculum for 3rd grade Roma students in Greece through a post-qualitative inquiry. Research setting is located in the region of Acharnai which is a municipality of Athens and research participants are twenty Roma students who attend the 3rd grade of the primary school. Third grade students are selected because they have started to shape a mathematics class student identity while at the same time negative beliefs about mathematics have not been established yet.

This paper aims at expressing, through the introduction of Venn diagrams, Roma students' subjectivity. It explores the lived experience of Roma students highlighting underlying links with mathematics academic knowledge in order to portray how the currere, the lived experience of the mathematics curriculum (Pinar, 2013), may replace the planned one. It traces alternative ways of learning and teaching process of mathematics in schools which has the potential to make sense to the Roma students and how students will actually gain voice through their mathematics involvement. Focusing on the interaction of Roma students with mathematics it is proposed an alternative way of approaching the mathematics curriculum reflecting on students' experiences and practices in various settings which act as sources for the development of mathematical knowledge. Roma students face difficulties related to their schooling in general and there are poor signs of improvement despite the efforts made by educational policies.

Research (non) methodology

Rhizomatic thought (Deleuze & Guattari, 1987) and writing orients the study of social interaction in educational contexts, to the deconstruction of arborescent models deterritorializing, thus, common assumptions that affect the educational process. Rhizomatic thought aims neither to provide a better research model nor to reject the conventional humanistic research but by creating multiple starting points (Colebrook, 2021) it is able to provide new insights and understandings of the ways a mathematics curriculum may be reconceptualised. "There are no points or positions in a rhizome, such as those found in a structure, tree, or root. There are only lines" (Deleuze & Guattari, 1980/1988, p. 8). A rhizome is a non-linear and non-hierarchical network which develops and evolves in contrast to the humanitarian tree and its influence in the field of education.

While researching rhizomatically, educational becoming and subjectivities at the level of people and situations involved, both interact as elements of a labyrinthine and incalculable (Lather, 2016) rhizome which consists of discontinuity, rupture and constantly emerging multiplicities. Research process and rhizomatic thinking have helped the researcher to think the 'to affect' and "to be affected" as two sides of the same coin of creating the canvas of a lived experience mathematics curriculum with Roma students. Rhizomatic thinking moves from a unified, conscious and rational epistemology of human consciousness to a relational ontology (Walsh, Böhme, & Wamsler, 2020).

This postqualitative research seeks to "test the limits of our knowledge" by using thinking with theory[1] (Jackson & Mazzei, 2012). Thinking with deconstruction (Derrida, 1967) as the theoretical framework for teaching and learning mathematics with Roma students, permits to work within and against interpretivism during data analysis. Deconstruction is not a method and cannot be transformed into one" (p. 3). For Derrida (1967/1973), a thinker with a method has already decided how to proceed and is simply a functionary of the method, not a thinker. As Derrida explained, deconstruction is not necessarily intentional—it is what "happens" (p. 89)—and categories like the research process, the interview, the field, data, data collection, and data analysis fall apart (St. Pierre, 2018).

During the research process Roma students' experiences values the context and the social construction of mathematics in relation to the meaning. This paper assumes an ethnomathematical rhizomatic position as a clear political position, because research promotes the preservation of the dignity of Roma students' cultural group. This political position rejects a colonialist approach which imposes only one way of thinking mathematically and produces widespread educational failure that causes suffering to children. The political position in mathematics education through an ethnomathematics rhizome assumes a respect for the knowledge produced in different contexts, which depends on the historical and cultural funds of knowledge of Roma students. Postqualitative inquiry does not try to measure this experience based on western mathematics, it is all done within the system and has value as such, which is a pure non-colonialist practice.

Intercultural orientation stands as an element of the quality of pedagogical practices (Abacioglu et al., 2020; Stathopoulou et al., 2012). The ethnomathematical rhizome as a pedagogical and philosophical act is basically an act open to the future development of children, which takes into account the current cultural and learning funds of knowledge of the child but remains open to the elements of the unpredictable, spontaneous and authentic logic. In that sense, the interculturally oriented curriculum development should use the different cultural and linguistic aspects that Roma students bring with them in order to strengthen their individual cognitive and socio-emotional development.

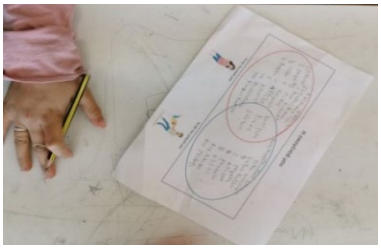
Postqualitative inquiry's immanence (St. Pierre, 2019) makes the research not to systematically repeat a preexisting research, reprocess to produce a recognizable result, but to experiment and offer something new and different that "might not be recognizable in existing structures of intelligibility" (St. Pierre, 2018). Deleuze (1968/1994, p. 136) explains that the thoughts and practices considered as authority of knowledge should be treated "every time as something which has not always existed, but begins, forces and under constraint". Research design in this case does not have pre-formed questions and methodology as the field under study unfolds in front of the researcher while experimenting, creating, becoming and re-orienting thought. Researcher's identity is shaped simultaneously with the research process.

[1] Jackson & Mazzei (2017) situate thinking with theory, "not as a method with a script but as a new analytic for qualitative inquiry. Every truth, Deleuze (1983) wrote, is of a time and a place; thus, we work within and against the truths of humanist, conventional, and interpretive forms of inquiry and analysis that have centered and dominated qualitative research texts and practices. We proceed with hesitation and a sense of instability, because as readers will see, there is no formula for thinking with theory: It is something that is to come; something that happens, paradoxically, in a moment that has already happened; something emergent, unpredictable, and always rethinkable and redoable" (p. 717).

In practice, the researcher approaches the subject through experimentation and lived experience, leaving open implications for new things that may arise in the process. The discursive breadths that result from lived experience are treated with transcendental empiricism but are not intended to establish universal truth about the process. The change takes place simultaneously with the research and the validity is developed through the eyes of the participants who experience the change in their daily practice. New ways of thinking can emerge through engaging with mathematical concepts, bringing about long-term and short-term changes in the daily lives of Roma students. Deleuze (1990/1995) suggests "never interpret experience and experiment (p. 87)" giving emphasis on the creative character of the research which is open to new "becomings" that seek to shape how both participants and the researcher are "becoming" in the assemblages of the research.

Research (non) data

Quite often students discuss to determine the relationship between them. The kind of relationships between them is a constant topic of discussion. They also discuss how these relationships arise through their family's history and their family tree. In order to illustrate these kinship relations in a tangible way, we decided to negotiate the Venn diagrams in collaboration with the students. First, the children were presented with an image of items that can be purchased from a grocery store and items that can be purchased from a greengrocer. Thus, two distinct sets of objects were formed, but also some objects emerged that exist in both stores. These objects are written at the intersection of the two sets.

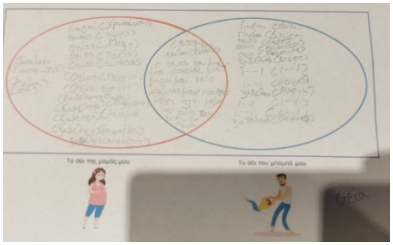


Picture 1: Representing family tree through Venn diagrams

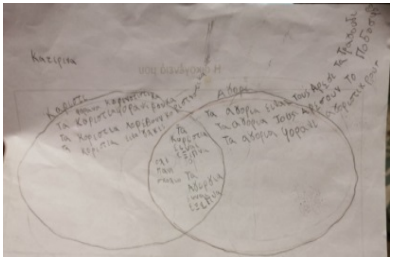
In order for the students to understand the meaning of the intersection and the union of the sets, they were presented with a simple diagram, which consists of all the relatives of the father and all the relatives of the mother. The intersection of these two sets includes the students themselves as well as their siblings. Students, after extensively completing their parents' family members, understand the essence of the Venn diagram, their practical utility in depicting their family relationships, and discover family members belonging to both groups.

In the discussions that take place in this context, Evangelia wonders "My father has one or two second cousins (brothers among themselves) one of whom has married my mother's sister and the other has married my mother's first cousin. Will their children be enrolled in both circles?"

Spyros states, "My father and mother are somewhat related. That is, my grandfather's brother adopted my mother. Nevertheless, they are not of the same blood. Shouldn't my parents belong to both circles as we children do?" At the same time, cultural issues emerge as the rest of the students realize that it is not "correct" to report publicly their family's incest issues. Students, thus, are beginning to hide those written in the intersection of the two sets in the Venn diagram by erasing some names.



Picture 2: Identifying persons who belong to the intersection



Picture 3: Thoughts about sex discrimination

This mathematics task gives children the opportunity to realize incest issues that concern their families, to consider whether it is right or not that this happens and whether they should reveal their family's secrets during the lesson. It is common in Roma society for marriages to take place in a closed circle. This certainly does not happen because the Roma do not want to alter their race as the prevailing view claims. In reality, the cause is that the prevailing culture avoids associating with them, thus leading to their marginalization and isolation as a group (Ciaian & Kancs, 2019). Consequently, teenagers in Roma community get usually married at the age of fourteen.

Zoe reports, "My godfather has married my mother's first cousin. My godfather has my father as his first cousin but I will not write him in the middle. I will write my godfather, without you knowing that he is my father's cousin. Because if they see it, they will kill me because I told tales about it". Katerina (who is a child with older sisters who have dropped out of school at the age of 12 in order to get married) makes a Venn diagram with the differences between boys and girls. At the intersection of the two sets there are the phrases "girls and boys are smart, they both go to school". In other words, we realize that the child is overwhelmed by thoughts about gender discrimination between boys and girls. Venn diagrams and mathematics become the occasion for her to be able to externalize these thoughts and to support her own point of view, that is, equality between boys and girls, in a more formal way.

Vasilis thinks directly about the similarities and differences that exist between their own tribe (Romanian-Vlachs or Rundarides) and the competing Roma group (Chalkidians) that exist in the same area noting as difference that "they steal, they live in dirty houses" while in the similarities he states that "they also go to school and believe in God". School and education therefore receive universal dimensions and are a common experience even for groups of people with conflicting interests. Education seems to bridge the gaps between rival social groups and be a common starting point for reconciliation.

Lakis continues to think about the research topic and he suggests comparing the hobbies that existed in past and today. He finds that the most important difference between the present and the past is the existence of electronic games and the internet. Considering similarities, he mentions the children's games such as hide and seek as well as hunting because "children always want to play in the countryside". He also mentions "In several countries some learn English, some German, some Greek but everyone does arithmetic (meaning Mathematics) because the numbers are the same for everyone. Everyone sells,

buys, counts and makes transactions". In other words, the student recognizes the universal dimension of mathematics, that mathematics is potentially a universal language of communication that unites rather than separates people, cultures and civilizations.

The above mentioned (non) data provide excuses that mathematics can become a vehicle for social improvement through occasions which permit re-examining individual's history and the historic present. A culturally oriented version of Venn diagrams for Roma students who identify applications of Venn diagrams in their real life illustrates an example of the mathematics rhizocurrere which challenges the idea of who can do mathematics. At the same time, they touch on issues of their recent personal and family history as well as issues of racial discrimination and gender issues. Such an approach to the mathematics curriculum creates discursive plateaus, which create the opportunity for personal human stories to emerge exposing teachers and students as producers of mathematics knowledge.

Conclusions

Rather than seeking a single, unified truth working according to an arboreal model of knowledge, this rhizocurrere promotes recognition of the contingent and temporary nature of what is discovered at those nodal meeting points. It also highlights that new knowledge moves in new, often unpredictable, directions, examining where our thinking may move along lines of flight. In effect, this alternative model acts as a canvas to try out new combinations of ideas. Thus, rhizomatic research culture is characterized by heterogeneity, multiplicity, proliferation, flexibility, non-linearity, connection and non-hierarchical networks.

This paper does not focus on identifying the truth of the Roma students but following Jackson's and Mazzei's (2012, p. 5) proposal, the research "is deliberate and transparent in what analytical questions are made possible by a specific theoretical concept and how the questions that we used to think with did not precede our analytic practice (as research questions might) but emerged in the middle of "plugging in".

The purpose of the paper is to create new discourse about the experiences of Roma students which may be related to mathematics and are powerful enough to turn the traditional mathematics curriculum into a vivid currere giving new impetus to the whole learning process. The research is not leveling in terms of developing conclusions that apply to all participants but reveals dimensions of living experiences, differences in way of thinking and living. Kumm and Berbary (2018) mention that "post-inquiry, as becoming, is often left open-ended or spilling over".

A teacher who applies the rhizome approach to an intercultural classroom will have the advantage of co-creating a teaching plan with his students that meets their particular (cultural, social and learning) needs which are not always clear in advance. In order to assess whether, on the one hand, the concepts of mathematics knowledge and, on the other, the deconstruction of student prejudices and common-sense imperialism have been achieved towards restoration of social justice. Dignity, recognition and reconciliation coexist as a transformation of and with coloniality rather than simply perpetuating it (Appelbaum, 2019). Mathematics becomes a tool for understanding the world that supports critical thinking and analysis. Roma students' interaction in the mathematics framework both with other students and the wider community creates alternative extensions on how students understand mathematics, themselves and how their world may be explored mathematically.

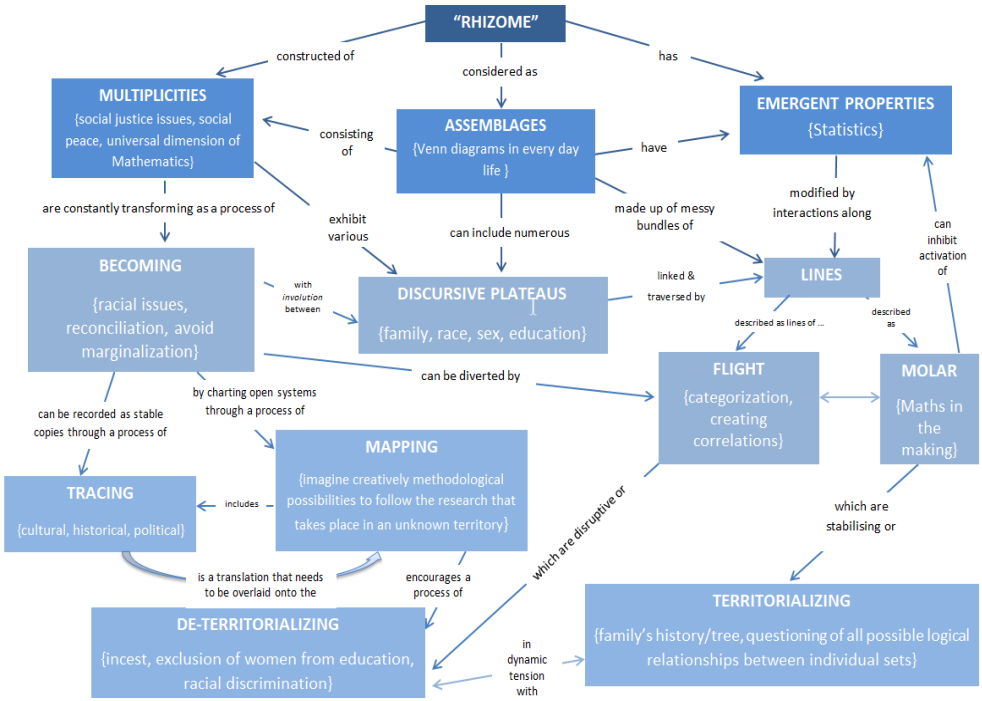


Figure 1: The rhizomatic development of the mathematics curriculum

This paper promotes a different view of mathematics curriculum through emerging contexts and relationships which show community's interests of what mathematics learning could be and develops students' experiences in connection with daily realities students face. The proposed rhizocurrere ensures that Roma students have access to mathematics in their 'language'. Although Roma students usually achieve poor mathematics knowledge, when a mathematics curriculum co-shaping process embodies their experiences ensuring thus their active participation, the learning outcomes become impressive.

The student's real education emerges when they are able to find ways to integrate individual issues by relating them to everyday life, in order to later continue self-regulated learning in the context of lifelong learning. Teachers through the development of such a rhizocurrere, pay special attention to the procedural side of knowledge, constantly updating their methods and leading students to unexpected cognitive fields arising from daily practice. Students develop a research culture trying to identify mathematics in situations which initially do not seem mathematical. They also develop critical thinking which will contribute to the creation of a better world through respect, understanding of intercultural differences and taking action as a consequence of their education.

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“I think it’s a smash hit”: Adding an audience to a critical mathematics education project

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Although critical mathematics education has been promoted as supporting learners to engage with social justice issues, there has been little research on how learners anticipate using mathematics to make a case for a change in behaviour by an audience. The aim of this study was to identify how small groups of preservice teachers discussed the kinds of mathematical representations needed to convince specific audiences to change their behaviour. It was found that the preservice teachers took into consideration their chosen audience’s background experiences and mathematical knowledge. As well as considering the kinds of representations which were most likely to be convincing for these audiences, the preservice teachers discussed whether their calculations should be available for scrutiny in their presentations.

Introduction

Critical mathematics education has as one of its aim to make students aware of the role of mathematics in social justice issues (Meaney & Lange, 2013). To achieve this aim, Frankenstein (1998) described four stages of what she labelled a *criticalmathematical literacy* curriculum: understanding the mathematics; understanding the mathematics of political knowledge; understanding the politics of mathematical knowledge; and understanding the politics of knowledge. In our project Learning about Teaching Argumentation for Critical Mathematics Education in multilingual classrooms (LATACME), one focus is on how preservice teachers can engage their future students by “facilitating the exploring and learning about the world through mathematics” (Lange & Meaney, 2019, p. 2). As part of developing their understandings about the mathematics of political knowledge, learners have to consider the needs of specific audiences, outside the classroom situation, when determining appropriate mathematical representations to include in their arguments about social justice issues:

> The purpose underlying all the calculations is to understand better the information and the arguments and to be able to question the decisions that were involved in choosing the numbers and the operations. (Frankenstein, 1998, p. 309)

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However, very little is known about how learners' knowledge of specific audiences affects their mathematical choices. This is partly because considering the needs of audiences, apart from examiners, is the domain of literacy subjects, rather than mathematics education. If literacy is connected to mathematics, it is usually in regard to the reading and writing skills needed to engage with mathematical ideas in textbooks, etc. (see, for example, Murray-Orr & Mitton-Kukner, 2017). In contrast, an interdisciplinary approach in which, for example, mathematics is used to argue the case for something, requires an understanding of how to write and interpret a persuasive text, thus blending mathematics with the literacy skill of persuasive writing. Williams et al. (2016) defined interdisciplinarity as involving two or more academic disciplines, integrated together in a teaching unit. Interdisciplinary approaches are usually based on a theme which has three essential components:

> (a) concepts should be appropriate and important to the individual disciplines,
> (b) interdisciplinary/integrated instruction should enhance the learning of the concepts and
> (c) the theme should provide a lens to recognize and understand larger issues and go beyond subject disciplines. (Williams et al., 2016, p. 20)

In the new Norwegian curriculum, implemented in 2020 (Utdanningsdirektoratet, 2019), interdisciplinarity is promoted, alongside developing students' democratic citizenship. However, in most research on interdisciplinary projects, mathematics is combined with science (see, for example, the overview provided by Williams et al., 2016). From our perspective, it is difficult to fulfill Frankenstein's (1998) requirement for criticalmathematical literacy's consideration of the mathematics of political knowledge without developing understandings about how mathematics is used to persuade an audience or manipulate its viewpoint.

In this article, we investigate preservice teachers' discussion in small groups about how to design a poster or brochure that would reduce the amount of cigarette butt litter, the theme we chose for the interdisciplinary project. To gain insights in the preservice teachers' understandings about the mathematics of political knowledge, we focus in this paper on how their awareness of their chosen audience's needs affected implicitly or explicitly the mathematical representations they included in their poster or brochure.

Using mathematics to argue for a change in behaviour

Research in mathematics education about how learners produce texts in which mathematics is used to argue for people to change their behaviour is uncommon, particularly where the focus is the learners' consideration of a specific audience's needs. Nevertheless, some research studies on critical mathematics education projects do touch on this. In a Norwegian study in which junior secondary students undertook a traffic investigation, they had to make an in-class presentation and write a letter to the local municipality about changing the existing road conditions to prevent serious accidents (Herheim & Rangnes, 2015). The teacher used the letter as a motivation for having the students determine whether wild sheep on the road was an issue serious enough to be included. The letter's audience affected the

students' mathematical thinking, in that they discussed how their collected data, should be presented to make a strong argument for raising the height of the crash barriers.

In her PhD project in the US, Pennell (2019) combined critical literacy with critical mathematics education in a study in which small groups of middle school students investigated various social justice issues. In one project about same-sex marriage, the students decided to set up a website that focused on the suicide rates of LGBTQ+ people in states which allowed same-sex marriage and those which did not. The three students in this group discussed how to present information so the audience would support same-sex marriage and so reduce the suicide rate. One student wanted to make more of the slight correlation between the data sets than the other students thought was appropriate. Pennell (2019) stated that the subsequent discussion about the importance of the message and its factual basis increased their understandings:

> This conversation further allowed the boys to reflect on their feelings about how information is displayed, how information can be manipulated (at worst) or structured (at best) to emphasize a point, and what language is necessary to get an argument across. (p. 84)

Although Pennell (2019) remained concerned about the use of exaggerated statistics to present the message, through for example the inclusion of a graph that over-emphasised a difference between suicide rates in states that did and those that did not allow same-sex marriage, she concluded that the students had addressed an authentic audience as was required in most language arts courses in the US. Having an authentic audience contributed to the students double-checking the correctness of their calculations and data.

Although these examples indicate that responding to an authentic audience did contribute to different kinds of conversations about critical mathematics education, the link between audience considerations and mathematical representations was not the focus of these studies. To provide more information about the impact of the audience, we investigate preservice teachers' discussions about the mathematical representations that they would include in a poster or brochure for changing people's behaviour.

Methods

The data came from one mathematics education workshop, designed and implemented by the second author, in the fourth year of a five-year Master teacher education course. It was part of the normal teaching programme and provided the preservice teachers with an introduction to possible research projects. It also allowed us to explore aspects of the LATACME project. In a survey of first-year preservice teacher, we had found that there was uncertain about aspects of implementing a project on air pollution that required students to engage with mathematical understandings in a real-world context. This workshop provided an opportunity to investigate preservice teachers' understandings about such projects at the end of their education.

One task in this workshop, which took about an hour, was to find out how the preservice teachers would tackle similar projects. This task was based on a virtual challenge started by

French young people of collecting cigarette butts in soft-drink bottles. A short video was shown of a young child discussing the dangers to the environment of cigarette butts (https://no-smoke.org/environmental-impact-of-cigarette-butts/). The preservice teachers were then given information about how on a 500 metre walk between the campus and the nearest light rail station, a bottle was completely filled with 254 cigarette butts. They were then asked to present a brochure or a poster that would use mathematics to convince an audience to reduce the amount of cigarette butts that ended up in the storm water drains ("Lag et argument med matematikk som vil overtale publikum til å redusere mengden sigarettstumper som blir skylt ned i avløpsvannet").

Four groups allowed their discussions to be audio-recorded. These groups had three (Group 1 and 4), four (Group 2) and five members (Group 3). Initially, all three authors read the transcripts and identified aspects of the discussion, which were loosely connected to Skovsmose's (1994) six entry points to reflective knowing. These entry points can identify the kinds of reflections that the preservice teachers engaged in about mathematics. However, it became clear that adding a requirement to choose an audience affected these reflections in ways not discussed by Skovsmose. In this paper, we do not discuss the mathematical arguments as such, but instead focus on the impact of adding a requirement to consider a specific audience.

Consequently, we focused on analysing the transcripts of the audio-recordings to identify how having to choose an audience affected the mathematical representations the preservice teachers included in their argument. Initially all three authors went through one transcript, identifying implicit or explicit references to the audience and connections to the mathematical representation to be included in their brochure or poster. After agreement was reached on what constituted a discussion (more than one statement) and an implicit and an explicit reference to audience, the first and third authors analysed the remaining transcripts. From this analysis, we then identified the foci that seemed to be brought out in discussions that connected audience needs with mathematical representations. These results are discussed in the next section.

Results and discussion

In this section, we begin by describing briefly the different audiences, before discussing the points raised in regard to how to incorporate mathematical representations into the presentations. Each group had extensive conversations, in particular Groups 2 and 4 whose discussions lasted for 42 and 33 minutes respectively. Nevertheless, the groups tended to raise similar issues in regard to why particular representations of mathematics should be included. These were to do with: the impact on the reader; and how much of the modelling, calculations or thinking should be included in the presentation. In the next sections, we describe the choice of audience and the points about the inclusion of the mathematics, with relevant examples from the transcripts.

Audience

Each of the four groups chose a different audience, sometimes as a result of an explicit discussion and at other times through an implicit assumption that there was an agreement on who the poster or brochure would be for. Group 1 explicitly chose the mayor of Bergen; Group 2 implicitly focused on those who throw away cigarette butts; Group 3 focused on parents who might worry that children could pick up and eat a cigarette butt; Group 4 discussed the managers of the main bus station and the areas around it, but with the intention of getting their support so that cigarette smokers could be approached to change their behaviour in regard to throwing away cigarette butts. Group 3 had the most extensive discussion about choosing an audience which seemed to stem from their own lack of knowledge about the polluting damage of cigarette butts.

G3S1: Jeg tipper det er veldig mange av de voksne som ikke bryr seg fordi de ikke vet.	G3S1: I guess there are a lot of adults who do not care because they do not know.
G3S5: Ja.	G3S5: Yes.
G3S1: Det var masse her jeg ikke visste.	G3S1: There was a lot here I did not know.

They went on to discuss how adults, which seemed to exclude themselves, were disinclined to listen to information about the environment because they felt bullied by climate change activists. Focussing on parents, grandparents and others who care about children would allow them to use their emotions to draw them in and then perhaps present the important environmental information.

G3S1: Ja. Vi må liksom ha en sånn der setning som vi får oppmerksomhet på da. Eller så fortsetter de å lese og da får de litt den der miljøgreia uten at de vet det.	G3S1: Yes. We must somehow have such a sentence where we get the attention then. Or so, they continue to read and then they get a little of that environmental thing without knowing it.

In each group, there were discussions about the mathematics that should be included in the poster/brochure presentations. As seemed to be the case in Herheim and Rangnes (2015), having a specific purpose for communicating to an audience did contribute to the kinds of mathematical discussions that occurred in the groups. The two main themes from these discussions are described in the next sections.

Making an impact

In considering the needs of their audience, the major concern was how to use mathematics to make a strong point that would convince the audience to change their behaviours. This seemed to be similar to the aim of the students in Pennell's (2019) study. However, our results provide other insights into different aspects of this choice. Sometimes, the discussions were about the visual impact that could be achieved with a poster or brochure that would result in the audience having a strong reaction to it. At other times, it was about finding numbers which an audience would consider relevant and so would contribute to changing behaviours.

In some groups, the need to make an impact with their presentation was implicit in the discussion. For example, Group 1 considered including a graph in their PowerPoint presentation. Their audience was the mayor of Bergen, but there is no discussion about whether the mayor would need information conveyed in a graph. Instead, it seems that implicitly they considered that a graph would look more formal and make the point they wanted to make more convincingly. Initially, they thought to include a bar graph but realised that it would only have one bar and hence not convey their point, so they then chose to include a line graph to show an increase of the number of cigarette butts against the amount of area, "Ja, at den bare øker med meterene. Så ser man bare at den stiger. Et linjediagram." ("Yes, it only increases with the metres. Then you see that it only rises. A line graph.") The impact that they wanted their audience to respond to was that the more area of Bergen, that was considered, the greater number of cigarette butts which would be found. Having a graph that showed a rise seemed to be considered a way of supporting the audience to see the urgency for doing something about it. However, the reasoning for the choice of a line graph is not discussed explicitly.

In the other groups, the focus seemed to be on finding a big number or on ways of presenting a big number, which would engage their audiences. For example, Group 2 focused on how to present "a big number" in a visually dramatic way. This seemed to be connected to their view that the (undefined) audience of their poster may not have the skills to interpret a lot of mathematics. However, they struggled to calculate a sufficiently large number that would allow them to compare, for example, the length of bottles or cigarette butts against the length of a light rail carriage or the height of a local mountain:

G2S1: Shit. Vi må ha litt høyere enn det, da.	G2S1: Shit. We must have a little higher than that, right.
G2S2: Vi må gjøre det litt mer dramatisk.	G2S2: We need to make it a little more dramatic.

Later they discussed how to situate the visual aspects into a coherent presentation:

G2S2: Fem bybanevogner, det.	G2S2: Five light rail cars, that.
G2S1: Fem bybanevogner! Per time!	G2S1: Five light rail cars! Per hour!
G2S3: Per time.	G2S3: Per hour.
G2S2: Fylles med røyk istedenfor mennesker.	G2S2: Filled with smoke instead of humans.
G2S1: Okei, det var litt kraftig, da.	G2S1: Okay, that was a little bit over the top, right.

The link to the audience is implicit but it is clear that shocking their audience in some way is viewed as important, if they are to affect the audience's behaviour towards the environment.

Group 4 also considered how to make an impact on their audience, who explicitly were the management of the bus station area, but implicitly seemed to be cigarette smokers. Within this discussion, they began to discuss whether the numbers would seem reasonable to the readers of their poster. They first decided in which area of Bergen, they should situate their efforts to change smokers' behaviours. In the end they chose the main bus station in Bergen which has a light rail stop and is close to the railway station. After some calculations of the area covered by the bus station light rail stop, Group 4 determined that there were a lot of cigarette butts in the area.

G4S1: Det er greit. Men det er heftig. Da er det titusen sigarettstumper på Bystasjonen. Okei. Da har vi det.	G4S1: It is okay. But that's a bit much. Then there are ten thousand cigarette butts at the bus station. Okay. Then we have it.
G4S2: Mhm.	G4S2: Mhm.
G4S1: Mer enn titusen. Det hørtes mye ut. Det hørtes veldig mye ut.	G4S1: More than ten thousand. It sounds a lot. It sounds like too much.
G4S2: Vi får regne på nytt da.	G4S2: We have to calculate again then.
G4S1: Eh, nå har jeg målt og. Skal vi se da.	G4S1: Eh, now I have measured too. Shall we see then.
G4S2: Det er jo, det stemmer jo for alle tallene.	G4S2: Yes, it's true for all the numbers.
G4S1: Nå har vi jo laget en veldig, dette her er en veldig forenklet modell. Det er jo derfor.	G4S1: Now we have made a very, this is a very simplified model. That's why.

Although they were initially happy with having such a large number to present to their audience, it soon raised concerns about whether the number was reasonable. This resulted in them checking their measurements and calculations, with the information provided earlier, to determine the rate of cigarette butts per square metre and multiply this by the area of the bus station. When the calculations seemed correct, they then considered how their simple model may have led to them obtaining an inappropriate number. Although not discussed explicitly, they seemed to fear their audience would not consider such a large number as being reasonable.

Similar discussions about the correctness of the calculations occurred in the other groups, but it was not always connected to concerns about whether their chosen audience would find the numbers too large. Instead, these discussions were more often to do with whether their peers and the teacher educator would expect some discussion of their calculations.

The amount of mathematics included in the presentation

The other main connection to the audience was in how the groups discussed what mathematics should be included in the presentation. This was related implicitly or explicitly to the knowledge and skills the audience might have.

Group 1 discussed using the greater Bergen area of 458 km^2 to determine how many cigarette butts could litter the area, if the density was the same as for the ones which had been collected. This then led them to similar concerns raised by Group 4, in the previous example, about whether their assumptions about the area were appropriate. They then considered both what areas would have more litter and those which would have less. Although they knew that the fjords around Bergen which were included in the 458 km^2 would also have cigarette butts, they decided to exclude this area because they could not be sure of how many there would be. They followed this discussion by thinking about who was their audience, "Men hvem er det vi vil liksom vise tallene våre til da? Er det kommunen eller er det?" ("But who are we to show our numbers to then? Is it the municipality, or is it?").

A similar discussion that linked determining an appropriate number to choosing an audience occurred in Group 4, where one preservice teacher stated, “Nei, jeg synes jo den er ganske bankers da, egentlig. Men det jeg lurer på er, hvem kan vi bruke disse tallene på?” (“No, I think it’s quite a banker then, really. But what I’m wondering is, who can we use these numbers on?”) Banker is an idiom which means that it is a good thing, in that it is “as safe as banks”. These discussions suggest that only the final number was worth sharing, which then led to the groups deciding who the numbers should be shared with.

There were also discussions about presenting numbers in ways that the audiences would understand. For example, Group 3 decided to give an approximate amount of cigarettes, for a particular area, because they considered it would be easier for the audience to relate to or for them to present.

G3S1: Kan vi ikke bare skrive over to hundre tusen?	G3S1: Can’t we just write over two hundred thousand?
G3S5: Ja.	G3S5: Yes.
G3S1: Det er litt sånn lettere å forholde seg til.	G3S1: It’s a little easier to relate to.

These discussions about how to present the numbers in a simple way could have been related to what they considered were the audience’s interests or skills in understanding mathematics. Group 2 discussed this explicitly as being about critical thinking skills.

G2S1: Også kan vi jo tenke at de som leser dette, de	G2S1: We can also think that those who read this, they
G2S2: Er dumme?	G2S2: Are stupid?
G2S1: De har ikke den der matematisk kritiske tenkningen på plass, så	G2S1: They do not have that mathematically critical thinking in place, so

In Group 4, the need for including more of their calculations so they could be checked by others came after they had decided that they had a strong argument to present with the numbers of cigarettes that they had calculated. A “smash hit” is a Norwegian idiom for something considered to be fantastic, something that will surely capture the audience’s attention.

G4S1: Jeg tenker det er en smash hit. Sånn, så kan folk stille spørsmål om hvorvidt det stemmer eller ikke etterpå. Sant, det handler jo om kritisk matematikk ikke sant? At her smeller vi i bordet med et tall og en modell også kan folk stille spørsmål ved hvor i all verden kommer det tallet fra.	G4S1: I think it’s a smash hit. That way, people can ask questions about whether it’s true or not afterwards. True, it’s about critical mathematics, right? That here we slam the table with a number and a model and then people can ask questions about where on earth that number comes from.
G4S2: Men, skal vi bare vise at vi regner, altså, regnemåten vår. Må vi ikke ta utregning med. Lag et	G4S2: But, should we only show that we calculate, that is, our way of calculating. Shouldn’t we include the calculation. Make a
G4S1: Jo, bruk løsningen din.	G4S1: Yes, use your solution.

Although they highlighted that including the final amount perhaps would open themselves up to people asking critical questions about where the amount came from, they did, in the end, decide to do just that.

Some of these discussions, whether explicit or implicit, about providing background to the total amounts, such as their calculations, did lead to some of the groups reflecting on how mathematics was used in what they read online or in news articles.

G2S1: Jeg skjønner jo nå at folk liksom lurer folk i artikler og sånn på nett og sånn, for det er jo veldig fristende å bare	G2S1: I understand now that people kind of trick people in articles and online, because it is very tempting to just
G2S2: Pynte på litt og	G2S2: Decorate a little and

These reflections led to considering how school students could made aware of this manipulation, "Det er en fin oppgave for elevene å gå tilbake til og se, og være litt kritisk til" (It's a nice task for students to go back to and see and be a little critical of). At least for members of this group, being involved in developing an argument using mathematics led to them considering how their own and their future students' understanding of the presentation of arguments using mathematics could be manipulated, particularly if the basis for calculations is not included. This was very similar to the point made by Pennell (2019) from her study of high school students tackling social justice themes.

Conclusion

One of the aims of critical mathematics education is to use mathematics to understand social justice issues as part of Frankenstein's (1998) understanding the mathematics of political knowledge. In this paper, we have considered how having preservice teachers consider the needs of a specific audience affected their development of an argument which used mathematics to change people's behaviour.

Williams et al. (2016) identified three components necessary for interdisciplinary projects. The theme to do with identifying an audience to convince to reduce cigarette butt litter did involve the preservice teachers integrating mathematics education with persuasive writing. Insights from the two subjects were combined when doing this project but led to considerations which would normally be outside of each of the individual subjects. Mathematics is not usually included in persuasive writing and writing persuasive arguments are not usually included in mathematics education. The addition of the requirement to choose and respond to the needs of a particular audience led to discussions about what was needed in order to persuade an audience as well as a need to check their calculations. Yet this combination was necessary in understanding how persuasive texts could mislead. For example, when Group 2 recognised that their numbers were not likely to withstand a critical scrutiny from others, there was a realisation that material that they were being presented with, such as information online, could also have been manipulated to make a specific point. Given that the Norwegian curriculum has both a focus on developing democratic citizenship

and interdisciplinary projects, critical mathematics education provides a fertile ground for inclusion in projects with these foci.

However, although all the groups were engaged with the task, the willingness to manipulate the mathematics to ensure that the audience had an emotional reaction, which was deemed important if they were to change behaviour, raises some issues. As Pennell (2019) found in her study, manipulation of numbers seemed to be accepted by some of the preservice teachers as appropriate if it ensured the desired result was achieved. It may be that wider acceptance of fake-news tactics has become normalised. This means that in discussions with preservice teachers about instigating such projects in their future classrooms, there is a need to explicitly discuss how to use the logic of mathematics, rather than just the inclusion of large numbers, in persuasive arguments.

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(Re)imagining spatialities for equity in mathematics education

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Contemporary discourse about the 'opening'/'closing' of schools and what is 'inside'/'outside' the curriculum potentially exacerbates existing inequities in mathematics education. This paper explores how different spatial imaginaries might advance or hinder efforts to deeply and systematically pursue equity. We use critical postcolonial thought for our (re)imaginings in the South African context. We argue that viewing the school, the mathematics curriculum, and language as nouned, bounded, spatial objects highlights what needs attention and for whom, but also points to the indelible, structural nature of exclusion. We propose a notion of spatiality as experienced encountering. This recognises all people and their practices as strategic agents, and emphasises relations between people, but also between the mind, body and Earth.

Introduction

Changes to our ways of being, necessitated by a worldwide health pandemic, has drawn attention to notions of space and mobility. Education discourse has focused on the 'opening'/'closing' of schools, and what should be 'inside'/'outside' the mathematics curriculum. Increasingly, educational inequities, notably, who can be said to be 'inside' or 'outside' of mathematics, are recognised. These inequities are not new, but are revealed in the process of troubling the rhetorical devices at play in discourses on education in relation to the pandemic, some of which serve to exacerbate the effects of exclusion. Thus, there is an urgent need to bring to this contemporary discourse, critical scholarship on access, power and equity, but also for the research community itself to think critically about assumptions and practices made in the name of inclusion.

This conceptual paper focuses on our thinking and language use about physical and conceptual spatialities, guided by two questions: What entry points exist for (re)imagining spatialities in mathematics education? How do these spatialities advance or hinder our efforts to deeply and systemically pursue equity? To respond, we focus on spatialities of mathematics curriculum, language in mathematics, and the school itself. Indeed, we embark on an ambitious project, noting that each aspect is deserving of deep interrogation in further writing. This paper thus acts as an early agenda, not for an action plan, but for spatialities of possibility in pursuit of what matters most in mathematics education, as we write from the

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purported 'postcolonial' context of South Africa. The concerns raised are both personal and political for us, as scholars writing from the experientialities and imaginaries of this context. However, in stating this as a reflexive positioning of our collaborations, we acknowledge our privileged positions, spatially, differentially, and corporeally relative to the postcolonial and the risk of reifying the very spatialities we seek to trouble.

To provoke our (re)imaginings of mathematics education, we embrace metaphors of land, cities and language. We deploy postcolonial thought from within critical theories of decoloniality, (eco)feminism, social-ecology, critical space theory, and related. Certainly, experiences of the colonial project and its ongoing remaking in successive global design projects, such as development, modernisation, and globalising capitalism, are diverse within and across postcolonial contexts (Mignolo, 2007, 2010). Yet, there are some commonalities in how these projects are operationalised and experienced in situated contexts that scholars thinking from these contexts are forced to confront (Bhan, 2019; Mignolo, 2007). Critical postcolonial thought is productive for our interest in spatialities. Firstly, thinking about physical and conceptual spatialities is central to the social construction of difference between people, and between people and the Earth, as a colonial strategy across global contexts. Secondly, while an 'outside' to the colonial project is not possible, this critical thought may offer us a glimpse from a temporary 'outside', in order to prize open, even if momentarily, alternative entry points to reimagining spatiality in our pursuit of equity in mathematics education. Thirdly, critical postcolonial thought brings into view ways of knowing, acting and being that have been marginalised in successive global imaginaries.

The objectification of space in global design projects

Postcolonial thought identifies the centrality in global design projects of imagining spatiality as an object contained within a boundary line, and solidified in nouning language. In order to frame an 'inside', that is the physical and conceptual 'territory' of empire, coloniality has needed 'exteriority' (Mignolo in Delgado et al., 2000; Mignolo, 2010). It has needed to invent an 'outside', which requires that the 'outside' and the 'other', "be brought into the frame" (Mignolo, 2010, p. 122), while also identifying the margins of that frame. In this sense, coloniality is "concerned" with the 'other', "even when not concerned" with the 'other' (Dlamini, 2020, p. 61). The latter has been achieved through an epistemology of universality, which masks the 'outside' and the naturalised power relations controlling the boundary line of difference.

Writing about the creation of the object, 'national park', in South Africa, Dlamini (2020) shows how the boundary line, physical (a fence) or conceptual (a land law), gives spatial form and content to this accepted 'reality'. These lines are fictions, yet they fix what and whom we expect to be in that space and what can happen there. Since a bounded object is contingent on an 'outside', defining content and participants requires non-content and non-participants, thus creating alterities and (re)enforcing hierarchies. This objectification privileges "roots" (Dlamini, 2020, p. 108); it "freezes" (p. 59) people spatially, thus fixing identities, commonly in racialised terms, in and across spaces (Dlamini, 2020; Green, 2020). It also interrupts and dispenses with historical flows – the "routes" (p. 108) – of people and

animals across space (Dlamini, 2020), demonising those that transgress these invented rules of the current global imaginary and constructing movement and boundary crossing as a 'risk' that needs to be managed and policed, physically or digitally (Mbembe, 2020). The securitisation of these risks is part of this invention and of the sense-making process of the 'real'. Since colonial difference masks the 'outside' and the 'other' (Mignolo, 2007), the bounded object potentially draws our attention to what is contained 'inside', not the 'outside'.

Imagining a bounded object also signals a 'start'/'end' in temporal space, with successive land laws in colonial and apartheid projects in South Africa ignoring and thus erasing previous histories, the "peopled past" (Dlamini, 2020, p.26). Language, particularly English as lingua franca, as the "companion of global designs" (Mignolo in Delgado et al., 2000, p. 12), is another device for establishing a temporal "hubris of zero point" (Mignolo, 2007, p. 159). In the past, this was achieved with the spatial operations of pen and paper in colonial bureaucratic audits by which 'life' began under colonisation and changed the place, experience, and future of the colonised (Ashcroft, 2014). It also was achieved in the codification of indigenous languages as objects; by converting interactive languaging practices "inscribed in your body into something that is 'outside' yourself on which you become dependent", this process changed people and the power relations between them (Mignolo in Delgado et al., 2000, p. 17).

In summary, imagining spatiality as a nouned, bounded object creates a fixed, naturalised 'reality' with clearly delineated content and participants to which our attention is drawn. Simultaneously, the necessary non-content and non-participants are frozen and hidden. This is a hierarchical structure in which power is invested through control of the line. Access in these terms then means crossing a policed line. Thus, it is a process and structure that manifests and maintains inequity.

Indeed, education, and 'Euromodern' mathematics and its use in scientific processes of objectifying human and non-human resources were key conceptual and institutional technologies for controlling and framing colonial 'territory' (Bishop, 1990; Green, 2020). We could argue that the 'real' of mathematics education and the contexts it locates to actualise its practices were operationalised through the reification of these imagined realities in the image of accepted global design projects. Indeed, this spatial imaginary is ubiquitous in mathematics (education) discourses, including in contemporary, instrumental debates about schools and mathematics curricula described in the introduction. For example, accepted mathematical practices include classification and objectification, and the formatting of its applications (Skovsmose, 1994) in maps and algorithms. Critiques of 'Verdinglichung' and its connections to the reification of mathematical objects and their nominalisation have been made to mathematics education, and associated with a way of forgetting (Swanson, 2017). This view invigorates binaries that frame mathematics education discourses, such as mind/body, everyday/mathematical, and Western/indigenous mathematics, and in prevailing triads such as mathematical concept/student/technology (Sinclair, 2021) or indeed amongst the oldest: mathematics/cognition/the child.

Objectified 'realities' in mathematics education

We now explore what the rhetoric of spatiality as bounded object might offer our thinking about equity in relation to school mathematics 'curriculum', 'language' for mathematics and the 'school' in South Africa[1]. First, we note that a description of context, of any length, cannot capture its complexity (Christie, 2020). How to represent socio-political context as an actor in mathematics education without essentialising is a recognised quandary (e.g., Valero, 2007). In addition, our description is limited to physical and conceptual realities that are shaped by their boundedness. Recognising the risks involved, we follow Valero (2007) by recruiting "existing literature and policy documents" (p. 227) for our contextual description. We stress that the product serves the purpose of advancing our thinking about spatial imaginaries for equity.

South Africa has a complex, 400-year history of colonial rule and its subsequent design projects. The spatialised relations established during this time became entrenched in the apartheid project that began in 1948 (Christie, 2020) with physical and legal boundaries marking lines of control. People and their identities became fixed, and racial linguistic classifications of 'white', 'black African', 'coloured', or 'Asian/Indian' came into form within bounded geographical spaces. Colonial era laws to control movement of 'black African' people took material form through the 'pass book' (Savage, 1986).

During this time, a hierarchy of bounded, segregated schools, mathematics curricula, and languages were developed. The state prioritised resources for the 'inside' of schools for participants classified as 'white' who spoke English or Afrikaans. Mathematics was important content for students being prepared separately for academic and skilled labour (Khuzwayo, 2005). Schools with 'black African' participants were tasked with producing unskilled labour, and thus mathematics was backgrounded and trivialised. Bounded African languages were to be used in primary school, with a switch to 50/50 English/Afrikaans in high school. In all schools, mathematics was tightly bounded. In fact, it was differentially presented as abstract/concrete based on the racial classification of the participant (Swanson, 1998).

This objectification of space historicising inequity provided an important focus for change in the transition to democracy, with legal changes intended to dissolve hierarchies and to weaken boundary lines. The Constitution recognises the 'other' in the form of equal rights to dignity, health, safety, water, sanitation, and basic education, and equal status for 11 languages, including nine African languages. Policy declared schools open to 'all races', paving the way for legal shifts and multilingualism. Funding policies used the geographic location of historically 'outside' schools to shape decisions around directing financial resources for basic infrastructure and fee waivers. In the development of one mathematics curriculum 'for all', People's Maths and ethnomathematics were brought to the table as recognition of the 'other' as a mathematics participant (e.g., Bopape, 1998).

[1] We focus on state schooling, constituting approximately about 94% of South African schools (Statistics South Africa, 2020). The private school system, which has its own hierarchy, is itself implicated in the constitution of inequity in this context.

Yet, almost thirty years into democracy, a hierarchy of bounded schools, mathematics curricula and languages is being reproduced, with existing boundary lines only slightly redrawn. Shifting the boundaries of imaginaries does not shift the location of schools geographically, nor the material base on which the content needs to be "developed" (Christie, 2020, p. 8). The participant/non-participant binary is no longer a simple race-based 'white'/'other', but a related, complex mix of racial, socio-economic, class and linguistic, geographical difference, well-described in the binary "fortified"/"exposed" schools (Christie, 2020, citing Teese & Polesel, 2003). While these objectified descriptions reify people and schools and do not recognise difference and how people navigate multiple spaces, they highlight potential spaces for necessary equity work. Crucially, since four-fifths of all schools are exposed schools, most students attend schools that cannot "catch up" to fortified schools (Christie, 2020, p. 8).

Policy change has removed racialised school boundaries, yet movement between schools has been one-directional, and only for some, into historically 'white', English-medium schools in legacy 'white' and wealthy suburbs (Soudien, 2004). Political compromises have allowed these schools to control their boundaries. Firstly, they can raise extra school fees to fortify these spaces with more mathematics teachers and commensurate smaller classes. Thus, these schools are closed to those who cannot afford the fees and/or the commute. The historically 'outside' schools cannot 'fortify' themselves in the same way, and 80% of participants rely on daily school feeding (Christie, 2020). Given the historical inequity, state funding directed at these spaces has not been sufficient; in 2018, 30% of schools had no running water and 20% inadequate sanitation (Christie, 2020). Also, fortified schools are closed to those who do not speak 'standard' English with a particular accent, adopt a certain demeanor and appearance, and play certain sports (Hunter, 2019). Students highlight how their bodies – their hairstyles, speech volume, and 'African' language use – are policed using pass book mentalities ("Sans Souci Girls' High School pupils protest", 2016).

1990's reform produced a Grade 1 to 9 mathematics curriculum 'for all', with a choice of "Mathematics" or "Mathematical Literacy" for Grades 10 to 12. Focusing on content such as "number" and "algebra" for "critical thinking", "problem solving", and "decision-making" (Department of Basic Education [DBE], 2011a, p. 8), the curricula show similarities to other contexts. Mathematics is defined as a "human activity", yet mathematical practices attributed to the 'other' are hidden in descriptions of "real life" contexts (p. 8). "Mathematics" and "Mathematical Literacy" participants will contribute as "citizen[s]" and "worker[s]", yet only the former can access university science, with the latter being "self-managing" (DBE, 2011b, p. 8).

Yet, most students are 'outside' the school mathematics curriculum. Policy prescribes a switch from mother tongue instruction in Grade 3 to English for mathematics instruction in Grade 4. Students at exposed schools are most likely to be those learning mathematics in a language they are still learning. School Mathematics may not be offered in Grades 10 to 12 at these schools. Of those who start school, approximately 12% will meet the 30% pass mark

of either Mathematical Literacy or Mathematics in Grade 12, with 3% of (mostly fortified) schools producing more Mathematics distinctions than the rest (Spaull, 2019). Data on what "work" (DBE, 2011a, p. 8), if anything, is open to mathematics non-participants *and* participants, suggests few opportunities for educational mobility. Almost half of South African youth are unemployed and are likely to remain so in their lifetime, and for those who have work, it is increasingly likely to be informal and precarious (Spaull, 2015).

The vast differences between fortified and exposed schools reflect how COVID-19 can only exacerbate inequity in South Africa. The ongoing effects of the pandemic require revisiting what is 'inside' the mathematics curriculum, with debates currently dominated by catch-up plans, predicting learning loss, and counting non-participation or dropout rates (e.g., Macupe, 2021; Pournara & Bowie, 2020). Yet, possibilities for virtual or safe physical boundary-crossing to schools and the mathematics curriculum are inequitable, whether resourced through the home or school. Crucially, the physical closure of schools, either completely or on a rotational basis to ensure safety, denies nine-million children access to daily school feeding.

This brief narrative reveals the ways in which historical, physical and conceptual spatialities of nouned, bounded objects have and continue to lay out maps of reality and of pre-determined futures for various peoples differentiated through their proximity or distance to colonial spheres of discourse and practice. In our pursuit of equity, this highlights, in this spatiality, what needs attention towards realizing even basic rights and for whom. Crucially, thinking in terms of critical spatial relations highlights the deep and indelibly structural nature of exclusion in this context and the challenge of overcoming the oppressions that this reality animates. Challenging the inequity entrenched within related colonial, global design and apartheid spatialities requires more than legally opening boundaries or providing more resources for some to 'catch up'. Rather, this pursuit requires a different spatial imaginary all together.

(Re)imaging spatialities as experienced encountering

In a counter move to the nouned, objectified spatialities of school, mathematics curriculum, suburban places, and non-participant 'others' of mathematics, we propose a process notion of spatiality, using the verbing language of *experienced encountering*. Imagining experienced encountering, especially in relation to mathematics education is a challenge, given how spatiality as a nouned, bounded object has been naturalised in this field. We use postcolonial thought as a guide as to where to forage for our agenda of (re)imagining. Since global design projects have subordinated the content and participants of an 'outside', we need to think *from* the 'outside' (Mignolo, 2007) as ethical allies with sub-alternated *peoples*, their *ways of moving* in the world, their *practices*, as well as their strategic *agency*. We need to act *relationally*: with people, and with the mind, body, spirit and Earth. We need to recognise *embodied (inter)acting*, *moving, journeying, changing*, *transienting*, *transgressing, differencing*, and *(em)powering*, not as risks to be removed, but as parts of an immediate life-world of the majority (Bhan, 2019; Mbembe, 2020).

We stress that our thinking does not involve a re-imagining of a romanticised, pre-colonial way of acting, moving and the relations defined by it. For the global imaginaries constituting our common world are characterised by "relations of authority, exclusion and inclusion, hegemony, partnership, sponsorship, appropriation between intellectuals, institutions in the metropole and those in the world periphery" (Connell, 2007, pp. viii–ix). Rather, it involves recognising *all* as participants, and as "infected" (Mignolo in Delgado et al., 2000, p. 11) by these projects, albeit differentially. This includes historically sub-alternated peoples and their motivated *(inter)action*. It also involves critique of the power relations invested in these interactions and their effects, such as the forced movements of people as a result of global/local conflict and climate change.

Experienced encountering in mathematics education

We now explore what the rhetoric of spatiality as experienced encountering might offer our pursuit of equity in mathematics education. We use verbing language: mathematics 'currere', 'languaging' and 'schooling' to provoke our imaginings. Our use of 'currere' draws on the work of Pinar (2011) in curriculum studies and its use in mathematics education (e.g., Wolfmeyer et al., 2017). Whereas 'curriculum' as noun generally identifies content to be learned, the verb 'currere', from the Latin word 'to run' a course, to journey, evokes learning as experiencing or living. In relational terms, students and mathematics-ing in schooling act as participating in the world.

We propose that mathematics currering in schooling involves experienced meaning-making for a student's (inter)actions in the world, where the social, cultural, political, and physical (both natural and human-made ecologies) co-exist. This notion includes the bounded spaces of buildings (school or home) that might provide the necessary security for students to live safely and with dignity. For us, experienced encountering involves safely moving in a world that is increasingly precarious. For some this might require practising schooling and mathematics-ing amidst the climate change related risks of water shortages, flooding, fires, and health crises (Gibson, 2020).

Encounterings in this world require expanding *who* is involved in mathematics currering: students, peers, scholars and educators in mathematics and other disciplines, caregivers, community groups, civil society organisations, the state, and health professionals. We find isiXhosa languaging useful for imagining these peopled (inter)actions. The two verbs 'ukufunda' (to read, learn or study) and 'ukufundisa' (to teach) (Kirsch & Skorge, 2010) recognise that all people bring to and learn in the encounterings in particular ways of knowing, acting, being, and languaging.

We argue that meaning-making in these encounterings involves an assemblage of practices. This includes critical mathematics education practices for understanding how mathematics acts in the world (Skovsmose, 1994), as well as critical information literacy for understanding how mathematical practices may enact an equitable world. Crucially, it involves the related sociological, ethical and political literacies for acting with *all* people in the world, and recognising difference, not so much as multiplicity, but as transienting as

characterising meaning-making. Meaning-making involves transdisciplinarity in which mathematics does not act to neutralise the political or render utilitarian the objects it recontextualises (Swanson, 2005). Lastly, meaning-making and (inter)actions involve languaging practices. We propose notions of languaging from postcolonial thought that strive to reinvent ways of using language that are the norm in multilingual contexts (e.g., Makoni & Pennycook, 2007). This heteroglossia reflects linguistic practices commonly referred to as 'translanguaging' in school mathematics in South Africa (e.g., Tyler, 2008). Here, languaging practices reflect people's flexible and agentic use of semiotic resources and meaning-making and (inter)acting as they move spatially. Such resources include registers, genres, and modes (written words, verbal talk, symbols, images, bodily movement and gesture, and touch). They include various language codes and accents, with languages not viewed as fixed, but as (inter)changing in (inter)acting, including transgressing practices.

Conclusions

We have presented the first steps of our journey to explore rhetorical devices for (re)imagining spatialities for equity in mathematics education. We acknowledge the limitations that (in)dwelling, in one context, on the metaphor of spatialities and the accepted 'realities' of mathematics education render. Yet, the two entry points we have explored, provoked by our reading of and (in)dwelling on critical postcolonial thought, suggests that such reflexive practice – self-reflexive and between epistemic positions – may be productive. Spatiality as a nouned, bounded object highlights pressure points in South African schools and mathematics curriculum for working towards realising even basic rights 'for all'. Yet, in highlighting how this imaginary, by mapping realities of difference, (re)produces inequity, we identify the need for a new spatial imaginary.

We propose, thus, spatialities of experienced encountering that recognise all people and their practices as strategic agents, and in the process make visible the importance of relations between people, but also between the mind, body and Earth. This renders embodied (inter)acting, journeying, changing, transgressing, as actions, not as risks to be controlled, but as ways of being-in-the-world for the majority. This (re)imagining requires further elaboration in an interactive process of in-depth engagement with critical postcolonial thought in relation to specific cases in South Africa. For the latter, we plan to begin by exploring experienced encounterings of schooling, mathematics-ing, and languaging amidst the climate change related risks of water shortages, flooding, fires, and health crises [as exemplified in the UKRI GCRF Water and Fire project (2019), on which one author is leading]. From this starting point, and following Bhan (2019), we aim to collaborate "incrementally from multiple locations" (p. 641) to explore how spatial (re)imaginings may emerge across contexts of precarity.

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Pedagogical imagination and prospective mathematics teachers' education

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The text presents a reflection on pedagogical imagination (PI) in the context of teacher education. A study group on mathematics education and inclusion was organised with students of a degree course in mathematics. After theoretical studies, the students were invited to imagine mathematics classes in an inclusive perspective. Among the imagined elements, we highlight: the school, students and classes. Based on this experience, we point out important aspects in the PI process including: theoretical study, teacher educator's role, common goals, environment that favours dialogue, and participants' previous experience. The pedagogical imagination in prospective teacher education allowed the prospective teachers to go beyond what is given and to be opened to what could be possible.

Pedagogical imagination in searching for possibilities

The concept of pedagogical imagination is presented in publications by Ole Skovsmose (Skovsmose & Borba, 2004; Skovsmose, 2009, 2011, 2015), which address methodological aspects of research that seeks to transform a situation.

Researching transformations implies that research also investigates possibilities. It is necessary to go beyond what is defined and can be observed, and to confront what is given with what could be different. It is important to search for *what is not, but could be*, which means to investigate alternatives and question what might appear as given.

To Skovsmose the idea of researching possibilities was motivated by an experience with doctoral students in the post-apartheid context in South Africa. The students wanted to follow scientific quality standards, but they also felt uncomfortable by researching the current situation, marked by the consequences left by the apartheid regime. How to research, for example, multicultural mathematics classrooms? It was difficult. The population was still completely segregated, separated into *white, black, coloured* and *Indians* neighbourhoods. Such separation still dominated the South African school context. One approach was to research what did not yet exist in schools, but it could become a reality.

Among the works of the doctoral students, Renuka Vithal's doctoral research (2003) can be highlighted. Vithal investigated whether and how mathematics education could

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contribute to a more democratic South Africa. She explored what would happen in a mathematics classroom, when trying to introduce what she called a social, cultural, political approach to the mathematics curriculum. In the research process, she questioned some criteria and presented the notions of current actual, imagined hypothetical, and arranged situations.

The situation before an educational experiment took place is called current situation (CS). Here one can identify aspects that can be changed. For example, the format of the classes, the method used, the engagement of the students. CS is an important point in the research, but it is also necessary to think about alternatives to this situation.

By imagining possibilities, one arrives at an imagined situation (IS). To get to IS, we reflect on the question: what would happen if something were done differently?

Proposing alternatives to the current situation, based on the imagined situations, leads to an arranged situation (AS). An AS is an alternative to the current situation, different as well from the imagined situation.

Connecting the vertices of the first triangle in Figure 1, we identify the processes involved in researching possibilities. The pedagogical imagination (PI) concerns the relationship between the current and the imagined situations. It is the process that makes it possible to create imagined situations, considering what could happen in a different way.

The relationship between the current and the arranged situation is the practical organization (PO). It is the organisation of actions with departure in the CS based on an IS, but simultaneously acknowledge what is possible giving the practical conditions.

Explorative reasoning (ER) is the process of addressing the imagined situation with departure in the arranged situation. It considers the viability of PI and the innovative elements of the PO. It is a critical interpretation of the two situations.

It is important to emphasise that the transformations through researching possibilities are continuous. This transformation is illustrated in the Figure 1.

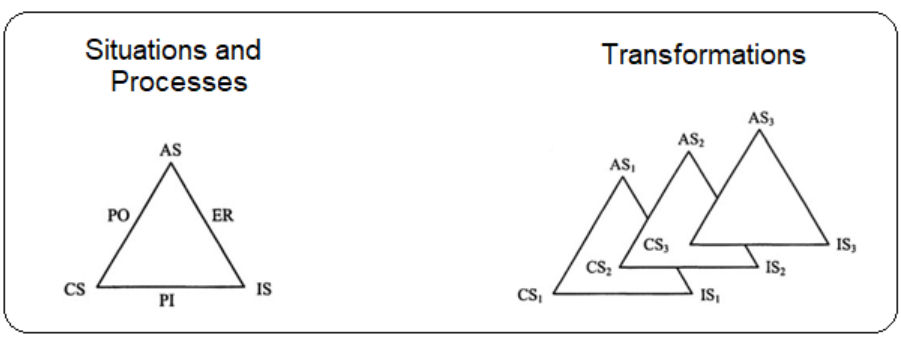

Figure 1: Researching possibilities (Skovsmose & Borba, 2004, p. 214, 221).

Transformations make integral part of the research process. The changes take place in the current, in the imagination, as well as in the arranged situations. In the following, we will concentrate on the process of pedagogical imagination based in a teacher education case.

Pedagogical imagination: Considerations

The concept of pedagogical imagination as proposed by Skovsmose is inspired by the conception of sociological imagination (Wright Mills, 1959). Sociological imagination is not directed towards given social facts, as being defined through a given social situation. It seeks to identify alternatives to this situation and to open up possibilities for change. This imagination allows us to realise that a given situation is not necessarily something fixed, but can be different.

Considering the educational context, Skovsmose presents the pedagogical imagination as a process that allows exploring possibilities for classroom situations. The imagination can have many different qualities, but it is always confronting the given situation. Both, in relation to research and to actual classroom practices, it is necessary to consider: "(1) what is actual, (2) what could be imagined, and (3) what could be tried out" (Skovsmose, 2011, p. 23). In order to explore educational possibilities, pedagogical imagination is crucial.

School practices involve many contrasts, including those of a socioeconomic nature. Researching possibilities involves imagining alternatives, which might lead to changes. However, it is important to acknowledge that sometimes only small changes are possible, but being small does not mean being irrelevant. Small changes in school education might not result a big social change. Even so, educational changes can make an important difference for many students (Skovsmose, 2011).

Pedagogical imagination does not come out of nothing. Concepts such as democracy, social justice and equity are some resources to take into account (Skovsmose, 2009). Such concepts are fertile ground for a pedagogical imagination. In the case we present here, the fertile ground was a discussion on equity and social justice related to the inclusion of people with disabilities in mathematics classes.

Pedagogical imagination and inclusive education: Reflections from a mathematics teacher education case

In Brazil, the education of people with disabilities is strongly regulated by legislation[1]. The increasing dissemination of the right to education has meant that socially underrepresented groups are present at school. As a result of these policies, more and more students with disabilities are attending regular schools. In 2018, there were about 1.2 million enrolments of students aged 4 to 17 years old to whom inclusive education policies are directed[2], and 92% of them were attending regular school (Censo da Educação Básica de 2018).

The presence of students with disabilities in schools has removed school community from its comfort zone. Among the many uncertainties, insecurities, conflicts and challenges that this community has faced, issues related to pedagogical actions are highlighted (Fernandes & Healy, 2015, our translation). Teachers often feel insecure to deal with the challenge of inclusion and to deal with diversity.

[1] More information of this subject can be found on Penteado et al. (2018).

[2] For public policy purposes, the target audience of Special Education is students with disabilities, global developmental disorder and high skills / intellectual giftedness (Decreto nº 7.611, 2011).

For Capellini and Fonseca (2017, p. 120), the difficulties presented in schools "show the need to confront discriminatory practices and create alternatives to overcome them" (our translation).

In this context, inclusive education is configured as an important theme in the education of teachers. It is necessary to think about ways that will lead teachers to consider the diversity present in the school and reflect on the teaching, learning and participation of different students in their classes.

We follow Skovsmose (2019) taking inclusive education as meetings amongst differences. Differences characterise the human beings, and differences live together and interacts with each other. In this sense, the school should be a space where an inclusive culture prevails, which considers and valorises differences of individuals, rather than consider differences as a being problematic. It should provide a curriculum accessible to all students, so that everyone learns (Capellini & Fonseca, 2017).

To address this issue, we organised a doctoral study aiming to understanding possibilities pedagogical imagination opens up for the mathematics teacher education, especially considering Inclusive Education. (The doctoral student is the first author of this paper, the supervisor is the second author.) The data production took place at an institution located in a city of São Paulo State, Brazil. An independent study group was organised with prospective mathematics teachers inspired by Murphy and Lick (1998). Independent groups do not have organisational goals. They are constituted by people with a common interest, seeking to develop personal knowledge. In our case, the common goal was to learn about inclusion of students with disabilities in mathematics classes.

All prospective mathematics teachers from the 2nd, 3rd and 4th years of the course were invited. The research aim and the proposed organisation for the study group were presented. They were also informed that the research had been approved by the University's Research Ethics Committee and that, among other things, their participation was voluntary and that they could cancel their participation at any time.

The group was constituted by 21 participants. Twelve weekly meetings were held divided into two parts. The Part 1 was dedicated to reading and discussing texts about inclusive education. The Part 2 took departure in the task for the prospective teachers to imagining a school situation including mathematics lessons in classrooms with at least one student with disability. Their imagination should be orientated by an inclusive perspective.

In Part 2, the participants were separated into four smaller groups, called G1, G2, G3, and G4. The teacher educator[3] participated in the smaller group meetings, recording the interactions in audio and video, making notes and participating in the discussions, by asking questions and making her own reflections. The proposal involved imagining the school context, the structural conditions, and those involved in the process.

[3] In this text, the term teacher educator is used to refer to the researcher, the first author of this article, as this was the role she exercised in the study group. This education action was done in the context of a doctoral research in progress at Unesp, under supervision of the second author.

The departure for the pedagogical imagination was to specify the school, the class, and the students for whom the classes would be planned. All participants brought school experiences, whether as students of basic education, as teachers or from internship experiences or university extension programs. Such experiences were essential when describing school spaces, available resources, rules to be followed, and people with whom they would work to. Many experiences influenced the descriptions.

The school descriptions include the choice of a fictitious school name, the location, the attended public, and whether it was a public or private school. They needed to think about how the physical space was, the institution's teaching concept, the mandatory written tests for assessment of the students. They also needed to imagine the physical organisation of the classroom: the organisation of the desks, maybe in rows, the blackboard quality, and the groups of students in the class. For each of these points, the prospective teachers should justify their choices, for instance by drawing on some memory of lived experience.

The prospective teachers also engaged in characterising the students. They detailed the difficulties, potentials, and characteristics of those with particular difference: someone with a mental or physical disability, with some learning difficulty, with family problems, someone very shy or very talkative. Such characterisations were based on their previous experiences, it could be with an autistic brother, a childhood friend in a wheelchair, a friend from college, etc.

To imagine the classes, the teacher educator suggested a mathematics content to be taught. The choice was made from pre-determined content of the official curriculum. They had to imagine what resources they would use, what activities would be implemented, how the room would be organised, and how the interaction among students would be.

Pedagogical imagination: A discussion on the case

Pedagogical imagination stimulates creativity and offers openness to think about possibilities. All groups recognised a feature of the current situation, as, for example, the presence of the head of the school to evaluate teachers' work in the classroom; the need to maintain discipline in class; the application of assessments according to predetermined standards, and the need to comply with the school curriculum.

However, all groups looked for ways to get beyond these situations. The imagined classes were participatory, investigative, and group-based. The prospective teachers thought of dynamic activities as a way to enhance the participation of all. The students talked to each other, walked around the class, and worked with manipulatives. The organisation of the chairs was no longer in rows, but gave space for students to work in groups. They also explored the idea of students sitting on the floor. Pedagogical imagination opened up possibilities for overcoming barriers with respect to: physical structure, organisation of classrooms, the communication pattern between students and teachers and among students, conventional evaluation mechanisms, among others.

All groups pointed out the need of tests in order to assess learning. Typically, tests are composed of well-defined questions presented in written format and appear as an imposition by school administrations or the government. Sometimes, the prospective teachers recognised that testing can be a form of intimidation and exclusion of students, especially if it is the only format of assessment.

The prospective teachers maintained the test, but included other means for assessing learning, such as engagement in the tasks. Regarding the tests, they thought of ways of favouring all students. For instance, for a blind student or one with motor difficulty, they offered the possibility to do the test orally; for an autistic student with hyper-focus on things from the sea, they elaborated questions that included this topic of interest.

For pedagogical imagination with a collective format as we did, collaboration mediated by dialogue was a crucial aspect. Such collaboration includes negotiations and deliberations. In our case, the imagination was a collective construction from beginning to end: from the presentation of the schools' conditions to the decision about the chosen evaluation methods. This aspect was evidenced by the participant Denise (all names are pseudonyms):

> Denise: In the part of thinking about activities, it was a matter of breaking down barriers in the mind, really. There was something I did not think about, but Dani talk, or Kátia, or Isabel. Then I thought: wow! How could I not think of that! And also the importance of sharing things too. When you put several heads together, you know?

The role of the teacher educator in this pedagogical imagination is also an aspect to be highlighted. Samira talks about this.

> Samira: I really like this more provocative thing, because it brings us to think. Every time you [teacher educator] provoke us in some way was very important, because it forces us to stop and think: Wait! It brings thoughts that maybe are in our subconscious. But it was important for us stay aware and try to change.

As Skovsmose (2015) considers, pedagogical imagination does not appear out of nothing. In this process, where the teacher educator's intentions, aims, and interventions were decisive. In our case, the goal was for the prospective teachers to think about mathematics classes in an inclusive perspective. With this aim, the teacher educator selected topics and texts discussed in Part 1 and made interventions at the Part 2 meetings of the study group.

An example was when one of the groups was thinking about strategies for working with statistics with a 6th year class of elementary school. In this class, there was a blind student, Josué. Participants were searching for ways to facilitate that everyone would be able to read the information contained in a three-dimensional graph. The graph was constructed by the students to represent the result of an investigation they carried out on their shoes number. The interaction among participants in the study group can be illustrated by the following transcript:

Denise: We have to think about the blind student.

Kátia: Maybe we could make the axes with glitter paper, because it is very rough. We could also, on the axis that has the shoe numbers, cut out the digit in EVA[4] with glitter and he could run his hand on it.

Danilo: Yes, we could think of using an EVA with glitter and one without glitter to differentiate.

Kátia: And so, we do the same for all groups, because it is more beautiful. Furthermore, we do not differentiate only one. More attractive. More colourful.

Denise: The good thing about EVA is because it is a cheap material. If it is not available at the school, one can buy it.

Teacher educator: Yes. But one thing I kept thinking ... You talked about cutting out the digits in EVA for the blind to read ...

Denise: Wow, guys! But is this how he reads?

Isabel: I do not know.

Kátia: I think so. I have seen selling ready-made EVA digits. I bought some other day, it is cheap.

Denise: But Kátia, it is not Braille.

Danilo: That is what I thought now. It is not Braille.

Kátia: That is right. This student reads in Braille.

Denise: Yes ... He feels the cut digit, but he does not read it.

Isabel: Absolutely!

Danilo: Wow! How could we not have thought of that?

The imagined strategy to make possible for Josué to read the information on the graphics was not really helpful. The group looked for a way for him to get access to information through hand touch. They tried to include him. However, how did Josué read? To reach this observation, an intervention of the teacher educator was necessary, inviting them to reflect.

The group realised that they would not know how to write the information in Braille. At this moment, the teacher educator introduced the reglete: an instrument similar to a ruler, used to write in Braille on a piece of paper using a kind of pen for puncture. It provided a new resource for the prospective teachers. As Denise concluded: "Then we will have a reglete." So, the reglete was used in this class.

The process of pedagogical imagination was based on negotiation through dialogue: one says something, other complements, reflects, argues. It was a dynamic process.

Table 1 highlights important aspects of the process of pedagogical imagination in the education of mathematics teachers, based on our case. These aspects were pointed out in the analysis of the interactions among the study group participants and the pedagogical imagination process they performed.

[4] Ethylene-vinyl acetate (EVA) is a synthetic foam that is widely used for crafts and schoolwork.

Aspect	Detailing
Theoretical Study	The role of theoretical study on inclusive education was shown during the conversations and actions of prospective teachers. E.g., when they were concerned about doing activities where everyone could read the graphs, even if all the graphs needed to be also in Braille. It shows the search for universal design for learning, a concept that was discussed in Part 1 of the study group.
Teacher educator role	Teacher educator is the person who guides the imagination process, selects texts of theoretical support, and intervenes in the discussions. The importance of her interventions was highlighted in Samira's speech. It was also in the transcripted dialogue, when the teacher educator led the participants to question how to provide Josué condition to read the graph.
Common theme/ goal	The fact that the participants had a common goal – mathematics classes from the inclusion perspective – was essential for the collaboration. Participants were always looking for strategies that would allow everyone to participate and learn.
Environment that favours dialogue	Due to its collective nature, dialogue was the way by which pedagogical imagination took place. Dialogic communication permeated interactions, negotiations, and deliberations for decisions. As Denise told, sometimes a person does not think about something, but some colleague comments invite for making new reflections. One learns together with others.
Consider participants' experiences	The whole process involved specificities and expectations for each context, which in addition to the theory studied, are influenced by the experiences of each one who participated. As we pointed out, all the imagined schools, students, and classes had characteristics of the participants' previous experiences.

Table 1: Aspects of the pedagogical imagination process in prospective teacher education

Final remarks

In this text we discuss the possibilities of pedagogical imagination in mathematics teacher education. Pedagogical imagination is the process that leads from a current situation to an imagined one. Our current situation was the fact that mathematics teachers find it difficult to cope with inclusive mathematics classes, particularly when it comes to classes with students with disabilities. In this process of pedagogical imagination, everything was imagined: schools, classes, and students. Everything was characterised in details, based on each participant's experiences.

From a current situation, possibilities to change it can be imagined, and this leads to an arranged situation. After a pedagogical imagination process, a current situation will never be the same. Imagination changes this reality.

Pedagogical imagination is free. It goes as far as the imagination allows. That is why the inputs are important: the theoretical support, the role of the teacher educator, and the interaction among the participants. Our case points to the importance of the collective aspect, of dialogue. It shows the richness of imagining with the others.

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Traces: Doing mathematics, and the mathematics that is done to us

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Carl Andre once wrote that "Art is what we do; culture is what is done to us." Might not the same be said about mathematics? In this study, I focus on "traces" to show how mathematics is both and simultaneously something we do and something that is done to us. In so doing, I articulate how this conceptualisation allows us to fully include the cultural historical dimension of mathematics while theorizing mathematics as doing, rather than something we teach, learn, know or understand.

Overview

We are familiar with the distinction between doing mathematics (when we solve problems, look for patterns, search for proof, create and apply methods, etc.), and learning about mathematics (when mathematics is a set of "facts", ideas and procedures to memorize or understand, and so on). Papert's (1972) went so far as to say:

> Being a mathematician is no more definable as 'knowing' a set of mathematical facts than being a poet is definable as knowing a set of linguistic facts. Some modern mathematical education reformers will give this statement a too easy assent with the comment: 'Yes, they must understand, not merely know'. But this misses the capital point that being a mathematician, again like being a poet, or a composer or an engineer, means *doing*, rather than knowing or understanding. (p. 249)

When I heard about artist Carl Andre's (2004) aphorism, "Art is what we do; culture is what is done to us" (p. 23), it made me think about these two dimensions in relation to mathematics. Mathematics as a practice implies both doing mathematics and mathematics being done to us. When working on a problem, there is a part where we exert control, freely decide what move we try, and navigate our way through. But we do this with concepts and rules established before us, that are then "imposed" (or "imposes themselves") on our activity. More so, one can argue that every step we take while doing mathematics also "frames" our next (possible) move. We are thus simultaneously both active and passive subjects in relation to mathematics.

In this paper I theoretically and empirically reflect on the relationship between what we do and what is done to us mathematically as a driving force in mathematics (education). I also present it as a way to include the cultural historical dimension of mathematics through an analysis of mathematical activity in terms of "traces" (Serre, 2002) to conceptualize how mathematics "reproduces" in time and space.

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Conceptualizations of mathematics, cognition and learning

In the 1990's many scholars in mathematics education were debating between the "cognitive" view of learning and teaching and the newly emerging "participative" conceptualization. One emphasized the active construction of mathematical understanding by individual students in the line of radical constructivism, while the other insisted on how learning and teaching are better viewed as taking part in certain practices historically, culturally and contextually situated. Naturally, many tried to overcome the conundrum. Sfard (1998) for example took a pragmatic approach, suggesting us both "metaphors" alternatively or in combination. One problem with this is that these conceptualizations rest on deeply incompatible, even competitive epistemologies. To elude the quandary, Sfard (2015) eventually develop her own theory, based on the notion of internal/external discourses.

But in the 2000's a new line of thinking started to take wind, so to speak, offering a radically different take on the matter by insisting on the "body" (see Sinclair & Freitas, 2019). Increasingly complex conceptualizations emerged, for example Roth's (2011) distinction between body and "flesh", or de Freitas and Sinclair's (2014) (none-human) "material bodies" agentivity. These theories often manage to address phenomena of interest to cognitive, participative or discursive approaches in their own light.

Of course, this is only a fraction of the theoretical work done in the last decades addressing these (and other) dichotomies. One can think for example about the writings of Radford and his theory of objectification, or Brown's uses of psychoanalysis. We are also familiar with Walkerdine's approach to subjectivity, and many might know of Pais' exploration of Hegel's philosophy. Another relevant if perhaps less notorious line of thinking can also be found in Luitel's notion of "im/pure knowledge", and some reflections on tools and technology in relation with the nature of doing/knowing mathematics also invited us to question our epistemologies.

As part of this tradition, my colleague Jérome Proulx and I began to develop in the 2010's our approach to mathematics education with a radical proposition of our own: to completely put aside all notions related to knowledge, knowing, teaching and learning (see, e.g., Maheux & Proulx 2015, 2017). The central notion for us is that of "doing" mathematics, a focus on mathematical activity that includes closely attending to (a) all the means by which people observably do mathematics (in writing, speech, gestures, with diagrams or various technologies and so on), and (b) the emergent nature mathematical activity (e.g., in "problem solving", which we describe as *problematizing mathematically*). One aspect that recently grasped our attention is the notion of "trace" as a mean to describe and explain for example the geographical-historical spread of mathematics as well as what ordinarily happen in a mathematics classroom. In this paper, I illustrate this conceptualization by a short analysis, while arguing that mathematics is both and simultaneously something we do and something that is done to us. Moreover, I argue that these qualities are essential to the *propagation* of mathematics when traces of mathematical activities trigger mathematical activity in response, which generates more traces, and so on.

Traces in/of mathematical activity

The concept of trace is coextensive with the living in general: As soon as there is referral to another or to something else, there is a trace
Jacques Derrida (2002, p. 127)

The above quotation by Jacques Derrida outlines the importance of "traces" in human experiences (and more). For Derrida, a trace is fundamentally some thing that makes present something other than itself. If I write $\frac{16}{64}$, what stands beside me is *not* a number (in itself), but marks of ink on paper that evokes the idea of a number I can also invoke by writing $\frac{1}{4}$, 0.25, or by saying "a quarter" and so on. None of these concretely observable traces contain the number and yet, they are (for me and you) a way for it to appear (and become part of an activity, should it only be that of recognizing the number)[1]. One famous study of this distinction is Magritte's painting *The treachery of images*, in which we see a pipe and sentence that reads "This is not a pipe" (see Foucault's 1983 book-length analysis). We can easily think of various analogues of this in relation to mathematics, for example:

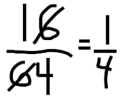

Figure 1. The treachery of traces of mathematical activity

A trace can be a written word, a drawing, a spoken sound, but also the alteration of something. Serre (2002) alerts us to the lack of attention given to traces, and un-exhaustively discuss four kinds of traces: (1) imprints, like a footprint in the sand, (2) after-effects, like the aftermath of a hurricane, (3) remanent, like a tiny quantity of poison or alcohol in the blood, and (4) inscriptions, like symbols and writings. It is important to insist on how the notion of trace differs from what we generally see in semiotics for example. Traces are not "signs" or "index", they do not "stand for" something (as in Pierce's theory), nor are they linguistically defined (as in de Saussure's approach). Semiology offers invaluable insights in relation to traces, and scholars in mathematics education explored many of its possibilities (Duval's theory of semiotics representations for example). But these approaches are rarely compatible with the epistemological posture I outlined in the previous section, in which metaphysical entitities are denied in the benefit of actual acts.

Closer to us, Bakhtin (e.g., 1981) conceptualizes the process of signification through which meaning is not only never pre-determined, but also always open, and attached not to signs but to full (chains of) utterances. Signs are intentional traces, but as traces what matters is the whole activity in which they are involved. Traces can then be seen as "operations" (Derrida, 1967, p. 71), and when Derrida says that "as soon as there is referral to another or

[1] As we will see, I am not suggesting here that numbers or other mathematical objects exist elsewhere, beyond our reference to them, but a full discussion of their emergent nature goes beyond what I can discuss here: see Maheux & Proulx (2014) for some insights. Similarly, a deeper philosophical discussion on the concept of trace is evidently in order.

to something else, there is a trace", it is the *act* of referring we should think about. More so, the operation goes in two directions (often at once): Traces are the product of some activity, and they also produce some activity. When I write $\frac{16}{64} = \frac{1}{4}$, I am generally mathematically active. The mark is a result of my mathematical thinking. And when I read the equation, I am also generally mathematically active, I am doing mathematics when I interpret it as an equality, two equivalent fractions, two ways of writing the same number, the result of a simplification, and so on. The word "generally" is important. It precisely reminds us that tracing or looking at $\frac{16}{64} = \frac{1}{4}$ does not necessarily imply doing mathematics. I need a special kind of orientation or better: I need to *do* certain kind of things with it.

In the following section, the analysis of a short fragment of a classroom lesson continues to present this perspective while bringing about aspects of how mathematics is both something we do and something that is done to us.

In the classroom

The point of philosophy is to start with something so simple as not to seem worth stating, and to end with something so paradoxical that no one will believe it.
Bertrand Russel (1918, p. 193)

The following episode takes place near the 20th minute of a lesson. Mary switches to a fresh page on the whiteboard, and as she speaks writes up in large characters "$\frac{16}{64}$":

Mary: Here I have sixteen over sixty-four, and I found that when I cross out the six's I get one fourth. Does this method work? Yes? Why?
Samy: Because you do four times six, it gives you sixty-four. If you do times six you get the same answer.
Mary: Okay, if you do one times…
Samy: Eh no, time 16.

Figure 2. Fragment of a classroom lesson

This provides us with a simple example of what we usually think in terms of how written traces come about in classroom mathematics activity. Mary uses the board to keep track of what she says, and the written traces apparently serve more or less the same purpose as the speech: make something available to students' consideration. The offer here seems to be the mathematical proposition of a "method" presented through an example, and students are asked to react. But *how* is this actually taking place?

If we slow down the production of Mary's "first" trace (in a series of many), we can appreciate its mathematical density. To make sense of what she writes and says, one needs to be able to use base 10 positional system for writing and naming numbers. These are, of course, cultural-historical constructs. The base 10 system draws on the work of Stevin (in 1585) who wrote a booklet on how "fractional numbers" could be calculated on by expressing them as sums of powers of ten. Stevin's proposition was then worked on, people talked and wrote about it: in France, a coma was added to separate the integer and the fractional part while the English preferred a dot. It is through all these discourses and inscriptions that this way of writing numbers finally (almost completely) took over the roman systems and made

its way to us. And of course, the idea certainly came to Stevin from reading or hearing about base 10 systems developed before him, the indo-arabic symbols for numbers, and so on.

The same goes for the history of fractional notation, also essential to make sense of what is going on in the classroom that day. We know writings by Diophantus (in Greece around 2000 BC) in which he talks about rational numbers, writings which Abu'l-Wafa (in Iraq around 960) knew and draws on for his work on certain kinds of "fractions". Bhaskara's writings (in India about 1150 BC) are also connected to them, and Al Kashi (in Iran around 1400) explicitly developed some of Abu'l-Wafa's ideas when he finally sets out most of the rules we use today to manipulate rational numbers in that form. From writings to writings, the mathematical idea occupies ground, it propagates in time and space, each occurrence/text having the potential to generate new mathematical activity that will keep the idea alive, and also develop it. These texts are all traces of mathematical activity left by those mathematicians, that provide a ground to work from. Not having to "start over" every time we do mathematics also implies that we need somehow to follow the steps of those who did mathematics before us. Their ideas, ways of thinking, the representations they came up with, their achievements, their mistakes, all that imposes itself on a mathematicians' work.

Forces are at play and affect us also because mathematically speaking, all ways of writing numbers are not equal. Some representations render specific mathematical work easy or difficult, necessary or impossible, relevant or useless, and so on. Think for example of computing $\frac{3}{10}+\frac{1}{8}$ versus $0.300+0.175$, or $\frac{1}{3}\times\frac{2}{5}$ versus $0.333\ldots\times 0.4$.[2] Forms of writing number can also make apparent or not some of their properties, and encourage us to think and work with them in specific ways (1.61803398875… might not say much, while $\frac{1+\sqrt{5}}{2}$ or $\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}$ are quite revealing, and inviting!). So by presenting to her students $\frac{16}{64}$ and $\frac{1}{4}$, Mary is not only introducing numbers, but also a certain way of thinking about them, of using them, and so on. The fractional writing alludes to the fact that $\frac{16}{64}$ is *two* numbers that combine to express *one*, that we can also write $\frac{1}{4}$ (or 0.25 for that matter), and so on. This can be seen as one of the ways mathematics *imposes itself* upon us. We never "directly access" mathematical ideas like numbers, but only encounter "traces" that are evocative, and these traces, shaped by history, are related to ways of thinking and ways of doing mathematically. They are not neutral manifestations, but tainted way to bring forth. So much so, could we say, that we dedicate years of study (e.g., at elementary school) to be able to make sense of them. This is also why we strongly react to the suggestion of crossing out the six: this is not how we use that form of writing, but it's also this form of writing that makes possible thinking we might do so. One might even argue that this possibility is itself imposed (as a possibility), and that is why students often came up with undesired ways of manipulating fractions: maybe the writing itself (and the fact that we do for multiplications) invites us to do $\frac{1}{2}+\frac{1}{4}=\frac{2}{6}$ by adding numerators and denominators?

[2]Another great example is how the Ancient Egyptian *had to* transform all fractions into a sum of unit fractions because that's the way their system worked ($\frac{3}{10}$ would be written as $\frac{1}{5}+\frac{1}{10}$), while we *can* express any rational number in an infinite number of ways ($\frac{3}{10}=\frac{30}{100}=\frac{15}{50}$ and so on).

Of course, such imposition does not mean there is no room to maneuver, on the contrary. When I hear or see $\frac{16}{64}$, I am not *forced* to do mathematics with it (some of my students quite often prove it), nor to work with it in a completely specific way. I *could*, for example, be looking for an equivalent fraction by first dividing $16 \div 64$ to find 0.25 and then write that as $\frac{25}{100}$, or do 0.25×8 to find 2, and thus $\frac{2}{8}$ and so on. I can even "misinterpret" what is in front of me, or make "little mistakes" when working with it: for example, saying "times six" when I mean "time sixteen", or thinking for a moment that "doing times six" has the same counter-effect as "crossing out" a six digit. What mathematics does is setting us up in a certain way (see Heidegger, 1953).

It gets more complicated because "instantiations" of mathematical ideas are not randomly found, but come from the active work of someone… who is also setting us up. Here, Mary is not only introducing numbers in a certain way, she also seems to be doing it for the purpose of illustrating a "method", and that is only as part of another endeavor (e.g., leading a math lesson) that is part of another (e.g., teaching junior high math to this group of teenagers) and so on. All these layers of activity are in a way piled up in her utterance, and part of what it comes to mean for us or for the students that day. The marks on the board and the words she spoke do not merely contain or in themselves assert all of these, but they are complicit to all (or some) of that. They at the least take part in the "normality" of all that is taking place, and even a relatively disruptive proposition such as simplifying fractions by removing numbers hardly goes against that. The mathematics done to us when we "do mathematics" in class is of a certain kind: we have a way of attending to mathematical traces, intentions and all kinds of procedures that are quite specific to school mathematical *genre* (as conceptualized by Bakthin for instance). Doing mathematics in school is not exactly the same everywhere of course, but still significantly differs from what professional mathematicians do at the university or in workplaces. The case of "weird fractions" such a $\frac{16}{64}$ is a good example of that. We can see in the literature mathematicians such as Boas (1973) or Stufflebeam (2013) work with them very differently than a teacher does (see, for example, Johnson, 1985). But again, it is only through such concrete actions and all the traces that they leave that ways of doing mathematics realize themselves.

The other aspect to consider here some more is how mathematical traces can generate mathematical activity in return. When we see something like $\frac{16}{64}$, we are not obligated to mathematically engage with it, but those of us who encountered that kind of writing over and over again almost instantly recognize it, and are somehow compelled to read it as a fraction, to "understand it". This is in a way related to how traces are always missing something: they are "negative spaces" that *we* need to fill. The necessity for interpretation, for signification, implies that we need to do mathematics for mathematics to be done to us. If I flip the pages of a book, and my eye caches numbers but I do nothing with them, or if I sit in the back of the classroom and play on my phone while I hear the teacher present something on the board, little seems to happen in terms of mathematics being done/to me. The book or the class could be on any subject, it would make no difference. But when I engage in mathematical "meaning making", when I begin to *do* mathematics, this is when it

all takes place. And more precisely, this is when *mathematics* takes place. For hundreds of years, Sumerian clay tablets were nothing but rugged rocks in the sand, until we began again to *respond* to them in reading (thinking mathematically) and writing (e.g., to explain the mathematics that has been done). Responding is a great way to describe what happens because it suggests both being affected by what we respond to, and affecting it by giving that something an actual signification. Whether saying "she is a number!" is a compliment or an insult actually rests not on the utterer, but on how we (e.g., the person called that way) respond to it ("and proud of it!", or "you should look at yourself!" or "what do you mean?", etc.). The response might have a verbal or gestural aspect to it, but we should also include here emotional responses for example, whether or not they are observable from the outside. Responses immediately offer themselves as something to be responded to, should it be by oneself (e.g., thoughts: this is what Husserl was so interested in) or others, in the case of observable responses.

Tangible responses-traces also have the advantage of being easier to follow. In the fragment above, we can see how Samy's utterance as a response to Mary's proposition. We can also recognize it as a mathematical response, meaning that Mary's utterance was "good enough" to generate some mathematical activity. Indeed, what Samy offers sounds to us as if he really attended to what Mary said and wrote, and what he articulates looks like the traces of some "mental" mathematical activity. In a way, his response is the result of mathematics being done to him, and we can pinpoint many aspects of his sentence that align with the kind of mathematics being done, and the writings and talk he was exposed to. His answer uses some of the numbers (names) used by Mary ("four", "sixty-four") and the idea of performing something on the numbers of the first fraction to obtain those of the second ("you get the same answer"). The presence of a "six" in Samy proposition ("you do four times six, it gives you sixty-four") is suggestive: Samy corrects himself ("eh no, times 16"), but the emphasis on the sixes in Mary's utterance could very well explain why he would think for a second that this might be the number he needed, thereby not only offering an answer to whether or not the method works ($\frac{16}{64}$ *is* the same as $\frac{1}{4}$ since…), but also why that is (i.e., removing the 6 is the opposite of multiplying by 6). And let's not forget that pragmatically, it is also by responding in such a way that Samy gives mathematical meaning to Mary's utterance[3]. And of course, Mary's response to Samy is doing just the same. From turn to turn, the trace on the board slowly grows, nourished by other traces of the ongoing mathematical activity. Figure 3 shows the board later on.

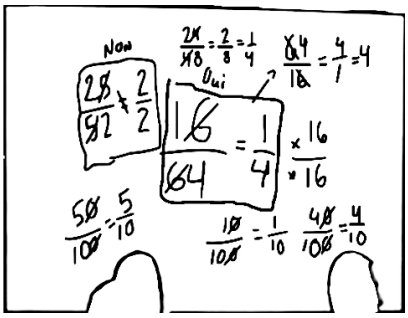

Figure 3. The board later on

[3] There is a famous sketch in which Costello "proves" that $28 \div 7 = 13$ and $13 \times 7 = 28$ (e.g., https://www.youtube.com/watch?v=xkbQDEXJy2k). Costello' writings are in many ways similar to what Mary was doing, but the answer to his performance is generally quite different!

A lot can be said about how traces on that board enabled and constrained the mathematics taking place. We see in the middle right a $\frac{\times 16}{\times 16}$ that followed Samy's correction, which leads to a discussion on why both fractions are equivalent. On the left, $\frac{25}{52} \neq \frac{2}{2}$ is what remains of the examination of a counter-example that also served to clearly distinguish two kinds of fractions: those that work with the method and those that don't (notice the rectangles drawn around both cases and the yes/no ("Oui", "Non") label added on top). The class then moved on to examine $\frac{64}{16} = \frac{4}{1}$ following a student reaction, and this case made it relevant to explain what the fraction $\frac{4}{1}$ meant (hence the added "= 4"), and why that fraction is not really different from the $\frac{64}{16}$ (hence the arrow). Next, Mary suggested to consider $\frac{24}{48} = \frac{2}{8}$ as another case of fractions that works, and a student pointed out that again, ultimately, the fraction $\frac{1}{4}$ was involved. Based on that, a discussion could have developed on whether all weird fractions are equivalent to $\frac{1}{4}$, but this was not the case. The class instead clarified the distinction between an equivalent fraction using smaller numbers (2 and 8 instead of 24 and 48) and the most reduced fraction (when no factors remain common to the numerator and the denominator). That discussion seems to have put some students on the path of new weird fractions in the form of $\frac{10}{100}$ (at the bottom), for which they actually could articulate why digits could be canceled out.

At that point, the board was full, and Mary had to switch to a fresh screen. The initial mark led to the production of so many other marks, all traces of and for mathematical activity, and there was no room left to write. The propagation illustrated in this spreading also connects with that of the mathematical ideas among the people in the room. More and more students contributed to the issue set up by Mary's opening proposition and, being express in such a publicly accessible manner, each of their ideas could reach all people in the room. Mary's "transcription" on the board of what students suggested continuously contributed to that: as a response, it validated them as relevant to the specific issue being addressed and to the whole of what was going on at the time (i.e. a mathematics lesson). But Mary's writing was also more akin to a *translation*: she interpreted the talk and offered specific mathematical ways to work with what she heard. For example, Samy's suggestions, in the fragment above, was turned into $\frac{\times 16}{\times 16}$ on the board, while Mary rephrased what Samy said as "ok ok so if you do time sixteen on bottom and on top we will get sixteen over sixty-four", clearly adding her own interpretation to his spoken words.

Moving on to a clean board, the traces of the first part of the classroom's work were no longer visible. But they were still very present, of course, and continued to affect what was going on. Mary even switched back between screens a few times. With today's technology, erasure becomes less permanent. In the past, traces of mathematical activity had to be carefully preserved. We know about Sumerian, Egyptian or Greek mathematics because some (often rare) texts were conserved. This is how the mathematical work of scribes and engineers could reach us, could be kept, taken on and developed. Nowadays, the iterability of traces almost creates the opposite problem: So much mathematical work is available that

contributions risk getting lost like drops in the sea. And on the other hand, it means that "mathematical mistakes" are also potentially more visible.

For the classroom, this raises questions. The mathematics we have done before is of course essential to the continuation of our mathematical activity today, should it only be in the traces it left "within us" (the memories we can recall, the ways of thinking we've become accustomed to, the methods we practiced and can now use, etc.). But the possibility for old traces of one's mathematical activity to come forth, be exposed, become other's object of attention is also a risk! We know that *all* mathematicians make mistakes now and then, and some of those are even famous… but we also live in a world where making mistakes is not as highly valued as we could wish (see for example Brown, 1993). And we could easily connect this with mathematical "trauma" (e.g., Lange & Meaney, 2011) that are other kinds of traces (sequels, scars) of mathematics being done/to us…

Last words: Doing mathematics, Husserl, Deleuze, and others

As I am running out of space, I want to conclude by highlighting how this study really rests on a very specific epistemology of mathematics. This perspective focuses on "doing mathematics" to the extent that what happens in school is no longer a matter of teaching and learning, but simply seen as opportunities for people to do mathematics together, and so doing encounter other people's work, and become familiar with mathematical ways of thinking, of doing, with mathematical ideas and so on (see again Maheux & Proulx, 2017). What we mean by the adjective "mathematical" here is crucial. Drawing on the conceptualization put forth by Husserl (see in Derrida, 1989) when he tells us that "geometry is on its way to its origin", we see mathematics as something that is still in the making, open-ended, boundless, and thus cannot really be defined (from the Latin *de-* "completely" *+finire* "to bound, limit"). Another way to see it is in Deleuze's (1994) infinite cycle of differences and repetitions in the course of which certain ways of being or doing take shape, but also continuously transform. Mathematics for us is a movement that goes in a way we can (here and now) recognize as mathematical (thereby contributing to what being mathematical "means"). A movement that leaves traces in its passage, spoors we are bound to follow, a direction only us can travel.

To all this, one might rightfully ask "but how do you consider the political and economic dimensions of school mathematics?" First, it is important to insist that I offer here a way to look at what is going on in school on an ordinary basis. I believe we can think of and examine various kind of classroom settings and management precisely asking "how is mathematics done (to us)" in this case? And I would suggest doing so not in hope of finding the right or the best way to do it, but rather in the spirit of appreciating potentials and advocate for diversity in the mathematical experiences offered to students. Politics is about empowerment, the possibility to do things. While reforming a school system or changing well established practices might sound like don Quixote's impossible dream, realizing that one actually *does* mathematics in a certain way, and that other ways of doing could be *included* without replacing everything is, in my view, a powerful move. Thinking in terms of doing

mathematics (how it is done, what it does, how it propagates and develop) rather than focusing only on "teaching" and "learning" can at least create the practical opportunity to enrich our discourses, and thus bring into light certain power to act essential political concerns and change, as articulated by Arendt for example.

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Time, speed, mathematics education and society: Questions that arise from assessment

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Time and emphasis on speed shape mathematics teaching, learning and assessment. Although they play an important role, this is not always reflected in research. I use two examples from the strictly timed and high-stakes tests of Greek National Exams in order to try and raise questions about these issues. How do time restrictions and how does emphasis on speed affect Mathematics education and assessment? How can the timed conditions of a mathematics tests affect students in different ways? What role does this emphasis play on the social and political dimension of mathematics education? I will attempt some initial reflections on these questions as well.

Why time?

In the last decades mathematics education research has tried to broaden its lenses. Social, cultural and political aspects are studied now in more ways and different points of focus have emerged. Mathematics education is being studied both as a social activity that is shaped by social conditions and as an activity that shapes our social reality. The study of the relation between mathematics education and our view of reality can be extended to our view of time.

Time is a dimension of mathematics education which is associated with many important aspects of it in teaching, learning and assessment. However, the importance that time plays in mathematics education and the emphasis that it has on speed are not adequately reflected in research. I will draw from the literature on the role of time in education, in order to raise and reflect on questions about the role of time in mathematics education and, especially, in mathematics assessment. How do time and emphasis on speed affect mathematics assessment? How can the timed conditions of mathematics tests affect students in different ways? Which questions does this study of time raise about the social and political dimension of mathematics education?

How does time shape education?

Time is a subjective experience as has been long analysed by philosophy, psychology and sociology of time (e.g., Adam, 1990; Meissner, 2007). In education, time has been studied already by Jean-Jacques Rousseau who "criticized time management in pedagogy" (Pourkos & Kontopodis, 2005, p. 252).

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> Even the most cursory look at contemporary school life reveals that everything is timed. It demonstrates that the activities and interactions of all its participants are choreographed to a symphony of buzzers and bells, timetables, schedules, and deadlines. Layer upon layer of such schedules form the structure of our education system. [...]. The beginning and end of lessons, the term, the school year, and the dates and times for tests and exams are some of the fixed points within which subjects, teachers' activities and pupils' expected progress are programmed (Ball et al., 1984; Delamont & Galton, 1986). Within the overall structure of finite time resources and nested timetables, activities are scheduled for pre-set durations. They are structured to follow certain sequences and to happen at a specific rate, at a particular time in the children's lives, over a fixed period, and for a set number of times (Adam, 1990, pp. 105–106).

The everyday school life of teacher and students is bound together by the time schedule. Every activity is very strictly organised in separate time slots, during which students and teachers are supposed to express a very specific behaviour. The time of students' play is very strictly separated from the time of the students' learning and this is signalled by the loud sound of the bell (Adam, 1990; Foucault, 1977).

How does time shape mathematics education?

Time has been studied specifically in mathematics education research, as well (Staats & Laster, 2019). In classrooms there is always a sense of hastiness towards the completion of the prescribed syllabus (e.g., Kollosche, 2017, p. 1010; Walkerdine, 2013, p. 240) and this can cause students to feel pressure and develop phenomena of math anxiety (Boaler, 2014; Engle, 2002) or mathematics refuse (Chronaki & Kollosche, 2019).

Not many aspects of time in mathematics education have been studied, like how teachers' waiting time in mathematics classroom discourse improves the length and quality of students' responses (Tobin, 1986) and what is the role of silence (Boistrup, 2010). But, important questions which have been raised by researchers on the role of time concerning the social dimension of mathematics education and assessment have not been studied adequately (Walen & Williams, 2002).

How does time shape mathematics assessment?

Assessment in education is commonly based on students' performance in given amounts of time. The time to be spent at each task is predetermined and students are pressured to perform at a certain pace (Adam, 1990).

Timed conditions of mathematics tests seem to have more negative effects on students than tests for other school subjects (Mollenkopf, 1950). Strictly timed conditions of mathematics tests may block the working memory, cause traumatic experiences to students and math anxiety (Boaler, 2014; Bosmans & De Smedt, 2015). Walen and Williams (2002) suggest that the type of anxiety when a student faces a mathematics test may be distinct from math anxiety or even test anxiety. Emphasis on speed in assessment affects students' view of mathematics, may emphasise blind memorising and makes them think that their object is not to "learn" mathematics, but to know how to "perform" mathematics (Boaler, 2014, pp. 470).

Researchers have compared the performance of different groups of students, according to their math anxiety levels, both in timed and in untimed conditions. Some studies suggest that timed mathematics tests have more negative effects on the performance of high-anxious students than low-anxious ones compared to untimed tests (Hembree, 1987; Tsui & Mazzocco, 2006; Gallagher, 1989; Plass & Hill, 1986; Onwuegbuzie & Seaman, 1995). However, there are other studies which suggest that they could not find statistically significant differences between students' performances under strictly timed or not conditions and, therefore, timed mathematics tests can be considered reliable (Mollenkopf, 1950; Bosmans & De Smedt, 2015; Kellogg et al., 1999). But these studies present methodological issues like small samples (although the one of Onwuegbuzie and Seaman has too). Moreover, all these studies are quantitative and they may miss discussing some important aspects of the effect of timed conditions on students. How does the timed condition of a test affect students in different ways and how can this change the characteristics of the test considerably? Does this raise equity issues (Walen & Williams, 2002)?

Studying these questions, I will examine two examples from timed mathematics tests. I will approach them in a theoretical consideration that regards mathematics assessment activities as acts that constitute discursive frameworks in which students and teachers develop subjective positions (Boistrup, 2010). Boistrup describes four types of assessment discourses in mathematics, one of those called "Do it quick and do it right" (p. 166). This type of discourse emphasises getting the work done fast and giving the right answer regardless of the mathematical processes that have to take place in-between. Developing an analysis on those mathematical processes and this discursive framework is important for our understanding of assessment.

Also, my theoretical approach views logical thinking as intertwined with time, namely that there is the dimension of logical time, described in Lacanian psychoanalysis (Lacan, 2006). When completing a test, the student is not alone, but in an interactive and dialogic situation with the designers of the test. In such situations the individual develops logical thinking which includes the dimension of developing thoughts in response to the Other's presence in time (Lacan, 1993). So, a student faces a mathematics test constantly negotiating the imaginary position of what they should do ahead of what they think that it is expected of them to do, namely in a dialectical and retroactive way and not as a static text.

Example 1: Possible paths in a timed test and luck

This example has to do with the Greek National Exams, which is a high-stakes, nation-wide, 3-hour exam that determines students' entry to higher education (also discussed in Makrakis, 2019). I will take as an example task Δ4 of the exams of June 2018:

> Let function $f(x) = 2e^{x-\alpha} - x^2, x \in \mathbb{R}$ & $\alpha > 1$.
>
> Δ4. If α=2 prove: $\int_2^3 f(x)\sqrt{x-2}dx > -\frac{32}{15}$ (Greek Ministry of Education, 2018)

Students have been trained according to their syllabus to know that there are six main ways for them to prove an inequality, which are shown in the first column of Table 1.

Proof method	Inequality for the function f in [2, 3]	$\int_2^3 f(x)\sqrt{x-2}dx > \cdots$
Monotonicity	$x \le 2 \Leftrightarrow f(x) \ge f(2) \Rightarrow \boxed{f(x) \ge -2}$	$\int_2^3 (-2)\sqrt{x-2}dx = -1,\overline{3}$
Extremum	$\boxed{f(x) \ge f(x_0)}$	-
Concavity and tangent line at $x_0 = 2$	Tangent for x=2: $y = -2x + 2 \rightarrow$ f convex in $[2,3] \Rightarrow \boxed{f(x) \ge -2x + 2}$	$\int_2^3 (-2x+2)\sqrt{x-2}dx = -2,1\overline{3}$
Concavity and tangent line at $x_0 = 3$	Tangent for x=3: $y = (2e-6)x - 4e + 9 \rightarrow$ f convex in $[2,3] \Rightarrow \boxed{f(x) \ge (2e-6)x - 4e + 9}$	$\int_2^3 [(2e-6)x - 4e + 9]\sqrt{x-2}dx \simeq -2,2253\ldots$
Mean value theorem and monotonicity of f′	$\exists \xi \in (2,x): f'(\xi) = \frac{f(x)+2}{x-2}$ and f′ increasing $\Rightarrow \boxed{f(x) > -4x + 6}$	$\int_2^3 (-4x+6)\sqrt{x-2}dx = -2,9\overline{3}$
Known inequality $e^x \ge x + 1$	$e^x \ge x + 1 \Rightarrow 2e^{x-2} \ge 2x - 2 \Rightarrow \boxed{f(x) \ge -x^2 + 2x - 2}$	$\int_2^3 (-x^2+2x-2)\sqrt{x-2}dx \simeq -2,419\ldots$

Table 1: Possible methods to prove an inequality

After constructing the initial inequality, students would have to multiply by $\sqrt{x-2}$, then to apply a known proposition in order for the integral to appear. This result might or might not lead to the desired inequality. The possible results of this are shown in the third column of Table 1.

Of all six initial methods, the only one that can be excluded early on is the one that uses the extremum, due to the monotonicity of the function, so only five remain. There is no rational way for a student to choose one or exclude one more. All five methods seem equally possible to solve the problem. So, a student cannot know beforehand which method can solve the problem.

Only two methods can, finally, solve the problem. These are the one that uses the monotonicity of the function (second row) and the one that uses the concavity of the function and its tangent line at $x_0 = 2$ (fourth row). The only one that produces the exact outcome is the last one and is, of course, the answer which the designers of the test anticipated that the students would give.

Since a student cannot know from the beginning which method to use so that to solve the problem, a student can choose only by luck. The probability of choosing a correct method is 40%, since there are five methods, of which only two produce the required outcome. A student can know if the method that he or she has chosen is correct only at the very end, namely after computing the last integral.

But a student that has chosen an incorrect method does not have the time to start using another one to solve the problem, because developing one of the methods takes very much time and these integrals require lots of arithmetical computations, as did the whole test. Mathematics tests of the Greek National Exams are, usually, so strictly timed that even experienced teachers would sometimes find it hard to complete them in time (Mavrogiannis, 2017). And no calculator, no graphic calculator and no other tool is allowed. Even if we suppose that a student could find the time to develop another method until the end, then the possibility of them choosing a correct one the second time is only 50% (two out of four). So, we may say that strictly timed math tests which emphasize speed can increase the factor of sheer luck in assessment.

Example 2: Possible paths in a timed test and how making a mistake can save you

Greek National Exams are considered so important that they are designed during the very night prior to the assessment by a secret and small group of people, in order for the test not to be leaked. This time pressure makes errors in the tests possible and they have happened sometimes, e.g., in 2003, 2006, 2011, 2013, 2014 and other years. Rhoades & Madaus (2003) name a category of errors in assessment tests as "latent", because they happen not just by a random mistake, but because of structural reasons that have to do with the designing of the text and educational policy.

Let us consider the following example from the Greek national exams of May 2003. Task 4 describes a function f. Then, question 4γ asks the students to prove that the function has at least one inflection point (Greek Ministry of Education, 2003).

This task has a serious mathematical error. It is not possible to prove that there is an inflection point with the given data. It can only be proven that there is a value x_0 with $f''(x_0) = 0$, but it cannot be proven that the second derivative is positive to its left and negative to its right, or vice versa. This can only prove a possible inflection point, which is not what the test asked. Finally, despite the obvious error, the commission of the exams decided to give full marks to the answer that proves only a possible inflection point.

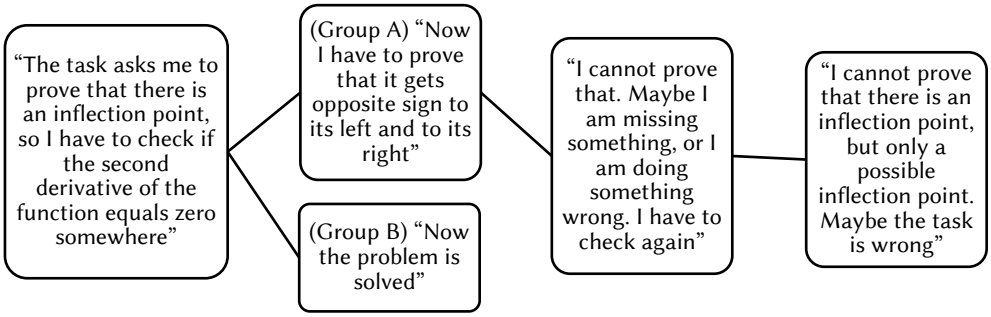

Figure 1: Possible approaches by the students

What would a student think when faced with such a task? We could sketch possible thought paths like in Figure 1. Group B mistakenly believe that the answer which they present is a correct one to the problem, as it is posed. Group A is the only one that have a mathematically correct approach to the problem. In the end, Groups A and B give exactly the same solution and obviously get the same marks. What is the difference? Group A needs much more time to think, compute and check all different paths that could solve the problem, as they know that they do not get the result which is asked. This, of course, can have a very harmful effect on their achievement at the other tasks of the test.

Group A follow a better approach as they know that what they have found is not a sufficient proof of what is asked, but they have a bigger chance to fail the test due to the amount of time that they have to spend. The fact that Group B is mistaken in thinking that their answer is correct, is the one that saves them a lot of time, and, consequently, this misrecognition gives them a bigger chance to succeed in the test. We are facing the paradoxical fact that, when solving this task, the better you are at doing mathematics, the worse you are probably expected to perform, because the test is timed.

Questions that the issue of time raises about the socio-political perspectives of mathematics education

We can say that strictly timed mathematics assessment tests have the problem of not assessing the content of mathematics as much as we may think. They increase the factor of luck considerably. Further, during a mathematics test, the development of different paths may result in very different performance outcomes, but these outcomes may not have to do with mathematics itself as much as we imagine.

Are these timed tests designed so strictly that the actual time to complete is inadequate by mistake or on purpose? If some of them are structured in this way by mistake, then how can a mathematics test designer calculate the time needed for a student to complete a task? Literature on this subject is very inadequate.

This ambiguity leads us to additional questions. Do mathematics education policy makers, test designers and teachers value speed as a skill of mathematical literacy to be assessed by a mathematics test? No official agent, that I know of, lists speed as a skill of mathematical literacy for the public education to teach and to assess, especially not OECD (2003), NCTM (2000), EU (The Council of European Union, 2018) or Ministries of Education (Greek Minister of Education, 2015).

If we do not consider speed a skill of mathematical literacy, then why do we assess it? As long as we assess speed, why is this not reflected in mathematics education research and why do we not talk about it or study it? Do we, actually, have a latent appreciation of speed as a skill of mathematical literacy but avoid talking about it? What does this void say about the social and political dimension of Mathematics education? Is there a critical stance on this issue?

Initial thoughts about some directions to be studied for answering these open questions include the following three. Firstly, Suurtamm et al. (2016) suggest that mathematics tests

which focus on the mathematical content may be longer and need more time, both from the students to complete and from the teachers to design, especially if they assess problem solving (Walen & Williams, 2002). Time and stress handling is not the only non-content factor that contributes to performance at mathematics tests (Webb, 1992), as Hembree (1987) studies and lists many of them, like test familiarity, the order of the tasks, praising, competition and others. Does large scale and high-stakes testing make it difficult for a test that focuses on mathematical content to be designed, completed and scored in a desirable time frame? Is the tendency towards standardised testing in mathematics related to that aspect?

Secondly, in mathematics education we prefer discursive norms and rituals which we are familiar with (Lundin & Christensen, 2017) and these make unclear what is mathematical and what is social. So, a student working at a certain familiar and desired pace and speed may seem as fitting a kind of ritual in our minds. Being quick at doing mathematics is sometimes attributed by society to power and masculinity (Bibby, 2010). Ultimately, mathematics education plays an important role for the formation of the subjects and citizens in contemporary neoliberal society (Valero, 2018) as governable subjects (Kollosche, 2018). Is the strictly-timed framework of mathematics education a contribution to the construction of citizens who always hurry and give emphasis on efficiency (Chronaki, 2017, p. 427) and progress (Llewellyn, 2016)?

Thirdly, research suggests that assessment sometimes does not emphasise the mathematical content, but the applicability of statistical tools with the goal to distribute students into a pre-given spectrum of performances (Webb, 1992). So, in the process of designing a test, the time limit becomes a tool for broadening this spectrum of performances and so delivering this goal more adequately. Is this a contribution to one of the social functions of education in a class society, namely to distribute students into the layers of the social division of labor (Althusser, 2014), with exclusion being a structural part of that function (Pais, 2014)?

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“I have started this new trend at the end of [students’] notebooks”: A case study of a mathematics teacher caught within a reproductive cycle of hierarchical cascading of power

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Underpinned by the Human Condition Theory, this case study casts light on how an Egyptian mathematics teacher navigates his instructional approach with regards to problem solving while operating within a centralised governance model of national schooling. The study investigates multiple layers of contextual power dynamics affecting the teacher’s perspective and instructional approach towards mathematical problem solving. Data were collected in the form of teacher focus groups. Results indicate that the teacher held a view of problem solving that approximated the conventional view in the literature, but he felt compelled to adopt procedures-oriented methods in his daily practice. To combat this situation, he instituted a “new trend” outside of his main teaching practice: “at the end of [students’] notebooks.”

Introduction

The evolution of teaching and learning in modern Egypt can be considered as a product of colonial, post-colonial, monarchical, Marxist, liberal and extremist influences (Makramalla, 2020). In this study, we explore how, resulting from this evolution, socio-cultural power dynamics seem to exist that govern teachers’ classroom practices. In particular, we focus on one Egyptian teacher who participated in an activity-based extended teacher focus group. We explore how the aforementioned activity revealed this teacher’s perception of problem solving as a concept and how he viewed his role in teaching problem solving in his classroom, which we choose to view as situated in the context of a schooling microculture and a wider societal microculture. The study of micro and macro socio-cultural power dynamics, how they are historically and politically rooted and their influence on a teacher’s choice are globally relevant and there may be transnational similarities in that regard.

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Problem solving in the Egyptian teaching and learning context

To combat the post-colonial socio-cultural divide in view of knowledge dissemination and knowledge acquisition, the Egyptian government took the radical decision in 1967 to adopt a one-size-fits-all curricular dissemination and integration scheme that was to be executed across all national curriculum schools irrespective of their history, type (public or private) or socio-cultural contextual positioning (Ibrahim & Hozayin, 2006).

Sayed (2006) reports that, in the interest of safeguarding against extremist stimuli, the school mathematics governance model in Egypt was designed to be highly controlled by multiple layers of authority, which in turn all center around the Ministry of Education (MOE). Over the past 30 years, this centrally governed uniform teaching and learning model has been heavily memorisation-oriented (Sika, 2001), a matter that has recently been much debated by researchers, development agencies and policy makers (Ibrahim, 2010).

Recently, the MOE has launched its instructional and curricular reform initiative (EDU 2.0), an initiative that aimed to underpin the mathematics curriculum in problem solving oriented practices (Discovery Education, 2019). This radical shift in the underpinned curricular ethos was implemented in a tight timeframe, a matter that stirred public debate (Al-Ashkar, 2019). Teachers reported not grasping what the reform was substantially about and how they were to adopt it. The wider work, from which this case study is derived, aimed at exploring how teachers related to problem solving as a teaching and learning construct in an attempt to better understand how, building on this relatedness, a problem solving oriented curriculum might practically be enacted. This case study is hence situated in the transition period between the release of the EDU 2.0 initiative and its actual implementation.

Theoretical framework

This study is theoretically underpinned by Arendt's (1958) conceptualisation of the Human Condition Theory. In her work, Arendt (1958) situated individuals as operating within the 'vita activa', the active life as we are given. Within this framework, she distinguished between three states of activity of a human condition: the condition of labour, the condition of work and the condition of action (Figure 1).

As evident in Figure 1, the condition of labour designates the individual to operate within a system of reproduction, in order to ensure one's own sustaining within the system (vita activa). The condition of work shows a state where the individual becomes a creator, making artefacts that could challenge the 'vita activa' framework wherein the individual is situated; these are artefacts that outlive their maker and have the potential of shifting the actual reality where they are embedded. While these two modes of operation concern the individual, the third one relates to the interrelatedness between individuals in actions that could break through the actual reality. People work together to make progress and change the society where they operate in as 'actors'.

In light of the presented context of teaching and learning mathematics in Egypt, the case study at hand concerns an Egyptian mathematics teacher as an individual operating within a 'vita activa' which is characterised by a multi-layered, hierarchically cascaded, centrally governed power dynamic. As will be elaborated further below, our socio-political contextual

analysis led us to situate the mathematics teacher within this system of centrality and uniformity as operating in the mode of the 'labourer'.

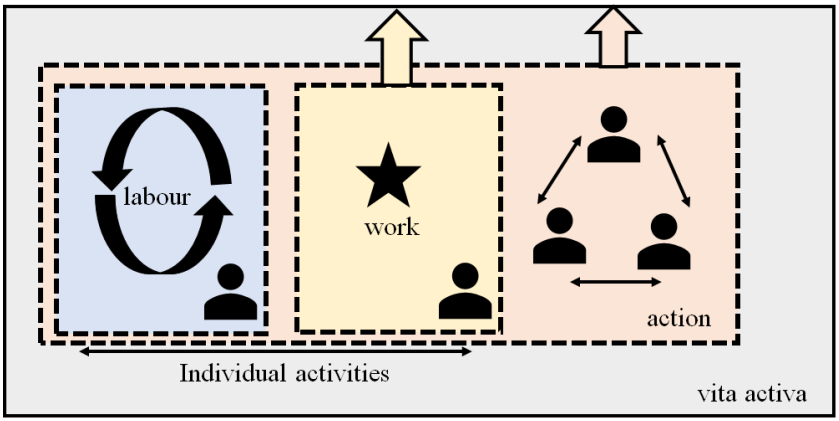

Figure 1: Perspective on three states of activity

As this study is particularly focused on teacher relatedness to mathematical problem solving, an activity that is recognised to be important for students' learning of mathematics at all levels of education in many countries internationally (e.g., Törner, Schoenfeld, & Reiss, 2007), we discuss also our conceptualisation of mathematical problem solving and respective tasks. According to Callejo and Vila (2008), a mathematical problem is defined as "a situation that proposes a mathematical question whose solution is not immediately accessible to the solver because he [or she] does not have an algorithm for relating the data with the unknown or a process that automatically relates the data with the conclusion" (p. 112). Building on this definition of a mathematical problem, we use the following definition of mathematical problem solving: "the activity that aims to generate a solution to a mathematical problem, that is, a response to the question in the problem that relates the data with the unknown" (Stylianides & Stylianides, 2014, p. 9).

The fact that a mathematical problem presents the solver with a situation for which there is no immediately obvious solution path implies that mathematical problem solving is 'cognitively demanding' (Henningsen & Stein, 1997) and cannot be reduced to memorisation or procedural forms of activity. We scoped our exploration of the teacher's perception of mathematical problem solving and its classroom integration around mathematical problem solving tasks, which we conceptualised drawing on Stein et al.'s (2000) Mathematical Task Analysis Guide. As shown in Figure 2, the Mathematical Task Analysis Guide distinguishes between two broad kinds tasks: those with lower-level cognitive demands ('memorisation' and 'procedures without connections' tasks) and those with higher-level cognitive demands ('procedures with connections' and 'doing mathematics' tasks). Memorisation tasks are at the bottom of the classification in terms of their level of cognitive demands while doing mathematics tasks are at the top.

Lower-Level Demands	Higher-Level Demands
Memorization • involve either reproducing previously learned facts, rules, formulae or definitions OR committing facts, rules, formulae or definitions to memory. • cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • are not ambiguous. Such tasks involve exact reproduction of previously-seen material and what is to be reproduced is clearly and directly stated. • have no connection to the concepts or meaning that underlie the facts, rules, formulae or definitions being learned or reproduced.	Procedures With Connections • focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections • are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • have no connection to the concepts or meaning that underlie the procedure being used. • are focused on producing correct answers rather than developing mathematical understanding. • require no explanations or explanations that focuses solely on describing the procedure that was used.	Doing Mathematics • require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • require students to explore and understand the nature of mathematical concepts, processes, or relationships. • demand self-monitoring or self-regulation of one's own cognitive processes. • require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Figure 2: Mathematical Task Analysis Guide (Stein et al., 2000, p. 16)

Building on Stein et al.'s (2000, p.16) classification of mathematical tasks (Figure 2) and on Schoenfeld's (1992) classification of mathematical activity as extending from 'memorisation' on the one end to 'problem solving' on the other, we mapped Stein et al.'s (2000) mathematical task types onto Schoenfeld's (1992) aforementioned spectrum as presented in Figure 3. In light of our adopted definition of mathematical problem solving and Stein et al.'s (2000) description of doing mathematics tasks, we refer to 'doing mathematics tasks' as 'problem solving tasks'.

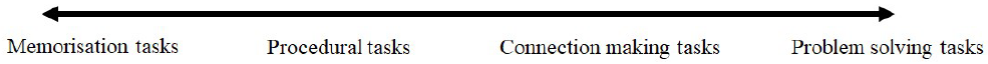

Figure 3: Spectrum of mathematical task types

The teacher as 'labourer' within a reproductive culture

The power dynamics that make up the 'vita activa' context where the Egyptian teacher is operating resemble those that have been described in the literature (Dewey, 2008/1932; Naguib, 2006) as a production and reproduction of a culture of autocracy. At every level of

the hierarchical ladder, the mono-system ensures that the same mathematics curriculum is practised in the same way across classroom contextual conditions. Being situated at the bottom of this hierarchical ladder, teachers are likely to feel trapped to reproduce this same culture of autocracy in their daily instruction (Figure 4). Naguib (2006) reported this course or action as being their only viable option, as implementing the prescribed form of instruction allows them to be socially and professionally recognised within a system that for many decades has fostered a societal expectation that reciting memorised knowledge is the main measure of success (Sika, 2001).

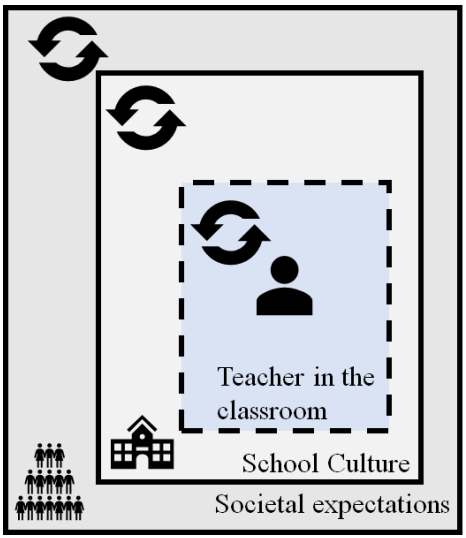

Figure 4: Situating the teacher as labourer within a reproductive culture

With this in mind, we view the Egyptian teacher as acting in the role of a 'labourer' (Arendt, 1958) within a culture of hierarchically reproduced forms of uniformity. Based on this choice of teacher positioning, teacher relatedness to problem solving classroom practices was explored, in order to address the following research question:

"How does an Egyptian mathematics teacher, who has been classified as operating in the position of a labourer within a system that for decades has been memorisation based, perceive mathematical problem solving tasks and the suitability of their integration in the local classroom?"

Research design

We present the individual case study (Yin, 2014) of one Egyptian mathematics teacher who operated in an Egyptian school governed by the wider centralised national agenda for schooling. Three mathematics teachers at this school participated, as part of a wider study, in a set of activities that were orchestrated in the setup of a focus group. The activities were

diversly designed to engage the teachers with a problem solving experience and reflection activities that related to the classroom implementation of problem solving tasks. The aim was to find out how teachers related to problem solving tasks with respect to the aforementioned classification of mathematical tasks (Stein et al., 2000), and how they related to their classroom implementation in light of their positioning within the wider power dynamic structure of centralisation.

The mathematics teacher whose case we present in this paper, Alan (pseudonym), has been teaching for the past 17 years at the same school, operating under a memorisation based national curriculum (pre-reform at the time of the study). Alan taught across all compulsory education year groups (years 1– 9). Despite being governed by the same contextual affordances and limitations as his colleagues were – such as lacking problem solving materials and having little autonomy and minimal classroom equipment – Alan stood out from his colleagues during the focus group as he expressed an interest in introducing his students to a different mathematical reality than the one in the pre-reform national curriculum (the national curriculum at the time of the study). We explore Alan's perception of mathematical problem solving along with the contextual affordances and limitations that he grappled with.

One of the focus group activities was task sorting (Friedrichsen & Dana, 2003). Each teacher was provided with a set of mathematical tasks that we had pre-classified according to the Mathematical Task Analysis Guide (Stein et al., 2000). The teachers were unaware of the task pre-classification and were asked to single out the tasks that they would consider to be as mathematical problem solving tasks. A subsequent discussion uncovered teachers' reasons for the chosen task classification. The teachers' contributions allowed us to uncover their relatedness to problem solving tasks. In a second round of the same activity, the teachers were asked to single out the tasks that they would adopt in their daily practice. The teachers' contributions in the subsequent discussion allowed us to better understand the micro-culture where the teachers were situated. It was important to uncover, based on the teachers' selections and explanations, how the micro-culture of the school was influenced by the wider national macro-culture and how, in turn, the micro-culture was influencing the teachers.

We used the constant comparative method for data coding (Glaser, 1995) to identify a set of features that the teachers associated with mathematical problem solving tasks in the first round of the task sorting activity. The *perceived problem solving task features* were then mapped against the features in the Mathematical Task Analysis Guide to examine the extent to which the teachers' perceived features of problem solving tasks aligned with those in the literature, notably Stein et al.'s (2000) 'doing mathematics' task category, which, as explained earlier, we (the researchers) associated with problem solving tasks. The data from the second round of the task sorting activity were also analysed from the perspective of the Mathematical Task Analysis Guide. Tasks that the teachers classified as well suited to the local context of their practice (the micro-culture of the school) were mapped back against the Mathematical Task Analysis Guide in an attempt to devise a code for reported *locally suitable task features. Locally suited task features* and *perceived problem solving task features* were finally contrasted against each other, in light of the literature guided framework (Stein et al., 2000).

Findings

The wider study from which this work is derived presents an elaborate account of the task features that this particular teacher related to mathematical problem solving. For the purposes of this paper, we chose to focus on mapping the features that this teacher associated with mathematical problem solving against the Mathematical Task Analysis Guide. As indicated in Figure 5, this teacher perceived problem solving tasks as mainly being tasks that establish connections to and between mathematical concepts, a feature that according to Stein et al. (2000) rather related to 'procedures with connection' tasks (Figure 2). The teacher distinguished between the act of making a connection to real life and the act of making a connection between mathematical concepts, as indicated by the following excerpts:

> "I think a mathematical problem solving task is a task where the student can make the connection between what he is learning and everyday life".
>
> "To me problem solving is when he can relate the mathematical concept, he [the student] is learning today with that, which he learnt in previous years".

The focus seemed mostly on the act of connection making.

The teacher's depiction of typical tasks utilised for daily instruction, however, were reportedly more procedural in nature (Figure 5). The teacher referred to a task that was "similar to something else the student was familiar with", "required a known procedure to solve", "has been solved before by the teacher".

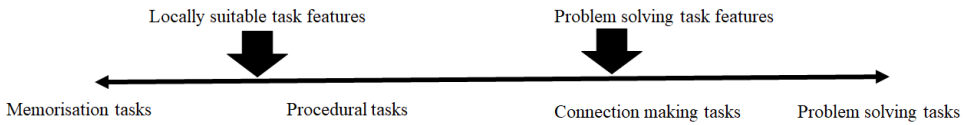

Figure 5: Mapping the main findings

The reflective part of the activity revealed that connection making mathematical tasks were the teacher's preference when it comes to classroom mathematical tasks. Nevertheless, as illustrated in Figure 5, the teacher also expressed the view that, within the school micro-culture it, was more suitable to adopt procedures-oriented tasks. The teacher expressed a feeling of being "trapped" in the system to adopt procedures-based mathematical tasks in the daily instruction, despite him preferring to adopt connection-making tasks. The school micro-culture endorsed memorisation-based instruction and this culture could not be neglected within the hierarchical execution of a reformed curriculum. The teacher seemed to find a way to balance this dynamic. To overcome this "trap", the teacher reported instituting a "new trend" in his daily practice: "I have started this new trend at the end of [students'] notebooks, where I write a problem that they can take home with them to think about. In the first five minutes of every session we have a conversation about what they have done with the problem. It's the part of the lesson that they like most."

Discussion

To discuss the teacher's so called "new trend", we build on our understanding of the contextual layers of macro-micro- power dynamics (Figure 3). Multiple layers of influence come into play affecting the teacher's own stance as well as classroom autonomy. In the following, we use the Human Condition Theory to untangle some of these layers.

The system expectation of the teacher

As indicated at the onset of this study, teachers operate as labourers within an organised and highly centralised system (Arendt, 1958; Naguib, 2006) where their value is determined by their ability to enact a replicated version of the national curriculum in their respective context of operation so as to prepare their students for a unified nationwide examination. In order to survive in this system, Alan has expressed the view of feeling "trapped". He feels obliged to adopt procedural mathematical tasks in line with the system's requirements. Alan, however, also reports a small window of opportunity to escape this trap. In the following, we analyse the findings by untangling the multiple vertical layers of this "trap", in an attempt to contextually situate Alan in this ecosystem of power dynamics.

The societal expectation of the teacher (macro)

As a result of operating in a memorisation-oriented teaching and learning culture, for multiple decades, over time societal expectations of schooling have been fixed around the act of certification (Hargreaves, 2006). As noted by Alan: "The parent does not send the child to school to learn. If they want their children to learn [skills] they can send them to summer camps. At the school, children should be prepared for the test." This in turn places additional demands on Alan to act (labour) in a way that sustains his existence within this system.

The school expectation of the teacher (micro)

Being immersed in a system and a society that places a high expectation on replicating the enactment of a memorisation-oriented mathematics experience in the classroom, in such a way that was nationally and societally accredited through certification, the wider study from which this work is derived addresses the role of the school as a micro-culture. It shows how the school, as a micro-culture of individuals working together, is positioned in such a way that it can either operate as an 'actor', presenting itself as a change agent to its stationed society (vita active), or as a 'labourer', yielding to the expectations of its stationed society in order to ensure its endurance (Figure 6).

In the first incident, the school acts as a change agent both within its micro-culture and to its stationed society. The former happens through fostering a local culture that appreciates problem solving thus counterbalancing the macro-culture, which appreciates memorisation, and producing local agents (teachers) that perceive mathematics accordingly. The latter happens through being stationed in its surrounding culture and operating with a dormant counter-culture that challenges the wider macro-culture. According to Alan, in response to the societal expectations of the context where the school is operating, the school is rather acting as a "certification institution". This adds a second layer of coercion on the teachers at

the school to teach for the test, thus acting as labourers within a wider macro-system of procedures-based instruction.

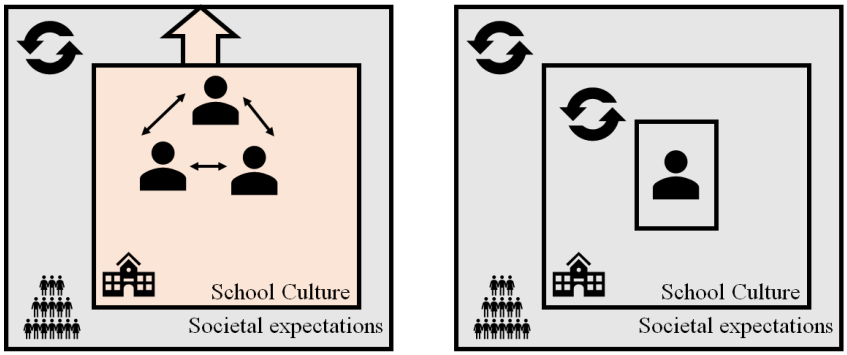

Figure 6: Two modes of operation for a school

The positionality of the teacher

Resulting from the aforementioned three levels of positioning, a teacher operating within this cascaded hierarchy, as discussed in the onset of this work, would be classified as a 'labourer'. Alan is indeed "trapped" to reproduce the same societally expected classroom experience. Yet, in accordance with his preference to teach connection-making mathematical tasks (Figure 5), he "instituted a new trend" that is reported to be the students' favourite part of the lesson. Alan finds a window of opportunity to use the first five minutes of the lesson and the end of the student notebooks to introduce his students to his preferred type of mathematics tasks, namely connection making tasks. Alan reports taking the initiative to search for and find extra-curricular materials to use in the "back of notebook" section.

In this way, it can be argued that Alan – within his scope of operation – is shifting away from the afore-ascribed position of the 'labourer' (Arendt, 1958) towards the position of a 'worker', creating artefacts that challenge the status quo. By searching for tools and materials and by utilising a tight framework to *create* a learning experience that is different from the one prescribed in the national context (vita activa), Alan counterbalances the hurdle of access to mathematical tasks. He also opposes to the contextual limitations to transfer an image of mathematics that is procedural in nature and presents to his students a learning experience of mathematics that relates to the act of connection making. It can therefore be argued that, by instituting this "new trend", Alan has placed himself in the position of a 'worker' (Figure 1).

Future research could involve conducting an in-depth interview with Alan in an attempt to understand what motivates him to counter-balance the system. The findings of this research could have implications for teacher professional development in high distance educational power systems, where teacher self motivation could be the only driving force to counter-balance a wider contextual dynamic. Tools and techniques could be set in place to empower a teacher, both with contextually suited materials to use and with an intrinsic drive to be a change agent.

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What may be a table in education research?

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Tabular representation of information can be traced to ancient times. They were and are used in a wide variety of ways. While every table is essentially a list, their interpretation varies from the simpler to the very complex. This aspect is explored in some detail by the present author. Based on the considerations, she suggests a number of ways of improving the study of tables in the classroom. Possible ideas of pedagogic consistency in the identification of difficulty of table use, and reduction of math anxiety are also touched upon.

Introduction

In this research, by a table will be meant any finite sequence of n-tuples that has been represented in table-form with at least two columns. This preliminary restriction is intended to avoid arbitrary finite sequences of things that can also be regarded as trivial tables.

Tables are everywhere, they have been used to keep records of transactions, record observations, facilitate calculations, represent arithmetic and algebraic operations and also to retrieve information. Babylonian multiplication tables on clay tablets are among the earliest tables in recorded history. Though lists containing information about pairs or triples of entities such as records of transactions go back to earlier times, tables for information are only a few centuries old.

Over the last few decades, tables have been put to far more interesting and complex uses across diverse disciplines. They have evolved radically in relation to representation, use, storage, retrieval, exploration, reasoning, and visualization. From a pedagogical perspective, they are taken for granted in the study of most subjects. Subjects such as computer science and statistics are relatively table-intensive, but comprehension of tables is assumed to be a matter of routine learning. Storage, retrieval, and related optimization problems are central to the study of databases from a machine, algorithmic, and software perspective. In most areas of soft computing, knowledge discovery from databases, and formal approaches to vagueness such as rough sets, tables are studied in different perspectives at considerable depth.

Despite such diversity in the interpretation and use of tables, they are not commonly explored from broader perspectives in school curricula, and only a handful of studies on related education research are known. When they are studied as in introductory chapters on statistics, the focus is usually on other aspects of the subject. In fact tables can be used to hide information from those without the skills or the time to read them. It is also known that difficulty with multiplication tables contributes to math anxiety or vice versa (Dowker et al., 2016; Luttenberger et al., 2018).

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In this research, aspects of the origin and use of tables are considered from a critical perspective, and a case is made for teaching about diverse interpretations and use of tables at different levels of learning. Further it is argued that an improved, broader approach to tables may help in reducing math anxiety of learners and teachers.

Tables for algebras, logic, and calculation

Tables for interpreting operations on a fixed finite set and those for calculation over the real numbers have certain similarities despite the relatively abstract nature of the former. However, they are used in diverse ways in universal/abstract algebra, logic, and related fields.

The interpretation of unary and binary operations on a finite set (that is reasonably small) can be easily represented as a table. But the actual properties satisfied by the algebra may not be directly readable from the table. Typically, tables play a complementary role in the study of abstract algebras. For example, a groupoid operation on the set S = {a, b, c} can be represented by Table 1.

$\odot$	a	b	c
a	a	c	c
b	b	b	a
c	a	b	c

Table 1: A groupoid

This way of representing operations is similar to that of multiplication tables in arithmetic (which is loosely taught as a partial algebra in schools in the present author's view).

Tables and graph theory

While every table can also be represented as a labeled graph satisfying certain properties in principle, such representation may be seen to be obscuring or sometimes improving the essential information present in a table. Further, a table can potentially be represented in non-unique ways using graphs based on perspective.

Graphs in turn are representable as relational systems, multi-sorted total algebras and single-sorted partial algebras (see Alberich et al., 1994; Chajda et al., 2015; Mani & Radeleczki, 2020). So the connection between tables and partial algebras capture related ontological content to an extent (from a mereo-ontological perspective as in Burkhardt et al. 2017). It may be noted that these are distinct from the multi-dimensional ontologies of math/statistical education research as for example in Pino-Fan et al. (2015) that do not refer to tables in a significantly different way.

Truth tables in logic

Some logics admit of truth tables or generalizations thereof. Set-valued valuations in particular can be used to handle relatively indeterministic situations (as in nmatrices). Truth tables in logic can be written for each logical operation/connective on a set of truth values (as for algebraic operations) or can also be interpreted by substitution when they are written schematically.

An example for the latter form for classical propositional logic is shown in Table 2 q and s need to be substituted by suitable sentences (built in accordance with formation rules) before reading from the table. Thus, for a given sentence q_0 with truth value 0 (false) and a sentence s_0 with truth value 1 (true), it follows that $s_0 \& q_0$ has truth value 0 from the table. This method does not explicitly assume set theoretic axioms or that the collection of all admissible formulas form a set. For details, the reader may refer to Magnus et al. (2020).

s	q	$s \& q$	$s \vee q$	$s \rightarrow q$	$s \leftrightarrow q$
1	1	1	1	1	1
1	0	0	1	0	0
0	1	0	1	1	0
0	0	0	0	1	1

Table 2: Truth table for PC connectives

Semantic tableaux or truth-trees are also used in proof theory and concern derivations using a method of breaking-down formulas to atomic ones. They can be used to check satisfiability of modal formulas, for decision procedures and proofs. Though these can be interpreted as tables regulated by rules, the connection is rather weak.

Tables for astronomical calculations

Tables for the purpose of computation have been extensively employed in positional astronomy for over 5000 years. The starting point of related geometric/physical models is the geocentric celestial sphere (Figure 1). Points on the surface of the sphere are mapped to positions of all stars, planets and other observable astronomical bodies. Further it is assumed that the abstract sphere rotates relative to the Earth along the North-South pole axis among other things. To represent points on the sphere, additional circles called the celestial equator (corresponding to an extension of the equatorial diameter) or the ecliptic (corresponding to positions of the Sun on a great circle) are used. Other local coordinate systems dependent on the position of the observer on the Earth have also been employed. All these required extensive use of special purpose trigonometric tables before the advent of logarithm based computations.

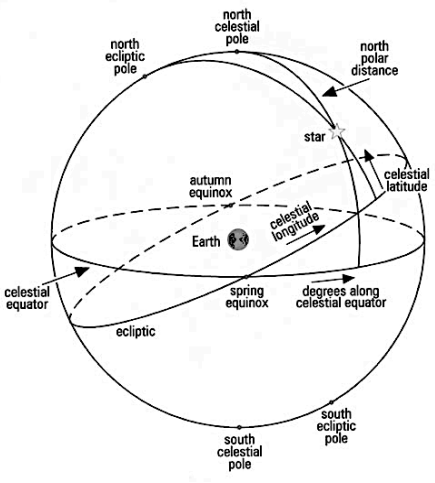

Figure 1: Celestial sphere (adapted from http://www.hps.cam.ac.uk/starry/mathtechniqueslrg.jpg)

An example of a table of sines, and tangents that dates to early sixteenth century (due to Regiomontanus; Rosenfield, 1988; Zinner, 1990) is presented in Figure 2. It may be noted this is part of the earliest systematic treatise on trigonometry.

The computation is not likely to be obvious to a modern reader because its use involves a few steps.

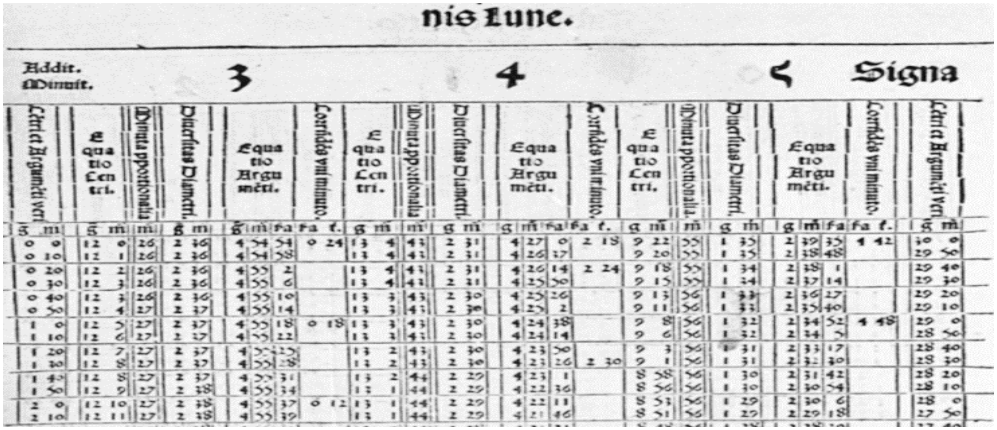

Figure 2: Trigonometric Tables, 1533 CE (adapted from www.hps.cam.ac.uk/starry)

The Rudolphine Tables (see Figure 3) due to Tycho Brahe and Johann Kepler (Swerdlow, 2000) that was published in 1627 permitted calculations (for the first time) of planetary positions at any given time in the past or future. The original book can be accessed from this web page: https://dibiki.ub.uni-kiel.de/viewer/resolver?urn=urn:nbn:de:gbv:8:2-1636149.

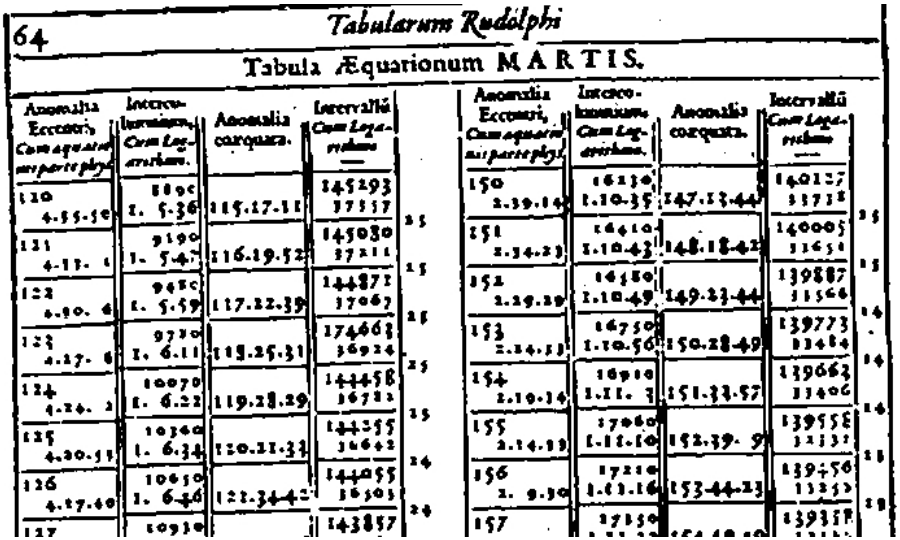

Figure 3: Rudolphine Tables, 1627CE

These tables were relatively accurate to 10 seconds, optimized on Kepler's elliptical heliocentric system, and involved use of logarithms.

Logarithmic calculations using tables were also developed during this time. The essential structure of these computational uses can be viewed in the perspectives described below.

Abstract forms

Logarithmic and trigonometric tables were used in computational contexts in accordance with the following abstract scheme:

- Problem: Compute the function $f: R^n \mapsto R$ at $(x_1, x_2, \dots, x_n) \in R^n$ subject to additional constraints.
- Step-1: Form a function $g \circ f(x_1, x_2, \dots, x_n)$ and simplify it. 'g' may be the identity function $gx = x$.
- Step-2: Read the value of the simplified parts from the table as per the if-then procedures.
- Step-3: Simplify
- Step-4: Transform the computed value as per suggested procedures (this may involve additional tables) and
- Step-5: Finally obtain the desired value of $f(x_1, x_2, \dots, x_n)$ to a degree of accuracy.

In the above, note that Step-2 also involves a search strategy. One has to visually converge to specific regions of the table to get the arguments (that may not be exact in absolute terms, but is so relative to the table). Essentially this step is about finding correct instances of an association relation between parts of the computational context and the table. Analogous aspects can be associated with Step-4.

Similar methods of computation are/were followed in contexts involving engineering and statistical tables. A clearer comparison with other types of table use can be made using this abstraction.

Tables in statistics

The history of the origin of tables for official statistics is somewhat known. At least in Europe, it can be dated to the 16th century (Lee, 1995; also https://www.york.ac.uk/depts/maths/histstat/).

Statistical tables arise from primary systematic sampling, other primary studies, data mining, information representation, secondary studies, and computations. The field is related to machine learning, computer science, data science and AI databases, and clear boundaries between these fields for features associated with tables are substantially different at times.

Typically, data collected in manual surveys needs to be preprocessed for storage in relational or other databases. This holds good for digital surveys that do not make use of suitable survey software. Sometimes data may need to be transcribed. The additional steps involved in these transformations can vary in complexity and the amount of decision-making required. Data from pilot studies are more likely to be harder because of the typically open-ended exploration involved. Experience in related activities can help students to develop reasoning skills in data-intensive activities and interest in related topics. Students do experience difficulty in undergraduate and higher stages of their career. This is reflected in

a number of research papers in fields involving the application of statistics of various forms. The replication crisis in certain fields may also be seen to be related to this (see for example Collaboration & Open Science, 2015).

Tables are only occasionally considered in the literature on statistics education research, and when they are then it is mostly about numeric tables, their construction, and interpretation from statistical perspectives (see for example Koschat, 2005; Pfannkuch & Wild, 2004). The five-step framework for interpreting tables and graphs in Kemp & Kissane (2010) is essentially about identifying different aspects of the table, its statistical context, quality indicators, the meaning of numbers, their differences, and identifying reasons for changes. It is intended for interpretation and not for construction or data presentation.

Tables in soft and hard computing

Softness and hardness of a computing context may or may not be expressed explicitly in data and specifically in table form. Sometimes the soft aspects are part of the interpretation aspect and sometimes these are kept in tables. Further, typically such representations can be done in a number of nonequivalent ways as for example in general rough sets. More efficient ways dependent on processing techniques may be discovered in the future – this statement should be read relative to ideas of embodied cognition rather than from static context-free perspectives. Such a state of affairs is already implicit in practices in the area, and aspects of this are explored in this section.

Data cleaning

The concept of *clean data* is always relative to the reasoning or computational context. Often more time is spent in the process of cleaning datasets than in other parts of the analysis. The result of such operations depends on loose principles, vague decisions, limited resources, familiarity with the context, and internal consistency of the dataset. Therefore, it is possible that different practitioners may produce non-unique datasets from the original.

Fixing time, for example, written in different ways in a column of a huge table may be done by actual uniformization or by eliminating entire rows that have improper formatting. Decisions relating to the scenario may have significant consequences.

While corrections of formatting inconsistency involve limited philosophical or meta-theoretic principles, situations relating to missing data, garbled data, outliers, wrong values, typos, extrapolation, supervenience, partial feature selection, partial feature extraction are very different. As a result it is not common for practitioners to arrive at similar end results after working on the same data dump — this has also to do with the lack of proper understanding of the philosophical and meta heuristics associated with the dataset in question. A substantial part of this is related to skills in reasoning with data, and their very generation or construction.

Specific problems like machine learning algorithms learning sexism, racism, and other forms of discrimination can also be said to be due to poor techniques of cleaning. Implicit in

this is the idea that a table is a form of expression (Haslanger et al., 2015; O'Neil, 2016) and not an immutable fact.

This diversity in interpretation of tables (and imperfect variations thereof) is however not matched in pedagogical practice in allied fields and should be of obvious concern.

Rules and Reasoning

A number of rules and patterns may be discovered from tables. For example,

- From a table of marks or grades obtained by class-X students in different subjects it may be deduced that *those with marks in the range [78, 99] in mathematics obtain at least 60 marks in physics.*
- From an instance of a few winning positions in a game of four-in-a-row, basic rules of the game maybe deduced. In fact, it is possible to find all possible winning combinations for a fixed table size.
- From a multiplication table of integers, some additional information about addition may be deduced. In fact additional rules depend on the language used at the meta level.

Because many of the rules discovered, and associated reasoning can be junk, finding appropriate ones requires considerable understanding of the context and application in the tabular framework. It can be argued that an early introduction to such reasoning can be useful for developing skills in rule discovery.

Abstract representations

The concept of information can also be defined in many not necessarily equivalent ways. In Mani (2020), the present author defined information as anything that alters or has the potential to alter a given context in a significant positive way within a semantic domain. This is among the most general definitions of the concept and can be related to interconnections between properties, discernibility and identity of objects. More specifically, a set of properties Q supervene on another set of properties T in a domain of discourse if there exist no two objects of the domain that differ on Q without differing on T. Let T be a set of mental properties, Q a set of physical properties, and the domain of discourse be restricted to some species. If the physical indiscernibility of any two entities relative to Q implies their mental indiscernibility relative to T, then Q supervenes on T.

Specific fields like general rough sets may impose additional conditions on the concept of information mentioned such as the possession of the property to potentially modify supervenience relations in the contexts, and of having formalizable approximations. While the concept of an information table (described below) as used in rough sets encodes a very special case of the above situation, it is nevertheless useful to capture all ideas of what a table might be.

In an information table, row names correspond to *objects*, column names to *attributes* and cell entries to *values assigned to the object-attribute pair*. This scheme works for all cases

including frequency, and contingency tables, and tables with complex partitions of columns or rows after a level. The last case does require some reformulation.

An *information table* $\mathcal{I}$, is a tuple of the form $\mathcal{I} = \langle \mathfrak{O}, \mathbb{A}, \{V_a : a \in \mathbb{A}\}, \{f_a : a \in \mathbb{A}\}\rangle$ with $\mathfrak{O}$, $\mathbb{A}$ and V_a being respectively sets of *Objects*, *Attributes,* and *Values* respectively. $f_a : \mathfrak{O} \mapsto \wp(V_a)$ being the valuation map associated with attribute $a \in \mathbb{A}$. Values may also be denoted by the binary function $v : \mathbb{A} \times \mathfrak{O} \mapsto \wp(V)$ defined by for any $a \in \mathbb{A}$ and $x \in \mathfrak{O}$, $v(a, x) = f_a(x)$.

An information table $\mathcal{I}$ is said to *deterministic* if and only if each object-attribute pair is associated with a singleton (that is essentially with a single value). In symbols,

It is said to be *indeterministic* $(\forall a \in \mathbb{A})(\forall x \in \mathfrak{O}) f_a(x)$ is a singleton (or incomplete) if it is not deterministic that is $(\exists a \in \mathbb{A})(\exists x \in \mathfrak{O})\ f_a(x)$ is not a singleton.

In Table 3, some information concerning evaluation of activities done by students in a class is tabulated. The table is complete, and it may be noted that the objects, attributes and associated value sets are as below. If the table is read in the default perspective, then student **F** obtained a A+ grade. **{E, F, C, K, L}** is the set of objects, **{Name, Novelty, Effort, Connections, Grade}** the set of attributes, and **{Medium, High, OK, Mild, A, A+, B+, 0, 1}** the set of values.

Name	Novelty	Effort	Model	Connections	Grade
E	Medium	Medium	1	OK	B+
F	High	OK	1	High	A+
C	High	Medium	0	Low	A
K	Medium	OK	1	OK	B+
L	High	Low	1	High	A+

Table 3: Activity Evaluation

Two-valued relations from a finite set F to another finite set K can be represented as Boolean matrix or as a table with entries being pairs of subsets of F and K in particular. Relations can also be derived from information tables through conditions of the following form: For any $x, w \in \mathfrak{O}$ and a subset B of $\mathbb{A}$, x is σ-related to w if and only if $(\boldsymbol{Q}a, b \in B)\Phi(v(a, x), v(b, w),)$ for some quantifier $\boldsymbol{Q}$ and formula Φ. The formulation is very general and a number of conditions can be written in the form. In symbols, $(\forall x, w \in \mathfrak{O})(\forall B \subseteq \mathbb{A})\sigma xw \leftrightarrow (\boldsymbol{Q}a, b \in B)\Phi(v(a, x), v(b, w),)$.

Other n-place relations and partial relations are also definable through extended versions of the formulas (including Boolean combinations and others). However, in practice most people do not bother to define or involve themselves with relatively complex formulas. *This contributes adversely to the quality of analysis in a number of situations including sociological ones.* Examples of such relations can be found in (Mani, 2018).

The point of this section is to assert that a wide spectrum of reasoning is possible naturally under minimal assumptions from tables. This increases dramatically when ideas of approximation or multi-valued or dialectical truth are brought in or when reasoning is

focused on mereological considerations. While such properties may be found in other classes of expression, they lack the deceptive simplicity and context relatedness of tables.

While the point of open-ended tasks (as specified in Yeo, 2015) in tabular frameworks can be argued for from examples such as those considered in Raval et al. (2020), a clearer understanding of the possibilities can be obtained from the abstractions of this section.

Math anxiety and tables

In the context of math anxiety, studies prove partly that in assessment situations, the internalized stereotype affects the perception of task difficulty and is related to increased strain and tension as well as decreased performance (Ertl et al., 2017; Luttenberger et al., 2018). Further over the course of childhood, this leads to avoidance of math, harmful learning behaviors, and lower performance. This is because the latter problem is typically associated with fear of the mathematical (whatever the cause may be). But the person experiencing math anxiety cannot be expected to experience it over the entire spectrum of mathematical activities. A complex geometrical problem that requires knowledge of trigonometry may induce far more anxiety than a study of tabular information. In other words, key parts of math anxiety refer to anxiety in the context of specific types of mathematics. Confidence in specific parts of mathematics can potentially help in overcoming the problem gradually. For these reasons open-ended study of different types of tables can be useful for eliminating math anxiety in people. While empirical studies supporting this very specific claim are wanting, closely related studies warrant it.

Bruner's (1961) framework, in particular, relates to the observation that if students learn to obtain knowledge of a subject through the use of their own mind, then they are likely to learn best. At a methodological level, in the context of math method courses in teacher training, this can be about emphasis on the use of manipulatives and concrete learning of the mathematical content, journal logs, small and whole group instruction and presentations, literature based activities, and practical experiences. A number of empirical studies (like Gresham, 2007) show that math anxiety can be reduced in math teachers through such a framework. Specifically, tables are regarded as commonplace objects that are everywhere. So they are unlikely to cause any additional anxiety when used in the situations mentioned, and can actually facilitate the mentioned methodology especially in the light of the possibilities discussed in this paper.

More for a consistent table pedagogy

Tables are typically deceptively simple and potentially hide a large amount of information. Discovery (or invention) of such information can involve a wide assortment of techniques that are used in multiple branches of science. From the overview of table use in different disciplines a critical understanding of what might be a consistent choice of goals for open-ended tasks can be indicated, and a well-designed diverse curriculum can help in appreciation of techniques and reasoning skills involved.

Data science, machine learning, soft computing, and AI are all used in diverse fields from relatively exact applied fields to the social sciences. But key skills relating to use of tables are not commonly taught in schools. This results in so-called skilled people mangling or distorting data during the data cleaning process. It is also often the case that such acts are not detected by anybody in associated work groups.

People in different walks of life use tables frequently. Typically, such usage is focused on the task at hand. For example, a Uber driver may keep track of time, earnings, and distance using tables on a notebook to keep a tab on any cheating by the parent company. But the driver may not bother to construct an improved table (or analyze the table in a way) that can potentially improve their own quality of life in additional ways. Apparently such restricted mindsets are likely constructed in some classrooms.

Conclusion

Teaching tables in relatively open-ended ways from lower classes can be most useful to foster critical thinking and promote smoother concept maturation over time. In this research a number of pointers to potential thrust areas, and the scope of such actions is explored, and further research is naturally motivated. It is also argued that an improved, broader approach to tables may help in reducing math anxiety of learners and teachers. The practice of *table study* can be a new subject essential to the practice of the relatively more exact, and inexact sciences. *Most importantly it is not restricted by specific viewpoints and is adaptable to multiple languages in both formal and less formal senses.*

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The problem of formation of moral values in the process of teaching mathematics

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The article discusses the problem of formation of moral values in the process of teaching mathematics. It reveals the fact that this process has a great potential for formation of both positive and negative moral values. We focus on the basic universal moral values - kindness, respect, love, dignity, duty, justice, tolerance, the purpose of life, happiness. We consider the problem of formation of moral values within the general approach to the problem and in the context of informatization of mathematical education. We suggest specific ways to prevent the formation of negative values and to encourage the formation of positive moral ones in the process of teaching mathematics. We find value-orientation teaching of mathematics as an approach to reach the goal.

Introduction

The formation of moral values is one of the main tasks of education. The possibilities of humanitarian school subjects in the formation of moral values of students are undoubtedly great. Literature and history may address to works depicting excellent artistic images or historical figures with excellent moral qualities of heroism, love or kindness, while the educational material of mathematics does not provide such opportunities. Nevertheless, the process of teaching mathematics also has a great potential for the formation of both positive and negative moral values (Mikaelian, 2011, 2015a, 2017; Yenokyan, 2019b).

Despite the fact that quite a lot of attention has been recently paid to the relationship between mathematics and ethics (Ernest, 2016, 2018; François, 2018; Mendick, 2006), the problem of formation of moral values in the process of teaching mathematics seems to be left out of this framework. Our works in this direction are published in Armenian and Russian (some of them are included to the list of references) and they are not available to a wide range of specialists. This publication will fill the gap. Due to space constraints, our wordings is shorter than we feel like, so we do not dwell on the problems of formation of the moral values of respect, justice, freedom, love, etc. It would also be interesting to demonstrate a practical presentation of value-orientated learning in the process of teaching mathematics, which is not carried out for the same reason.

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We have been guided by the following principles in presenting the results of our research on the forming of moral values through the process of teaching mathematics.

a) First of all, designate good as a basic moral value, the possibility of forming good and prevent evil in the process of teaching mathematics and their connection with other moral values. This topic has been extensively studied by various authors. b) Present the moral values formed in the process of teaching mathematics, interesting for the MES and perhaps not previously studied by other authors. From this perspective, we examined the moral values of duty and virtue. In addition, in the case of duty, we were guided only by Kant's ideas about this moral category, and we considered the value of virtue in the context of historical development, taking into account the philosophical approaches of different times. c) Present the relationship and dependence on worldview and various philosophical ideas that the formation of moral values in the process of teaching mathematics has. The paragraph on the meaning of life and happiness in our work is presented with this approach. d) To present the problem of formation of moral values in the context of modern approaches of mathematical education: humanization and informatization.

Good and evil in the process of teaching mathematics

Good and evil are the most important moral values. Although there are different opinions about good and evil in the nature of a person (Kant, 1965; Mikaelian, 2011), there is a prevailing idea of their formation through education in all approaches. The process of teaching mathematics occupies a special place in the formation of the values of good and evil of students.

Firstly, good is a boon, that is, beneficial. Mathematics education is a boon. It is a boon as a component of education; a boon in terms of its applied purpose, in terms of the role it plays in the functions of formation and development of intelligence, in the mental processes of students. However, experience shows that the vast majority of students do not take advantage of this boon. Moreover, a mathematical lesson for them becomes a repulsive, unpleasant environment or evil (Ernest, 2018; Mikaelian, 2011). In the process of teaching mathematics, it is difficult to carry out the formation of such a sign that characterizes good as tolerance. In their works Mikaelian (2015a) and Yenokyan (2016) examine the psychological and moral consequences of a teacher's intolerance for a student's mistake, their delay in response and other actions and ways to avoid them.

As a rule, students with high academic performance are given manifestation of kindness on the part of the teacher, whereas it is the weak students who need this but do not receive it. Even one encouraging word by the teacher while solving a simple exercise can draw such a student into the learning process. The teacher's behaviour should not contain such signs of evil as rancour, revenge, suppression of someone else's will, coercion, ill will, pressure, hatred or revenge (Mikaelian, 2015a). Mikaelian (2011) also discusses the formation of such qualities inherent in good as benevolence, respect, compassion, lack of rancour and revenge. Yenokyan (2019a) discusses the issue of preventing the qualities inherent in evil in the process of teaching mathematics.

The moral duty in the process of teaching mathematics

Duty is the ability to recognize one's own moral responsibility. The role of education is great in the awareness, formation and implementation of duty. Mathematics also has necessary educational potential in this direction, the identification of which was carried out in Mikaelian (2011) where we consider a person's duty in relation to themselves and others. Kant (1965) identifies three types of duty in relation to themselves in which a person acts as a living being, a spiritual being or a moral being.

The duty of a person towards oneself, as a living being, primarily implies the obligation to maintain existence for which they try to satisfy their vital needs. The process of teaching mathematics pushes a person to intellectual actions and helps to strengthen willpower, properties that contribute to the satisfaction of life's needs. At the same time, long-term mental work, work with an imaginary world, which is inherent in those engaged in mathematics, is aimed at keeping a person from communicating with nature and people, that is, from the natural essence of a person. That can lead to poor health and contradicts the duty of attitude towards oneself as a living being. Duty to oneself, as a living being, also implies the preservation of the human species, for which a person creates a family, gives birth to children, takes care of them and raises them. Presumably, the process of teaching mathematics, while increasing the intellectual abilities of a person, does not contribute to the fulfilment of this duty. It does not contribute to the realization of communication with people, which is also a duty in relation to oneself as a living being.

According to Kant (1980) a person's duty towards oneself as a spiritual being is the development of spiritual, mental and physical forces. Among the spiritual values, Kant first notes mathematics. However, mathematics not only constitutes the most important part of the human spiritual world, but also makes a significant contribution to the development of other essential components of the human spiritual world - science and technology, acts as an indispensable tool for the formation of various branches of art, as an important means of forming beauty in them. Mathematical education plays an important role in the formation and development of thinking, imagination, memory, attention, will and other mental processes, and these in turn contribute to the development of spiritual forces (Mikaelian, 2015b, 2019, 2020; Ernest, 2020; Francois, 2019).

Mikaelian, (2011) considers the issue of manifestations of a person's moral duty towards oneself in the process of teaching mathematics- moral self-knowledge as the first dictate of duty to oneself and conscience as the ability to assess one's own activities, to see oneself as an inner judge. It also examines the relationship between the process of teaching mathematics with falsehood, servility, avarice - vices that lead to a violation of duty towards oneself as a moral being. In relation to the third, mathematics has different roles. In everyday matters it leads to stinginess; in the intellectual sphere it leads to formation of extravagance and in the professional sphere it leads to the formation of generosity.

Kant distinguishes between two types of duty in relation to others- love and respect. Mikaelian (2011) discusses the manifestation of love and respect in the processes of teaching

mathematics. The author mainly shows that the competitive nature of mathematical activity can hinder the formation of a common duty of love in relation to another – philanthropy (Mikaelian, 2018a). The process of teaching mathematics has many opportunities for the manifestation of charity, gratitude and compassion as debts of love towards others. In this process may also appear the vices of ingratitude, envy and malice in the two-sided subjective teacher–student or student–student relations. These vices are contrary to the duty of love. A teacher has to be skilful to lead the process in the necessary direction.

Mikaelian (2011) considers the problem of the manifestation of a duty of respect towards others in the process of teaching mathematics in two-way subjective relations of teacher–student and student–student. It discusses the possibilities of manifestation in this process of arrogance, ridicule and slander, vices contradicting the duty of respect towards others. We need to be especially careful about the duty of respect. Indeed, while neglect of the duty of love does not harm anyone, disregard of the duty of respect insults one's dignity.

Feeling of respect is closely related to that of love. Just as in the physical world, bodies are in harmony due to the balance of the forces of attraction and repulsion, harmony is carried out using the balance of the forces of attraction and repulsion and moral harmony between people is the result of the balance of these forces of attraction and repulsion - love and respect (Kant,1980). In the process of teaching mathematics, moral harmony between the student and the teacher of mathematics, based on mutual love and respect, is realized only with students with good academic performance (Mikaelian, 2011).

Virtue and vice in the process of teaching maths

The first doctrine of virtue was developed by *Aristotle* (Aristotle, 1983). He viewed virtue as a golden mean between two extreme vices - excess and lack. Thus, courage is something between madness and cowardice; generosity in relation to material goods is something between extravagance and stinginess; truthfulness is something between of boasting and hypocrisy, wit is something between buffoonery and rudeness. Friendliness is something between quarrelsomeness and quarrelsomeness, bashfulness is something between shamelessness and timidity, etc. Mikaelian (2011) considers the possibilities of applying these approaches of Aristotle in the process of teaching mathematics. It is shown that they can enable the mathematics teacher to keep the learning process from unnecessary extremes or the danger of the formation of vices. There are drawings that characterize the manifestations of Aristotelian virtues in the process of teaching mathematics, the accuracy of which has been experimentally confirmed. For example, Figure 1 is a representation of the formation of the Aristotelian concept of the virtue of courage. The drawing conventionally depicts areas of cowardice, courage and madness. The marked layer shows to which of these areas math education pushes the student. We see that mathematics education is inherent in keeping a person from madness, not pushing for courage and generating a sense towards of fear of danger.

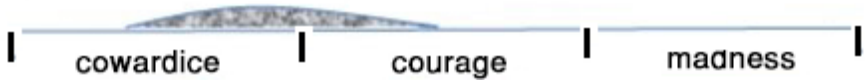

Figure 1: Courage and mathematics

The next group of virtues comprises the "radical" virtues of ancient Greece: moderation, courage, wisdom and justice. Mikaelian (2011) discusses the influence of mathematical education on the process of formation of these moral qualities in students. Mikaelian (2011) also discusses the role of mathematics education in the formation of the basic theological virtues of hope, faith and love. For the formation of all these virtues, as well as the corresponding vices, mathematics has a great educational potential, the manifestation and prevention of which depends on the activities of the teacher.

Let us turn to *Solov'ev's* understanding of virtue and the role of mathematical education in its formation. According to Solovyov (2012) three qualities shame, compassion and cowardice lie in the basis of virtues. In the process of teaching mathematics, manifestations of Solov'ev's understanding of shame and cowardice are often encountered, while compassion is manifested in exceptional cases.

B. Franklin (Franklin, 1956) also turns to virtues. He finds that the virtue of human activity has to be measured by its usefulness for achieving success and identifies a number of virtues, three of which are considered the main ones. There are hard work, accurate fulfilment of monetary obligations and frugality. In the formation of all these virtues, mathematics education plays a significant positive role (Mikaelian, 2011).

Meaning, purpose of life, happiness and the process of teaching mathematics.

The meaning of life is one of the highest moral values, which is related to the determination of the ultimate goal of existence, the purpose of humanity as a biological species, as individuals. The meaning of life is one of the main worldview concepts that are of great importance for the formation of the spiritual and moral appearance of a person. Can the process of teaching mathematics contribute to the formation of this moral value? The answer to the question depends on our approaches to this concept: what do we consider the meaning of life? Different philosophical directions answer this question in different ways.

If we proceed from the ideas of *hedonism*, then the meaning of life is limited to pleasure. In (Mikaelian, 2015b) it is shown that pleasure can arise both from the identification of the internal structure of mathematical material, and from the use of mathematics in solving problems that have appeared in different academic subjects. In both cases, pleasure is associated with beauty. In the first case, we are dealing with signs of mathematical beauty in the form of intellectual search, finding, and discovery, and in the second case with signs of applicability (Mikaelian, 2019).

Utilitarianism is premised on the fact that the meaning of life involves obtaining benefits (a variant of egoism) or giving benefits (a variant of altruism). Mikaelian (2018a) shows that the process of teaching mathematics is mainly selfish. It is aimed at obtaining benefits. In

relation to others or by a pro-social nature, mathematical activity is not altruistic, but sadistic. Consequently, within the framework of general education, it mainly leads to competition. In addition, the benefits of mathematics education are obvious from the point of view of spiritual development and the formation of the students' worldview, the achievement of professional success in the future, and so on (Ernest, 2020; Mikaelian, 2020a).

Let us turn to the problem in detail from the standpoint of *eudemonism* where the meaning of life is limited to happiness. There are various moral approaches to happiness that do not satisfy the logical requirements of the definition of the concept, nevertheless they make it possible to evaluate human happiness, especially those signs that are necessary to achieve happiness. According to one of these approaches (Balashov, 2008) in order to be happy, a person must have physical health and material wealth and all this must be enveloped in love and creativity (a mixture of happiness). In the mixture of happiness, we also include positive guidelines for life and give the following formula for happiness in table 1 (Mikaelian, 2011).

The material side of happiness		The spiritual side of happiness		Blend of happiness
Physical health	Material and social wealth	Spiritual health	Spiritual wealth	Positive guidelines for life, creativity

Table 1: The formula of happiness

How much does mathematial education contribute to human happiness? Is the child happy in the mathematical class at school? Paul Ernst (2018) rightly believes that most of these children are unhappy. Moreover, for students who are not interested in mathematics, a mathematics lesson turns into a place of imprisonment (Mikaelian, 2011). How does the process of teaching mathematics affect the formation of each component of happiness? A. Schopenhauer finds that nine-tenths of human happiness depends on physical health (Schopenhauer, 1990). Mathematical education contributes to the acquisition of material wealth and in terms of acquiring social wealth, it can have a double meaning: it plays a positive role in cooperative relationships and even plays a negative role in family, friendship and similar relationships (Mikaelian, 2011). On the formation of mental health, as a component of moral purity, mathematical education can have both a positive and a negative impact (Mikaelian, 2011; Ernest, 2018). As for mental wealth, mathematical education has only a positive impact. It contributes to the formation and development of intelligence, thinking, imagination, attention, memory and volitional qualities. It also plays an important role in the formation of various aesthetic values (Mikaelian, 2015b, 2019).

Mathematics education can have a double meaning in shaping a blend of happiness. The influence on the formation of optimism and faith is mostly positive, and on the formation of love of life, cheerfulness and a cheerful mood it is mostly negative. For students who are able to grasp mathematical material, that is the patterns in it and their beauty, mathematics is an inexhaustible source of love and hope, while for students deprived of such abilities, it brings only hatred and torment (Mikaelian, 2011; Ernest, 2018). The same picture applies to creativity.

The possibilities are similar to those of physical education at school. Only these two school subjects provide students with inexhaustible opportunities for creativity and unfortunately restricted to some students (Mikaelian, 2011).

The meaning of life is usually associated with the problem of setting goals and their implementation. Two main goals are associated with the problems of self-preserving and preserving the human species. The first problem is solved through professional activity, while the second — through the creation of family. In the former mathematical education plays a positive role, but in the latter its role can be negative (Mikaelian, 2011).

The problem of formation of moral values in the context of modern approaches to mathematical education

One of the modern approaches to the development of mathematics education is its *humanization*: humanistic approach in the process of teaching mathematics. One of the ways to implement this approach, together with the knowledge of objects and phenomena of the surrounding world, is the formation of values and value orientations of students. In the diversity of values, the most important place is occupied by moral values that for the most part become the cause of value orientations and determine human behaviour. As noted, the process of teaching mathematics has great potential for the formation of moral values, the identification of which becomes one of the ways to humanize mathematics education (Mikaelian & Yenokyan, 2016).

In the works (Mikaelian, 2020b; Yenokyan, 2020) the problem of formation of moral values in the context of *informatization* of mathematical education is considered. In particular, the formation of the values of tolerance and fairness in the conditions of informatization of mathematical education is considered. The authors propose specific ways of solving the problem.

One of the modern trends in the development of mathematical education is *historicization*: inclusion of individual episodes of history in the process of teaching mathematics, which can also be successfully applied in the formation of moral values. In (Yenokyan, 2015) the question of the formation of moral values through teaching mathematical problems in the Armenian educational environment of the 7th-19th centuries is considered. It is shown that in the mentioned period and especially in the 7th century, the Armenian mathematician Anania Shirakatsi and other authors created mathematical problems in which situations with moral content are widespread, which contribute to the formation of moral values.

It is extremely difficult to turn the issue of the formation of values, moral values in particular, into the goal of a specific educational process, since it is not valued as knowledge or as a skill and is not controlled. Its implementation is due to the professional and moral approaches of the teacher. However, even if desired, the mathematics teacher is not able to implement it, since textbooks and methodological literature do not aimed at the goal. To solve the problem, we propose to apply value-oriented mathematics teaching (Mikaelian,

2018a,b; Yenokyan, 2018b). Together with the formation of knowledge, a value-oriented approach becomes the goal of lessons and the task of value-formation. In (Mikaelian & Mkrtchyan, (2020), the effectiveness of the use of elements of logic in the organization of value-oriented teaching is shown, and in (Mkrtchyan & Yenokyan 2016) a value-oriented approach is applied in the process of learning the elements of logic. In (Yenokyan, 2018a) a value-oriented approach is applied in high school in organizing function learning.

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Inclusion, tolerance and dialogue in mathematics classes

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This paper presents a reflection related to the concepts of inclusion and tolerance in mathematics classes where the differences amongst the students are acknowledged. In a context permeated by tensions, the complexity of teaching and learning processes becomes more evident to try promoting education for all. In order to contribute to a better understanding of practices based on the concept of equity, this paper discusses aspects relating to inclusion, tolerance, and dialogue. Teaching and learning practices based on dialogue can contribute to the fight against oppressive practices, thus favouring tolerance, cooperation, and construction of equity, essentials elements for the inclusion of any student in mathematics classes.

Introduction

This article presents a reflection related to the concepts of inclusion and tolerance in mathematics classes where differences between students are recognized, highlighting aspects that contribute to teaching and learning mathematics guided by the concept of equity.

In the field of education, inclusion can be understood as a commitment to favor underrepresented groups in society, people without possibilities for development in schools as it traditionally presents themselves. In this sense, inclusive education is related to ideas of equity and social justice.

Equity differs from equality, recognizing the different means and conditions for distinct groups or people to have access to the same rights. Promote equal access to resources, qualified teachers, and pedagogical support for all students, regardless of their differences, are actions that may be understood as ways to guarantee the same opportunities, but that is not true.

Students from different social groups, with different skills, needs and backgrounds, do not have the same conditions of access to the same rights. For example, a deaf student who uses sign language, in a class without the presence of an interpreter of sign language, will certainly be harmed. Thus, support is necessary for this student to have the same conditions as his colleagues.

Ladson-Billings (2009) also argues that, when using the term equality to refer to learning opportunities, the history of different cultural groups and the ways in which the organization of society has influenced the opportunities offered are neglected to different groups of people. Thus, it emphasizes the importance of recognizing and valuing differences when dealing with issues concerning the chances for all students to be able to achieve academic success.

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Equity includes the recognition of inequities present in society and at school, from different cultures, different social groups, different skills, and the different knowledge that students bring to the classroom (Faustino, 2018). Thus, it assumes that different opportunities need to be created so that different people have access to the same right.

Silva et al (2017) state that in order to promote equity, it is important to ensure the quality of education, enabling students to access and stay in school, as well as the development of citizenship and critical knowledge. The authors argue that issues of social justice, social inequalities, as well as socioeconomic disparities, should relate to equity promoting practices.

Regarding to mathematics education, some authors emphasize the potential of mathematics education in identifying inequities present in society and in establishing a pillar for strengthening struggles for social justice. Struggles for all human beings, regardless of social class, race, gender, ability, religion, need to have access to the same rights. (Frankenstein, 1983; Gutstein, 2006; Skovsmose, 2012)

In research in Mathematics Education the term equity has been mostly related to inclusion, since in societies the cultural diversity is increasing, it is necessary to incorporate including practices that enable students to learn independently of pre-established standards (Bishop, Tan, & Barkatsas, 2015).

The reflections proposed in this article are supported by concerns related to education and social justice with a focus on teaching and learning mathematics in school environments. The discussions presented are based in the experience of inclusive education in Brazilian schools. We defend the idea that an education based on concepts of equity and tolerance can be favored by an approach that favors dialogical communication.

In what follows we present the theoretical perspective related to the concepts of inclusion and tolerance. Then, we present a discussion of aspects related to the dialogue that favored equity in mathematics classes. We conclude by highlighting the importance of dialogical interaction for the practice of tolerance, as well as for the cooperation and construction of equity in educational practices.

An inclusion perspective

Based on conceptions related to human rights, including education progresses with an ideal of equity and presupposes an education centered on the student and their skills and that fully meets their educational needs so that everyone can learn.

This perspective is aimed at all students, nonetheless benefits mainly the students that had been historically excluded or on the margins of the society such as, for example, blacks, elderly, disability and LGBTQ[1] people, as well as, all people who do not have the same education opportunities as the majority of the population.

In Brazil, most of the concerns of inclusive education are related to schooling for people with disabilities due to the historic exclusion suffered by this group during a long period and the fight for access and democratization of education. Since 2008, the introduction of new educational polices related to inclusive education guarantee the access of students with disabilities in regular schools. Thus, the current discussion is directing to permanence of

[1] LGBTQ is s an acronym for lesbian, gay, bisexual, transgender, and queer people.

these students, as well as, students from other underrepresented groups. The reflections presented in this paper are directing to this context.

The presence of students with disabilities in regular schools opens space for questions, positions, doubts, discussions and reconstruction of practices that have not contributed to the inclusion process, as well as to the development of concepts and actions compatible with the school's inclusive proposal. Uncertainties arise since, although there are attractive things related to the inclusion process, there are also problematic aspects.

> Apparently, it is a praiseworthy thing to work for inclusion in an educational domain. There does not seem to be any need of justifying an inclusive education. It seems by itself an attractive thing to do. The question is just how to do it. The point to be stressed here, however, is that inclusion is also a contested concept. (Figueiras, Healy, & Skovsmose, 2016, p. 16)

These authors see the word inclusion as a contested concept. That is a concept that can have different modes of interpretation, as well as operate in different discourses. Since it has no defined meanings, it can represent social, economic, religious, political, and cultural controversies. Thus, they believe that one of the ways to explore the contested nature of inclusive education is through the notion of deficiencialism.

Marcone (2015) calls deficiencialism the expression that refers to the construction of disability in terms of standards of normality. To reinforce the disability argument as an invention with an ideal of normality as a parameter, the author sought to view disability as an experience, something that we are subject to and not a previously established condition.

To try to answer some of the questions that moved him, Marcone (2015) sought inspiration in anti-colonial readings and in post-colonial theories, which discusses the ways in which relations between Western countries and their eastern colonies were established. Based on these conceptions, Marcone defined deficiencialism. For the author, disability is something invented by a group that sees itself as normal and puts the group of people with disability like a distinct of them, causing tension throughout an idea of dichotomy, disguised by politically correct speeches.

In general, deficiencialism refers to some groups as disabled and provides opinions on what this group is and is not capable of doing. This is a conception frequently identified in mathematics teaching practices aimed at students with disabilities and confronting such a conception requires a change in the understanding of disabilities.

In this sense, Skovsmose (2019), suggests seeing the term disability throughout the notion of difference, and thus, one can interpret inclusive education as education based on differences. For the author, differences define one of the main characteristics of the human condition and are seen in all spheres of life.

To consider such a conception is to consider the construction of education that supports the learning of all students. Thus, a curriculum designed with the blind student's skills in mind, for example, is still suitable for students with perfect vision. That is, it is possible to think in a school of differences.

Thus, inclusive education can be considered as an education that seeks value differences and not an education that tries to include the different in some standard of normality. For Skovsmose (2019), this conception allows us to interpret inclusive education as new ways of providing a meeting amongst differences.

Tolerance in the meeting amongst differences

Moura (2020) understands encounter as a human ability to be with the other (or others) experiencing a collaborative relationship, which suggests movement, action to discover, to be aware of new things. The author stresses that this relationship is not always possible, since even sharing the same physical space, it is common for people to be at different levels of understanding and abilities. In this sense, the meetings become responsible for promoting a collaborative relationship in living with each other. One of the ways to build such a relationship is through the practice of tolerance.

> Falo da tolerância como virtude da convivência humana. Falo, por isso mesmo, da qualidade básica a ser forjada por nós e aprendida pela assunção de sua significação ética – qualidade de conviver com o diferente. Com o diferente, não com o inferior (Freire, 2014, p. 25–26)
>
> I speak of tolerance as a virtue of human coexistence. I speak, for this same reason, of the basic quality to be forged by us and learned throughout the assumption of their ethical meaning – the quality of living with the different. With the different, not with the inferior (author's translation)

In Portuguese, the word tolerate is understood in common sense as a synonym for abide in its sense of resisting something painful or standing firm in the face of an uncomfortable situation. Tolerance in this sense is as if the tolerant did a certain favor to the tolerated, forgiving the supposed inferiority of the tolerated in a benevolent way.

It is common that the presence of students with disabilities in regular schools is understood in this way. Working with these students, in general, is considered praiseworthy and worthy of admiration, precisely because of the belief that they lack something. In this perspective, tolerating different students means accepting them in the school space, but not worrying about developing practices that actually include these students in the teaching and learning processes, since allowing them to be present at school is already seen as a great favor.

The concept of coexistence with the different and the acceptance of this is a point to be highlighted in the ideas of Paulo Freire, which is implicit in all his understanding of education. Freire (2014) breaks with the tolerance paradigm as a virtue of superiority when affirming tolerance as a virtue of human coexistence and not of an individual.

In this perspective, tolerance has a mutual commitment developed through coexistence amongst people, based on human relationships. In other words, the person is tolerant not because he or she is superior, but because he or she recognizes in the other person someone who has a condition different from his or hers.

> A tolerância genuína, por outro lado, não exige de mim que concorde com aquele ou aquela a quem tolero ou também não me pede que a estime ou o estime. O que a tolerância autêntica demanda de mim é que respeite o diferente, seus sonhos, suas ideias, suas opções, seus gostos, que não o negue só porque é diferente. O que a tolerância legítima termina por ensinar é que, na sua experiência, aprendo com o diferente. (Freire, 2014, p. 26)
>
> Genuine tolerance, on the other hand, does not require me to agree with the one who I tolerate or also do not ask me to estimate him or her. What authentic tolerance demands from me is that I respect the different, their dreams, their ideas, their options, their tastes, that I do not deny it just because he or she is different. What legitimate tolerance ends me up teaching is that, in its experience, I learn from the different. (author's translation)

In other words, tolerating does not mean agreeing with another, but it requires respect for the different to the point that you can learn from them. Thus, tolerance values all knowledge and skills, which enable the construction of new knowledge.

In this way, I realize a collaborative relationship in which tolerance is practiced, as a relationship in which the other is recognized as a different person, with whom I can learn. In this sense, Moura (2020, p. 194) understands meetings as a meeting between two or more people, who recognize the differences from each other and are open to the new learning possibilities that can arise from that meeting.

This meeting does not happen spontaneously and without any setbacks and, like any human relationship, it is full of tensions. For Freire (2014) tension is an essential element for knowledge, and it is creative since it destabilizes positions imposed as rules or "absolute truths", in addition to stimulating curiosity, questioning relations of power and oppression.

Thus, the meeting between the different invites us to reflect on the absolute positions that do not contribute to us being tolerant, that is so that we are open to learning from each other.

Like Freire (2014), I believe in tensions as something that contributes to knowledge. The history of the education of people with disabilities in Brazil and all the legislation that supports the presence of these students in regular schools today was only possible due to questions about the power relations and oppressions experienced by these students, which made it possible to search for spaces in schools the right to education and other rights that allow their full participation in life in society, like any citizen.

All achievements by students with disabilities in Brazil, in particular their presence in regular schools, have contributed to new questions about education and new practices, especially with regard to forms of teaching and learning.

To think inclusive education in terms of encounters amongst the different is to think of an education that practices tolerance through the recognition and appreciation of the difference of each one and that sees the possibility of learning from the different. Thus, it is necessary that new possibilities regarding the teaching and learning processes of mathematics be discussed.

Among several possibilities, Skovsmose (2019) highlights Landscapes of Investigation as a proposal that invites encounters between differences in mathematics classes. Such a proposal involves the organization of learning environments that favor ways of communication that challenge the exercise paradigm (Skovsmose, 2000). The typical pattern of communication is dialogue understood as a conversation aimed at learning. Through it, each participant has the opportunity to expose their ideas and defend their point of view, in order to collaborate with collective thinking and the creation of new perspectives.

In what follows are present, a discussion on aspects related to the dialogue that favored inclusion and tolerance in mathematics classes.

Dialogue in mathematics classes

Dialogue is understood as fundamental to establish critical perspectives on mathematics and for the cooperative construction of new mathematical ideas. Dialogue is seen as an element that favors a collaborative learning process and opens spaces for epistemic and socio-political criticism (Moura, 2020).

An essential aspect of the dialogue is the promotion of equity in terms of communication, valuing a horizontal relationship between those involved, and is not influenced by the roles or conditions of teachers and students.

Alrø and Skovsmose (2004) emphasize that promoting equity does not imply denying diversity and differences. For the authors, this refers "[...] to ways of dealing with diversity and difference, and the principal concept is fairness. Fairness does not only refer to emotional aspects, it also refers to the way the content matter of the dialogue is dealt with" (p. 124).

In other words, promoting equity includes dealing with differences and that must be understood as a respectful relationship between people who are partners in carrying out a joint action. Thus, the strength of the arguments must be centered on their importance for the investigation carried out and not because this argument comes from a position that has more or less power in a given relationship.

The authors point out that such an approach is not free of risks due to the higher rate of unpredictability. There are uncertainties when you want to know what the other person thinks, even you suspecting something, you are not sure what the answers will be.

Handle with risks can also be related to student's feelings. For example, they may feel uncomfortable during the investigation process when they have a contested or rejected opinion by others involved. On the other hand, they can feel happy when the shared perspective helps in the investigation. The unpredictability in this context can also mean new possibilities for learning, contributing to autonomy during the process, knowing that it can happen in different ways.

For Milani (2015) "dialogar é estar com o outro, é mover-se em direção ao outro, ao interessar-se pelo o que outro diz" (p. 203). In this movement to go where the other is, the author presents three elements that she considers essential in a dialogical interaction: *active listening*, *estrangement* and *decentring*.

Active listening involves asking questions and giving non-verbal support (watchful eye; body directed at who speak; a facial expression that shows interest; nodding in the affirmative) to the speaker while seeking to understand their perspective.

Estrangement, on the other hand, can occur when there is a difference between ways of thinking, for example when the teacher thinks of a strategy for solving an exercise, and a student presents another strategy completely different and unexpected. Faced with a situation like this, the teacher can choose to ignore it, insisting on their perspective, which can decrease the chances of dialogical interaction. Or, the teacher can consider the situation that causes strangeness, trying to understand how the student is thinking. This action of understanding the other, going to where he is, is understood by Milani (2015) by *decentring*, which is linked to understanding the point of view of the other who speaks.

Alrø and Skovsmose (2004) call attention to the difficulty of valuing dialogical interactions in a traditional mathematics class, based on solving exercises. In this model, classes are guided by the textbook in which the teacher presents some ideas and teaches techniques necessary for solving exercises that are also found in the book. There are no spaces for questioning or justification about the relevance of exercises that, in general, present only one correct answer.

Skovsmose (2000) refers to this model as teaching based on the exercise paradigm and argues that it should be contrasted by an investigative approach. He states that such an approach is one that has the potential for dialogical communication to occur in the classroom and presents the propose of Landscapes of Investigation as a possibility in this perspective.

Investigating according to the purpose of Landscapes of Investigation includes collectivity and collaboration and for that to happen it is necessary that students get involved through an invitation and not as something imposed. The lines of investigation take shape from the exploration of diverse perspectives giving rise to insights into the approach to a problem. However, those involved must be prepared to renounce a perspective, that is to say, to analyze what would happen if it were not maintained, not placing it as something unquestionable.

The teacher is still responsible for planning, mediation, decisions about the appropriate methodologies for the mathematical contents and for assessment strategies. However, when trying to understand the student's perspectives, the teacher is no longer the only one to have something to teach.

During the dialogic interaction, the students are no longer who just learns and the teacher is no longer the one who just teaches. While learning, students also can teach teachers. The teacher's displacement to understand the student's perspectives may contribute to overcoming the vertical relationship between teacher and students (Faustino, 2018).

Thereby, teachers have the opportunity to learn from students and about students. This movement towards overcoming the vertical relationship during classes, allow students to share their perspectives, their differences, their understandings of the world, their skills, and ways of expressing themselves.

When moving to understand the student's perspectives, the teacher opens up to the reframing of ideas and positions, respecting the several ways of learning and paths to be followed. The methodological and assessment strategies start to also consider the student's perspectives and the way they experience the world. This movement is essential for the construction of equity.

Concluding remarks

In order to contribute to a mathematics education that values differences, Skovsmose (2019) proposes that Inclusive Education be thought of as new ways of providing encounters amongst differences. In this sense, these encounters are understood as a human ability to be with the other (or others) experiencing a dialogue relationship, which considers a movement, an action of discovering, an act of being aware of new things.

To think inclusive education in terms of encounters amongst the differences is to think of an education that practices tolerance in its genuine sense, that is, that recognizes and values the difference of each one and that sees the possibility of learning from the different.

The propose of Landscapes of Investigation shows itself as an invitation to encounter differences in mathematics classes and makes room for dialogical interactions to appear. The characteristics present in the dialogical interaction – active listening, estrangement, and

decentring – contribute to the emergence and maintenance of the dialogue and are identified when there is an interest in understanding what the other says.

In this sense, the potential of dialogic interactions is emphasized for a cooperative relationship in which people act at the level of equity, since it allows differences to be respected, through the action of wanting to know what the other means, regardless of who this other is, their skills or position held during an interaction.

Through dialogical interaction, new meetings are facilitated in the classroom. Teachers learn with students and their differences, making this meeting not just a sharing of the same environment, but a movement to see the other, to want to be together favoring the practice of tolerance, as well as cooperation and the construction of equity, essential elements for the inclusion of all students in mathematics classes.

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Framing school mathematics challenges inside and outside Missouri metropolitan areas

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We report the results of an interview study conducted with leaders in 50 districts across the U.S. state of Missouri, which focused on what they identified as—and how they framed—their districts' most salient mathematics-related problems. Our results point to meaningful differences related to whether leaders work within metropolitan areas in terms of the type of problems they identify and how they frame them. Leaders in non-metropolitan districts appear more likely to define and frame problems around improvement in standardized test scores, whereas leaders in metropolitan areas are more likely—though not guaranteed—to define problems in terms of equity and the ways that students experience school mathematics. Related factors include economic resources, mathematics leadership, and commitment to inquiry-based pedagogy.

In our work, we have been interested in initiating partnerships with school districts centered on "problems of practice" (Penuel et al., 2015) related to mathematics. Our efforts have spanned a variety of contexts, from small districts in rural communities to large districts in urban settings. In the course of our initial listening and investigating what such problems of practice might be, we became especially attuned to what district leaders identified as problems and how they framed those problems. As prior work has shown, the problems that leaders identify and how they frame them have consequences—for both policy implementation and the ways that teachers respond (Coburn, 2006). But we also began to wonder whether leaders' identification and framing of problems varied between different types of communities and settings.

A context of reform

Our study is against a backdrop of "reform" in the U.S. over the last 30 years with respect to both mathematics and broader accountability policies. Regarding the former, there is a strong "school mathematics tradition" (Cobb et al., 1992) of mathematics teachers demonstrating procedures for solving classes of problems, asking students to replicate those procedures,

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and assessing for correctness. For decades, researchers and practitioners in mathematics education have been working to counter the inertia of the school mathematics tradition by describing and enacting teaching that fosters things like inquiry, reasoning, discourse, sense-making, and conceptual understanding (National Council of Teachers of Mathematics [NCTM], 2014).

The common rationale for advocating such a different pedagogical approach is at least twofold. First, it affords the necessary "opportunities to learn" on which research on learning in mathematics classrooms over the years has converged. A second argument is that it affords more equitable opportunities to learn (and fosters more positive experience with) mathematics (NCTM, 2014). Scholars have raised critical questions about the extent to which such "mainstream" equity-oriented practices can disrupt the history of which populations have been best served by school mathematics (e.g., Barajas-López & Larnell, 2019), but they are generally viewed as playing a part in increasing *access* to opportunities to experience success in mathematics—a necessary (though insufficient) condition for equity (Rubel, 2017).

Beyond the few pockets of success that have been achieved in enacting more equitable, high-quality mathematics teaching, the U.S. has been largely unsuccessful in effecting change in how we do school mathematics at scale (Hiebert, 2013). "Reform" language may have made it into circulation, but much of what research in the learning sciences and mathematics education has yielded about supporting mathematics learning has had little real impact or lasting influence in many school settings.

Accountability reforms, on the other hand, *have* been remarkably "successful"—successful in pursuing the broader neoliberal agenda of commodifying access to education (Connell, 2013) through standards and testing policies by which students can be categorized and schools—particularly those serving poor communities of color—can be deemed "failing." All of this helps to place the blame on teachers and public institutions (and the communities themselves) and to frame market-based reforms as the solution. These reforms have resulted in shifting discourses and building infrastructure around problems manufactured by setting standardized measures of proficiency (Kitchen et al., 2016), perhaps most acutely in mathematics (Pais, 2014).

Because the school mathematics tradition is (seemingly) more amenable to teaching directly to tested procedural proficiencies, many of the responses to accountability pressures run counter to the vision for learning and teaching described above (e.g., NCTM, 2014). This has been especially true—and especially detrimental—in schools serving student populations that are poor and comprised predominantly of African American students or other students of color (Davis & Martin, 2008). Still, those responses structure the current policy climate in which teachers and others work to achieve particular goals—however narrowly defined—for students' mathematics learning and therefore require our attention if we hope to improve mathematics instruction at a wider scale.

Orienting perspectives

Attending to discourse(s)

Our study is rooted in an assumption that the policy discourses—"language use around a particular socially and historically situated topic, which [...] corresponds to a particular position and perspective"—district leaders employ to define problems and strategies for addressing those problems can be consequential for maintaining or transforming institutional cultures (Bertrand et al., 2015, p. 3). We view the discourses employed by individuals in education institutions—especially those in officially designated positions of leadership—as both reflecting and constructing power relations (Anderson & Mungal, 2015). Whether about mathematics (e.g., the "math wars" and "back to basics" movements) or accountability policies (e.g., proficiency labels for children, "failing" schools), these discourses are drawn and appropriated to varying extents from broader narratives "floating around" in the world (Sfard & Prusak, 2005, p. 18), and are more likely to be taken up when accompanied by public pressure—to which school and district leaders have always been vulnerable. And these discourses help construct well-defined "problems" around which leaders—who certainly do not want to be caught helming "failing" institutions—can chart a clear path toward "improvement."

Attending to framing

School improvement efforts are (at least implicitly) rooted in the identification of a problem (Penuel et al., 2013). As Stone (1989) explained, problems are defined through *causal stories*, or narratives that actors craft to present certain conditions as (a) caused by human actions and (b) responsive to human intervention. In other words, causal stories define a condition as a problem by placing blame and portraying particular elements of the problem as amenable to policy. Thus, what we are interested in is leaders' identification and *framing* of problems (Coburn, 2006; Penuel et al., 2013). Scholars have considered how framing as a discursive act can be deployed strategically to influence the perspectives and experiences of others (Benford & Snow, 2000).

In school districts, the discursive frames that leaders deploy to define and motivate action around problems can vary with respect to responses to the current accountability policy climate. In their examination of urban district leaders' discourses, Jackson et al. (2018) found that leaders' ways of framing the problem of poor outcomes on state standardized tests often differed depending on their office. Those in leadership offices (responsible for supervising principals) typically employed an instructional *management* frame, which leads to simply reconfiguring human resources in order to produce higher test scores, whereas leaders in curriculum and instruction offices (responsible for teacher support) more likely employed an instructional *improvement* frame, through which they focused on supporting teachers' learning and growth. Jackson et al.'s (2018) characterization of leaders' differing frames emerged from particular contexts—leaders from multiple offices within urban districts, framing the problem of poor student outcomes. But there could be other types of problems of interest, and there are certainly other types of districts, including small, rural districts that may not have leaders beyond a superintendent, let alone multiple district offices.

Attending to geography

Location is important because it shapes access to opportunities and resources, conferring "advantages and disadvantages to the persons and organizations whose fortunes are linked to specific places" (Logan, 1978, p. 414). In one of the earliest works investigating the "geography of opportunity" in education, Tate (2008) examined educational outcomes in two places—Dallas and St. Louis—and found that students from low-income neighborhoods were disconnected from the resources associated with the city's industrial centers. Specific to mathematics education, Tate and Hogrebe (2015) investigated how relationships between poverty, teaching and fiscal contexts, and Algebra I test scores vary between Missouri school districts, finding more statistically significant relationships around the large urban areas.

Methods

Oriented by the previously summarized literature, in this section we describe our methods for answering the following research questions: 1) Do leaders' identification and framing of problems differ with respect to districts' size and proximity to metropolitan centers? 2) If so, what factors are related?

Sampling and participants

We conducted our study in Missouri, a state located in the middle of the U.S. and home to more than 6 million people (the country's 18th most populous). On opposite borders are the state's two urban centers—Kansas City to the west and St. Louis to the east, with their surrounding metropolitan areas, approximately 2.1 million and 2.8 million people, respectively. At the other end of the density spectrum, about a quarter of the state's population live in rural communities.

We purposefully sampled a range of districts with respect to population and proximity to metropolitan centers. We classified each Missouri municipality in two ways—by proximity to metropolitan centers and by population size. Table 1 presents the size range and the total and sampled numbers of Missouri municipalities in each of six classifications. Within each classification, we randomly ordered the municipalities and determined the school district the municipality's residents attended.

Classification	Population range	Total in state	Interviewed
Urban Metro	> 200,000	2	2
Large Metro	25,000 – 200,000	21	11
Small Metro	< 25,000	262	10
Large Non-Metro	25,000 – 200,000	6	5
Small Non-Metro	2,500 – 24,999	93	12
Rural Non-Metro	< 2,500	655	10
Total		1,039	50

Table 1: District sample and classifications.

In each district, we identified the leader tasked with overseeing mathematics curriculum and instruction. We looked to see whether a district-wide mathematics coordinator was listed on the district's website. If not, we contacted the general curriculum coordinator. If neither such role was listed, we contacted the superintendent. In each case, we asked whether someone else was more directly responsible for mathematics curriculum and instruction. In total, we interviewed 4 superintendents, 17 assistant superintendents of curriculum and instruction (or equivalent), 25 other district leaders in curriculum and instruction (18 of whom were in roles specific to mathematics), and four secondary mathematics teachers. Smaller and/or nonmetro districts were less likely to have mathematics-specific district leaders or, in some cases, even leaders directly responsible for curriculum and instruction.

Data collection

Interviewees participated in semi-structured, 45-minute, audio-recorded interviews. The majority (45) were conducted in 2018, with 5 in 2017. We asked "are there currently any math-related challenges in the district that have been made an explicit focus?" with follow-up questions regarding perceptions of the source of the challenge (or problem) and how the district was currently addressing it. Additionally, we included follow-up probes about any other district mathematics goals, the curriculum in use, and whether a particular instructional approach was being promoted in the district.

To supplement the interview data, we also collected contextual information from governmental websites that could be related to leaders' identification and framing of problems. We collected 2018 data for each district regarding student achievement, student racial and economic demographics, and per-pupil expenditures.

With respect to student achievement, we collected rates of proficiency on federally mandated state standardized tests for grades three through eight. For each district we calculated the percentage of scores categorized as either "proficient" or "advanced" (and not "basic" or "below basic") across all grades combined and subtracted the state average from that percentage. We constructed the variable in this way because we conjectured that concern about achievement (and thus the problems that leaders identify) might be relative to a district's standing compared to other districts.

Similarly, we suspected that leaders' problem identification and framing may be related to or influenced by the economic resources available in the district. As an indicator of that factor, we gathered average 2018 per-pupil expenditures for each district.

Metro and/or larger districts had more racially and ethnically diverse student populations. Besides the two urban metropolitan districts, non-metropolitan districts, on average, had a larger percentage of their student population that qualified for free or reduced-price lunch than their metropolitan counterparts. And yet, per-pupil expenditures were greater, on average, in metropolitan districts.

Analysis

Elsewhere we have described the qualitative analysis through which we categorized what participants identified as mathematics-related problems and how they framed those problems (Munter et al., 2020). In that analysis, we identified three main types of problems described across the 50 interviews: *student outcomes* (e.g., student achievement; n=35), *student experiences* (e.g., enjoyment of mathematics; n=12), and *equity* (e.g., "achievement gaps"; n=3). Expanding Jackson et al.'s (2018) work, we also categorized each leader's framing as "strictly management," "strictly learning," or "both." In this paper, we focus on our quantitative analysis of the relations between those types and framings of problems and aspects of the leaders' districts.

As a reminder, our research questions were Do leaders' identification and framing of problems differ with respect to districts' size and proximity to metropolitan centers and, If so, what factors are related? Given the dominant discourses nationally about achievement on state standardized tests described previously, we were interested in whether the prevalence of outcomes-related problems compared to other types of problems (i.e., student experiences, equity) varied by geographical region and context. Of course, it is conceivable that leaders in districts whose achievement rates had been significantly below the state average may have been more likely to identify outcomes-related problems than those with higher rates of proficiency. Thus, we employed logistic regression to pursue our questions, as such models are appropriate for dichotomous dependent variables (i.e., outcomes-related problem or other problem) and allowed us to control for districts' standing in terms of proficiency on the state's most recent annual standardized test, and to examine possible relations to other factors.

We constructed two sets of models. In the first, we estimated the likelihood of leaders identifying an outcomes-related problem rather than another type of problem. The second set of models estimated the likelihood that leaders employed a strictly management framing of outcomes-related problems. To answer the first research question, for both sets of models, controlling for the district's difference from the state average in standardized test proficiency rate, we estimated the relation of the dependent variables—problem *type* in the first set of models and problem *framing* in the second—to (a) whether their district was in a metropolitan or non-metropolitan area, and then (b) size of enrollment, with a third model (c) including both to determine which was more strongly related to the type of problem or framing.

For the second research question, we repeated the two sets of models and included three additional contextual variables. As an indicator of districts' economic resources and support, we included per-pupil expenditures. Motivated by Jackson et al.'s (2018) finding that ways of framing problems can differ with respect to district leaders' roles, we included a dummy variable for whether districts had an officially designated mathematics leader (who we would have interviewed), as we conjectured that there could be differences related to mathematics specificity of leaders' foci and backgrounds. Last, we imagined that the extent to which districts were trying to break from the "school mathematics tradition" (Cobb et al., 1992) could influence problem identification and framing. We therefore included a dummy variable

indicating whether, based on leaders' interview responses, districts were pursuing an instructional vision aligned with an inquiry-oriented approach promoted by reform efforts in mathematics education (e.g., NCTM, 2014) rather than a more conventional, direct instruction approach or no particular approach at all.

Variable	1a	1b	1c	1d	1e
Metropolitan area	-2.20**(0.78)		-1.88*(0.82)	-0.41 (1.05)	
Enrolment (in thousands)		-0.11*(0.05)	-0.06 (0.06)		
Per-pupil expenditures (in thousands)				-0.47 (0.29)	-0.52*(0.25)
District math leader				-2.33*(1.12)	-2.51*(1.05)
Commitment to inquiry-based pedagogy				-2.49*(1.17)	-2.60*(1.13)
Difference from state avg test proficiency	-0.06*(0.03)	-0.07*(0.03)	-0.06*(0.03)	-0.10*(0.05)	-0.11*(0.05)
Constant	2.26***(0.64)	1.87**(0.55)	2.54***(0.70)	8.45*(3.53)	8.96**(3.32)
Pseudo R-squared	*0.254*	*0.307*	*0.359*	*0.445*	*0.442*

Table 2: Estimated coefficients (and standard errors) of first set of models predicting an outcomes-related problem ($N = 50$; $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$).

Results

Tables 2 and 3 list the results of our logistic regression analyses. As shown in Table 2, controlling for percentages of students scoring "proficient" or "advanced" on 2018 state standardized mathematics tests compared to the state average, we found that leaders in metropolitan districts (1a) or larger districts (1b) were less likely to identify outcomes-related problems than were their counterparts in non-metropolitan or smaller districts. Including both variables in the model (1c), however, suggests that metropolitan proximity is more strongly related to the type of problem identified.

Our second set of models (in Table 3) revealed a similar pattern, although it was size (model 2c) that was more strongly related to the dependent variable. Specifically, leaders in smaller districts were more likely to employ a strictly management frame of outcomes-related problems than a strictly learning frame or a combination.

The other models in Tables 2 and 3 attend to our second research question about additional factors. After dropping the now-non-significant metro variable in Model 1d, Model 1e suggests that the identification of outcomes-related problems was negatively related to all of the additional factors, including per-pupil expenditures, whether there was a district math leader, and whether the district was committed to inquiry-based pedagogy.

As Model 2d shows, per-pupil expenditures and math leadership were not related to type of framing of outcomes-related problems. However, the commitment to inquiry-based pedagogy was omitted from this model, because none of the leaders who expressed such a commitment employed a strictly management framing.

Variable	2a	2b	2c	2d
Metropolitan area	-1.978** (0.735)		-1.301 (0.816)	
Enrolment (in thousands)		-0.29* (0.12)	-0.24* (0.12)	-0.25+ (0.14)
Per-pupil expenditures (in thousands)				-0.29 (0.22)
District math leader				-1.22 (1.31)
Difference from state avg test proficiency	-0.019 (0.029)	-0.029 (0.029)	-0.030 (0.033)	-0.04 (0.04)
Constant	0.099 (0.388)	0.675 (0.505)	0.932+ (0.540)	3.98 (2.51)
Pseudo R-squared	*0.146*	*0.232*	*0.274*	*0.310*

Table 3: Estimated coefficients (and standard errors) of second set of models predicting a strictly management framing of outcomes-related problems ($N = 50$; $^{**}p < 0.01$; $^{*}p < 0.05$; $^{+}p < 0.10$).

Discussion

We found that leaders from non-metropolitan districts disproportionately described outcomes-related problems, and that leaders from smaller districts were more likely to employ a strictly "management" framing of those problems. This suggests that in smaller districts geographically beyond the state's metropolitan centers, discourses of accountability and standardized test achievement predominate when it comes to identifying and framing mathematics challenges. This raises concerns that smaller, non-metro communities may, even more than their larger, metro counterparts, be victims of the "corporate model of schooling," including orienting school communities to interventionist approaches to raising performance scores that reduce the roles of students and teachers to that of data production and communicating economy-based narratives about the purposes of school (mathematics) (Pais, 2014).

Our investigation of additional factors pointed to possible explanations for the metro-nonmetro difference. That per-pupil expenditures were related to problem type suggests that the very personnel or programs that metro districts can afford may push them to attend to matters beyond accountability expectations. One expenditure in those communities may be district leadership tasked specifically with focusing on mathematics learning and teaching in the district, as our analysis found that the presence of such leaders was related to the type of problem identified. If this is the case, the implications for smaller districts cannot be that they, too, should hire a district mathematics leader, as they likely lack the necessary

resources. Rather, the responsibility lies with those in power—the policy makers and enforcers of the state—to make changes in expectations and/or forms of support to smaller communities outside metro areas in ways that invite emergent local concerns (Chronaki & Lazaridou, 2019) so that issues beyond achievement outcomes are brought into focus.

Our findings also pointed to a relation between districts' commitment to inquiry-based pedagogy and the type and framing of problem identified. That none of the leaders in districts with such a commitment employed a strictly management framing raises concerns that resources devoted to addressing outcomes-related problems—particularly with a strictly management framing—may only perpetuate the "school mathematics tradition" (Cobb et al., 1992) and that this may be especially true in smaller, non-metropolitan communities.

Our findings have important implications for not just state officials and their responsibility to account for the impact of their policies, but also researchers hoping to engage in partnerships with practitioners. Attending to our potential partners' discourses in identifying problems of mutual interest may reveal deeper challenges related to power, geography, and resources.

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Mathematical concepts and methodological dilemmas: Research on the movement of thought and body

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This paper explores some of the methodological dilemmas that emerge when researching mathematics learning processes with pre-service elementary teachers, whilst focusing on the genetic and material dimensions of mathematical concepts. Case study data from a project situated in a UK University is used to help elaborate and unravel these dilemmas. The discussion focuses on recent writing about the nature of mathematical concepts, merging philosophical, sociological and cognitive perspectives, with the aim of unpacking the complex ways in which 'movement' of thought and body plays a pivotal role.

Introduction

This paper seeks to explore the methodological implications which arose when I sought to engage in research with elementary pre-service teachers, where I aimed to encourage them to step out of the known, the familiar, and the learned, to bring forth an 'inventive' and 'speculative' dimension to their thinking about the nature of mathematical concepts. Although many other researchers have pursued kindred aims, under the general rubric of 'constructivist learning', my own conviction was that creative speculation in mathematics demanded leaving the secure fold of established and respected theoretical frameworks and methodological paradigms for mathematics education research, and the need to venture further afield into ontological considerations from within the philosophy of mathematics.

The context for the research is a UK teacher education program where elementary teachers are trained for mathematics teaching; and although these future teachers arrive with plenty of cultural capital and insight into children and learning, they are ill-prepared to dive into complex texts about classical philosophical questions like: what is a mathematical concept? What is the nature of mathematics? They are more readily attuned to sociological and psychoanalytic theories of socio-cultural identity, which is a mainstay of the education training at the institution, furnishing them with strong insights into the political framing of education. And they are equally handy with constructivist theories of learning, which are also a mainstay, whereby the common-sense notion that learning should be student-centred, collaborative and emerge through a process of discovery resonates with their intuited notions of childhood education, often gained through their own 'apprenticeship' of (mathematics) education (Lortie, 2002). Despite the fact that such learning theories implicitly

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carry philosophical assumptions about the nature of mind, body and concept, it remains very challenging for most of us to philosophically engage directly with the specificity of mathematical practices and concepts. We can train teachers to problematize 'mathematics education' but the aim here is the more challenging prospect of problematizing *mathematics itself on its own terms.*

With that in mind, this paper discusses how attempts were made with participants of this research study, to explore the question: what is a mathematical concept? This discussion and exploration of mathematical concepts from a more philosophical perspective, will be enriched through engagement with some recent compelling approaches, and a discussion of how the ideas of posthumanist philosopher Karen Barad might help us to open up teacher training in new directions. The paper explores how this approach is able to support the means to awaken the student teachers' possibility to re-imagine fundamental mathematical concepts in new ways, and to think more radically about how inventive, generative and speculative habits and practices are actually the *ontological* source of mathematical concepts. In embracing the opportunities this provides for all involved, I re-turn to the learning of geometrical mathematical concepts at the elementary phase of schooling, and document Barad's (2007) inherent demand for a fundamental re-working of ontology and epistemology to align with the ethical and thus implications for methodology within the research process. This will involve assessing the implications of such an approach to mathematics education, along with challenges which ensue in taking up a posthumanist theoretical framework in research in this area. I will use some data from a case study to help elaborate the possibilities such a framework offers, which may not be so evident through aforementioned pedagogical frameworks which are underpinned by commonly established and recognisable theories. I will also unpack the methodological dilemmas/challenges that emerge while working with pre-service teachers in this vein. I focus on the concept of circle, and share data where pre-service teachers encountered circles as socio-material events, emphasizing the dynamic and genetic matter-concept mixture that was in process during the events.

What is a mathematical concept?

To unfold this discussion, I will firstly share some recent compelling theoretical approaches to the question of 'what is a mathematical concept?' Each offers fresh insights, which range from the social construction of concepts; the role of the body in concept-development, to anthropological and ecological approaches.

According to Barwell and Abtahi (2017) language and wider discourses through "thoughts, ideas, mental representations" (p. 178) operate to define what 'concept' is within mathematics. They expose a fascinating polarised binary produced within dominant discourses within the Canadian (British Columbian) media where a dichotomy of 'back to basics' and 'discovery' (investigative) learning is produced, where 'concept' becomes objectified within the latter through mediated representations of what mathematical concepts are. Through an embodied cognition enactivist approach, Coles (2017), aims to

attend to the ecological as more than purely language-based, and carries a prime focus on mathematical concepts as relational, with particular attention to similarities, differences, and comparisons. These are not seen as determinate; they are always produced through acting on discernible relations and differences within an event or encounter rather than on a mediated representation or awareness of objects.

Coles (2017) moves away from an object-oriented approach where for example, a child might label (through the use of language) individual objects within a set, whilst developing understanding of early number concepts. For Coles (2017), a move away from such a 'counting world' with an emphasis on concepts as fixed objects towards one which attends to mathematical concepts as 'relations', enables a move to a 'measurement world' after Bass (2015, cited by Coles, 2017). Through an embodied encounter, the concept, for example the cardinality of number, can be brought forth through a relational encounter. An example Coles (2017) gives, is the use of Cuisenaire rods, with each different colour representing a different proportionate length. When two or more different coloured rods are physically placed together, this embodied act enacts a naturally occurring comparison, through a focus on the relationship. Here, number is perceived not as a collection of objects but as a "dynamic relation". Numbers are brought into existence through the action of placing rods against each other. There is some sense of production or generation, and through a relational perspective, it may also be suggestive of a sense of 'indeterminacy'. What this does is to move away from purely linguistic practices and moves to focus on mathematical practices which through the relational is folded into a complex material milieu.

Through enactivist embodiment, Coles (2017) desires to challenge the classical divide between subject-object or knower and known through the phenomenon which produces the mathematical concept or relation. However, he does acknowledge that this is difficult, as we are often inherently grounded in a binary between knower and known, something which can be seen operating within the earlier example of the mediated mathematical 'concept' as highlighted by Barwell and Abtahi (2017). In order to surmount this, Coles (2017) turns to the use of illusions, in the tradition of Maturana, a key enactivist thinker. Here, the phenomenon requires a consideration of context, as well as the relations that are borne out of it. Objects can only be perceived through their relationships, thus in this way concepts are treated as objects here. There is a suggestion of subject and object being seen as emerging out of each other therefore there may be some sense of a commingling. Specifically, for Coles (2017, p. 221) "Relations are at once material (arising from a consideration of objects) and abstract." They are not seen to exist in the objects or in the human mind. These relations, thus 'concepts' arise from the interactions of the human with the world and each other.

For Nemirovsky (2017), taking an anthropological turn highlights the inherent nature of the mathematical concept as being multi-faceted and situated in material practices. He critiques the classic Aristotelian image of 'concept' through a 'classification' model of predicates, already essentialised, and with a lack of historicity, thus knowledge of production. Furthermore, he suggests a nature/culture binary is produced through "taxonomies driven

by relations of power and enforcements" (pp. 252–253) where anything falling between essentialised boundaries is afforded lower status. Nemirovsky moves away from this static, classification model and offers instead the crystalline image of 'concept' as that which is "inspired by growth, decay and individuation" (p. 253). For Nemirovsky, this means concepts are always under formation; they are relational, always in consort with something else and of the material. In doing so, he foregrounds both the potentiality of the senses; i.e. a concept meshes all the senses of a virtual. Nemirovsky is also drawing on Deleuze here, to argue that the virtual alludes to a non-deterministic "footprinting" of a multitude of possibilities. This idea of meshing suggests something beyond a mere linkage of ideas within concept-formation. Through the work of Ingold (2015), Nemirovsky invites us to question the rigidity and determinate nature of mathematical concepts, and to address their historicity through attention to the multitude of relational ways in which they might have been formed.

De Freitas and Sinclair (2014, 2017) operate within a posthumanist and specifically 'Inclusive Materialist' framework, which draws on key Baradian ideas. They are committed to concepts as "aesthetic-political" acts and similarly to Nemirovsky, are particularly keen to avoid traditional notions of mathematical concepts as being static, essentialised and inanimate, set apart from our physical world, to be unproblematically 'discovered'. This is part of an agenda to show how concepts live in the world, and to avoid a "double-standard ontology" (de Freitas & Sinclair, 2017, p. 76). They consider how concepts are often linked within mathematics by way of logical connectives; the pervasive act of "ordering" mathematical concepts based on a 'logical' sequence within school curricula can however be challenged, once we start problematizing concepts. De Freitas and Sinclair (2014, 2017) draw on the work of Gilles Châtelet to present several propositions about mathematical concepts. In contrast to Barwell and Abtahi (2017), they problematise the idea of 'concepts' as metaphors, representations or mental constructs abstracted from the material world (where abstraction might be treated as a subtraction of nuisance or secondary qualities). For de Freitas and Sinclair, the discussion of 'abstract' concepts carries an ontological challenge, one where concepts connect with both the virtual and the actual within an immanent world without transcendent ideal mathematical forms.

Within the framework of Inclusive Materialism, the active and yet undetermined 'virtual' is imbued within matter, as multiplicities of what de Freitas and Sinclair (2017) call 'quivering' or indeterminate potentialities, any one of which can result in an instantiation through the actualisation that occurs from the latency within the matter. This latency is not an essentialised mathematical form within or of the matter, but rather the mathematical concept is always becoming through "the mobility, vibration, potentiality and indeterminacy" (p. 78).

My interest is in how these approaches are helpful for working closely with mathematical concepts and adult learners training to become elementary teachers; each draws attention to material-mathematical practices and to indeterminacy. I am working with a theoretical framework that is kindred with those that have been discussed above and case study data will now be explored through this milieu.

Concepts as apparatus

I work with the ideas of philosopher of science Karen Barad on 'Agential Realism' (2007) where concepts arise from specific material arrangements. Barad invokes the reader to engage with an epistemological-ontological-ethical framework (Barad, 2007, p. 26), where the ethical and "response-ability" is considered within all aspects of the research process. This results in an inherent demand for a fundamental re-working of the traditional framing of ontology and epistemology, where the ethical is brought into play and the material is afforded agency. Along with methodology, each are equally imbricated within an 'ethically' oriented entanglement. For Barad (2003), these entanglements are material arrangements or apparatus; I work within this framework to develop the idea of the 'mathematical body'. This is created through an assemblage which specifies a distinct material arrangement, one which is relational, contingent on the actors in the assemblage, within the becoming of a new mixture of concept and matter. By actors, this means a diverse set of agents, human and otherwise - the human, the compass, the straight edge, as well as the concept itself as a configuration of forces. This aligns with Barad's theoretical stance that 'agency' for all actors, human and non-human, is produced through the becoming of the 'concept'; therefore, unlike for Coles (2017), where the human 'interacts' and acts upon the material, agency for Barad (2007) is not pre-invested in any of the actors prior to the assemblage in its becoming.

My approach aligns with those previously discussed, specifically in an attempt to think more broadly about concept formation, and one where the human is to be decentred. As Coles (2017) notes, philosophically, and methodologically, this can be difficult as it creates a deeply ontological challenge. The following discussion of data attempts to explore some of these complexities.

Case study data: The circle de/composed

From a methodological perspective, as a researcher whose research contexts have always consisted of classrooms, which are fully involved with doings and not just sayings, it seemed natural to make the move from the comfort of socio-cultural paradigms where much research relies on standard observation protocols, or the common employment of interviews and questionnaires as research tools. As well as from the demands of allegiance to a qualitative and often interpretative inquiry paradigm, where that which is the "social" and of 'knowing' through language is taken for granted. Those researching beyond qualitative inquiry paradigms, such as de Freitas and Sinclair (2014) and Barad (2003, 2007) offer a particular challenge to the in-built Cartesian dualism which gives precedence to mind over body within much education research. They argue such inquiry can only ever be 'representationalist'. Posthumanist philosophers challenge the lack of attention to the materiality of the human body itself as well as that of 'more-than-human' actors. (Barad, 2007; Coole & Frost, 2010).

The research project discussed here aimed to encourage pre-service trainee teachers to step out of the known and bring forth an 'inventive' and 'speculative' dimension to their thinking, perhaps to access an indeterminate potentiality through the virtual (de Freitas &

Sinclair, 2014; Nemirovsky, 2017), and to crack open the very notion of circle and other concepts. In other words, concepts broke from the 'known properties' attached to the shape, as static onto-assumptions about their essential quality, and became unfolding events, which were dynamically involved in the becoming of concepts. Together, we tried to attend to that which was eclipsed by language-use in learning, which might have otherwise hindered our capacity to leave behind deeply entrenched existing ontological and epistemological assumptions. Barad's requirement to work within an epistemological-ontological-ethical framework demands all elements to be ethically considered as an imbricated whole; this results in a fundamental shift in thinking, where methodology is not a separate and distinct feature of the research process, of already fixed and pre-determined means guided by a research paradigm with 'already decided' apparatus which perceives already existent concept(s). That would typically suggest engagement in empirical acts where the concept is supposedly brought forth, to be discovered/recognized by the learner in this tried and tested constructivist way.

However, to work within Barad's theoretical framework, it is necessary to think with the onto-epistemology; where no boundaries exist between the theory and the methodology for Barad, and her theory of 'agential realism' encounters concepts in experimental apparatus and design through a realisation of this framework. Hers is an attempt to better engage with the force of the conceptual on the plane of the material, and vice versa. Barad's attention, as a philosopher of science, shifts from purely discursive practices to apparatuses as material-discursive practices, and attention to the importance of experiments where concept-matter mixtures mutate over time. For Barad, Physics philosopher Bohr designs experiments of consequential significance, whereby concepts and apparatus commingle (de Freitas, 2017). These are also experiments through which the very distinction between the social and scientific, culture and nature is constituted. Therefore, method and theory cannot consist only of general epistemological schemes underpinned by beliefs grounded in 'mediation' between language and object but must grapple with the aberrant dynamic movement of thought-matter, incorporating the singular complexities inherent in this kind of eco-sophical development (de Freitas & Sinclair, 2014).

In taking up the call of Barad (2007) to examine complex activities for how they reach outside of language, and to take up her theoretical approach of agential realism, I sought to engage in 'teaching design experiments' as a methodological approach. The first and unfortunately misguided staging post on this journey, was to fold in the body to recognise its capacity for cognisance as this offered some scope to further consider the empirical acts which might create access to spaces where students might leave behind old and worn pre-conceived ideas about their understandings of the nature of mathematical concepts as fixed objects mediated through metaphors or language (Coles, 2017; de Freitas & Sinclair, 2017). For research purposes, I hoped this would open up more-than-human and more-than-discursive ways of doing mathematics; to harness the potential of the materials and their engagement with the physical and the material to surface relations, as promoted by Coles (2017). A series of activities were designed, which asked the students to engage in circle-

formations using a variety of media and importantly, their bodies. The activities were directed, instructional and though there was some scope for varying outputs, the focus was on the decomposition of circles. I had a realisation that this methodological approach did not align with the call by Barad (2007) to 'speculate', 'generate', 'come into being'. The specific material arrangement or 'apparatus' ultimately centered the human learner, by necessity as having the agency to act upon inert matter in the manner of Coles (2017); but perhaps more importantly, these tasks were all decomposition tasks, so the circle was given in materiality, as much as it was given conceptually, and I realized I was still centering the familiar, known, and learned – the experiment was focused on decomposition rather than composition.

This was uncharted territory for me as researcher and although both de/composition tasks are helpful, I note that my Baradian tendency had me convinced that if I was to move into the realm of generativity and to enable the 'becoming' of circle, it was necessary to engage in a different way. That is, to listen to Barad's call to move away from a relativist approach which inherently separates observer and observed, with the observer (invariably human) having privileged agency, thus challenge my still-entrenched human-centric view of 'concept'. Viewing the 'mathematical concept' as an agentive actor produced through an intra-action required me to move this concept into a space of speculation and indeterminacy where the specific material arrangement or 'apparatus' required careful consideration. For Barad, 'apparatus' here is not to be seen as that which is independent of the observer (subject) and the observed (object); an independent measuring tool bridging a perceived 'gap' and mediating between the two. Here, the epistemological-ontological-ethical needed to be brought into play as all actors (human and non-human) are constituted through the 'apparatus' therefore this embroiled nature necessitated observer and observed becoming an integral part of the specific material arrangement or apparatus. It was not possible to think about any one of these aspects in isolation; this had clear methodological implications and implications for the production of and analysis of data within the research inquiry undertaken within this theoretical framework.

I needed to perhaps connect with Nemirovsky's proposition (2017) to consider an approach akin to that of Ingold's 'wayfaring', of one where concepts are made over time with no particular or determinate intent; they 'become' (2015). Or perhaps to think of the event of circle creation as one where virtual 'singularities' destabilize the making of circles in generative ways, so that the truly novel emerges. It was necessary for me to create teaching design experiments where mobility, creation and the 'virtual' had opportunities to come into being. The 'virtual' as a multiplicity of possibilities, in the vein of Chatelet's promulgation (de Freitas & Sinclair, 2017), required a loosening of tutor direction and an opening up of playful and dynamic activity which harnessed the use of tools, environment as well as the unfinished human body, for production and generativity through the relational (Barad, 2003; de Freitas & Sinclair, 2014; Coles, 2017). A much less prescribed atmosphere where the students were asked no more than to produce what they conceived of as the concept of 'circle', with a wide variety of media freely available, yielded some interesting results.

Cracking open deeply established associations of exploration of circle as related to its (fixed and determinate) properties provided some striking results.

One such result involves Mimi. Mathematical objects operate according to logical constraints and ontological possibilities. In considering the circle, De Freitas and Sinclair (2017) focus on both an actualization of the virtual and a realization of the possible, which together yield 'new-ness'. The latter corresponds with a definition of 'circle' which satisfies the constraints of the locus of all points being equi-distant from its centre. This highlights how the requirement to abide by the onto/logical necessity of this definition demands compliance with a set of points which are determined by the constraints of the static nature of this definition. However, there is also another kind of determination, whose definition differs in that it is a dynamic one, and of the material rather than the logical. One which involves an actualizing of the virtual dimensions of the matter through the event of producing a continuous curve, one which involves action, mobility and thus generates. Mimi surprisingly engages with both these static and dynamic definitions. Firstly, she realizes the possible of 'circle' by conforming to the set of points determined by the constraints of being in relation to a fixed centre, through the creation of a circle-like figure by cutting out of folded paper and further continues in this vein by using the circumference of this paper circle-like figure to form a further string circle which is glued just inside the outer circumference. However, she then proceeds to create a circle which does not follow this onto/logical definition. She uses a length of string to work her way out from a central inside. In connecting with the growing 'area' of her circle, there is a material physicality and mobility imbricated fully within the string, paper and glue she works with or which perhaps works her; where the mathematical concept is 'becoming' through "...the mobility, vibration, potentiality and indeterminacy..." (de Freitas & Sinclair, 2017, p. 78) within the materiality of the forming circle. Mimi and her circle are produced through Barad's (2007, p. 147) 'field of possibilities' where statements and subjects emerge. Barad describes this field as being dynamic and full of contingent multiplicity. When asked how she produced the inner-circle, Mimi shrugs and ventures:

> Mimi: I'm not really sure. I just started here (points to first point of string she has glued down in the centre) then just – I don't know, just went like this! (Traces a circular winding action with her right index finger in the air).

She does not say it but implies that the string led her on an ontological journey of 'new-ness' which even she did not know the map for – the concept of circle was new, not simply recollected or recognized. The point is not simply that she learned a new way of thinking about circles. My approach affirms that fact, but my turn to posthumanist theories opens up this event so that the very ontology of the concept is at stake. In other words, philosophically examining this activity through the lens of posthumanisn, allows us to radicalize the way that mathematical concepts become concepts. In disrupting notions of agency and how this comes into being and for which actors within a specific material arrangement or apparatus (Barad, 2007), we are able to break up and destabilize institutional mathematics, allowing us

to critique curriculum. My posthumanist philosophical analysis puts Mimi into a much broader milieu of matter-concept mixtures, actually empowering her by looking more broadly. This journey relates to the 'inherent mobility within the circle' to which de Freitas and Sinclair (2017, p. 81) point. The circle is very much as they say 'materially produced'. Mimi's (inner) *circling event* eventually does result in the production of the form of circle; however, during production this is full of indeterminacy; the material is still within the 'virtual', quivering with a multitude of potentialities with all the possibilities this may bring for Baradian ontic-semantic boundaries within this specific material-discursive arrangement or apparatus.

Conclusion

Research focusing on the teaching of elementary level mathematics with student teachers is interesting and complex. These students come at once as enthusiastic apprentices (to the endeavour of teaching mathematics) and yet often strangely alienated from mathematics itself. Undertaking research focused on conceptual mathematics is a risky endeavour, given the affect and fear and the strong desires at work. For these reasons, I have sought alternative theoretical approaches and new kinds of experimental practices that would allow me to take up the many complex processes by which mathematical concepts become mathematical concepts.

If we wish for student (pre-service) teachers to engage with an ontological challenge such as this, we need to look to research to stimulate a professional conversation, where a more philosophical slant is brought into our teacher education programs. Conversations are needed which might open up and challenge deeply entrenched notions of the nature of mathematics as a discipline as well as the nature of mathematics as a curriculum subject, in order to challenge the essentialised taxonomies highlighted by Nemirovsky (2017). This may involve recognising that this is uncharted and at times difficult territory, where regulatory bodies with deeply-entrenched ontologies may need to be navigated. Furthermore, at a time when the current pandemic has at times disrupted the dominant grand narratives of socio-cultural theories by bringing into sharp focus and problematizing our dependence on language-focused pedagogies in online learning, there may be an appetite to return to questions about the materiality of mathematics pedagogy, and reconsider the ontological nature of mathematical concepts. Barad (2007) offers something distinctively different and appealing. However, hers is a highly theoretical framework and I continue to work on the personal challenge this presents to connect with research such as mine.

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Framing mathematics-related policy problems: Discourses linking school and district leaders to state policy

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The discourses reflected in policy structure how education is talked about, understood, and enacted in schools. This paper employs critical discourse analysis to explore how dominant discourses perpetuated by state policy penetrate school and district leaders' framing of mathematics-related policy problems. The analysis revealed that leaders employed neoliberal discourses, discourses of inclusion and diversity, and deficit discourses in ways that may inadvertently maintain educational inequality.

How a policy problem is *framed* is important because it assigns responsibility and legitimizes some solutions and not others (Benford & Snow, 2000). And, since leaders are most successful in influencing framing activity and the direction of work (Coburn, 2006; Penuel et al., 2013), this paper investigates how leaders frame problems in their schools and districts. To date, research on problem framing in education has focused on how frames are negotiated within a particular school or district. Attention to context has often been limited to school conditions, ignoring the influence of the broader sociopolitical environment. This is important, as research suggests national and state policy and parents and community members influence leaders' meaning making and policy responses (e.g., Turner, 2015). That is, leaders draw on broader discourses about education, including those from policy, in framing problems.

However, there is scarce research investigating these discourses—the language, practices, and identities reflected in talk about policy problems—especially with respect to mathematics education. Attending specifically to mathematics is important, as mathematics receives considerably more attention by policymakers (Spillane & Hopkins, 2013). As one of the two most tested subjects (U.S. Department of Education, 2015), mathematics is especially vulnerable to neoliberal discourses of standardization and accountability (Martin, 2013). Further, mathematics reflects and perpetuates broader racialized narratives about who is (and is not) good at mathematics (Martin, 2009), and by drawing on these discourses, leaders contribute to—or undermine—children's access to learning opportunities.

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Consistent with critical discourse studies, we argue that research on the discourses reflected in leaders' problem framings provide insights into the ways schools legitimatize and reproduce inequity (Fairclough, 2015). To this end, we investigate how leaders in one U.S. school district frame mathematics-related policy problems. Specifically, we ask: What discourses are reflected in leaders' framing of mathematics-related problems? How do these discourses manifest or differ from those in state policy? How are these discourses employed in leaders' framing, and what purposes do they serve?

Conceptual framings

Frames are discursive devices actors invoke to strategically shape others' sensemaking and to mobilize them to take action (Benford & Snow, 2000). Frames identify conditions as problematic, assign responsibility, and privilege certain policy solutions (Stone, 2012). Because actors intentionally invoke frames that connect to the interests, values, and beliefs of those they hope to mobilize (Snow et al., 1986), school and district leaders hoping to define a certain condition as problematic or advance a particular solution might—with varying degrees of intentionality—employ dominant discourses. Therefore, this paper seeks to investigate the discourses leaders employ in their problem framing, and the purposes these discourses serve in identifying problems, casting blame, and legitimizing solutions.

Critical discourse analysis (CDA) argues that discourses are ideologically shaped by power relations (Fairclough, 2015). For school and district leaders then, their problem framing activity operates within broader sociopolitical contexts characterized by discourses competing for power. As such, leaders construct and negotiate frames that operate within and between discourses and the ideologies they embody, or at least those that are most powerful. Frames also take up and exert power through discourses. That is, frames that align or resonate with dominant ideologies are more likely to be taken up, serving to reproduce structures that sustain dominant interests. By bringing together CDA and framing, this study seeks to expose the dominant discourses employed in leaders' problem framing and how they empower or establish limits on practice.

Discourses in education

Under the guise of free-market capitalism, policies and reforms of the last several decades have increasingly focused on meeting the needs of the economy (Apple, 2006). Mathematics education in particular has served a neoliberal agenda through, for example, its commodification of learners and mathematics knowledge in service of military and national security (Martin, 2013). These discourses of economic productivity and efficiency have brought standardization and high-stakes accountability testing to the forefront (Au, 2016). Though such policies have been framed as supporting *all* students (Martin, 2003), this focus on accountability has been especially harmful for historically marginalized students (Davis & Martin, 2008).

Language like "all children" and "no child left behind" has been particularly powerful in uniting policymakers and educators around market-based school reforms (Lenhoff & Ulmer, 2016). While simplistically ideal, discourses of inclusion promote colorblindness (Martin, 2013) and marginalize the needs of historically underserved students (Martin, 2003). Inclusion has been linked to calls for diversity, which, more recently, has paired with neoliberal projects, including those that commodify non-white[1] racial identities, to focus exclusively on the positive learning outcomes for *all* students (Goldstein Hode & Meisenback, 2016). By connecting increased access to education for marginalized groups to goals that serve white interests, the neoliberal case for diversity promotes interest convergence (Bell, 1980) and detracts from more race-conscious practices that work to address past discrimination (Rhoads et al., 2005).

Implicit in accountability policies that fail to address structural racism and economic inequalities yet expect equality of outcomes (Darling-Hammond, 2007) are deficit discourses that construct students of color and low-income students and their families as the causes of academic failure (Valencia, 2010). As an alternative to a student deficit discourse, leaders may also draw upon a *teacher* deficit discourse, emphasizing the shortcomings of teachers and their instruction (Turner, 2015). However, like student deficit discourses, these reinforce problematic structures and deflect attention away from the district's potential role in the reproduction of systemic racism and inequality.

Context and methods

Brookside School District (BSD, a pseudonym) is a rural school district in the U.S. state of Missouri. It serves about 1500 students, approximately 80% of whom are white and 80% identified as free or reduced lunch (an indicator of socioeconomic status)[2]. In recent years, BSD has performed below the state average on standardized tests, with significant disparities between white and Black students. To address some of these concerns, in 2018, the district convened a Diversity Task Force (DTF), which was headed by the district superintendent and included community members. Since then, the state initiated a "Targeted Support and Improvement School Plan" for the middle school (grades 6–8) for the 2019–2022 school years, due to low achievement scores in mathematics and English language arts among Black students and students with individualized education plans. Missouri began identifying "Target" schools in 2018 as a response to the passage of the Every Student Succeeds Act (ESSA) in 2015.

Employing CDA, we examined the discourses reflected in leaders' problem framing and in state policy. For the latter, we analyzed the Missouri Consolidated State Plan (2019) for meeting ESSA, paying particular attention to the policies for Targeted Support and Improvement Schools. We analyzed for leaders' framings as emerging from three semi-

[1] We follow Gotanda (1991) in capitalizing Black and not capitalizing white.

[2] To protect confidentiality, we do not include additional demographic data.

structured interviews, conducted in 2020, with the middle school principal (Patty), the district superintendent (Stacy), and the district chief academic officer (Celia). These semi-structured interviews were approximately one hour long and covered a range of topics including their responsibilities, their perspectives on perceived challenges related to mathematics and equity, and the district's current equity initiatives.

Analysis began with reading through the interview transcripts to identify sections where leaders discussed math-related policy problems, including when they articulated the causes and solutions for such problems. These excerpts were then subjected to finer analysis where we coded for different discourses. We specifically attended to neoliberal discourses and discourses of inclusion and diversity because BSD leaders were facing pressures from the state to improve test scores, not just for historically marginalized students, but for all students. Initial analysis also revealed the prevalence of deficit discourses in leaders' talk. While we specifically attended to the aforementioned discourses, we also looked for others that might not fit these categories.

Then, we closely analyzed the discourses employed in leaders' framing, particularly attending to what Fairclough (2011) describes as *discourses* (ways of representing) and *style* (ways of being). Specifically, we examined the discourse or ways of representing by asking: What problems were presented? How were problems defined? Why are the problems significant? How were problems addressed? How was mathematics defined? We also examined style or ways of being: How did problems position students, families, and teachers? Who or what was the cause of the problem? Who were solutions for? How were students and their academic and mathematical abilities described? After, we looked for patterns across the different ways that discourses were employed in leaders' framing and the purposes they served. These emerged from the data as we compared and contrasted the ways these discourses were used to justify and legitimize problem frames, particularly those that served to maintain dominant ideologies and power relations. We took a similar approach in analyzing the state policy, particularly looking for continuities and contradictions between the policy and interviews.

Findings

Both Missouri policy and BSD leaders employed neoliberal discourses to identify problems centered on standardized test scores. Though Missouri policy drew upon structural inequity discourses to implicate a lack of access as the cause, this was undermined by deficit discourses. BSD leaders also employed deficit discourses, casting blame on students and their families, as well as teachers and their instruction. Both, however, ignored and deflected blame away from potentially problematic structures. Intertwined were also discourses of inclusion and diversity, which delegitimized solutions specifically for Black students. In the following sub-sections, we explain the various purposes these discourses served in leaders' problem framing.

Discourses identify and define problems

Missouri policy to ESSA employed neoliberal discourses to identify achievement-focused problems. In particular, Missouri's "first strategic goal is for all students to graduate from high school ready for college, career, and life," including "reducing by half the rate at which students fail to achieve proficiency" on state standardized tests (p. 17). Similar discourses were reflected in all BSD leaders' identification of achievement as an important problem. Stacy (the superintendent), for example, said that "academic achievement, of course, is always a focus and even outside of just state assessment data." Here, "of course" and "always" suggest that this problem is assumed and persistent, while "even outside" indicates that standardized test scores are accepted as a routine and legitimate measure of learning. Such language reflects the powerful hold neoliberal discourses have on the district and what Stacy considers problematic.

In addition to "improving outcomes for all students" (p. 2), Missouri tracks different subgroups to "close educational achievement gaps" (p. 10). These "subgroups" refer to different student populations, including, for example, white, Black, and Hispanic students, and not necessarily student groups that the state has identified as being historically underserved. By focusing on "all" students, Missouri policy draws on discourses of inclusion that reinforce a colorblind perspective and marginalize the needs of historically underserved students. Simultaneously, by focusing on achievement gaps, Missouri policy perpetuates the narrative that mathematics is a discipline in which those successful are white, middle-class (Martin et al., 2017).

Such problems and discourses were also reflected in BSD leaders' framing. For example, Patty (the middle school principal) explained that "our focus right now is, of course, to narrow the gap in skill achievement for our math students, but students overall, not just math but ELA as well, to help get us out of that Target status, but to get our kids where they need to be to be successful." Here, Patty suggested that the "gap" is important to address because of the state's labeling of the school as Target, and framed the problem centrally around students of color and their "achievement" on tests. However, reliance on test scores narrowly defines what counts as mathematics and, as Adiredja and Louie (2020) have argued, fails to acknowledge students' out-of-school knowledge. And by framing the achievement of students of color against that of white students, Patty participated in "gap-gazing" (Gutiérrez, 2008) that perpetuates the narrative of Black students as underperforming (Martin et al., 2017).

Discourses assign (and deflect) blame

Discourses not only identify and define problems, they also assign blame. Employing structural inequity discourses, which focus on the role of inequitable social structures in student achievement (Bertrand et al., 2015), Missouri policy attributed the cause of poor achievement to a lack of access. For example, to support the goal that all Missouri students graduate college and career ready is "access, opportunity, equity," which ensures that "best practices and quality programs must be available to every child from preschool through postsecondary" (p. 6). Thus, Missouri policy implies that the cause of poor achievement is

that not "every" student is afforded access to quality schools and opportunities. Little attention, however, is given to the structural and institutional forces that create and maintain such inequities in access.

Further, the structural inequity discourses employed in Missouri policy were weakened alongside the deficit discourses also present. The Target Schools policy defines schools as not meeting quality indicators by identifying those with "consistently underperforming subgroups" (p. 30). Rather than, for example, positioning students as not being afforded access to quality educational opportunities, students were instead described as "chronically low-performing" (p. 29). By invoking a disease metaphor (Stone, 2012), Missouri policy naturalizes students as academically unsuccessful and centers students' perceived internal deficits as the cause for poor achievement.

BSD leaders also ignored problematic structures when drawing on deficit discourses that cast blame on students and their families. Patty, for example, explained that Black students' test scores have been low because "I think some of that is just, not just, but is socioeconomic-based in our community. We've lost so much industry here. Our SES is much lower than it used to be." More specifically, she explained that:

> Because what I see with our Black students in particular is that they, if the students that come out of pretty strong foundational homes, whatever that structure looks like, they've got a good home base tend to see the value in math, or they have an intrinsic interest in math, and those are the ones that are successful.

By implying that the problem could be connected to an economic depression in the broader community, Patty framed the problem around issues of class instead of race. She, however, did not give a justification for why poverty might more negatively affect Black students than white students (nor did any other leader who framed the problem in a similar way). Patty then went on to dismiss these structural arguments by suggesting that Black students are not successful in mathematics because they either are not intrinsically interested or do not come from a "strong foundational" home. By drawing on deficit discourses, Patty not only attributed the cause of the problem to Black students and their families, but also obscured institutional racism (Martin, 2013) and other structural issues, such as a lack of jobs in the community.

In addition to placing blame on students and their families, leaders also attributed responsibility to teachers. Celia (the chief academic officer), for example, explained that Black students' test scores are low because:

> Teachers have really focused on more of a traditional approach and not really trying to see and meet the needs of every, you know. This way doesn't always work for kids and how can we kind of change our teaching or instruction to kind of meet those needs…Our elementary teachers are not really comfortable with math.

Celia explained that teachers' "traditional" instruction and lack of mathematics content knowledge do not meet students' needs. By focusing on instructional practices, she, on the one hand, places responsibility for the problem within the institution (i.e., on teachers' instruction). On the other hand, by only focusing on teachers and teaching, Celia ignored

the possibility of her own and others' complicity and deflected responsibility for the problem by obscuring what role the district might be playing in perpetuating racial inequality (Turner, 2015). Intertwined in the teacher-deficit discourses are also discourses of inclusion. By attending to teaching practices that "meet the needs of every" student, Celia ignored why teachers' current instruction more negatively affects Black students than white students and precludes any solutions that would specifically meet the needs of Black students.

Discourses (de)legitimize solutions

Finally, discourses legitimize some solutions and not others. To ensure that "all children have equitable access to opportunities that prepare them for success in school and in life" (Missouri Department of Elementary & Secondary Education, 2019, p. 6), Missouri policy offered accountability-focused solutions. In particular, the state administers reports with achievement data, and "these reports can and do drive improvement for all students, helping to close educational achievement gaps" (p. 10). That is, the Target Schools policy identifies schools with "chronically low-performing subgroups" (p. 29) and compels them to improve, through carrots-and-sticks accountability, so that "all" students achieve. While such policies might be "successful" in incentivizing schools to focus on outcomes, which indeed are the problems BSD leaders identified, Missouri gives little guidance on how schools can actually improve achievement, particularly for their historically marginalized students.

BSD leaders took up this charge by convening a Diversity Task Force (DTF) with the local community, whose main initiative has been a mentoring program. Originally, the mentoring program was designed specifically for Black students, but this resulted in community backlash from (mostly) white parents. Celia explained that "I think just in our community. I think we have to kind of open it up to everybody […] We tried to kind of target that and bringing in some speakers and things and it didn't seem to go over as well." In response, BSD leaders and the DTF decided to "open it up" by using chronic attendance as a qualifying criterion. Stacy explained that:

> We're going to use chronic attendance as our qualifying characteristic which we just decided, kind of really solidifying that because we don't want to be so exclusive that we're not including other students just because they're not Black or just because they're not fitting what we say, you know, this one area. And so we know that chronic attendance is related to academics, is related to behavior, is related to parent engagement. And so all of the students that we feel that we could reach and touch fit that mold and the likelihood that we're going to reach students that are a minority population is significant we feel.

Here, Stacy explained that the DTF's mentoring program originally targeted Black students, one of the subgroups the state alerted them to, but in response to community backlash, retreated to discourses of inclusion, employing some of the very same language Missouri policy provided about attendance. In another part of the interview, Stacy explained that chronic attendance (absenteeism) refers to a state indicator for students that miss more than 10% of school days. Indeed, the 90/90 attendance principle—"90 percent of the students must be in attendance 90 percent of the time" (p. 7)—is one of the performance indicators for

the School Improvement Program. This suggests that Stacy, and the district, used state policy to target most of their Black students without being "exclusive."

Though this problem framing is well-intentioned, by employing discourses of inclusion, including those from Missouri policy, solutions originally focused on Black students' learning were translated into that of all students. And, by connecting achievement to attendance, and attendance to student behavior and parent involvement, Stacy suggested that Black students are not successful in mathematics because they are not well behaved and have disengaged parents. This framing, reflecting both deficit discourses and discourses of inclusion, materialized into a mentoring event where a "diverse" set of speakers—a white, female alumna and female high school student of color—presented "to any middle school student" about work ethic.

Discussion

Missouri policy employed neoliberal discourses to identify achievement problems and accountability solutions to address a lack of access. Such structural inequity discourses were, however, undermined by deficit discourses that positioned students as underperforming and the cause of poor achievement. Simultaneously, discourses of inclusion and diversity centered the importance of "all" students achieving, which marginalized the needs of those historically underserved. As seen in this analysis, these broader discourses make their way into leaders' problem framing activity, with varying levels of intentionality. On the one hand, leaders seemed to be fairly aware of the hold neoliberal discourses had on what they considered problematic. Such discourses, however, were difficult to interrupt, especially as there were few alternative discourses available to identify different problems, including, for example, those related to equity beyond disparities in achievement. On the other hand, leaders seemed to be unintentionally employing deficit discourses, yet these still creeped in. Meanwhile, discourses of inclusion were, at times, strategically invoked, especially because they held currency in the community, as was the case of using chronic absenteeism to target Black students while avoiding being "exclusive."

CDA explains that because discourses are linked to the distribution of resources (Gee, 2014), those reflected in leaders' problem framing have the potential to transform or maintain power relations. More common in this analysis were discourses that appeared to uphold, or at least not challenge, educational inequities. For example, while Missouri policy did implicate inequitable access, it did not explicitly discuss the role of structural and institutional forces in maintaining said inequities. Without such discourses available to be taken up, BSD leaders' problem framing ignored solutions that would address any problematic structures. Related, discourses of inclusion and diversity legitimized an "inclusive" mentoring program, which manifested in social practice as a case of interest convergence (Bell, 1980) where Black students were only afforded social goods (mentoring) if white students were as well. Despite wanting to improve mathematics instruction for Black students, discourses creeped into BSD leaders' problem framing in ways that maintained dominant interests.

An important implication here is that the discourses reflected in state and national policies can influence how local leaders frame policy problems. Such discourses limit or expand how problems are defined and what solutions are possible, serving to maintain or challenge inequality. Critical awareness of the discourses available and powerful in the broader sociopolitical environment, and the implications they have for policy solutions and how students are positioned, may allow leaders to more intentionally address educational inequalities and improve mathematics teaching for students, especially those historically marginalized. However, because many of these discourses are perpetuated by broader state and federal policies, advancing equity in mathematics education must necessarily involve change at an institutional level.

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Students, agency and mathematical subjectivity

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This paper reports on a study of 18 Swedish year 1 students' (7–8 years) work with mathematics textbooks analysed according to the concept of agency. The empirical data consisted of video material, students' representations and mathematics textbooks. The result showed that some exercises enable agency, and some do not. Also, students' opportunities for agency are affected by the notion that, according to the students, mathematical symbols are the resource that should be used to be considered successful in mathematics. A conclusion from this is that the textbook needs to be used consciously, offering different learning situations based on both opportunities for agency and multimodal aspects to provide all students learning situations that benefit both learning and the opportunity to discover themselves as mathematical individuals.

Introduction

In mathematics education, the textbook is a widespread learning resource (Mullis, Martin, Foy, & Arora, 2012). According to TIMSS (Trends in International Math-ematics and Science Study) and other comparative international studies, the use of mathematics textbooks is higher in the Baltic and Nordic countries than in other parts of the world (Grevholm, 2017). Regardless of how much the teacher plans and stages the teaching, the students' mathematics teaching often involves individual work with mathematics textbooks. Students are thus assumed to be able to work individually with their mathematics textbook.

This study built on multimodal social semiotics (Kress, 2010), where communication in different resources or *modes* (Kress, 2010) such as images, mathematical symbols and writing is studied. From this perspective, the student's work with the mathematics textbook is understood as the student's *meaning making* (e.g., Kress, 2010) working with the mathematics textbook. First this study sheds light on what the textbooks are designed to offer students and, second, the students' meaning-making working with the mathematics textbook. The purpose of what the textbook is designed to offer is understood as the learning resource's meaning potential, a relational concept (van Leeuwen, 2005).

To deepen the understanding of students' meaning making the concept of *agency* was used. Agency refers to an individual's possibility to take active participation (Bezemer & Kress, 2016). In this study, the concept is used based on the student's opportunity to participate when working with mathematics textbooks. Students' opportunities to take agency is then discussed concerning the concept of mathematical subjectivity (Palmer, 2009), referring to that an individual *becomes* mathematical in different situations rather than *is* mathematical, and that the concept concerns "how to understand oneself and to be

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understood by others in relation to mathematics" (my translation, Palmer, 2009, p. 17). This means that mathematical subjectivity is connected to meaning making and actions.

Thus, the aim of the study is to discuss how students' meaning making when working with mathematics textbooks can be understood from the concept of agency. The analysis has been guided by the following research questions: Which meaning potentials are designed into the exercises? How do the students make meaning when working with the textbooks? What opportunities for students' agency is possible when working with the mathematics textbook?

Literature review

Research on agency linked to students' mathematics learning is based on different theoretical perspectives. Louie (2019) describes learning as socially constructed by studying mathematics teaching of five American elementary school teachers through discourse analysis. She used the term *agency discourse* and highlighted agency to achieve equality. Louie concluded that agency can be the path to all students' right to discover a rich mathematics education but must be adapted to different student groups, focusing on vulnerable groups. Louie emphasized the importance of supporting teachers in this development work without blaming the teaching staff. Discourses are also concentrated on by Norén (2011). She starts from Foucault's power perspective and studied linguistic discourses for multilingual students in year eight and national mathematics tests. One conclusion that was drawn was that an essential factor in solving text problems consisted of the possibility of being able to switch between different linguistic discourses and students' ability to take agency in the test situation.

The agency concept is used from cognitive as well as social perspectives on learning. In a method development study based on social semiotics and Halliday's functional linguistics, Morgan (2016) used the term *human agency* in a discourse analysis of how mathematics teaching enables agency linked to mathematics learning. Alshwaikh and Morgan (2013) used the term *learner agency* in a multimodal discourse analysis of Palestinian mathematics textbooks, focusing on the type of activity the student is expected to engage in and the choices available. Björklund Boistrup (2010) used agency from a design-oriented perspective, similar to this study. She studied assessment documents in mathematics teaching communication through a case study in year 4 (7–8 years), where the opportunities for students' agency and learning were in focus. The results showed that the students' agency increased if the teachers showed interest in what the students showed knowledge about, instead of assuming an assessing role where the students' performance was valued in praise or dissatisfaction. She also concluded that restrictions on which modes are offered to students might limit students' opportunities for agency.

Conceptual framework: Meaning potential and agency

The concepts used in this study builds on a social semiotic approach referring to the individual's meaning making in a social and cultural context where social processes create conditions for learning and the individual make meaning of information (Kress, 2010).

A meaning potential (van Leeuwen, 2005) is understood as an offer or as opportunities and limitations. The concept of meaning potential originates from the concept of affordance developed in social semiotics to a relational discovery, which implies that a meaning potential only exists when individuals and resources meet. Although there may be more given meaning potentials linked to an individual's encounter with a resource, different individuals can also focus on different meaning potentials (van Leeuwen, 2005). The mathematics textbook, is designed to offer a specific meaning potential, the textbook author's purpose with the exercise, or the *designed meaning*. In this study, a comparison between (1) the mathematics textbook's designed meanings and (2) the student's meaning making in her/his work was made. In the student's encounter with this potential meaning, space for agency can arise.

The concept of agency is understood as the individual's active participation and the ability to act independently (Bezemer & Kress, 2016). The individual's temporary involvement in different environments, for example, the student's work with mathematics textbook, and the individual's response to this can be understood as the individual's capacity to make choices and be linked to activity and passivity (Björklund Boistrup, 2010). Kress (2010) writes that meaning involves a creator, which connects to active participation or agency. In this study, the concept of agency is directed towards the student's opportunity for agency in her/his individual work with the mathematics textbook.

Methodology

Video transcripts from 18 Year 1 (aged 7–8) students were collected chosen out of a convenience sample. In order to getting to know the students one week was spent in the class before the data collection. Here, my 12 years of experience as a compulsory teacher, came into use. I sat with one child at a time in a room next to the classroom which made it possible to focus on the students' meaning making in detail. This approach gave me the opportunity to ask the child questions while working with the textbook.

The video material consists of 450 minutes of film, approximately 25 minutes per child, with a range from 19 to 44 minutes. A tablet was used for documentation., Tablets were used in the students' everyday teaching and therefore did not mean any special focus from the students. The tablet was placed obliquely above me and the child. This allowed both the child, the textbook, and me to be seen in the video. The child started working on the exercise on her/his own. After some time, I asked questions of an investigative nature, such as "Can you tell me how you went along on this side?", "How did you know how to work with this exercise?", or "I saw that you did something with this image here, can you show me?" If the child had difficulties getting started with the exercise, I provided support in the form of questions such as "Can you use the images to solve the exercise?".

The choice of textbook series was guided by the textbooks used in the class, which is well known in Sweden. It consists of two tracks: Favoritmatematik (Favourite Mathematics) 1A and 1B and Mera favoritmatematik (More Favourite Mathematics) 1A (Ristola, Tapaninaho, & Tirronen, 2012a, 2012b; Haapaniemi, Mörsky, Tikkanen, Vehmas & Voima, 2013). Mera

favoritmatematik is considered a more challenging textbook. The exercises were chosen based on the results of a quantitative study (Norberg, 2021). They should address subtraction as an arithmetic operation, the design of the exercises should be commonly used, show breadth according to how the different modes were used, and the mathematical content should not be new for the students. The exercises (see Figures 1–7) were colour-copied and handed out to the child, one at a time.

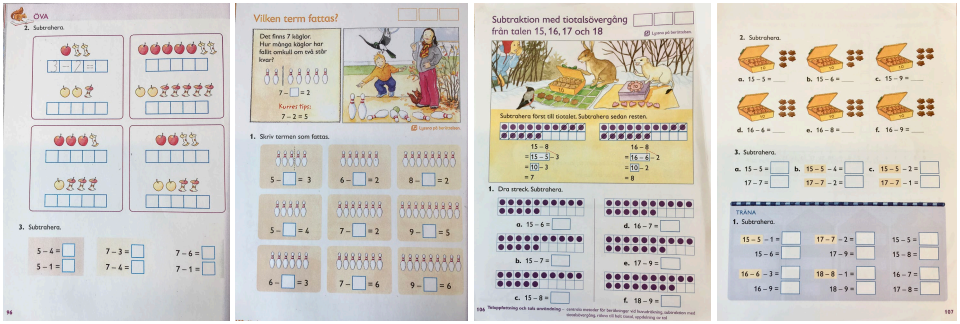

Figure 1 (from left to right): Apples and cores (Ristola, Tapaninaho & Tirronen, 2012a, p. 96).

Figure 2: Bowling pins (Haapaniemi, Mörsky, Tikkanen, Vehmas & Voima, 2013, p. 150).

Figure 3: The hare and the ermine (Ristola, Tapaninaho & Tirronen, 2012b, p. 106).

Figure 4: Gingerbread cookies (Ristola, Tapaninaho & Tirronen, 2012b, p. 107).

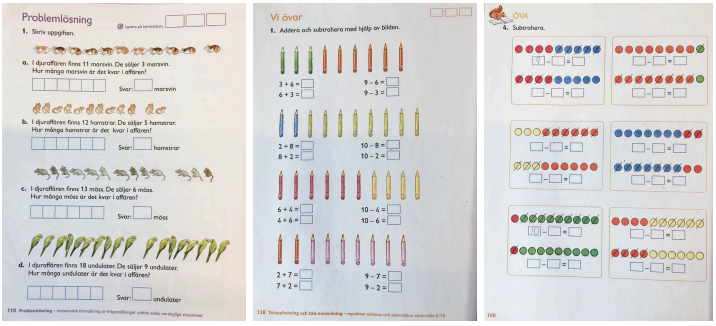

Figure 5 (from left to right): Petstore (Ristola, Tapaninaho & Tirronen, 2012b, p. 110).

Figure 6: Pencils (Ristola, Tapaninaho & Tirronen, 2012a, p. 138).

Figure 7: Dots (Ristola, Tapaninaho & Tirronen, 2012a, p. 140).

Illustrator: Rajamäki, M. With publisher's permission.

Framework for analysis

An analysis in three steps was conducted in order to understand the students' meaning making. First, a textbook analysis of the exercises was made to capture the designed meanings (the purpose). Second, the video material and the students' representations (their

answers) were analysed. Third, the designed meanings and the students' meaning making of the exercises were analysed according to the concept of agency.

The textbook exercises were analysed in the textbook analysis, referring to the research question 1: *Which meaning potentials are designed into the exercises?* The exercises' mathematical content and the mode or modes that carry information for solving the exercises were studied using the teacher's guide to answer how subtraction is addressed. Carpenter and Moser's (1982) categorisation was used to document subtraction types: *joining, separating, part-part-whole, comparison, equalizing-add on and equalizing-take away.* Examples in Figure 1; for instance, the exercise named 2 consists of a separating situation, the apples have been eaten. The exercise named 3 involves subtraction without a specific subtraction type; the information does not contain a specific subtraction type but subtraction in general.

In the *video material with the students' representations*, research question 2: *How do the students make meaning when working with the textbooks?* guided the coding of the data. To analyse the video transcripts, they were first transcribed based on different modes and using three headings: speech, image, and body language. In the image column, the students' use of images and cases in which they drew an image to support their calculations were documented. The students' representations (on the copied papers) were used as support for this analysis. In the next step of the analysis, I noted whether the child solved the exercises as designed, according to the subtraction content. After that, the transcripts were coded by reading through the material several times and highlighted using a colour code. Then condensed meanings were summarised in a matrix, from which various categories about how the student's made meaning when working with the textbook emerged.

To further deepen the understanding of students' meaning making when working with mathematics textbooks, *an analysis was also made based on the concept of agency* and the question: What opportunities for students' agency is possible when working with the mathematics textbook? Interest was directed towards the student's space to act independently and to choose different ways to make meaning when working with the mathematics textbook. The textbook exercises and students' meaning making were analysed based on the concept of agency or student's independent participation (Bezemer & Kress, 2016) and what opportunity for agency is allowed to the student when working with the mathematics textbook. For the textbook exercises, I searched for opportunities for agency concerning an imaginary work with the exercise based on the sentence offered. Next, I searched for whether the student's meaning meant opportunities to act independently and choosing how to make meaning when working with the exercise.

An example is given here of how the analysis of the *textbook exercises* was carried out on the basis of the exercise "Dots" (see Figure 7) to concretise. The subtraction type is separating, as dots are crossed out. The exercise's designed meaning was that the information needed to solve the tasks is found in the images and answers are made with mathematical symbols. With the help of the concept of agency, this could be understood as meaning that the exercises did not offer opportunities for agency for the students in that the design of the exercise does not allow different working methods.

An example of the analysis of *students' meaning making* is the students' work with the exercise "Gingerbread cookies" (see Figure 4). The students worked in different ways with that exercise. Some of the students based solely on the mathematical symbols in their work, some used images and mathematical symbols, and a couple of students drew images which they used to support the calculation. This example shows that the design of the exercise provided an opportunity for students' agency.

Results: Students' possibilities to take agency when working with mathematics textbooks

The data shows examples of exercises that enable agency and exercises that do not when students are working with them. The data also show that students' opportunities for agency are affected by the notion that mathematical symbols is the mode that should be used if the student should be considered successful in mathematics.

When exercises enable student's agency

The analysis showed that there are exercises where the design of the mathematics book provides opportunities for agency and the opposite. Exercises designed to enable agency are designed so that the student can choose working methods, and this can be done by selecting the order in which the modes are used or deciding which modes to use.

An example of an exercise that enables student's agency is *"The hare and the ermine"* (see Figure 3). The subtraction type in this exercise is separating and bridging ten. The student can choose between starting their meaning making through the image mode or the mathematical symbols mode. The students who began their meaning making from the images started by crossing out the number of dots to be subtracted and then counted the remainder and finally wrote mathematical symbols on the line. The students who began their meaning making by starting from the mathematical symbols made a calculation using the mathematical symbols and then crossed out dots or did not use the image at all. In the former way of working, the image can be a support for the calculation. In contrast, in the latter, the image is used as an additional representation for the student's subtraction calculation.

Another example where the exercise enables student's agency is seen in the exercise "Pens" (see Figure 6). The subtraction type here is part-part-whole, pencils of different colours are subtracted. Here the student can choose between using the image in the calculation or basing her/his meaning solely on the mathematical symbols. The data showed that some of the students used only the mathematical symbols, while others also used the images.

In exercises that enable agency, the students can thus, to a greater extent, decide for themselves how they want to work with the exercise, agency is possible. Here, the students' own choices can form conditions for which learning situation they want to face by choosing the rules of procedure or the mode they wish to use in their meaning making.

When exercises limit student's agency

The analysis of data shows the occurrence of exercises whose design limits students' ability to take agency. Such exercises are designed so that a specific way of working with the book is required to solve the exercise based on its designed meaning. It may be necessary that the work takes place in a particular order or is based on a specific mode and that the work is then done with a specific mode or certain modes.

An example of an exercise where *the work needs to be done in a particular order* is in the exercise "Apples and cores" (see Figure 1). The designed meaning is that the student first uses the information in the solved example and then solves the tasks in the same way. The image is to be interpreted as a sequence of events, and the calculation is written in empty boxes using mathematical symbols. The results showed that the students worked on this exercise by first using the example. Then they started from the image and interpreted it in different ways. Some of the students interpreted the image as first; for example, there were three apples, and then someone ate an apple. Others interpreted the apples as one term and the apple cores as the other, and used the image as a static image. Finally, they wrote a mathematical symbol in each box to make the calculation. So, some of the students interpreted the image as a change take from-situation and some did not.

An example of an exercise where the student's meaning making is based *on a specific mode* is in the exercise "Dots" (see Figure 7). The exercise is designed so that the student's meaning making is done based on the mode image. The designed meaning is that the student's meaning making should be based on the images, and then the answers are represented with the help of mathematical symbols in the empty boxes. When the students worked on this exercise, they started by looking at the image, counting the number of dots and crossed out dots. The subtraction calculations were then made using that information, and answers written in mathematical symbols.

In exercises that limit agency, students are, to a lesser extent, given the opportunity to decide for themselves how they want to work with the exercise. The learning situation is shaped by the fact that the work takes place in a particular order or based on a specific mode and that the work is then done with a specific mode or certain modes.

Students' opportunities for agency and expressions about the subject of mathematics

The results also showed that some students expressed a notion that mathematical symbols are "better" to use than the other modes. Choosing the mathematical symbols mode to a greater extent over other modes was described by the students as something that shows that they are successful in mathematics, demonstrated in both words and actions. Based on the concept of agency, it is made clear that the student's opportunity for agency is affected by the notion that students who are successful in mathematics do not use the images but start from the mathematical symbols. This is expressed by several students, both students who use the image and those who do not use the image to solve the tasks. In an answer to my question about whether it is possible to use the cones to count in the exercise in Figure 2 "Bowling cones", one of the students answered: "Yes, but you do not have to if you are good

at math". The space for agency that the design sometimes entails, such as in the exercise with the cones, shrinks from the prevailing notion that it is more desirable to solve the tasks without using the images. So, although it is possible to choose to work with the images, there is a resistance to this as it is not a desirable way to solve the tasks.

Discussion and concluding remarks

The results based on the agency concept showed examples of exercises that do not enable students' agency. The exercises can be based on a designed rule of procedure that states how the different modes are to be used to solve the exercise according to the designed meaning. If, on the other hand, the design allows for different rules of procedure, students' agency is made possible. To a greater extent, the student can then decide for herself how she wants to work with the exercise. Here, the student's own choice could shape the conditions for which learning situation the student encounters. However, the results showed that in such exercises, there was a tendency for students to choose mathematical symbols for their meaning making over other modes. In the following, I discuss this from two different aspects.

First, meaning making starting from mathematical symbols can contribute to the student being able to miss out on a learning situation that can support the conceptual understanding of the mathematical content. For example, the image mode may show a subtraction type not captured in mathematical symbols (see for instance Figure 7 "Dots"). However, working with the mathematics textbook based on mathematical symbols can also mean that the student already possesses the knowledge required in the exercise. This makes the situation complex and, it is important not to assume that a student who works with the mathematics textbook based on the mathematical symbols automatically belongs to the latter group of students.

Secondly, students' tendency to start from the mathematical symbols to a greater extent can contribute to students already at the age of 7–8 not understand themselves as mathematical individuals. If the student cannot meet the notion of choosing mathematical symbols over other modes, it can help shape an identity as "non-mathematical". This can be compared with what Palmer (2009) describes as mathematical subjectivity with the meaning of *becoming* mathematical as an individual rather than *being* mathematical and that this is related to the students meaning making. The way students' make meaning when working with textbooks and how they value their own, and other's work, affect their understanding of themselves as mathematical or not. Mathematics teaching should be based on the assumption that all students are experiencing themselves as mathematical individuals.

If learning situations where all students at the age of 7–8 understand themselves as mathematical individuals could be created, the chances increase that students develop identities as mathematical individuals. Of course, the way to get there is complex, and the work with the mathematics textbook is only part of this identity creation. However, the mathematics textbook is widely used in mathematics teaching. It is in many ways, structured as a question-and-answer textbook, and the answers are almost always in demand in mathematical symbols mode. It is not surprising that the students develop a notion that mathematical symbols are the most important mode. Here, a question arises as to whether

the students' descriptions linked to discovering themselves as mathematical individuals or not would look different if the mathematics textbooks were designed in a way that to a greater extent recognizes students' displayed knowledge through various modes. This can be compared with Björklund Boistrup's (2010) conclusion that restrictions in which modes are offered can limit students' opportunities for agency.

A question that arose in this study was which represented knowledge is recognized in mathematics textbooks for year 1. The results showed that mathematical symbols have a superior position already in year 1. If representations through modes other than mathematical symbols are recognized to a greater extent, more students would have the opportunity to discover themselves as mathematical individuals. This supports the students' further mathematics learning and the possibility that they continue to see themselves as mathematical individuals. The design of the teaching determines which agency is created for the students (Kress, 2010). With a good foundation in early mathematics teaching, it is easier to continue working towards education where more students develop identities as mathematical individuals.

Based on the reasoning given, an interpretation could be that the possibility of agency in the student's individual work with the mathematics textbook combined with mathematics textbooks that better utilize the student's represented knowledge through different modes would be desirable. Agency has been described as desirable in most studies (Björklund Boistrup, 2010; Louie, 2019; Norén, 2011). Though, a mathematics textbook that opens up to students' agency would automatically lose clarity regarding the designed meaning. Suppose the student has great opportunities to choose working methods. In that case, this may mean that parts of the exercises' design can be deselected and thus indirectly that certain mathematical content is deselected. This is because some modes are better suited than others for specific information; for example, a subtraction type cannot be displayed using only mathematical symbols. Based on this, a mathematics textbook with increasing opportunities for agency would mean a textbook that is, to a lesser extent, suitable for individual work since the teacher is needed in order to highlight the mathematical content.

A conclusion is that the work with mathematics textbooks for primary school's youngest students should be done with great consciousness. A suggested design based on this is mathematics teaching that requires representations in different modes and two different learning situations. (1) One type of learning situation offering opportunities for agency where the working method is not based on individual work, and (2) another type of learning situation where the student can work individually based on clearly designed offers to consolidate content already known to the student. These two different kinds of learning situations could benefit learning situations where all students learn and also discover themselves as mathematical individuals.

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A cultural and glocalized approach to the maker movement: An ethnomathematical perspective

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The maker movement has faced constant criticism for not considering the existing making practices in the context of people of diverse cultural and ethnic groups. Nonetheless, it is quintessential that the teaching and learning of STEM be done in terms of situating it in diverse socio-cultural contexts. This situation has informed the current study to attempt a cultural and glocalized form of approaching 'making' through the lens of ethnomathematics. By combining methods from Gerdes, Bishop and Vygotsky in the sphere of ethnographic, design-based and activity inquiries, this conceptual paper conceptualizes what doing STEM through the existing making practices in the context of people of diverse cultural and ethnic groups is and how that can also supplement, inform and centre stage the global practices of conventional making. Thus, a conceptual framework for cultural and glocalized approach to the maker movement in the lens of ethnomathematics and its sister disciplines is developed.

Introduction

Makerspaces are learner-centric physical areas where individuals develop ideas, experiment, tinker, design, and create artifacts using digital and physical tools to represent their learning (see Pocock, 2016) in computational thinking, STEM, etc. To this, makerspaces have helped to refute the notion that STEM is about following some set of rules and principles which are abstract. Though, Dougherty (2012) asserts that the term maker is universal and core to human identity and Wing (2011) defined computational thinking (CT) to suggest that to teach or learn CT computers are not necessarily needed and also CT entails one's ability to design solutions that can be executed by a computer, a human, or a combination of both. And again, the National Council of Teachers of Mathematics (NCTM, 1991) and other researchers have highlighted, in their guidelines, the importance of building connections between mathematics and students' personal lives and cultures (see D'Ambrosio, 1990; Gay, 2000; Ladson-Billings, 1995; Rosa & Orey, 2003). There has been consistent criticism against the maker movement, stemming from the fact that it does not include the existing making practices in the context of people of diverse cultural and ethnic groups, therefore, the doing of STEM and CT through the existing making practices in cultural and diverse contexts or supplementing them with the conventional making practices is not a focus (see Tan & Calabrease-Barton, 2018; Blikstein, 2020). For example, Dougherty is of the view that there

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may exists no making in such diverse cultural or ethnic backgrounds which may evidence, inform, supplement and centre stage the practices of conventional maker movement hence, there is the need to export what goes on in a makerspace into such locations (see Britton, 2015). Again, in a 2013 keynote address at Stanford's FabLearn Conference on digital fabrication in education, Leah Buechley (2013) described the MAKE organization as being focused on a narrow range of maker activities (primarily robotics, electronics, and vehicles) and even a narrower range of makers, with 85 percent of its magazine covers featuring white boys and men. Hereafter, it can be concluded that the activities of makerspaces in the United States and Europe have promoted STEM and CT as a neutral and culturally-free discipline removed from social values, diverse cultural and ethnic contexts (see D'Ambrosio, 1990). Of concern is how Western "exceptionalism" has a profound influence on non-Western education, in the sense that indigenous and non-conventional forms of STEM pedagogical approaches continue to be marginalized. These have resulted in school pupils, particularly in rural cultural communities, experiencing schooling as a one-way border crossing (Giroux, 1992), a situation that requires them to leave their home culture at the school gate and move into the alien culture of the school. However, as the maker movement is situated in the constructionist theory (Papert, 1993a), educators are helped to understand how different media can be used to express and transform multiple ideas in different contexts (Ackermann, 2001). Thus, the constructionists process could take many forms and shapes, in that, children might want to program a computer or build robots in ways that might violate the canonical rules of professional-coding and engineering-productively and creatively going against the grain of well-established practices in these technical fields, resulting in epistemological pluralism (Turkle & Papert, 1991). For example, how the maker of the Tlingit basket came up with the design that depicts a great understanding of Euclidean geometry is a puzzle (Mukhopadhyay, 2008). In Civil's (2016) analysis of a seamstress' practice, she noted that to make the pattern for a skirt, the seamstress made a quarter of a circle in such a way that it showed the circle as the locus of all points equidistant from a central point. Yet, these seamstress and Tlingit basket makers did not have a course in formal geometry, nor talk about Euclidean Geometry in their making. Similarly, In lieu of these STEM in such non-conventional makerspaces, I hypothesize the focus of making practices in diverse ethnic and cultural contexts (indigenous; cultural; craft; ethno making-see Blikstein, 2020; Bang & Barajas-López, 2018) as a form of making spaces (Tucker-Raymond & Gravel, 2019) by attempting a glocalized form of makerspaces that is situated in ethnomathematics (Ascher, 1991; Millroy, 1992; Gerdes, 1985). This serves as an avenue to increase and focalize research in the maker movement on the existing making in diverse ethnic and cultural context. In essence, this conceptual paper conceptualizes what doing STEM through the existing making practices in the context of people of diverse cultural and ethnic groups is and how that can also supplement, inform and centre stage the global practices of conventional maker movement.

Cultural making and ethnomathematics

D'Ambrosio (1990) defines *ethno* in the socio-cultural context as language, jargon, and codes of behavior, myths, and symbols as Blikstein (2020) defines making as a more general type of practice than only 3D printing, robotics, etc., enacted on basic or advanced materials. Combining Blikstein's (2020) and D'Ambrosio's (1990) definitions of "making" and "ethno", ethnomaking or cultural making may refer to a more general type of practice, enacted on basic or advanced materials, either with no or high technology within a specific cultural context which may evidence the doing of STEM. Ethnomaking comprises activities, materials, practices, and themes that are attuned to a more specific group, culture, or region (such as basket making, pottery, electronics upcycling, woodworking, or costume making) (Blikstein, 2020). An ethnomathematician will see ethnomaking as an avenue to discover the STEM concepts in the making of these activities, practices and materials. This process is to help reconstruct or 'unfreeze' STEM thoughts that are 'hidden' or 'frozen' in old techniques (see Gerdes, 1986). From an emic and etic perspective (Rosa & Orey, 2010), the artisan, who imitates a known production technique, is not doing STEM (emic). However, the artisan(s) who discovered the technique was/were thinking STEM (etic). An example is the geometric visualization of the weaver expressed through actions and materials (Ascher, 1994). As she puts it: ...the carpenter definitely is dealing with a mathematical idea; the mathematician who [arbitrarily decided to trisect an angle only with ruler and compass] was dealing with an idea. They are both important but they are different. And, they are linked (D'Ambrosio & Ascher, 1994, p. 38). It is as precise as an illustration in a schoolbook on geometry (Mukhopadhyay, 2008).

Glocalized making and ethnomathematics

For Branigan-Pipe (2016), the principles that shape the 21st Century Makerspace learning environments are those same principles that have guided the indigenous people for ages. For example, the first robot to walk the earth was a bronze giant called Talos, created by Hephaestus, the Greek god of invention. Inventing, making, tinkering and designing are indigenous practices (Gutiérrez, 2015). Glocalized making is the process of taking characteristic elements from the maker movement and infusing them into existing making practices in diverse culture and ethnic contexts. Such forms of making are supposed to be locally and globally relevant. Glocalized making supports historicized experiences as the making approach and practice by encouraging both indigenous and conventional makers to develop an understanding of the interconnecting relationship among ideology, power, and culture and rejects any claim to universal foundations for truth and culture, as well as any claim to objectivity (see Leistyna & Woodrum, 1996) in the maker movement. In glocalized making, the maker movement does not see the knowledge and practices of communities of diverse cultures and ethnic backgrounds as without maker culture and hence, awaiting the conventional maker movement deposits—what Freire (1970/2000) coined, "the 'banking' concept of education" (p. 72). Glocalization is a concept originally coined by business circles

and it means to create products for the global market, but customized to suit local cultures and tastes. The fundamental question of the glocalized making is about the compromise between what is already there in the culture, the practices, the materials, and the new elements from the maker movement that teachers or designers want to bring (see Blikstein, 2020). Glocalized making is to produce innovative ways of thinking and reasoning. Teachers can be engaged in the designing of 3-D and 2-D mathematical manipulatives and lessons, based on cultural making to anchor their pedagogical and conceptual knowledge. Also, teachers can be trained to introduce light up fashion and electronics into existing cultural making of indigenous textiles.

Conceptualizing cultural and glocalized making

A major problem with mathematics education in contemporary society is its overwhelming bias towards a Western orientation in its topics and research paradigm. A search for new approaches and methodologies is necessary to record historical forms of mathematical ideas that occur in different cultural contexts and to take advantage of the emerging globalization of business, science, religion, art, music, and other aspects of culture (Orey & Rosa, 2019). Numerous studies have demonstrated the sophisticated mathematical ideas and procedures that often include geometric principles in craftwork, architectural concepts, and practices in the activities and artifacts developed by many indigenous, local, and vernacular cultures (Eglash et al., 2006). Thus, ethnomodelling (Rosa & Orey, 2010) and ethnocomputing (Eglash et al., 2006). However, as noted by Rosa and Orey (2010), modelling is an essential tool for ethnomathematics. But when we create a model for a cultural artifact or practice, it is hard to know if we are capturing the right aspects; whether the model is accurately reflecting the mathematical ideas or practices of the artisan who made it, or imposing mathematical content external to the indigenous cognitive repertoire (Babbitt et al., 2012). Again, the mere pointing to a photograph of some intricate basket or monumental pyramid is not sufficient for engaging children or developing their mathematics skills. (Babbitt et al., 2012). Hence, the use of computational media to help address these challenges thus, ethnocomputing (Babbitt et al., 2012). However, ethnocomputing can be seen as an approach that might have subtracted the cultural aspect of cultural material through technology and computationalization. Again, as a criticism to ethnocomputing, what is known as a major challenge in the educational system in many communities of colour and low income is the inequality of educational resources which includes access to computers and other ICT materials. For example, in one experimental study of ethnocomputing (see Babbit et al., 2015), it can be seen that the study computationalized the cultural making of Adinkra symbols from a rural Ghanaian community using computers and, implemented them in a Ghanaian city classroom. Could this be attributed to the fact that electricity and computers were not available in classrooms of such rural Adinkra making community? This is meant that students whose ideal cultural making was computational-ized might have limited access to their transformed cultural making practice. As a note, “we have been programming for just a few decades, but people have been making for millennia” (Blikstein, 2020, p. 116). Hence,

why not focus on the cultural context and what goes on or glocalized such cultural making? Thus, focusing on the mathematics/STEM identifiable in the cultural making practices and/or supplementing it with conventional maker practices leads to a new approach and methodology which is necessary to record historical forms of mathematical/STEM ideas that occur in different cultural contexts, and to take advantage of the emerging globalization of business, science, religion, art, music and other aspects of culture thus, the need for cultural and glocalized making. Also, Mukhopadhyay (2008) stated, "given that throughout the known history of humankind people have always created music and dance, designed household objects, adorned their bodies with paints and designs, it is not unusual for anthropologists and others to examine where patterns come from" (p. 64). Franz Boas (1927/1955) wrote that: "No people […] however hard their lives may be, spend all their time, all their energies in the acquisition of food and shelter […] Even the poorest tribes have produced work that gives them aesthetic pleasure […] [they] devote much of their energy to the creation of works of beauty" (p. 9). No matter how diverse the ideals may be, the general character of the enjoyment of beauty is of the same order everywhere. This realization points to the link between indigenous craftwork making and formal mathematics, computations, engineering, etc. namely an instinctive fascination with patterns. The knowledge of an indigenous "craft" – basket weaving, canoe making, for example – is complex and situated in cultural values and everyday living. For example, the construction of canoes for a non-Western group using indigenous material evolves over a substantial period of time incorporating cultural traditions of design, understanding of engineering, and adaptation to the environment. To an individual who does not belong to the community, the canoes, although functional and efficient, may look crude and primitive. If the individual happens to be trained in, and used to, only technologically-sophisticated building tools, the process of construction might seem to him/her as simplistic and rudimentary. As a consequence, both the process and the product of creating artefacts are not recognized as a cognitively complex process. Hence, an obvious reason why the maker movement does not contend such forms of making. In this regard, cultural and glocalized making draw on studies employing ethnomathematics, ethnomodelling and ethnocomputing as a framework to understand the computations and mathematics in the existing making practices in diverse cultural and ethnic contexts (Eglash et al., 2006; D'Ambrosio, 1990), and how those is supplemented to operate within the conventional maker movement. For example, through ethnocomputing, local designs have been analyzed as forms and the applications of symmetrical classifications from crystallography to indigenous textile patterns (Eglash at al., 2006) and these can be supplemented with electronics to produce e-textiles, thus glocalized making. On the other hand, ethnomathematics can use cultural and glocalized making as a tool to help reconstruct or 'unfreeze' the mathematical thinking that is 'hidden' or 'frozen' in old techniques, like, e.g., that of basket making. In ethnomodelling, Rosa and Orey (2006) modelled the mathematical knowledge that lace makers in the northeast of Brazil use to make patterns that have mathematical concepts not associated with traditional geometric principles. Here, cultural making seeks to rather observe and interpret the making processes from the lace

makers to identify the mathematical concepts. Again, glocalized making would empower the Brazilian lacemaker and students to introduce the concept of computer and electronic programming into such cultural making (see Jayathritha & Kafai, 2019). Again, mediated activity through the lace making can be derived for teachers to be engaged in the designing of 3-D and 2-D mathematical manipulatives to anchor their pedagogical and conceptual knowledge through a reflection of such cultural making in the conventional maker world, hence glocalized making. I define these connections as cultural and glocalized making, which is the act of identifying the making practices in diverse cultural and ethnic contexts to identify STEM, and also supplementing, informing and centrestaging it with practices of conventional maker movement. In this context, Figure 1 shows a conceptualisation of cultural and glocalized making, as the intersection of three fields of knowledge: cultural anthropology, ethnomathematics, and the maker movement.

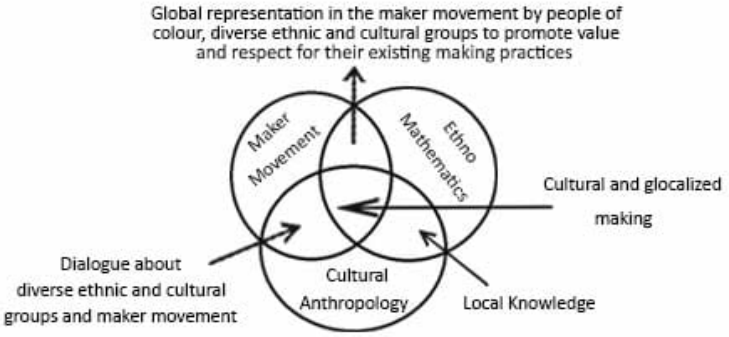

Figure 1: Cultural and glocalized making as the intersecting region of three knowledge fields

From Figure 1, the intersection between the maker movement and ethnomathematics relates to the global representation of diverse ethnic and cultural groups in the maker movement and hence, respect and the valorization of the tacit knowledge (Rosa & Orey, 2007). Also, traditions found in diverse contexts, and often the students therein, will be enabled to uncover the mathematical/STEM concepts hidden in the making of cultural artifacts and supplement these with practices of conventional maker movement. Therefore, it becomes necessary to begin by using sociocultural contexts, realities, and interests or unique needs of students and not mere enforcement of a rigid set of external curricular rules and values with often decontextualized activities (Rosa & Orey, 2007). This approach brings about the supplement, and centre stage between the maker movement and cultural anthropology in order to reach critical transitivity (Freire, 1998). According to Rosa and Orey (2007), diverse forms of local knowledge develop the context, source, and form for what is found in the intersection between mathematics and cultural anthropology and occurs when members of distinct cultural groups use it to solve problems faced in their own contexts. It also becomes a profound body of knowledge often built up by these members over time and across generations of living in close contact with their own historical, social, cultural, and

natural environment (D'Ambrosio, 1990). This context uses a definition of cultural making as the uncovering, unfreezing, and supplementing mathematical ideas, notions, procedures, and practices in which the prefix ethno relates to the specific mathematical knowledge possessed by the members of distinct cultural groups, where ethnomathematics adds cultural perspectives to the mathematics discovered through making. Hence, it results in addition of cultural perspectives also to the maker movement. In the glocalized making process, global mathematical knowledge through maker movement must supplement and be adapted to local mathematics in the making practices in diverse cultural and ethnic contexts.

Methodology for recognizing or identifying STEM through cultural making

In this section, I propose a demonstration of making in the cultural and glocalized form as a pedagogical approach that challenges the conventional way of thinking or doing mathematics/STEM in the maker movement. This approach is also a form of an ethnographic and design-based inquiry where researchers participate in cultural making activities.

The Gerdes (1985) approach

Gerdes (1985) looked to the geometrical forms and patterns of traditional objects like baskets, mats, pots, houses, fish traps, etc. and posed the question: why do these material products possess the form they have? In order to answer this question, he learned the usual production techniques and tried to vary the forms. By this process of rediscovering the mathematical thinking hidden in these baskets and fish traps – and in other traditional production techniques – the future teachers personally feel stimulated to reconsider the value of their cultural heritage: in fact, geometrical thinking was seen as not alien to their culture. This "unfreezing of culturally frozen mathematics" can serve, in many ways, as a starting point and source of inspiration for doing and elaborating other interesting mathematics. Another method for identifying the STEM in cultural making is by using Bishop's (1988) mathematical activities model. This model consists of identifying how counting, locating, measuring, designing, playing, and explaining is done in the cultural making process.

The glocalized method

"Unfreezing frozen mathematics" through cultural making forces mathematicians and philosophers to reflect on the relationship between geometrical thinking and material production, between doing mathematics and technology (see Gerdes, 1985b). The latter enforcement, thus the relationship between doing mathematics and technology focalizes cultural making in the lens of conventional making to eradicate how technology and computations have subtracted the cultural aspect of maker movement. Hence, through the ethnographic studies by using the Gerdes (1985), there lies the opportunity for design research through socio-cultural theory grounded in mediated activity derived from Vygotsky (1978) for teachers to be engaged in the designing of 3-D and 2-D mathematical manipulatives, robotics and electronic programming to anchor their pedagogical and conceptual knowledge through a reflection of such cultural making in the conventional

maker world. This creates an opportunity to use such cultural making practices in a globalized context, which is glocalization.

Conclusion and final considerations

It is apparent, to this end, to assert that maker movement has sidelined existing making practices in the context of people of diverse cultural and ethnic groups whereas mathematics education considers these aspects. In order for this situation to be reconciled, the teaching and learning of mathematics need to be reconsidered. In essence, there must be a link between culture and making as a pedagogical approach that is informed by ethnomathematics. For example, teachers can be engaged in the designing of 3-D and 2-D mathematical manipulatives, robots, electronics programming and lessons that are based on cultural making to accentuate students' pedagogical and conceptual knowledge. This situation provides the basis or the starting point, as well as a source of inspiration, for doing and elaborating mathematics and not merely learning about it. The proposed cultural and glocalized pedagogical approach for teaching and learning mathematics in this paper serves as a basis to reconsider the conventional way of thinking or learning mathematics in the maker movement. It is evident that the approach is particularly helpful in the sense that students are able to formulate their own concepts by situating them within their cultural contexts and also linked to conventional and other global practices. Through the combined use of methods from Gerdes (1985), Bishop (1988) and Vygotsky (1978), in the sphere of ethnographic studies and design-based research, studies in this area of research will reveal how the cultural and glocalized approach can supplement, inform, and centre stage the global practices of conventional making to the maker movement when situated in ethnomathematics.

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Voices and texts in Swedish mathematics teacher education

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Instead of getting access to the diversity of perspectives in mathematics education, prospective mathematics teachers read secondary sources of research providing a local or skewed image of the field. It then becomes important to see what is assumed scholarly sufficient for inclusion as texts for prospective mathematics teachers. The literature from a mathematics teacher programme, in Sweden, was submitted to an analysis of authors' occupation and country of residence as well as what types of texts the prospective teachers read. Through these texts, the prospective teachers were invited to participate in a local practice-based intellectual conversation. Besides risking not engaging with the forefront of research, there is a risk that prospective teachers are not sufficiently invited to challenge the educational philosophies with which they are accustomed.

Introduction

Different voices participate in mathematics teacher education. Voices from teacher educators, researchers and teachers in the field. Authors who write the compulsory literature for prospective mathematics teachers are counted as intellectuals with authority to speak about mathematics teaching and learning (Alvunger & Wahlström, 2018). Sayed et al. (2017) studied mathematics teacher education in South African universities to see who these writers were and what intellectual debates the prospective teachers were invited to through their course literature. They found that the prospective teachers got limited access to the multiple understandings of Africa's complex educational discourses and they concluded that "there is a need to provide intellectual fora for different actors to come together and think deeply and regularly about the meanings and processes which saturate their worlds" (p. 86). Sayed et al. describe this as a process of expanding imaginations, which involves rethinking what counts as relevant scholarship. To expand imaginations is described as a process of decolonisation where voices from different countries and cultures are given authority as intellectuals, which by extension offers access to competing perspectives from the intellectual debate. Inspired by Sayed et al.'s work, the present study begins with an interest in what texts the prospective teachers read and who the authors of these texts are.

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What it means to expand imaginations can, of course vary; it could be to get access to influences from other countries, for example insights in educational philosophies underlying materials or methods that we might import (Kaiser & Blömeke, 2013). Competing, or at least different, perspectives are more often found in research literature than in the specifically written course literature (Alvunger & Wahlström, 2018). In the present study, we study the course literature and its authors in one programme for Swedish mathematics teachers to identify the prospective teachers' opportunities to expand imaginations. In this, we study what counts as relevant scholarship for prospective teachers in Sweden and the opportunities they have for a diverse understanding of educational philosophies.

The intellectual arena for prospective mathematics teachers needs to be broader than the culture of which they are already a part. In a plea for more cultural awareness in research of mathematics education, Andrews (2016) shows how mathematics education is deeply rooted in the culture within which it is carried out. Culture is here understood as the "covert culture", the way of life of the people that shapes mathematics education in a specific context (Knipping, 2003). International large scale assessments call attention to successful countries, creating a search for cross-national pedagogic excellence when it could be more reasonable to search for understandings of cultural differences (Andrews, 2016). Different cultures bring different educational philosophies which grounds mathematics education, and these philosophies needs to be taken into account in order to understand teaching methods and materials from different contexts (Kaiser & Blömeke, 2013). In multicultural educational settings, teachers need to be able to recognise cultural conflicts in interactions with students and parents form other cultures (Bishop, 1994). On the other hand, mathematics education, being situated within a culture, could imply a need for more locally situated mathematics teacher education, similar to what Sayed et al. (2017) highlight when they ask for more African authors in African teacher education. From the perspective of the programme studied here, which enjoys the privilege of being a European mathematics teacher education in a European country, the need however differs from a country where education has been imposed by a colonising power, such as South Africa. In Sweden, there is rather a need for access to competing or diverse perspectives of mathematics education to challenge set ways of thinking.

It is essential that prospective, professional, mathematics teachers get access to an academic intellectual arena. Swedish mathematics teacher education is based on mostly secondary sources of reference where the content is presented from a given perspective, strongly normative, instead of being discussed from different perspectives, as it is in research publications – and more so for prospective primary teachers (Alvunger & Wahlström, 2018; Pansell & Jatko Kraft, forthcoming). Swedish prospective mathematics teachers are immersed in the ways of mathematics teaching rather than getting access to its theoretical grounds (Asami-Johansson et al., 2020). Theoretical perspectives of mathematics education are, in Sweden, not always visible (Österling, 2021), which implies a need for access to research literature and theoretical grounds for the ways of mathematics education. On the other hand, in all educational systems there is a process of didactic transposition where elements of knowledge are translated between institutions. In this process, sometimes the rationale of

the knowledge gets lost (Bosch & Gascòn, 2014). Yet even if prospective mathematics teachers need access to the content taught in teacher education, it is undesirable for all readings to be reconstructed normative texts. This would strengthen what Asami-Johansson et al. (2020) describe as "immersion in the ways of mathematics teaching" with little visibility of the theoretical underpinnings. As Alvunger and Wahlström (2018) also conclude, this would enhance the possibility for well-informed professional teachers with scientific grounds for their practices.

The present study aims to explore the writers and texts of Swedish mathematics teacher education. That is, the agents and products of the transposition in this particular case. In this, we search for the prospective teachers' possibilities to expand their imaginations into other cultural settings as well as different levels of academia. This is why we asked the following questions: "Who are the authors in the mathematics education courses?" and "What types of texts do the prospective teachers read?"

Methodology

The process of transposition is a central idea of ATD (the anthropological theory of didactics) where educational practices centrally is seen as part of a culture, one or more academic contexts, an educational system and more, all called institutions (Bosch & Gascòn, 2014). The theoretical grounds for the present study are, consequently, found within ATD. A central idea in ATD is that academic knowledge transforms through these different institutions on its way to teachers and students; it undergoes what in ATD is called a didactic transposition (Chevallard, 2006). People who think about education, constituting the noosphere, rewrite, or transpose, scholarly knowledge so it will be accessible for students, in this study prospective teachers. In this study, the authors of course literature are taken to represent the agents of the noosphere. These agents author different types of texts which thereby privilege practices of and/or perspectives on mathematics teaching. These texts can be normative through presenting teaching methods as best practice, but they can also be reflective and critical, presenting different perspectives.

The compulsory course literature is a central part of higher education since it is specifically chosen to cover course contents. The teacher programme in the present study is for prospective teachers of grades 4-6. The prospective teachers take four 7.5 ECTS courses in mathematics education: Teaching and learning of (a) number sense and arithmetic, (b) geometry, (c) relationships and change; and a course in curriculum studies, assessment and grading. The data for the present study is the compulsory literature in all four courses in mathematics education. From books, only the chapters included in reading instructions were selected as data.

Inspired by Sayed et al. (2017) we made an overview of the literature listing year of publication, number of pages, authors' names and affiliations. The literature generally being secondary sources drew our attention to the references in the course literature. To be able to say something about the primary sources behind the course literature, we generated a similar overview of all authors and texts in the reference lists in the course literature of the first course on number sense and arithmetic.

Source	Course a	Course b	Course c	Course b	References in course a
No of texts	50	37	39	21	868
No of authors	24	32	39	27	465

Table 1: Overview of included texts

The agents of the noosphere, the authors, were categorised by their occupation at the time of publication and their country of residence. Some authors' occupation was already known, others were found in short descriptions of the authors, in the texts. Finally, for some authors we had to search for the information on, for example, university webpages. This means that an author can be interpreted as a Swedish teacher educator even if s/he is now researcher, living in Germany. Authors in the course literature were in many cases familiar to us which made it possible to categorise them as both researchers and teacher educators. Authors in the primary sources were in many cases researchers who may be teacher educators as well, yet this was not always possible for us to determine. We chose to include only certain information, wherefore the number in the author categories may not be entirely accurate.

The products of the noosphere, the kinds of texts the prospective teachers read were divided into the categories: (i) Books, chapters or papers written specifically for (prospective) teachers; (ii) Curricular materials, including textbooks, lessons or teacher guides; (iii) Policy texts, for example, national curricula or other texts authored by some of the national agencies connected to the educational system; and (iv) Scientific publications, such as scientific journal papers and scientific book chapters.

Findings

The overview of texts made it possible to say different things about who the agents and products of the noosphere were. The texts were relatively new, 90% of the texts had publication dates between 2000 and 2019. For the references, 50% were published between 2000 and 2019. About two thirds of the authors were female in the course literature. In the references from the first course there were about 40% female authors. Only six texts were authored by an agency, for example the Agency of Education, in the references from the first course 18 texts were written by an agency. We will now focus on the authors' occupation and country of residence, and what types of texts they wrote.

The authors' occupation

There were authors who were teacher educators only. There were also authors who were teacher educators as well as researchers. There was consequently a need for a diagram showing overlapping categories, wherefore the results are displayed as Venn-diagrams. In Figure 1, the categories are displayed in a circle with overlapping fields when there was a documented overlap.

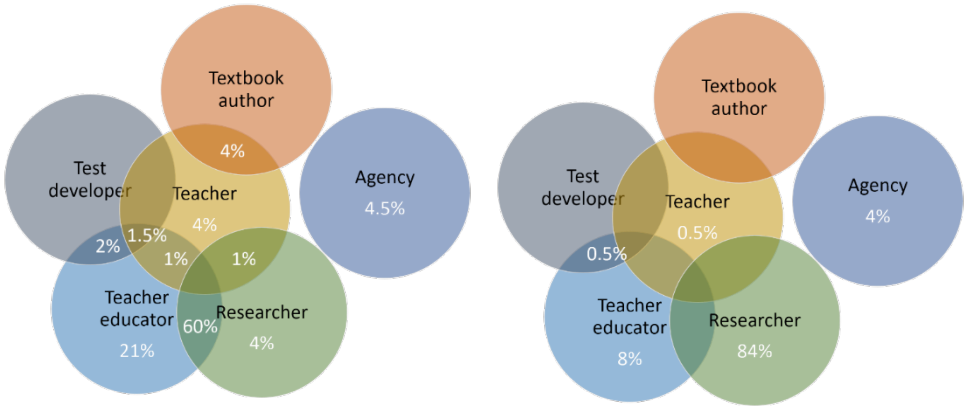

Figure 1: The authors' occupation, in percentage of total amount of authors the two selections respectively. Authors of course literature to the left (101 in total). Authors of references in the course literature to the right (464 in total). A number, not the size of overlap, represents authors with double occupations.

The authors of these mathematics teacher education course materials are mainly teacher educators. The majority of them were also researchers. The researchers most often wrote about their research or about something connected to their research. However, there were also examples when researchers wrote about something other than their research, this as the case in about one fourth of the texts. One example of this is when the author wrote about the same things after as before their PhD. Such examples give the impression that researchers sometimes write more in their role as teacher educator than as a researcher. Despite what the researchers wrote about, it is clear that the prototypical author in this teacher education programme was a researcher and to some extent a teacher educator.

The authors' country of residence

To show the course material authors' country of residence we have displayed the number of authors from a specific country on a schematic world map. As a comparison, we displayed the country of residence of authors of the publications referred to in the first course. Placing the authors on a world map gives an image of how different parts of the world that were represented in these texts and consequently how the prospective teachers were offered possibilities to expand their imagination into other cultures. To simplify this image the countries of residence have been grouped. European authors are presented in Swedish, Nordic and other European countries. African and Oceanian countries are grouped per continent. Asian countries are grouped in Russian, Middle East and other Asian countries. In North America, both Canadian authors and authors from USA are displayed.

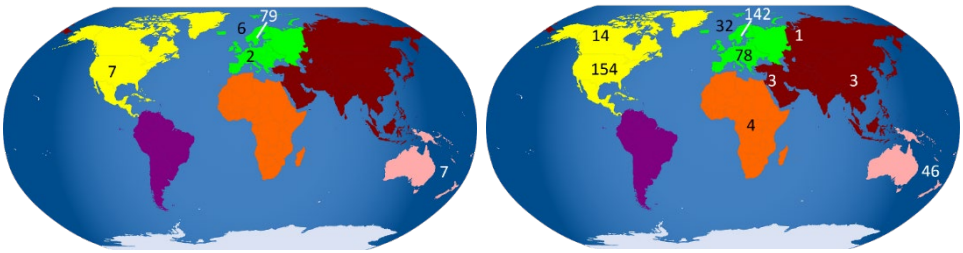

Figure 2: The authors' country of residence displayed on the world map.[1] To the left, the country of residence of authors for all four courses are displayed. To the right, the country of residence of authors of the references of the first course are displayed.

There was a concentration of authors from Sweden along with some authors from the other Nordic countries. Other than that, there was some authors from USA and the Oceanic countries. Looking into the references from the first course, one third of the authors came from Sweden (142) and about as many comes from USA (154). The last third comes from the Nordic countries (32), Europe (37) and Oceania (46). Except from two texts in German and twelve texts in Norwegian, the texts are in Swedish or English, about half each. The number of authors from Asia and Africa are negligible. No authors came from South America in either sample. In this mathematics teacher education programme it is clear that intellectuals are European, North American or Oceanian. The prospective teachers got little opportunity to read and by extension expand their imagination into Asian, African or South American educational cultures or practices.

Types of texts

The texts were mainly publications for teachers. Some of them were books written specifically for mathematics teacher education; some of them papers written for teachers with a diverse content, short reports, research or even teachers describing successful lessons. The publications for teachers were more or less normative describing how mathematics teaching should be done. An example of this was statements about students having to understand before they learn a procedure. In Figure 3 we show by percentage how large a part of the total references were publications for teachers/teacher education, curricular materials, policy texts or scientific publications. We also showed what type of texts that was referred by the texts in the first course.

The prospective teachers read almost exclusively texts written specifically for teacher education or teachers in general. Scientific publications were only a few. The lighter staples do, however, show that the scientific publications were present in the publications for teachers even though almost as many references also were publications for teachers. It is clear that the prospective teachers read research through the voice of someone, most often a Swedish researcher. They read transposed versions of research, of which Swedish researchers wrote 48 out of 259.

[1] Image from OpenClipart-Vectors on Pixabay

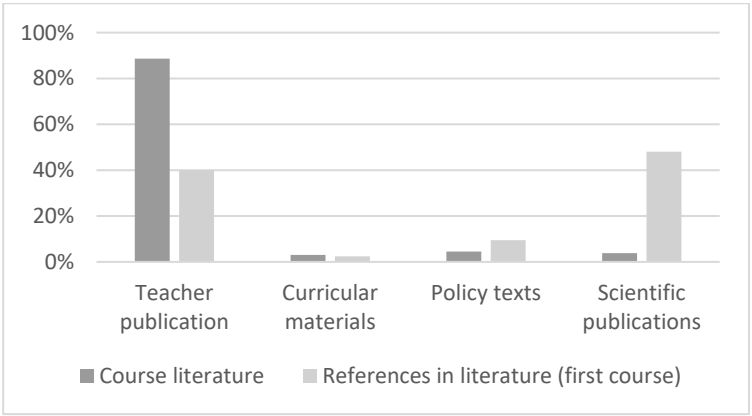

Figure 3: Type of text. The dark staple show what type of texts the prospective teachers read (145 in total). The lighter staple show what type of texts that was referred in the first course (554 in total).

Discussion

We posed questions about who authored course literature in this specific mathematics teacher education programme in Sweden and what kind of texts they wrote. In short, the answer is that Swedish researchers and teacher educators write publications specifically written for teachers and teacher education where they, at least in the first course, refer to one third Swedish and one third North American together with some European and Oceanian authors, all of whom had written research publication and/or publications for teachers. The prospective teachers' access to research was indirect, through what the course literature referred to, and even then, 40% of these references were publications for teachers. The intellectuals of the noosphere of this teacher education were, for the most, researchers mostly from what Bishop (1994) would call Western countries. English speaking countries dominated, there were only a few references written in another language than English or Swedish and all references from the first course were written in European languages. The prospective mathematics teachers got little access to cultural influences from either Asia, Africa or South America.

The studied mathematics teacher education programme offers what Sayed et al. (2017) claim that South African prospective teachers need more of: access to their local context of mathematics education. That is, if you leave out Swedish minority groups such as Sámi and Roma people, who are not represented in the literature. Rather, our findings indicate that the Swedish prospective teachers in this programme primarily had access to research transposed to their local context; research done in the same context or in the USA. Even if we take into account the total number of publications of educational research produced in the USA compared to other countries[2], there is still an over-representation of American research.

[2] https://www.scimagojr.com/countryrank.php?category=3304

There are however, other countries than USA affecting Swedish teaching practices, for example countries from where many have migrated. Despite this, there are only a handful authors from African and Middle East countries and none from South American countries. Other countries having direct effect on Swedish teaching practices are countries with high scores in PISA assessments since curricular materials are imported from countries such as Singapore (Andrews, 2016). Production of educational research in both Africa and the Middle East are scarce compared to USA[3]. Even if it still does not compare to USA, Singapore does have a more flourishing educational research milieu, not the least in comparative studies (see also Kaiser & Blömeke, 2013). Japan is another example of a country productive in educational research yet invisible in our findings. A more diverse understanding of educational systems from different cultures should extend teachers' opportunities to view, and understand, imported curricular materials or claims from immigrant parents or students. Connecting back to Sayed et al.'s (2017) study where decolonising is foregrounded, there is a possibility to contribute to the decolonisation of mathematics education. As long as "Western" countries maintain Western mathematics education, we also maintain colonisation of the mind. Opening up to diverse views on mathematics education, we also enhance possibilities for prospective mathematics teachers to view their own educational system and its cultural underpinnings critically.

Our findings confirm Alvunger and Wahlström's (2018) claim that Swedish prospective teachers read secondary sources of reference. We also show that the sources of this secondary literature are only slightly more research publications than secondary resources. Prospective teachers thus read research through the eyes of Swedish researchers who are familiar with the culture and educational philosophies within which prospective teachers will work. If these researchers transposed research from different cultures it would enable the prospective teachers to expand their imaginations as well as relating both research and different educational philosophies to the traditions of the Swedish school. The problem was that the literature, written by Swedish researchers and teacher educators relied on texts, many with secondary sources of research also written by Swedish researchers. This way, the traditions and set ways of Swedish mathematics becomes established as truths. Using researchers as authors give the literature a status of being scientific. Given our finding that only half the sources are scientific publications, the literature gives an illusion of being scientific, when it is rather a merge between research and texts written for teachers, which Alvunger and Wahlström (2018) describe as normative texts. The prospective teachers are left with the challenge to recognise what is research and what is an opinion or a tradition.

Our findings show that the studied mathematics teacher education did not offer diverse perspectives of theoretical underpinnings nor diverse educational philosophies. We argue that a scientifically grounded education requires prospective teachers to be invited, not only to read, but also to interpret research. Such education would enhance prospective mathematics teachers to become well-informed professional teachers with scientifically

[3] https://www.scimagojr.com/countryrank.php?category=3304

grounded teaching practices (Alvunger & Wahlström, 2018). However, our analysis only show who wrote the course literature and of what kinds of texts it consisted. A deeper analysis of how different theoretical underpinnings and educational perspectives are covered and contrasted in a teacher education would give a more nuanced understanding of what possibilities the prospective teachers are offered to expand their imaginations and challenge the set ways of teaching of their culture.

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Indigenization, decolonization, and reconciliation in mathematics education

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The aim of the study was to explore mathematics teacher educators' views on indigenization, decolonization and reconciliation in Nepalese classroom context. Three universities teachers' educators were selected purposively. A qualitative-interpretive inquiry was used as a research method to conduct in-depth semi-structured interviews with three teacher educators. The interview data was transcribed and read several times, for constructing themes and subthemes. The major themes of the study were: local mathematics in indigenous knowledge, indigenous for meaningful mathematical concept, indigenization for ownership of learning, decolonization as an empowerment, and reconciliation in the emerging context.

Introduction

The term indigenous knowledge (IK) describes local, culturally specific knowledge unique to a certain population at a place. IK is often depicted as being alive, in current use, and transmitted orally (Simonds & Christopher, 2013). IK may come from the development field to describe, for example, agricultural methods or uses for botanicals that come from local knowledge (Simonds & Christopher, 2013).

Decolonization requires teachers to know Indigenous worldview to illustrate the connections among reconciliation efforts, decolonization, education, and the science of story work (Macmath & Hall, 2018). Similarly, an approach might be employed not only in social studies contexts, but also in the sciences and mathematics in the provision of notions such as re-inhabiting, decolonizing, Indigenizing, and reconciling to work with marginalized rural students (Lowan-Trudeau, 2017). Decolonization, however, is a broader concept that encompasses actions and processes that counteract, reverse, or terminate all of these phenomena. Claiming that decolonization was a partial and multi-layered process, comprising different elements, causes and consequences, still requires us to situate the phenomenon in time and space (Collins, 2016).

IK involves all students, teachers and community members. Indigenization as a discourse through academic writing and research emerged the early 2000s (Mihesuah & Wilson, 2004) and has increased in scope and depth to become a globally transformative movement.

Reconciliation is improving the relationship between indigenous students and non-indigenous students in the classroom. It helps to rebuild trust, avoid blame each other.

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Similarly, Willie Ermine (2007) argues about the ways in which these processes are related, explaining that reconciling Indigenous and Western worldviews. Reconciliation is complex; it means different things to different people. Canada's Royal Commission on Aboriginal Peoples (1996) outlined a straightforward definition more than twenty years ago: "Canada must dispense with all notions of superiority, assimilation, and subordination and develop a new relationship with Aboriginal peoples based on sharing, mutual recognition, respect, and responsibility" (Battiste, 2013, p. 26).

Decolonization in mathematics education

Indigenous education has to be linked to "antiracism and social justice" (Government of British Columbia, 2015a, p. 13). Reconciliation requires coming to terms with schooling as a tool of colonialism that has harmed indigenous people. The school system was designed to maintain and impose colonial language, culture, and identity (Battiste, 2013, p. 30). Bringing indigenous culture into the school systems without first decolonizing relationships can be used to mask racial logic as the bringing in of indigenous culture does not acknowledge white settler privilege, colonial injustices, and practices of erasure (Battiste, 2013). Decolonization requires a "deconstruction and a reconstruction" (Battiste, 2013, p. 10). The indigenous renaissance has deconstructed and discredited the traditional Eurocentric views of indigenous peoples and their heritage as exotic objects that have nothing to do with knowledge, science, or progress. However, it has not displaced the educational empire of European knowledge (Battiste & Henderson, 2009).

Indigenous pedagogy

Indigenous pedagogy seems essential to teaching mathematics for meaningful understanding. It may support students in valuing their mathematics learning experiences. Similarly, the British Columbia Ministry of Education identified seven principles necessary to "integrate Aboriginal world views and knowledges" and their rationale (BC Ministry of Education, 2013, p. 3). A teacher should integrate IK in mathematics curriculum with teaching activities. He or she can relate the mathematical concepts to local materials and cultural artefacts. Teachers should foster positive relationships with diverse students, between students, and individual to individual.

Integrating indigenous knowledge

The framework of learning for integrating IK supports integration of IK with mathematics. It guides teaching and learning to enhance students' learning in mathematics. One of such frameworks is The First Peoples Principles of Learning (FPPL) implemented in BC curriculum (FNESC, 2011). This focuses on action, reflection, integration and connection of mathematics contents to local knowledge. So, it is useful in our diverse cultural context.

Mathematics teachers can use local learning materials for giving meaningful mathematical concepts. For instance, teachers can follow the FPPL while integrating IK in

mathematics depending on local social and cultural context. However, there is less practice of integrating IK and pedagogy in mathematics classroom in Nepal and many other places.

The purpose of this study was to explore the mathematics educators' perception and practice of indigenization, decolonization and reconciliation in Nepalese classroom context. The research question was: How do teachers perceive and practice of indigenization, decolonization and Reconciliation in mathematics class room?

Review of literature

From Indigenous perspectives, indigenization of the academy refers to the meaningful inclusion of Indigenous knowledge(s), in the everyday fabric of the institution from policies to practices across all levels, not just in curriculum. Marlene Brant Castellano (2014) envisions Indigenized education to mean that every subject at every level is examined to consider how and to what extent current content and pedagogy reflect the presence of Indigenous/Aboriginal peoples and the valid contribution of Indigenous knowledge. Mihesuah and Wilson's (2004) Indigenizing the Academy: Transforming scholarship and empowering communities is one of the first texts put forward by Indigenous scholars who reflect on what it meant to indigenize the academy. Those students and scholars re-claiming claiming their Aboriginal identity as a decolonization process (Pidgeon, 2016).

Teachers can practice mathematics teaching and learning through integrating IK and pedagogy in the classroom. In doing this, students can more easily perceive the mathematical concepts and ideas. These practices can help to evolve ownership of the subject. We can deconstruct mathematics pedagogy and knowledge and integrate the indigenous knowledge. Similarly, Pidgeon (2016) concluded that non-Aboriginal peoples must take responsibility and be part of their own decolonizing process and move towards reconciliation. In an Indigenized institution, Indigenous peoples remain empowered in their self-determination and cultural integrity. I want to know how best the teachers helps to indigenize the classrooms. Teaching may be effective and meaningful if teachers connect mathematical concepts with local knowledge and cultures. In our context, every mathematics students and teacher might evolve their own identity through decolonization process. However, it is challenging due to the dominance of western colonization. I think indigenization focuses on study of aboriginal students. Indeed, reconciliation will require mutual learning from, between, and across Indigenous and Western knowledge systems, without privileging Western knowledge, or appropriating IK through training and education. It must respect and recognize the diversity of Indigenous approaches (Levac et al., 2018).

It is important to acknowledge indigenous students through reconciliation with Greenwood's pillars of decolonization and reinhabitation (Lowan-Trudeau, 2017). There are two underlying assumptions of the policies on cultural diversity and culturally inclusive learning: (a) individual differences caused by various socio-cultural factors have to be managed in terms of individual equity and (b) individual stakeholders are required to have the self-awareness ability to participate in cultural diversity. These assumptions are completely opposite to Indigenous collectively homogeneous approaches to learning. In IK

contexts, individual needs are aligned with community as collective consciousness through interrelatedness and interconnectedness (Dreamson, Thomas, Hong, & Kim, 2017). In my opinion, we can evolve mathematics according to the needs of our society.

It is believed that indigenous knowledge, in general, can be used to promote the teaching of mathematics in multicultural classes. We can use game for the identification of mathematics concepts on enjoyable way and it develops Spontaneous interaction, association of competition, enjoyment and recreation amongst students as they communicate their activities to fellow participants (Nkopodi & Mosimege, 2009).

A dialogic process develops strong community partnerships, creates opportunities for cultural exchange between stakeholders, and supports their equal involvement in planning, delivering and evaluating learning and teaching (Dreamson, Thomas, Hong, & Kim, 2017). The concepts of interrelatedness and interconnectedness determine Indigenous holistic pedagogies and form the metaphysical foundations for understanding interculturality, culturally inclusive learning is more than ensuring equal participation of students from culturally diverse backgrounds because equity is culturally bound.

Some Balinese cultural elements: ceniga, the yard size based on asta kosala-kosali, and also the pattern of Legong dancer position were used in teaching suitable plane figure geometry and practical formulas are taught to the students (Wijayanti, Sunardi, Tirta, Margaretha & Wijaya, 2019).

Mediation plays a significant role in learning mathematics. It suggests that children learn from others and society through active interactions and participation in activities in groups. Scaffolding and guidance are necessary for learners. “Vygotsky described the Zone of Proximal Development (ZPD) as a distance between child’s ability in independent problem solving and potential ability of problem-solving with guidance” (Panthi & Belbase, 2017, p. 4). Habermassian technical, practical and emancipatory interest emphasized three ways of knowing are empirical-analytic, historical hermeneutic and emancipatory. This theory focuses on contents, meaning making and critical thinking. I employed this theory in my study. The task of empirical analytic science incorporates technical cognitive interest; historical-hermeneutic science incorporates practical interest, and the approach of critically oriented science incorporates the emancipatory cognitive interest (Habermas, 1972, as cited in Grundy, 1987, p. 10). The practical interest is defined as a fundamental interest in understanding environments through interaction based upon a consensual interpretation of meaning (Grundy, 1987). Historical-hermeneutic knowing is meaning making from historical records or literary.

Research method

I employed a qualitative-interpretive inquiry as research method. ‘Interpretation’ within a hermeneutic study is not an easily described data analysis method. Van Manen (1990) explicate interpretation as a “free act of seeing meaning” (p. 79) within the data. I chose three teacher educators from three universities was purposefully selected for each university in the Kathmandu district. I employed open ended interview questions; each interview lasted

about 35 minutes. I transcribed the recorded text data into Nepali and then into English. I read transcription in several times and applied thematic analysis to constructs themes and subthemes. A hermeneutic process of interpretation required reflexivity with questions on, data collected and its interpretation (Berger, 2015). I used pseudonyms of my participants as Chintan, Manan and Utkhanan for anonymity.

Results and discussion

Five themes emerged from text data – local mathematics in indigenous knowledge, indigenous for meaningful mathematical concept, indigenization for ownership of learning, decolonization as an empowerment, and reconciliation in the emerging context. I discussed each theme by connecting to praxis, as the interplay between theory and practice (Panthi, Luitel & Belbase, 2018) as follows:

Local mathematics in indigenous knowledge

I asked first question to my participants, what is your views about indigenous knowledge of mathematics? The following excerpts are samples of participants' views on this question:

Chintan: IK is a vague term. It includes students' prior knowledge, we integrate IK on western mathematics for knowing the concepts clearly and it is necessary for learning mathematics. We can use the indigenous mathematics representation as a basis for learning western mathematics. If we develop the base of IK of current mathematics, it helps to develop the concepts and meaning understanding of the subjects. Moreover, it supports a new form of learning concepts and coherence of learning mathematics. It is an essential for us. It comes more or less in our classroom practice (Interview, 13th May, 2020). He emphasizes that IK is a basis for meaningful understanding of modern mathematics.

Manan: IK is that which includes matters of agricultural, Industries, poultry, metal works, jewellery, transaction which includes exchange system gives the concepts profit and loss, counting system which includes measurement. He focuses on creating meaningful environment and teachers' gives task according to students' interest. We think whether it is creating mathematical reasoning or not, it is the intentionally creating but not naturally evolved (Interview, 14th May, 2020). The very existence of all things – mountains and waterholes, plants and animals, ourselves and our languages, the way we live – is evidence of the actions of the creator ancestors (Love, Moore, & Warburton, 2017).

Chintan: The dominance of western knowledge exists in our mathematics. If you ask question to the students, they can share their experiences. Teacher: What is a circle and circular shape? Student: circle is a two dimensional figure with fixed point and fixed distance from that point, such as bottom of Theki, Sikka, Nanglo (Nepalese Local name of indigenous objects). It is easy to learn mathematics integrating indigenous knowledge. We emphasis to develop thinking knowledge of our students. We can ask questions about cylinder, cylindrical shape such Theki, Dhungro, Madal (Nepalese Local name of indigenous objects) etc. teacher can create an environment to discuss about things which the students have already knowing

objects in the classroom and link with mathematical concepts. (Interview, 13th May, 2020). He argues that students give reasoning mathematical concepts to the local objects.

Using indigenous ways of knowing in research methods is different from using or benefiting from indigenous or cultural knowledge per se. Nonetheless, the use of Indigenous ways of knowing to better understand a topic—to make an impact on eliminating health disparities, for instance—may lead to the exposure of IK and the challenges we have raised in this essay (Cochran et al., 2008).

Indigenous knowledge for meaningful mathematical concept

I asked second question to my participants, how do you use indigenous knowledge for meaningful mathematical concept? The participants' views on this question have been presented in the following excerpts:

Chintan: IK is necessary for early knowledge, easy to form concepts, coherent learning, develop ownership learning, students friendly learning, respectful learning, cultural respect learning, shared learning, discussing learning, western approaches such as inquiry based approached, constructivist approach, project based approached support to investigate Indigenous knowledge. It is easy to integrate IK through progressive approach in the classroom and vice-versa indigenous approaches (Interview, 13th May, 2020). He viewed that IK is valuable for meaningful and effective learning.

Manan: We should promote IK which can use for understanding the power, nature and application in mathematics. We can apply it if it supports to understand abstractness of mathematics. We should provide knowledge of mathematics to the students for making competent and skilful in the world. We should preserve indigenous ways of knowing (Interview, 13th May, 2020). It is used in modern mathematics for enjoyment on meaningful learning. The most significant aspect of decolonization is a process of historical and ideological change (Collins, 2016) in mathematics teaching and learning.

Reconciliation in the emerging context

I asked third question to my participants, what is your views about reconciliation in the emerging context of mathematics education in practice? The participants' views have been presented as:

Chintan: Reconciliation is process of building relationships, making different mathematical concepts as whole content and making compatible. It is the relationship between knowledge level and pedagogical level, it is also a two-way relation between progressive approaches and indigenous approaches. There is a relationship between IK and western knowledge. There is a friendly relationship between students and teacher. It is easy to transform learning. It emphasizes the relationship from individual to individual, teacher to students, knowledge to practice (Interview, 13th May, 2020). It focuses on building a relationship between teachers to students, students to students, students to society, teacher to society, individual to individual. The reconciliation is changes outside remedial relations

between Indigenous and non-Indigenous peoples in terms of past destruction (Lowan-Trudeau, 2017) of cultures and land. Manan and Utkhanan supports this view.

Decolonization as an empowerment

I asked fourth question to my participants, what is your views about decolonization as empowerment in mathematics from local practice? A sample of participants' views have been presented in the following excerpt:

Chintan: The concepts of decolonization come from Latin America. It is the reclaiming, reviving, challenging, making on practice of native languages as a medium of instruction, cultures, approaches, knowledge of the mathematical concepts. It is a response, reject of colonial ideas. It is similar with anti-colonial thoughts. However, there are two type of colonization such as physical and ideological. We discuss here with ideological colonization or mind colonization. We are guided by others, we have no new thoughts of our own. We have colonized thoughts. If we develop own ideas in mathematics knowledge, that can be a decolonizing practice. We can practice own practice such as Gurukul (traditional teaching) practice. It is challenging on our structure (Interview, 13th May, 2020). He focuses on deconstruct mathematics knowledge and pedagogy. Decolonization focuses on a practical (local context) or intellectual problems (Collins, 2016) in mathematics teaching and learning. Chinn (2007) found that critical professional development employing decolonizing methodologies articulated by indigenous researchers Abbott and Smith has the potential to raise teachers' awareness of the connections among personal and place-based experiences, cultural practices and values, and teaching and learning.

Indigenization for Ownership of Learning

I asked fifth question to my participants, what is your views about indigenization for ownership of learning mathematics in local practice? The following excerpts show the participants' views:

Manan: We cannot say that our mathematics is western mathematics because anyone can add new knowledge. For instance, base 10 system is Hindu mathematics. So, mathematics is common domain where it includes mathematics of every cultures. For instance, Euclid can collect mathematics in thirteen elements where it contains mathematics of various cultures such as Eastern and Western. Mathematics is a large bag. It is expanding different time periods in different places. Parallel lines, perpendicular lines, straight lines are included in eastern mathematics and even in western mathematics knowledge. (Interview, 14th May, 2020). He highlighted on our gurukul system which focuses on rote memorization and different mathematical concepts, He believes that modern mathematics includes IK and it is a large bag. We can also connect indigenous mathematics and pedagogy on modern mathematics for making practical in our cultural context.

Furthermore, Mana said: Indigenous Knowledge depends on for what, for why. Do we practice IK our children? We focus on value of mathematics. It is better to use to connect our mathematics for learning easily, procedural understanding, increasing motivation level and

raising questioning. Other beauties of mathematics is abstract reasoning. There are some challenges for teaching logarithmic functions, exponential functions using IK (Interview, 14th May, 2020).

Utkhanan: We synchronize IK into modern mathematics. We should not detach IK from modern mathematics. We learn mathematics from our own ways. We adapt with new knowledge relate to our indigenous knowledge. We can use story telling method, telling poem, telling song, wring genre etc to develop the concepts of mathematics. However, Chandrakala Devi Dhananjaya, who is the first mathematician, she wrote poem related about mathematics. She did not generate meaning. She believes in gaining knowledge by rote memorization. We can use indigenous pedagogy in mathematics classroom (Interview, 14th May, 2020). He views that we can use various strategies to construct generate mathematics knowledge meaningfully associated with local knowledge such as mathematics on temple, artefacts, Doko, Ring, Tauwa, Ping, Nepali musical instruments.

Insightful thoughts

IK is a bases for western mathematics learning meaningfully. It is an essential for developing new knowledge and pedagogies. It evolves ownership of teaching and learning mathematics. Teachers synchronize IK of knowing into modern mathematics. For instance, they can show Nanglo, Sikka for the concepts of circle and they can also demonstrate dhungro, Theki for the concepts of cylinder. Use of IK makes mathematics meaningful and developing ownerships. Indigenization is the process of reflection of current content and pedagogy in terms of indigenous knowledge. It is also a reflection practice between teachers to students, between students, individual to individual in the classroom. Furthermore, decolonization is the reclaiming, revive, challenging, making on practice on own languages, cultures, approaches, knowledge of the mathematical concepts. Similarly, reconciliation is process of building relationship, making different mathematical concepts to whole, making compatible.

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Dis-affirming mathematics education practices: An edutopía in Colombia

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The process of building an academic community of mathematics educators, concerned with issues of social, cultural and political nature within a developing country like Colombia is a hard endeavour, that demands to address and embodying critical stances in several ways. This paper reports and analyses the experiences of a Colombian research group in mathematics education (EdUtopía) are discussed, in order to elucidate some of the theoretical and methodologies findings achieved as a group, as well as the challenges and consequences that arise when the paradigms of a conservative academic community are called into question. Three aspects of the group's journey are highlighted: their integration process, the initiatives and forms of organization they have had up to now, and the impact and reception of their work.

The desire of make a contribution to the formation of critical citizens for a democratic and peaceful society, while working as mathematics teacher is a complex matter in a country like Colombia, since the national research tradition in the field is still ill-grounded and fundamentally guided by a diffusionist conception of science development, that assumes a colonial scheme of center-periphery (Matharan, 2016), in which local researchers role is reduced to receive, import, translate and apply notions, theories and methods created by others coming from a "civilized" society. As a result of such conception, the majority of the mathematics education (EM) research in our country focuses its efforts on teaching decontextualized content, with practical purposes of merely improving scores on international standardised tests and mainly under cognitive approaches[1].

We want to share our attempts to break the referred colonial scheme. To do so, this text reports experiences of our research group in mathematics education (EdUtopía), that works since the last 12 years under approaches of social, cultural and political nature. We analyse

[1] We refer to Flórez & Céspedes (2019), García (2014), Jaramillo (2014), León (2014), Ortiz (2014) and Gómez (2000) as previous overviews of the colombian community of mathematics education researchers.

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the theoretical and methodologies that have been used, as well as the challenges and consequences that our displacements of tradition implied. This text presents three aspects of EdUtopía's becoming: our integration process, the initiatives and forms of organization we have had so far, and the impact and reception of our work within the Colombian educational community.

We contend that studying the trajectory of a research group in mathematics education that runs counter to a hegemonic trend serves to create strategies, sub-versions and alternatives, by seeking a mathematical education that have in its horizon a critical democracy with social justice, in pursuit of a well-being that counterbalance individualistic and economic rationalities

De-formation

When the research group was constituted in 2009, all the members of the EdUtopía research group were mathematics teacher educators at the Universidad Distrital Francisco José de Caldas (UDFJC), in its undergraduate and graduate programmes. In particular. The undergraduate program provides a university degree in education, and is considered atypical when compared with other mathematics teacher training programmes in Colombia, due to its intended classroom work methodology (oriented towards problem solving) and also for its curricular structure. Such training programme is, in itself, a curricular research project, organised by four problematic/thematic cores that aim to train teachers-as-researchers in a wholistic way, distancing itself from other programmes whose teacher training interests are more mathematically oriented.

The program problematic/thematic cores are: School Mathematics, Advanced Mathematical Thinking, Professional Contexts and Teaching Practice. Through each one of them, the program is driven with/towards a high social and political sensibility. It is important to highlight that this social sense used to be fostered in a general way by teacher educators with a human sciences background, linked to the Professional Contexts core, who were not necessarily trained in mathematics. It was not clear how the pre-service teachers managed to relate mathematics to their own experience. This gap led to the possibility of creating EdUtopía as research group, aiming to reflect on those issues from a Critical Mathematical Education perspective.

EdUtopía began as a group of colleagues and friends, with relatively similar initial backgrounds –mathematicians or mathematics educators– from public universities in the capital of Colombia, with research interests in mathematics education and with an inclination towards socio-cultural and political aspects of mathematics teacher education: ethnomathematics, the history of mathematics, the philosophy of mathematics, the didactics of mental operations, the incorporation of Information and Communication Technologies (ICTs) in mathematics education. Some of our previous work experiences were developed around segregated communities, where we realise that social issues emerging from grassroots organisational processes in communities were not addressable from the cognitive approaches, because such approaches imagined improvement of education only as a matter

of improving teaching. The approach to social issues, the need for collaborative work that support communities' agendas, as well as the willingness to think about citizenship education within the mathematics classroom, forged our interests.

This kick-starter point brings us to a new field of work that became a line of research within the UDFJC. We initially took references in the ideas of researchers such as Alan Bishop, Ubiratan D'Ambrosio, Orlando Fals Borda, Paulo Freire, Ole Skovsmose and Paola Valero. As the EdUtopía research group, we discussed statements from critical mathematics education such as "the social precedes the mathematical", "everything educational (or everything human) is social" or "mathematics is a non-neutral knowledge", which we still find revealing and challenging for the field of teacher education. Thus, in the EdUtopía research group:

- We began to incorporate ideas into the different training spaces of the undergraduate or graduate courses that each one was in charge of, even when such spaces were fundamentally disciplinary in mathematics, forcing us to make it clear that the socio-political training of future mathematics teachers it is not a question of context, but structural and linked to mathematics itself.
- We sought alternative methodological horizons of work, different from those that were customary in research groups (v.gr. the use of control and experimental groups with rigid planning), due to our inclination towards contemporary methods of social research and our disengagement with positivist research.
- We discuss ideas with other research groups at the UDFJC and other universities, with the aim of contrasting views on the objects of research, incorporating a closer relationship between objects and subjects of research, employing more reflexive methods and having purposes that could be more sensitive of the social impacts of research.
- We directed last year student's projects, based on a principle of horizontality in which pre-service teacher and teacher educators were academic peers, not directors and supervisors. We worked on concrete problems experienced by the future teachers when they made their practicum and internships
- We interacted with in-service teachers, from different parts of the country, having a direct impact on classrooms and their contexts, reporting results in different sets of publications.
- Former pre-service teachers created their own collectives of teachers on critical perspectives, maintaining spaces for communication with Edutopía.
- We continued graduate studies as personal initiatives, at master's and doctoral level, according to our own specific theoretical interests within the social, cultural and political aspects of mathematics education.

One of the first places we created as a group was a reading seminar. This seminar was later joined by students interested in participating in the group's activities. When one of our colleagues ran an elective course on Ethnomathematics, we went on a field trip to the region

of Tierradentro, one of the settlements of the Nasa indigenous community. We did not go there to teach or to impose agendas or methods, but rather to learn about their forms of organisation and interaction through assemblies, which is highly influential to our own group structure.

After a couple of months, we proposed a research project entitled "Learning milieus as a Critical Mathematics Education proposal in the professional development of in-service mathematics teachers" and worked with a group of mathematics teachers from a public school in Bogotá, the Colegio Distrital Paulo Freire. On this experience:

> The group of teachers met weekly for a year, to discuss and reflect on the educational practices of many teachers at school, and on how these practices reduces mathematical knowledge only to the disciplinary, prioritising the use of algorithms and detaching such knowledge from any context. (Leal & Torres, 2011, p. 8, our translation).

In addition to the final report of the project, several of the reflections that emerged from these meetings were subsequently published by the participating teachers, with whom learning milieus were constructed to address social problems perceived in the school (Ángel & Camelo, 2010; Cardozo, Chaparro, & Mancera, 2010; Leal & Torres, 2011; Mancera-Ortiz, Camelo & González-Alvarado, 2015; Sánchez & Torres, 2009). Years later, these teachers still remember the experience with the EdUtopía research group and point to it as the most significant among those they have lived through, due to the lasting impact on their classrooms, the ways of participation that were provided and the type of collaborative relationship that was built there.

Up to this point there were no stumbling blocks. However, the invention of new forms of group organisation also brought with it many challenges and disagreements, failures, discarded experiments, and a certain loss of momentum. These inventions and its pitfalls are related in the following section.

Dis-organisation

Our ways of working have changed substantially during these 13 years. Although we initially assumed the usual dynamics of collectively executing a project (Leal & Torres, 2011), later we incorporated criticism of the way we carry out our own work and ventured into various forms of group production: reading seminars on theoretical foundations of critical mathematics education, joint writing and cross-review exercises, which produced some texts (see Sánchez & Torres, 2009; Mancera-Ortiz, Camelo, & González-Alvarado, 2015). In 2017 we decided to hold a series of remote meetings with colleagues from other groups and universities to discuss cross-cutting issues of importance for critical perspectives. In each meeting we addressed a particular topic, sharing elements from personal research experiences and then exchanging the reflections we have made on the subject, seeking to establish affinities, differences or even divergences between positions.

There were initiatives that finally did not materialise, such as the writing of the proceedings of the remote meetings, the hosting of academic events in alternative spaces and formats, and the establishment of cooperative alliances with research groups from another

country. In the period from 2015 to 2020, the group adopted a new strategy in order to provide support to the doctoral studies that four of us started: we undertook exercises of active listening, collaboration, commented on individual productions, as well as work in subgroups (Parra et al., 2017; Marcone et al., 2019).

Now more recently, after getting our doctoral degrees, we have promoted academic meetings between master's students that we advise in our universities (Universities of Cauca and UDFJC). In these meetings, our current and former students have interacted as academic peers of colleagues in other universities, a circumstance that has allowed exchanges of ideas and positions on mathematics education from their own teaching experiences.

It is possible to identify common elements in all the forms of work that we have undertaken: a) the horizontality, b) the plurality of individual sensibilities and interests, c) the intense debate that the proposals entail, d) the fraternal spirit in carrying out the debates, and e) the back and forth between theory and practice.

Our commitment to *horizontality* questions the endogamy and hierarchy that we observe as habitual and established in training and research spaces of Colombian mathematics education. We decided not to embrace working and organisational schemes in which a "principal" researcher decides the group's research agenda and acts as its spokesperson. Our working sessions can include the presence of undergraduate, postgraduate students and invited colleagues, seeking to take care to assign the same responsibilities and times for the construction of arguments. This way of working demands more time in decision-making, because we work and discuss until we reach consensus and collaborations. Leaderships are transitory and always limited to specific and circumstantial activities. Another consequence of this absence of a centralised spokesperson is that there is practically no presence in institutionalised or administrative spaces. Membership and permanence in the group is not seen as a tool for obtaining institutional rewards (which in Colombia ensures workload time or funding for research projects).

Regarding the *plurality of individual sensibilities/interests*, it is important to mention that EdUtopia does not operate as a study group dedicated to the work of any particular author or pre-established concept. The criterion of relevance to discuss or not a topic or author does not include anything than its potential to address social concerns about mathematics and its teaching. EdUtopia's current research agenda is the result of a combination of interests and approaches that go through the philosophy of mathematics, ethnomathematics, the philosophy of difference, the constitution of subjectivities and mathematical modelling. The most visible consequence of this combination is a kind of conceptual and methodological indefiniteness, which, far from being a symptom of stylistic dilettantism or eclecticism, is a sign of openness and intellectual curiosity. We assume ourselves to be akin to, but not pigeonholed with, socio-cultural and political approaches to mathematics education. EdUtopía is conceived as an agora, an open stage in which to communicate our changing concerns.

The notion of critique is central to the group not only as a theoretical notion, but also as a working precept. The *intense* (and sometimes merciless) *debate* of postulates and arguments

has characterised us, to the point of creating an internal dynamic that could not be followed by some members who chose to leave the group. We seek to embody critique in terms not only of elaborating critiques, or to address situations of crises in the practices of mathematics or mathematics education in the contexts in which we are in the world, but also as an awareness towards the limits and conditionings of research practice itself.

For us, critique comprises, among other things, the attitude of problematising, questioning, illuminating the blindspots of what appears to us as mathematics educators as normal, good, necessary, neutral and unproblematic. When a member wants to work on an paper or a project, they know that they can rely on the others to find weaknesses and omissions in the theoretical and methodological decisions. The passage of time and several episodes of confrontation and conflict have led us to assume our fierce character, not as something automatically harmful, but as a mechanism to elaborate our ideas.

The *fraternal spirit* is the flip side of the feisty character. Members have established relationships of friendship, support and affection that have helped us to strengthen each other and to deal with the heat of certain discussions in a depersonalised way, as well as to establish a space for companionship in the face of the loneliness and competitiveness of our professional practice as academics. Life's adversities have also been a space for expressing solidarity among us.

Our conception of critique as a working value is also visible in our actions to subject the academic discussions of a theoretical-methodological nature to the contrast with school practices. This implies *a two-way exercise*, where practice and theory are mutually challenged. We have made this exercise concrete through a frequent search for articulation with international and local academic communities, where we invite colleagues from other groups, backgrounds and workplaces to debate our ideas, as a way to counteract the atomisation and insularity of academic productions. We have experiences of collaboration with rural and urban teachers' collectives for periods of more than 10 years, without the need to be executing a funded project or even without the aim of producing an academic paper. These collaborations are given under the premise of considering the teacher as an intellectual pair who can and should raise theoretical reflections, which is why we have co-written articles and papers with in-service teachers. Conversely, we also participate in academic spaces in which we try to address tensions and concerns arising from teaching practice, under the premise that theory must deal with the here and now of educational realities.

Dis-approval

Assuming our affinity with socio-cultural and political approaches to mathematics education has allowed us to be critical of theories that systematically ignore "those at the bottom of the social fabric". Such systematicity is an inconvenient practice, since it has served to invisibilise, hide or disfigure the reality of the social, cultural and political problems experienced in Colombian society (and in other developing ones). This affinity goes hand in hand with the idea of thinking the right to education under the conception of its possible

recipients, assumed as "those from below", those from the periphery, the inferior ones, or, as Galeano (1971) would say, "the nobodies". They are part of the new social, racial and gender segregation, since our school system has been built taking as a reference its addressees – workers, poor, peasants, blacks, etc, but assumed as inferior in the power-knowing-being scale.

Aligned to that, we consider that we must be cautious to not fall into the historical dynamic in which the State has served, as a tool or instrument, to exercise class hegemony (Haya de la Torre, 1994). It is time that, among all of us, we give ourselves the opportunity to think about our countries, and this implies accepting that the educational demands and needs of today's population are different from those that the classical educational offer has met. In this respect, Tenti-Fanfani (2005) mentions:

> Today, the educational demands and needs of the population are different. Differences of all kinds (ethnic, cultural, social, gender, etc.) tend to be asserted and seen as legitimate, and different aspirations cannot be satisfied by a simple expansion of classical educational provision. (p. 20, our translation).

From this perspective, questions arise such as: what does it mean to meet the educational demands and needs –in mathematics education– of the current Colombian population? In what ways does mathematics education facilitate or impede the construction of a critical democracy in Colombian society? What forms of work emerge in/among the communities[2] with whom we share?

Meeting these demands has led us to have a constant struggle against the status quo of mathematics education in Colombia, which maintains that any educational proposal must focus its interest on school mathematics content established as common, generalised, standardised, and susceptible to being measured by international tests; this content favours the handling of procedures and algorithms deprived of context and sense, even without noticing who will be the recipients of such an educational proposal. In contrast, within the group we consider the social to be a fundamental aspect of learning mathematics. We understand the social as the possibility of bringing –with the students– problematic situations associated with the micro/macro context of those who learn, to be studied collectively within the educational institution and supported by mathematics, in order to interpret and re-interpret such situations.

That insight led us to another consideration: in order to pose situations of interest to learners, we must know who our students are, which brings with it an inquiry into the contexts in which they find themselves, perform and live together. Such knowledge is not a superficial or geographical matter; it also refers to the deep social, cultural and political relationships and interweavings that embody multiple historical moments.

Therefore, we distanced ourselves from positions that consider students as universal and cognitive subjects, who by the fact of being in a classroom are ready to learn mathematical knowledge. On the contrary, we understand that our students are more than a homogenised

[2] Groups of in-service and pre-service teachers, indigenous people, students and parents across the country.

body, that they are in constant relationship with others, that they are subjects with likes, dislikes, desires and dispositions, who sometimes distance themselves from the act of knowing from the academic mathematical logic.

We consider that our starting point must take into account the knowledge of our students and their social, cultural, historical and political context that conditions their lives, from which we must raise, as teachers, specific problems or topics that emphasise the responsibility of students as citizens. This is concretised in the possibility of participating in an informed way in the making of decisions that affect us all, making use of mathematical tools and in the pursuit of social justice.

The interest in the relation between mathematics and social context has placed us in a dual scenario, as we assume a permanent confrontation of arguments between those who consider viable approaches to socio-cultural and political approaches to mathematics education such as ours (approval) and, at the same time, with those who reject it (disapproval).

It is enough to see how many of the teachers who are in the process of continuing their training process (in postgraduate studies, for example) see themselves identified with our approach, given their daily experiences at the schools[3]. We are not unaware that there has been a certain "boom" among Colombian mathematics educators, either by approaching the socio-cultural and political perspectives of mathematics education, or by trying to give arguments such as "all didactics is social", or "all mathematics is social", given that they admit that their usual approaches have lacked this analysis, and realise that questions about the critical and citizenship education being done through mathematics are increasingly emerging with force.

Convergent statements from school teachers contrasts with the positions of academic colleagues in our national community, who state that the social is important (as a meaningless catch-all phrase) but who claim that it is our only duty to focus on disciplinary and didactic mathematical knowledge, keeping disciplinary knowledge in a preponderant status in the educational process, insofar as it is necessary and sufficient to empower those who learn it. There are others who position themselves by pointing out that the social aspect is important, but optional for the mathematics teacher, placing the social aspect in the background, even in a condition of elusive and/or postponable, as if it were not a constituent dimension of educational action. Is it visible how re-emerges the traditional perspective of knowledge as disembodied, universal and hegemonic, which colonises and superimposes itself on other knowledge forms.

Dis-considerations

In the previous sections we have recounted our attempts to do practice and research in mathematics education in unconventional ways that escape the narrow frameworks of a

[3] For example, they share our view that students are much more than cognitive subjects, and that it is therefore important to consider the social, cultural and political context that surrounds them.

tradition present in academic and professional bodies. This search is motivated by a conscious and public stance on our social agency as mathematics educators and has led us to question aspects that are taken as natural and resolved, such as the relationships with the groups that are investigated, the particular conceptions about the very purpose of the field of mathematics education and the role of the researcher. Our stance on these aspects has enabled us to learn and achieve things that would have been unattainable within the established models of research practice. At the same time, we have had to deal with the consequences of going against the tides.

With regard to relations with the groups being researched, in contrast to certain forms of academic extractivism, where communities are approached to obtain data for imposed and circumstantial research, we have chosen to: a) have more direct, in-depth and continuous contact with the different actors involved in the research, b) to visit their territories guided by them, recognising situations that they consider as critical, and c) to make them participants in the decisions on the topics, methods and results of the research, calling for collective reflections on the impact of the experience on the transformation of the chosen situations.

We believe that conceiving the field of mathematics education as devoted to the finding and reporting of "successful" experiences that can demonstrate progress in the quest for the improvement of teaching conveys the assumption of a non-existent homogeneity of the students and their educational realities and, ultimately, implies a delimitation of mathematics education that we find exhausted and inappropriate for realities such as Colombia's. We have responded to this reduction with a vindication of the heterogeneity of the individuals, interests and living conditions that inhabit the communities. In turn, this has led us to recognise the usefulness of reporting all experiences, whether or not they fit into the established concept of "success", since they reflect the issues that challenge us and leave communities aware of the challenges to be taken on.

Finally, in relation to the role of the academic, the established tradition naturalises a unilateral and omnipotent role in the decisions, reflections and writing of research. Such a role places the researcher in a struggle for visibility and positioning, which is evident, for example, in calls for funding, awards, and institutional recognition. Subverting the expected prominence for the researcher, we implemented collaborative research and writing exercises authored by teachers and university students. We also decided to abstain from participating in institutional calls and research measurements, which has allowed us to open up to broad work agendas, not subordinated to pre-established terms of reference. This determination also allows us to do the research we want to do and not the trendy research that is applauded by the fashions of the moment, or just the research that is being paid by funding agencies.

This text intends to foster a discussion with MES colleagues on the singular experiences that they have lived in their own path, namely about the ways in which research groups are i) de-formed, ii) dis- organized and iii) dis-approved within their local communities. Such discussion can help the MES community to reflect about the conditions of possibility for research in mathematics education, and how some working notions (v.gr. *horizontality*,

plurality, etc.) can emerge. In short, we believe that to take a critical stance on the officially accepted practice of research is to take up a struggle for authenticity and independence.

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Altered traits of alumni from a collaborative learning environment

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A rural STEAM centre aims at developing responsibility in children, the courage to create projects and the competency to work together in this collaborative learning environment. Here children have freedom and responsibility to plan what they work on, develop mastery in Mathematics through projects in electronics and programming, work individually or with peers, etc. These factors appear to support develop a positive attitude towards Mathematics in children at the centre. This paper is based on reflections of alumni two years after graduating from the centre. What factors do they still appreciate? We examine both the skills and attitudes they developed, retain and apply in their further education and life. What are the altered traits alumni retain when they transition from such centres to mainstream education?

Context and introduction

C3STREAM Land (C3 is *C*onscious for Self, *C*onscious for Others, *C*onscious for Environment, STREAM= STEAM+ R (Research), henceforth referred to as C3SL) are rural STEAM

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centres in Tamil Nadu in India. This paper follows the alumni of the centre at Udavi school, one of the outreach schools of Auroville. Auroville is a universal township that works towards human unity and engages with the villages around Auroville. Most of the children attending the school come from Edayanchavadi village.

Udavi school aspires towards the holistic development of the child and follows the Tamil Nadu state board syllabus. C3SL works with 80 children from 6th to 10th intensively for 5 hrs/week for all their Mathematics classes at Udavi. The STREAM centres have been in operation in the school for the last 5 years. In demographics, the occupation of parents of the children is unskilled labour (35%), skilled labour (55%) and salaried workers (10%).

The aim of the C3SL centres is to develop the qualities of responsibility, equality and the courage to create alternatives. At C3SL centres the children learn to take responsibility of their learning and have freedom to plan and set their goals each week. They can choose to work individually, with peers across grades in multi-grade environments or with facilitators. They have access to Mathematics materials, strategy games, puzzles that help them engage with mathematics and play games. They have access to computers where they program in Scratch, Geogebra and Alice and also 3D modelling and printing. They also have access to electronics, Makey-Makey, robotics and other materials interacting with engineers who work in the industry. These help children not only address their curriculum, but also create projects that demonstrate their mastery on topics learned. The use of programming to develop mathematical thinking at C3SL has been documented before (Ranganathan, 2015, pp. 339–346).

In India the lack of interest in education is attributed as the number one reason for children to drop out of high school (Government of India, 2018, pp. 126). Among the subjects Mathematics is considered the most difficult subject requiring special attention not just in India, but across the world. While both literature (Köller, Baumert and Schnabel, 2001, pp. 448–470) mainstream teacher experiences suggests that the interest in Mathematics deteriorates as children get to higher grades at C3SL there was an improvement in attitude towards Mathematics in children from 7^{th} to 9^{th} grade from when they were in 5^{th} grade. We examined the factors based on the interventions mentioned above earlier (Ganesan, 2019, pp. 894–898) when children were part of C3SL and had access to it.

In this paper we look at the following research questions:

1. How alumni reflect now on the factors attributed to the improved attitude of children towards Mathematics when they were at C3SL. Specifically:
   - the responsibility and freedom given to plan their own work
   - creating projects physically and with programming on mathematics, electronics, robotics, etc.
   - choice to work alone or with peers
2. If alumni found their experience at C3SL useful and what skills they learnt that they have applied in higher education.
3. Given the diversity of what they experienced, other factors they valued at C3SL.

4. If the experience at C3SL resulted in altered traits in alumni in determining their choices in higher education and how they perceive they would like to learn.

Altered traits are defined as a new practice or attitude that endures beyond the environment that helped create it (Goleman & Davidson, 2017, p. 8)

Philosophies underlying C3SL

The philosophy underlying the approach for C3SL is based on the principles of progressive and constructivist thinkers like Jerome Bruner in the United States, Sri Aurobindo and Mukunda in India and many others briefly described here.

The theoretical framework of the work at C3SL is based on the three principles of true education by Sri Aurobindo (1921, pp. 18):

- Nothing can be taught
- The mind needs to be consulted in its own growth
- From near to far

The first principle can be linked to the constructivist theory that knowledge cannot be forced into the mind of a child, nor can a child be moulded or hammered into the form desired by the adult. That the teacher can guide, support and encourage a child in the process of learning, enabling them to evolve towards perfection.

It further indicates that each human being is in their own path of discovery and progress. This is also recognized in the National Education Policy (Government of India, 2020, p. 12) which states that 'knowledge is a deep-seated treasure and education helps in its manifestation as the perfection which is already within an individual.' We would like to examine how the children have continued their journey and what role, if any, C3SL had in it.

The second principle indicates that the child needs to be consulted in his/her learning. This is done at C3SL as the children to plan what they want to work on (choosing a plan to work on) and how they want to work on it (self-work, peer-work, creating projects, etc). The children have this freedom and responsibility and we examine what the alumni felt about this.

The third principle is to work from near too far. To work from what is tangible and accessible to children to what is abstract to them. The children work on projects they care about, materials they can access and manipulate and move towards abstract ideas. They also create projects and programs at C3SL that take abstract ideas in Mathematics and convert them into visual projects that make them concrete.

Constructivist Education Theory (Bruner, 1960) is in line with the philosophy that knowledge is not delivered into the learner (whether child or adult) but recreated by the learner on his or her own. Children actively construct their knowledge by connecting new knowledge to what they already know.

Contemporary thought is also aligned with these ideas, Mukunda (2009) describes the three aspects of learning that are relevant to schools – conceptual knowledge, procedural knowledge and higher order reasoning. Conceptual knowledge (and change), she states, greatly benefit from constructivist approaches. Procedural learning benefits from learning to

program a computer to do the procedure. Higher order skills benefit from problem solving methodology.

While earlier curricular frameworks in India had already suggested shift from 'useful' capabilities to understanding and application the NEP 2020 states clearly:

> Pedagogy must evolve to make education more experiential, holistic, integrated, inquiry-driven, discovery-oriented, learner-centred, discussion-based, flexible, and, of course, enjoyable.

In the context of the nation-wide conversations captured by the NEP the interventions being carried out at C3SL of creating experiential, learner-centred and enjoyable collaborative learning environment that challenges children to show their mastery by creating projects is relevant. Equally important is how these interventions are perceived - noticed, appreciated and retained by the children themselves over time.

Methodology of the research

Selection of alumni

We use purposive sampling focusing on one batch of alumni of C3SL at Udavi School who had been at the centre in their 8th and 9th grades and were now in other schools in their 12th grade. This batch was selected because we were looking for a sample of students who have been away from C3SL for at least 2 years and had also experienced it for at least 2 years. The alumni are following the streams of humanities (arts), commerce and science with a predominant number in commerce.

This is a qualitative case study and the data collection consisted of

A group sharing of the nine (of thirteen) alumni on what they are doing and what they have retained and used from their experience at C3SL.

This was followed by a review of the intention of each of the 20 questions of the survey.

The children then filled out the survey individually.

Four children who could not attend the group sharing filled out the survey remotely without (a or b).

The primary data collection for this research was 'c' and 'd' and 'a' was the secondary data collection used for cross verification.

The survey was created with the intention of having both a qualitative (Linkert scale) question linked to a descriptive one that would clarify the choice with details and reflections. The selected Alumni were in 12^{th} grade and it was expected that they would be able to respond to these questions in simple English in writing. We felt that this would give them the time to reflect and respond rather than take interviews with us.

The survey had both broad questions like usefulness of C3SL in their life, or their memories of C3SL, as well as pointed questions on specific interventions of C3SL. We expected the descriptive questions to bring out the diversity of responses of how they engaged in with C3SL in line with the specific teachers experience of the children's engagement at C3SL. We also included questions to verify if indeed their current

environments were different from what they experienced at C3SL, how they coped with the change and what they preferred and more importantly why.

The conversation and the responses were analysed to understand what the children have found special, useful and what continues to be useful and impacts them about C3SL. The descriptive responses allowed us to check for factual consistency with their teachers and our records e.g., of what children they had created in C3SL. While we focus on the three interventions all the questions and responses of the survey is available online (C3SL, 2020) for review. The children know C3SL by its former name STEM Land and this is the name used in the raw data for children. We, however, continue to use C3SL to retain the flow of this paper.

Survey and responses

We had noted factors or interventions that had improved the children's attitude towards Mathematics when they were at C3SL. We reviewed these with the alumni to see if they still found these interventions significant.

Intervention: The freedom given to plan their work

At C3SL, we believe children are responsible for their learning and for their growth. Children create a plan of what they are going to learn each week. They are assessed each week on the goals they work towards.

To understand whether children prefer to plan what they want to study or prefer that the teachers plan for them we asked two questions, *'Do you get the freedom to choose what you want to learn in the school you are in now?'* and if the response was negative *'Would you prefer to work with freedom of choosing what you want to learn or not?'*

Except two students all of them said that they don't have the freedom to choose what they want to learn in the schools they study now and all of the students said they prefer to work with freedom of choosing what they want to learn. One of the children specifically said that he wants to have his freedom to choose what he wants to study, but he can't because of society since the importance is given to marks rather than knowledge.

We have also asked children *'What are the difficulties you face switching from the system we follow in C3SL to other school environment?'* Children said that they perceived the system in C3SL as having the freedom to choose what they want to do, access to resources needed and being allowed to interact with everyone. This made C3SL a joyful environment. They felt not having such an environment limits learning as they are not allowed to use resources even though they are there, nor find support when needed. They have also said that they had to memorize everything, had to always study and don't have any other activities. One child remarked 'By choosing to learn I dedicate more in learning.' [By choosing to learn I am more dedicated in learning].

Intervention: Creating projects physically and with programming on mathematics, electronics, robotics, etc.

In C3SL children are encouraged to create projects to demonstrate their mastery of mathematical concepts. Children learn programming early through peer learning and with facilitators. Children come up with their own idea on projects based on what they learned in their chapter these could be physical projects e.g., building a game or in software e.g., creating a visualization of a concept or an interactive project or game.

We asked *'Did doing projects in Scratch, Geogebra and Alice help you in any way?'* All of them said doing projects using Scratch, Geogebra and Alice helped them to remember the concepts in Mathematics. All the children said that doing projects helped them in many ways such as to improve their speed in Mathematics and is easy to understand Mathematical concepts. One of the students said that doing a project in Scratch helped her to learn graph, and computer programming. One of the children said that Scratch helped him get a different perspective, Geogebra to learn geometry. They still remembered the projects they made and owned their work. In response to their current status all the children said that they don't get the resources, time and support from staff in the school they are currently attending for such activities. One of the children said that she feels more comfortable working with computers when compared to the other children in the school she is currently pursuing.

We asked them about projects not directly linked to academics that some of them worked on 'Were you able to learn mathematics concepts when you work on electronics, robotics and Big-shot? 1 – less, 5 – neutral, 10 – more.' All the children were positive or neutral about this and in the descriptive responses four of them said they were able to learn mathematical concepts while working on electronics, robotics and Big-shot cameras.

Intervention: Choice of working individually or with peers

At C3SL children have a choice of working individually or in groups. This allows for individual learning as well as peer-to-peer learning. In addition, in a week there are some multi-grade classes which allows children to work across grades. At C3SL the children learn about themselves and how they like to learn in response to the question, *'How useful was peer learning?'* one of them indicated that they liked to work alone, another was neutral about it the rest preferred to work with peers.

Further we asked children *'Do you continue to learn with peers in the other schools?'* Most of the children have said within the classroom they can't continue peer learning as they don't have an environment where they can communicate with other students or share freely with teachers.

In response to *'Have you taught anything you learnt from STEM land to your friends in other schools?'* Almost all children indicated that they had taught something to others they mentioned using Ubuntu, programming in Scratch and Alice, html, practical geometry, solving the Rubik's cube. Further, in response to an earlier question on application of what they had learned children shared that they are helping their friends and classmates

understand concepts through alternative methods and taught puzzles and Rubik's cube to others.

Usefulness of C3SL

We asked the children to rate and answer 'Was C3SL useful for you (on a scale of 1 to 10) and if so in what way? 1 – Not Useful, 5 – Useful and 10 – Very Useful.'

Figure 1 indicates all students felt that C3SL was useful for them. In the descriptive response thirteen children indicated that C3SL helped them learn programming and Mathematics through Scratch, Alice, puzzles and games. One child reflected that he used to look for answers to the questions, but his experience at C3SL has helped him look instead at the methods of solving a question. One child said that it is useful for his higher studies specially in Mathematics and Physics. Another said it is useful for him to learn animations and electronics. Yet another said it is useful to learn new things. Each child brought forward a different facet of C3SL based on their own interest and what they explored at C3SL indicating that each found something he or she had explored based on the diversity and freedom given to them.

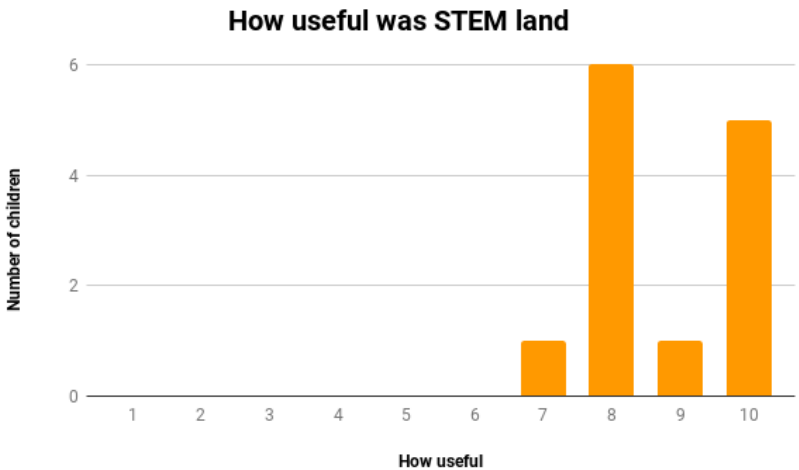

Figure 1: Rating of 'Was C3SL useful for you (on a scale of 1 to 10)?'

What they retained and its application in higher education

We asked a question *'How much are you able to retain what you have learnt in C3SL? (1 - Not Retain, 5 – Retain, 10 – Retain Very Well)'.* Most children felt they were able to retain concepts they learned in Mathematics as well as what they learned in programming through Scratch, Geogebra, Alice, Goanimate as well as working on hardware such as Arduino, Mindstorms, Makey Makey which are programming related or Mathematics related.

To the question *'How much (of this) are you able to apply in your higher studies?'* the children wrote that they felt that they found Mathematics related subjects such as Accounts, Economics as well as Computer Science related subjects Computer Application and Computer Science easy due to the similarity with what they learned in C3SL. Children also mentioned that they are helping their friends/classmates in understanding concepts through alternative methods in which they had expertise. One of the children mentioned that he learned lots of shortcuts in Mathematics and is able to apply them in his higher studies. In the conversation before the written survey one child even mentioned that she was among the only two girls in a class of fifteen in a Computer Science and she was topping the class.

We also asked a question *'What course are you pursuing?'* Most of the children who have graduated from C3SL have opted for Mathematics related fields such as Commerce, Science, Computer Science and Biology with Mathematics. One student who took humanities (Arts) took up Computer Science as an elective.

'How are you doing in Mathematics as compared with other subjects? 1-10.' Out of thirteen students seven of them said they are doing well in Mathematics as compared to other subjects. Two students mentioned that Mathematics helped them to score more marks in other subjects like Accounts and Physics. The remaining students did not have Mathematics as a specific subject in their course. Others indicated that they did not have Mathematics but Computer Science they do well in.

Interventions noticed by children

We looked across the questions on aspects noticed by children about the environment created at C3SL and how they felt there. In a response to *'One thing you feel proud of being in C3SL'* a child wrote 'freedom, relationship, learning, understanding among student and staff'. In response to the transition to mainstream schooling a child wrote 'we enjoyed the environment in C3SL but now we are totally away from happiness.' On the response to the question *'Did playing games help you learn Mathematics?'* the children had mixed responses to whether playing games helped in Mathematics as such, however, one child wrote, 'yes! that made time to relax my mind', similarly in response to *'How interested were you in solving the weekly puzzles? 1-10'* while most children said they were not interested again we had a different child write, 'I felt my mind get relaxed and very interested.' The diversity provided at C3SL for differing tastes of children allowed each child to find something they enjoyed illustrated in *'Share your memories of STEM land.'*

> I loved playing games and getting the freedom to choose what I want to learn. I very much enjoyed the (leadership) workshops. Because I learnt a lot about myself and my class mates.

The children were sharing about the leadership/stewardship workshops we had offered children to make choices from their possibility (universal values they care about) rather than socialized fear. Another shared, 'I liked Stewardship that gone (was held) weekly once with our team staff. I enjoyed it and I am still following my stand and fear.'

The diversity of what touched the children in C3SL *'One thing you feel proud of being in C3SL'* four students said that learning programming and being able to create projects made

them feel proud at STEM land. Two students mentioned that soldering and electronics kits made them feel proud of STEM land. Three students have said that they feel proud of being a STEM land student. They have also mentioned that STEM land gave the additional knowledge apart from text books and that they feel proud of the teaching method being part of a buddy system, having freedom to choose what they want to study, exposure to games and logical puzzles. One child captured the environment as 'freedom, relationship, learning, understanding among student(s) and staff(s)'.

Conclusions

Improvement of interest in Mathematics with grade is an important goal of Mathematics education across the world. The interest of children in Mathematics had improved in children at C3SL due to the interventions such as:

- the responsibility and freedom given to plan their own work
- creating projects physically and with programming on mathematics, electronics, robotics, etc.
- choice to work alone or with peers

while children were in C3SL. This research found that alumni even after two years after they graduated from C3SL and no longer experiencing such an environment perceived these interventions as formative to their understanding of how they like to learn, that it made it environment joyful and made them responsible.

All of them felt that they would like to continue plan their own work as they did at C3SL and choose how they want to engage with others. Even through most were not allowed to engage with peers within their classrooms now, they were supporting peers and had taught something they had mastered at C3SL to others beyond classrooms.

Each of them brought forward a different facet of C3SL they engaged with some in response to specific questions on projects, programming, electronics, games and puzzles and some as a response to generic questions about their experience e.g., leadership. A few of them specifically brought up the leadership and how it continues to play a role in their lives allow them to work from possibilities (universal values they care about) and transcending socialized fear.

We observed that the children had retained skills not only in Mathematics, but also in Programming, Electronics, etc. Further they had developed a positive attitude towards these and most had taken up Programming in their higher education even if they had chosen Arts as their discipline of choice across gender.

The children not only understood what they had learned, but also had clarity in the development of logical thinking, strategies and multiple methodologies of solving questions. They also shared that they found the learning environment joyful.

We see that interventions at C3SL had not only short-term effects on improving attitude of children towards Mathematics, but also longer-term impacts in their attitude towards how they understand themselves and how they learn. We term these as altered traits, indicating

the impact of the few years they engaged in C3SL continues to have in their lives moving from one 'among the crowd' to 'standing out of the crowd'.

We believe these interventions if introduced as part of Mathematics classrooms could support children taking responsibility for their learning as well as create supportive environments for their peers. This is relevant not only in the National Education Policy 2020 for India, but also for children across the world to use Mathematics and STREAM not only as skill development, but of developing responsibility both towards themselves and others.

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The role of space in mathematics education

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Throughout my everyday life as Mathematics teacher I have been thinking about the conditions I put my students through. At some point the question about the role played by 'space' emerged. In this paper, I pursue the conceptualization of 'space' and, in particular, I try to understand the role played by this notion in the development of mathematics education in the remote teaching context. My intention is to conceptualize experiences offered to students based on my own professional context. At first, I explain the context where my question emerged, then I share the comprehension of 'space' I am assuming. I also share a panorama of some research in Mathematics Education, and, finally, I set up some questions to the community, as well as, I set a research agenda.

Forewords

First of all, I conceptualize the difference I am assuming between mathematics education starting with lower case or capital letter. By the latter, I mean the research field, the area of inquiry. By the lower case, I mean the everyday practice of a Mathematics teacher, the actions that take place in relations where the intention is teaching and learning something called Mathematics. In the title and in the scope of this paper, I am referring to the second case.

The main concern here is the category 'space' and how this concept becomes present in the education process, particularly in the teaching of Mathematics. The context is the remote teaching implemented in Brazil during the pandemic. It is not my intention to draw a panorama of the education during this time in this country, but to highlight a case study as exemplary of a kind of situation faced in these days. The situation becomes clearer in the section 'Some exemplary cases'.

As many teachers all around the world, I myself had to deal with some challenges and uncertainty during 2020. The pandemic brought upon humankind a brand-new obstacle for us to face. It is not possible – at least desirable – to list these obstacles as they are the same to everyone. Instead, I picture a scenario where I live to bring some common ground to this argument. Writing about my particular context, in Brazil, this text intends to share some thoughts about the role played by mathematics education in this scenario.

The remote teaching I am referring to was implemented in Brazil in the first semester of 2020. In the institution where I worked during that year, it was implemented in March. It consisted mainly of using Google Classroom to send to students activities covering the content expected in a grade. This was combined with online meetings, in the time when the

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class took place, or at some other time scheduled with students, and also with the use of WhatsApp. In addition, to substitute the live meetings some teachers used recorded video lessons with related tasks, shared in the Classroom, and assisted students following a schedule.

Despite all the efforts to engage students in this remote teaching, each month a greater number of students were late with their activities. Following the situations of these students in the Help Students Service, provided by the institution, I saw that a considerable number were leaving the remote activities to help their families in maintenance duties.

This situation it is not a surprise in itself. Considering the socio-economic conditions in which these students live, it becomes clear that helping the families with the maintenance activities is a duty of the children since a young age. The responsibilities increase as they grow up. At 15 to 17 years old, if they are not at school, they have to take part in the main source of funding of these families. In the next section I present an overview of this scenario.

Some exemplary cases

During 2020 I worked, as Mathematics teacher, in an institution that offers simultaneously technical education – integrated in high school level – and undergraduate courses. The students at high school level have the possibility to live in the campus, while attend two courses: Agroecology or Farming, both full-time – because they have to cover the national standards to the high school level in half-period and attend technical training in the other half. Because of the possibility of living in the campus, many students are attracted from faraway cities and come from very different backgrounds. The accommodation is the main criteria for the final decision of some parents to send their young to the institution – the students are 15 to 17 years old, the legal age in Brazil is 18 years old.

Both courses can be completed in 3 years, if the students pass all curriculum components first time. During this time, the routine of the students starts at 7:30am with breakfast. The classes start at 8:00am and finish at 5:15pm, with 1h15min for lunch at 12:00am. All the meals, including two coffee breaks, are offered without cost. The funding comes from the Federal Government and the agricultural products produced in practical classes are used to complement the meals. This routine begins on Mondays and repeats until Fridays when the students leave the campus and go back to their families during the weekend.

Because of the pandemic situation, students were sent home in the middle of March. After 15 days of interruption in classes, these returned in remote mode as an option to students who wanted to continue their studies. Over the year, while the scenario got worse and worse, the remote classes became mandatory and all were required to return to their studies. This decision added 2 problems in this scenario: (i) It was necessary to think about curriculum adaptation because in the same class there were students following the course since the beginning of remote education and students who started when it became mandatory, and (ii) a number of the students that were not attending the classes, not because they did not want to, but because they did not have the conditions to.

To help the major group of the students that did not have access to the necessary technologies for remote education (e.g., internet access and laptops), the institution started programs to lend mobiles and laptops, as well as offered scholarships that could be exclusively used to pay for mobile internet access. But, these programs could not reach all the students. There was not a sufficient number of devices to meet every request and there were students that live on farms or in villages where there is no mobile signal to access remote classes.

Besides all the technological issues, another concern came to be the relationship with the family. I could refer to the lack of an appropriate place to study, or the absence of support, but the main concern is the social role played by these students. While living in the campus, the young were spared from the work at the family place, a condition that was no longer available during the pandemic time. So, the students had to conciliate a full-time course with some duties in the family place. These duties include, but are not restrict to, taking care of children, attending informal jobs to help the family financially, and dealing with agricultural activities.

It is not surprising – but very concerning – that the rate of students leaving the classes or with results less than expected, was very high. These cases, together with the pedagogical issues related to teaching remotely and concerns about the learning of those students with full possibilities to attend the remote education, put the institution in a very complex position.

Among all these issues I came to understand that the space where these students were living – homes with some level of lockdown, younger siblings and older relatives to care for, duties in maintenance – did not allow remote learning to take place. Engaging these students was not a question about 'contextualization'. The issue was not to make these students see the Mathematics in their everyday life. As I was realising, the Mathematics became present only when none of these other things were happening. I mean, the education process that in pre-pandemic times was the main concern of these students, in pandemic times, was dropped. It took place only in the time they could save from everyday life activities.

My understanding turned into a concern when as I was reading the Students Support Service report. About one of my 16-year-old student, who was not attending the remote classes, it was written:

> […] a home visit was carried out to understand the situation experienced by the student (father with knee injury that prevents work activities and mother undergoing medical follow-up after cancer treatment). The carrying out of agricultural activities is under the responsibility of the student. They have dairy cattle and all management is divided between him and his younger sister. They report that the internet is shared by a group of residents and they are the last residence and when it is peak time (after 6pm) there is no way to use it, access becomes slow. We suggested printed activities and the student reiterated that he prefers to fail and do it all over again in the next year. He believed that they would return to face-to-face classes soon, and as he accumulated so many activities he did not see the means to resume. (Report about student A).

At this point, it seemed to me that, to this particular student, he had been denied access to the education process. The activities that were proposed by remote education could not fit into the space he was living in. The demands placed by his social relations were not accounted for the scheduled activities; on one hand because he was attending a full-time course, on the other hand because he was no longer living in the campus, but in a space that was not structured around the curriculum. His access to the education process also was denied when the institution insisted on bringing him back to the ongoing process. The reality of this student made claims to other ways of being.

As pointed out by Skovsmose (2005), some learning obstacles are the result of being subject to an absurd policy. I mean, some processes of exclusion can be hidden in the way a policy is dressed up. What else could the remote education be, seen from a perspective based in the everyday life of student A? The 'absurd' is not a characteristic of the remote education in itself, only in contrast with the space where this student lives might one state this value judgement.

This same question could be raised based on 16-year-old student B's report. About him it was written:

> There is no internet at home, installing one access point by radio has a very high cost. No cellular network. He lives in the farm zone at 25km from the centre of nearby city. He gets in touch only when he is working at his sister's house (planting onions). He expressed concern about not been promoted to the next grade. I asked about the possibility of contacting the Municipality Education Department of the nearby city to provide space to take the classes, but the student claims that he has a logistical problem, they are without a car and he uses only a motorbike (without license) to get around. I did not feel the willpower on the part of the student to seek a solution to his problem. He knew about the Digital Aid Notice, but he didn't sign up. (Report about student B).

Instead of pursuing a value judgement of remote education, I take the road of asking myself what could a mathematics education do to deal with this matter from a non-colonizing perspective. I mean, how could I analyse this situation without normalizing it. Neither a perspective centred in institutional curriculum and a formal process of education that blames the student, nor an everyday life-based perspective that denies the values of a formal education.

My concern might be formulated in these following questions: How does education occupy the space that children and families live in? How does everyday life make itself present in the education process of these children? What is the relation between education and space? It is in response to this last one, that I wrote this paper.

Milton Santos' concept of space

Milton Santos was a Brazilian thinker, very important to the Geography field. His work has become relevant in different research areas. His is best known for his theorization around Space and Territory. As a researcher he went beyond theorization about 'oppressive political economic and colonial relations that have shaped Brazil. Within his writings and practices

he also, crucially, fought for and found promise in counter-rational, non-hegemonic values and knowledges placed at risk by global capital.' (Melgaço & Prouse, 2017, p. 2).

The notion of 'space' raised in Santos' work involves at the same time the form (the objects in the space), the function of these objects (the action that takes place in relations to that objects), and the structure (the social movement around those objects), that is, Santos' proposal is to understand space as process and product of social relations.

> The space would be a set of objects and relations that take place on these objects; not among then specifically, but for which they serve as intermediaries. Objects help to establish a series of relationships. Space is the result of the action of human kind on their own space, intermediated by natural and artificial objects. (Santos, 2014, p. 78, my translation).

From this perspective, the notion of 'space' is produced from multiple determinations whose origin are in different levels and numerous scales, from simple places until the international dimension. Space, as mentioned in the quote above, are not only the landscape, but the localized combination of social variables. The international dimension might be understood as the process of globalization read from a Marxist perspective.

Therefore, space cannot be studied as if material objects have in themselves their determination. It must also take into consideration an object's social function and the structure that surrounds it. Each object produced is full of symbolism, meant to impose the idea of a content and a value that, in fact, it does not have. The meaning of the object is warped by appearance. To Santos (2012, p. 59), '[...] Things are already born full of symbolism, representativeness, intentionality designed to impose the idea of a content and a value that, in reality, they do not have. Its meaning is deformed by its appearance'.

The analysis, in Santos' perspective, starts from the matter itself and reaches the abstraction considering not only the individual reason, but the concrete of things. In his analyses proposal, the form introduces us to the material object, its current function guides us to the process that raised it, and the process guides us to the social structure that gives meaning to that object in a social context.

Taking a panoramic picture of the exemplary case using Santos' perspective, I might affirm that the remote education, as an object materialised in the everyday life of the students in this case, brought in its determination some socio-economic categories historically produced. I mean, remote education was not only a task that the students must accomplish to realise some grade for their course. It was also a way of life, socially determined, into which the students had to fit. This structure related to the object imposed a globalized pattern of living.

When the determination of remote education encountered the determination of the students, a fight among structures took place. At this point it was expected of the students to decide. The Students Support Service expected that this globalized way of living fitted the ongoing routine of the students. The students waited for the moment when they will have space to carry out two separate foregrounds: one during the week at the campus, the other at the weekend in their families.

Connecting this perspective based in Santos' work with a critical read provided by Skovsmose's point of view, I need to look to the dispositions of these students to try understand their engagement or non-engagement in remote classes. In this process it is necessary to consider the place where the students were, imposed on them, by the material determination, ways of meaning production and roles that must be played. This determination is historically constituted and governed by socio-economic categories. I mean, no decision taken by these students was an individual decision.

Either by the decision in itself or by the possibilities seen by the students, the totality of the social relations, materialised in the space, guided their foregrounds.

Corroborating Skovsmose (2005), what was the point in studying Mathematics when in their place, Mathematics was not for them? In other words, the foregrounds of these students had been defined by the role played by space in the mathematics education, not by their background. It is so, because the space – form, structure and content – drove these children in one way that no longer included remote education.

In the reports quoted early, Students A and B were viewing their currently space as a provisory one. The students hoped for a future where they might go back to the campus and restore their missed everyday life. This landscape is the space where education is allowed to exist, only there. But, at that moment, the space where they were, full of material determinations, guided their disposition to ways of living where such things as online meetings and scheduled activities could not exist.

Someone might say that the notion of foregrounds is sufficient to analyse the situation. To those people I will express my response as following: analyse the role played by 'space' in the determination of those foregrounds. My focus is on the space, not only in the relations that take place in the space.

At this point, I might say that the remote teaching has highlighted the difference among students. Has been also in the spot the fact of space, in Santos' terms, most be considerate. In the classroom the background of the students is easily erased, and a normative mathematics education takes place. As a sanitised landscape, the classroom imposed a globalized structure over a group of young people who live in such different spaces. During this colonizing process, these individual differences were hidden to promote equality, at least from one perspective, the colonialist perspective.

In my experience, the context of remote teaching failed to hide individual differences and mathematics education was inserted into that space. There, the normative education process is put under another set of rules; instead of dictating, it is ruled under everyday life categories.

Connections with Mathematics Education

As I note, Santos' conception of 'space' is widely acknowledged in Geography. I choose to use his approach in my first steps on the concept of 'space' because of the theoretical support it provides. But I also initiate an overview of works in Mathematics Education that have been, at some point, dealing with 'space' as a concept. This overview involves scanning some

exemplary research; it is not a state-of-the-art review. To proceed I look at some repositories guided by the keywords 'space' and 'mathematics education'. In this section I share some results of this overview.

To Martin and Larnell (2017), their perspective, on urban Mathematics Education, was raised as a response to the idea of urban as a space marked by a profound disorganization, in which mathematics education research, practice and policy are mainly concerned with reorganization of students, schools and community. That is, to these authors urban Mathematics Education goes beyond geographical context. It is imbricated into lives within its diversity in socio-cultural contexts.

In the Brazilian context there are some exemplary works. Moraes and Garnica (2016), bring an analysis of how space has been contextualized in some research, including Mathematics Education and History fields. As underlined by the authors, there are works, for example Arrais (2004), Campos (2012), and Certeau (2000), that understand space as a materialization of social relations, as well as the result of appropriations through symbolic investment that share value judgement in cities. To Moraes and Garnica (2016, p. 84), "spaces are not a neutral, they are created and creators, physical and discursive elaborations, actions and intentions. Spaces shape and are shaped by subjectivities".

Severino-Filho and Silva (2021) establish a characterization of space, based on the work of Certeau (2000), from considerations about intercultural relations that take place in teacher education spaces that work in intercultural contexts with indigenous peoples. For these authors, space is a social-cultural context that emerges from the act of inhabiting a place. Assuming this concept and theorizing about education process, the authors characterize the 'formal education space' and the 'socio-educational space'. This second one is produced by the student community and inhabited by a student at different stages of their life. In this 'socio-educational space' takes place "the different dimensions of non-school education and, consequently, the generation, acquisition, and diffusion of specific knowledge for each space" (Severino-Filho & Silva, 2021, p. 189). Regarding 'formal education space', the authors state that "our school, in its traditional structure, is a space historically reproduced, or imported, with minor adaptations, of European culture. So that the teacher's culture subdues the students' culture differences and silences their previous knowledge, believing that finally, one can replace them with standard school knowledge" (Severino-Filho & Silva, 2021, p. 190).

Inspired by Said (1990), a set of works theorises concepts directly related to the notion of 'space' – despite the fact that this word is not used as a keyword in these works. Marcone (2015) conceptualizes the notion of *deficiencialism* as a way to refer to the relation among people and space in the social context. To him, the category "disabled" people emerges from a power relation in which someone emphasises some characteristic that, in that particular context, in reference to a given pattern of humankind, another person is not able to respond as expected. In other words, the disability results from the relation with the space that these people occupy.

Bose and Marcone (2019) examine the relation among children, place and education. From the observation of a railway platform in Mumbai suburban, the authors theorise about social

justice, equity and the redefinition of young students' foregrounds. When the authors bring Skovsmose (2005) to the discussion, they throw light on some of my concerns. The authors state: "Barriers for learning are often individualised and those obstacles are considered as part of a social and cultural background attached to the person (see Skovsmose, 2005). [...] The fault, then, is on the individual and not on the system." (p. 5).

Unlike India, Brazil is not a caste-based context, but it is not out of the realm of the stratification created by capitalist social structure. When children are not allowed to play their role in this society, as in the case of the students of mine, they also have their foregrounds limited.

So, at this point of my exploration, I regard my practice as contributing to the fragmentation of the foregrounds of my students. In the previous sections of this text, I look at the remote teaching in my context from a point of view centred in the student everyday life. Here, I see it from my perspective as a teacher. Collaborating on the implementation of remote teaching and proposing activities centred in online learning or adapted to paper, I ignored a very important point of the process. In both cases, the teaching was centred in academic Mathematics and there was no place for this category in the space where my students were learning. Because of this lack of legitimacy, the everyday life of the students took over the learning process.

Highlighting this point at least two main concerns become clear: (i) the role played by education in the families; (ii) what kind of education I was proposing to my students. None of these are simple questions. None of these can be treated lightly, given the risk of falling into dichotomies or blaming families or teachers. The risk could be aligned with the discussion promoted by Skovsmose (2005) over classic and progressive racism. As in that case, I have to drive my attention to the social structure that guides the education process.

By searching for problems in the families or in the student attitudes, what remains hidden is the fact that access to education, granted by the Brazilian Constitution, has been denied to these students. I might also hypothesise that homeschooling, an education process not regulated in Brazil, has been promoted under the guise of remote education. Both of these points guide me to discuss the political implications of the questions raised in this paper. For the moment, I maintain my focus on conceptualising space and use these points to develop my research agenda.

What this brief review shows me is that the concept of 'space' has been studied from different perspectives in Mathematics Education. Some of these perspectives assume a characterization that converges to the understanding of space as a product of social practices that occurs in the relation of communities and the place where they live. This understanding seems to converge to Santos' proposal presented in the previous section of this paper.

Now, with this understanding, the question is: what can I do to get over this colonizer relation between education and space? In the following section I raise some possibilities instead of pointing to a final direction.

Forwards-looking

It is necessary to take into consideration that during the period referred to in my description of exemplary cases, I had 140 students registered in my classes. Many of them could attend the remote education without problems. I also had to deal with psychological and socio-economic issues experienced by additional students. The cases highlighted in this text, as I wrote in the subtitle, are exemplary and serve my intention of discussing the concept 'space'.

In relation to the education process it is possible to conclude that Student A and B were dealing with learning obstacles, in Skovsmose (2005) terms. To these students, remote education, unlike as expected by the institution, was not an alternative to continue their classes during the pandemic. All the options provided by the teacher and institution failed in the same point. It was not taken into consideration the social structure interlocked in the objects and places made available to students.

To build a research agenda from that experience and continue thinking about space as a philosophical concept, I raised some perspectives and questions. Bringing together the concerns of Martin and Larnell (2017), I ask: How might mathematics education in the future be even more responsive to issues of access and opportunity in the context of such forces and global urbanism? When it is clear that not everyone is able to follow together today's endeavour, like remote education and quality in the access to technology. It is the global urbanization the only way? Should assistance policies and practices take part in this concern?

Putting space within the scope of an investigation inevitably leads us to talk about the process of colonization. Access to technology in countries like Brazil is a way of maintaining power relationships between the global north and south. Therefore, an education process mainly based in technology, poorly distributed among the society, is a strong instrument to perpetuate power relations that characterise colonizing process.

In a similar direction of Martin and Larnell (2017), my intentions in studying space and mathematics education is driven, in this moment, to the reality of Rural Education. In the same sense of these authors, I am not defining the rural space by its geographical location. So, my intention is to elaborate an approach that could be moved from Rural Education, to Intercultural Education, as in Severino-Filho and Silva (2021), without redefinitions of theorized concepts.

Seems likely that the firsts steps in this research agenda guide me to 2 key-points: It is necessary enlarge my definition of mathematics education; clearly I am addressing issues that are not included in the statement 'everyday practice of a Mathematics teacher', it may be necessary understand this practices in its material, socio-historical and political faces. The second key-point is dealing with the interrogation 'which are the implications to Mathematics Education when a researcher takes this concern in mind?'; a mathematics education concerned mainly with the standards was a cause of a fragmentation of the foregrounds of my students. Particularly, when we look to the interaction between places aligned with the global urbanization and places marginalized in this process, some

mathematics education practices corroborate to the maintenance of this power relation. Mathematics Education, as the research field that deal with these practices, is a solid ground to rise this question.

To the question "what is the relation between education and space?" a provisory answer might consider how the education process is dressed up, as pointed by Skovsmose (2005). In its current way, education hides the requirement of a normalized space. I mean, the formal education process does not only need a student and their desire to be included in this process. A globalized (and very sanitised) standard of space has been imposed within the education process. Particularly with the remote education implemented in Brazil during the pandemic, this standard promoted learning obstacles for a considerable number of students.

The theoretical support offered by Santos (2012) is seen to be suitable to this endeavour. It brings the possibility to carry out socio-economic relationships in the same time allowed for work with objects, places and landscape. At the moment this concern seems necessary to me. In the next steps, I hope to pursue in discussion in Mathematics Education and produce some possibilities and suggestions to teachers.

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Mathematics as a social practice? Antagonisms as a conceptual tool for examining discourses

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This paper discusses the notion of antagonism as a conceptual tool for examining discourses in mathematics education, with a focus on the political dimensions of discourse. Drawing upon a short university classroom interaction about a visual proof, I suggest that the discourse of mathematics as a product and the discourse of mathematics as a social practice are in an antagonistic relation. I illustrate this relation by exploring how each discourse tries to fix the meaning of concepts like proof, formal proof, and mathematics. Finally, I explore how antagonisms can describe political aspects of discourses, by discussing the limits of both hegemonic discourses and counter-discourses, as well as the space for challenging hegemonic discourses.

Mathematics: A product or a social practice?

Mathematics education has been largely framed by what we might call "mathematics as a product", namely the idea that students need to learn the product of academic mathematicians' labour. However, as mathematics education took a social turn (Lerman, 2000), we notice an embracement of epistemological and theoretical frameworks which highlight the social character of learning. At the same time, the focus towards non-academic mathematics (e.g., ethnomathematics) also contributes to the development of a discourse which conceptualises mathematics as a human activity (e.g., Pais, 2013). Thus, we can identify two discourses which conceptualise mathematics in roughly two opposing ways: the discourse of mathematics as the product of mathematicians' labor and the discourse of mathematics as a social practice (of students, mathematicians, or communities).

Valero (2018) maintains that the practices and meanings of mathematics education are not fixed, but there is rather a constant struggle with regards to their validity and legitimisation. The relation between looking at, teaching, and learning mathematics as if it were a social practice versus as if it were a product is an example of this kind of struggles. This paper aims to explore the relation between the two discourses, by discussing how the notion of antagonism (Laclau & Mouffe, 2001) can be used as a conceptual tool, which allows consideration for the political aspects of discourse. I illustrate this idea by using the notion of antagonism to analyse a short interaction in a mathematics university classroom, where a visual proof is presented and discussed. I argue that the discourse of mathematics as a

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product and the discourse of mathematics as a social activity are in an antagonistic relation, each trying to fix the meaning of concepts like proof, formal proof, and mathematics. The paper is organised as follows: I first draw upon the work of Laclau and Mouffe to outline the role of antagonisms in their discourse theory; I then continue with the context, the presentation and the analysis of the interaction; and I finally discuss how looking at the interplay between the two discourses as an antagonistic relationship helps highlight the political aspects of the discourses.

Antagonisms in the Discourse Analysis theory of Laclau and Mouffe

There is a growing body of mathematics education research, which uses discourse analysis as a theoretical, epistemological, and methodological framework. In discourse theories, language is not a mere description of social reality; instead, language plays a role in constituting social reality, or, in other words, the ways in which we talk about things give meaning to them. To illustrate, the case of a student suggesting that 1 is a solution of the equation x+1=3 could be described as them experimenting with mathematical equations, not having mastered the equation solving algorithm, lacking previous knowledge, not being careful, learning by making a mistake, or it could even not be the focus of anyone's attention. Following Doxiadis' (2011) Foucauldian conceptualisation of discourse, all these discourses refer to the very same event, but make different identities available for students, produce different kinds of knowledge about the student and their mathematical learning, and guide the teacher's, students', and researchers' actions in different ways, thus creating different power relations.

Laclau and Mouffe (2001) conceptualize discourses as partial fixations of meaning. They maintain that "any discourse is constituted as an attempt to dominate the field of discursivity, to arrest the flow of differences, to construct a centre" (p. 112). Different discourses struggle to fix the meaning of privileged points called floating signifiers, while points whose meaning has been crystalized within a discourse are called nodal points (Jørgensen & Phillips, 2011). For example, learning is a floating signifier, in the sense that different learning theories, such as behaviourism and social constructivism, struggle to fix its meaning. Learning can also be a nodal point within a particular discourse, such as a constructivist discourse. However, the meaning of "learning" shall never be permanently fixed: even if a discourse (for example, social constructivism) becomes hegemonic, this fixation is never final, as other discourses will eventually strive to contribute to, challenge, or deconstruct this meaning.

This is where the concept of antagonism becomes relevant. Antagonisms occur when discourses collide, struggling to fix the meaning of floating signifiers, or privileged points within the field of discursivity (Jørgensen & Phillips, 2011). For example, in mathematics education discourses, mathematics is a floating signifier, in the sense that different discourses struggle to fix its meaning and its limits. An ethnomathematical discourse and a cognitive theory discourse would (typically) fill the meaning of mathematics differently, thus depicting mathematics education differently. Nevertheless, as antagonistic discourses attempt to dominate the field of discursivity, meaning can never be fixed (Laclau & Mouffe,

2001). While antagonisms can be resolved through a hegemonic intervention, a perspective which has earned social consensus (Jørgensen & Phillips, 2011), this is only a temporary state. Domination of the field of discursivity can never be fully achieved.

The importance of antagonisms as a concept in Laclau and Mouffe's theory lies with the idea that the political is based on antagonisms and is constitutive of human societies (Mouffe, 2005). This is an ontological statement which asserts that it is through discourses and the antagonisms between them that meaning is produced. Stavrakakis (1997) maintains that this theory aims to move from a reductionist logic towards a logic of articulation: "since social identities do not arise from the (class or other) essence of (individual or collective) subjects, they can only be a product of construction, a construction at the level of discourse, a construction that articulates heterogeneous elements and gives them new meaning" (p. 24; my translation). Therefore, the political is constitutive of mathematics and mathematics education: all things mathematical do not exist in an a priori sense; mathematics and mathematics education are/become as a result of antagonisms between discourses.

Antagonism of discourses in a university mathematics classroom

Interaction

In this section, I use the notion of antagonism to analyse a short interaction between the professor and a student in a university "Introduction to Proofs" classroom. I collected these data as part of my doctorate thesis, titled "Political Aspects of Mathematics in two Undergraduate Mathematics Courses". The interaction discussed here is about the formality of a proof and I chose this piece of data because of its potential to highlight the notion of antagonism between two discourses.

Proofs and visual proofs in mathematics education literature

Proofs are considered a vital part of academic mathematics. Proofs do not only verify the truthfulness of a mathematical statement, but have a variety of other functions, including explanation, systematisation, discovery, and communication (de Villiers, 1990). From a socio-political perspective, it is important to consider the role of proofs as devices which can have the final say about a mathematical statement: if something has been proven mathematically, there is no doubt that it is true (e.g., Gutiérrez, 2013). In this way, not only proving but also mathematics itself are constructed as a type of knowledge which cannot be challenged.

The proof discussed here is what we may call a visual argument/proof (e.g., Alsina & Nelsen, 2010), although the professor himself does not use the word "visual" to describe it. While there seems to be a consensus that visual representations are heuristic tools both in the philosophy of mathematics and in mathematics education, their status as proofs or even as parts of proofs is contested (Hanna & Sidoli, 2007). Hanna and Sidoli (2007) talk about the different perspectives regarding the role of visualisation in proofs and distinguish between three conceptualisations of visual representations: as adjuncts to proofs, as an integral part of proofs, and as proofs themselves. We can see this tension (or antagonism between discourses about the status of visual proofs) in how Alsina and Nelsen's (2010) paper "An

Invitation to Proofs without Words" refers to "visual proofs" as "pictures or diagrams that help the reader see why a particular mathematical statement may be true, and also to see how one might begin to go about proving it true" (p. 118). Although Alsina and Nelsen name these visual arguments "proofs" and argue about their importance in mathematical practice, they also describe them as a starting point for proving the theorem, indicating that the proof starts after the visual argument has taken place.

The interaction and its context

The interaction happens in a large class of approximately 150 students. The course is an "Introduction to Proofs" course in a University in Ontario (Canada). Introduction to Proofs courses are typical 1st year courses in North America for students who wish to follow a mathematics-related discipline. Most students in this course aspire to follow a computer science program and in order to enter it, they need to achieve a high grade in the proofs course.

The interaction happens in the chapter of cardinality, which is a formal mathematical concept for describing the "size" of a set (including infinite sets). Here, the class focuses on proving that the cardinality of the naturals is the same as the cardinality of the cartesian product of the integers with the integers (i.e., Z×Z). An alternative description of this statement is that the infinite set whose elements are all pairs between the numbers $0, \pm1, \pm2, \pm3, \ldots$ (e.g., $(-2, 5), (5, -2), (3, 1), \ldots$) has exactly as many elements as the set of natural numbers (i.e., $\{0, 1, 2, 3, \ldots\}$). To prove this statement, the professor presents the visual proof shown in Figure 1, which is based on identifying a spiral representation of all elements of Z×Z, so as to consider them as elements of a sequence (i.e., constructing a bijection from N to Z×Z).

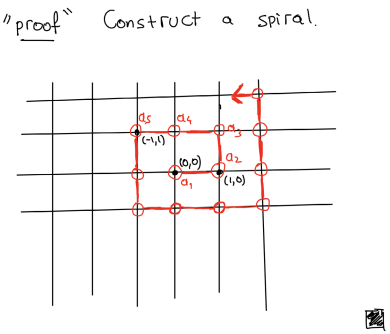

Figure 1: Visual proof (Recreation of the picture that I took of the board)

Before the following interaction happens, the professor has orally suggested that the proof is not a formal one, while – as can be seen in the figure – the word proof itself has been written in quotation marks on the board (indicating that it might not be a "proper" proof). Regarding the reason that this (visual) proof is presented in the class, the professor mentions that "a formal proof of this is annoying" and that the way in which the proof is

written in the textbook might be difficult to understand. After the presentation of this (visual) proof has been completed, the professor asks students if they have any questions and the following interaction happens:

Student: Is this considered a formal proof?

Professor: This is not considered a formal proof, uhm, partly because it's- ok. Here's what [inaudible] Could you find the tenth element of this sequence? And to put it another way, if I asked you to find the tenth element and I asked someone else to find the tenth element in this room, would you come up with the same tenth element?

Student: [definitively] Yes.

Professor: So, that's pretty good. If the answer is "yes", that means that there is a pretty good common understanding of what's happening here. Uhm, ah, we could write that a little more formally, like, take one step to the right, then one step up. Then you go left one more than you've gone, go right one more than you've gone. And, like, I could explain that, like, robotically. But it's not clear that it's going to be easy to understand. Or that it will be, like, you have a better understanding of it. Uhm, yeah, we typically avoid presenting the complete formal proof of this, because it hides what's actually going on.

In this interaction, a student asks if the (visual) proof given by the professor is considered a formal proof. The professor categorically responds that it is not. As he proceeds to explain why this is the case, he asks the student back, whether the proposed sequence is presented in a way that any two people would find the same tenth element. The student responds with a definite "yes", which is a response that does not support the professor's argument regarding the informality of the proof. The professor accepts this as there being a "pretty good common understanding" and then suggests that this proof could be written more formally (through a more "robotic" description of the steps) but it then would hinder understanding.

The two discourses and the antagonism between them

The above interaction can be read as two discourses being in antagonism: mathematics as a social practice and mathematics as the product of an established discipline. In this section, I will first describe how I see each of the two discourses as being present in the interaction and I will then discuss the antagonistic character of their relation.

The first of the two discourses conceptualises mathematics as a social practice. This refers to mathematics being done in and through interactions between people. In the example, we can see the discourse of mathematics as a social practice in the following aspects: the discursive construction of a "good" proof as one which facilitates understanding (e.g., as shown in the sentence: "we typically avoid presenting the complete formal proof of this, because it hides what's actually going on"); the very fact that a visual proof is presented in order to make the idea of the argument clear and communicable; and the need for people to have a shared understanding of the argument (e.g., as shown in "would you come up with the same tenth element?"). In the Proofs class, the discourse of mathematics as a social

practice does not only appear in this interaction; it is also evident in multiple events, including when the professor encourages collaboration and communication between the students, invites students to share their disagreements and discomforts with the presented ideas, engages students in a "collecting data – making conjectures – proving the conjecture" process, emphasises the existence of a diversity of thinking about mathematics among different people, and emphasises the need for mathematical arguments to be understood by their audience(s). In all these events, mathematics is conceptualised as a social practice.

The second discourse is about mathematics as the product of an established discipline. In the interaction, we can see the manifestation of this discourse in the clarity with which a visual proof does not quite qualify as a formal proof and maybe not even as a proof. This clarity implies that the standards of what constitutes a proof and a formal proof are created outside the walls of the classroom, independently of the professor's and students' interactions (e.g., passive voice in "this is not considered a formal proof"). These standards are the result of the practices and norms of the mathematical community, which are in turn reflected in the academic requirements set for novice mathematicians and computer scientists by mathematics departments. In this context, it is the mathematical result (i.e., theorem, proof, etc.) expressed according to the traditional norms and rules, which is of most importance. A formal expression of this result can verify that the result is correct, can be tested, and is expressed in a way in which no misunderstanding may occur. In this discourse, while mathematics as a product might be socially created, it is also reified, taking, in a sense, a life of its own.

The first point which I wish to highlight about how the two discourses are in an antagonistic relation is that by antagonism, I do not refer to the disagreement between the professor and the student regarding the preciseness of the algorithm's presentation; in other words, here, we do not have two people (the professor and the student) antagonising each other. The two discourses are not each voiced by the professor and student respectively; instead, both discourses are voiced by the professor himself. For example, we can see both discourses present in the question "if I asked you to find the tenth element and I asked someone else to find the tenth element in this room, would you come up with the same tenth element?". Here, the idea that a formal proof leaves no space for ambiguity refers to proof as a product; however, the enunciation of it in terms of "actual" people having an "actual" conversation and agreeing or disagreeing highlights mathematics as a social practice.

By suggesting that the two discourses are in an antagonistic relation, I intend to highlight that the concepts of proof, formal proof, and mathematics become floating signifiers, whose meaning is attempted to be fixed by the two discourses. Regarding "proof" as a floating signifier, we see that the professor orally refers to the visual argument as being a proof, but he uses quotation marks when writing down the word proof in the notes. This difference between the oral and written practice can be interpreted in terms of the presence of the two aforementioned discourses. On the one hand, the discourse of mathematics as a social practice attempts to conceptualise proofs as including well-articulated visual arguments. On the other hand, the discourse of mathematics as an established practice attempts to

conceptualise proofs as a formal construct in which each proposition follows from the previous one. The two discourses are differently privileged in the professor's written and oral speech. Mathematics as the product of mathematicians' work is privileged in the professor's written speech, in which the quotation marks indicate that the visual argument is similar to a proof but does not quite qualify as one. Mathematics as a social practice is privileged in the professor's oral speech, in which he never challenges the status of the proof as a proof. The differently privileged manifestation of the two discourses in the oral and written speech can be understood in terms of the different functions of the oral and written speech: in his oral speech, the professor aims to help students understand an argument, while his written speech aims (among other purposes) to act as notes for students to study. In the example, the two discourses collide with each other, while struggling to "fix" the meaning of "proof" in different ways.

With regards to "formal proof" as a floating signifier, the discourse of mathematics as a social activity tries to fix the meaning of the formality of a proof in terms of the existence of a "common understanding". In the interaction, the student suggests and the professor accepts that there is a common understanding regarding the tenth element of the sequence. Yet, this does not result in the recognition of the proof as formal. In terms of collision of discourses, we can argue that there seems to be "something more" about the formality of a proof which the discourse of mathematics as a social practice cannot quite challenge. This "more" is created by the discourse of mathematics as an established product, which is at a privileged position for defining what is formal. Formality – as developed historically – involves much more than the disappearance of the possibility of a mistaken representation, such as compliance with strict linguistic rules. To use Laclau and Mouffe's (2001) concepts, the discourse of mathematics as an established practice is *hegemonic*, when it comes to formality of proofs. Playing a bit more with the metaphor of collision in a visual way, I would suggest that as the discourse of mathematics as a social practice attempts to fix the meaning of formality, it hits the "wall" of mathematics as an established practice. The wall is not unbreakable, but it is extremely hard to break.

As far as mathematics as a floating signifier is concerned, it is notable that the word mathematics is never mentioned in the above incident. Nevertheless, I argue that there is an implicit antagonism about its meaning in the above interaction. As proof has developed to be the golden standard of mathematics and formal proof has developed to be the golden standard of proofs, antagonisms regarding the meaning of "proof" and "formal proof" are also antagonisms about mathematics. In other words, what constitutes "mathematics", "academic mathematics", or "formal mathematics" are at stake here. There is no a priori meaning of these concepts; instead their meaning is constantly negotiated and produced through antagonisms between discourses, which are historically, culturally, and politically situated.

As the two discourses are in an antagonistic relation in the professor's articulation, we can see the limits of each discourse. Laclau and Mouffe (2001) write that "[a]ntagonism, far from being an objective relation, is a relation wherein the limits of every objectivity are shown – in the sense in which Wittgenstein used to say that what cannot be said can be

shown" (p. 125). In the example above, we can see the limits of the discourse about mathematics being a social practice when it comes to the universality of understanding: if universality of understanding fails, then the discourse of mathematics as a social activity fails, too. We can also see the limits of the same discourse in the non-negotiability of whether a visual argument (in the context of cardinality) constitutes a formal proof. On the other side of the antagonistic relation, mathematics as an established practice fails when its formal presentation hinders the meaning which it attempts to purify.

Mathematics as a product vs. a social activity: An antagonistic relation

Besides this course, the antagonism between the discourse of mathematics as a social practice and mathematics as a product is also present more generally in mathematics education without being necessarily framed as antagonistic. In this last section of this paper, I argue that the concept of antagonism is able to capture the relation between the two discourses, in a way which highlights its political aspects. My argument runs as follows: I describe how the two discourses (social practice and product) are evident in mathematics education; I continue by examining and critiquing an alternative narrative which captures their relation as a relation of progress; I then explore how this relation can be understood as an antagonistic one; and I conclude by suggesting that the notion of antagonism is useful for highlighting political aspects of discourses.

For quite some time, in mathematics education, the idea that mathematics is what mathematicians do has been hegemonic. Within what might be called "the social turn" in mathematics education, we see the gradual prevalence of a counter-discourse that suggests that mathematics is a social practice; a discourse that conceptualises meaning, thinking, and reasoning (both in the classroom and in mathematicians' work) as products of a social activity (Lerman, 2001). Contrary to Platonic views which claim that mathematics is somewhere out there (not in our tools, actions, or discussions), the discourse of mathematics as a social practice conceptualises mathematics as happening through people's communication and actions. At the same time, however, the discourse of mathematics as a product preserves much of its hegemony in mathematics education. We can see this discourse in two aspects: first, academic mathematics bears a hegemonic status: its truth, once produced, is not negotiable (Gutiérrez, 2013). Second, as schooling has a function of distributing students into different social positions (e.g., Pais, 2013), the institutional framework is instrumental in reinforcing the discourse of mathematics as a product. For example, "the kinds of mathematics that we teach" is described in curricula across the world through specific outcomes and objectives that the students are required to meet (Abtahi, 2020). Similarly, standardized assessment is enabled through a discourse of mathematics as a product: every student is assessed on the same piece of knowledge, which has been created outside the classroom walls.

One way of describing the relation between the two discourses is through a story of progress: mathematics education used to focus on mathematics as a product, but it has gradually turned towards focusing on the social elements of doing mathematics. In this story,

mathematics as a social practice is framed as a set of more recent moments in a linear progressive path followed by a mathematics education which constantly becomes better, more participatory, and more equitable. In this way, mathematics as a social practice is seen as contributing to mathematics education constantly becoming "better", more participatory, and more inclusive, mitigating or challenging the impersonal, inequitable character of ("old") mathematics. This narrative of progress runs through mathematics education which, both as research and practice, appears to be committed to straightforward progress (Lllewellyn, 2015). Deriving from Laclau and Mouffe's (2001) view on the impossibility of meaning fixation, we can see the fragility of this approach: even if we agreed that making mathematics more activity based contributes to a betterment of mathematics education, there is no reason to assume that at some point, through this practice, mathematics education will become fully participatory or equitable. In my view, this narrative of progress fails to address how the "old practice" is still here, hegemonic, trying to absorb the "new" practice.

The concept of antagonisms (Laclau & Mouffe, 2001) offers an alternative reading of this relation which suggests that both discourses co-exist in mathematics education, while each of them tries to fix the meaning of mathematics; the result of this collision is unfixed. As attempted with the analysis of the short interaction about the visual proof, the conceptualisation of antagonism can be helpful for addressing the limits of both hegemonic discourses and counter-discourses, as well as the space for challenging hegemonic discourses. In the example of the visual proof, the discourse of mathematics as an established practice was hegemonic but we can recognise its limits (e.g., it sometimes hinders understanding) as well as the ways in which the discourse might be challenged, through discourses which value people's understanding and interactions. At the same time, an antagonism-oriented analysis makes visible the limits of the ability of the counter-discourse (i.e., mathematics as a social practice) to challenge the established practice of mathematics as a product.

While there seems to be a wide agreement in mathematics education about mathematics being a "social practice", this discourse seems to often oversee the hegemony and impact of mathematics as a product. In other words, I suggest that the discourse of mathematics as a social practice (within institutions in which students are expected to master a specific material, be examined in it, and be accepted or not in a program based on their performance) falls very short in terms of challenging the existence of a knowledge which will not change, regardless of what the people talking about it think or do. In the example from the Proofs course, the discourse of mathematics as a social practice cannot be fully actualised as it collides with the discourse of mathematics as a product. Looking at mathematics education research and practice more generally, I want to conclude this paper with the following questions. The discourses of mathematics as a product and a social practice both exist in literature and there is a general urge to move from the former to the latter. What are the ideological functions of mathematics education discourses which highlight the social without addressing its shortcomings? Can an antagonism-oriented view of the two discourses' relation highlight how mathematics education research is political and how its

constructs (e.g., the discourses about what mathematics is) are products of and agents in power relations?

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Students dealing with tasks aiming at model- and context-oriented reflections: An explorative investigation

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Generally, reflections on mathematical concepts seem to be important to acquire mathematical knowledge at school. In theoretical works focusing on how mathematical education can contribute to societal participation and active citizenship, model- and context-oriented reflections play an essential role. This paper presents a study in which 8th grade learners were encouraged to perform both types of reflections through selected tasks. By means of interviews, reflection processes at the student level were made accessible. Results from a first, explorative analysis of the interviews are reported.

Introduction: Reflection for mathematical literacy

In this section, the theoretical background of the investigation is presented. Studies and concepts from selected authors will be cited to point out which expectations are associated by the demand for reflection. Within the framework of these concepts some empirical studies have been carried out. A brief look at them shows that there is a gap regarding the learners' individual reflection-behaviors. The present study addresses this gap.

In discourses that specifically deal with a contribution of mathematics education for societal participation and development of society, reflecting plays a particularly important role. In this perspective, reflecting relates not only on mathematical structures, but targets moreover on relations between mathematical concepts and settings in which they are used. In German-speaking countries, discourses about mathematics education for active citizenship relate to the concept of "Bildung" (education) or "Allgemeinbildung" (general education) (Heymann, 2003), whereas in international context such discourses relate to various perspectives of *mathematical literacy* (Jablonka, 2003) or *mathemacy* (e.g., Skovsmose, 1998). For a more detailed description of the German terms "Bildung" and "Allgemeinbildung" and their relations to mathematical literacy, see Vohns (2017, pp. 968–970).

Roland Fischer's approach to "Allgemeinbildung" faces the division of labour in our society. Not everyone can be an expert in every field. Rather, a person is an expert in one field and at the same time a layperson on all the other subjects. For laypeople the capacity of communication with experts, judging and taking decisions for one's own concerns would be crucial. Fischer attributes the preparation for the role of the laypeople to (higher)

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secondary education and deduces that this would require a focus on *basic knowledge* and *reflection* (Fischer, 2001, 2012). In Fischer's scientific surroundings, a number of works have emerged in which his concept was related more concretely to mathematics teaching. These are, in particular, reports on the experiences of teachers who have designed their own lessons on certain contents according to Fischer's concept. In the research project "reflecting in mathematics classrooms" (Schneider, 2019), in which I am involved, the perspective of the researchers is not at the same time that of a teacher; here, discrepancy between the ideal and the reality of reflecting in mathematics classes is focussed and ways of overcoming this gap are developed. In the conceptual definitions of reflecting, Schneider distinguishes four types of reflection, with model- and context-oriented reflection each designating a separate type. The conceptual definitions will be discussed in more detail in the next section, since the investigations shown in this paper will refer to them.

Ole Skovsmose (1998) focusses *mathematical literacy* or *mathemacy* in the light of education to democracy in a technological highly developed society. In such a society *formatting power of mathematics* takes action when reality is built through mathematical concepts and models. Different types of reflection should contribute to become conscious about this formatting power. Skovsmose is the first to outline the types of model- and context-oriented-reflection. He specifies that "a model-oriented reflection has the relationship between mathematics and an extra-mathematical reality as its object" (p. 199). This relationship involves simplifications that become necessary with any modelling, where certain aspects of reality are taken up while others are neglected. Further, such reflections can face the reliability of model-outputs or the validity of a model. In context-oriented-reflection, mathematical models are analysed from a different perspective:

> A modelling can, however, also be considered from a more general perspective with a different interest. The guiding questions could be: What is the actual purpose of carrying out the modelling? What, in fact, is the political and social function of applying mathematics to a certain situation? A reflection guided by such questions can be called a *context-oriented reflection*. (Skovsmose, 1998, p. 199)

With model- and context-oriented reflection, he describes two different perspectives for analysing a mathematical model, especially models used in social contexts. Skovsmose carried out various classroom projects in which students were confronted with uses of mathematics in their environment and were encouraged to reflect about these uses on different levels, not only model- and context-oriented ones.

Katja Lengnink's (2005) approach focuses on "a certain level of mathematical literacy, which allows reflecting and assessing mathematical processes important in every day live" (p. 246). For this purpose, she describes four categories of reflection, combining Skovsmose's model- and context-oriented reflection into one category. This is plausible, as these two types of reflection are not so clearly distinguishable from each other when it comes to formulate concrete questions about models for pupils. With Schneider's (2019) specifications, model-oriented reflection is more clearly distinguished from context-oriented reflection, as the latter no longer refers to a specific model for one particular situation, but to a particular mathematical concept which finds application in various situations.

Christian Büscher and Susanne Prediger (2019) face a *reflective side* of Mathematical Literacy and refer to the OECD definition of the PISA Mathematics Framework (p. 198). They conceptualize the process of reflective learning as "Conceptual Activity" according to Vergnaud (p. 203). Based on tasks to *statistical measures* that students work on in groups during an interview, they construct different possible paths of reflective learning. Their tasks invite to model- and context-oriented reflections, although Büscher and Prediger do not distinguish between any different types of reflection; but unlike the studies mentioned above, they investigate reflection processes empirically on the learners' side.

Preliminary investigation boundaries and research questions

Beside the rich theoretical work on reflection, partly outlined above, there is a gap in empirical work. Büscher and Prediger contribute to filling this gap by examining students' reflection processes on *statistical measures* in more detail. In the other empirical studies mentioned above, teachers and researchers were one and the same person and encouraged their students to reflect by means of (sometimes) large classroom projects. In the project "Reflecting in mathematics classrooms" (Schneider, 2019), the perspectives of researchers and teachers are separated in personnel. Nevertheless, the focus remains on an entire mathematics class. The research study described here targets more on the learner level. Reflection processes of students are to be investigated on the basis of tasks that can be worked on largely independently of what is happening in the classroom and that are specifically aimed at model- or context-oriented reflections. These two types of reflection are focussed on because they seem to be particularly significant for the goals aimed at in the theoretical conceptions. In Fischer's terms, a layperson, unlike a mathematical expert, will need to assess the use of mathematics in a particular context and make decisions about it. Outside the mathematics classroom and after school, the lay mathematician is unlikely to encounter situations in which it is important to get to the bottom of inner-mathematical contexts. Rather, it will be important to focus on connections between mathematical models and the situation they are supposed to describe or standardise, as well as to become aware of possible effects that underlie the mathematization of a situation. There is no doubt that knowledge of inner-mathematical relationships is important for this purpose. In the theoretical concepts, transfer is hoped for at the end: a transfer of knowledge that emerges from mathematics-oriented reflection, but especially from model- and context-oriented reflection. In addition, a transfer of the model- and context-oriented reflection *processes* themselves is desirable, since there will be new types of models and situations in which the previously acquired knowledge must be *transferred* to a new topic. In order to take a closer look at the processes of reflection among students, the study is oriented towards the conceptual definitions of the project "Reflecting in mathematics classrooms":

> *Reflection* (related to the learning of mathematics in school) means thinking about characteristics, connections, relationships, effects or meanings that cannot directly be read [off] from the given fact. (Schneider, 2019, p. 4)

With this definition, the difficulty remains in assessing what cannot be "directly read off from the given fact" by an individual student. If the student already knows about that particular characteristic, connection, effect…, it might be directly readable for him or her. On the other hand, for someone who does not yet know about particular relationships, reflections are probably necessary when it is required to become aware of these relationships. Model-oriented reflection is set as:

> Thinking about relations between mathematical concepts and inner-mathematical, but above all extra-mathematical situations. The focus on reflection is on the relationship between mathematics and the world, on thinking about mathematical models, usually of extra-mathematical situations and their fit, limits, effects, implicit assumptions for the concrete inner-mathematical or extra-mathematical situation. (Schneider, 2019, p. 4)

Model-oriented reflection is here referring to a concrete model for one particular situation. In contrast, context-oriented reflection is not related to a model for a particular situation, but to a certain mathematical concept and its effects in different contexts, of our world and society, in which it is used (Schneider, 2019).

The conducted study aims to gain insights about reflection processes on the level of learners, when working on tasks which aim at model- or context-oriented reflections. The following research questions are meant to be addressed: How do students deal with such tasks? Where is model- or context-oriented reflection recognizable? To what extent can different ways in dealing with the task contribute to or prevent requested reflections? Such different ways can be shaped by taking into account aspects of the context, by the use of mathematical knowledge and skills, or by possible conventions from the mathematics classroom. In order to address these questions, I conducted interviews with pairs of 8th grade students.

Investigative approach: Tasks and interviews

In what follows, two concrete tasks are presented, one of which aims at model-oriented reflection, the other at context-oriented reflection in the sense of Schneider's (2019) definition. Subsequently, it will be outlined how, together with the tasks, reflections and insights into such reflections are intended to be fostered by conducting interviews with pairs of students. In the next section the findings of a first explorative evaluation of the interviews are shown.

Task 1, headed "Baustelle" (construction site) focusses on model-oriented reflection:

> Das Bauunternehmen Heeg beschäftigt 60 Arbeiter(innen) auf verschiedenen Baustellen. Am Einfamilienhaus der Winters bauen 3 Arbeiter(innen). Sie werden voraussichtlich in 40 Arbeitstagen mit dem Rohbau fertig sein. Der Sohn der Winters rechnet ein bisschen herum und sagt: „Herr Heeg sollte alle 60 seiner Leute an unserer Baustelle einsetzen, dann wäre unser Rohbau in 2 Tagen fertig." Was meint ihr dazu?
>
> The construction company Heeg employs 60 workers on various construction sites. 3 workers are building the Winter's family home. They expect to finish the shell in 40 working days. The Winter's son does some calculations and says: "Mr. Heeg should use all 60 of his people on our construction site, then our shell would be finished in 2 days." What do you think about this?
>
> (Task 1, original wording above, below translated by C.P.)

Tasks with contexts like this are common in Austrian Textbooks. However, such tasks usually aim at the calculative goal; e.g., how long do x workers need for the construction. The fit of the model or possible limits for the descripted situation of the task are usually not considered by the questioning of these tasks. If questions about the fit or limits are in principle faded out in classrooms, then the focus on practising calculations can thwart ambitions that, according to Fischer or Lengnink would correspond to a demand for "Allgemeinbildung". At setting this task, students are assumed to have conceptual knowledge and calculation skills related to inverse proportions. The obviously implausible outcome "60 workers – 2 days" should act as a stimulus to take a closer look at this modelling. Finally, the students are asked about their thoughts on this, leaving open a variety of directions for consideration. The relationship between model and modelled situation might focus on the one hand on the limits of modelling, on the other hand also on the extent to which such modelling is generally suitable.

Task 2, headed "Prozente – geht es auch ohne?" (Percentages – dispensables?) aims at context-oriented reflection:

> Das mathematische Konzept Prozent (%) wird in vielen verschiedenen Situationen verwendet. Zum Beispiel in folgender Aussage: „Die Wahlbeteiligung in Österreich an der Wahl des europäischen Parlaments 2019 lag bei 59,8%." Was wäre, wenn es dieses mathematische Konzept Prozent (%) nicht gäbe?
> (Assignment „Prozente – geht es auch ohne?", original wording)
>
> The mathematical concept of percentages (%) is used in many different situations. For example, in the following statement: "The voter turnout in Austria in the 2019 European Parliament elections was 59.8%." What if this mathematical concept of percentages (%) did not exist?
> (Task 2, original wording above, below translated by C.P.)

Percentages are a form of representation for relative frequency. In this respect, they are interchangeable with other forms of representation. However, the concept of relative frequency itself is not interchangeable. Effects of this concept with respect to different contexts can refer on the one hand to the percentage representation, on the other hand to relative frequency itself. The decision to specify an exemplary context in this task was made firstly to provide a broader field of contexts, as the most obvious one for students might be the commercial context, and secondly to indicate a considerable advantage of representations of relative frequency. The context of the election of the European Parliament was chosen, as it took place a few months before the interviews and the comparison of shares in different basic quantities by means of percentages is advantageous here, since EU member states participated with quite different numbers of eligible voters.

The interviews were conducted with pairs of students. Working on a task in the context of a conversation with a classmate should favour a natural and spontaneous exchange about results and thoughts that led to them. The presence of the interviewer provided the opportunity to intervene spontaneously to ask for more detailed explanations or to introduce other perspectives. The tasks themselves may stimulate reflection, but leisure and time must be provided for it. A reflection-stimulating environment can be fostered by not aiming for a

predefined product but being interested in the students' personal reflections. This survey is based on the following features: Students themselves decided who will work together as partners; the interest in the thoughts and individual working processes was emphasised in the introductory phase of the interview; the students were given paper and pencil for notes but were not required to create a written product. The interviews were documented by means of audio recording. The tasks were worked on one after the other. First, each participant received the task for model-oriented-reflection. For the following five minutes, individual work was scheduled to grasp and first deal with the task. Second, the two partners exchanged ideas and worked on the task together; the interviewer intervened as little as possible but asked for explanations of statements or stimulated alternative directions of reflection. Third, the task for context-oriented-reflection was worked on in the same way. At the end, the interviewer asked for a kind of review of the tasks and students' work progress, in terms of affective feelings.

First findings from an explorative analysis of interviews

An initial analysis of nine interviews on the tasks quoted above provides interesting insights into students' handling and reflecting processes. In task 1 (construction side), aspects from the context are considered by all pairs, but play quite different roles in their considerations. That and how differently fruitful such contexts can be taken up for reflections is to be shown here exemplarily on the basis of three different pairs attending the same class (8^{th} grade).

Gina and Hannah (all names changed) reproduced the son's result in using a table, where they carry out the arithmetic operations underlying inverse proportions. Both picked up the improbability of this outcome for the construction site context. They soon noticed a difference in the requirement of this task, compared to their maths lessons. Using different aspects from the context, they implicitly criticised the model, for example, in using the fact that building materials have to dry. In the process, however, they only explicitly referred to the specific result "60 workers – 2 days" and it is not clear to what extent the modelling itself was challenged here:

196	Gina:	Aber wenn man jetzt z.B. Beton macht. Der muss ja auch noch fest werden.
197	Hannah:	Ja! Das heißt.
198	Gina:	(Pause, 1 Sek.) Das heißt, es funktioniert nicht.
199	Hannah:	Das mit den 2 Tag ist ja (Pause, 1 Sek). Es kann ja, wenn man sagt, okay die brauchen 40 Tage. Aber jetzt nicht, weil so wenig arbeiten, sondern nur weil es trocknen muss und weil sie, keine Ahnung {verschluckt: was noch}.
202	Gina:	Ja, schon auch weil so wenig arbeiten, oder?
203	Hannah:	Ja. Ja, auch. Aber ich glaube, dass das normal ist.
204	Gina:	Ich glaube, dass das mit 3 Leut

205 Hannah: Dann kann das ja net in 2 Tag fertig sein. Sagen wir mal, sie würden rein theoretisch, sie würden 20 Tage brauchen, wenn sie so, einfach nur weil es so trocknen muss und das alles. Dann geht das ja auch mit 60 Leuten nicht in 2 Tag, nur weil die jetzt zu 60 sind.

(Original wording above, translated by C.P. below)

196 Gina: But if you make concrete now, for example. It still has to harden.
197 Hannah: Yes! That means.
198 Gina: (Pause, 1 sec.) That means it doesn't work.
199 Hannah: The thing with the 2 days is (pause, 1 sec). It can be, if you say, okay they need 40 days. But now not because so few people are working, but only because it must dry and because they, no idea {gulps: what else}.
202 Gina: Yes, but also because there are working so few, right?
203 Hannah: Yes. Yeah, too. But I believe that's normal.
204 Gina: -I believe that the 3 people-
205 Hannah: (at the same time:) -Then it can't be done in 2 days.- Let's say they would, purely theoretically, they would need 20 days, if they, simply because it must dry and all that. Then it can't be done in 2 days with 60 people, just because there are 60 of them now.

A different trend becomes visible with Igor and Janis. Igor noted in the individual work that it would cost more and take longer because of the drying, but they did not address these limitations in relation to the model when they exchanged ideas. Rather, they focussed on inner-mathematical considerations, especially because Igor and Janis followed the son's results in different ways of calculating with Janis rounding the numbers inappropriately.

Kolja and Leo indicated arithmetical operations underlying relations of inverse proportionality, by which the son's result can be reproduced. While Gina and Hannah made productive use of the difference they expressed about the conventions of their mathematics lessons, such conventions seem to be a hindrance for Kolja and Leo. Kolja noted in his individual work that the son's calculation is valid "if all workers work with the same effort", and Leo, when asked, explained to the interviewer in similar terms that inverse proportion applies under this condition. Yet, during their work there was an inhibition to use aspects from the context to evaluate the model. Kolja tried to introduce such aspects into the discussion, but this was rejected by Leo, who pointed out that this would not be suitable for a "mathematical task". The impression that this can be attributed due to conventions of mathematics teaching is reinforced by statements at the end of the processing of this task (see line 270, below) and in the final phase of the interview:

491: Kolja: Wenn man das in der Schularbeit haben würde, so wenn fünf Seiten da sind in einer Stunde, oder in den 50 Minuten halt, dann würde man nicht so lange drüber nachdenken. Man würde das halt einfach ausrechnen.

(Original wording above, translated by C.P. below)

491: Kolja: If one would have that in the test, so if five pages are there to be done within an hour, or within the 50 minutes, then one would not think so long about it. One would simply calculate that.

Only through interventions, the discussion was led to the context of the construction site, and Kolja and Leo emphasised its relation to the model. Leo expressed a very interesting consideration about the modelling done by the son:

153: Leo: es stimmt (Pause, 2 Sek) nur, weil zwanzig Mal so viele Leute sind, heißt das nicht, dass sie zwanzig Mal schneller sein, aber
155: Kolja: aber wie soll man sonst aufs Ergebnis kommen?
156: Leo: Mhm (bejahend)
157: Kolja: Ich mein halt
158: Leo: Ich mein, dass sie schneller sind ist klar, aber dass sie nur in zwei Tagen fertig werden, eigentlich wenn man drüber nachdenkt
160: Kolja: Geht das nicht
161: L: Das ist ja nicht die Geschwindigkeit mit zwanzig multipliziert. Sondern nur die Arbeiter.

(Original wording above, translated by C.P. below)

153: Leo: it's true (pause, 2 sec) just because there are twenty times as many people doesn't mean that they will be twenty times as fast, but
155: Kolja: but how else should you get a result?
156: Leo: Mhm (affirmative)
157: Kolja: I just mean
158: Leo: I mean it's clear they are faster, but that they are finished in only two days, actually, if one thinks about it
160: Kolja: it's not possible
161: Leo: That is not the speed multiplied by twenty. But only the workers.

This statement, however, did not help them criticise this form of determining the duration of the necessary work as inappropriate. Rather, they agreed that the result determined by the son is not realistic and considered other aspects of the context, ultimately clueless about how to work on the task:

252 Leo: Ja es gibt so viele Antwortmöglichkeiten, eigentlich. (Pause, 13 Sek)
253 Kolja: Ich hab gar keine hmh (lächelnd)
254 Leo: Ich weiß auch gar nicht mehr weiter
255 Kolja: Ich hab auch keinen Plan mehr
256 Interviewer: Was findet ihr denn so schwierig? Beim Beantworten?
257 Leo: Ja
258 Kolja: Weil wir nicht wissen, wie wir es begründen sollen
259 Leo: Weil, du hast, wenn du
260 Interviewer: Was, was wollt ihr denn begründen?
261 Kolja: Warums net geht. Weil wir

262	Leo:	Ja wir wissen jetzt, dass es eigentlich nicht möglich ist, nur weil man zwanzig Mal so viele Arbeiter hat, dass man zwanzig Mal so schnell ist
264	Kolja:	Ja
265	Leo:	Aber, wir wissen jetzt nicht wieso eigentlich
266	Kolja:	ja
267	Leo:	was wir da jetzt als Antwort nehmen könnten, wieso das nicht so ist.
268	Kolja:	wir müssen halt eine Variante finden, wo alles zusammenpasst, so dass es kein Schlupfloch mehr gibt, dann haben wir die richtige. Hmh (lächelnd). Hm (feststellend).
270	Leo:	Ja wenn das jetzt im Mathematikunterricht gewesen wäre, hätte ich es mit der Tabelle mit der indirekten Proportionalität ausgerechnet

(Original wording above, translated by C.P. below)

252	Leo:	Yes, there are so many possible answers, actually. (Pause, 13 sec)
253	Kolja:	I don't have any hmh (smiling).
254	Leo:	I also don't know any further.
255	Kolja:	I have no plan either.
256	Interviewer:	What do you find so difficult? When answering?
257	Leo:	Yeah
258	Kolja:	Because we don't know how to justify it.
259	Leo:	Because, you have, if you
260	Interviewer:	What, what do you want to justify?
261	Kolja:	Why it doesn't work. Because we
262	Leo:	Yes, we know now, that it is actually not possible, just because you have twenty times as many workers, that you are not twenty times as fast.
264	Kolja:	Yeah
265	Leo:	But we don't know now why actually.
266	Kolja:	Yes
267	Leo:	what we could take as an answer now why that's not the case.
268	Kolja:	we just have to find a version, where everything fits together, so that there is no more loophole, then we have the right one. Hmh (smiling). Hm (stating).
270	Leo:	Yes, if this had been in math class, I would have calculated it with the table for indirect proportionality.

Task 2 (Percentages – dispensable?) showed that the context of the election of the European Parliament is still too foreign for 8th grade learners to use it for in-depth reflections. However, students found suitable contexts for themselves to make different considerations, above all using the commercial context with price reductions and value added taxes. This enabled them to work out important aspects of the concept of percentages. Two interview excerpts shall illustrate that below, the first of the two pairs is already familiar to the reader.

During the individual work of Gina and Hanna, Gina suggested that percentages can be used to apply equal shares to different basic quantities. She seemed to have become aware of this only during the individual work phase, where she expressed the idea that a discount

could also be indicated in absolute numbers. In conversation with Hannah, she found more suitable words for this, compared to the individual written work, and Hannah could follow her explanation well. In addition, Hannah contributed an alternative way of representing relative frequency by means of a fraction:

287	Gina:	Das ist ja z.B. wenn man jetzt bei Kleidung. Und da sagt man jetzt minus 25 Prozent auf das und das. Dann ist es natürlich egal, ob das Leibl jetzt 10 Euro oder 30 Euro kostet, das sind immer minus 25 Prozent.
290	Hannah:	Ja.
291	Gina:	Und das heißt, das ist dann irgendwie schon praktisch, gäh? Weil 100 Prozent kommt ja darauf an von was man jetzt ausgeht.
293	Hannah:	Ja, sicher. Das ist immer anders.
294	Gina:	(Pause, 2 Sek) Also Prozent sind schon für was gut. (lacht)
295	Hannah:	Ja, die sind schon für was gut. Ja, doch das macht schon Sinn. Das ist so ein einheitliches Ding, was man auf Dinge, auf Objekte, übertragen kann. Zum Beispiel eben wenn man so Sachen verkauft, was leichter ist. Weil du kannst net sagen, ich meine du kannst schon, das ist ein Viertel vom Preis weniger.
299	Gina:	Ja. (Pause, 1 Sek) Aber ist nicht ein Viertel vom Preis, weil das habe ich eben auch da unten irgendwo hingeschrieben, also im Endeffekt läuft das ja wieder aufs Gleiche raus.
301	Hannah:	Es läuft aufs Gleiche raus. Aber es ist mit Prozent, glaube ich, einfach leichter.
302	Gina:	(Pause, 2 Sek) Ja, aber so ein Bruch und Prozent hängen ja zusammen. Und eigentlich haben wir die Dezimalzahlen. Glaube ich halt. (lacht)

(Original wording above, translated by C.P. below)

287	Gina:	That is, for example, if one with clothes now. And there one says now minus 25 percent on this and that. Then, of course, it doesn't matter if the shirt costs 10 euros or 30 euros, it's always minus 25 percent.
290	Hannah:	Yes.
291	Gina:	And that means it is somehow practical, isn't it? Because 100 percent depends on what you start out from now.
293	Hannah:	Yeah, sure. That is always different.
294	Gina:	(Pause, 2 sec) So percent is really good for something. (laughs)
295	Hannah:	Yeah, it is really good for something. Yes, that makes sense. That is such a uniform thing, which you can transfer to things, to objects. For example, if one sells stuff or so, which is easier. Because you can't say, I mean you can, that's a quarter off the price.
299	Gina:	Yes. (Pause, 1 sec) But isn't a quarter of the price, because that's what I just wrote down there somewhere, so in the end it comes back to the same thing.
301	Hannah:	It comes out to the same. But it's just easier with percent, I believe.

302 Gina: (Pause, 2 sec) Yes, but a fraction and percent are related. And actually we have the decimal numbers. I believe. (laughs)

Mark's and Nicolo's reflections addressed the fact that looking at a relative share can produce very different perspectives depending on what it refers to:

376 Mark: Oder auch im Sport, wenn sie manchmal so sagen, also das sagen sie nicht, aber rein theoretisch, wenn du sagst jemand ist zwanzig Prozent schneller als wie der vorherige

378 Nicolo: (flüstert) Ja, aber das sagst du net.

379 Mark: Ja, aber jetzt rein theoretisch. Also, im Kopf merkst du nachher schon, der war schon VIEL schneller.

381 Nicolo: Mhm (bejahend).

382 Mark: Als wie wenn du nachher die Zahlen siehst, dass der irgendwie fünf Sekunden schneller war oder so. Wenn du die Prozent hörst, wirkt das natürlich gleich viel mehr. (Pause, 2 Sek) Und ist eben auch bei den Rabatten, dass wenn da steht das ist zehn Prozent, denkst du dir gleich, das ist nicht so wenig. Aber dabei ist das gar nicht so viel.

(Original wording above, translated by C.P. below)

376 Mark: Or also in sports, when they sometimes say things like that, well they don't say that, but purely theoretically, if you say someone is twenty percent faster than the previous one

378 Nicolo: (whispers) Yes, but you don't say that.

379 Mark: Yes, but purely theoretically now. So, in the mind you notice, that was MUCH faster.

381 Nicolo: Mhm (affirmative).

382 Mark: Like when you see the numbers afterwards, that he was somehow five seconds faster or something. When you hear the percentages, of course, it has a much higher impact. (Pause, 2 sec) And it's the same with discounts, that if it says that's ten percent, you immediately think to yourself, that's not little. But in fact it's not that much.

Moreover, the interviews reveal that learners have difficulties with inner-mathematical connections between percentages, fractions, and decimals (see Gina and Hannah, line 302) and that this partly hinders context-oriented considerations.

Summary and outlook

Based on theoretical concepts from the first two sections, the question arose how learners deal with tasks that target specific types of reflection. Through interviews interesting insights to reflection processes emerge on three different aspects.

First, the extra-mathematical context can be differently fruitful for reflections. Depending on how learners are able to deal with it, it can contribute to relevant reflections (such as the drying for Gina and Hannah in task 1, discounts and sports in task 2) or lead to getting lost in reflections of the context while the mathematics gets out of focus. Second, deficient

mathematical knowledge and skills can partly hinder model- and context-oriented reflection. Resulting discussions of inner-mathematical relationships can be quite fruitful, but at the same time the level of model- and context-oriented reflection may be left out (e.g., Igor and Janis). Thirdly, presumed conventions from mathematics classes become visible. A focus on fast and strict processing of clearly solvable tasks may limit the view of mathematical models, as in the case of Leo and Kolja, but Gina and Hannah show that this does not necessarily have to be the case. With regard to task 1, it should be examined whether a formulation that presents the selective result "60 workers - 2 days" less prominently encourages the learners to reflect more on the model as a whole and its limitations.

Overall, the first explorative approach to the analysis of the interviews allows to describe peculiarities that seem to inhibit reflections on the one hand. On the other hand, students conducted a variety of relevant reflections. Further analysis of the interviews should focus more specifically on the task processing as a whole, less at isolated passages for deeper insights regarding model- and context-oriented reflection processes. Follow-up interviews with the same students on other tasks with different mathematical contents are planned. The search is on for model- or context-oriented reflection processes across specific contents and tasks.

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Normative modelling as a paradigm of the formatting power of mathematics: Educational value and learning environments

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Mathematical modelling can be differentiated into a descriptive and a normative prototype: While the first reproduces realistic phenomena by means of mathematics, the second represents the use of mathematics in order to shape reality. The describing function of mathematics nourishes common beliefs of mathematics, whereas normative modelling leads to the discovery of characteristics as subjectivity, ambiguity and dependency on human influence. The educational value of teaching normative modelling has several aspects: broadening the understanding of mathematical modelling, promoting citizen empowerment and initiating a reflection about the role of mathematics in society. This potential is illustrated by suggesting concrete learning environments that enable the discovery of normative modelling in civic issues.

Citizen empowerment as motivation

> *Applied mathematics are only interesting and really indispensable for general knowledge, when real life examples illustrate the functioning of mathematical modelling (Winter, 1995, p. 38, translation by the authors).*

The application of mathematics in various contexts is a necessary condition for its usefulness but is not a sufficient one for its educational value. Mathematical modelling as the act of correlating reality with mathematics is often taught in order to prove the presence of mathematics in everyday life. Exemplary context should not be limited to the objective that students can cope with this specific situation. Furthermore, it is indispensable from the perspective of promoting citizen empowerment that they discover how and wherefore mathematics is used and which impacts has this modelling for reality. More than being able to calculate their income tax, students should know the main determining decisions behind this tax model and not confuse this arbitrary variable setting with physical constants. This use of modelling represents a concrete paradigm for the formatting power of mathematics and is a motivation for an awareness rising: Recognising which aspects of a mathematical modelling in a socially relevant context can be changed and in which way reveals possible courses of action and enables a public debate on desirable transformations.

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This paper begins with the differentiation between a descriptive and a normative function of modelling. As shown afterwards, the latter dispose of important educational potential for a broader understanding of mathematical modelling as well as for its application in social, economic or political contexts. Finally, the design of exemplary learning environments gives the opportunity to discuss how normative modelling can be taught and what insights students have acquired in previous implementations.

Characterisation of normative modelling

Davis and Hersh (1986, p. 115) enumerate three functions of mathematics: descriptive, predictive and prescriptive. This enumeration can be complemented and transferred to models: There are models that describe, that predict, that prescribe and that explain (Henn, 2002, p. 6).

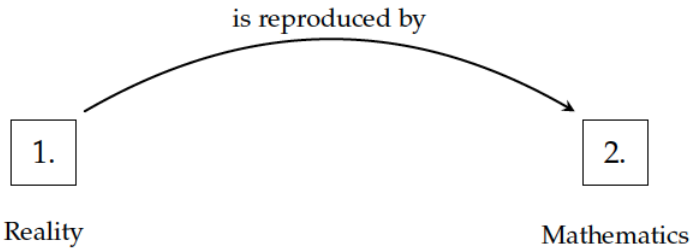

Figure 1: Scheme of descriptive modelling

Nevertheless, the first three can be summarised as the result of descriptive modelling as they have a certain approach in common: 'Descriptive' relates to the fact that all three models are based on a mathematisation of a real-life phenomenon (Fig. 1). The apple had fallen before Newton formulated his law of gravity. The Arctic Ice would have melted and would continue to melt even without a mathematical description or prediction. Freudenthal (1978) speaks of models as "after images" or "plaster cast" (p. 131). The mathematical modelling serves as a reproduction for a better processing, analysis and understanding of the original phenomenon. Consequently, the motivation of the descriptive mathematical modelling constitutes the following subcategories of descriptive, explanatory and predictive modelling.

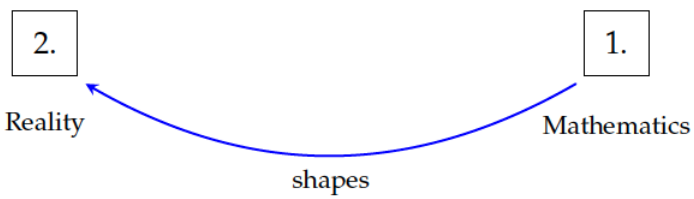

Figure 2: Scheme of normative modelling (Pohlkamp & Heitzer, 2020b, p. 218)

In the case of normative modelling, mathematics is however used to design standards and rules in order to shape or even create reality (Fig. 2). Hence, the reference to formatting norms explains the denomination although the term prescriptive modelling would be a synonym (Greefrath & Vorhölter, 2016, p. 9). For further illustration of this type of modelling, Freudenthal (1978) uses the metaphor of the "knitting pattern" (p. 131), the comparison with playing rules is by experience particularly suitable for students. One example of the formatting power of normative modelling is the income taxes: You can only pay taxes, once a tax rate has been defined.

Nevertheless, such an example could contribute to misconceptions: Of course, a given tax rate also describes the amount due and a citizen can predict one's tax debt with help of the mathematical function. The categorisation 'descriptive vs. normative' refers more to the intention of the original modelling process than to the handling of the resulted model. That is why we prefer to attribute these adjectives only to 'modelling' and normally not to 'model'. Furthermore, this dichotomy is based on idealised prototypes. Often aspects of both types interfere in realistic contexts: In particular, the normative modelling of a tax rate is not totally independent from a descriptive report of individual and public finances: On the one hand everyone should still be able to live from the money remaining after taxes, on the other hand the state needs a certain amount to offer its services.

For the purpose of analysing mathematics (education) from a social and political perspective Skovsmose (1998) formulated "the thesis of the *formatting power of mathematics*: Social phenomena are structured and eventually constituted by mathematics" (p. 197, emphasis in original). Normative modelling may not be the only manifestation of the formatting power of mathematics, but it represents a paradigm making insightful foci accessible. In particular, the difference between applied mathematics in natural versus social sciences can be drawn. In addition, somebody is actively formulating or varying the conditions of social interaction during the process of normative modelling by using mathematics. In this context, the human influence on the formatting power becomes obvious. Even if mathematics should in general not be reduced to being a tool, the perspective on the modeller leads to a critical characterisation of the process itself. Normative modelling is not unambiguous, but subjective as well as debatable (Pohlkamp, 2020). Thus, other persons could get a totally different outcome depending on their modelling decision. In a democratic society there should not be one autocratic modeller, instead the modelling should be established on a social consensus, Davis speaks of a social contract:

> The 'contract' metaphor is a useful phrase to designate the interplay between people and their mathematics and to make the point that mathematizations are the work of man, constructed according to human will (Davis, 1988, p. 12).

This reference to the Enlightenment indicates the educational value of sensitising to this use of mathematics: It makes a difference whether you deal with a descriptive or a normative modelling. In contrast to the laws of physics you can easily dispute and change some parameters in the model of the tax income. Even though we have used the same instance for normative modelling until now, other examples document the significance of mathematical

norms behind civic issues: the definition of poverty (Vohns, 2017, pp. 972–976), proportional representation, business valuation, allocation of resources, air pollution thresholds …

Normative modelling constitutes a valuable instrument for precising and analysing the way in which mathematics is used in the set-up of human society.

Educational value of normative modelling

Hereinafter, the educational value of discussing normative modelling with mathematic students will be outlined. For that matter, the understanding of education expressed here follows the German speaking tradition integrating "aspects of societal participation and active citizenship as important goals of formation" (Vohns, 2017, p. 970). Similarly, Apple (1992) differentiates between a functional and a critical mathematical literacy: The first comprises the skills needed to be well adapted for everyday life and to contribute to the economy; however, the ladder includes in a broader understanding the ability to challenge the existing social order. In this regard, knowledge about normative modelling enables to understand the mathematical mechanisms behind certain social constructions. Even if many modellings are very complex, a reflection of exemplary authentic contexts offers a framework for a civic engagement: Participating in a public debate as future citizens means that students are able to name and question the relevant mathematical aspects and methods.

While modelling has been implemented as a mathematical competence in most curricula of German language, it can be shown that normative modelling remains a blind spot, because the definition of mathematical modelling is often restricted to a descriptive function. At the same time, analysing the formatting power of mathematics can easily be discussed with students via the modelling discourse. Moreover, studying normative modelling addresses specific sub-competencies of modelling. In the process of normative modelling the steps of mathematising, interpreting and validating (Greefrath & Vorhölter, 2016, p. 19) are crucial. Thereby, modellers take at first contestable decisions – is a fee fair if everyone pays the same absolute amount (e.g., broadcast license fee in Germany) in contrast to the same percentage (e.g., day-fines as sentence) or would it even be thinkable if richer persons paid relatively more (e.g., progressive income tax)? Then they can check the consequences of their premisses and decisions by measuring the effects in reality and by comparing alternative models. The multitude and arbitrariness of the way mathematics is used to shape reality become obvious. Certainly, ignoring the air resistance is a considerable human interference in formulating the formulas of free fall. But an experiment proceeds the modelling which results can be verified with empiric data. Yet in the normative case, there is no best approximation to the original phenomenon; neither is there a limitation whether the formula of the tax income should be bases on a linear, a proportional or still exponential function.

Doing justice to the normative type when teaching modelling is tied to complementary learning targets. The general objective of *explaining the formatting power of mathematics by means of normative modelling* comprehends amongst others the following targets (Pohlkamp, 2020):

1. *Exemplifying prototypic characteristics of normative modelling*
   Discovering the abovementioned characteristics as ambiguity and subjectivity facilitates to deconstruct the myths about Mathematics that Hersh (1991) identified. This initiates a reflection of mathematics, its use and role in society as another positive impulse.
2. *Outlining premises, decisions and alternatives*
   The task is to encourage critical minds. Confronted with supposedly completed models, a responsible citizen should be qualified to detect other options and especially be careful if they are being told that there is not an alternative.
3. *Judging normative modellings as well as the possibility of judgment*
   While comparing different outcomes given the same cause for the normative modelling, students encounter the same challenges as during the modelling. As there cannot be a benchmark as exactness of reproduction, the choice of quality criteria is subjective and arbitrary as well. Whether a tax model is fair depends on your understanding of fairness.

The ideal long-term objective is *distinguishing descriptive and normative elements in authentic modellings* by taking into account that the prototypes rarely occur in purity. Due to the larger dissemination of descriptive modelling these learning targets express the plea that an examination of the normative type should be integrated in learning mathematics before a direct juxtaposition. Exposing the misconception of mathematics as a tool of neutral reproduction goes along with a search for decisions in so called descriptive modellings that are debatable and maybe biased.

There are good reasons to address basic knowledge and the ability of reflection rather than technical skills in order to empower citizens' participation in complex discourses (Fischer, 2012, pp. 13–14). Nevertheless, *constructing an own solution to realistic problems by retracing the process of normative modelling* is a deliberate approach. Thus, students are sensitised for the formatting power because they experience the variety of possible decisions. Whereas most citizen are consumers of models originated of a normative intention, the producers' standpoint is yet instructive according to the following analogy: Empirical psychological results of an online simulation have shown that once learners have produced fake news on their own, they are more critical when facing up misinformation afterwards (Roozenbeek & van der Linden, 2019).

In the same way that playing rules determine sportive and/or gaming activities, normative modelling contributes to a quantified regulation whenever a political, economic or interpersonal situation seem to call for it. Taking into consideration that paradigmatic applications emanate from social sciences, you can differentiate the perception of the position of mathematics in the canon of school subjects. Not only in a general perspective, but also for vocational preparation it is important that students round off their vision of mathematics traditionally associated within the STEM-disciplines. However, this can contribute to misconceptions. After a workshop introducing normative modelling one student formulated the lesson learned "Mathematics exists everywhere" (transl. by the

authors). This overregularisation of the existence of mathematics beyond natural science is a manifestation of an all-embracing jurisdiction of mathematics (Kollosche, 2017, p. 641).

But overall, learning targets around normative modelling can be linked to encouraging attitudes of a critical citizenship. It is a technique of domination when "producers of normative models like to behave as though their models were well-determined and unique" (Freudenthal, 1978, p. 135). Therefore, it is emancipatory for students to investigate the often hidden mechanism in the construction and functioning of the societal reality.

Insights within exemplary learning environments

Introducing normative modelling to students can take place on a fundamental level:

> Mara calls for a taxi to the train station. Halfway, her friend Nele is joining her. The whole ride costs 12 euros. How should they split the costs? (Sjuts, 2009, p. 191, translation and adaption by the authors).

Apart from obvious answers – Mara pays 12 euros (she called for the taxi and didn't need to share the ride), each defrays 6 euros (each one reached her destination) – the following solutions surprise students the most: Mara pays 8 euros and Nele 4 euros (Mara travelled twice as far) vs. Mara contributes 9 euros and Nele 3 euros (Mara pays the first half distance on her own, while they are sharing the second one equally). In a mathematical argumentation most students do not expect such results revealing principal characteristics of normative modelling: ambiguity (several logical solutions without a calculation error), subjectivity (it is one's interest to pay the less possible) and the necessity of a debate (you can't agree to disagree as the taxi driver must be paid). As *pars pro toto* for questions of allocation, the citizen empowerment is pertinent: Not knowing the alternatives is a disadvantage for one of the girls. One constraint is however, that some students are not aware of the political dimension, a difference of one euro is too insignificant. The other consists in the simple calculation for which reason no modelling competencies are stimulated.

An appropriate context of retracing a complex modelling process with students is proportional representation; starting with discussing proportionality vs. integrity, students remodel different methods of apportionment and form an opinion about current reforms of the election system (Pohlkamp & Heitzer, 2020b). Exploring normative modelling can also be limited to vary parameters of a given model. When students define a tax income threshold in a digital visualisation and dynamically examine consequences, the arbitrariness of any decision becomes obvious (Pohlkamp & Heitzer 2020a, pp. 291–292). In face of the malleability within one model, critical insisting on the technical background of civic issues is worth it whereas references to mathematical argumentations are normally intended to conclude a public debate.

Analysing economic models is in particular educational because they appear more objective than political or social contexts. In fact, interest-calculation is part of most mathematics curricula without referring to its normativity. This topic is treated as an unalterable formula although anthropologically there has been a different view on interests forbidden or condemned by most religions.

Business valuation is another economic issue widely spread in media. An open activity would be to let students compare financial statement data from e.g., the *big five* tech companies. In order to nominate the most valuable company they have to select data they consider relevant, translate values of different unities in one point scale and to weigh these results. If intended, business vocabulary like the difference between revenue and profit can be familiarised for general knowledge. From the point of normative modelling, it is more important that students deduce the prototypical characteristics from their own procedure and reflect possible judgements by comparing the different models. Even if most models generated by students are far from realistic models, it arises a critical potential. In a try-out of this activity most of the students' models integrated the company's staff number and often prioritised it with a high weight. Verifying that the staff number is irrelevant in common authentic models leads as far as denouncing the role of human in capitalism.

Economic models are good examples that not only the choice of a model exhibits normativity, but also the choice of data within a model. The product of the number of a company's outstanding shares and the value of a share defines the market capitalisation, common referent for business valuation. Since the share value changes nearly daily, the day of reference is important information and the market capitalisation a volatile value. This relativises frequent headlines on which company (mostly one of the *big five*) is currently the most valuable.

An authentic and accessible other context to promote both modelling competencies and statistical literacy is to deal with real data about the Sea Ice (Fetterer et al., 2017). At first glance describing the past and predicting the future development of the surface via regression seem to be a classical descriptive modelling. Of course, this visualisation (Fig. 3) represents the conclusion of a students' activity to examine the data, but it is the moment when an analysis with respect to normative aspects becomes relevant. Taking a deeper look, it makes a huge difference on which statistical measure the model is based. Extrapolating from the surface maxima of the last years you could even claim that the change is marginal. Students reconstruct how climate change deniers would select data to support their messages devoid a detectable calculation error. The normativity lies on the one hand on the arbitrariness of the choice, on the other hand, the visualisation shapes how dramatic we perceive the described phenomenon. This contradicts the common belief that the description by mathematical means is *per se* neutral. Such a modelling perspective cannot only be applied to the data processing, but also to the collection of data: By measuring the surface it must be distinguished between the area and the extent of the sea ice. Further critical questions could for example extend to: the difference between surface and volume (balancing between the effort of data collecting or rather of modelling and the accuracy of the outcome), the assumption of a linear regression (and its restrained validity on a larger interval), the neglection of aggravating aspects (effects of the climate crises are interdependent). All are consequences of human decisions in the data-based modelling process. Provided that both prototypes of modelling are familiar, the examination of this context initiates the discovery of a hybrid form next to learning of the climate crisis.

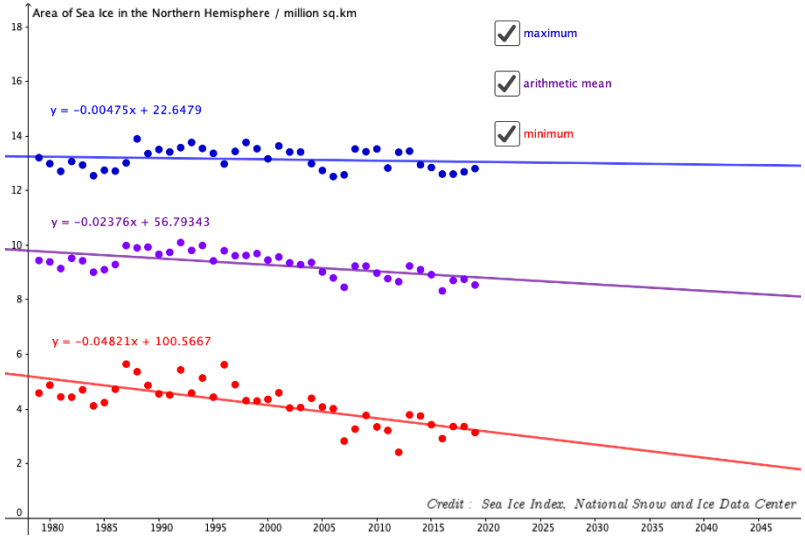

Figure 3: Modelling the development of the sea ice surface

These different teaching ideas give an example in which form and by means of which issues normative modelling can be concretised for students. Learning in the context is always tied to the meta theme of normative modelling embodying the formatting power of mathematics and the human influence on its use.

Concluding reflection with respect to common beliefs about mathematics

In extracurricular workshops the functionality of almost all learning environments mentioned above could be tested, usually they were combined with a lesson about the normative modelling of apportionment methods. On the basis of written answers by the students (n=67), the learning results could be contrasted with the primary targets. When asked to "explain normative modelling in [their] own words" students are able to summarise principal characteristics of normative modelling as ambiguity, shaping of rules/reality, subjectivity. Some aspects are illustrated by their own metaphors (e.g., set screw, stencil) or examples (e.g., demarcation between countries, wage increase). However, the following statements are typical for a certain unease (transl. by the authors):

> Mathematics is used to develop a human-made model. The set screws can be selected freely. This is not natural.
>
> [Normative Modelling is the] mathematical background in reality and the penury of math for describing situations.

The reflection of normative modelling comes into conflict with internalised beliefs about mathematics. The first quotation insists that mathematics is an invariable system that humans aren't allowed to change. In the second one, although the limits of mathematics are recognised ("penury"), the domination of the describing function of mathematics persists.

Both wordings have in common that mathematics is first and foremost associated with a neutral tool. In addition, there is a tendency to reduce mathematics to calculation in order to rescue the concept of unity and objectivity and to blame the context for any unexpected and irritating effects:

> [Normative Modelling is a] context interfering into mathematics.

Overall, students can explain normative modelling, its mode of operation and its significance and they wish to learn further about it. Despite this positive evaluation, students attribute traits to mathematics that are quite antithetical to normative modelling. This tension revealed via the learning environments initiates a critical reflection about the formatting power of normative modelling that has been pointed out by most of the students:

> One determines something by means of mathematics in order to apply it to reality.

Even if a certain vision of mathematics seems to be deep-seated, normative modelling has shown to be an opportunity to discuss a use of mathematics that is relatively unknown and yet so relevant: Sensitised to the normative use of mathematics, citizens are no longer misled by the idea that mathematical arguments in political and social context are neutral and irrevocable. Only the knowledge about this changeability enables a reaction to social orders based on mathematisations: If one cannot fight descriptively modeled natural laws, one can fight social constructs as products of rather arbitrary normative modeling. The latter are also omnipresent in the human world:

> We are born into a world with so many instances of prescriptive [normative] mathematics in place that we are hardly aware of them, and, once they are pointed out, we can hardly imagine the world working without them. (Davis & Hersh, 1986, p. 120)

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Designing curricum to acknowledge quantitative, sociocultural, critical, and spatial ways of knowing in mathematics teacher education

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The novel framework described in this manuscript was designed by the researchers to make explicit the intersection of socio-cultural, critical, quantitative, and spatial ways of knowing the world, which are inherent in mathematics education and quantitative learning environments more broadly. This work builds on the Teaching Mathematics for Spatial Justice (TMSpJ) framework of Rubel, Hall-Wieckert, and Lim to consider how TMSpJ can be incorporated into mathematics teacher education, with particular attention given to interrogating the spatial realities constructed by the educational system in rural areas. We suggest that our framework can be a powerful guide for designing learning experiences for pre-service mathematics teachers (PST). As part of this proceeding, we will first define the framework and then share a statistics unit for PST that incorporates the critical aspects of knowing the world.

Introduction

The social and historical dimensions of reality have long been important when considering the teaching and learning of mathematics in a social context (Lerman, 2000), and more recently political dimensions have been highlighted in the sociopolitical (Gutiérrez, 2013). However, a dimension that has been somewhat neglected is that of the spatial aspect. In the introduction of Edward Soja's (2010) book *Seeking Social Justice*, he describes social (in)justice as "an integral and formative component of justice itself, a vital part of how justice and injustice are socially constructed and evolve over time" (p. 1). Spatial justice, for the purpose of our research, considers individual experiences defined by space, and how circumstances are determined by geographies that are socially constructed. Larnell and Bullock (2018) created a socio-spatial framework for urban mathematics education claiming a spatial turn in mathematics education. A recent push in that direction has been spear-headed by Rubel and her team in their work on conceptualizing *Teaching Mathematics for Spatial Justice* (TMSpJ; Rubel, Hall-Wieckert, & Lim, 2017, 2016; Rubel, Lim, Hall-Wieckert, & Sullivan, 2016). In their work, they developed and implemented lessons involving spatial justice themes that intersected with critical pedagogy. Their work was situated in a mathematics course for social justice in high school and undergraduate classes serving

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predominantly underserved students in New York City (Rubel et al., 2016). This work shows the potential that considering spatial justice in mathematics education can provide tools and design heuristics that encourage more socially focused opportunities in mathematics classrooms.

Drawing from the seminal work of Rubel and company (Rubel et al., 2017; Rubel et al., 2016; Rubel et al., 2016), our work incorporates the notion of spatial justice in two fundamentally different ways. One way is by developing an epistemological framework for the teaching and learning of mathematics that explicitly acknowledges sociocultural, critical, quantitative, and spatial ways of knowing the world drawing from Soja's work (2010). The second way is by investigating how such an epistemological framework could be used in designing curriculum for pre-service mathematics teachers, whereas the activity would also provide a critical lens for the pre-service teachers to make sense of their future classroom environments.

We assert that spatial ways of knowing are particularly relevant and important to consider in pre-service teacher education because education systems themselves are based on geopolitical boundaries. We used a design approach (Cobb et al., 2003) to focus on pragmatic problems in learning ecologies, specifically how to incorporate spatial justice meaningfully into the preparation of mathematics teachers. As design work is iterative, through the process of delving into the literature and applying our theories in practice we refined our problem into the following specific research question we consider in this paper. *How can the design of learning experiences for PSTs be framed to create opportunities for PSTs to interrogate systems that they are a part of using practices from quantitative, critical, sociocultural, and spatial ways of knowing the world?*

Teaching mathematics for spatial justice

TMSpJ was developed by considering place-based education in conjunction with Critical Mathematics Education to create a way of thinking about teaching mathematics that drew upon spatial perspectives in relation to reading and writing the world (Rubel et al., 2016). To that end, we began by considering how to incorporate Rubel, Hall-Wieckert, and Lim's (2016) design heuristics in our teacher education program. The guiding questions they used to initially designing their work include:

- How does the system work?
- What is the system's geography?
- Who participates, where, and how often?
- How is the system experienced in peoples' everyday lives?
- Is the system fair? How could it be transformed to be fairer? (Rubel et al., 2016, p. 564)

The obstacle, when considering our work, was in thinking of how to communicate the design heuristics we were using to PSTs who had little to no prior experiences with the practices of critical literacy or interrogating systems. The design heuristics developed by Rubel, Hall-Wieckert, and Lim (2016) was explicitly situated in a critical lens drawing upon

the work of critical scholars (Freire, 1970; Freire & Macedo, 1987; Gutstein, 2003, 2006). However, the nuances of the ontological and epistemological stances inherent in working within a critical lens are not made explicit in the guiding questions, so that someone lacking familiarity with the perspective may struggle when engaging in the work. In considering Soja's (2010) work we zoomed out to consider the ontological perspective he discusses and then zoomed in to an epistemological perspective to consider the design of curriculum for PSTs incorporating spatial justice.

Ways of knowing framework

The epistemological ways of knowing our world are constituted by socially, historically, and spatially situated discourses. In striving to define an epistemological perspective and considering socio-cultural theories of learning (Boaler & Greeno, 2000; Lave & Wenger, 1991; Lerman, 2000) in mathematics education, we began to realize that in our design process we were struggling to identify what specifically the practices were for someone to investigate a system where all four ways of knowing are incorporated that are the focus of our work.

Our framework is based on the notion that there are many different ways of knowing or coming to know the world. Some ways of knowing are privileged in society and by institutions, whereas others are often marginalized, silenced, or discredited. We acknowledge that our framework is not comprehensive of all the ways of knowing the world, instead, we focus explicitly on four important ways of knowing (socio-cultural, critical, quantitative, and spatial) in quantitative learning environments that have been considered and investigated in various combinations by scholars in the past (d'Ambrosio, 1985; Frankenstein, 2009; Gutstein, 2006; Lerman, 2000; Rubel et al., 2016; Skovsmose, 1994).

Ways of Knowing	Quantitative	Socio-cultural	Critical	Spatial
Key Practices	– Measurement – Collecting data – Modeling the world through quantification – Shaping the world through quantification – Problem-solving through quantification and abstraction	– Using funds of knowledge of communities to make sense of the world – Considering the ways different communities of practice come to know the world – Drawing upon the assets of a community	– Reading and writing the world – Making sense of social structures – Transforming social structures – Problematizing structural causes of (in)justice	– Considering how space/social structures shape/are shaped by one another – Understanding that spatial/social structures are historical – Interrogating the political space, one is situated in – Reflecting on and transforming the physical space one is situated in

Table 1: Ways of Knowing Framework

When establishing our Ways of Knowing (WoK) Framework we felt as if all four aspects were essential to create a comprehensive understanding of issues related to social inequities. The strength of our framework is that the interconnectedness between the ways of knowing is central and provides a lens that allows social constructs to be fully examined.

Guiding objectives for designing curriculum for teacher education

In working to design curriculum, our specific audience was pre-service elementary teachers (E-PSTs) in a content and methods class. To engage in all four aspects of the WoK Framework we described, we began our design work by considering learning objectives for each way of knowing for the unit we would design for E-PSTs.

In the sections that follow, we discuss how we developed the learning objectives for a unit by consulting past research and our work to provide a descriptive example of the process of building curriculum from the WoK framework for others. We also provide an illustrative example of an activity that highlights how we designed the curriculum to incorporate all the aspects of the framework to interrogate aspects of the education system.

Quantitative

For the unit we designed, we focused specifically on statistics content and practices. There are some crucial differences between mathematics and statistics that make it particularly powerful for interrogating the world. One of those elements is that context is central to the discipline of statistics (Cobb & Moore, 1997; Wild & Pfannkuch, 1999) and the statistical inquiry process that is at the heart of what is recommended for statistics instruction in K-12 education. Another difference between mathematics and statistics is the focus on variability. There must be variability in data to use statistics, statistical questions are those that anticipate variability, and variability is one of the main aspects of a distribution we try to measure to describe it and to do inferential statistics. The Guidelines for Assessment and Instruction in Statistical Education (GAISE) framework recommends beginning with a focus on variability within a group before considering variability between groups and co-variability (Franklin et al., 2007). Building off the GAISE framework the Statistical Education of Teachers (SET) document further discusses the statistical preparation of teacher including specific recommendations by grade level (Franklin et al., 2015). Because the focus of our work is on elementary teachers it made sense to work from the SET recommendations to determine the learning objectives for the quantitative ways of knowing aspect of the framework, which focus on the content, pedagogical content and horizon knowledge for teaching statistics in elementary grade levels (grades 1-5). Based on these goals the design of our instructional unit focused predominantly on content knowledge. In the content course, only four weeks can be dedicated to the topic of statistics, and for the methods course, the statistics content is predominantly in one unit.

Spatial

For considering the spatial components of our unit we drew from past work of Rubel, Hall-Wieckert, Lim (2017). In their work with high school and community college students, they looked at how to bring issues of spatial justice into the mathematics classroom. Though our

population is pre-service teachers they are also undergraduates and are mostly of similar age as the students in Rubel, Hall-Wieckert, and Lim's work, therefore we believe the practices translate well. In developing a set of analytic codes to investigate how the students in their study engaged with the spatial tools they provided they creating a coding scheme that was a mix of a map-reading framework from Morris (2013) and a statistical graph literacy framework from Shaughnessy (2007). Soja (2010) in his work described three different levels of spatial understanding, macro, meso, and micro geographical elements. Macro looks at the political aspects of space, meso, how processes evolve in a spatial context and the uneven development of space, and micro focuses on the inequalities created through decisions made at an institutional level. For our planned activities, we focused on the meso-geographical level and how that impacted the student's educational experience.

Designed curriculum for teacher education

With the creation of the TMSpJ curriculum, using the WoK Framework we had five focused goals for the students.

- Use descriptive statistics and graphs to consider schools as an aggregate.
- Use statistical investigations to interrogate claims.
- Create a final project that brought the tenets of this work together.
- Use spatial representations of data to investigate statistical questions.
- Create statistical questions to guide an investigation of educational data.

Quantitative (Statistics)	Spatial
1. Develop the necessary content knowledge and statistical reasoning skills to implement the recommended statistics topics for elementary-grade students 2. Develop the content knowledge associated with the middle school–level statistics content 3. Develop an understanding of how statistical concepts in middle grades build on content developed in elementary-grade levels 4. Develop an understanding of how statistical content in elementary grades is connected to other subject areas in elementary grades	1. Recognize and interpret components of a map 2. Use maps to compare two places 3. Identify a pattern in a map related to more than two places toward building a spatial argument 4. Interpret causes of a spatial pattern 5. Make connections between the context and the spatial pattern
Critical	**Sociocultural**
1. Interrogate structural elements of the education system 2. Interrogate one's subjectivity concerning perceptions of education and the education system	1. Consider the culture of a class or school 2. Consider the cultures of people that are part of the community a school is situated in and serves 3. Reflect on one's own culture and subjectivity.

Table 2: Learning objectives for statistics unit situated in the Ways of Knowing Framework.

Designing a unit using the ways of knowing framework

Working from the focused goals we then sought to flesh out the learning activities for the unit. Brief descriptions of each of the major activities in the unit can be found in Table 3. The overarching thought behind our design was to start with students focusing on thinking about themselves, then zooming out to think about their class, then the community surrounding the institution, and finally to the state and possibly beyond. In this way, we were able to start by meeting the student where they were at and then expand into more and more technical statistical ideas while also zooming out in terms of the context under consideration. The context of the educational system was at the core of all the investigations that were designed for the unit to situate the unit in a context of interest and relevance to the students who were PST's.

Identity, Census, and Statistics
This activity has students begin by students responding to the prompt, "Write a short description (4-6 sentences) of how you would describe your identity to someone else," Students are asked to synthesize their descriptions into five words. Students reflect on what is gained and lost by synthesizing their descriptions into words. As a class, all students aggregate their five words and are asked to then organize the words in a way that helps them to respond to the question of what the identity of students in the class is. Once students have completed their organization of the word data they are then asked to write a brief description of their class based on the word data that they then share. Students then reflect on how their descriptions of themselves compared to their class descriptions and on what is gained and lost by moving from their descriptions to descriptions of the class. The activity is connected to the practice of statistics including what counts as data, how data is collected, and that statistics focus on aggregates, which are related to samples, sampling, and making an inference.
Statistical Investigations: Questions and Data
This activity introduces students to the statistical investigation cycle from the GAISE framework (Franklin et al., 2007) and developing their understanding of the question and collecting data elements of the cycle. Students take a survey before class to collect data. During class, students are presented with the main types of variables (categorical: ordinal and nominal; quantitative: discrete and continuous) and provided examples of each. They are then asked to identify the variable types from the survey. Then the different types of questions (research, statistical, and analysis) are discussed and more specific criteria are presented of what makes a good statistical question. Students are then asked to come up with a statistical question they could ask given the data they have from the survey.
Statistical Investigations: Analysis
For this activity, students explore data on the schools (School Report Card) near the institution that the students are attending. The students begin by exploring several categorical variables on the school report cards and considering how those variables are measured, what they tell them about the school, what they don't tell them about the school, how the data can be organized and visualized, and how to ask and answer statistical questions using categorical variables. Students then move on to considering quantitative variables in the context of school report card data and the main ideas considered on categorical variables.

Statistical Investigations: Interpretation and Interrogation
Students are provided a statistical argument presented by the superintendent of the state educational authority. They were asked to consider the questions: "What do you notice? What are you left wondering? What message is the person who created the infographic trying to convey?" The argument gets at the difference between how the mean and median measure the center of a distribution and how outliers impact them in the context of income data. To help highlight the issue, students then look at a dataset from the American Communities survey that is a representative sample of the region around the institution and investigate the question of what the typical income of people in the region is. Students use the analysis techniques they learned in the previous activity to then respond to the question and connect what they learn back to the data argument they were considering earlier and interrogate the argument. Students are then shown two different types of spatial data analysis tools one from the state educational authority (https://gdacreporting.ondemand.sas.com/srcfinance) and one from the Opportunity Atlas (https://www.opportunityatlas.org). Opportunity Atlas allows individuals to view the micro-level of a community, metrics that the program affords are measures such as income, commuting zone, and job growth. The program allows you to input a zip code within the United States and through various representations consider the make-up of unique regions. Students are then asked to use the tools to consider the following question about the context of income: "What trends do you see in the average teacher pay based on location? Are those trends mirrored in the opportunity data? Explain." Students then present their explorations.
Considering Space
To learn about spatial ways of knowing more specifically students are presented with descriptions of what spatial means, spatial relationships to consider, and what spatial justice is. Students are then given three different mapping tools to investigate the region around the institution in relation to the schools they were investigating with the school report card data from earlier. One map shows the school district boundaries. Another map shows the actual location of the school buildings. The final map allows students to investigate various "opportunity" variables. Students were asked to consider what the maps helped tell them about the schools and what information they don't tell them.
Comparing Distributions
Building on students' understanding of statistical investigations and applying their knowledge, they construct an investigation comparing two or more groups on a quantitative variable. Students learn how to compare distribution, which measures to focus on, and how to interpret findings. To continue to build the students' understanding, both quantitative and critical, we chose to use the Common Online Data Analysis Platform (CODAP) software, with a constructed data set related to the target area. CODAP allows individuals to work from a set of data, choosing specific variables, creating different visual representations with ease. CODAP allows an individual to drag and drop self-selected variables. The reason we chose to include a program to construct the representation, was that our focus for meeting the needs of the WoK was on the analysis of graphs, not the construction of the graph.

Putting it all Together: Statistical Investigation Project
This activity is a culminating project meant to pull together all the ideas that were focused on in the unit and provide the student an opportunity to show what they have learned. For the project, students are given a data set that contains information from the school report cards they were considering earlier but this time for all the schools in the state. Students are asked to come up with a statistical question that requires them to compare two distributions and then do a statistical investigation to respond to the question. They are then asked to delve deeper by investigating some pattern or outcome they observed in their statistical investigation using spatial data tools and other data sources they explored during the unit. For the final culminating activity our overarching goals included: – Quantitative: Focus on graphing and descriptive statistics, describing univariate categorical and quantitative variables, informally compare categorical and quantitative variables between groups. – Socio-cultural: school culture, student backgrounds, student lived experiences. – Critical: structural issues in access to education – Spatial: location of educational opportunities, spatial distribution of opportunities, goods, and people, spatial boundaries of educational systems and policies. Students then created 15-minute screen capture videos presenting their investigations and deeper dives into the context.

Table 3: Descriptions of main designed activities for statistics unit

Conclusion

Through our work, we realize that this work is first and foremost necessary, but not simple. Our students, when engaging in activities supported and defined by our framework were able to consider situations through a lens that may have been novel to them. In focusing on specific aspects of the framework and then bringing it all together in the Statistical Investigation Project, we allowed the students to consider the quantitative, spatial, critical, and socio-cultural space in which they reside with diligent consideration. Through our Ways of Knowing Framework, in tandem with the tenets of Teaching Mathematics for Spatial Justice, we strived to promote a mindset that encourages individual growth that we hope is lifelong. Our goal throughout this research project has been to broaden the understanding of all individuals that together create a community, namely in rural settings. Based on our research we advocate that the use of our framework can be a powerful guide for designing learning experiences for pre-service mathematics teachers (PST).

Education and access to opportunities often determine trajectories established for students, especially in mathematics. In using the proposed framework to work with pre-service teachers, we have seen our students begin to gain insight into multiple ways of knowing that we hope will help them to better understand and support their future students. We hypothesize through such experiences the lens through which they see and understand the world becomes inclusive, allowing their ways of knowing to look at the social, critical, and spatial challenges. When considering how the uneven development of geography

impacts the opportunities of students in various communities, it is essential to equip pre-service teachers to critically examine and validate those practices that fail to provide equal opportunity.

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Cultural artefacts and mathematics: Connecting home and school

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Cultural artefacts are anything constructed by people within a group that reflects the cultural identity of that group of people. A culturally contextual teaching and learning approach supports learners by engaging them in an investigation of a topic in a cultural setting. In this study, four mathematics teachers and thirty-two students from grade six to ten participated. Students were provided a rich environment to explore mathematical ideas embedded in cultural artifacts found in the out-of-school environment. The mathematical ideas embedded in these cultural artifacts enabled students to develop mathematical ideas beyond the four walls of the classroom. Both teachers and students reported that the cultural artefacts facilitate an opportunity to explore mathematical ideas in a cultural setting and helped them to connect mathematical ideas located in the out-of-school environment with formal mathematics.

Introduction

Cultural artefacts are any object created by the people of any cultural group. These artefacts reflect the cultural identity of those groups of people. There are different types of cultural artefacts including dresses, houses, utensils, baskets, ornaments, paintings, designs, and so on. All these artefacts give information about the culture of their creators and the ideas associated with these objects (Pradhan, 2020). The study of cultural artefacts and the ethnomathematical ideas embedded in them are the sources for mathematics teaching and learning. Cultural artefacts provide rich opportunities to connect ethnomathematical ideas to the school curriculum. The pedagogy which inculcates the ideas, knowledge and experience of a certain group of people, who create different cultural artefacts that tends to preserve their social and cultural identity is called ethnopedagogy. Thus, the culturally contextualized pedagogy allows students to learn the mathematical ideas and knowledge of that time from the perspective of that particular group. Bonotto (2007) viewed that "the connection between real-world and classroom mathematics is not easy because the two contexts differ significantly" (p. 187). However, mathematics teaching and learning could be more interesting and effective if an appropriate connection is made between students' out-of-school activities and experiences and school mathematics. The connection with students' everyday activities and experiences seem to be more appropriate. It is widely acknowledged that the inclusion of ethnomathematical perspectives in school mathematics values students' cultural backgrounds and experiences. The observation of mathematical ideas embedded in

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different cultural artefacts located in the surroundings of students' cultural contexts would motivate students to see mathematics as relevant to their lives outside the classroom (Adam, 2004).

Students construct mathematical knowledge with the help of their prior knowledge, experience, and active participation in their environmental activities. The cultural activities of the children include many implicit mathematical ideas. These activities provide an opportunity to connect mathematical ideas embedded in the out-of-school contexts with school mathematics lesson. Francois, Mafra, Fantinato and Vandendriessche (2018) highlighted that the integration of learners' out-of-school knowledge is a stepping-stone for acquiring new knowledge. However, in Nepal, our school mathematics curriculum and teacher training are largely ignoring learners' cultural activities and their ethno-mathematical ideas (Pradhan, 2017; Ezeif, 2002). This is not only an issue with in Nepal, since Rosa and Gavarrete (2017) also observe that children's ethnomathematical knowledge and learning approaches are not taken into consideration in the formal school mathematics curricula. The mathematical ideas embedded in cultural artefacts and students' experiences should be blended with formal mathematics in the classroom. In this paper, I explore the mathematical ideas and knowledge embedded in cultural artefacts and in doing so address the following research questions.

- What mathematical ideas and knowledge are embedded in cultural artefacts?
- How might the mathematical ideas embedded in cultural artefacts be connected and used to understand school mathematics?

Framework for connecting home and school

It is now well understood that each student brings a unique set of knowledge, skills, and experiences to a new learning situation. Various research has identified that cultural artefacts and embedded ethnomathematical ideas help learners to develop formal mathematical ideas and knowledge (Pradhan, 2020). Paulo Freire (1970) suggested that "children cultural capital, the knowledge children bring to school from their home and cultural environment, should be welcomed and utilized in school for teaching and knowledge building process" (as cited in Stringer, Christensen & Baldwin, 2010, p. 24). In this way, students' out-of-school knowledge embedded in different cultural artefacts is celebrated and utilized as a pedagogical tool in the construction of mathematical meaning.

Constructivism is a widely supported educational theory that rests on the idea that students create their knowledge in the context of their own experiences (Fosnot, 1996). It focuses on students, being actively engaged in doing rather than passively engaged in receiving knowledge. The development of student's ability to construct knowledge requires apprenticeship into culturally specific cognitive and social practices. The cognitive development of a child's increasing mastery over the culturally determined developmental tasks is mediated by social agents. Vygotsky (1978) argued that an understanding of how knowledge develops requires an understanding of the social and historical origins of

knowledge and changes in that knowledge. Vygotsky (1978) metaphorically described the Zone of Proximal Development (ZPD) as the difference between a child's performance and that child's performance when guided by experts. This framework argues that learners are actively engaged in an activity when they are based on their interest, constructive investigation, and collaborative learning. Learners acquire knowledge about their culture and history from their encounters with adults and peers.

The framework developed in this study shows how the cultural artefacts and ethnomathematical ideas observed in children's out-of-school contexts can be a mediating tool to construct mathematical meaning. My argument in this study is that the cultural context in which children are situated is a rich environment to generate and distribute mathematical knowledge. Therefore, cultural artefacts are the background knowledge for the learner to develop foreground knowledge (Vithal & Skovsmose, 1997), a series of mathematical concepts and ideas to be perceived. In view of this, Lakoff and Nunez (2000) also considered children familiar context and experience to be the source knowledge for the development of abstract concepts (target knowledge) of mathematics. If the cultural artefacts found in the out-of-school environment are seriously observed, the learners can unfold various mathematical ideas and knowledge hidden in cultural artefacts. My framework in this study argues that ethnomathematical ideas embedded in the cultural artefacts helps to foster school mathematical knowledge in effective and meaningful ways.

Methodology for connecting home and school

My research observed the ethnomathematical ideas and knowledge embedded in cultural artefacts. In order to make a plan to approach data recording before entering the field, I prepared interview guidelines and observation protocols for teachers and students so that it would be easier for me to generate the data where in the field (Creswell, 2014). I collected data through examining documents, observing behavior, and interviewing participants both students and teachers. I carefully recorded all possible conversations with the help of the video camera and took field notes as much as I could. The data generated with the students' communities reflect how they are rich in ethnomathematical ideas and knowledge. In my study, the cultural artefacts in the students' community and their ways of understanding the natural phenomena, and their ethnomathematical knowledge were analyzed using the notion of pluralism. While conducting this research, I continuously addressed the research questions exploring the mathematical ideas embedded in the cultural artefacts and their use in teaching and learning school mathematics. The following procedure was adopted to connect the mathematical ideas and knowledge embedded in cultural artefacts to formal school mathematics.

Selection of Study Location: One of teacher participants of my study and I together visited some temples before selecting the study location where we observed the possibilities for incorporation in a mathematics lesson. Ultimately, we decided on Old Guheshwory temple situated at Tarakeshwor Municipality of Kathmandu District as the study location for the

field trip. The temple, Old Guheshwory, is a prehistoric temple established centuries ago. There are many monuments and artefacts present in and around the temple.

Selection of Student Participants: Thirty-two students were selected for this study. Out of them, eight students were selected from each grade of 6 to 8 (basic level) and four students were selected from grade 9 and 10 (secondary level) each. The student participants of this study were selected from a secondary public school of Kathmandu District.

Student Awareness Program (SAP) for Field Trip: Before the visit to the study location, the researcher presented the possibilities of mathematical ideas embedded in the cultural artefacts and encouraged them to explore. I had designed the SAP intending to encourage them to connect real-life situations and school mathematics. The SAP was conducted before going on the field trip and lasted about one hour. The thirty-two students and four mathematics teachers participated in this program.

Formation of Collaborative Group: Thirty-two students were divided into four different groups. Each group included 8 members from five different grades: 2 students from grade 6 to 8 each and one from 9 and 10. The group thus formed was heterogeneous and this provided an opportunity for collaborative learning. The group was formed in such a way that they represented students from all grade levels with the aim of providing a collaborative learning environment. Four teachers were assigned to facilitate each group.

Study Tools and Materials: Students were encouraged to explore mathematical ideas through observation of different artefacts surrounding the temple. They were provided measuring tape, paper, pencil, and other instruments by which they would explore and verify different mathematical properties. Students were also requested to write down their feelings and perceptions of the field trip, focusing on the mathematical ideas learned and the ways of gaining knowledge through the ethnopedagogy.

Lesson Design for the Field-trip: After reaching the study location, the students were first provided a common task. All the students were requested to observe different cultural artefacts surrounding the temple and note the mathematical objects they could identify. The teachers' role was to develop a lesson, monitor and provide feedback when students were engaged in the task. I along with the teachers, had to develop the tasks for the four groups of students after viewing the scenario in the temple premises. Four major open-ended tasks were: to identify a center of the circle, to identify a type of quadrilateral by measuring the sides of the terrace, to explore the concept of symmetry, and to identify an axis of reflection. Then the designed tasks were given to each group.

Data Management: This study was based on primary data collected through the observation of cultural artefacts and monuments, interviews with teachers and students. I reviewed all of the data gathered from the multiple sources (Creswell, 2014) and then organized them into categories or themes that cut across all of the data sources. After observing the data, I linked them with many possible theories to interpret them. I triangulated the data, and the theoretical closures and gave meaning to my findings.

Maintaining Ethical Issues: I ensured anonymity and confidentiality to all research participants and briefed them on how the data were going to be used and protected. As my study was intended to explore mathematical ideas of cultural artefacts embedded in the temple, it was ensured that the cultural and religious identity of the participants were highly respected and recognized.

Connecting cultural artefacts and school mathematics

Cultural artefacts represent the social and cultural knowledge and identity of the people of a particular community. Such cultural artefacts are rich in mathematical facts and principles and are source for mathematical concepts. Many mathematical ideas are embedded in those cultural artefacts. The result of this paper is based on the empirical research carried out in the course of my Ph.D. study. I had designed a lesson for the exploration of mathematical ideas outside the classroom with the collaboration of teacher participants. I had assumed the existence of the mathematical ideas beyond the four walls of the classroom and the students' mathematical competencies would be enhanced with the help of out-of-school mathematics.

After we reached the temple premises, we let the students observe the temple, monuments, and its surroundings. I asked the participant students to note down the geometrical objects and the probable mathematical concept that they could identify in and around the temple. After some time of observation, each group was requested to present their observations and findings. The students explored mathematical ideas embedded in different artefacts inside the temple premises. I observed that the students actively participated in the learning process. Each student was engaged in different activities in their ways. Some of them were taking data with measuring tools, while some were comparing those data to determine if they were symmetrical or not, and others them were using verification of different geometrical objects.

Each group involved in the common task serially presented their findings. The common observation made by the students were the 2D shapes of the objects like triangle, square, quadrilateral, circle, oval, octagon, trapezium, parallelogram, and rectangle embedded in the cultural artifacts. They also observed 3D shapes like hemisphere, cone, frustum, sphere, cube, cuboid, cylinder, prism, and pyramid. They sorted out the concepts of concentric circles, transformation, reflection, rotation, symmetry, pattern, and tessellations. The identification of concentric circles used to form such impressive artefacts also gives us a hint on how these mathematical ideas were created at that time which is far from today's formal studies based on bookish knowledge. Consistency and homogeneity are found in the construction of cultural artifacts. Therefore, we can see that the people used implicit mathematical ideas in such creations. Each artefact was precisely constructed resulting in the design of the artefacts. One artefact had a cuboid overlapped by other cuboids but of smaller size continuing till the top point was formed. It gave a shape like that of a pyramid. Also, there was an artefact with an octagon shape satisfying every property of a regular octagon.

Student participants also reported that the images that were carved in the doors, ceilings, windows, and other places were completely symmetrical. The artefacts which possessed bilateral symmetry seemed to have an axis of reflection which acted as a mirror giving the same image as the part which formed the complete view of those artefacts. The carving of different objects, in woods and stones were systematically carved. If observed with reference to any line, we get each part of the object that has the same reflected image. Every artefact that was considered to be a part of the symmetrical design was really attractive.

Image 1: Teacher assigning the task to students

One group of students was assigned a task to identify the center of the circular surface of the *Chaitya*. It possessed different geometrical shapes along with a circular face. The excitement in students was increasing. And probably, they were trying to figure out the mathematics present around the Chaitya itself as they were making an angle to angle view of it. This required physical materials like measuring tapes, ropes, markers, etc. which were provided to the group. They were left to accomplish of the task for about 15 minutes. Students were struggling to find the dimension of the circular surface of the *Chaitya*. They were trying to use a compass but this did not work out because of the larger size of the circular surface. Finally, they came to the teacher and said:

Image 2: Students engagement in the task

Student: Sir, we found out the center sir.
Teacher: Wow, what did you do children? Can you share that with me?
Students: yes sir!
Teacher: Please tell me the process.
Students: Sir, first we took a point in the circumference of the circle and put one end of the measuring tape in that point. Then, we loosely held the other side of the measuring tape and moved the tape along the circumference until we found the tightest point of the measuring tape. This probably was the longest chord, the diameter. Hence we marked the diameter and calculated the position of its midpoint. This is the center of that circular side.
Teacher: Wonderful! This is a great procedure, my children. I am really happy with you!
Student: Thank you, sir.

Image 3 depicted the procedures followed and the brief report presented by the student participants to solve the assignment given to them.

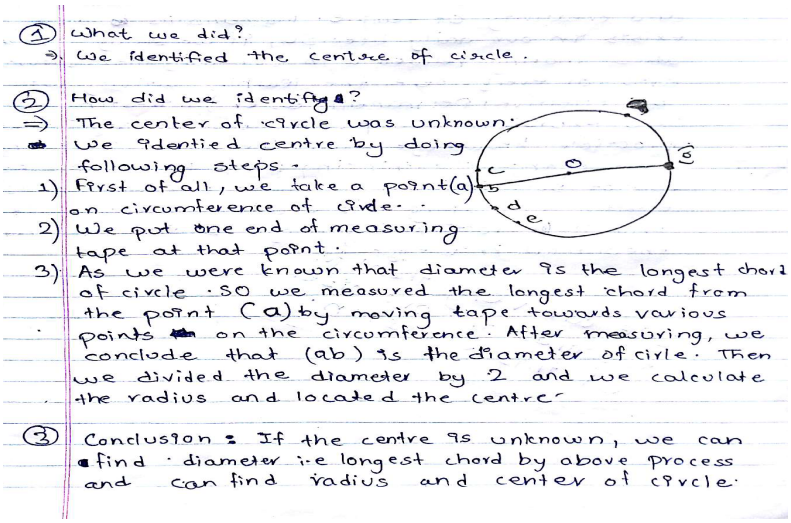

Image 3: Students' Presentation of the Assigned Task

Their report shows that they had used empirical knowledge to solve the mathematical problem of finding the center of the circle in the case of non-given other dimensions of a circle. As a researcher, I observed their presentation to their teacher facilitator. I observed their facial expressions which seemed to have a shining face, enjoying each step of their work. I also observed their behaviours. They were showing active participation and involvement in the task. All the students were involved as active participants in a group. Then I had a brief conversation with the students:

Researcher: Students, I appreciate your achievement!

Students: Thanks, sir!

Researcher: What did you learn today?

Students: Sir, we enjoyed today's practical work. We got that opportunity to work out the way that is not even in our textbook.

Researcher: For example.

Student: Sir we figured out the way of determining the center of a given circle in a way that is not mentioned in the textbook. You probably have noticed that in our presentation.

Researcher: Yes! Then what do you think about this project? Would it contribute to the learning of those textbook materials?

Student: No doubt about that. We internalized the mathematical concepts. We touched those mathematical ideas with our senses. We are never forgetting these learned items all our life.

Researcher: Thank you for your wonderful compliments.

Students: Especial thanks goes to you sir as you made all this provision.

I was really happy to see these students doing their tasks so eagerly. I could feel their excitement and enhanced facial expressions. The heterogeneous nature of the group involved in the task provided an opportunity to learn collaboratively. This framework provides an opportunity to learn mathematics concepts and ideas, even of a higher order, that are embedded in the cultural artefacts. The observation of cultural artefacts from a mathematical point of view contributed to the development of a positive attitude towards the subject. It also helped to elaborate mathematical concepts and develop knowledge to tackle non-routine problems.

The wonderful mathematical ideas and concepts were found in the observation of different artefacts. The mathematical ideas embedded in the out-of-school context and the pedagogy used in their cultural setting could be a powerful tool for the teaching and learning of school mathematics. The culture-friendly pedagogy provides students with the opportunity to explore the mathematical ideas embedded in different cultural arts and artifacts. Regarding the students' views on the ethnopedagogy, one of the student participants say:

> It is our first trip of this kind. I had never imagined that mathematics could be learned without a textbook and worksheets beyond the classroom. We learned a lot of mathematical ideas and became able to explore mathematical ideas embedded in the cultural artifacts.

From the observation in the field and the interviews with the students, I found that ethnopedagogy is an effective approache for the teaching and learning of mathematics. This approach for teaching mathematics provided an opportunity to learn mathematical concepts embedded in the cultural arts and artifacts. This approach created an environment for the students to construct mathematical knowledge and develop ideas in their own ways. With this connection, other research participants said:

> I never thought that mathematics can be learned without a textbook. With the observation of different cultural artefacts in the temple and its premises, we find different mathematics objects. We verified the mathematical facts and properties with the measuring instruments and calculating their dimensions. I learned different mathematical ideas with joyful moments and enjoyed a lot with this approach.

The constructivist framework was followed during different stages in my study. The students were given a rich learning environment and allowed to create their meaning by providing different tasks during teaching and learning through the use of ethnopedagogy. As my theoretical orientation about knowledge generation is based on the premise of a constructivist philosophy, the children construct mathematical knowledge as a result of active participation in their social context. In this vein, Vygotsky (1978) argued that cultural practices and resources mediate children in the process of development of thinking and can help them to learn school mathematics. Many things in the out-of-school environment can be connected to the teaching and learning of school mathematics. In my observation in the field, the study of different cultural artefacts plays a significant role in the teaching and learning of mathematics. The connection of students' familiar context in the process of teaching mathematical content provides a rich opportunity for the learners. The incorporation of an

ethnomathematical approach in the school mathematics classroom encourages students to learn and develop a positive attitude towards mathematics.

Concluding remarks

Mathematics is a pan-cultural phenomenon. Each culture has developed its mathematical system based on their everyday needs. There are lots of mathematical ideas present in the out-of-school environment if analyzed seriously. The observation of the cultural artefacts and the mathematical ideas embedded in those artefacts motivated and encouraged the students to explore more mathematics. They may also develop a positive attitude towards school mathematics. The study of mathematical ideas embedded in cultural artefacts helps to create ample opportunity to develop mathematical knowledge beyond the four walls of a classroom. The field-trip approach can be used as a pedagogical tool for the teaching and learning of mathematics. Their report shows that they had used empirical knowledge to solve the mathematical problem of finding the center of the circle in the case of non-given other dimensions of a circle. This approach provides students with the opportunity to learn mathematics in their own way and to develop mathematical ideas without the textbook and beyond the classroom. The collaborative learning framework involving heterogeneous group members from grades six to ten provides an opportunity to learn mathematics concepts, even at the higher level. Students from the junior classes also explored different mathematical ideas with the help of senior peers. It is concluded that students' knowledge construction ability requires apprenticeship into culturally specific cognitive and social practices.

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Narcissus and Echo, content and learning, meanings and belongings: a decolonial possibility in school spaces

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The school system constitutes a hierarchical, colonial and exclusive space that maintains an unequal society and that reinforces privileges of some over others. Content and learning ideas are central to their configuration. With this scenario, we produced in this essay a possibility for school spaces, from a decolonial reading of Grada Kilomba, in his interpretative performance of the Greek mythology Narcissus and Echo. Images, sketches and clippings of texts are constituents of this narrative. An alternative, in the midst of what happens in this school system, can be constructed from a decolonial discussion of education (mathematics), through the ideas of meanings and belongings.

While I write,
I am not the 'other' but the self,
not the object but the subject.
Grada Kilomba

Grada Kilomba, a Portuguese artist based in Germany, held in 2017 an exhibition at the São Paulo Biennial with the following title: Illusions Vol. I Narcissus and Echo. In an artistic way of revisiting and reinventing some myths that constitute relationships and affections in our contemporary society, Kilomba invites to produce other relationships and other affects, which serve as inspiration in the writing of this narrative. It is an invitation to a decolonial production that problematizes two protagonist "actors" in the school: the contents and the learning. Through compositions with the performance of Grada Kilomba, in the midst of images, clippings of her writings and doodles, we produced a narrative and other possibilities to interrogate school mathematics contents and learning.

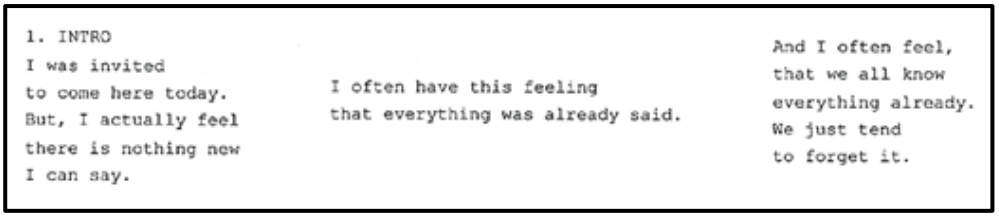

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Image 1: World invitation (Adapted from Desobediências poéticas de G. Kilomba, 2019, *Pinoteca de São Paulo*. p. 62)

Narcissus

In the 16th century, Caravaggio painted a narcissus in love with his image. In the spell given by Nemesis, the goddess of judgment, this young man should love someone who would never love him back. With few possibilities, one and only one was present in his life: to love his own image.

Image 2: Narcissus (from Narciso, Caravaggio de M. Imbroisi; S. Martins, 2019. *História das Artes*. In: https://www.historiadasartes.com/sala-dos-professores/narciso-caravaggio)

Grada Kilomba writes:

PART II
VIII.
Narcissus,
Narcissus became
a metaphor
for someone who takes itself
and its own body,
as the object of love.

Narcissism,
narcissism is the
love directed towards
the image of oneself,
it is the excessive admiration
of one's self appearance,
and the incapacity to love
or acknowledge others
as objects of love.

Image 3: Narcissus (Desobediências poéticas by G. Kilomba, 2019, *Pinoteca de São Paulo*. p. 71)

Unfolding this reading of Narcissus, we have a contemporary society at different levels and intensities, with subjects manufactured under this logic. Going a step further, the Myth of Narcissus does not only refer to people, but also to systems and, in our reading, to the school system. This hierarchical, colonial and exclusionary system that maintains an unequal society and that reinforces privileges of some people over others (Quijano, 2005; Walsh, 2013; Tamayo, 2017; Giraldo and Fernandes, 2019). It is evident that these inequalities are produced in different ways and scales. An example would be a small city in the global north, in a rich

country, compared to a small city in the global south, in a poor country. However, when producing a global reading of our society, we see that it is strongly marked by economic inequalities, structured from binarisms, exclusions, misery, wars, climate crises, etc. Just as an example, COVID-19 is a symptom of this global society. It's not a problem. The global economy is not in crisis due to the pandemic. It is the economic, social, political and climatic relations between humans and non-humans that produce this symptom of our global society (Lugones, 2014).

Image 4: Narcissus and school (Desobediências poéticas by G. Kilomba, 2019, *Pinoteca de São Paulo*. p. 55)

In the school system, mathematics occupies a prominent place, sometimes considered the most valued discipline that ultimately defines who will be excluded or included (Bishop, 1999; Walsh 2009). Mathematics in the classroom is constituted by contents ordered in a hierarchical manner, linearly, from simpler to more complex processes. Mathematics in the classroom is often closed on itself. Some things can be said, through predefined rules and processes. Other things cannot be said, because there is no definition, axiom, theorem, corollary or property that allows to speak. School mathematics often seeks a fixation of its own image.

Image 5: Narcissus as a school system (Desobediências poéticas de G. Kilomba, 2019, *Pinoteca de São Paulo*. p. 48)

Sometimes, assessment practices materialize at school through content and the students' learning of this content. Content and learning assume a unique place in the discussion about education, compose the matrix of a colonial process, it constitutes political strategies for a system closed in itself, a system passionate about its image, constituted by eurocentric patterns, which overlap its modes of know everyone else (Quijano, 2005). The limit of the school, and of the whole school system, is making students learn content in the hope that, with that, it will be possible to build another society. Hope is an affection of stagnation, dissatisfaction as a structure of enjoyment and paralysis of subjects in schools.

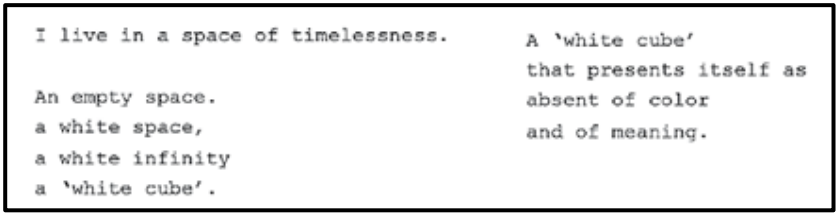

Image 6: Space (Desobediências poéticas de G. Kilomba, 2019, *Pinoteca de São Paulo*. p. 75)

Echo

But the story of Narcissus cannot be told without the story of Echo, as Grada Kilomba warns us.

Image 7: Echo and Narcissus (Desobediências poéticas by G. Kilomba, 2019, *Pinoteca de São Paulo*. p. 50)

One day, Hera, goddess who protects marriage and fertility, was looking for her husband Zeus, god of gods, sovereign, who was having fun with nymphs. Echo, in an attempt to protect Zeus, engaged Hera in a long conversation, so that Zeus had time to escape. Faced with this attitude of Echo, Hera cursed the nymph with the following spell: Echo, who loved to speak, could now just repeat the last words she heard. The nymph would never say a word of her own again. Soon, Echo fell in love with Narcissus and, being unable to speak a word,

she lived in hiding looking and contemplating her passion. One day, given an opportunity by Narcissus' speech, Echo repeated his words, corresponding to his expectations. Composing with Grada Kilomba:

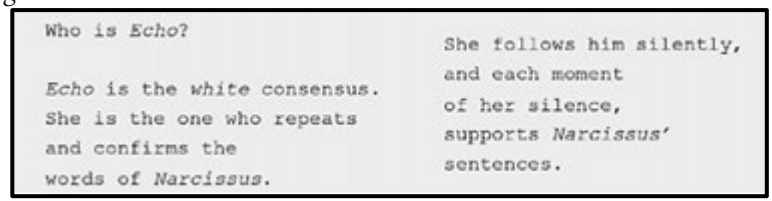

Image 8: Echo (Desobediências poéticas by G. Kilomba, 2019, *Pinoteca de São Paulo*. p. 76)

Echo authorizes Narcissus' speech. She sustains and maintains Narcissus' wishes. In our reading, Echo is constituted in the evaluations that take place in the mathematics classrooms, evaluations of the students' learning that reproduce the consensus projected by the system and that in the end make the system stays as it is, maintaining its current power structure. The school system produces some success and excessive failure and operates with no voice, no questioning from those who live and follow the rules and make other movements unfeasible. Echo, too, constitutes, in our reading, the external comparative evaluations of school systems, which are carried out in a naive movement in search for quality in education, in a comparison between students from different countries, even though this assumption is never made explicit by the students and by the international assessment systems.

Learning assessment in mathematics is a research theme in the area of Mathematics Education, which presents different possibilities for teachers to carry out other assessments in the classroom, such as an assessment as a research practice (Buriasco; Ferreira; Ciani, 2009, Esteban, 2001), an assessment for learning (Fernandes, 2009), the investigation of students' errors, as pedagogical possibilities for the teacher's work (Borasi, 1985, Viola dos Santos; Buriasco, 2008). However, all these practices are limited by the learning of students' content, or even the production of other dynamics in the classroom. They operate in a logic of improving the classroom, as well as the system, the progress and development of students and their knowledge in schools. Echo, that is, the evaluations that take place in schools have little impact on structural changes in our society. As we read in Grada Kilomba:

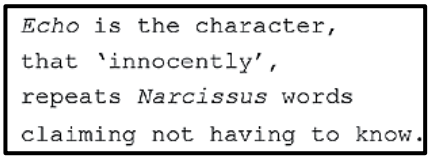

Image 9: Echo (Desobediências poéticas by G. Kilomba, 2019, *Pinoteca de São Paulo*. p. 76)

The colonial project of (mathematics) education

Narcissus and Echo, according to our read of Grada Kilomba, help us to problematize the mathematics classroom and the assessments that take place there. Our problematization focuses on the content and learning. Perhaps, these are the strongest products of a colonial project[1] and the organization of a society that has the school as a space for the reproduction of its logic, affections and knowledge.

In our contemporary society, fabricated by surveillance capitalism, increasingly violent in relation to the ways of coexistence between humans and non-humans, we have a school that is often constituted as a hierarchical, binary and exclusionary system (Quijano, 2005; Walsh, 2013; Tamayo, 2017; Giraldo and Fernandes, 2019). Some characteristics of the contemporary school education appear democratic in access, in opportunity, in quality and in the production of knowledge for an entire development of the human being, assuming a character of universality. However, we are still guided by the assumptions of a (mathematics) education project that highlights traits of Eurocentrism and operates with the effects of coloniality (Quijano, 2005).

In this school, subjects are sometimes constituted as objects, such as those in which "reality is defined by others and identities are created by others" (Kilomba, 2019, p. 28). With Narcissus and Echo, we read this school as a place where it operates in reproduction and repetition actions, cast by rules, regulations and guidelines, designed by a strategic system that only reflects its images.

Concerning the [epistemological and hegemonic] (mathematical) knowledge that constitutes this school, Walsh (2009) states that it often feeds and maintains the oppressive structures of society, due to the conditions of production and because it is impossible for students to propose authentic explanations to the reality that surrounds them. They also reduce their possibilities of problematizing structures and inequalities that make up their lives and that of their families. In this way, school denies differences and operates as a place of exclusion, even when students learn mathematical content "well". In terms of Grada Kilomba:

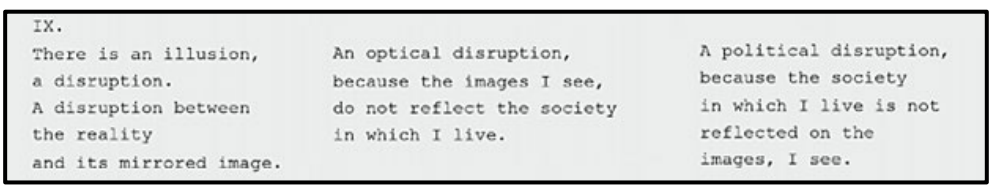

Image 10: Disruption (Desobediências poéticas by G. Kilomba, 2019, *Pinoteca de São Paulo*. p. 72)

[1] I speak of a colonial project in the direction of that opera with effects of coloniality. By coloniality, with Quijano (1997), I understand it as something that transcends particularities and maintains the logic of historical colonialism relations, which does not disappear with the end of the colonial experience, with independence.

Our argument is that the school is limited to discussions about content and learning. Other discussions unfold from this, but content and learning are still central. This limitation causes the school system to be constituted in a narcissistic way and that the evaluations that take place in the school spaces are reinforcement for the maintenance of this system, like the nymph Echo.

Discussions about different ways of organizing ourselves with nature, about the relationship between humans and non-humans, the affections and feelings that move us little to deal with global problems, such as: misery, climatic crises, deforestation of our forests, pollution of our rivers, the neglecting of refugee crises, the extermination of indigenous populations, the prejudices still very present in our relations, the idea of women still as a subordinate, submissive and violated human being by the idea of men, etc. are still little moved in spaces schoolchildren. Often, they can even be inspirations for some projects, but the final target is the mathematical content and the learning of it by the students. Again, in resonance with Grada,

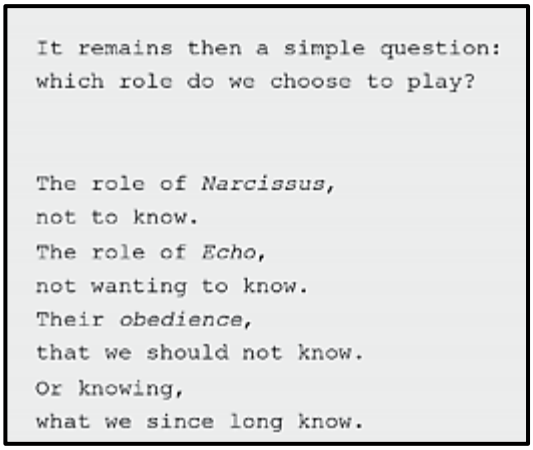

Image 11: Know (Desobediências poéticas de G. Kilomba, 2019, *Pinoteca de São Paulo*. p. 77)

For (mathematical) educations with decolonial attitudes: Meanings and belongings

An alternative to this scenario would be producing educational spaces based on the concepts of meanings and belongings. Not an alternative, in the direction of replacing what happens in schools. Much less as a salvation. An alternative that can be combined with what happens in schools, but that produces a process of deconstruction of these educational systems and, with that, possibilities for the construction of other affects, logics and knowledge. It is an alternative with a decolonial attitude, which puts ideas like improvement, progress, development on hold. It produces tensions, sparks, other effects with schools in a colonial project (mathematics) education. From Walsh, we assume the term decolonial in the direction

of not eliminating or silencing the colonial, but acting on the fissures as a place of production of possibilities, in becoming. That is,

> I do not intend to simply disarm, undo or reverse the colonial; move from a colonial to a non-colonial moment, as if it were possible that the employers' traces would cease to exist. The intention, instead, is to accentuate and provoke a positioning - a continuous posture and attitude – to transgress, intervene, insurgent and influence. (Walsh, 2009, pp. 14–15).

We speak of possibilities as an invitation to production/action, as "insurgent practices that fracture modernity/coloniality and make possible other ways of being, thinking, knowing, feeling, existing and living with" (Walsh, 2013, p. 19). Operating in a space of possibilities is "[...] creating another way of understanding, another way of articulating knowledge, practices, [and] subjects" (Santos, 2007, p. 39), who "have the right to define their own realities, to establish their own identities, to name their stories" (Kilomba, 2019, p. 28).

In this article, in the form of a narrative, we displace and discuss the characteristics of mathematics education in the school system with a decolonial attitude (Walsh, 2013). We do not intend to explaining a research object, present a theoretical framework, follow a methodological procedure and with that, present some considerations from an analytical movement. Our production, our theorizing process, invents itself in a narrative that unfolds some considerations, provokes some affections, and invites the reader to produce with us. It is not about presenting closed arguments, but invitations. It is not about producing explanations, but rather about bringing researchers into processes of inventing concerns. For this reason, we use Grada Kilomba's performances with photos, aphorisms, poems and writings. Some attitudes of decolonial productions in mathematics education that come close to our essay may be the works of Carolina Tamayo (2017) and Victor Giraldo and Filipe Fernandes (2019).

Meanings[2] produced by students, which are invented in their stories and life experiences, as a possibility to produce economic, political and cultural organizations in schools. A school, a mathematics classroom built on the basis of a political project in which the whole community is part of it and where the processes of producing meanings of the members of that community constitute a point of reference for this political project. Of course, that knowledge constructed and systematized by society, can also be part of this political project, but not as a centrality. For example, the idea of making decisions in different situations can be a starting point for building activities in which students can produce knowledge, narratives and affections. Making decisions does not constitute content, but rather an invitation, as a process of producing meanings of students and teachers (Lins, 2005; Viola dos Santos; Barbosa; Linardi, 2018; Paro; Viola do Santos, 2019).

Belonging could be another idea in the production of schools amid decolonial attitudes. It is not a matter of focusing on students' learning of certain processes of producing meanings, nor of evaluating those processes. It is about producing, carrying out maintenance,

[2] Meanings is a notion of Model of Semantic Fields (Lins, 2001, 2005, 2012; Oliveira, Lins, 2002). Just one draft, /.../ 'Meaning' is characterised as what a person actually says about an object, in a given situation (activity); but it is not everything that a person could eventually say about that object (Oliveira, Lins, 2002, p. 2)

deconstructing certain processes of belonging of students and teachers in certain educational spaces. Economic, political and cultural conditions are central to the discussion and problematization of processes of belonging. Collectivity and uniqueness of students and teachers are two sides of a coin that are tense at all times. The demands that provoke movements in schools (processes of producing meanings and processes of belonging of students and teachers) are directly related to the lives of these students and teachers, in an attempt to produce readings from the global issues of our society.

Wars, hunger, climatic crises, dehumanization, that is, human and non-human relationships in the Anthropocene, can constitute a possibility for the production of meanings and belongings in school spaces. The verb to learn would be just one more in schools, in compositions along with the verbs to imagine, to dream, to smile, to cry, to move, to create, to relate ...

Narcissus and Echo, school and evaluations, students and teachers, families and communities, human and non-human, in crossings that produce other space-time materials, are invented in other modes of life production, always in an invitation, between meanings and belongings.

Image 12: Producing (Desobediências poéticas by G. Kilomba, 2019, *Pinoteca de São Paulo*. p. 58)

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Developing STEM identity: Beyond STEM content knowledge in an informal STEM club

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In this study we seek to expand the boundaries of mathematics education research. We explored a group of women's experiences attending an extracurricular, informal STEM (science, technology, engineering, mathematics) club in the 1990s. Using oral history methods, we learned that women's experiences in life and the club contributed to their identity development. These women have allowed themselves to be, or keep searching for, who they want to be. As a collection, these stories evidence the potential of informal learning spaces as spaces for identity development that are usually absent in school mathematics. As identity development is crucial to develop critical and independent thinkers, this study is a call for creating spaces where underrepresented learners, like women (Wood, 2020), can reach their fullest potential.

As mathematics teacher educators (MTEs), we have partnered with the founder of a long-running extracurricular, informal girls STEM club (GSC) on a broad research agenda to document, celebrate, and analyse the experiences of the founder, club leaders, and girls who have participated in the clubs. The GSCs were initiated in the United States in 1994 by a parent who was dismayed to hear her 10-year-old daughter opt herself out of attending a magnet school because "math is hard" (Personal communication, October 20, 2020). This mom knew that math was not hard for her daughter, who had great test scores and grades. Her daughter seemed to be sliding into not wanting to appear smart, and she wanted to do something to stop it. She started the first GSC at her daughter's school for her and her friends in fifth and sixth grades. For 25 years, this mom shared resources from robotics, engineering, science, mathematics, and computer science, as well as support for implementing activities and developing funding sources and club policies to help GSCs spread all over the world in 28 states and 9 countries (e.g., Nigeria, Vietnam, Australia, and England) to enhance girls' interest and confidence with STEM in an informal, afterschool learning environment.

Leaders run the clubs for girls in grades K-12, ages five to 18, and plan the activities, which may be completed in a single club meeting or be extended projects over a semester. Leaders are facilitators, rather than the one who holds the answers and gives grades, as they work to create environments and activities that support girls to take risks and develop

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positive feelings and dispositions towards STEM. For example, as girls, the women in our study explored activities related to the stock market, dissecting owl pellets, meeting women who work in STEM careers, and assisting a veterinarian.

In this study, we aimed to document memories of women's experiences attending the first GSC in 1994. We used oral history (Shopes, 2011) as a data collection and analysis method. We consider that these women's stories provide insights into the influence of informal learning spaces on supporting women's identities. We investigate the following questions: What stories do participants share about themselves and the GSC? What insights can we (MTEs) gain from learning about the participants' identities?

Perspectives and theoretical framework

Mathematics education and informal learning spaces

Mathematics education is a field that emerged at the intersection of mathematics and psychology (Kilpatrick, 2014) that has broadened its research topics and practices over time (Stinson & Walshaw, 2017). Although the focus in mathematics education research has been the "didactic triad" (Valero, 2010, p. LX) of the relationship between mathematics, teaching, and learning, researchers have called for expanding research topics outside of the didactic triad (Ernest, 1998; Valero, 2010). Examples of the expansion of mathematics education research topics are the research program called ethnomathematics (D'Ambrosio, 1985) and mathematics in informal learning spaces (Nemirovsky, Kelton, & Civil, 2017; Nunes et al., 1993).

Informal learning spaces and GSCs

Extracurricular, informal learning spaces like GSCs provide opportunities that can sustain girls' early interests in STEM disciplines through their later years of schooling in various ways. Sadler et al. (2012), for example, suggested that programs designed to support school-aged girls' interest in mathematics and science can increase their interest in choosing STEM-related career paths at the end of high school. Exposing girls to a variety of female role models in STEM learning and careers allows girls to see that they are worthy of representation in STEM fields (Anderson & Cavallaro, 2002). Hands-on activities that excite, spark curiosity, and connect school-day lessons to their everyday lives can increase girls' interests and shape their identities in STEM areas (e.g., Chen et al., 2011; Holmes et al., 2012; Bell, Lewenstein, & Shouse, 2009; Tyler-Wood et al., 2012). Other positive academic impacts can include improving STEM content knowledge, enhancing motivation and engagement in STEM learning, and improving academic performance (Chittum et al., 2017; Krishnamyrthi, Ballard, & Noam, 2014; McCreedy & Dierking, 2013; Moreno et al., 2016; Sahin, 2013). Girls may also find the unique characteristics of the informal learning spaces to be beneficial, such as having freedom to follow their own interests and values, feeling less pressure from school academic requirements or standardized testing, and working in collaborative groups (McCreedy & Dierking, 2013).

Identity of girls

The construct of identity can provide insights into experiences of learning mathematics in all learning contexts and for all learners. For our study of women who participated in the first GSC, we draw insight from Darragh's (2016) definition of identity

> as the performance and the recognition of the self. It exists in the moment of the performance and as it is recognised. We perform ourselves—be it by telling stories, joining groups, acting in a particular way at a particular time, positioning ourselves and others within wider societal discourses…Furthermore, identity is a result of the process of identifying, whether this is self-identification or identification by others. This view of identity keeps in mind the audience at all times as the ultimate identifier and enables us to consider the ways in which power is exerted in this recognition (p. 29).

The identity that results from the process of identifying may not be desirable. The identity performance acts of the past influence the identity performance acts of the present and future.

Methods

Oral history in mathematics education

Oral history is a qualitative methodology that has been broadly used by sociologists, anthropologists, and historians that is linked to memory and orality (Garnica, 2011).

> Oral history is understood as both an act of memory and an inherently subjective account of the past. Interviews record what an interviewer draws out, what the interviewee remembers, what he or she chooses to tell, and how he or she understands what happened, not the unmediated 'facts' of what happened in the past. An interview, therefore, renders an interpretation of the past that itself requires interpretation (Shopes, 2011, p. 452).

Although oral history methods are not widely used in mathematics education research, these methods have been used in mathematics education research to explore the history of training, practices of mathematics teachers, and teaching artifacts (Gomes, 2019). When mathematics education researchers use oral history, they carefully prepare for the interview by defining the focus of the inquiry, collecting and studying background information, and developing plans to cultivate rapport with participants (Shopes, 2011). Although the women in our study will describe the same events, the subjective nature of oral history causes each to tell a different story (Garnica, 2011).

Participants and research team

Participants in this study were nine women who were members of the original GSC. Now in their mid- to late-thirties, the women participated in the first GSC in 1994-1995, when they were in fifth and sixth grades. They went to the same elementary school, which was located in a large subdivision neighbourhood outside a large metropolitan area in the United States. Seven of the participants lived in the neighbourhood when they attended GSC. All the women have obtained at least bachelor's degrees, and all are employed outside the home.

The research team consisted of one faculty member, a research associate, four doctoral students, two undergraduate students, and the founder of GSCs. While all members of the research team contributed to the development of our data collection tools, the research associate and doctoral students conducted all focus group meetings and interviews with participants. Two or three researchers were present for each meeting, which were all conducted over Zoom.

Contexts that inform the oral history data collection and analysis

We collected data in the fall of 2020, consistent with oral history methods (Shopes, 2011), which required collecting and studying background information about the time and place of the first GSC. The clubs' founder worked with a local historian to develop the local context from the mid-1990s (personal communication, October 14, 2020). The school was in a county just outside of a large metropolitan area in the United States. This school district had a reputation as having the best schools in the metropolitan area. Many people worked for the Federal government, making the population generally well-educated, middle- to upper-middle-class, and somewhat transient. The population around this school, however, was more middle- and working class, and rooted in the area, with many students graduating with friends from their kindergarten class. In the 1990s, the town's population grew 25% from 16,200 to 21,600, while the demographics shifted from 77% White, 10% Black, and 9% Hispanic to 58% White, 10% Black, and 26% Hispanic. During this time of suburban sprawl, town leaders sought to bring in more white-collar, professional office jobs, while the influx of Hispanic families brought more day laborers to the community and English language learners into the school. A large neighbourhood included family-friendly amenities such as the school and a pool, to which children could safely walk with their neighbourhood friends. However, lack of transportation for out-of-school activities prevented students who could not walk home or be picked up from participating. The school's principal acknowledged a risk of potential controversy over single-gender activities, but she gave permission for GSC anyway.

Data collection activities

Building from our understanding of the time and place of the original club, we developed a survey to elicit participants' experiences (Dewey, 1938/1998) related to their STEM learning and interests in both formal school and informal extracurricular settings, spanning their time in the club to the present. Fourteen of the original 41 GSC participants completed a survey, and nine agreed to participate in a follow-up focus group meeting and an individual interview. To accommodate participants' schedules, we arranged three focus group meetings of two, three, and four participants.

From the survey data, we created research memos to document emerging themes that influenced topics for our focus group interviews. To cultivate rapport with the women, who had never met us, we combed the survey data for details with which we resonated. For each of the three focus groups, one member of our research team found a personal photo that

represented an area of resonance with the women in that focus group. For example, three of the four women in the first focus group had children, so we shared a photo of one of us with a daughter in the garden. The three women in the second focus group each had a son and a daughter, so the researcher shared a photo of herself with her own son and daughter. The two women in the third focus group both enjoyed being outside in nature, so the researcher shared a photo of herself by a lake.

Focus groups were scheduled based on participants' availability. One researcher welcomed the women and outlined the goals for the meeting. She introduced herself with her chosen photo. To help trigger memories from their time in GSC (Salandim, 2019), a researcher asked the participants to introduce themselves using the following prompt:

> Introduce yourself to us as if you are entering the GSC so we can conjure an image of you as a girl anticipating the afternoon. I will give you an example from when I was in 5th grade. I will use some "juicy" words so hopefully you will be able to conjure me as a girl. "My name is Mary Francis, and I go by the whole name. I am thinking of when I was in 5th grade and going to the GAA Club, which is a girls' athletics club. I just came from Mr. Smith's class, have arrived at the bowling alley, and have switched out my own wood and leather clogs for the bowling alley shoes. I am looking forward to seeing my friends Lori and Nancy and hope I am on their team, but I am NOT looking forward to Molly Brooks telling me how to bowl."

Participants were encouraged to build on or add to each other's introductions. Afterward, a researcher shared our rationale and goals for the study. She described the individual interview process and asked the women to bring a photo that conveyed something related to their GSC experiences, past, present, or future. The focus group ended with the women sharing what they would say today to their little girl selves.

Focus group recordings were transcribed and reviewed as primary source research to inform each individual interview outline (Shopes, 2011). The researchers conducting individual interviews worked to develop skills in feminist oral history interview methods, learning to listen beyond the words shared and to probe for additional detail (Anderson & Jack, 1991). Prompts included the following, with specific prompts and questions developed from the survey and focus group data: (a) Tell us about the picture; (b) Tell us about the school and about being a girl in the school; (c) How did you decide to pursue your current career? (d) How were your experiences in your class and GSC similar or different? (e) How do you recognize a good math learner? (f) Would you say that you were(not) a strong mathematics learner? Two or three researchers were present for each interview, which were conducted over Zoom.

Analysis

In oral history methods, analysis of primary and secondary resources is completed before the oral history interview outline is created, thus shaping the individual interviews (Shopes, 2011). Here, analysis was completed after each data collection activity to inform the next data collection activity for the oral history story findings. As we set out to understand the role of GSC in participants' lives, the founder provided background information regarding

her motivations for starting GSC. Therefore, our survey questions focused on memories of GSC, feelings toward mathematics and science both in formal and informal learning spaces, activities engaged in with families/children, post-secondary experiences, and career choices. Analysis of the survey data indicated the women are more than the sum of their formal and informal education experiences and career choices, which led the research team to develop a strategy to connect with the participants as fellow women who are also connected to GSCs, rather than as stereotypical intimidating researchers. As mathematics teacher educators, we chose to analyze the survey data for *resonances* (Conle, 1996) in order to find correspondences on a very personal level, that might evoke an emotional dimension, as we needed to quickly develop a trusting relationship with these women to whom we were strangers. Focus group transcripts were analyzed using a feminist oral history listening framework (Anderson & Jack, 1991) to find areas to probe further in the individual interviews, such as missing or incomplete information, feelings and understandings of experiences, and our own areas of confusion or discomfort. After transcribing the individual interviews, we read the transcripts and lightly edited them to document the stories the women told (Shopes, 2011). While each woman defined the plot of her own story, we shaped their stories initially through questions we asked and the prompts we posed, as well as through our editing process.

In oral history research, the stories that emerged from the lightly edited transcripts are the findings. Next, we used Darragh's (2016) identity definition to understand instances of self-identification and identification of and by others in the stories.

Preliminary findings

We share preliminary story findings for two of the women, Jo and Amanda. To respect the space limit, we represent each women's findings using three paragraphs. The first paragraph is a summary of the woman's story, and the following paragraphs are excerpts of their stories.

Jo's story: A continuous search for confidence and a career path

Jo is a white middle-class mother, wife, and daughter who portrayed herself as searching for a career path, wanting to help people, and working on her confidence. When referring to others, Jo would make explicit her value for others' opinions; Jo will always listen and learn from observing others while running, playing with her children, or interacting with her husband. Jo remembered enjoying being with other girls at GSC as she could be herself. In a mathematics classroom, she would not raise her hand and would lead the boys to respond to the teacher's questions. Although, as an adult, she reported feeling confident around male colleagues, Jo sees value in girls being with only girls in clubs such as the GSC, especially to boost their confidence.

> I think it's such a positive, like has such a positive impact on these girls' lives, like they you know, maybe they are shy like I was and, and don't speak out a lot in class or because, because I was never one to raise my hand and answer a question. And so maybe it helps them with, you know, their confidence.

Two of her role models are women, her mom and sister, and she sees them as strong and successful women who allowed themselves to search for career paths that made them happy. Yet, Jo sees herself as opposites to her role models in terms of personality. Despite her shyness and lack of confidence, Jo never gives up on her career dreams. Across her adult life, Jo has had different jobs and been in places where she was the only woman and in other places where women have surrounded her. She has never felt inferior with males, but recognized differences when in predominately women's spaces. Jo valued being with other girls at GSC, and she wants her daughter to experience something similar.

Jo is afraid of failing people. Jo does not know where she got so scared of failure; rationally, she has never failed, but thinking of failing is always in her mind. "I don't want to fail, but sometimes you have to; how else would you learn?" Jo's inside voice tries to rationalize a feeling that overwhelms her. Her memories of being at GSC are pleasant as she felt confident in those spaces. As an adult, she is glad she has her running friends, husband, and role models alongside her. Their voices become an umbrella she opens and uses as a shelter to shake off her shyness in her pursuit of a career path that fulfills her.

Amanda's story: Living the present moment and loving people

Amanda is a white middle-class woman and daughter who told the story of being comfortable with uncertainty, living with minimal possessions, and feeling fortunate with her life. In her search for a career path, she was attracted to theatre, yet others let her know that she was not good enough to be an actress. However, she enjoyed the theatre people. Amanda was fortunate to have a female teacher who helped her see herself in the theatre field but doing a different job: Stage manager. She graduated from a very competitive program and loved her job. Her social network in her hometown and job was diverse; Amanda enjoyed being with people with different backgrounds, cultures, and identities. Amanda portrayed herself as a passionate worker, and a human being who enjoys nature and few possessions.

Amanda's stories portrayed her as an optimist and someone who lived a minimalist-esque life: "I don't own anything; I don't even have an apartment [...] I literally own what I can fit in my car." Her lifestyle has prepared her to be grateful for whatever life brings her. For instance, during the pandemic, she has enjoyed family and the nature surrounding her. Although she does not know if she will have a job in the future, she still plans to enjoy what she has in the present moment. Amanda's low needs and passion for life contrast with typical ways in which women are positioned by society. Amanda does not manage a house or children, but rather people in the performance industry. "I basically manage. I manage people and I manage what's happening in the show."

Managing people could sound as though Amanda's heart is cold and selfish. Yet, Amanda is full of love. Her parents encouraged Amanda's love for people since she was little. They supported her, exposed her to people from different parts of the world, and showed so much trust in her, that in middle school, she was often by herself in her parents' house with a group of male friends. Amanda recognized gender differences as a girl and an adult. For instance, when answering a teacher's question at the school, she described how girls would stop

themselves from responding, "I just think like stereotypically they [girls] were like 'oh the guys are gonna know it' or whatever." As an adult, Amanda felt confident enough to address any issue or question, in front of a male or female audience; she showed herself sure enough to be whoever she wanted to be, silencing societal demands.

Discussion

We aimed to answer the following research questions: What stories do participants share about themselves and the GSC? What insights can we (MTEs) gain from learning about the participants' identities? These women had non-traditional roles in society and told hopeful stories of themselves. They are both searching or performing themselves to others in a way that makes them feel fulfilled. Jo is still searching for her career path, while Amanda is proud of performing a role that could be considered by society as non-traditional for women. As MTEs, we learned from their stories that informal learning spaces have the potential to allow girls to perform an identity different from the one they perform at school. The two participants showed evidence of performing different selves in different places: Being quiet and shy in school, while chatty and confident in the GSC. For instance, in school, both participants perceived themselves as not needing to answer the teacher's questions as they portrayed the boys in class as capable of doing it. This was different in the GSC where, as girls, the women felt confident enough to raise their hands and speak up. Now adult women, they comfortably contribute, lead, and work with male colleagues in their work lives. Although we cannot claim any strong relationship between the participants' confidence as adults and their GSC experiences, there is something MTEs can learn from pedagogical approaches used in informal learning spaces. Participants enjoyed their experiences in the club, seeing their experiences as confidence boosting. Because the participants described themselves as performing other selves in such spaces, we think that informal learning spaces allow for identity development.

As MTEs, we found the process of collecting and creating the women's stories, and then analyzing the stories for self-identification and identification by others, to reveal complex relationships between factors that we had not previously considered to play a role in mathematics learning. Because MTEs usually study the relationship between mathematics, teaching, and learning (Valero, 2010), we have joined others who have claimed that informal learning spaces seem to promote identity development (Krishnamyrthi, Ballard, & Noam, 2014; McCreedy & Dierking, 2013). We see a need for MTEs to study practices that others use in informal learning spaces and then bring those practices into academic contexts. We envision school spaces as nurturing all learners' identities, in addition to nurturing their content knowledge. We are eager to continue this work.

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Interpretations of meanings in mathematics education and students with disabilities on higher school

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The article is an excerpt from a doctoral research and aims to understand about meanings that students with disabilities attribute to studies in mathematics. It is proposed to discuss concepts of Critical Mathematics Education, with interpretations of experiences of meaning according to the proposals of four concepts: foreground, intentionality, personal meaning and action-image. For this, the data production is composed of semi-structured interview with a higher education student. The results suggest the interpretations that the student attributes to studies in mathematics, relate to the socio-political dimension of education and can be described in terms of the concepts covered, with perceptions that reflect possibilities, impossibilities, priorities and confidence in the future.

Background

In Brazil, concerns directed at the purpose of including students with disabilities in educational systems, start with public policies, which are consequences of conquests of struggles and social movements. Today, the entry and permanence of this group of students at the university, make up some of the challenges that require the development of actions to assist this process, such as discussions that look for to facilitate the academic survival of students. Furthermore, as Skovsmose (2017) clarifies, it's not common that university studies address critical reflections in the socio-political context in mathematics. Therefore, it is important that in higher education, students have the opportunity to reflect the socio-political dimension of Mathematics Education, in order to promote their own engagement in studies.

The data presented in this article is part of a doctoral research (Roncato, 2021), conducted in Brazil, and report concerns with teaching and learning of mathematics. In this context, I investigate the following question: what interpretation of meaning in Mathematics Education are carried out in terms of foreground, intentionality, personal meaning and action-image? Understanding student's interpretations about studies in mathematics can reveal social and cultural aspects of learning and teaching, bringing contributions to debates in Critical Mathematics Education.

To start our studies, were invited nine students with disabilities, who were studying in Higher Education. The only requirement in relation to the course they attended was that there be mathematics classes in the curriculum. They are, therefore, students with various disabilities who take varied courses, as shown in the table below:

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Name	Disability	Course
Carolina	Robinow Syndrome	Pedagogy
Júlia	Fragile X Syndrome	Human Resources
Manuela	Fragile X Syndrome	Human Resources
Flor	Auditory processing disorder	Pedagogy
Bela So	Attention deficit disorder	Pedagogy
Polly	Deafness	Brazilian Sign Language
Lívy	Cerebral Palsy	Psychology
Lary	Low Vision	Psychology
Juba	Cerebral Palsy	Radio, Television, Internet

Table 1: The Students

Defining the participants, it was the moment to elaborate the questions that would be realized. In Skovsmose (2014a), Vollstedt (2011) and Kollosche (2017), the necessary and comfortable inspiration was found to start the discussions. The intention was to offer guests moments of reflection on the presence of mathematics in their lives, in dialogues about the topics: *stories of my life; mathematics in my life; mathematics studied in higher education; mathematics in my future.*

The interviews were divided into two moments: the first with the completion of the inquiries and, in the second, the presentation to the participants of the transcription of the results. For the inquiries, different forms of interaction were used, composed of: use of figures, photographs, sounds, clippings from newspapers and magazines, various objects, words or phrases and images of tasks of mathematics referring to the mathematical contents studied by them. These strategies had the objective to promoting accessibility and benefit communication. For example, were presented for the participants with a variation of figures, objects and photographs of animals[1]. Then they should choose ones that best represented what they thought and felt about mathematics (Figure 1):

Figure 1: Mathematics in my life. (Created by author)

This strategy model worked as an element that triggered the discussions, expanding the conversations that, in an interview in a questionnaire format, might not have the same effect.

For the analysis, we sought to transcribe the data according to the moments that indicated the experiences of meanings in Mathematics Education, interpreted according to four

[1] David Kollosche in his research uses this idea relating mathematical feelings to an animal. For more information see Kollosche (2017).

concepts: foreground, intentionality, personal meaning and action-image. This is because there are suspicions that these concepts may influence student engagement in mathematics studies, constituting some possibilities for interpretations of meanings. With Ole Skovsmose, we sought the understandings of foreground and intentionality; Maike Vollstedt contributed the concept of personal meaning; as for the concept of image-action, it was built from the understandings of self-concept, auto-exclusion and microexclusion.

Dialogues with the four concepts

Skovsmose (2014b) identifies in Mathematics Education concerns in face of a variety of conditions in which the teaching and learning of mathematics occur. It is based on dialogue and involves notions such as, for example, autonomy, freedom and social justice. Of these concerns, we can mention the concepts, such as: foreground, inclusion, landscapes of investigation, matemacy, mathematics in action, representativeness, meaning, intentionality, among others.

Thus, it is important to discuss the context in which students' learning occurs, understanding that some of them may recognize that studies in mathematics are fundamental for their professional future; others may only intend to achieve good exam results; and, still, there are those who are not interested in mathematics, because they are not engaged in the execution of the tasks of this discipline. In this case, Skovsmose (2018, p. 766) argues that "when students do not realize the meaning of what they are doing in the mathematics classroom, it may be due to the fact that they cannot connect it to the future". The author, then, proposes that the basis for the construction of interpretations of meaning in Mathematics Education, is the engagement of students in the context of the mathematics classroom and suggests, even though giving interpretations of meaning "through a foreground can reveal a socio-political formation of students' meaning experiences" (p. 766).

According to the purposes of Skovsmose (2014a), foreground is a process directed to the future of the person (or group of people), considering several aspects, such as: the possibilities that the political, social and economic situations provide for the person; what she, in these situations, considers possible to achieve; realities or illusions, hopes and dream conquests. For this reason, it is not static and can be reworked. And it represents aspirations and desires, defeats or achievements. It doesn't mean getting to a certain place in the future, but recognizing that you can get there. In addition, the author understands that foregrounds are multiple and built according to individual or group experiences.

As for intentionality, Skovsmose (2014a) suggests a reinterpretation of the concept, including social structures, with economic, political, cultural and discursive factors, understanding, therefore, that when addressing the interpretation of a student's intentionality when the focus is on studies, it is important to consider the social, his/her real life. Intentionality is to be directed to the choice of a student to perform or not perform the mathematical tasks proposed by the teacher, for example; also has to do with the reason, decision in this execution: perhaps the student decides positively to recognize in the learning of this content, an important element in the composition your professional future; or maybe he likes math and wants to improve his school performance, achieving good results in exams.

Thus, the intentionality to carry out school tasks or not, is conditioned to the student's commitment to learn this or that mathematical content. If the discussions focus on the school environment, a student's priorities tend to reveal the reasons that drive him or her away from studies, his intentionality when dealing with mathematics studies. The personal meaning that the student attributes to studies in mathematics

Vollstedt (2011) conceptualizes personal meaning as being subjective, expressiveness, object and action when the focus is on the student's mathematical learning. For her, a mathematical content is understood by the student if it is admitted as relevant personally. In this case, the study of the construction of personal meaning is important, since it can reveal potentialities and affinities evidenced by students with respect to mathematical studies, the relevance for each one in engaging with learning situations. It can be rebuilt according to each person's school and social life experiences. Thus, personal meaning is linked to the relevance that disciplinary contexts and tasks have for the student.

We can understand that personal meaning is linked to classroom situations in the moments when these contents and tasks are presented, to the experiences previously experienced when the focus is on learning, on the expectations of each one, has to do with intentionality and with the foreground. I also admit a relationship with the student's image of himself, bringing him closer to or away from his studies.

The authors Alrø and Skovsmose (2004), when proposing learning as action, open discussions conceptualizing zooming-in and zooming-out, understanding the former as the approach of students in learning, which develops from their engagement in tasks, intentionality of them. For the second case, students not engagement themselves in tasks, they drive away, the decision is not focused on studying or learning.

From this conceptualization, there are several elements that interfere in the engagement with the studies. One of them may be a student's image of himself. By image I understand how the perception that instigates, that pushes forward towards studies; or that drive the students away, repudiating everything that concerns mathematics. This image can be positive (it instigates, pushes for studies), it can be negative (it repudiates, it drives away) and it can be reconstructed in face of the possibilities and opportunities in face of a certain circumstance. The action-image concept is built from the influence of other concepts: microexclusion, self-concept and auto-exclusion. Carneiro, Martinelli and Sisto (2003, p. 428) present comparisons between students' difficulties directed to learning and self-concept, admitting that: "self-concept has been pointed out as one of the influencers in this process", as the student experiences successful experiences or school failure, according to the curricular component. In other words, a student may have a bad self-concept for one discipline and a good one for another.

Self-concept is the idea that the person makes of himself, a construct that influences self-esteem and that in some cases, becomes an obstacle when the context is turned to studies. We can understand how the student's image of himself, which can move him, both moving him away and bringing him closer to his studies.

The distancing of students in mathematics, the exclusion that many students impose, is conceptualized by Kollosche (2017) as auto-exclusion from mathematics, thus defining: "auto-exclusion from mathematics education can be conceptualized as the exclusion from the mathematical discourse by the learner" (p. 38). Therefore, Kollosche clarifies that auto-exclusion "is not merely the result of psychological dispositions of the individual, which could then be changed by pedagogical intervention, but that self-exclusion is created in the interplay of the individual and the social environment" (p. 39). For the understanding of microexclusions, Faustino et al. (2018, p. 900) clarify that "microexclusions are subtle practices, carried out consciously or not, which tend to isolate the individual in a given environment". If a student cannot keep up with the learning pace of other colleagues, he can isolate himself; or even when he feels inferior and afraid of his colleagues in the face of a lack of understanding of disciplinary tasks.

I consider that microexclusions, self-concept and auto-exclusion tend to interfere with the student's image of himself, when the focus is on studies in mathematics. If this image is negative, intentionality may not be directed towards engagement in studies, distancing the student from learning; if, on the contrary, the image is positive, it can promote a movement of approach, one directed towards learning.

Thus, what moves the student towards mathematical studies (or repels), his willingness to act in relation to these studies, energy, the possibility of boosting his own learning, performance, has to do with the image that it makes itself that directed to these mathematical studies. Action is an active part of learning, it is enthusiasm, the energy that acts on the student, moving him to studies. Therefore, it is possible to explore the interpretations of personal meaning in view of the student's image of himself, of the foreground, of intentionality. In addition, Skovsmose (2017, p. 18) asks "what could Critical Mathematics Education mean for different groups of students?", including students with disabilities. The challenge, then, is to propose advances in Critical Mathematical Education, dialoguing with interpretations of meaning, in the possible encounters of the four concepts.

For the article, the reports of a participant in the study, the relations that she establishes between the mathematics studied in higher education, professional practice and life itself are presented.

I'm afraid of math, I'm afraid of spiders

The way the person interprets the possibilities and impossibilities, joys and frustrations, opportunities, obstacles, hopes and fears, has to do with the experiences already lived and the influence of factors such as social, economic, political and cultural. In this sense, Skovsmose (2014a) relates the notion of foreground to the way in which the person experiences the experiences around him. So, knowing childhood, family aspects, cultural background and some detail of the life of people, can reveal tendencies to the foreground that, according to purpose Skovsmose (2014a), "the person's background it refers to everything she has ever lived, while the foreground refers to everything that may happen to her" (p. 35).

Who is Julia?

Júlia is student and at the time of the interview she was studying Human Resources. She reported that she had a happy childhood, surrounded by friends and games. Now, if there are any conversation related to mathematics in matters between friends, she considers that the conversation will be difficult to continue. She adds that she is afraid of mathematics, with its giant numbers and formulas:

> Student: Mathematics is something I don't get along with very well [...], I find it difficult to understand [...] mathematical operation, giant numbers, formulas.

Then, the student compared mathematics with the feelings she has for the spiders:

> Student: I am terrified of the spider, the spider has several legs that remind me of mathematics, with various accounts.

It is observed in Julia's words, a certain fear and detachment from everything related to mathematics, since she admits to be a discipline that is difficult to understand, having difficulties in expressing a positive relationship with this knowledge in higher education.

> Student: At university I never liked it. But it was not always so. At school things were easier and I felt better, because I could understand, easier.

When Julia saw on the table some math tasks, she was learning in her university classes, she frowned in an expression of disliking, as if to exclude herself, admitting that the content was too difficult for her, that she did not understand. And she completed by making references to the sad moments experienced in classes, pointing out that she was humiliated by colleagues in the face of the difficulties she had in carrying out the tasks. Such an attitude can lead the student to move away from studies, promoting a negative image of himself/herself when it comes to engaging in tasks:

> Student: For me, mathematics is something impossible to understand, very complicated and leaves me in a somewhat lost situation.

The image that the person makes of himself (or herself) is a construct that influences self-esteem, becoming, in some cases, an obstacle if that image for negative and, at other times, when positive, instigates, pushes towards something. In this case, Júlia considers that mathematics for her is something difficult to understand and that leaves her isolated, "half lost", perhaps an obstacle that makes it difficult to arrive somewhere in the future.

According to the understanding of Skovsmose (2014a), foreground is related to several aspects, among them, realities, illusions, hopes, possibilities or impossibilities. When asked about her wishes and expectations for the future, possible achievements, the student said she wants to have autonomy in life, be financially independent, have a good job and admitted that mathematics will be present in this future, only in ordinary financial operations, like shopping at the supermarket.

However, faced with the impossibilities of understanding mathematics, Júlia opted for alternatives, composing a new future that would make her happy. So, days after the interview, at our new meeting, Júlia said she had given up on the Human Resources course because, according to her, the course was permeated by "a lot of mathematics with giant numbers".

It appeared that her decision revolved around social influences and the promotion of the moment she was in. Thus, the student re-elaborated possibilities towards the future, then finding a reason to direct her to tomorrow: she opted for another course. This establishment of new possibilities for the future opened some paths for her, finding the reasons that lead to studies, a new intentionality. Given this context, Júlia invited me to participate in a research about Fashion (the new course she was studding). From the answers to the questions, the task that she should perform involves mathematical elements, with data collection, construction and interpretation of graphs and tables.

Although at the beginning of the study, Julia demonstrated a departure from mathematics, now, in the reworking of her future, she confessed that mathematics seems simpler to her, demonstrating that she likes it: "but this math is cool, without the giant numbers". She seems satisfied and happy in the face of the new learning in mathematics that were now available.

Skovsmose (2014a) is attentive to the existence of a close relationship between intentionality and foreground. The reasons that lead Julia to the new studies indicate a directionality, a being directed towards, an intentionality and, in this case, it concerns her foreground. The situation that drives her is guided by intentions, priorities, motives. According to purpose Skovsmose et al. (2009, p. 238), "the meaning of mathematical education is not only linked to the understanding of mathematical concepts, but also to the foreground of students and is related to the intentions for learning", with the reasons, with the affinities and, therefore, point to learning.

In addition, for Vollstedt (2011) it is essential to understand the universe that surrounds the student and the personal relevance that the studies have for him (or her), revealing potential and affinities that are evidenced by the students with regard to mathematical studies, being reconstructed according to the interpretations of each person's school and social life experiences. Possibly a reason, an intentionality in contact with knowledge, an engagement directed to learning. For Júlia, the re-elaboration of the reasons that propel her to studies may also reveal a re-elaboration of the personal meaning directed to mathematics studies.

Some discussions

Now, I want to present my views on interpretations of meaning in terms of these four concepts: foreground, intentionality, personal meaning and action-image.

On our first meeting, Julia showed a real aversion to everything that contained mathematics. Several times she cited the math of accounts, giant numbers and formulas, even comparing it with the dread she feels for spiders. As she herself stated, she never liked mathematics at the university, proposing that it was not relevant to the study. Still, she recognized the possibility of using mathematics in ordinary financial transactions, such as grocery shopping. The presence of mathematics is summarized in quantifications and numbers, in additions and other mathematical operations. There are suspicions that the image of herself takes her away from her studies. She dropped out of the Human Resources course, justifying that it contained a lot of math and giant numbers, demonstrating fears and obstacles, an irrelevant study.

It turns out that foreground is not static and can be reworked, as well as the choices we make throughout life, which can change. Júlia changed, having new priorities and, in this journey, took possession of cultural, social and economic factors from the moment that was revealed before her, in other direction to a new course. Intentionality can be reorganized, under the influence of various aspects, such as opportunities, desires, hopes, expectations and possibilities for the future. In the new studies, Júlia takes mathematics classes, but, as she said, it is a good mathematics and, therefore, she reorganizes her intentionality by addressing the possibilities of achievements. Apparently, the new studies are providing greater involvement of Julia in the educational situation, favouring her engagement in the mathematical context. This fact also contributed to her reconstructing personal meaning, highlighting the relevance of mathematics for new professional achievements. Still, when she declared that this mathematics was good, there are signs of an approximation with the studies, with the action-image being reinterpreted by approximation in the face of the change of reference.

More reflections

In this article, I proposed discussions seeking to understand what meanings a student with a disability who is studying higher education attributes to studies in mathematics. The paths taken were determined considering what I admit as one of the fundamental factors to reflect on mathematical learning, with interpretations of meanings attributed to the studies. My inquiries were guided by an understanding of the four concepts: foreground, intentionality, personal meaning and action-image.

I also follow Skovsmose's arguments (2018, p. 766) when admitting that "students' experiences of meaning have to do with how they see their future opportunities in life". In this case, the author proposes that the meaning is, first of all, an experience and highlights that "when students do not realize the meaning of what they are doing in the mathematics classroom, it may be due to the fact that they cannot connect it. you to the future" (p. 769).

Resuming Skovsmose's (2018) understandings, it seems that Julia did not understand the meaning of what she was doing in math classes when she was studding at the first course. At the first moment of the interview, Júlia admitted that there was no connection between the mathematics studied and the future. However, from the moment that she seeks other expectations for the future, there are signs of optimism and confidence in herself directed towards studies. The data suggest that the interpretations of the meaning experiences with students with disabilities attribute to studies, relate to the socio-political dimension and can be described in terms of foreground, intentionality, personal meaning and image-action.

Then, for Júlia, the experiences of meaning can be interpreted connecting to the proposals of the four concepts: Foreground as it is reworked and linked to the expectations of professional and personal life; Intentionality, as reorganized with reference to the professional future; Personal Meaning, as reconstructed for relevant to new studies; action-image, which is reinterpreted as an approximation to the new study references. It is possible that Júlia found reasons for studying mathematics, from socio-political influence in her life.

In Skovsmose (2018), the author approximates the concepts of foreground and intentionality, when it is including sociopolitical contexts in interpretations of meaning, when the focus is on studies in mathematics. Thus, I understand that these interpretations relate to the way the student perceives the possibilities, opportunities, longings, expectations, hopes, obstacles, aims, decision, motive, joys or sorrows related to the future. It is related to priorities that studies have for the student, can reveal affinities and potentials, the personal meaning. The student's image of herself or himself, directing him or her intention to the learning process, also receives sociopolitical influences. As a consequence, I consider the four concepts (foreground, intentionality, personal meaning and action-image), can connecting with each other in the student's learning experiences. I don't mean to say that this is the only path, motive, purpose towards learning, but it is one of the.

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Gendered narratives shaping whole-class discussions: Who is invited to present which kinds of solutions?

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In problem-based lessons, teachers invite students to present their ideas to the class. If multiple students arrive at a solution that the teacher wishes be shared, whom do teachers select to present a particular solution and why? Across provided categories of higher-achieving and lower-achieving girls and boys, teacher participants selected boys and girls roughly evenly. However, participants more often indicated intent to invite a lower-achieving girl to present a direct-model solution. On the other hand, participants more often indicated intent to invite a higher-achieving boy to present a more mathematically sophisticated solution. Written justifications for these intentions reveal gendered storylines about mathematics and illustrate an additional way those storylines shape classroom instruction.

Introduction

Problem-based lesson formats often begin with the posing of a mathematical task, allot time for problem-solving, and culminate in a teacher-facilitated whole-class discussion in which specific students are invited to present their work (e.g., Shimizu, 1999; Stein et al., 2008). Inviting students to present mathematical work to their peers encourages the sharing and evaluation of ideas, facilitates an emphasis on reasoning, and distributes both mathematical agency and authority (Gresalfi & Cobb, 2006). In general, when a teacher prompts a student to present ideas to the whole class, this is a public invitation to take a position, ostensibly of a noteworthy mathematical idea. Thus, students' participation in a whole-class discussion is a way of taking up and enacting status-oriented positions a site where social negotiation of "whom we become (in relation to mathematics or otherwise)" and "what we do in the moment (our being)" in classrooms occurs (Black & Radovic, 2018, p. 270).

Social understandings of prompting a student to present ideas to the whole class likely depend on classroom norms (Cobb et al., 2009). In some classrooms, inviting a student to present their ideas might signal the teacher's public assignment of competence (Cohen & Lotan, 1997). In other instances, inviting a student to present a less sophisticated

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mathematical idea (or error) might signal other, less desirable positions. If multiple students arrive at a specific solution that the teacher wishes be shared, whom do teachers select to present a particular solution and why? In particular, we focus on teachers' considerations related to how teachers perceive the intersection of mathematics ability with gender.

Theoretical framework

We acknowledge our problematic reproduction of inaccurate binary notions of gender. Our view of gender is, instead, as socially constructed and performative, rather than as a biological or fixed state of being. Our analysis refers to a context in Israel in which teachers' predominant social understanding of gender in schools is as binary and fixed. Although gender-based differences in school mathematics achievement have narrowed over time, differences in self-concept in mathematics persist, around the world (Sax et al., 2015). We interpret the tendency of girls toward lower self-concept in mathematics as an outcome of social systems that position mathematics as masculine and reinforce boys as better suited for it, rather than evidence of deficiencies innate to girls (Mendick, 2005). All people participate in these social systems, and classrooms are sites in which gender inequalities are re/produced, with parents and teachers playing special roles (Mendick, 2005).

Self-concept in mathematics, or how people see themselves relative to mathematics, is shaped in part by gendered narratives that circulate, are told and retold, and get re/enacted in social interactions in classrooms (Nasir & Shah, 2011). Some are dominant narratives and reinforce social hierarchies. For example, a dominant narrative is that success of high-achieving girls in mathematics is a product of their diligence, whereas boys' success is a product of "bold" problem-solving (Lubienski et al., 2021). A second, related narrative is that mathematical ability is innate, more often among boys (Mendick, 2005). These are but two examples of interrelated narratives that comprise unequal binary oppositions -- like objective/subjective, analytic/emotional, certain/fleeting, innately able/ hard-working, confident/lacking confidence-- wherein one in every pair is more valued *and* associated with masculinity (Mendick, 2005). Although literature in our field often frames these narratives instead as individually-held beliefs, we agree with Parks (2010) and others who problematize that such a framing then implies interventions to change individuals' beliefs rather than a need for broader counternarratives that can challenge those dominant ones.

Related research

Recommendations from teacher education literature

We identified in teacher education literature (TEL) five principles that can guide teachers in how to select and sequence ideas to share. First is "to build a mathematically coherent storyline" (Smith & Stein, 2011, p. 44), in alignment with the lesson's goals. That is, sets (and sequences) of solutions generate *mathematical storylines* by drawing attention to specific mathematical connections. Selecting a sequence of solutions depends on which storyline better aligns with the lesson's mathematical goals. A second principle is around making

mathematics accessible. TEL describes *attending to accessibility* in terms of progressions increasing from concrete to abstract, specific to general, brute force to efficient, or common to unique (Shimizu, 1999; Smith & Stein, 2011). TEL recommends that teachers select and sequence solutions along one of these dimensions and sequence them in an increasing progression (Smith & Stein, 2011).

A third principle privileges, instead, variety in terms of the representations used (numerical, graphical, figurate, formulaic, etc), problem-solving heuristic engaged (solve a smaller problem, work backwards, look for patterns, etc.), or even solutions that produce different results (Stein et al., 2008). A *diverse set of solutions* could be seen to support the contrasting of solutions towards arriving at consensus around which solution is most reasonable or most efficient (Inoue, 2011). Variety would suggest more opportunities for finding more connections across diverse approaches (Larsson & Ryve, 2011). Furthermore, prioritizing or highlighting a diverse set of solutions could be seen to promote values of democratic society in supporting participation around rational, collaborative reasoning. A fourth principle addresses *strategic inclusion of errors*. Engaging errors can deepen mathematical understanding by clarifying the problem situation and normalizes error-making as a natural and inevitable part of doing mathematics (Brodie, 2014). An erroneous solution could resolve misunderstandings, to then enable "work on developing more successful ways of tackling the problem" (Stein et al., 2008, p. 329).

Finally, a fifth principle is around *social considerations* of whom to select as *authors* of solutions to present their ideas. Smith & Stein (2011) suggest that teachers distribute opportunities to present ideas equally, to equally distribute opportunities to "demonstrate their competence and to gain confidence in their abilities (p. 49). Some refer to strategic selection of shy students, or those who have experienced less success in mathematics (Inoue et al., 2019). Groth (2015) cautions that inviting a student to present a "less developed idea" might backfire and ultimately dissuade them from further participation (pp. 14–15).

The principles overlap in various ways. For example, including a particular error might be part of an intended mathematical storyline, a way to scaffold thinking, or a way to position a particular student as having erred. Similarly, a particular diverse set of solutions might also tell an associated mathematical story. However, these five principles demand negotiation, and in many cases, commitments to one principle over another likely guide teachers' in-the-moment decision-making. Alongside recommendations from TEL is research literature that examines teachers' practices, in terms of how they negotiate this broader set of principles.

Classroom-based research

Recommendations in TEL and their interpretations by teachers are often divergent; in this case, around teachers' alternate understandings of the nature and role of classroom discussions (Kosko et al., 2014). Research among teachers, in both U.S. and Australia, for example, shows a foremost emphasis on accessibility considerations (Livy et al., 2017; Meikle, 2014; Tyminski et al., 2014). Most participants of these three studies (pre-service teachers)

indicated that they would sequence presentation of solutions in a progressive order, beginning with what they considered the weakest (incorrect, most incomplete, or direct model) and then progress to what they viewed as the strongest (correct, complete, abstract). In all three studies, participants justified their choice of these progressions in terms of an assumption that such progressions best support students who are struggling. Hewitt (2020) found that in-service elementary teachers attend to accessibility, in terms of the potential of a particular strategy to support engagement. In addition, Hewitt found that teachers attend regularly to mathematical as well as social considerations, paying attention to attending to unequal patterns of participation.

We seek to add to this research by posing the following research question: *With respect to students labelled as lower and higher-achieving girls and boys, whom do teachers select to present which kind of solution and why?*

Research Context

We conducted this study in northern Israel, a context characterized by diversity and straddling a rural/urban divide. Forty-two Israeli teachers participated, in the context of a university-based course; most (38) identify as women and most (37) identify as Palestinian/Arab-Israelis (P/AI). P/AI, unlike those in the West Bank and Gaza, hold citizenship, form nearly one-quarter of Israel's citizenry, and are a slight population-majority in Israel's north. This socially-constructed category encompasses a diversity with ethnic, geographical, linguistic, political, socioeconomic, and religious dimensions. P/AI women, like all women, contend with local patriarchal systems, as well as broader, structural marginalization of women in Israeli society (Yiftachel, 2009). Nevertheless, among P/AIs, girls tend to outperform boys on state and international mathematics tests, at elementary and secondary levels (Rapp, 2015), reversing trends among Israel's majority. Interviews with P/AI women mathematics teachers reveal narratives that girls' excellence and achievement in school mathematics is a result of girls' diligence and understood to be socially valuable (Rubel & Ehrenfeld, 2020; Sabbah & Heyd-Metzuyanim, 2020). Socioeconomic value of this success is tempered, however, by a prevalent routing to teacher education, rather than STEM fields, perpetuated within communities and by majority institutional arrangements (Agbaria & Pinson, 2013).

Methods

We posed this version of the Handshake Problem:

> There were 9 players at the first basketball practice of the season. The coach told the players to introduce themselves by shaking hands with one another. Assuming that every player shook hands exactly one time with every other player, how many handshakes occurred?

We provided participants with eight solutions (Figure 1) and prompted them to interpret the mathematical thinking underlying each. We draw attention here (because of their dominance in the results) to solution H, an organized direct-model, and C, one of two

inductive approaches but with a geometric representation. We asked participants to consider a hypothetical upper-track 7th grade class of 32 students and that each solution was produced by four students: higher-achieving/lower-achieving girl or boy. We asked participants to indicate a) which three solutions they would choose to be presented to the class, b) in what sequence, and for each designated solution, c) which student they would choose to present each from among the four categories and why.

A (Additive, no answer). The first player shakes hands with the other eight players. The second shakes seven players' hands, and so on. The number of handshakes decreases in 1 for every other player, until the eight player, who has only one hand to shake. The ninth player does not have any hand to shake. $8 + 7 + \ldots + 1$

B (Inductive, tabular, correct answer). In case of 2 players there is one handshake. In case of 3 players there are three handshakes. In case of 4 players there are six handshakes. So I think that $2 + 1 = 3$ is the next result of the number of handshakes (for 3 players). Then $3 + 3 = 6$ is the subsequent number of handshakes (for 4 players). My rule is 'the next number of handshakes = number of players + number of handshakes'.

Number of players	Number of handshakes
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36

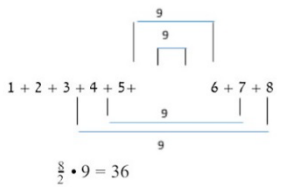
The answer is 36 handshakes

C (Inductive, geometric, no answer).

3 players – 3 handshakes
4 players – 6 handshakes
5 players – 10 handshakes

D (Multiplicative, error). I multiplied 8 in 9 and got 72 handshakes.

E (Additive, correct answer).



F (Multiplicative, story, correct answer). Every player shakes the hands of the other players – 9·8=72. But only half of it, because each handshake involves two players. So the answer is 36 handshakes.

G (Multiplicative, error). 9·9=81

H (Direct model, no answer).

(8 handshakes)	1-2	1-3	1-4	1-5	1-6	1-7	1-8	1-9
(7 handshakes)	2-3	2-4	2-5	2-6	2-7	2-8	2-9	
(6 handshakes)	3-4	3-5	3-6	3-7	3-8	3-9		
(5 handshakes)	4-5	4-6	4-7	4-8	4-9			
(4 handshakes)	5-6	5-7	5-8	5-9				
(3 handshakes)	6-7	6-8	6-9					
(2 handshakes)	7-8	7-9						
(1 handshake)	8-9							

Figure 1: Provided Solutions

Analysis

We tabulated the selections, according to position (Table 1) and the assignment of each solution by gender and mathematics achievement (Table 2). We compiled the written justifications for each solution and analyzed those justifications using qualitative thematic analysis (Braun & Clarke, 2006). We coded a justification as including a gendered narrative if it included statements with girls/boys as the subject of the sentence, e.g., "girls are more creative than boys" (P23) or "boys usually find outstanding solutions more than girls do" (P2). We grouped similar narratives into categories. We identified statements that elaborated the teacher's sense of the potential impact of inviting a particular student to present an idea, for that student or others and again, grouped similar statements into categories. We repeated the coding process in multiple iterations and resolved disagreements through discussion.

Results

Participants frequently chose C, H, or one of the errors (D or G) (Table 1). Those who selected H, assigned it to first or second position, explaining that it is the most accessible, comprehensive, and confirms the correct answer. Those who chose C assigned it nearly always as the last in the sequence, explaining that it is "non-routine" and "creative." Participants commonly cited accessibility considerations in justifying such a sequencing, beginning with the direct model or an error and ending with an inductive solution translated into geometric terms. Overall, participants selected boys or girls roughly evenly overall (Table 2). Two gender-related patterns are evident: Participants commonly invited 1) a lower-achieving girl to present H and 2) a higher-achieving boy to present C.

Solution	1st choice	2nd choice	3rd choice	Total
A Additive	2	9	0	11 (26%)
B Inductive, Tabular	0	2	4	6 (14%)
C Inductive, Geometric	0	4	30	34 (81%)
D or G Errors	21	8	0	29 (69%)
E Multiplicative, Exp.	1	3	1	5 (12%)
F Multiplicative, story	2	6	1	9 (21%)
H Direct model	16	10	6	32 (76%)

Table 1: Selection of Solutions in Sequence

Of the 32 who chose Solution H, 23 chose a girl, and nearly always (19, 59%) a lower-achieving girl. Participants indicated a range of narratives: several about girls' mathematical weaknesses (e.g., "Girls usually choose the long and safe way" P27) and several about girls' presentation skills (e.g., "She will be able to simplify the explanation in friendly terms" P25). However, the most frequently repeated narrative (by 16 participants) addressed an intended outcome, that inviting a lower-achieving girl would attend to the girl's lack of self-confidence, self-concept, and rate of participation. As an example, P23 said, "it is known that girls generally have low self-confidence, I wanted to give her support and for her to experience success, especially since this is clear and correct, and in addition, this can encourage the rest of the girls with low self-confidence."

In contrast, of the 34 who chose C, 22 indicated that they would invite a boy to present it, nearly always (17) a higher-achieving boy. This time, the narratives were around boys' mathematical reasoning strengths or communication skills. Participants attributed characteristics to these boys such as "outstanding," "creative," and "special". For example, P26 explained that "on the basis of experience, boys like busy-work less than girls, and mostly try to find other ways to solve a problem. In addition, boys have better spatial abilities than girls, so this solution is better suited to a boy." In contrast to solution H, few (3) told narratives about the potential contribution of inviting a higher-achieving boy to present C,

for the student or for other class members. When, on the other hand, participants assigned solution C to higher-achieving girls (this occurred much less often), they tended, again, to explain this as an opportunity to remedy those girls' lack of self-confidence. For example, P8 explained "Girls don't have self-confidence, and are shy, so I chose a girl because boys have self-confidence, to go to the board and to talk in front of the class."

	B-Low	G-Low	B- or G-Low	B-High	G-High	B- or G-High	Total n (%)
A Additive	4	6	1	0	0	0	11 (26%)
B Inductive, Tabular	1	0	0	2	3	0	6 (14%)
C Inductive, Geometric	5	2	0	17	6	4	34 (81%)
D or G Errors	5	3	1	10	7	3	29 (69%)
E Multiplicative, Exp.	3	0	0	2	0	0	5 (12%)
F Multiplicative, story	2	1	1	1	4	0	9 (21%)
H Direct model	4	19	2.5*	2	4	0.5*	32 (76%)
Total	24	31	5.5	34	24	7.5	126

*Value of 0.5 resulted from one participant indicating all 4 student categories

Table 2: Assignment of selected solutions by gender and math status

Discussion

Our results suggest that social considerations play a significant role in teachers' decision-making about whom to invite to present which kind of solution for a whole-class discussion. Many indicated a preference for a direct model solution and that they would invite a lower-achieving girl to present it. In nearly all cases, they designated direct-model H as first or second to be presented. If there is a classroom norm around supporting accessibility by always beginning with the easiest example or least sophisticated solution, as prior studies indicate (Livy et al., 2017; Meikle, 2014), then inviting a lower-achieving girl to present first or early on signals to the class (and to the girl herself) that her solution holds a low value. Inviting a lower-achieving girl to present her mathematical ideas as a way to attend to her perceived low self-confidence would likely undermine that intention. Instead, this arrangement likely reinforces any positions of low self-confidence and might ultimately dissuade them from further participation.

With respect to the more sophisticated solution C, most participants chose this to be included and more often indicated that they would invite a higher-achieving boy to present C. Here, too, the sequencing is significant, as in most cases, participants sequenced C to be last. Again, if there is a classroom norm around the sequencing of examples or ideas, as the prior research indicates, then being invited to present later in the sequence likely signals the value of this solution to the class, as most sophisticated. These findings support (and are

supported by) the dominant narrative that boys' excellence in mathematics is a function of their mathematical talent.

Although this was not a study aimed to study gender-bias specifically among P/AI teachers, because nearly all of the participants identify as P/AI, we add an interpretation of these results with respect to this particular socio-political context. Prior research locates a trend that girls among P/AI people outperform boys in school mathematics and speculates underlying contributing factors, from a lack of school-based opportunities in Israel's Arab school sector other than mathematics, an agentic response by P/AI girls and women for self-realization or to challenge underestimation of their capabilities (Rubel & Ehrenfeld, 2020). This study's results suggest a preliminary picture of how classroom socialization processes might reinforce the status quo. These processes likely contribute to how, on the one hand, higher-achieving P/AI girls mostly pursue feminized (and lower paying) professions and, on the other hand, lower-achieving P/AI girls struggle to access higher education or secure employment (see Haj-Yahya et al., 2018).

Two groups of students figure significantly less prominently in these results: higher-achieving girls and lower-achieving boys. The tendency here of public positioning of higher-achieving boys as exceptional (more often than higher-achieving girls) likely compounds the narrative that girls succeed because of diligence rather than brilliance. This is consequential in the local context: although P/AI girls tend to out-participate and out-score boys in K-12 mathematics, they largely continue towards teacher education rather than STEM disciplines or careers. At the same time, although participants indicate that they view invitations to a student to present an idea on the public floor as a resource for confidence-building, their assignments suggested their view of this resource as predominantly for lower-achieving girls. We are curious to further explore narratives about lower-achieving boys to interpret why they figure less often in these results.

An important limitation to this study is that it is oriented around a single mathematics task. We did not specify to participants a learning goal for the hypothetical lesson or imagined whole-class discussion. In addition, we have flattened social diversity into two social constructs (perceived gender and ability) and separated those into artificial binary distinctions. These variables do not operate in binary terms and intersect with other social markers, such as race, language status, physical appearance, social popularity, and more. Nevertheless, these results illuminate how circulating dominant narratives about gender and mathematics shape teachers' decision-making regarding whom to invite to present which kind of solution on the classroom's public floor. In the life of a classroom, consequences of those decisions likely include the reinforcing of those dominant narratives and the gendered hierarchies they represent.

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Reflections on professional growth within the field of mathematics education

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Being involved in a teaching development project as university teachers and researchers of mathematics education, we started to critically reflect on professional growth within our specific situation. We developed our own understanding of professional growth based on the subject-scientific approach. We reject a limited understanding of professional growth that equates being professional with being skilful in fulfilling an occupation. For us, the concept of professional growth must necessarily reflect socio-political conditions, which also includes mathematics and mathematics education as academic disciplines. Our reflections have led us to question the relationship between professional practice and research in the process of professional growth.

Introduction: Critical considerations on professional growth

The following reflections are formulated from a specific vantage point: We are at the same time in the position of being researchers and university teachers in the field of mathematics education at a German university. This is a coupling of professional responsibilities, that may be regarded as typical for teacher educators (see, e.g., Adler et al., 2005). Our research focuses on mathematics education at university, which may be considered as atypical in our context. Also, our biographies can be related to the German mathematics education community in terms of typical and atypical aspects: Our employment condition is quite typical, we both have fixed-term contracts. To maintain our 'employability', we need to show our development in both research and university teaching according to the norms prevalent in these two professional fields. We both have quite different educational biographies[1], neither of which is typical within the community. It is inconclusive whether this puts us in a more privileged position.

The starting point for our considerations in this contribution is a critical reflection on our participation in a teaching development project that aimed at strengthen inquiry-based

[1] One of us has a diploma in psychology and a straightforward educational biography. The other one has a working-class background, and not straightforward educational biography with a diploma in mathematics and a master's degree in mathematics education.

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mathematics education in higher education[2]. We see this development project as an effort to build up a form of professional development as a shared endeavour. In doing so, professional development should go beyond framing university teachers as in need of being developed by researchers. As a framework, this project works with an adapted form of the Community of Practice approach (Lave & Wenger, 1991). The Community of Practice approach serves as a reference for quite a number of endeavours in the field of the professional development of (university) teachers – that, quite often, intend to fabricate 'better' teaching practices. Within this logic, teachers serve as a tool for the provision of (ever changing) ideas of 'quality' education. In the following, we do not understand theories on teaching and learning mathematics nor theories on professional growth in terms of a fabrication logic[3], but rather as theories that enable us to reflect. To us, reflection goes beyond being a tool for just generating 'better' practice, but considers the societal and political constitution of the respective concerns. The project we were involved in emphasised the relationship between professional development of university teachers and inquiry into the field of mathematics education. This focus encouraged us to reflect on the relationship between teaching and research. In our context, the leitmotifs of *education through research* [Bildung durch Wissenschaft] and *unity of teaching and research* [Einheit von Forschung und Lehre] are often referred to (for a critical review see Ricken, 2007). Certainly, these refer to an ideal image of university education and research, which as such is not or has never been fully realised, and it is doubtful to what extent these statements are only (still) used as empty phrases. But then again, these leitmotifs offer us a legitimation to deliberately explore the relationship between research and teaching and its influence on our professional practice as researchers and university teachers in the field of mathematics in higher education. Furthermore, we see a potential for mathematics education at university (both in research and in the further development of the teaching of this field) in reclaiming and interpreting these leitmotifs in relation to our professional interests.

The relationship between research and teaching is negotiated in international discourse under the heading of teaching-research nexus[4] (Simons & Elen, 2007). There is a wide variety of ways to interpret this nexus. In the following, we do not argue per se for adopting our specific cultural interpretation, such as the leitmotifs *education through research* and *unity of teaching and research*. Rather, we argue for considering this nexus (in its respective culture-

[2] Partnership for Learning and Teaching in University Mathematics (PLATINUM): https://platinum.uia.no/

[3] We use 'fabrication logic' as a metaphor for summarising homologies in the structures of diverse requirements and rationales (in dealing with those requirements) that we encounter in our context - which often hastily smooth out contradictions and try to resolve them unilaterally with respect to what is possible within the given. This metaphor helps us to question our own thinking with regard to the extent to which we are compliant with such rationales. It offers the possibility of deliberately relate to such rationale.

[4] It is often understood in a very one-sided way, as good research leads to good teaching. We argue for also considering the other direction.

specific connotations) and possibilities of interpretation when it comes to reflect its potentials in relation to professional interests. We have formulated questions in the discussion section that could aid the reflection and interpretation of this nexus also in other local contexts and positions.

To us, the term professional[5] goes beyond being skilful in fulfilling an occupation, it always entails a responsibility for a human society (Combe & Helsper, 1996) even though this argument is lost in current discourses about the role of university education (Banscherus, 2015). To act in a professional manner and professional knowledge thus go beyond their reference to an academic discipline. Being professional is more than just expertise, it always also contains the political. It requires a political commitment to the concerns of the respective profession. For us, these are the concerns of education and research, in general, and in the field of mathematics education, in particular.

In our daily experience, efforts in the further development of the teaching profession often follow a hegemonic thinking in the structural organisation of the development process that are based on a hierarchy between experts from an academic field and university teachers thought of as laypersons that are supposedly in need of being developed (Dreier, 2006 on resemblances of schooling, as an institutionalised developmental process, and therapeutic setting and the role of knowledge). Contrary to the trend of decoupling educational and research responsibilities, for example, in a separation of professional development efforts of these two areas, we argue that there is a potential for the further development of research and teaching in its double self-referentiality (*doppelte Selbstbezogenheit*; Reinmann, 2019). The object of research and our position each have a self-referentiality:

The object of our research and of the inquiry in the professional development we participate in is a mathematics education course at university for future teachers. The course presents mathematics education as an academic discipline rather than teaching a mere methods course. Under the header of the leitmotif *education through research*, the course shall introduce the academic discipline of mathematics education, prepare the students for their future profession and contribute to their personal development. The leitmotif *education through research* obliges higher education institutions to establish a link between research and teaching: Teaching shall be conducted in a research-oriented way[6] (Reinmann, 2019).

[5] The conception of 'professional' always relates to the actual local formation of the historical bloc. The traditional conception of 'being professional' in our context refers to a model of comparably strong responsibilities of the state for the social sphere compared to the currently, within neoliberalism rising, market-based regulation. Within the traditional idea, 'professions' are independent institutions that are concerned with questions related to the social. The emphasis on the social responsibility is still in use (even though it is also in our context contested) to justify financing and accountability models other than the market-based ones. To us, reclaiming such an idea of 'professionalism' in our context holds the potential to present affiliable arguments that are counter to an individualising, abridged, neoliberal notion of professionalism.

[6] Both leitmotifs, *education through research* and *unity of research and teaching* are connected. Actually, research should also benefit from teaching. However, this aspect of the *unity of research and teaching* is often lost.

Our inquiry object thus has a specific form of education as its object, which in turn has to do with research in the academic discipline mathematics education. The first self-reference refers to the implicit object of the inquiry: Not only teaching and learning, but also research itself belongs to the inquiry object.

The persons involved in research and that are part of the field that is the object of the research endeavour, are the same in our context, e.g., as researchers in the field of mathematics education and as university teachers to be researched concerning their interaction with students. These roles are not clearly separated (thus always have a share in the educational practice to be researched): Those, in turn, who initially belong to the research object as university teachers, are themselves active in research at the same time. A separation between research and practice, as well as experts and laypersons to be developed, is misleading within the second self-referentiality: a personal involvement in the field of being inquired into, and thus being placed in a position of being the one, who is inquiring, and being part of the research object at the same time.

If we acknowledge the double self-referentiality as being constitutive for the context in which we operate, it does not allow for a supposedly external and neutral standpoint towards the research object. But, often only taking up a neutral stance towards the inquiry object is taken to be appropriate for a research endeavour into one's own teaching and learning. At best, an external shall judge one's development, or the progress displayed in the teaching arrangement. At least, a researcher shall distance her-/himself from one's own personal involvement e.g., through applicating supposedly objective measures to evaluate a supposed development, either referring to an enhancement of teaching quality or regarding one's own professional development. In short: One's own involvement is conceived as a confounding variable and potentials within the double self-referentiality are denied.

However, we see our involvement and being affected as an opportunity for broadening our personal and the general professional knowledge base on teaching and learning in tertiary mathematics education. This led us to the question: How can we understand professional growth without relying on a supposedly external and/or neutral point of view as a benchmark and with considering the societal and political constitution of our professional concerns?

Our understanding of professional growth

We conceive professional growth as extending one's own space of action possibilities[7] (described below) in teaching-learning relations with/in a community of practice. This conceptualisation is inspired by the subject-scientific approach (Holzkamp, 1995, 2013; Dreier, 1999; Ludwig, 2003) and integrates the idea of a situated development within a community of practice (Jaworski, 2004; Lave & Wegner, 1991).

[7] Action possibility is an analytical category. The analytical categories offered by the subject-scientific approach "conceptualise the mediation between the vital necessities of sustaining the societal system as a whole and these necessities on the subjective level of the discrete individuals" (Holzkamp, 2013, p. 20).

Within the subject-scientific approach, human beings – and therefore learners, teachers and researchers - are perceived as "producers of the life conditions to which they are simultaneously subject" (Holzkamp, 2013, p. 20). It points to consider the conditions, which in our context are specific learning-teaching conditions as well as the conditions of doing research in mathematics education, and it underlines the possibilities of the subject - learners, teachers and researchers - to influence these conditions. The analytical category action possibilities (*Handlungsmöglichkeiten*) grasps possibilities and hindrances to act in and on specific conditions from the standpoint of the subject. Central of thinking in terms of action possibilities is the twofold possibility (*doppelte Möglichkeit*): To either reproduce restrictive conditions or the (however small[8]) possibility to extend established practices and alter socio political conditions. Conditions do not fixate nor determine reasoning, rather conditions are integrated into subjective reasoning with the form of societal-mediated meanings that constitute a space of action possibilities. The subjective-available space of action possibilities is not fixed, but can be extended through conscious engagement. In consequence, we conceptualise professional growth as an extension of the subjective-available space of action possibilities and emphasise the social dimension of this extension process. In alliance with others, it is possible to seize more opportunities for actions and participate in changing conditions that are constraining one's envisioned practices. Dreier (1999) relates this to community processes:

> The fundamental human duality between acting within the existing limits of social practice and extending its scope of possibilities is grounded in a similar duality of modes of participation [in a community], i.e. of participation in the reproduction of the current state of affairs or of contributing to change it so that participants may extend their degree of disposal over the social practice. (p. 6)

Two aspects are important in the formulation "in alliance with others": Firstly, that one can question one's own ways of thinking and involvement together with others. And secondly, that one can act together to change things. However, this concern can also be torpedoed by constantly propagating pragmatic solutions and thus creating the impression that there is no need to alter conditions.

To us, an inquiry stance towards teaching and learning means to think in alternatives. In consequence, we do not take current conditions, approaches and theories to teaching-learning mathematics (education) for granted, but question these for its obstructive components and our possibilities to think beyond the narrow horizon of current practices. This objective can be related to an emancipatory objective of academic work that is also of key importance for building up a professional knowledge base (Langemeyer, 2020). Within teaching-learning relations, as well as expert-layperson relations, we often act in a restricted

[8] This follows the basic assumption that in antagonistic class conditions, the attempt to gain more control over conditions is always accompanied by the risk of getting in conflict with the agents of power and provoking restrictions.

manner, in a modality of alignment with or subjection to given obstructive structures and its logics[9] – e.g., following a logic that places the teacher in a position of being responsible to fabricate learning and the learners in a position of being in need of further development. But education and professional growth can also be thought of as cooperative activity of being directed towards extending one's own space of action possibilities, which also includes extending one's own control over restrictive professional conditions, e.g., in our context teaching-learning relations and research conditions. In alliance with others, it entails the possibility of overcoming obstructions. Research could provide terms for reflecting[10], terms that foster the process of seeking self-understanding for the professional task of teaching. To serve as a reference within the professional knowledge base, we share the conviction, that research in mathematics education needs to integrate social theory and cannot disregard broader societal conditions in the interpretation of phenomena that can be found in teaching-learning. For example, in speaking about our professional growth, we cannot disregard the prevalent norms and interpretations of the research-teaching-nexus. To disregard broader societal conditions would conceal the Political by reducing mathematics education to the development and implementation of teaching tools and strategies and thus constrict mathematics education research.

To interconnect research activity in mathematics education and the teaching and learning of mathematics (education) with each other within a community of practice entails the potential of developing and building on theory that integrates several standpoints of the teaching-learning relations. These standpoints hold the anchor points for the reflective task of decentering from one's own viewpoint and together, in alliance with others, develop a meta-standpoint.

The collective practice can be understood as supporting an ongoing (self-)understanding process that takes place between the community members and the socio-political conditions in which the professional work is situated. This includes the "reflection of social requirements and conditions in an attempt to (re-)establish self-understanding in individual situations of action and to be able to act in a competent [/professional] manner" (Ludwig, 2003, p. 1, translation by author). The basis for a collegial self-understanding process is the shared socio-political conditions. But how these are perceived and what priorities are assigned by the community members varies. For development with/in a community, it is necessary to understand and penetrate these available perspectives. "Seeking (self-)understanding" refers to the attempt to gain knowledge about and trace one's own personal and structural entanglement in contradictory situations. In such situations people participate in practices which run counter to their desired practices. By striving for (self-)understanding, we attempt to gain more disposal over our research and teaching practices. Rihm (2006) points out that in our routinised daily work, we often just interpret situations within the

[9] Safekeeping at the cost of the (re-)production of restrictive conditions.

[10] This requires not just any theory, but necessarily requires theory that doesn't back or conceal the restrictive structures within the professional knowledge base.

horizon of the typical space of action possibilities of our daily practices – in accordance with professional knowledge base. This means we unquestioningly accept quite a number of aspects of typical ways of working in our professional community. An illustrative example for acting against one's own intentions is detailed in the story of Mrs. Smith that can be found in Cabral & Baldino (2019a). Furthermore, Cabral & Baldino (2019b) reveal that it is compliant in the (re-)production of undesired practices to integrate restrictive conditions in the professional knowledge base by interpreting restrictive conditions as being necessary for the learning process. But "to understand"[11] means to gain knowledge about and trace one's own and structural entanglements in contradictory situations, in which we participate in practices counter to our desired practices. Seeking understanding, therefore, means to widen one's own view going beyond the horizon of our everyday entanglement.

This kind of (self-)understanding goes beyond a merely introspective and individual way of progressing (Rhim, 2006): A cooperative de-centering from a personal viewpoint allows to take a meta-standpoint that makes it possible to recognise the interrelation of different research, teaching and learning practices prevalent in society and the collegial group. Reinmann (2019) points out that due to the double self-referentiality, a de-centred reflexivity is necessary in order to adequately address the object of research in higher education teaching and learning. This stands in contrast to other forms of reflection that are intended to ensure an evaluation of one's own performance aimed at an increase in teaching quality. This reflexive distance not only allows to identify supporting and obstructive conditions, but also to recognise potentials of altering conditions.

These potentials can relate to mathematics education research and teaching. As described above, in our context it is not possible to make a precise distinction between research and teaching. In addition, we see a potential for professional growth in the double self-referentiality. We would claim that a professional growth process, conceptualised such as we have described above, is able to contribute to an expansion of the general professional knowledge base. Deliberately, we do not want to outline this possibility in this contribution on the basis of describing any improvements in teaching practices, 'proving' how helpful our reflection has been for the betterment of actual teaching practices. To us, it is impossible to write an "Hollywood-style happy end" (Cabral & Baldino, 2019a, p. 33) without talking down the political and dilute our professional interests.

Discussion: Mathematics education as professional field

We conceptualised professional growth as an extension of the subjective-available space of action possibilities that is constituted by societal-mediated meanings which in turn reflect social conditions. To act professionally means to take responsibility to society into account and goes beyond a mere reference to the academic discipline as a resource for knowledge

[11] "Interpreting" is then not opposed to "understanding"; rather, "understanding" simultaneously suspends "interpreting" in itself and transcends it (Holzkamp, 1985, p. 395).

and skills and as an occupational field. To us, the academic discipline, which is the object of professional action, is part of our social conditions.

Expanding and actively shaping our space of action possibilities thus also means the possibility of altering and reshaping conditions. We agree that also the content is political (Gutiérrez, 2013, pp. 9–11). In relation to our academic field, mathematics education, this also means demanding a "say" on questions such as "What is mathematics about?" or "How is the social relevance of mathematics, or a specific mathematics object, to be valued?". In our understanding, professional responsibility and taking a stance for professional interests requires what Gutiérrez (2013) terms "political conocimiento"[12].

In our experience, mathematics and research on the education of mathematics are often taken for granted as exclusively external conditions of professional action. In our context, it is particularly striking that such an assumption does not hold. We argued that considering the double self-referentiality is crucial with respect to our position. In addition, we would question such an externalising construction of relating academic disciplines and professional practices to each other. The academic disciplines of mathematics and education shape the teaching of mathematics (education), but the relation is often interpreted in a one-sided manner: Being skilful in teaching mathematics is equated to fabricating learning environments that transmit and (re-)produce a specific understanding of mathematics. Teaching mathematics (education) certainly is always fabricating an understanding about mathematics and theories on learning. Fabrication logic cannot be ignored or rejected. But it is necessary to think about and go beyond this fabrication process. So, any conception of being professional needs to integrate these aspects. But, a one-sided interpretation of the relation between the academic disciplines and the professional task of education leads to a skewness that restrict the possibilities of its further development.

Looking at the relationship of academic disciplines and professional practices from both sides: First, coming from the perspective of academic disciplines, Schraube and Marvakis (2016, 2019), Holzkamp (2004) and Biesta (2015), besides others, argue against a diminution of teaching-learning theories. In principle, theories can help in a process of reflection that looks beyond what is already possible. However, it is problematic when teaching-learning theories only consider and remain within the boundaries of the currently given, e.g., by integrating restrictive conditions as being quasi-natural conditions for teaching and learning, or conceptualising learning as a process that is only able of reproduction. First, it restricts one's own thinking about alternatives. Second, theory is a professional basis for legitimating practices. Therefore, practices that potentially look beyond the currently given can be dismissed as illegitimate with reference to narrowed theories. This implies a responsibility for the academic discipline of mathematics education: Providing knowledge for university teachers, regardless of whether they are also active in research in this discipline or not,

[12] Her concept is anchored in a different context and theoretical traditions based thereon. Despite all overlaps in content, we have therefore decided to anchor our argumentation in concepts and theories that are familiar to our context.

means not only providing pragmatic knowledge. It also means providing theories that allow for further development (going beyond what is currently given) and make this visible. Second, coming from the perspective of professional practice, we can note, that the right and opportunity to have a say also depend on the status and understanding of the discipline: If mathematics education is only understood as and presents itself as a promoter for mathematics and service provider for educational policy, then its professional responsibility of furthering mathematical practices for a human society is pushed aside and perishes.

Finally, we would like to emphasise that our reflections have emerged from our context. We have interpreted the teaching-research nexus from our German perspective. In doing so, we are of course not free of the logics prevalent within our field of study. With these reflections on professional growth we wanted to demonstrate a form of thinking in alternatives that, in our context, seems to be suitable to challenge an understanding of professional growth that externalises benchmark criteria.

In our opinion, our understanding of professional growth holds the potential to be generalised, even though it is not directly transferable. So, we would like to conclude our reflections with thinking about implications also for mathematics educators who are working in other contexts and positions. We summarised them in the following questions:

- How can the relationship between research and teaching within the field of mathematics education be thought of in other local contexts?
- What specific leitmotifs and referentialities are relevant for the context and how do these constitute research and teaching in relation to each other?
- Which interpretations of leitmotifs and what referentialities are acknowledged, hold legitimacy, or are ignored? What do they offer for professional growth within the field of mathematics education?

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So you think you're a math person: Understanding cognitive dimensions of stereotypes in mathematics

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Research consistently documents a "math person" stereotype that positions white and male students as most mathematically able, and so reproduces gender and racial inequity in the field. Recent work suggests that this stereotype also includes how "math people" think, learn, and problem-solve, alongside how they look. This exploratory study combines survey and interview methods to examine if certain stereotyped cognitive attributes (such as a student's preferred problem-solving style) influence students' and teachers' perceptions of other students as more or less mathematically able. Initial results support the hypothesis, with interview data also illustrating how the cognitive component interacts with broader stereotypes to shape students' experiences and define them as belonging or not in math classrooms and careers.

Introduction and literature

If you study or teach mathematics it's hard not to hear the phrase "math person," whether in the classroom, the literature, or just day-to-day conversation (Devlin, 2000; Heller, 2016). Unsurprisingly, the "math person" label is not simply a throwaway one. It carries real meaning for how individuals view themselves and others as belonging – or not belonging – in mathematics classes and institutions.

Stereotypes are socially legitimized and reproduced beliefs that people in certain social groups have, or are more likely to have, certain characteristics (Steele, 2010). In the case of "math people," the stereotype is a consequence of narratives that intertwine individuals' *mathematical* identity (their sense of competency and belonging in mathematics) with other social group identities such as race and gender (McGee, 2015; Steele, 2010). These narratives become embedded in institutions as well as individuals, legitimizing practices that position white and male students as more mathematically able than female, Black, and/or Brown students (Kim et al., 2018).

From middle school to university in the United States, Black and female students are less likely than their white male peers to be placed in and take high-level math courses, independent of their academic record (Faulkner et al., 2014; Francis et al., 2019; Treisman, 1992). Women, and especially women of color, consistently experience feelings of "not fitting" in STEM classrooms unless they conform to a narrow and constricting idea of what women in mathematics "are like" (Kim et al., 2018). They are also more likely to experience

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social math identity dissonance, where individuals do not think they are "math people" even though they believe others perceive them this way (Heller, 2016). The "math person" stereotype and the narratives behind it also attack students' own sense of mathematical competence, and can depress their academic performance (Spencer et al., 2016; Steele, 2010).

These effects can occur even in situations where there is equal gender and/or racial representation (Kim et al., 2018). In short, pervasive stereotypes of white male mathematical superiority create both structural and personal barriers that traditionally-excluded students in math classrooms must contend with. While students can and do fight back, building communities and "robust" mathematical identities that support them in defying the stereotype, they are overall less likely to develop the positive mathematical identity necessary for any student's persistence in the field (Boaler, 2002; Heller, 2016; McGee, 2015; McGee & Martin, 2011)

While the racial and gender components of the stereotype are well documented, there may also a *cognitive* component that "math people" *think* in certain ways as well as *look* certain ways. Much of the prior cited work, as well as many general-audience pieces about mathematics in society, note the association of "math people" with words like "logical," "objective," or "rational" (Burton, 1999; Devlin, 2000; Lakatos, 1976). Additionally, recent work shows that presentations of how one must think and problem-solve in mathematics can directly lead to students feeling inclusion or exclusion.

In a study of AP Calculus courses in California, Boaler and Greeno (2000) found that the methods of thinking and problem-solving presented by math teachers affected students' feelings of belonging in mathematics. In classrooms where math was a "closed" field where single, right answers were found through inflexible methods, students who enjoyed thinking more critically, or debating multiple answers, felt out of place and reported that mathematics "wasn't right for them" (Boaler & Greeno, 2000). On the other hand, "open" classrooms that encouraged multiple methods of inquiry supported a greater diversity of students to grow positive mathematical identities. The fact that students experienced inclusion or exclusion based on a "cognitive" identity, and the common association of "math people" with "logical" or "objective" ways of thinking, suggests the possibility that the "math person" stereotype may also be a stereotype about how mathematicians *think* – as well as how they look.

This leads to the following research questions exploring the potential existence of a cognitive component of the "math person" stereotype:

1. Do students and teachers evaluate the likelihood of other students' interest and success in math differently, based on characteristics signaling different forms of cognitive identity?
2. Do students experience math education differently based on their cognitive identities?

If there is a cognitive component to the "math person" stereotype, we would expect school experiences to reflect this in some way. Students who identify more with the stereotypical way of thinking would be more likely to be perceived as "math people," while

students *not* so identifying would be less likely to receive the label. If there isn't a cognitive component to the stereotype, then a student's preferred way of thinking would not affect their internal or external perception as a "math person."

The first step in studying possible cognitive components of the "math person" stereotype is to operationalize those components. Drawing on the literature, I propose the following list of possible cognitive stereotype components. Math people:

1. think logically (vs. intuitively) (Burton, 1999; Devlin, 2000),
2. prefer certainty and problems with definitive answers (vs. uncertainty and problems with multiple competing answers) (Boaler & Greeno, 2000; Lakatos, 1976),
3. prefer to be told unquestionable facts (vs. debating or critiquing knowledge) (Boaler, 2002).

I hypothesize that the first component of each of these pairs forms a stereotype of how "math people" think. That is, they define a "way of thinking" that is socially perceived as being necessary or important to enjoying and succeeding in mathematics, while the opposite "way of thinking" is not useful for doing math or is even an active hindrance.

If the stereotypical "way of thinking" were somehow more necessary for doing mathematics than other ways of thinking, then it wouldn't be stereotypical – it would just be generally necessary for doing math. However, it cannot be over-emphasized that *none* of the hypothesized stereotyped characteristics *actually are* more necessary to doing mathematics than their opposites. While mathematical inquiry frequently is logical, and math research questions frequently have definitive answers, mathematics throughout history has also relied on intuition, debate, and intense questioning of assumptions. Researchers, math educators, and mathematicians all agree on this point: mathematics is as much of a contested, living, uncertain field as any other, and intuition, uncertainty, and debate characterize mathematical work just as much as logic, certainty and "truth" (see, e.g., Klein, 1982; Burton, 1999; and Lakatos, 1976).

Methods

This study employs mixed methods to shed as much light as possible on the potential existence of a "way of thinking" stereotype. A vignette-type survey of students at Swarthmore College and teachers in the Philadelphia area was used to assess if the presence or absence of the stereotyped way of thinking affected a student's external perception as a "math person." The survey was followed by interviews with volunteers from both populations to explore in more detail where the "math person" stereotype is encountered and how it impacts educational experiences.

The survey consisted of four sections: an introduction obtaining informed consent and asking if the participant was a teacher or student; a vignette presenting a hypothetical student profile, followed by questions about that student; questions about respondent attitude towards and experience in mathematics; and lastly demographic questions. Students were recruited using social media posts, and messages through departmental mailing lists.

Teachers who had graduated from Swarthmore College or who had hosted Swarthmore students in their classrooms for fieldwork were contacted through Swarthmore's alumni network and asked to share the survey with their own contacts. Through the college Department of Educational Studies, the survey was also distributed to teacher groups and organizations in the Philadelphia area.

The vignette was the central analytic tool of the survey. The vignette described a hypothetical student and varied three aspects of the student's identity: their gender, their interests and habits, and their "way of thinking." Each aspect had two poles: one hypothesized to be a part of the "math person" stereotype, the other not. Vignette names were chosen from the 20 most common names for both White and Black high-school-aged children in the U.S., though this does not prevent the respondent from making assumptions about the student's racial identity (U.S. Social Security Administration, 2019).

Each respondent read a vignette version that randomized the factors presented (stereotypical/not stereotypical for each). They were not told that the project was specifically studying stereotypes in math. They were next asked to predict the student's interest and success in five school subjects (math, natural science, foreign language, social studies, and English) on a five-point Likert-type scale. Response patterns to the post-vignette questions would then ideally capture the salience of each factor in shaping perceptions of the student. Although imperfect, by asking for reactions to a concrete situation the vignette provided a more robust construct than directly asking respondents about their beliefs (Bryman, 2004). The factors, with text, are reproduced in Table 1.

Gender functioned as a control variable. Given significant evidence that "math people" are stereotyped as male, if no differences in hypothetical student perception were observed based on gender, the survey may not be capturing authentic reactions. *Way of Thinking* was the key variable, as it operationalized the cognitive components hypothesized to be a part of the "math person" stereotype. *Personality* separated a student's specific "way of thinking" from any associations the survey-taker might have between ways of thinking and a student's more general personality and interests.

The survey was built using Qualtrics XM and distributed electronically. Question order was randomized when applicable. Written consent for the use and sharing of the data was collected at the beginning of the survey, and the survey would not allow completion unless consent was given. The survey opened on December 2nd, 2018 and closed on February 6th, 2019. Data was analyzed using the open-source statistical computing tool R.

The final question of the survey asked if the respondent was willing to participate in an interview. Student interview participants were selected semi-randomly to vary class year and subjects studied. To maximize data relevance, teacher interview participants primarily taught mathematics and were varied across grades taught. In total five students and three teachers were interviewed. Questions focused on participants' own conception of the "math person" stereotype, if the stereotype was justified, and their experiences of encountering the stereotype in school and work.

Factor	Hypothesized Stereotyped pole	Hypothesized Non-stereotyped pole
Gender	Male	Female
Personality ("Academic" or "Creative")	"Academic" personality "[Student] is on the quiz bowl team and goes to debate club when [she/he] has time. [Her/his] room is usually tidy and she/he has separate notebooks for each class."	"Creative" personality "[Student] sings in the school choir and performs with the poetry club when [she/he] has time. [Her/his] room is usually messy and she/he keeps all her/his school notes in one large binder."
Way of Thinking ("Objective" or "Subjective")	"Objective" way "[Student] has strong opinions and defends them with reasoned arguments. [He/she] enjoys finding right and wrong answers to questions, and [he/she] believes that objectively correct answers exist in most cases. [She/he] prefers classes where the teacher spends most of the period lecturing, and in class [she/he] asks challenging questions that build on the information being presented."	"Subjective" way "[Student] has strong opinions and defends them with references to [her/his] personal experience. [He/she] enjoys finding multiple answers to the same question, and [she/he] believes that objectively correct answers don't exist in most cases. [He/she] prefers classes where the teacher spends most of the period on independent projects, and in class [she/he] asks challenging questions that critique the information being presented."

Table 1: Stereotype factors selected for analysis, with vignette text.

Interviews were semi-structured and lasted approximately one hour each. Written and verbal consent for the recording of the interview, and the use and analysis of the transcripts, were collected at the beginning of each interview. The interviews were transcribed by hand and Atlas.ti was used for coding.

Results

One hundred and thirty Swarthmore students and eighty-two teachers responded to the survey. A majority of respondents identified as women (73%) and a majority identified as white (60%). Students were more racially diverse than teachers, and the racial distribution of students was generally representative of the student body at Swarthmore College (Swarthmore College, 2018). Students spanned class years evenly, but 78% of majors were in social or natural science. Most students and teachers were upper-middle or upper class. English literature, mathematics, and social studies were the most common primary subjects for teachers. Three-fifths of teachers were not alumni of Swarthmore College. On average students were slightly less likely to identify as math people than teachers, and slightly less likely to enjoy doing math.

Teacher and student responses to the vignette battery varied based on the vignette presented. Overall, the more that the vignette student possessed a hypothesized stereotyped identity, the more that student was perceived as likely to be interested in and successful in mathematics (Figure 1). Students had higher variation in responses and appeared to be more sensitive to vignette factors than teachers, while for both groups the hypothetical vignette student's perceived interest in math was more affected by vignette factors than perceived success (Figure 1).

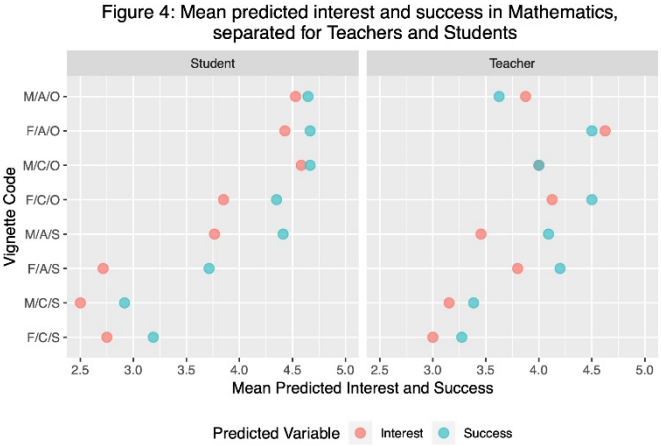

Figure 1: Mean predicted Interest and Success for the hypothetical vignette student, by vignette factor (n = 212). Key: M/F = male/female, A/C = academic/creative, O/S = objective/subjective.

Linear models were constructed to examine which of the vignette factors had the greatest effect independent of the other two. The salience of "Way of Thinking" was maintained in the models for both teachers and students (Figure 2; Figure 3). Few other recorded variables had a visible effect on predicted interest or success for either group; the only ones to have a significant effect were teachers' enjoyment of doing math, and teacher respondent gender (Figure 3).

The survey data supports the hypothesis that the "math person" stereotype contains a cognitive component. The more stereotypical traits the hypothetical student had (being male, being academic, or having an objective way of thinking), the more they were judged to be interested in *and* to be successful in mathematics (Figure 1). The apparent effect of vignette

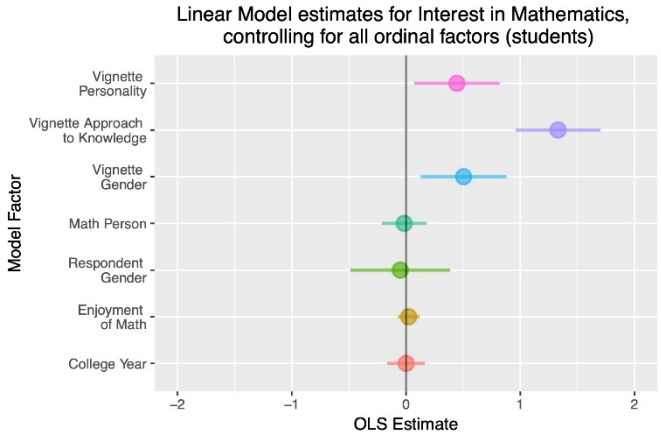

Figure 2: Linear model slope estimates using an ordinary least-squares method for the influence of seven ordinal factors on student-predicted hypothetical student interest in mathematics (n = 130).

factors on respondent answers is corroborated by the linear models, which show "Way of Thinking" (labeled in the figure as "Approach to Knowledge") to be the factor associated with the greatest change in student and teacher perceptions, followed in salience by "Personality," then "Gender" (Figure 2; Figure 3). While the latter two factors have a substantive effect on student perceptions, neither influences teacher perceptions. This suggests that the teachers in the survey pool are more resistant to stereotypes around mathematics, possibly due to their greater experience with math as compared to the students. Or perhaps they are just careful in their responses. Either way direct observation would be necessary to validate this apparent pattern of teachers' less-stereotypical perception of students.

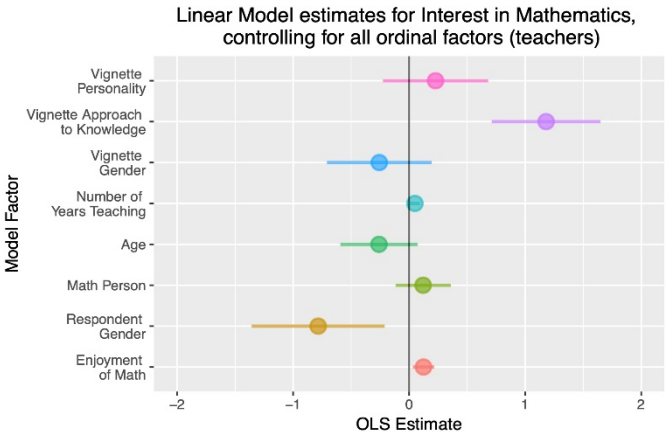

Figure 3: Linear Model Slope estimates using an ordinary least-squares method for the influence of eight ordinal factors on teacher-predicted hypothetical student interest in mathematics (n = 82).

Overall, we conclude that for this survey sample, the "way of thinking" factor appears to have robust effects on a student's external perception as interested and likely to succeed in mathematics. However, the non-random survey sample and the relative inapplicability of linear models to this data make stronger conclusions impossible.

Data from the interviews corroborate these findings, but also describe a "math person" stereotype that is richer and more subtle than the tested survey factors. The wealth of insight provided by the interviews into the stereotype was much greater than can be analyzed in this paper; I will thus focus on the three most relevant themes.

Firstly, "way of thinking" was described by students as being either a meaningful factor in their experience, or as a part of the "math person" stereotype. Words like "ordered" and "logical" were used to describe this way of thinking. These descriptions corroborate the survey, and offer evidence that students compare themselves (or are compared) to this way of thinking as they build their mathematical identity.

> Student 4: I, it really just seems like a way of thinking…the words that come to mind are *logical,* um, like an ordered way of thinking…
>
> Student 5: I feel like the question that I was always asked…were like, 'if you like xyz then you should pursue this career,' and I always remember one of the questions being 'if you like knowing that there are concrete answers to questions, you should go into math or science.'

Secondly, both students and teachers mentioned *other* factors when describing "math people," alongside way of thinking, race, and gender. These included an ability to calculate quickly and a strong interest in math to the exclusion of other pursuits.

> Student 2: I mean, working within the framework of stereotypical math people, I think it's generally people who are numerically inclined, would rather think about things in terms of numbers than in terms of words, and people who usually do math quickly, um, and who do it accurately, like, the first time around…
>
> Student 5: I know the things that immediately come to mind are really not true, like I always imagine like a math person, in quotes to be like, if I'm honest visualizing a white guy sitting like at a desk with frazzled hair, and has been working on a single problem for like days, like generally introverted I would say, and not a great communicator with other people (laughs)…

As seen in the second quote, interviewees were frequently aware that the factors they were describing were stereotypical, and they did not necessarily believe that these stereotypes were accurate portrayals of mathematics or mathematicians. These findings also give insight into how the stereotype is reproduced in practice: for example, an ability to calculate quickly will aid performance on typical school assessments, and so reinforce the student's identification as a "math person." In fact, both students and teachers were often aware of this chain of events, and directly connected factors of the "math person" stereotype to the math abilities that are valued and privileged by typical U.S. mathematics curricula.

> Student 4: I self-identify as creative, um, which I think, had I maybe been in a different sort of math setting, would have been more valued, um, but for the majority of math classes I took it was really just memorization and problem solving, and never proofs or the cooler kind of jumping around stuff, yeah…
>
> Student 3: I think there's people who have like test anxiety and stuff, so they do bad in math classes, um, and people who can't do math quickly, and so do we consider them to be bad at math?...But if you think about high-level mathematics, I mean, you're sitting with a problem for a long time and figuring it out doesn't make you any less good. So I don't know….but I think people will never go and become higher mathematicians being told they're whole life that they're bad at math because they're slow…

Lastly, perhaps the most striking finding was that all but one participant (a teacher) did not identify as a math person. This included two other math teachers and two students completing STEM majors. However, many *had* identified as "math people" in the past, before other experiences caused them to change their identification.

> Student 4: I think in seventh grade when I got to go into higher math…I was just having fun, I would stay after school and do math, I was like 'Yeah, I guess I am a math person,' then I got into the science-y high school…I actually do remember taking a hit to my confidence when I only got in to the second lowest [track], I was like 'Oh, I guess I'm not that good at math…

The participants' math histories suggest that on top of the direct factors comprising the "math person" stereotype, there is also an "infallibility" component: after either a) experiences of difficulty or failure in mathematics, or b) comparison to students more capable in/more interested in mathematics than themselves, students drop their "math person" identity. One cannot be a math person if one struggles in math.

Discussion

Although not complete, our data begins to paint a better picture of the "math person" stereotype. Survey data supports the hypothesis that "math people" are stereotyped as orderly, logical people with an "objective" way of thinking (Figure 1). They calculate quickly, are more rational than creative, and don't appear to struggle to do math. Interviews add that the stereotype is reproduced in the U.S. both by discourse around who likes math and why, and by norms of teaching and assessment that link quick calculation and infallibility to school success. Being a "math person" is, however, a fickle belonging, as failure or struggle can quickly lead to losing the label. In short, the cognitive stereotype carries social power that sorts students into and out of mathematics through feelings of exclusion and not being "good enough."

The limitations of a one-year project constrained the design of this study, including a non-representative survey sample, no opportunity to use fieldwork or trial surveys to inform the final survey design, and time for only eight interviews. While the findings are robust given the available data, they are not safely generalizable and require validation with a better-tested survey battery and larger dataset. I also rely on accurate self-reporting by participants, and so would benefit from confirmation through more direct methods (Jerolmack & Khan, 2014). Finally, the survey sample size required that the "Gender" variable be a strict binary, and that gender and race could not both be included as variables alongside "Personality" and "Way of Thinking." Given that the math person stereotype is deeply racialized, any future work must study the interactions of the cognitive factors with a students' racial identity as well.

Should these results be reproducible, they have implications for both further research and educational practice. The individual effects of the "way of thinking" component on student self-perception described in interviews mirror prior findings on racial and gender stereotypes, as the component affects students' mathematical identities and their perception of their belonging in mathematics (Boaler, 2002; McGee, 2015). Thus a "way of thinking" stereotype can theoretically be combated using interventions already proven to help address other stereotypes (e.g., Steele, 2010). Stereotypes thrive when they are unarticulated and unchallenged, meaning that the first step to dismantling the stereotype is to articulate it and call it out in all its forms.

But stereotypes are only symptoms of larger social narratives around belonging and ability. They can also operate unconsciously, a fact echoed by how interviewed students largely rejected the stereotype, but on the survey student respondents upheld it over all factors (Spencer et al., 2016). Creating math classrooms that support all students requires

tackling those larger structures. We must ask how "way of thinking" is used in constructing legitimate belonging in mathematics, and explore in more detail how this stereotype is communicated – and enforced – to students. Given the history of concepts like "ability" being weaponized to uphold white and male superiority, we should also explore how the cognitive stereotype is used to support the construction of Black and female students as "less able" in mathematical spaces and so legitimize their exclusion, even in the absence of explicit racism and misogyny. Ideas like the cognitive stereotype may be providing a convenient colorblind excuse for patterns like those observed by Faulkner et al. (2014) and Francis et al. (2019).

Lastly, since the cognitive stereotype affects students' persistence in math, the stereotype goes beyond student experience to actively shape the composition of larger mathematical communities. Building on work by researchers such as Boaler (2002) and Lakatos (1976), future work can explore how the "way of thinking" component of the stereotype affects mathematical communities, and especially ask if the stereotype has consequences for how legitimate mathematical knowledge is produced – and what knowledges and skills are considered "mathematical" in the first place.

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“Minoritised mathematics students are motivated by gratitude”: An analysis of storylines in Norwegian public media

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As part of a larger participatory research project, we examine storylines about mathematics education and students from minoritised cultures and/or languages in Norwegian news media. We identify some of the positionings construed in this public discourse to consider what kinds of positionings are made available in relationship to education and mathematics education. In particular, we focus on one storyline we found, “minoritised mathematics students are motivated by gratitude” because we hope to engage the MES community in exploring it with us.

In this paper, we investigate storylines about youth from minoritised cultures and/or languages in Norwegian news media to identify some positionings made available in this public discourse. News media concurrently reflects and influences public opinion on mathematics and mathematics education, and about migrated and Indigenous youth. We note here that we are aware that any wording that points to the youth whose storylines we are most interested in should be used with caution and great respect. The possibilities—nondominant, minority, othered, non-Norwegian, multicultural, etc.—all rest on attributions that are not always consensual, and they imply problematic power relationships or may feel stigmatising or unfamiliar to members of the groups themselves. In this paper, we use the word minoritised, though we recognise it too may be problematic.

Our investigation of news media launches the beginning of a longitudinal, participatory research project with teachers, administrators, community members, youth, and families to understand and enact the kinds of storylines that position for example newly arrived students and Indigenous youth in asset-based ways. We are motivated by the fact that these students’ opportunities to learn mathematics are often diminished in comparison to their majority group peers, especially in mathematics and mathematics education (Källberg, 2018;

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Ryan, 2019; Udir, 2018). We also know that the positioning presented in news media can affect individual students' and groups of students' identities (Mendick, 2005; Wagner, 2019), the relations with and expectations of mathematics education and thereby opportunities for mathematics learning and life choices.

Storylines

According to positioning theory, people interpret their experiences through storylines – through "lived stories for which told stories already exist" (Harré, 2012, p. 198) such as for instance a coach/athlete storyline. Davies and Harré (1999) pointed out that the multiple storylines at play "are organized through conversations around various poles, such as events, characters, and moral dilemmas. Cultural stereotypes like nurse/patient, conductor/orchestra, mother/son may be called on as a resource" (p. 39).

Storylines make positions available, which could be either accepted or resisted. For example, a parent could position himself as a teacher in a teacher/student storyline when trying to help his child with homework. The child could resist by rejecting this positioning and try to interact within a different storyline. Hence storylines are negotiable; they are reciprocal and contingent (Wagner & Herbel-Eisenmann, 2009).

Storylines about mathematics and mathematics education in public media

Recent scholarship has begun to identify storylines present in news media. One storyline says that *mathematics equips society* in the pursuit of economic growth and national prosperity (Herbel-Eisenmann et al., 2016). This storyline positions students and their mathematical achievements as national commodities, valued by means of global ranking systems such as PISA and TIMMS (Lange & Meaney, 2018). The exchange value of the commodity is a high rated workforce—a STEM workforce. Yasukawa (2019) described how these rankings may translate into national pride or shame. In the context of Norwegian newspaper analyses, Lange and Meaney (2018) showed how this storyline ramifies into political rhetoric built on the 'schoolification' of the Norwegian public barnehage (kindergarten) for the sake of equipping the nation.

In contrast to this storyline that positions countries as competitors, the storyline *mathematics equips the individual* positions individuals as combatants in pursuit of social and economic advancement (Wagner, 2019), or as citizens equipped for collective action (Jablonka, 2003; Rodney, Rouleau, & Sinclair, 2016).

Storylines in Norway

Although the media storylines discussed in the previous section apply to many contexts, different countries also have ideals that might be more particular to that context. In Scandinavian societies, as for example reflected in the Norwegian national curriculum, qualities such as social justice, equity, and equal opportunities are emphasised. The national curriculum takes a 'mathematics for all' approach (Nortvedt, 2018). As stated in the

introduction, however, such ideals in education and mathematics education, may not have been realised for minoritised groups.

For us, the research context is part of the data because it provides one source of potential storylines. What people say about a country and its inhabitants are storylines. There are some historic (and historically shifting) storylines about minorities/immigrants and non-Norwegian peoples/nations of Norway. As our findings show, some may be seen as fulfilling the storyline: the groups are useful parts of the labour force and necessary for the growth of Norway; hardworking; the groups' traditional ways of living that make use of natural resources inaccessible by others; the trade and handicrafts of the groups are praised. Others are less positive: the cultural characteristics, identity and languages of the peoples are unwanted; the groups are prospering at the cost of others; the groups are not complying to norms of the society; etc.

According to the Norwegian Government decisions, Norwegian society is historically composed of seven peoples: the Norwegian people, the Kven and Sami peoples in the North, the Forrest Finns in the South, and the non-territorial Romani, Rom and Jews. These minorities have suffered from various injustices and assimilation policies over the last centuries, often justified by some of the storylines mentioned above. E.g., for Kven and Sami students, use of their mother tongue was not permitted in schools, not even outside the classroom (Engen, 2010). After the Second World War, minorities in Norway slowly started to take back their rights and place in the Norwegian society, an ongoing and nonhomogeneous process. The assimilation storyline hence still exists and is among other things made present by how authors communicate in media articles. From the 1970s migrant workers from Pakistan, Turkey, and Morocco (Reisel, Hermansen, & Kindt, 2019) were invited to Norway to supplement the workforce. Migrants (in Norwegian "innvandrer", and later "migrant"), their children and their children's children got their own categories in Norwegian population statistics in the years to come (Søbye, 2014). The recent diversification of migration channels such as work permit programmes, increased mobility due to the EU enlargement, student migration, and family reunions have impacted Norwegian demography. Like elsewhere, the most recent decades in Norway have been characterised by global migration and displacement events due to conflicts, violence, political instability and climate change (IOM, 2019).

Methodology

This research is part of the MIM project which investigates educational possibilities and desires in Norwegian contexts, particularly focused on mathematics education in times of societal changes and movements. Languages may be the most obvious challenge in diverse mathematics classrooms, but they connect to cultural differences and conventional characteristics of the discipline. Indigenous communities have experienced linguistic and other challenges for decades as a result of colonisation. Such tensions are now appearing in "ordinary" Norwegian classrooms because tensions in education are intensified by language and cultural differences in times of large migration (Cenoz & Gorter, 2010). These tensions

are reflected in public news media. They are local but reflect global trends. News media reflects these trends and thus reifies them as public storylines, which impacts students' potential positionings.

We studied text-based mass media sources that acknowledge Redaktørplakaten, an ethical codex for publishers in Norway: including daily newspapers, weekly or monthly journals, tabloids, etc. We focused on articles published from January 2003 to September 2020 to include the time in which a new national syllabus was formed and launched in 2006**,** and the discussion leading to the launch. A librarian supported our search of the Norwegian database Atekst (http://retriever.no)**,** to identify articles that included words from each of three groups that represent the categories shown below with their groups of words also translated into English:

A) Indigenous and migrational contexts: urfolk (Indigenous people), minoriteter (minorities), migrasjon (migration), innvandring (immigration), Samer (Sami people), Kvener (Kven people), Skogfinner (Forest Finns), Tater/romani and Rom (Romani), Jøder (Jews), flerspråklighet (multilingual), flerkulturell (multicultural), mangfold (diversity)
B) Education: utdanning (education), skole (school), ungdomskole (upper elementary school), videregående (high school), opplæring (teaching), pedagogikk (pedagogy), didaktikk (didactics), klasse (class), klasserom (classroom), lærer (teacher), lærerstudent (student), vurdering (assessment), karakterer (grades)
C) Mathematics: matematikk (mathematics), matte (math), matematikkdidaktikk (mathematics didactics), realfag (science), naturfag (science), økonomi (economy), statistikk (statistics), programmering (coding), geometri (geometry), algebra.

We are motivated by positioning theory and the notion that minoritised bodies are highly visible since they are seen as out of place in the dominant society (Puwar, 2004) to ask the following questions:

1. What ontologies of minoritised bodies are portrayed in the news media articles?
2. How are these ontologies enacted in the context of mathematics education?
3. What ontologies of the mathematics do these storylines imply?

We narrowed our original 1896 articles to 501 after removing articles deemed irrelevant to our research focus and then selected 96 articles for further analysis because they were more centrally related to the three questions and offered some rough answers to them:

1. The media portrayed minoritised bodies as gendered, vailed, dark, masses, wilful, lost, helpless, drop-outs, lazy, sloppy, and unruly.
2. The media implicated gatekeeping, success, failure, gratitude, opportunity, hard work, rights, benevolence, language, culture, sufficiency and hope as constructing ontologies.
3. The mathematics portrayed in these relationships was neutral, formal (Western), cultural and a(nti)-mathematical.

The storylines related to mathematics education and minoritised groups of transcultural students were often implicit and scattered across the articles. We envisioned the articles as collages in which the authors assembled images that they thought suitable to put together to make up a full collage, or in other words to tell the article story. An article may not explicitly connect mathematics, the minoritised group and education, but all three of these things occurred in the article. In other words, the author of the articles had chosen to knit these images together.

Due to the reciprocity of positioning, to identify the positioning of students from minoritised groups required us to look also at how individuals from dominant groups were positioned. We read the excerpts several times and preliminarily coded in an iterative process, based on the positionings we found in relation to the search words. The process was conducted both jointly among us and individually which allowed us to compare and refine our coding. Finally, we grouped the excerpts according to the coding and re-read the excerpts with the purpose of formulating propositions that articulated the storylines. Some storylines we identified in the media include *Minoritised mathematics students are motivated by gratitudee, Mathematics is language- and culture-neutral,* and *Minoritised groups' mathematics achievements are linked to culture and gender.*

Focusing on the *gratitude* storyline

In this paper, we focus on the storyline for which we are most interested in our MES colleagues' feedback: *minoritised mathematics students are motivated by gratitude.* The storyline is construed tacitly in positionings related to obligation, gratitude and benevolence.

This storyline relates to a storyline identified by Schwöbel-Patel & Ozkaramanli (2017) as *the grateful immigrant* which imposes certain societal behaviours, expectations and obligations such as willingness to work hard, gratitude to the host nation and unwillingness to be a burden to the state resources. The model minoritised student or grateful immigrant is expected to take up positions that involve excelling in education, contributing to labour, display vulnerability and weakness, and ultimately to undertake the benevolence and superiority of the host country's culture (Thiruselvam, 2019).

The binary structure of gratitude and benevolence materialises, for example, on the 17th of May, the Norwegian Day of Constitution, when it is a common practice that selected students hold speeches at their schools. In the excerpt below the newspaper article reports on how an immigrant girl exclaims gratitude for her life and the opportunity to go to school in Norway as part of the celebrations. The immigrant girl's exclamation makes available a benevolent position for the dominant societal group to inhabit, since gratitude is a response to benevolence (Higgins, 2004).

> «Kjære alle sammen», begynner hun, «jeg er så takknemlig! Takknemlig for at jeg får gå på skole her i Norge.» --- 17.mai - arrangementer. Talen er en av mange der elever uttrykker takknemlighet for livet i Norge og i [...] kommune.

> "Dear all", she begins, "I am so grateful! Grateful that I get to go to school here in Norway." --- May 17 - events. The speech is one of many where students express their gratitude for the life in Norway and in […] municipality. (our translation)

However, Schwöbel-Patel & Ozkaramanli (2017) note that the binary structure of gratitude and benevolence may come at a price, which can position minoritised groups as indebted. The debt can be paid off by loyalty and contribution to the majority society, which the excerpt below reflects.

> I hendene holder hun en hvit konvolutt og en rose. Inni ligger et dokument som forteller om femmere i matematikk, naturfag, norsk muntlig og samfunnsfag, og seks på muntlig eksamen i naturfag. - Norge har gitt oss en trygg plass å være. Da må vi vise Norge respekt tilbake. Jeg kan ikke sitte rolig og vente. Det er bare jeg som kan hjelpe barna til å bli gode mennesker, til å få gode jobber som kan hjelpe Norge.
>
> In her hands she holds a white envelope and a rose. Inside is a document that tells about grade five in mathematics, science, Norwegian oral and social studies, and grade six in oral exams in science. - Norway has given us a safe place to be. Then we must show Norway respect back. I cannot sit still and wait. Only I can help my children to become good people, to get good jobs that will help Norway. (our translation)

The excerpt above tells a story about a refugee mother who has graduated from the Norwegian school system with high scores. Her motivation is to be able to support her children at school so that they eventually will "get good jobs that will help Norway".

In the excerpt below the students are positioned as alienated from the majority society because they identify a need (to promise) to learn Norwegian culture and language to be able to learn mathematics and then "do great". Concurrently the media endorses the storyline *the dominant language and culture are keys to learning mathematics* since possession of the Norwegian language and culture will allow the students to understand mathematics.

> VIL BYGGE NORGE: Bare vi lærer om kulturen her og kan nok norsk til å forstå matte, skal vi klare oss kjempebra, lover NN[1] (28), NN (17) og NN (16).
>
> WANTS TO BUILD NORWAY: As long as we learn about the culture here and know enough Norwegian to understand math, we will do great, promise NN (28), NN (17) and NN (16). (our translation)

What appears to be of the essence is to understand mathematics—language and culture are merely means to that end. The value that the nation state can add to the students is their mathematical performance (Lange & Meaney, 2014). It is by means of that value that the students can pay off their debt, because mathematical knowledge allows them to contribute to the majority society by "doing great" and by helping to build it. The notion of value added by mathematical knowledge which is present in the storyline *minoritised mathematics students are motivated by gratitude* shows that this storyline overlaps with the two storylines that Wagner (2019) highlighted namely, *mathematics equips society* and *mathematics equips individuals*.

[1] We anonymised people's names in the media and blinded with initials NN

According to the gratitude storyline the best way for minoritised youth to achieve what is expected or obliged is to resolve alienation, become integrated and be prosperous in the majority society (Ahmed, 2008). A storyline about the happy, successful immigrant is present in our data material. In connection to that storyline and the storyline *minoritised mathematics students are motivated by gratitude*, the value that doing well in mathematics adds to minoritised people's contribution to society is in their potential to function as a source of success, as for instance mentioned in the excerpt above. Consequently, when minoritised students excel, mathematics becomes a signifier or explanation of (the expected/obliged) success. For instance, in the excerpt below the newspaper article author reported about an immigrant girl's excellent grades, not mentioning specific subjects, but pointing out that her proud father is a mathematics teacher.

> Syv seksere, seks femmere og én firer er karakterer som gjør NN, til daglig matte- lærer på Sandefjord videregående skole, til en stolt pappa.
>
> Seven sixes, six fives and one four are grades that make NN, a daily math teacher at Sandefjord upper secondary school, a proud father. (our translation)

The storyline emphasises that to succeed and pay their debt of gratitude, minoritised students must work harder and be more ambitious than students from other groups—an effort that is expected of them as part of the storyline about the grateful immigrant (Schwöbel-Patel & Ozkaramanli, 2017). In addition, minoritised students may not just be positioned as indebted to the majority society, but also to their families and communities.

> Barn av innvandrere føler ofte at de skylder foreldrene å gjøre en innsats, siden mor og far ofret mye for å bygge et nytt liv i Norge, ifølge en studie fra 2010.
>
> Children of immigrants often feel that they owe it to their parents to make an effort, since their mother and father sacrificed a lot to build a new life in Norway, according to a study from 2010. (our translation)

The author of the excerpt below positions immigrant youth as hard working and as having high ambitions, which may be a manifestation of the storyline about the grateful immigrant.

> Innvandrerungdom har høyere ambisjoner enn resten av elevene. Men de sliter på skolen. Én av tre klarer ikke elementær matte.
>
> Immigrant youth have higher ambitions than the rest of the students. But they struggle at school. One in three fails basic math. (our translation)

According to the author, despite the extra efforts most immigrant students fail to learn mathematics. In the context of this storyline one may draw the conclusion that they fail to fulfil their obligations—to allow mathematics to equip themselves and society. This has implications for the foregrounds of minoritised students.

Some excerpts indicate that some minoritised students reject their obligation to learn (mathematics). Consequently, they do not comply to the storyline about the grateful, hardworking immigrant. Those minoritised people's unwillingness to contribute to the wellbeing of the majority society is seen as troublesome (Ahmed, 2008). The excerpt below

illustrates how immigrant youth become positioned as lazy and/or not understanding/disrespecting the rules of the dominant society.

> Vi ønsker at de får gode arbeidsvaner. Når vi sier at skolen begynner klokken 08, betyr det at de må være der da, ikke klokken 09 eller 10.
>
> We want them [immigrants] to get good work habits. When we say that school starts at 08, it means that they must be here at that time, not at 09 or 10. (our translation)

In Norway immigrant girls proceed to higher education more often than do adolescents of Norwegian descent (Reisel, Hermansen & Kindt, 2019). In addition, they more often chose education strands that comprise science and mathematics to become for instance physicians or engineers. Hence, girls from minoritised groups may be seen as better than boys at fulfilling their obligations towards the majority society. In fact, they are so successful that the Norwegian prime minister positions them as part of the picture of "the Norwegian dream".

> Hun er ikke den eneste jenta fra denne skolen som går ut med nesten bare toppkarakterer. For fem år siden ble to minoritetsjenter med til sammen 36 seksere trukket fram i nyttårstalen til statsminister Jens Stoltenberg, som «et bilde av den norske drømmen».
>
> She is not the only girl from this school who leaves with almost only top grades. Five years ago, two minority girls with a total of 36 sixes were highlighted in the New Year's speech to Prime Minister Jens Stoltenberg, as "a picture of the Norwegian dream". (our translation)

Herbel-Eisenman et al. (2016) identified a storyline present in the public realm which implies that the main goal of mathematics education is to produce a STEM workforce. This is how mathematics equips society (Wagner, 2019) and how the mathematical knowledge of minoritised youth can materialise the "Norwegian dream".

Conclusion and Implications

Our inquiry into the storylines about youth from minoritised groups in the Norwegian text-based mass media sources was compelled by a desire to understand some of the storylines and positionings available in this context. This exploration is the beginning of a longitudinal project, in which we are collaborating with teachers, administrators, community members, youth and families, to understand what storylines they would like to have made available to them in the teaching and learning of mathematics. Drawing on a participatory design, these storylines can then be used to imagine new positionings and practices in mathematics classrooms and in schools. We started with an analysis of media not to say these *are* the storylines, but rather to sensitise ourselves to some of the existing storylines and, thus, to recognise storylines already potentially available and shaping positionings.

Although we found a range of storylines in this analysis, we explored one particular storyline that *minoritised mathematics students are motivated by gratitude*. Our investigation looked at articles relating to Indigenous and immigrant groups, but this storyline positions immigrants as the ones expected to be grateful. Similar to previous findings, this storyline may be understood to say that as one puts economic growth and national prosperity first,

immigrants are supposed to work hard and contribute to the labour force. The storyline suggests they should be thankful to Norway for allowing them to live and go to school there. In our conversations with students, we will want to find out what this storyline about thankfulness might mean for them. Positioning theory reminds us, that any positionings and storylines are negotiable. For example, the dominant members in society cannot expect to impose their benevolence and their expectations of gratitude on those who are positioned outside of the dominant member group unless the minoritised members aspire to be part of the majority society (Berman, 1999). This is why paying attention to existing storylines can be useful for imagining new and different storylines, positionings and practices that are more humane and equitable than the ones that might be currently available.

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Opening the university for seniors: A possibility for inclusive mathematics education

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This paper presents research results on mathematics education with seniors. The research took place in a program entitled Universidade Aberta à Terceira Idade (UnATI, Open University to Seniors). The aim of UnATI is to open the university to the neighborhood community offering activities organized based on the knowledge produced by the university. Our research group has been investigating the contribution of mathematics education in such program. Our main concern is social inclusion in a general perspective. The results allow us to identify the contribution in aspects such as: social representation of mathematics, quality of life, intergenerational relationship. Each one of these aspects are detailed in the text.

Introduction

Research with seniors took place in a program entitled Universidade Aberta à Terceira Idade (UnATI) (Open University to Seniors). The aim of UnATI is to open the university to the neighbourhood community offering activities specially organized for it based on the knowledge produced by the university. In Brazil, according to the Elderly Statute[1], elderly people are those aged over 60 years.

Since 2011, the Épura research group[2] has been investigating the contribution of Mathematics Education in such program. We have been running two different activities with seniors: *Encontro com a Matemática* (*Meeting with Mathematics*) and *Encontro com a informática* (*Meeting with informatics*). The meetings took plac every second week during the school year. They took place in the department of Mathematics Education at Universidade Estadual Paulista (Unesp). The research data were produced during these meetings. We have used video, notes and interviews. In this paper we concentrate on data produced in the *Encontro com a Matemática.*

[1] Law No. 10,741, of October 1, 2003, is intended to regulate the guaranteed rights of people aged over 60 years.

[2] https://igce.rc.unesp.br/#!/departamentos/educacao-matematica/grupos-de-pesquisa/epura/apresentacao

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Universidade Aberta à Terceira Idade (UnATI)

The Universidade Aberta à Terceira Idade (UnATI) is an extension program coordinated by the Pró-reitoria de Extensão Universitária e Cultura (PROEX). It is running simultaneously in 20 campuses of Unesp[3]. Each campus has the autonomy to develop the activities according to the demand of its community and to the research area developed in the campus.

UnATI at Rio Claro has been mainly associated to the Physical Education Department. In 2011 Mathematics Education started a partnership with physical education promoting meetings in which were proposed activities as games, challenges and exploratory tasks. The team responsible for carrying out these activities were constituted by researchers from the university, Master and PhD students from the graduate program in Mathematics Education and undergraduate students in Mathematics. During the first years the meetings were entitled *Conversas sobre Matemática* (*Conversations about Mathematics*) changing to *Encontro com a Matemática* (*Meeting with Mathematics*) in the last years[4].

This article presents results of the analysis of three activities of the *Encontro com a Matemática.* This analysis highlights the contributions of the development of Mathematics activities with elderly people.

Encontro com a matemática

This activity took place in the laboratory of Mathematics Education of the Department of Mathematics Education at Unesp. It was announced in social media and e-mail lists. There was no need for prior registration in order to participate. It was sufficient to come to the laboratory at the agreed time with disposition to talk about Mathematics. At every meeting, a new subject was approached together with the participants, like; exploring the Tangram; exploring a calculator; math magic; forms in mirrors; reading and discussing numeric and graphic information; Mathematics Bingo.

The participant's school level was remarkably diverse. Some had never attended school and others had college degrees. Some had a problematic relationship with Mathematics as a result of failing school due to Mathematics, and others had sympathized with Mathematics, but never had the opportunity to study. Those who think of Mathematics with a positive perspective saw the *Encontro* as a chance to make a dream come true. Luis (64), one of the participants, revealed to us a bit about the condition of the senior. He tells:

> [A minha trajetória escolar] foi sofrida porque morava no sítio. Naquele tempo, não tinha como, a gente precisava andar sete quilômetros [para chegar à escola]. E sete quilômetros para voltar [da escola para casa]. Ia e voltava a pé. Chuva, sol. Eu gostava de lá. [Na escola] eu gostava da professora, dos amigos. Aprender [a] ler, aprender [a] escrever, aprender [a] fazer contas [de Matemática]. Isso tudo era bom, mas era sofrido chegar [à escola]. Estudei até o terceiro ano. Depois, a outra [escola] era mais longe, daí não dava para ir. [Ent]. (Lima, 2015, pp. 104–105)

[3] Unesp has 24 campi.

[4] More details are in Lima (2015) and Scagion (2018).

> [My scholar trajectory] was suffering because I lived in a farm in the countryside. At that time, there was no possibility, we needed to walk seven kilometers [to reach the school]. I would come and go walking. Rain, sun. I liked there. [At school] I liked the teacher, the friends. Learning [to] read, learning [to] write, learning [to] make calculations [of Mathematics]. All that was good but suffered to reach [the school]. I studied until the third grade. After that, the other [school] was too far away, then I could not go. (Lima, 2015, pp. 104–105, our translation)

The meetings were mediated by a dialogical and investigative approach according to Alrø and Skovsmose (2004) and Freire (2011). University team and seniors shared experiences and perspectives. Nothing was imposed on the other. Based on this interaction it was possible to establish a perspective on seniors as "human beings that can produce knowledge on many subjects, including Mathematics by means of collaboration" (Lima et al., 2019, p. 1336).

In the following, we bring details of some activities developed during the *Encontro com Matemática*.

Exploring the tangram

This was one of the most appreciated activities by the participants because of the challenge involved in it. At first, it was presented some historical aspect of Tangram. It was told about legends. After that, was disposed a set of pieces for each participant and we started a conversation about the relationship between the Tangram pieces. It was proposed many challenges of image composition using the pieces as illustrated in Image 1 (Lima, 2015, p. 152).

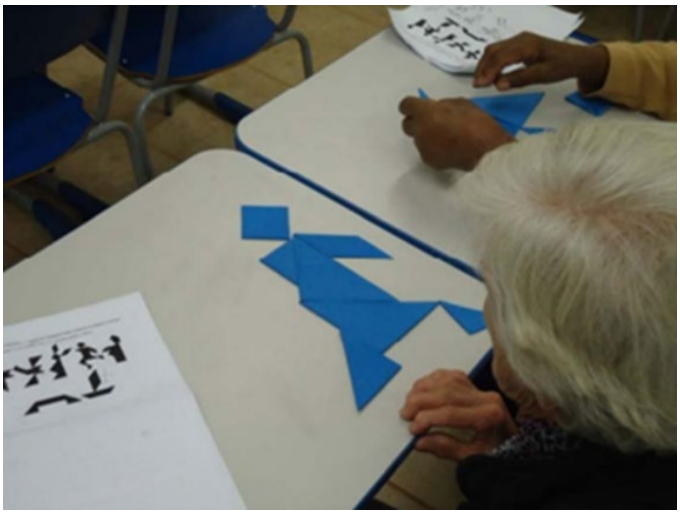

Image 1: Lady manipulating a Tangram

Reading and discussing numeric and graphic information

This activity was also highly successful among the seniors. It involved a conversation about news that are circulated in terms of numeric and graphic information. The activity could

start with a video followed by a discussion on the message one could get from the video. Figure 1[5] illustrate points that were discussed about the skewed graphics. Another example is a conversation on fake news. This happened in 2018 during the president election period in Brazil. We talked about the veracity of the shared news on social media.

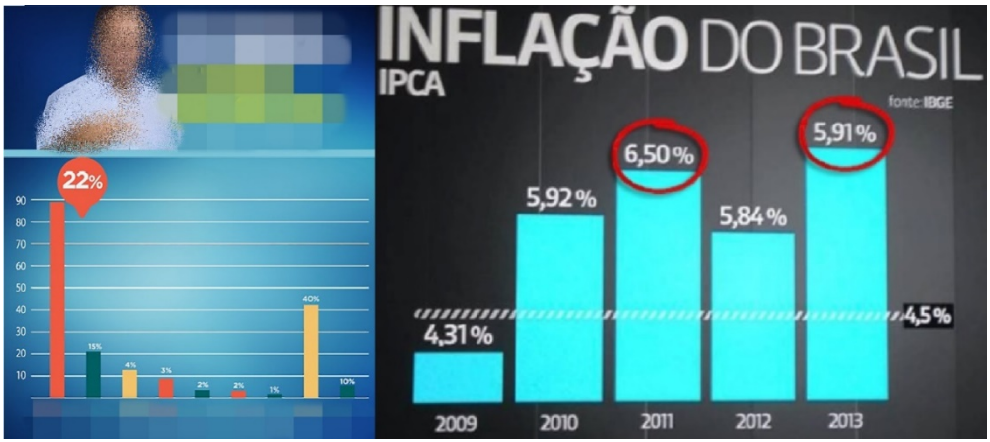

Figure 1: Deceptive Graphics

Mathematics of Bingo

The Bingo is a well-known game by the senior community. It is something so simple that does not require high-cost material. They also like it because there are prizes and a spirit of healthy competitiveness.

Image 2: Overview of Bingo day

[5] These graphics can be visited on http://www.semanaon.com.br/coluna/3/553/o-jornalismo-que-se-apresenta-aos-brasileiros and https://noticias.uol.com.br/politica/eleicoes/2018/noticias/2018/05/30/psdb-sp-divulga-grafico-desproporcional-de-doria-e-tira-do-ar-apos-criticas.htm. Last access in 03/24/21.

In our meetings, Bingo was transformed into an activity that involved mental calculation, mathematical curiosities, and popular knowledge. For example: the number two was called by *the only number that is even and prime*; the number 45 was mentioned as *three quarters of the minutes of an hour*; 54 by *four dozen and a half*, and 51 was called *a good idea*, referring to the name of a popular cachaça. It was necessary to have a calm rhythm that most of the seniors could figure out what number was called. There were prizes for different achievement. The most important was when one completed first all the numbers in a card as shown in image 3. During the whole game was stablished a very friendly atmosphere. Image 2 presents an overview of what this environment was like.

Image 3: lady playing Bingo

Contribution from the *Encontro com a Matemática*

Through these years, the *Encontro com a Matemática* has become a learning environment for the seniors and for the university team. The research carried out on this environment has identified contribution in aspects such as: social representation about Mathematics, quality of life, intergenerational relationship.

The first aspect is related to Scagion's (2018) research (first author of this paper) that identified the social representations of Mathematics presented by seniors. Scagion was inspired by social representation theory as elaborated by Serge Moscovici who states that social representation is:

> a modality of particular knowledge that has the function of elaborating behaviours and communication among individuals, and that in turn also contributes to the processes of formatting behaviour and orientation of social communications (Moscovici, 1978, p. 77).

According to Jodelet (1985), a social representation is "a manner to interpret and think of our daily reality, a way of social knowledge" (p. 360).

Scagion interviewed seniors who attended the *Encontros* to ask about schooling, daily life, work, quality of life, and future. The analysis identified the following representations:

Mathematics is everything; Mathematics helps in the quality of life; it is good for the elderly to understand Mathematics; the relationship with Mathematics change over time and Mathematics is for few people (Scagion, 2018, p. 8).

> Alcides (84[6]) Eu gostaria de aprender muita coisa. [...] É o que estou fazendo, aprendendo alguma coisa que nunca aprendi, aqui no curso que nós fizemos, e acho que isso vai além, mas tem aquela história do tempo. (Scagion, 2018, p. 87)
>
> Alcides (84) I would like to learn many things. [...] This is what I am doing here, learning something that I have never learned. Here at the course that we attend, and I think it goes beyond, but there is always that story about time. (Scagion, 2018, p. 87, our translation)
>
> Deisy (80) A minha relação com a Matemática é confortável, porque eu consegui vencê-la, antes era um tabu, que não me deixava satisfeita. Eu tinha que decorá-la e não compreende-la, agora eu a compreendo e a aceito. (Scagion, 2018, p. 79)
>
> Deisy (80) My relationship with Mathematics is comfortable because I was able to beat it. Previously it [Mathematics] was a taboo, that did not make me satisfied. I had to memorize it and did not understand it. Now I can understand and accept it. (Scagion, 2018, p. 79, our translation)

A second aspect in the *Encontros* is related to a quality of life. Life quality is related with material as well as emotional well-being. It depends on the good interpersonal relationships and opportunities of participating in society. This aspect is a result of the research which is presented in detail in Lima (2015) and Lima et al. (2019). The data were produced during the *Encontro com a Matemática*[7] by means of notes, interviews and video. The analysis

> reveals an active engagement of the women and men in the project as well as provides some indication about contributions to the seniors. About their engagement, we considered: a) their interest in discussing the subjects during the meetings; b) their persistence to perform mathematics tasks on their own; c) their argumentation about the ideas showed; and d) their movement of sharing the activities with other people who were not participating in the project. In relation to contribution to senior people, we considered manifestations that expressed: a) improvement of cognitive aspects; b) opportunity for social interaction; and c) the possibility of learning new subjects related to mathematics. (Lima et al., 2019, p. 1332, our translation)

The way the *Encontros* were organized favoured elements connected to quality of life. They could interact with a theme, Mathematics, which was for many of them the reason for evading the school when they were young. In the *Encontros* they had the experience of success. Social integration is essential for the improvement of quality of life. Tasks which stimulated oral presentation, logical reasoning and the dialogical interaction were important approaches. Let the participants tell us what they felt.

> Davi (67) [...] abriu a nossa mente [dele e da esposa], abriu o nosso raciocínio. A gente começou a trabalhar com a mente. Entendeu? A, realmente, querer ver e tentar fazer, de ver pronto. Quando eu não consigo resolver alguma coisa, ou, quando você me pergunta alguma coisa assim muito rápida, dá uma batedeira aqui na cabeça. Sabe? É uma sensação difícil de explicar.

[6] age

[7] Lima data production was from 2011 to 2012 and that time the activity was called *Conversas sobre Matemática* (Conversations about Mathematics).

> E eu sinto isso não só nas suas aulas [faz referência às outras atividades do AtivaMente desenvolvidas pelo grupo da Educação Física], que é um sinal que eu estou colocando alguma coisa na cachola, não só nas suas aulas, como em outras aulas, que a gente ocupe a mente, a memória. Para mim, estudar Matemática aqui com você é um pouco descômodo, porque mexe comigo. Entende? Mas o que conta é que é muito benéfica. Porque tem que movimentar a cabeça, porque é uma questão de raciocínio essas atividades. (Lima, 2015, p. 93)
>
> Davi (67) [...] opened our mind [him and his wife], opened our reasoning. We started to work with the mind. Did you understand? To really want to see and to try out, to see it done. When I cannot solve something, or, when you ask me something very fast, it mixes everything in the head. You know? It is a difficult sensation to explain. And I feel it noy only in your classes [he refers to other activities of AtivaMente developed by the Physical Education group], which is a signal that I am putting something into the box [head], not only in your classes, as in other classes, that we put our mind into work, the memory. To me, studying Mathematics here with you cause a little discomfort because shake me. You understand me? But what counts is that it is greatly beneficial because we must move on our mind. Because these activities have to do with logic reasoning. (Lima, 2015, p. 93, our translation).

And Ju (60) reinforces this contribution:

> A gente, às vezes, fica sem lembrar algumas coisas, sem memorizar. Então, com a ajuda de vocês, a Matemática tem ajudado. Eu memorizo bem números. Pelo fato de estar sempre utilizando telefone, números diversos e o Bingo [Matemático] ajuda muito. Resumindo, todas as atividades da Matemática são boas. Por exemplo, aquelas de blocos de madeira [Blocos Lógicos] foi excelente [...]. Tem algumas coisas que você passa a pensar em relação ao que você ouviu e então a gente acaba também colocando no dia-a-dia da gente. Isso melhora a qualidade da vida da gente. No memorizar, no se concentrar, se organizar. (Lima, 2015, p. 95)
>
> We, sometimes, cannot remember things, without memorizing. So, with your help, Mathematics has helped. I memorize the numbers well. By the fact of always using the telephone, many numbers, and the Bingo [Mathematics] helps a lot. Summarizing, all the Mathematics activities are good. For example, the wooden blocks [Logic Blocks] were excellent [...]. There are some things you start to relate with what we heard and then we end up putting it into our daily lives. It improves our quality of life. In memorizing, concentrating, organizing. (Lima, 2015, p. 95, our translation).

Davi continues,

> A gente [a esposa e ele] gostou de tudo. Inclusive, desafiei o meu neto a fazer um Tangram [montar algumas figuras com o Tangram] e ele não conseguiu sozinho. Foi muito bom, porque eu expliquei a ele como se fazia para encaixar as peças. Ele prestou atenção, quando eu falei que tinha que olhar as peças para formar os lados que eram iguais e as figuras que as peças podiam formar. Assim, dava para fazer o todo [montar a figura que se pretendia] (Lima, 2015, p. 154)
>
> We [his wife and him] liked everything. So much so that, I challenged my grandson to make a Tangram [to set up some figures with the Tangram] and he could not do it by himself. It was very good because I explained to him how it should be done to fit the pieces. He paid attention, when I said he had to look to the pieces to make the side that were equal and the figures that

> the pieces could form. So, we could make the full piece [to set up the expected figure] (Lima, 2015, p. 154, our translation).

A third aspect to be highlighted is the intergenerational relationship occurred in a *Encontro com a Matemática*. We understand intergenerationality as an interaction between groups of different generations, relating experiences, opinions, cultures and knowledge. According to Hatton-Yeo and Ohsako (2001) intergenerationality is a vehicle to an exchange of knowledge between generations looking for individual and social benefits.

In the UnATI activities, young university students and seniors shared life experiences and established bonds. Intergenerationality provides a resignification of old age, both by seniors and young people. According to Mendes, Leandro and Lopes (2017), spaces of intergenerational interaction can be beneficial for the establishment of bonds and social practices that provide inclusion.

Final considerations

Encontro com a Matemática departure from the perspective of promoting the appreciation of senior's life, which means right to education, health and enjoyment. During the meetings, it was imperative that the environment was encouraging for everyone to feel free to share their ideas, doubts and life experience. This was achieved through dialogic communication.

By this article we hope to show the importance of working with Mathematics Education for seniors in an inclusive perspective. The *Encontro com a Matemática* starts from the perspective of promoting the value of life for the elderly, which means the right to education, health and entertainment. It is organized as a space for activities that rely on dialogue between young and seniors. During the meetings, it was essential that the environment was stimulating so that everyone felt free to share their ideas, doubts and life experiences. This was achieved through dialogic communication. Communication during the performance of activities provided reflections on meeting differences, the need to promote quality of life for the elderly, concern with accessibility.

Intergenerationality contributes to all the publics involved and the social relationships of these people. For the elderly, it contributes to promoting quality of life. On the other hand, it contributes to the reinterpretation of young people based on reflections on the interaction with the elderly. Young people can think about their future as an elderly person. For society in general, the contribution can come as valuing the elderly and reducing prejudice and violence.

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Making a math that was more (than rote work): Turn-of-the-century math education reform in the United States

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Math education researchers have long advocated for progressive pedagogical methods centering students' active production of their own math learning. In this historical paper I reconsider progressive pedagogy in relation to teachers by locating it in the emerging research university of the turn-of-the-20th-century United States. As education and math professors developed research-oriented departments, they justified their extrication from teaching duties and other "rote" tasks through the ideal of the "active producer" of knowledge. And yet, despite professors' removal from the work of teaching, they sought to intervene on high school math teachers–who were presumably doing and enforcing "rote work"–with progressive pedagogy so that students could emulate the "active [university] producer."

Introduction

Mathematics education research has advocated progressive pedagogical practices focusing on student agency and activity–in contrast to "rote" and "mechanical" learning–since its very beginnings. While the effort to provide pedagogical recommendations based on student agency and activity is often portrayed as scientific and novel, these pedagogical recommendations are an almost 200-year-old tradition of education researchers, teacher educators and textbook writers, dating back in the United States to Samuel Read Hall's *Lectures on School-Keeping* (1830).

And yet, the emphasis on student agency and activity is widely held to have never been achieved in the "reality" of school mathematics apart from research, particularly amongst its proponents. According to mathematics education research, math teachers continue to deliver rote instruction in math. How do we explain the persistence of this tradition in the face of its reputation for failure?

Critiques of progressive pedagogical recommendations and math education research more generally have often explained the endurance of the "active child" and "math for all" as a case of researchers falling prey to capitalist ideologies or having an emotional attachment to reform. In this paper I focus on the material interests of university math and education researchers in maintaining their position as university experts. While "material interests" certainly includes salaries, here I focus on professors' interest in differentiating

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their work from the work of grade school teachers. This is both rhetorical, in the sense that professors write about how their work is better than that of teachers, and literal, in that professors have an interest in not having to do the day-to-day work of public-school math teachers who teach children en masse.

Through a historical approach approximating "research on research" (Pais & Valero, 2012), I revisit the emergence of U.S. research universities in the second half of the 1800s. I argue that pedagogical progressivism in math education–as it was adopted by both mathematicians and education professors–was part of an effort to secure non-teaching jobs with expertise over teaching. The idea that researchers who don't teach children know more about teaching than people who do was a historical achievement. Professors leveraged progressive pedagogical recommendations in constructing teachers as inevitably "rote" by nature of being teachers. They were thus in need of intervention from non-teachers who also happened to be the paragons of active knowledge production in the university.

Methods and theoretical perspectives

As a part of the broader social and socipolitical turns in mathematics education (Lerman, 2000; Valero, 2004), critical mathematics education researchers have subjected mathematics education research itself to the critical gaze through "research on research" (Pais & Valero, 2012). This has included interrogations of school math and math education research's participation in broader projects of white institutional control and white supremacy (Martin, 2013); imperialism (Bishop, 1990); settler colonialism (Gutiérrez, 2017) and capitalism (Pais, 2014). The ideal of the "active child" has come under critique for its inherent classism, sexism and ableism (Lubienski, 2000; Walkerdine, 1989; Yolcu, 2017), for being mythological (Valero, 2002), and for constituting a mechanism of control (Popkewitz, 2004).

Relationships between mathematics education researchers and teachers have received less critical attention. Chronaki (2004) and Darragh (2017) have argued that the gaze of the researcher maintains a sense of superiority to teachers and even colonizes teachers' activities. Montecino and Valero (2017) take the approach of analysing how OECD and UNESCO discuss teachers, arguing that these discourses control teachers' subjectivity. I take a similar approach by considering publications emanating from university professors, analysing how university professors portrayed their own work and that of teachers. I contextualize these portrayals in material shifts in the structure of university and grade school teaching.

My approach draws on the recent resurgence of historical research in mathematics education scholarship (Karp & Furinghetti, 2016). In particular, I build on the work of scholars using historical methods to re-consider math education research's origins, goals, and role in broader political projects (e.g., Bullock, 2013; Lundin, 2012; Tröhler, 2013; Yolcu, 2017). I focus specifically on the turn of the 20th century because high school teachers' and university researchers' work had been relatively similar up until that time, but was rapidly differentiated, as I explain in the paper. As a case of the broader global expansion of public education and university systems, I consider the development of research universities in the

United States, specifically Johns Hopkins and the University of Chicago.[1] While the U.S. case is indeed a specific one, the focus on the evolution of a particular country's research institutions illuminates local relations between education systems in ways that are reflective of education systems around the world. In order to achieve this, my methology involved drawing connections across relatively internal histories of turn of the century universities and grade schools. I also analysed key reports and publications relevant to mathematics education and, as a means of contextualizing mathematics education, drew from the literature on the growth of common schooling and normal schools across content areas.

As I show, as Johns Hopkins and then the University of Chicago established themselves as research institutions in the image of the German university, professors in mathematics and education alike were organizing for their own removal from teaching anyone but graduate students. These organizing efforts–as a part of a broader move against work that could be considered "rote"–occurred under the auspices of the academic ideal of the "active producer" of knowledge. It was in this context that mathematics and education professors recommended that students be made to emulate the "active producer" of knowledge as well. In pursuing "active production" and a larger distinction between their work and the work of teachers, professors portrayed grade school teachers as doing and enforcing "rote" work, particularly in their prescription of progressive methods emphasizing the child's agency and activity. As grade school teachers were increasingly subject to criticism from professors, they were also subject to the bureaucratization concomitant with the massive expansion of public school systems.

High school professors

In contrast to common (elementary) school teachers, American high school math teachers were an elite group until the late 1800s. High school attendance was reserved for an elite few, and high school teachers were often men trained at the university rather than women trained at normal schools. Labaree describes that in the case of Philadelphia's Central High School, teachers were originally referred to as "professor," had dramatically larger salaries than common school teachers, and governed the school collectively without oversight (Labaree, 1986). While few high school teachers attended National Education Association (NEA) conferences, they were initially lumped into the "higher education" department with university professors.

Vice versa, the work of the university mathematician was more similar to that of the high school math teacher than it is today. University mathematicians were employed in their capacity for teaching rather than doing research (Roberts, 1997). Indeed, American mathematicians had yet to contribute substantially to the burgeoning European "pure" or modern mathematics scene, and most of the higher-level work was done for the military or by engineers. The category of "mathematician" was also larger and consisted of anyone who

[1] At the time, education in the United States consisted of common (elementary) schools, high schools and academies and normal schools for teacher training and universities.

practiced math regardless of their place of employment. This included teachers, applied math practitioners working for the federal government, amateurs and even hobbyists or "puzzlers" (Abrams, 2020).

That said, status differences between college professors and high school teachers–particularly those working in public schools rather than private academies–existed early on. High school teachers detected the "contempt with which many college men regarded high schools," and "rarely asked professors to their meetings" in the NEA. Referring specifically to the 1870s and 1880s, NEA historian Wesley emphasizes that there was no "golden period in which public school people [teachers and administrators alike] gratefully accepted the educational leadership of college and university professors" (Wesley, 1957, p. 69).

To the degree that college mathematicians and high school teachers enjoyed similarities in their work, this would change in the last quarter of the nineteenth century. On the one hand, high schools were rapidly expanding, and thus high school teaching lost the status that came with scarcity. As the ranks of high school teachers grew, they became subject to the bureaucratization of school systems. On the other hand, U.S. mathematicians would come to focus on the production of their own, original research in emulation of mathematicians in Germany, France and Italy. New, "modern" or "pure" developments were increasingly abstract and removed from application, even as they took on the familiar "foundations" of mathematics in arithmetic and geometry. As the next section explores, U.S. mathematicians would begin to organize for relief from the relatively "rote" work of undergraduate teaching in favor of training graduate students and free time for research.

Inducting away from teaching

Johns Hopkins would serve as the eminent example of the new American research university until the mid-1890s. Johns Hopkins' first president, Daniel Coit Gilman, who began his term in 1876, concerned himself with the development of postgraduate programs that would prepare one to do research. This demanded faculty equipped to train graduate students in the research process; requiring both proven capacity and the free time to keep up with new developments. Johns Hopkins trustee George William Brown argued that university faculty should "be teachers in the largest sense, that is, should have the ability and the leisure too, to add something by their writings and discoveries to the world's stock of literature and science" (French, 1946, p. 7). Ultimately Gilman would relieve research faculty from the standard teaching assignment of fifteen to twenty hours a week. Adjuncts, assistants (to be pulled from the graduate students) and visiting professors would help provide undergraduate instruction (Parshall & Rowe, 1994, p. 57).

Gilman hired 61-year-old British algebraist James Joseph Sylvester for Johns Hopkin's first professorship of mathematics on the basis of his research credentials. Amidst the increasing popularity of the experimental or "inductive" natural sciences conducted in the "laboratory," Sylvester would argue that, despite the deductive presentation of mathematical research, the process of actively *doing* math was one of inducting generalizations from specific examples; it was experimental. Indeed, Sylvester's approach to his famed work in

the Theory of Invariants depended on the extensive computation of examples. Ultimately he was able to hand off this relatively rote aspect of his work to dedicated graduate students (Parshall & Rowe, 1994, Chapters 2-3).

Sylvester was hired despite his poor teaching record with undergraduate students, and yet his record for inspiring mathematical producers was impeccable. Sylvester's first student, one of the very first research mathematicians trained in the U.S., would recall that "the professor broke every rule and canon of the Normal Schools and Pedagogy, yet was the most inspiring teacher conceivable" (Halsted, 1895, p. 206). Sylvester's lecturing style reflected the joys of mathematical invention; he would follow the train of any thought that came to him rather than teaching a curriculum. He would have students work on open problems toward the goal of presenting "laboratory reports." (Parshall and Rowe, 1994, p. 86). In short, Sylvester taught his students to "*do* mathematics actively, not merely study it passively" … they had to "get their hands dirty" (Parshall and Rowe, 1994, pp. 82, 106).

Under Gilman's leadership and in Sylvester's mathematics department, the role of the university professor was reimagined as first and foremost the role of the "active producer," and their most important instructional role was to facilitate future mathematicians in embodying it. Sylvester's first student explains Sylvester's belief that "without unceasing original research and published original work there could be no real university teaching, and that any university professor who, without such a basis, pretended to be a good teacher, was, consciously or unconsciously, a selfish fraud" (Halsted, 1895, p. 204).

Across the next ten years, American mathematicians would increasingly follow in the footsteps of their European counterparts in their focus on "pure mathematics," abstract and apart from application. In 1888, the founding of the American Mathematical Society –in emulation of the London Mathematical Society– formalized the collective interests of U.S. university mathematicians in focusing on pure research, rather than teaching or application. And yet, university mathematicians would not hesitate to make recommendations to grade school education, despite their ever-increasing removal from it.

High schools were expanding astronomically in the United States. While there were 325 high schools in 1860, by 1890 there were approximately 2,536. In contrast to the college prep academies tied to the colleges by entrance exams, the new high schools had started relatively beholden to the (largely job-training) needs of the communities that they were located in (Wesley, 1957, pp. 64–66). Concerned about the development of the high schools outside of their control, the National Education Association assembled a committee consisting mainly of college professors and private school principals in order to administer organization to the burgeoning array of secondary school options.

The process of generating what would ultimately be the Committee of Ten Report quickly extended out of the typical terrain of curriculum recommendations and into pedagogy. Under the leadership of Sylvester's successor at Johns Hopkins, Simon Newcomb, the Math Committee of the Committee of Ten recommended increased abstraction in arithmetic as a means of avoiding learning myriad rote procedures (NEA, 1894). For example, applied work

in "commercial mathematics" was rejected for relying excessively on rote learning. And yet, it was also recommended that abstractions be motivated "inductively" through concrete examples.

To conclude, math professors, within the broader emergence of the U.S. research university, were increasingly organizing for time for "pure" research devoid of rote computation. The new ethic of the "active producer," who taught by example and collaboration, was developed in contrast to the relatively "rote" work of teaching the canon. Even as math professors became more removed from teaching anyone who was not training to be a math professor themselves, they sought to extend the reach of the "active producer" ideal and avoidance of rote work to grade school math education.

Finding the foundation

The ideal of the student as an active producer, in the image of the new university researchers, was also institutionalized through the newly developing chairs and departments of pedagogy. John Dewey famously embraced students' self-directed learning in activity, which he conceptualized in contrast to teacher-directed, and thus rote, schoolwork. Dewey arrived at the re-established and Rockefeller-financed University of Chicago in 1894. The university's president was gearing Chicago to overtake Johns Hopkins as the premier U.S. research university. At the same time, he sought to expand the reaches of the university into teacher education (which was still largely undertaken at normal schools) and the operations of the Chicago public school district. Dewey was just the person to help.

As head of the departments of philosophy and pedagogy, Dewey argued that the best way to avoid students doing rote work was for education professors to engage in an academic search for the "foundations" of education; psychologically, historically, and sociologically. However, rather than experimentation, this involved applying a pseudo-historical, racist theory of human progress (Fallace, 2009). Dewey gave central importance to students re-enacting the "stages of civilization." In order to re-enact these stages, students were to engage in historic manual occupations toward active re-discovery of concepts within the context of work. By revisiting the supposed social and economic foundations of civilization, the study of education would take content "out of the abstractness forced upon it for purposes of its own convenience of study and put into its concrete connections with the rest of the world" (Dewey, 1986, p. 287). Ironically, this project of taking content out of "forced" abstractness would be led by professors–who traded in abstractions–rather than teachers.

In asserting theoretical foundations as a privileged means of educational improvement, Dewey, among other university professors who did not work in grade school education, asserted themselves as having improved knowledge of how to teach over teachers themselves. Indeed, as Dewey declared, "without insight into the psychological structure and activities of the individual, the educative process will, therefore, be haphazard and arbitrary" (Dewey, 1897, p. 85). In the current state of education "far too much of the stimulus and control proceeds from the teacher, because of neglect of the idea of the school as a form of social life" (p. 88). In practice, progressive pedagogy's orientation around "foundations" relocated

the locus of educational knowledge from the teacher to the university, which Dewey argued at length was the correct location for such investigation (Dewey, 1896).

The active producers

Leadership of the U.S. mathematics community also migrated from Johns Hopkins to the University of Chicago under department head Eliakim Hastings Moore. The Chicago department aggressively pursued pure mathematics, ironically reaching new heights of abstraction through a focus on the very "foundations" of mathematics, including arithmetic and geometry. Indeed, mathematicians at the University of Chicago had ceased to think of the mathematicians of the prior generation, who had pursued applied work in fields such as astronomy, as mathematicians at all (Roberts, 1997, p. 112). Professors were lured to Chicago with the understanding that they would work exclusively in the graduate school and not take on undergraduate teaching responsibilities (Boyer, 2015, p. 81). Applications and undergraduate teaching fell under the category of rote work and were not conducive to "active production."

American mathematicians engaging in pure mathematics became absorbed in the axiomatic description of systems (such as geometry or arithmetic) or objects (such as a group). For example, instead of thinking of the integers as the relatively concrete list of numbers {…-2, -1, 0, 1, 2}, they could be reconsidered as a particular case of an abstract group, to be defined by a minimal and independent number of relations between its elements rather than by what those elements are. In this they owed to the work of European mathematicians such as David Hilbert and Felix Klein, who had trained much of the University of Chicago's math department. And yet, American mathematicians took this to a new extreme, seeking to cleanse their systematizations from relatively concrete mathematical "objects" in favor of their abstract properties (Moore, 1903).

As Moore's math department purged themselves of the concrete as a means of becoming active producers, Moore took up arms for mathematics education reforms promising to facilitate students in doing the same. This was, however, through the opposite approach of engaging students in the concrete. In his famed exiting speech as president of the American Mathematical Society, "On the foundations of mathematics," Moore advocated for the laboratory method and correlation between math and science as a means of engaging students in "the practical sides of mathematics." He also recommended the "diminution of emphasis on the systematic and formal sides of mathematics"; students should study "the subject itself, and not the words, either printed or oral, of any authority on the subject" (Moore, 1903, pp. 405, 419). Students would learn to "be sure in a naïve and elementary way," and mathematics education would then be imbued with the "spirit of research (pp. 406, 411). Rather than learning through the direct instruction of the teacher, "in most cases, much of the proof should be secured by the research work of the students themselves" (p. 419).

Conundrums

In this paper, I demonstrated that rather than being a natural state of affairs, current relations between teachers and mathematics education professors (and the professoriate more generally) are a historical achievement, in which progressive pedagogy played a large role. Progressive pedagogy portrays not only an ideal for children's "active learning, but also emulates university researchers' self-image and fight for work conditions in juxtaposition to the "rote" situation of teachers. Professors argued that, by nature of not being teachers, they knew more about teaching than teachers, because they were liberated from the presumed "rote"-ness of being a teacher. Ironically, they also recommended that teachers leverage the concrete as a privileged means for students to become active producers, even as they retreated increasingly into the abstract themselves.

Rather than being a problem that can be solved by improved theoretical perspectives, I would argue that the researchers' "colonizing gaze" on the teacher is a constituent part of the structure and material conditions of university teacher education and mathematics education research. More work is needed in understanding the relationship between researchers and teachers through the perspective of work conditions and class relations. This includes questions of how we portray teachers and researchers as differentially able to reflect and know about teaching, which has consequences for who gets paid to take free time for reflection. This in turn has consequences for the improvement of teaching.

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Supporting orality and computational thinking in mathematics

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The pedagogy of The Algebra Project introduces mathematics concepts to students through experiential activities that students then analytically examine using a combination of both informal and formalized language. In this paper, we identify how cultural forms of orality can be supported within this discursive approach, while also introducing computational thinking activities that are particularly constructive. We argue that when cultural forms of orality are left unacknowledged and unexplored, they can lead to issues that have a negative effect on both student engagement and comprehension. But when explicitly supported in the specific ways that are outlined here, new opportunities arise for student-led creative and pragmatic inquiries that have the potential to deepen the level of student engagement and comprehension.

The process of schooling fails many mathematics students by overlooking opportunities to engage or blatantly rejecting their orality, a cultural asset of many youth. Yet mathematics education has long identified the importance for student-student discourse in learning mathematics and promoted pedagogical strategies such as small group and whole group discourse routines (NCTM, 2014). Both Piaget and Vygotsky make central the importance of interaction in their theories of learning (Steffe & Thompson, 2000), yet the 5-step curricular process (Moses & Cobb, 2001) developed by The Algebra Project (AP) is among few pedagogical practices that intentionally bridge from experientially grounded, ordinary discourse to that regimented discourse (Quine, 1981) of the mathematics discipline. AP's work cycle provides multiple opportunities for learners to move between internalization and externalization (Papert, 1990), bridging the partial stories of each of the sociocultural and the constructivist perspectives on learning.

In this paper we report our approach to instruction in mathematics and computer science that engages students' orality grounded in the historical work of AP, arguing our approach develops both mathematical reasoning and computational thinking. We present a developmental cycle that builds from student assets of culturally rich orality to develop mathematical abstractions and literacy. The development cycle engages children's propensity to imagine, create, reason, and discuss.

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The paradigm of orality to literacy, instead of orality vs. literacy

Our research addresses the manner in which schools fail many young people influenced by deeply meaningful cultural forms of orality. By itself, the experience of orality does not need to be treated as an educational deficit; rather, when seen through the appropriate pedagogical lens, orality can be treated as an asset toward producing a rich discursive academic environment. Our argument is that orality can provide constructive building blocks to literacy within various disciplines unless it is ignored or mistakenly pitted as an incompatible stumbling block to literacy.

This second position is such as that taken by Orr (1987) in a well-known book, *Twice as Less*, "For students whose first language is BEV [Black English Vernacular], language can be a barrier to success in mathematics and science" (p. 9). By examining schoolwork, Orr determined the use of nonstandard English led to misunderstandings. Orr suggested a connection between students' nonstandard use of particular "prepositions and conjunctions that in standard English distinguish certain quantitative ideas" (p. 9) and their misunderstandings of certain quantitative relations, resulting in "a lack of distinction between addition and multiplication and between subtraction and division and thus to a confusion between 'twice' and 'half'" (p. 13), for example.

Orr's argument is firmly in the camp that views features of certain language patterns within African American cultures as an impediment to learning, reflecting the deficit paradigm that views orality as having negative implications (at least academically) on particular students. In *Orality and Literacy*, Ong (2002) provided an important counter to this way of thinking. He identified African Americans as a "dominantly oral culture" (p. 43). And although not focused on African American culture, he addressed commonality across primarily and secondarily oral cultures. For Ong, critiques of phrases such as "twice as less" exemplify a misconception that sees patterns of orality as being prelogical or illogical "in the sense that oral folk do not understand causal relationships" (p. 56).

Ong (2002) made the case that there are specific and necessary imperatives in oral cultures that do not follow certain textual patterns, reasons for which do not rule out logical and sophisticated conceptualizations as claimed by Orr (1987). In this line of reasoning, Ong (2002) states that,

> The elements of orally based thought and expression tend to be not so much simple integers as clusters of integers, such as parallel terms or phrases or clauses, antithetical terms or phrases or clauses, epithets. Oral folk prefer, especially in formal discourse, not the soldier, but the brave soldier; not the princess, but the beautiful princess; not the oak, but the sturdy oak. Oral expression thus carries a load of epithets and other formulary baggage which high literacy rejects as cumbersome and tiresomely redundant because of its aggregative weight. (p. 38)

So, the word "twice" used with "less" may be no more a misunderstanding of how subtraction differs from multiplication than how the use of double negatives imply that the speaker doesn't understand the difference between a negative and a positive. In the phrase, "twice as less," just like in the phrase, "ain't no sunshine," the first word may be there to confer a greater emphasis and importance on the second part of the phrase.

Ong (2002) extended his critique of views like those presented by Orr,

> You cannot without serious and disabling distortion describe a primary phenomenon [like orality] by starting with a subsequent secondary phenomenon [like a math concept] and paring away the differences. Indeed, starting backwards in this way—putting the cart before the horse—you can never become aware of the real differences at all. (p. 12)

In other words, if you fail to understand or acknowledge the key role that orality plays within some cultures, unnecessarily conflicted assumptions can result, negatively affecting the students involved. One must make note that orality, in Ong's words,

> can be quite sophisticated and in its own way reflective... To assume that oral peoples are essentially unintelligent, that their mental processes are 'crude', is the kind of thinking that for centuries brought scholars to assume falsely that because the Homeric poems are so skilful, they must be basically written compositions. (pp. 55–56)

Moreover, we believe deficit views toward oral cultures also feeds into an anti-constructivist viewpoint, in that it sees the context of orality as something that needs to be removed to reach some type of false *tabula rasa* goal, instead of constructively building upon oral conceptions in the quest to help the student become more literate.

In our approach, orality provides a perfectly suitable starting point for a type of informal mathematical discourse with the students that leads to a more formal, literate discourse over time. This approach follows W. V. Quine's (1981) treatment of constructivist mathematics learning as grounded in language discourse, with mathematics as an especially rich conceptual language that provides connections to both informal and formal conceptualizations. Ong (2002) explains that orality is often more "situational rather than abstract" (p. 48), and in our research, we've seen the need to incorporate or create experiential curricular material that has an appropriate situational context like that produced within the Algebra Project curriculum. In this pursuit, we incorporate constructivist computational thinking and programming activities that utilize enactive and iconic elements to provide youth the opportunity to explore mathematical ideas in imaginative and creative ways.

The Algebra Project pedagogy and the five-step curricular process

For some thirty odd years, the Algebra Project (AP) and Bob Moses, its founder, have struggled with issues of what to teach and how to teach in order to raise the floor of math literacy for those students most disenfranchised in the U.S. public school system. AP developed and refined its culturally centered approach to mathematics education in schools with majority African American populations in southern districts like Jackson, Mississippi and Atlanta, Georgia, as well as in urban districts like Chicago, Illinois and San Francisco, California. The culturally based pedagogy they developed draws from the critical role that social facilitation and social identity serves in communities of color, where collaborative models are especially prominent. This is evident, for example, in the "call and response" cultural patterns that make their way into the classroom, but it is also connected to the need to cooperatively and collectively solve persistent challenges in resource-poor communities. Because of this, AP has found that this population of students is more receptive to collaborative learning models than to the more individualistic and competitive approach that is the standard in U.S. schools.

The collaborative approach developed by AP involves students working together in small groups that report out to the rest of the class as they make progress. The breakout groups help develop a sense of ownership over the ideas the students address in the groups, and they help facilitate giving different students various leading roles on a rotating basis within the group, using their orality to help them develop a sense of the importance of their agency and their voice. AP's 5-step curricular process gives students many opportunities for this type of positive affect when studying mathematical concepts, and we are finding the same to be true when we integrate concepts from computer science. The AP curricula that emerged, and most importantly, the project's curricular process, is the synthesis of three distinct lines of thought: (1) experiential learning, (2) agency first through student voice, and (3) the regimentation of ordinary discourse—seeing children's orality as an asset in their learning.

Experiential learning as mathematization

From its inception, Moses' development of AP has been continuously informed by his participation in the Civil Rights struggle in Mississippi and the community organizing tradition which arose out of it. "The Algebra Project is first and foremost an organizing project—a community organizing project—rather than a traditional program of school reform" (Moses & Cobb, 2001, p. 18). So how AP elicits the classroom participation of students who have been convinced that they cannot do mathematics is of prime importance. What would a good community organizer do in a community feeling powerless? They would encourage a sense of shared agency within which to develop a sense of empowerment. Doing this in a school setting implies the need for a domain for doing mathematics that involves a shared concrete experience that students can collectively work together to address. Learning mathematics in this model primarily consists of students mathematizing the events of their shared world, their communities, the places where they felt most expert in the company of their peers. This perspective over time developed strong connections to the experiential learning models of Piaget, Dewey, Lewin, and Kolb.

The basic sequence was simple. Collectively, students engaged in some physical activity designed to be intrinsically interesting. They tried it. They thought and talked about it. They came up with ways to understand and improve the experience. And finally went back to play and experiment with it again. This was the basic experiential learning cycle built into AP curriculum units: collective activity around a shared event, reflection upon the event, a conceptualization of key aspects of the event, and finally an application of the learned concepts to begin another cycle of experiential learning.

The regimentation of ordinary discourse

Moses was also deeply influenced by the mathematical logician W. V. Quine. Quine's (1981) perspective that the foundations of mathematics—arithmetic, elementary logic, and elementary set theory—begin in the regimentation and structuring of ordinary discourse fit naturally into a classroom practice grounded in the natural language discourse patterns of students, i.e., their various cultural forms of orality. Students' ordinary discourse was

referred to as People-Talk because that was the way ordinary people talked among themselves when not a part of a specialized academic or professional group. The structuring of ordinary discourse was identified as Feature-Talk. Feature-Talk served to bridge the gap between the everyday discourse of students and the abstract symbolic representation of the conceptual language of mathematics. Mathematics—like other artificial languages, e.g., coding—is a language that is read and written but never spoken. Feature-Talk provides a means for students to read and write abstract symbolic representation (equations, inequalities) in an interpretative and hence meaningful fashion.

How the voiceless find their voice: Community organizing in the classroom

The primary tool of community organizing for the Student Non-Violent Coordinating Committee (SNCC) in the Mississippi theatre was first and foremost the meeting. The meetings were where the voiceless found their voice. Community members, the share-croppers, first met in small groups and focused on the concerns that they brought with them. Next, the small groups brought their concerns to larger groups through the voices of the sharecroppers who first raised them. This was a practice that Ella Baker brought to SNCC, SNCC brought the practice to the Mississippi Delta, and AP brought the practice to the mathematics classroom.

By first meeting in small group and discussing the shared experience of the class, students who thought they had nothing to contribute to the mathematical conversation found they did have thoughts and opinions that they could share and indeed were eager to share. Discourse in a mathematics classroom may not come easy at first, but once it gets started it can be hard to stop, like the way static friction is harder to overcome than kinetic friction. As an empirical matter, we have seen that once students get engaged around a shared experience they find intrinsically interesting, they are both willing and able to discuss, investigate, and even entertain conjectures about the whats, hows and whys of the event. Students in AP classrooms don't just talk about the event; they capture what they find most important in pictures and in text. The typical AP classroom is plastered with chart paper covering the wall. Students, typically in small groups or teams, capture their thoughts about their experience on chart paper, publish it by hanging it on the wall and report out to the whole class on its contents. As with the community meetings of Mississippi sharecroppers in the 1960s, this process is wholly owned by the students themselves.

The development cycle of voice – agency – identity through the 5-step process

There is a dynamic here which is meant to capture the conceptual growth and development of students who move through the 5-step curricular process. The students begin together with a (1) shared concrete experience. Because all students have access to thoughts and opinions about this concrete event, all students have a place and a voice at the table. The students (2) draw pictures of the event, write, and speak about the event first at the intuitive level of everyday discourse, (3) People Talk. The 5-step process engages the strengths that students bring both from their natural language abilities and from the natural informal logic that is

embedded in the language that they speak. They then engage in (4) Feature-Talk through the consideration of those features or attributes of the event that they consider most salient and interesting. From an identification of important features, they move on to consider how those features are related. And finally, they try to capture these features and their relations in (5) iconic or symbolic representations of their own construction. These representations are set to the task of problem solving and are refined by students until they can effectively handle the same representational tasks using the conventional symbols of mathematics.

Within the 5-step curricular process, mathematics is generated and learned as a collective enterprise. Students come to understand the notion that the knowledge they build is not just for themselves, because it ultimately benefits their team and their entire class. To achieve this collectivism, the AP follows what is now a fairly well-known trend in discourse rich mathematics classrooms, with addition of specific details that draw upon orality and build both individual and collective voice. Mathematical work follows a pattern of individual thinking (production), small group work (publication), and finally whole group discussion (peer-review).

Computational thinking within the orality to literacy paradigm

To the AP's 5-step curricular process we add computational thinking (CT) activities and skills. These activities introduce collaborative technological tools and their created artifacts to the classroom in constructive ways. A CT process is outlined as defining the problem by decomposing it, solving the problem by recognizing solvable patterns in the problem, and analyzing and understanding the impact of a problem's solution. Thus, CT includes a set of skills we describe as decomposition, pattern recognition, abstraction, algorithmic thinking, and analysis of impact.

In the AP curricular process, the second step is for students to create models of a shared experience through a drawing, graph, or other representation. While doing this, students can identify subcomponents and/or subsystems within their model using the CT skill of *decomposition*. As students move into the People Talk, students use the CT skill *pattern recognition* identifying patterns in the subcomponents. During Feature Talk, AP's aim to discover and define features abstractly parallels the CT skill of *abstraction*. The final stage of the AP curricular process, representation, provides opportunity for the next CT skill, examining symbols *algorithmically* to address how the symbolic form can be used to produce various outcomes. This CT stage involves using technology to programmatically and algorithmically examine abstract ideas using technical and computational tools, such as a microworld. This is an important added stage to AP's curricular process, allowing for further examination and analysis of an abstraction's *impact*.

The developmental cycle and the microworld

Learning a new idea, from a constructionist (Papert, 1990) viewpoint, is an active process characterized by a developmental learning cycle. When the learning process occurs within this cycle, a person is actively integrating the concept they are learning (or constructing) into their own broader intellectual understanding of a related domain. This active engagement

from the learner is critical to the learning process and is exemplified as the student does the work of exploring, analyzing, and probing a new idea that emerges from a concrete experience until it becomes familiar enough to be abstracted or generalized for them so that they can apply the concept appropriately in whatever task is subsequently given to them. The student's exploration, analysis, and examination that makes up the student's intellectual work is, by and large, an internalized process that is aided by different types of educational resources. Figure 1 gives a picture of how we see those resources playing their part when the developmental cycle of the AP along with CT interventions are fully in place.

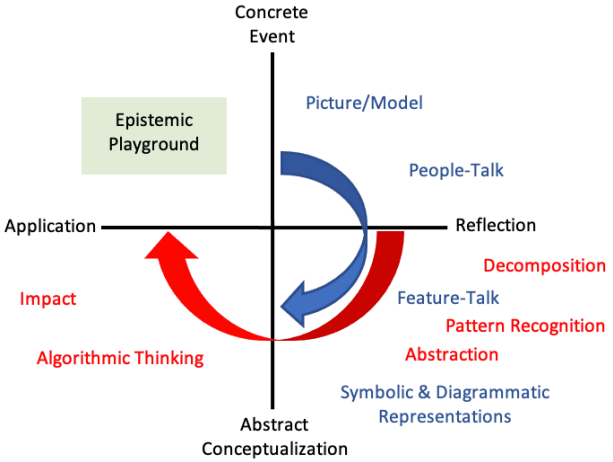

Figure 1. Math and computational thinking developmental learning cycle

The introduction to the conceptual material starts with a concrete event and modeling that event in a physical way, such as through a picture, chart, or graph. This opens the door to informal and formal discourse about the event that directs the student into progressively deepening reflections. CT activities are introduced during these reflections that assist the student in representing the mathematical features abstractly and symbolically. In this way, these steps offer students a bridge from a concrete external event to something that involves internalized conceptual understandings. When students are not provided with such a bridge, we believe many students fail to build an appropriate intellectual scaffolding a mathematical concept may require.

Even though building an internalized abstract conceptualization is often the goal of academic instruction, it is insufficient if it is not ultimately developed into a form that can be articulated and/or applied by the student in a subsequent activity. In other words, the student needs to be able to take what they have internalized, and then externalize it in an appropriate way. And for this to happen, the student needs to be able to put what they have learned into a systematic and algorithmic form. That is to say that the student needs the ability to move through a set of steps that shows how the concepts that they have learned can produce various types of impacts. From a constructionist viewpoint, this latter stage of the learning process is no less important than the former stage, because once again, if the student is not an actively engaged participant in this latter stage, then even the best instructions given may not lead to a successful outcome. And we believe that actively engaging the student in this latter process requires allowing the student to play with the

concepts they are learning, and this means allowing them to engage their imagination and creativity during this part of the learning process.

Going straight into a testing phase after learning some new abstract idea is not the best way to help students get a firm grip on concepts that may be difficult to thoroughly digest. Students need an appropriate opportunity to chew on an idea rigorously before being tested on how well they have digested it. Therefore, our final quadrant in the map of the developmental learning cycle is the epistemic playground. This playground is the place where a student is given the opportunity to explore and experiment with the ramifications of the new concepts that they have constructed. We believe all learners need this, but in many educational settings, this type of activity is not provided. For some students, these explorations might involve entirely internalized cognitive musings, and as such, they are able to chew on the ideas without the help of the teacher. But we believe that with an appropriate microworld, the playground can be an externalized activity that makes it easier for the teacher to actively instruct the students on how to engage in this critical final stage of the developmental cycle.

We, like Papert (1980), define a microworld to be a digital environment where students have tools that they can use in creative ways to explore concepts related to a specific conceptual domain. If the domain is fractions, then students in the playground can creatively arrange fractions in various ways and see what happens to their properties and values as they move numbers around. If the domain is geometric shapes, then the students have tools which allow them to create various lines, shapes and angles in ways that allow them to test the geometric principles of the geometric forms they have just constructed. A microworld might involve programming, and it might not. For instance, Papert argues that Logo is a microworld because it allows its users to easily create simple structures, test ideas, and get meaningful feedback without needing to know many of the programming language's details or commands. On the other hand, a language like BASIC, and programming environments using BASIC, are not microworlds because even though BASIC is not a complex programming language, specialized and sophisticated knowledge is required before a student can use it to create simple structures, test ideas and get meaningful feedback.

We believe that adding CT activities in the specific ways we have been outlining can help engage learners in the full developmental cycle shown in the above diagram when learning mathematics. The cycle involves internalization and externalization, reflection and application, discourse and reasoning, rigorous analysis and abstraction, as well as imaginative and creative play. An interesting feature about the developmental cycle as we have outlined it, is that it starts with a shared concrete event, and when it progresses all the way to the epistemic playground students are able to engage in explorations and experiments that can also be shared as concrete events with other students. The developmental cycle begins with a community of learners sharing things through their various cultural forms of orality, and it ends that very way as well.

Conclusion

In schools across the world, students come with various backgrounds and cultural influences. When educational practices disadvantage certain cultural influences over others unnecessarily, the result is a type of cultural chauvinism that has no place in a multicultural society. And what our research further suggests is that culturally insensitive educational practices are also a missed opportunity to engage in an effective educational developmental cycle. When a student's own particular forms of cultural orality are treated as an essential part of a discourse rich curricular process, such as the one from AP that we have presented here, the result is a learning environment that has fewer barriers to the student becoming engaged. And greater engagement in the informal dialogue (People Talk) opens the door to stronger engagement in the formal dialogue (Feature Talk) when the student begins the steps toward decomposition, pattern recognition, abstraction, and understanding symbolic representations.

Papert (1980) indicated that the cultural context plays an important role in an individual's educational development, and he argued that this broader sociocultural component must be addressed in the basic constructivist paradigm:

> All builders need materials to build with. Where I am at variance with Piaget is in the role I attribute to the surrounding cultures as a source of these materials. In some cases, the culture supplies them in abundance, thus facilitating constructive Piagetian learning. For example, the fact that so many important things (knives and forks, mothers and fathers, shoes and socks) come in pairs is a 'material' for the construction of an intuitive sense of number. But in many cases where Piaget would explain the slower development of a particular concept by its greater complexity or formality, I see the critical factor as the relative poverty of the culture in those materials that would make the concept simple and concrete. In yet other cases the culture may provide materials but block their use. (pp. 7–8)

The poverty that Papert is talking about here is not in the culture, per se, but instead in the materials, or tools, available to the learner to appropriately build upon the conceptual underpinnings that they start with, to develop intellectual structures that represent literacy in any particular field. In this paper, we have argued that when existing advanced technologies, such as microworlds, are properly used, they can indeed serve as constructive materials that students can use in more creative and imaginative ways. The creative and imaginative approach brings to mind the idea of an intellectual sandbox where an engaged mind is encouraged to play with an idea until they have begun to develop a sense of familiarity and ownership over that idea.

In this paper we are arguing for a learning paradigm that involves a particular type of developmental cycle. The cycle starts with a concrete event or activity that can be shared amongst a community of learners. And then within that learning community a discourse begins that is informed by rich and cultural expressions of orality that increase engagement and emphasize the importance of every student's voice being heard. From there, a process of decomposition, pattern recognition, and abstraction follows. When these stages involve the use of computational thinking paradigms, students can interact with the formalisms and abstractions they have been learning about within a technological and programmatic microworld. The microworld serves as a conceptual playground where the students can

externalize and experiment with the concepts to better understand and digest them. And this playground ultimately leads the students to making new constructions that can become as concrete to them as the original event that set this cycle into motion.

This cycle involves both externalized and internalized processes that work together. Without a balance between these two types of engagement, the developmental cycle is incomplete. In our view, many, if not most, students are faced with an incomplete developmental cycle when it comes to mathematics instruction, as well as other formal areas of instruction. And students are often unsuccessful because of this. When true balance is achieved in this learning paradigm of the developmental cycle, students enter what Ackermann (1990) called a cognitive dance:

> My claim is that both 'diving in' and 'backing up' are equally important in getting such a cognitive dance going. How could anyone learn from their experience as long as they are totally embedded in it? There comes a time when one needs to translate the experience into a description or a model. Once built, the description gains a life of its own, and can be addressed as if it were 'not me.' From then on, a new cycle can begin, because as soon as the dialogue gets started (between me and my artefact), the stage is set for new and deeper connectedness and understanding. (p. 10)

To truly serve students from diverse backgrounds in modern classrooms, we need to provide them with the entire developmental cycle, which involves shared community building activities and events, the embrace of orality supporting informal discourse, rich decomposition and abstraction developing formalized discourse, and finally a sandbox where ideas about formalism and algorithms can be turned into intellectual constructs with which to play. *In short, children must have a mathematical learning environment in which they imagine, create, reason, and discuss framed in a cycle of abstraction, formalization, and language regimentation that begins and ends with the concrete.*

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Diversity and teacher students' positioning in research: Nuances that matter

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How to educate future mathematics teachers to deal with the challenges of diversity in mathematics classrooms continues to be a central concern for practice and research. Through researching the research on mathematics teacher education that addresses equity, justice and diversity in initial teacher education, three different positionings of teacher students were identified: the teacher student as unachieved, recipient and agentic. The nuances in how research positions teacher students are important in themselves, since they allow to challenge implicit assumptions about opportunities to develop a multifaceted sensibility towards diversity in teacher education.

Introduction

In many countries there is an increased attention to equity and inclusion, given the systematic documented failure or deficiencies of some students. Since many students in classrooms are "diverse" —in contrast with the idea of a relatively homogenous student body—, mathematics teachers need to be able to attend to and manage such "diversity" for creating possibilities "for all" to engage in mathematical learning and thus produce expected results. Such possibility of access to learning is seen as a matter of personal and social importance due to the democratic and economic repercussions of not achieving expected levels of competence in mathematics. These desires are often expressed in national educational policies, in the curricular documents in mathematics and in the regulations for (mathematics) teacher education. These are also some of the concerns and arguments that the research concerned with social justice in mathematics education have voiced (Yolcu, 2019).

For the concrete practices of mathematics teacher education, the recognition of "diversity" poses the demand that teacher students need to gain qualifications during their initial training to be prepared to manage diversity when they become teachers. But how such demand is in fact realized in teacher education is an open question. As one turns to research, it is possible to say that most studies on equity, inclusion and diversity are conducted in schools where practicing teachers deal with the multiple challenges of different pupils and their mathematical learning, as Yolcu (2019) confirms. The research often points to

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implications for teacher education as part of their conclusions. However, the research on mathematics teacher education itself addressing diversity is scarce.

Previous socio-political research on initial teacher education (e.g., Nolan, 2011; Wager, 2012) almost exclusively takes the perspective of the teacher educators and prioritizes organisational issues of teacher education or the curricular contents. The main assumption is that teacher students need to be prepared for their future profession and are therefore offered courses, which should educate them to be socially aware and just teachers. In other words, the attention towards the effects of teacher education for managing diversity overshadows the fact that diversity may well be already vividly present in the practices of teacher education and among their participants, and not only on its outcomes.

As a way to start turning the gaze towards the practices of initial teacher education as a site for unfolding the meanings of diversity, equity and inclusion, we examine research in the setting of mathematics teacher education programs that explicitly focus on the political predicaments of "diversity" in a broad sense. That is, empirical research that prioritizes political issues as central elements of initial mathematics teacher education. We examine how teacher students are positioned in the research to cast light on the effects of such positionings in building particular notions of who the teacher student is becoming and the possibilities of action regarding diversity in mathematics education. We pursue the questions of how is research positioning the teacher student with respect to "diversity" and how is research configuring what counts as "diversity" in mathematics teacher education.

Researching positioning in research

This paper presents a study that uses a *researching research* (Pais & Valero, 2012) strategy on literature on mathematics teacher education addressing issues of diversity, inclusion and social justice. Researching research as a methodological strategy is not equivalent to a literature review that maps and describes existing results. Instead, it analyses how research itself builds particular ways of conceiving of its objects and thus shapes what is possible (or not) to see and say concerning a given problem. Thus, inspired in Foucaults' (1973) discussions on the emergence of human sciences and on critical studies of education (Popkewitz & Brennan, 1998), this strategy points to the very political nature of research, in this case, how teacher education research itself not only captures and represents the world but actively effects power by shaping its concepts, objects, limits and practical possibilities.

The notion of *positioning* is an important tool in our analysis. Positioning in socio-linguistic and cultural psychology terms refers to a discursive phenomenon through which individuals assign to themselves or others a series of attributes (e.g., Davies & Harré, 1990) through the performance of actions (Potter & Wetherell, 1987). In some mathematics education research, positioning is used as an indicator of how power relations organise discourses and how individuals both take up and get assigned different attributes as a consequence of discourses articulated in the interactions of teaching and learning in

classrooms (e.g., Herbel-Eisenmann et al., 2010). The frequently used way of doing this analysis is focusing on the linguistic characteristics of informants' utterances to interpret them in terms of position and positioning.

In our analysis we take a different approach to identify positionings. Taking a Foucaultian stance on discourse to analyse published research texts invites us to conceptualize the positioning of teacher students in terms of the attributes assigned to them in the statements of truth that emerge in the texts. The overall guiding questions were: What is being said about the teacher students and how are they being portrayed? In other words, what are the texts stating about teacher students and diversity? Drawing on Valero and Knijnik (2015) we distinguish three levels of analysis to move from the texts to the identification of positionings: (1) We focus on the level of the *sentences* in the texts that say something about the teacher students, diversity and their relationship. (2) We identify the meaning of the sentences as they express repetitions and differences on what is said about the teacher students, diversity and the relationship between those two. This leads to the identification of *enunciations*. (3) Finally, we identify the larger stories or *statements of truth* that the enunciations express about the teacher students, diversity and their relationship. It is in these statements where we locate the positionings of the teacher students in the discourse unfolded in the research texts.

The empirical material studied is a set of 34[1] published papers in international research journals in English, in the period 2008–2020. The criteria for the selection of studies were (1) that papers should be situated in the context of mathematics teacher education, and focus especially on issues tightly connected to mathematics teacher education, such as course content or projects, documents (curricula, international reports, etc.), teacher educators' work, teaching and experiences, or student teaching; (2) that they contained an empirical component such as interviews, surveys, observations, texts of various kinds, etc., i.e. theoretical/discussion papers were excluded; and (3) that they had a focus on diversity, equity and inclusion, meaning that either an overall aim of the study concerns diversity, or that specific research questions have this focus.

A general characterization of the material shows that 20 papers were about teacher students' learning and their involvement to some extent. The remaining 14 focused on teacher educators' work and reflections, and analyses of course literature and syllabuses. The papers are written by researchers in different geographical locations and the concerns for diversity include race and socioeconomic differences as a matter of social justice in mathematics teacher education; issues of language and cultural differences among teacher students; and the attention to individual needs and dis(ability), language and ethnicity.

[1] Out of the 34 papers that constitute our data set, we only include 21 in the list of references for the sake of space. These 21 texts best illustrate the findings and are marked with * before them.

Positionings of teacher students and diversity

The analysis of the research texts allowed us to, from within the discourse, distinguish three distinct statements regarding teacher students and diversity. We start by formulating the statement and the series of enunciations that compose each positioning. We then illustrate with fragments from the texts the nuances in the attributes that are associated to each positioning.

The unachieved teacher student

Teacher students have deficiencies related to their capacities to be teachers and to their own perception of diversity. The teacher students need to be developed to acquire new desired capacities —beliefs, moral stances, desires. They have to learn more —knowledge and strategies— to be able to fulfill the task of a mathematics teacher that can handle students' diversity. Knowledge of diversity is insufficient but a precondition for future action. The responsibility of succeeding in this process is on the students' shoulders. Diversity is encapsulated in forms of knowledge, beliefs and moral stances.

Many mathematics teacher students do not have a capacity to deal with diversity. That capacity is related to their knowledge of mathematics and how they conceptualise issues of diversity, equity and social justice (Max, 2017), as "there is a symbiotic relationship between difficulties with mathematical content and teaching for social justice" (Garii & Appova, 2013, p. 206). But also, teacher students' beliefs (Simic-Muller, 2015), reflections (Yow, 2012), self-awareness (Yow, 2012), values and ethical standing to appreciate (Goodson-Espy et al., 2016), recognize and act to change (Guerra & An, 2016).

Teacher students need to gain knowledge about diversity as part of their education to be able to cope with the realities and demands of mathematics classrooms. They have to change "conceptions of teaching mathematics for social justice" (Jong & Jackson, 2016, p. 28) and increase their awareness about equity in teaching. Even though they understand both the mathematics and the social justice content, "their ability to integrate both in a single lesson [can be] both inadequate and flawed" (Garii & Appova, 2013, p. 206). Hence, "it is the responsibility of teacher education programs to address these misconceptions and challenge the deficit beliefs" (Simic-Muller, 2015, p. 33) held by teacher students. There is a need to "know more about how [teacher students] conceptualize their role and ability to create equitable environments for their future students" and to study "what considerations [teacher students] make when conceptualizing equitable environments" (Max, 2017, p. 288).

However, the overall responsibility for change lies on the teacher student, because it is him/her who needs to learn the content of teacher education. Therefore, teacher educators need to provide tasks and "assignments that require PSTs to explore social justice problems involving mathematics in a global context" (Goodson-Espy et al., 2016, p. 792) and "examine how well teacher-candidates are able to incorporate [ideas of diversity and] in what ways do the teacher-candidates attempt to approach mathematics content through [diversity] lenses" (Garii & Appova, 2013, p. 200). Importantly, "teacher educators should prepare PTs to decrease oppressive practices and increase liberative [mathematics practices]" (Yow, 2012,

p. 84), and set goals of moral order so that "PSTs [...] come to appreciate individuals within their own communities as valid practitioners of mathematics; and [...] come to understand the responsibility that they, their school districts, and others bear to ensure that all students have equitable opportunities to learn mathematics" (Goodson-Espy et al., 2016, p. 791).

The recipient teacher student

Teacher students are the recipient of teacher educators' actions to provide exposure to different ideas and notions of diversity. Diversity is a content to be learned, but it is the responsibility of teacher education educators and programs to offer opportunities for teacher students to deal with situations where diversity is important. The preparation is not achieved just through knowledge but also through what teacher education programs allow participants —teacher educators and teacher students— to do. Diversity is conceived as the exposure to different ideas on how to deal with pupils' differences.

Teacher educators and teacher education programs bear the responsibility to offer opportunities for teacher students to deal with issues of diversity in mathematics education (Mintos, 2019; Essien, 2014), inclusive education (Guðjónsdóttir & Óskarsdóttir, 2019) and multilingual contexts (Essien, 2014). Even though teacher students "must develop certain processes to teach mathematics through a lens of equity", teacher educators "might support PSTs to acquire knowledge, scrutinize their beliefs and emotions, and develop interpersonal communication" (Vomvoridi-Ivanovic & McLeman, 2015, p. 86), and be aware of "different views [in the organisation] on how to prepare teacher students for work in inclusive school settings" (Guðjónsdóttir & Óskarsdóttir, 2019, p. 2), which, for instance can happen through rehearsals of instruction and "in-the-moment coaching" (Averill et al., 2016, p. 500). In order for teacher students to develop awareness of equity in teaching, "ample time to discuss authentic issues in diversity [is required]" hence, there is need for teacher educators "to be aware of their own oppressive and liberative teaching practices in doing such work" (Yow, 2012, p. 83).

For this to be achieved, teacher educators need to reflect on "what should happen in teachers' education programs" (Chitera, 2011, p. 236) in order to prepare teacher students for work in diverse contexts. Should, for example, teacher educators "use local languages as they prepare the prospective teachers?" (Chitera, 2011, p. 236). In offering opportunities for teacher students to teach mathematics in multilingual contexts it is of importance how teacher educators pay attention to different "facets of mathematics teacher training [...] and how [these facets] can inform pre-service teacher education" (Essien, 2014, p. 64).

There are "different views [among teacher educators] on how to prepare teacher students for work in inclusive school settings" (Guðjónsdóttir & Óskarsdóttir, 2019, p. 2), and teacher education courses focus on various aspects of diversity, such as disabilities, multilingualism, social justice and on creating learning spaces for all. Hence, if there is no consensus and "teacher educators have different views on how to prepare teacher students to work in inclusive settings" (Guðjónsdóttir & Óskarsdóttir, 2019, p. 11) this can lead to inconsistencies in conceptualisations of inclusion. This, in turn, "can result in confusion and uncertainty

among student teachers. Some might believe that it is not their responsibility to teach all learners and others that there is a special 'toolbox' to use for certain types of learners" (Guðjónsdóttir & Óskarsdóttir, 2019, p. 8).

Teacher educators' awareness of the context of teacher students' is of importance in offering opportunities to integrate urgent social justice issues in mathematics teacher education, for instance through "focusing on how [the educator] could live out [her] values of social justice [concerning HIV & AIDS] more fully in [her] own practice as a Mathematics teacher educator" (van Laren, 2011, p. 338). Teacher educators have, over the years, gained experiences from teaching diverse contexts that inform the structures and contents in the program, such as language diversity. Therefore, educators need to be aware "of the issues that teachers face and about which teachers might need to be sensitized" (Thompson et al., 2016, p. 7); to take responsibility for "address[ing] the language diversity of their students" and to take into consideration "the role of mathematics teacher educators in this preparation" (Thompson et al., 2016, p. 122). By engaging teacher students in "experiences of culturally responsive mathematics teaching that they could take with them into their own teaching" (Averill et al., 2009, p. 162) and by providing opportunities to interact "with students from various backgrounds in the practicum experience, pre-service teachers will also be able to learn how race, socioeconomic class, and gender could affect a youth's decision-making and development" (Wang & Apraiz, 2018, p. 41). Learning through experiences from practicum will "serve the student teacher well when he or she teaches a group of culturally diverse students" (Wang & Apraiz, 2018, p. 41).

The agentic teacher student

Teacher students and teacher educators are equal active participants in constructing and experiencing what diversity may mean to develop a different sensibility towards pupils' differences. Diversity is framed as a sensibility to identify, think about and challenge the many sides and faces of students' and pupils' differences.

Teacher students' previous experiences (Jackson & Jong, 2017) and opinions of how diversity is foregrounded in the program (Çelik, 2018) are central in making sense of diversity in teacher education. Through increasing awareness and collaborative work (Goodson-Espy et al., 2016), and in negotiating dominant educational discourses (Hossain et al., 2013; Farnsworth, 2010) teacher students are agents in constructing and experiencing diversity in mathematics education.

By inviting teacher students to critically reflect on their "own experiences as mathematics learners, the influence of their past mathematics teachers, and their own conceptions about how mathematics should be taught" (Jackson & Jong, 2017, p. 67), they are also invited to challenge and rethink issues of diversity. Teacher students contribute and co-construct meanings of diversity through critical reflections "on issues of equity in the mathematics classroom" (Jackson & Jong, 2017, p. 79). These responses may "not necessarily [be] the desired or ideal" (Jackson & Jong, 2017, p. 71). However, in inviting teacher students to also reflect on "how often the teacher training program provides them with opportunities

for learning how to teach to diverse groups of students" (Çelik, 2018, p. 14), and, in constructing meanings of issues of diversity, the discourses are open for negotiation.

A "challenge for teacher education is to find ways to support pre-service teachers in negotiating these Discourses in ways that engage critical reflection and support a kind of inner dialogue around such contradictions" (Farnsworth, 2010, p. 1485). However, one way can be to listen to what they say. Teacher students contribute with experiences "You don't ask any questions because you are scared that, er, you, you insult the teacher by asking him or her 'why?" (student voice in Hossain et al., 2013, p. 44) and bring to the context of teacher education ways to challenge previous experiences "[…] and what I understand here is that, as a teacher, you need to know all the ways because the, the student might understand one way better. Whereas where I'm coming from, the teacher teaches whichever one he's [sic] comfortable with, and that's it" (student voice in Hossain et al., 2013, p. 44). So, as [teacher students] "are thinking about equitable issues in their emerging practice in terms of access and power" (Max, 2017, p. 293), there is need for teacher education programs to integrate "equity-based teaching strategies in methods courses and allow for opportunities for preservice teachers to think about equity in terms of equality and how they can structure their practices to best support the diverse populations they will inevitably encounter" (Max, 2017, p. 293).

Nuances of positioning that matter

Researching the research in the 34 selected papers allowed us to identify three different statements and related enunciations concerning how the teacher students are positioned in the texts and how notions of diversity is connected to the actual positioning.

The first positioning of the teacher student as *unachieved* does not only point to the notion of lacks in teacher students that need to be filled, but also to the individual responsibility of the teacher student to fill his/her own lacks. In this positioning diversity is also connected to the lack that has to be filled, that is, the main idea navigating in the research is the focus on what "has to be learned". The second positioning of the teacher student as *recipient* has a slightly different turn which acknowledges the responsibility of teacher education —the educators, structures, programs and organisations of the curriculum— to provide opportunities for teacher students to fill their lacks. Notions of diversity do not really differ in these two positionings. What differs is whose actions that are in focus: the lacking teacher student's or the providing teacher educator's. The third positioning of teacher students as *agentic* seems to turn the gaze to the multiple differences among teacher students and teacher educators as a very same site of exploration and source of engagement with diversity. The idea of diversity as a sensibility to notice difference starts from the very same situation and conditions of teacher education. It is not necessarily a lack that needs to be filled for a future practice, but rather a field to explore, truths to negotiate and structures to challenge.

A general observation is that the statements do not coincide with "types of research" that allowed us to categorise the 34 papers in discrete categories. Although some statements

appear strongly in some papers, more than one statement may appear in the same text. This means that there is no reason to characterise some research as intentionally positioning teacher students as *unachieved, recipient* or *agentic*. However, there are few of the 34 studies that involve the teacher student to generate a sensibility where the teacher student is not an object of learning or of a teacher education recipient, but that the teacher student is an agentic player. Instead, what we have seen in this analysis is that the teacher students are kind of set in a "passive" role of active knowledge acquisitors rather than an active position of critical participants, and diversity is a content to be learned.

Diversity is defined and approached in different ways, and this may have implications for how the teacher student is positioned in research. Approaching diversity as a subject to learn or as sensibility to notice difference hence opens for discussions about future teachers' autonomy and awareness of the multifaceted nature of teaching and learning of mathematics. We can see that there is a risk that teacher students are expected to learn to know how to teach in diverse classrooms. But they are not taken as agents that could promote a sensibility to possible changes and see multiple differences in students that may be present in a situation. They are not invited to think critically, and to see the diversity that they themselves are a part of. There is also a risk that teacher students stay in a place of "secure knowledge" while diversity is multifaceted, like a diamond and its multiple refractions that cast light, rather than like a simple cube.

If we consider that these small nuances matter, we as researchers could consider conducting research that looks critically at more than the teacher students' (lack) of (acquisition) of knowledge, but instead at the process of seeing them as capable to develop new changing sensibilities. Positioning the teacher student as agentic requires research where the teacher students are invited into a multifaceted discourse, practice and experience of diversity to be sensible to the nuances. It can be that this indeed happens in many instances of teacher education, but the research has so far not explored this type of position. Research seems to be more interested in attending to the political demands of increasing knowledge in mathematics teacher education, while other very relevant dimensions of diversity may be underprioritized.

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Researching representation of diversity in mathematics pedagogical texts: Methodological considerations

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National exams in mathematics are important pedagogical texts which have been published annually for decades. They provide an interesting corpus for a project investigating the development of representation of diversity in mathematics tasks. Also, there are many reasons for including several diversity dimensions when researching diversity, e.g., gender, ethnicity, religion, functionality, sexuality, and class. Based on previous research on diversity in pedagogical texts, I discuss some of the methodological considerations that must be made when designing such a project.

Introduction

The aim of this article is to discuss some methodological considerations needed when investigating the development of representation of diversity in pedagogical texts over time. As these methodological questions are often strongly dependent on context, I will discuss one example: mathematics exams in Norway. While many details will be different in other contexts, many of the main ideas may still be useful.

Mathematics is traditionally a subject preoccupied with tasks (Walls, 2005). National exams have a special role: their tasks are published by the government, and are studied by teachers as signals of what should be prioritized (Burkhardt & Schoenfeld, 2018). Earlier exams are often used to prepare new groups of students for exams (Andresen et al., 2017). Rubel and McCloskey (2021) argues that exams most likely also influence teachers in how they include contextualization.

Exams are promising sources for studying change over time, as they are annual events with a long history. The national exam at the end of compulsory school in Norway has been arranged every year since 1962. In this period, Norwegian society (as many others) has been through profound changes in terms of diversity, with more gender equality, more ethnic diversity, and less discrimination of the LGBT population, to mention just three examples. The role of diversity in exams has probably also changed. *How* diversity has changed over time, however, is a matter for research.

I acknowledge my position as a CIS-gendered, male, white, urban professor of mathematics education to be a position of power, which leads to blind spots when

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researching power issues that never affected me personally. My identity as gay and atheist have given some personal experiences of oppression. In my research, an interest in LGBT issues has gradually evolved into a more general interest in diversity. Analyses of data should preferably be performed by a group with diverse perspectives.

There are many possible approaches in mathematics education to combat oppression and injustice, including e.g., using mathematics to investigate injustice, or involving the local community in making mathematics relevant. This article is based on the premise that representation is important and that being made invisible is harmful. Representation is certainly not a panacea for social justice, but one of a range of factors that need to be researched.

Theoretical background

The importance of representation

Rubel and McCloskey (2021) argues that there are four rationales for contextualization of mathematics: 1) support the learning of mathematics, 2) motivate students, 3) support the teaching of functional literacy (both for private and for professional life), 4) support the teaching of critical literacy. Seeing oneself depicted may give a feeling of being included and accepted as opposed to excluded and rejected. Problems that feel relevant to one's own situation may lead to experiencing mathematics as relevant. Moreover, seeing oneself depicted in different contexts may serve to expand or limit one's foreground (one's image of one's own opportunities) (Rubel & McCloskey, 2021). In addition to this function as a 'mirror', diversity in texts also functions as 'curricular windows' (Luecke, 2011), letting children get to know minorities that are not physically present in the particular classroom or school.

Diversity dimensions

Many diversity dimensions could be discussed. I have chosen to include gender, ethnicity, culture, religion, geography, functionality, sexuality, and class. Age is excluded for reasons of space. I want to look at how researchers have studied the dimensions in exam tasks in mathematics when it comes to representation. Many other aspects of mathematics' relation to the different diversity dimensions could be examined, for instance different groups' relation to the subject and reasons for this. Such aspects are outside the scope of this article.

There are many reasons to investigate several such dimensions at the same time. Firstly, intersectionality research illuminates the danger of looking at diversity dimensions one at a time (Bowleg, 2008). The experiences of black women are not simply a product of the experiences of blacks and the experiences of women. Secondly, the different dimensions may well be treated differently in the exam tasks, and I would like to compare how they change over time. Thirdly, it is possible that some dimensions are rarely or not at all visible in the exam tasks, which makes it difficult to base a project on such dimensions alone. If diversity

dimensions are treated one by one, there is a clear danger that some dimensions will never be researched.

Bowleg (2008) notes that when doing intersectionality research, researchers must commit to a transdisciplinary approach, and they must examine "any contradictions or tensions relevant to these intersections" (p. 318).

I have argued that to provide an overview, we should include many dimensions and study long-term development. As some dimensions may rarely occur, we need to analyse a large sample of the exam tasks. Therefore, the analytical tool needs to be rather simple and with little nuance, for instance by just counting the numbers of occurrences of certain 'diversity markers'. This leads us to the topic of 'tokenism'.

Tokenism

The exclusion of a group from a text can be remedied by turning some people into members of that group. This can be done by changing the names or appearances of people in the text, or the information given about them. Such an approach will be criticized as *tokenism*, that is superficial inclusion without changing the underlying structures. While famously used in cases of women with non-traditional jobs (Kanter, 1977), tokenism is also used of people included in texts to improve the appearance of diversity. If a text is written from a white, middle class, straight perspective, it does not magically include more perspectives even if some names or skin colours are changed. Groups' everyday lives and their experiences need to influence the texts.

Stereotypes and normativities

Consistently placing people from a culture in contexts particular to that culture, will rightly be criticised for preserving stereotypes. This is the opposite of tokenism, the inclusion of people from a culture without changing the context. In some cases, tasks can even uphold pejorative stereotypes (Rubel & McCloskey, 2021).

Normativities, for instance heteronormativity, is relevant in two directions. On the one hand, a lack of representation contributes to upholding the normativities and the oppression they contribute to. On the other hand, such normativities mean that characters not described as belonging to a minority, are automatically seen as being part of the majority. In addition to analysing how many occurrences there are of the minorities and the majorities, the many characters that do not have a description are also relevant, as these may unconsciously be seen as part of the majority.

Luminescent or invisible

Ohnstad (1992) coined the concept pair *luminescent or invisible* to describe the dilemma a lesbian woman faces when deciding whether to come out: when coming out, the sexual orientation is treated as the most visible aspect of the person's identity, which feels unsafe, but when not coming out, that aspect of the identity stays invisible. I consider this concept pair useful for the discussion of diversity in exams (see below).

Different diversity dimensions

Despite my commitment to intersectionality, in this part, I will look at my choice of diversity dimensions one by one and provide examples of how each has been researched, as relevant articles where intersectionality is invoked, are rare. Even Macintyre and Hamilton (2010), analysing textbooks for representations of gender, ethnicity, disability and sexual orientation, are unable to analyse them together, as disability and sexual orientation were invisible in the data.

Norwegian research will be included when available. Where I found little research on exam tasks, I refer to research on textbooks as well, as both are pedagogical texts, albeit in different settings. As I look at many dimensions, each discussion must be concise.

Gender

Boaler (1997) claimed that earlier, "mathematics textbooks dealt only with male experiences" (p. 287). Among recent studies are Norén and Boistrup (2016), who counted boys and girls in Swedish mathematics textbooks for 8th and 9th grade, and analysed their roles. There are more boys, they are more active and more often have jobs, while girls to a larger degree are caregivers in the home. Using somewhat similar methods, other studies give a similar picture in different countries: Palestine (Karama, 2020), Brazil (Pereira, 2019), Greece (Chassapis, 2010), Britain (Macintyre & Hamilton, 2010) and the US (Piatek-Jimenez et al., 2014), finding that males are more mathematical and have more – and different – careers.

Ethnicity

Through the last 50 years, Norway became a more multicultural and multireligious society. International research on race/ethnicity is therefore increasingly relevant also for Norway. Analysing the ethnicity and activity of persons in tasks, Piatek-Jimenez et al. (2014) found that in US textbooks, "whites are portrayed as being more mathematical and more active and are shown in more careers than minorities" (p. 55), while Pereira (2019), in a study of Brazilian exams, found that nearly all characters could be socially perceived as white. In Norway, Flottorp and Poorgholam (2003) analysed mathematics textbooks from a multicultural perspective, and showed that Asian or African names and looks were underrepresented.

Culture

Flottorp and Poorgholam (2003) searched for ways in which Norwegian textbooks mirrored a multicultural reality. They found that tasks connected to buying food often fit with a traditional Norwegian cuisine, with little oil, rice, garlic, or olives. Holiday destinations were often the cabin or Mallorca. When places in other parts of the world were mentioned, it was often economically important places. Currencies mentioned were European currencies or US dollars. A Norwegian way of living, with a nuclear family in a house, was often portrayed. Mathematicians mentioned were often Western.

Fyhn and Nystad (2013) took another approach. They analysed how the 2009 Norwegian mathematics exams (of the same series of exams mentioned above) reflected four key Sami values. One of the values included were Sami traditional knowledge, in which knowledge is seen as a process, not a product. Therefore, tasks with process focus would be considered in harmony with Sami values regardless of the tasks' contexts. They concluded that the exam reflected Sami values to a certain degree, but that more emphasis on modelling would improve the fit with Sami values.

Religion

In Flottorp and Poorgholam (2003), little religion was found in textbooks, and when included, it was often Christianity. Few other studies have investigated the religious content in mathematics textbooks or exams. Yilmaz and Ozyigit (2017) studied one Turkish high school mathematics textbook from each of three eras, and found that religious contexts varied over time, with no instances found in the newest textbooks.

Geography – "urban bias"

Textbooks in many subjects have been criticized for an *urban bias*, reflecting authors' urban context, ignoring the rural way of life (Nawani, 2010). Fyhn and Nystad (2013) mentions that in the 1990s, Norwegian exams were criticised for the same. For instance, tasks on tram timetables may be of little interests to students with no access to trams. An urban bias would most likely lead to fewer tasks on rural activities such as farming and fishing, and more tasks on urban activities.

Functionality

In previous decades, attitudes towards functionality have changed, and visibility in society has increased. Through the Salamanca statement (UNESCO, 1994), disabled people's right to be included in school have been stressed. It does not seem, though, that this visibility has reached textbooks to the same degree. Macintyre and Hamilton (2010) looked for representation of disability in Scottish mathematics textbooks, but found nothing, neither in text nor images. Hardin and Hardin (2004) found that US Physical Education textbooks rarely included photographs of persons with disabilities. Hodkinson (2012) studied resources for primary-aged students in England. Very few included any mention of disabilities. Both Hardin and Hardin and Hodkinson combined a count of photographs or mentions with a more detailed study of the instances where disabled people were visible. In both studies, portrayals were stereotypical.

Sexuality

Family structures have also changed throughout the period. For instance, in Norway, gay sex was forbidden until 1972, while same-sex marriage was introduced in 2009. Dubbs (2016) notes two ways queer perspectives could be included in mathematics education: by including queers or by using the mathematics to investigate inequalities. Macintyre and Hamilton (2010) found no representation of sexual orientation in Scottish mathematics textbooks.

Smestad (2018) showed that while lack of inclusion was the normal finding in other countries, in Norway, LGB issues are more often included in textbooks in some subjects (not including mathematics). In some of the research finding a lack of inclusion, researchers go on to analyse in which parts of the textbooks it would have been reasonable to include queer perspectives.

Class

The last diversity dimension I will include, is class. The research I have found is not concerned with the representation of people from different classes as much as with how the tasks' contexts are relevant to the students. Macintyre and Hamilton (2010) point out that different social groups have different expectations and experiences to draw on. Harper (n.d.) gives a convincing example of an assessment task that "unfairly relies on assumptions that middle-class students would make", such as people normally working five days a week. Flottorp and Poorgholam (2003) found tasks asking students to calculate the area of their own room, thus assuming they all had one.

Chisholm (2018) calls for more historical – and comparative – studies of representation in textbooks, while Niehaus (2018) recommends a mixed-method approach, to "first lay out an overview of how certain diversity-related categories are represented before going into more depth through a text-based content analysis" (p. 334).

Discussion

It does seem that in most of the diversity dimensions, researchers have been counting representation as part of their analysis. These counts in themselves have given rise to interesting conclusions – not least in pointing out how certain groups are excluded. However, researchers tend to want to go beyond this, to analyse the roles and contexts in more detail. In this way, research has not only shown that girls are underrepresented in Swedish textbooks, but also that they are given a less active role. Research has also shown the portrayal of the disabled to be stereotypical. An analysis where inclusion is identified, can therefore both give an important count and a material to discuss in more detail. In Table 1, I provide a summary of ideas/examples of what to look for.

Höhne and Heerdegen (2018) point out that it is common to use a binary view of gender in such research. Even if we use a more inclusive gender concept, there are problems with assigning gender to persons based on names and appearances. However, the point is not to assign gender, but to analyse whether the attributes persons in such tasks are given, contributes to the perpetuation of gender norms. Thus, if persons with traditional male names are overrepresented in tasks, that is still an interesting finding.

Similarly, when counting 'ethnicities', we do not accept that there are clear boundaries between ethnicities, but rather analyse how different ethnicity markers are represented. Such markers could be whether the names are traditional Norwegian or not, or phenotype (perhaps skin colour), as in Pereira (2019).

Dimension	Count	Further analyses
Gender	Males/females/other (based on names, pronouns, and appearances)	What they do and how they are positioned
Ethnicity	Ethnicities (based on names and appearances)	What they do and how they are positioned
Culture	Places (European vs. non-European). Food. Currencies.	Looking for examples of "immigrant culture"
Religion	Religions	How they are portrayed
Geography	Place names and activities (rural/urban)	
Functionality	People with apparent disability vs. people with apparent non-disability.	How the two groups are portrayed.
Sexuality	Contexts having to do with families (traditional vs non-traditional). Mentions of minority sexualities.	
Class	(Occupations, large amounts of money)	Class-specific assumptions

Table 1: Suggested points to look for.

Relevant markers when it comes to class could include which jobs people have, the amounts of money they handle, or class-specific occupations or hobbies. Operationalisation would be difficult, however, as the markers not only varies across (and probably within) countries, but also over time. At which point in time did buying a bicycle turn from being a luxury to becoming normal in Norway, for instance? The research cited above suggests that in the case of class, analyses of middle-class assumptions are more fruitful than a counting of representations.

I have argued that counting representation provides both valuable quantifications and material for further analysis. Some aspects complicate this. As pointed out in research on queer representation, it is also of interest to identify contexts in which queer people could naturally be included, but where they (we) are not. In Fyhn and Nystad (2013)'s work on Sami values in exams, the issue is not to look at representation, as tasks that are in harmony with Sami values do not have to include Sami representation. If we just identify instances of representation and then analyse these, key perspectives are lost.

In addition, the dimensions are different. Some diversity markers are visible, others are not. Probably, more persons in tasks will have an explicit gender or ethnicity, either in texts or images, than a sexuality or religion. If there is a tradition of not being explicit about, say, people's sexuality, functionality or religion, the first occurrence would probably stand out and seem distracting – even luminescent (Ohnstad, 1992). By giving an overview over a long period of time, we will shed light on such traditions.

Another issue is fragmentation; We want to avoid ending up with a series of results only concerning one diversity dimension at a time. Every person's identity includes several

dimensions, and simple contexts, such as having your own room, touches upon several dimensions at once: it may be connected to class, the urban/rural divide, and culture. As pointed out by Bowleg (2008), an important phase of an intersectional analysis is to examine "any contradictions or tensions relevant to these intersections" (p. 318). In the context of exam tasks, this would entail studying the diversity contexts together. Regarding the counts, pivot tables should be made and studied. If representation of different ethnicities and functionalities is present, we can easily check whether disabled people tend to be portrayed as white.

More controversially, we may ask whether all dimensions are equally important, considering other societal developments. I will argue that a major equity issue in Norwegian schools is that working class immigrants are disadvantaged in mathematics, in terms of economic factors (e.g., no PC at home), lower quality education, and language demands of the tasks (Bjørnset et al., 2020). Including more tasks set in rural settings may increase the disadvantage of an already disadvantaged group. While diversity in general benefits all, each effort does not necessarily benefit all equitably.

Counting occurrences of different diversity markers helps study inclusion on one level. There is a risk that the diversity markers are just tokenism. There is also the thorny issue of tokenism and stereotypes. If including a person of Chinese appearance and name, should they go skiing (would not that be tokenism?) or doing something central to Chinese culture (would not that be perpetuating stereotypes?) Also, just including farm contexts do not assure that a rural child will feel included, as not all rural children have an interest in farms. For real diversity to occur, there must be diversity in the portrayal of Chinese, and of other identities. No such diversity is possible if the number of occurrences is low. Thus, counting still gives important information.

Concluding suggestion

I have considered some methodological considerations needed when investigating the development of the representation of diversity in mathematics exams over time. Although some operationalisations needed are difficult, it seems feasible and fruitful to analyse the whole corpus of mathematics exams (in Norway's case: 60 years of exams) based on counting occurrences of inclusion (Table 1). This could provide an overview of the development of the exams in terms of diversity.

Such an overview would not cover all important perspectives. However, it would be a foundation for further, more detailed analyses, of selected (combinations of) diversity dimensions and time periods. Cross-country quantitative analyses are feasible, to allow comparisons and then the further cooperation on analyses of interesting differences or similarities. This would take careful work on the operationalisations, however, as differences between classes, for instance, are not the same for all countries. Such an analysis would be in line with recommendations from Chisholm (2018) and Niehaus (2018) and provide new opportunities for insights.

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Taking a visibility perspective on gendering in secondary school mathematics

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The theoretical perspective of visibility has been used in organisation studies to understand how participation can be constructed differently for minority and majority groups. This paper combines liberal and poststructural approaches to understanding who is made visible, and how, in mathematics practices where choice aligns with gender. I re-analyse existing data from case studies of English schools with high mathematics participation for girls aged 16-18. Within these accounts of girls' experiences, prevailing discourses are those that render boys' struggles and successes highly visible. Quiet effort serves as an invisible background that is ascribed to girls and to maturity. In contrast, the hypervisible girl achiever is understood through a single aspect of identity – mathematical success without struggle or support.

Introduction

With recent constraints on classroom access and data collection, it has been instructive to focus on existing data from a different theoretical perspective, allowing for the possibility of forming new conjectures with a different explanatory empirical reach (Inglis & Foster, 2018). This paper focuses on a re-analysis of some data from a series of case studies (Smith & Golding, 2018) that examined accounts of practice in five English schools or colleges whose participation rates showed that they were successful in recruiting girls into pre-university mathematics. This is in the context of an education system where students aged 16 choose a specialist education pathway and, if they have already achieved a threshold performance, may study no mathematics. In 2019, 54% of girls studied no named mathematics course aged 16-18. Among the two-thirds of students choosing a pre-university 'A level' course, 20% of girls chose mathematics as one of their subjects compared to 37% of boys.

The original study and thematic analysis were designed to focus on the characteristics of the selected schools in terms of identifying any (1) intentional strategies addressing girls' participation or (2) aspects of mathematics pedagogy, careers or teacher guidance that support girls' participation, and how these were conceived, operationalised and evaluated by teachers and girls? The re-analysis focuses on a third research question:

What messages are current in the school culture about who does mathematics?

There is an argument that girls' participation in mathematics is limited by the absence of role models and messages – that if you cannot see it you cannot be it – but much research into subject choice suggests that simply seeing and hearing a message is not enough.

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Students themselves are critical receivers of media tropes such as 'Girls can do maths too' (Mendick et al., 2007); while exhortations can remain as imagined futures if there is no family "science capital" that makes pathways compatible with lived aspirations (Archer et al., 2014). My previous research (Smith, 2010, 2020) has focused on how individuals are positioned within educational discourses as construing themselves, in ways that are socially and institutionally intelligible, as adolescents who express themselves through their choice (or not) of mathematics. These complexities of messaging and identity performance have also been recognised in theoretical work in gender and organisation studies, notably within Lewis and Simpson' (2010) work on surface and deep levels of analysis within studies of workplace voice and visibility. This paper discusses bringing these theoretical perspectives from organisation studies to the original data.

Visibility; invisibility and hypervisibility

A theoretical perspective that studies voice and visibility is not new. It starts from an assumption that differences should be valued and asks 'Where are the voices or the people who embody those differences and why they are missing?' It points out the damage to institutions of missing those voices and also to individuals. People's workplace credibility and self-definition rely on being perceived in a way that they desire, so organisational studies should examine for whom and why this is not happening (Buchanan & Settles, 2019). Lewis and Simpson (2010) characterise this strand of visibility research as aiming for reformation rather than transformation of existing structures. It is based in a liberal feminist worldview (that is, one conceptualising equality as freedom for all) and hence it seeks to fill gaps caused by the absence of important voices and to reconfigure the reception of those voices to create more neutral grounds for participation. So far, so good and I will return to insights from such studies. However, Lewis and Simpson oppose this "surface-level" strand to a "deeper" theorisation that examines the silences in organisations and what they achieve. This strand is based instead on a poststructural worldview that identity – including gender identity - is performed and regulated in discourse (Butler, 2004; Rose, 1998). In such research, the invisibilities of women in undergraduate mathematics (Rodd & Bartholomew, 2006) or of Black girls in STEM-promotional materials (Brink & Stobbe, 2009) are understood as ongoing processes of rendering invisible or of diverting the gaze, rather than as omissions. The silences we should examine are not only those that are imposed on, or chosen by, minority group members but also the silences that are taken-for-granted. These constitute the norm and they position some of us – often White, men, straight and older – as dominant, both through our invisibility within a background culture and also as spectators of those who differ from that constructed norm.

The work done by Simpson and Lewis in recognising these two theoretical strands is important, but their characterisation as 'surface' or 'deep' misses the pragmatic nature of much of education research: although we owe it to the future to transform our education systems, we also have a reforming goal to make a difference to the students who are studying

now. Education research could use poststructural theoretical insights that question the processes of rendering visible or invisible to suggest changes within current institutional practices, or at least to understand better the ways that these structure participation for a variety of participants with multiple identities.

One important concept that emerges from the more liberal strand of visibility research is that of hypervisibility, describing a process where a person's multiple allegiances and positions are projected as a single identity. In this form, the condition for becoming visible as a worker is to become an embodiment of difference, requiring their behaviour to represent the category. This can present as tokenism for those in minority groups and exposes people to continued surveillance for belonging (Buchanan & Settles, 2019). Hypervisibility is also identified as a strategy for defining the norm. Organisation studies within university mathematics departments identify the hypervisible normative scientist as highly-focused and performative (Brink & Stobbe, 2009; Jaremus et al., 2020).

A useful insight from poststructuralism is that visibility is re-constructed at many levels, ranging from cultural and institutional practices to peer interactions and self-talk: "social structures are the medium and the outcome of the actions they recursively organise" (Brink & Stobbe, 2009, p. 453). Discourses circulate power, and 'coming to know' who you are and what is expected of you is not only repressive but also productive. This productive perspective gives meaning to repeated findings that people may act to render *themselve*s invisible or hypervisible as a way of erecting barriers, reducing discomfort, inviting or defending against scrutiny (Rabelo & Mahalingam, 2019; Smith, 2010). Although visibility processes are available to all, the conditions of using them are asymmetric, in particular in relation to which identities or practices can be enacted 'in the background', without attention. There are risks for minority groups in both exposure and disappearance (Lewis & Simpson, 2010).

Although it originated in employment studies, this discursive theoretical framing of visibility resonates with the complex and contested performances of identity in 14-18 classrooms. These are helpfully understood as ways that power circulates through framing young people's knowledge of themselves and others, inscribing who can act in what way and in what conditions of visibility. While overall discourses tend towards reproducing dominant practices, moments of alterity exist and discourses do change, both locally and globally.

The 'surface' and 'deep' approaches also bring different methodological priorities. Visibility is often approached through collecting personal accounts. Primacy is given to experience: experiences such as your voice, your body or your work being unseen or neglected. Focusing on commonalities between accounts may reveal or give substance to a theme that has previously been ignored - such as the experiences or knowledge of a minority group. On the other hand, a poststructural view of discourse argues that this is not enough: experiences are given meaning by discourse and dominant discourses function despite and even maybe because of the ways that some constructions of knowledge are ignored or sidelined. Focusing

on what is the same about minority group accounts will re-enact this spotlight on taken-for-granted knowledge. Instead, purposefully studying *within* group variation may point to factors that trouble dominant narratives (Leyra, 2017; Gholson, 2016). In this context, exploring the multiple different ways in which one can be a girl choosing mathematics can show the multiple points of tension or alignment that lead away from choosing mathematics.

The study

The data from the original study was collected in four 11–18 schools and one 16–18 college. The focus was on the proportion of girls studying any A Levels who chose mathematics as one of their 3–4 subjects, as evidenced by national school data published between 2012 and 2016. It is known that girls' mathematics participation is higher in schools that are independent, academically selective or single-sex (Vidal Rodeiro, 2007) so after ranking the data, those sites were excluded, as well as small schools. We then purposefully sought a mixture of school types and sociogeographic settings. Six sites were approached and five agreed, selected from the eighth to tenth deciles for participation over the previous 2 years.

The original study was carried out by two researchers who collected the following data in each site:

- Participation data by gender and examination grades for 3 years, permitting analysis of the attainment profile of the girls who chose mathematics.
- Focus group interview of 3–5 mathematics teachers to explore their strategies for retaining girls in mathematics
- Focus groups of girls (7 in total; with 18 girls before choosing A level, and 29 who had chosen mathematics)
- 1–2 observed mathematics lessons: field notes focused on local factors proposed by teachers as promoting participation.
- Interview with lead teacher in subsequent year

The current reanalysis of this data by one of the researchers focuses on 23 texts consisting of field notes (4), focus groups (13) and interviews (6). Focus groups had been chosen as a data collection method to allow negotiation and collective agreement on accounts of strategies and experiences. The focus group schedule included prompts such as 'Does anyone think differently?' 'How do others react to that?' with the aim of soliciting diverse ways of accounting for individual choices (Mendick et al., 2007).

One purposeful decision was to focus the enquiry on girls' choices and experiences as of interest per se and not in comparison to boys'. When selecting the sample, for instance, the chosen metric highlighted participation among girls, not the relative composition of teaching groups. There were no focus groups for boys and the questions asked only about effects on girls. Two exceptions were asking the teachers to describe any school initiatives for boys that challenged traditional patterns of subject choice (after asking the same for girls) and asking the students explicitly if there were "any times in this school when people have implied that mathematics […] is more of a boys' subject than a girls' subject?" At the time of

data collection (2015) neither the researchers nor the participants challenged that binary framing of gender, which I would now rephrase to remove the direct comparison. As the subsequent analysis indicates, binaries are powerful discursive organisers of meaning in classrooms.

The data was analysed by coding sections of transcribed text in repeated passes. The thematic coding from the previous study (based on strategies, pedagogies and effects) was removed, although as one of the original researchers I am necessarily informed by meanings elucidated through that initial process. The analysis is both literal and interpretive (Mason, 2017) in that I use lexicogrammatic elements of the text in order to construct a version of what I take the data to signify. Throughout, the aim is to consider what positions and knowledges are made intelligible in the discourse in this setting.

The analytic framework was conceived around three dimensions, each of which was filled-in with detailed subcodes. The first of these is visibility, based on the analytic question 'Who are the subjects, and how are they positioned?' Coding was based on a combination of thematic prominence in the text and words such as 'notice' or 'stand out' that describe public attention. Subcodes included visible teacher enjoyment, boys' success and struggle, choice-making, hypervisible achievers. Initially there was no code for invisibility: in part because what is not mentioned cannot readily be coded and also because the nature of the interview questions made girls' participation a focus. The second dimension was the boy-girl gender binary. Text which explicitly included this binary was subcoded as to how this was ascribed relevance by the speaker: either 'an effect', 'no effect', 'a balanced effect' or 'traditional v fact' (for extracts where a stereotypical effect was simultaneously voiced and challenged). The third dimension was differences constructed in language. A systematic search was used to identify comparative words (e.g., 'but', 'more than') as well as juxtapositions of phrases that imply comparison through near repetition (e.g., "even if you don't like it [maths], you need it"). These instances were then categorised as subcodes such as 'like/ need', 'alone/supported', 'White British/Asian'. The whole data set was then recoded for any instance related to any part of these emergent themes, whether including a comparison word or not. That allowed categorisation into differences that appeared predominantly as binaries and those that require multiple possibilities. There were two of the latter: school subjects and home-culture. Although mathematics was the subject of interest in the interview, a range of subjects were drawn on for comparison. Home culture is considered by both girls and teachers to be relevant for participation; the speaker's own ethnicity and White British ethnicity were used as reference points for comparison but not exclusive ones.

Findings

Three aspects are reported in this paper: who and what are rendered visible, invisible or hypervisible in accounts of mathematics experience. These have been chosen for interest rather than as representative of the whole data set. They are findings that were highlighted particularly by the methodological approach.

Visibility

One notable subcode that emerged from the analysis was the visibility of boys' success and struggle in mathematics. This was coded in 13 of the 23 texts and across sites. As said above, boys' participation was deliberately made less visible in the questions and so its presence in the text shows how readily the structure of school discourse allows discussion to turn to comparison. One aspect of this was that boys were described as actively – and publicly - engaged in deploying for their own learning the classroom resources such as time, attention and ladders created by attainment setting. The extracts below illustrate this aspect of visibility, with T/S indicating teacher or student speakers and sites indicated as A–E.

TB: For us, the issue is how do we motivate boys, because the girls achieve more, so the squeaky wheel gets the grease – we're forever trying to think of ways to support boys more, to make it more equitable.

SC: I think it's more evident in maths. You like notice them more. I think the girls are more comfortable in their own sets. Like I think the boys want to go higher – like get higher in the set.

SA: It's only like a couple more. So it's not…obvious that there's more. But its obvious than they're more like…they talk more in the lesson.

This last comment concluded a focus group discussion about whether there were actually more boys in the top set (of 15 girls and 16 boys) or there simply appeared to be more. Together these comments suggest that mathematics lessons in these sites would be recognisable to many teachers of adolescents as largely led by vocalised contributions from boys ('squeaky wheels') with quieter, but still effective, participation by girls. Note that in school B the teacher describes them as achieving more. This prevalence of public talk by young men is identified through undergraduate mathematics (Ernest et al., 2019) as a factor in hiding girls' competence.

This visibility works to reinscribe men and boys as default mathematicians. Even in situations such as these schools where this norm was understood and challenged as an outdated stereotype, it touches on lived experience for boys and girls:

TD: My boys are really embarrassed about being in set 7: they just don't want to write it on their books, they try to distance themselves from it.

SC: You could argue that those…that's because there's a lot of pressure on boys to be all macho, so they… [others: Yeah, Yeah] …they express how well they've done. [Yeah] So the stereotype is even towards the boys, and that could be affecting which girls do it. Because they don't want to be… [the only girl] … boastful like the boys.

The second extract above introduces another aspect of the visibility of boys. The speaker's gap is filled in by another student with "the only girl". Completing sentences suggests a shared understanding and here that is the construction that boys' participation in mathematics is as part of the group while girls would participate alone.

Invisibility

Although there was no initial coding for invisibility, it is interesting to examine a few instances in which omissions were significant or contrasts were overturned. The first of these concludes a teacher discussion of maturity, and suggests that, by the age of 16, girls' achievement and conscientiousness are what is required in mathematics:

> TB: (Girls are) more used to working, generally, unless you've got really conscientious boys in the class, so the girls set the standards and expectations.

Here girls are taken to represent the invisible norm while similarly diligent boys would stand out as visible "really conscientious" participants.

There was just one response in which boys became invisible. In the school context, it is widely recognised that girls are more likely to study Psychology and not Physics. Implicitly, then, the references to 'they' and 'students' in the following refer to girls, rendering their circumstances universal:

> TA: I said, oh, we're starting mechanics next week. And some of them just go, ohh – because they automatically just have that perception of it's Physics. And I'm like, no; actually you will enjoy it. And they do when they get into it. And most students actually really enjoy mechanics. But I think with statistics a lot of our students do things that link ... Psychology.

The final example (Figure 1) challenges the idea that A level classrooms are always more feminised or inclusive spaces. It is an extract from field notes on a lesson on statistics in which a man teacher sought to remind students of the word 'consistent' in relation to narrowly spread data, asking twice 'What is the C-word?'. 'The C-word' is a common euphemism for a swearword referring to female genitalia.

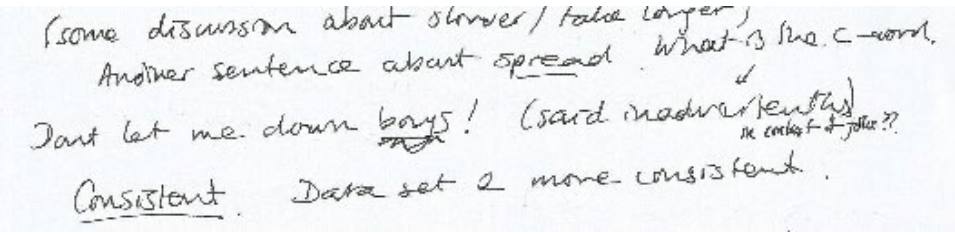

Figure 1. Extract from field notes

The point here is not that a teacher made a poor or unlucky choice of words, but that the way that he could then restore the pedagogic sense of the interaction was to call on "boys" to engage with him both as a teacher and personally ("don't let me down"). The 7 girls in the room looked at their desks until the teacher, with apparent relief, eventually received the answer he needed. The girls were rendered invisible by the intrusion of a wider sexualised discourse that excluded them – temporarily but very effectively – from hearing or replying to the teacher's intended pedagogy.

Hypervisibility

One of the positions that was available in mathematics classroom discourse was that of the hypervisible girl achiever. In two sites, both teachers and students referred to particular girl students who were good at mathematics, suggesting that others would recognise the person. This was coded as hypervisibility when that position focused on a single aspect of identity, and in both cases this aspect was achievement with effort but not struggle:

SC: She's ridiculously clever and she's just getting As in all of the tests. And she doesn't even need to worry about anything because she's so good at it. And she doesn't find that workload too much, whereas I think I would do.

SD: In maths there's sometimes a brick wall you have to climb over – unless you're Michelle of course! (laughs)

Michelle: I don't always find it easy – and anyway [teacher] says I'll find it harder when I get to university, but that's part of the fun of it.

As seen in the gender research, the hypervisible member of a minority group represents the possibility of participation but allows for a very narrow drawing of lines as to what counts as a girl mathematician. Both students who introduce the hypervisible girl achiever suggest that this identify is uncomplicated ("just", "all", "doesn't even need to worry", "unless you are… of course!") and distance themselves from that. Those in the position may seek to diffuse this. In her quote above, Michelle (a pseudonym) uses three qualifiers (not always, anyway, part of) and calls on their teacher as a liked authority and to future unknowns to complicate the way she has been presented. Those who are not hypervisible are also positioned in the discourse, but as spectators aware of how participation can be threatened and the work that is needed to maintain it alongside friendships and other relations. Thus in one of these schools a woman teacher suggests that "Girls I think often feel the need to be really, really good at [mathematics]" and the student focus group agrees that "Unfortunately most girls I'd say would probably just… they want to escape it rather than fight it…the stereotypes."

Discussion

The intention of this paper has been to highlight some findings from a process of analysing student and teacher accounts through the perspective of visibility. Perhaps the most significant finding is the confirmation that, even in sites that demonstrably promote girls' participation, and where the data collection focuses on this effort, the prevailing discourses are those that render boys' struggles and successes in mathematics classrooms highly visible. Students and teachers give prominence to meanings that honour individual choice and reject explicit gender stereotypes. Nevertheless, their discourse shows that institutional practices tend to defend the status quo. Boys' social visibility in the mathematics classroom surfaces in the data as their use of airtime, the valuing of their efforts to compete and their own problematising of their presence in lower sets. This naturalises a sense of belonging in mathematics. As other research has recognised, "the process of doing mathematics is

constructed as masculine" (Mendick et al., 2007, p. 22). What these findings add is that this construction remains dominant in settings which do in fact have high girls' participation.

It is worth returning to Lewis and Simpson's (2010) notions of surface and deep strands of theoretical thinking. The surface approach asks 'Where are girls voices and why are they missing?' The findings from mixed settings suggest that girls' voices are present but they are largely inscribing mathematics as a context for performances of masculinity in which girls are observers and/or out-of-the-spotlight actors. The deep approach asks 'What is achieved by this silence or invisibility?' The invisibility traced in these findings is girls' invisibility in the competitive and performative aspects of classroom practice that form social relationships. One effect is that of constructing their successful participation as private diligence. This is an example of how invisibility can circulate power productively; they are seen by teachers as setting standards of good effort and compliant behaviour. A second effect is that they can be excluded from public mathematical talk, particularly in situations in which gender becomes highlighted, perhaps informally. These are not distinct effects. Ernest et al. (2019)'s study of undergraduate classrooms showed that women's "hidden competence", evidenced by their participation in private group talk and informal explanatory 'sidetalk' during plenaries, was not matched by an equal contribution to public talk and was not reflected in how their achievements were recognised by peers or tutors. In this study, the effects combine to suggest that girls are isolated when studying mathematics, "the only girl". This position of isolation does not match the high participation rates in these schools, and further analysis is required to investigate how group participation is made intelligible by the binaries presented in the discourse.

One aspect of this may be the protective shadows cast by the positioning of hypervisible girl achievers as students whose identities are distilled to a single requirement of achievement in mathematics. This means that other students can position themselves as having multiple identities – being both girls and mathematicians – in a way that is complicated but unremarkable. Hypervisibility is not necessarily comfortable. Being accepted as a girl mathematician entails meeting high expectations in a setting where you are not visible as one of the girls. Brinke and Stobbe (2009, p. 465) call this the paradox of invisibility: "that visibility and invisibility as gendered practices and practising gender are part of the same organizational reality." They identify this paradox in the workplace and in academia, and these small-scale findings suggest it is present in schools.

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Ethical thinking and programming

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There is an increasing prevalence of programming in mathematics education. In this paper, we focus on ethical aspects of programming by first presenting a literature review. Three interrelated topics are identified in the review: reasons to include ethics in programming, code literacy, and facilitating ethical thinking. These three topics are then used to reflect on ethical thinking and programming in three tasks developed by a lower secondary school teacher. We argue that the context of programming tasks such as climate change, sustainability, and decision-making of self-driving cars, can facilitate students' ethical thinking alongside their learning of how to program.

Introduction

Programming is increasingly influencing society in invisible ways: what kind of information and advertisements people receive on social media, what people find through search engines, who get treated in hospitals or gain access to vaccines, terms of insurances, access to loans, etc. O'Neil (2016) denoted mathematics-based algorithms as weapons of math destruction if they contribute to destructive decisions, for example, increasing inequality in society through biased mathematisation of a problem. She further argued that this threatens democracy and emphasised that transparency and disclosure are crucial because the invisible algorithms steer human behaviour favouring certain interests. Programming and algorithms do not possess any morality; humans embed values into the algorithms. Human choices in mathematical modelling and programming affect whether values such as profit should trump fairness. O'Neill highlighted that if we treat mathematical models, data, and algorithms as neutral, we "abdicate our responsibility", and she argued for bringing "fairness and accountability to the age of data" (p. 218).

Artificial Intelligence (AI), where machines are programmed to learn, think, and act like humans, is of particular concern because decisions may be based on machine learning, without humans having the last say. Examples of related public debates include Tay – the hateful Microsoft bot, self-driving cars, and warfare based on AI, where one discussion concerns who is responsible if something goes wrong. Rhim et al. (2020) compared Korean and Canadian respondents' preferences on what ethical principles should be guiding for self-

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driving cars in imagined car crashes. They concluded that there are some cultural differences in ethical reasoning, as Canadian repondents tended to favour decisions minimising casualties, while Korean favoured decisions that followed traffic rules. Buolamwini (2016) emphasised that due to algorithmic biases and the social implications and harms that AI can cause, an awareness of "who codes", "how we code", and "why we code" is crucial. Colman et al. (2018) underlined that since 1980, "more and more problems from more and more diverse fields are being treated as "computable" problems; which means they are being tackled in algorithmic manner" (p. 8), even if the problems are wicked or unsolvable. As critical citizens, it becomes essential to question whether all problems are computable and how results need to be treated with care.

With these concerns in mind, it is important to remember the wonders digital algorithms and AI have helped achieve, for example, improved medical care, smartphones, and a range of other mathematics and programming-based technologies. However, many wonders are accompanied by concerns and constitute challenging dilemmas. These may be consequences of digital technologies applied in new contexts, for instance, face recognition applied in surveillance of citizens and data collected by Google and Facebook. It can also be dilemmas that are part of the technology itself, such as the question of who is responsible for fatal accidents caused by self-driving cars. AI is often accused of including black box decisions, i.e., that it is impossible to know what exactly led to the specific decision. This implies a lack of transparency and thereby, uncertainty associated with the quality of decisions (NENT, 2020).

The development of digital technology has long shown a mismatch between who gains from it and who are burdened (e.g., technology contributing to climate change). Digital technology impacts individuals (e.g., smartphones increase individuals' possibilities, but data on personal searches and likes are collected and traded) and society (e.g., how a good life and a good society is understood and the easy spread of fake news). Taken together, the development of programming and digital technology constitute a range of ethical dilemmas connected to its applications, power relations, and personal and societal implications.

According to Balanskat et al. (2017), computational thinking, programming, or coding was included in at least 20 national curriculums in Europe in 2017. This is likely to increase because of the EU's policy document *Digital agenda for Europa* (http://ec.europa.eu/digital-agenda), and how programming is becoming more evident in national policy documents (Bocconi et al., 2018). Programming has become a significant part of the Norwegian curriculum in mathematics (MER, 2020). It is also included in natural science, music, arts and design, and as an elective subject in primary and secondary school. An aim of the elective subject is to develop ethical awareness and critical thinking regarding processes and uses of technology. Relatedly, an aim of the mathematics curriculum is to develop an understanding of technology that can help students make "responsible life choices" (MER, 2019, p. 3). Based on these guidelines from the national curriculum, the focus in this paper on technology, programming and ethics can be regarded as an important part of mathematics education.

Investigating ethical thinking when programming is a rather new research field within mathematics education. In this paper, we start exploring what such ethical thinking might be. We do this by first conducting a literature review to identify key topics in the research that addresses ethical thinking and programming in education. Secondly, we present three programming tasks and discuss them in light of the identified topics in the review.

Clarification of concepts

Computational thinking is the overarching concept in which programming and algorithms constitute key components. Computational thinking involves systematic steps to solve problems and find solutions, and programming is often required to execute these solutions (Morris et al., 2017). Balanskat and Engelhardt (2015) described programming as "the process of developing and implementing various sets of instructions to enable a computer to perform a certain task, solve problems, and provide human interactivity" (p. 6). Normally, such instruction sets are called algorithms, which Colman et al. (2018) defined as instructive steps "used to organise, calculate, control, shape, and sometimes predict outcomes" (p. 8). These steps are expressed through a code, understood as a computer programme's digital language, which carries out the algorithms in practice (Nygård, 2018). Bocconi et al. (2018) denoted computational thinking as "a thought process entailed in designing solutions that can be executed by a computer, a human, or a combination of both" (p. 7). In this paper, we add ethical thinking to this thought process.

Ethical thinking can be described as an "assemblage of skills and cognitive processes related to determining how to act ethically and how to think through ethical choices and scenarios" (Schrier, 2015, p. 394). Ethical dilemmas involve situations where there is no obvious choice between two or more conflicting scenarios. For instance, the programming of self-driving cars need to implement decision-making on who to hit in case of an inevitable accident (Seoane-Pardo, 2016, 2018; Rhim et al., 2020). Introducing students to ethical decision-making while learning programming can help prepare them for these situations. Ethical awareness is then relevant and can include understanding moral dilemmas, identifying perspectives, interpretation of situations, and imagining causes and effects of decisions (Schrier, 2015).

Three ethical approaches are of particular relevance in ethical decision-making. *Virtue ethics* involves moral characteristics and intention behind actions, *deontological ethics* deals with the rules guiding the moral evaluation of the action, while *consequential ethics* emphasises the consequences of actions (Hursthouse & Pettigrove, 2016). Decision-making models can be of value when focusing on ethical thinking in educational contexts. Schrier (2015) described identifying and being aware of ethical issues, of evaluation and judging, of focusing on specific goals, and of implementing action as important characteristics of how to deal with the process of ethical considerations. These elements are relevant to include when teachers facilitate programming with students.

Methods

A literature review was conducted to gain insight into research on ethical thinking and programming in education. Two databases were used, Eric and Scopus. The search words were coding, programming, ethic*, ethical issue, ethical concerns, ethical dilemmas, education, school, learning, teaching, classroom, and education system. The search was limited to full-text articles from 2010–2020. A total of 255 texts were identified, but the majority of these were not relevant (e.g., the words *coding* or *programming* did not refer to computer programming). Ultimately, 25 of the 255 articles were scrutinized, and several of these are referred to in this paper. A more comprehensive literature review could have included other search words, other search engines, conference proceedings, and books. We categorised the included literature in terms of three identified topics: reasons to include ethics in education on programming, code literacy and ethical thinking, and facilitating ethical thinking.

We apply these topics when discussing three tasks developed by Max, a mathematics, science, and programming teacher in lower-secondary school. She developed several tasks for an elective subject, programming. The three tasks presented in this paper are chosen because they somehow involve ethical aspects. The first task involves the ethics of self-driving cars. The second task concerns how drones can monitor and be a tool to counteract climate change. The third task is on how drones and programming can help create a better future. Empirical data from her classroom are not available yet, but that does not displace the purpose of the discussion of the tasks in this paper: to investigate potentials for facilitating students' ethical thinking.

Ethical thinking and programming in education research

In the following, we present the three core topics identified in the literature review.

Topic 1: Reasons to include ethics in education on programming

Colman et al. (2018) and Dufva and Dufva (2016) argued that ethical reflections should be included when coding is part of teaching. In line with O'Neil (2016), Dufva and Dufva (2016) stressed that coding is not value-free and reflects both "conscious and subliminal values of the programmer" (p. 98). They claimed that teaching programming is often restricted to mechanical learning and suggested that education should include discussions of the ramifications of technologies. They further argued that programming could be seen as a "complex man-made tool for shaping the world" (p. 107), changing our behaviour and practices. This resembles Skovsmose's (1994) concept of the formatting power of mathematics, which denotes the role mathematics has in shaping society, often in invisible ways. Programming and mathematics often form the basis of technological structures, processes, and innovations. Ravn and Skovsmose (2019) highlighted how mathematics-based actions, often formed through technology that incorporates mathematics, should be ethically sound. They use Google's search engines as an example of a mathematics driven technology

where mathematical operations and algorithms are included in a complex network of operations that needs ethical considerations.

Colman et al. (2018) emphasised the ethical challenge connected to big data. Digital citizens are users of Internet and social media, such as Google and Twitter, and at the same time, providers of data that influence the same citizens through invisible algorithms developed to ensure profit. Ethics and societal implications should, according to Burton et al. (2017) and Rodríguez García et al. (2020), be key elements in computer engineering courses and school subjects involving AI.

Kules (2016, 2018) reasoned that ethical issues like diversity and equity should be addressed when learning programming. He described a curriculum for helping (graduate and undergraduate) students "recognize, analyze and take action when they encounter these issues" (2018, p. 31). The curriculum was part of a course teaching JavaScript and Python and constituted learning outcomes targeting to situate programming in the broader social structures. This was facilitated by students reading relevant literature and by weekly reflective discussions concerning topics such as coding for the social good and forms of inequity, for instance how algorithms determine prison sentences and how search engines reinforce existing social and cultural biases on attitudes towards women. The findings showed that students demonstrated abilities to identify and examine ethical issues in programming. This explicit focus on ethical issues associated with programming brings an awareness to programming as a non-neutral activity that involves ways of impacting society.

Reasons for adding ethics to learning programming thus include understanding that programming is not necessarily value-free, that digital technology is shaping the world, also in ways that are unfair and destructive, and that big data and artificial intelligence are particular topics that require ethical reflection.

Topic 2: Code literacy and ethical thinking

To participate in discussions on ethics, Dufva and Dufva (2016) proposed code literacy to be recognised as "understanding of the code and its intentions and context" (p. 98). They connect these discussions to Freire's (2014) ideas of "forming rather than training students" (p. 107), as training students in coding is not enough to form a critical mindset of digital technologies' potential implications. Dufva and Dufva (2016) suggested that education and code literacy should include the following: *learning a programming language*, *understanding the structure and intelligence of code, and how code can solve problems* (through being a transformative tool, making the world (supposedly) a better place). Further, *placing coding in its cultural context* (how mindsets, ideologies and trends influence the code), *understanding ways code affects society and individuals* (through game obsession or hierarchical systems that affect people's everyday lives, e.g., through the Internet and political (mis)uses of code), and *identifying power issues involved in coding* (e.g., Google's personalised search results, possible misuse of data for surveillance purposes, hacking damages, but also consumer power and other possibilities for changes). Lastly, *learning to use code as a way of self-*

expression (by deconstruction main concepts or exercise creativity). The purposes thus include both basic principles and logics of coding, as well as the more intertwined ethical issues involving coding and individuals/society. Code literacy is therefore, according to Dufva and Dufva, about understanding and living in a digitalised world.

Summing up, ethical reflection and ethical decision-making is argued to be part of code literacy. Coding can thus facilitate ethical awareness, reasoning, and reflection related to ethical decision-making in real-world problems, in line with Schrier's (2015) education goals for ethical thinking through computer games.

Topic 3: Facilitating ethical thinking

Several frameworks for ethical thinking related to education and programming have been highlighted. Burton et al. (2017) suggested that students should learn about ethical theories to apply them as tools for reflecting on case studies. They showed how deontological ethics ("What are the right rules"), utilitarianism ("What is the greatest possible good for the greatest number?"), and virtue ethics ("who should I be?") provide different questions and, thus, different modes of ethical thinking relevant to the problem at hand (p. 26). They suggested a mix of the three approaches when deciding how a robot should behave or how to target the audience when advertising.

Ladwig and Schwieger (2020) suggested the following ethical decision-making model for graduate courses in computer programming: *describe possible ethical dilemmas, identify stakeholders, outline options on how stakeholders are affected, and make a decision.* They suggested, amongst others, that students discuss ethical perspectives regarding algorithms for self-driving cars concerning choices in a potential collision. Seoane-Pardo (2016, 2018) argued that ethics is an essential part of computational thinking. Seoane-Pardo's developed teaching and learning lessons on self-driving cars for students (age 14–16) on programming and ethical reflection. The lessons were developed to facilitate various ways of ethical thinking in decision-making: consequential approaches – minimising casualties, minimising own risk or minimising economic cost, deontological approach – protecting those who follow the rules, and non-deterministic approach – mimicking random behaviour (Seoane-Pardo, 2018). He noted that AI programming is too advanced for these students, but pointed to tools developed in Arduino and Scratch which are designed for students to program ethical decision-making situations with self-driving cars.

Estevez et al. (2019) showed how an AI workshop with high school students changed the students' perception of ethical issues. The workshop was about developing code and discussing AI properties and limitations. After the workshop, the students were less concerned about dangers presented in sci-fi movies and more concerned about AI regulations to prevent bad usage of data due to privacy and freedom perspectives.

Taken together, ethical thinking and learning can be facilitated through working with real-world problems while applying ethical theories, how decision options affect stakeholders differently, and drawing attention to ethical dilemmas.

Three programming tasks

We now turn to three programming tasks designed by a lower secondary teacher, Max. These are discussed in terms of potentials for ethical reflections, referring to findings in the literature presented above.

First task: Self-driving cars

As part of the task involving self-driving cars, Max made a pamphlet with some information. One of the paragraphs reads as follows:

> In the development of self-driving cars, systems and services must be developed. Programming is an important element to achieve this, but merely developing technology is not sufficient. Students must reflect on the possibilities and challenges this technology presents and take a stand on associated ethical issues.

It is stated that AI technology in self-driving cars includes both possibilities and ethical challenges. This implies that such AI technology is not value-free and is therefore a reason for including ethics in computational thinking, in line with Burton et al. (2017) and Rodríguez García et al. (2020). The task encourages reflections on ethical issues which later are connected to the programmer's role:

> Machines do not have ethics and morals; they do as they are programmed. If a self-driving car is in a situation where someone has to die, it must choose. How should we decide how it should act, and will we buy a car that is programmed to kill us?

The task suggests that the teacher understands code literacy as something more than just technical aspects, as it calls for ethical awareness and reasoning on decision making associated with a real-world problem, in line with Dufva and Dufva's understanding of code literacy (2016).

The task facilitates ethical thinking through bringing attention to choices of a self-driving car, and that a choice may have stakeholder consequences. It signifies an ethical dilemma concerning who must die in a specific situation. If a driver in a regular car encounters an accident, decisions are often based on intuitive reactions. In a self-driving car, the decision on whether to crash into, say, a pedestrian or a car, is taken in advance through decisions in the code. Reflecting on these ethical dilemmas is what Ladwig and Schwieger (2020) and Seoane-Pardo (2016, 2018) promote. This implies that various ethics theories are relevant for drawing attention to ethical questions such as what the dilemmas are, who the stakeholders are, what are the decision options, and what makes machines different from people in decision-making situations. The task does not explicitly address all these aspects, but it can be done by the teacher or students. Attention can be drawn to different stakeholders: the driver, others in the traffic, people involved in the car production, and authorities responsible for traffic safety. Decisions taken in the coding will affect these stakeholders differently. For instance, governments aiming to reduce traffic deaths may prefer the choice of minimum deaths, while it may pay off for the producer to let the buyer of the car choose preferred algorithms.

Second and third task: Sustainability and climate change

> *Task 2:* Have you ever wondered how we can use drones to stop climate change? Use your creative skills to find a solution where drones can be used for this purpose. Present creative suggestions on solutions by programming a mini-drone so that it conducts an operation that can help stop global warming.
> *Task 3*: Mother Teresa says that sometimes we think that what we do is just a drop in the ocean. But the ocean would be smaller without that drop. Create a product that can be programmed either block or text-based, and which can help make the world a better place to live for children in the future. Keywords: sustainability, energy, the sea, entrepreneurship, farming, technology, etc.

In these extracts from two different tasks, the students are asked to contribute to make the world a better place for future children. Preventing global warming and making the world a better place are ethical topics in themselves and give reason for ethical reflections. The tasks suggest that drones can have an essential role in monitoring climate change indicators in remote places. They could partly replace fossil transportation, plant seeds in agriculture, replant forests after deforestation and in post-fire areas, etc., and directly contribute to potentially less CO_2 emissions. This implies that ethics can be embedded in the programming context and not necessarily connected to whether the specific choices of coding are ethical.

We did not find examples of programming in education that aims to improve societal problems in our literature review. Max' tasks, however, connect programming with ethical concerns for future children and society and may raise equity issues in the process of solving the task, in line with what Kules (2016, 2018) called for.

Identifying opportunities for uses of drones, and being able to programme for these uses, can facilitate ethical thinking related to drones' more challenging aspects, like how they are used in (illegal) surveillance, warfare, etc. Ethical contexts can be understood as part of what Dufva and Dufva (2016) proposed as code literacy in terms of intentions and the context of the code. In these real-world tasks, the potentially *positive* implications of digital technologies are emphasised. No ethical dilemmas are suggested, but dilemmas associated for intance with misuse of drones or with climate change can be raised during class by the teacher or by students.

These tasks are somewhat different from task 1, as they do not specifically request ethical thinking in line with what is described in topic 3. However, they may facilitate ethical thinking implicitly as the context of the tasks is ethical (doing something good) and they may introduce ethical dilemmas. This implies that the ethical thinking is not primarily on the code level, but connected to the application of the device or to connecting issues. The ethical thinking has thus a weaker link to code literacy, as covered in topic 2. The teacher probably had an ethics-based reason to include these particular themes, which may be related to what we found on topic 1, but the reason is not explicitly expressed in the tasks.

Concluding remarks

Findings from the literature review demonstrate the relevance of ethical thinking in relation to programming. We identified three topics from the literature review: reasons to include ethical thinking in education on programming, code literacy and ethical thinking, and how to facilitate ethical thinking in programming. We used these topics to identify relevant qualities and further potentials of three programming tasks. The three tasks added valuable perspectives that where not documented in the literature review. The review reveals that relevant empirical examples are scarce. Our next step is thus to do empirical research on ethical thinking and programming in the classroom.

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Why (mathematics) education in a democracy must be critical education

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Education, as a fixed component of society, has the task of forming democratic principles and values in addition to subject-specific goals. Critical competencies must be promoted in every school subject. In this paper, teaching principles of critical education are derived from the requirements of democracy. Furthermore, it is argued that mathematics education has a special role to play here, but that, at the same time, mathematics' unique characteristics may explain why it is still no matter of course to integrate critical aspects into this subject.

Introduction

What is the main purpose of education? What do we, as educators, want our students to learn? What skills do we want them to acquire? These questions are by no means easy to answer and have triggered debates in various disciplines. If you look at these questions with Klafki's (2007) understanding of education, one reason for the recurring debates about the goals of education might be the ever-changing demands on the educational system as our societies are exposed to constant change: 'education issues are social issues' (p. 49, own translation). He did not want to claim that the educational system must be oriented to the basic structures of society but that education systems as an integrated part of society have the opportunity and the task of assessing and helping to shape social developments. In this sense, education is a fixed component of society, and at the same time, society is a fixed component of education. On this background, we should dare a closer look at the interplay of (mathematics) education and society at a general level.

Most Western societies are built on democratic structures and values. However, the concept of democracy is multidimensional and cannot be easily explained. The large number of different definitions and descriptions of democracy suggests that it is by no means a fixed construct but rather a fundamental form of coexistence that follows certain basic ideas and values. Aguilar and Zavaleta (2012) tried to analyze and sum up different interpretations of the concept of democracy, which were used in journals for mathematics education. They identified that democracy not only has a political (free elections), a juridical (equal rights), and an economic (distribution of cultural and natural goods) dimension but, most importantly, a socio-cultural one. Other definitions highlight this aspect of democracy as well: Democracy is also characterized by creating conditions for the development of human dignity, equal opportunities, and emancipation, including that different ways of thinking and different

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opinions are not just tolerated but welcomed. This means that democracy thrives on the citizens' individual contributions and that everyone can and should contribute to a functional democracy. This already points to the fact that a democracy is always in kind of a process and not a fixed condition. West (2005) described it with reference to the use of the word – for him, democracy is 'more a verb than a noun' (p. 68), as this would better describe the process in which a democracy constantly finds itself. A democratic society must, therefore, continuously work to remain one, since it will always have to face new challenges – be it through processes of globalization, immigration, or budding ideologies that are represented by political extremists.

But how can we best ensure to develop our society in a democratic manner for a better living in the future for all citizens? How can we prepare our youth to get involved in the development process in the best possible way and thus help shape their future? How can we accomplish that they are becoming active citizens that can make decisions after thoroughly weighing up all options and then taking responsibility for them? As the former secretary-general of the United Nations Kofi Annan stated in a speech at a world conference in 1998: 'No one is born a good citizen; no nation is born a democracy. Rather both are processes that continue to evolve over a lifetime. Young people must be included from birth.' Education, therefore, plays an essential role. Dewey (1916/1997) was one of the advocates who already insisted at the beginning of the 20th century that a democratic society can never function without an educated citizenry. People need to be informed to participate intelligently within private or political life. The question remains how such education has to look like if it is to meet these requirements.

Dewey and others criticized education systems that do not in some way work for the purpose of maintaining democracy, but mainly to attract the best-trained workers in order to be and remain economically competitive and to position themselves in the international market. In the same manner, Giroux and McLaren (1989) argued that it is not enough that schools only provide skills and knowledge that are necessary for successfully competing in the world market. With this kind of education, the system is producing workers instead of citizens.

Another criticized mode of education is when schools are considered a place where knowledge is simply transferred into the students' heads and where students should simply remember the pre-structured and presented information. Already at the beginning of the 20th century, the German educator Kühnel (1916/1950) criticized that the (teaching) goals in school are always material in nature, that everything revolves around the teaching of this material, and that the entire demand on school is governed by it. Thus, school makes its students dependent – 'dependent in judgment, dependent in feeling, dependent in decisions and also in action – […] it makes the students wait […] for the stimulus to do something' (Kühnel, 1924, p. 96, own translation). Freire (1970/2014) called this kind of education the 'banking' concept. He described it as an educational concept where 'knowledge is a gift bestowed by those who consider themselves knowledgeable upon those whom they consider to know nothing' (p. 72). How should students learn to actively participate in society if they

are trained to be passive by listening instead of participating, receiving pre-structured information without thinking for themselves? Hytten (2008) stated that despite the fact that, as mentioned above, democracy is in a state of constant change, it 'is taught as if it is a fixed, static system passed on to younger generations' (p. 337). She indicated that the school system – as it is – teaches students that 'right answers are rewarded far more than good questions, critical thought, imagination, or creativity' (p. 339). Such an approach of imparting knowledge can never meet the requirements to turn students into responsible citizens who can actively participate in society and critically reflect on the decisions made by our representatives. We cannot expect our students to develop democratic values if an authoritarian style is used in class. That already indicates how closely education is linked to social systems and that they cannot be considered separately.

If we genuinely perceive students as citizens-in-the-making and not as future workers or objects to be fed with pre-structured knowledge that should only be trained to give 'right' answers, then, I will argue, *education in a democracy can be nothing other than critical education.* I start out with what can be understood under critical education and elaborate on the principles that should be fulfilled in such a type of education and on how these contribute to educate democratic citizens and to counteract non-democratic educational principles. I then turn to the point that mathematics education has a special role within this debate and why it might be particularly difficult to bring a critical aspect into this subject. Moreover, this demand seems to be more crucial than ever, considering the new challenges to a democracy that our fast-moving and globalized times bring with them, which I will discuss in more detail thereafter.

Being critical in education

At first, it would be necessary to clarify what can be understood by the word *critical* and how we can understand it in connection to education. A glance at the Oxford Dictionary (Oxford University Press, 2020) reveals five different meanings. First of all, it defines being critical as 'expressing disproval'. This meaning of the word usually has a rather negative connotation. In connection with education, this definition would fall short and would degrade education and school to a nagger about social processes. The description of 'critical' in the context of education, which seems to be the most relevant one, is only found in fourth place in the dictionary.[1] Here, being critical also means 'making careful judgments' – 'involving making fair, careful judgments about the good and bad qualities of someone or something'. The use of the term also depends on the historical background the word might carry. The concept of critique appears very prominently in Kant's *Critique of Pure Reason*, where he attempted to bring together rationalism and empiricism through his critical philosophical reflection. In doing so, he attributed great importance to the use of reason,

[1] The second, third, and fifth definition of the word seems to be less relevant in the school context. Further definitions of the word "critical" within Oxford dictionary: 'important', 'serious/dangerous', 'of art/music/books etc.'

which, according to him, includes the ability to draw conclusions and to examine oneself. The term was interpreted differently when it appeared within *critical theory*, where the meaning is understood as a further development of Marxist theory that focuses primarily on social aspects and aims at exposing its mechanisms of domination and oppression and questioning its ideologies. In the context of education in a democracy, the latter interpretation seems to form a good basis, although I would argue that at least some aspects of Kant's perception of critique would fit within a critical education as well.

The critical education approach evolved from various theoretical concepts like critical theory, with its representatives of the Frankfurt School like Horkheimer, Adorno, and Habermas, or *critical pedagogy* with theorists like Freire, Giroux, and Klafki. There is no general definition of the concept. I understand critical education as an education that enables students to critically examine aspects that surround them (whether private or societal). For that purpose, they should be able to understand the background of these circumstances, identify relevant information as such, recognize the possible (possibly veiled) interests of the involved actors, place the situation in a larger context, and argue how they draw their well-founded conclusions after comparing them with possible other ones. They are then capable of taking responsibility for their decisions derived from their conclusions, and these should, if necessary, lead them to take action. This makes critical education an active education rather than a passive one.

Following the definition and derived from the above requirements for maintaining a democratic society, certain guiding principles need to be followed to genuinely foster critical education. I will argue how these principles meet the demanded requirements of education in a democratic manner and counteract the other forms of education (that are criticized).

Content-related principles

Including relevant extra-school contexts. If school functions as an integrated part of society and vice versa, it must not act as a kind of parallel world in which no reference is made to real-life beyond school. Rather, it must directly include current or past social events and developments in everyday teaching. First of all, this also ensures that the school keeps track of important societal developments and does not just fulfill its mission to prepare students for their future jobs. As Dewey (1997) already pointed out:

> There is the standing danger that the material of formal instruction will be merely the subject matter of the schools, isolated from the subject matter of life-experience. […] When the acquiring of information […] do[es] not influence the formation of a social disposition, ordinary vital experience fails to gain in meaning, while schooling, in so far, creates only 'sharps' in learning—that is, egoistic specialists. (p. 13)

Students should thus gain a general understanding of societal processes, including different ways of looking at them, as they are dealt with in different school subjects. This should enable them to better recognize the relations between these societal topics.

Critically reflecting on school subject matter. It is just as necessary to talk about and critically reflect on the subject matter of the schools too, as they are always value-loaded. As

Skovsmose (1985) summed it up, students need to develop a 'critical distance' towards content taught in school. Teachers should include discussions about interests or assumptions, as well as about the limits of a subject, into class. Otherwise, it can easily happen that things are taught as if they were the only 'right' thing leaving no space for other perspectives – which makes one think more of methods of dictatorship than of a democracy. If an educational practice is to make students only listen and remember, they will not be able (or would not feel the urge) to question the information imparted when a teacher introduces new knowledge. In that case, they would just as well accept wrong as right. Kühnel (1950) already pointed out that 'the mechanical word memory is completely non-judgmental at first. […] A child learns right and wrong just as easily' (p. 141, own translation).

As education is supposed to be so much more than the mere teaching of subject matter, evolving a critical gaze on curriculum contents as well as on contents evolving extra-curricular, seems to form a major principle of it and of preparing the democratic citizen.

Learning-environment related principles

Dialogical teacher-student relationship. In addition to the content, it is essential *how* this content is conveyed. Within critical education, the teacher-student communication should not be one-sided (teacher reports and explains to the students), but students' contributions and opinions should be included in the lessons. This may require a rethinking of the teacher's role, as within such a pedagogy, 'the teacher is no longer merely the-one-who-teaches, but one who is himself taught in dialogue with the students, who in turn while being taught also teach. They become jointly responsible for a process in which all grow' (Freire, 1970/2014, p. 80). Therefore, lessons need to be carried out more in a form of dialogue, where students are also welcomed to integrate their former experiences into the learning processes. In this way, students are somehow involved in designing the educational process. They learn early that individual contributions are important, that their questions and points of view are taken seriously, and thus also begin to take responsibility for their own learning process.

Focusing on the process, not only on results. As becoming critical is not a competence that can simply be passed on by mere information, students must practice it themselves in different situations. Therefore, the classroom atmosphere should be designed to allow for an open, critical exchange on in- as well as on out-of-school topics. Questions and opposing views should be dealt with openly and perceived as enrichment in everybody's learning process. Critical reasoning, even if not always successful, should get far more rewarded than quickly getting solutions without knowing why these are considered the right ones. The focus is thus on *how* and *why* a solution comes about rather than the solution itself.

Practicing democratic principles. Pluralism, as a guiding principle in democratic societies, values citizens in their diversity and does not limit their opinions. A democracy allows for different attitudes, views, interests, or beliefs – indeed, it thrives on them. Therefore, the opportunity to openly exchange and discuss ideas and former experiences in an educational environment not only makes school more relevant to students, but allows them to practice important democratic principles. They learn to tolerate other opinions from their classmates,

which possibly leads them to rethink their own points of view. At the same time, they acquire argumentative skills when they justify their standpoint and defend it against others. Thus, this kind of learning environment might lead to the activation of various thought and reflection processes. In such a way of conceiving education, the classroom becomes a place where the learning process is collectively managed and driven forward, by bringing in a variety of opinions and bringing reasoned arguments for and against them: where students realize that their voices count and thus step out of their passive roles as mere listeners, where they are encouraged to actively participate in class, and where they can take responsibility for their own learning process as they realize that participation can bring about change (even if only on a small scale).[2]

Therefore, following these principles might form a fundamental basis for educating critical democratic citizens. However, it is also clear that the concept of critical education can only fulfill its purpose holistically if these principles are incorporated into all school subjects. In doing so, no subject should be allowed to evade its responsibility. Otherwise, it quickly appears as if there are subjects that either have nothing to contribute to the development of a critical competence (begging the question of these subjects then still have any justification at all in this understanding of education) or as if there are subjects where one does not need critical competences at all. Some have already pointed out the enormous importance of mathematics for the education of critical citizens since numbers play an essential role in political debates and other social issues and are often used to legitimize decisions. Moreover, the use of mathematics might even create realities.

Being critical in mathematics education

The pioneers in this respect were Skovsmose and Frankenstein, who quite simultaneously – in the mid-1980s – developed a concept of *critical mathematics education* (CME). Skovsmose (1994) drew our attention to the *formatting power* of mathematics, as realities can be created by the use of mathematics. He advocated that students should realize the formatting power instead of being unquestioningly controlled by it. Frankenstein (1989), based on Freire's pedagogy, used a concept of CME to question facts that are taken for granted with mathematics and wanted to show that the use of mathematics can by no means be considered neutral. Based on these ideas, many other projects were developed, which use the content in mathematics lessons in a critical manner to point out social injustices and actively counteract them (Gutstein, 2006), link culture and mathematics (e.g., D'Ambrosio, 1985; Powell & Frankenstein, 1997) or offer different perspectives on climate change (e.g., Coles et al., 2013). Although there have been great movements towards a form of mathematics education that

[2] I am aware that this does not mean that young people will not become actively involved later on without this kind of education. However, first, letting them deal with these democratic principles and societal topics already in school, can make it much easier for them to recognize personally important issues later, and second, it might pave the way for students who would otherwise not get the idea of becoming actively involved.

integrates critical aspects, it still appears to be exceptional in everyday teaching than a matter of course (at least within German-speaking countries). The fact that one does rather think of incorporating critical aspects within other subjects than mathematics, such as political education, geography, or history, might be due to specific characteristics of the subject:

First, more than any other science, mathematics is considered neutral and objective – a description of reality free of interpretation. Mathematics and numbers suggest universality and indisputability and make facts seem impersonal and neutral. With this attribution, it seems almost absurd to critically question mathematical facts (i.e., to look at contents in class also with regard to interest and benefit) or even to critically examine social events (which may also be represented by numbers) through mathematical glasses. If we encounter numbers in any theme, be it at school, in our private lives, or in the media, we quickly get the feeling that we are presented with unquestionable facts – one would have much less of an idea to criticize them than to criticize a verbal statement that says the same thing without numbers. This still powerful myth makes it tedious to establish critical education as a guiding principle in mathematics classes. However, it is precisely because it is easier to be convinced of numbers without critically reflecting on them that critical mathematics education is essential. That a blind trust in numbers is quite problematic within democratic societies, as numbers are often used to justify regulations, is shown in Porter (1996):

> A decision made by the numbers […] has at least the appearance of being fair and impersonal. Scientific objectivity thus provides an answer to a moral demand for impartiality and fairness. Quantification is a way of making decisions without seeming to decide. (p. 8)

What makes critical (mathematics) education even more important is the fact that we have now mutated into an 'information society' in which it is easy to access information from all over the world in real-time, and we are constantly surrounded by them. Hence, it has become more and more challenging to know which information is to trust. De kerrsmaker and Roets (2017) pointed out that people do not easily adjust their judgments if they get to know that what they first believed in has been proven to be wrong. It is a matter of cognitive abilities like reasoning, problem-solving, or understanding how willing people are to rethink their views based on misleading information (and, of course, also a matter of will to put these abilities to use). Gordon (2018) argued that critical thinking is a key capacity to combat misinformation in the news and that education plays a key role in developing such critical skills. That it is essential to train such skills in mathematics as well is shown by Willingham (2007) who stated that critical thinking could not function if there is no appropriate expertise in the background – which he called the domain knowledge. So, if we want students to understand and question, for example, whether the numbers presented in a political debate can be correct and what purpose they might serve, mathematical knowledge must also be present in the background (here, for example, interpreting graphics, handling large numbers, a basic understanding of mathematical models, ...). On the other hand, pure mathematical knowledge is not enough either – the critical examination of where, why, and how mathematical representations are used has to be trained too to be applied in

different situations: 'Just as it makes no sense to try to teach factual content without giving students opportunities to practice using it, it also makes no sense to try to teach critical thinking devoid of factual content' (p. 10). That is another reason why teaching only specialist knowledge in each subject is not sufficient.

This brings us to the second unique characteristic of mathematics education: no other subject than mathematics can do without any reference to social matters and worldly topics – these are usually even included in the curriculum. Geography deals with processes of globalization, languages require an essay on a socially relevant topic, or biology discusses diseases of affluence. However, in mathematics, teachers could focus on teaching mathematical rules and procedures without embedding them in non-mathematical contexts (and they did in specific times and places). An explicit reference to extra-curricular themes is really necessary only for very few fields of (school) mathematics (an exception would be financial mathematics). Of course, this does not mean that extra-school contexts automatically lead to critical education, but this at least provides a basis from which to start. It is also not enough to include real contexts into the classroom through dressed-up word problems that are supposed to help understand mathematics better but usually distort reality very broadly. Kühnel (1950), in his demand for a new way of arithmetic teaching, already pointed out that such an instruction can never fulfill higher educational goals:

> Arithmetic must no longer remain and end in itself but should become a means to pursue higher purposes. The higher purposes, however, can be no other than the grasping of reality, which confronts us in spirit and nature, and the promotion of culture [...]. The purpose of arithmetic is to provide the basis for a mathematical understanding of the things and phenomena of nature and human life. (pp. 67–69, own translation)

Especially when time is short and the pressure to get students through standardized exams is high, these higher educational goals are pushed into the background – mathematics lessons get along wonderfully without including them. A development towards a critical education, especially in mathematics teaching, is a great challenge and probably requires a rethinking that starts at the institutional level so that overall conditions are created in which teachers can act accordingly.

These demands are not really new, and it may seem that they are self-evident demands on a school system in a democratic country. However, it is apparent that in the past, in many areas within education, nothing appropriate has happened to meet these demands in the long term. Furthermore, the developments of the last few years (or even of the last decade) indicate that the educational focus is on entirely different aspects than enhancing critical education and democratic values. The growing importance and spread of standardized tests in schools show their effects directly in the classroom. Pupils' achievement is primarily measured only in their success on these tests, as they often serve as the key to higher education. Developments in the national school systems are initiated based on these standardized test results (e.g., A-level exams, PISA). In Austria, for instance, test performances are compared on a national level and schools that scored poorly are given a mandatory mandate by the school ministry to work with outside teams to develop the school so that a better

result is achieved on the next test. What happens is obvious: The focus in the classroom is on preparing students in the best possible way so that they perform well on these tests. Hence, what skills students actually evolve during their time in school (which is up to 13 years) gets less important than achieving high scores. Therefore, both teachers' and students' focus and source of pressure are clear – providing or getting the knowledge to pass these tests. Developing other skills will take a back seat and can be seen as an addition if there is some time left. Others have pointed out this problem as well. For example, White and Cooper (2015) demanded that 'instead of a system of education that valorizes test scores over authentic knowledge, what is needed is a system of education where student [sic] are free to explore, to investigate, to think and learn; in short, to become "critical thinkers"' (p. 17).

These developments show that the requirement of critical (mathematics) education is by no means outdated or something that can be taken for granted. Especially now, when we are constantly surrounded by information and, due to the strong global connections, events somewhere around the world can also have a direct impact on processes in our own country, it is more important than ever for our youth to develop a critical competence. Therefore, we as educators must not tire of positioning this as a central educational goal where everyone within the educational system needs to participate regardless of the subject. For, in the end, the most central aspect of a democracy is the informed citizen.

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Ethical mathematics awareness in students' big data decision making

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In this paper, we investigate adolescent students' ethical mathematics awareness as they reason through interview tasks that provoke them to make a decision based upon the results of data analytics. Drawing on multiple resources, including ethical principles from numerous STEM and business disciplines, we introduce an analytic framework for documenting the ethical mathematics awareness of adolescent students. Our findings indicate that 1) students justify their decision using a variety of ethical principles, and 2) half the students' decisions benefitted their hypothetical employer and half protected society. We conclude the paper with a discussion of implications for designing future instruction to support students' growth in ethical mathematics awareness.

Scholars in mathematics education maintain mathematics is "well integrated into the technological, industrial, military, economic and political systems of the present 'Westernized' world" (D'Ambrosio, 1998, p. 67). According to Lengnink (2005) the systemic influence of mathematics is a result of its embeddedness in the technology that has come to shape our world by unconsciously affecting our thinking and behavior. Unfortunately, the assumed objectivity of mathematics often generates "dehumanized thinking" which lends itself to "instrumentalism […] ethics-free governance and social practices" (Ernest, 2018, p. 205). Benjamin (2019) appropriately labels this phenomenon *The New Jim Code*, referring to the use of technologies and, by default, mathematics to reproduce existing inequities. As such, the effects of mathematics and technology on human behavior and well-being require that mathematics become a morally and ethically grounded discipline.

Literature on the detrimental effects of mathematics on society has been expanding since the onset of globalization (D'Ignazio et al., 2020; Wheelan, 2014). O'Neil (2016) describes the discriminatory algorithms and models that have come to organize, manage, and make

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decisions about human lives. These mathematical models, intended to increase productivity and reduce human error, have replaced human decision-making processes in areas such as loan disbursement, insurance qualification, policing, and political campaigns. Because such algorithms are built using statistical correlations as proxies for desired information, groups who have been historically targeted for their perceived socially unacceptable behavior become trapped in a vicious cycle of self-fulfilling prophecy. In 1973, Paulo Freire warned that the influx of new technologies had the potential to develop massified consciousness, or an individual's flawed belief that humans act through free choice alone rather than by choice and manipulation. Individuals with massified consciousness participate in their own domination by uncritically accepting media reports rather than through their critical reflection on the world. To overcome such thinking, Freire (1970) introduced a pedagogy aimed at developing, critical consciousness, whereby an individual learns to "perceive social, political and economic contradictions and to take action against the oppressive elements of reality" (Freire, 1970, p. 36). Mason (1986) claims that it is the responsibility of educators to ensure that citizens possess the "intellectual skills to deal with information" including the ability to read, write, reason, and calculate (p. 10). Given the intimate role that mathematics plays in politics and policy, civic life, Information and Systems Technology, as well as the number of mathematics majors who go on to work in related industries, we argue that mathematics education should play a considerable role in preparing students for their ethical responsibilities as well.

The majority of the literature on ethics in mathematics is theoretical and/or philosophical. Few empirical studies exist which incorporate ethics into mathematics learning; the majority of these exist in the realm of Critical/Social Justice mathematics (Frankenstein, 1983; Gutstein, 2006; Kokka, 2020; Rubel, 2017). In this paper, we explore the ethical mathematics awareness of 13 middle and high school students of racial and economic privilege by analyzing their responses to tasks that involve making decisions with the results of data analytics. For this exploration, we developed an Ethical Reasoning in Mathematics (ERiM) analytic framework that is specific to data science. In this paper, we present the ERiM framework and use it to analyze adolescents' ethical mathematics reasoning.

Methodology

Our theoretical perspective is situated within the Critical Mathematics education (CM) program elaborated by Frankenstein (1983) and Shor (1993). The complementary goals of CM are to a) promote students' capacity for critical reflection so that they can question mathematical arguments, models and representations and b) inspire students' agency to act in ways that use mathematical communication to liberate, rather than disenfranchise, groups. We define the term *critical mathematics consciousness* (CMC) as a content-specific form of critical consciousness (Freire, 1970; 1973) that involves the awareness of the role that mathematics plays in disenfranchising or liberating oppressed groups in society and the willingness and commitment to act. CMC involves three different types of awareness (Stephan et al., 2021):

- *Sociopolitical Mathematics Awareness* that mathematics is used to model and interpret the real world and can be used to make decisions both at the individual and systemic levels that may be oppressive or liberatory.
- *Ethical Mathematics Awareness* that human beings do mathematics; thus there are potential ethical dilemmas and implications of mathematical work.
- *Communicative Mathematics Awareness* that mathematical communication has the power to educate and mis-educate society and encourage the masses to act in certain ways.

Like Freire, we view critical mathematics consciousness as something that can be developed with critical pedagogies.

Analytic framework

Ethical mathematics awareness refers to being aware of ethical implications that may or may not be considered in the process of making mathematical decisions, such as violations of privacy, questions of who owns the mathematical data, and bias in the data, to name a few. To create the Ethical Principles Framework in Table 1, we consulted a variety of resources including philosophy and mathematics education articles. While the general principles expressed in these resources were useful, we also suspected that more nuanced principles pertaining to the STEM professions might be found in materials published by professional organizations for engineering, data analytics, artificial intelligence, computing and information, science, and statistics. Since many of our interview questions employed a business scenario as the context for making decisions, we also explored resources from business domains including marketing, accounting, and finance.

The Ethical Principles Framework does not represent a finite list of principles, but rather a compilation of the most relevant principles for ethical reasoning in the intersection of STEM and business professions. One point of elaboration involves the distinction between fairness and discrimination. Fairness involves attempts to avoid biased decisions and thus unequal treatment towards human beings. Discrimination refers specifically to the unfairness committed upon already-oppressed groups in society.

Participants

The work presented in this article was conducted as the first step in a Design Research Project (Cobb et al., 2003) that focuses on Designing for Critical Mathematics Consciousness (CMC) among students of privilege. Interviews were conducted *primarily* with 14–16-year-old students with economic and racial privilege to a) understand their CMC and b) determine viable contextual problems that may provoke critical mathematics consciousness. The Design Research Team consists of two mathematics educators, two STEM educators, one doctoral student and one assessment professional. We interviewed thirteen students, nine of whom identified as male and four as female. Nine of the students matriculated at Hill High

Charter[1] high school, three from a Lakeview Charter middle school and one from Paradise Bluffs, a private, middle school. Thirty percent of Hill High Charter's students perform at grade level in mathematics and the student population is 65% White, 22% Black and 5% Hispanic. Sixty-three percent of Lakeview Charter's students perform at grade level in mathematics and the student population is 70% White, 13% Black, and 5% Hispanic. Finally, eighty percent of Paradise Bluffs' students perform at grade level in mathematics and the student population is 75% White, 13% Black, and 1% Hispanic.

Ethical Principles	Considerations	Professional Domains Addressing the Principle
Privacy	How does respect for freedom or personal autonomy apply? Is confidentiality required? Is consent needed or obtained? Is privacy violated?	NSPE*, JMU ASA, ACM/AI
Fairness (equality)	What is the fair or just thing to do? Is there fair access to systems that are created?	ACM, AIB, Data Analytics, JMU
Accuracy	Is data reliable and accurate?	NSPE, AIB, ACM/AI, Science, Data Analytics
Accountability	Who is accountable? Have they communicated the data in a misleading manner? Is the source reliable?	
Property	Whose data is it to sell? Who owns the data?	Data Analytics
Loyalty	Is the decision/activity loyal to the organization (e.g., makes profit, keeps ideas within organization, does not use organization's ideas to make money elsewhere)	AIB, JMU
Accessibility	What information does an organization have the right to access about people? Who has access to this data? User/buyer?	Data Analytics
Algorithm Bias	Are algorithms objective? Do algorithms (un)knowingly discriminate against individuals or groups?	AI
Transparency	Are the codes for algorithms readily available for inspection?	ACM/AI, Science, ASA
Ecological	Has the impact on humans and ecosystems been considered?	ACM/AI, NSPE
Employment	Will the decision/activity harm an individual's or group's employment status?	AI
Discrimination	Has the decision/activity avoided negative effects on oppressed societal groups?	Data Analytics, ACM/AI, AIB

*James Madison University Ethical Reasoning in Action (JMU); American of Statistics Association (ASA); Resnik (1998)-Science; Chessell (2014)-Data Analytics; National Society for Professional Engineers (NSPE); Association for Computing Machinery (ACM); Artificial Intelligence (AI); International Business (AIB).

Table 1. Ethical Mathematics Reasoning Framework

[1] All student and school names are pseudonyms.

Method

Interviews lasted from 25-45 minutes and took place at the student's school in a private setting during a non-academic class period such as physical education. The interviewer began each session with general talk, attempting to build rapport with the student, and then posed five tasks. Due to the brevity of the paper, we only elaborate on students' ethical awareness on the Great Groceries task below.

Great Groceries is one of the biggest successes in American grocery chains. Impressively, Great Groceries reported $14.74 million in quarterly profits last year. You report to the CEO for Great Groceries. One of his goals for 2020 is to increase profits.

Your Management Team, based on extensive data analysis, comes up with the following three recommendations for increasing profits:

***Option A*:** Tiered Membership Fee to shop at the store. Offer tiered membership fees, the idea being that richer customers buy more products. The Tiered-Membership Program would increase profits by 1.8%.

	Member's Yearly Income Level	Membership Fee per Year
Upper Tier	$150,000 and up	free
Middle Tier	$70,000-$150,000	$50
Lower Tier	$30,000-$70,000	$75
Sub Tier	Under $30,000	$100

***Option B*:** Raise the price on the five top selling items by $1.50 each, increasing profits by 0.5%.

Option C: Put more digital, self-serve checkout stands in their stores to decrease the number of human workers, raising profits by 3.5%.

Which option would you recommend to your CEO, if any? Explain. Why didn't you suggest Option A, B, or C?

The options were designed intentionally to create a potential ethical dilemma for students between choosing an option that placed profits over people. Option A may prompt students to recognize issues of economic classism and structural oppression by imposing a shopping fee on the poorest customers. Option B, in our view, posed the least harm to people but also raised the least profit. Option C would generate the most profit but also eliminate jobs. When a student made a decision, the interviewer posed follow up questions to elicit their justifications. Each interview was transcribed and three team members used the Ethical Mathematics Reasoning Framework to categorize each student's responses independently. Once coding was completed, the three team members met to compare results and achieve consensus.

Findings

We organize the findings around two main themes. First, there were two main concerns that informed students' decisions: Consequences for a) humans and b) profits. Second, there were a variety of ethical principles invoked by students as they made their decisions.

Impact on Humans or Profit?

Of the thirteen students who answered the Great Groceries question, six picked the option that they considered to have the least harmful impact on humans. Two students indicated that they would choose option C because they felt their personal employment status depended upon recommending the option that would bring in the largest profit; these students indicated that if they did not have to report to the CEO, they would choose the least harmful option. The remaining five students chose the option they perceived to be best for the business, in terms of increase in profits.

Jamal's reasoning is one example of instances where students anticipated how their decision might negatively impact humans.

> Jamal: I pick option B [be]cause like, um, option C taking people's jobs didn't sound right and because people need money too, to help their families. And option A, just helping the richer customers is, it's not really cool.

For Jamal, Option B offered the best possible outcome for people "because it doesn't seem like it's doing anything bad…" However, Option A was not fair because some people, most notably the rich, were getting something for free while others had to "pay a fine."

While five students eliminated Option C, seven other students articulated their loyalty to the company by choosing it. For example, Collin recognized the impact that choosing Option C would have on humans, yet made his decision based upon what yielded the highest profit.

> Collin: It's kind of hard because you're also putting so many people [out of] the jobs. But that would also mean you wouldn't have to pay them. You wouldn't have to pay them at all. So yes, your profit would go up. So I guess, C.
>
> Int: All right, how come you didn't do option B?
>
> Collin: Because if the top five selling goods are already selling at their price and you bump it up a dollar 50 maybe those people won't want to buy it anymore.

For Collin, his commitment was to increase profit for the company, despite recognizing the negative impact on employment of the workers. He continued to rationalize his decision by arguing that minimizing the number of workers would decrease salary expenses, raising profits even more. Even when asked if he did not have a CEO to report to, Collin still chose Option C.

Jade, on the other hand, changed her decision when given the opportunity not to report to the CEO. Her initial decision was Option C, possibly due to her belief that her primary loyalty was to her employer even though it conflicted with her ethical beliefs.

> Jade: I think the last option probably would be the best just based on how this one is 1.8% this is 0.5 and this is 3.5 but at the same time…although this raises profits it does put a lot of people out of jobs, which also isn't a good thing for the rest of everyone…Um, I don't think that's [Option B] a bad one, but it doesn't have the best increase of profit. And at the same time it does make things more expensive. So you might lose customers.

Jade: [*Interviewer gives her the choice not to report to the CEO*] Oh yeah, I would probably say the first one then because, um, well you're not forcing everyone to pay an extra dollar 50 and, if you can afford the membership, then you'll get the membership…You don't have to worry about like the moral part of it. Um employment, making sure people get paid and can support themselves.

In the exchanges above, Jade recognized the impact that choosing Option C has on the livelihood of humans due to losing their means of supporting themselves. However, her loyalty to the CEO and the company compelled her to make an "amoral" decision. Although she did not recognize the negative impact of Option A on families that live in poverty, she nevertheless, chose what, to her, was the least hurtful option when the possibility of not reporting to the CEO was posed.

In summary, about half of the students we interviewed made their decision by considering the impact that it would make on human beings, while roughly half remained loyal to the company. Two of those seven however, changed to a less harmful Option when relieved of the criterion to report to the CEO. As we will see in the next section, students evoked several ethical principles as they decided which option to recommend.

Other ethical principles

In the previous section, we documented the end result of students' reasoning on Great Groceries and learned that the number of students making choices based primarily upon negative consequences for humans was about equal to the number who considered the positive effects on the company's profits. There were six ethical principles evoked by students as they justified their decision: Employment, Fairness, Loyalty, Accuracy, Discrimination, and Privacy.

Eight of the thirteen students mentioned the impact that their decision would have on *employment*. Five of those eight argued that choosing Option C would negatively affect people whose means of supporting themselves would be compromised. Another student, Miya, did not choose Option C, but her reference to employment actually focused on the negative impact that unemployment has on the economy, not the family. Finally, two students who chose Option C commented on employment arguing either that people would lose jobs but profits would go up (Collin above) or did not acknowledge the impact on workers at all, just increase in profit.

Six students mentioned *fairness* in their justifications saying that it was not fair that some people had to pay more than others just to get in the store. Some of the students' arguments indicated that they noticed it was the poor who were being made to pay more than the rich and exhibited empathy for the unfairness. Only one student made connections to systemic discrimination in this Option which we identify as the Discrimination principle.

Of the six students who used *loyalty* to justify their decision, not surprisingly, all of them chose the Option that they perceived would generate the highest profit.

Four students questioned the *accuracy* of the profit predictions made in some of the options. Accuracy refers to questioning the data both in terms of reliability and validity.

Although the students did not refer to the data that was used to create these options, they did question the viability of the options, or as Miya said: "I don't like Option A. It just doesn't, they're just *thinking*. They don't have the math for it. They can't just say that." In other words, Miya suggested that the Option creators were merely conjecturing (thinking) rather than using data (math). In terms of *privacy*, only one student mentioned this principle in regards to Option A. Ella argued that she would choose Option A, over Option C, only if the customers' plans could be kept private so that they were not aware of the higher fee they may be paying (implying a fairness issue).

Only one student made his decision (Option B) based upon recognizing what he perceived to be *discriminatory* practices in two of the options.

> Tyrone: Definitely not option A…So if even if the idea worked out, you're cheating out the lower class who only received $30,000, and they have to pay a membership fee of $100…that's a lot of money and no one would really spend that much money that's in poverty on their tier with $100. And it's kinda like taxes, wealthier, I know a lot of popular companies don't have to spend money on taxes, but they're the ones who have the most. Option B [is] the most fair route I would go because everything else is just like morally incorrect. That's mean, morally incorrect because you're taxing, um, poverty. And then option C…I don't want to decrease jobs because people still need jobs. And, um, I know a lot of the grocery store occupation is like located with people in poverty, high-schoolers and retirement, people in retirement. So I don't think it's fair to any of those groups because they're the ones most troubled…

We categorized Tyrone's reasoning as recognizing the discrimination that could occur as the result of carrying out Options A and C. Tyrone attended to the oppression that would be inflicted (fees; taxes) on already oppressed groups of people (people in poverty) by systems (wealthy companies; governments). Not only did he foresee the negative impact of his decision in the Great Groceries Dilemma but also recognized systemic injustices that are sometimes perpetuated by the taxation system in his country.

In summary, the students employed a variety of ethical principles as they deliberated the impact of their business decisions. Students almost evenly split on staying loyal to the company by increasing profits or making decisions that would not impact the livelihood of citizens negatively. Attending to the employment of citizens was the top ethical principle evoked by students. It should be noted that two students questioned the ethics of Option B by considering which particular items are best sellers. For example, Miya and Greta thought it would be unethical to raise the prices on what were likely top sellers at the time of the interview (the COVID-19 pandemic), toilet paper and hand sanitizer.

Conclusion

Our analysis indicated that middle and high school students have indeed developed quite a repertoire of ethical principles and employ them as they consider situations that potentially call for empathy and subsequent action. At least half of the students chose options that they

perceived would have the least negative consequences for humans with one student noticing potential systemic discrimination. The students who exhibited loyalty to the company and its profits either did not notice the negative potential outcomes of the other choices or felt obligated to choose profits as a part of their job. Or as Tony put it, "when you're running a business, it's about profit." We see problems such as Great Groceries as rich contexts for discussions about the accuracy of the data-driven options, issues of fairness and discrimination and whether profits or the safety and livelihood of citizens should drive data-based decision making.

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Understanding practices in an interdisciplinary group from a case study

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Mathematics Education Researchers (MERs) contribute to the growth of mathematics education when joining interdisciplinary groups. However, we know little about ways of work and discourse within such groups. We explored practices reported by an MER during a semi-structured interview about her interdisciplinary group experiences. A grounded theory analysis resulted in identifying a collection of MERs' practices in interdisciplinary groups. We argue that when MERs are a part of interdisciplinary groups, they engage in practices that involve ways of being, operating, and interacting. MERs' awareness of practices in interdisciplinary groups contributes to the evolution of mathematics education as a field.

Williams et al. (2016) defined disciplinarity as a "phenomenon" involving "specialization" of work and discourse (p. 4). Members of interdisciplinary groups are practitioners of different disciplines who exhibit specialized forms of work and discourse, also called practices (Hyland, 2000; Williams et al., 2016). Prior definitions of practice include descriptions of methods for functioning within a group or community (e.g., Cobb & Yackel, 1996; McIntyre, 1984; Schön, 1983; Wenger, 1998). Based on existing literature, we describe practices as established or emergent ways of *being*, *operating*, and *interacting* with others during a collective activity (Suazo-Flores et al., 2021, para. 4). Ways of *being* refer to an individual's conceptualization of self as part of a group. Ways of *operating* refer to ways the group engages in an activity (e.g., regularity of meetings, participation expectations). Ways of *interacting* refer to the development and communication of a group's discourse norms.

Given that researchers use certain practices unique to their discipline, mathematics education researchers (MERs) may face challenges when interacting with researchers from other disciplines. For example, Bruce et al. (2017) described the academic challenge of framing a research idea within an interdisciplinary research team "in ways that permit all potential research collaborators to identify and situate themselves" (p. 158). Goos and Bennison (2018) described physical and institutional challenges such as traveling to different

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disciplinary meeting venues and ensuring that interdisciplinary work contributed to achieving tenure. These reports describe considerations MERs might take up as they contemplate joining interdisciplinary groups. Building from our previous work, where we analyzed practices reported interdisciplinary groups' published research articles (Suazo-Flores et al., 2021), we continued exploring practices by studying a MER's experience in an interdisciplinary group.

Mathematics education as a discipline

MERs' practices draw from the discipline of mathematics education comprised by mathematics and psychology (Kilpatrick, 2014; Stinson & Walshaw, 2017). Described as the study of relationships between mathematics and human beings (Dörfler, 2003, p. 147), the discipline of mathematics education was developed in response to behaviorism (Kilpatrick, 2014). Early MERs were disciplinary experts in mathematics, psychology, education, and linguistics and theorized about mathematics curriculum, teaching, and learning. Currently, MERs often have degrees or specializations in mathematics education and research the traditional areas (e.g., mathematics curriculum, teaching, and learning). Additionally, MERs explore mathematics as a cognitive activity (Gómez, 2000; Kilpatrick, 2014; Valero, 2010) that involves human beings' minds and bodies (de Freitas, 2014), identities (Darragh, 2016), and sociopolitical and cultural surroundings (Gutiérrez, 2013; Valero, 2010). As the discipline of mathematics education continues to evolve, it enables and constrains MERs through sanctioned practices and inquiry domains. Valero (2010) described the boundaries of the discipline as moving to align with "ethical commitments" (p. LXXXV) of MERs' work and what the work can offer us and society. One way of expanding the boundaries of mathematics education research is to explore the products and processes involved in research practices (Ernest, 1998). An example of a product of interdisciplinary mathematics education research is the answers obtained from studying a topic using interdisciplinary lenses (e.g., Bruce et al., 2017). MERs' experiences in interdisciplinary groups are a special phenomenon, which is the goal of this study.

Mathematics education, interdisciplinary research, and practices

MERs collaborations with researchers from other disciplines to solve complex problems have expanded mathematics education research boundaries. Four examples from published articles are provided here. Bruce et al. (2017) explored how different disciplines (psychology, mathematics, and neuroscience) understand and study spatial reasoning. This work expanded mathematics education research boundaries by illustrating spatial reasoning's educational significance from multiple disciplinary perspectives. Goos and Bennison (2018) documented mathematicians' and mathematics educators' work in designing teacher education curriculum. The authors described collaborations between mathematicians and mathematics educators that provided opportunities to integrate mathematics content and pedagogy and resulted in curricula that benefited teacher candidates. Biology education researchers worked with an MER to study undergraduate biology students' graphing

practices in two different environments (Gardner et al., 2021). This collaboration added new insight into the common research problem of promoting graphing practices in new ways in mathematics education. Krummheuer et al. (2013) advanced the understanding of children's creativity when working with mathematical problems drawing from a socio-constructivist approach and a psychoanalytic perspective. The researchers used attachment theory to explore a student's activity when engaged with mathematical problems in a cooperative learning situation. Their findings described the working processes of creative children and expanded this to include the psychodynamic background's influence.

As interdisciplinary groups contribute to the evolution of mathematics education as a discipline, there is a need to explore practices used in interdisciplinary groups to support future MERs' interdisciplinary endeavors. Suazo-Flores et al. (2021) explored practices reported in published research articles. Findings included descriptions of three practices used in interdisciplinary collaborations: "working towards research interests," "cultivating trust and open-mindedness", and "understanding of institutional support." Because the aim of the published articles, used as data, was to communicate findings rather than to describe interdisciplinary practices, the practices identified were initial and tentative. This paper extends our prior work to illustrate practices experienced by MERs by asking: What are some representative practices described by an MER who participated in an interdisciplinary group?

Methods and analysis

This qualitative study is part of a larger project where we interviewed five MERs who were part of different interdisciplinary groups. We conducted semi-structured interviews (Kvale, 1996) to understand the practices that were a part of MERs' experiences in such groups. We present a case study (Flyvbjerg, 2011) of one of the five MERs (pseudonym Amelia), who was the leader of an interdisciplinary group. The transcript from Amelia's semi-structured interview constitutes the data for this study.

We used grounded theory (Charmaz, 2005) to analyze Amelia's descriptions of working in an interdisciplinary group. The analysis was done in three phases. In the first phase, we used our definition of practice as a conceptual framework to identify instances where Amelia described practices either she or the interdisciplinary group used. Each instance was coded as *being*, *interacting*, or *operating*. For example, the following excerpt was coded as an instance of a practice fitting into the *being* category.

> Amelia: For me, it's been a wonderful growing experience. So I don't feel like I've lost anything because I still have my life in my discipline, and I have a much enriched and expanded life as well by having these experiences that I didn't realize I was going to get.

Amelia referred to her identity as an MER noting, "I still have my life in my discipline." She also recognized that her work in the interdisciplinary group had "enriched and expanded" her life. We identified this as a way of *being* because it provided evidence of her evolving identity as an MER through her work with people from other disciplines.

In the second phase, we used instances from the transcript to refine our descriptions and definitions of the *being*, *interacting*, and *operating* categories. We also created phrases that summarized the practices identified in each category. For instance, we created the phrase "perception of self as a leader in a discipline and compared to other disciplines" for the above piece of transcript. This analytical process allowed us to enhance our definition of practices as ways of *being*, *interacting*, and *operating* and identify a collection of practices within each category (findings reported in Suazo-Flores et al., in press). Once the revised definitions and collections of practices reached a saturation point, the third phase of analysis began. In this final phase, we used the revised definitions and collections of practices to re-code our data corpus, identifying and describing practices reported by Amelia.

Findings

Five representative practices are presented as findings for this research. We describe these as representative practices because they embody and allow us to communicate the main elements of the *being*, *interacting*, and *operating* categories. For the *being* and *interacting* categories we present one practice and three from the *operating* category. The five practices described in this report do not represent all possible practices.

Ways of being, view of others' roles

Practices in the *being* category refer to Amelia's description of her individual identity as a part of the group or how she perceived others as a part of a group (Suazo-Flores et al., in press). The practice includes being a member of the group and being in the sphere of the group's work. The following transcript excerpt was coded as evidence in the *being* category and assigned the phrase "view of others' roles."

> Amelia: So I wanted this to be a truly collaborative project where even though, you know obviously Chris [pseudonym] and I had in mind what we wanted to do, it wasn't nailed down. There had to be space there for everyone to contribute and to recognize complementary expertise. No one, no one knows everything. So that's, that was one thing I looked at as well.

Amelia referred to her identity as a leader in developing a proposal for the project and the identity of others within the group as collaborators. She intentionally wanted to make spaces for her colleagues' voices and create a "truly collaborative project." She also described her views of others in the group by recognizing that everyone in the group had different knowledge and expertise when she said, "No one knows everythin." We interpreted Amelia's words as evidence of how she conceptualized colleagues' roles in the interdisciplinary group.

Ways of interacting, build trust/respect

The practices in the *interacting* category refer to Amelia describing ways in which she and others in the interdisciplinary group: (1) developed and communicated standards for discourse related to the group or (2) negotiated the meanings of ideas, frameworks, representations, or phrases. The following transcript was identified as a practice in the

interacting category and assigned the phrase "build trust/respect." Amelia expressed that her work in the interdisciplinary group "gave [her] opportunities to get to know people and to break down some of the barriers and the mistrust that had existed certainly in [place] for a very long period of time." Amelia explained that "barriers" and "mistrust" interfered with people from different disciplines working together. We interpreted this excerpt as the group breaking down barriers by negotiating and reconstructing ideas, allowing people to start developing common understandings.

Ways of operating

The practices in the *operating* category refer to Amelia recounting ways of doing within the group. Included in this category are structures that allowed the group to conduct work like holding regular meetings, promoting a sense of community, identifying leadership roles, or stating that ideas like "open-mindedness" were important. *Operating* also includes understanding policies and actions from an outside group or institution that indicate the work's value.

Having a common purpose

One practice identified in the *operating* category was assigned the phrase "having a common purpose," which could take the form of a common goal or a research topic.

> Amelia: The goal was to drive a major improvement in the quality of mathematics and science teachers by supporting new preservice programs in which faculty schools or departments of science, mathematics, and education collaborate on course design and delivery combining content and pedagogy. So that mathematics and science are taught as dynamic, forward looking, and collaborative human endeavours. So the whole purpose of the program is to get mathematicians and scientists working together with mathematics educators and science educators. Because the way secondary teachers are prepared in [place] is you come to the university, you learn the mathematics in the mathematics department, and at some other time, time you go to the opposite side of the campus and you learn how to teach the mathematics. And these two groups of people never talk to each other.

In the quote, Amelia recounted the goal that motivated the creation of the interdisciplinary endeavor; developing integrated curricula between mathematicians, scientists, mathematics educators, and science educators). Even though potential interdisciplinary group members might not know each other, there was a common purpose that motivated them to work as a part of the group.

Intentional team building

A second practice in the *operating* category was described as "intentional team building." Amelia reported that her interdisciplinary group had understandings that allowed the group members to work with autonomy and in collaboration. These understandings encouraged the members of the group to work together as they used and shared existing experiences.

> Amelia: Everyone had tried something already to do something different in so that first meeting we started, well when we wrote the proposal actually, created a menu. So we were about menus and repertoires, not about mandates. So that was the menu that we've been trying. And then we each picked something from that menu that we decided to try this approach next year and then we swapped and learned from each other.

A way of collaborating within the interdisciplinary group was to create "menus" instead of "mandates." This approach acknowledged the value of everyone's knowledge and previous experiences, which allowed them to share and build from each other. Group members tried new ideas and shared their experiences with the redefined approach. The sharing of ideas allowed the interdisciplinary group to learn from each other and learn about working together as a team.

Understanding external constraints

A third practice in the *operating* category was assigned the phrase "understanding external constraints." The way a group is positioned within and between other groups or institutions can afford or constrain work progress. For example, group/institutional policies could indicate the value of work being done by providing meeting space. In contrast, group/institutional policies could hinder interdisciplinary work by rejecting research because it uses unfamiliar methods for analysis. Amelia reported that it was important for her interdisciplinary group to be aware of institutional norms, make sense of the influence of these norms, and develop ways to operate within these norms.

> Amelia: So I think our project was not so much about let's have this grand design we'll design this incredibly sophisticated expensive program and online packages and so on. And it was about thinking about the small changes that we can actually make that are not going to require layer upon layer upon layer of [institutional] approvals which we know is going to take a long time.

Amelia described how knowing how institutions operated allowed her to identify ways to work within those institutions. She knew that significant changes would require approvals from outside of the group. These institutional policies might discourage people involved in the interdisciplinary group's work. She explained that the group focused on building from what people were already doing and making small changes that could result in significant improvements—this way of collaborating allowed the members to see their work in the interdisciplinary group as attainable and meaningful.

Although practices were identified in the *being*, *interacting*, and *operating* categories, we found that the categories were not mutually exclusive. Practices can align with the description of more than one category. For example, "understanding external constraints" can describe a way of *operating* and may include ways of *interacting* as people make sense of policies that exist with different groups. Practices in one category can also relate to or influence practices in another category. Like the prior example, "being open-minded" can describe a way of *operating*, but may include ways of *interacting* as people learn what it means to be open-minded. Below is a quote showing evidence of a practice that could be

classified in the *being* and *operating* categories. We identified this practice as "view of self in a discipline" and "having common goals or common research interests."

> Amelia: This project was not meant to be a research project but I was determined to build research into it because I wanted to understand, "How does the interdisciplinary piece work?" So you know the chief scientist, all he wanted was new approaches to teacher education but we thought, "No, let's research this and try to work out what's going on here." So, and I think that all the team members found it very, very stimulating to be asked these kinds of questions.

In this excerpt, we see evidence of practices in the *being* and *operating* categories. Amelia shared her personal views of her role in the interdisciplinary project: "I was determined to build research into it." She also reported how group members felt motivated to interact with each other around common goals and research.

Discussion and conclusion

We have identified representative practices in the *being*, *interacting*, and *operating* categories. Practices in the *being* category referred to MERs describing their view of themselves and others in the interdisciplinary group. Practices in the *interacting* category referred to ways of negotiating meaning of ideas and developing communication standards that allowed the group to collaborate. Practices in the *operating* category referred to mechanisms and group members' ways of doing in the group to work together despite coming from different disciplines with specialized forms of work and discourse and navigating institutional constraints or policy.

The grounded theory approach allowed us to explore the categories of *being*, *interacting*, and *operating* further. We have expanded our original work (Suazo-Flores et al., 2021) by providing representative practices in each category from an MER's experience in an interdisciplinary group. For instance, we now see the practice of "working towards research interests" as part of the practices in the *operating* category. "Having a common purpose", "intentional team building", and "understanding external constraints" were identified as additional practices in the *operating* category. Moreover, we now see "understanding institutional support" as an example of "understanding external constraints" and part of the practices in the *operating* category. The practice of "cultivating trust and open-mindedness" relates to researchers *interacting* in an interdisciplinary group in ways that allowed group participants to express themselves freely and consider the perspectives of others. We also found evidence that the categories are not mutually exclusive, which is evidence that understandings of the categories is still emerging.

One limitation of this study was the disproportionate number of practices identified in the *operating* category. The findings of our prior work (Suazo-Flores et al., 2021) resulted in practices that mainly fit with the *operating* category. These results influenced our interview protocol questions for this research and resulted in more opportunities for the participant to describe practices related to the *operating* category. A second limitation of this study is the interview data and analysis belonging to one case. We are analyzing more data to add more evidence from new cases. Future studies should explore practices in other interdisciplinary

groups or include observations of group members' interactions to expand our understanding of practices in the *being* and *interacting* categories.

As evidenced by our previous work (Suazo-Flores et al., 2021) and this paper, interdisciplinary groups involve MERs to developing practices in the *being*, *interacting*, and *operating* categories. MERs would benefit from being conscious of how they see themselves and how they see others as part of a group. For Amelia, it was important to preserve her identity as an MER. Amelia was able to direct her work to still contribute to her discipline while working in the interdisciplinary group. She also understood that no one knew everything in the group, which encouraged all group members to use and share their expertise and learn from each other. MERs also need to understand and identify ways to work within and among other groups or institutions. Amelia's group focused on making small and meaningful changes within the institutional structures. This allowed everyone in the interdisciplinary group to operate and feel sustained.

One of the benefits of MERs joining researchers from other disciplines is to solve more complex problems and expand mathematics education research boundaries. This study has shared evidence that MERs working in interdisciplinary groups also grow as individuals. Amelia recognized how much she learned from working with people from other disciplines and expanded her professional network. In reflecting on her experiences in interdisciplinary groups, Amelia noted:

> Amelia: [This interdisciplinary work] means everything. It really means everything. I still can't believe this is where my career has ended up [...] there is something that we all care about, that might be a bit more important than each of us individually. That's the best thing that's ever happened to me in my life.

Amelia's interdisciplinary interactions allowed her to grow professionally as an MER and as a person, as evidenced when she said, "That's the best thing that's ever happened to me in my life."

Interdisciplinary work provides opportunities to expand the boundaries of mathematics education research. Identifying and developing interdisciplinary practices is one way to allow MERs and the discipline to grow. As MERs identify ways to feel sustained and free to pursue their research endeavors outside of the didactic triad (Valero, 2010), the field of mathematics education will continue extending its research boundaries. We showed evidence that when MERs are part of interdisciplinary groups, they engage in practices that involve ways of *being*, *operating*, and *interacting*. To feel sustained in interdisciplinary interactions, MERs need to be aware of who they are and how they see others in the group, negotiate and develop discourse standards, and identify ways to collaborate within and among their institutions and groups. Our research explores MERs' experiences in interdisciplinary groups and contributes a set of practices that MERs might use in their future interdisciplinary interactions.

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School mathematics as a tool for spreading religious fundamentalism: The case of 'Vedic mathematics' in India

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There is a widely held belief that mathematics and school mathematics curriculum are apolitical even though critical thinkers in mathematics education have challenged this claim and pointed to how the larger socio-political context determines what mathematics is, what mathematics should be taught in school and how it should be taught. Often the political nature of mathematics is hidden from us. This paper seeks to argue that while holding on to the belief that mathematics is apolitical, there is an attempt by a certain section in India to use school mathematics curriculum as a tool to spread Hindu religious fundamentalism and that Vedic mathematics becomes an ideal component for achieving this. The paper also seeks to argue that Vedic mathematics would alienate not just the religious minorities but also those from the marginalized castes.

Introduction

This paper addressed to the MES community in the first place and to all committed to the cause of building a more egalitarian world, arises out of a serious anxiety that in an increasingly globalized academic world, in our interest to be inclusive, we might unwittingly promote voices from geopolitical margins, unaware of the fact that some of these are voices of the locally oppressive forces. This is truly a complex issue given that power and hierarchy operate form multiple social structures or institutions which may or may not intersect and hence it is not possible to classify a person clearly as either an oppressor or an oppressed-one could be both. Moreover, operating in the globalized academic world, what is visible to us immediately are the global hierarchies such as the north-south divide, racism, white supremacy, eurocentrism, and so on. So, for example seen from outside, India would be visible as a developing, low-income country, perhaps also as an ancient civilization with a rich history, including significant achievements in mathematics, available in the ancient Indian language Sanskrit. Caste as a significant structure in India, its relationship to language, the rise of Hindu religious fundamentalism and the systematic effort on the part of the Hindu fundamentalist to distort history, including the history of mathematics in India are perhaps less likely to be visible. One reason why hierarchies present in the geopolitical margins, are invisible in the globalized academia is because, typically those who represent

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the geopolitical margins are the beneficiaries of and hence blind to the existence of the local hierarchies. There is therefore an urgent need to seek out alternate perspectives that challenge the dominant voices from marginalized geopolitical locations. And there is also a responsibility on the part of the scholars from geopolitical margins to foreground the voices of those who are marginalized in their location. I would like to discuss the case of Vedic mathematics both as a possible example of the above situation and as an instance of how school mathematics becomes a tool for spreading religious fanaticism in India.

Mathematics is generally believed to be apolitical, partly because of the prevalent notion that there is only one mathematics which has applications in several contexts. Even though this belief has been challenged by critical thinkers (who argue that mathematics, like any other body of knowledge is socially constructed and is shaped by the beliefs and values of a community of practitioners who bring a form of mathematics into existence), outside the small community engaged in mathematics education research, the dominant notions about mathematics as apolitical and universal continue to prevail. Mathematics education, with its dubious reputation of acting as a gate keeper to the socio-economically marginalized learners, is well recognised as deeply political, serving the interest of the socially and structurally privileged. However, in the Indian context with very little research in mathematics education, both mathematics and mathematics education continue to be seen as apolitical. Against this backdrop, there is an attempt by Hindu right-wing elements to use mathematics as a political tool to promote religious fundamentalism in India by incorporating 'Vedic mathematics' in school curriculum. The term 'Vedic mathematics' invokes images of mathematics and mathematicians of ancient India, of a form of mathematics that has something to do with the Vedas and written in Sanskrit. Seen from outside India 'Vedic mathematics' could well be an instance of ethnomathematics and seen from within the country, it could instil pride in India in the minds of the learner. I would discuss what is 'Vedic mathematics' and what is the politics behind incorporating 'Vedic mathematics' in school curriculum. Using mathematics as a political tool has parallels in other contexts. Vithal and Skovsmose (1997) discuss in detail how, in the name of cultural diversity and ethnomathematics, an inferior form of mathematics was taught to students coming from the bottom of the hierarchy, during the apartheid regime in South Africa.

A myth called 'Vedic mathematics'

In the year 1965 a book titled *Vedic Mathematics* was posthumously published under the name of a monk Bharati Krishna Tirthaji, who was a Shankaracharya (religious head) of Govardhana Peetham in Puri, Odisha. The book was brought out by Motilal Banarsidass publishers. Bharati Krishna Tirthaji (whose name at the time of his birth and till he became a monk was Venkataraman) studied mathematics and secured a MA in mathematics from American College of Sciences in Rochester, New York, writing the examinations from Mumbai before he took the religious path. The book, available in English, consists of 16 sutras or simple mathematical formulae. While the Shankaracharya claimed that these formulae are part of a parishishta (supplementary text / appendix) to Atharvaveda, this claim

could not be established. Dani, a professor of mathematics who has been writing consistently to dispel the myth of Vedic mathematics says that K.S.Shukla, a renowned scholar of ancient Indian mathematic, on one occasion personally met the Shankaracharya and requested him to point out the sutras in the parishishta of the Atharvaveda, only to be told by the Shankaracharya that the 16 sutras demonstrated by him 'occurred in his own parishishta and not any other' and even this supplementary text alluded to by Shankaracharya does not seem to exist in Sanskrit (Dani, 1993). Clearly, the fact that the sutras are not found in any appendix to Atharvaveda must have attracted a lot of attention even at the time of publication of the book because the general editor to the book V. S. Agrawala and Manjula Trivedi, who has written the note 'My beloved Gurudeva' on the Shankaracharya, feel compelled to address this issue. Agrawala tries to explain this away by saying the word Veda, according to the Shankaracharya himself, refers to 'the fountainhead and illimitable storehouse of all knowledge' and since 'the Vedas as traditionally accepted in India as the repository of all knowledge should be and not what they are in human possession' (Tirthaji, 1965, p. 6). Manjula Trivedi says 'Obviously these formulae are not to be found in the present recensions of Atharvaveda; they were actually reconstructed, on the basis of intuitive revelation, from materials scattered here and there in the Atharvaveda' (Tirthaji, 1965, p. x).

The issue is not about the validity of the 16 formulae. They are valid and provide quicker methods to solve some problems and are at the level that they could be incorporated in school mathematics. The formulae have been compared to similar work by Trachtenberg and others. Dani says

> the material in the swamiji's book looks like a compilation of tricks in elementary arithmetic and algebra, discovered by some intelligent hobbyists. What it contains are ways of working out some classes of computations; in certain special cases these ways turn out to be faster than the standard school-book methods (Dani, 1993, pp. 1579–1580).

The issue at hand about *Vedic Mathematics* is not the mathematical content, but the adjective 'Vedic' attached to it. The natural question that arises is, why did the Shankaracharya claim that the sutras are found in an appendix to Atharvaveda when they are not and why was the book titled *Vedic Mathematics* instead of something like *A Bunch of Formulae* or *Some Interesting Mathematical Shortcuts*? Does this have anything to do with the rise of Hindu Nationalism in India, the status of Veda in Hindu religion, the status of Shankaracharya, the position of Hinduism vis-à-vis other religions in India?

Vedic Mathematics and the agenda of Hindu Supremacy

The book *Vedic Mathematics* appears in post-independent India, in the year 1965, five years after the author's death. With the enduring contributions of the mathematical genius Srinivasa Ramanujan, with the contributions of several other notable mathematicians of the 19th and 20th century, with the work of P. C. Mahalanobis and C. R. Rao in statistics, with the establishment of premier research institutes in post independent India, there was a lot of hope for India to make lasting contributions to modern mathematics and statistics when the book *Vedic Mathematics* was in the making. Research on Ancient Indian mathematics also

led to important publications such as the volumes *History of Hindu Mathematics* by Dutta and Narain (1962), *Ancient Indian Mathematics and Vedha* by Gurjar (1947), the papers by T. A. Saraswathi Amma (1959, 1961) on geometry in ancient and medieval India and more, apart from the works of W. E. Clark (1930), G. R. Kaye (1915). At least 11 doctoral theses were written on the history of mathematics before 1960 (Gupta, 1993). Therefore, in post-independent India, still young at the time the Shankaracharya was working towards the book, there was much to look forward to both by way of researching the past achievements in mathematics and making a mark in modern mathematics. Then why did the Shankaracharya write a book of 16 formulae and call it *Vedic Mathematics*?

One could try and find an answer to this question from the preface to the book by the Shankaracharya, where it becomes clear that the Shankaracharya's concern is more with establishing the supremacy of Vedas. Towards this end, he begins by first reinterpreting the term 'Vedas' to mean a repository of all knowledge across all disciplines that existed in the past, that exists now and that could ever exist. This interpretation would establish the supremacy of Vedas and the work left for the committed servants of the religion is to demonstrate this to the world. It is important to note that the period when the Shankaracharya invests himself with this work is also the period when Hindu Nationalism is on the rise in India. In the early 20th century, Hindutva emerges as a strong force seeking to establish the superiority of Hinduism over western superiority and challenging the spread of Islam and Christianity in India. Shankaracharya's work in mathematics is in response to the poor representation of ancient Indian mathematical knowledge by western scholars. In the words of the Shankaracharya:

> And the contemptuous or, at best patronising attitude adopted by some so-called Orientalists, Indologists, antiquarians, research scholars etc., who condemned, or light-heartedly nay, irresponsibly, frivolously and flippantly dismissed, several abstruse-looking and recondite parts of the Vedas as "sheer-nonsense"-or as "infant-humanity's prattle", and so on, merely added fuel to the fire (so to speak) and further confirmed and strengthened our resolute determination to unravel the too-long hidden mysteries of philosophy and science contained in ancient India's Vedic lore, with the consequence that, after eight years of concentrated contemplation in forest solitude, we were at long last able to recover the long lost keys which alone could unlock the portals thereof. (Tirthaji, 1965, pp. xiv–xv)

So, his aim was to 'demonstrate' that all the western mathematical knowledge is already found in the Vedas. He does this by saying that he developed strong intuitions in mathematics after eight years of concentrated contemplation and discovered the 16 formulae in a parishishta or appendix of the Atharvaveda. These 16 sutras according to him, can solve the toughest mathematical problems that the present day advanced western mathematics is trying hard to solve spending a lot of time and energy. He also lists 11 topics in the western mathematics and says all these are covered by the 16 sutras and declares 'there is no part of mathematics pure or applied, which is beyond their jurisdiction' and adds that if one spends 2 to 3 hours per day for one year all the mathematics that one learns in western universities spending 16 to 20 years.

To sum up, this is a clear case in which a religious head who commands a lot of respect among the Hindus comes up with 16 formulae which can make some computations simpler and wilfully makes false climes namely, they are part of the Vedas and they are as good as all the mathematics taught in the western universities today, and the Vedic methods are so superior that they can be learnt in one twentieth of the time it takes to learn western mathematics.

These claims have met with serious criticism from some of the well-established mathematicians and scientists from India and abroad (Dani, 1993, 2001, 2007, 2012; Raju, 2014a, 2014b; Narlikar, 2003; Bal, 2010; Singh, 2020; Plofker, 2009; Shukla, 1991). Some of the leading mathematicians from the country signed and sent a letter to the National Council of Education Research and Training saying what passes off as Vedic mathematics is 'Neither Vedic Nor Mathematics'. But these criticisms have had very little impact on the spread of Vedic mathematics in India.

Vedic Mathematics: A meeting point for religious and economic right wing

Vedic mathematics started getting introduced in school once Bhartiya Janata Party (BJP), a Hindu right wing political party came to power in four states in India in the year 1991 (Jayaraman, 1997). In the last two decades there have been a flurry of activities around Vedic mathematics. Just a search in google scholar will lead one to hundreds of research publications by Indian scholars which are based on Vedic mathematics, in computer science. In contrast Vedic mathematics has not received any attention from the international academic community if we leave out a handful of books and articles by authors from outside India.

At the school level, Vedic mathematics lends itself for promoting commercial activities. There are online classes for children on Vedic mathematics, offline classes that combine Vedic mathematics with Abacus, books with title like *Vedic Math Made Easy*, *Vedic Math Workbook* for children and so on. Among the several other additional mathematics classes that urban middle-class children are forced to attend, apart from the mathematics they learn in school, Vedic mathematics is one. And the books and online classes build their own myths in addition to what is already present in the very idea of Vedic mathematics. Here are a few examples:

- Vedic math is also called Speed Maths. It is a very age-old system of mathematics. It was taught in ashrams in olden days & holds good for improving your math (aptitude) skills in modern day.
- This Vedic mathematics course aims to promote the traditional knowledge of Mathematics mastered by the mathematicians of ancient India.
- Vedic Maths is the World's fastest calculating system and was originated in ancient India. It enables performing calculations 10 times faster than the conventional method. It is an elaboration of the 16 sutras (and 13 sub-sutras) derived from Vedic calculating system as mentioned in the ancient Vedas (particularly 'The Atharva Veda') of India.

Interestingly, a few of the books also say that the tricks have nothing to do with Vedas though the book is called *Vedic Mathematics* or say that Vedic mathematics is another name for speed mathematics.

Vedic mathematics also receives regular of press coverage, partly because, ministers from the ruling BJP at the centre and from some state governments headed by BJP, make statements about introducing Vedic mathematics in school curriculum while speaking about the new Education Policy introduced in 2020. One such press report is about a call that a grade XII student, Usman Saifi, received from the Prime Minister Mr. Narendra Modi for topping the board examination; the prime minister congratulation Usman asks him to learn Vedic mathematics and teach his friends (TOI, 2020). There are websites dedicated to Vedic mathematics, YouTube videos, journals, seminars, conferences and offer for a career in Vedic mathematics. In all these references to Vedic mathematics, for some Vedic stands for ancient Vedas and for some others say it just stands for a set of computational tricks 10 times faster than standard methods.

Right wing political outfits such as Rashtriya Swayamsevak Sangh (RSS) have been making consistent demand that Vedic mathematics be incorporated in school curriculum. It is difficult to imagine if such a demand would be meaningful had the book been titled differently or if any other book offering 16 efficient formulas for fast computations would have achieved the cult status that Vedic mathematics has achieved.

A method behind the myth of Vedic mathematics

It would be naïve to think that the Shankaracharya came up with idea of naming the book 'Vedic Mathematics' to simply glorify the Vedic past. Both the content and the title of the book have significant role to play in fanning Hindu fanaticism. Manufacturing a glorious (mythical) past is part of the efforts by the fascists Hindutva forces to undermine what they would call 'western' knowledge across disciplines. They have also made false claims about Indian achievements in science and have attempted to rewrite the history of India with no regard for evidence. In fact, in the year 2014, Prime Minister Narendra Modi claimed that ancient India already had knowledge of plastic surgery and assisted reproductive technologies (Rahman, 2014). The strategy fascists use is the same: systematically create myths, false narratives, fictitious accounts, and base all claims about the superiority of Vedic knowledge on them, spread these among the common people, particularly among the Hindus, create a sense of having been wronged by the Islamic and colonial rulers and finally issue a call to set the history right. The power of the claims lies precisely in the fact that there is no way one can challenge them using reason or evidence- the discourse is not pitched at the level of academic engagement for one to counter them using logic and material evidence. In fact, the 'western' academic approach itself would be considered suspect.

In this scheme of things, it does not matter that what Vedic mathematics provides is a just a set of efficient algorithms that are consistent with what is being taught in schools now. The objective is not to replace the mathematics taught in school by ancient Indian mathematics; instead, it is to use the so-called Vedic mathematics to advance the Hindu

communal agenda. Imposing a set of formulae as Vedic mathematics, amounts to making a symbolic assertion about the superiority of Vedic knowledge.

The Shankaracharya achieves exactly this. In his own words,

> Eversince (i.e., since several decades ago), we have been carrying on an incessant and strenuous campaign for the India-wide diffusion of all this scientific knowledge, by means of lectures, blackboard- demonstrations, regular classes and so on in schools, colleges, universities etc., all over the country and have been astounding our audiences everywhere with the wonders and marvels not to say, miracles of Indian Vedic mathematics. (Tirthaji, 1965, p. xv).

RSS affiliated Vidhya Bharati schools which cater to 3.4 million students from economically marginalized background from across the country teach five additional subjects apart from the subjects prescribed by central or state board of education to which the school is affiliated. Vedic mathematics is one of the five additional subjects. RSS and its associates make regular demand that school curriculum should be changed to align with Vedic knowledge. In 2002 when the BJP came to power at the centre, it changed the history curriculum and textbooks with no regard to academic rigour or to the constitutional values of secularism and harmony (Gohain, 2002; Tapar, 2002; Roy, 2002). Several states ruled by the BJP have revised their school curriculum and in the case of mathematics, they have introduced Vedic mathematics.

Though some of the prominent mathematicians have argued that there is nothing Vedic about Vedic mathematics and some of the books on Vedic mathematics brought out by private publishers also say the same, Hindutva outfits continue to propagate the myth about Vedic mathematics. Here is a sample from the book *Fundamentals and Applications of Vedic Mathematics* brought out by the Delhi State Council for Education Research and Training (Teotia, 2014):

> Vedic mathematics forms part of Jyotish Shastra which is one of the six parts of Vedangas. The Jyotish Shastra or Astronomy is made up of three parts called Skandas. A Skanda means the big branch of a tree shooting out of the trunk. (p. 3)
>
> The "Vedic mathematics" is called so because of its origin from Vedas. To be more specific, it has originated from "Atharva Vedas" the fourth Veda. "Atharva Veda" deals with the branches like Engineering, Mathematics, sculpture, Medicine, and all other sciences with which we are today aware of. (p. 7)
>
> Vedic scholars did not use figures for big numbers in their numerical notation. Instead, they preferred to use the Sanskrit alphabets, with each alphabet constituting a number. Several mantras, in fact, denote numbers; that includes the famed Gayatri Mantra, which adds to 108 when decoded. (p. 8)

This book not only adds its own twist to the Vedic connection, but it also makes a direct link between mathematics and religion by referring to mantras. It must be noted that the famous Gayathri mantra can be chanted by only males and even among them only brahmins can chant the whole of it.

Vedic mathematics and symbolic violence

The need for imposing Vedic knowledge arises only if there are those who might not be inclined to take interest in it. Ancient Indian mathematics had significant contributions from Buddhist and Jain traditions. Islamic art, architecture, music, literature, mathematics, and science have contributed richly to the cultures in India. Historically, apart from the dominant classical tradition in mathematics, there were also vernacular traditions in mathematics brought into existence by those who were engaged in productive labour. The name 'Vedic mathematics', will not invoke images of any of these. Caste is fundamental to Hinduism – there is no way that one can be a Hindu without belonging to one caste or the other. Caste system is a hierarchical system with brahmins at the top of the hierarchy. Access to Vedas was not equally available to all Hindus. Vedas were written/chanted in Sanskrit, a language accessible only to brahmin men and perhaps the kings while others used Prakrit. Dalits, who were/are treated as untouchables had no right to even hear the chanting of Vedas. Similarly, those who belong to Islam or Christianity had no access to Vedas. The same goes for several tribal groups who do not even identify as Hindus. The so called 'Vedic mathematics' therefore stands for a form of mathematics accessible only to those who had the right to read Sanskrit, namely the brahmin men and few other powerful elite men. The act of imposing a set of 16 formulas as 'Vedic mathematics' on everyone would therefore be hegemonic.

It is in the name of Vedic religion that people were divided hierarchically into castes. It is in the name of Vedas that Dalits were treated as untouchables, subjected to humiliation, structural violence, poverty and denied their basic human rights. What pride can they take it Vedic mathematics? What fascination would Vedic mathematics hold for students from non-brahmin castes while mathematical knowledge embedded in their traditional caste-based work has no place in it? Why would Muslims and Christians in India – many of whom converted in the hope of escaping the caste-based violence and humiliation- want to take pride in Vedic mathematics when they face systematic attack from the Hindu right wing? Wouldn't inclusion of Vedic mathematics in school curriculum alienate these students and is that not a form of symbolic violence?

Conclusion

Mathematics education in India is desperately in need of systematic research to understand the factors that are responsible for the underrepresentation of Dalits, tribals, religious minorities, and women in mathematics; for the socio-economically marginalized, mathematics functions as a gatekeeper, ensuring that they continue to provide cheap labour and this needs to be studied and addressed. Similarly, there is an urgent need to document alternate forms of mathematical knowledge embedded in various forms of traditional work and local culture as part of people's knowledge of mathematics. Rather than urging the government to take these up seriously, the Hindutva forces call for the introduction of Vedic mathematics in school curriculum with a clear intension of spreading Hindu religious fundamentalism. Even as they belittle western knowledge, they use whatever recognition

Vedic mathematics receives form the west to their advantage. Given this, the international academic community committed to social justice has the dual responsibility of being inclusive of the geopolitically underrepresented on the one hand, and sensitive to the local hierarchies on the other, in order to stay true its commitment.

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Critical envisioning of embodiment in mathematics teaching

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A body of scholarship on embodied mathematics learning has demonstrated the process of mathematics learning that is inseparable from learner bodies. Thus far, the scholarship on embodied mathematics learning has made limited connections with the critical conceptualization of embodiment that allows us to see the history and power behind mathematization of bodies. How can teacher candidates come to see embodiment in mathematics learning through critical lenses? In this study, we consider the possibility of utilizing digital illustrated stories based on ethnographic findings on (im)migrant families' intergenerational embodied mathematics learning. We present preliminary findings to illuminate how teacher candidates' discourses became heterogenous when they made meaning of the details in the designed illustrated story.

Toward critical conceptualization of embodiment

In mathematics education, there has been a body of work that highlighted mathematical thinking and body in light of gesture production, bodily coordination and mobility (e.g., Abrahamson & Sánchez-García, 2016; Alibali & Nathan, 2012; Chronaki, 2019; Hall & Nemirovsky, 2012; Hwang & Roth, 2011; Lee, 2015; Ma, 2017; Nemirovsky et al., 1998). Epistemologies and ontologies supporting this body of embodiment literatures in mathematics education range; for instance, ecological dynamics (as seen in Abrahamson & Sánchez-García, 2016), material phenomenology (as seen in Hwang & Roth, 2011), and distributed cognition (as seen in Ma, 2017). de Freitas and Sinclair (2013) drew from new materialism to emphasize assemblages of diverse materialities and learner bodies. Scholarship on embodied mathematics learning altogether has demonstrated the process of mathematics learning that is inseparable from learner bodies.

In a slightly different vein of scholarship, learner body was considered in relation to the development of culturally specific forms and functions as seem in an Indigenous numerical system observed among Oksapmin children and youth in Papua New Guinea in Saxe's (2012) study. Historical and longitudinal accounts of the Oksapmin people's 27-body-part counting system illuminate how the counting system has been reproduced and altered along with the shift in political, economic, and educational macro systems in Papua New Guinea (Saxe, 2012).

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We seek to further advance this line of research by shedding light on how certain bodies are forced to be *hidden* in the public space of learning, how the mobilities of certain bodies can be restricted or liberated, and how such negotiation of bodies interact with the stories and histories of the learner. In the study focused on (im)migrant families' embodied multiplication algorithm (Takeuchi, 2018), both children and mothers were hiding this algorithm because it differed significantly from the mainstream norm. They were hiding what was passed on intergenerationally, through which they conformed themselves into the mainstream. Thus far, the scholarship on embodied mathematics learning has made limited connections with the critical conceptualization of body or embodiment. However, critical conceptualization of embodiment is crucial in seeing the history, power and politics behind mathematization of bodies (Takeuchi & Dadkhahfard, 2019; Takeuchi & Aquino Ishihara, 2021). In considering norms and hiddenness, we conceptualize body guided by queer theory (Ahmed, 2006; Butler, 1993, 2015) that brings forth the tangled relationships among body, norms and power. *Othered* bodies or *unintelligible* bodies that deviate from the norm can be tacitly excluded from materialization and be treated as *abject* (Butler, 1993) — that is masked and hidden. Power is exercised over non-normative bodies through materialization and abjection of bodies. Such othered and unintelligible bodies can become less extended and less mobile in social space and when such bodies are constrained in their mobility, they turn to "the body that is 'out of place'" (Ahmed, 2006, p. 140). Resistance to this exclusion can be actualized by making visible the hidden bodies as a performative act, as proposed by Butler (2015): "bodies in their plurality lay claim to the public, find and produce the public through seizing and reconfiguring the matter of material environments" (p. 71). In the following section, we discuss the possibility of digital illustrated stories as a potential tool to make the invisible visible and queer the normative mathematics teaching practices.

The possibility of digital illustrated stories to make the invisible visible

The use of an alternative medium of research communication has been discussed in relation to its power for displacing and disrupting colonizing knowledge-making practices (Kayumova et al., 2018). Researchers have employed various forms of artistic renderings to communicate theory with the general public as seen in digital storytelling (Lambert, 2013) and graphic novels and comics (Curnow & Vea, 2020). Our goals of utilizing digital illustrated stories are aligned with such collective efforts to evoke the power of artistic renderings for public communication — in the context of this study, public communication of theory with teacher candidates. Toward this goal, we utilized visual art in the form of research-informed illustrated story. In our project we focused on interrogating dominant narratives and deficit perspectives with teacher candidates by centralizing the unseen story and silenced voices of historically marginalized groups. Our project holds a similar vision discussed in Chronaki's (2019) study where creative choreography was used with teacher candidates to enact and reconstruct their pedagogical moment of teaching (or not teaching) the concept of area to children in the early years. Chronaki (2019) proposes "an affective bodying with concepts" (p. 329) as a reconstructive pedagogy of early mathematics education. In our study, we

explore if the visual medium of graphic design in the form of designed illustrated story could offer unique affordances to facilitate experiences and dialogues on equity and learning with teacher candidates.

Methodology

Our study was carried out in the three interrelated phases. We designed the illustrated story in Phase II, based on the findings of Phase I. We then used the designed illustrated story in Phase III in the context of teacher education.

Phase I: Ethnographic study on embodied mathematics in an (im)migrant community

A year-long ethnographic study was conducted on in-school and out-of-school mathematics learning for Filipino (im)migrant community in an urban city of Japan that is increasingly becoming linguistically and ethnically diverse. In preparation for Phase II, we identified key themes from the analysis of the following data: 1) semi-structured interviews with 12 Filipina women who were living and raising children in Japan, 2) semi-structured interviews with nine elementary school-aged children. Filipina women in this study came to work in Japan and the majority (10 of 12 participants) stayed in Japan after marrying Japanese men. At the time of study, Filipino was one of the major and the fastest growing ethnic groups in Japan. Since the late 1970s, Filipina women have been coming to Japan to fill the bride shortage in farm village areas, to work as entertainers in urban cities, and, more recently, to work as nurses, caregivers and English language teachers and tutors. All of the Filipina mothers interviewed said that they were from a big family of lower socioeconomic status and that financially supporting their family was their main motivation for coming to Japan. Parental involvement in children's school learning was reported as limited, partially because Filipina women in this study felt they did not know the curriculum and pedagogy in Japanese schools. The immigration policy of Japan grants citizenship by parentage: children who are born into a family of a Japanese father or a Japanese mother are granted citizenship. Some of the child participants did not meet these criteria for citizenship and therefore were unsure whether they would continue to be granted for their residency. Because of this context, we use the term, "(im)migrant" community to refer to our research participants in Phase I.

Phase II: Designing a digital illustrated story.

The goal of this phase was to design an illustrated story based on the analyses of ethnographic observation data and interview data carried out in Phase I. Both authors repeatedly discussed the storylines and images based on our analyses and we decided to visualize a story of two migrant children (May and Ryan, pseudonyms) whose parents taught non-dominant method of mathematics (i.e., finger multiplication method depicted in Takeuchi, 2018). The theme of the story reflects key findings from Phase I about how May and Ryan experienced learning of multiplication differently at home and in school. The illustrated story showcases how these migrant students had to hide what was taught at school in fear of the teacher's gaze and their mothers' instruction not to share the finger multiplication method at school.

We decided to create an open-ended illustrated story to invite participants' storytelling and collectively created the stories that continue after the last frame illustrated. We also decided to create an illustrated story with no text in order to communicate with young learners from diverse linguistic backgrounds in our future projects. Author 2 (Shima Dadkhahfard) who has a background in both graphic design and picture book illustration used her skills to digitally illustrate the story. The design of the digital illustrated story was carried out in iterative cycles by attending to the feedback shared by the participants.

Phase III: Using the designed illustrated story

We used the illustrated story designed in Phase II in a teacher education course and facilitated the conversation about mathematics teaching, equity, and power. The Phase III of the study was conducted in a teacher education program located in a city in Canada. All the teacher candidates were recruited from a Year 1 teacher education course and they were specialized in early childhood education and in elementary school education. All the teacher candidates were English language speakers and were born and raised in Canada. A total of 22 teacher candidates participated in the study and five of them were racialized teacher candidates. Written responses were collected during in-class activity and follow-up interviews were conducted with five teacher candidates. Each interview lasted approximately 90 minutes. Interviews elicited how teacher candidates interpreted the illustrated story and whether and how they connected or did not connect with the story (as a teacher candidate or as a student). They were also asked to discuss mathematics pedagogy depicted in the illustrated story in relation to their future teaching. During the interview, teacher candidates sketched and drew images of how they would continue the story. All the interviews were audio- and video-recorded and to ensure anonymity of teacher candidates, video did not capture faces their participants (video was mostly used to capture gestures and drawings). Soon after each interview, we have written detailed field notes and interview data was fully transcribed.

Analysis

We first analysed how the participants interpreted the illustrated story and the amount of detail they paid attention to in the illustrations. In particular, we examined whether the participants attended to the illustrated differences and marginalization in classroom mathematics learning for the students and families depicted in the story (e.g., a parent is instructing the child not to share the finger multiplication strategy at school). We used this analysis to gauge how the participants attached value to the use of body parts in mathematics classrooms. We draw from Bakhtin's (1981) notion of "authoritative discourse and an internally persuasive discourse" (p. 342). Bakhtin depicts authoritative discourse as "the word of the fathers" (p. 342) that is *a prior* discourse demanding to bind and obey. It is a form of discourse that does not merge with others and remains "sharply demarcated, compact and inert" (p. 343). In contrast, internally persuasive discourse is "half-ours and half-someone else's" (p. 345). It is not finite and open to "ever newer ways to mean" (p. 346), and to be

unfolded through its contact with other voices. Authoritative discourse and internally persuasive discourse are not dichotomous; dialectical struggles between them characterize the process of our ideological becoming. In our analysis, we examined whether and how the voices of a teacher, as a student, and as a character depicted in the story, came to be in contact with each other. In doing so, we gauge the interactions between authoritative discourse and internally persuasive discourse that is more heterogeneous (Rosebery et al., 2010) and open with the possibilities for new meanings.

This paper's central focus is on how the teacher candidate participants made sense of the illustrated story that was designed based on a study on (im)migrant families' use of finger multiplication algorithm at home and at school. In this paper, we examine discourses around embodiment in mathematics classrooms in relation to the participants' interpretations of the illustrated story and their described relationships with mathematics.

Results

Authoritative discourse on embodiment in mathematics teaching

Authoritative discourse — a prior discourse that remains demarcated and solid —treated any method that involves body parts as something to be discouraged or replaced in mathematics classrooms. Such discourse was not questioned or challenged by some participants even when they engaged in meaning-making around details in the illustrated story. For example, the authoritative discourse manifested in the following quote from Alissa's interview that was constructed mostly from the standpoint of a teacher. It positions what parents taught at home with fingers (that was depicted in the illustrated story) as something to be "deconstructed" or replaced with an online resource widely used in the local school board for in-school mathematics learning. In this discourse, the finger multiplication method was predetermined to be devalued as a wrong method. The topics of emphasized parts of the following quotes are about finger multiplication method taught at home.

> I think it's very clear, like, in students when they're getting that support at home, um, in compared to when they're not. […] if you're able to supply them with the right resources (…) So for example, with math, right, um, if the parent... *for this child who's, like, being taught the wrong answer, but like, on your fingers, then giving them something like Mathletics Program,* right? That'll teach them correct methods, it'll give them lots of practice. So, I think *giving them the right resources to kind of deconstruct their teaching methods, their parent's teaching methods.*

Similar discourse was observed in Betty's interview, although it is expressed in a more nuanced manner. Instead of prohibiting the use of finger or replacing it without understanding it (as expressed by Alissa), Betty emphasized the importance of understanding why students were using their fingers. But as a teacher, Betty would teach her students that "there is more than just the fingers" by introducing other approaches. The following quote from the interview shows another instance of authoritative discourse expressed from the standpoint of teacher.

> *I think I would ask them why they think the fingers are right first.* 'Cause kind of seeing, like, where they are at, like, in terms of their understanding is really important, 'cause kind of determines how you can help them moving forward and then demonstrating, like, through other activities how they could approach the solution. So, giving them, like, a more hands-on approach and showing them, like, okay, like, let's take these blocks and group them in nine groups of seven and then we can keep telling them, like (...) kind of count and see, like, what the answer is *so that they know, like, there's more than just the fingers.*

The teacher candidates' interpretation of the illustrated story was mainly from the perspective of the future teacher and did not interact with perspectives as a student or as migrant student characters depicted in the illustrated story. Authoritative discourse was reproduced around the norms on the use of body parts for classroom mathematics learning.

Differences in participants' meaning making of the details in the illustration

For the cases where we observed authoritative discourse, we realized the lack of details in the illustrated story that the participants interpreted. In contrast, heterogenous discourse was observed when the participants engaged in making-meanings around details in the illustrated story. The details the participants paid attention to (or did not pay attention to) are summarized as follows: 1) the spatial set-up of the classroom (e.g., individual worksheet, the teacher standing in front of students, memorization of multiplication facts), 2) differences among students (e.g., colour, lines separating main characters from the rest), 3) inter-generational teaching of the finger multiplication method, and 4) the gesture of shushing by a parent and the gesture of hiding by the child.

For example, some participants made meanings of diversity and differences by paying attention to different colours allocated to main characters and their homes (that made contrast with other characters). The following quotes represents how colour schemes helped Carol to make meaning of the diversity and differences.

> I understand that the color makes them different (...) It seems that their lives are way more colorful, like, they (...) I guess now that I understand, like, the cultural aspect, like, *they're coming from different countries, they're bringing so much more, because their homes are colorful, like, their mothers have so much color. Like, they're bringing much more than just themselves.*

Figure 1: A frame from the designed illustrated story (illustration by Dadkhahfard).

Another detail in the illustrated story that was interpreted in Dana's interview was the bubble that represents the tradition of the finger method of multiplication. As she pointed out the bubble showing the tradition of the finger method in the illustrated story, Dana said, "this is a reference to the girl when she was younger. So, it must mean her mom taught it to her like that as well, so it's kind of like the traditional aspect of it." There was another layer of detail in the illustrated story that engaged Emma in a heterogenous conversation around the gesture of shushing. The following quote from Emma's interview shows how she interpreted the gesture of shushing by a parent of the main character as counting on the fingers is not acceptable at school.

> It's kind of interesting to me about the *mom shushing her here*. So, it's obviously *a pretty known thing that they don't like to count on their fingers*. Yeah, trying to tell the kid not to show them, I guess. Yeah, the *school doesn't like it.*

In the following section, we examine how interpreting and making meanings of these details in the illustrated story engaged participants in heterogenous discourse.

Heterogenous discourse to challenge the norm

Heterogenous discourse was identified when the participants engaged in interpreting and making meaning of the illustrated story not only from the standpoint of a teacher but also speaking as a student, as a girl, as the character that merged with speaking as a teacher. Carol started her conversation by stating that, "first and foremost I feel like my experience as a woman and like specially math, is like girls are not great at math and so I always had that idea that I was not good at math." She then shared her own story when she was a student at high school. She explained how she was discouraged by her high school teacher in the mathematics classroom:

> I had that experience, I would even count on my fingers doing, like, math in high school, and some high school teachers would be, like, "What are you doing? Like, that's so, like, childish. Like, you should have passed that." But sometimes it's nice to, like, physically, like, understand what you're doing.

The following quote in Carol's interview is heterogeneous in the sense that the standpoint is not merely of a teacher but also of a student. Such discourse helped challenge and disrupt normative practices in mathematics classrooms for some of the teacher candidates. Reflecting on how the negative experiences *as a student* who was ashamed of the use of fingers, Carol said:

> I think, when you've been discouraged yourself, you kind of want to, like, combat those, like (...) Yeah. And don't let some student to feel discouraged because they got the right answer, but they did it in the wrong way, because I don't feel like there's a wrong way. So, yeah, I guess just, like, building confidence in students is super important, and so, that's what I want to do. (laughs)

This quote was spoken from her standpoint as a teacher but also spoken from the standpoint as a student who used to be discouraged in using body parts in mathematics classrooms.

Such heterogenous discourse was similarly observed in Dana's interview. From *the perspective of the characters* in the illustrated story, Dana explained the important role of teacher in understanding the background and history of students and their parents in the following quote:

> I feel like if she [the teacher in the illustrated story] was able to understand where she [the student in the illustrated story] is coming from, or where her parents are coming from, she [the teacher in the illustrated story] could validate their learning by saying, 'yes, this is how you do it. As long as you have a skill and you can apply it, should be fine.

Dana's use of deixis (Hanks, 2009) indicates how she was using the illustrated story and thinking with the characters. Similar to Carol, Dana questioned normative mathematics classroom practices that could lead to students' shame of using body parts. She suggested an alternative, "it's important to allow students to represent their understanding in more than one way" as she envisioned her future teaching.

Discussion

In our study, the illustrated story was developed as a medium to evoke conversations with teachers, teacher candidates and K-12 students around the norms in mathematics classrooms and the power of such norms that could constrain equitable access to mathematics learning. In this paper, we discussed both the possibilities and limitations in facilitating the teacher candidate participants' ways of interrogating the normative practices in mathematics classrooms, through the designed illustrated story. In the teacher candidates' discourse, what Bakhtin (1981) termed as authoritative discourse was manifested, a voice of an authoritative teacher who perpetuates the norm around body and mathematics learning. At the same time, when the participants made meaning of details in the illustration, the discourses came to be heteroglossia that "represents the co-existence of socio-ideological contradictions" (p. 291) and helped the participants to question normative practices in mathematics classrooms.

Our preliminary findings call for advancing critical conceptualization of embodiment in mathematics education, that could bring forth the intertwined relationships among body, norms and power (Ahmed, 2006; Butler, 1993). The participants in this study, even when they questioned the norm, did not explicitly address the issue of race, body and power in mathematics learning. This limitation could be partially due to the fact that our illustration did not make visible race and racism. We are currently in the process of redesigning the illustrated story to evoke conversations on the relationships among race, body, norms and power in mathematics education, with a wide range of audiences including teachers, teacher candidates and parents and children.

Overall, in the context of teacher education, this paper adds to the body of scholarship in embodiment and mathematics learning by illuminating the possibilities of using the research-informed illustrated story for interrogating the norms around body in mathematics classrooms, in the context of teacher education.

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Statistical literacy of Quilombola girls: The importance of considering funds of knowledge

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Quilombola communities are ethnic groups comprised of descendants of enslaved Africans, defined by criteria of self-attribution and validation. Becoming Quilombola women involves a sociocultural construction intersected by issues of gender, class and race, which demands the mobilization of funds of knowledge to organize and maintain life in their territories. This doctoral thesis project aims to analyse Quilombola girls' actions and reflections which relate their funds of knowledge and statistical knowledge. Theoretical approach is based on Gal's perspective of statistical literacy, and critical mathematics education. The study will be conducted in a Quilombola community in Brazilian northeastern region. Methodology will explore how statistical investigative cycle approach can empower girls towards to critical read and write the world.

Introduction

Quilombolas are "ethnic-racial groups according to self-attribution criteria, with their own historical trajectory, endowed with specific territorial relations, with a presumption of black ancestry related to resistance to the historical oppression suffered" (Decree nº 4.887, 2003). In Brazil, the rights of Quilombolas are guaranteed by the Constitution of the Federative Republic of Brazil (1988), as well as Quilombola school education that is provided for the National Curriculum Guidelines (Ministério da Educação, 2013a) and other national official documents. Despite these legal achievements, the preservation of the Quilombola cultural legacy is still not guaranteed.

Our study covers the issue of anti-racist mathematics school education in a Quilombola context, based on theoretical perspectives and research studies which problematize issues related to race, power, equity and social justice in mathematics education.

The perspective of critical mathematics education is committed to social justice and democracy, relying on critical education, represented more markedly by Freire (1972/1990), who advocates the transformation of reality and oppression by oppressed people themselves, through knowledge of reality, formulations, reflection-action-reflection and dialogic interactions.

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According to Forner et al. (2017), "even though Freire did not think about mathematics when developing his legacy, several of his elements have a direct relationship with the development of mathematics" (p. 755). Thus, in critical mathematics education there are categories that are concepts examined by Freire, such as criticality, dialogue, democracy, reading the world, autonomy, which are very important for our study and essential for the work developed with Quilombola girls in the realization of an investigation in your community/family.

Emerged in the 1980s, critical mathematics education highlights the political aspects of mathematics education, suggesting that teaching should be based on dialogicity, horizontality between teachers and students and the use of mathematical knowledge in reading the world, through critical inquiries, aimed at promoting changes for students and their social contexts (Frankenstein, 1998; Skovsmose, 2007).

Our study raises questions and reflections on the relationship between critical mathematics education, Quilombola school education and anti-racist education. When we relate these three educational perspectives, we launch propositions which can confront the questionable and combatable processes of selection, exclusion and segregation common to the mathematics education of people in vulnerable situations (Teixeira, 2014), especially those belonging to the black communities, including those in which people of Quilombola ethnic identity participate.

The study will be carried out in the Onze Negras Quilombola Community, located in the Northeast of Brazil and which is marked by female leadership (Cabo de Santo Agostinho, 2007). The study aims to analyse data and reflections produced in the Quilombola girls' statistical investigations about women's daily practices to organize and maintain lives in their territories. These investigations will be carried out from stages of the investigative cycle (Wild & Pfannkuch, 1999) in the perspective of statistical literacy (Gal, 2002). The approach to data production will also be problematized by the ideas of Funds of Knowledge (González et al., 2005; Gutstein, 2007).

Critical mathematics education, funds of knowledge, statistical literacy and empowerment the knowledge of black Quilombola women

Critical mathematics education explicitly considers the political, social, democratic and emancipatory dimensions of mathematical knowledge (Frankenstein, 1998; Gutstein, 2006; Skovsmose, 2001; Valero, 2017; Valoyes-Chavez, 2017). This approach helps us to see mathematics as a mediator to be utilized by people to organize thoughts and actions, to critically understand the socio-political aspects of their lives, and to improve their own cognitive, procedural and social skills.

The perspective of critical mathematics education can potentialize an enhancement of cultural experiences, practices and knowledge for the work of the mathematics educator, whose essential principle is the knowledge built and embodied in people and communities, which Gutstein (2007) calls original and sociohistorical:

> It involves how people understand their lives, their communities, power relationships, and their society. We also mean the cultural knowledge people have, including their languages and the ways in which they make sense of their experiences. Some refer to this as "indigenous knowledge," "traditional knowledge," "popular knowledge," or "informal knowledge" (p. 110).

González (2005) argues that critical mathematics education should value the so-called funds of knowledge, which consist of bodies of knowledge, practices, skills and strategies existing in each family or community, which include mathematical knowledge. When investigating knowledge backgrounds, they associated them with teaching through an ethnographic project, which foresaw pedagogical changes in the teaching of mathematics, giving more meaning, encouragement and improvement in the quality of school learning.

In the context of our project, the Quilombola women of Onze Negras community are owners and generators of funds of knowledge, which they mobilize to organize, nurture and maintain lives. They are responsible for a vast number of practices and knowledge which are resistant to historical erasure (Carneiro, 2005; Njeri, 2019). Such knowledge cannot be ignored by the institutions that deal with those people, because such inconsideration perpetuates the epistemicide arising from colonialism, which is defined by Carneiro (2005) as:

> one of the most effective and lasting instruments of ethnic/racial domination, due to the negation of the legitimacy of knowledge forms, knowledge produced by the dominated groups and, consequently, its members as subjects of knowledge (p. 96).

Skovsmose (2001, 2007) argues that for mathematics education to comply with democratic standards, it cannot be at the service of a power system that discriminates people by gender, race and social status, undermining their empowerment for citizen action.

According to Valoyes-Chávez (2017) "racial ideologies achieve fundamental functions in the reproduction of racial inequalities in the access to mathematical knowledge and in the construction of students' mathematical identities" (p. 147). Thus, racial ideologies favour the naturalization of failures in relation to the domain of mathematical content conveyed in the teaching process of black students. In the field of mathematics education, it is essential to question how racial tensions are communicated in mathematics teaching processes.

Skovsmose (2007) emphasises that the students' socio-political environments must be considered essential for the teaching and learning processes of mathematics. That kind of educational approach might help decentralizing the hegemonic power of teachers' interests, because is based on an important democratic principle in which teachers and students play different but equally important roles in dialogue. Therefore, mathematics educators must continue "in search of ways to reconcile their classroom practice with struggles for social change" (Frankenstein, 1998, p. 116).

Gal (2002) argues that statistical literacy may be necessary for people and their communities in different ways, making them more aware of trends and phenomena of social and personal importance, as well as enabling them to face opportunities, public debates or actions in community. He proposes a theoretical model of statistical literacy involving elements of knowledge, which are literacy skills, statistical knowledge, mathematical knowledge, and dispositional elements, which relate to people's critical stance, beliefs and

attitudes (Gal, 2002, pp. 2–3). In this research project, we focus on Gal's perspective which emphasises the critical elements of statistical literacy. In addition, we will explore possibilities to challenge students to be co-investigators and question hegemonic ideologies, their contradictions and falsehoods, providing transformative learning experiences (Frankenstein, 1998). Thus, the perspective of statistical literacy is a process that can provide the basis for a critical social participation of individuals, not only with technical and conceptual knowledge of statistics, but also with subjective knowledge of individuals (Gal, 2002).

Monteiro and Carvalho (2021) state that "this perspective of statistical literacy, which promotes critical interpretations of this statistical information, is a very important social need, since it helps people to exercise their citizenship to the full" (p. 613).

In light of the statistical literacy model (Gal, 2002) and the perspective of Critical Mathematics Education (Frankenstein, 1998; Gutstein, 2006; Skovsmose, 2007; Valero, 2017; Valoyes-Chavez, 2017) we propose this study, in which girls of a traditional Quilombola community participate in activities within the investigative cycle, in which they will produce data on the knowledge of Quilombola women.

About Quilombos and Quilombola people

In Brazil, from the 16th to the 19th century, there were movements of struggle and Quilombola resistance against slavery, exploitation and the death of indigenous and African peoples. Quilombos were formed, territories organized and instituted with the objective of ensuring the existence of the insurgents. It is in this historical context that ethnic identity is also forged, which carries the power and strength to exist and resist in situations of profound dehumanization.

The Quilombola communities are comprised of people descended from a civilization which was originally prosperous in different ways. A large population that was disrooted from their mother continent, was slavered, and suffered genocide in the exile, but despite all managed to keep in their bodies, words and culture, the organization of Quilombola nations, *terreiros* (African-based collective religious spaces), movements and samba schools (Nascimento, 2002).

Currently, in Brazil and in other corners of the world, colonialism is maintained through racism that remains entrenched in social structures, which can be revealed in statistical data presented by several sources of information, in journalistic news and in the general media. Such statistical data reflect socioeconomic inequalities between different racial and ethnic groups, especially when referring to the reality of black women.

Our study is politically situated, when it turns to the reality of Quilombola women, in an investigation carried out by girls from this ethnic-racial group, as becoming a black and Quilombola woman requires learning that involves the struggle against a context of slavery, patriarchal tradition and classist, perpetuated by the oppression, exploitation and subordination of black bodies (Ribeiro, 2017).

According to Gomes (2020), when Quilombola women produce knowledge "they promote their emancipation and come to understand their vital force in the construction of

Quilombola identities, which assert themselves through daily struggles against all sorts of oppression" (Gomes, 2020, p. 11). This fight for the affirmation of their identities, for the organization, nutrition and maintenance of their people's lives takes place in different territories, with women as major agents in facing the socioeconomic problems of their communities.

Despite advances in the field of law, Quilombola communities and social movements are permanently intensifying their resistance and struggles in order to overcome challenging situations. Education and epistemic productions are important action fronts for this political purpose. In Brazil, Law N° 10.639 (2003) stands out, which regulates the mandatory teaching of Afro-Brazilian History and Culture in the Basic Education curriculum, the National Guidelines for Education in Ethnic-Racial Relations (Ministério da Educação, 2013b), the Guidelines for Education Escolar Quilombola (Ministério da Educação, 2013c), the Statute of Racial Equality (Law 12.288, 2010), among other laws and policies carried out by the "Movimento Negro Educador" (Gomes, 2017).

It is known that every community possesses beliefs, practices, and knowledge, including mathematical knowledge. In this sense, we believe that there is a long history of deletion, silencing or intentional epistemic neglect of the Quilombola knowledge funds, which we seek to address in this research.

Research paths

Quilombola communities are historically sources of knowledge and sociocultural practices, while resist and struggle against domination, exploitation and scarcity of material resources for subsistence. Therefore, the "Onze Negras" Quilombola Community, whose leadership is female, is also based on this same purpose of organization, maintenance of its members and combating different forms of injustice, through the mobilization of ancestral knowledge cared for by older people and respected by the youngest.

The project will propose to a group Quilombola girls to be engaged in an investigative cycle process (Wild & Pfannkuch, 1999; Santana & Cazorla, 2020) which will produce statistical data related to funds of knowledge mobilized daily by Quilombola women to maintain life in their territory (Figure 1).

The research method will involve three main procedures. Initially, it will be developed analysis of educational documents which guide the teaching of mathematics and statistical topics at local Quilombola school. This documental analysis aims to identify which pedagogical approaches are suggested, as well as how are related to Quilombola culture and alignment with the curriculum guidelines for Quilombola school education (Ministério da Educação, 2013c).

A second methodological procedure will be associated with semi-structured interviews carried out with voluntary Quilombola women participants to identify funds of knowledge which may possibly involve the investigation carried out by female Quilombola students in the experience of the investigative cycle. One possible topic might be about food and traditional recipes. For example, the Mesa Ancestral (2019) published a report on traditional

Quilombola gastronomy in which a leader of Onze Negras community explained the knowledge taught by older Quilombola people to overcome hunger, as well as to enjoy food and to promote health:

> When I was a child, for example, I learned that satisfying hunger depends on knowledge and getting hands-on. (...) We were worth more than there was in the garden, so it was quite common for my grandmother to take advantage of everything there. (...) Our great-grandparents taught us a lot that most people who live in big cities only hear from a doctor, for example, put crushed eggshell on top of the food (Mesa Ancestral, 2019).

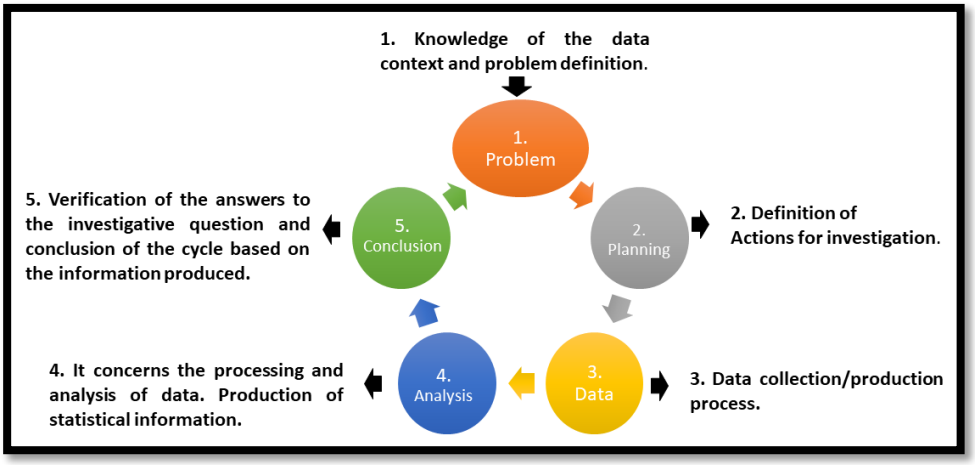

Figure 1: Investigative Cycle. Adapted from Wild and Pfannkuch (1999).

A third procedure will be related to pedagogical meetings between the researcher with Quilombola girls to reflect and experience the statistical investigative cycle. The topic to be investigated will be related to cultural communitarian practices. The girls will be involved in a cooperative problematization in order to choose such a topic. The statistical data collected by the participants will be analysed and represented with the collaboration of researchers.

As part of research data analysis, we will identify and problematize knowledge and dispositional elements of statistical literacy (Gal, 2002) mobilized by the group of girls. These reflections will involve content analysis carried out by the researchers. The main results and reflections will be socialized with the community.

Preliminary considerations

Our study in progress has driven us to more questions and more reflections in mathematics education field, especially when we establish a relationship with anti-racist education and Quilombola school education. What mathematics education is being practiced in the classroom, from an anti-racist perspective? What parameters, references and educational

practices in mathematics education are being built, taking into account education for ethnic-racial relations, provided for in Law 10.639 (2003)? Does the mathematics education experienced in Quilombola schools respect and reflect the culture of these people as provided for in the educational guidelines for Quilombola school education?

We know that cultural rights are integral to human rights. When access to cultural assets and the historical past of a people is denied, the relationship of domination and the practice of violation of human rights is perpetuated. Now, for Quilombola people, access to the past and ancestral knowledge means reflecting on oppression and claiming for rights and reparations.

Therefore, we believe that Quilombola girls, when reflecting on the statistical information produced in their investigative actions on the funds of knowledge and Quilombola women's sociocultural practices to organize and maintain community life, will possibly present dispositional elements related to racism, sexism and the ideas of emancipation. We also believe that critical statistical research, from the perspective of statistical literacy, is one of the possible ways of strengthening students from ethnic-racial communities, which, together with other educational ways, contributes to social change.

With our study, we also wish to contribute to the debate with the community of researchers in mathematics education, suggesting a critical analysis of the tensions historically built-in power relations, knowledge – especially mathematical knowledge – and Quilombola ethnic-racial communities.

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Drawing upon Mi'kmaw pedagogies

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This paper draws upon conversations with Mi'kmaw (Indigenous) elementary school teachers in a Mi'kmaw controlled school in what we now call Nova Scotia, Canada, as they reflect upon their students' drawings of math class. As part of a larger research project, we invited Mi'kmaw students to draw pictures of themselves doing math, and pictures of themselves in math class. These drawings served as a prompt for their teachers to reflect upon the pedagogical practices revealed in the drawings. The resulting conversations provide an illustration of the pedagogy these teachers enact in their classroom spaces and their tacit beliefs and values about learning. The discussions with these teachers reveal the deeply relational pedagogy that is an integral part of this school community striving to decolonize education.

Introduction

We could never have anticipated the insights offered by classroom teachers during a recent circle conversation in their school, a Mi'kmaw controlled community school in Unama'ki (Cape Breton Island, Nova Scotia, Canada). The teachers were part of a larger research study examining ways to decolonize mathematics teaching and learning through centering Mi'kmaw ways of knowing, being, and doing—L'nuita'simk. We invited them to attend an after-school session to discuss the drawings that their students had made of their mathematics classroom. In one drawing, students were asked to illustrate themselves "doing math" and, in another, to draw "my math class". The students from kindergarten through to grade 4 were given the same drawing tasks. The drawings were intended to make the classroom visible through the children's eyes and to help teachers and researchers see what students find meaningful in learning mathematics. What we did not anticipate was the rich conversation that arose when teachers were asked to comment on the drawings and what level of insight the teachers provided for their teaching and learning. This article describes the unfolding teacher conversation through an enactivist lens where individual and collective knowings co-evolve and emerge simultaneously. Their pedagogical practices revealed in the discussion highlight the ways in which they are working to create a truly Mi'kmaw space for learning.

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The MATH project

The conversation circle described in this paper is a piece of a larger research project called Moving Achievement Together Holistically (MATH) that draws upon Lunney Borden's (2010) framework for decolonizing mathematics teaching and learning for Mi'kmaw children. The framework considers elements that can support Mi'kmaw children in mathematics learning through meaningful personal connections that draw from the importance of cultural connections, connecting learning experiences with community values, rooting pedagogical practices in Mi'kmaw ways of learning, and drawing upon the structures of Mi'kmaw language to inform and transform mathematics teaching and learning. The MATH project particularly focuses on implementing pedagogical practices and learning experiences that are rooted in L'nuita'simk that draw from the framework's focus on language and values. Two key guiding principles inform the work: verbing mathematical experiences and rooting experiences in spatial reasoning. In this work we are interested in learning how the implementation of this framework can support Mi'kmaw students' learning and achievement while also attending to their developing identities as Mi'kmaq.

The school we worked in for this aspect of the project, is a Mi'kmaw community run school that is part of the Mi'kmaw Kina'matnewey (MK) collective. In Canada, Indigenous education is a federal responsibility even though all other education falls to provincial authorities. Historically each individual Indigenous community had to negotiate education agreements with the federal government. In 1997, the Mi'kmaw communities in Nova Scotia formed MK as a way to collectively negotiate education agreements, giving jurisdictional control to each community while also having collective shared services to advance the educational interests of all communities (Paul et al., 2017). While MK schools are quite successful with respect to high graduation rates, there is still considerable work to be done to improve mathematics achievement. Our work aims to address an MK identified goal of increasing student achievement and supporting teacher professional learning in the area of numeracy.

Decolonizing education

We make a conscious decision in this paper to not focus on deficit discourses that point to "gaps" for Indigenous children in Canada or the negative impacts of colonial systems of education (Gutiérrez, 2008). We take as shared that these issues exist, that the impacts of colonialism are real and devastating, and that mathematics teaching and learning remain complicit in the colonial project (Nicol et al., 2012; Stavro & Miller, 2017). We choose instead to focus our attention, as we do in our research, on how we can identify and describe healing spaces where Indigenous children can thrive.

We adhere to the idea that asset-based approaches to mathematics education support and affirm children's identity and help them to develop as doers of mathematics (Celedon-Pattichis et al., 2018). Teacher beliefs about children are key in an asset-based classroom where high expectations and cultural affirmation are integral to children's success (Castagno & Brayboy, 2008; Ladson-Billings, 2014).

Battiste (2010), a Mi'kmaw scholar, describes learning from an Indigenous perspective, as a process of nourishing the learning spirit stating:

> What guides our learning (beyond family, community, and Elders) is spirit, our own learning spirits who travel with us and guide us along our earth walk, offering us guidance, inspiration, and quiet unrealized potential to be who we are. In Aboriginal thought, the Spirit enters this earth walk with a purpose for being here and with specific gifts for fulfilling that purpose. In effect, the learning Spirit has a Learning Spirit. It has a hunger and a thirst for learning, and along that path it leads us to discern what is useful for us to know and what is not. Our individual gifts for fulfilling our purpose are expressed in ourselves, in our growing talents, and in our emerging or shifting interests. These gifts often manifest themselves in surprise and in joy. That time of learning has often been called a 'wondrous' time and lasts a lifetime. (Battiste, 2010, p. 15)

Our team

Our team has a long-standing relationship with the school and the teachers in the school, as well as with the children. They are very familiar with seeing us in their school and get excited when we bring fun math activities to work on. Lisa, of settler descent, previously taught within this school and many of the teachers are her former students. Kyla, who is Mi'kmaw, serves as a mathematics coordinator and coach for MK. Ellen, of settler descent, worked as a research assistant and outreach coordinator with Lisa for years before taking a faculty position, allowing her to interact with the teachers and the students for years both within and outside of school. Evan, also a settler, became part of the team when he joined the Faculty of Education at StFX and quickly became connected to the school. All but one of the teachers are Mi'kmaw and from the community. They all completed their Bachelor of Education degree at StFX and several were enrolled in or had recently completed graduate degrees and programs at StFX. The one non-Indigenous teacher had been working in various roles in the school for years. While all four authors were present on the day we collected the drawings, only Ellen and Lisa were available to discuss the drawings with the teachers which they did several months after the drawings had been collected.

Methodology and theoretical framing

For this work we draw upon Indigenous research methodologies (IRMs) integrated with complexity theory as we engage in collaborative meaning making alongside teachers. Indigenous research methodologies are rooted in a desire to decolonize research and honour community voices in a way that challenges typical approaches to research (Smith, 1999). Typically, IRMs focus on the Rs of research, namely notions of respect, relevance, responsibility, and reciprocity (Kirkness & Barnhardt, 1991), as well as reverence and relationality (Archibald, 2008; Kovach, 2009). It is important to recognize that our work is made possible by the deep connections we have to the community and the school, as well as an on-going relationship rooted in reciprocity. Our work in this space is guided by the Mi'kmaw concept mawikinutimatimk (coming together to learn together) that invites all participants to bring

their gifts and knowledge to the circle knowing that we all have things to share and things we can learn from one another (Lunney Borden & Wagner, 2013). We find alignment between our IRMs rooted in mawikinutimatimk and complexity theory.

For the purpose of this paper, we draw upon complexity theory which provides a fitting theoretical framework from which to view this research and, specifically, the teacher conversation that forms the body of data for analysis. Mason (2008) notes that complexity theory "concerns itself with environments, organizations, or systems that are complex in the sense that very large numbers of constituent elements or agents are connected to and interacting with each other in many different ways" (p. 6). Our classroom-based research brings together students, teachers, administrators, and researchers in complex interaction as co-evolving and co-implicated agents, intent on clarifying individual conceptions as much as seeking collective knowledge about what it means to learn mathematics. Complexity theory supports our view that these ways of knowing "take shape simultaneously" (Davis, 1996, p. 5) and problematizes the distinction between teacher and learner as each play an important role in knowledge creation. Davis and Simmt (2003) explain that complexity theory focuses on a range of nested learning systems, which "includes the co-implicated processes of individual sense-making and collective knowledge-generation" (p. 142). This is of particular relevance to this study as we look to children's drawings first, as insights from the individual into what it means to learn mathematics; and subsequently, to teachers' interpretations of these drawings as insights into their classroom and pedagogy.

As Kieren (1995) asserts, our research

> must trace the patterns of mathematical activity and understanding as it occurs, must look at the mechanisms and beliefs by which persons act mathematically, must attempt to account for the ways in which the environment occasions or creates space for personal mathematical activities and must account for the interactions and conversation through which mathematical activity occurs and by which it is bounded.

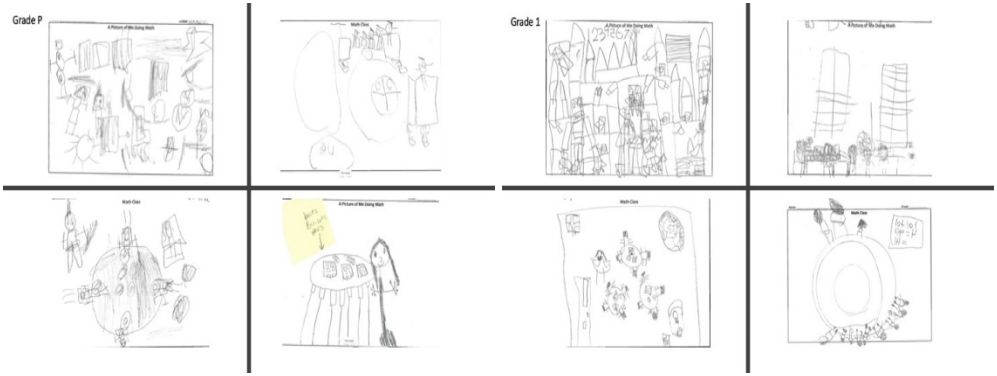

Figure 1: Grades Primary and 1 drawings

Inviting the teachers to join us in the analysis of children's drawings of "My mathematics class" and "Me doing mathematics" through conversation provides insights into the types of

mathematical activity enacted by students and teachers, the environment created by the teacher and within which the students engage in mathematical activity and their classroom interactions. Through this process, we begin to reveal the pedagogical values of the teachers and their shared understanding of what mathematics teaching and learning looks like in their school.

On the day we collected the data analyzed in this paper, Lisa and Ellen had been to the school to work in the classrooms with teachers, a part of the reciprocity of the work. The teachers met with Lisa and Ellen after school to view the drawings as a group and discuss what was seen in the drawings. There were five teachers and the vice-principal (VP) present. We will refer to the teachers as Teacher P who taught grade Primary (Kindergarten), Teacher 1 who taught grade 1, Teacher 2A and Teacher 2B who taught the two grade 2 classes, and Teacher 3 who taught grade 3. Only Teacher 2B is non-Mi'kmaw, all other teachers and the VP were from the community. We spent about 30 minutes reviewing the drawings from the grades P to 3 classes, as teachers commented on what they saw in the drawings. Figures 1 and 2 are selected samples of drawings from the children by grade level. We then invited the teachers to share their ideas about what the collection of drawings revealed for them. In the next section we share some of the teachers' insights.

Figure 2: Grades 2 and 3 drawings

Teacher insights

We noted several themes in what the teachers discussed as they reflected upon the drawings. These included the ways in which their pedagogical choices, in particular the use of learning centres, demonstrated the importance of play and movement, the need to develop students' independence, and the need to honour children's ways of knowing and learning. After we had viewed the drawings we invited teacher reflection. The following transcript is their initial reactions.

The teachers talked about how they made use of learning centres in their math class as a practice that established a clear routine for the children and provided the sort of pedagogical approach that allowed them to support students while building independence.

Lisa: So, we're just wondering what you see in them, or if you notice anything, or if anything stands out to you.

Teacher P: We all do centres.

Lisa: We all do centres, for sure.

Teacher 2B: They seem happy. Like no one seems sad in the pictures.

Lisa: Yeah.

Teacher 1: Busy, when I think they're not learning anything, [giggling] they like it though. I find they learn a lot more at centres than anything. Different ways. They like different stuff; they like to make stuff, the grade ones, different kinds of centres that spark their interest.

Teacher P: Well, AYR very quiet then math is very loud and fun. (laughter)

Lisa: Your class definitely, Teacher P, talked a lot about like, playing with Lego and BeeBots and they didn't actually even understand what we meant by "math." Cause they were just like, like so we had to kind of prompt them, right? Remember like, to think about okay well, what kinds of things do you do when you're learning about numbers or shapes or, and then they got into talking about coding the BeeBots and playing with Lego, but they actually didn't really have a concept of "math"… which is beautiful. (laughter)

Teacher P: They don't think they're doing math, but we are. Like even playing with the, when you saw the tables like partitioning, you see three, you see the two and one. (pause) And there's a lot of movement.

Teacher 2A: Yeah, centres stood out for me.

In this initial conversation of the pedagogical routine of using centres, we see how the teachers regard this as way to learn through play and to allow for choice and different ways of engaging with mathematical concepts. Both Teacher P and Teacher1 share how they want students to learn through playing and through exploring a variety of tasks, without even recognizing that it is mathematics. Later, Teacher 1 asked Lisa if this is something that happens in other schools. When Lisa shared that she often gets resistance from teachers when doing PD because some teachers see this approach as requiring too much work, the teachers seemed surprised that anyone would believe that. Lisa asked what it was about learning centres that they enjoyed as it was clearly a whole school approach that the elementary teachers were employing.

Analysis

In reviewing these pieces of the transcript, we want to highlight some pieces that we believe highlight the pedagogical values we see as we spend time in the school. In particular, we note that teachers have a shared approach to their pedagogy, that they believe in their students and strive to support their learning in diverse ways, that they value play and active engagement in learning, and that they work to make school an extension of home and community that embraces a Mi'kmaw way of being.

A shared pedagogical approach

There is considerable consistency amongst the teachers with respect to what they value when teaching mathematics. The use of learning centres is consistently implemented from the earliest days in school and continued throughout the elementary years (K to 4). We are aware that this vision did not happen by accident, the leadership team at the school made a conscious effort over several years to build the capacity for teachers to work in this way. This involved professional learning opportunities, restructuring physical space, and working collaboratively on this vision. It is clear that the teachers understand this to be their way of working in this school and their efforts are rewarded as they see the benefits to their students. In fact, it is so a part of how they work that they question how anyone could work in any other way.

They do not see planning for this sort of active engagement as a burden, rather they see this as the most beneficial way to engage all learners and provide a variety of experiences to support the many ways in which children might want or need to learn. Often, they talked about how this approach allowed them to work one-on-one with students or in small groups to build conceptual understanding while the other students were engaged in centres that they could do independently.

> Teacher 2B: I like if you're teaching something harder, you don't – like today I was doing regrouping. And there's no way that I could have done that with all of them right? So, I just had the kids at the table I was at. So, all the other ones, may have been doing something they could do I was okay with just focusing on these kids. I can't be over here but, everyone is going to get their turn with me here eventually.

They saw this pedagogical approach to supporting students as a crucial part of their work and believed that doing centres allowed them to effectively support students. At one point, the VP shared an experience from her practicum in another school, where the mentor teacher refused to allow her to do centres, arguing that the students would not be able to do them. Hearing this, the teachers pushed back against this idea, arguing that resistance comes from not having tried this approach, because they believed that if other teachers tried it, they would love it.

> Teacher P: I think just getting up and moving is really good for them. They're not in one spot learning and they learn in so many different ways, right? So, when you do centres, you can touch those different ways of learning, especially the hands on and then.
>
> Teacher 2B: And it's a change, it's not like the same thing all afternoon.
>
> Teacher P: And they know to get up, go to the right direction, the routine, I guess.
>
> Teacher 2A: It's really routine.

Furthermore, Teacher 2B pointed out that "they've been doing it since Kindergarten" and Teacher 2A pointed out that with this approach "the kids are managing themselves," which is rooted in a fundamental belief that their students are competent and can be independent learners.

Believing in students

Throughout the conversation, teachers repeatedly talked about the "different ways" in which students were invited to engage in learning. Over and over again, they came back to this idea and it was evident that they saw this as a way to allow students to find their own strengths through these different types of engagement. In the transcript excerpt provided above, it was acknowledged that the teachers' planning for students included "tapping into all the intelligence and stuff we have" and "you can touch those different ways of learning", built on a belief that all students can learn mathematics demonstrating their high expectations for students. The view of students as competent and independent learners was further exemplified in statements and agreement about students thriving within the centre routines: "they know what to do". As the teachers viewed and made sense of the drawings, they repeatedly highlighted the students' skills and connections, as well as the retention of mathematical content and recollection of activities.

Value of play and active engagement

There is a sense of joyfulness in the ways in which teachers talked about the learning experiences in math, as Teacher P stated, "math is very loud and fun!" The fun they describe often is connected to a belief that math can be learned through play. They expressed the many ways in which they chose to design tasks for centre activities that involved play and offered that students can come to know through play stating "they don't even know they're learning" and "they don't think they're doing math, but we are." This approach evokes Battiste (2010) notion of joy and wonder as key aspects of Indigenous learning. Indeed, the laughter and camaraderie throughout the conversation paralleled the joy teachers strive for in their classrooms as well.

Tied to this sense of wonder and joy is also a respect for children's desire to move and explore the world. As Teacher P stated, "there's a lot of movement" in the drawings, reflecting the importance of movement in the classroom. This again contributes to the joy children experience in the classroom.

L'nuita'simk is home

The sense joy is evident in the faces of children in their own drawings. Teacher 2B noted that everyone looked happy and this is something that we, the authors and many of the teachers took for granted. It was only when it was pointed out to us by others while sharing this work at a research conference that it was made remarkable. Happiness is what we see every day in the school. Happiness, laughter, joy are all a part of what it is to be Mi'kmaw and this school exemplifies this daily. As we analysed this data and engaged in the writing of this paper, Lisa shared a story from earlier in her academic life when she was discussing her recently completed Master's thesis in 2001 with her grade 11 students. She recalled asking "What is it that makes our school different?" to which one student shared, "Miss, it's home!" Decades later, this school is still working every day to create this sense of home for their students. This is a key part of L'nuita'simk.

Concluding thoughts

Through reflecting together on students' drawings, we have drawn out the pedagogy that exists in this Mi'kmaw elementary school. As systems grapple with ways to improve learning outcomes for Indigenous youth, to Indigenize and decolonize classrooms and programs, we see in this example a place to truly be Mi'kmaw. So often, we are asked to talk about what this looks like and the expectations are frequently rooted in stereotypical views of Indigenous Peoples. It need not be that way. By beginning with asset-based beliefs about children and a worldview rooted in community ways of knowing, being and doing – L'nuita'simk - we see that it is possible to transform education for Indigenous children in a way that celebrates who they are and imagines who they might be.

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Materials for teaching mathematics to upper secondary migrant or minority students

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The multicultural nature of society influences education in many countries. Support for teachers is available, but mostly for languages and social sciences. Therefore, many mathematics teachers feel a pressing need for materials supporting them in teaching in multicultural classrooms. A lot of mathematics teaching materials exist for different settings, though most of those do not take migrant students or multicultural settings into consideration. Very few have been created specifically for migrants, but those are mostly for younger students in elementary grades. Following the guidelines developed in a Czech-Austrian collaboration project, we therefore want to propose mathematics teaching units for upper secondary migrant students.

Theoretical background

A lot of mathematics teachers work in socially and ethically heterogeneous classrooms. Many of them feel not sufficiently prepared to deal with a classroom context of pupils having a migrant or minority background, coming from countries or parts of the society with different cultures and different languages (Moraová, Novotná, & Favilli, 2018). Mathematics teachers in all grades feel the necessity of training and materials which reflect the needs of their students in terms of linguistic and cultural differences. There is a fairly large body of literature dealing with language issues in teaching mathematics to migrant students (e.g., Prediger et al., 2018; Gutiérrez, 2002), or with various teaching strategies (e.g., Moschkovich, 2010; Bose, 2021). There is – thankfully – also a growing body of literature advising researchers, curriculum authors, textbook developers etc. about taking an equity approach that does not stop at language, but takes cultural, socio-political and gender issues into consideration as well (e.g., Gutiérrez, 2013; Leder, 2019; Moschkovich 2013). A good overview of that literature has been provided by Civil (2020) and Barwell et al. (2016), who also clarify the terms and outline various research directions. Despite that large body of literature, not a lot of actual concrete teaching materials exist in this area, and what is there is mainly for younger students in elementary and lower secondary grades. So it is particularly important for upper secondary mathematics teachers to be able to design suitable teaching units (or modify existing units) that allow students to see reality from different perspectives, and also to develop greater self-knowledge and awareness of cultural differences and how they can

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advance one's own learning. Such units should be carefully designed, following the principles of culturally responsive teaching and equity pedagogy. Novotná, Ulovec and Moraová (2020) developed criteria that can be applied to such teaching units for them to be appropriate to be used in culturally heterogeneous classrooms. These criteria ask for teaching units to (inter alia)

- foster tolerance and help to overcome linguistic and cultural differences,
- be of interest for both minority and majority pupils,
- value diversity,
- refer to environments familiar to all pupils,
- make use of different cultural backgrounds as "funds of knowledge",
- use a variety of contexts, names and places from multiple cultures to allow children to better be able to relate to them,
- avoid cultural stereotypes and prejudice,
- use culturally responsive teaching and equity pedagogy.

Even though mathematics is often seen as abstract matter, particularly to upper secondary students, it could become somewhat closer to them and their reality by using experiences from their own lives. The use of everyday objects like ornaments or electric appliances gives the subject an affective aspect and an emotional dimension not to be neglected, and can be a source of increased creativity.

The concept of Substantial learning environment (SLE) was introduced by Wittmann (1995). It is an educational environment meeting several criteria: It has a simple starting point; it represents objectives, contents, and principles of teaching mathematics at a certain level; it is a rich source of mathematical activities; it is flexible and can be adapted to the special conditions of a classroom; it integrates mathematical, psychological and pedagogical aspects of teaching mathematics.

SLE forms a rich field for empirical research. The presented paper focuses on one aspect of the educational domain. It shows that SLE supports teaching in culturally and socially heterogeneous classes and students' and teachers' creativity. SLE also motivates students to understand the importance of mathematics for all domains of life regardless their heterogeneity. The piloting of the presented teaching units (amongst a number of other ones) justifies that it has the potential of meeting the needs of students attending schools at present with their cultural and linguistic diversity. The use of objects from everyday life gives mathematics an important affective dimension.

Substantial learning environments also support creativity in mathematics lessons – both on the teachers' and the students' part – and can be used to create teaching units for multicultural classrooms. Being given the cultural background, teachers are motivated to use their creative potential to look for the mathematics that can be discovered and taught in the particular environment.

Methodology

Following the above-mentioned criteria, we propose materials for three teaching units in mathematics for culturally heterogeneous classrooms in upper secondary school. We shall describe the first unit in depth, and give a short synopsis of the other units.

For the development process of the list of criteria described above, we refer to Ulovec, Novotná and Moraová (2021) and Novotná, Moraová and Ulovec (2021). The development process of the original teaching units on which our proposed units below are based is described in depth by Favilli (2015). There, a three-step development was followed: Each teaching unit was proposed and piloted by one group of authors (a team of linguists and mathematics educators), tested unchanged by a group of mathematics teachers and teacher educators, then modified, and the modified version was piloted again by yet another group of mathematics teachers and teacher educators. Since we based our proposal on three units that went through such a rigorous development process, we did not feel the need to go through that whole process with our units again. We did take the three original units, modified them according to the above-mentioned criteria, and then piloted them in a regular school setting, as described in detail at the unit below.

Teaching unit example 1: Parabolas to cook with

Introduction

This unit is based on the idea that ecology and the environment are topics of interest for students of many cultural backgrounds. Also, it can be a good activity to start a discussion about why not everywhere in the world people can prepare food in the way and with the means that we are used to in so-called industrialized countries. The concept of this unit was developed by Pointner (2016).

The unit is intended to be used for secondary school grade 7 (age of students: 17 years). It can either be used as a 1 lesson activity (to introduce parabolas and some of their properties), or as a 2-3 lesson activity if the proofs are also included.

Description of the teaching unit

The unit can start with a discussion activity to think about cooking with solar energy. Either the students come up with a picture of a "solar cooker" (e.g. from the Internet), or the teacher introduces the picture from the teaching materials. The students can then discuss about why this "solar cooker" actually works.

The remaining lesson is mainly a teacher-centred activity where certain properties of a parabola are introduced. Depending on the time, interest of students, and mathematical level of the class, the proofs can either be worked on by the students, introduced by the teacher, being used as study material or exam preparation, or skipped entirely.

The lesson can end with a brainstorming activity about other applications of parabolas or paraboloids (satellite TV, dishes for communication and radar, etc.).

Teaching materials for this unit

Introduction

When you think about cooking with solar energy, you probably imagine something like this:

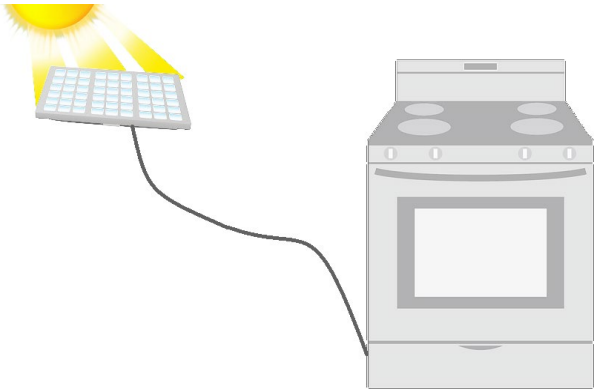

Figure 1: Oven powered by solar energy (drewn by ourselves using MS Paint)

Solar cells (photovoltaic cells) are used to convert sunlight into electricity, which can then be used to power electric devices such as ovens. In many places of the world, however, this is not a practical or affordable solution.

A much simpler way of cooking with solar energy looks like this:

Figure 2: Solar cooker (picture by Solar Power Nepal e.V.; this society is dissolved by now)

But how and why does this work? Let's look at it from a mathematical point of view.

This device, in its basic form, is a paraboloid. A paraboloid, in turn, is a surface that can be thought of as a generalized version of a parabola. We will come back to this a little later. Let's first have a look at the basic form of a parabola.

What is a parabola?

Definition: A parabola is the set of all points in a plane that have the same distance from both a given point F and a given line l. The point F is called the *focal point*, the line l is called the *directrix*.

$$par = \{X \in \mathbb{R}^2 \mid \overline{XF} = \overline{Xl}\}$$

This definition does at first glance not give us a lot of insight into what a parabola actually looks like. It does, however, give us a way to construct a parabola: We start with a line l and a point F (which is not on the line). We call the distance $\overline{Fl} = p$. p is also called the *parameter of the parabola*. Then we pick an arbitrary number $d > \frac{p}{2}$ and draw a parallel line with a distance d from l. We then set the compass to distance d, put the needle point into F, draw a circle, and intersect it with the line that we just drew:

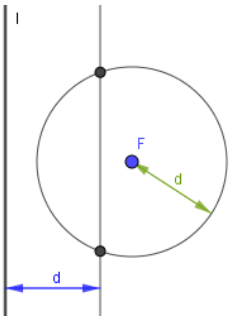

Figure 3: Parabola construction, step 1

We have now constructed two points that have the same distance from F and l, i.e. by its definition, two points of the parabola.

If we pick other arbitrary values for d with $d > \frac{p}{2}$, we can get more points of the parabola. The finished construction looks like this:

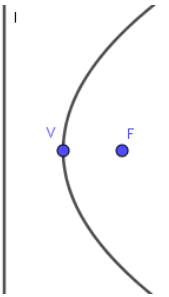

Figure 4: Parabola construction, step 2

The point V which is closest to the line l (and exactly half way between F and l) is called the vertex.

Properties of a parabola

We have not started cooking yet! Let us therefore see what happens if a ray (e.g. of sunlight) is coming in from the open side of the parabola, parallel to the x-Axis, and is reflected at the inside of the parabola (which means it is reflected at the tangent to the parabola at the point where the ray hits the parabola).

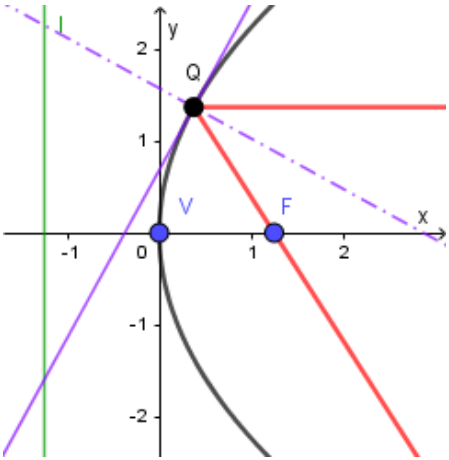

Figure 5: Reflection of a ray of light at a parabola

The reflected ray passes directly through the focal point F. This is true for any ray coming in parallel to the x-Axis. If many parallel rays come in, the situation looks like this:

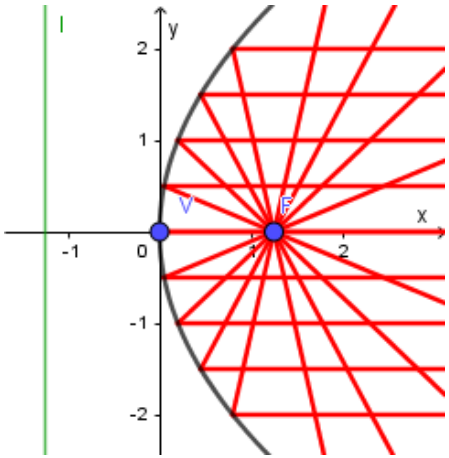

Figure 6: Reflection of numerous rays of light at a parabola

From a parabola to a paraboloid

Now of course we cannot cook with a parabola, which is a curve. But we can get a surface by rotating a parabola around an axis of rotation, thereby forming a so-called surface of revolution. If we take a standard parabola and rotate it around the x-Axis, we get a paraboloid, which also has the property that all rays coming in parallel to the axis of rotation are reflected into the focal point. Now we can take any reflective material (sheet metal, tinfoil, …) and form this surface with it, and we get a mirror that reflects all the incoming sunlight (and thermal radiation) into one point. If we place a pot or other cookware in the position of the focal point, the content gets heated. This is exactly how a solar cooker works. Depending on the actual construction (and of course if the sun is shining), a solar cooker can reach temperatures of 150 °C to 180 °C.

Piloting of the teaching unit

Given the current situation, the unit had to be piloted in a distance learning setting. This was done in a culturally heterogeneous 7th grade class (11 out of 24 students had a first language that was not the language of instruction) in Vienna by a teacher with 5 years teaching experience. Students were asked to find videos about solar cookers in the internet, and a surprising variety of videos were found, which provided ample material for discussions. The links to all videos were put on the Moodle page for the class. Some of the videos were shown during the teaching unit. The unit was then conducted as planned and described above (in distance learning mode using Moodle). After the teaching unit, online interviews were conducted with the teacher and with 3 students (two of which had a migrant background). The teacher reported that students were very motivated during this unit, and that lively discussions (in the chat) took place. Students particularly debated about why electricity is not sufficiently provided to certain parts of the world, about fair distribution of resources, and about how this method of cooking is done for fun by "western" nationals e.g. in Canada or the USA, but as a survival strategy in other parts of the world (as demonstrated by some of the videos). The teacher also reported that students had difficulties with going from the parabola in two dimensions to the paraboloid in three dimensions, and that she would recommend covering this step more in detail than it currently is. Student A (female, with Turkish roots) found the topic very interesting, and particularly liked the fact that students were asked to look for videos ("not something we usually do in mathematics", she commented). She also commented that the graphic representations that were given during the unit (in GeoGebra) helped her a lot to understand the different steps. Student B (female, no migrant background) enjoyed the discussions in the chat but regretted that the discussion could not be done in a presence setting ("in normal teaching", she said). She also reported having difficulties with "going from the plane to the space just like that", but enjoyed that "we could see that math can be used, like, for real". Student C (male, with Afghani roots) was very engaged during the chat discussion, and particularly wondered why the "fun cooking" videos from USA always showed male cooks while the "real life daily cooking" videos from Africa or Central America showed female cooks only. He suggested having a discussion

about gender roles in a future lesson. He commented on enjoying the brainstorming activity at the end of the lesson where he provided some of the inputs about other real-life applications of paraboloids.

In general, the piloting showed that the motivational effect of the material was fairly high both for migrant and non-migrant students, the hoped-for equity discussions actually exceeded our expectations, but that the step from two to three dimensions has to be improved and be gone through in more detail.

Teaching unit example 2: Ornaments

The simple starting point in this case is a number of ornaments whose origin is in different cultures, with the intention of allowing minority pupils to be heard, to present ornaments typical for their culture or home, to break the wall between home and school, between mathematics naturally used at home and mathematics used at school (Meaney & Lange, 2012).

Ornaments undoubtedly meet the criteria of a substantial learning environment. They offer a rich source of mathematics (they can be used when teaching tens of different topics and areas) and at the same time allow the introduction of culturally heterogeneous contexts, show that very different cultures have very different but equally elaborate and intriguing ornaments, and provide space for creativity – drawing ornaments, tiling, making tessellations, bringing photographs and images from home and using them as a background to mathematics and art activity. For details, see Moraová, Novotná and Favilli (2018).

Teaching unit example 3: Magic squares

The aim of this unit is to have the students work at the same time on decimal numeration and on the use of the language of instruction, in writing and speaking in mathematics, both in terms of vocabulary and explaining one's reasoning. It is also to allow verbal exchanges about written and oral numeration used yesterday and today in various countries and to highlight the input of other civilizations to the construction of mathematics in Europe. This unit is based on this ancient magic square discovered in 1956.

The proposal concerns arithmetical concepts. The work context refers to the decrypting of an old magic square discovered in China. The activity aims at revisiting already known concepts in order to improve students' knowledge and to explicitly express certain fundamental properties, as well as at developing symbol sense. In other words, it promotes a metacognitive analysis of concepts in Arithmetic to clarify the relation "natural number – symbolic representation", and therefore the relation "digit – position – value" and finally the "order of magnitude" concept. Hence it faces the cognitive obstacle corresponding to the epistemological obstacle when passing from number concept to numeral concept. In addition, the proposal gives rise to significant opportunities for introducing both the demonstration activity and the algebraic thought, and for thinking on geometrical questions. The learning context is suitable to promote the development of language skills in understanding and writing a text and in explaining plans, strategies and solutions. For details, see Favilli (2015).

Summary and outlook

The piloting described in this paper and other results already published (Novotná, Ulovec, & Moraová, 2020) showed that these and other teaching units based on the inclusive criteria list above are motivating to both migrant and non-migrant students, and that they also elicited debates in the classroom about equity that went beyond the mathematical content of the lessons.

There have been several projects focusing on the effects of linguistic and cultural heterogeneity in classrooms from different perspectives. But the question of suitable materials for multicultural classrooms in the school practice, their recommended form, availability etc., still remains. To get to more useful answers, following the recommendations of Alrø, Skovsmose and Valero (2003), and following the structure of Barwell et al. (2016), we plan to analyse real teaching episodes that are carefully planned together with the involved teachers, observe them in multicultural classrooms, and include interviews with school leaders, teachers and students on a broader scale. Also, we will continue to explore possible ICT support (which we started doing in Ulovec and Novotná 2021).

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In between feminisms and contradictions and inventions and mathematics education

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In this paper, we come up with some problems and discussion related to feminism, contradiction, inventions and mathematics educations. Our attempt is to create some sparkles/contradictions/ questions without answers through hypotheses in which mathematical ideas are discussed. In this sense, using/producing identity categories is still an obstacle (sometimes necessary) for research that falls within these paths. The mathematics, in many ways, is identity. Maybe the institutionalized mathematics is the big bad guy to feminisms studies in mathematics education. Feminism could be a political strategy not to deal with research problems of mathematics education, but a possibility to produce relationalities beyond the binaries. It is necessary and urgent to produce others mathematics.

Introduction

This essay is created from a doctoral research project by the first author, which is being supervised by the second author, in the Graduate Program in Mathematics Education, at the Federal University of Mato Grosso do Sul, Brazil. In our research project, we intend to articulate some feminist discussions with the field of mathematics education, given the importance of the theme and the scarcity of these studies, although we may find some research on gender and mathematics education.

As part of our research process, we weave some questions, as the ones below, but also the questions that will be made throughout this article, which constitute tensions that move us throughout this project. The questions are the following: Is opting for a feminism that fights for equality among women and men a way for scenes of gender violence not to happen? Is it possible that feminism is just a binary issue of gender (male and female), disconnected from other aspects such as class, race, sexuality, etc.? How the training courses for teachers in mathematics (teachers, employees, students, curriculum, physical structure, etc.) positions them in face of gender issues and their intersectionality (class, race, sexuality, ...)? What movements are formed within these courses on these issues?

Here, a first explanation is required on how we see these studies. We strongly believe that current feminist studies and gender studies have a lot in common, but they are distinct from each other.

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On the one hand, gender studies focus more on differences in performance in math activities between men and women, or on denouncing women's characterizations in a way inferior to men, or even on explanations of how there is a process of exclusion of women in mathematics situations in relation to men. On the other hand, feminist studies tend to explore other issues, such as: the superficial binaries between sex/gender; the uses of mathematics that operate in the logic of the excluded third and thereby strengthen relations of exclusion, domination and patriarchy; violence and exclusion, even if in subliminal forms, that occur in school spaces or in the formation of mathematics teachers[1].

One of the reasons for making this distinction is the studies of the so-called third wave of feminism as an invitation to think about feminist studies beyond the idea of identity that the very notion of gender evokes (Hollanda, 2019), encompassing aspects such as technology and nature, that may (or may not) go through gender issues.

Feminist philosopher Donna Haraway (2000) endorses this idea in her text 'Cyborg Manifesto: science, technology and socialist feminism in the late twentieth century'. She uses the idea of the cyborg – a cybernetic organism, a hybrid of machine and organism, a creature of social reality as well as fiction – to say that, instead of operating through categories (even and mainly gender), we must surrender to the 'pleasure of the confusion of borders, as well as in favor of responsibility in its construction' (p. 37). Also according to the author, 'the cyborg is a creature of a post-gender world: he has no commitment to bisexuality, to pre-Oedipal symbiosis, to non-alienated work' (p. 38), and as he is ours ontology, its philosophy must be embraced.

We agree with Butler (2011), when she tells us that

> Obviously, the political task is not to refuse representational politics — as if we could. The juridical structures of language and politics constitute the contemporary field of power; hence, there is no position outside this field, but only a critical genealogy of its own legitimating practices. As such, the critical point of departure is the historical present, as Marx put it. And *the task is to formulate within this constituted frame a critique of the categories of identity that contemporary juridical structures engender, naturalize, and immobilize.* (Butler, 2011, p. 8)

Currently, feminist movements in our society are varied, different and even contradictory, and a series of adjectives and radicals appear together with the word feminism to designate different movements: black feminism, decolonial feminism, radical feminism, transfeminism, etc.

Just to highlight one of these movements, and some of its developments, we mention black feminism, which, according to Haider (2019) had one of its most influential texts, 'A Black Feminist Statement', created by a group of black lesbian activists formed in Boston named Combahee River Collective (CRC). These militants were dissatisfied with the ongoing revolutionary socialism project, arguing that it had been corrupted by Racism and Sexism on the Left. This movement was one of the precursors of the idea of intersectionality, an idea that also permeates decolonial feminism. (Hollanda, 2019).

[1] This is a deepening point for our project as a whole. We emphasize that gender studies and mathematics education and feminist studies and mathematics education are not concurrent or even contradictory. However, these works may focus and be structured in ways that differ in some aspects, and this fact may be interesting for our research.

One of the contradictions of feminist movements is that presented by feminists who criticize those dedicated to gender studies, as they understand that this denomination of gender studies hides the one who is the true subject/object of studies, the woman, since this is usually denied or marginalized by an androcentric culture. (Louro, 1995). We question ourselves, by looking at this contradiction, whether taking 'the woman' as a unit of analysis (and here we also question whether this is possible) would not be to reinforce the power structures through which violence is perpetuated.

Despite the differences and contradictions, we agree with Haraway (1995) when he argues that 'There is no single feminist point of view because our maps require too many dimensions for this metaphor to serve to fix our views' (p. 32), but that perhaps is the search for other explanations of the world.

In this scenario, in this paper we problematize three situations, with the production of problems in naturalized situations of our everyday life. We produced some sparkles/contradictions/questions without answers. Our intention is to bring these questions to try to produce some ways to make another mathematics education, beyond the process of teaching and learning mathematics, maybe as a political-pedagogical strategy to invent other relationalities and other schools. Mathematics Education, always in plural, as a possibility to problematize central discussions/tensions in our society, such feminism, is vital.

We will show three scenarios in which we discuss some feminist statements, specifically the feminism that commented before. With all this narrative we produce some problems to Mathematics Education research, specifically, our research work.

Machismo mata, feminismo salva

Figure 1: A poster during a protest (RUA, 2020)

This photograph was taken from RUA (2020) at a protest in Brazil, in which women took to the streets to ask for justice in a possible case of rape by a social influencer[2]. We will not go deep into this case, which certainly deserves to be discussed carefully, but rather we will take

[2] For further information about this case, see https://theintercept.com/2020/11/03/influencer-mariana-ferrer-estupro-culposo.

a look at the possibility of production with the words contained in the poster wielded by one of these protesters: MACHISMO MATA. FEMINISMO SALVA!

Surprisingly, in the process of producing this article, we came across a linguistic barrier: the lack of a translation for the word *machismo* in the English language. In Portuguese, our native language, the word has different uses and meanings, and it is part of our daily lives in such an extent that its definition is often unnecessary. Looking for similar words or expressions, in the English language, we come across the expressions *male chauvinism* and *sexism*.

Guttman (2013) conducted a study whose objective was to study the history of the terms *macho* and *machismo*, since, according to the author, these terms have many discrepancies in relation to their uses in Social Sciences. Among the author's notes, is that 'machismo under discussion here cannot be reduced to a coherent set of sexist ideas; it is not mere male chauvinism.' (Gutmann, 2013, p. 72)

Far from wanting to arrive at a definition of the word 'machismo', we can understand it, for the purposes of this article, as a process in which men and women, with marked identities, build themselves in affections that promote patriarchy, hierarchization and a submission between them. We may see this process happening from culturally accepted affections, unfortunately for a long time, until today, such as in small sentences, actions, clothes, movie scenes, excerpts from books and why not, also in mathematics and in the educational spaces in which it is worked. We believe that this understanding meets the understanding described in the excerpt below.

> a system of representations-domination that uses the argument of sex, thus mystifying the relations between men and women, reducing them to hierarchical sexes, divided into dominant pole and dominated pole that confirm each other in a situation of objects. [...]. Thus, machismo represents (articulates real and imaginary relationships) this domination of men over women in society. (Drumont, 1980, p. 82, our translation)

A natural tendency, with this in sight, is to identify (again, by operating in the logic of identification) *machismo* as the villain, since patriarchy, hierarchization and submission of women to men are associated with it. This can be seen in the phrase *machismo* kills, which could have many different interpretations: *machismo* can literally kill,[3] but it also, in a metaphorical way, kills opportunities, freedom, equal rights, dignity, both men and women.

Thus, feminism seems to be a salvation (feminism saves) in a context in which machismo presents itself as a villain. In other words, from all the violence (physical-mental) caused by machismo and that constitutes our society, electing a movement (in this case, feminism) to save everyone seems to be the only possible path. Apparently, we keep operating in the logic of identification: we identify a problem-villain (machismo) and also a salvation (feminism), as suggested in the poster.

When considering this issue, we ask ourselves: will feminism, in different configurations, be able to solve our problems? Wouldn't identifying a villain and pointing out salvation be to continue operating in a patriarchal logic of the world?

[3] Here in Brazil, in the year 2020, about five women were killed a day from femicide.

An exemplary example in a teacher training course in mathematics

Figure 2: A poster found in a women's bathroom

The poster in Figure 2 was in the women's bathroom at the Institute of Mathematics, Federal University of Mato Grosso do Sul, the university where the Graduate Program in Mathematics Education of which we are part is located. We chose to bring the photo of the poster in its original version, and then its transcript as follows:

> What do you prefer? A dirty or clean, organized or disorganized environment? Have you turned on the light? Turn off when exiting. / Do not throw paper and tampons into the toilet and do not leave them exposed in the trash. / Do not leave splashes on the toilet seat and floor. / After washing your hands, make sure that the tap is completely closed. / After using the toilet, flush. / Do not leave residues inside the sink: toothpaste, makeup, hair strands, etc. / Throw trash in the trash. / Be kind and act good practices, cooperation and good habits, it generates a clean and pleasant environment for everyone. (Our translation of the poster text)

The statements of this pamphlet placed in the women's bathroom, which apparently intend to 'kindness and good practices', can be unfolded in some questions: who put that paper there? Is there a paper in the men's bathroom that gives tips on how to be kind and exercise good practices, or is this an exclusive recommendation for women? Why leaving an absorbent exposed in a trash can is a practice that must be watched over? Does this have anything to do with the process of erasing menstruation, which, as pointed out by Louro (1999), is also related to the erasure of women's sexuality in school environments? What intra-actions does this pamphlet operate on math teachers and future math teachers? How do these bodies resist?

These questions are configured as possible tensions of how practices that exclude and that hierarchize bodies are subtly imposed on our daily actions. It is not surprising that some people argue that these statements do not address issues of gender and patriarchy and that they are only suggestions for maintaining the hygiene of the women's bathroom and the personal hygiene of women.

At the XIV SESEMAT (Seminário Sul-mato-grossense de Educação Matemática), held in a virtual environment at the Federal University of Mato Grosso do Sul, we presented our research project, and as part of the presentation, we exposed the photograph of the pamphlet of the bathroom and the previous questions.[4]

Surprisingly, one of those responsible for creating and fixing the pamphlet in the women's bathroom, a student in the Bachelor's and Mathematics course, was present during this presentation. She asked the floor, at the end, to tell us that she had no idea that the pamphlet had a sexist aspect – a conclusion she drew by herself after seeing the presentation – and that she was concerned to see how a simple pamphlet could interfere and produce specific ways of being in the world. In addition, she went on to tell us that the pamphlet was designed based on constant complaints that some employees of the cleaning sector of the building made, which involved the items they listed in the pamphlet, such as the use of makeup, and of the tampons exposed in the trash.

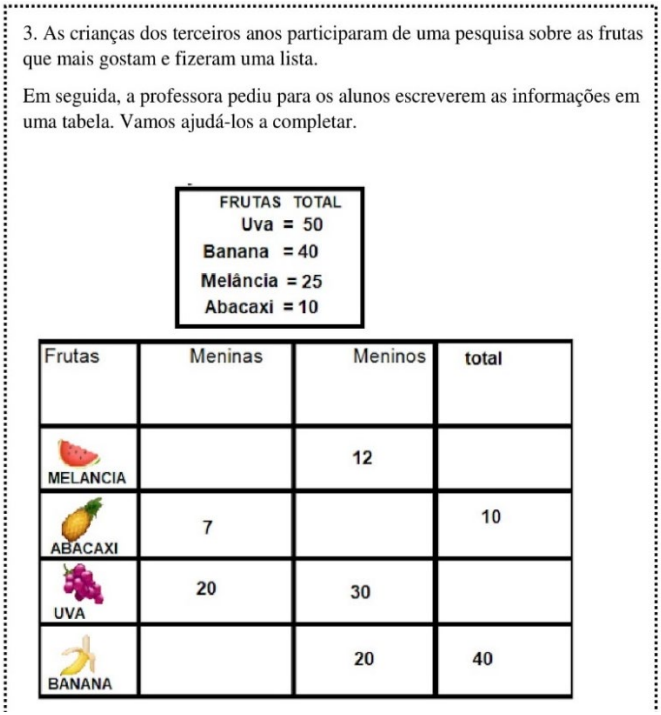
This example is exemplary because it involves a tangle of questions that, due to the complexity of each one of them, run the risk of being analyzed in a simplistic way, such as our initial inclination when asking questions in our research project. In this tangle, there are gender issues, such as the issues that were raised, but also issues of social class, since the demand was from the cleaning employees themselves, who felt that something could be done to help them in their daily tasks. On the other hand, there is also a structural *machismo* that makes us naturalize certain habits, such as hiding menstruation, and, because it is structural, it crosses everyone who is part of the daily life of this course, and it would be interesting to discuss.

This poster leads us to ask the following questions: how can we lead discussions that do not fall into the trap of a simplistic analysis of complex issues? How could we create questions that problematize practices taken as natural (in this case, the erasure of menstruation) and

[4] For further information on this paper presentation, see Viana (2020a).

that produce violence in the bodies of future mathematics teachers? How to create questions that do not claim specific and fixed ways of doing research? How to create questions without the means to do so? Feminism is (or could be) a thematic research of mathematics education?

Paradoxes



3. As crianças dos terceiros anos participaram de uma pesquisa sobre as frutas que mais gostam e fizeram uma lista.

Em seguida, a professora pediu para os alunos escreverem as informações em uma tabela. Vamos ajudá-los a completar.

FRUTAS TOTAL
Uva = 50
Banana = 40
Melância = 25
Abacaxi = 10

Frutas	Meninas	Meninos	total
MELANCIA		12	
ABACAXI	7		10
UVA	20	30	
BANANA		20	40

Figure 3: A 'math' exercise

The exercise above is a 'math' activity prepared for children aged 8 and 9. The statement says: 'Third-year children participated in a survey on the fruits they like best and made a list. Then, the teacher asked the students to write the information on a table. We will help them to complete it'. Right after the statement, we can see a small table with the names of some fruits, followed by the total amount. The fruits in the order they appear in this small table are: grape = 50, banana = 40, watermelon = 25, pineapple = 10. In the larger table, in the first column, the fruits are described in the following order: watermelon, pineapple, grape and banana. The second column, whose title is girls, tells us that 7 girls opted for pineapple and 20 girls opted for grapes. In the third column, which is the boys' column, 12 boys opted for watermelon, 30 boys opted for grapes and 20 boys opted for bananas. The fourth and last column is the column that indicates the total number of students in each row of the table.

In this simple math exercise, there is a process of constituting binary identities - male and female - and simply erasing other possibilities. It is an activity proposed for children between 8 and 9 years old, apparently without any kind of prejudice. However, there is a veiled process of binarization established in a mathematical problem. By setting girls and boys as a category of analysis, it is being admitted that in the world there are only boys and girls, categories that possibly were determined by biological characteristics.

Following the paradox in this situation: on the one hand, the political importance of constituting identities (in this case, men and women), since in this way we managed to fight for more representativeness of a class that has historically been subjugated to another. Speaking more directly, women or the woman category must first exist in order for their claims to be heard. In our case, some of these claims are for more equal wages, valuing jobs that (purposely or not) are unpaid and are seen as female (maternity, domestic services, care, among others), linguistic issues (in Portuguese we face some problems because it is a binary language), etc.

On the other side of this paradox, there is the implication that, by establishing an identity, we are leaving out subjects, and with this, we are promoting new processes of exclusion. What some feminist movements have done is to fight for an expansion of the attributes that guide these identities; however, there are still minimum characteristics that need to be met so that the subjects themselves can claim this expansion, which feeds into the paradox.

In this sense, the book 'Gender Relations, Mathematics Education and Discourse' (Souza & Fonseca, 2010) dedicates a chapter in discussing an identity issue that we consider urgent, although it is not within the scope of our studies: how a male superiority for mathematics is built in and by the school. According to the authors,

> Confessions of 'embarrassments' and announcements of 'behaving' refer to ways of 'being a woman' and 'being a man' in our society. Such 'embarrassments' and 'behaving' are publicized in the media. They appear in multiple practices of women and men, becoming the subject of anecdotes, and they are in bar conversations, they are on truck bumpers, in string literature, in soap operas, television, on the internet, in advertisements... and at school! Finally, 'reason' (to which 'Mathematics' would be linked) is proclaimed in prose and verse as masculine; and unreason (which separates and moves away from 'Mathematics') is characterized as feminine. (Souza & Fonseca, 2010, p. 14, our translation).

We understand that fighting for representation is an operational term within a political process that seeks to extend visibility and legitimacy to women as political subjects. The representation is the normative function of a language that would reveal or distort what is considered true about the category of women.

When we look again at Figure 3, we see an example of this argument: it is important that we talk about the category of women, especially in situations where we need to guarantee their rights, but in a mathematical exercise whose objective seems to us to be the organization of data in a table, using men and women as the category of analysis for these data. It seems to feed the idea that men and women are naturally different, even when choosing a fruit of their choice.

We cannot fail to mention that behind this representation/identification policy there is a psychoanalytic desire for the maintenance of these structures. It is worth remembering that the search for identity is not an inherent characteristic of the human being, but, on the contrary, they are 'socially instituted and maintained norms of intelligibility' (Butler, 2011, p. 23) for the very maintenance of the concept of identity.

Therefore, we ask ourselves: Is it possible the existence of a mathematics education without a subject? What about if we consider feminism as a political strategy to constitute new math class, in which the idea of identity is replaced from the center of the educational process? Maybe, the invention is a possibility.

Some room to produce a feminist mathematics education

Firstly, it is not a question of building a research school or a theory in the midst of discussions of mathematics education, such as, for example, of Critical Mathematics Education. Nor is it about compartmentalizing feminist studies among those who study feminism and its relations with mathematics education, as what happend with Inclusion Mathematics Education group, specifically in Brazil. In our view, it is a possibility to produce ground, spaces, material and discursive practices by which we can talk about hierarchization, binarism, patriarchism, and exclusion/inclusion could be problematized in a mathematics education.

On the one hand, we think is necessary to bring the representation idea to, for instance, deal with the boys and girls mathematics exercises. On the other hand, we think is necessary to produce new concepts beyond the binary, including mathematical concepts to word with the students. These concepts can be produced in the classroom, by teachers, but also by students if these issues and concerns are problematized with them.

In this sense, it seems powerful to use the exercise contained in Figure 3 to trigger a classroom discussion about the need of dividing the table into boys and girls. Questions could be asked to the children and then adding other columns to the table. This exercise could also be brought into a teacher training course to discuss how some exercises emphasize some normative divisions of language, which hide processes that produce differences between men and women.

Another possibility is to label the columns of the table in Figure 3 – instead of the man and the women' categories presented – the 'humans, non-humans and others' categories. What would the children respond to that? Are there some differences between these three columns? What is 'others'? Is it possible to work this with children of 8 and 9 years old? Why not?

Perhaps, in spaces where discussions like these were held, posters like the ones in Figure 2 would not make sense, as the bathrooms would also be divided into the categories 'humans', 'non-humans' and 'others'. And the claims in Figure 1 could also be somehow shifted to make room for new claims for a world structured around these other categories.

Thus, operating with discussions between feminism and contradictions and inventions and mathematical education can be a political strategy to affect the relations between humans, non-humans and others in our contemporary society, as the possibilities presented above.

In conclusion, here we bring some final questions: how institutionalized mathematics in teacher training courses corroborate with this ideal of the Enlightenment Man, leaving women out once again? In what way are colonialities (and their intersectionalities) and their resistances played out by teachers, employees, students and other people who are involved in this course? *Machismo* kills and feminism saves? Is representation an obstacle for feminist studies in mathematics education? Does mathematics constitute an obstacle for feminist studies in mathematics education? (Viana, 2020b)

All these questions bring some possibilities to think about feminism and mathematics education. It isn't questions looking for answers, but just statements for beyond questions and answers. They are statements which proclaim inventions, as the ones suggested above.

If we let ourselves think about feminisms and contradictions and inventions and mathematics educations through symptoms and not problems, one possibility is to ask questions as a process of theorization. A research movement that produces with what is yet to come. Maybe, feminism with a political strategy isn't meant to deal with research problems of mathematics education, but a possibility to produce other relations between human and non-humans, relationships beyond binaries. Maybe it is necessary and urgent to produce other mathematics.

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Children, pets and statistical investigation: A sociocritical dialogue

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We describe the theoretical framework, development, and reflection of a master's research. We seek to understand: "What statistical competencies (statistical literacy, statistical reasoning, and statistical thinking) do 6-year-olds in a 1st-grade class of elementary school constitute when conducting a statistical investigation?" The Data production was performed through a set of tasks, developed by the researcher and the children. In this paper we present the investigation context "Mistreatment of the pet dogs". In our analysis, we found that the tasks and the methodology chosen contributed to developing of competencies such as statistical literacy, statistical reasoning, and statistical thinking and allowed the children to reflect on the social justice.

Introduction

This paper presents a statistical investigation about the "Mistreatment of the pet dogs" developed by first-grade students in an elementary school. This paper draws of the master research developed by Mendes (2020), under the supervision of Vilas Bôas in the Professional Postgraduate Master's Degree Programme in Education: Teacher Qualification for Basic Education of the University of Uberaba, Uberlândia, Brazil. The participants were 28 children, aged about 6 years old. The aim of the project was to engage the children in a statistics project, by collecting data, organising them in tables, and displaying them in a graph. The Investigation Context was the types of mistreatments that can be inflicted on pet dogs.

Our aim for the project was to show that statistics education can be focused on social praxis, involving a theme that encompasses real problems and situations from a contextualised perspective, through a reflective and critical investigation. The question of research aimed to identify and understand which statistical competencies (literacy, statistical reasoning, and statistical thinking) do 6-year-olds attending a 1st-grade class of the elementary school develop when conducting a statistical investigation. As a general objective, we chose to investigate the development of statistical competences in this group of children.

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Fundamentals of the theory: Statistics education, the research's theoretical rerspective

Whatever the area or object of study of the researcher, he/she may use statistical concepts. Daily newspapers are rich in graphs, indexes, and comparative analyses of all species. In the Guidelines for Assessment and Instruction in Statistics Education (GAISE; Bargagliotti, Franklin, Arnold, Gould, Johnson, Perez, & Spangler, 2007), statistics is described as important in many professional activities such as providing nutritional information and establishing the safety and efficacy of medicines among others. Frankenstein (1987), cited by Borba and Skovsmose (2001), states that the incorrect use of mathematical information leads to discrimination and proposes the use of mathematical problems inserted in social situations as a way to empower students through mathematical tools that will enable them to have a critical view of the world. Given the importance of statistics in everyday life, it can play fulfil the role, outlined by Frankenstein to overcome discrimination and empower students.

Thus, mathematical and statistical data can contribute to important debates in society, which can provide a way of reading of the world. In this way, a broader view of everyday situations, can "become part of the language with which political, technological, and administrative suggestions are presented" (Borba & Skovsmose, 2001, p. 127), and can contribute to social justice, equality, and autonomy.

We believe that engage well-designed statistical projects can fulfil the characteristics of critical education. Our approach to critical statistics education is in alignment with Campos (2017, p. 125), who described the objective of teaching statistics as always needing to be accompanied by the objective of developing students' criticality. This must accompany their engagement in political and social issues, relevant to their democratic citizenship and who fight for social justice in a humanised and non-estranged environment.

Campos et al. (2011) propose that it is crucial to teach in an environment that provides research and reflection on problems related to the students' daily lives. Thus, students are called to take responsibility for the information, understanding and reflecting and then drawing their conclusions about the results obtained. In terms of statistics education, Campos et al. (2011, p. 14), stated that students should "be prepared to raise problems of interest, formulate questions [...], collect data, [...], reflect, discuss and critically analyse the results." Consequently, Rumsey (2002), Garfield (1998), Chance (2002), and DelMas (2002) argue that students need to gain and make use of three essential statistical competencies: statistical literacy, statistical reasoning, and statistical thinking.

Garfield (2002) sees statistical literacy as the understanding of terminology, symbols, and the skills to interpret graphs and tables. The author also explains that statistical reasoning is related to the way individuals' reason with statistical ideas, that is how statistical information makes sense to them. For Chance (2002), students who developed statistical thinking can think beyond the problem raised; they can seek beyond the information they are offered. For Chance, statistical thinking is an ability to see globally, to understand the process altogether.

Process, Cycle, Phase of a statistical investigation

Lopes (2004) proposes that the investigative perspective should form the basis for the teaching and learning process because students would experience the conception and analysis of data. Lopes highlights five steps that make up the statistical investigation process: Defining the question; Collecting data; Representing data; Interpreting data, and Making decisions. The researcher highlights that, by taking these steps, students are able to build skills that will help them deal with everyday statistical concepts, thus favoring the development of important skills such as: Statistical Literacy, Statistical Reasoning and Statistical Thinking. GAISE (Bargagliotti et al., 2007) names these five steps as Phases of the Statistical Method and classified them into four other stages:

> I. Formulate Questions: clarify the problem at hand; formulate one (or more) questions that can be answered with data; II. Collect Data: design a plan to collect appropriate data; employ the plan to collect the data; III. Analyze Data: select appropriate graphical and numerical methods; use these methods to analyze the data; IV. Interpret Results: interpret the analysis; relate the interpretation to the original question. (Bargagliotti et al., 2007, p. 11).

Although those authors address different aspects of a statistical investigation, they seem to converge to the same actions. To carry out a statistical investigation both Lopes (2004) and Bargagliotti et al. (2007) suggest: Formulating questions to be investigated; Elaborating hypotheses; Choosing sample and collection instruments; Collecting data; Interpreting and representing them.

Investigation contexts

Choosing the contexts for statistical investigations comes with its own criteria. For example, Alrø and Skovsmose (2010) wrote, "conducting an investigation means abandoning the comforts of certainty and getting carried away by curiosity" (p. 123).

Campos (2017, p. 113) describes an Investigation Context, as a "set of tasks, which are, first of all, a context of development of knowledge, skills, and, therefore, a space for data production." As in the landscape of investigation proposed by Alrø and Skovsmose (2010), in the Investigation Contexts, students are responsible for the investigative process. Thus, the research contexts are the means and places in which data production takes place. It is the space where children carry out the search for information to solve, understand and analyse the task propositions.

Methodoloy: Procedures of the research and analysis

For a better understanding of the research problem, a qualitative investigation was carried out in the participant mode. The research was conducted in the second half of 2019 weekly meetings at the Municipal Elementary School Mario Alves de Araújo Silva[1]. The institution is maintained by the Municipality of Uberlandia and administered through the Municipal

[1] The Municipal Department of Education authorized the disclosure of the name of the school for the purpose of publishing the research

Department of Education. Twenty-eight children (aged between 6 and 7 years) attending a 1st-grade class at elementary school participated in the research. We developed our research in four stages: i) bibliographic study; ii) elaborating tasks performed in each phase of the statistical investigation; iii) data production developing the investigation contexts, and iv) search result analysis.

We review the theoretical framework of Statistical Education present in doctoral theses, master's theses, books that discuss Statistical Education, jornal of articles and event proceedings in the area of Statistical Education and Mathematics Education.

The data production was carried out with participant observation and the researcher's intervention by proposing a set of tasks, developed in the Investigation Contexts, which were elaborated from the theoretical framework. The themes addressed were: what is my pet; what is the care and mistreatment of the pet dog.

While planning the investigation context and development of tasks in the classroom, we used different resources from the children's environment that allowed them to recognise various kinds of information that involved the statistical investigation. We used different instruments to collect research data: audio and video recordings of the task development and we took field notes.

The data production of the research happened during the realization of the Investigation Context. For the development of the tasks, we offered the children situations, in which they experimented: resolution strategies, dialogued with their colleagues, tested and verified their ideas and communicated their reasoning to the colleagues and the researcher. From the dialogues with the children, we organized different ways to collect, organize and represent data, as the investigation Contexts were developed.

For data analysis of the research, we chose to triangulate the data from the student, the teacher, and the researcher and look at the field diary alongside the audio/video recordings. Throughout the we developed 12 Investigation Contexts, including 28 tasks that were divided into three themes, "What kind of pet do you have?" "Care for the pet dog," and "Mistreatment of the pet dogs." The statistical investigation developed in the first two themes involved the classes at the school, and the third theme went beyond the school walls, involving children's families and friends. The investigation context happened during two weeks, for a total of seven hours, and aimed to promote dialogue and alert children about the mistreatment of pet dogs, stimulate their awareness of the problem and evoke their critical consciousness to this problem, and combine statistical development skills with social justice issues.

The investigation context: "Mistreatment of pet dogs"

In Brazil, every day, we hear about numerous cases of animal abuse, understood as violence and physical disrespect against animals, for example, "abandoning; beating; striking; mutilating; poisoning; keeping trapped in chains permanently; keeping in small and unsanitary places; not sheltering from the sun, rain, and cold; denying them water and food daily; denying veterinary assistance, and others" (Dias, 2000, pp. 156–157).

Guimarães (2019) describes the teaching of statistics as having investigation as its structuring axis, in which autonomous appropriation of knowledge contributes to reflective practices about the world. For this reason, the tasks of this investigation context were proposed in such a way that it would allow children to investigate autonomously and freely. Besides, it favoured the interaction between students and social practices, approaching another field of knowledge, contributing to interdisciplinary learning.

Task 1: Establishing the research theme

According to the requirements for engaging in an investigative task, children must recognise the situation and the challenge involving it. Thus, to ask the question "What kind of mistreatment of pet dogs have you ever seen?", we created an environment that provided dialogue on the subject. We played two movies/cartoons,[2] "The Abandoned Dog" and the "Kitbull," for the children. The objective was show scenes of social injustice to sensitise and enhance the dialogue on the subject in such a way that the subject would emerge more easily and allow children to remember facts already witnessed regarding animal mistreatment.

Task 2: Data collection

Once we established with the students what to investigate, we decided with the group which population would be investigated. The children interviewed five people among their family members and neighbourhood friends. The children were instructed to carry out the following procedures at home: choose five people to answer the survey, tabulate the answers and fill in the table.

The objective was to take outside the school, through a statistical investigation, an alert to a topic of great social injustice: cruelty, chaining, beating, abandoning, not providing help, lack of hygiene, lack of food, dirty place, and tick-infested animals, as shown in Figure 1. As a data collection tool, we used an investigation sheet[3] with these variables.

The investigative question choice was: "Which animal mistreatment have you ever seen?" With this question, we contemplate what the BNCC (2017) guides, bringing discussions on topics of social urgency, based on ethical, democratic, sustainable and solidary principles.

Task 3: Data organisation (2 hours)

Each child received a table, Figure 2, made by the researcher to register the data collected. During the explanation of how to execute the activity, we sought to establish a dialogue with the children so that it was possible to make them aware of the importance of organising the data collected with an investigative look at the results. The children collected and tabulated data without the teacher's or the researcher's intervention in an individual and autonomous way, which are essential characteristics for the formation of critical citizens.

[2] The drawings can be found at the following addresses: https://www.youtube.com/watch?v=AZS5cgybKcI and https://www.youtube.com/watch?v=rFiZEm0ww8k

[3] Term chosen by the children, since the class associated the word investigation throughout the research as a way to discover something that is not known.

Figure 1: Investigation sheet on mistreatment (Mendes, 2020, p. 140)

Figure 2: Table mistreatment of puppies (Mendes, 2020, p. 141)

Task 4: Doing a pictogram (3 hour-class)

The children constructed the graph, Figure 3, using the images cut from the investigation sheet, considering only the images registered in the frequency table of the collected data.

We invited the children to go to the schoolyard, arranged them in a circle, and placed a cardboard representing the graph in the middle, and explained that each child would put on it the images they had cut from the investigation sheets.

Figure 3: Children building the graph (Mendes, 2020, p. 141)

After everyone completed the graph, we explained that we had built a pictogram. Once it got too big, we proposed replacing ten small pieces of paper (chips) with a large one to make the pictogram more visible. The dialogue below shows how it happened:

Pesquisadora: nossa! nosso gráfico ficou enorme, como vamos fazer para contar?
Estudante M: contando um por um
Pesquisadora: mas vão demorar demais, como podemos fazer isso mais rápido?
Estudante L: escrevendo na tabela

Pesquisadora:	ah! Sim! Mas antes de escrevermos na tabela, para nosso gráfico ficar mais fácil de enxergar, o que vocês acham de a cada 10 fichinhas dessas pequeninhas, trocarmos por uma grande igual a essa
Estudantes:	como assim?
Estudante L:	escrevendo na tabela
Pesquisadora:	por exemplo, a cada 10 fichinhas pequenas dessa de "crueldade", nós vamos trocar por essa ficha maior aqui.
Researcher:	wow! our graph is huge, how are we going to count?
Student M:	counting one by one
Researcher:	but it will take too long, how can we do this faster
Student L:	writing on the table
Researcher:	oh! yes, but before we write in the table, to make our chart easier to see, what do you think about every 10 chips of these little ones, we change it for a big one like this one
Students:	I did not understand.
Student L:	writing on the table
Researcher:	for example, for every 10 small tokens of this "cruelty", we'll exchange it for this larger one here

In the above dialogue, without conceptual rigor, we worked on the statistical reasoning competency in the specificity presented by Garfield and Gal (1999), that is, reasoning about data representation, which is the understanding of how each type of graph is appropriate to represent a set of data.

This Investigation Context provided students with an initial development of skills necessary for their growth as citizens, in the construction of democracy and citizenship in the search for equity and social justice, which, according to Gerardo (2008), implies looking at the around us and knowing how to interpret what we saw, using a powerful tool – knowledge.

Final considerations

With the development of the tasks of this investigation context, it was possible to go through the four phases of the statistical method: the first phase, which includes the survey of the problem, the second phase comprising data collection, and the third phase of the statistical method select the appropriate graph and numerical methods for data analysis. We explored subjects such as data organisation and representation, as cited in GAISE (Bargagliotti et al., 2007).

The investigation questions started with the researcher and as the children developed new knowledge, they began to elaborate questions and make generalizations, expanding the research universe. The GAISE report (Bargagliotti et al., 2007) points out that planning a study and formulating questions for data collection are tasks that significantly contribute to the development of critical statistical analysis.

The data production was facilitated through questions to the children: for example, “What theme will be researched?”; “How do you search for data?”; “What will be part of the research?”; “How this data will be collected and tabulated?”; “How to present and interpret them?”.

In the course of carrying out the tasks in this Research Context, we observed that the children developed skills to: choose the topic to be investigated (task 1); collect data through the investigation list (task 2); tabulate the data and organize them (task 3); build representations for your data in the pictogram chart (task 4). The results analyzed show that through the tasks of the Research Context, the children showed initial manifestations of statistical skills.

We observe that statistical literacy is present when children are able to understand some statistical terminology and when reading textual information, graphs and tables. During the execution of tasks 2 and 3, we observed that when we handed the children the table and the investigation list, they were already familiar with those terms and already knew how to perform data collection by themselves. With this, we realised that statistical literacy competence was already in development, since they demonstrated an understanding of the statistical terminologies (Garfield, 2002).

In dialogue with the children about the construction of the pictogram (task 4), it was possible to reason about the data. Initially and simply the children worked their reasoning statistical, since “reasoning about data representation, which addresses understanding how the graphs can be modified to represent the data better” (Campos, 2017, p. 98).

Although this investigation context did not approach statistical thinking competence, we can observe that this competence existed, even if implicitly, as by encouraging children to think of a way to collect data (task 2), we prompted them to find an action strategy, which, according to Wodewotzki and Jacobini (2009), pervades statistical thinking.

Thus, we can affirm that 6-year-olds can develop, even initially, statistical competencies such as literacy, statistical reasoning, and statistical thinking when conducting a statistical investigation.

When developing the tasks presented in the investigation context, we observed that they provided for aspects that went beyond statistics education, such as moments when we used resources to open discussions on the topic of animal cruelty, with videos and discussions with children. In this environment, they had space to speak, discuss, expand their communication skills, and dialogue on the theme chosen and the phases of the statistical investigation. The conception of Statistical Education that we assume is directed towards social action, as, “valuing attitudes aimed at social praxis, students engage with the community, transforming classroom reflections into action.” (Campos et al., 2011, p. 12).

The social interactions enhanced by research are of fundamental importance for the construction of a reflective and critical citizen, they are: carrying out a Statistical Investigation (collecting, organizing, analyzing and presenting data); know and recognize the social use of numbers; express their ideas about the mistreatment of Pets topics discussed

in the conversation circles and in the activities; working in groups collaboratively; live with colleagues respecting the individual differences of each one.

It is important to emphasise that for this we must propose tasks that address themes of the children's interests, that are planned based on curriculum guidelines aimed at teaching statistics and that, during its development, the children are made protagonists of the process, which may enable them, as stated by Lopes (2004), to experience its entirety from conception to data analysis.

We emphasise that the aim was to develop a study that would enable children to experience a researchers' role in the statistical process, since they come to school full of curiosity and questions. Thus, we should motivate them, encourage existing curiosities, and provoke others so that the process/cycle/phase of a statistical investigation develops. Here is our contribution to reflection on the development of statistical skills and their role in the formation of critical citizens.

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TIMSS and the World Bank: Mathematics education for human capital and consumerism

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In this paper I sustain the mathematics education community's decades-long critique of international competitive assessments with my historical-contextual analysis of the IEA's TIMSS. Using perspectives on the globalization of education, I highlight TIMSS' association with the World Bank to display the assessment's motivation: increasing the people throughout the world who enter the global economy as both wage laborers and lifelong consumers. In assembling and discussing economic data, it appears that those countries participating in TIMSS do in fact increase their participation in the global economy. Such motivations and achievements should give the mathematical academic community pause as we consider engagement with TIMSS and similar globalizing efforts in mathematics education.

Introduction

Our community knows all too well the international competitive assessments in mathematics education, including the International Association for the Evaluation of Educational Achievement's (IEA) Trends in International Mathematics and Science Study (TIMSS) and the Organization for Economic Cooperation and Development's (OECD) Programme for International Student Assessment (PISA). Cogent critiques among our community of mathematics educators began as early as 40 years ago with, e.g., Freudenthal (1975). He emphasized many points including IEA's lack of engagement with mathematicians and mathematics educators and the dubious conclusions often made from the data. The present project aims to take up these conversations of critiquing TIMSS yet again and specifically by associating the project to its broader policy objectives and especially via entanglement with the World Bank.

The specifics to the critique I present here, as an historical-contextual approach, relate to other critiques of the globalization of mathematics education put forth in the community, e.g., Ernest (2009). He frames his discussion on globalization and mathematics education with the dominating discourse of the knowledge economy (an ideology central to the World Bank's activities). He suggests an "export" of Eurocentric models of education and higher education across the globe and, specific to my focus, a suggested "appropriation effect" via the international competitive assessments, as in:

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> [Projects] can also be focused on applying some predetermined framework as in international assessment and comparative studies (e.g., SIMS, TIMSS, PISA) typical of scientific paradigm research. In both of these types of research there is what I shall term the *appropriation effect*. In this, locally gathered knowledge from 'developing' countries is appropriated for academic and other uses in 'developed' countries. (pp. 4–5)

In his critique, the outcomes of these assessments are suggested to have significant impact in altering the mathematics curricula throughout the globe. In my critique, I will suggest that outcomes from TIMSS and the like exactly service the broader vision of society that its associates (the World Bank and, more generally, the globe's power-elite) desire. Namely, TIMSS activity is associated with increasing the people across the world who have entered the global economy as wage laborers and lifelong consumers.

This research presentation contains a focused argument on TIMSS using a historical-contextual method of analysis. This is one component to a broader research project in which I engage discussion of the impact of these international competitions. I refer to the phenomena as "assessment spread" and use both contextual-historical methods as well as content analysis of released assessment items for TIMSS, PISA, and (forthcoming) other activities in the globalization of mathematics education. The presentation herein offers one slice of my present work, namely the historical-contextual analysis of the IEA's TIMSS.

Globalization and education

Regarding globalization of education, Spring (2014) offers three theoretical frameworks for interpreting the global rise of mass schooling: world culture, world systems/post colonialist and culturalist. The international competitive assessments reflect world culture theory more heavily than it does the others. World culture theory insists "all cultures are slowly integrating into a single global culture" (p. 7) yet begs the question: what are the prominent assumptions and ideologies within said monoculture? By invoking world culture theory with critical engagement, I turn the theory on its head in suggesting that the key ideologies present are a global monoculture that embraces capitalism and, specifically, viewing people as wage laborers who are also lifelong consumers. In this way I am using world culture theory differently than others: some world culture theorists suggest an evolutionary process whereby cultural practices (mostly Eurocentric) dominate because they are uncritically perceived to be better (Spring, 2014). I flip the theory by viewing the emergent monoculture through a critical lens focused on the possibility that the culture's survival requires such dominance. In other words, the ideology of consumerism and human capital theory must spread for participants in its cultural practices to remain as such. As evidence of this requirement, take middle class US citizens as deeply committed practitioners of the consumerist culture. They need cheaper electronics in the hopes that these will deliver happiness (axiomatic to the consumerist ideology). The cheaper goods require additional peoples of the world to enter the culture of human capital, as factory workers making personal computers and cellular phones, for example.

Critical scholars of mathematics education have clearly documented the human capital embedded within mathematics education. My own work (Wolfmeyer, 2014) analyzes the social network of US national math education to reveal a consistent human capital theme amongst the individuals and organizations responsible for math education policy. Gutstein (2006) highlights where this exists in the US National Council of Teachers of Mathematics (NCTM) 1989 content standards: the four "societal goals" of math education, include economic growth and the development of human capital.

> Today, economic survival and growth are dependent on new factories established to produce complex products and services with very short market cycles…Traditional notions of basic mathematical competence have been outstripped by ever-higher expectations of the skills and knowledge of workers; new methods of production demand a technologically competent workforce… Businesses no longer seek workers with strong backs, clever hands, and "shopkeeper" arithmetic skills (NCTM, 1989, pp. 3-4).

NCTM reinforces their commitment to human capital theory by suggesting mathematical literacy for all is an "economic necessity" rather than, say, a requirement for a participatory democracy. I argue that NCTM's commitment to human capital theory and market economies, as seen here, also implies a commitment to the consumerist ideology.

These suggestions to associate dominant mathematics education practices with both human capital and consumerism is made stronger by analysing the globalization of mathematics education via the IEA's TIMSS. As revealed in the next section, TIMSS association to the World Bank and my historical-contextual approach reveals the assessment's engagement with furthering participation in the global economy. The data will reveal that TIMSS participating countries also increased their income levels per capita during their period of involvement. I am very careful not to suggest that TIMSS participation *caused* this circumstance; however, it certainly associates to it.

From 10 to 60+, plus the World Bank

What began as the participation of 10 wealthy nations (all from Europe except the US and Israel) now includes participating countries from Africa, Asia, Central America, South America, Asia, Oceania, Europe, and North America. The participating countries in TIMSS also now include economic diversity, with several countries having lower income categories. In the span of time since its pilot study in 1959 to the most recent iteration in 2019, the IEA's mathematics assessment has also included perhaps its most significant player: the World Bank. Since the 1999 iteration, the World Bank has been listed as a primary funder for the project.

As a primary context for TIMSS, the World Bank and its involvement in education deserves our community's careful attention. Looking first at their own documents on education, we see their motivations laid bare. The World Bank and their networked actors put forth the suggestion of a knowledge economy as the key for emerging economies in so-called "developing nations." This appears in multiple World Bank publications, as in *Building knowledge economies: Advanced strategies for development* (World Bank, 2007). Similar

arguments are put forth by other organizations, such as the global economic elite (as represented by the OECD) who declare an education for human capital among the wealthy nations as the primary educational imperative, as in Keeley (2007).

Existing literature on the globalization of education provides sustained critique of the World Bank's involvement in education. Spring (2004) offers poignant analysis to the framing in which the World Bank takes educational action:

> [Their] educational ideology contains a particular vision about how society should be organized. For many people, this vision is just assumed to be a necessary part of the advancement of world societies. It is an image of the good society that is often unquestioned because of its promise of economic abundance for all […]. *As envisioned by the World Bank, a good society is one based on the mass production of consumer goods within a global economy.* Each region or nation contributes to mass production through factory and agricultural goods. The production of agricultural goods is done on large corporate farms or plantations. Small family agricultural units are replaced by large units with factory-like organization. Workers in these larger units are trained for specialized roles and work in corporate teams. Those who previously worked on family farms either work on corporate agricultural units or move to urban centers. The migration of the population from rural to urban centers follows the pattern that occurred in Western industrialized nations in the 19th and early 20th centuries. Displaced rural workers provide the labor supply for new industrial concerns. In addition, gender equality and low fertility rates free women from the home to increase the supply of labor […]. What is valued is constant economic growth, which requires a continuous development of new products and increased consumption. There is no definable end to this process. When does a society reach a point when it does not need economic growth? Is there a point of stasis? Or is it an endless cycle of developing new products and creating consumer demand? *Of course, from the viewpoint of the World Bank, the problem is that many countries have not reached a high enough level of economic development to participate in the mass consumer society. The role of education is to help them make this leap* (pp. 40–41, emphasis added).

As Spring continues, he provides examples of the types of educational activities that the World Bank has enacted over time. These include a variety of curriculum, funding and loans, and assessment programs. In this presentation I am pointing to specifically one of these, namely the World Bank's funding of and entanglement with TIMSS. It's clear that TIMSS has been enacting the World Bank's educational ideology since at least 1999 when they acknowledged World Bank as a significant funding source.

I next merge this contextual understanding, that TIMSS engages with the World Bank's education and global society vision, with a historical approach to the participating countries in TIMSS. I compiled lists of the participating countries over time and cross-checked these with World Bank historical data on Gross National Income (GNI) per capita. They indicate the countries over time that have been brought into IEA's assessment practice in mathematics assessment. After the pilot study in 1959, we have the FIMS (First International Mathematics Study) of 1964, the SIMS (second) of 1980-1982, the TIMSS (third and to now including science) of 1995, and finally, the renaming of the practice to TIMSS (Trends in International Mathematics and Science Study) and its iteration via a 4 year cycle every year

with the last one completed in 2019. Table 1 below includes the countries participating in the four-year iterative assessment between 1995 and 2007 but an additional variable presented: the World Bank's analytical classification for each country based on the Gross National Income per capita.

1995	1999	2003	2007
Argentina+++	Australia++++	Argentina+++	Algeria++
Australia++++	Belgium++++	Armenia++	Armenia++
Austria++++	Bulgaria++	Australia++++	Australia++++
Belgium++++	Canada++++	Bahrain++++	Austria++++
Bulgaria++	Chile+++	Belgium++++	Bahrain++++
Canada++++	Chinese Taipei++++	Botswana+++	Bosnia and
Colombia++	Cyprus++++	Bulgaria++	Herzegovina++
Cyprus++++	Czech Republic+++	Canada++++	Botswana+++
Czech Republic+++	England++++	Chile+++	Bulgaria+++
Denmark++++	Finland++++	Chinese Taipei++++	Canada++++
England++++	Hong Kong SAR++++	Cyprus++++	Chinese Taipei++++
France++++	Hungary +++	Egypt++	Colombia++
Germany++++	Indonesia+	England++++	Cyprus++++
Greece+++	Iran++	Estonia+++	Czech Republic++++
Hong Kong++++	Israel++++	Ghana+	Denmark++++
Hungary+++	Italy++++	Hong Kong SAR++++	Egypt++
Iceland++++	Japan++++	Hungary +++	El Salvador++
Indonesia++	Jordan+++	Indonesia++	England++++
Iran++	Korea++	Iran++	Georgia++
Ireland++++	Latvia++	Israel++++	Germany++++
Israel++++	Lithuania++	Italy ++++	Ghana+
Italy++++	Macedonia++	Japan++++	Hong Kong SAR++++
Japan++++	Malaysia+++	Jordan++	Hungary ++++
Korea++++	Moldova+	Korea++++	Indonesia++
Kuwait++++	Morocco++	Latvia+++	Iran++
Latvia++	Netherlands++++	Lebanon+++	Israel++++
Lithuania++	New Zealand++++	Lithuania+++	Italy ++++
Mexico+++	Philippines++	Macedonia++	Japan++++
Netherlands++++	Romania++	Malaysia+++	Jordan++
New Zealand++++	Russian Federation++	Moldova+	Kazakhstan+++
Norway++++	Singapore++++	Morocco+	Korea++++
Philippines++	Slovak Republic+++	Netherlands++++	Kuwait++++
Portugal++++	Slovenia++++	New Zealand++++	Latvia+++
Romania++	South Africa+++	Norway++++	Lebanon+++
Russian	Thailand++	Palestinian National	Lithuania+++
Federation++	Tunisia++	Authority*	Malaysia+++
Scotland++++	Turkey++	Philippines++	Malta++++
Singapore++++	USA++++	Romania++	Mongolia++
Slovak Republic++		Russian Federation++	Morocco++

1995	1999	2003	2007
Slovenia+++		Saudia Arabia+++	Netherlands++++
South Africa+++		Scotland++++	New Zealand++++
Spain++++		Serbia*	Norway++++
Sweden++++		Singapore++++	Oman++++
Switzerland++++		Slovak Republic+++	Palestinian National
Thailand++		Slovenia++++	Authority*
USA++++		South Africa++	Qatar++++
		Spain++++	Romania+++
		Sweden++++	Russian Federation+++
		Syria++	Saudi Arabia++++
		Tunisia++	Scotland++++
		USA++++	Serbia+++
		Yemen+	Singapore++++
			Slovak Republic++++
			Slovenia++++
			Spain++++
			Sweden++++
			Syria++
			Thailand++
			Tunisia++
			Turkey+++
			Ukraine++
			United Arab
			Emirates++++
			USA++++
			Yemen+

Table 1. Countries participating in TIMSS and World Bank's analytical classification for each country

I use the economic classification to make arguments about the significance of the assessment spread after displaying the tables of participating countries. In this table and a second table (not included here due to space limitations but displaying the countries that participated 2011 through 2019), the categories of economic classification include High Income, Upper middle income, Lower middle income, and Low income are displayed as ++++, +++, ++, and +, respectively. Thresholds for these categories vary slightly from year to year and increase over time due to inflation of the US dollar. The thresholds for 2019's analytical classifications were: Low income as GNI per capita less than $1035 US, Lower middle between $1036 and $4045 US, Upper middle income between $4046 and $12535 US, and High income above $12535. Sources to create these tables include the IEA (n.d.) website with lists of participating countries and World Bank Group (n.d.), a data set with historical classifications according to the economic groups listed above.

The tables reveal several interesting phenomena after careful consideration of the years and involvement over time. The simple and clear statement that this assessment has spread

across the globe throughout the years cannot be underestimated. What began as an initial pilot study to compare a handful of countries is now a consistent practice with about 61 countries participating on average in the last four iterations. And increasingly the participating countries include more diversity with respect to income categorization. It is the latter point that deserves further explication of meaning as it relates significantly to the World Bank's vision for education and global society.

Looking at the diversity of GNI per capita among the countries in the 1995 TIMSS, we see a majority of participating countries with high levels of income yet also seven that are upper middle and 11 that are lower middle income. In the next few iterations of TIMSS, the numbers of lower middle income increases (to 13 participating countries in years 1999 and 2003) and the participating countries include those with lower income now as well (two countries in 1999 and 4 in 2003). The pattern of increasing economic diversity holds steady as the cycles move on, but interestingly toward the later years we begin to see fewer numbers in the lower and lower middle income categories. In the last few years, the spread of countries has not continued to amass new participating countries similar to the rate of spread we saw earlier in time. However, here is the striking fact presented in this data: taking together the full list of participating countries in TIMSS cycles, we have 84 in total, many of these were countries with high income classifications yet some were not. *Exactly 25 countries from among all participating increased their GNI per capita income category as classified by the World Bank during the years of participation in TIMSS.* Only two of these (South Korea and South Africa) went down in classification and came back up during the years of TIMSS. The remaining 23 entered the TIMSS process in a lower income category and improved during the years they were participating in TIMSS, many holding steady at their higher income levels in the last few cycles of TIMSS. Of those countries that entered in less than "high income" and *did not* increase their incomes (18 countries total), 10 of these were one-time participants in TIMSS and don't have sustained participation in TIMSS to associate to income levels. Thus, the 23 participating countries increasing income levels more than outweighs the 8 countries that did not. As another way of looking at this data, of the 84 participating countries exactly 6 began participation as classified "low income" with exactly 5 of these increasing their income category during the years of TIMSS participation.

The increase in income level corresponds exactly to Spring's description of the World Bank's global vision. A country that increases its per capita income levels over time means that more people in the country are entering the wage labor market and have the ability to consume goods and services in market economies. When the World Bank looks at data for countries like these, it sees its vision realized: more people entering the global marketplace as both the human capital that markets need and the consumers in an ever-increasing demand for products.

Looking specifically at the TIMSS participating countries, we see an increase in GNI per capita for these 21 countries equating to *25% of all participating countries in TIMSS.* Furthermore, the majority of high-income countries hold steady at the high income level. This is remarkable data, the World Bank is surely pleased to see that countries participating

in its activities are increasing the numbers of people entering the wage labor market and increasing demands for global consumption. The World Bank and IEA might even make a leap in suggesting that the TIMSS mathematics assessment causes new mathematics education practices that in turn enable a country's citizens to enter the global economy more readily. However, such an assertion is poor mathematics. In other words, I am not pointing to any cause-and-effect relationship between TIMSS participation and the increase in income levels of participating countries; I am pointing to the association between the two. A nation-state's choice to participate is likely indicative of several other actions they are taking, many of them additionally with support of the World Bank, to increase their engagement with the global economy. At the very least, participating in TIMMS represents a country's willingness to engage in the ideology that Spring describes and it should come as no surprise, then, that the majority of countries who could increase their income levels actually did. Although there can be no suggestion of cause and effect between TIMSS and increasing global economic activity, it will serve useful to indicate further how the two are associated. This appears with additional components in my broader research project on mathematics education assessment spread across the globe, specifically the accompanying content analysis of TIMSS released items.

Discussion

In presenting this slice of my analysis of the global assessment spread of mathematics education, I aim to reinvigorate discussion and critique of these practices. The data on GNI per income for TIMSS participating countries perfectly corresponds to the World Bank's hopes for global society. This project resonates with previous critiques of the power-elites globalization of mathematics education and is but one contribution I aim to offer. Time permitting, in the presentation I can share initial findings related to my content analysis of TIMSS. Using Harouni (2015) as a framework, coding released items from TIMSS presents further research that confirm the entanglement of TIMSS with World Bank's vision. Namely, the content analysis of TIMSS reveals that the overwhelming usage of mathematics portrayed by the TIMSS practice is only of one type: commercial-administrative. By asserting the commitments and underlying assumptions in global mathematics education assessment spread, I hope that the community can continue to reject these assumptions assertively and with confidence in argumentation. Presenting these critiques also aims to motivate alternative discussions on the possibilities in globalizing mathematics differently, e.g., Appelbaum & Gerofsky (2013) pathways toward alterglobalizing mathematics education.

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Critiquing Bourdieu: Implications for doing critical pedagogy in the mathematics classroom

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Bourdieu's ideas are highly influential amongst critical mathematics education theorists, and yet there is little consensus on how they might be applied to classroom practice. This paper develops a critique of Bourdieu's theoretical tools that offers greater insight for practitioners. We argue that a greater understanding of the dialectic between the use value and exchange value of school mathematics can help in exposing and challenging exploitative practices. Through applying our critique to the 'Teaching Mathematics for Social Justice' research project, we generate further insight that will inform others committed to doing critical pedagogy in the mathematics classroom.

Introduction

The popularity of Bourdieu's theories amongst mathematics education researchers was evident in the high turnout at a recent Critical Mathematics Education Working Group session at a day conference of the British Society for Research into Learning Mathematics (BSRLM) which offered an opportunity to discuss how these theories might inform classroom practice (Wright, 2020a). Whilst there was consensus that Bourdieu's theoretical tools can help to explain the exploitative nature of school mathematics, and the associated reproduction of inequities in educational outcomes, there was less agreement on how to use them in developing practical strategies to challenge the status quo. This paper explores how Bourdieu's theories might be developed and built upon to inform the classroom practices of those committed to challenging the exploitation of mathematics learners from marginalised groups.

Bourdieu's theoretical tools applied to education

Bourdieu argues that one of the primary functions of schooling is to reproduce existing power relations within society by affording systemic advantage to students from already-dominant groups over those in less dominant positions (Bourdieu & Passeron, 1990). He singles out mathematics as playing a key role in this function:

> Often with a psychological brutality that nothing can attenuate, the school institution lays down its final judgements and its verdicts, from which there is no appeal, ranking all students in a unique hierarchy of all forms of excellence, nowadays dominated by a single discipline, mathematics. (Bourdieu, 1998, p. 28)

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School mathematics serves as a critical filter in regulating who has access to more prestigious higher education courses and better-paid employment (Jorgensen, 2016). The persistent and strong association between socio-economic background and success in school mathematics is a manifestation of how mathematics functions as a tool of stratification or 'gatekeeper' (Gates, 2003) in excluding marginalised groups in society from accessing broader forms of economic, cultural and academic capital.

Bourdieu highlights how the reproductive function of schooling relies on a misrecognition of the school as a meritocracy in which those who succeed do so as a result of their natural ability or giftedness, thus hiding the systemic advantage (or capital) that is afforded to students from more privileged backgrounds (Bourdieu & Passeron, 1990). This explains the common belief that mathematical ability is innate and the predominance in many countries, including England, of 'setting' (or grouping students by prior attainment), despite contrary research evidence suggesting this has little impact on educational outcomes (Francis et al., 2017). The act of 'symbolic violence' involves children and families from marginalised groups accepting, and contributing towards, their occupation of lower or dominated positions in the field by assuming responsibility for their own lack of success (Bourdieu & Passeron, 1990).

Bourdieu (1986) describes how some social and cultural resources, i.e., 'symbolic capital', are more highly valued and legitimised by the school. He argues that families from dominating class positions accumulate higher levels of social capital, which they invest in helping their children develop a 'mathematical habitus' that aligns more closely with the values of the school, e.g., through consciously initiating counting games to prepare them for mathematics lessons. This gives students from more privileged backgrounds an advantage in accumulating the 'mathematical capital' that helps them to identify as mathematics learners and to take advantage of opportunities presented in the classroom (Williams & Choudry, 2016).

Whilst Bourdieu's theoretical tools offer critical insight into the structural conditions that produce and reproduce inequalities, they are limited in identifying ways for classroom practitioners to disrupt or even transform such cyclical structures (Wright, 2020a). Bourdieu and Passeron (1990) pose the following paradox in highlighting the dilemma faced by teachers who wish to challenge the exploitation faced by students from marginalised groups in their classrooms:

> Either you believe I'm not lying when I tell you education is violence and my teaching is legitimate, so you can't believe me; or you believe I'm lying and my teaching is legitimate, so you still can't believe what I say when I tell you it is violence. (p. 12)

By acting as its agents, such teachers themselves might be seen as endorsing the system they wish to call into question. How, then, might teachers and educators wishing to fight injustices and challenge inequities within the mathematics classroom do so within a system that appears to be inherently stratified? One approach that has been advocated is to help students from less privileged backgrounds realign their habitus with the middle-class values recognised by the school (Jorgensen, 2016). However, Bourdieu contends that the structuring

of habitus and symbolic capital functions to (re)produce positions in the field, meaning 'success' in mathematics for the few is always held relative to the failure of others. He would therefore argue that this approach enables some students to join the dominant group and share in the exploitation of others, whilst giving credence to the claim that school is a genuine meritocracy (Bourdieu & Passeron, 1990).

Gutiérrez (2008) warns of the dangers of 'gap gazing', highlighting how mathematics education researchers often focus too narrowly on closing attainment gaps between different groups, whilst ignoring issues of learners' identity and power/agency. This serves to reinforce deficit thinking and negative narratives about students from marginalised communities. Those advocating for equity and social justice must aim to expose and challenge systemic injustices as well as looking for strategies that help marginalised students become more successful. Understanding the reproductive function of education might well be a precondition for appreciating its potentially transformative function (Wright, 2020a). In the following sections, we develop Bourdieu's analysis in relation to school mathematics with a view to identifying actions that might be taken by those wishing to disrupt its reproductive nature.

Relating 'use value' and 'exchange value' to teaching and learning mathematics

Williams and Choudry (2016) argue that understanding mathematical knowledge as 'mathematical capital' focuses on the 'exchange value' of school mathematics, i.e., the extent to which mathematics knowledge/qualifications can provide access to positions of power and the means to dominate others. They argue that attention should also be given to the 'use value' of school mathematics, i.e., its application to modelling/solving real-life problems (functional knowledge), as well as the pleasure/ satisfaction of doing mathematics (consumption of knowledge). This reflects what equity-minded teachers often see as the two most important but contradictory aims in supporting students from marginalised groups in their classes (Wright, 2016): enhancing their enjoyment of, and engagement with mathematics through providing meaningful/relevant problem-solving activities (promoting the 'use value' of school mathematics); and helping them to attain the grades in school examinations that will broaden, rather than restrict future life opportunities (promoting 'exchange value').

Black, Choudry, Howker, Phillips, Swanson, and Williams (2021) draw on the praxis of 'Funds of Knowledge' (FOK) to address the above contradiction with implications for developing critical pedagogy in mathematics education. The FOK approach emphasizes how knowledge and experiences that students bring with them from their home/community environments offer rich resources for the development of a school curriculum which establishes meaningful connections with their lives beyond school (Moll, Amanti, Neff, & Gonzalez, 2009). It challenges normative assumptions that children from marginalised communities are in some way 'deficient' and highlights how funds that learners might contribute towards learning are often ignored by the teachers/school.

Black et al. (2021) offer a critique of the FOK approach, whilst also recognising its critical aspects (in that it the rejects a deficit view of marginalised learners). They highlight how FOK does not necessarily challenge the process of capital exchange but can serve to transform resources learners bring with them from their community primarily into resources with exchange value within the educational field. Hence, 'domesticated' approaches to FOK may allow some marginalised students to gain greater access to the formal curriculum without challenging the hegemonic practices of schooling that maintain the marginalisation of others. This critique aligns with Freire (1972) who argues that emphasis should be placed on empowering learners to use the resources they bring with them to raise their consciousness, i.e., become more aware of structural inequalities so that they may challenge them, rather than merely to achieve success within a formal curriculum. He sees the role of the 'educator' as enabling learners to develop collective agency so that they begin to see oppression as a limiting situation that they can transform. In mathematics education, we argue that teachers and educators should go beyond enhancing the mathematical capital of marginalised students and instead seek ways to expose and challenge common assumptions and myths, e.g., that the mathematics classroom is a level playing field, which serve to disguise the arbitrary characteristics of mathematical capital (Williams & Choudry, 2016). By challenging dominant discourses, the exploitative aspects of school mathematics can be exposed and undermined.

The above critique suggests a need to reconceptualise FOK as a 'cultural commodity' that can be used to mobilise collective power to challenge exploitative practices associated with school mathematics (Black, et al., 2021). This recognises the dynamic unity of both use value and exchange value (capital) involved in coming to know mathematics. The knowledge, competences and academic identities that allow learners to gain school qualifications offer exchange value in the form of mathematics capital and this capital also has more immediate exchange value in that it enables individuals to acquire the power needed to manage power relations within the educational field for their own benefit (Williams, 2016). Additionally, the use value of knowledge, competences and academic identities can be interpreted narrowly, as preparing individual students for future study or employment (meeting economic needs), or more widely as fulfilling an individual's social, political and recreational needs. From a Freirean perspective, use value can also be seen in terms of meeting the collective needs of the community (or humanity), through mobilising solidarity with those who are exploited and engaging in collective action to challenge oppression. Black et al critique 'domesticated' versions of FOK that aim to help individual learners accumulate mathematical capital for private gain, rather than mobilise funds for public good. The appropriation by the school of the resources that students bring with them from their home/community means that the potential use value of these resources, particularly relating to its critical use in challenging oppression, may not be realised.

This critique of domesticated versions of FOK highlights a potential contradiction between enabling individual learners to accumulate power within the educational field (exchange value) and addressing the collective needs of the community (use value). Developing students'

knowledge and identities as a 'cultural commodity' involves harnessing this contradiction through taking actions that enable marginalised students to become successful in school mathematics whilst also challenging the conditions that result in their marginalisation. In seeking such actions, teachers and educators must take account of 'dark FOK' (which we refer to as 'difficult FOK'), i.e., knowledge/experiences directly related to exploitation that may be painful or challenging to confront, rather than focusing on 'light' FOK, i.e., positive experiences (Zipin, 2009). Similarly, teachers/educators need to engage with the full range of emotional experiences that students experience in their struggle against oppression.

Application of theory to practice

In this section, we apply some of the concepts discussed above to the Teaching Maths for Social Justice (TMSJ) research project. Whilst the project was not consciously designed around a FOK approach, we highlight below how considering the dialectic between use value and exchange value (see previous section) helps to generate new insight from the project findings and to inform others wishing to do critical pedagogy in the mathematics classroom. The methodology and findings from the TMSJ project have been reported in detail elsewhere (Wright, 2016, 2017, 2020b). Here we focus on those aspects of the project we consider most relevant to our argument.

The project was a collaboration between an academic researcher (the first author of this paper) and five teacher researchers (Anna, Brian, George, Rebecca and Sarah – all pseudonyms), based in four different multi-ethnic and non-selective schools in deprived areas of London. All four schools had a relatively high proportion of students in the following categories: those eligible for free school meals, those speaking English as an additional language, and those identified as having special education needs. The project involved three participatory action research cycles, spread over the 2013/14 academic year, in which the research group planned, tried out in the classroom during 'research lessons', and evaluated a series of teaching ideas, approaches and activities. The findings were generated from the action research cycles through an iterative process that included various methods of data collection (including surveys, interviews, research journals) and analysis (including reflective discussions, planning/evaluation meetings, thematic analysis, coding).

The research group adopted the following conceptualisation of teaching mathematics for social justice, drawn from critical mathematics education research literature (e.g., Gutstein, 2006; Skovsmose, 2011) that teacher researchers were encouraged to read:

1. Promote engagement through collaborative/discursive problem-solving approaches.
2. Provide meaningful activities by drawing on learners' real-life experiences.
3. Use mathematical inquiries to enable learners to better understand their own situations.
4. Develop learners' agency through facilitating mathematical investigations.
5. Develop a critical understanding of the nature/position of school mathematics.

These five aims appear to relate closely to the notion of the cultural commodity presented earlier. Taken in isolation, aim 2 might suggest a domesticated approach to FOK that focuses

on exchange value and ignores systemic causes of marginalisation. However, aims 2, 3 and 5, taken together, offer the possibility of utilising resources students bring with them from their community to expose conditions that result in their marginalisation. Aims 1 and 4, alone, might suggest a focus on accumulating mathematical capital through generating powerful knowledge for the individual. However, as noted above, such powerful knowledge is also vital for mobilising the collective power needed to challenge exploitative practices.

The teacher researchers articulated how this conceptualisation resonated with their reasons for becoming mathematics teachers in the first place. However, they initially perceived these aims to be in direct conflict with the need to enable their students, particularly those from marginalised groups, to achieve the grades in terminal school exams that will enhance future life opportunities and choices (reflecting the exchange value of school mathematics). The perceived conflict between the exchange value and use value of mathematics generated significant discomfort amongst teacher researchers in relation to their identities as teachers.

> Whilst other teachers would probably love to do this stuff [TMSJ aims], they don't have the confidence that the children would get the grades that they need to get in half term tests. That is the single main constraint, I think. And also, unless teachers have read up on this way of teaching, they won't necessarily trust that it will get long-term results either. They'll just think that it's a very risky strategy. (Anna, Interview 2)

The teacher researchers highlighted constraints they faced in adopting alternative practices that promote the use value of mathematics. These include the lack of time and easily accessible resources, the pressure to get through a rigid scheme of work, and a culture of performativity in schools that discouraged risk taking in the classroom. For instance, senior managers would often undertake 'learning walks' that involved short and unannounced visits to classrooms to make 'on-the-spot' judgements about the quality of teaching and learning. However, the mutual support provided by the research group gave teacher researchers the confidence to take risks in their classrooms and to begin to put into practice the five TMSJ aims. They tried out a series of classroom activities that drew on students' real-life experiences and linked these to mathematical concepts and social justice issues. For instance they instigated a debate on how to distribute a fixed amount of money as wages amongst a group of workers (e.g., doctor, nurse, cleaner, …), taking into account the value attached to each person's work, and how to display/measure the resultant levels of inequality (e.g., using Lorenz curves/Gini coefficient). Through jointly planning and evaluating activities in research group meetings, they became more aware of strong links between particular social justice issues and mathematical concepts, making it easier to identify opportunities in the scheme of work to introduce TMSJ activities.

> It's given me the confidence to step off the scheme of work treadmill, of getting through different topics or chapters, and actually saying: 'Well, these topics, say cumulative frequency, or percentages, I can fit these within a project on something to do with these kids' world, or to do with our world as a whole'. (Brian, Interview 3)

The teacher researchers described how providing students with learning experiences with greater relevance to their lives outside school led to significant increases in the engagement and enjoyment of students, particularly those from marginalised communities, not just during the research lessons, but with school mathematics in general. This relates to the notion of individual use value. Providing more meaningful contexts enabled students to gain a deeper understanding of mathematical concepts whilst increasing their motivation to learn more formal/abstract mathematical procedures needed for achieving success in school mathematics.

Many of the activities tried out in research lessons drew on positive FOK, e.g., the 'Election' activity introduced students to the mathematics behind different methods of counting votes by exploring how these could be used to agree the class's 'favourite film of all time'. However, other activities drew more on difficult FOK, e.g., by tackling issues around debt and pay-day loans. The teacher researchers recognised the need to tackle these issues sensitively as they directly affected the families of some students in the class. They also reported how these activities provided opportunities to develop a greater appreciation of the struggles some students faced on a day-to-day basis and how these impact on learning. Building empathy between teachers/students seems essential for enhancing the collective use value of school mathematics.

Whilst the teacher researchers decided the focus for many of the activities, for the 'Making a Change' activity, they provided an opportunity for students to explore their own issue. Students were encouraged to choose an issue of interest to their group, to identify a change they would like to see made, to use mathematics to construct an argument in support of this change and to present their findings to other students in the class. The teacher researchers were surprised by the overwhelmingly positive response to the activity and the strength of students' arguments for change, particularly from those previously identified as low attaining or disengaged.

> I liked the presentation as I got to do something that I felt strongly about. It gave me a chance to express how I feel, also including maths to support my presentation. (Student in Rebecca's Year 9, set 3 of 4, survey response to 'Making a Change' activity)

The agency demonstrated by students in using mathematics to argue for the public good, in some cases around issues relating to systemic causes of inequity (e.g., one group chose to investigate favouritism exhibited by teachers towards students whilst another group explored discrimination experienced by disabled people), highlights the potential for developing the collective use value of school mathematics.

The teacher researchers acknowledged the opportunity provided by the research group to engage with academic research, and to use this in challenging dominant discourses and to reflect critically on their own practice and that of others. They began to question their previous beliefs, e.g., that equality could be realised merely by working harder to help marginalised students gain greater success. They started to recognise systemic causes of inequality and barriers to learning, e.g., they became increasingly aware of the damaging effects of setting (grouping students by prior attainment), which was used in all four schools. They also became conscious of their tendency to avoid trying out innovative teaching approaches and ideas with lower-attaining students and began to appreciate the importance of providing

all students with engaging and empowering learning opportunities. Indeed, they noted how the ideas tried out during research lessons had the greatest impact on students from marginalised backgrounds and those previously alienated from school mathematics.

> What struck me through incorporating TMSJ activities into my lessons was primarily how engaged students became, particularly low-attaining students who had previously hated maths. (Anna, final report)

The project suggests it is possible to harness the apparent contradiction between the exchange value and use value of school mathematics, by enhancing the engagement and achievement of students from marginalised backgrounds, and, at the same time, developing their appreciation of how mathematics can be used to solve real-life problems, understand their social/political/economic situations and to begin to develop arguments that challenge the conditions leading to their marginalisation. This helps to explain the significant interest in the project amongst colleagues in all four schools, as evidenced by requests for teacher researchers to share their findings in department meetings and to incorporate their ideas in developing schemes of work. The clear appeal to others of revealing the use/exchange value dialectic highlights the potential of methods adopted in the project for legitimising the teaching approaches and exposing/disrupting exploitative aspects of school mathematics on a wider scale.

Conclusion

Applying the concept of 'cultural commodity' to the TMSJ project demonstrates how the school mathematics curriculum can provide a site for collective struggle and for taking practical actions for the public good, whilst also recognising it as a site for the accumulation of capital. In attempting to put into practice the conceptualisation of TMSJ outlined above, teacher researchers enacted several changes that, whilst not necessarily revolutionary, might be seen as stepping stones towards disrupting the reproductive function of school mathematics, which restricts the accumulation of mathematics capital to students from privileged backgrounds. These included enhancing students' agency and identities as mathematics learners (particularly those from marginalised groups) through seeing themselves in the activities (Gutiérrez, 2008), and becoming conscious of systemic causes of marginalisation, e.g., the stratification caused by setting. A broad conceptualisation of TMSJ, which harnesses the dialectic between use value and exchange value, is essential as, given the high-stakes nature of school mathematics, teachers/schools will not buy into the ideas and approaches that promote the former unless they also clearly embrace the latter.

Teachers need to be ready to become 'activists' (Gutstein, 2006) in developing empathy and building solidarity with students and marginalised communities and taking practical actions that expose and challenge the conditions that lead to their oppression (Freire, 1972). This is likely to be daunting for many teachers as it will require care and attention in tackling sensitive issues and may unearth raw emotions. It is also likely to encounter resistance, as the reproductive function of education serves to protect the scarcity of resources that hold exchange value so that privileged groups can maintain their dominant position (Bourdieu & Passeron, 1990). Educators should support teachers in this common struggle by establishing

networks of teachers that can provide the mutual support needed to encourage teachers to take risks and overcome constraints in the mathematics classroom. They also have a vital role to play in facilitating teachers' engagement with critical research literature that encourages them to challenge dominant discourses and interrogate existing practice.

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Recognising the benefits of progressive pedagogies for promoting equity and social justice in the mathematics classroom

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School closures arising from the COVID-19 have highlighted inequities in society, as well as schooling, and hence provide an opportunity to re-engage with the debate over the mathematics curriculum. We argue that progressive pedagogies are an essential part of a socially just mathematics curriculum, and that making these pedagogies more visible to learners can address concerns that they might further marginalise students from disadvantaged backgrounds. We draw on the findings from the Visible Maths Pedagogy research project that outlines strategies that can be successfully used for making the teacher's pedagogic rationale explicit to learners.

Introduction

The recent COVID-19 pandemic has served to heighten public awareness of existing inequities and injustices within education, and more widely in society, as their consequences have become more visible. Marmott et al. (2020) describe how the pandemic has disproportionately affected those from disadvantaged social groups in England, amplifying inequalities that have grown significantly over the past decade. They highlight how "shockingly high COVID-19 mortality rates among British people who self-identify as Black, Bangladeshi, Pakistani and Indian" can be largely attributed to living in more deprived areas and crowded housing, and being exposed to greater risks of contracting the virus at work, conditions which "are themselves the result of longstanding inequalities and structural racism" (p. 6). After emerging from the pandemic, they warn against attempting to re-establish the status-quo that existed before, with its extensive inequalities in income, health and wellbeing.

School closures resulting from the pandemic in England, beginning in March 2020 and January 2021, and the shift to online teaching, had a disproportionately large impact on the attainment of students from disadvantaged backgrounds, with their families having significantly less time and resources than others to support their learning (Muller & Goldenberg, 2020). This was evident in the differing levels of access to resources and participation rates amongst social groups. More than three times as many families from low-

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income households as high-income families reported their children not having adequate access to the electronic devices needed for studying (Montacute & Cullinane, 2021), and teachers from the most deprived schools reported a much higher proportion of students without an adequate Internet connection (The Sutton Trust, 2021). Children from wealthier families were reported as being twice as likely as others to participate in daily live or recorded online teaching sessions, and to spend much more time learning. Twice as many teachers in the most deprived schools (as those in affluent schools) reported a significant drop in the quality of work received from students (Cullinane & Montacute, 2020).

Hodgen et al. (2020) highlight how the closures of schools, and the transfer of mathematics teaching to online platforms, resulted in significantly lower participation and engagement rates amongst low-attaining and disadvantaged students. They warn that this is likely to further increase gaps in mathematics attainment between these groups and other students. They found that the emergency responses of schools to the pandemic (in contrast to well-planned distance learning approaches) resulted in remote teaching that reduced the level of scaffolding and support provided, and severely restricted mathematical learning experiences, particularly those of low-attaining students. There were very limited opportunities for students to receive feedback, interact with teachers and other students, discuss their mathematical ideas with others, and to engage with metacognitive tasks.

Whilst COVID-19 is likely to exacerbate inequalities in educational outcomes, we should not forget that things were far from perfect before the pandemic struck. Whilst collaborative and discursive learning experiences (as highlighted above) were noticeably absent during the recent school closures, they were often sadly lacking from many mathematics classrooms prior to this. Many students experienced school mathematics as passively learning rules and procedures without any clear purpose, with few opportunities to work collaboratively, and encountering content which they found boring and irrelevant (Nardi & Steward, 2003). High levels of alienation and disengagement amongst some learners have been accompanied by a persistent and strong association between students' socio-economic status and their level of mathematical achievement and participation (Boaler, Altendorf, & Kent, 2011). Since school mathematics acts as a critical filter in determining access to higher education and better-paid jobs, these differences in achievement serve to limit social mobility and reproduce patterns of inequality in society (Jorgensen, 2016).

The shortcomings of the remote mathematics teaching approaches imposed on teachers through the emergency response to the school closures therefore highlights the need to re-evaluate existing practices in mathematics teaching. Marmott et al. (2020, p. 4) argue that, in emerging from the recent lockdown in England:

> There is an urgent need to do things differently, to build a society based on the principles of social justice; to reduce inequalities of income and wealth; to build a wellbeing economy that puts achievement of health and wellbeing, rather than narrow economic goals, at the heart of government strategy; to build a society that responds to the climate crisis at the same time as achieving greater health equity.

We offer a similar argument, from a mathematics education perspective, that the COVID-19 pandemic offers an opportunity to re-engage with the debate over the mathematics curriculum and to renew calls for a genuinely engaging and empowering mathematics curriculum that challenges the inequities and injustices that have become increasingly apparent in schools, and which continue to plague our society.

Progressive pedagogies, equity and social justice

This paper focuses on the adoption of what we refer to as 'progressive' pedagogies. By this, we mean teaching approaches that: embrace collaboration and discussion amongst students, involve posing rich and challenging open-ended problems that require students to make decisions about the direction their learning takes, recognise multiple solutions to problems, encourage students to articulate their reasoning, and welcome errors and misconceptions as learning opportunities (Boaler, 2008; Swan, 2006). Progressive pedagogies are accepted by many authors as an essential element of a mathematics curriculum that promotes equity and social justice. Evidence shows that they motivate and engage a wider range of learners, not just those with a predisposition towards learning mathematics that is fostered through their privileged upbringing, and they provide the mathematical agency that is needed by learners to solve problems they may come across in their future lives (Bartell et al., 2017; Gutstein, 2006; Skovsmose, 2011; Xenofontos, Fraser, Priestley, & Priestley, 2020).

Tensions and contradictions in curriculum reform over recent decades have been evident in the debate, and often heated exchanges (particularly in the US and UK), between equity-minded mathematics educators and those, from more conservative ideological positions, who advocate more traditional teaching approaches (Schoenfeld, 2004; Wright, 2012). Perhaps more worrying is the recent growth in popularity of teacher-led pedagogies advocated by those who claim to be motivated by concerns for equity. These approaches are often justified by claims that the relatively unstructured nature of progressive pedagogies renders them invisible to learners. Thus, students from less wealthy backgrounds, who generally find it more difficult to decipher the rules of the game in the mathematics classroom (Bernstein, 2000), are likely to be further disadvantaged by progressive pedagogies, for which it is not always clear what students must do to be successful (Lubienski, 2004).

Teacher-centred approaches that have become increasingly popular in recent years include Direct/Explicit Instruction in the US (Doabler & Fien, 2013; Rosenshine, 2012) and Mathematics Mastery in the UK (Drury, 2018). These approaches share the premise that effective learning depends upon teachers presenting concepts in highly structured and unambiguous ways. The use of carefully selected examples, demonstrating small conceptual steps, is designed to avoid cognitive over-load and to ensure students draw correct inferences. Whilst Mathematics Mastery has been associated with modest improvements in overall attainment (Jerrim & Vignoles, 2015), students are given little opportunity to develop agency through engaging with open-ended problem-solving tasks. Direct Instruction, which is often targeted at lower-attaining students or those from disadvantaged backgrounds, places greater emphasis on maintaining a fast pace, regular guided practice and the routine

use of testing (Doabler & Fien, 2013; Rosenshine, 2012). It shares many of the problems associated with other teacher-led pedagogies including the disengagement and disempowerment of learners (Ewing, 2011; Gutstein, 2006; Nardi & Steward, 2003).

Muller and Young (2019) have called for a renewed emphasis on 'powerful knowledge', drawing on Bernstein's (2000) notion of abstract, specialised and coherent knowledge that enables learners to extend their horizons. However, they highlight how teacher-centred approaches often focus on a narrow interpretation of curriculum as a list of topics that lacks coherence and ignores issues of pedagogy. They argue that powerful knowledge should be more than an assortment of isolated propositions and must include an understanding of 'disciplinary meaning', i.e., how propositions become accepted within the discipline. For school mathematics to be empowering, therefore, students need to develop an appreciation of how new knowledge is generated by mathematicians and how this becomes accepted through argumentation and debate amongst peers (Ernest, 1991). This means providing students with experiences that reflect the processes mathematicians go through in solving real-life problems, including: working collaboratively, following new lines of inquiry, posing problems, making conjectures, making assumptions to simplify the problem, deciding which tools/methods to use, interpretating/explaining/justifying solutions to others (Mason, Burton, & Stacey, 1985). Progressive pedagogies aim to provide precisely those experiences described above and should therefore be regarded as invaluable for students in generating powerful mathematical knowledge.

However, there remains the risk that students from less privileged backgrounds might be further disadvantaged by the invisibility of progressive pedagogies (Lubienski, 2004). Rather than using this as an excuse for adopting teacher-centred pedagogies, we argue that those seeking a genuinely equitable and empowering mathematics curriculum should adopt progressive pedagogies whilst seeking strategies for making these more visible to learners. This is the aim of the project described below.

The Visible Maths Pedagogy (VMP) research project

The Visible Maths Pedagogy (VMP) research project was a collaboration between an academic researcher (Pete – the first author of this paper) and two teacher researchers (Alba and Tiago – the co-authors) based in Stoke Newington School (a non-selective and ethnically-diverse secondary school in London with an above-average proportion of students from disadvantaged backgrounds). The mathematics department already made extensive use of progressive pedagogies, had recently developed its scheme of work to include more open-ended rich tasks, and was in the process of moving towards mixed-attainment teaching groups. The research project aimed to explore strategies for making progressive pedagogies more visible to learners and their impact on students' mathematical engagement and achievement. The methodology and findings from the project have been reported elsewhere (Wright, Carvalho, & Fejzo, 2020; Wright, Fejzo, & Carvalho, 2020). In this paper we describe some of the strategies that were developed, and draw on selected findings from the project,

to highlight the benefits of making progressive pedagogies more visible for promoting equity and social justice in the mathematics classroom.

The strategies devised by the research team focused on prompting discussions with students around the pedagogic rationale for the progressive teaching approaches adopted. During cycle 2, for example, the teacher researchers asked students to present their solutions to an open-ended problem to the rest of the class, whilst using 'scribing' to write down exactly what each student said, regardless of whether it was correct or not. The teacher researchers then asked follow-up questions that aimed to draw out any ambiguities/errors in students' solutions. In order to prompt students to consider the reasons for adopting this teaching approach, i.e. enabling students to decide whether a solution is acceptable and to recognize errors and misconceptions for themselves, the teacher researchers then prompted a whole-class discussion by asking questions of the form: 'Why do you think I asked you to …?'

During Cycle 3, students were again asked to present their own solutions to an open-ended problem to others. This was followed by a whole-class discussion in which the teacher researchers facilitated agreement amongst students on what a 'model solution' would look like, which was then copied down by all students and used as a reference point for solving similar problems. During Cycle 4, students were provided with a series of questions, e.g., 'What is the question asking me? What information do I already have?' These questions were contained within a text box, printed on card, and laminated (hence the teaching approach was referred to as 'boxing up'). Again, the strategies for making the pedagogic rationale more visible included prompting a discussion around the purpose of using these approaches. For instance, for 'boxing up' the teacher researchers asked: 'Why is this useful? What does this question allow you to do?' However, during Cycles 3 and 4, the teacher researchers also made use of a card sort strategy that involved providing students with a series of statements, some of which were considered primary reasons for using each teaching approach, others potentially valid reasons not considered primary, and the remainder invalid reasons. Students were asked to discuss which statements they thought most closely reflected the teacher's reasons, and to arrange the statements in order with these at the top. For the 'boxing up' approach, the following six statements were used: 'So I can make a plan to help me', 'So I can identify the key information in the question' (both primary reasons); 'So I can share my ideas with other students', 'So I can recognise similarities and differences between problems to solve a problem' (both potentially valid reasons); 'So I can work through all the problems more quickly', 'So I can focus on my work without being distracted by others' (both invalid reasons).

Findings

In this section, we present selected findings from the project that are most relevant to the argument in this paper. More fuller accounts of the project findings, together with details of the research methodology and analytical framework, are published elsewhere (Wright, Carvalho, & Fejzo, 2020; Wright, Fejzo, & Carvalho, 2020).

The strategies were tried out during 'research lessons' as part of four action research cycles carried out over two academic years between 2017 and 2019. The first two cycles (completed during the first year of the project) involved working with two mixed-attainment Year 7 (age 11–12) classes taught by Tiago and Alba. Cycles 3 and 4 involved two mixed-attainment Year 8 classes containing some, but not all, of the students who participated in the first year (the classes were reorganised at the end of Year 7). As part of the evaluation of the strategies, Tiago and Alba conducted interviews, shortly after the final three research lessons, with 3 students from each of their own classes. The interview questions focused on exploring the extent to which students appreciated the teacher's pedagogical rationale and how to engage with progressive teaching approaches in achieving mathematical success. As we were particularly interested in the impact of the strategies on disadvantaged students, these students were chosen from those identified as 'pupil premium', a measure of socio-economic deprivation used to allocate additional resources to schools in England.

Only two students, Keira (in Alba's class) and Neal (in Tiago's class), were selected to be interviewed in both Year 7 and Year 8. Both students were of black Afro-Caribbean heritage (note black Afro-Caribbean boys were identified by the school as an under-performing group). Keira was of average attainment (compared with others in the school), engaged well in lessons, and was generally articulate, although she sometimes struggled with accessing questions because of literacy/comprehension difficulties. Neal was of below-average attainment and was a more reluctant learner. He lacked confidence and often needed encouragement to engage with activities. Both students were more comfortable with closed tasks than open tasks. We have chosen to present data from the interviews of these two students to exemplify the findings from the project. Whilst the responses of Keira and Neal reflected those of other students interviewed, the impact of the project on Keira and Neal in the second year was more noticeable than others. This might be because the interviews provided further opportunities for both students to reflect on the teacher's pedagogic rationale.

Most students, including those from disadvantaged backgrounds, exhibited high levels of engagement and enjoyment, and described themselves as being successful during the research lessons. Initially, most students attributed this enjoyment/success to factors commonly associated with teacher-led approaches, e.g., completing large amounts of work, and getting answers correct. However, by the end of the project, there appeared to be a small shift towards attributing enjoyment/success to engaging with the progressive teaching approaches employed during the research lessons:

> I think I did really good, because I was, like, annotating in my work […] when you were telling us to annotate […] everything you did, like, I was doing as well. (Keira, Cycle 2)

Neal began to see collaborating with others as integral to his own success:

> Well, I did pretty good […] because me and [another student] […] we did our own separate question. And then after we just worked it out together, after, to see if how we got the same answer, and then what method we did and see what's the easier method. (Cycle 2)

Both students appeared to enjoy the opportunity to engage with, and solve, more challenging problems:

> I liked […] the questions were […] well, some of them were easy, but then some of them were tricky, I did like a lot of working out for it. (Neal, Cycle 2)

> I enjoyed it because […] for me, I like learning about […] I like getting pushed. So, when you were asking us questions, and it, like, it helped us, like, push ourselves. (Keira, cycle 4)

At the start of the project, it was common for students to misinterpret the teacher's intentions in employing progressive pedagogies, often viewing these as attempts to seek compliant behaviour from students. However, over the course of the project, students exhibited a growing appreciation of the teacher's pedagogic rationale and were more likely to accurately identify and articulate the primary reasons given by teachers for adopting progressive approaches.

During Cycle 2, for example, Keira identified correctly that the main purpose of the 'scribing' approach was to enable students to identify errors for themselves:

> And by writing everything we've said, that will help, not just, like, the person, it will show everyone, like, where it went wrong. Instead of, like, you telling us, and that, we can learn from our mistakes.

Whilst Neal did not mention errors or misconceptions. Instead, he referred to the purpose of 'scribing' as encouraging students to focus on the method for solving a problem rather than the answer (which might be considered a valid reason but not identified by the teacher researchers as a primary reason):

> But then it doesn't really matter if it's correct or not […] their working out might be correct, but it's just that they, maybe, done something wrong at the end.

During cycle 3, Keira correctly identified a primary purpose of the 'model solution' approach as encouraging students to discuss and compare each other's ideas:

> So, like, um […] if we're working in partners, and it's like we're deciding on a method to use, we can say 'this one is more efficient to use because of this'.

Neal also identified (correctly) primary purposes for the 'model solution' approach, i.e., to promote mathematical communication and independent problem solving:

> And then someone who has the correct answer could explain, like, how they got the answer, and put it into, like, more detail. […] If you get something like similar, like, you could just flip back and like check 'Oh, how did you do that? How do I do […] how do I answer the question?' Yeah.

Similarly, during Cycle 4, both students clearly articulated the main purposes of the 'boxing up' approach, i.e., to enable students to identify the key information they need to solve a problem and to appreciate the value of planning:

> And so, like, say if you're stuck, and you're like 'OK, so I don't know what the question's asking me', you can look at the green box and see: 'OK, look what I have already, what is it […] what is the question about? What is the important key information about the question?' (Keira)

> Like planning's very important […] you always have to plan before you start the work because then you, like, don't rush through it quickly. Because it wouldn't really, like, clearly make sense to you. (Neal)

Conclusion

In this paper, we have argued that the recent COVID-19 pandemic has highlighted the need for a genuinely engaging and empowering mathematics curriculum that can address issues of equity and social justice in the mathematics classroom. We have outlined how progressive pedagogies are an essential component of such a curriculum as they provide learners with the awareness of disciplinary meaning that they need to develop powerful mathematical knowledge. The findings from the VMP research project reaffirm other research findings (e.g., Boaler, 2008; Gutstein, 2006) that demonstrate the benefits of progressive pedagogies in enhancing the motivation and engagement of a wider range of students, particularly those from disadvantaged backgrounds, and enabling them to experience greater success in mathematics lessons. The project highlights strategies that are effective in making progressive pedagogies more visible to learners, by providing students with a greater appreciation of their teacher's pedagogic rationale. Some authors have claimed that progressive pedagogies can lead to students from less privileged backgrounds being further disadvantaged (Lubienski, 2004). This paper makes a significant contribution to the debate around equitable mathematics teaching approaches through highlighting how this risk can be avoided by making progressive pedagogies more visible to learners.

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Minimising mathematical anxiety in teaching mathematics and assessing student's work

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This paper builds up a theoretical perspective and supports a possibility of creating a special assessment environment for students, where mathematical knowledge and understanding can be assessed with a reduced number of external psychological factors that may affect such assessment. A concept of a zone with minimal effect of anxiety is introduced and described. Students' successful work on extending the zone by means of a carefully selected chain of questions, where some questions only are part of a real assessment, allows students to reconsider their attitudes towards mathematics and assist teachers to identify some students' main learning difficulties as of psychological character. Further suggestions about developing and investigating special assessment environments are outlined and discussed.

Introduction

Assessment plays a central role in the teaching and learning of mathematics. However, a variety of factors can moderate successful learning of mathematical content before any kind of formal assessment takes place. Among others, they include anxiety in general terms, which often transforms into mathematical anxiety related specifically to the learning and studying of mathematics at school and university. Furthermore, it can affect assessment in the way where the actual results may not necessarily reflect the level of mathematical knowledge some students may possess (Stobart, 2008). The COVID-19 pandemic exposed this situation in multiple directions. While the total number of people in Australia affected by COVID-19 in one way or another can be seen as moderate in comparison with many other countries (e.g., many European countries and USA), anxiety among people in general and those studying mathematics in particular, has remained high in 2020. For the latter category of people, assessment risks connected with mathematical anxiety as the result of anxiety in society due to COVID-19 have also increased in 2020.

The paper presents a theoretical perspective on minimising mathematical anxiety in teaching mathematics and within assessment environments. Ashcraft, Krause, and Hopko (2007) define mathematical anxiety as the negative emotional reaction experienced by some individuals when put in situations that require mathematical reasoning or problem solving. These feelings can in turn affect mathematical performance. In particular, its assessment component. Reflecting on assessment's past outcomes and future perspectives Kilpatrick (1992) wrote:

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> Today, E. L. Thorndike, the ardent positivist and trailblazing psychologist, almost seems justified in his faith; everything that exists can be measured – in some fashion. The 20th century has produced an assessment practice in education that is dominated the world over by psychometrics, the measurement of psyche. The challenge for the 21st century, as far as mathematics educators are concerned, is to produce an assessment practice that does more than measure a person's mind and then assign that mind a treatment. We need to understand how people, not apart from but embedded in their cultures, come to use mathematics in different social settings and how we can create a mathematics instruction that helps them use it better, more rewardingly, and more responsibly. (p. 43)

In this view, the development of assessment should address the progress of mathematics education as research and teaching domain. While sharing this point, but not having intentions for measuring one's mind or measuring mathematical anxiety, (e.g., Hopko, Mahadevan, Bare, and Hunt, 2003), the paper aims to build up theoretical foundations and methodological support for a possibility of creating a special assessment environment for mathematical works of students. An environment, where students' mathematical knowledge and understanding can be assessed with a reduced number of external psychological factors that may affect such assessment and compromise its validity and reliability. I am not claiming, to the least extent, that mathematical anxiety is something stable or unchanged and, for example, can be used as the best predictor of mathematical skills or mathematical abilities. I acknowledge instead that this is a socially developing symptom that can be recognised far beyond mathematics classrooms. Research based estimations give approximately 17% of the population that can be classified as high in mathematical anxiety (Ashcraft, Krause, & Hopko, 2007).

Theoretical perspective

My theoretical perspective is derived from constructivism, mathematical anxiety issues, and assessment methodology. I take into account that current learning perspectives incorporate three important assumptions (Anthony, 1996):

- Learning is a process of knowledge construction, not of knowledge recording or absorption.
- Learning is knowledge-dependent; people use current knowledge to construct new knowledge.
- The learner is aware of the processes of cognition and can control and regulate them.

From a constructivist perspective (von Glaserfeld, 1984), it is easier for a student, under appropriate arrangement of teaching, to act as an architect, to reveal the truth and construct new knowledge, than to learn ready-made knowledge without understanding its origin, meaning and interrelations (Davis, 1991). In other words, "learning is a process of construction in which the students themselves have to be the primary actors" (von Glaserfeld, 1984). Different forms of inquiry activities can help to minimise mathematical anxiety in teaching. One of the well-known forms of such activities is open problems. For open problems I follow Arsac, Germain and Mante (1988) characterisation which is based on the following:

- The statement of the problem is short, so that it can be easily understood, it fosters some discovery and all students are able to start the solution process.
- The statement of the problem does not suggest the method of solution, or the solution on itself, but it creates a situation stimulating the production of conjectures.
- The problem is set in a conceptual domain which students are familiar with.

Thus, students are able to master the situation rather quickly and get involved in attempts of conjecturing, planning solution paths and finding counter-examples in a reasonable time. I recognise, however, that a teacher is a person who has to regulate directions of students' work and adapt it to the lesson's needs. I also take into account that "the open problems promote the devolution of responsibility from the teacher to students" (Furinghetti and Paola, 2003, p. 399). For teacher's role to control any learning situation, Mercer's idea (1995) of "the sensitive, supportive intervention of a teacher in the progress of a learner, who is actively involved in some specific task, but who is not quite able to manage the task alone" helps to minimise mathematical anxiety (p. 74). Also, keeping a balance between students' collaborative and individual work is of great importance to address the issues of mathematical anxiety. It is also important, whenever possible, to make all efforts and find individual motivation for students to be successful in their studies.

Students with low achievement and learning difficulties in mathematics form a substantial part of all students. According to Magne (2003) "low achievement is a social construct. It is not a fact but a human interpretation of relations between the individual and his environment" (p. 9). Difficulties in learning mathematics are often seen through relationships between mathematical anxiety and mathematical competence (Hembree, 1990; Faust, Ashcraft & Fleck, 1996; Ma, 1999). Mathematical anxiety can vary from apprehension or dislike, worry to genuine fear or dread (McLeod, 1994) and includes more specific behavioural issues such as tension, frustration, helplessness, distress and mental disorganisation. Faust (1992) argued that mathematical anxiety fits the classic definition of a phobia. Miller and Bichsel (2004) pointed out that mathematical anxiety was the strongest predictor of mathematical performance. For example, Ashcraft, Kirk, and Hopko (1998) examined mathematical anxiety and mathematical competence in a study that administered a standardized mathematical achievement test. There are common regulations which allow students (in particular, students with special needs) to keep short breaks during the time tests/exams take place. Such regulations are designed to address difficulties some students may experience. However, the influence of anxiety during the assessment period cannot be resolved through such arrangements and needs to be investigated further.

I consider mathematical anxiety that affects students' performances in assessment items in connection with cognitive styles that coordinate functioning of cognitive processes (Gardner et al., 1959). According to Guilford (1980), cognitive styles can be characterised as intellectual organising functions that initiate and control psychological activity of the individual. The shift of the balance in the direction of uncontrolled cognitive styles brings to the activation of different components of mathematical anxiety whose significant

accumulation within a short time interval leads to the blockage of those components of intellectual activity on the metacognitive level which are responsible for the creative regulation of the intellectual behaviour. As a result, students can demonstrate partial or full inability to work on questions from the assessment item. I distinguish three common situations that are classified according to the extent to which the students' work on assessment pieces can be affected.

- Student's performance is entirely affected by mathematical anxiety issues. Working on assessment questions represents sporadically made attempts with most of writing to be non-relevant to the assessment materials.
- Mathematical anxiety issues considerably affect student's performance. Although, it depends on particular assessment questions. Working on assessment questions represents a mixed type of work, where correct writing intertwines with non-relevant notes.
- Student's performance is marginally affected by mathematical anxiety issues, if it is affected at all.

As mentioned above, despite the low score students from the first two categories are likely to receive as their assessment result, the question about their knowledge of the corresponding mathematical content remains unanswered. One of possible directions is in developing a "free of anxiety" type of assessment environments, one of which is presented below.

I continue with a few definitions to describe in more detail the structure of the theoretical construction I have developed. I call *a zone with minimal effect of anxiety* the content knowledge that remains stably accessible by an individual having mathematical anxiety issues. Through a carefully selected chain of questions, this zone can be extended so that its extension will contribute to the gradual growth of confidence of that individual. The design of a chain of questions must have the following features:

- An increasing level of difficulty of questions with the final ones being equivalent to the level of difficulty of questions in standard assessment items. Some consecutive questions may be of approximately the same level of difficulty.
- There are no hints provided. Altogether, this is a set of connected or non-connected to each other tasks within a smaller topic that helps students to gain more confidence during their work on assessment items.
- Not all questions are subject to assessment. Students are provided with information about available marks for a whole piece of assessment or for groups of questions, not for the separate questions. Again, it aims to contribute to supporting students' confidence at the highest level possible.

Such sets of specially designed questions serve as a didactical tool that allows switching students' intellectual activity to an area where they can feel themselves comfortable up to a certain level. In special terms, it helps to unblock students' intellectual activity on the metacognitive level which improper functioning under the conditions of mathematical

anxiety may cause a significant decline in the results of students' assessment. The role of the principle of likes and dislikes, i.e., what I do well, I like to do it, in a competitive way, public way, under time constraints, etc., should not be underestimated in these circumstances. The presence of different questions from the same topic has a multiple meaning: firstly, it aims to bring students' attention to a constructivist point of view, where a number of questions from the same part of the course content may stimulate students' positive transition from the zone with minimal effect of anxiety to more challenging situations; secondly, it aims to divert students' attention from the anxiety issues since they (students) are no longer focussed on one particular question they may struggle with. However, they (anxiety issues) disperse on a number of questions instead, with some of them being perceived positively by students (in particular those within the zone with minimal effect of anxiety). Thus, minimising their possible negative influence.

Below I present an example of assessment material related to proof of the infinity of primes, which is suitable for discussion with Grade 11 and 12 students in Australia and first year undergraduate students. At first students are expected to work through the Erdős proof (provided to students) and then answer a number of questions on the topic. Full proof proposed by Erdős can be found, for example, in Aigner and Ziegler (2001). The special feature of this proof is that the construction leading to a contradiction is not the result of direct conclusions from the assumption and given conditions. This is a separate construction where the initial assumption and several implications from it are taken into account to find a way to come to a contradiction. Availability of easy-to-follow parts of the proof along with harder ones gives a "perfect landscape" in attempt to overcome psychological factors. Another feature of this example is a possibility to activate working memory – ability to hold a mental representation of information in mind while simultaneously engaged in other mental processes (Baddeley, 1986). Questions that represent the content beyond the lowest level of difficulty related to the topic (Q1) or accept analogical reproduction (Q3 and Q6) are not assessed. Questions Q1-Q3 form a zone with minimal effect of anxiety, while questions Q5, Q7 and Q9 appeal to working memory. Information in the brackets after each question is not available for students, i.e., students are not aware which questions are assessed and which are not.

Example: Questions for assessment (on the base of the Erdős proof of the infinity of primes)

1. What numbers are prime? Are all natural numbers prime? What can you say about 87? Is it prime? (Not assessed)
2. What is common between $\sum_{n=1}^{\infty} \frac{1}{n}$ and $\sum_{p-all\ primes} \frac{1}{p}$? (Part of assessment)
3. What is the initial assumption of the Erdős proof? (Not assessed)
4. Can you explain in your own words what a convergent series is? Graphical representations can be used if required. (Part of assessment)

5. When we select $\frac{1}{2}$ there must be a natural number k such that $\sum_{i\geq k+1}\frac{1}{p_i} < \frac{1}{2}$. Why do we choose $\frac{1}{2}$ here? What may happen if we replace $\frac{1}{2}$, say, with $\frac{1}{3}$? (Part of assessment)
6. What do the numbers N_b and N_s mean in the proof? (Not assessed)
7. N is listed as arbitrary in the proof. Why is the particular value $N = 2^{2k+2}$ important? How can we use it? (Part of assessment)
8. Describe in your own words how a proof scheme is built up in the Erdős proof. (Part of assessment)
9. Why can we represent every $n \leq N$which has only small prime divisors in the form $n = a_n b_n^2$, where $a_n$is the square-free part? (Part of assessment)
10. Give some reasons why you think a contradiction can be achieved. (Part of assessment)

Concluding remarks

In this paper I have attempted to provide a theoretical background for teaching mathematics and creating a special assessment environment to address the needs of students with high levels of mathematical anxiety. The ideas discussed in the paper can be useful for the variety of mathematical topics studied in Grade 11 and 12 as well as at undergraduate level. They can be extended to lower school grades or used for extra curriculum activities where assessment is still in place. The concept of a zone with minimal effect of anxiety can be incorporated into different assessment methodology used for different categories of students (e.g., Black et al., 2003). The consequences of mathematical anxiety can be further specified and investigated to address specific components such as tension, frustration, helplessness, distress and mental disorganisation. Such specific behavioural issues do not form a unitary emotional scale or symptom and further investigation in this direction is required. This would allow obtaining a more complex picture in understanding how assessment can be organised in the most effective way with regard to specific behavioural issues. There is a possibility that some assessment results may remain low in the new assessment environment. In particular, if some students only answer those questions that are within the zone with minimal effect of anxiety. I would suggest interpreting such results as those that are still more reliable since they provide more information in comparison to standard assessment materials and may give more evidence that for some students their learning difficulties lie in the area of conceptual understanding of basic mathematical ideas rather than linked with mathematical anxiety. I would like to emphasize that the proposed theoretical model highlights the central role of a teacher as a representative and main facilitator of the "free of anxiety" environment in the classroom which is the main prerequisite for creating and supporting a "free of anxiety" assessment environment.

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